Relational Algebra

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Basic Concepts

Five basic operators on tables:

- 1. Select σ_{θ}
- 2. Project ⊓_L
- 3. Union U
- 4. Difference \
- 5. cross product ×

Cont...

Input: one table $(\sigma_{\theta}, \Pi_{L})$, or

two tables $(\cup, \setminus, \times)$

Output: a table

The operators U, \, \ are the usual set operators

$$T_1 \cup T_2 = \{t : t \in T_1 \text{ or } t \in T_2\}$$
// Recall a table is a set of tuples, so no repeated tuples allowed.

$$T_1 \setminus T_2 = \{t : t \in T_1 \text{ and } t \notin T_2\}$$

$$T_1 \times T_2 = \{\langle a_1, ..., a_k, b_1, ..., b_l \rangle : \langle a_1, ..., a_k \rangle \in T_1 \text{ and } \langle b_1, ..., b_l \rangle \in T_2\}$$

For $T_1 \cup T_2$ and $T_1 \setminus T_2$ the tables T_1, T_2 must be compatible: same number of attributes, and corresponding attributes must have same domains.

Exercise

Suppose T_1 has n tuples and T_2 has m tuples.

- How many tuples in $T_1 \times T_2$?
- How many tuples in T₁∪T₂?
- How many tuples in $T_1 \setminus T_2$?

Thus, the cross product can result into large tables.

Answer

- Suppose T₁ has n tuples and T₂ has m tuples.
- How many tuples in $T_1 \times T_2$? Answer = n*m
- How many tuples in $T_1 \cup T_2$? Answer = between max(n,m) and n+m
- How many tuples in T₁ \ T₂? Answer: between 0 and n

Thus, the cross product can result into large tables.

Project - □

- Projection list L = list of attributes
- $\Pi_L(T)$ = the table consisting of the columns of T whose names are listed in L

$Project - \Pi$ (Example 1)

TABLE	S				TAB]	LE SP	(shipments)
sno	sname	status	city		sno	pno	qty
S1	Smith	20	Londo:	n	S1	P1	300
S2	Jones	10	Paris		S1	P2	200
S3	Blake	30	Paris		S1	Р3	400
S4	Clarke	20	Londo:	n	S1	P4	200
S5	Adams	30	Athen	S	S1	P5	100
					S1	P6	100
TABLE	P				S2	P1	300
pno	pname	color	weight	city	S2	P2	400
P1	nut	red	12	London	S3	P2	200
P2	bolt	green	17	Paris	S4	P2	200
Р3	screw	blue	17	Rome	S4	P4	300
P4	screw	red	14	London	S4	P5	400
P5	cam	blue	12	Paris			
P6	cog	red	19	London			

 Π sno, status(S) = ?

Project - □ (Example 1 - Answer)

TABLE	S			
sno	sname	status	city	
S1	Smith	20	London	
S2	Jones	10	Paris	
S3	Blake	30	Paris	
S4	Clarke	20	London	
S5	Adams	30	Athens	

TABLE P

pno	pname	color	weight	city
P1	nut	red	12	London
P2	bolt	green	17	Paris
Р3	screw	blue	17	Rome
P4	screw	red	14	London
P5	cam	blue	12	Paris
P6	cog	red	19	London

```
$\mathbb{\Pi}$sno, status(S) = $\text{S1 20} $\text{S2 10} $\text{S3 30} $\text{S4 20} $\text{S5 30}$
```

```
TABLE SP
          (shipments)
sno pno
          qty
    Р1
          300
S1
S1
          200
          400
S1
    Р3
S1
          200
    Ρ4
S1
    Р5
          100
          100
S1
    Р6
S2
          300
S2
    Ρ2
          400
S3
          200
S4
          200
          300
S4
    Ρ4
S4
    Р5
          400
```

Project $-\Pi$ (Example 2)

	TABLE	S				TABI	LE SP	(shipments)
	sno	sname	status	city		sno	pno	qty
	S1	Smith	20	Londo	n	S1	P1	300
	S2	Jones	10	Paris		S1	P2	200
	S3	Blake	30	Paris		S1	Р3	400
	S4	Clarke	20	Londo:	n	S1	P4	200
	S5	Adams	30	Athen	S	S1	P5	100
						S1	P6	100
	TABLE	P				S2	P1	300
	pno	pname	color	weight	city	S2	P2	400
	P1	nut	red	12	London	S3	P2	200
	P2	bolt	green	17	Paris	S4	P2	200
	Р3	screw	blue	17	Rome	S4	P4	300
	P4	screw	red	14	London	S4	P5	400
	P5	cam	blue	12	Paris			
	P6	cog	red	19	London			
$\Pi_{ extstyle $	o,statu	S	1 20 2 10		Π status(S)	= ?		
			3 30					
			4 20					
		S.	5 30					

Project - □ (Example 2 - Answer)

TABLE	S				TABI	E SP	(shipments)
sno	sname	status	city		sno	pno	qty
S1	Smith	20	Londo	n	S1	P1	300
S2	Jones	10	Paris		S1	P2	200
S3	Blake	30	Paris		S1	Р3	400
S4	Clarke	20	Londo	n	S1	P4	200
S5	Adams	30	Athen	S	S1	P5	100
					S1	P6	100
TABLE	P				S2	P1	300
pno	pname	color	weight	city	S2	P2	400
P1	nut	red	12	London	S3	P2	200
P2	bolt	green	17	Paris	S4	P2	200
Р3	screw	blue	17	Rome	S4	P4	300
P4	screw	red	14	London	S4	P5	400
P5	cam	blue	12	Paris			
P6	cog	red	19	London			
o,status	s(S) = S	1 20		π	2.0		
, , , , , , , , , , , , , , , , , , , ,	_			LI status(S) =			
					30		
	\$1 \$2 \$3 \$4 \$5 TABLE pno P1 P2 P3 P4 P5 P6	sno sname S1 Smith S2 Jones S3 Blake S4 Clarke S5 Adams TABLE P pno pname P1 nut P2 bolt P3 screw P4 screw P5 cam P6 cog 7, status(S) = S3 S3 S4	snosnamestatusS1Smith20S2Jones10S3Blake30S4Clarke20S5Adams30 TABLE P pno pname color P1 nut red P2 bolt green P3 screw blue P4 screw red P5 cam blue P6 cog red	sno sname status city S1 Smith 20 Londo S2 Jones 10 Paris S3 Blake 30 Paris S4 Clarke 20 Londo S5 Adams 30 Athen TABLE P pno pname color weight P1 nut red 12 P2 bolt green 17 P3 screw blue 17 P4 screw red 14 P5 cam blue 12 P6 cog red 19 o, status(S) = S1 20 S2 10 S3 30 S4 20	sno sname status city S1 Smith 20 London S2 Jones 10 Paris S3 Blake 30 Paris S4 Clarke 20 London S5 Adams 30 Athens TABLE P pno pname color weight city P1 nut red 12 London P2 bolt green 17 Paris P3 screw blue 17 Rome P4 screw red 14 London P5 cam blue 12 Paris P6 cog red 19 London O, status(S) = S1 20 S2 10 S3 30 S4 20 S3 30 S4 20 S4 20 S4 20 Sa I Single Single	sno sname status city sno S1 Smith 20 London S1 S2 Jones 10 Paris S1 S3 Blake 30 Paris S1 S4 Clarke 20 London S1 S5 Adams 30 Athens S1 TABLE P S2 S2 pno pname color weight city S2 P1 nut red 12 London S3 P2 bolt green 17 Paris S4 P3 screw blue 17 Rome S4 P4 screw red 14 London S4 P5 cam blue 12 Paris P6 cog red 19 London O, status (S) = S1 20 S2 10 30 30	sno sname status city sno pno S1 Smith 20 London S1 P1 S2 Jones 10 Paris S1 P2 S3 Blake 30 Paris S1 P3 S4 Clarke 20 London S1 P4 S5 Adams 30 Athens S1 P4 S5 Adams 30 Athens S1 P5 S1 P5 S1 P6 P7 P5 P5 P6 P7 P7 P2 P1 P2 P3 Screw P4 P4 P4 P5 P2 P3 P4 P4 P4 P4 P5 P2 P2 P3 P4 P4 P4 P4 P4 P4 P4 P4 P4

select - σ_{θ}

Selection condition θ . Can:

- Simple: X # Y or X # c, where X and Y are attributes, c constant value, and # in {=, <, <=, >, >=, !=}
- Composite: (NOT θ_1), (θ_1 AND θ_2), (θ_1 OR θ_2), where θ_1 and θ_2 are selection conditions

 $\sigma_{\theta}(T)$ = all tuples of T satisfying condition θ

select - σ_{θ} (Example 1)

TABLE	S				TABI	LE SP	(shipments)
sno	sname	status	city		sno	pno	qty
S1	Smith	20	Londo	n	S1	P1	300
S2	Jones	10	Paris		S1	P2	200
S3	Blake	30	Paris		S1	Р3	400
S4	Clarke	20	Londo	n	S1	P4	200
S5	Adams	30	Athen	S	S1	P5	100
					S1	P6	100
TABLE	P				S2	P1	300
pno	pname	color	weight	city	S2	P2	400
P1	nut	red	12	London	S3	P2	200
P2	bolt	green	17	Paris	S4	P2	200
Р3	screw	blue	17	Rome	S4	P4	300
P4	screw	red	14	London	S4	P5	400
P5	cam	blue	12	Paris			
P6	cog	red	19	London			

 σ status>20(S) = ?

select - σ_{θ} (Example 1 - Answer)

(shipments)

TABLE	S				TABI	E SP
sno	sname	status	city		sno	pno
S1	Smith	20	Londo	Ω	S1	P1
S2	Jones	10	Paris		S1	P2
S3	Blake	30	Paris		S1	Р3
S4	Clarke	20	Londo	n	S1	P4
S5	Adams	30	Athen	S	S1	P5
					S1	P6
TABLE	P				S2	P1
pno	pname	color	weight	city	S2	P2
P1	nut	red	12	London	S3	P2
P2	bolt	green	17	Paris	S4	P2
Р3	screw	blue	17	Rome	S4	P4
P4	screw	red	14	London	S4	P5
P5	cam	blue	12	Paris		
Р6	cog	red	19	London		
o status>20	S	3 Blake		Paris		
	S	5 Adam:	s 30	Athens		

select - σ_{θ} (Example 2)

	TABLE	S				TABI	E SP	(shipments)
	sno	sname	status	city		sno	pno	qty
	S1	Smith	20	Londo	n	S1	P1	300
	S2	Jones	10	Paris		S1	P2	200
	S3	Blake	30	Paris		S1	Р3	400
	S4	Clarke	20	Londo	n	S1	P4	200
	S5	Adams	30	Athen	S	S1	P5	100
						S1	P6	100
	TABLE	P				S2	P1	300
	pno	pname	color	weight	city	S2	P2	400
	P1	nut	red	12	London	S3	P2	200
	P2	bolt	green	17	Paris	S4	P2	200
	Р3	screw	blue	17	Rome	S4	P4	300
	P4	screw	red	14	London	S4	P5	400
	P5	cam	blue	12	Paris			
	P6	cog	red	19	London			
O sta	atus>20	(S) =	S3 Blak S5 Adam		Paris Athens	$\Pi_{ extsf{S}}$	no (C	status>20(S))

= ?

select - σ_{θ} (Example 2)

TABLE	S				TABI	E SP	(shipments)	
sno	sname	status	city		sno	pno	qty	
S1	Smith	20	Londo	n	S1	P1	300	
S2	Jones	10	Paris		S1	P2	200	
S3	Blake	30	Paris		S1	Р3	400	
S4	Clarke	20	Londo	n	S1	P4	200	
S5	Adams	30	Athen	S	S1	P5	100	
					S1	Р6	100	
TABLE	P				S2	P1	300	
pno	pname	color	weight	city	S2	P2	400	
P1	nut	red	12	London	S3	P2	200	
P2	bolt	green	17	Paris	S4	P2	200	
Р3	screw	blue	17	Rome	S4	P4	300	
P4	screw	red	14	London	S4	P5	400	
P5	cam	blue	12	Paris				
Р6	cog	red	19	London				
O status>20	•	S3 Bla S5 Adan		Paris Athens	$\Pi_{ extsf{S}}$	no (o	status>20(S)) =	S3 S5

cross product - x (Example 1)

TABLE	S				TABI	E SP	(shipments)
sno	sname	status	city		sno	pno	qty
S1	Smith	20	Londo	n	S1	P1	300
S2	Jones	10	Paris		S1	P2	200
S3	Blake	30	Paris		S1	Р3	400
S4	Clarke	20	Londo	n	S1	P4	200
S5	Adams	30	Athen	S	S1	P5	100
					S1	P6	100
TABLE	P				S2	P1	300
pno	pname	color	weight	city	S2	P2	400
P1	nut	red	12	London	S3	P2	200
P2	bolt	green	17	Paris	S4	P2	200
Р3	screw	blue	17	Rome	S4	P4	300
P4	screw	red	14	London	S4	P5	400
P5	cam	blue	12	Paris			
P6	cog	red	19	London			

 $S \times P = ?$

cross product - × (Example 1)

TABLE	S					TABI	E SP	(shipme	nts)					
sno	sname	status	city			sno	pno	qty						
S1	Smith	20	Londo	n		S1	P1	300						
S2	Jones	10	Paris			S1	P2	200						
S3	Blake	30	Paris			S1	Р3	400						
S4	Clarke	20	Londo	n		S1	P4	200						
S5	Adams	30	Athen	S		S1	P5	100						
						S1	Р6	100						
TABLE	P					s2	P1	300						
pno	pname	color	weight	city		S2	P2	400						
- P1	nut	red	12	London		s3	P2	200						
P2	bolt	green	17	Paris		S4	P2	200						
Р3	screw	blue	17	Rome		S4	P4	300						
P4	screw	red	14	London		S4	P5	400						
P5	cam	blue	12	Paris	Ç∨D	= (3	0 +11n	100)						
P6	cog	red	19	London	S1	Smith	_	London		P1	nut	red	12	London
					S1	Smith		London		P2	bolt	green	17	Paris
												green	• •	
					S1	Smith	20	London		P6	cog	red	19	London
					S2	Jones	10	Paris		P1	nut	red	12	London
					S2	Jones	10	Paris		P2	bolt	green	17	Paris
						• • • •		• • • • • • •	• • •	• • •	• • • • • • • • • • • • • • • • • • • •	• • • • • •	•	
					 S5	Adams	30	Athens	• • •	 Рб	cog	red	19	London
					$\mathcal{S}\mathcal{S}$	Auaiiis	50	ACHEHS		TO	cog	IEU	19	допаон

Derived Operators

- **0-join**: $T_1 \bowtie_T T_2 = \sigma_{\theta}(T_1 \times T_2)$ where each comparison $X_1 \# X_2$ in θ is between an attribute X_1 of T_1 and X_2 of T_2 .
- Equijoin: θ-join using only '=' comparisons
- Natural join: $T_1^*T_2$ = the equijoin of the two tables such that $\theta = (X=X \text{ AND } Y=Y \text{ AND}...)$

where X, Y,.. are the common attributes of the two tables. Moreover, only **one copy** of each common column is included in the output table.

Note: In all comparisons (X # Y), if either of X and Y is **NULL**, then (X # Y) evaluates to **false**.

θ -join- \bowtie (Example 1)

TABLE	S				TABI	E SP	(shipments)
sno	sname	status	city		sno	pno	qty
S1	Smith	20	Londo	n	S1	P1	300
S2	Jones	10	Paris		S1	P2	200
S3	Blake	30	Paris		S1	Р3	400
S4	Clarke	20	Londo	n	S1	P4	200
S5	Adams	30	Athen	S	S1	P5	100
					S1	P6	100
TABLE	P				S2	P1	300
pno	pname	color	weight	city	S2	P2	400
P1	nut	red	12	London	S3	P2	200
P2	bolt	green	17	Paris	S4	P2	200
Р3	screw	blue	17	Rome	S4	P4	300
P4	screw	red	14	London	S4	P5	400
P5	cam	blue	12	Paris			
P6	cog	red	19	London			
1 :1:1	D - 0						

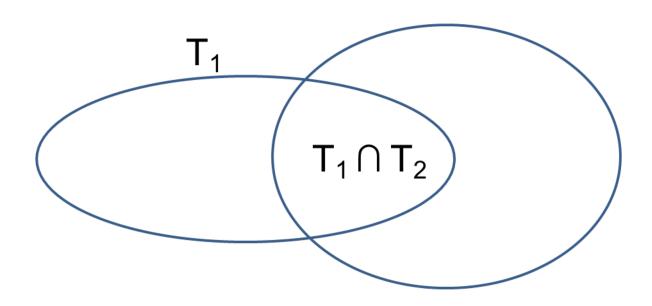
S \bowtie tity=city P = ?

θ -join- \bowtie_{θ} (Example 1)

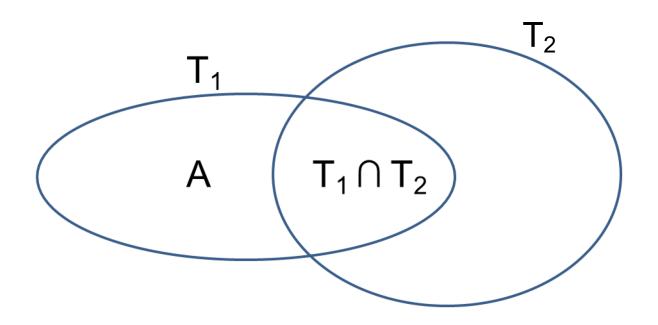
TABLE	S					TABL	E SP	(sh	ipments))	
sno	sname	status	city			sno	pno	qty	7		
S1	Smith	20	Londo	n		S1	P1	300)		
S2	Jones	10	Paris			S1	P2	200)		
s3	Blake	30	Paris			S1	Р3	400)		
S4	Clarke	20	Londo	n		S1	P4	200)		
S5	Adams	30	Athen	S		S1	P5	100)		
						S1	P6	100)		
TABLE	P					S2	P1	300)		
pno	pname	color	weight	city	У	S2	P2	400)		
P1	nut	red	12	Lond	lon	s3	P2	200)		
P2	bolt	green	17	Pari	S	S4	P2	200)		
Р3	screw	blue	17	Rome	2	S4	P4	300)		
P4	screw	red	14	Lond	lon	S4	P5	400)		
P5	cam	blue	12	Pari	S						
P6	cog	red	19	Lond	lon						
⊠ city=city	P = S1	Smit	th 20		London	P1	nut		red	12	London
, ,	S1	Smit	th 20		London	P4	scr	∋w	red	14	London
	S1	Smit	th 20		London	P6	cog		red	19	London
	S2	Jone	es 10		Paris	P2	bol.	t	green	17	Paris
	S2	Jone	es 10		Paris	P5	cam		blue	12	Paris
	S3	Bla	ke 30		Paris	P2	bol.	t	green	17	Paris

• Set intersection: $T_1 \cap T_2 = ?$

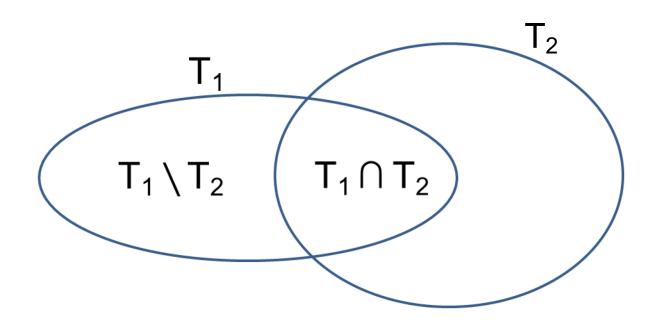
• Set intersection: $T_1 \cap T_2 = ?$



• Set intersection: $T_1 \cap T_2 = T_1 \setminus A = ?$



• Set intersection: $T_1 \cap T_2 = T_1 \setminus (T_1 \setminus T_2) = ?$



Exercise

TABLE	S				TABI	LE SP	(shipments)
sno	sname	status	city		sno	pno	qty
S1	Smith	20	Londo	n	S1	P1	300
S2	Jones	10	Paris		S1	P2	200
S3	Blake	30	Paris		S1	Р3	400
S4	Clarke	20	Londo	n	S1	P4	200
S5	Adams	30	Athen	S	S1	P5	100
					S1	P6	100
TABLE	P				S2	P1	300
pno	pname	color	weight	city	S2	P2	400
P1	nut	red	12	London	S3	P2	200
P2	bolt	green	17	Paris	S4	P2	200
Р3	screw	blue	17	Rome	S4	P4	300
P4	screw	red	14	London	S4	P5	400
P5	cam	blue	12	Paris			
Р6	coq	red	19	London			

Queries:

- Get all suppliers located in London
- Get all locations of suppliers
- Get names of suppliers who supplied P2
- Get names of suppliers who supplied some green parts
- Get names of suppliers who didn't supply any green parts

Exercise - Answers

TABLE	S				TABI	E SP	(shipments)
sno	sname	status	city		sno	pno	qty
S1	Smith	20	Londo	n	S1	P1	300
S2	Jones	10	Paris		S1	P2	200
S3	Blake	30	Paris		S1	Р3	400
S4	Clarke	20	Londo	n	S1	P4	200
S5	Adams	30	Athen	S	S1	P5	100
					S1	P6	100
TABLE	P				S2	P1	300
pno	pname	color	weight	city	S2	P2	400
P1	nut	red	12	London	S3	P2	200
P2	bolt	green	17	Paris	S4	P2	200
Р3	screw	blue	17	Rome	S4	P4	300
P4	screw	red	14	London	S4	P5	400
P5	cam	blue	12	Paris			
P6	coq	red	19	London			

Queries:

- Get all suppliers located in London $Q1 = \sigma_{city="London"}(S)$
- Get all locations of suppliers Q2 = ⊓_{city}(S)
- Get names of suppliers who supplied P2 Q3 = $\Pi_{\text{sname}}(\sigma_{\text{pno="P2"}}(S^*SP))$
- Get names of suppliers who supplied some green parts Q4 = Π_{sname}(σ_{color}="green" (S*SP*P))
- Get names of suppliers who didn't supply any green parts Q5 = ∏_{sname}(S) \ Q4

Division operation (derived)

- T1(X,Y) ÷ T2(Y) = all tuples x such that (x, y) is in T1, for every tuple y in T2
- The two tables must have the structure shown above, otherwise division operation does not work:

The attributes of T2 must be a subset of those of T1.

Cont...

The value x1 is the only X-value in that is paired in T1 with every Y-value in T2.

TABLE	S				TAB]	LE SP	(shipments)
sno	sname	status	city		sno	pno	qty
S1	Smith	20	Londo	n	S1	P1	300
S2	Jones	10	Paris		S1	P2	200
S3	Blake	30	Paris		S1	Р3	400
S4	Clarke	20	Londo	Ω	S1	P4	200
S5	Adams	30	Athen	S	S1	P5	100
					S1	P6	100
TABLE	P				S2	P1	300
pno	pname	color	weight	city	S2	P2	400
P1	nut	red	12	London	S3	P2	200
P2	bolt	green	17	Paris	S4	P2	200
Р3	screw	blue	17	Rome	S4	P4	300
P4	screw	red	14	London	S4	P5	400
P5	cam	blue	12	Paris			
P6	cog	red	19	London			

Get sno's of suppliers who have supplied **all** parts = ?

TABLE	S				TABI	E SP	(shipments)
sno	sname	status	city		sno	pno	qty
S1	Smith	20	Londo	n	S1	P1	300
S2	Jones	10	Paris		S1	P2	200
S3	Blake	30	Paris		S1	Р3	400
S4	Clarke	20	Londo	n	S1	P4	200
S5	Adams	30	Athens	S	S1	P5	100
					S1	P6	100
TABLE	P				S2	P1	300
pno	pname	color	weight	city	S2	P2	400
P1	nut	red	12	London	S3	P2	200
P2	bolt	green	17	Paris	S4	P2	200
Р3	screw	blue	17	Rome	S4	P4	300
P4	screw	red	14	London	S4	P5	400
P5	cam	blue	12	Paris			
P6	coq	red	19	London			

Get sno's of suppliers who have supplied **all** parts = $\Pi_{sno,pno}(SP) \div \Pi_{pno}(P)$

TABLE	S				TABI	E SP	(shipments)
sno	sname	status	city		sno	pno	qty
S1	Smith	20	Londo	Ω	S1	P1	300
S2	Jones	10	Paris		S1	P2	200
S3	Blake	30	Paris		S1	Р3	400
S4	Clarke	20	Londo	n	S1	P4	200
S5	Adams	30	Athen	S	S1	P5	100
					S1	P6	100
TABLE	P				S2	P1	300
pno	pname	color	weight	city	S2	P2	400
P1	nut	red	12	London	S3	P2	200
P2	bolt	green	17	Paris	S4	P2	200
Р3	screw	blue	17	Rome	S4	P4	300
P4	screw	red	14	London	S4	P5	400
P5	cam	blue	12	Paris			
P6	cog	red	19	London			

Sequences of Operations and RENAME-ing

RedOnly $\leftarrow \sigma_{color='red'}(\mathbf{P})$ // naming

Result $\leftarrow \Pi_{sno}(SP * RedOnly)$ // using the new name

Sequences of Operations and RENAME-ing

ρT(R): renaming table R as T (same attribute names)

Next, assume R has n attributes

- ρ(B1,...,Bn)(R) : renaming attributes of R as (B1,...,Bn)
- ρ T(B1,...,Bn)(R): renaming both the table R and its attributes

TABLE	S				TABI	E SP	(shipments)
sno	sname	status	city		sno	pno	qty
S1	Smith	20	Londo:	n	S1	P1	300
S2	Jones	10	Paris		S1	P2	200
S3	Blake	30	Paris		S1	Р3	400
S4	Clarke	20	Londo	n	S1	P4	200
S5	Adams	30	Athen	S	S1	P5	100
					S1	Р6	100
TABLE	P				S2	P1	300
pno	pname	color	weight	city	S2	P2	400
P1	nut	red	12	London	S3	P2	200
P2	bolt	green	17	Paris	S4	P2	200
Р3	screw	blue	17	Rome	S4	P4	300
P4	screw	red	14	London	S4	P5	400
P5	cam	blue	12	Paris			
Р6	cog	red	19	London			

Sequences of Operations and RENAME-ing

Snew $\leftarrow \rho_{(sno,sname,status,scity)}(S)$ // S.city is called scity in Snew Join1 \leftarrow Snew * SP // natural join on sno Join2 \leftarrow Join1 * P // natural join only on pno, city is not common

Result $\leftarrow \Pi_{\text{sname,pname,qty}}(\text{Join2})$

Exercise

- Show that ÷ is indeed a derived operation.
- We compute $T1(X, Y) \div T2(Y)$ in three steps

```
Q1 <-- \Pi_X(T1)  // the X-values in T1  
Q2 <-- \Pi_X(Q1×T2 \T1)  // the X-values in T1 not paired  // ...with every Y-value in T2  
Q3 <-- Q1 \ Q2  // Q3 is T1(X, Y) ÷ T2(Y)
```

Quick Note About SQL: The 5 basic operations

```
\begin{split} & \Pi_{A,B}(R) \text{: } \text{SELECT A, B FROM R;} \\ & \sigma_{\theta}(R) \text{: } \text{SELECT * FROM R WHERE } \Theta \text{;} \\ & R \text{ U S: } \text{(SELECT * FROM R) UNION } \\ & \text{(SELECT * FROM S);} \\ & R - S \text{: } \text{(SELECT * FROM R) EXCEPT } \\ & \text{(SELECT * FROM S);} \\ & R \text{ x S: } \text{SELECT * FROM R, S;} \end{split}
```