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2015

18th Annual High School Mathematical Contest in Modeling (HiMCM) Summary Sheet (Please attach a copy of this page to your Solution Paper.)

Team Control Number: 6083

Problem Chosen: A

This paper seeks to outline a solution to minimize congestion at highway lane closures. The underlying metrics used in developing the model are “fairness” and “efficiency”. We measured fairness as the standard deviation of the average speed of cars in a merge zone; thus, a “fair” system is one in which all cars take approximately the same time to pass through. Efficiency is calculated as the average speed of cars in the merge zone, so a high number signifies a high efficiency.

For our model, we decided to expand upon the previously existing zipper merging system, in which cars occupy both lanes before the lane closure and alternate into the open lane. While keeping the two aforementioned metrics in mind, we then analytically determined the speed at which cars should enter a “merging zone” based on levels of congestion. This notion of a dynamic speed limit is known as “speed harmonization”, a technique employed in some European countries. Our model shows that rather than entering the merge zone at the speed limit (65 mph), cars should enter the zone at around 50 mph during times of traffic congestion. Although seemingly counterintuitive, this helps alleviate the congestion. To test our analytical model, an agent based model was created using NetLogo. This NetLogo model simulates traffic around a highway lane closure and allows for analysis of the effects of several variables such as the speed limit, distance of the merge warning sign, and even the number of lanes affected.

The final solution incorporates both the zipper merging system and speed harmonization. First, drivers should be instructed to merge as late as possible and follow the zipper pattern when merging. Second, an adaptive speed limit zone should be created near lane closures by placing an electronic speed limit sign before the zone starts. The speed limit can be adjusted based on the traffic in order to balance the flow through the affected area.

Overall, this solution will ease congestion around lane closure zones and reduce problems that are associated with heavy traffic such as road rage, wasted fuel, and travel delay.

Director of the Department of Transportation,

As you know, traffic congestion is a significant issue that adversely affects many roads. In our study, we attempted to develop a method to refine the flow of vehicles near lane closures, as these areas frequently cause traffic congestion. At the same time, we wanted to ensure that this system was fair for all drivers. Based on our findings, there are specific guideline and signage changes we strongly recommend you adopt.

Firstly, we suggest that something must be done to influence drivers to merge later rather than earlier. While it is understandable that many drivers merge early because they fear not doing so will result in risky maneuvers, this practice imbalances the distribution of vehicles between the two lanes and often times results in traffic jams. On the other hand, late lane merging allows both lanes to be optimally used and thus improves efficiency in traffic flow. There are several approaches which can be taken to eliminate early merging. You could update driver education materials or employ an outreach campaign, delivering widespread messages to publicize the benefits of late merging as identified in our paper. You could also reposition and re-word merge warning road signs to discourage early merging.

In addition, we recommend that you implement speed harmonization, which is the inclusion of dynamic speed limits through electronic road signs. By setting up a detector at the sign announcing the lane closure and another half a mile ahead, we can count how many cars pass by this distance in a given time, giving us a measured “congestion”. Based on varying degrees of congestion, we can manipulate the speed limit to optimize traffic flow near the lane closure.

Our methods should be tested before widespread implementation; after all, this is a rather large change to our transportation systems. Perhaps you could start with select merge zones with a history of congestion. Based on preliminary field testing, the solution could be further refined and then promoted nationwide.

The adoption of these recommendations will undoubtedly alleviate traffic congestion near lane closures. In turn, this will provide a plethora of benefits, including higher fuel efficiency for cars, reduced road rage, faster arrival times, and even greater economic production.

Thank you,
Team #6083

Introduction

Traffic congestion can result in a multitude of detrimental effects, including late arrivals for many commuters, wasted fuel from idle cars, and road rage. One common situation where this phenomenon occurs is during lane merging. For example, in a typical highway (65 mph speed limit) lane merging, a two-lane road will converge into a one-lane road. Since each person drives differently, drivers in the ending lane will perform varying maneuvers to merge into the other lane. After seeing a sign indicating an ending lane, some drivers will immediately switch lanes, while others will wait to merge later. In addition to considering the various behaviors of merging drivers, it is equally important to recognize the actions of the drivers in the remaining lane. The combination of all these variations which exist within the system ultimately results in disrupted traffic flow. Thus, this paper seeks to model and describe a universal method for two-lane highway merging that minimizes congestion. Furthermore, this analysis is applied to alternate situations, such as merging on a three-lane highway or on a slower two-lane secondary road.

Assumptions and Justifications

- Assumption 1:** Cars enter the model at the speed limit.
Justification: People commuting want to get to their destination, so they tend to drive at about the suggested speed limit if there are no obstructions. Therefore, cars entering the model are traveling at the speed limit set for the road.
- Assumption 2:** Cars adhere to the two-second rule.
Justification: Drivers wish to maintain a safe following distance to the car in front of them. One widely followed guideline is the two-second rule, in which drivers stay at least two seconds behind the vehicle directly in front of them (Crimson Concrete).
- Assumption 3:** All cars are the same dimensions, 6 feet wide and 16 feet long.
Justification: This is based on average midsize sedan dimension data retrieved from USA Today (Woodyard). These dimensions do not heavily affect the model, but note that this does not account for buses, cars with trailers, or other large vehicles.
- Assumption 4:** When the driver is aware that a two-lane road is being reduced to a one-lane road and there is an open space in the remaining road for a car from the closing lane to merge into, the car will merge.

| | |
|-----------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Justification: | Afraid that they will have to make a risky maneuver further down the road, most drivers will take advantage and merge when they see an open space. |
| Assumption 5: | Signaling to switch lanes helps with merging. |
| Justification: | A car signal lets other drivers know the intentions of the signaling car. |
| Assumption 6: | All cars have the same acceleration and deceleration. |
| Justification: | Constant acceleration and deceleration values are simplifying assumptions for the model; although cars will realistically have varying acceleration and deceleration speeds, the majority of these differences are small and have no significant effect. |
| Assumption 7: | Drivers are unaware of lane merges until they see an indicating road sign. In reality, some drivers familiar with the road will merge far before seeing any indication to merge, but it is difficult to determine which drivers will act this way. Therefore, as a simplifying assumption, all drivers are unfamiliar with the road in our model. |
| Assumption 8: | Drivers have a reaction time of 0.75 seconds. |
| Justification: | This is based on the average reaction time data retrieved from multiple online sources. Drivers will need time to react before they hit the brake, which influences the delay time. |
| Assumption 9: | In all calculations, friction and air resistance were neglected. |
| Justification: | Friction and air resistance are two factors that affect all distance, speed, and time calculations; however, they do not significantly affect the calculations. Removing both of them greatly simplifies the model. |

Definitions

The “efficiency” of the model is measured by the average speed of cars in the zone, which is proportional to the average time in the zone. Higher average speed means greater efficiency.

“Fairness” is measured by the deviation of the speeds of the cars. Lower deviation from the average speed means greater fairness.

Basic Car Model

Analysis of Driver Behavior

While merging from a closing right lane into the left lane, drivers exhibit a variety of behaviors that have certain consequences on the flow of traffic:

- When they merge far before the right lane ends in order to avoid hassle, it causes congestion in the left lane, since traffic is meant to fully utilize the length of both lanes for as long as possible.
- When the right lane drivers merge wherever there is space, it requires left lane drivers to slow down to make space for them, causing a backlog of traffic as all cars in the left lane must now slow down to accommodate for the merging.
- While merging as late as possible is more efficient because the cars in the left lane to slow down the least, it can lead to other issues. If a driver waits until the last moment to merge but is unable to find a spot, he/she must slow down to a complete stop because the road is closing. This decreases amount of space that all of the driver in the right lane have to merge, and also requires the cars in the left lane to slow down considerably to allow the stationary car to merge. Late merging may also result in a higher chance of car accidents as the time to merge reduces, people are less able to find possible openings to merge.

All of these behaviors also have implications in causing road rage. In general, drivers become irritable when roads are congested or when they feel like traffic passage is unfair. This leads drivers who are:

- “Side-zoomers”, those that merge as late as possible, and often cause inflammatory remarks. Surrounding cars and cars that are being passed in the slow lane will feel that it is unfair which may increase chances of road rage.
- “Early mergers”, those that merge immediately because they feel that it is not fair to those in the slow lane. They can potentially cause large traffic delays, which may increase the chance of road rage along with the chance of unfairness.

We decided that road rage can best be controlled by mitigating the root cause of a number of scenarios that cause drivers to become irritated, namely traffic delays. Therefore, our model seeks to make traffic as efficient and fair as possible.

Analytical Merging Model

There has been extensive research conducted on the methods of merging that result in reduced traffic, and the zipper method appears to be one of the best ways for cars to efficiently enter a new lane without causing a traffic flow disturbance (Johnson). Hypothetically, cars should be able to merge into the new lane without causing a traffic jam if drivers in the left lane travel at a constant speed, maintain a large space ahead of them, and allow drivers in the right lane to speed up and merge as their lane closes (*Figure 1*). Drivers in the right lane should merge as late as possible so both lanes are used for the maximum amount of time. Afterwards, the cars in the single lane can accelerate to the speed limit and continue on their way. Our first step was to model this pre-existing zipper method and determine the relationship between the speeds of cars, before and after the merge.

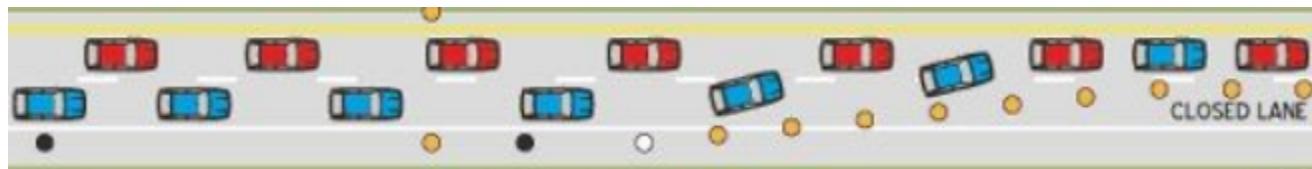
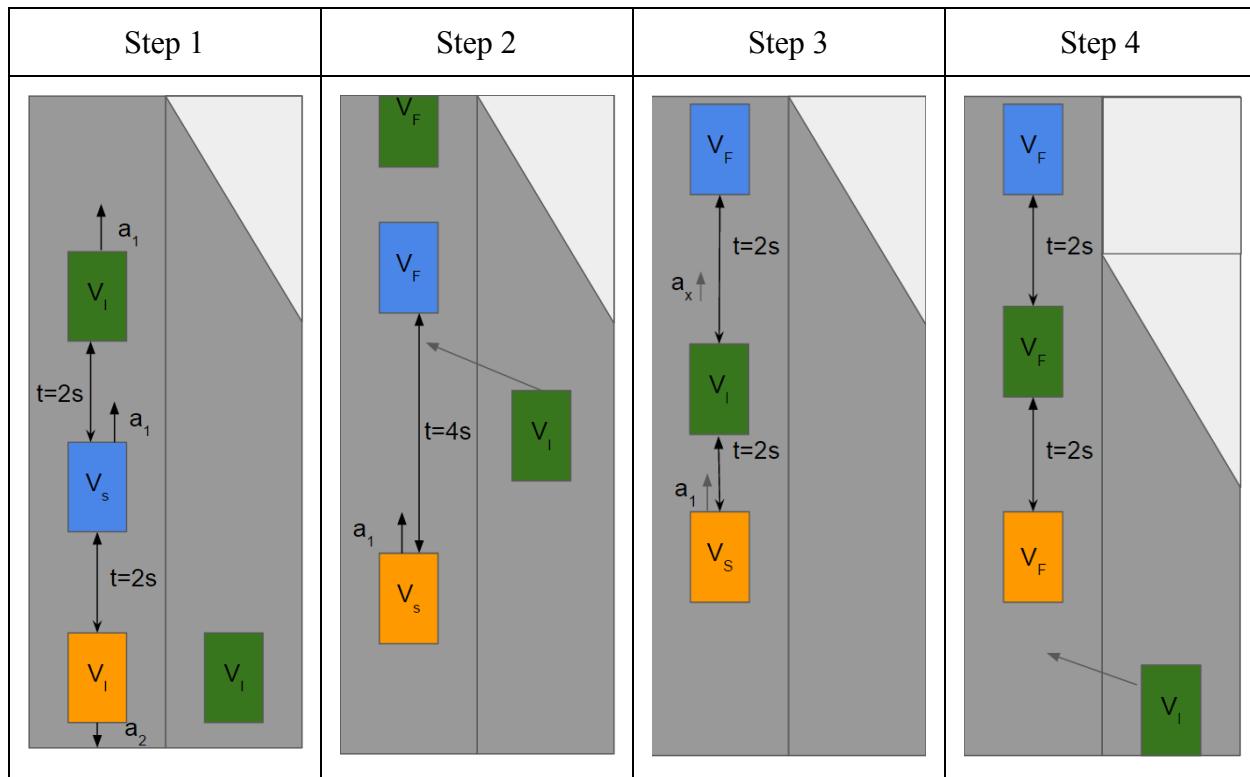


Figure 1: Zipper merging. Blue cars from the closing lane merge with the red cars in the open lane in an alternating pattern. This method is most efficient for reducing traffic flow disturbance.



Step 1

The Blue car has just finished slowing down to allow a car ahead of it to merge and is about to accelerate at a_1 ft/s to reach the speed limit V_F ft/s; the current Blue car speed is represented by V_S ft/s. Both the Orange and Green cars are travelling at the initial speed prior to entering the merging zone, V_I ft/s.

Step 2

Now the Blue car begins accelerating to V_F ft/s and the Orange car simultaneously begins decelerating at a_2 ft/s² in order to create enough space for the Green car to merge. From various online sources, the average acceleration and average braking deceleration was calculated (Baricella, Road Safety News). For the analytic model, we designated $a_1 = 11.4829$ ft/s² and $a_2 = -21.458$ ft/s², although in our simulation model, the acceleration rates are adjustable. Most driving guidelines recommend that drivers keep 2 seconds behind the car ahead of them (Crimson Concrete); therefore, the distance traveled by the Blue car minus the distance traveled by the Orange car needs to be at least $2(V_F + V_S)$, which takes into account both the distance between the Blue and merging Green cars, and the distance between the merging Green and Orange cars. Using one-dimensional physics laws, and letting the final speed be V_F ft/s for the Blue car, and the final speed be V_S ft/s for the Orange car we found:

$$d_{top} = \frac{V_F^2 - V_S^2}{2a_1} \quad d_{bottom} = \frac{V_S^2 - V_I^2}{2a_2}$$

$$d = d_{bottom} - d_{top}$$

In order to maintain the following distance of 2 seconds, 2 seconds in front and 2 seconds behind the merging car need to be created. Thus, $d = (\frac{V_S + V_F}{2})(4 - x)$ is used to calculate the distance needed to be created, where x is the current following distance in seconds. For example, if the current following distance is 2, then an additional 2 seconds needs to be created such that there exists a 4 second distance between the Blue car and the Orange car so that the Green car can merge in and still maintain the 2 second following distance buffer in front and behind. Thus, the distance created for a car merging into the left lane must be equal to at least 4 seconds.

$$\frac{V_F^2 - V_S^2}{2a_1} - \frac{V_S^2 - V_I^2}{2a_2} = (\frac{V_S + V_F}{2})(4 - x) \quad (\text{Equation 1})$$

Step 3

The Green car merges into the left lane, behind the Blue car. The Orange car accelerates at a rate of $a_2 \text{ ft/s}^2$ and Green cars immediately begin accelerating at a rate of $a_x \text{ ft/s}^2$ such that $a_x > a_2$ until they both reach a speed of $V_F \text{ ft/s}$ and the 2 second boundaries still exists between the Blue car and the Green car, and between the Green car and the Orange car. Because we assumed that drivers will want to abide the two-second rule, we can assume that merging cars will properly accelerate to maintain the 2 second buffer between the car in front and in back. This means that the acceleration of merging cars will exceed the acceleration of the cars in the left lane.

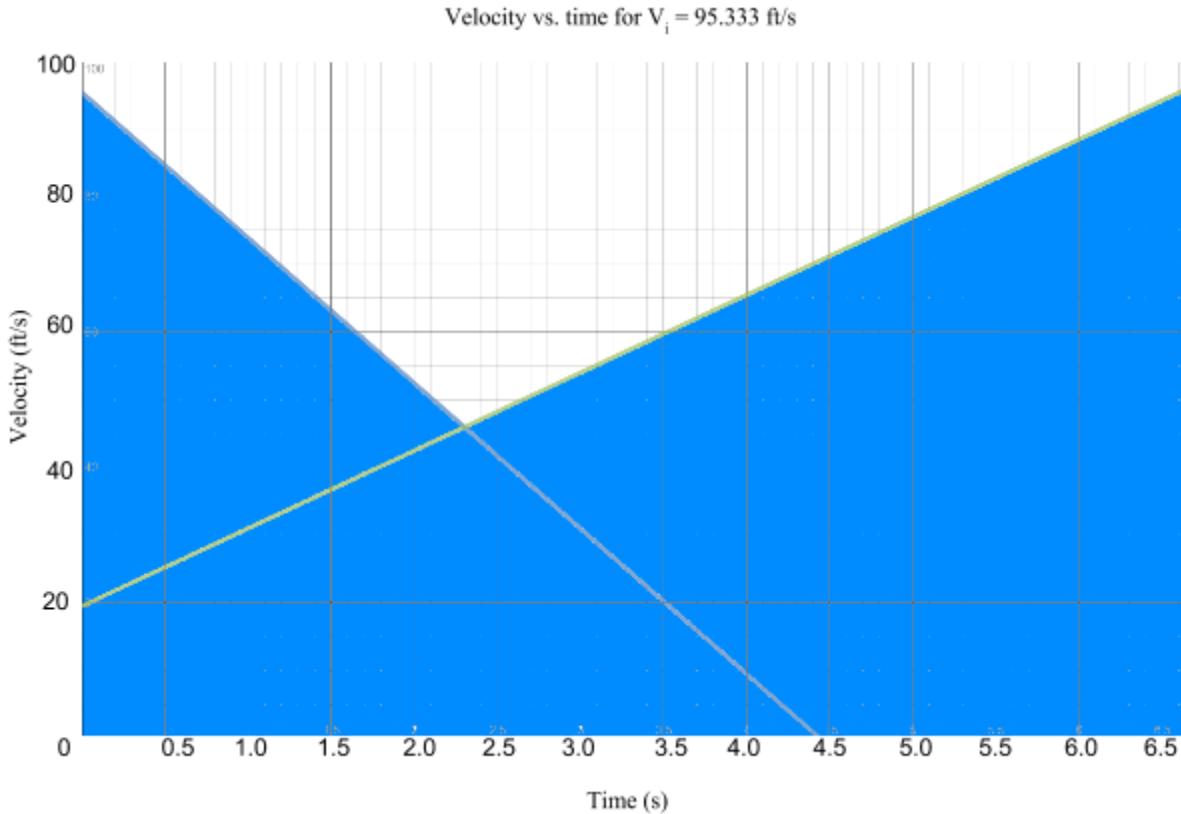
Step 4

Now that all the cars resume their speed at the speed limit, they are all driving at the original speed they entered at. These cars have all effectively left the congestion zone. A new car will begin to merge behind the Orange car after the acceleration to $V_F \text{ ft/s}$ starts and will merge once the Orange car reaches $V_F \text{ ft/s}$. This process continues in the same alternating pattern (zipper merging).

Analysis of Speed over Time

Using this merging model, we now have several ways to adjust the car movements so that they are optimized. Using a V_F of 95.333 ft/s (65 mph), which is the standard speed limit on US highways and a following time of 2 seconds, we wanted to see how the speed of the Orange car varied over time, with different V_I .

We started by examining the speed over time of a Orange car (a car getting passed) with a V_I of 95.333 ft/s. When the Green car signals to merge, the Orange car immediately begins decelerating at -21.458 ft/s^2 until it reaches V_S , which we solved for using Equation 1 and found to be 45.931 ft/s. This occurs at $t = 2.302 \text{ s}$, and marks the transition from step 1 to step 2. Next, the car begins accelerating at 11.4829 ft/s^2 until it reaches the speed limit of $V_F = 95.333 \text{ ft/s}$, at which point it is in step 3. This occurs at $t = 6.605 \text{ s}$.



$$y = -21.458t + 95.333 \text{ from } t = 0 \text{ to } t = 2.302$$

$$y = 11.4829t + 19.492 \text{ from } t = 2.302 \text{ to } t = 6.605$$

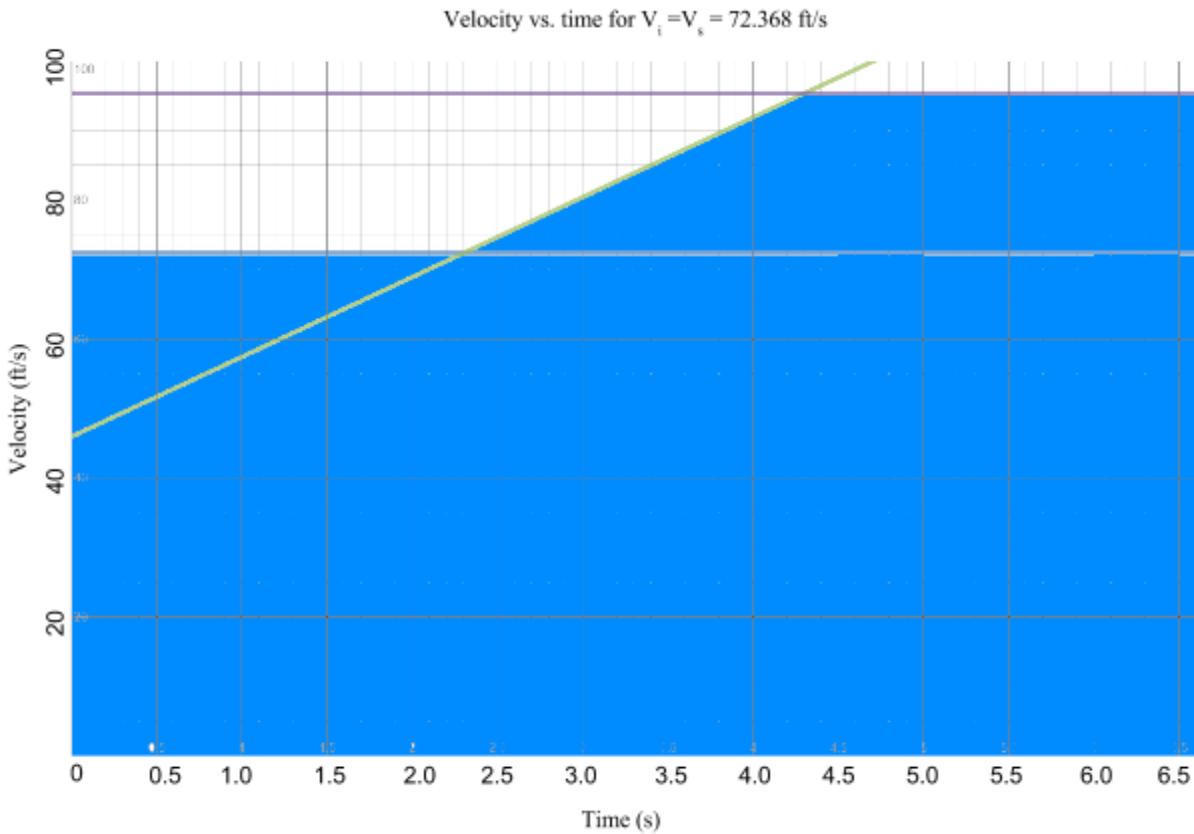
Noting the steep dip from $t = 0$ to $t = 2.302$ where the speed is reduced by more than half, it was deduced that decreasing the slope to 0 would increase the overall average speed over the entire interval because there would be no slow down ($V_s \Rightarrow V_1$), and therefore make the cars' passage through the closure more efficient. To model this, we created a second graph of the Orange car's speed over time, with $V_1 = V_s$, so that the Orange car does not slow down, merely allowing the Blue car's acceleration to create a gap large enough for the Green car to merge.

Taking into account this alteration, the equation solving for V_s becomes:

$$\frac{V_f^2 - V_s^2}{2 a_1} = \left(\frac{V_s + V_f}{2}\right)(4 - x)$$

$$V_I = V_S = \frac{-a_1(4-x) + \sqrt{(a_1(4-x))^2 - 4(a_1 V_f(4-x) - V_f^2)}}{2}$$

Once again we used a V_F of 95.333 ft/s and a following time of 2 seconds, and solved for a V_S of 72.368 ft/s . Therefore, the car maintains a speed of 72.368 ft/s until it reaches step 2, at $t = 2.302$. At this point, it begins accelerating at 11.4829 ft/s^2 until it reaches the speed limit of $V_F = 95.333 \text{ ft/s}$. Since V_S is much faster in this graph, the car reaches the speed limit of 95.333 ft/s much quicker, at $t = 4.302$. It then continues at this speed until the end of the time interval, at $t = 6.605$.



$$y = 72.368 \text{ from } x = 0 \text{ to } x = 2.302$$

$$y = 11.483x + 19.492 \text{ from } x = 2.302 \text{ to } 4.302$$

$$y = 95.333 \text{ from } x = 4.302 \text{ to } 6.605$$

In order to compare the efficiency of these two graphs, we integrated both of them and divided by the time interval to find the average speed.

| $V_I = 95.333 \text{ ft/s}$ | $V_I = V_S = 72.368 \text{ ft/s}$ |
|---------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------|
| $\int_0^{6.6} V_{current}(t) dt = 477.499 \text{ ft over 6.6 seconds, or } 72.348333 \text{ ft/s on average}$ | $\int_0^{6.6} V_{optimal}(t) dt = 553.806 \text{ ft over 6.6 seconds, or } 83.910 \text{ ft/s on average}$ |

Since the first graph has a much lower average speed, we can conclude that zipper merging at $V_I = 95.333 \text{ ft/s}$ is slower on average than zipper merging when $V_I = V_S = 72.368 \text{ ft/s}$. Therefore, it may be beneficial to lower the speed limit before a lane closure to allow zipper merging to occur more efficiently. We can effectively say that when $V_I = V_S$ an optimal recommended speed is generated that increases overall average speed.

However, the first graph does not account for the “lag” created by braking nor the reaction time needed to brake for the left lane drivers. This lag caused by braking only further supports the conclusion that shows that temporarily lowering the speed limit for a small interval of road length is a solution to traffic jams. In reality, the average speed over the merging interval is much lower 72.34833 ft/s calculated value because as more cars enter, the more the traffic delay increases, concordantly causing the average speed to decrease. The traffic delay increases because cars are spending longer and longer in V_S instead of instantaneously accelerating after reaching minimum speed. Because the optimal speed $V_S = 72.368 \text{ ft/s}$ allows for an interaction between the two lanes where the cars do not need to slow down for other merging cars, there should exist no traffic delay. The lag is nonexistent if all cars are passing through a certain interval at a rate of 1 car per 2 seconds.

Using our equations to model the current situation where there is maximum congestion and maximum speed (cars are evenly spaced 2 seconds apart and every car is initially traveling at 95.333 ft/s) we found that once the cars reach the lane closure, the merging will cause traffic to backup because the cars in the left lane need to slow down significantly in order for the cars in the right lane to merge properly, causing a wave-like compression of traffic. We decided to investigate this delay further.

Lag calculation

If each car is 2 seconds behind the one in front of it (maximum congestion), and traveling at some speed above the optimal speed, the cars will need to brake to let the cars from the right lane to merge. The time taken to brake coupled with each driver’s reaction time is what creates a wave-like compression of lag (*Figure 2*)

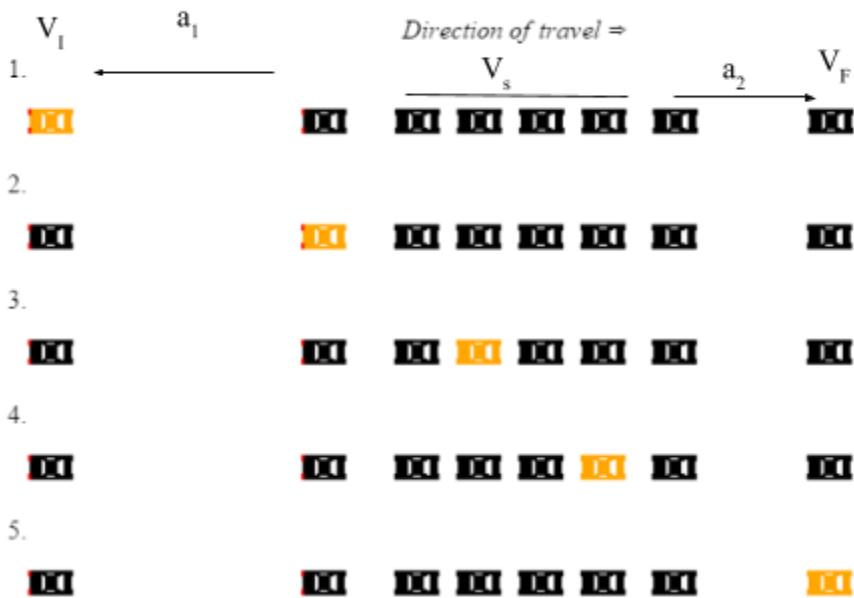


Figure 2 : Diagram of how wave compression occurs. When the cars at the front brake, it causes all the subsequent cars to slow down as well. This wave-like motion is cyclic and can potentially disrupt traffic flow.

If the lag is great enough, a traffic jam may occur. The purpose of this analysis is to calculate specifically how much lag results on a 65 mph highway lane closure during a period of heavy congestion (each car is two seconds apart).

The first car to reach the lane closure experiences no lag because no slow cars are in front of it. However, when the as soon as a car in the left lane brakes to allow a car in the right lane to merge over, it will cause every subsequent car behind it to brake to maintain spacing. This braking causes every car behind it to travel at a slower speed. Because the following distance remains at 190.6 feet (2 seconds * 95.333 ft/s) and the cars are now traveling at a slower speed, cars behind the first will have to spend a longer time driving up to the previous spot where the car in front was, where the merge previously occurred (*Figure 3*).

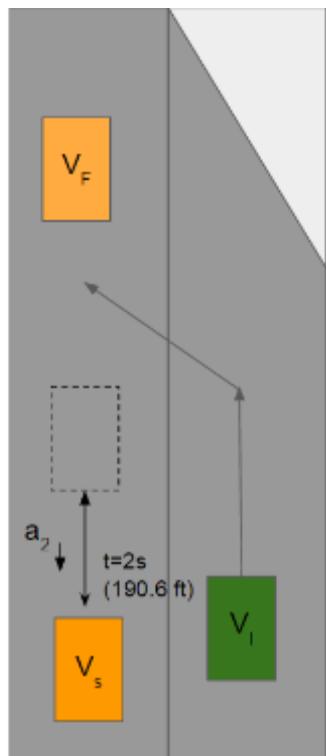


Figure 3: Cars from the right lane merge with the left lane. Rapid deceleration of cars on the left lane to allow for 4 seconds of follow distance causes congestion.

To factor in reaction time, 0.75 seconds was used (Following Distances and Road Crashes). This means that

the driver will take 0.75 seconds before he/she will start to apply the brakes. This delay in braking will shorten the following distance briefly. To calculate this reduction in following distance, the speeds for both the car in front and the car behind were integrated:

The front car will travel at 95.333 ft/s and decelerate at -21.458 ft/s² until it reaches 45.931 ft/s from t = 0 to t = 2.302 seconds (car in front starts braking at t = 0 and starts to accelerate away after 2.302 seconds).

$$\int_0^{2.302} V_{front\ car}(t) dt = 162.595 \text{ feet traveled}$$

The back car will travel at 95.333 ft/s from t = 0 to t = 0.75 seconds (Car behind does not apply brakes until 0.75s) and decelerates at -21.458 ft/s² from t = 0.75 to t = 2.302 seconds (62.0240 ft/s at t = 2.302 seconds).

$$\int_0^{0.75} V_{back\ car}(t) dt = 193.609 \text{ feet traveled}$$

By subtracting these integration values, we find that the distance between the two cars decreased by 31.014 feet. Subtracting this value from the initial following distance of 190.609 feet gives us a new following distance of 159.586 feet.

The time needed to travel the remaining distance is dependent upon the rate at which the Back car decelerates down to 45.9307 ft/s, the speed required to allow for a car to merge into the lane. To find the possible range of values that the possible delay times could fall in, the minimum and maximum possible times were calculated. For the minimum time, first an 119.103 feet are traversed at a rate of 62.024 feet per second which means it takes 1.920 seconds and then the remaining 40.483 ft are traveled at a decelerating rate of -21.458 ft/s², such that by the end of the 159.586 feet, the speed of the car is 45.931 ft/s, the speed necessary for merging. This yields a minimum time of 1.920 s + 0.75 s = 2.67 seconds to clear the 159.586 feet. For the maximum possible time, the back car continues to decelerate from 62.024 ft/s to 45.9307 ft/s for 0.75 seconds. During this deceleration, the car will travel 40.483 ft and 119.103 feet remain to close. For the remaining 119.103 feet, the car drives at a rate of 45.9307 ft/s, which means it takes 2.593 seconds to travel this distance. This calculation yields a maximum time of 0.75 s + 2.593 s = 3.343 seconds to clear the 159.586 feet.

Driving at a slower speed for the same distance of 190.6 feet between the cars means it takes a time of anywhere between 2.67 and 3.343 seconds. However, in optimal conditions, there should be no lag, thus cars can reach the previous location of the car in front of it in 2 seconds. This

means that there is a deviation from the 2 second optimal time of 0.67 to 1.343 seconds per merge at max congestion and 65 mph. This small lag quickly stacks up additively as cars pass over time. Consequently, when the left lane starts to congest, the right merging lane will also slow down and become congested, because it takes longer for a 4 second window to open up for cars to merge over. Both lanes will travel slowly if the speed at which the cars are traveling exceeds the optimum speed. Over a duration of a mere 10 minutes, at a rate of 30 cars per minute in each lane, 3.350 - 6.715 minutes of traffic can potentially build up from merging inefficiently. Over the course of a long rush hour period (4:30 pm to 7:30 pm) a car could potentially spend an estimated one to two hours stuck in traffic due to the congestion caused by the lane closure.

Speed Harmonization

Speed harmonization is a method to relieve traffic congestion that is common in Europe. By measuring the flow of cars among a specific section of a road in real time using sensors, an electronic speed limit sign a few hundred feet before the lane closure will change accordingly. Having a dynamic speed limit allows for flexibility when conditions change. For example, if the highway is more congested and has a very high density of cars, the speed limit will be lower so that the zipper merging method will be effective. However, during less populated hours, the speed limit may be higher. By adjusting the speed at which cars travel, a more stable traffic flow is achieved, thereby relieving congestion and optimizing efficiency.

This harmonic speed limit is found by calculating the optimal speed such that $V_I = V_S$ and that cars in the left lane do not have to slow down to allow cars on the right to merge.

Our model incorporates the design of speed harmonization near lane closures on both highways and on inner city roads. Since traffic congestion occurs frequently in these areas, we can apply speed harmonization to regulate the flow of cars and improve upon the zipper merging system.

NetLogo Agent Based Model

In order to corroborate the analytic model, an agent based model was created in NetLogo. This model incorporates the assumptions about individual driver behaviors and parameters. The behaviors in the code are listed below.

1. The car moves forward at its current speed.
2. If the car is moving slower than the speed limit and there is not another car within the ‘following distance’ in front of it, the car speeds up.
3. If there is a car within the ‘following distance’, the car slows down.
4. If the car is in the lane that ends and has passed the lane closure sign, they will merge if adjacent to an available space, defined by ‘following distance’. *This does not contradict the zipper model, in which there is not enough space to merge without the other lane slowing down.*
5. If the car is getting close to the lane closure, they will slow down and turn on their turn signal.
6. Cars in the remaining lane will slow down if a car in front of it in the other lane has its signal on. This allows the other car to merge more easily.

The environment is a 200 patch tall map with a road running up the center. Each patch is the width of a highway lane, 12 feet (Interstate). Cars travel from the bottom to the top, after being randomly placed in a lane. The cars are sent randomly at a constant probability (one of the variable inputs) and always enter the model traveling at the harmonic speed limit, and will accelerate back to the original speed limit after merging. The lane closure sign is denoted as a yellow patch at the side of the road, and the lane closure is denoted by a red zone in one lane. The positions of the sign and the lane closure can be changed in the model. All distances, speeds, times, and accelerations are in terms of patches and ticks (1 patch= 12 ft, 1 tick = 0.1 seconds).

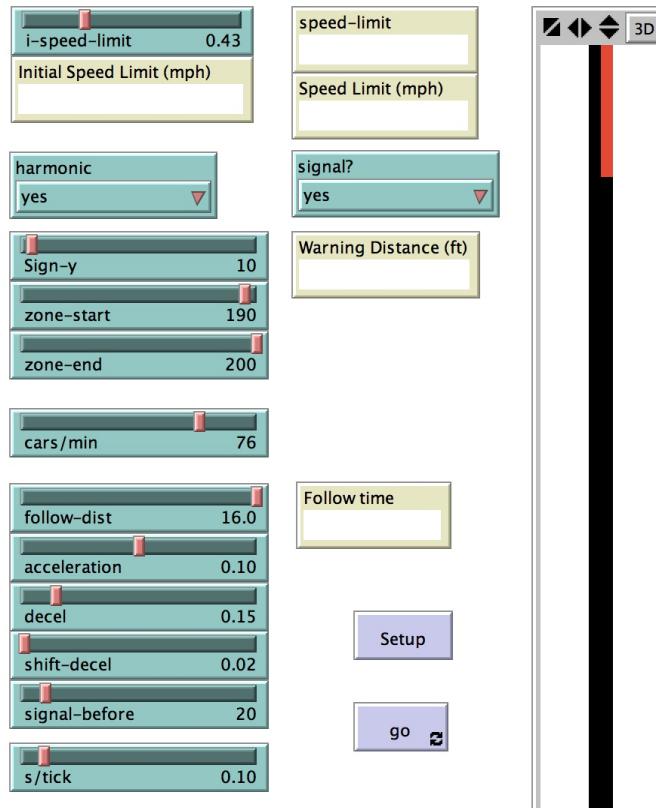
Two lanes merging to one lane

Using the metric for changing speed limit based on the average following distance (in patches), this model sets up two lanes, one of which closes. The model measures the number of cars in the the road between the sign and the lane closure and calculates the average following distance between the cars. The harmonic speed limit (initial speed entering system) is then derived from this number, the approximate acceleration of the cars, and the speed limit, V_F , using the equation:

$$V_I = \frac{-a_1(4-x) + \sqrt{(a_1(4-x))^2 - 4(a_1 V_f(4-x) - V_f^2)}}{2} \text{ where } x \text{ is following distance in seconds.}$$

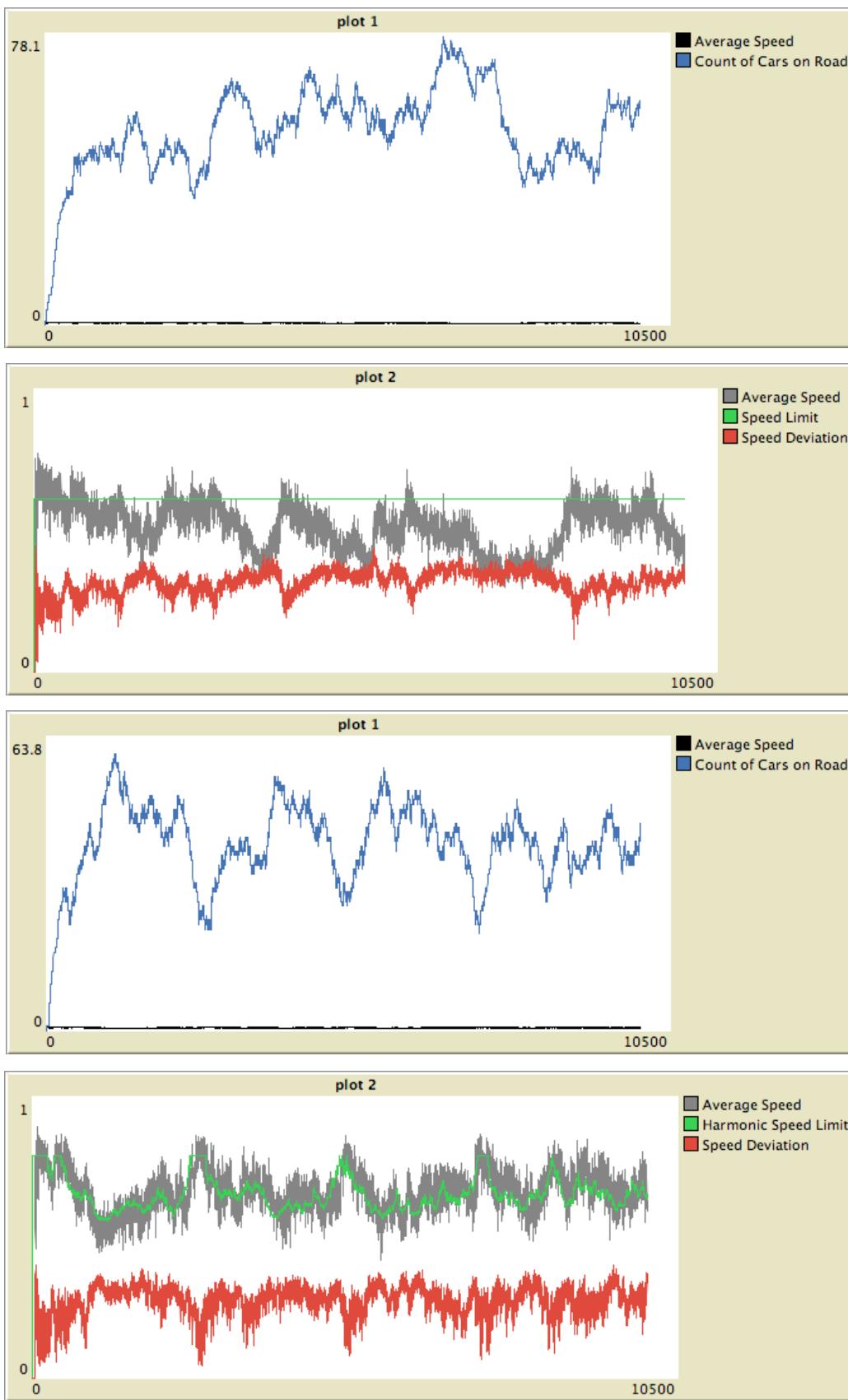
The “efficiency” of the model is measured by the average speed of cars in the zone, which is proportional to the average time in the zone. Higher average speed means greater efficiency.

“Fairness” is measured by the deviation of the speeds of the cars. Lower deviation from the average speed means greater fairness.



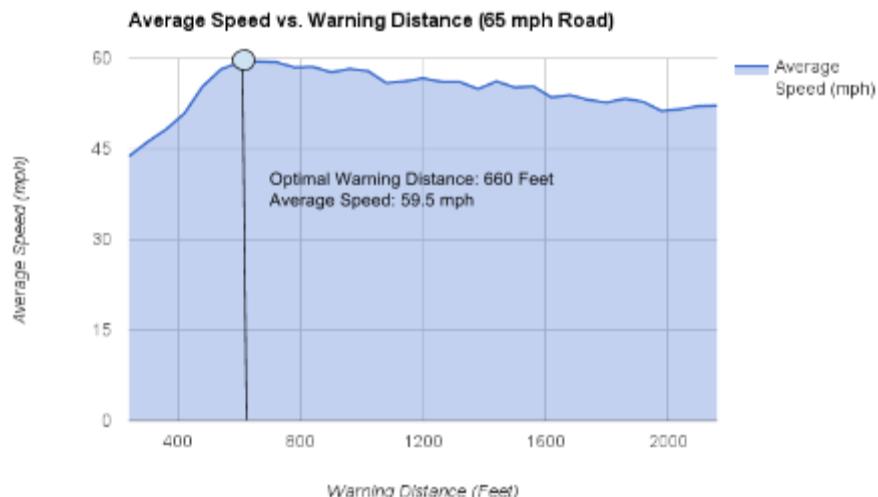
There are several inputs for this model, the values of which were chosen based on the analysis of driver behavior and car statistics. After setting the location of the sign and setting the driver parameters (such as following distance), the simulation runs for 10000 ticks (1000 seconds) and outputs two plots (examples shown below). In the plots below, the first two are without speed harmonization, and the second two incorporate it. According to these plots, the harmonization allows cars to maintain a higher speed and results in a more efficient and fair merge (The average speed for the second set is higher while the deviation is lower). At a speed of 65 mph, not only is the average speed lower than the average speed when a harmonic speed limit (50 mph) is set, but also the count of cars on the road is higher overall, indicating a higher level of congestion.

Also note that the speed limit decreases (down to about 0.61 patches per tick, or 50 mph) when there is a large influx of cars and increases (back to 0.79 patches per tick, or 65 mph) when there are fewer cars. This confirms that the analytic model is reliable for predicting the optimal speed limit, which is then used as the harmonic speed limit.

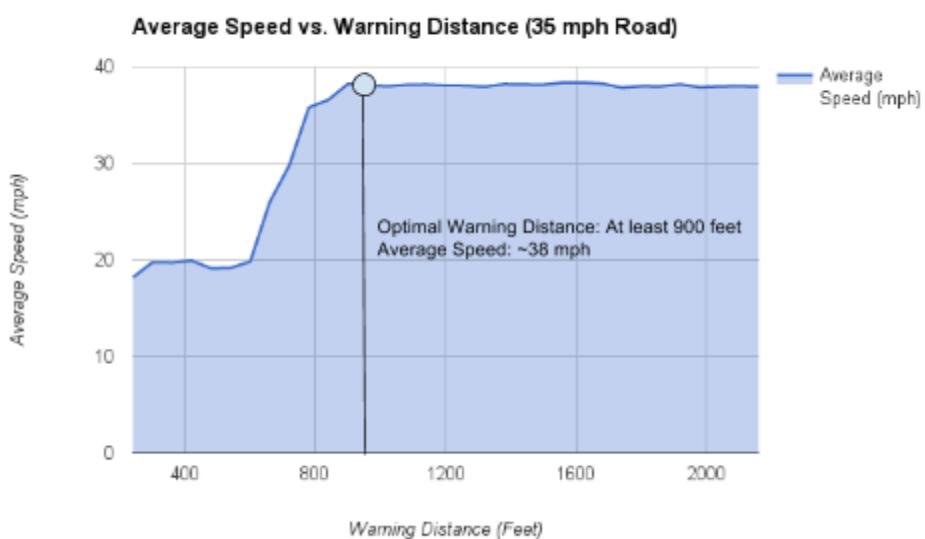


The model is then iterated several hundred times to test for the optimal sign placement relative to the lane closure (the difference between the two is the “warning distance”).

For a 65 mph highway, the optimum efficiency occurs when the lane merge warning sign is placed 660 feet before the lane merge. If placed further away, drivers merge too early. If placed closer, the drivers are not prepared enough and large amounts of congestion occur.

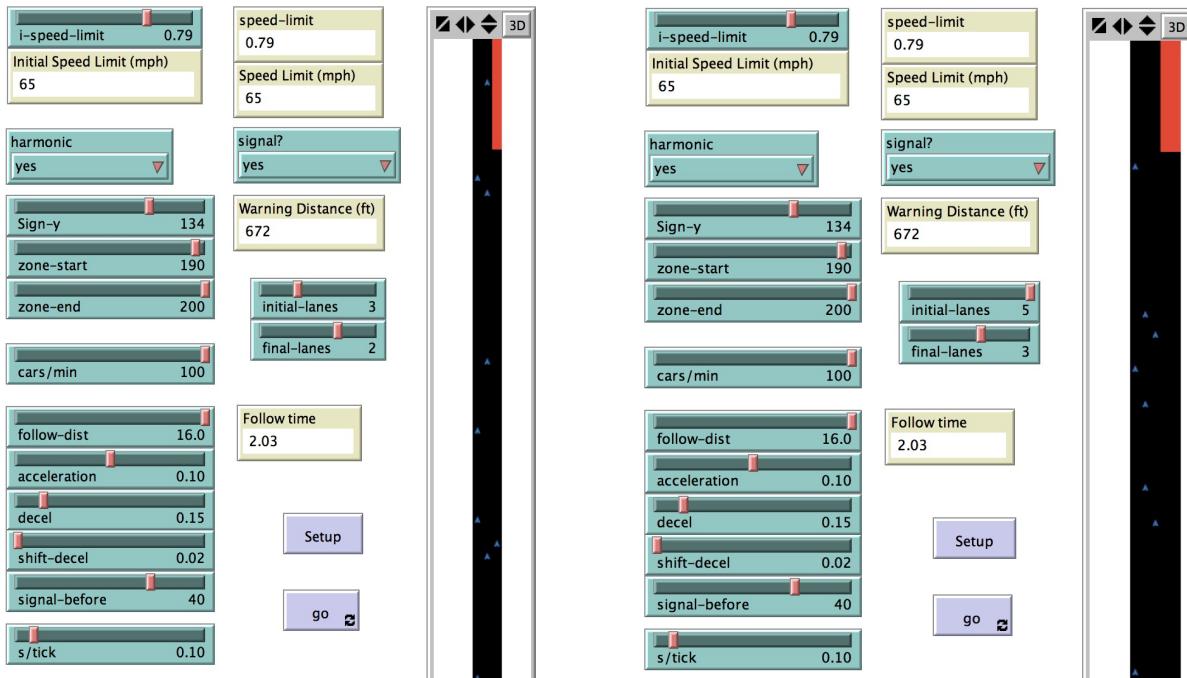


On a slower secondary road, with a speed limit of 35 mph, the lane merging sign should be placed at least 900 feet before the lane merge. At this speed, moving it further away does not seem to cause more congestion, but placing it closer tends to cause much more congestion.



Merging More Lanes

This model adds an option to change the number of initial and final lanes (examples shown: 3 lanes merge to 2 lanes, 5 lanes merge to 3 lanes).

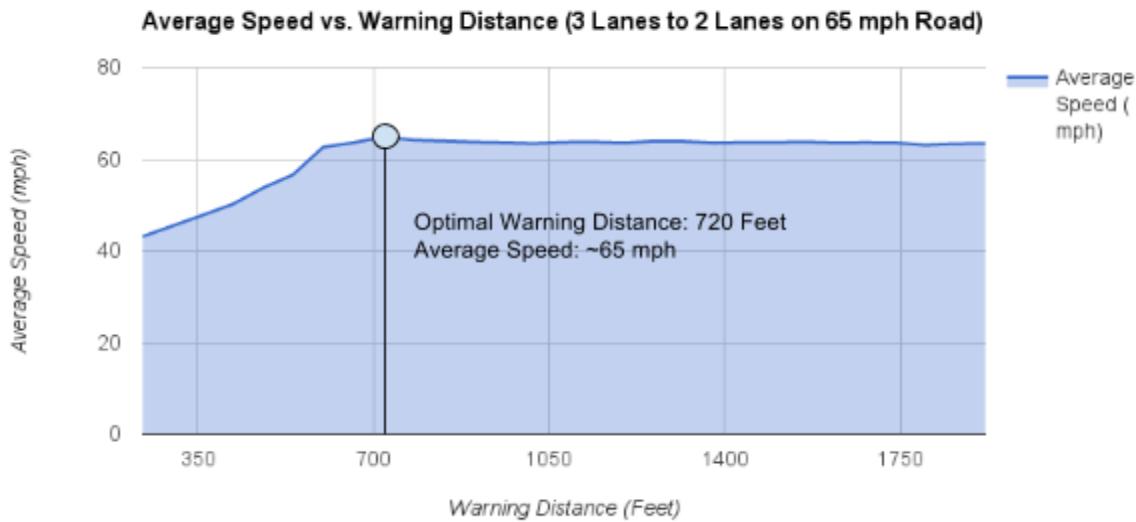


Using the same method and behaviors as before with the two lane model, other configurations are tested for optimal warning distance.

Three Lane Road is Reduced to Two Lanes

Even with the addition of an extra lane, the same concept of zipper merging can be applied. The drivers in this model will follow the same behaviors as with the two lane merge, with one exception: drivers in the center lane will switch to the other open lane to make space for merging drivers.

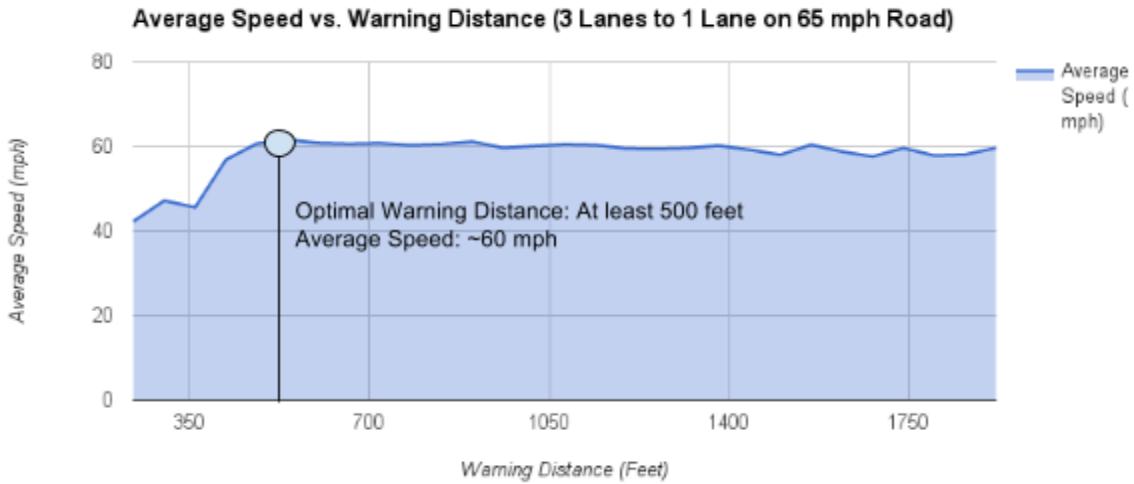
On a three lane road that loses one lane, the merge warning sign should be placed at least 720 feet from the lane merge. Similar to the slower two lane road, increasing this distance does not have much of an effect, while decreasing the distance reduces the efficiency significantly.



Three Lane Road is Reduced to One Lane

Once again, the same zipper merging method is applied to the system. Drivers will follow the same previous behavior models to merge into a single lane.

On a three lane road that loses two lanes, the merge warning sign should be placed at least 500 feet from the lane merge. Note that this closure causes extremely large delays if there are more cars in the model than can be accommodated in one lane.



Changes to Driver Guidelines and Signage

Driver Guidelines

If our method becomes standardized, drivers must become comfortable with the process of zipper merging and speed harmonization. This can be taught to current drivers through public service announcements and to new drivers by updating the driver's education curriculum. Firstly, drivers should be taught to merge as late as possible so the zipper system can be employed.

Conversely, drivers should let those in the closing lane merge as late as possible. It is important to know that this is not "cutting", but simply an efficient allocation of both lanes. Furthermore, claims that this system is "unfair" can also be repealed by illustrating how the zipper system works, as it allows cars to fairly alternate into the open lane.

Heavy emphasis should also be placed on recognizing merging road signs and speed harmonization limits. This way, drivers will know when is the appropriate time to merge. In order for traffic to flow smoothly, all drivers have a responsibility to adhere to the rules.

Signage

According to our NetLogo simulations, the maximum traffic efficiency for a two lane 65 mph merge zone occurs when cars begin merging 660 feet from the lane closure. Therefore, we encourage the Department of Highway Safety to post "Merge Ahead" signs approximately 660 feet before the right lane merges with the left lane. For alternative lane closure situations, the NetLogo model provides information as to where to put these signs. In addition, to promote the practice of zipper merging, these signs should also tell drivers to merge late rather than early, and to take turns merging.

We also recommend that the electronic speed limit sign be placed anywhere from 0.25 to 0.5 miles away from the merging lanes. Placing the sign further means that the cars are travelling slow for too long, while placing it too close would potentially cause congestion from the sudden slow down. After the lanes have merged, we suggest posting the regular speed limit sign immediately so that cars can resume traveling at faster speeds and don't congest the cars that haven't yet merged.

Strengths

- **The NetLogo model is highly adaptable for sensitivity analysis.** NetLogo is an agent based modeling program capable of testing multiple conditions due to its interactivity.

For our model, many changes can be made on specific parameters such as car acceleration values or the regular speed limit.

- **The model satisfies our initial definitions of “fair” and “efficient”.** By using both the zipper merge and speed harmonization, drivers will have relatively high average speeds with low standard deviation throughout the merge zone.
- **The NetLogo simulation verifies our analytic model.** For example, when analytically calculating the optimal speed limit during periods of heavy congestion, we determined this value to be around 51 mph. Later, when using the NetLogo model, simulation trials determined the same optimal speed limit. This helped to confirm the results of our analytic model.

Weaknesses

- **The model assumes all drivers and cars are uniform.** In reality, drivers will have unique behaviors based on their character. Since it is difficult to predict how each driver reacts differently to a situation, we made all drivers safe and intelligible. Likewise, there are too many existing car models to incorporate specifications. For the sake of simplicity, we had to reduce all cars to a uniform type with the same acceleration and deceleration values, inherently decreasing the realism of our model.
- **The model does not account for some notable circumstances.** The NetLogo model is unable to incorporate factors such as the weather or texting drivers. In reality, these factors will influence the dynamic of the merge zone, taking away from the accuracy of our model.
- **Adding new signs can be costly.** An electronic variable sign can cost upwards of \$100,000 (Moeur). Furthermore, manual costs of labor in repositioning existing merge signs may accumulate quickly. In addition, constantly monitoring traffic patterns may be too high maintenance. Instead, monitoring traffic patterns only during rush hours of the day may be more effective and less resource consuming.

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