Team 3192

Losing objects is an inevitability of human existence. We lose objects, and occasionally, we lose ourselves.

In the first section of the problem, we were tasked with finding a lost object in Hopkinton State Park with only a pen flashlight at our disposal and the sun quickly setting. We constructed four different scenarios that tailor our model to an individual's search, depending on what the pre-existing conditions exist. Our four models take into account various contingencies, considering whether or not the individual knows generally where the object is, and whether or not the individual stayed on the path. In situations where the individual stayed on the path, we aimed to maximize efficiency by minimizing redundant edge traversal through the use of Euler Circuits and Paths. If the individual knew a general area in which the object may be, we instead maximized area, increasing the probability that the object would be found by using an Archimedean Spiral, which has been shown to be the most efficient search pattern for an open area. We established 100%, 100%, 70.46%, and 2.21% certainty that our object would be found for each of the four scenarios. In the scenario with the the lowest probability, the low level of certainty is due to the complete lack of parameters for search; in this case, we maximized the area covered.

The second part of the problem asked us to devise a model for finding a jogger who has gotten lost at night in Fort Ord, using the pen flashlight as our only light source. We determined that the jogger's intention of going on a 5.0 mile run meant that he or she is no more than 5.5 miles deep into the park, with 0.5 miles accounting for any wandering on the part of the jogger. For this scenario, we constructed two models: one that accounts for when the jogger is stationary and another for when the jogger is mobile. If the jogger is unconscious (and therefore stationary), the searcher must travel along the Euler path that extends a maximum of 5.5 miles in trails into the park depending on the jogger's point of origin. If conscious, the jogger could rationally decide to remain in the same position, wander frantically, or wander out and return to the same starting point. Because we did not have any basis to accurately predict the behavior of the jogger, we made certain assumptions about the jogger's path of travel. We once again used Eulerization to calculate the optimal search path to find a jogger moving in the park, based on his or her starting area. If this area-specific search along the Euler path is unsuccessful, then we decided that the searcher should travel along the most popular and prominent paths in the entire park. The combination of both methods greatly increases the efficiency and likelihood of finding the lost jogger, regardless of whether the jogger is conscious or unconscious.

While there were several limitations, such as the exclusion of terrain from our consideration, we believe that we have developed an optimal model with the given information that satisfies our logical assumptions. Our plan encompasses a variety of plausible scenarios,while minimizing total search time and maximizing probability of success.

Introduction

Looking for lost objects is a common task, but one with general variation in efficiency. Sometimes we know the general location of the object, and other times, we simply have no idea. In context of Problem B, we are searching for a small object in Hopkinton State Park, and for a possibly unconscious jogger in Fort Ord Public Lands.

Hopkinton State Park is located in Hopkinton, Massachusetts and features hiking trails, among other recreational activities, over its 1450 acre area. The area of study is a portion of Hopkinton State Park and encompasses Hopkinton Reservoir, and a swimming pool. We are searching for a small object, such as a ring, which was left somewhere in the park.

Fort Ord Public Lands in Monterey County, California, is a former US Army post, now turned into a nature reserve with biking and jogging trails. Live rounds and explosives may still remain in the park, considering the area was used for military purposes for over 60 years. The area of study encompasses a large space of unexploded ordnance, and features a combination of paved roads and trails that are used for hiking, biking, and equestrian activities.

In our paper, we have split the problem into two sections, B1 and B2. Problem B1 refers to the search for a small object in Hopkinton State Park, and Problem B2 refers to the problem of finding a jogger on Fort Ord Public Land. We relied on the unique properties of Euler paths and Archimedean spirals to comprehensively search the selected areas and achieve optimal efficiency.

Assumptions and Justifications

Problem B1

Certain assumptions were clarified in the problem itself, and we used the given information as justification to determine:

- If the pen light flashlight is shined on the object, the object will be found.
- The average person walks at 4 miles per hour, and the time allotted for the search is two hours. Therefore, the distance covered by the individual searching will be roughly 8 miles.

In addition to the assumptions stipulated by the given problem, we determined that the pen light flashlight has the ability to illuminate a circle with a radius of five feet. The area illuminated by the flashlight can vary, depending on the angle of the flashlight to the ground, as well as the intensity of the light. Full-size incandescent flashlights produce 15 to 20 lumens, a measure of how much total light the source generates, on average, and high-quality pen flashlights can generate up to 24 lumens of light. Because the individual can turn while searching, the area illuminated will be circular.

In cases where our model determines an individual should begin their search in a particular location, we did not factor in the travel time from where the individual currently stood to the location where he or she would begin the search. Search, by definition, means "to look into or over carefully or thoroughly in an effort to find or discover something," and therefore, travel time does not qualify as "searching." Thus, the individual has the full two hours to search, because the search time begins when they arrive at the specified location. Because of the 15 parking locations scattered around the area of the park, it is not unreasonable for the individual to drive to the parking location closest to his or her starting location of the search.

There is a possibility that the small object being searched for has fallen into either

Hopkinton Reservoir or the swimming pool nearby. We disregarded those cases, given the low likelihood of retrieval, as our flashlight would be insufficient to find and to retrieve the small object in water. Therefore, we excluded the area covered by bodies of water in our calculation of the total area of the park, since we would not be searching there.

Additionally, we assumed that the individual searching for the object would be able to follow the path specified by the model. For instance, if our model indicated that the person should walk in a spiral, where the distance between two corresponding loops of the spiral was 10 feet, then the individual would be able to follow the pattern. Because we were unable to determine whether any conditions in the park would hamper the individual's search, we assumed that the individual can traverse the park's grounds, uninhibited by any topographical conditions such as marshes, swamps, ravines, and other geographical features that would hamper the path of the search. The contour map of the area does not show grossly steep features, so we assumed that the individual would have no difficulty in traversing the park in terms of altitude.

Additionally, we assume paths are 10 feet in width and will be completely illuminated by the pen flashlight.

Problem B2

For the problem of finding the lost jogger we made several assumptions. Because joggers typically run on paths, we assumed that the jogger did not stray from the trail. Path or trail refers to roads and hiking trails. Therefore, in our search for the jogger, we only considered the trail. Moreover, we considered the safety of the searcher in our decision to not stray from the path. Since the search is being conducted at night, the searcher himself may get lost if our model dictates he wander around the park. The safety of the searcher and most probable behavior of the jogger justifies this assumption. The park features several sensitive habitat areas and privately-owned areas that explicitly tell the jogger "Do not enter." These locations provide additional caution to the jogger to avoid straying from the path. Also, mountain lions appear occasionally in the park. Straying from the path could potentially cause increased danger to the jogger, and thus act as an additional incentive for the jogger to stay on the path.

The problem stipulates that the jogger was planning on running and returning to his starting location within 5 miles. It is unclear whether the jogger was in the middle of his jog when he got lost, or if he got lost after completing his five mile jog. Therefore, we establish a 5.5 mile distance limitation, from the jogger's starting point to his furthest probable location. The 0.5 mile addition to the 5 mile jogging distance accounts for the contingency in which the jogger has completed the 5 mile jog and is now wandering along the paths.

In this scenario, we make the same assumption that the pen flashlight is able to illuminate a radius of 5 feet, as we had done earlier in problem B1. Moreover, we assume that if the light is shined upon the lost jogger, then the jogger will be found. Given that the searcher is actively looking for the individual, it is reasonable to conclude that if the searcher catches sight of the lost jogger, then the jogger will be discovered.

The problem does not indicate whether or not we know the jogger's location for starting the jog. However, based on assumptions we made, we can conclude the jogger's starting location. Because the jogger most likely drove to the park to begin his jog, we determined that his starting location was one of the seven parking lots. Even if the parking lot is not his starting location for his jog, it is a known point where the jogger started his direction of travel. Moreover, because the park closes half an hour after sunset, and it is now dark, we can reasonably conclude that there will be no cars in the parking lot--except for the car of the lost jogger. Thus, we can pinpoint from which parking lot the jogger began traveling. We constructed a model that used one of the parking lots as an example for conducting the search; this approach could easily be replicated for the other parking lots using the methodology we have outlined.

We planned for two contingencies in this model, based on whether the jogger is conscious or unconscious. If the jogger was assumed to be unconscious, we assumed he or she remained stationary throughout the search. However, if the jogger is conscious, then we presumed movement; if the jogger consciously decided to stay in one location, then we would use the unconscious model. Predicting the behavior of the jogger relies on many factors, including personality and the jogger's familiarity with the wilderness. There are several courses of action that the lost jogger could decide to pursue. These are factors we are unable to assess, as we do not know the jogger's individual traits, so we could not conclusively presume what actions the jogger would take. Thus, we assumed that if the jogger was conscious, then he would be moving. Logically, we can presume, the jogger would not be moving to evade the searcher, however.

Unlike problem B1 where there was a time constraint given, problem B2 provides no time constraint for the search to be conducted. Since temperatures in Salinas, California, where the park is located tend to be mild around late fall, with temperatures ranging from a low of 40 degrees Fahrenheit to a high of 60 degrees Fahrenheit, the jogger is unlikely to suffer frostbite. Also, because humans can survive three to five days without water, and 21 days without food, we can reasonably assume that the hiker will be alive for at least 36 hours under these conditions, allowing ample time for the search to be conducted.

In the case that the jogger is conscious, we made certain assumptions about his path of travel. In his disoriented state, he would most likely choose to continue walking. When confronted with multiple roads, he would be more likely to choose the road that he is more familiar with. The roads more familiar to him would depend, but would more likely be the trails of the park that are frequented the most often by the various patrons of the park. Therefore, in order to narrow down what can be an extremely and unrealistically elaborate search , during which our lost jogger's condition could seriously deteriorate, trails which receive no traffic were excluded from our analysis for the dynamic search.

Model

Problem B1

Area of Hopkinton Park

The area of the portion of Hopkinton Park that we studied was 0.6852 square miles. We computed the area by importing the image into Adobe Photoshop, and using the magnetic lasso tool to outline the yellow portion of the park, excluding the water and surroundings. Factoring in the conversion scale provided, then we expanded the image so that the adjusted image size of the map encompassed an area that was one mile wide, and a mile and a half long. From there, we filled in the selected area with pink, and colored in the surroundings green. Using the histogram tools, we found the total number of pixels for the image, 656,042, and the number of pixels in the selected area, 299,679. Because the length and width of the entire map was one mile by one and a half miles, the area of the map was 1.5 square miles. Multiplying the entire area of the map by the pixel proportion, 299,679/656,042 yielded an area of 0.6852 square miles for the portion of the park we are studying. For additional information, please refer to Appendix D.

$$\frac{1.5\ miles}{656{,}042\ pixels} = \frac{x\ miles}{299{,}609\ pixels}$$

Length of Trails in Hopkinton Park

All the trails within the selection of Hopkinton Park were measured with the Pixel Proportion Method. Using the magnetic lasso tool found in Adobe Photoshop, we selected all the paved roads, hiking trails, and nature trails within the park. Then we "stroked" the selected lengths, meaning that a red line eight pixels wide traced all the paths. For ease of computation, the lengthy paved road at the far left of the map was excluded, because its inclusion would generate gross inefficiency in the model and because of the low probability the object would be found at that location, considering it lies on the periphery of the map, and time is of the essence. We determined the number of pixels the red line took up, which was 54,386, and then divided by eight to end up with 6798.2312 pixels, so that a line one pixel wide spanned all the paths in the park. From there, we set up a proportion, with the pixel width of the adjusted park image, 830 pixels, over the length of the park image in miles, 1.25 miles, set equal to 6798.2312 pixels, the number of pixels the trails spanned, over X, the total length of the trails we sought to find.

$$\frac{1.25\ miles}{830\ pixels} = \frac{x\ miles}{54{,}386/_{8}\ pixels}$$

The reason that the number of miles spanning the park's adjusted image width was different for finding the area of the park and finding the length of the trails is because we used different pictures in our calculations--but still made sure to set the lengths equal to the scale provided on the map. Solving that proportion yields a total trail length of 10.2383 miles. Please refer to Appendix E for further information.

Scenarios

For this model, we planned for various scenarios. If the individual knows the general area where the object might be, such as a picnic table, parking lot, or pavilion, for instance, it would be most efficient if he or she travelled back and searched these areas, where he or she believes the object would most likely be first. Also, whether the individual stayed on the park's trails, which included paved roads, as well as hiking and nature trails, or whether he or she wandered off into other areas of the park would influence which areas of the park should be searched. With these considerations in mind, we developed four scenarios:

1. General area of object's location known, stayed on trails
2. General area of object's location known, did not stay on trails
3. General area of object's location unknown, stayed on trails
4. General area of object's location unknown, did not stay on trails

Scenario 1

Because we assume for these scenarios that the individual stayed on the trails throughout their wanderings in the park, we only need to examine the length of the trails, and not all the area of the park. We divided up the entire area of paths into three sections of the park, assuming that a person would only spend their time in one section of the park, and from there, assuming that the person is aware of the general location where they spent their time. In the odd case that the individual loses the ring right on the border between regions, then the individual still has time to search the other regions because the time taken to search each region is approximately one hour.

To maximize the coverage, we used the concept of Euler path, where an individual goes to every edge once, and only once, defining edge as the part of the trail that is in between two nodes, which are either a starting or ending part of a trail (intersection). Each node has a degree, which is the number of edges extending from the node. Euler paths only exist if they have two--and only two--vertices of odd degree. In our model, we simplified the path of trails by creating a digraph, a visual representation of a graph using only edges to represent the trails and nodes to represent the intersection of paths. Based on this simplification, we discovered there were no naturally existing Euler paths in the partitioned regions. We eulerized each of the partitioned areas by artificially creating a path between nodes of odd degree so that there were only two vertices of odd degree in the end, establishing an Euler path. These artificial paths, in reality, represent a person going backwards on the same trail. By distance measurements of each path segment, this system minimizes redundancy, or going over the same path twice, by minimizing the distance the individual would retrace. In this way, we visit all of the trails most efficiently.

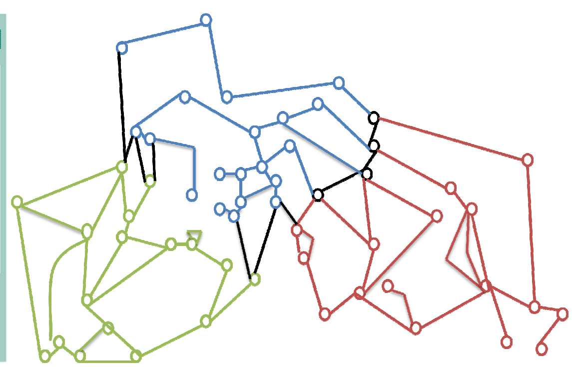
To briefly summarize in the context of the problem, eulerization means converting a undirected connected graph to an Euler Path by adding additional paths to the graph to reduce the amount of odd vertices to the point that the graph only has two vertices of odd degree. These additional paths allow the searcher to backtrack on a path already traveled once. For the sake of our model, optimal eulerization is the model that minimizes the total distance of backtracking.

For all three regions, we created Euler paths, not Euler circuits, the difference being that with a path, an individual would go through all edges without ending up at the vertex of origin. Mathematically, the starting point is one of the odd vertices, and the ending point would be the other odd vertex. We avoided making an Euler circuit, which would end at the starting point, because this method would create unnecessary restrictions and impede efficiency.

<u>Graphs</u>

The following graphs depict the partitioning of Regions A (green), B (blue), and C (red). For the sake of clarity, all of these graphs were merged into one. To see the graph simplification of each region only, see Appendix A Figures 2, 4, and 6.

Note: The black nodes and lines denote shared nodes and lines. For example, in the series of black nodes and lines in between regions B and C, these nodes and lines are both counted in each region.



Region A - Eulerization

There were 12 vertices of odd degree, so we had to eliminate 10 vertices of odd degree, which meant that we had to establish at least five redundant paths, in order for there to be an Euler path. Establishing a redundant path means that the individual would have to retrace their steps along the path in the opposite direction. See Appendix A Figure 2 and Appendix C Table 2 for clarification.

Region B - Eulerization

In Region B, there were also 12 vertices of odd degree. However, there were instances when the odd vertices were not directly connected to each other, meaning the only way to reach one odd vertex from another odd vertex would be to travel along multiple edges. For a more detailed explanation of our eulerization method for this region, see Appendix A Figure 4 and Appendix C Table 2.

Region C - Eulerization

There were 12 vertices in region C, and we needed 5 additional edges to eulerize the additional region. Using a method extremely similar to region A, we created an Euler path in region C. See Appendix A Figure 6 and Appendix C Table 2.

Scenario 1 Optimization Method

For all cases, the optimal search path is an Euler path which minimizes the aggregate distance of redundant paths. In order to do this, we developed a basic algorithm:

We first assumed that it is impossible to blaze a new path between unconnected nodes on the map.

- Connect one odd vertex/node to an adjacent odd vertex/node. Exhaust these possibilities by connecting all adjacent odd vertices possible. Continue to do this until there are two odd vertices remaining.
- If more than 2 odd vertices exist and said vertex is not adjacent to another odd vertex, take the following actions:
- If the odd vertex has exactly one even vertex en route to another odd vertex, create redundant paths from the first odd to even vertex and from the even vertex to the second odd vertex.
- If the odd vertex has at least 2 even vertices en route to another odd vertex, allow this odd vertex to be the starting/ending point of the Euler path.
- The remaining point will be the starting/ending point of the Euler path.

This algorithm minimizes the amount of redundancy by prioritizing adjacent connection above all else, which ensures the least amount of paths, and therefore minimizes redundant distance. Afterwards, we used the distance data collected for each path to calculate various models that incorporated different redundant paths, and discovered that these setups minimized the distance of all possible redundant routes.

Scenario 2

For this scenario, because the general area of the object's location is known, and because the individual did not stay on the trails, we must consider area. The most efficient pattern of searching was found to be a spiral, to cover the greatest area in the least time. Under the assumption that the flashlight shines a radius of five feet, we devised an Archimedean spiral, which according to studies done by Gal and Langetepe, maximizes the distance under this scenario.

This way, the distance between θ and θ+2pi equals ten, allowing us to cover all the area that the individual walks over with the flashlight. Because the distance between the loops of an Archimedean spiral is always ten, this method yields a consistent and comprehensive search pattern.

To develop the equation for this spiral, we used a polar equation:

$$r(\theta) = \frac{10}{2\pi}\theta = \frac{5}{\pi}\theta$$

10 represents the diameter that flashlight illuminates.

Since we know the individual can walk for two hours at rate of four miles per hour, he or she can cover a total of eight miles, or 5280×8 miles, or a total of 42,240 feet. We know the arc length is 42,280 feet, and we do not know θ, so we set:

$$\int_0^x \sqrt{\frac{25}{\pi^2}\theta^2 + \frac{25}{\pi^2}}\, d\theta$$

Equal to 42,240 feet, resulting in:

$$\int_0^x \sqrt{\frac{25}{\pi^2}\theta^2 + \frac{25}{\pi^2}}\, d\theta = 42{,}240$$

We find that θ=230.3772 radians, which is as far as an individual would be able to go in two hours following the search pattern of the Archimedean spiral. Substituting θ=230.3772 back into the original equation,

$$r(\theta) = \frac{10}{2\pi}\theta = \frac{5}{\pi}\theta$$

We find r=366.6568 feet, which is the radius of the area covered using this search pattern. In this way, the Archimedean spiral search pattern will cover an area whose diameter is the equivalent of approximately the length two football fields.

If the individual has two locations in mind of where the lost object may be found, he or she would first begin searching at one of the likely locations, and start looking for the object with the pen flashlight an Archimedean spiral. After one hour of searching, if the object is not found, then the individual would move to the other hypothesized location where the object may be found. Here, instead of one large spiral, there would be two spirals. Because the individual is to devote one hour to each mini-spiral, we set the original Archimedean spiral equation's arch-length equal to 4×5280 miles, and following the same method as mentioned in the previous two paragraphs, each mini-spiral yields an angle of 162.8923 radians, which corresponds to a radius of 259.2511 feet when θ is substituted into the original equation.

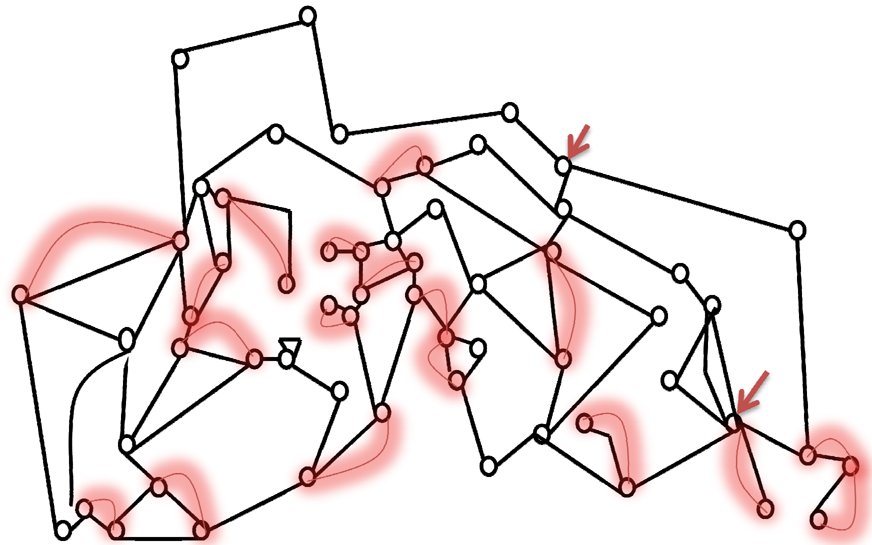
This approach could be generalized to any number of potential search locations, where the radius and the area covered by the individual in each mini-spiral would progressively become smaller as the number of potential locations increased, but the total area of the park searched would remain the same, that is 8×5280 feet, or 42,240 feet. For a visualization of the Archimedean graph, see Appendix A Figure 7.

Scenario 3

In this scenario where the individual stays on the path for the duration of his or her stay in the park, but does not know the general area where he or she dropped the object, we used a method similar to Scenario 1. Because the probability of finding the object is equal along each edge, given that the individual has no idea where he may have dropped the item, and since no edge is more likely to yield the discovery of the object than any other edge, the individual would start at the vertex of odd degree that is closest from them. With regards to the construction of the efficient path, we took measures to avoid traveling along redundant edges for as long as possible.

We create a preliminary path by tracing an edge from a starting vertex of odd degree, progressing as far as possible without retracing any area. Our criteria for choosing which paths to pursue were that when confronted with an intersection of multiple paths, we chose the path that extends the farthest. For instance, with a fork in the road, if going left yields a trail of 0.17 miles and turning right yields a path of 0.68 miles, we would turn right. Such a scenario is called a "turn." If this edge leads to a vertex, which has no untraversed edge except for the one used to enter the location, then do not pursue it. This is a "dead end" and will add redundancy to our model, decreasing the path distance our individual will be able to search in two hours. Using these guidelines, we predict what the next two steps are for the individual to travel. If, within two steps, this path leads to a dead end, then do not pursue the longest path that extends from your current vertex. Select the next longest edge, and if you would not reach a dead end within two steps from your current location, pursue that path of travel. Once you have reached a vertex that has no edges that would not lead to dead ends in two "turns," select the longest edge to follow. While this introduces redundancy, this is necessary to eulerize the graph, which would be able to traverse every edge exactly once, the most efficient method for searching.

From this point, the individual would begin using Hierholzer's algorithm to determine his or her path of travel. Hierholzer's algorithm instructs the person searching to choose any starting vertex on an Eulerian graph, and follow a trail of edges from that vertex until he or she returns to the vertex of origin. It is immaterial whether the individual chooses the shortest path or the longest to return to the original vertex; the algorithm works regardless. Once a path that begins and ends at the same vertex has been established, called a circuit, look for any vertices along the circuit that have unused edges. Because our web of trails has been eulerized, it is impossible to get stuck at a point other than at the vertex of origin, as the even degree of all the other vertices means that when an edge crosses a vertex, there is another edge that has not been travelled through, extending from that same vertex. Starting at one of these unused vertices, make another circuit with the unused edges. Add this circuit to the originally constructed circuit. Using Hierholzer's algorithm results in an Euler circuit over the graph. The eulerization of the total park is as follows:



The nodes with arrows point to them represent the odd nodes.

The individual is able to progress along 7.214 miles with no redundancy. This distance was calculated using the path outlined by a mostly greedy algorithm. For further explanation of this particular algorithm, see Appendix F. At the 7.214 mile mark, however, the individual reaches a dead end, and must backtrack, causing redundancy. For more information regarding the eulerization of the graph, see Appendix A Figures 8, 9, and 10.

Scenario 4

In this scenario, the individual searching for the lost object did not wander exclusively on the trails in the park and does not know in what general area he or she may have lost the object—but does know that the object is in the park somewhere. Because of these factors, it would make no difference where the individual began their search, so starting at his or her present location would be as good a starting place as any. We use the same method of searching as we employed in scenario 2, with the Archimedean spiral, covering an area that is 366.6568 feet in radius.

This situation is extremely unlikely--that the individual would have absolutely no recollection of where their object may have been lost. Because each area of the park has the exact same likelihood as another other area of containing the lost object, there is no advantageous location of begin searching. As a result, the likelihood of finding the object is very low.

Probability

Scenario 1

In this scenario, the general area of the object's location is known, and the individual did not stay on the trail. As long as the individual is sure the object is in that general location, there is a 100% probability that the object will be found within the two hours. The restriction for this to be true is the fact that the individual must be able to pinpoint the general location of the object in either region A, B, or C. If not, then this scenario does not provide the proper search pattern for the individual to use; see scenario 3 instead.

Scenario 2

For this scenario, because the general area of the object's location is known, and because the individual did not stay on the trails, the probability of finding the object is 100%--as long as the individual's prediction of the general area where the object should be is accurate-- that being, encompassed by the 422,400 square feet surrounding the starting point. Because the Archimedean spiral encompasses such a large area, we can assume that if the individual's assessment of where the object was last seen or would most likely be found is correct, then the object will be found. Therefore, if these assumptions are correct, the probability of finding the object is 1.

In the event that the individual strayed from the path and the general location of the object is known, but the possible locations span an area larger than 422,400 square feet, the same method of the Archimedean spiral would be employed, simply until the two hour limit was up. In this case, it is impossible to search all of the possible locations, for the sum of the areas of the possible locations adds up to more than we can feasibly search within our time constraints and with our flashlight. The probability of finding the lost item would then be 422400 divided by the total area in square feet of possible object locations.

Scenario 3

Using the method for finding an object if the general area is unknown and the individual stayed on the trail, the individual is able to cover at least 7.214 miles without repeating any traversed land. Because the total length of all the trails is 10.2382 miles, there is a 70.4616% probability that the small object will be found.

Scenario 4

For this scenario, because the general area of the object's location is unknown, and because the individual did not stay on the trails, we must consider area. The area encompassed by a person travelling at 4 miles per hour in an Archimedean spiral would be the arch length of the spiral, 8 miles, multiplied 10 feet, as that is the width of the spiral. This multiplication yields 0.01515 square miles. Because the total area of the park is 0.6852 square miles, there is a 2.2145% probability that the object will be found when the general area of the object's location is unknown. In this unlikely but unfortunate scenario, the individual has absolutely no recollection of where he or she has been in the entire park. Given that the ring could be found anywhere in an area of 0.6852 miles, the likelihood of finding the ring within two hours is very small.

Problem B2

Fort Ords Public Lands Park

In order to find the lost jogger in the Fort Ord Public Lands Park, we employed a plan consisting of two methods to have a good chance of finding the jogger. We created two scenarios, one where the jogger is stationary, and another where the jogger is moving. The first method aims to specifically modify the search to the area by finding the trails the jogger would take after entering the park,. The jogger would have entered near a parking lot where the jogger parked his or her car. Using eulerization, the searcher can optimize the distance covered and minimize time used. If these trails are exhausted and the jogger is still not found, the searcher then switches to the second method, which maps the trails with the highest traffic and walks along the Euler path created from the model.

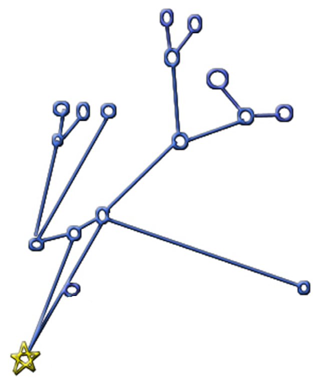
Situation 1-Unconscious Jogger

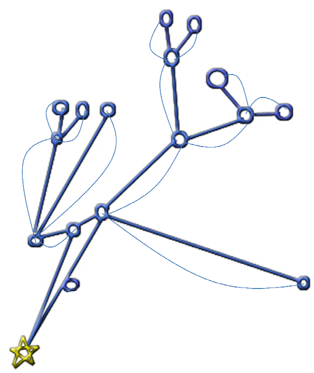
Because the jogger is unconscious, we are searching for a stationary target. His location should not be more than 5.5 miles from the parking lot, because he went on the jog with the intention of running 5 miles. Most likely, the jogger went on the run with the intention of coming back, so he would only have progressed 2.5 miles into the park. However, we have planned for the scenario where the jogger is 5.5 miles into the park, accounting for some frantic wandering on the part of the jogger. Based upon our assumptions, we began at the parking lot where the jogger's car is located, tracing out all the possible paths of distance less than 5.5 miles accessible from the parking lot. The exact distances of each path were found using Adobe Photoshop, the histogram tool, and then proportions, as has been used in Problem B1. From there, we constructed a graph, placing nodes with the park's trails intersections and drawing edges to represent each trail's path. Using Adobe Photoshop, we truncated the paths to precisely 5.5 miles in path distance from the parking lot.

There are seven parking lots in Fort Ord, and to apply this strategy to other parking lots would involve the same methodology. To see our numbering for these parking lots, see Appendix B Figure 3. Given the starting location of the jogger is at one of the parking lots, and assuming that the jogger has progressed no more than 5.5 miles from the parking lot, there is a 100% probability that the stationary jogger will be found, because all the paths within the area will be traversed, and there is no pressing time constraint placed on the rescuer. However, the searcher still seeks to find the jogger in the least amount of time. To accomplish this task, we employed eulerization so that the traversal of all the trails within the 5.5 mile path distance is optimized, and thus the most efficient method of covering the length of all the trails where the unconscious jogger may be is pinpointed.

To demonstrate this method, here is procedure for searching for the jogger, if he began from parking lot 1:

Non-Eulerized Graph Eulerized Graph





Note: Star indicates location of Parking Lot 1

This is a simplified graph of Figure 1(in Appendix B). On the left, the non-eulerized graph theoretically shows a route with no redundancies whatsoever in trying to find the lost jogger. This graph represents the optimal way to travel all edges from the start of one of the seven parking lots. In the scenario, the parking lot is denoted with a star. The ending point will also be at the star.

However, the non-eulerized graph is, by nature, not the most efficient path of search. Therefore, there is no pure optimal path that can guarantee finding the lost jogger based on where the jogger's car was parked. The most optimal way to eulerize the graph is to minimize redundancy, or the back-tracking of steps. After several models of eulerization, the graph on the right was discovered to be the best eulerization (using the Scenario 1 Optimization Method). Using the Pixel Proportion Method, the distance calculated to traverse for parking lot 1's eulerized graph is 11.8 miles.

Note: This is just for one of the seven parking lot methods. The same method to calculate the parking lot method for the other six can be employed in the exact same manner, using the total path distance of 5.5 along any branch of the different parking lot paths. This example was shown for clarity of the reader.

<u>Situation 2-Conscious Jogger</u>

If the jogger is conscious, we no longer assume that the jogger would only be within 5.5 miles of the parking lot. Theoretically, given enough time after the reporting of the incident, the jogger could be anywhere within the boundaries of the park. To search the entire park would clearly be unfeasible, considering the aggregate length is about 40 miles as determined by the Pixel Proportion Method, assuming there would be no backtracking whatsoever to get to all of the trails. Including backtracking, the total eulerization would likely be well over 55 miles. Given the time constraints associated with locating the jogger before his or her health seriously deteriorates, the map of trails necessitates simplification.

Graph Simplification by Popularity

In order to still maintain a good chance of finding the jogger while cutting down on the search time, we devised a diagram that only included the most popular trails. We used data of how frequently certain trails are used in Fort Ord. Using the data of the hikers' usage of the park's trails, found from the 2010 Bicycle-Equestrian Trails Assistance (BETA) Observation report, we determined that 45 trails are never used by people on foot. We disregarded the unused trails and grouped the high-traffic trails together. The complete diagrams can be found in Appendix B Figure 2.

Jogger Prioritization

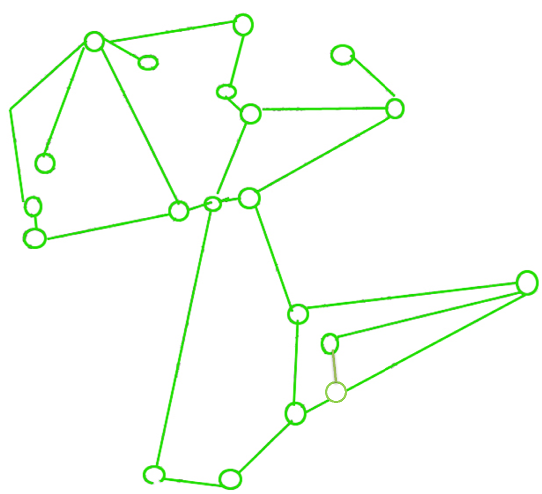
Assuming the jogger is not totally disoriented, he or she will adhere to the following rules:

1. The jogger knows he or she is lost and is actively trying to escape the park.
2. The jogger will prioritize more prominent roads (those delineated in a solid black line) more often than hiking trails (dotted lines), because prominent roads lead to parking lots.

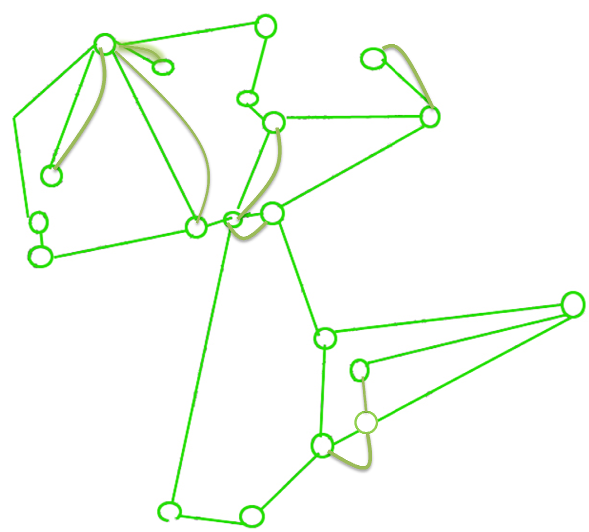
Composite Graphs

Combining the graphs produced, both by Popularity Simplification and Jogger Prioritization creates the simplified map of all trails, as follows:

Non-Eulerized Graph



Eulerized Graph



The graph on the left is the simplification without eulerization, and therefore, no redundancies. The graph on the right is the optimized eulerization with minimal redundancies using the Scenario 1 Optimization Method. Since this is an Euler Cycle, not an Euler Path, any node is a starting point. The total distance of the graph on the right is 30.05 miles, which takes approximately 7.5 hours assuming the searcher traverses the park at a constant 4 mph. The Popular Trail Method will remain the same for all seven Parking Lot Methods.

Parking Lot Method versus Popular Trail Method

The parking lot scenario, in general, proves to be the more specific and time-efficient method of finding the lost jogger. Therefore, the Parking Lot Method should be tried first. Once this method is exhausted, the searcher should proceed to the nearest node on the Popular Trail Map to begin the second phase of searching – the Popular Trail Method.

In terms of time allotment, we offer an example which begins by employing the same method as the Parking Lot Method at parking lot 1. In this scenario, the search begins at 7:30 PM. If the searcher proceeds through the course, a total of 11.8 miles, at a constant pace of 4 mph, the searcher will finish the Parking Lot Method for Parking Lot 1 in 177 minutes, or 2.95 hours. Then, at 10:27 PM, the searcher will progress to the Popular Trail Method. Then for approximately the next 7.5 hours (the total distance of the Popular Trail Method is 30.05 miles; 30.05/4mph=7.5125 hours), the searcher will follow the Popular Trail Method map. If at any point the searcher discovers the lost jogger along the path, the searcher will stop the search and take the jogger back to the parking lot where he or she started from.

Note: for other scenarios with other parking lots, the starting nodes will vary, but the Popular Trail Method map will remain the same. In other words, different parking lot methods will link into different starting points on the Euler path of the Popular Trail Method.

Limitations

Problem B1

Our model for finding a small object in Hopkinton State Park does not factor in the changes in elevation present in the park. As a result, taking into consideration elevation would change the total area of the park, because it would no longer be a flat surface, and would change the distance and area that we would be able to cover using our search patterns. However, this change in elevation is relatively minor; we are not considering a mountainous region. For this reason, our model is able to represent the situation on the ground with a reasonable level of accuracy.

Although this was discussed in our assumptions, the fact that we assume our model's paths are all accessible to the searcher is a serious limitation. In real life, our spiral path may be obstructed by large trees, or by inaccessible barriers, such as gates. Furthermore, we disregarded bodies of water in our model because of limited resources for finding and retrieving the object. The possibility of losing the object in water is not nonexistent, but considering our available time and our allowed use of only the pen flashlight, our model optimizes our resources for a search on land.

Speaking of resources, our model bears the limitation that we are only able to use the pen flashlight. In certain instances, using the pen flashlight in combination with other resources would be more productive. For instance, if we were searching for a metal object, such as a ring, a metal detector in conjunction with the pen flashlight would be ideal and would aid greatly in the probability of finding the ring. The chance of a person carrying around a metal detector in case they lose a small metallic object while in a park is relatively small, and thus would have little real-world applicability.

One major flaw in the design of the problem itself is the assumption that if light shines upon an object, then that object will be found. For practical modeling purposes, this assumption is certainly necessary; however, in real life, it is possible--if not likely--that the searcher may overlook the object, given that he is walking at 4 miles per hour and frantically scanning the area.

Moreover, due to the restrictions stipulated in the problem, there is only one person searching for the object when in reality, multiple people could be looking for the object, increasing area covered, and thus increasing the probability that the object will be found. Also, searching for a small object in dim lighting alone, and armed only with a pen flashlight is a dangerous pursuit. It may simply be more efficient to return to the park the following morning and search at that time. However, for this model, we adhered to the given instructions--that the sun was setting soon and thus we had a time limit in place for our search.

One of the great intangibles of searching for objects is the individual may remember certain aspects of where he or she last saw the object as the search progresses, when something the searcher sees triggers his or her memory, and leads to the revelation of information that would aid the search. Our model is unable to take this into account, because the occurrence of this phenomenon is unpredictable and because the timing of this phenomenon, if it occurs, would vary greatly. Therefore, in the case that an individual suddenly remembers information about the lost object in the middle of the search, we advise for them to relocate themselves to the location where they now believe most likely contains the object, and begin their search there, using either the Archimedean spiral or the scenario 1 method, depending on whether or not the individual stayed on the trail or wandered off from it.

Problem B2

Our model of finding the jogger in Fort Ord public lands did not factor in elevation of the land the searcher and the lost jogger would be travelling through. Adding in elevation would slightly alter the distances the searcher and the jogger would be able to cover because of the uneven surface of the land. Because we assumed that the land was flat when calculating the length of the trails, we underestimated the true length of the trail. These are reasonable assumptions, however, and because our methodology remains consistent throughout the model, we can still guarantee a reasonable level of accuracy.

We assumed that the lost jogger would stay on the path. While this is not a heavy assumption by any means, it is possible that the jogger may have wandered off the trails, considering he or she may be exhausted and delirious. In such a case, our model would be unable to locate the lost jogger. Moreover, we ignored the trails that received no traffic in our analysis. However, in reality, if the jogger is attempting to determine which path to pursue in the pitch black darkness, he or she will be unable to notice the distinguishing patterns of each trail and therefore the assumption that he or she will choose the path more travelled may be inaccurate. In such a case, it would be more likely for the jogger to stop wandering and simply stay put, allowing for us to use the unconscious model.

Our model, while comprehensive in the area it is able to search, requires an extensive amount of time to find the jogger. In our assumptions however, we have set aside 36 hours, believing that the jogger is able to survive for at least that duration of time without food or water. We focused more so on covering the greatest amount of area to increase the likelihood of finding the lost jogger, since time was a parameter that allowed us a reasonable level of flexibility.

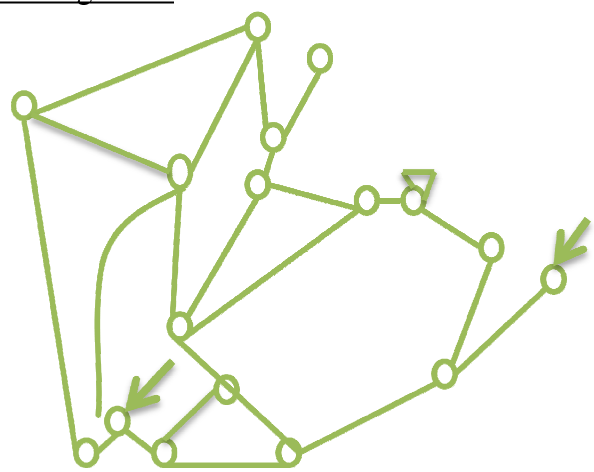
One of the major limitations of our model is that if we know for a fact that the jogger is conscious and moving, there is no direct way for us to search for the jogger; we must first pursue the search path for the contingency that the jogger is unconscious. While this appears to be a major limitation, in reality, it is highly dubious that the searcher will know whether the jogger is conscious or unconscious from the start. Therefore, searching for the jogger as if the jogger were unconscious would be the logical first step, which the course of action our model prescribes.

Perhaps the greatest limitation to our model is that it does not take into account heuristics, and would not be conducive to the insertion of new information to change the search pattern. This is a limitation imposed by the problem itself, considering we had limited knowledge of the individual, and that there was no real-time mechanism to update the model on the search's progression and success. Adding heuristics to the model would increase efficiency and probability of finding the jogger, as the model would be able to adapt to a variety of changing conditions. However, given the information available, our model manages to determine an efficient pathway to search for the lost jogger, whether he or she is conscious or unconscious.

Conclusion

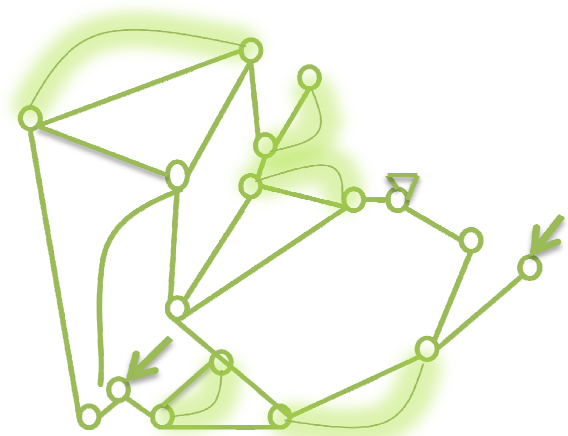
For both Problems B1 and B2, our model develops a case-by-case strategy which takes into account the various scenarios one may be confronted with when attempting to find a lost object or a missing person. In the case of a lost object, there are four possible scenarios, and each of our strategies seeks to maximize the probability of finding the lost object within the two hour time constraint through search patterns derived from the Archimedean spiral, optimal traversals of an Euler Path or Cycle resulting from the eulerization of the parks' areas, and the Hierholzer's Algorithm. For the lost jogger problem, two possible situations were identified: either the jogger is in motion, or the jogger is stationary. By taking this into account, our model becomes flexible enough to adjust upon receiving new information about the jogger's condition. It is our belief that by creating specific methodologies, our model can successfully handle all possible scenarios. The prospect of finding a person or lost object in the midst of miles of trails, seems daunting--not even taking into account the limitation of a lone searcher and only a penlight. Our model manages to yield strong probabilities of finding the lost object, or the missing person, despite the difficulties that confront it.

Appendix A: Eulerization of Problem B1 - Regions A, B, and C
SCENARIO 1
Figure 1: Original Graph of Region A:



The arrows represent the starting point and the ending point at which the individual conducts the search.

Figure 2: Eulerization of Region A:



The glowing green arcs represent the redundant paths. It's important to note that the curved lines do not actually represent a new trail being blazed into Hopkinton, but to visually represents the redundancy. In reality, the individual will simply walk on the same path between the same two vertices - just in a different direction.

Figure 3: Original Graph of Region B:

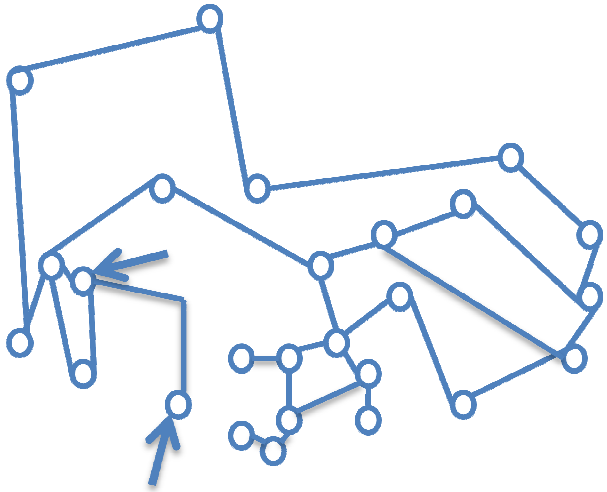


Figure 4: Eulerization of Region B:

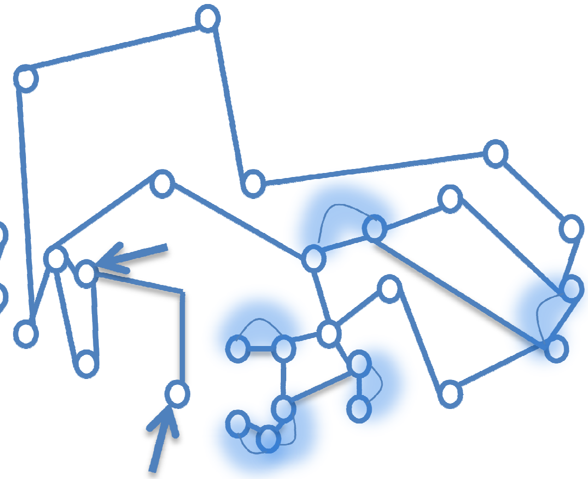


Figure 5: Original Graph of Region C:

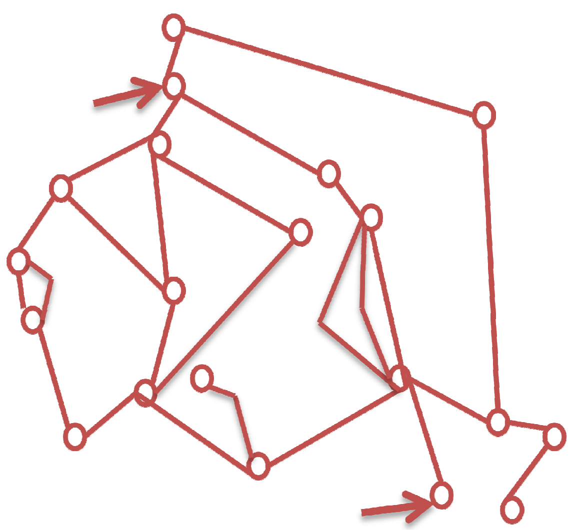
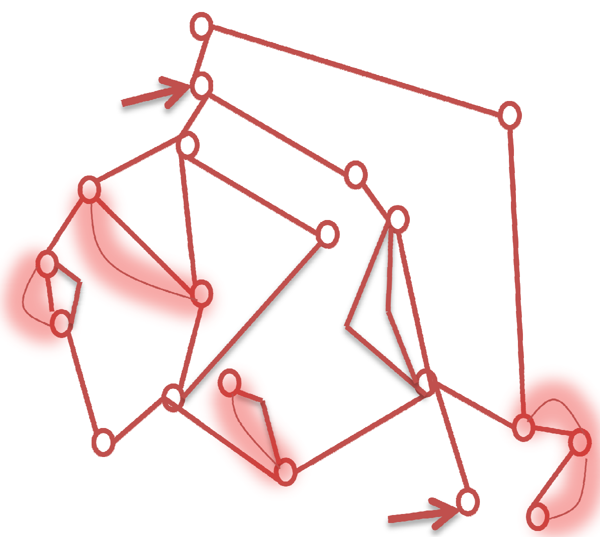
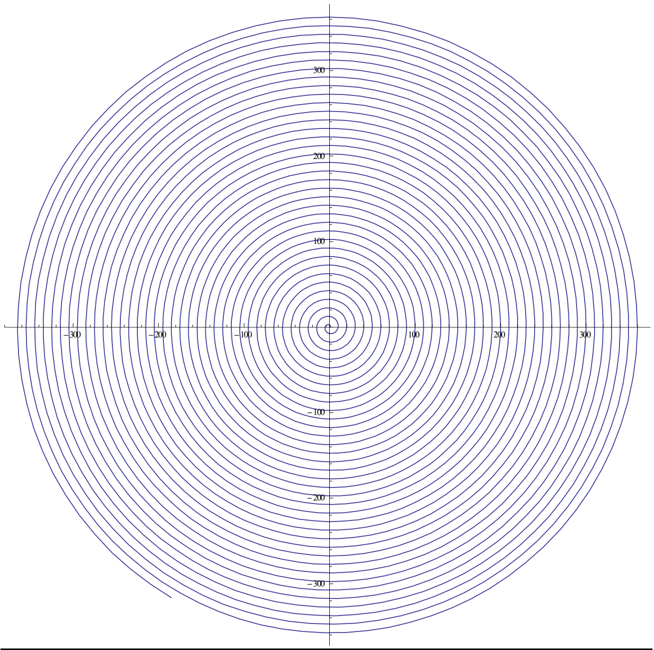


Figure 6: Eulerization of Region C:



SCENARIO 2:
Figure 7: Graph of Archimedean Spiral:



SCENARIO 3:
Figure 8: Original Map of Entire Park:

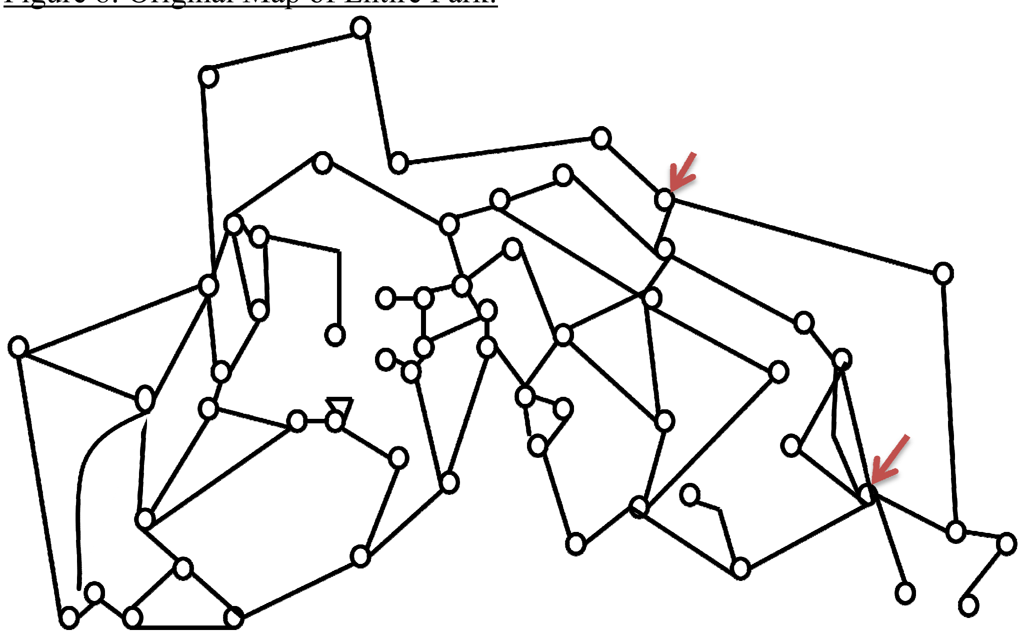


Figure 9: Eulerization of Entire Park:

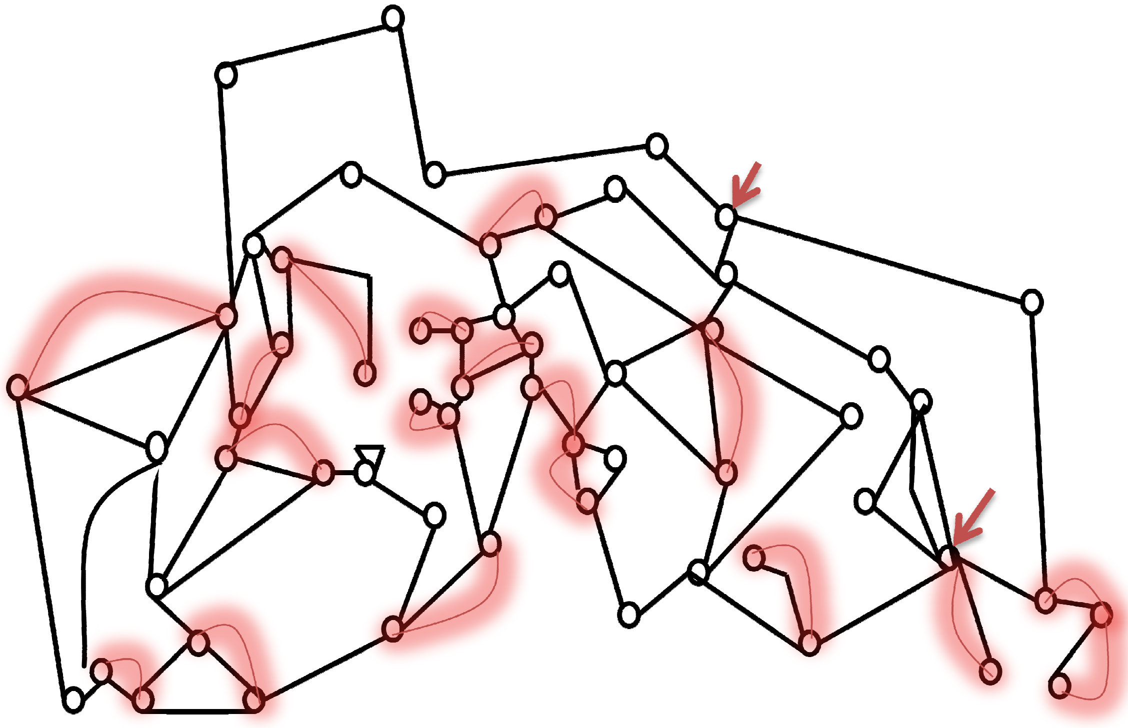
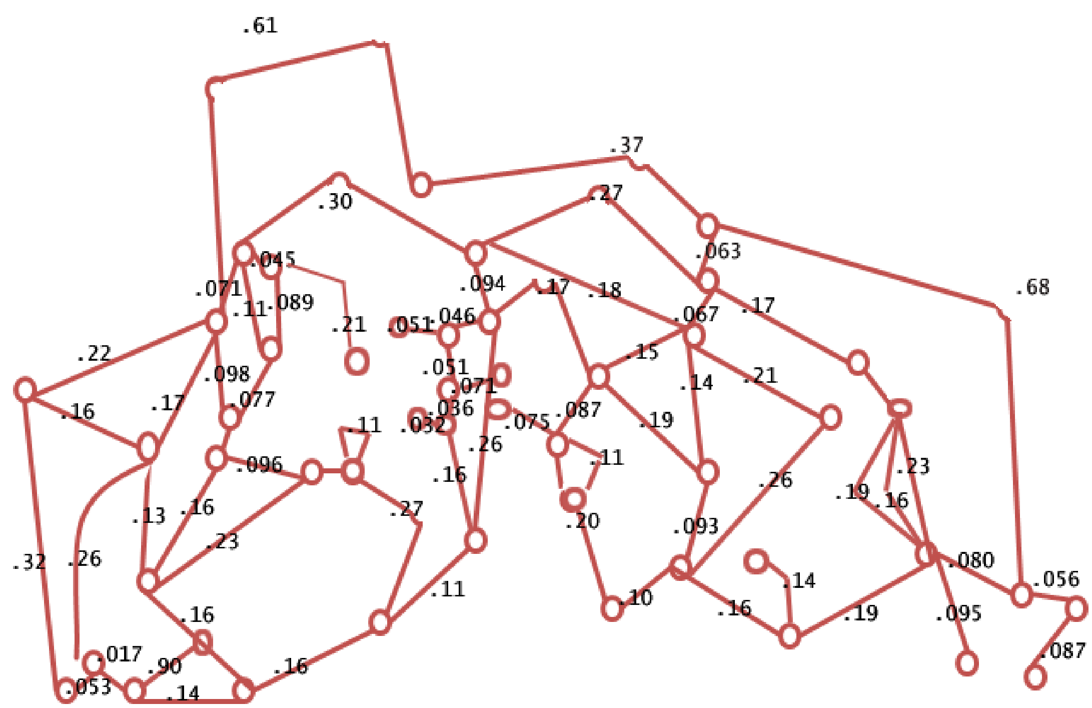
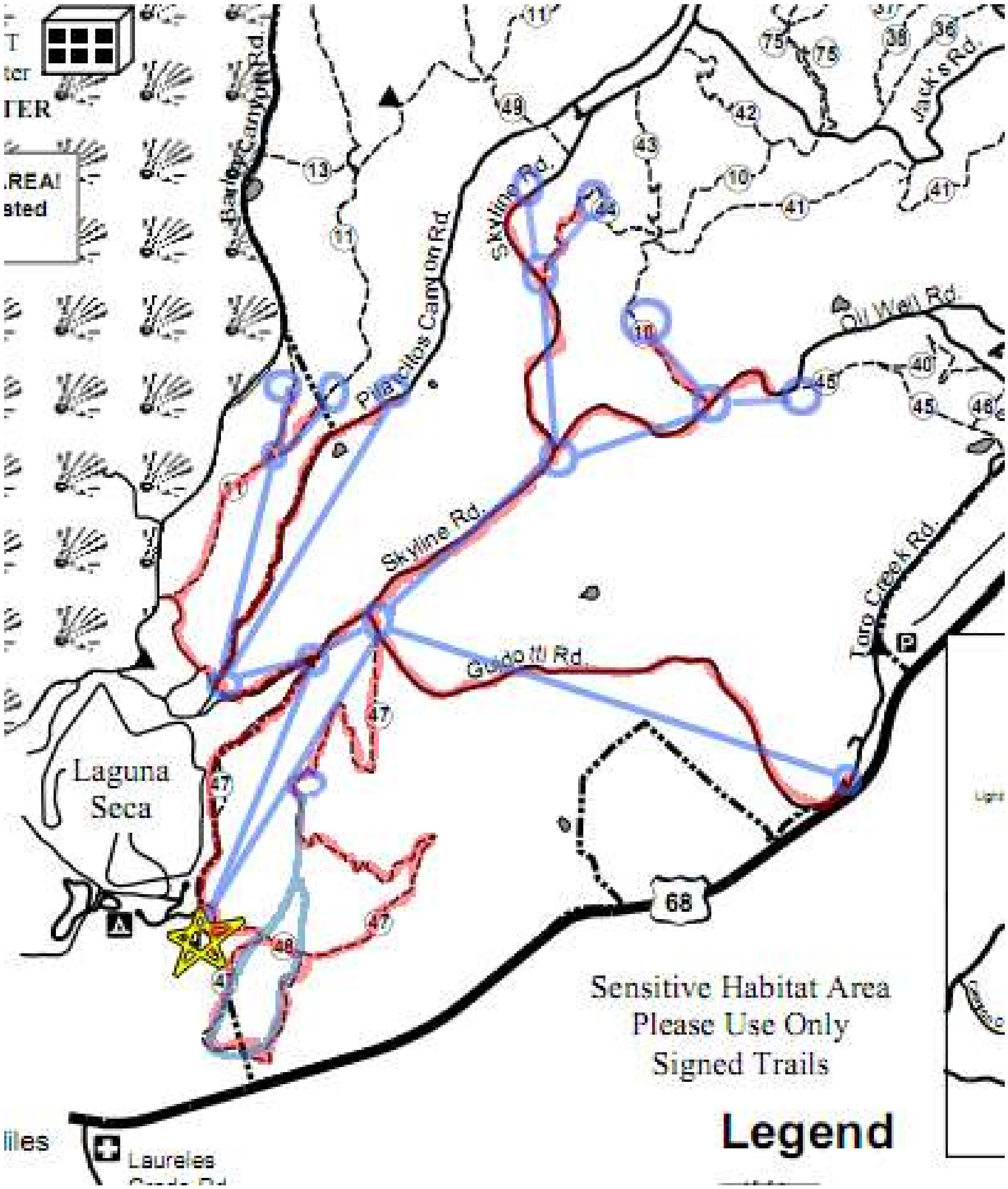


Figure 10: Full Park Map Showing Distance for Each Segment:



Appendix B

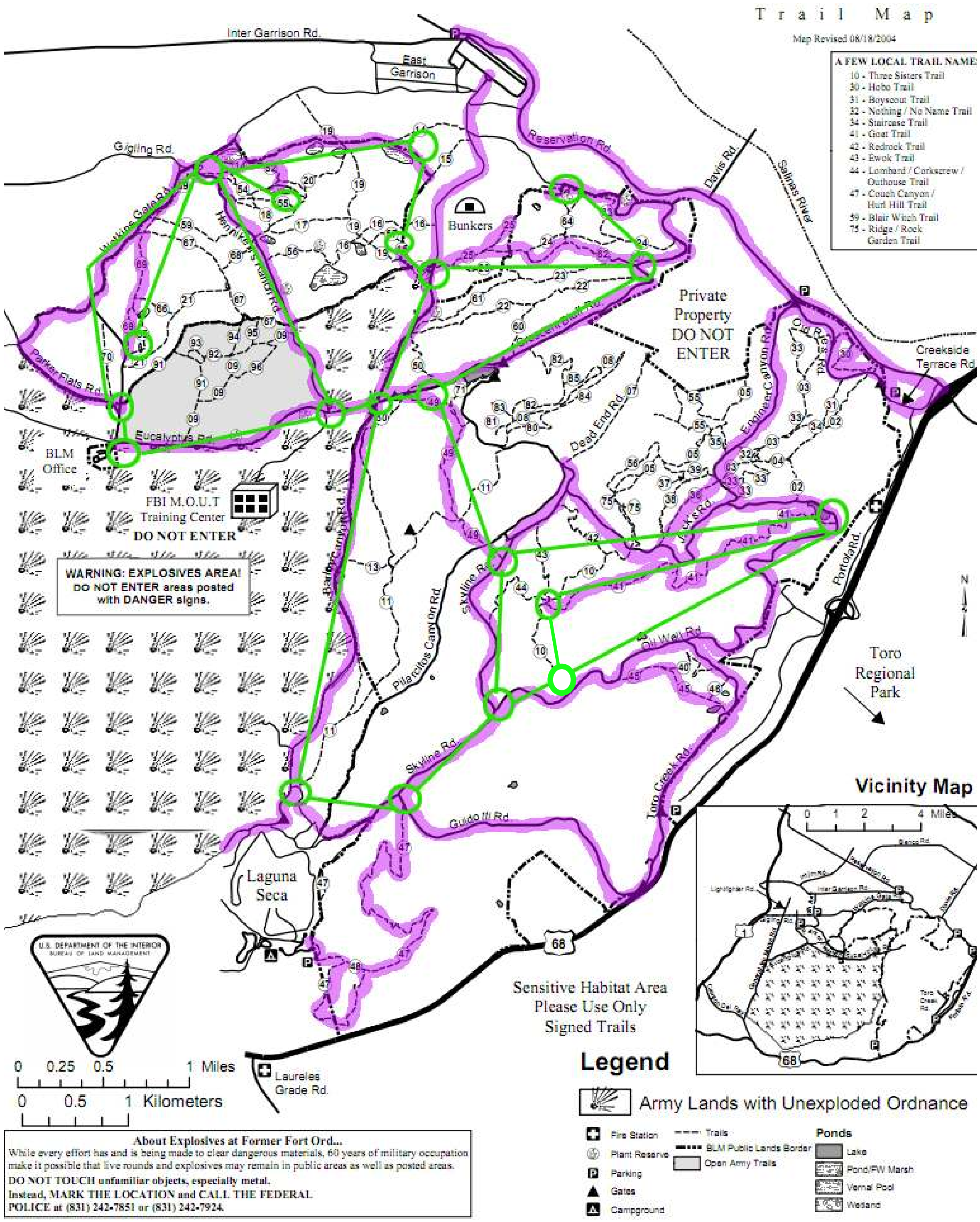
Figure 1: Graph Simplification of Parking Lot 1 Trail Method:



Notes:

- The Star represents the 1st Parking Lot, where the searcher initiates the search.
- The red lines indicate the extent of going 5.5 miles on each branch of trails stemming off from the 1st Parking Lot.
- The blue lines represent the graph simplification of the red lines. For ease of eulerization, the blue graph was used in the model.

Figure 2: Graph Simplification of Popular Trail Method:

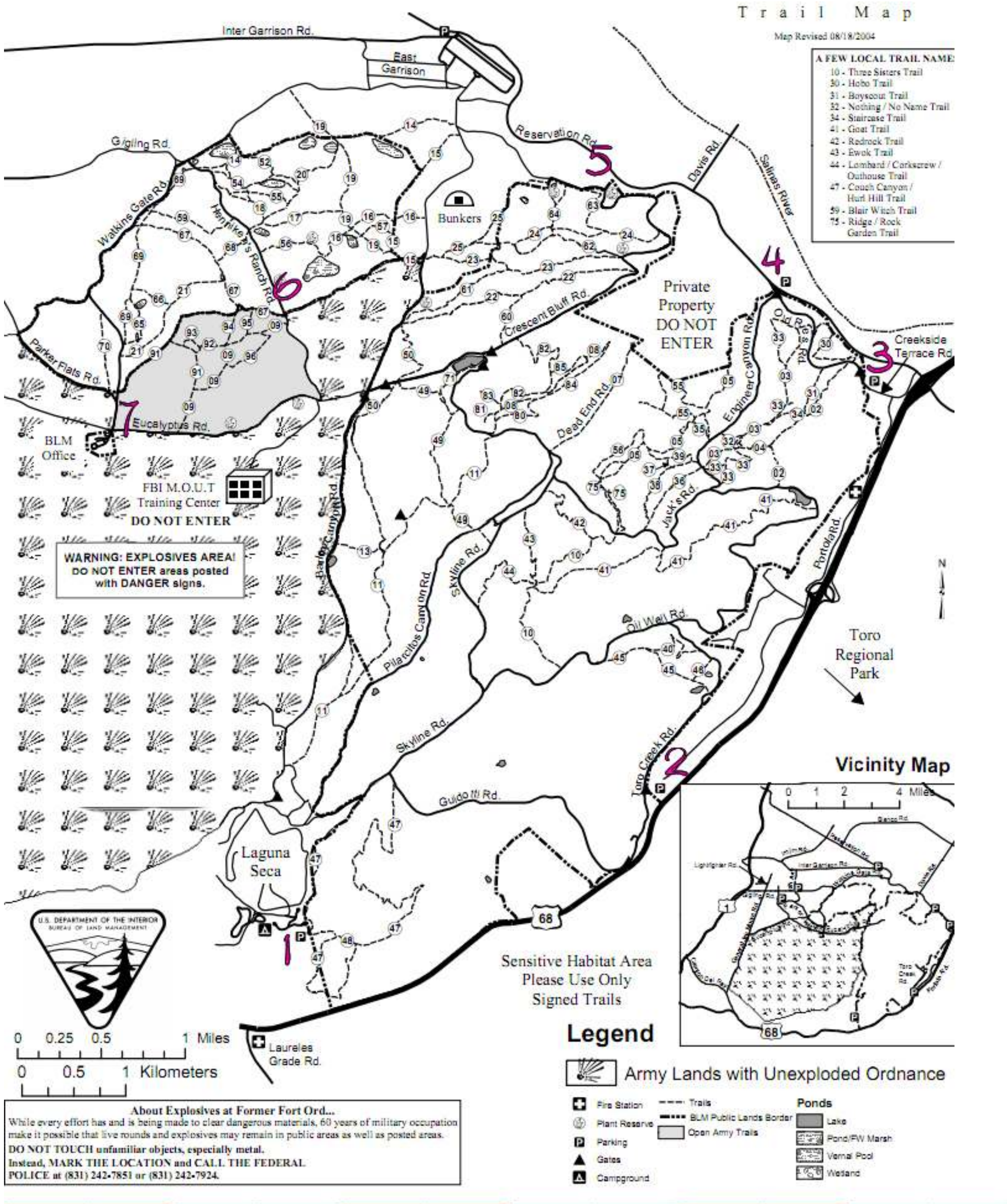


Notes:

- All nodes are potential starting places depending on the proximity to the individual parking lot methods.

- The purple lines indicate the most popular trails.
- The green lines indicate the final simplification of both the popular trails and the jogger prioritization framework.

Figure 3: Numbering of Parking Lots:



Appendix C: Data Tables for Problem B1 Scenario 1

Table 1:

Pixels and Distance (miles) of Each Edge Created by Trail Paths

Pixels	Miles	Rounded to 2 SigFigs
88	0.016566265	0.017
168	0.031626506	0.032
191	0.035956325	0.036
237	0.044615964	0.045
246	0.046310241	0.046
269	0.05064006	0.051
270	0.050828313	0.051
279	0.05252259	0.053
297	0.055911145	0.056
335	0.063064759	0.063
351	0.066076807	0.066
378	0.071159639	0.071
379	0.071347892	0.071
398	0.074924699	0.075
409	0.076995482	0.077
427	0.080384036	0.080
462	0.086972892	0.087
463	0.087161145	0.087
473	0.089043675	0.089
478	0.08998494	0.090
493	0.092808735	0.093
497	0.093561747	0.094
502	0.094503012	0.095
511	0.096197289	0.096
522	0.098268072	0.098
545	0.102597892	0.10
561	0.10560994	0.11
589	0.110881024	0.11
595	0.112010542	0.11
603	0.113516566	0.11
677	0.127447289	0.13
733	0.137989458	0.14
745	0.140248494	0.14
748	0.140813253	0.14

776	0.146084337	0.15	
827	0.155685241	0.16	
829	0.156061747	0.16	
832	0.156626506	0.16	
841	0.158320783	0.16	
852	0.160391566	0.16	
865	0.162838855	0.16	
869	0.163591867	0.16	
897	0.168862952	0.17	
914	0.172063253	0.17	
928	0.174698795	0.17	
975	0.183546687	0.18	
994	0.187123494	0.19	
995	0.187311747	0.19	
1031	0.194088855	0.19	
1064	0.200301205	0.20	
1109	0.20877259	0.21	
1134	0.213478916	0.21	
1168	0.219879518	0.22	
1225	0.23060994	0.23	
1239	0.233245482	0.23	
1357	0.255459337	0.26	
1370	0.257906627	0.26	
1392	0.262048193	0.26	
1428	0.268825301	0.27	
1447	0.272402108	0.27	
1580	0.297439759	0.30	
1712	0.322289157	0.32	
1950	0.367093373	0.37	
3232	0.608433735	0.61	
3635	0.685299699	0.68	
54386	10.23832831	10.238	Total

- These values were obtained by using the Pixel Proportion Method, as mentioned in the paper. Individual segments were traced and the number of pixels contained in each segment was counted. By using proportions, the appropriate distances in miles and their respective rounded distances were obtained.

- To illustrate the calculation, a path made up of 3635 pixels is .68 miles long, because 3635 divided by 8 (the thickness of the pixel) yields a length of 454.375 pixels. This is set equal to the predetermined proportion (found from the adjusted image's pixels and scale in miles).

$$\frac{x\ miles}{4.54.375\ pixels} = \frac{5/_4\ miles}{830\ pixels}$$

Solving the proportion yields:

$$x = 0.68\ miles$$

- The total number of pixels and the total distance of the paths is found by adding all of the individual paths.

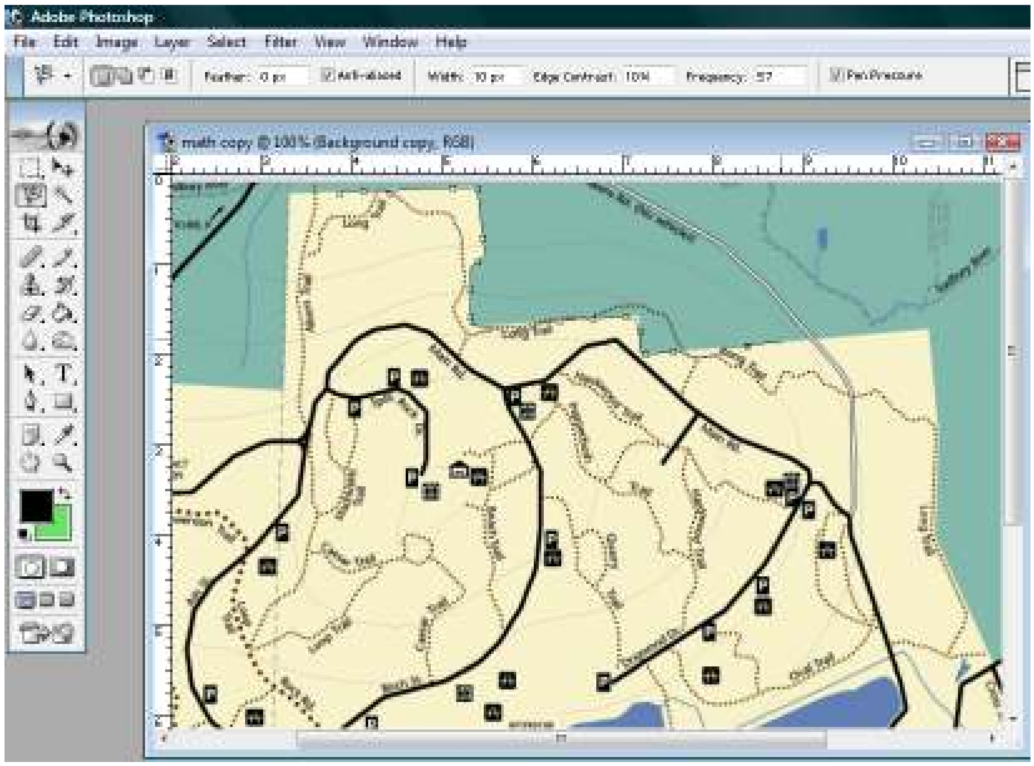
Table 2:
Distance (miles) of Each Edge Created from Trail Paths by Region

Green (A)	Blue (B)	Red (C)
0.016566265	0.031626506	0.055911145
0.05252259	0.035956325	0.080384036
0.076995482	0.044615964	0.087161145
0.08998494	0.046310241	0.094503012
0.096197289	0.05064006	0.110881024
0.098268072	0.050828313	0.113516566
0.112010542	0.063064759	0.140248494
0.113516566	0.066076807	0.140813253
0.127447289	0.071159639	0.156626506
0.137989458	0.071347892	0.158320783
0.158320783	0.089043675	0.160391566
0.160391566	0.094503012	0.162838855
0.162838855	0.112010542	0.163591867
0.163591867	0.146084337	0.187311747
0.168862952	0.168862952	0.194088855
0.219879518	0.183546687	0.23060994
0.23060994	0.213478916	0.268825301
0.262048193	0.272402108	0.608433735
0.268825301	0.297439759	0.685299699
0.322289157	0.367093373	
	0.608433735	
0.219879518	0.031626506	0.140248494
0.076995482	0.035956325	0.187311747
0.096197289	0.050828313	0.110881024
0.08998494	0.071159639	0.055911145

0.160391566	0.094503012	0.087161145	
total	**Total**	**total**	**total (all paths+retraces)**
3.682605422	3.368599398	4.380271084	11.4314759
TIME TAKEN	**TIME TAKEN**	**TIME TAKEN**	
0.920651355	0.842149849	1.095067771	% of One Hour
55.23908133	50.52899096	65.70406627	Minutes

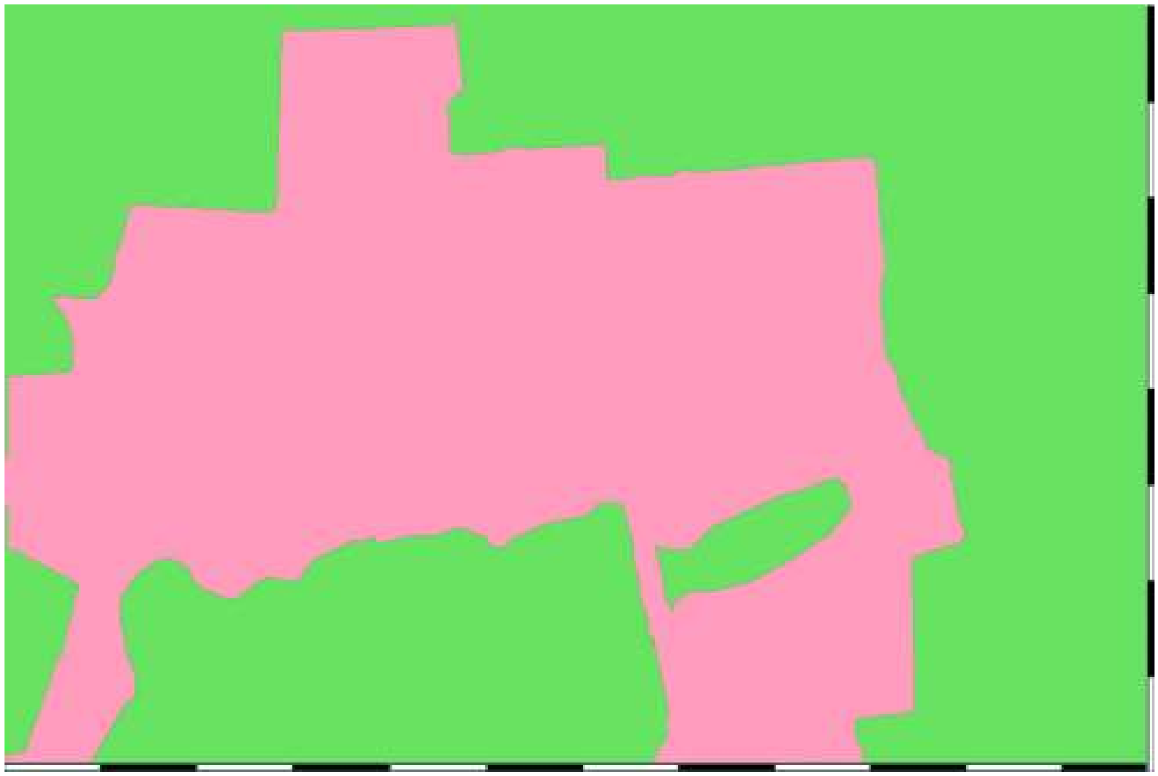
The values in this table are simply the distances taken from Table 1, separated by region. The cells highlighted in yellow indicate paths that needed to be retraced in order for the graph to be eulerized. The total length of each region in miles was found by adding each individual path that made up the region, in addition to the retraced paths, as indicated by the yellow cells. Since the individual travels at a speed of 4 miles per hour, we divide the total length in miles by 4 to determine the time it takes to completely travel a region. This leaves us with the number of hours in the form of a percentage, which can easily be multiplied by 60 to determine the number of minutes that must be allotted.

<u>Appendix D: Determining Area of Park in Problem B1</u>
First, we take the original map of Hopkinton.



The edge of the yellow park region is traced by the magnetic lasso tool. A selection is made, and the park area is filled in with pink.

The selection is inversed, and the surrounding area is filled in with green. A scale from the original image is translated onto the new image so that a proportion between pixels and images can be set up to determine area of the entire park. The adjusted park image is 1 mile by 1.5 mile (each black and white block=1/4 mile).

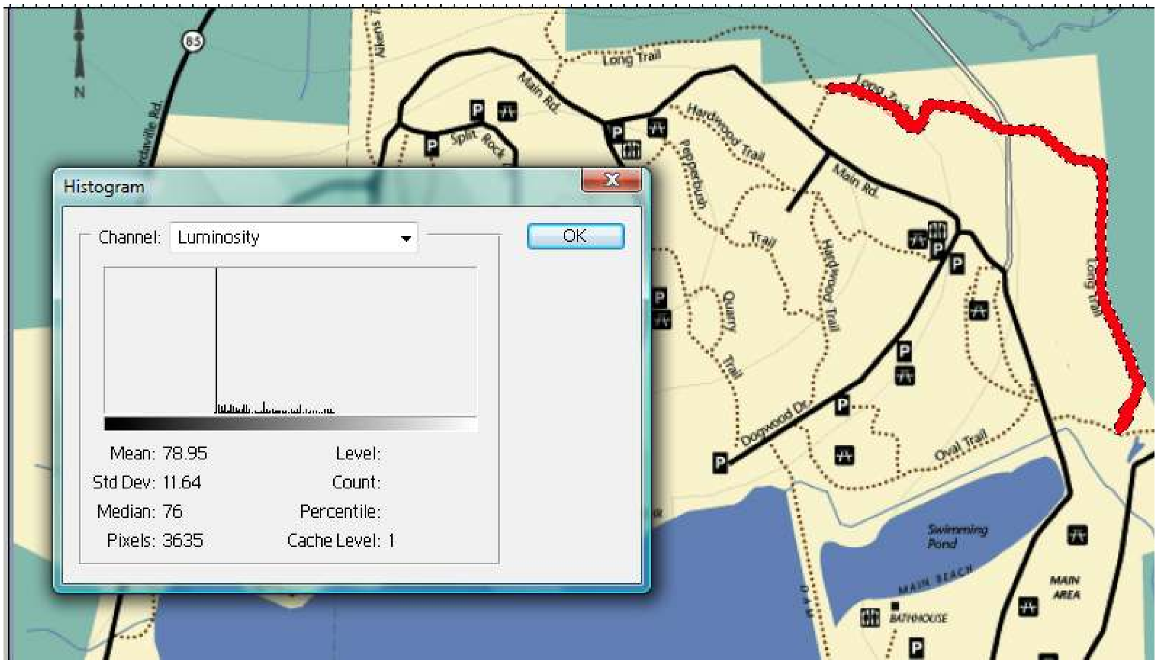


Using the histogram tools, we found the total number of pixels for the image, 656,042, and the number of pixels in the selected area, 299,679. Because the length and width of the entire map was one mile by one and a half miles, the area of the map was 1.5 square miles. Multiplying entire area of the map by the pixel proportion, 299,679/656,042 yielded an area of 0.6852 square miles of the portion of the park we are studying.

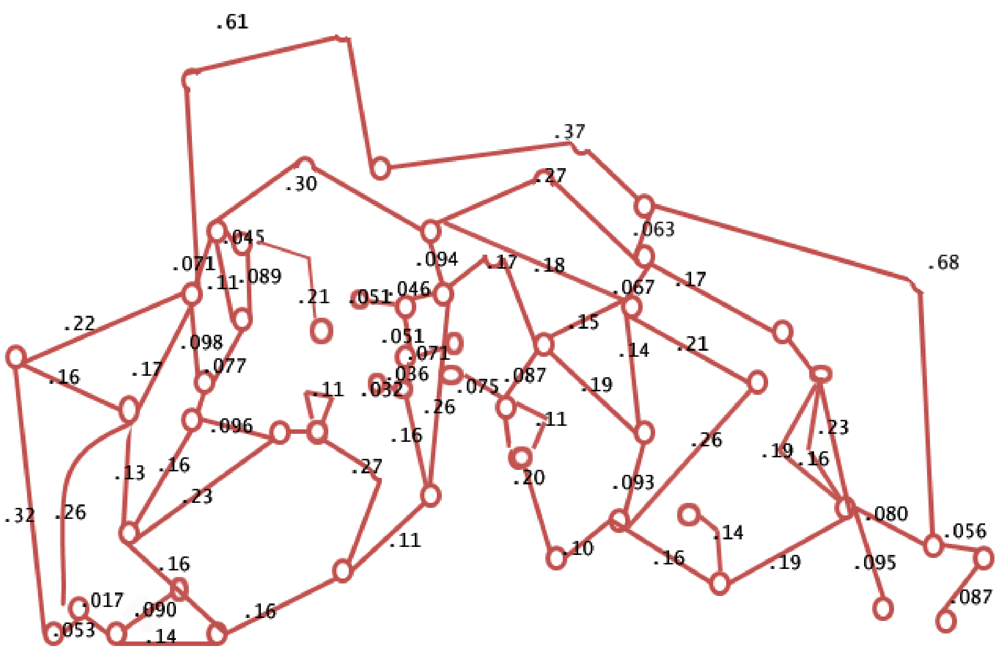
Appendix E: Determining Length of All Trails and Individual Trails in Problem B1
Using the magnetic lasso tool, each of the paths was stroked with a red pen. A selection was made, and similar to the method stated in Appendix D, the pixels were used to determine the total length (not area, this time) of the trails.

We set up a proportion, with the pixel width of the adjusted park image, 830 pixels, over the length of the park image in miles, 1.25 miles set equal to 6798.2511, the number of pixels the trails spanned, over X, the total length of the trails we sought to find. Solving that proportion yields a total trail length of 10.2383 miles.

To find the distance of individual trails, each specific path was traced out on Photoshop, and the Pixel Proportion Method and the Histogram tool (which provides the number of pixels) was employed.



The individual path distances were taken and put into a data table, as found in Appendix C, and then translated onto an undirected, connected graph for easy viewing, visualization, and eulerization.



Appendix F: Greedy Algorithm Explanation

A greedy algorithm is one which always selects the locally optimal choice. In terms of graph traversal, this means that the algorithm will always select from the edges available at its current vertex the edge with the best value for the quantity being optimized. In this case, the algorithm selects the longest edge, as we wish to maximize distance. However, greedy algorithms are not guaranteed to produce an optimal solution in all cases. As such, and keeping in mind that we also wish to minimize redundancy, we modify the algorithm being used in this case to be greedy in all instances except for when the longest edge leads to a dead end within two iterations of the standard greedy algorithm, meaning that we would be forced to traverse a redundant edge.

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