Team #824

Isabella Zhang, Carol Yin, Jenny Shan, Alan He

October 18, 2016

Table of Contents

1 Summary

1.1 Problem 1

1.1.1. Interpretation of Problem

When running a presidential campaign and aiming to win the election, it is essential for us to divide our limited financial resources strategically to yield increase in popularity and gain support from the majority. In order to achieve that, we decided to interpret the problem as evaluating every state's general preference and finding corresponding strategies to maximize our candidate's popularity in each state.

We decided to use the poll results as a relatively accurate reflection on the popularity of both candidates, learning which candidate do people tend to vote for and how certain they are about their choices. Grouping the poll results by states, we can observe the general preference of each state. According to the different situations in each state, we will then determine the specific financial strategies for publicizing our candidate using the total utility curve and learning from the past elections.

However, what we present is only a simplified model of the problem. We made certain assumptions (such as our opponent's strategies have no influence on us and the election result perfectly reflects the preference of the majority) to make the complex problem more manageable for us. We also used data from the past elections to help us analyze the situation, which may cause some inaccuracies in the model. There are also more factors influencing the result of presidential election that we did not take into consideration due to the limit of time, resource and capability.

1.1.2. Techniques and Methods

When establishing the mathematical model to find out the optimal strategy for dividing financial resources, we used categorization, total utility curve and statistical methods such as standard deviation and bell curve to get the solution we wanted.

1.1.3. Conclusion

We classified all states into three categories according to three different types of change in our candidate's popularity after the campaign invests expenditure, using total utility curve to determine general strategies of dividing money resources to each of the category. We then developed the idea of "relative significance of devoting expenditure", helping us design more specific strategies maximizing the efficiency of spending expense.

1.2 Problem 2

1.2.1 Interpretation of Problem

To construct a traffic flow model for ambulances in the crowded Manhattan area, it is essential to consider factors like the amount of cars, the distance of a path, and the speed for a vehicle on various streets and roads. Therefore, we decided to first calculate the speed of an ambulance in different roads, and then determine the amount of time an ambulance needs to go through them. Finally, we can provide the fastest route for the ambulance driver according to different road conditions when we combined the road segments that require the shortest time.

1.2.2 Techniques and Methods

Our model is based on the theory of traffic flow, which is the relationship between the well-function of the road and the maximization of efficiency for vehicles. In order to minimize the time an ambulance take for travelling, we use the Greenshield's formula, the Greenberg's formula, and the Underwood's formula that calculates speeds at roads with different amount of traffic load. We also incorporate the theory of uninterrupted traffic flow because ambulances can ignore red lights to save them more time.

1.2.3 Conclusion

We realized the request by our model mainly through separating the calculation for speed according to different density of traffic on various road segment. In this way, the estimation for speed is more accurate. Eventually, the route that we suggest through our model will be the shortest one but will also have relatively low amount of traffic so that the ambulances can pick up the injured people more quickly with more safety guarantee.

2. Problem 1: Design a strategy to divide our candidate's campaign finance resources to maximize the probability of success nationally.

2.1. Introduction

It is essential for each presidential campaign to design an optimal plan for using expenditure, for they only have a limited amount of total financial resources and wish to use this money to gain the highest popularity for winning the election. Thus, it is important for us to mathematically find out the relationship between campaign's expenditure and candidate's popularity that is shown in polls. In the following sections, we will talk about how interpreting the polls and applying our model will help develop an optimal financial strategy.

2.2. Model

2.2.1. Introduction of the Model

When we first looked at this problem, we thought that our campaign's financial resources should be divided according to our candidate's popularity in each state. On the one hand, we should spend the most resources on ambivalent states, especially the ones favoring our candidate a bit more over our opponent (e.g. a state where our candidate has the popularity of 60%). On the other hand, we can spend less on the states where our candidate's popularity is either extremely high or extremely low since we are almost sure about whether this state will vote for me or not and no matter how much money we input, the outcome won't change. Based on this fundamental idea, we developed a detailed and fluid model, considering various factors including the number of electors in each state, the deviations of our collected data, and the total utility that helps us find the most efficient way of distributing resources to maximize our candidate's votes specifically to the situation of each state.

2.2.2. Assumptions and Justifications

- The actions of the other candidate throughout the entire election will not influence our candidate's popularity;
- People who waive have a certitude of 0% in choosing either the candidate;
- Voters who participate in the poll have a preference for either our candidate or our opponent;
- The state electors' votes reflect the preference of most individuals within the state;
- All 50 states adopt "winner take all" system in the election process;
- Expenditure for increasing votes is mainly used on advertisement and publication;
- The general party preference of the states has not changed over the past eight years;
- Since we are unable to find an accurate function representing the relationship between popularity and financial input, we use specific data from the 2008 presidential election to help us build the model. Here, we assume that Barack Obama adopted the optimal financial strategies;
- The change in popularity caused by different inputs obeys the principle of diminishing marginal utility.

2.2.3. Definitions

- Proportion of electors in each state (D_a): the percentage of electors in each state, calculated by the number of electors in that state over the total number of elector in the whole nation.
- Choice of candidate: if the voter votes for our candidate, we mark this choice as 1 unit.
- Certitude of choice (C_a): how certain people are of their choices.
- Relative popularity (P_a): the relative probability each state will vote for our candidate. Here, we set the state with highest average popularity as unit 1 (100%), and all the other states' relative certainties are calculated by $P_a/P_{max} \times 100\%$
- Significance of spending expenditure (N_a): how necessary it is to spend financial resources on each state, calculated by P_a (average popularity of our candidate in each state) $\times$ D_a (proportion of electors in that state).
- Relative significant of spending expenditure (S_a): the relative significance of spending expenditure on each state in this election. Here, we set the state with highest significance as unit 1 (100%), and all the other states' relative significances are calculated by $N_a/N_{max} \times 100\%$
- Expenditure (M): how much money the campaign needs to spend on each state to gain the highest increase in popularity (the *x*-value of the vertex of the totality utility graph).

2.2.4. Clear Explanation of the Solution

Step 1. According to the results of the poll, calculate the possibility of each individual voting for our candidate.

Case 1: If the person is voting for our candidate, and his or her certitude of that choice is $C_{us}\%$, we mark this individual probability

$$X_n\% = C_{us}\% \times 1 \qquad (1)$$

Case 2: If the person is voting for our opponent, and his or her certitude of that choice is $C_{oppo}\%$, we mark this individual probability

$$X_n\% = (1 - C_{oppo}\%) \times 1 \qquad (2)$$

Step 2. Calculate the overall popularity of our candidate in each state:

(a) Eliminate the extreme values of $X_n\%$:

If most values of $X_n\%$ are clustered in a certain range, we need to eliminate the values that fall more than 1.5 times the interquartile range above the third quartile or below the first quartile of the data to prevent them from skewing the data to either side.

(b) Calculate each state's average possibility to vote for our candidate:

$$B_a\% = \frac{X_1\%+X_2\%+X_3\%+\ldots+X_n\%}{n} \times 100\% \quad (3)$$

$X_n\%$ is the possibility for an individual in that state to support our candidate, and n is the number of voters in the state after we eliminated the extreme values. We can use the formula above to calculate the mean value of individual possibility to vote for our candidate, $B_a\%$, which represents the state's overall preference in the election.

(c) Determine each state's overall possibility to vote for our candidate, P_a, by calculating the standard deviation for $\{X_1\%, X_2\%, X_3\%, \ldots, X_n\%\}$:

Let the standard deviation be σ_a.

$$\sigma_a = \sqrt{\sum (X_n\% - B_a\%)^2 \cdot \frac{1}{n}} \quad (4)$$

$$Z_n = \frac{X_n\% - B_a\%}{\sigma} \quad (5)$$

$$\overline{Z} = \frac{Z_1+Z_2+Z_3+\ldots+Z_n}{n} \quad (6)$$

We use Z score and σ_a to check whether or not the average possibility we calculated in the last step can represent the overall preference of all the voters in that state.

Case 1: If the absolute value of Z score is smaller than 2, our model is representative enough. In this case, $P_a\% = B_a\%$.

Case 2: If the absolute value of Z score is greater than 2, our model is not representative enough. Therefore, we use the median value $X_{median}\%$ of $\{X_1\%, X_2\%, X_3\%, \ldots, X_n\%\}$ to represent the overall preference. In this case, $P_a\% = X_{median}\%$

Step 3. Calculate the proportion of each state's number of electors to the total number of electors of the nation.

$$D_a\% = \frac{d_a}{D_a} \times 100\% \quad (7)$$

In the formula above, D_a represents the total number of electors in the U.S., and d_a represents the number of electors that state has. Thus, we can calculate $D_a\%$, which is the proportion of the state's number of electors to the total number of electors of the nation.

Step 4. Calculate the significance of spending expenditure in each state.

$$N_a\% = P_a\% \times D_a\% \qquad (8)$$

We calculate $N_a\%$, the significance of spending expenditure of each state, by multiplying $P_a\%$, each state's overall possibility to vote for our candidate, with $D_a\%$, each state's proportion of electors.

Step 5. Calculate the relative significance of spending expenditure in each state.

Let $S_a\%$ be the relative significance of spending expenditure on each state, we calculate it as:

$$S_a\% = \frac{N_a\%}{N_{max}\%} \text{ x } 100\% \qquad (9)$$

Where we get $N_a\%$ from our last step and $N_{max}\%$ represents the greatest significance of spending expenditure among all states. Therefore, $S_a\%$ represents the relative significance of spending expenditure on each state.

Step 6. Using the Total Utility Curve to determine the strategy of maximizing efficiency in spending expenditure on each state.

In order to make our campaign's financial strategy the most efficient, we need to consider the relationship between our input (financial resources) and output (increase in popularity). Here, we employs an economic concept: the total utility. The total utility measures additional satisfactions gained from increase of consumption. In the specific case of presidential election, we see the popularity as a measure of satisfaction toward each candidate, and the amount of financial resources the campaign devote in publicizing its candidate as the indicator of consumption. The total utility curve is shown below as Figure 1.

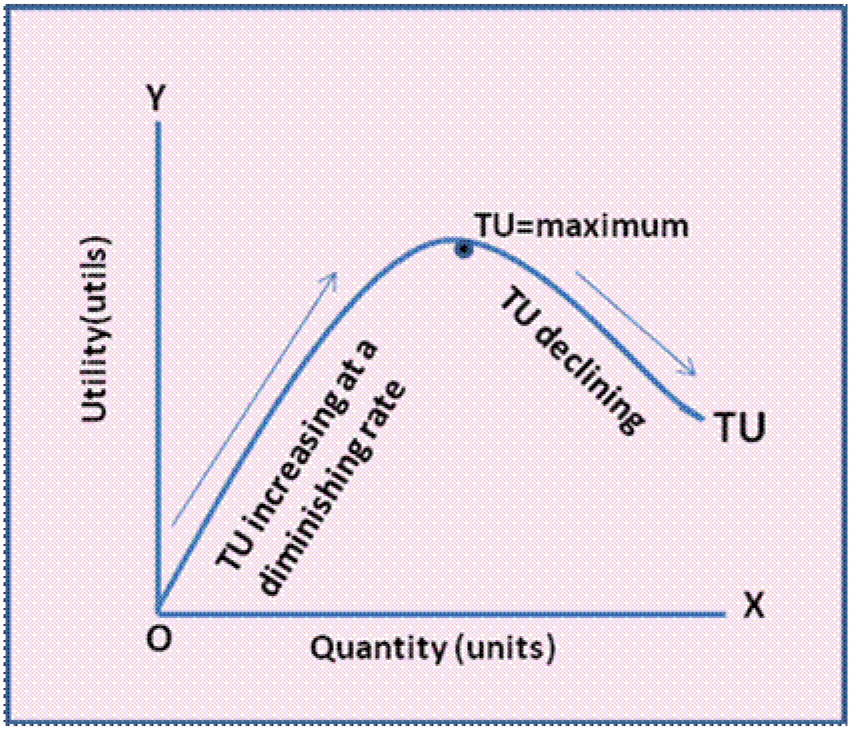


Figure 1 Total Utility Curve

It is obvious that the total utility curve represents a quadratic function. After the utility reaches its maximum, instead of continue increasing, the curve starts to shape downward. If we see it in our specific case, such fact can be explained easily: if the campaign fails to spend enough money in publicizing its candidate, the popularity may not reach its potential maximum; however, if the campaign spends too much money, it not only wastes financial resources, but may also cause a decrease in the candidate's popularity, for the overexposure may make the voters grow tired of the candidate. Hence, in order to determine the optimal financial strategy, we need to find the exact point where the popularity reaches its maximum. Since the situations are different in all 50 states, we need to create 50 different curves, one for each state.

In the total utility curve, the x-axis represents the amount of financial resources the campaign spends on publicizing the candidate, while the y-axis represents the corresponding popularity the candidate gain throughout the time. The y-intercept on the curve should be the initial overall popularity in each state, P_a% (as indicated by the poll). The x-value of the curve's vertex represents the money the campaign needs to devote on that specific state, and the corresponding y-value is the maximum popularity the candidate gains after spending those money. At this point, the financial resources are used the most effectively.

However, it is beyond our capability to create such a total utility function only with variables. Hence, we simplified it to a quadratic equation and used the data from the presidential election of 2008 to help us calculate different slope values and build the model. We used the data of 2008 election primarily because we thought the results of 2012 election may be influenced by Obama's performance in his first presidency, and the data earlier than that may no longer be representative for nowaday situation. In order to make our conclusion clearer, we categorized all 50 states into three major categories:

1. The states where our candidate's popularity maintained under 50% both before and after spending any expenditure;
2. The states where our candidate's popularity started from under 50% but reached over 50% after the campaign spending expenditure;
3. The states where our candidate's popularity maintained over 50% both before and after spending any expenditure.

The more specific calculation and corresponding justification are shown in the next section.

2.2.5. Calculation and Justification Based on Past Data

In this section, we used the data we found on the presidential election of 2008 to help us calculate changing coefficient and justify our model. We chose the data from 2008 mainly because we do not want Obama's first presidency to influence the voter's preference. We also assume that president Obama utilized the most efficient financial strategies in his campaign.

Table 1 State Categorization based on Coefficient of Changing Utility

State	Change in Popularity (%)	Money (Million Dollar)	Calculated Coefficient of Changing Utility	Corresponding Category
Alabama	-2	0.264945	28.4917116	1
Alaska	1	0.134686	-55.12582344	1
Arizona	1	1.451358	-0.4747346134	1
Arkansas	N/A	N/A	N/A	N/A
California	7	5.569829	-0.2256390777	3

Colorado	4	10.262617	-0.03797901951	2
Connecticut	9	0.730335	-16.87323214	3
Delaware	N/A	N/A	N/A	N/A
Florida	2	36.863622	-0.001471749806	2
Georgia	7	4.105888	-0.4152253338	1
Hawaii	N/A	N/A	N/A	N/A
Idaho	3	0.007638	-51423.53721	1
Illinois	9	0.023319	-16550.94001	3
Indiana	4	16.66379	-0.01440497217	2
Iowa	3	13.261774	-0.01705760328	3
Kansas	N/A	N/A	N/A	N/A
Kentucky	0	0.183738	0	1
Louisiana	N/A	N/A	N/A	N/A
Maine	N/A	N/A	N/A	N/A
Maryland	N/A	N/A	N/A	N/A
Michigan	6	12.149631	-0.04064667961	3
Massachusetts	9	0.046839	-4102.298711	3
Minnesota	4	3.004408	-0.4431412433	2
Mississippi	N/A	N/A	N/A	N/A
Missouri	0	11.232569	0	1
Montana	4	1.685525	-1.407957658	1
Nebraska	N/A	N/A	N/A	N/A
Nevada	4	9.165698	-0.04761336812	3
New Hampshire	0	10.826488	0	3
New Jersey	3	0	∞	3
North Carolina	1	15.131705	-0.004367413057	2
New Mexico	16	4.017049	-0.9915296922	2
New York	13	1.148016	-9.863872967	2
North Dakota	N/A	N/A	N/A	N/A

Ohio	4	25.739865	-0.006037365403	2
Oklahoma	10	0.163515	-374.0113679	1
Oregon	N/A	N/A	N/A	N/A
Pennsylvania	3	39.686115	-0.001904776751	3
Rhode Island	N/A	N/A	N/A	N/A
South Carolina	N/A	N/A	N/A	N/A
South Dakota	4	0.61139	-10.7009745	1
Tennessee	N/A	N/A	N/A	N/A
Texas	N/A	N/A	N/A	N/A
Utah	12	0.297645	-135.4515762	1
Vermont	13	0	∞	3
Virginia	4	25.444889	-0.006178155998	2
Washington	N/A	N/A	N/A	N/A
West Virginia	-3	1.18759	2.127101387	1
Wisconsin	5	12.408878	-0.03247169618	3
Wyoming	1	0	∞	1

*Note: We did not find the data on popularity or expenditure for some states, so we marked them N/A. According to our source, President Obama did not spend any money on states of New Jersey, Vermont and Wyoming, yet those three states all experienced an increase in popularity, making the changing coefficient ∞. However, we doubted if the data was inaccurate, so it is highly unlikely for the candidate to spend nothing on any state.

In order to make the function easier to model, we decided to see the total utility curve as a quadratic function. In this way, it became possible for us to find a formula expressing the relationship between financial expenditure invested and change in popularity in each state. As we stated in the last section, the x-axis of the curve represents the amount of financial resources the campaign spends on publicizing the candidate, while the y-axis represents the corresponding popularity the candidate gain throughout the time. The data we collected from presidential election of 2008 rendered us two points on each curve: the y-intercept $(0, P_a)$, indicating each state's initial possibility to vote for our candidate, and its vertex (M, V_a), representing the final popularity our candidate got after investing the expenditures. According to the vertex formula of quadratic equation, we can represent the curve as $y = a(x - b)^2 + P_a$. In this way, if we plug in the two points we got, we can calculate the coefficient a for each state's function.

Take Connecticut for instance, we know the point (0, 52) and (0.73, 61) on the curve. We then plug in the two points into the equation $y = a(x - b)^2 + P_a$. Get:

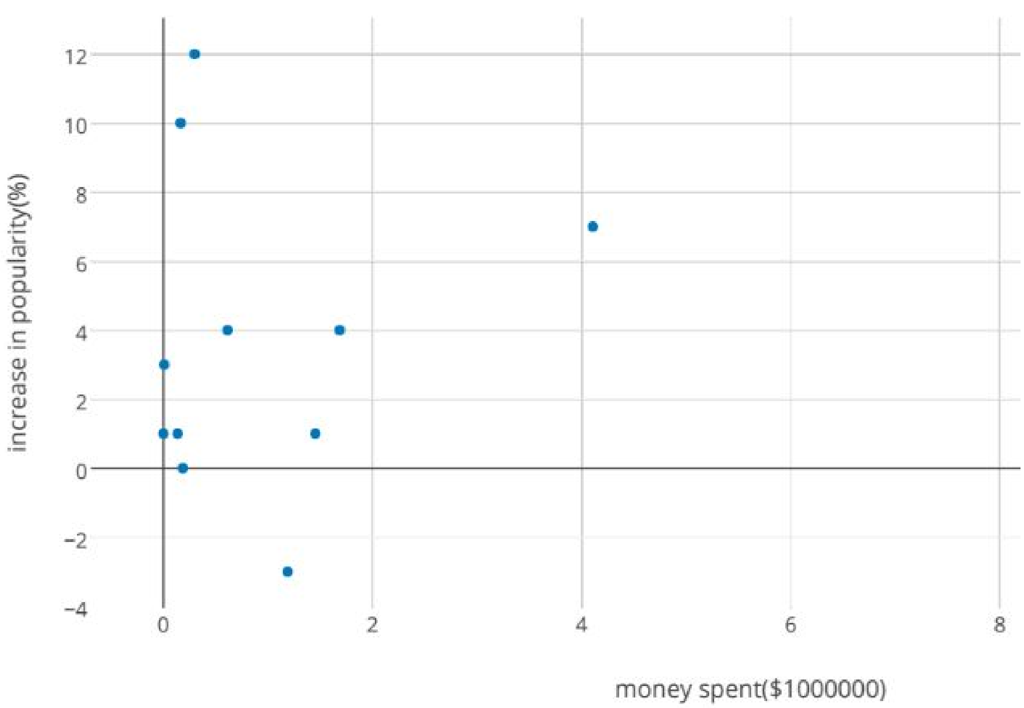
$$61 = a\,(0 - 0.73)^2 + 52 \qquad (10)$$

$$a = -16.87 \qquad (11)$$

The coefficient a shows the popularity's rate of change responding to the changes in expenditure the campaign devotes. As we know the initial popularity of our candidate in each state, we are thus able to use the coefficient to help us calculate how much financial resources (or the percentage of financial resources) the campaign needs to spend on each state to yield final preference over 50%. Our calculation and categorization are as the following:

Category 1. The states where our candidate's popularity maintained under 50% both before and after spending any expenditure.

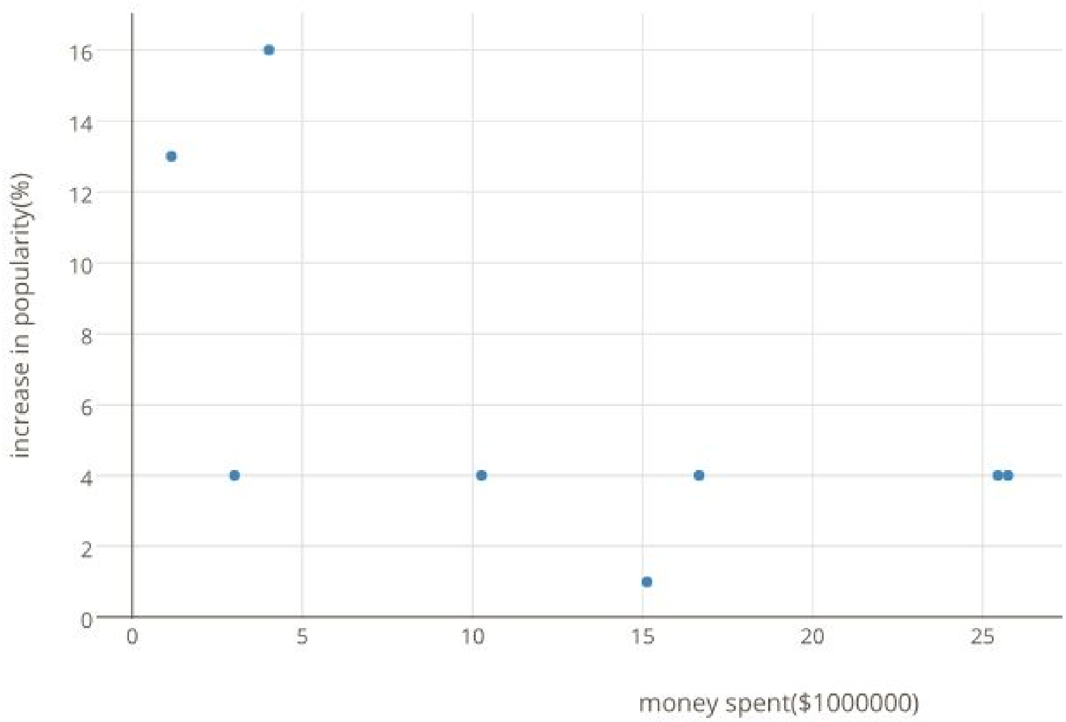
Figure 2 Change in Popularity vs. Expenditure Spent for Category 1



This category includes 13 states (AL, AK, AZ, GA, ID, KY, MO, MT, OK, SD, UT, WV and WY). The mean value of the coefficient *a* is -8.25, with a standard deviation of 3.8.

Category 2. The states where our candidate's popularity started from under 50% but reached over 50% after the campaign spending expenditure;

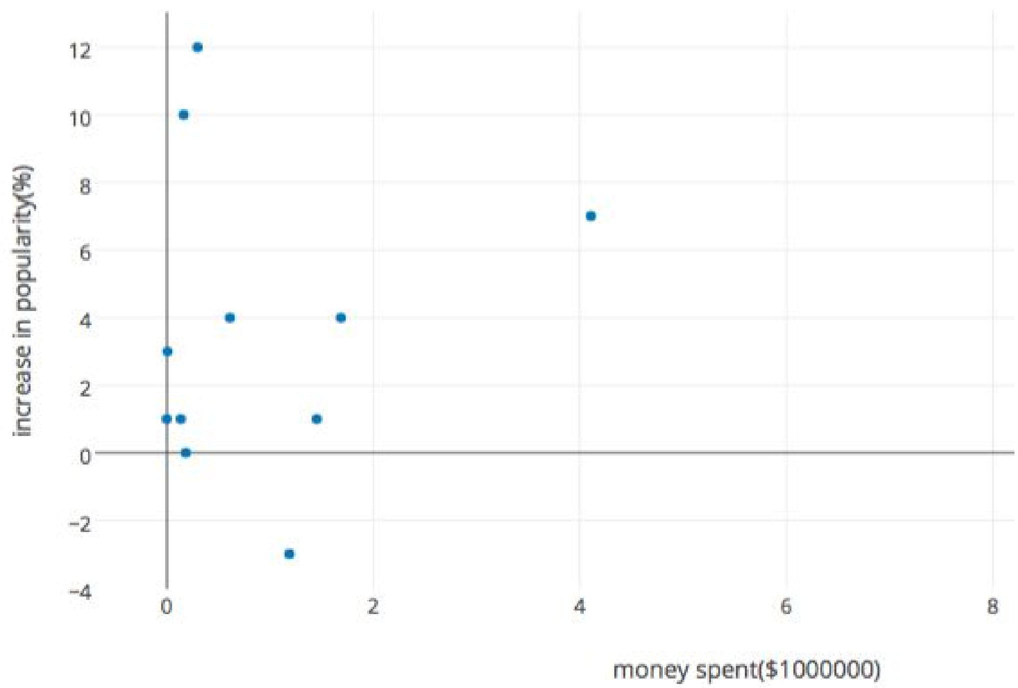
Figure 3 Change in Popularity vs. Expenditure Spent for Category 2



This category includes 9 states (CO, FL, IN, MN, NC, NM, NY, OH and VA). The mean value of the coefficient *a* is -1.26, with a standard deviation of 0.4.

Category 3. The states where our candidate's popularity maintained over 50% both before and after spending any expenditure.

Figure 4 Change in Popularity vs. Expenditure Spent for Category 3



This category includes 12 states (CA, CT, IL, IA, MI, MA, NV, NH, NJ, PA, VT and WI). The mean value of the coefficient *a* is -2.15, with a standard deviation of 6.1.

Table 2 Justification of our categorization

Catagories	Actual Expenditure	Our Model
always below 50%	8.39%	8.78%
below 50% to above 50%	55.08%	57.51%
always above 50%	36.53%	33.70%

According to our source, the total expenditure of President Obama's presidential campaign in 2008 was \$251,062,476. We classify all the states into three categories as formerly elaborated. The spend in category 1, 2, and 3 is respectively \$91,724,973, \$138,275,961, and \$21,061,542. We calculated the proportion of spending in each category to the total expenditure.

For example, the percentage of spending in category 1 can be calculated as:

$$\frac{21061542}{251062476} \times 100\% = 8.39\% \qquad (12)$$

The coefficient *a* of the total utility curve represents the popularity's rate of change, helping us to determine the optimal strategy of dividing financial resources. The coefficient of category 1, 2, and 3 is -8.25, -1.26 and -2.15 respectively. Since the utility curve concaves down, we know that the value of *a* should always be negative. Thus, we can take the absolute value of *a* to determine how fast the function increases. The larger the absolute value of *a*, the steeper of the curve, and the faster it reaches its vertex. Therefore, larger absolute value of *a* represents less expense, showing that the slope is inversely proportional to the amount of spending. This conclusion we drew also makes sense in reality. The candidate tends to invest most money on the ambivalent state, hoping to win them to their side. They also need to spend some expenditure on the states that they appear to be winning, wishing to ensure that lead to help them gain the presidency. They may not want to spend so much resources on the states that have long been supporting the opponent party, for even if they invest a lot of money, it is still highly unlikely for them to win those states.

In order to show that inverse relationship between coefficient *a* and amount of expense, we use the reciprocals of *a*'s absolute values to represent the need of expenditure. The sum of reciprocals is calculated to be $\frac{1}{8.25}+\frac{1}{1.26}+\frac{1}{2.15}\approx 1.38$. We then calculate the proportion of each category's slope's reciprocal to the sum.

For example, in case 1, the percentage can be calculated as:

$$\frac{\frac{1}{8.25}}{\frac{1}{8.25}+\frac{1}{1.26}+\frac{1}{2.15}} \times 100\% = 8.78\% \qquad (13)$$

In this example, the deviation between the prediction of our model and the real data of presidential election in 2008 is

$$(8.78\% - 8.39\%) \div 8.78\% \times 100\% = 4.4\% \qquad (14)$$

which is within our acceptable range of $\pm 5\%$. Hence, the categorization in our model is relatively accurate, and our method of calculating coefficient *a* is justifiable. All the calculation results we get are shown in Table 2.

Table 3 Justification within each category

Category	States	Relative Significance of Investing	Proportion of Expenditure	Difference

Category 1	Michigan	4.26%	7.67%	- 3.41%
	California	2.26%	5.35%	- 3.09%
	Wisconsin	8.05%	11.92%	- 3.87%
Category 2	Florida	2.68%	2.88%	- 0.20%
	North Carolina	13.40%	11.82%	+ 1.58%
	New Mexico	3.74%	3.14%	+ 0.60%
Category 3	West Virginia	6.33%	5.64%	- 0.67%
	South Dakota	3.38%	2.90%	-0.48%
	Montana	3.55%	8.00%	+4.45%

For each category, we calculated the relative significance of spending expenditure on each state and the actual proportion of expense Obama's campaign spent during the election. The relative significance is calculated by comparing S_a with the sum of S of all other states in the same category (see explanation below); the proportion of expenditure is the expense spent on the state divided by the total expense in the same category. We listed three examples from each category to show that the differences are generally kept under 5%.

The relative significance of investing is calculated as:

$$S_a = P_a\% \times D_a\% \qquad (15)$$

$$Relative\ significance = \frac{S_a}{S_a+S_b+S_{c+...+S_n}} \times 100\% \qquad (16)$$

Take Florida as our example,

$$S_{Florida} = 49\% \times 2.65\% = 1.30\% \qquad (17)$$

$$Relative\ significance = \frac{1.30\%}{1.30\%+4.48\%+...+5.64\%} \times 100\% = 2.68\% \qquad (18)$$

So the relative significance of Florida in Category 2 is 2.68%.

Financially, the proportion of the expenditure is

$$\frac{\$368678}{\$128013344} \times 100\% = 2.88\% \qquad (19)$$

The difference between the relative significance and the proportion of the expenditure is

$$2.88\% - 2.68\% = 0.2\ \% \qquad (20)$$

which falls under 5% limit, proving our model is justifiable.

Through the calculation above, we compare the relative significance of investing (which is essentially our prediction) with the proportion of expenditure (which is the reality). It is clear in Table 3 that our model is adequate in dividing the financial resources among different states, for it successfully keeps the error between prediction and reality within 5%.

2.2.6. Conclusion

To determine the optimal strategy of dividing financial resources for the presidential campaign, we first need to classify all the states into three categories based on past elections and expectations: one with the majority supporting our candidate no matter we invest expenditures or not, one with the majority supporting our opponent no matter we invest expenditures or not, and one with majority changing their minds to support our candidate after the campaign devoted money for publication. According to our model, the campaign should spend around 33% of expenditure to ensure the candidate's lead in the supporting states, devote around 58% of money trying to gain popularity from the ambivalent states, while investing around 9% on the states that had long tended to favor the opponent candidate. As for how to invest expenditures most effectively within the categories, our model suggests the campaign to calculate the "relative significance of spending money". We believe that both the overall popularity of each state and the proportion of electors that state have both contributed to this relative significance.

2.2.7. Sensitivity Analysis

Uncertainty in our model's output may be attributed to the following anomalies:

- A voter may report inaccurate information about his or her preference of candidate and corresponding possibility in the poll.
- The electors final votes may not perfectly reflect the preference of voters in the state.
- Our opponent may adopt strategies to undermine our performance.
- The total utility may not effectively correlate to the efficiency of our strategy. Even if we maximize the total utility, we still cannot find out the optimal strategy of dividing expenditures.
- The data we gathered for calculation and justification may be insufficient or inaccurate.

2.2.8. Strengths and Weaknesses

Strengths:

- Fluidity: Our model is fluid and adjustable to suit any circumstances as long as enough information and financial resources is provided for the presidential campaign;
- Tractability: Our model is easy to analyze and apply, and the calculation is easy to follow.
- Generalizability: Our model fits multiple situations and circumstances given the amount of variables it contains.
- Precision: Our model can offer an overall accurate prediction of the most efficient financial strategies in presidential campaigns.

Weaknesses:

- The reliance on the data from 2008 may cause our model to be inaccurate, for President Obama did not necessarily adopted the optimal financial strategies, and the circumstances may also changed throughout the years.
- The process of presidential election is over simplified. Certain assumptions make our model fail to be truly realistic.

2.3. Citations

1. Ayron, Lee. "Presidential Polls Over Time." *The New York Times*. N.p., 2 Nov. 2008. Web. 17 Oct. 2016. http://elections.nytimes.com/2008/president/whos-ahead/polling/index.html?src=tp#AL_46
2. "Distribution of Electoral Votes." N.p., 10 Dec. 2010. Web. 15 Oct. 2016. https://www.archives.gov/federal-register/electoral-college/allocation.html
3. "Election Tracker: Ad Spending." *CNN*. N.p., n.d. Web. 16 Oct. 2016.
4. Gabaix, Xavier, and David Laibson. "The Seven Properties of Good Models 1." Scholar.harvard. N.p., n.d. Web. 12 Oct. 2016.
5. "Total Utility." N.p., n.d. Web. 14 Oct. 2016. http://book.transtutors.com/cmsimg/27082011-economics187.gif

3. Problem 2: Design the most efficient ambulance routes from hospitals to crash sites.

3.1. Introduction

Manhattan is a big city with massive amount of cars in it. However, it takes so long for the ambulance to get to the destination when there are vehicle collisions happen. The longer it takes for the ambulance to arrive, the higher risks that people will meet. Therefore, we are designated by the mayor of New York City to manage a more efficient way of ambulance routes to avoid factors that might limit the time. We achieve this goal by analyzing the data we have for the recent collision data in Manhattan and consider about these influential factors to find an efficient route. When we are constructing the mathematical model, we use the theory of uninterrupted traffic flow and find the most efficient route for any ambulance starting from various hospitals.

3.2. Model

3.2.1. Introduction of the Model

We build our model that maximizes the efficiency of ambulances based on the traffic flow models. Because ambulance can pass an intersection although there is a red light, we consider the traffic flow for ambulances as an uninterrupted flow. First, we calculate the density of the traffic flow. Secondly, we calculate the speed according to the value of the density on a segment of road in three situations: when density of the flow is big, when density of the flow is moderate, and when density of the flow is small. Then, we use the average speed to calculate the time an ambulance needs to arrive at the place where the accident happens. In order to minimize the time, we divide the route into segments according to left and right turns and find the minimum for the combination of time an ambulance needs to arrive at the final destination through each road segment.

3.2.2. Assumptions and Justifications

- Ambulances in each hospital are abundant if there are multiple needs;
- The traffic in New York City follows the model of traffic flow;
- An ambulance can ignore traffic lights and some traffic laws to save more time;
- All the roads in Manhattan have rectangular shapes;
- At each segment of road (before a right or left turn into another road), the average speed is constant throughout the segment.

3.2.3. Definition

Q (number of cars/1hr): The number of cars passing an intersecting surface of a road in a given period of time

K (number of cars/1km): The number of cars passing an intersecting surface of a road in a given length

V (km/1hr): The displacement of a vehicle in a given period of time

K_j (number of cars/1km): The maximum number of cars passing an intersecting surface of a road in a given length. When $K = K_J = K_{max}$, $K = K_J(1- \frac{V}{V_f})$, $V = 0$.

V_f (km/1hr): The maximum of the displacement of a vehicle in a given period of time where the traffic density reaches zero. When $V = V_f = V_{max}$, $K = 0$.

K_m (number of cars/1km): The number of cars passing an intersecting surface of a road in a given length when the traffic flow (Q) reaches its maximum value.

V_m (km/1hr): The displacement of the vehicle when the traffic flow (Q) is at maximum value

D_n (km): Total distance from a nearby hospital to the destination

3.2.4. Clear Explanation of the Solution

Step 1. Calculate K according to the relationship between K and Q.

According to the flow (Q) – density (K) model,

$$Q = V_f(K - \frac{K^2}{K_j}) \qquad (1)$$

we know the flow (Q) of a section of the road, the maximum capacity of the road(K_j), and the highest limit speed (V_f) when there is almost no traffic. Therefore, we can calculate the K value for a given road.

Step 2. Calculate the speed for each road according to the value of K for each road that an ambulance will pass.

Case 1: When K's value is moderate, according to the Greenshield model,

$$V = V_f(1 - \frac{K}{K_j}) \qquad (2)$$

Case 2: When K's value is relatively big, according to the Greensberg model,

$$V = V_m ln(\frac{K_j}{K}) \qquad (3)$$

Case 3: When K's value is relatively small, according to the Underwood model,

$$V = V_f e^{\frac{-K}{K_m}} \qquad (4)$$

Step 3. *Calculate the distance from the nearest hospital to the destination by using the differences in longitude and latitude.*

Let the Hospital's position be (a, b), and let the destination's position be (a_1, b_1) that:

a = latitude, b = longitude of the hospital; a_1=latitude, b_1= longitude of the destination

Distance: $D = (\frac{|a_1 - a|}{0.009} + \frac{|b_1 - b|}{0.012})$ (km) $\qquad$ (5)

Step 4. *Calculate the time an ambulance needs to arrive at the destination for various possible routes.*

Because there are various ways for an ambulance to travel from the starting point (hospital) to the destination, we can write out all the combinations of road segments that make up of the route:

$$D = d_1 + d_2 + d_3 + \ldots + d_n = d_a + d_b + d_c + \ldots + d_z = \ldots \quad (6)$$

$$t_{first} = \frac{d_1}{v_1} + \frac{d_2}{v2} + \frac{d_3}{v_3} + \ldots + \frac{d_n}{v_n} \quad (7)$$

$$t_{second} = \frac{d_a}{v_a} + \frac{d_b}{v_b} + \frac{d_c}{v_c} + \ldots + \frac{d_z}{v_z} \quad (8)$$

$$t_{third} = \ldots \quad (9)$$

Find the T_{min} for the combination of paths that takes the least time, which is the optimal route for the ambulance.

3.2.5 Conclusion

Our model uses the three models in traffic flow to calculate the speed of an ambulance in different situations where it is driven on different road conditions. By knowing the traffic flow amount of each road segment in a given time, we can calculate the time it needs to drive through its length. Then, we can add up the time an ambulance needs for the each segment that eventually makes up the path from the hospital to its destination. Through this method, we can figure out the fastest path as we find out a combination of road segments that takes the least amount of total time.

3.2.6 Sensitivity Analysis

Because not all roads in NYC are distributed in a rectangular shape, there are some errors regarding the method to calculate the distance of route between the hospital and the destination. In addition, there are accidents happening on the road that the ambulance passes and they will affect the time of arrival, because we can't predict such unexpected events to happen. In addition, some government policies of limiting vehicles will also influence our choice of path.

3.2.7 Strengths and Weaknesses

Strengths:

- Delicacy: Our model incorporate the method of calculating the specific flow, speed, and density of the traffic system. So it is detailed.
- Accuracy: Because our model recognizes that the three formulae for calculating speed fit into roads with different traffic density, it is well-rounded and carefully planned out to make sure that the calculated data is representative enough for the real data.
- Feasibility: Our model is very realistic because since the blocks in Manhattan are arranged regularly into rectangles, the method in step 4 is easy to realize.
- Generalization: Our model fits into almost every block of Manhattan because it is designed for the entire Manhattan.

Weakness:

- Our model doesn't take into account the unpredictable events happening around Manhattan, including parades or a sudden traffic jam.
- Our model hasn't been justified by huge amounts of real data.
- Our model lacks a method to calculate the distance for roads that fail to follow the rectangular blocks.
- At different time of a day, the traffic load might be different. But we fail to consider the effect of time.