AoCMM Competition Entry #791

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Summary

Problem 1

Interpretation of Contest Problem

We have interpreted the question as an optimisation problem where we are trying to find out how much money any candidate's campaign must spend to win the state vote.

Overview of Methods

Using a Swing state for building our model, we have used normal distributions to represent the vote of the people. We accounted for an initial bias in the state, as well as opponent spending.

Statement of Conclusions

Using approximate data, we developed formulas to calculate the least amount of money a candidate would need to spend to win any state.

Problem 2

Interpretation of Contest Problem

We approached this problem by trying to find the most optimal route from the hospital to the crash by minimising the time it takes for the ambulance to arrive.

Overview of Methods

In our model, we have developed algorithms which account for the possibly of more than one free hospital in the area, no free ambulances, priority assessment of different crashes, roadworks, traffic jams, and the most optimal hospital.

Statement of Conclusions

By converting the map of Manhattan to a weighted directed graph, and assigning weights to each edge, we were able to easily calculate the shortest path from any hospital to the crash site using our algorithms.

Contents

Modeling – Problem 1

Question:
You are running a presidential campaign in the United States as one of two candidates, and have the results from current polls indicating how well you are doing compared to your opponent in each state. The polls include the following information about each person who has taken the poll: which candidate they will vote for and how certain they are of their choice. Determine a strategy to divide your candidate's campaign finance resources to maximize their probability of success nationally.

1.1 Introduction of the model

1.1.1 Approach

When dealing with elections, and peoples votes, we are inevitably dealing with human emotions and perceptions. As such, it is incredibly hard to accurately predict human behaviour, as it is influenced by an incredible amount of factors. However, through our investigation, we have found certain correlations between candidate actions and spending and the amount of votes they received. We used these to develop our model. Overall, we have taken a statistical approach to the problem, using the ideas of normal distribution to predict the amount of votes.

1.2 Assumptions and Justifications

- When dealing with normal distributions, we consider only a shift in the mean, not a slanting of the graph.
- In swing states, each candidate has 50% of the votes.
- The amount of money that is spent on the people is the only deciding factor in the voters decision process.
- We assume both candidates are essentially identical in the way they appeal to voters.

- Only Democratic and Republican candidates will be considered.
- We assume that this is the final race, where only 2 candidates remain.
- The media cover the election fairly, and no bias is present.

1.3 Explanation of the Model

When considering the historical evidence from past elections in the United States, we find that certain states always vote for one candidate, based on their party. We can split these into "Red" states and "Blue" states, which are Republican and Democratic respectively. Swing states are states where the votes for either party are almost identical.

1.3.1 General Idea

Number of votes

We started considering our problem by looking at a single state. At the start, let us assume that this is a swing state where 50% of the population votes for either candidate. We can draw a normal distribution for the population of this state:

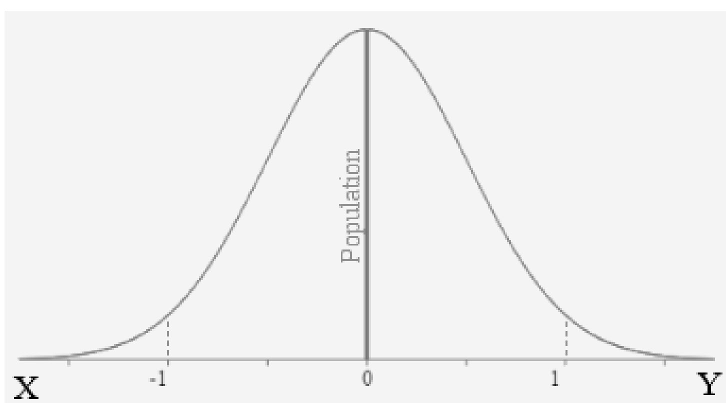

Figure 1.1: Distribution of the population

Where we have the confidence of the voter on the x axis. We will consider this normal distribution from -1 to 1, where $100x$ is the percentage confidence of the voter for their candidate. This general form is given by the equation:

$$f(x\,|\,\mu,\sigma^2) = \frac{1}{\sqrt{2\sigma^2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}} \tag{1.1}$$

Here, σ is the standard deviation, and μ is the mean. For this example, we consider the standard deviation to be 0.5 and the mean to be 0. When applying this model to a real life scenario, these values would be adjusted to match the condition of the state based on real life data and other predictions. In our model, candidate X represents a Democratic nominee and candidate Y represents a Republican nominee.

We can find the total number of votes that a candidate has by finding the area under the curve in the interval $-1 \leq x \leq 0$ for a Democratic candidate, and $0 \leq x \leq 1$ for a Republican candidate. For candidate X who is a Democrat, this could be expressed as:

$$V_x = \int_{-1}^{0} f(x\,|\,\mu,\sigma^2)dx \tag{1.2}$$

By spending money, the candidate wins more votes. By increasing the number of votes for a particular candidate, the mean of our distribution shifts to his/her favour. If we imagine that candidate X spent money, and received more votes, we would expect to see the following:

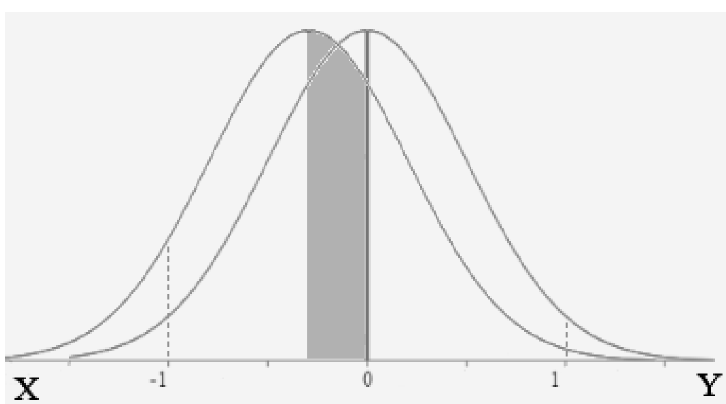

Figure 1.2: Voter opinion after spending money

The gain in votes for candidate X is the shaded area under the curve. We can again find the area using Formula 2.2. To find the increase in votes, we subtract the previous amount of voters from the new one:

$$V_{new} = \int_{-1}^{0} f(x\,|\,\mu_1,\,\sigma^2)dx - \int_{-1}^{0} f(x\,|\,\mu_2,\,\sigma^2)dx \tag{1.3}$$

Where μ_1 and μ_2 are the new and old means respectively.

Wining number of votes

For this model, we considered a "winning" number of votes to mean that a candidate has a 10 percentage point advantage over their opponent. For a Democratic candidate D, the state is given by the following:

$$S_D \begin{cases} Win, & \left[\int_{-1}^{0} f(x\,|\,\mu,\,\sigma^2)dx\right] \geq \left[\int_{0}^{1} f(x\,|\,\mu,\,\sigma^2)dx\right] + 10 \\ Lose, & \left[\int_{-1}^{0} f(x\,|\,\mu,\,\sigma^2)dx\right] < \left[\int_{0}^{1} f(x\,|\,\mu,\,\sigma^2)dx\right] + 10 \end{cases} \tag{1.4}$$

For a Republican Candidate R, this would be

$$S_R \begin{cases} Win, & \left[\int_{0}^{1} f(x\,|\,\mu,\,\sigma^2)dx\right] \geq \left[\int_{-1}^{0} f(x\,|\,\mu,\,\sigma^2)dx\right] + 10 \\ Lose, & \left[\int_{0}^{1} f(x\,|\,\mu,\,\sigma^2)dx\right] < \left[\int_{-1}^{0} f(x\,|\,\mu,\,\sigma^2)dx\right] + 10 \end{cases} \tag{1.5}$$

This gives us the winning and loosing conditions for any candidate in our model.

1.4 Division of monetary resources

In reality, Democrat and Republican voters tend to come from different backgrounds, socio-economic groups, etc. A voter decides who to vote for based on the values of the attributes that make him up. These could be predicted with enough real-world data, but is incredibly complex for a mathematical model. We have thus assumed that all voters are the same, and only decide who to vote for based on the money that candidate spends.

To know how much the candidate needs to spend in each state, we can look at how much money it costs to gain one vote. According to CNN, these values are roughly \$2.43 for Democrats, and \$2.14 for Republicans [1]. However, to account for inflation, we have estimated that any candidate needs \$2.50 per vote. This

[1] http://edition.cnn.com/2016/03/02/politics/super-tuesday-cost-of-votes/

means that the money a candidate will need to gain 1 percentage point of the population is:

$$m_x = 2.5\left(\frac{population}{100}\right) \tag{1.6}$$

However, this is in the case where there is no opponent spending. If a candidate spends enough money to gain 10% of the population, and a rival candidate also spends the same amount of money, the overall amount of money cancels out, and the mean stays 0. This means that the number of votes gained by candidate X can be summed up with the following formula

$$V_x = \left\lfloor \frac{m_x}{2.5} - \frac{m_y}{2.5} \right\rfloor \tag{1.7}$$

where m_x and m_y is the amount of money that candidate X and candidate Y spent, respectively, and V_x is the number of people that will vote for candidate X. To find V_y, we would simply switch m_x and m_y.

We can rearrange this equation to tell us how much money a candidate needs to spend to gain a certain number of votes (V):

$$m_x = \lceil 2.5V + m_y \rceil \tag{1.8}$$

For example, if candidate Y is spending $\$2,000,000$, and candidate X wants to gain 10% of the population in a state of 10 million people, we get the following equation:

$$m_x = \left\lceil 2.5\left(\frac{10,000,000}{10}\right) + 2,000,000 \right\rceil$$

and

$$m_x = \lceil 4,500,000 \rceil = \$4,500,000$$

Candidate X must spend 4.5 million dollars to get the vote of 10% of that states population.

Biased states

Most states in America are not swing states, and it is clear which side they will vote for based on the past record. With some of these states, it is irrelevant how much money is spent by the candidate as the state has always voted for one side or the other. For example, if candidate X is Democratic, he/she will not increase her chance of winning nationally by spending money in Texas, which has always voted for the Republican Party[2]. This is because of the socio–economic background of the state and its population.

In other words, if the state has a record of voting for a party that the candidate is *NOT* a member of, no money should be spent in that state by that candidate.

On the other hand, if the state has a record of voting for the party that the candidate is a member of, the candidate should spend the remainder of his/her money after winning the Swing states.

1.4.1 Media and advertising

The things that a given campaign should spend money on in a given state is given strongly by the amount of people that it would influence. In other words, the media type in which a certain amount of money should be spent should be proportional to the amount of ad revenue from each of these media types, as we can assume that ad revenue is proportional to the size of the audience.

[2] *https://en.wikipedia.org/wiki/Red_states_and_blue_states*

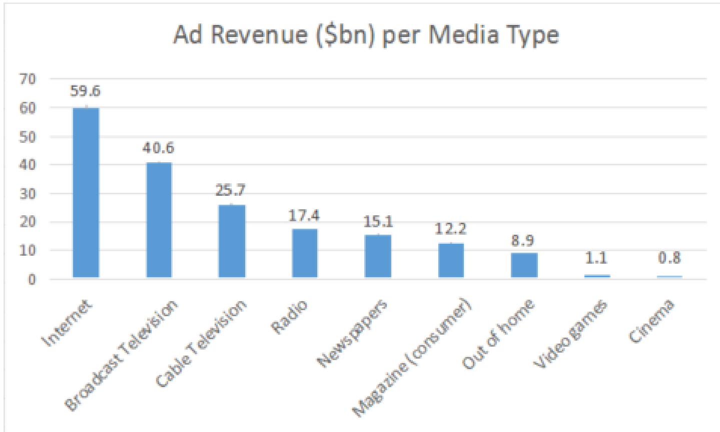

Figure 1.3: Ad revenue in $bn in the US

SOURCE: https://www.statista.com/statistics/272500/advertising-revenue-in-the-us-by-media/

If we find the percentage ad revenue for each of these media types, we get the following table:

Media	**Percentage Revenue**
Internet	32.86
Broadcast Television	22.38
Cable Television	14.17
Radio	9.59
Newspapers	8.32
Magazine (consumer)	6.72
Out of home	4.91
Video games	0.61
Cinema	0.44

However, a presidential candidate will hardly use video games, and similar sources of ad revenue, to advertise their campaign. We can thus reduce this table to the following:

Media	Percentage of Money to Spend
Internet	36.63
Broadcast Television	24.28
Cable Television	15.36
Radio	10.41
Newspapers	9.03
Out of home	4.29

These are roughly the percentages that the campaign should spend on the different media to advertise their candidate. In other words, if the campaign has m amount of money to spend on advertising, they should spend $0.3663m$ on the Internet, $0.2428m$ on Broadcast Television, etc.

These values are a rough estimate based on the national revenue, and would need to be changed to be representative on a per–state basis.

1.5 Summary

In summary, we assume that money is the only influencing factor during the election process. As such, the amount of money spent by a campaign correlates with the amount of votes for that candidate. Furthermore, we assume that the percentage of how sure people are they will vote for a particular candidate is spread in the form of a normal distribution. By spending more money, the mean of the normal distribution curve shifts in favour of that candidate, and he gains more votes; the area under the graph in the relevant interval increases.

The things the candidate should spend money on depend on the state, but there is an overall trend in the media that influence the most people.

1.6 Sensitivity Analysis

It is very difficult to come up with a clear and accurate mathematical model (even with abundant real life data) about expected human behaviour, especially about a presidential campaign. An incredible amount of factors play into the decision process for people who are voting for their candidate, and assuming that money is the *only* influencing factor is fundamentally flawed.

1.7 Strengths and Weaknesses

Strengths

- Portability: This model is easy to apply to different scenarios; not only the presidential election, but also other voting situations
- Ease of Use: The model doesn't involve complicated equations and is thus fast to apply

Weaknesses

- A lot of modifying and data gathering is necessary to obtain an accurate answer from this model.
- This model doesn't take into account the order in which elections happen in the states in the US.
- This model doesn't very well incorporate the different geographical areas, and the availability of the different media in them.

Modeling – Problem 2

Question:
Manhattan alone experiences over 50,000 vehicle collisions every year. Even worse, the response time of ambulances to crash sites still sits at over 9 minutes. As a result, the mayor of New York City has designated your team to manage more efficient ambulance routes from surrounding hospitals to crash sites and back. Take into account areas with more frequent accidents and consider all factors including number of ambulances needed, traffic jams, distance, etc.

2.1 Introduction of the model

As can be shown by a quick internet search, the phrase ”ambulance response times” can be the cause of a large furore. Long ambulance response times cause dismay among the population, whilst short response times are taken very positively by people all around. However, in more crowded areas, it is sometimes hard to keep a reasonable response time.

2.1.1 Approach

We have chosen to approach this problem by setting our main goal as reducing the time it would take an ambulance to reach a site of the traffic accident. We looked at this problem by constructing a weighted directed graph of a part of Manhattan, roughly the size of 150 acres. We have then used a path finding algorithm to find the best path, and optimised our results by changing the properties of our graph.

2.2 Assumptions and Justifications

For this problem, we made the following assumptions:

- There is no time lag between an accident happening and an ambulance being dispatched to the place of the accident
- Since Manhattan is the centre of New York, we assume that the traffic on the roads stays the same at any time during the day (ie. the number of vehicles is roughly the same)

- Before being dispatched, the ambulance has all the information about the roads in Manhattan (including roadworks, traffic jams etc.). Once the ambulance is dispatched, none of the road conditions change.
- All the accidents are similar in their nature
- Once an ambulance sets off, it is not possible to change the destination
- A hospital has exactly as many free spaces as they have ambulances
- Only the E.M.S. owned ambulances positioned in hospitals, roughly 490 of them[1], will be considered. This means that an ambulance has to return to the hospital from which it departed.
- Both ways (from and to the hospital) will be considered

With enough primary data, we could be able to account for these assumptions in our model. However, we were unable to find data of a reasonable enough quality, and thus decided to not include the above in the model.

2.3 Explanation of the Model

2.3.1 General idea

To model our problem, we need to be able to navigate ourselves around Manhattan. We decided to pick a part of Manhattan bound by the East 57^{th} Street, East 60^{th} Street, 6^{th} Avenue and Park Avenue. This is roughly the area of 150 acres, or $0.6km^2$. The map can be seen in Figure 3.1:

[1] *http://www.nytimes.com/1993/10/31/nyregion/ambulance-service-a-user-s-guide.html?pagewanted=all*

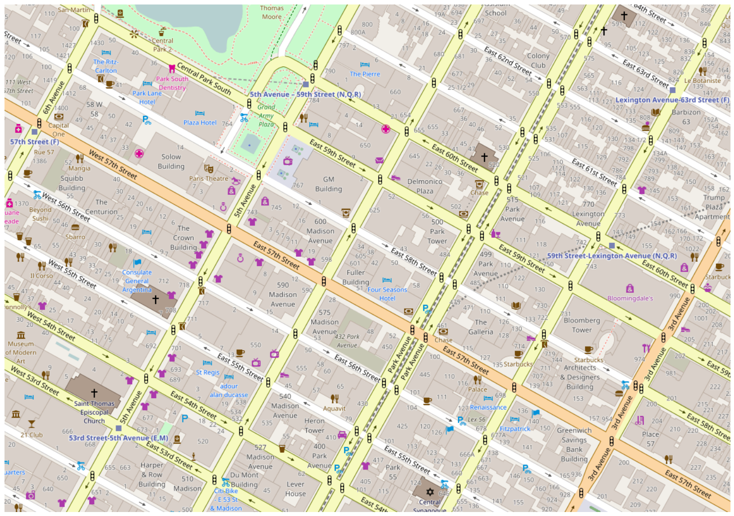

Figure 2.1: A part of Manhattan

To tackle the problem, we have converted this map of Manhattan into a graph with every intersection being a node, and the incoming roads being the edges. One way roads were accounted for by using directed edges. We thus obtain:

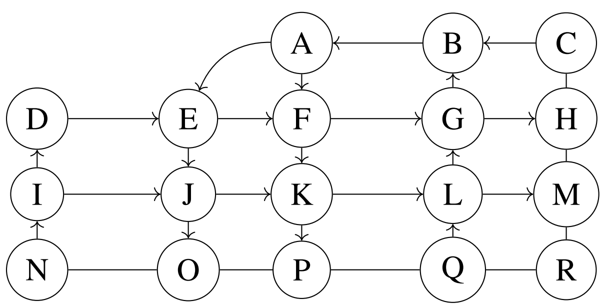

Figure 2.2: Directed Graph of the Manhattan streets

We then found the locations of the accidents in Figure 3.3:

Figure 2.3: An example of the location of accidents

We then estimated their location using the rules below:

- **If the accident happened at an intersection:**
  We considered the intersection the location of the accident

- **If the accident happened on a one-way road:**
  We considered the location of the accident to be the initial node. For example, if an accident happened on the edge $I \longrightarrow J$, we considered the location of the accident to be I.

- **If the accident happened on a two way road:**
  The location of the accident was the node that it was closer to.

2.3.2 Distance Analysis

To find the optimal path through the graph, we need to know the distances between each of the nodes. However, this isn't very relevant when talking about ambulance response times. For an ambulance, the priority of arriving at the accident fast is bigger than using the shortest distance to get there.

We decided to use time as the primary form of distance measurement. To find the time it takes to get from any one node x to any node y was calculated as the following:

$$x \xrightarrow{t} y = \frac{d}{v} \tag{2.1}$$

Where $d = distance$ and $v = velocity$. We have measured the distance of each edge as the distance from one intersection to another.

To make the calculation more precise, we removed decimal places by using the ceiling function:

$$x \xrightarrow{t} y = \left\lceil \frac{d}{v} \right\rceil \tag{2.2}$$

Note: All the measurements are in metric units.

The ceiling function is more appropriate than the floor function to account for any short delays.

2.3.3 Influencing Factors

Speed limits

According to the research paper ”Time saved with high speed driving of ambulances.” [2], the average time saved for an ambulance that drives at a higher than average speed in a metropolitan area is 2.9 minutes. This is due to the fact that ambulances will not stop at traffic lights, and have the right of way on intersections. We didn't consider the 2.9 minute bonus, as we account for it by the ”immediate” change from one edge to another. Furthermore, we need to find the speed at which the ambulance generally travels. The general rule tends to be that the speed of the ambulance is equal to 50% [3]over the speed limit:

$$v_{ambulance} = v_{limit} * 1.5 \tag{2.3}$$

This is the speed of the ambulance that we used in calculations. To find the speed limit on any edge, we used NYC Speed Limit Registry [4]

[2] *”Time saved with high speed driving of ambulances.”*Petzll K,Petzll J,Jansson J,Nordstrm G

[3] http://www.dailymail.co.uk/news/article-1247796/Drive-ambulance-fast-youre-fired-The-NHS-diktat-paramedics-just-10mph-speed-limit.html

[4] *http://www.nyc.gov/html/dot/downloads/pdf/current-pre-vision-zero-speed-limit-maps.pdf*

Traffic

We also took traffic into account. Since we previously assumed that the amount of vehicles on the road is constant, we didn't have to take the time of the accident into account; however, we did consider possible traffic jams. To find the speed of the ambulance through a traffic jam, we modify our previous formula:

$$v_{ambulance} = v_{traffic} * 1.5 \tag{2.4}$$

This gives us a good estimate of the speed of the ambulance in different conditions. However, if the traffic is stationary, we assume that the ambulance moves with an at least the walking pace of $1.4ms^{-1}$.

Roadworks

In the case of roadworks, we considered two cases:

1. **The road is closed:**
   We simply don't draw the edge on the graph.

2. **There is a new speed limit:**
   In this case, we just use equation 3.3 to find the speed of the ambulance. If there is a traffic jam, we use equation 3.4.

2.3.4 Return Journey

So far, we have only considered the journey of the ambulance to the site of the accident. This is arguably more important than the return journey as the paramedics will already be able to stabilise the victims of the accident on the way to the hospital. However, in the case of some crashes, the paramedics don't have the adequate equipment to help the patient, and it is important to get them to the hospital in the fastest way possible.

2.3.5 Application

Initial Journey

We can now calculate the times for each edge. An example calculation would go as follows:

```
INPUT speed_limit
INPUT speed_of_traffic
INPUT street_length

IF !roadworks:
    speed = speed_limit*1.5
ELSE:
    IF traffic_jam:
        speed = speed_of_traffic*1.5
    ELSE:
        speed = speed_limit*1.5

time = CEILING[distance/speed]
```

For the edge between nodes D and E, this would yield:

$$D \xrightarrow{t} E = \left\lceil \frac{252}{17.8816} \right\rceil = 15 \tag{2.5}$$

It would thus take the ambulance 15 seconds to go from node D to node E.

We can populate the rest of the graph with the time values by applying this algorithm to obtain Figure 3.4:

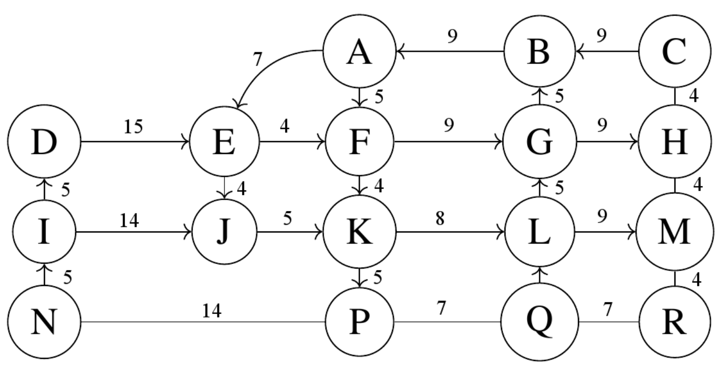

Figure 2.4: Weighted Directed Graph

This is a weighted, directed graph. However, from Figure 3.1, we can see that there are hospitals on nodes B and J, so we will mark them on the graph with red and blue. Furthermore, let us assume that an accident happened on node R (marked in green). For finding the shortest path from a starting node to any other node, we used Dijkstra's Algorithm. The source code is in the Appendix.

This allows us to find the shortest path from each hospital to the crash location. If we apply this algorithm to Figure 3.5, we get the following result:

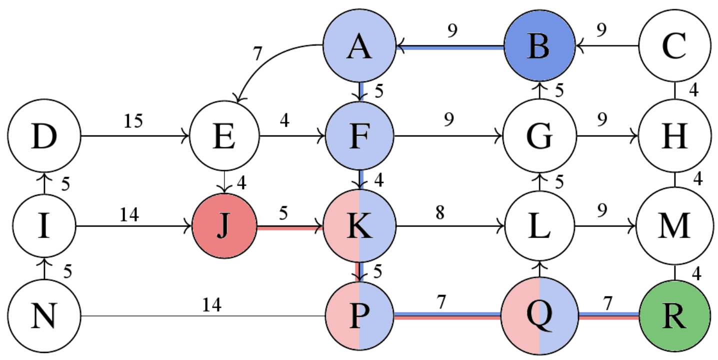

Figure 2.5: The graph after applying Dijkstra's Algorithm

The paths taken by the algorithm were the following:

$$J \xrightarrow{t_{min}} R = 5+5+7+7 = 24 \tag{2.6}$$

$$B \xrightarrow{t_{min}} R = 9+5+4+5+7+7 = 37 \tag{2.7}$$

If only the forward journey is considered important, it is better to send an ambulance from J rather than from B, as

$$J \xrightarrow{t_{min}} R \leq B \xrightarrow{t_{min}} R \tag{2.8}$$

Return journey

To calculate the way back, we use the same model as before, but this time we set the accident location (Y) to be the initial node and the hospital to be the destination node X. This gives us the equation:

$$t_{return} = X \xrightarrow{t} Y \tag{2.9}$$

We then defined a formula for the total time it takes an ambulance from hospital X to deliver a patient from an accident Y:

$$t_{total} = [X \xrightarrow{t} Y] + [Y \xrightarrow{t} X] \tag{2.10}$$

Now, applying Dijkstra's Algorithm on Figure 3.5 to find the return journey, we get the following graph:

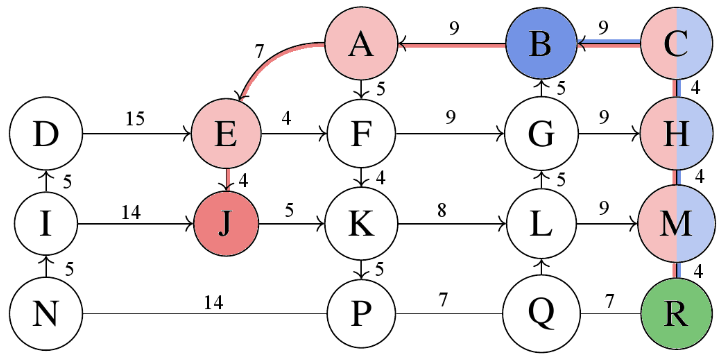

Figure 2.6: Return Routes

This gives us the following time values for the return journey:

$$R \xrightarrow{t} B = 4+4+4+9 = 21 \tag{2.11}$$

and

$$R \xrightarrow{t} J = 4+4+4+9+9+7+4 = 41 \tag{2.12}$$

As you can see, whilst it was faster to get to the location of the crash (R), from hospital J, it is faster to come back from R to hospital B. We can find the total time for each hospital to be:

$$B \xrightarrow{t} R \xrightarrow{t} B = [9+5+4+5+7+7]+[4+4+4+9] = 58 \tag{2.13}$$

$$J \xrightarrow{t} R \xrightarrow{t} J = [5+5+7+7]+[4+4+4+9+9+7+4] = 65 \tag{2.14}$$

This means that it is better overall to send an ambulance from hospital B as

$$[B \xrightarrow{t} R \xrightarrow{t} B] \leq [J \xrightarrow{t} R \xrightarrow{t} J] \tag{2.15}$$

2.3.6 Throughput limitation

Ambulance problem

We also noticed it is possible for a hospital to have no free ambulances available. In that case, we decided to send an ambulance to the place of the accident from the next closest hospital. Since we also assumed that a hospital has exactly as many free spaces as they have ambulances, an ambulance *must* return to the hospital it was dispatched from.

Priority Assessment

We also noticed a case where a hospital has only one free ambulance, but needs to cater to two or more accident areas. This proved to be too hard to include in the model in its full form, as we wanted to include all the following factors:

- The number of people injured
- The age of the people involved
- The concentration of people in the area of the intersection
- How busy the road is
- The severity of the injuries
- The number of fatalities
- The possible inclusions of important government officials

It was not possible for us to explore this further in depth due to the time constraints. We have thus assumed there is no time lag between the hospital getting a call and dispatching an ambulance, and that once an ambulance is dispatched, it cannot change direction. However, in the rare case that a call is received at exactly the same time, we included the following in our model:

```
IF time_to_x < time_to_y:
    destination = x
ELSE:
    destination = y
```

```
IF time_to_x == time_to_y:
    destination = random
```

This means that the hospitals cater to the closest accident, and choose one at random when they are the same time distance away.

No free ambulances

If there are no free ambulances in the closest hospital, the next closest hospital is used. If there are no free ambulances anywhere, the accident is put on hold until an ambulance is free; then, a normal procedure is followed.

2.3.7 Summary

We can summarize our entire model using the below code, assuming the graph representation of Manhattan has already been computed:

```
INPUT graph_representation
INPUT hospitals_in_manhattan
INPUT accident_location

FOR x in Number of Hospitals:
    time_there = Dijkstra(graph_representation, hospital[x])
    time_back = Disjktra(graph_representation, accident[x])

    total_time = time_there + time_back

    IF total_time < minimum_time:
    minimum_time = hospital[x]

IF total_time > minimum_time AND total_time < second_time:
second_time = hospital[x]

    IF free_ambulances_in_manhattan == 0:
        WAIT 1
                IF free_ambulances_at_closest_hospital == 0:
        USE second_time

INPUT number_of_crashes

IF number_of_crashes != 1:
    IF time_of_call_1 == time_of_call_2:
        IF time_to_x < time_to_y:
            destination = x
        ELSE:
            destination = y

        IF time_to_x == time_to_y:
            destination = random
```

This decides which hospital is suitable for dispatching an ambulance.

2.4 Sensitivity Analysis

The errors in our model can be attributed to the following factors:

- The approximation of the location of the crash, rather than its precise geographical location
- The estimate of the speed of the ambulance in traffic zones

2.5 Strengths and Weaknesses

Strengths

- Fluidity: By changing a few initial variables, this model is able to be adapted to work in different conditions.
- Reliability: This model is able to adapt to different scenarios, and work in many of them.

Weaknesses

- This model is slow to implement, as it requires that the map be converted to a graph, which can be time and resource intensive.

Appendix

3.1 Problem 1

3.1.1 Disjkstra's Algorithm Source Code

```
function Dijkstra(Graph, source):

create vertex set Q

// Initialization
for each vertex v in Graph:

// Unknown distance from source to v
dist[v]  INFINITY
// Previous node in optimal path from source
prev[v]  UNDEFINED
// All nodes initially in Q (unvisited nodes)
add v to Q

dist[source]  0    // Distance from source to source

while Q is not empty:

// Source node will be selected first
u  vertex in Q with min dist[u]

if u is target:
terminate

remove u from Q

// where v is still in Q.
for each neighbor v of u:
alt  dist[u] + length(u, v)
```

```
// A shorter path to v has been found
if alt < dist[v]:
dist[v]  alt
prev[v]  u

return dist[], prev[]

S = empty sequence
u = target

// Construct the shortest path
while prev[u] is defined:

// Push the vertex onto the stack
insert u at the beginning of S
// Traverse from target to source
u  prev[u]

// Push the source onto the stack
insert u at the beginning of S
```

Source: $https://en.wikipedia.org/wiki/Dijkstra's\_algorithm$

Bibliography

In order:

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