Our objective is to mathematically model the behavior of the Untied States national public debt over the next eight years. We need to predict the value of the national debt for each year between 2009 and 2017 and be able to adjust our prediction based on different circumstances. Our model accounts for the change in national public debt based on various factors, such as government spending, previous debt, national debt interest rate, and the gross domestic product (GDP).

From our general model, we can create several extensions, allowing us to vary the parameters from which it is comprised. Different values for each parameter allow us to develop differential equations that model the change in the debt year-to-year. Since the data is based on yearly figures, we use Euler's Method to solve these equations numerically. The different extensions that we create come from varying the interest and GDP growth rates. In addition, we develop two different spending policies, which we call the "Eisenhower plan" and the "Reagan plan". The Eisenhower plan represents federal expenditures that are 18% of GDP, while the Reagan plan represents federal expenditures that are 22.4% of GDP. These two plans are demonstrative of the administrations with lowest and highest spending as a percentage of the GDP on average since World War II.

Our results show that the most efficient combination for the reduction of the national debt is low interest, high GDP growth rate, and low spending. Since it is impossible to directly control the GDP growth and interest rates, we focus on these as market circumstances upon which to build the extensions of our model, and see what effect the different spending plans have on our forecast. Our model shows that, in order for the national debt as a percentage of GDP to be lowered, the federal government must adopt a spending plan with a low expenditure during an economic situation that avoids unfavorable extrema in GDP growth and interest rates.

National Debt and National Crisis

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I. Problem Restatement

The national debt, or public debt, is the amount of debt held by the United States Federal Government, excluding any intergovernmental debt (United States Public Debt). The national debt depends on various factors, such as government expenditures, income, and the interest rates on the borrowed money.

Our objective is to create a model that can help explain the national debt, and use it to forecast the amount of debt from the year 2009 to 2017 given at least two alternative government spending and tax plans. We must also determine the degree to which each parameter in our model affects the outcome. In order to create this model, we will consider various sources of historical information and determine how each variable has affected debt and related factors in the past.

II. General Assumptions and Justifications

1. The United States Government borrows money from various sources in order to pay off whatever expenditures are not covered by the federal income.

    While a branch of the federal government does print money, this money must be a negligible source of income; otherwise, the United States would undergo hyperinflation.

2. The Government must pay interest on the money that it owes.

    Banks and other lenders expect an interest payment as an incentive to lend money.

3. There is no United States national debt ceiling.

    The debt ceiling is the maximum debt allowed by law. In practice, government elevates this value to accommodate the growing debt (Welna, 2006). Therefore, we can assume that if the debt approaches the ceiling, a new ceiling will be made to accommodate it. In reality, it is as if there is no debt ceiling.

4. All sources of federal income other than taxation are negligible.

    The government earns a minor income from selling certain goods and services. However, these sources are not large enough to be significant. The problem description itself states that the federal income is composed primarily of taxes. In making this assumption, we simplify our model, since other sources of income would be superfluous variables.

5. No major and completely unpredictable events will occur that greatly impact the world.

    We assume that such upheavals as colliding with an asteroid or finding extraterrestrial life will not occur. It is impossible to provide for these in our model, so we must take it as given that nothing will cause an abrupt change in the economy.

6. The gross domestic product (GDP) is an indicator of the state of the economy (Barnes, 2008).

    The GDP is the total value of all goods and services produced by a country in a given year. Essentially, it measures the wealth that has been produced by the country. After adjustment for inflation, the GDP tends to grow more rapidly during economic booms and more slowly during recessions. It is useful to consider the GDP as an economic indicator because expressing the national debt as a fraction of the GDP is more meaningful than expressing it in dollars.

7. The government's debt and its annual expenditures can be meaningfully expressed as fractions of the GDP.

    Without knowledge of the nation's wealth at any given time, the figures for the national

debt and the expenditure have little meaning. Looking at these quantities relative to the GDP gives a concept of their magnitude in the context of the economy at the time. It also helps account for inflation.

8. The total tax revenue in a given year is equal to 19.5 percent of that year's GDP.

   This is known as Hauser's Law. Although controversial, this assertion is strongly supported by market data (Ranson, 2008). The factors that influence total tax revenue such as the tax rate, tax brackets, median house income, and the unemployment rate change over time, but Hauser's Law has been shown to continually and accurately approximate net revenue regardless of these factors. Therefore, if we assume that Hauser's Law is true, we would be able to remove many variables without any loss of accuracy to our model.

We make additional assumptions that are specific to each of the extensions of our model. These assumptions involve changing the parameters that predict the future of the economy, namely GDP and interest rates. More detail will be given as the assumptions are adopted.

III. Task I: Our Model of the National Debt

a) General Model Overview

Our model for forecasting the national debt begins with the general concept that the national debt at time *t* is equal to the sum of the initial national public debt, the interest on that debt, and the federal expenditure during that year subtracted by the total tax revenue. Thus, the national debt can be represented by equation [1].

$$D(t) = D(t-1) + G_{sp}(t) - T(t) + (D(t-1) * I(t)) \quad [1]$$

In this equation, $D(t)$ represents the debt in a given year in billions of dollars, $G_{sp}(t)$ represents government expenditures in billions of dollars per year, $T(t)$ represents tax revenue in billions of dollars per year, and $I(t)$ represents the interest rate on government borrowing in a given year, as a value between 0 and 1 (representing rates between 0% and 100%, respectively). The time t is measured in years and $t = 0$ corresponds with the beginning of the year 2008. Another important fact is that our function is defined only over the positive integers, in order to take into account the fact that the data supporting this model is presented in discrete one-year segments.

Now that we have an equation representing the national debt, we consider the following general equation:

$$f(t) = \frac{\Delta f}{\Delta t} + f(t-1)$$

This equation holds true for any function *f* defined over such discrete intervals. We can then restate it as:

$$\frac{\Delta f}{\Delta t} = f(t) - f(t-1) \quad [2]$$

Next, we combine equations [1] and [2] in order to arrive at:

$$\frac{\Delta D}{\Delta t} = G_{sp}(t) - T(t) + (D(t-1) * I(t)) \quad [3]$$

Applying Hauser's Law to equation [3] allows us to replace $T(t)$ with a multiple of GDP in billions of dollars which is represented by $y(t)$:

$$\frac{\Delta D}{\Delta t} = G_{sp}(t) - .195 * y(t) + (D(t-1) * I(t)) \quad [4]$$

This final equation represents our base model, which allows us to forecast the change in the national debt based on three different variables. These variables are the national debt interest rate $I(t)$, gross domestic product $y(t)$, and the federal expenditure $G_{sp}(t)$. We account for these different variables in the following extensions of our base model. In addition to this, these different extensions allow for us to determine the sensitivity of the national debt to each of our three different variables.

b) Extension I

Assumptions and Justifications

1. The GDP growth rate from 2006-2020 will remain at an average constant of 2.9% per year.

> Although it would be best for us to consider in our model the individual factors that affect the GDP growth rate per year, they are beyond the reach of this model. However, we found a study showing that the average GDP growth rate will be 2.9% per year (Foresight 2020, 2006).

2. The national debt interest rate during the years 2009-2017 will remain at an average constant of 5.18%.

> The national debt interest rate fluctuates greatly every month even though the national debt stays constant because of how the National Treasury sells and redeems securities. Some securities are sold at much lower interest rates than others. In addition to this, the securities mature after various time periods. The difference between the interest rates on the securities redeemed and the securities sold in a given month account for wildly fluctuating interest rate (Frequently Asked Questions About the National Debt, 2008). Even though it would make our model more accurate to account for the interest rate every month, we cannot predict what that interest rate will be. Therefore, we use an average of the national debt interest rate over the past eight years which comes out to be 5.18% (Average Interest Rates on US Treasury Securities, 2008).

Model

Our assumptions have shown that both GDP and the national debt interest rate cannot be easily forecast. In this extension, we use the GDP growth rate of 2.9% per year and the national debt interest rate of 5.18% in order to develop an equation relating the national debt to the federal expenditure. This relationship is useful because the federal expenditure represents the only variable in this model that the government fully controls.

We use the value of GDP growth in the following differential equations in order to determine a function for the GDP with respect to time.

$$\frac{dy}{dt} = .029 \quad y$$

$$\frac{1}{y}\,dy \quad = .029 \quad dt$$

$$\int \frac{1}{y} dy \quad = \int .029 \quad dt$$

$$\ln(\ y\ ) = .029 \quad t + C_1$$

$$y \ = \ e^{.029 \ \ t + C_1}$$

$$y \ = \ C_2 e^{.029 \ \ t}$$

Since $y(0)$ is equal to the GDP at the beginning of the year 2008 and $y(0) = C_2$, we can use our data to find that C_2 = 14,000 billion dollars (Gross Domestic Product). Then we can substitute y and I into equation [4] and arrive at:

$$\frac{\Delta D}{\Delta t} = G_{sp}(t) - .195 * 14000e^{.029t} + .0518D(t-1)$$
$$= G_{sp}(t) - 2730e^{.029t} + .0518D(t-1)$$

This extension represents the most realistic extension of our model because it uses constants that agree with current trends. The next several extensions will use extremes of both national debt interest rate and GDP growth rate in order to demonstrate some other possibilities. This equation will be used in Section IV in order to forecast the national debt in the year 2017.

c) Extension II

Part a

Assumptions and Justifications

1. The GDP growth rate from 2006-2020 will remain at an average constant of 5.1% per year.

    An analysis of previous GDP growth rates from 1947 to 2007 showed that for consecutive eight year periods, the greatest average GDP growth rate is 5.1% (National Income and Product Accounts Table, 2008). Therefore, we use this value to create a model to show the effect of high GDP growth on the national debt.

2. The national debt interest rate during the years 2009-2017 will remain at an average constant of 5.18%.

    See Extension I Assumptions and Justifications

Model

In this extension, we use the GDP growth rate of 5.1% per year and the national debt interest rate of 5.18% in order to develop an equation relating the national debt to the federal expenditure. Again, the only variable that the government is in control of is the federal expenditure. We use the same procedures as for Extension I in order to arrive at:

$$\frac{\Delta D}{\Delta t} = G_{sp}(t) - 2730e^{0.0611t} + 0.0518D(t-1)$$

This extension details a model for the national debt based on the assumption of a high average GDP growth rate. This model will be used in part IV to forecast the national debt in 2017.

Part b

Assumptions and Justifications

1. The GDP growth rate from 2006-2020 will remain at an average constant of 2.4% per year.

    An analysis of previous GDP growth rates from 1947 to 2007 showed that for consecutive eight year periods, the least average GDP growth rate is 2.4% (National Income and Product Accounts Table, 2008). Therefore, we use this value in order to create a model to show the effect of low GDP growth on the national debt.

2. The national debt interest rate during the years 2009-2017 will remain at an average constant of 5.18%.

    See Extension I Assumptions and Justifications

Model

In this extension, we use the GDP growth rate of 2.4% per year and the national debt interest rate of 5.18% in order to develop an equation relating the national debt to the federal expenditure. Again, the only variable that the government is in control of is the federal expenditure. We use the same procedures as for Extension I in order to arrive at:

$$\frac{\Delta D}{\Delta t} = G_{sp}(t) - 2730e^{0.024t} + 0.0518D(t-1)$$

This extension details a model for the national debt based on the assumption of a low average GDP growth rate. This model will be used in part IV to forecast the national debt in 2017. In addition, our forecasts in Part IV will determine the sensitivity of the national debt to changes in GDP growth rate.

d) Extension III

Part a

Assumptions and Justifications

1. The GDP growth rate from 2006-2020 will remain at an average constant of 2.9% per year.

    See Extension I Assumptions and Justifications

2. The national debt interest rate during the years 2009-2017 will remain at an average constant of 4.01%.

    Based on the historical monthly interest rate data for the past eight years, we determined that the lowest interest rate was 4.01% (Average Interest Rates on US Treasury Securities, 2008). Since this is the lowest interest rate in the past eight years, we decided that it would make a reasonable lower bound for interest rates. Therefore, we use this value to create a model to show the effect of low national debt interest rates on the national debt.

Model

In this extension, we use the GDP growth rate of 2.9% per year and the national debt interest rate of 4.01% in order to develop an equation relating the national debt to the federal expenditure. Again, the only variable that the government is in control of is the federal expenditure. We use the same procedures as for Extension I in order to arrive at:

$$\frac{\Delta D}{\Delta t} = G_{sp}(t) - 2730e^{0.029t} + 0.0401D(t-1)$$

This extension details a model for the national debt based on the assumption of a low national debt interest rate. This model will be used in part IV to forecast the national debt in 2017.

Part b

Assumptions and Justifications

1. The GDP growth rate from 2006-2020 will remain at an average constant of 2.9% per year.

    See Extension I Assumptions and Justifications

2. The national debt interest rate during the years 2009-2017 will remain at an average constant of 6.64%.

    Based on the historical interest rate data for the past eight years, we determined that the highest interest rate was 6.641% (Average Interest Rates on US Treasury Securities, 2008). Since this is the highest interest rate in the past eight years, we decided that it

would make a reasonable upper bound for interest rates. Therefore, we use this value to create a model to show the effect of high national debt interest rates on the national debt.

Model

In this extension, we use the GDP growth rate of 2.9% per year and the national debt interest rate of 6.64% in order to develop an equation relating the national debt to the federal expenditure. Again, the only variable that the government is in control of is the federal expenditure. We use the same procedures as for Extension I in order to arrive at:

$$\frac{\Delta D}{\Delta t} = G_{sp}(t) - 2730e^{0.029t} + 0.0664D(t-1)$$

This extension details a model for the national debt based on the assumption of a high national debt interest rate. This model will be used in part IV to forecast the national debt in 2017. In addition, our forecasts in part IV will demonstrate how sensitive the national debt is to national debt interest rate changes.

d) Extension IV

We used four pairings of the GDP growth rates from Extension II and the national debt interest rates from Extension III. These four additional models show the effect on national debt at the extremes of both conditions. For assumptions and justifications of the values, see previous extensions.

Part a

Model

In this extension, we use the GDP growth rate of 5.1% per year and the national debt interest rate of 4.01% in order to develop an equation relating the national debt to the federal expenditure. We use the same procedures as for Extension I in order to arrive at:

$$\frac{\Delta D}{\Delta t} = G_{sp}(t) - 2730e^{0.061t} + 0.0401D(t-1)$$

This extension details a model for the national debt based on the assumption of a high GDP growth rate and a low national debt interest rate. This model will be used in part IV to forecast the national debt in 2017.

Part b

Model

In this extension, we use the GDP growth rate of 5.1% per year and the national debt interest rate of 6.64% in order to develop an equation relating the national debt to the federal expenditure. We use the same procedures as for Extension I in order to arrive at:

$$\frac{\Delta D}{\Delta t} = G_{sp}(t) - 2730e^{0.061t} + 0.0664D(t-1)$$

This extension details a model for the national debt based on the assumption of a high GDP growth rate and a high national debt interest rate. This model will be used in part IV to forecast the national debt in 2017.

Part c

Model

In this extension, we use the GDP growth rate of 2.4% per year and the national debt interest rate of 4.01% in order to develop an equation relating the national debt to the federal expenditure. We use the same procedures as for Extension I in order to arrive at:

$$\frac{\Delta D}{\Delta t} = G_{sp}(t) - 2730e^{0.024t} + 0.0401D(t-1)$$

This extension details a model for the national debt based on the assumption of a low GDP growth rate and a low national debt interest rate. This model will be used in part IV to forecast the national debt in 2017.

Part d

Model

In this extension, we use the GDP growth rate of 2.4% per year and the national debt interest rate of 6.64% in order to develop an equation relating the national debt to the federal expenditure. We use the same procedures as for Extension I in order to arrive at:

$$\frac{\Delta D}{\Delta t} = G_{sp}(t) - 2730e^{0.024t} + 0.06.64D(t-1)$$

This extension details a model for the national debt based on the assumption of a low GDP growth rate and a high national debt interest rate. This model will be used in part IV to forecast the national debt in 2017.

IV. Task II: Two Alternative Plans

Of the three parameters of our model, the GDP and the interest rate are future conditions beyond the government's control. These conditions are covered by the four extensions. But now, in order to get an idea of how the government can behave to change the national debt, we create two plans that represent the two extremes of government spending. Seeing how our model responds to these plans, the federal administration may choose to lean toward one of the two extremes as its fiscal policy. Those two extremes are low expenditure and high expenditure.

To define the two plans, we look at the history of federal expenditure since 1953, as a fraction of the GDP (Office of Management and Budget, 2004). We calculate the mean of the yearly ratios of expenditure to GDP during each president's administration (see Table 1). This shows that, relative to the GDP, Dwight Eisenhower spent the least, and Ronald Reagan spent the most. We use these two presidents' expenditures for the two plans: the Eisenhower low-expenditure plan and the Reagan high-expenditure plan.

Table 1

President	Term	Average yearly expenditure (% of GDP)
Eisenhower	1953-1960	18.0
Kennedy	1961-1962	18.6
Johnson	1963-1968	18.5
Nixon	1959-1963	18.5
Ford	1974-1976	20.5
Carter	1977-1980	20.8
Reagan	1981-1988	22.4
H. W. Bush	1989-1992	21.9
Clinton	1993-2000	19.9

We first consider how the Eisenhower and Reagan plans would fare in extension I. We assume that the government spending is constant, *relative to the GDP*. Thus, since Eisenhower's administration spent an average of 18.0% of the GDP each year, government spending from 2009 to 2017 can be given by:

$$G_{sp}(t) = .180 * 14000e^{.029t}$$
$$G_{sp}(t) = 2520e^{.029t}$$

Substituting this function for government spending into the model for extension I, we get:

$$\frac{\Delta D}{\Delta t} = G_{sp}(t) - 2730e^{.029t} + .0518D(t-1)$$
$$= 2520e^{.029t} - 2730e^{.029t} + .0518D(t-1)$$
$$= .0518D(t-1) - 210e^{.029t}$$

We then use Euler's method, which tells us that:

$$D(t) = D(t-1) + \left.\frac{\Delta D}{\Delta t}\right|_t$$

Since the national debt at the beginning of 2008 was $D(0) = 9200$, Euler's method can be used to find the national debt for the next nine years, as shown in the following calculations:

$$D(1) = 9200 + (.0518 * 9200) - 210e^{.029} = 9500$$
$$\frac{D(1)}{y(1)} = \frac{9500}{14000e^{.029}} = .659 \qquad \text{(2009 projection)}$$

$$D(2) = 9500 + (.0518 * 9500) - 210e^{.058} = 9700$$
$$\frac{D(2)}{y(2)} = \frac{9800}{14000e^{.058}} = .6606 \qquad \text{(2010 projection)}$$

$$D(3) = 9700 + (.0518 * 9700) - 210e^{.087} = 10000$$
$$\frac{D(3)}{y(3)} = \frac{10100}{14000e^{.087}} = .6613 \qquad \text{(2011 projection)}$$

. . .
. . .
. . .

$$D(9) = 11500 + (.0518 * 11500) - 210e^{.261} = 11800$$
$$\frac{D(9)}{y(9)} = \frac{11900}{14000e^{.261}} = .655 \qquad \text{(2017 projection)}$$

So extension I predicts that, under the Eisenhower plan, the national debt will be 11,800 billion dollars, or 65.5 percent of the GDP, in 2017. This is a higher dollar value, but a lower fraction of the GDP, than in 2008. At face value, the Eisenhower plan seems to be able to slow the rise of the national debt.

Next, we put the Reagan expenditure plan through extension I. In the same way that we derived a function for Eisenhower's spending, we find the function for Reagan's spending to be:

$$G_{sp}(t) = 3136e^{.029t}$$

We put the two government expenditure functions from the Eisenhower plan and the Reagan plan into all of the extensions of our model. The results are summarized in the following two tables.

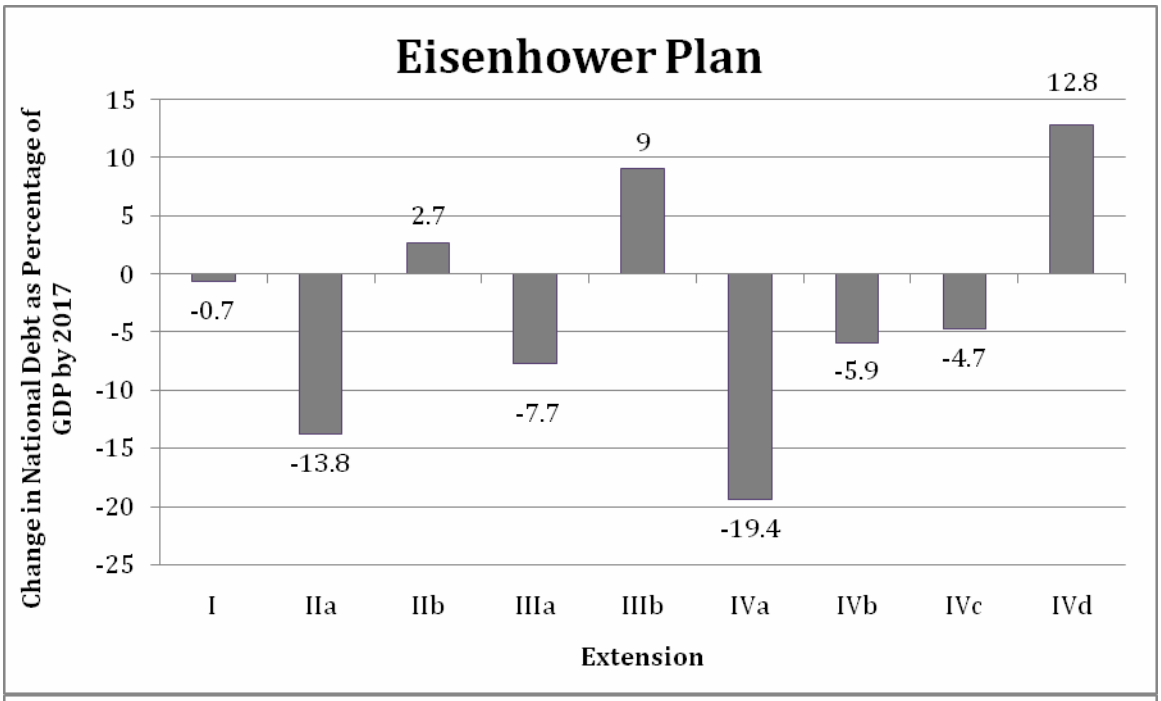
NATIONAL DEBT IN BILLIONS OF DOLLARS

		2008	2009	2010	2011	2012	2013	2014	2015	2016	2017
I	Eisenhower	9200	9460	9728	10003	10285	10575	10873	11179	11493	11816
	Reagan	9200	10095	11048	12063	13144	14294	15517	16819	18202	19672
IIa	Eisenhower	9200	9456	9713	9971	10230	10489	10747	11004	11258	11509
	Reagan	9200	10104	11077	12124	13250	14460	15760	17157	18656	20265
IIb	Eisenhower	9200	9461	9731	10010	10297	10594	10900	11216	11543	11880
	Reagan	9200	10092	11041	12049	13120	14258	15465	16747	18106	19548
IIIa	Eisenhower	9200	9353	9505	9657	9809	9959	10109	10257	10403	10548
	Reagan	9200	9987	10818	11694	12619	13595	14623	15707	16848	18051
IIIb	Eisenhower	9200	9595	10009	10445	10902	11384	11890	12422	12982	13571
	Reagan	9200	10229	11338	12534	13822	15209	16702	18309	20037	21894
IVa	Eisenhower	9200	9348	9490	9626	9755	9875	9986	10086	10174	10250
	Reagan	9200	9996	10847	11755	12724	13758	14861	16037	17291	18627
IVb	Eisenhower	9200	9590	9994	10413	10847	11296	11761	12242	12739	13252
	Reagan	9200	10238	11368	12595	13930	15379	16951	18657	20506	22510
IVc	Eisenhower	9200	9354	9509	9664	9821	9978	10135	10293	10451	10610
	Reagan	9200	9985	10811	11681	12596	13559	14572	15636	16755	17931
IVd	Eisenhower	9200	9596	10013	10452	10915	11403	11917	12460	13033	13638
	Reagan	9200	10227	11332	12520	13799	15173	16649	18235	19938	21765

NATIONAL DEBT IN PERCENT OF GDP

		2008	2009	2010	2011	2012	2013	2014	2015	2016	2017
I	Eisenhower	65.7	65.6	65.6	65.5	65.4	65.3	65.3	65.2	65.1	65.0
	Reagan	65.7	70.0	74.5	79.0	83.6	88.3	93.1	98.1	103.1	108.2
IIa	Eisenhower	65.7	64.2	62.6	61.1	59.6	58.1	56.5	55.0	53.5	51.9
	Reagan	65.7	68.6	71.4	74.3	77.2	80.0	82.9	85.8	88.6	91.5
IIb	Eisenhower	65.7	66.0	66.3	66.5	66.8	67.1	67.4	67.7	68.0	68.4
	Reagan	65.7	70.4	75.2	80.1	85.1	90.3	95.7	101.1	106.7	112.5
IIIa	Eisenhower	65.7	64.9	64.1	63.2	62.4	61.5	60.7	59.8	58.9	58.0
	Reagan	65.7	69.3	72.9	76.6	80.3	84.0	87.8	91.6	95.4	99.3
IIIb	Eisenhower	65.7	66.6	67.5	68.4	69.3	70.3	71.4	72.4	73.5	74.7
	Reagan	65.7	71.0	76.4	82.1	87.9	94.0	100.3	106.8	113.5	120.5
IVa	Eisenhower	65.7	63.5	61.2	59.0	56.8	54.7	52.5	50.4	48.3	46.3
	Reagan	65.7	67.9	70.0	72.1	74.1	76.2	78.2	80.2	82.1	84.1
IVb	Eisenhower	65.7	65.1	64.5	63.8	63.2	62.5	61.9	61.2	60.5	59.8
	Reagan	65.7	69.5	73.3	77.2	81.1	85.1	89.2	93.3	97.4	101.6
IVc	Eisenhower	65.7	65.2	64.7	64.2	63.7	63.2	62.7	62.2	61.6	61.1
	Reagan	65.7	69.6	73.6	77.6	81.7	85.9	90.1	94.4	98.8	103.2
IVd	Eisenhower	65.7	66.9	68.2	69.5	70.8	72.2	73.7	75.2	76.8	78.5
	Reagan	65.7	71.3	77.1	83.2	89.5	96.1	103.0	110.1	117.5	125.3

V. Results and Conclusions



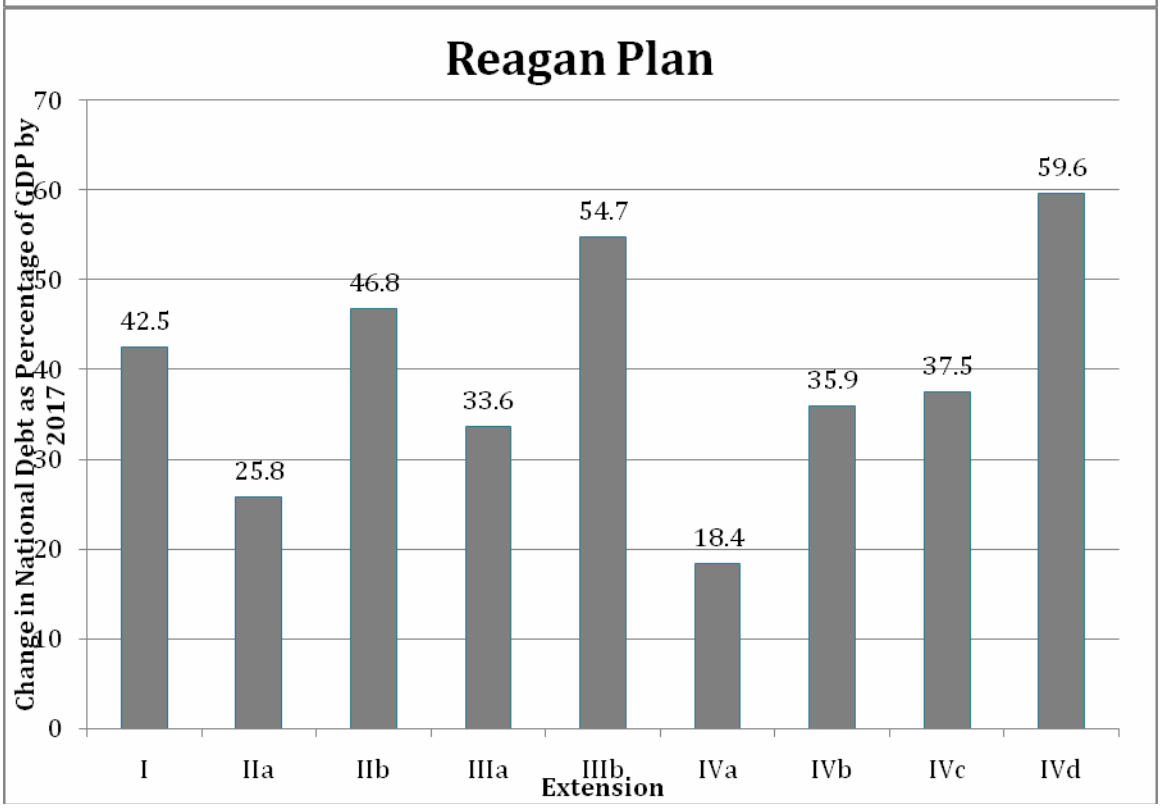


Figure 1

In order to obtain our results, we used the four extensions developed from our general model, combined with our two spending plans. We wanted to find how we could lower the national debt and the national debt as a percentage of GDP. The optimal case that lowers the national debt as a percentage of GDP is a low interest rate, high GDP growth rate, and low federal expenditures. As expected, the least effective case is the opposite. However, the interest and GDP growth rates are impossible to control, so we must consider how spending, the factor we can control, affects the results under different circumstances. Under the Reagan plan, with high federal expenditures, national debt as a percentage of GDP always increases. However, under the Eisenhower plan, national public debt as a percentage of GDP decreases by 2017, except in extreme situations such as the low GDP growth of extension IIb or the high interest rates of extension IIIb. Under either plan, national debt in dollars increased every year during the years 2009-2017. In summary, our model states that in order to lower the national debt as a percentage of GDP in the next eight years, the government must enact a more conservative spending plan. In addition, we found that, while the national debt never decreased in dollars for 2009-2017 in any of the extensions, it would decrease to zero if we let our forecast run to the year 2149 for extension I, the year 2051 for extension IIa, and the year 2066 for extension IIIa. Although our model may not be accurate enough to predict the debt for the next 60 years, it suggests that a low expenditure plan may make it possible to completely pay off the national debt.

The different extensions also allowed us to test the sensitivity of our model to the different parameters. Figure 1 shows the change of national debt as a percentage of the GDP from our forecast. A negative value means that the national debt decreased in relation to the GDP by 2017 in our forecast. In order to show the relationship between each parameter and our forecast, we considered the observed extrema of each parameter on its own (extensions II and III) and which parameter was stronger when they were taken together (extension IV), all in relation to what we considered "normal" market conditions (extension I). In "normal" conditions, the Eisenhower plan reduced the national debt as a percentage of the GDP by .7% and the Reagan plan increased it by 42.5% by 2017. In extension IIa (high GDP growth), the Eisenhower plan reduced the same percentage by 13.8% and the Reagan plan increased it by 25.8%. In extension IIIa (low interest rate), the Eisenhower plan reduced this percentage by 7.7% and the Reagan plan increased it by 33.6%. With both plans, higher GDP growth affected the model more than lower interest rates, which is intuitive, as a higher GDP created more tax revenue according to our model. By similar reasoning, comparing the unfavorable extrema (extensions IIb and IIIc) shows that high interest causes poorer debt performance than low GDP growth. This also agrees with intuition, as high interest rates cumulate, whereas poor GDP simply means poor tax revenue. Finally, considering the mixed shows that a good GDP with high interest (extension IVb) creates better performance for the Eisenhower plan and the opposite is true (extension IVc creates better performance) for the Reagan plan.

VI. Strengths and Weaknesses

Strengths:

1. Due to the way we built each extension from the basic model, we can easily vary the different parameters and perform Euler's Method on the new extension. For example, if we wanted to test a different method for predicting the GDP, we could substitute it in for the current exponential model.

2. Over large periods of time, we take into account perturbations in the global and national markets in that they should average out over the course of about a decade.

Weaknesses:

1. Our model assumes Hauser's Law. Although it is strongly supported by market data, it is highly controversial and may be shown to be invalid.

2. Since our model assumes that the effects of perturbations average out over time, we must use a large time period, and the results are less reliable for the intermediate years.

3. All of the extensions to our model assumed that GDP grew at a constant percentage of itself yearly and that the interest rate was constant over a decade. Although we believed that these values averaged out, a more advanced model could take a possibly more accurate prediction into account.

VII. Letter to the President

Dear Mr. President:

Congratulations on being elected! We know that you are inheriting a host of difficult issues, one of which is the rapidly increasing national debt. After thorough analysis of U.S. economic history and national debt, we have devised a model that accounts for the change in national public debt, based on government spending, previous debt, interest rate, and the gross domestic product (GDP). After we constructed this model, we created several different extensions of the model that allowed us to change the parameters upon which it depended. Our tests involved different combinations of GDP growth rates, interest rates and spending plans. We hope that the results of our model will help you make the crucial budget decisions with which you will be faced over the next four years.

We considered two different spending plans, which we coined the "Eisenhower plan" and the "Reagan plan." In the former, we considered a yearly federal expenditure of 18% of the GDP, the average rate of spending during the Eisenhower administration and the lowest such rate of all presidential administrations since the conclusion of World War II. For the latter, we considered an expenditure of 22.4%, the average during the Reagan administration and the highest rate during the same period.

The results of our forecasts confirmed what may seem intuitive: the most efficient combination for the reduction of the national debt is low interest, high GDP growth rate, and low spending. Obviously, it is impossible to directly affect the GDP growth rate and interest rates. However, keeping our model in mind can help guide decisions on how much to regulate spending. We are aware that there are many crucial government programs, and we are not advising that they all be cut. However, it is vital to evaluate the trade-off between the benefits of spending and its effect on our debt. Even with the Eisenhower plan, we found that some extreme market conditions, like low GDP growth rate or high interest rates, caused the national debt as a percentage of GDP to rise; however, our other scenarios showed that the Eisenhower plan lowered national debt as a fraction of GDP. On the other hand, the Reagan plan increased the national debt as a percentage of GDP in every situation. In other words, it is very difficult to lower the national debt as a percentage of GDP, and even more difficult to lower it in nominal dollars, especially within eight years.

We acknowledge that our model depends on the assumption of Hauser's Law, which states that tax revenue will stay at approximately 19.5% of the GDP regardless of the tax rate. This is a fairly controversial statement, which may be unpopular with many, but there are ample data to support the assertion. We hope that our model is of use to you and to your advisors.

Best,
Team 2159

Works Cited

A GRAPH REPRESENTATION OF A BASIC MACROECONOMIC SCHEME: THE IS-LM MODEL. (2001). Retrieved November 15, 2008, from http://www.economicswebinstitute.org/essays/is-lm2.htm

An Analysis of the Presidents Who Are Responsible for the Borrowing. (2008, September 21). Retrieved November 15, 2008, from United States National Debt: http://www.cedarcomm.com/~stevelm1/usdebt.htm

Average Interest Rates on US Treasury Securities. (2008, November 6). Retrieved November 15, 2008, from Treasury Direct: http://www.treasurydirect.gov/govt/rates/pd/avg/avg.htm

Barnes. (n.d.). *Economic Indicators: Gross Domestic Product (GDP)*. Retrieved November 15, 2008, from Investopedia: http://www.investopedia.com/university/releases/gdp.asp

Budget of the United States Government. (2004). Retrieved November 15, 2008, from http://www.cfo.doe.gov/budget/04budget/content/appendix/hist.pdf

Budget of the United States Government. (2008, January 24). Retrieved November 15, 2008, from GPO Access: http://www.gpoaccess.gov/usbudget/browse.html

Economic Statistics. (n.d.). Retrieved November 15, 2008, from The White House: http://www.whitehouse.gov/fsbr/esbr.htm

Foresight 2020. (2006). *The Economist* , 10-15.

Frequently Asked Questions About the National Debt. (2008). Retrieved November 15, 2008, from The Muser: http://www.greatreality.com/DebtFAQ.htm#OweWild

Gross Domestic Product. (n.d.). Retrieved November 15, 2008, from Wikipedia: http://en.wikipedia.org/wiki/Gross_domestic_product

National Income and Product Accounts Table. (2008, October 30). Retrieved November 15, 2008, from Bureau of Economic Analysis: http://www.bea.gov/national/nipaweb/TableView.asp?SelectedTable=1&ViewSeries=NO&Java=no&Request3Place=N&3Place=N&FromView=YES&Freq=Year&FirstYear=1930&LastYear=2008&3Place=N&Update=Update&JavaBox=no#

Ranson, D. (2008, May 20). *You Can't Soak It Rich*. Retrieved November 15, 2008, from The Wall Street Journal: http://online.wsj.com/article/SB121124460502305693.html

The Debt to the Penny and Who Holds It. (2008, November 14). Retrieved November 15, 2008, from Treasury Direct: http://www.treasurydirect.gov/NP/BPDLogin?application=np

United States Public Debt. (n.d.). Retrieved November 2008, 2008, from Wikipedia: http://en.wikipedia.org/wiki/United_States_public_debt

Welna, D. (2006, March 16). *Congress Sets New Federal Debt Limit: $9 Trillion*. Retrieved November 15, 2008, from NPR: http://www.npr.org/templates/story/story.php?storyId=5282521