HIGH SCHOOL MATHEMATICAL CONTEST IN MODELING

OUTSTANDING PAPERS

HiMCM

November 2005

The contest offers students the opportunity to compete in a team setting using applied mathematics in the solving of real-world problems.

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Editor's Comments

This is our eighth HiMCM Special Issue. Since space does not permit printing all of the nine National Outstanding papers, this special section includes the summaries from seven papers and abridged versions of two. We emphasize that the selection of these two does not imply that they are superior to the other Outstanding papers. We also wish to emphasize that the papers were not written with publication in mind. Given the 36 hours that the teams had to work on the problems and prepare their papers, it is remarkable how much was accomplished and how well written many of the papers are.

Contest Director's Article

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The High School Mathematical Contest in Modeling (HiMCM) completed its eighth year in excellent fashion. The mathematical and modeling ability of students and faculty advisors is very evident in the professional submissions and work being done. The contest is still moving ahead, growing with a positive first derivative, and consistent with our positive experiences from previous HiMCM contests.

This year the contest consisted of 253 teams (an increase of 25 teams over last year) from 51 institutions (an increase of 8 institutions over last year). These institutions were from eighteen states and from the Hong Kong International School, China, and Australia. This year, we again charged a registration fee of $45 for the first team and $25 for each additional team from the same institution.

The teams accomplished the vision of our founders by providing *unique* and *creative mathematical* solutions to complex open-ended real-world problems. This year the students had a choice of two problems.

Problem A: Modeling Ocean Bottom Topography

BACKGROUND:

A marine survey ship maps ocean depth by using sonar to reflect a sound pulse off the ocean floor. **Figure A** shows the ship's location at B on the surface of the ocean. The sonar apparatus aboard the ship is capable of emitting sound pulses in an arc measuring from 2 to 30 degrees. In two dimensions this arc is shown within **Figure A** triangle by ∠ABC, and the emanating sound pulses are displayed by the dashed lines and the solid lines BA and BC.

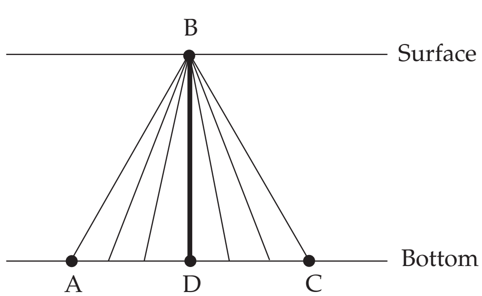


Figure A

When a sonar sound pulse hits the bottom of the ocean, the pulse is reflected off the ocean bottom the same way a billiard ball is reflected off a pool table; that is, the angle of incidence α equals the angle of reflection β as illustrated in **Figure B**. Since the ship is moving when the sound pulse is emitted, it will pick up a reflected sound pulse at location F in this picture. The actual depth of the water is the length of BD in **Figure A**.

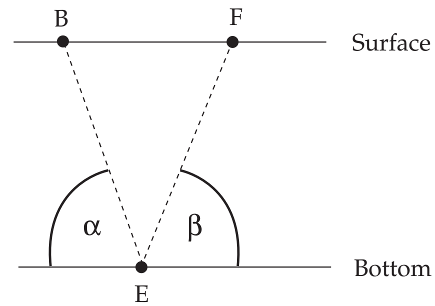


Figure B

USEFUL INFORMATION:

Oceanography vessels usually travel at a speed of 2 m/s while Navy vessels travel at 20 m/s. The sonar apparatus aboard these ships is capable of emitting sound pulses in an arc measuring from 2 to 30 degrees. The typical speed at which a sonar sound pulse is emitted is 1500 m/s.

Devise a model for mapping the topography of the ocean bottom. Write a letter to the science editor of your local paper summarizing your findings.

Problem B: Gas Prices, Inventory, National Disasters, and the Mighty Dollar.

It appears from the economic reports that the world uses gasoline on a very short supply and demand scale. The impact of any storm, let alone Hurricane Katrina, affects the costs at the pumps too quickly. Let's restrict our study to the continental United States.

Over the past six years, Canada has been the leading foreign supplier of oil to the United States, including both crude and refined oil products. (*Petroleum Supply Monthly*, Table S3 - Crude Oil and Petroleum Product Imports, 1988-Present. See page 5 for Canadian exports to the United States.)

- Canada was the largest foreign supplier of oil to the United States again in 2004, for the sixth year running (from 1999, when the country displaced Venezuela, to 2004 inclusive).
- In 2002, Canada supplied the United States with 17 percent of its crude and refined oil imports—more than any other foreign supplier at over 1.9 million barrels per day.
- Western Canadian crude oil is imported principally by the U.S. Midwest and the Rocky Mountain states.
- Eastern Canada's offshore oil is imported principally by the U.S. East Coast states, and even by some Gulf Coast states.

Many refiners are buying enough to serve motorists' current needs, but not enough to rebuild stocks. "They are looking to buy the oil when they need it," according to *The Washington Post*. "When they are uncertain about the future, they hold back" (*The Washington Post:* Crude Oil Imports to U.S. Slow With War 3/31/03).

Build a better model for the oil industry for its use and consumption in the United States that is fair to both the business and the consumer. You can build your model based on a *peak* day.

Write a letter to the President's energy advisor summarizing your findings.

Commendation: All students and advisors are congratulated for their varied and creative mathematical efforts. Of the 253 teams, 149 submitted solutions to the A problem and 94 to the B problem. The thirty-six continuous hours to work on the problem provided for quality papers; teams are commended for the overall quality of their work.

Many teams had female members. There were 341 female participants on the 253 teams. There were 949 total participants, so the females made up approximately 36% of the total participation, showing this competition is for both genders. Teams again proved to the judges that they had "fun" with their chosen problems, demonstrating research initiative and creativity in their solutions. This year's effort was a success!

Judging: We ran three regional sites in December 2005. Each site judged papers for both problems A and B. The papers judged at each regional site may or may not have been from their respective region. Papers were judged as Outstanding, Meritorious, Honorable Mention, and Successful Participant. All finalist papers for the Regional Outstanding award were sent to the National Judging. For example, eight papers may be discussed at a Regional Final and only four selected as Regional Outstanding but all eight papers are judged for the National Outstanding. Papers receive the higher of the two awards. The National Judging chooses the "*best of the best*" as National Outstanding. The National Judges commended the regional judges for their efforts and found the results were very consistent. We feel that this regional structure provides a good structure for the future as the contest continues to grow.

Judging Results:

NATIONAL & REGIONAL COMBINED RESULTS

Problem	**National Outstanding***	Regional Outstanding Only	Meritorious	Honorable Mention	Successful Participant	Total
A	**6**	7	47	61	28	**149**
B	**3**	4	33	33	21	**94**
Total	**9**	11	80	94	49	**253**

General Judging Comments: The judge's commentaries provide specific comments on the solutions to each problem. As contest director and head judge, I would like to speak generally about solutions from a judge's point of view. Papers need to be coherent, concise, and clear. Students need to restate the problem in their own words so that the judges can determine the focus of the paper. Papers that explain the development of the model, assumptions, and its solutions and then support the findings mathematically generally do quite well. Modeling assumptions need to be listed and justified but only those that come to bear on the solution (that can be part of simplifying the model). Laundry lists of assumptions that are never referred to in the context of the model development are not considered relevant and deter from the paper's quality. The mathematical model needs to be clearly developed and all variables that are used need to be well defined. Thinking outside of the "box" is also considered important by judges. This varies from problem to problem but usually includes model extensions or sensitivity analysis of the solution to the teams' inputs. Students need to attempt to validate their model even if by numerical example or intuition. A clear conclusion and answers to specific scenario questions are all key components. The strengths and weakness section is where the team can reflect on their solution and comment on the model's strengths and weaknesses. Attention to detail and proofreading the paper prior to final submission are also important since the judges look for clarity and style.

CONTEST FACTS:

Facts from the 8th Annual Contest:

- A wide range of schools/teams competed including thirty-three teams from Hong Kong (21), China (8), and Australia (4).
- The 253 teams represented 51 institutions.
- There were 949 student participants, 608 (64%) male and 341 (36%) female.
- Schools from eighteen states participated in this year's contest.

THE FUTURE:

The contest, which attempts to give the under-represented an opportunity to compete and achieve success in mathematics, appears well on its way in meeting this important mission.

We continue to strive to improve the contest and we want the contest to grow. Again, any school/team can enter, as there will be no restrictions on the number of schools or the numbers of teams from a school entering. A regional judging structure will be established based on the number of teams.

These are exciting times for our high school students. Mathematics continues to be more than learning skills and operations. Mathematics is a language that involves our daily lives. Applying the mathematical principles that one learns is the key to future success. The ability to recognize problems, formulate a mathematical model, use technology, and communicate and reflect on one's work are keys to success. Students learn confidence by tackling ill-defined problems and working together to generate a solution. Applying mathematics is a team sport!

Advisors need only be a motivator and facilitator. They should encourage students to be creative and imaginative. It is not the technique used but the process that discovers how assumptions drive the techniques that is fundamental. Let students practice to be problem solvers. Let me encourage all high school mathematics faculty to get involved, encourage your students, make mathematics relevant, and open the doors to success.

Mathematical modeling is an art and a science. Teach your students through modeling to think critically, communicate efficiently, and be confident, competent problem solvers for this new century.

CONTEST DATES:

Mark your calendars early: the next HiMCM will be held in November 2006. Registrations are due in October 2006. Teams will have a consecutive 36-hour block within the contest window to complete the problem. Teams can register via the worldwide Web at www.comap.com.

HiMCM Judges' Commentary

Problem A

Students could easily get into this problem because of the figures provided that included a simple isosceles triangle. Almost 60% of the teams chose this problem.

However, ocean floors are not flat, so the provided figure is an oversimplification. Students were expected to transition from this simple, smooth triangle to a "rough" floor (or undulating ocean floor) thus considering the slope and angles as a variable.

Overall the student papers in this problem were much better than those in the other problem. The modeling contest format was also more closely adhered to in the write-ups. However, most summaries and letters were poorly written. It seemed that they might have been written before the model, analysis, and conclusions were completed.

The letters for the science editor were poor. These letters were to be written for a scientific editor and not to a mathematics professor. They needed to be in English and quickly get to the point. The letters were supposed explain how the teams planned to model the ocean floor and why theirs was a good model. Any results or conclusions with numerical values could easily have been included. Many letters read like technical reports or were too vague to be helpful.

The judges were impressed with the creativity, quality of the analysis, and the writing by most of the teams who chose problem A. Teams appeared to use the Web to find information about SONAR and other facts bearing on the problem. It was not imperative for teams to pick a location in the ocean in order to model the bottom.

However, most judges felt even the Outstanding teams' models lacked realism about the ocean bottom and other objects that SONAR may pick up. Judges expected to see some sensitivity analysis as to how the model was affected by key variables. Judges wanted to see an attempt at validation even if it was through a few numerical calculations that were compared to possible reality.

One of the items that discriminated the better papers was careful modeling of "rough" ocean floor.

Problem B

Students struggled with problem B. It was either data overload or oversimplification that caused most of the problems identified by the judges. Many teams merely found the number of gallons in the U.S. reserve and found a "reasonable" increase and suggested this ratio increase. They assumed that the increase in the reserve would cap costs. There was no mathematical modeling used to get to either this ratio or an adequate discussion on how this would improve the problem within natural disasters. Since little modeling was used in this type of analysis, a majority of these papers were ranked lower in comparison to those that performed some adequate mathematical modeling.

Judges expected to see some discussion and analysis of crude oil and its refinement as well as a discussion of supply versus demand for oil. From here, the impact of a natural disaster could be discussed and modeled. A discussion of the variability within the model was also an element judges hoped to see. Very few papers had any elements in all these areas.

Most letters to the Energy Czar were poorly written. The letter needs to briefly restate the problem and briefly state the conclusion. It should state the purpose, facts, and results well enough to encourage reading the main body of the paper.

COMMENT ABOUT COMPUTER GENERATED SOLUTIONS:

Many papers used computer code. Computer codes used to implement mathematical expressions can be a good modeling tool. However, the judges expect to see an algorithm or flow chart from which the code was developed. Successful teams provided some explanation or guide to their algorithm(s)—a step-by-step procedure for the judges to follow. The code may only be read for the papers that reach the final rounds. The results of any simulation need to be well explained and sensitivity analysis preformed. For example, consider a flip of a fair coin. Here is the algorithm:

INPUT: Random number, number of trials

OUTPUT: Heads or tails

Step 1: Initialize all counters.

Step 2: Generate a random number between 0 and 1.

Step 3: Choose an interval for heads, like [0.0.5]. If the random number falls in this interval, the flip is a head. Otherwise the flip is a tail.

Step 4: Record the result as a head or a tail.

Step 5: Count the number of trials and increment: Count = Count + 1.

An algorithm such as this is expected in the body of the paper with the code as an appendix.

COMMENTS ABOUT GRAPHS:

Judges found many graphs that were not labeled nor explained. Many graphs did not appear to convey information used by the teams. All graphs need a verbal explanation of what the team expects the reader (judge) to gain (or see) from the graph. Legends, labels, and points of interest need to be clearly visible and understandable, even if hand written. Graphs taken from articles should be referenced and annotated.

General Comment from Judges:

Summaries: These are still, for the most part, the weakest parts of papers. These should be written after the solution is found. They should contain results and not details. They should include the "bottom-line" and the key ideas used in obtaining the solution.

They should include the particular questions addressed and their answers. Teams should consider a brief three paragraph approach: *restate the problem* in their own words, a short description of *their method to model and solution* to the problem (without giving any mathematical expressions), and the *conclusions* providing the numerical answers in context.

Restatement of the Problem: Problem restatements are important for teams to move from the general case to the specific case. They allow teams to refine their thinking to give their model uniqueness and a creative touch.

Assumptions/Justifications: Teams should list only those assumptions that are vital to the building and simplifying of their mathematical model. Assumptions should not be a reiteration of facts given in the problem description. Assumptions are variables (issues) acting or not acting on the problem. Every assumption should have a justification. We do not want to see "smoke screens" in the hopes that some items listed are what judges are looking to see. Variables chosen need to be listed with notation and be well defined.

Model: Teams need to show a ***clear link*** between the assumptions they listed and the building of their model or models.

Model Testing: Model testing is not the same as testing arithmetic. Teams need to compare results or attempt to verify (even with common sense) their results.

Teams that use a computer simulation must provide a clear step-by-step algorithm. Lots of runs and related analysis are required when using a simulation. Sensitivity analysis should be done in order to see how sensitive the simulation is to the model's key parameters.

Conclusions: This section deals with more than just results. Conclusions might also include speculations, extensions, and generalizations. This is where all scenario specific questions should be answered. Teams should ask themselves what other questions would be interesting if they had more time and then tell the judges about their ideas.

Strengths and Weaknesses: Teams should be open and honest here. What could the team have done better?

References: Teams may use references to assist in their modeling. However, they must also ***reference the source*** of their assistance. It is required of the team to show how the model was built and why it is the model chosen. Teams are reminded that only ***inanimate resources*** may be used. Teams cannot call upon real estate agents, bankers, hotel managers, or any other real person to obtain information related to the problem.

Adherence to Rules: Teams are reminded that detailed rules and regulations are posted on the COMAP site. Teams are reminded that they may use only ***inanimate sources*** to obtain information. Teams are reminded that the ***36-hour time limit is a consecutive 36 hours.***

MathModels.org

It is highly recommended that participants in this contest as well as future participants take a look at the new modeling Website, www.mathmodels.org, which has a wealth of information and resources.

Problem A Summary: Illinois Mathematics and Science Academy

Advisor: Steven Condie

Team Members: Amy Chen, Esther Shyu, Yifan Sun, Cindy Wang

We developed a model that accurately maps the ocean floor two-dimensionally through sound waves. In one scenario, using the theory that sound travels as a beam and reflects off the ocean floor at an angle corresponding to the "billiard ball effect," we developed a recursive model to estimate such values while sailing. Three ships are used in this model. One ship, an oceanography vessel, travels at 2 m/s in a line. It emits a sequence of frequencies unique to emitting angles. The other two ships are naval vessels, traveling at up to 20 m/s and constantly changing directions to detect the emitted frequencies. By dropping anchors, we find the first two values for depth, which can then be used to find the first estimated change in depth, or angle of slant. These are the only two times that the anchor is dropped. Afterwards, we use the recursive model to, knowing which frequency is detected at what distance from the oceanography vessel, estimate height and slant angle values. These values can then be used to model in two dimensions the sea floor, theoretically to infinity.

In another scenario, using the rules of longitudinal wave physics, we acknowledge that once sound reaches a surface, it reflects off the point of contact in a series of concentric circles. Thus after a sound signal is emitted at a specific frequency and angle, by knowing the speed of the boat, the speed of sound in water, and the amount of time elapsed between the sound emission and echo retrieval, we can find the water depth at a specified location.

Problem A Summary: Maggie Walker Governor's School

Advisor: John Barnes

Team Members: Simha Mummalaneni, Yuan Rao, Xun Zhou, Angela Zhu

We devised a model to map the topography of the ocean floor, given sonar apparatuses capable of emitting pulses at 1500 m/s in a 2 to 30 degree arc from oceanography vessels traveling at 2 m/s or Navy vessels traveling at 20 m/s.

Assumptions were made regarding sound propagation underwater, including homogeneity of medium and simplification of wave theory due to the concentrated downward intensities of downward pulse projections. Our approach was to not model sound using particle motion, but wave mechanics. This gave us a more accurate depiction, and one more in line with real-world applications. The challenge was addressing the complications that spherical waves cause. However, we overcame this by using narrowed beams of sound. We built our model up from a one-beam stationary sonar system to a multi-beam scanning system. Moreover, we devised a strategy for coordinating logistics to preserve detail in the generated contour map while maximizing efficiency. A wave propagation model and wavefront interaction model were created to generalize the behavior of sound waves underwater. The model was then numerically interpreted using Python Source Code generated data. The graphical data provided interpretations of the wavefront behavior when reflected from a theoretical planar ocean bottom and a sinusoidal ocean floor using various step sizes.

The primary strengths of our model are flexibility, insensitivity, and simplicity. Because our model uses two scans, one general and one detailed, it is able to analyze simple surfaces efficiently and complicated terrains effectively. Applying the general scan first makes our model insensitive to minor details in the ocean floor's surface and keeps the model simple. The main weaknesses of the model are due to its dependence on assumptions, namely the simplification of the wave theory. Also, it was unable to accurately account for certain terrain landforms of the ocean bottom. However, these problems were negligible in comparison to the flexibility, insensitivity, and simplicity of the model.

Problem A Summary:
Maggie Walker Governor's School

Advisor: John Barnes

Team Members: Jonathan Giuffrida, Daniel Lacker, Steven Li, Palmer Mebane

This model is separated into two sections; the first governs the data that a mapping vessel receives from echoes off the sea floor—the intensity and the bearing. The second describes the method that the sonar system uses to translate the data into a map of the terrain. The model is also divided into two- and three-dimensional functions and applications.

Most of the assumptions in this model are simplifications of waves and 3D surfaces. Most notably, the ocean floor is assumed to directly reflect sound waves that strike it, so that the incidence angle is equal to the reflection angle. Another assumption is that the vessel collects data on the intensity and direction of incoming sound waves.

The first part of the model is completed from an omniscient perspective—the layout of the sea floor is known. Here, the model focuses on finding the points that reflect sound waves back to the ship as it moves on the surface. These are the points that furnish the ship's crew with data to map the surface. The second part of the model describes the method for converting these data into a map. Testing is accomplished by both parts; the ocean floor is defined as a function, the omniscient part of the model predicts the data the vessel collects, and the data are translated into a map of the ocean floor consistent with the function. The equations are fully described in two and in three dimensions. They are then applied and tested only in two dimensional space because computing power is insufficient for extending calculations to a third dimension, though the basic principles behind 2-D sonar also apply in 3-D.

The model is able to find the position of any point on the surface of the ocean floor by knowing the intensity and bearing of an incoming echo. Most notably, this model is capable of calculating data points received by the sonar system for any function describing the ocean floor, so that the model simulates both the collection and the application of sonar data.

Strengths of this model are primarily based on its inclusion of the first part, which models the reflection of sound waves off the ocean floor independent of the vessel. Also, given accurate input, the model will map the sea floor with almost perfect accuracy. A limitation of this model is the infeasibility of modeling the reflection of sound waves off a 3-D surface so that they return to the moving ship, but this is due to a lack of calculating power and not to flaws or oversights in the model. The most obvious improvement would be to solve the pending equations describing the motion of sound waves reflecting off 3-D surfaces.

Problem A Summary:
Maggie Walker Governor's School

Advisor: John Barnes

Team Members: Helen Han, Yu-Sung Huang, Nick Kitten, Mike Taylor

Through copious research, we have developed a powerful model where resolution is only dependent on hardware and which allows marine geologists to map the ocean floor efficiently and accurately.

By examining the two-dimensional triangle formed by the boat's path and the paths of the sonar before and after reflection, the distance to the reflected object can be determined knowing only the variables of travel time of the boat and wave, the velocity of the boat and wave, and the angle between the path of travel of the boat and the incoming sonar. Using that angle and the angle it projects onto the horizontal plane, the depth and relative x- and y-coordinates of the target can be calculated using a spherical coordinate system. By adding those coordinates to the coordinates of the boat when it receives the wave, the absolute coordinates of a data point with depth D are known. A boat that makes several passes across an area of ocean collects these points at set time intervals. The data can be plotted on a three-dimensional graph and then interpolated to create an approximation of the ocean bottom's surface.

The total time a wave travels before detection must be known in order to calculate the distance from the boat to the reflecting point. One of our most important findings is a unique method of encoding a virtual timestamp into the frequency of the sonar. In this model, the frequency is increased from a lower limit to an upper limit at a fixed rate per second, at which point it drops to the lowest frequency and repeats, thus allowing the wave to be identified.

Using a three-dimensional expansion of the Lagrange Interpolating Polynomial, discrete data points collected in the grid system were used to estimate values at other points on the grid system. This interpolation algorithm uses a linear combination of existing points with different weights to find approximations of values on the ocean floor. In this algorithm, strict application of theory was used to ensure the interpolation is unbiased and optimal, minimizing errors in approximations.

To test validity of the model, the difference between the volume under the approximated ocean floor and the volume under the true ocean floor divided by the area of the rectangle the volume covers serves as the error comparison for the surface generated by the model. Strengths include the model's strong, unbiased interpolation algorithm, its easy application to raw data for detected sonar pulses, and its innovation in assigning a frequency timestamp to identify waves. Weaknesses include its lack of consideration for Sound Velocity Profile in water and the presence of marine life.

Problem A Summary: Shanghai Foreign Language School

Advisor: Pan Liqun

Team Members: Gu Liyi, Cheng Tao, Shen Yichen, Gu Yizun

We first consider how one ship detects and maps the ocean bottom topography. We use a kind of detecting system that is proved to be both efficient and precise. We assume that the ship emits sound pulses periodically when sailing forward, and it emits sound pulses at moments t, $t+i$, $t+2i$... respectively. The ship only receives pulses in the intervals. Furthermore, we make the assumption that through the latest technology, the arrival angle and the elapsed time from emission to receipt of the sound pulse beams are detectable. The case in which the arrival angle is not detectable is discussed in the part of More Considerations.

According to these assumptions, we analyze every piece of received information. We can determine a series of dots in a line, the depth of which is labeled as 'D'. We then can determine the depths of reflection points of received sound pulses, which provide an initial topography with many blank spaces. The blank spaces are slopes that are too steep for the sonar receptor to receive reflected pulses from. We also discuss how to determine the depths of these blind spots minutely. With this method, we can obtain the depth of *every* point of the ocean bottom. Taking advantage of the software Matlab, we can easily generate 3-D topography and hypsographic maps.

In terms of model testing and mapping precision analysis, we present two complex ocean bottom models and map the topography with our method. The simulation results are very satisfying. Moreover, we've made further considerations such as the effects of the refraction of sound pulses due to increasing density of seawater with the ocean depth, the sway of the ship because of wave and wind, etc, making our model more scientific and precise.

Problem B Summary: Hong Kong International School

Advisor: William Stork

Team Members: Aditya Balasubramanian, Dawn Ho, Dominic Wu, Rebecca Yim

The objective of our models is to stabilize oil prices in the United States, so that it is fair to both consumers and the oil industry. It benefits the oil industries in that our models allow the oil prices to increase naturally yet help the consumers by reducing fluctuations so that there will not be sudden spikes in oil prices. Natural oil price increase is possibly caused by a decrease in world supply by oil depletion, but the fluctuations can be minimized by always having a reserved supply of oil to keep supply above demand, thus eliminating sudden price increases. Model One serves as the minimum amount of money the United States must expend to meet demands for crude oil in the country so that activities can run normally. However, in this model the supply is equal to the demand of the U.S., with nothing left over in reserves. This model is susceptible to oil price fluctuations because after a significant event, gas prices usually rise. Oil industries decrease supply after significant events, which in turn increases prices, to ensure that they will still earn a profit even if refineries buy less due to fear of not making a profit from consumers as well. Other significant events such as wars destroy oil drills or refineries, decreasing supply.

To counter the effect of significant events and meet U.S. demands, Model Two projects the cost of oil the U.S. pays so that there is always an excess of oil in the Strategic Petroleum Reserve (SPR) to supply the country during significant events. The U.S. purchases extra oil to increase stock of the SPR and create a buffer. With sufficient supplies of oil, prices for consumers will not be too high since the supply would be higher than demand. This favors only the consumer needs. Although it allows increase in oil prices due to oil depletion, the model counters fluctuations by increasing the stockpile in the SPR. This model is slightly unrealistic because the amount the U.S. pays every year for oil increases quickly.

In order to account for business needs, we took the average of Models One and Two to make Model Three. This benefits consumers and businesses equally, while allowing less government intervention. In this model, consumers are less affected by increased prices, businesses benefit from significant events, and the government does not have to pay as much to stabilize the oil economy.

Model Four shows the time when the world oil supply will not be able to fulfill the demands of the world and the U.S. The world's reserves will be depleted in about 70 years. This model shows that the U.S. would need to turn to alternative resources of energy by the year 2020 otherwise the demand will not be met and normal functioning of the country will not be maintained. Since world population is increasing exponentially, world consumption of oil is likely to increase quickly. Model Four helps the U.S. plan for the year when the world demand is greater than the supply. The model uses Hubbert's peak theory where the oil supply increases slowly and then peaks at about 2010 before it decreases to complete a bell curve. This model then is a maximum for our model while model one is the minimum.

The U.S. would need to expend:

$$M = 1964.42(x - 1985)^2 - 20882.3(x - 1985) + 224460,$$

where M = money used and x = year in order to maintain minimum normal activity. However, the US should expend:

$$A = 1963(x - 1985)^2 - 20647(x - 1985) + 227177$$

Where A = Average amount of money and x = year to create an excess in the SPR as a buffer for significant events.

The maximum that the US should expend depends on the world oil supply, which will be lower than world demand by 2020. Thus oil prices will be too high for the U.S. to afford enough oil to maintain the reserve and normal functioning. That's why the US should consider switching to alternative energy by 2020.

Problem B Summary: Hong Kong International School

Advisor: William Stork

Team Members: Phillip Bak, Henry Chu, Ivan Li, Michael Suen

There are numerable factors that affect the oil market in large ways, in the case of this proposal: natural disaster, political conflict, regional importers and exporters, quality of oil, and alternate energy sources. However, the U.S.'s disregard for potential long-term influence is detrimental; the U.S. Energy Information Administration places the remaining worldwide reserve of oil at only 44.6 years worth. America's dependence on limited sources of oil and its emphasis on increased production as opposed to decreased consumption could damage the economy.

Therefore, the problem demands a new model that will attempt to act as a buffer against extreme fluctuation in oil price, while maintaining fairness between the industry and the consumer. The model will also aim to regulate America's dependence on foreign oil, maximizing domestic refinements, all the while monitoring consumption to ensure efficiency and sustainability.

For simplicity, certain educated assumptions must be made. Most importantly, oil must be assumed to never be in excess of supply for demand, meaning oil prices will increase exponentially, and the demand for oil will never drop (until none is left). Due to principles of economics, oil companies are assumed to not hold an oil reserve. Normal price fluctuation must be assumed to be negligible compared to crisis fluctuation. Other possible variables must be assumed to be negligible.

With these assumptions, our model presents three responses: (1) establishment of a National Oil Bank, (2) encouragement of oil companies to improve refinement techniques, and (3) supporting the investment of alternate energy options.

The National Oil Bank forces oil storage by suppliers, which are easily accessed in comparison to Strategic Petroleum Reserve (SPR). In addition, a tax on the annual revenue of suppliers and a sales tax on consumers will maintain the program. The National Oil Bank will aim to counter the adverse impact of crises. Encouraging oil companies to research refinement capabilities will improve their ability to process lower quality crude oil. As a result, the range of supplies will increase, encouraging the self-reliance of the U.S. oil industry. The supply of oil will be more consistent, and less prone to supply shocks. By promoting the development of alternative energy sources, the government can reduce consumption, and provide a substitute for oil. As a result, the energy industry will maintain relative stability, providing sustainability for the future.

Although we found no actual cases to support our theories, the proposals are viable and doable with our model; it addresses current problems, namely America's dependence on limited foreign oil, its attachment to sweet crude oil, and the emphasis on increased production as opposed to decreased consumption. In essence, the proposed model attempts to shift America away from an oil-dependent economy, and broaden the energy market for a more stable future economy.

Problem A Paper: Illinois Mathematics and Science Academy

Advisor: Steven Condie

Team Members: Ari Blumenthal, Jason Edes, Nathaniel Steinsultz, Scott Zager

Since we are given an approximate speed of sound in water of 1500 m/s, we can use the time between sending and receiving a signal to calculate distance traveled by a sound wave. The objective is to use this information in combination with measurements of the ship's distance traveled to calculate the ocean floor depth at any given point. A set of these measurements yields a topographical map.

INITIAL ASSUMPTIONS WITH EXPLANATIONS

1. The boat is buoyant. If a boat lacks buoyancy, it sinks. If this happened, the crew could find the depth of the ocean with a barometer and thermometer. This would oversimplify the problem.
2. There is no external sound interference. In the ocean, other objects can produce sounds that might interfere with our calculations. We did not include these because doing so adds uncontrollable variables
3. The speed of sound in water is a constant 1500 m/s. This speed is dependent on things such as temperature, pressure, and salinity. We do not have the tools to measure them, and therefore cannot include them in our model.
4. Sea level is constant. We measure depth as the distance between ocean bottom and sea level. However, sea level changes over time; so unless we map the entire ocean, we would get inaccurate values. We cannot calculate sea level without measurements, which we don't have.
5. The ship is equipped for single-beam mapping. Multi-beam mapping oversimplifies the problem by using several beams of differing frequencies to map a whole swath of ocean floor. Because there are many data points for a single section of floor, with each beam distinguishable by frequency, we could make a more accurate map by comparing error ranges for each of the beams.
6. The boat does not bob up-and-down or side-to-side. If it does, the arc of a pulse is erratic, skewing our results with a variable we are not given.
7. The equipment can accurately measure the boat's changes in position and time— for instance, a GPS system and a digital clock. Our model relies on accurate inputs of distance and time.
8. The Earth is locally flat. Accounting for curvature requires non-Euclidean geometry, since the Gaussian curvature of a sphere is not 0 and the Earth is nearly spherical. Thus, none of our geometry would work. Plus, because the Earth is large, its curvature has little effect on our results, on the order of 10^{-8} m.

INITIAL EFFORTS

First we realized that we could treat the pulse as an infinite number of beams expanding from a single point (where the boat catches only one of these beams). We then thought that we could model ocean depth by using the distance traveled above the surface to catch an individual beam of a pulse arc, as shown in Figure 1.

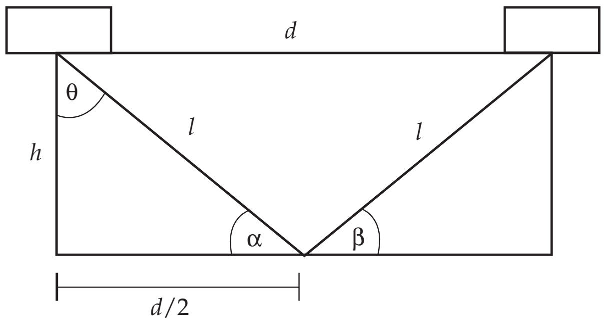


Figure 1

We can then use the equation

$$l = \frac{1500t}{2} \tag{1}$$

where l is distance traveled by a pulse until it hits the floor and t is the time between sending and receiving signals. We are given that $\alpha = \beta$. By the Pythagorean theorem:

$$h^2 + \left(\frac{d}{2}\right)^2 = l^2 \tag{2}$$

This assumes a flat floor. With an irregular floor, the received beams may not be those that bounced halfway along the ship's distance traveled d. We cannot assume away the irregularities because the problem is too simple: one could send a pulse without moving and measure the time t for it to return. Since the only piece of arc one then receives is a beam that travels straight down, $h = \frac{1500t}{2}$ would give depth at every point.

REVISED MODEL

We were discouraged by the floor's irregularity until we realized that we could approximate the floor with an acceptable range of error. We devised a new model that gives two estimates, one too deep and one too shallow, and averaged them. We began by drawing a figure with all the variables (Figure 2).

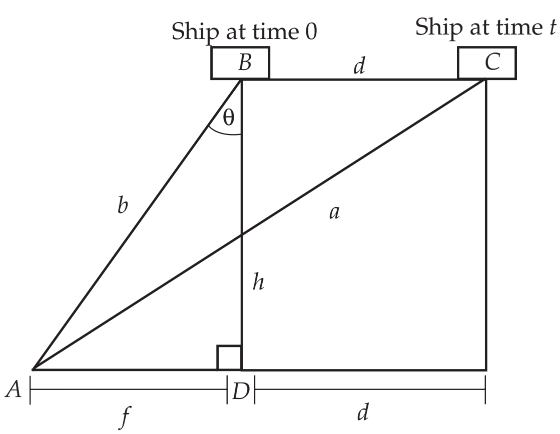


Figure 2

In Figure 2, the ship starts at B at time 0, where it emits a sonar pulse. The ship travels to C, d meters away. When the ship reaches C, the pulse has reached A, reflected off an angled portion of floor, and traveled to C, a total of $a + b$ meters. Since $\angle ADB$ is a right angle, $f = b \sin \theta$ and $h = b \cos \theta$..

The sonar pulse is emitted at a velocity of 1500 m/s, and we know that $d_s = vt$, where d_s is distance traveled by the pulse, v is velocity of the pulse, and t is the time for the pulse to go from B to A to C. From these we find:

$$a + b = 1500t, \text{ or } a = 1500t - b \tag{3}$$

These calculations yield a revised image in which each variable can be expressed in terms of b, d, t, and θ (see Figure 3).

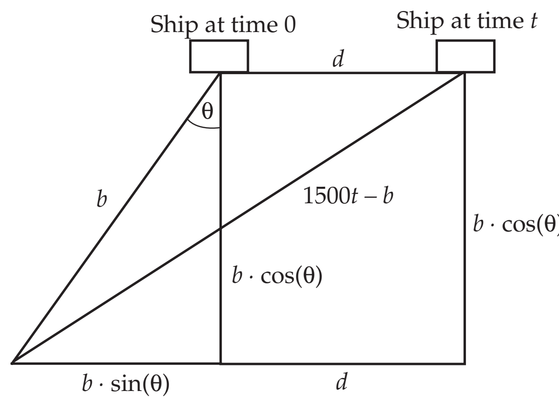


Figure 3

By using the Pythagorean theorem and the equations just stated, we found b when given d, t, and θ, all of which can be measured on the boat.

$$(f + d)^2 + h^2 = a^2 \tag{4}$$

$$(b \sin \theta + d)^2 + (b \cos \theta)^2 = (1500t - b)^2 \tag{5}$$

We then solved equation (5) for b to get

$$b = \frac{1500^2 t^2 - d^2}{2d \sin \theta + 3000t} \tag{6}$$

We then solved for h:

$$h = b \cos \theta \tag{7}$$

It must be cautioned that the same time recorded and distance traveled could result from a host of sound paths, giving many values for height (see Figure 4).

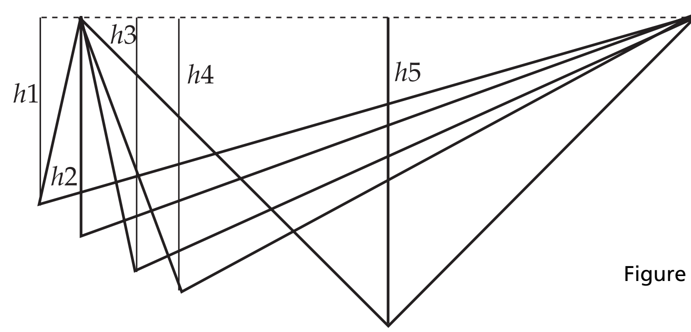


Figure 4

The difficulty here is that we cannot be sure how to interpret a received signal, but we could make educated guesses. Say we emit a pulse 30° wide. The received beam could have bounced off a flat part of the floor and formed an isosceles triangle, as in Figure 1. But, it could have hit a slanted portion of floor and been skewed, as in Figure 2.

Maximum depth will always be at the altitude to the base of the isosceles triangle whose two congruent sides are the paths of sonar beams reflected off a point with slope 0. The sum of these two sides is the distance the beam traveled, as can be calculated from the time between the beam's emission and receipt of its reflection.

The alternative is the skewed triangles in Figure 4. To skew the triangle, you shorten one of the sides. Since the sum of the two sides is constant, they must rotate upwards, again as in Figure 4.

If a ship at *B* (Figure 5) fires a sonar beam, then the left edge of the expanding wave can take the path of ΔABC and arrive at the same point, at the same time, as the other edge of the wave that travels *BDC*. Clearly, the altitude (which is a depth estimate) is smaller in the skewed triangle than in the isosceles triangle. However, we must make sure that the base angles of the isosceles triangle assumed to be received by the ship is within the range of the beam width (2° to 30°).

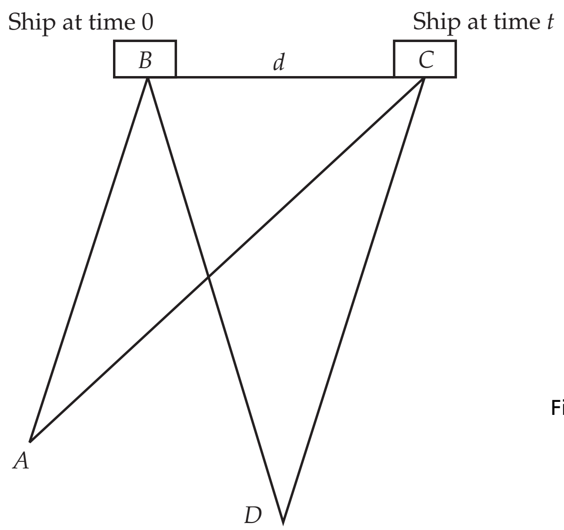


Figure 5

To receive the reflection that takes an isosceles triangle path, the boat must travel distance *d* (Figure 5) in the time it takes the beam to go from *B* to *D* to *C*. To make sure that no matter what, the largest isosceles triangle possible within the arc of the beam will reach the surface in front of the boat, we minimize arc width at 2°.

We can calculate the distance (*D*) the beam travels along the isosceles triangle, the time (*t*) for the beam to be received after it is emitted, and the horizontal distance the beam travels (*x*). Therefore, we can calculate a speed (*v*) that the boat should travel. These calculations are shown below (*d* is ocean depth, which need not be known).

$$D = \frac{2d}{\cos\left(\frac{2^\circ}{2}\right)} = \frac{2d}{\cos(1^\circ)} = 1500t \tag{8}$$

$$t = \frac{2d}{1500\cos(1)} \tag{9}$$

$$\frac{L}{6} = d\tan(2^\circ/2), \text{ or } L = 2d\tan(1^\circ) \tag{10}$$

$$v = \frac{L}{t} = \frac{2d}{1500\cos(1^\circ)} = 1500\tan(1^\circ)\cos(1^\circ) \approx 26.179 \tag{11}$$

Since the required velocity is more than 20m/s, the boat cannot receive the beam. Thus, any isosceles triangle that the boat receives must be within the arc of the beam since the boat can't move fast enough to receive the beam from the very edge of the arc.

IMPORTANT CAUTION

Our model estimates average depth over an interval that can vary with the input data. If all the values produced by our algorithm were compiled into a map, they would appear as a series of horizontal lines at varying depths. However well this model works in rendering ocean depth, it yields little information about the slope of the ocean floor.

EXAMPLES WITH VESSEL SPEED *V* AND TIME *T* BETWEEN PULSE EMISSION AND RECEPTION

Scenario 1: $v = 20$ m/s and $t = 3.5$ sec.

First we find minimum depth for $\theta = 1°$. We know that

$$b = \frac{1500^2 \cdot 3.5^2 - (20 \cdot 3.5)^2}{2(20\cdot 3.5)\sin 1^\circ + 3000 \cdot 3.5} \approx 2624.472$$

By substituting for *b* in equation (7), we find $h = 2624.472\cos(1°) \approx 2624.073$.

Maximum depth can be found using equations (1) and (2): $l = \frac{1500 \cdot 3.5}{2} = 2625$.

We can substitute *l*'s value into equation (2) to get a maximum of about 2624.767 for *h*.

We average the minimum and maximum values of *h* to get 2624.42 m.

This depth estimate is for an area of the floor off of which the beams reflected. This area can be calculated by forming a right triangle with top small angle of half the arc width angle (in this case, 1°) and vertical side of *d*, which is the depth estimate. The bottom horizontal side is a radius of the circle the beam covers, calculated from $\tan(1°) = \frac{r}{2624.42}$ to get $r \approx 45.81$.

Therefore, over an area of radius of 45.81m, the average depth is about 2624.42m.

Scenario 2: Next we calculate an average depth when $\theta = 15°$. We know that $b = \frac{1500^2 \cdot 3.5^2 - (20\cdot 3.5)^2}{2(20\cdot 3.5)\sin 15^\circ + 3000\cdot 3.5} \approx 2623.628$.

Substituting this value of *b* into equation (7), we obtain $h \approx 2534.230$.

Then we use the same isosceles triangle calculations to get 2624.767 m for the maximum. Again, we average the minimum and maximum depths to get 2579.5m over an area with radius of $r \approx 691.175$ m, calculated from $\tan(15°) = \frac{r}{2579.5}$.

PING INTERVAL

The ship should send a ping when it receives one in order to reduce the danger of receiving multiple signals at one time, potentially out of order. This is because every beam reaches the surface at approximately the same time, so that when we receive a signal, the rest of the arc should have cleared, and we can safely fire another pulse.

WHAT BEAM ANGLE TO USE

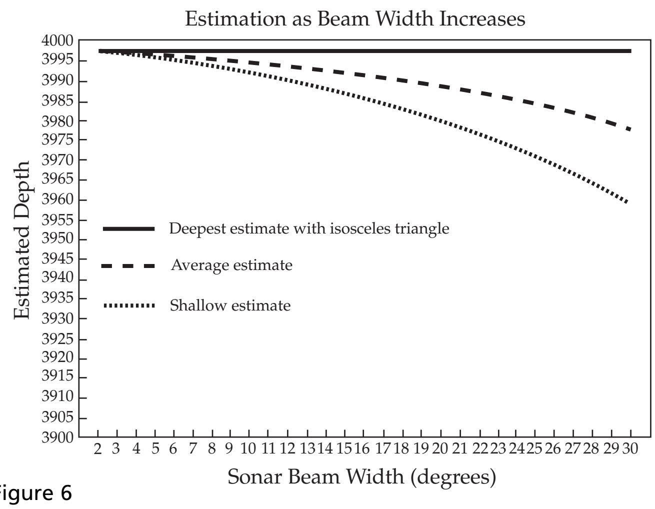


Figure 6

Figure 6 was generated from a Peri script that used our algorithm to calculate depth estimates from boat velocity of 20 m/s and time between beam emission and reception of 5.33 seconds. The beam arc width was incremented by 1° from 2° to 30°.

Figure 6 shows that, as beam width increases, the difference between shallow (scalene triangle) estimate and deep (isosceles triangle) estimate grows. Thus, error margin grows as beam width increases. A 2° arc should be used to get an accurate mapping.

This raises a question: Why use a 30° beam? It might seem that a boat could go faster with the 30° beam, as a pulse covers more area. However, the boat cannot go fast enough to take advantage; it could travel at maximum speed while firing a 2° beam and still get good data. This was shown in equation (11) when we calculated a boat speed needed to catch the widest isosceles triangle beam of a 2° arc: 26.279 m/s. For the sake of quality, the boat should travel slowly so that it can send multiple beams within a 1 km range, instead of moving quickly and collecting a few data points within that range.

STRENGTHS OF OUR MODEL

- Our model allows us to calculate the relatively small margin of error for our approximation, which is valuable in real-world measurements.
- Since our ship fires a pulse upon receiving the echo from its last emission, the estimate applies to the ocean floor at every point over which the ship passes.
- The equipment is common: clock, GPS system, and single-beam sonar system (cheaper and more common than its multi-beam counterpart).
- We account for ocean floor irregularity. Although this introduced error, we felt that it was worthwhile because it added drastically to our model's applicability.

WEAKNESSES OF OUR MODEL

- Our assumptions are many. Some have minimal effect, but some are critical.
- Without a boat and the required equipment, we have no way to test our model.
- Single-beam sonar is useless for a vessel with a multi-beam system because the latter can map the ocean floor with greater accuracy and speed.
- We assume that pressure, salinity, and temperature do not affect sonar speed.
- The model yields approximate depths, so that small juts and valleys are averaged out, and not represented in our final data.

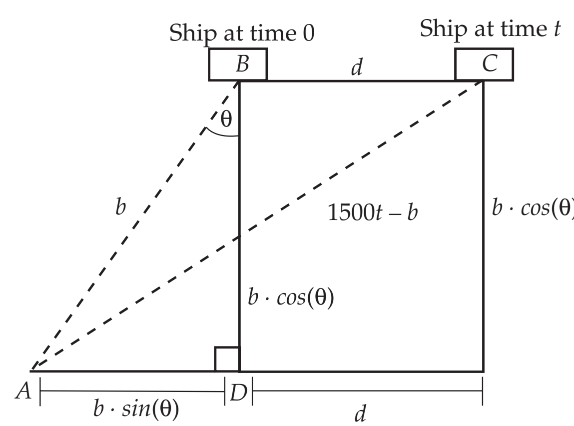
ENVIRONMENTAL CONCERNS

For humane reasons, we feel that the sonar beams should be kept as small as possible because the larger the arc of a beam, the more likely it is to encounter animals such as dolphins, porpoises, and sea lions that use sonar communication. According to "Cumulative Sperm Whale Bone Damage and the Bends," such sound pulses can distress whales so much that they surface too quickly, causing decompression sickness.

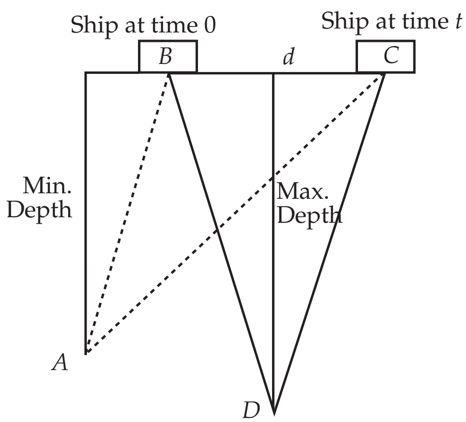
LETTER TO NEWSPAPER SCIENCE EDITOR

We developed a way to model the topographic features of the ocean floor with small margin of error. Our algorithm uses single-beam sonar to estimate depth based on the minimum and maximum possible depths. We realized that we could treat the expanding pulse generated by a sonar system as an infinite number of individual beams originating from a single point, only one of which is received at the surface. The task then became to determine the path traveled by a beam based only on the time and place of emission and reception.

A single beam can travel the same distance but reach different depths. In order to find the minimum possible depth, we calculated the distance traveled by the beam, and found:
$h = b \cdot \cos\theta = \dfrac{1500^2 \cdot t^2 - d^2}{2 \cdot d \sin\theta + 3000 \cdot t} \cdot \cos\theta.$



Model to determine Minimum Depth



Comparison of Minimum Depth and Maximum Depth

The maximum depth that fits the received reflection is the case where the beam travels along an isosceles triangle and bounces off the bottom as if the bottom were flat. This makes the question simple because you can create a right triangle where the hypotenuse is half the total distance a beam travels (which can be calculated from the time difference between emission and reception). From this you can calculate the maximum possible depth by $\left(\frac{1500t}{2}\right)^2 + \left(\frac{d}{2}\right)^2 = t^2$.

We calculate the minimum (the scalene triangle on the left) and the maximum (the isosceles triangle) values so that we can average the two and thereby account for any case of the beam path, since we can't tell whether the bottom is sloped or flat. For a high quality mapping, the ship should move slowly so that it can send and collect more data points per kilometer. The only concern with speed is that if the ship moves too quickly it may outrun the sonar beam. However, it can be shown that to outrun the beam, the ship would have to move faster than 20 meters per second, which is the fastest our ships can travel. Should a high quality mapping be required, the ship may use a small beam width and move slowly. If the mapping must be done quickly, the ship must move faster, but we then lose quality. Therefore, our model is flexible.

RESOURCES USED

1. Geometer's Sketchpad. Computer Software. http://www.keypress.com/sketchpad/index.php.
2. L-3 Communications Seabeam Instruments. 2000. *Multibeam Sonar Theory of Operation*. East Walpole, MA: L-3 Communications Seabeam Instruments.
3. Misner, Charles W., Kip S. Thome, John Archibald Wheeler. 1970. *Gravitation*. New York: W. H. Freeman and Company.
4. Moore, Michael J., and Early, Greg A. 2004. Cumulative Sperm Whale Bone Damage and the Bends. *Science Magazine*, 24 December, 2215.

Problem B Paper: Dubuque Hempstead High School

Advisor: Karen Weires

Team Members: Larkin Accinelli, Jamin Hitchcock, Stephen Longfield, Nick Ruhter

1.0 RESTATEMENT

We set out to determine to what extent economic and environmental variables impact the supply and price of oil and gasoline. After assessing these factors, we created a model to analyze their effects and provide means of controlling them in a way that is fair to consumers and business.

1.1 VARIABLES

We selected four main variables.

- US Refinery Production: The US refines over 17 million barrels of crude oil daily to make about 314,160,000 gallons of gasoline. US refineries make 98% of US gasoline. Temporary refinery closings strain gasoline supplies and cause price increases.
- Exchange rates: With the majority of US crude oil purchased from foreign markets, the strength of the US dollar is crucial to keeping energy prices low and constant. An unfavorable exchange rate can radically alter prices.
- Supply of Crude Oil: If international disputes, natural disasters, or environmental reasons curtail supply, it could dramatically affect prices.
- Long-run Demand Schedule: As population and energy demand grow, the Long-run Demand Curve shifts upward, stressing supplies because of scarcity.

1.2 CONSTANTS

We held several factors constant to provide a legitimate method of testing the variables.

- Tariffs: It is difficult to predict future tariffs. Moreover, the creation of NAFTA has lowered the number of tariffs that apply to this problem.
- Short-term Demand: After looking at demand over several years, we found that, despite small fluctuations, overall demand is fairly stable.
- Driving Habits: We chose to keep these constant for comparison purposes. We feel that overall gas consumption is constant, with vacation times equating to slower market periods. Also, holding driving habits constant makes comparing natural disasters and maintenance closures more accurate.

1.3 ASSUMPTIONS

Our assumptions fall into two categories.

1.3A INTERNATIONAL AND TRADE ASSUMPTIONS

- Free trade: Due to NAFTA and Canadian oil importation, we assume free trade.
- Civil Unrest: For the sake of simplicity and continuity, we assume no civil unrest.

- Importation of Crude oil: We assume that all domestic crude oil goes to US refineries. This gives us a consistent number of barrels for crude oil importation.

1.3 B DOMESTIC ASSUMPTIONS

- New Technology: Due to the speculative nature of this variable, we factored no new technology into our model.
- Depressions and Hyperinflation: In the case of either, the importance of oil is negligible compared to other pressing economic matters, and demand drops. Therefore, we chose to eliminate the chance of either occurring.
- Government Stability: Since the future of government is speculative, we eliminated the chance of radical changes in government towards the oil industry.
- Lack of Collusion: We assume no collusion in the oil industry. Collusion makes it difficult to determine market prices based on supply and demand.

2.0 METHOD

We evaluated the four variables independently. When analyzing each, we employed the economic doctrine of *ceteris paribus* by holding the other three constant.

For each variable, we developed a plan for government and the oil industry to follow in order to ensure a stable market price. We decided that a constant price range—as opposed to drastic price changes—is beneficial to consumers and businesses. We then demonstrated that our method decreases price fluctuations.

2.1 FAIR EQUILIBRIUM PRICE

Our research found that the price of a barrel of oil is about \$50. About 60% of the price of gas accounts for crude oil, and there are 42 gallons in a barrel. From $\frac{50}{42\times 0.6} = 1.98$, we get a gas price for firms to break even in terms of crude oil. By raising the price to \$2, we ensure a small profit for the firms (which, of course, have profits from other parts of their business). Therefore, the price is fair for consumers and refineries.

2.2 REFINERY PRODUCTION

Refinery production is influenced by natural disasters and maintenance closings. We assumed that at any given time, a constant number of refineries are closed for maintenance. We defined the number of active refineries as 149 when no disasters occur. We then averaged the production capacities of the 149 refineries. For simplicity, we assumed that each refinery is independent of the others.

For analysis, we built a model of 150 refineries, each producing 300 barrels of refined oil a week. In the sixth week, we had a hurricane hit 15 refineries, which produced nothing for two weeks. In the third week after the hurricane (week nine), these 15 were at half production; by week ten all were at full production. We then found the total supply of refined oil for the first 11 weeks. From this, we found the change in price in each week from the equilibrium price (price at full supply and normal demand).

To better control price, we developed a model for government and the oil industry. First, the government builds a reserve by buying a certain amount during weeks one and two. This increases gasoline demand and price slightly. After these two weeks, the government trades the gasoline in its reserves for new gasoline from the refineries, maintaining reserves while allowing demand to fall back to normal. (The trade prevents deterioration over time.) When the hurricane hits (week six), the firms that were closed receive equal shares of gasoline from government reserves during weeks six and seven. This marginally decreases the change in price due to the hurricane. After these two weeks, reserves are depleted, yet the refineries that were hit are at half production. In week nine, all refineries are at full production. At this time, the government begins to build up their reserves, again raising price slightly. The cycle then begins again.

After some calculation, we found that without reserves, the equilibrium price of \$2.00 would rise by \$0.22 when the hurricane hits and then decline to \$2.11 for a week. However, with reserves, the price would rise by \$0.11 for the first two weeks and fall to \$2.00 after that. When the hurricane hits, the price would rise to \$2.11 for three weeks.

In comparing the two situations, we discovered that the one with reserves had less drastic price changes than the one without reserves. We decided that it is better to use a system of reserves to protect against sudden and drastic changes in production.

2.3 EXCHANGE RATES

Exchange rates can alter gasoline price by changing the price of imported oil. For example, if the dollar depreciates, refineries can purchase less crude oil with their money. In order to keep the supply of gasoline constant, the refineries must pay more, increasing costs and thus increasing the price of gasoline.

We considered two ways to control exchange rates:

- Use of monetary policy to offset whatever causes a change in exchange rates, and then use of fiscal policy to offset the monetary policy.
- Use of the reserves built up in the manner described in Section 2.1.

2.3A MONETARY AND FISCAL POLICY

If the dollar depreciates, the government can use monetary policy to offset this. First, the Fed would raise interest rates, driving up foreign investment. This would increase demand for the dollar in the international market, causing the dollar to appreciate and offsetting the initial decrease in its value. The same holds if the dollar appreciates.

2.3B PROBLEMS

We had to resolve certain problems with this plan.

1. The monetary policy may contradict goals of full employment and low inflation. Therefore, the government must enact fiscal policy opposite to the monetary policy. For example, the monetary policy above is a tight money policy, for higher interest rates lower domestic investment and decrease

aggregate demand (total demand of the economy), causing a recession. To offset this, the government can employ an expansionary policy of either higher government spending or lower taxes. These policies would increase aggregate demand back to a level of full employment. The same applies to lower interest rates and inflation. Here, we make two assumptions to keep the model workable. One, we assume that government policies have instantaneous effect. Two, we hold all other things equal so that nothing else affects the exchange rate. (See **Figure 1.**)

2. We also assume that the economy is full employment GDP unless monetary and fiscal policies alter it. Therefore, there is no need to employ the policies for any reason other than to alter exchange rates.

2.3C USE RESERVES

Another option is for government to use reserves that were built up as described in Section 2.1. With no disasters, the government can use reserves to prevent a price increase. In this scenario, refineries buy gasoline from the government instead of importing it. Here, we assume that the exchange rate is constant in order to simplify oil distribution. Though a ban on importing may decrease demand for the Canadian dollar (causing the US dollar to appreciate), we found that overall effect on trade is negligible.

2.4 SUPPLY OF CRUDE OIL

If a primary source stops supplying the US with oil, each refinery loses a certain amount of oil. As shown by our equations, a loss in a major supplier drastically effects US production. We determined that if the loss is no more than 11% of total US production for one and a half weeks, the new, smaller reserves can be used to control price and replenish supply. Any higher percentage, or longer time period, however, necessitates tapping into the larger government reserves.

2.5 LONG-RUN DEMAND

We accounted for the last variable by deciding that over time, new technology that allows the refineries to produce more gas from less oil, or to produce more gas in less time, will emerge. If it does not, the refineries can compensate by buying more refined oil from other countries. Both of these increase long-run supply and long-run demand, keeping price steady. (See **Figure 2.**)

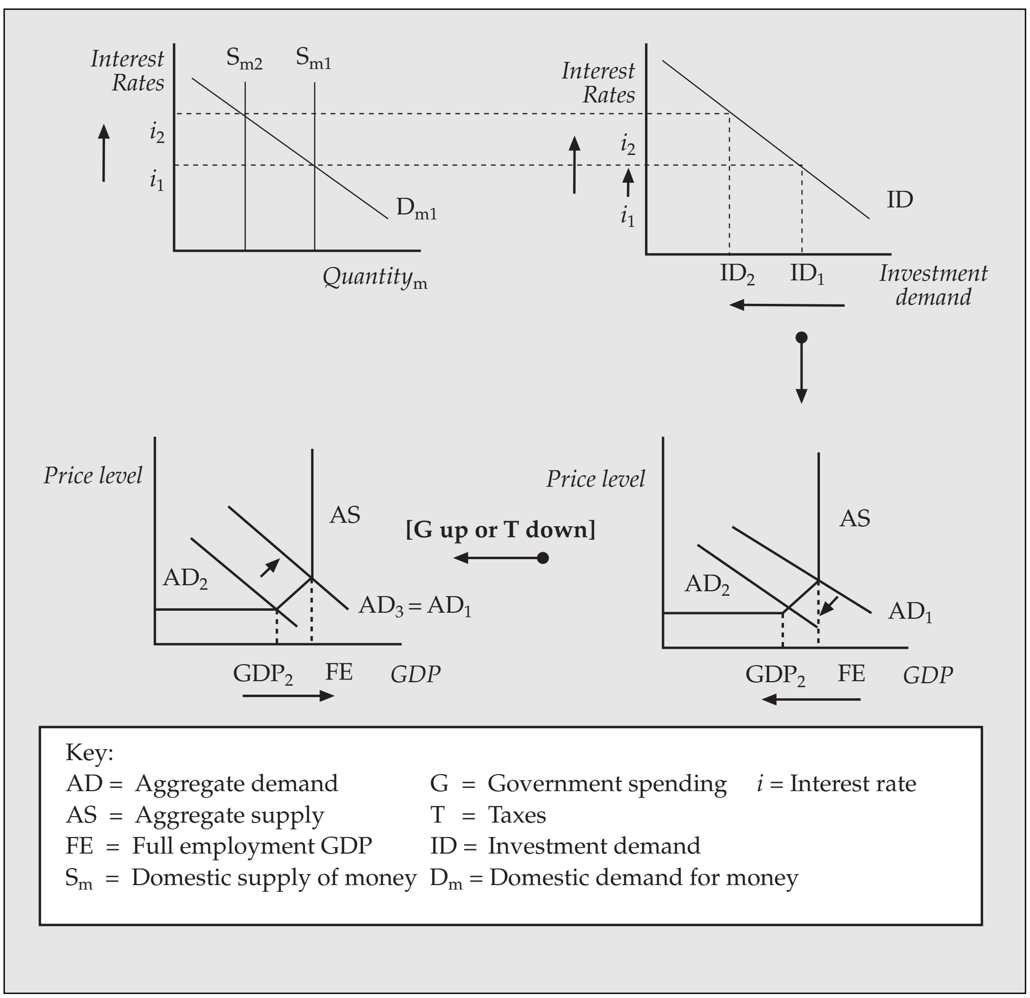


Figure 1.

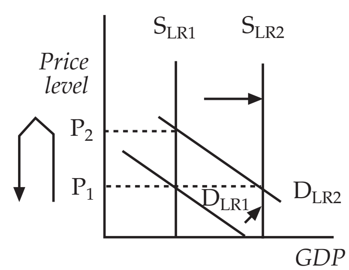


Figure 2.

3.0 THE MODEL

From the above analyses, we found that the most important factor in our model is government reserves. We applied the model to actual US data. From research, we learned that there are about 149 US refineries running at full capacity during a non-closure period. The total amount of oil refined in a week was averaged to get 800758.8 barrels/week/refinery. The reserves per refinery were collected over a 5-week period of 26971.1 to obtain a total reserve per refinery of 134,355.5. When a refinery closes, the government taps the refineries' reserves. We split the model into four time periods for the refinery production variable. Phase 1 is stockpiling of short-term reserves for five weeks. Phase 2 is a general market period, with maintenance of reserves and full production. Phase 3 occurs if a refinery shuts down for w weeks, and the government taps short-term reserves. Phase 4 is a rebuilding period. The closed refineries are at half production and all others are at full production with no use of reserves. This phase lasts 5–w weeks. In our model, not all reserves are always depleted. In these instances, the following Phase 1 lasts w weeks to replenish the supply. Therefore, the government should build up a reserve for such a situation. If a situation in excess of the planned scenario occurs, the government can tap into its long- term reserve.

While production is normal, if exchange rates change, the government can either use reserves or employ fiscal and monetary policy to swing exchange rates back to the base rate. When using reserves, the government replaces the imported oil that the companies have stopped. This keeps gasoline price constant until the exchange rate returns to normal. If the other route is chosen, government should alter interest rates to maintain a normal exchange rate and then use fiscal policy to offset the monetary policy.

If a country decreases or cuts off oil to the US, the government can do one of two things. If the supply is less than 10% of the total supply for one and a half or fewer weeks, the short-term reserves can be used. Any longer or larger proportion necessitates the large government reserves being used. Of course, at this time, exchange rates are constant and no refineries are closed.

The government and the oil industry must watch for a growing demand without any new technology. If this occurs, industry must buy foreign refined oil to build up supply. Otherwise, new technology will increase supply enough to offset increasing demand.

4.0 COMPUTER PROGRAM

To compare the reserve plan to the current system, we developed a program to simulate their effects on prices. The program models random supply shortages. The probability of refinery problems is based on historical data.

Each week there is no shortage the government stores gas in the reserves. This amount is found by multiplying the average amount to be supplied to the market by the probability of a shortage. If there is a shortage, the difference between the normal supply and the amount available is taken from reserves and sent to the market. If prolonged shortages use up reserves, the supply is limited to the production of the working refineries until government can rebuild its reserves. The amount of price change is found by dividing the potential production price times the equilibrium price by actual production. The price for the no-reserve model is calculated the same way.

The price is calculated over a period of 1000 weeks to create enough data to get a good average price as well as a good idea of the maximum and minimum prices that can occur. These values were used to compare price stability and actual cost to consumers.

After running the program we found that the reserve model kept prices more stable than the model without reserves. The reserve model also saved consumers money.

5.0 FINAL ANALYSIS

We realize that our model cannot satisfy everyone because, after all, economics is finding the best configuration of limited materials to obtain maximum satisfaction. Under our model both producer and consumer wants are satisfied better than if the market were left alone.

5.1 STRENGTHS

- Since we tested four variables and based decisions on economic principles, the data accurately represent the US oil industry.
- The model shows the benefits of a controlled reserve system. We tested the model with the combination of economic theory and computer programming. In both cases the results favor the reserve system.
- Besides testing the four variables, we have shown multiple outcomes for each scenario and diagramed methods of controlling the price of gas.

5.2 WEAKNESSES

- The model is based on economic theory that isolates variables. It is based on ideal conditions and *ceterus peribus*, which never occur in the real world. We felt that, without these theories, the problem is too speculative.
- The model does not account for situations when more than 10% of US oil refining is shut down or during a depression or hyperinflation. We feel that these are unlikely to occur and would only skew results.
- The model does not address the possibility of collusion. In real oligopolies, businesses collude and charge prices that our model cannot predict.

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7.0 EQUATION VARIABLES

n = Number of oil refineries in the US

$\overline{B}$ = Barrels/week/refinery

x = Probability that four refineries are shutdown for 5 weeks each

w = Number of weeks x is shut down, $0 < w < 5$

B_s = Number of barrels per refinery saved per week

P = Price at equilibrium

P_{BU} = Price per gallon of gas as the reserve is building

P_M = Price per gallon of gas when the reserve is filled and turn-over is smooth

P_{d_1} = Price per gallon of gas with x refineries at 0% production and reserve being used

P_{d_2} = Price per gallon of gas with x refineries at 50% production and reserve not used

P_x = Price of a gallon of gas at a given supply

S_x = Supply of gasoline on the market

P_E = Equilibrium price of gasoline

8.0 EQUATIONS DEALING WITH RESERVES

$P_{BU} = \dfrac{Pn\overline{B}}{n\left(\overline{B} - \dfrac{B_S}{5}\right)}$ for w weeks

$P_M = \dfrac{Pn\overline{B}}{n\overline{B}}$, $\therefore P_M = P$ for 0–42 weeks

$P_{d_1} = \dfrac{Pn\overline{B}}{(n-1)\overline{B} + \dfrac{nB_S}{5}}$ for w weeks, $0 \le w \le 5$ if 1 refinery shuts down

$P_{d_2} = \dfrac{Pn\overline{B}}{(n-1)\overline{B} + \dfrac{xnB_S}{5}}$ for 5–w weeks, $0 \le w \le 5$

$P_x = \dfrac{\overline{BnP_E}}{S_X}$

$w\,\overline{B} = \dfrac{NB_S}{5}$ x 5

$n = 149$

$\overline{B} = 800758.8$

$x = 20/52$

$l = 5$

P = \$2.00/gallon of gas

$P_{BU} = \dfrac{200(149)800758.8}{149\left(800758.8 - \dfrac{26871.1}{5}\right)}$ = \$2.01

$P_M = \dfrac{200(149)800758.8}{149(800758.8)}$ = \$2.00

$P_{d_1} = \dfrac{200(149)800758.8}{(149-6)800758.8 + \dfrac{149(26871.1)}{5}}$ = \$2.07

$P_{d_1} = \dfrac{200(149)800758.8}{(149-1)800758.8 + \dfrac{1(800758.8)}{2}}$ = \$2.01

9.0 EQUATIONS DEALING WITHOUT RESERVES

$P_{BU} = \dfrac{Pn\overline{B}}{n\overline{B}} = P_E$ when the government is building up reserves

$P_M = \dfrac{Pn\overline{B}}{n\overline{B}} = P_E$ when reserves are being maintained

$P_{d_1} = \dfrac{Pn\overline{B}}{(n-1)\overline{B}}$ when some refineries are shut down

$P_{d_2} = \dfrac{Pn\overline{B}}{(n-1)\overline{B} + \dfrac{x\overline{B}}{2}}$ when afflicted refineries are at half production

P_{BU} = \$2.00

P_M = \$2.00

$P_{d_1} = \dfrac{2(149)800758.8}{(149-6)800758.8}$ = \$2.08

10.0 EQUATIONS DEALING WITH CRUDE OIL SUPPLY

y = Number of barrels not supplied per week per refinery

$(n)B_S = lyn, \therefore B_S = ln$

$\frac{ny}{7 \text{ days}} = 1.9$ million barrels per day

$y = 89261.7$

$P_{BU} = \frac{Pn\bar{B}}{n\left(\bar{B} - \frac{B_S}{5}\right)}$ when reserves are being built

$P_M = P_E$ when reserves are being maintained

$P_D = \frac{Pn\bar{B}}{n(\bar{B} - y)}$ when oil supply is being restricted

11.0 LETTER TO THE PRESIDENT'S ENERGY ADVISOR

We are pleased to bring to your attention a mathematical model that we believe will improve the United States of America's fluctuating oil prices. Our model creates an oil reserve to be used to control severe price swings that result from natural disasters, maintenance closing of refineries, and depreciation of the dollar.

The proposed model and computer program take into account the four main components of price deviations: variations in refinery production, exchange rates, supply of crude oil, and long-run demand. By slowly building up a reserve supply, this one easily drawn upon, and carefully using changes in interest rates when necessary, the federal government can control oil retail prices in a way that is fair to both consumer and producer.

After modeling an economy similar to that of the US for 1000 weeks, we found that under our model gas prices varied by only \$0.03. Under an economy similar to the current US situation prices varied by over \$0.14.

Thenceforth, we recommend that you reevaluate the energy crisis in the US and seriously consider this report.