HIGH SCHOOL MATHEMATICAL CONTEST IN MODELING

OUTSTANDING PAPERS

HiMCM

The contest offers students the opportunity to compete in a team setting using applied mathematics in the solving of real-world problems.

November 2009

Additional support provided by the National Council of Teachers of Mathematics (NCTM),
the Mathematical Association of America (MAA),
and the Institute for Operations Research and Management Sciences (INFORMS).

Editor's Comments

This is our twelfth HiMCM special issue. Since space does not permit printing all eight National Outstanding papers, this special section includes the summaries from six papers and abridged versions of two. We emphasize that the selection of these two does not imply that they are superior to the other Outstanding papers. We also wish to emphasize that the papers were not written with publication in mind. Given the thirty-six hours that teams have to work on the problems and prepare their papers, it is remarkable how much they accomplished and how well written many of the papers are. The unabridged papers from all National and Regional Outstanding teams are on the 2009 HiMCM CD-ROM, which is available from COMAP. ❑

Contest Director's Article

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The High School Mathematical Contest in Modeling (HiMCM) completed its twelfth year in excellent fashion. The mathematical and modeling ability of students, and faculty advisors, is very evident in the professional submissions and work being done. The contest is still moving ahead, growing with a positive first derivative, and consistent with our positive experiences from previous HiMCM contests.

This year the contest consisted of 277 teams from 199 schools in the United States and 78 international or foreign schools. These institutions are from seventeen states and seven different countries. This year, we again charged a registration fee of $75.

The teams accomplished the vision of our founders by providing unique and creative mathematical solutions to complex open-ended real-world problems. This year the students had a choice of two problems.

Problem A: Water, Water Everywhere

Fresh water is the limiting constraint for development in much of the United States. Devise an effective, feasible, and cost-efficient national water strategy for 2010 to meet the projected needs of the United States in 2025. In particular, address storage and movement, de-salinization, and conservation as some of the possible components of your strategy. Consider economic, physical, cultural, and environmental effects. Provide a position paper for the United States Congress outlining your approach, its costs, and why it is the best choice for the nation.

Problem B: Tsunami ("Wipe Out!")

Recent events have reminded us about the devastating effects of distant or underwater earthquakes. Build a model that compares the devastation of various-sized earthquakes and their resulting tsunamis on the following cities: San Francisco, CA; Hilo, HI; New Orleans, LA; Charleston, SC; New York, NY; Boston, MA; and any city of your choice. Prepare an article for the local newspaper that explains what you discovered in your model about one of these cities.

Commendation: All students and advisors are congratulated for their varied and creative mathematical efforts. Of the 277 teams, 125 submitted solutions to Problem A and 152 to Problem B. The thirty-six continuous hours to work on the problem provided for quality papers; teams are commended for the overall quality of their work.

Many teams had female members. There were 438 female participants on the 277 teams. There were 1121 total participants, so females made up over 39% of the total participation, showing this competition is for all students. (This percent is almost triple the percent of woman in other math competitions.) There was at least one female on most of the teams and 18% of the teams were all female (50 teams).

Teams again proved to the judges that they had "fun" with their chosen problems, demonstrating research initiative and creativity in their solutions. This year's effort was a success!

Judging: We ran three regional sites in December 2009. Each site judged papers for problems A and B. The papers judged at each regional site may or may not have been from their respective regions. Papers were judged as Outstanding, Meritorious, Honorable Mention, and Successful Participant. All finalist papers for the Regional Outstanding award were sent to the National Judging. For example, eight papers may be discussed at a Regional Final and only four selected as Regional Outstanding but all eight papers are judged for the National Outstanding. Papers receive the higher of the two awards. The national judging chooses the "best of the best" as National Outstanding. The National Judges commended the regional judges for their efforts and found the results were very consistent. We feel that this regional structure provides a good structure for the future as the contest grows.

Judging Results:

NATIONAL AND REGIONAL COMBINED RESULTS

Problem	National Outstanding		Regional Outstanding		Meritorious		Honorable Mention		Successful Participant		Total
	#	%	#	%	#	%	#	%	#	%	
A	**4**	3.2%	5	4%	32	25.8%	49	39.5%	35	28.26%	**125**
B	**4**	2.63%	10	6.58%	28	18.42%	63	41.47%	47	30.9%	**152**
Total	**8**	2.8%	15	5.4%	60	21.66%	112	40.4%	82	29.6%	**277**

National Outstanding Teams

Davis Senior High School, Davis, CA
Illinois Mathematics and Science Academy, Aurora, IL (two teams)
International School of Duesseldorf, Duesseldorf, Germany
Maggie Walker Governor's School, Richmond, VA
Mills Godwin High School, Richmond, VA
Shanghai Foreign Language School Affiliated to SISU, Shanghai, China
The Ellis School, Pittsburgh, PA

Regional Outstanding Teams

Castilleja School, Palo Alto, CA
Central Academy, Des Moines, IA
Charter School of Wilmington, Wilmington, DE
Dubuque Hempstead High School, Dubuque, IA
Hong Kong International School, Tai Tam, Hong Kong (four teams)
Illinois Mathematics and Science Academy, Aurora, IL
Maggie Walker Governor's School, Richmond, VA (two teams)
Mills Godwin High School, Richmond, VA (two teams)
Shanghai Foreign Language School Affiliated to SISU, Shanghai, China
The Ellis School, Pittsburgh, PA

NCTM Standards: Many of us have read the NCTM standards and clearly realize the mapping of this contest to the NCTM 9–12 mathematics standards. This contest provides a vehicle for using mathematics to build models to represent and to understand real world behavior in a quantitative way. It enables student teams to look for patterns and think logically about mathematics and its role in our lives. Perhaps in a future *Consortium* article we will dissect a problem (paper) and map the standards into it.

General Judging Comments: The judge's commentaries provide specific comments on the solutions to each problem. As contest director and head judge, I would like to speak generally about solutions from a judge's point of view. Papers need to be coherent, concise, and clear. Students need to restate the problem in their own words so that the judges can determine the focus of the paper. Papers that explain the development of the model, assumptions, and its solutions, and then support the findings mathematically, generally do quite well. Modeling assumptions need to be listed and justified, but only those that come to bear on the solution (that can be part of simplifying the model). Laundry lists of assumptions that are never referred to in the context of the model development are not considered relevant and deter from a paper's quality. The mathematical model must be clearly developed, and all variables that are used must be well defined. Thinking outside of the box is also considered important by judges. This varies from problem to problem, but usually includes model extensions or sensitivity analysis of the solution to the team's inputs. Students must attempt to validate their model, even if by numerical example or intuition. A clear conclusion and answers to specific scenario questions are all key components. The strengths and weakness section is where the team can reflect on their solution and comment on the model's strengths and weaknesses. Attention to detail and proofreading the paper prior to final submission are also important, since the judges look for clarity and style. Citations are very important within the paper as well as either a reference or bibliography page at the end. We encourage citations within the paper that deal directly with use of data and figures, graphs, or tables. We noticed an increased use of Wikipedia; teams should realize that, although useful, the information might not be accurate. Teams should acknowledge this.

CONTEST FACTS:

Facts from the Twelfth Annual Contest:

- Wide range of schools/teams competed including teams from Hong Kong, China, and Korea.

- The 277 teams representing US and International institutions.

- There were 1121 student participants, 683 (61%) male and 438 (39%) female. There were 50 all-female teams.

- Schools from only seventeen states participated in this year's contest.

THE FUTURE:

The contest, which attempts to give the under-represented an opportunity to compete and achieve success in mathematics, appears well on its way in meeting this important goal.

We continue to strive to improve the contest, and we want the contest to grow. Any school/team can enter, as there are no restrictions on the number of schools or the numbers of teams from a school. A regional judging structure is established based on the number of teams.

These are exciting times for our high school students. Mathematics continues to be more than learning skills and operations. Mathematics is a language that involves our daily lives. Applying the mathematical principles that one learns is a key to future success. The abilities to recognize problems, formulate a mathematical model, use technology, and communicate and reflect on one's work are keys to success. Students gain confidence by tackling ill-defined problems and working together to generate a solution. Applying mathematics is a team sport!

Advisors need only be motivators and facilitators. They should encourage students to be creative and imaginative. It is not the technique used but the process that discovers how assumptions drive the techniques that is fundamental. Let students practice to be problem solvers. Let me encourage all high school mathematics faculty to get involved, encourage your students, make mathematics relevant, and open the doors to success.

Mathematical modeling is an art and a science. Teach your students through modeling to think critically, communicate effectively, and be confident, competent problem solvers for this new century.

CONTEST DATES:

Mark your calendars early: The next HiMCM will be held in November 2010. Registrations are due in October 2010. Teams will have a consecutive thirty-six-hour block within the contest window to complete the problem. Teams can register via the Web at www.comap.com.

MathModels.org

We strongly recommend that participants in this contest as well as prospective participants take a look at the new modeling Website, www.mathmodels.org, which has a wealth of information and resources.

HiMCM Judges' Commentary

Problem A

Professor Dave H. Olwell, Naval Post-Graduate School

This problem is of particular importance to the western United States, where the author resides. Originally proposed as a problem for the college level MCM, the problem was selected for the HiMCM contest by the contest directors because it can be addressed with high school mathematics.

This raises the question, what are reasonable expectations for a high school team, in 36 hours, to model this situation. As a regional head judge and one of the national judges, and as the problem author, I offer the following insights.

First (and this comment is made each year), if the problem asks for a letter, news article, or position paper, the student group must provide one if it hopes to be recognized.

Second, the problem statement explicitly called out three components to be addressed. They were storage and movement; desalinization; and conservation. Addressing all three greatly increases the chance of recognition.

The three components above have interplay based on the cost of methods used to implement them. Conservation is generally cost-effective but is usually not sufficient. Storage and movement of ground water can be cost-effective if not too many vertical obstacles exist and if a supply is reasonably close. However, diversion of water can have adverse environmental effects on fisheries and other habitats. Desalinization is very expensive and the expenses grow the farther (horizontally and vertically) the water must be moved. It is, however, potentially a nearly unlimited source. It does also present environmental issues of residue and heat disposal, and of the effects of whatever energy source powers the desalinization plant.

The better papers this year attempted to present frameworks for choosing solutions in this trade-space. The mathematics required to do this are very accessible at the high school level; most demand, supply, and cost calculations can be done at the pre-calculus level. It is in essence a network-modeling problem with arcs having capacities and costs.

The challenge is that a national solution of a high-resolution network model would necessarily be very complex. But the problem did not call for a national solution; it called for a national strategy. Such a strategy could provide an appropriate framework for local decisions, which are often easier.

One approach is a greedy algorithm. For a given location, one calculates the demand and current supply, and eliminates any gap choosing the lowest cost means available until it is exhausted, and then moves to the next lowest. On a local scale, this is very tractable. On a regional or national scale, it becomes more complex as different localities demand the limited low cost resources: who gets priority?

There were many strengths in this year's papers. Almost all the papers did a reasonable job of estimating the demand for water in 2025, some with very high resolution. Several of the papers did excellent jobs modeling the cost of moving water (both horizontally and vertically) and of storage. There were good discussions of the costs and environmental effects of desalinization (although I don't recall anyone mentioning that water produced at sea level must be pumped uphill to almost all consumers at a cost.) There were many good discussions of conservation strategies and costs.

A few of the papers did outstanding jobs of representing their strategies graphically.

There were some notable patterns of weakness. Many papers only considered the familiar household usage and conservation, not recognizing that household use is a minor fraction of overall national demand. Many did not look at all three facets (conservation, desalinization, movement). Some did not include the position paper. Others forced the use of calculus in their solution, although it was not really appropriate. On the other hand, some papers offered no mathematical treatment of the problem at all.

Student groups should remember that the problems posed in these contests are not going to have a unique solution—they are designed not to have one, in fact. Students should remember that general high school mathematics are adequate to the task at hand. What we are looking for is evidence of good modeling of the problem with these tools, and then discussion of the implications of the model and its solution(s).

Good luck in 2010!

Problem B

William P. Fox, HiMCM Contest Director

This problem's statement was concise but clearly had elements for the students to consider. Students should have clearly defined what they considered "devastation." The teams that did so we were to articulate the similarities and differences between cities. Most students completed most of the required tasks. Many did not pick a city of their choice to analyze. Almost all teams had the letter to the newspaper, but few did what we would call an excellent job of concisely telling their story. Thus, teams should ensure that that they complete and include all the required tasks in their submission.

The executive summaries for the most part were either absent or poorly written. This has been ongoing since the beginning of the contest. Faculty advisors should spend some time with their teams and advise to write a good summary. Many summaries read like technical reports or were too vague to be helpful. Summaries need to contain the results of the model as well as brief explanation of the problem. The executive summary should entice the reader (in our case the judge) to read the paper.

The letter to the newspaper should have been a concise explanation of the modeling results that included (1) devastation caused by various sized tsunami that could hit the city; (2) some discussion on preparedness; (3) a brief description or statement of the potential impacts; and (4) the approximate cost. The ability to summarize and present information is critical in real life and real jobs. Again, many teams failed to complete this in their submission.

The judges felt the first critical task was to define devastation and define a metric or measures that they could use to measure such devastation. Many teams chose costs or area under water as a possible metric for devastation.

Few teams, if any, did any sensitivity analysis on relationships between earthquake size and tsunami size, although some teams did an outstanding job here.

We found that many of the assumptions and research for the problem were very good. Teams did some history of earthquakes and tsunamis for their modeling efforts. We encourage teams who take data from other sources or graphics to *include the reference at that point* as well as on a reference page at the end. We saw the use of data from blogs and Wikipedia—such information can be suspect, and we encourage teams to obtain data and information from reliable sources.

There were a wide variety of approaches used from simple algebra through simulation models. We found many simulation models were never well explained nor were flow charts used. It was as if these techniques were a black box. As models, they should be explained as to what they do and why they could be used in the scenario.

One issue was with significant digits. The models built by the teams were in dollars of some magnitude. Yet numerical values were presented to (at times) many decimal places. Clearly, this was not necessary.

General Comments from Judges:

Computer generated solutions: Many papers used computer code. Computer code used to implement mathematical expressions can be a good modeling tool. However, the judges expect to see an algorithm or flow chart from which the code was developed. Successful teams provided some explanation or guide to their algorithm(s)—a step-by-step procedure for the judges to follow. Code may only be read for the papers that reach the final rounds, but not unless the code is accompanied by a good algorithm in words. The results of any simulation need to be well explained and sensitivity analysis preformed. For example, consider a flip of a fair coin. Here is a general algorithm:

INPUT: Random number, number of trials

OUTPUT: Heads or tails

Step 1: Initialize all counters.

Step 2: Generate a random number between 0 and 1.

Step 3: Choose an interval for heads, like [0, 0.5]. If the random number falls in this interval, the flip is a head. Otherwise the flip is a tail.

Step 4: Record the result as a head or a tail.

Step 5: Count the number of trials and increment: Count = Count + 1.

An algorithm such as this is expected in the body of the paper with the code as an appendix.

Graphs: Judges found many graphs that were not labeled nor explained. Many graphs did not appear to convey information used by the teams. All graphs need a verbal explanation of what the team expects the reader (judge) to understand (or see) from the graph. Legends, labels, and points of interest need to be clearly visible and understandable, even if hand written. *Graphs taken from other sources should be referenced and annotated.*

Summaries: These are still, for the most part, the weakest parts of papers. These should be written after the solution is found. They should contain results and not details. They should include the "bottom line" and the key ideas used in obtaining the solution. They should include the particular questions addressed and their answers. Teams should consider a brief, three-paragraph approach: a *restatement of the problem* in their own words, a short description of *their method and solution* to the problem (without giving any mathematical expressions), and the *conclusions* that provide the numerical answers in context.

Restatement of the Problem: Problem restatements are important for teams to move from the general case to the specific case. They allow teams to refine their thinking to give their model uniqueness and a creative touch.

Assumptions/Justifications: Teams should list only those assumptions that are vital to building and simplifying their mathematical model. Assumptions should not be a reiteration of facts given in the problem statement. Assumptions are variables (issues) acting or not acting on the problem. Every assumption should have a justification. We do not want to see "smoke screens" in the hopes that some items listed are what judges want see. Variables chosen need to be listed with notation and be well defined.

Model: Teams need to show a *clear link* between the assumptions they listed and the building of their model or models. Too often models and / or equations appeared without any model building effort. Equations taken from other sources should be referenced. The team is required to show how the model was built and why it is the model chosen. Teams should not throw out several model forms hoping to "wow" the judges; this does not work. We prefer to see sound modeling based on good reasoning.

Model Testing: Model testing is not the same as testing arithmetic. Teams need to compare results or attempt to verify (even with common sense) their results. Teams that use a computer simulation must provide a clear step-by-step algorithm. Lots of runs and related analysis are required when using a simulation. Sensitivity analysis should be done in order to see how sensitive the simulation is to the model's key parameters. Teams that relate their models to real data are to be complimented.

Conclusions: This section deals with more than just results. Conclusions might also include speculations, extensions, and generalizations. This is where all scenario specific questions should be answered. Teams should ask themselves what other questions would be interesting if they had more time and then tell the judges about their ideas.

Strengths and Weaknesses: Teams should be open and honest here. What could the team have done better?

References: Teams may use references to assist in their modeling. However, they must also *reference the source* of their assistance. Teams are reminded that only *inanimate resources* may be used. Teams cannot call upon real estate agents, bankers, hotel managers, or any other real person to obtain information related to the problem. References should be cited where used and not just listed in the back of the paper. Teams should also have a reference list or bibliography in the back of the paper.

Adherence to Rules: Teams are reminded that detailed rules and regulations are posted on the COMAP site. Teams are reminded that they may use only *inanimate sources* to obtain information. Teams are reminded that the *thirty-six-hour time limit is a consecutive thirty-six hours*.

Problem B (Going Green) Author's Comments

David H. Olwell

Chair, Department of Systems Engineering
Naval Postgraduate School

This problem was written to provide high school student teams a chance to make a significant contribution to the discussion about an ongoing national issue, using only high school mathematics. It was deliberately open-ended to allow both the creativity of the student and the widest possible set of mathematical techniques to come to bear.

As the author, I imagined that an Outstanding paper would have several characteristics. First, and most importantly, it would address all of the requirements identified in the problem statement. In particular, it would identify how much carbon dioxide needed to be removed, propose one or more approaches, show that they were feasible and effective, determine the costs, and address minimizing the impact. And, as particularly required, it would have a well-written, persuasive, short summary letter for Congress.

I imagined that a variety of approaches were possible, and that some of them were capacity constrained. I thought a "greedy algorithm" would be the easiest and likely best student approach. That is, each possible removal option had a cost per metric ton for removing carbon dioxide and an upper limit on how much the method could reasonably handle. For example, forestry approaches had a cost per metric ton removed that could be estimated, and an upper bound on how many acres could be planted and irrigated. Algae biofuel methods, as another example, had net costs and upper bounds as to acreage and water available. And so on. Most of the options that had capacity constraints, and a submission that did not address this, usually had difficulty showing that the approach was feasible.

A reasonable approach would be to find the lowest cost method and select as much as available, then the next lowest cost method, and so on, until the appropriate amount of carbon had been removed. This also had the advantage of allowing simple models to be used in a powerful way.

As I read the submissions, I was struck by how many of them failed to include the short summary paper to Congress. Such a submission was automatically excluded from consideration as an Outstanding paper. The lesson learned for subsequent teams is not a new one, but it bears repeating: Read the problem! If the problem statement asks for a letter or report to summarize your results, make sure you include one!

The paper that was chosen National Outstanding followed an excellent, straightforward approach. First, it estimated the amount of carbon dioxide that needed to be removed, above what was currently removed, to obtain neutrality. It considered four options for sequestration of carbon, and provided plausible cost estimates and capacity limits for each. The team determined a solution and provided a very well-written letter to Congress. The paper was well illustrated.

The mathematical techniques chosen were employed correctly and appropriate to the problem. The team did not try to artificially force an inappropriate technique into their paper.

This paper is an excellent example of the power of simple, well-thought-out mathematical models for analysis of complicated problems. This team, from the Ellis School in Pittsburg, Pennsylvania, is commended for their excellent work under time pressure. Their parents and teachers can be very proud of the quality of thought displayed: The students showed that they have received an excellent education.

Problem A Summary: Shanghai Foreign Language School

Advisor: Pan Liqun Sun Yue

Team Members: Xintong Huang, Qingyan Lu, Yizhou Shen, Yixin Wang

The ultimate goal of our whole modeling is to devise national water strategies mainly concerned with six aspects: conservation, efficiency, markets, collaboration, improved technology and interagency coordination increase. Also, we predict the trend of its development in the future. Thus we build five models.

The first model predicts the fresh water withdrawals of the United States at a state level from 2010 to 2025. We apply regression analysis to the data of state-level fresh water withdrawals. We have taken an appropriate level of accuracy based on the usage of the data.

The second model is committed to the plan of water transfer. We have leveled each region in terms of its water shortage degree. Also, we have built a model of max spanning tree to get the shortest route of transfer. We have drawn on the experience of Chinese water transferring project to calculate the capital cost of the project.

The third model estimates the desalination plant construction and the processing cost by establishing a sequence.

The fourth model simulates the water price rise to find the U.S. water price cap.

We have also researched measures of Supervisory Control and Data Acquisition (SCADA) system, water purification, remote sensing techniques and Geographic Information System to relieve the water shortage before 2025.

The fifth model is devoted to the economic, physical, cultural and environmental impact of different measures discussed above by using Analytic Hierarchy Process (AHP).

Finally, we have figured out an action plan of U.S. National water strategies to achieve our common ultimate goal in WATER 2025.

In a nutshell, we have adopted five models in distinctive thoughts, covering predictions of all times, to the ideal simulations and predictions.

Problem A Summary: Illinois Mathematics and Science Academy

Advisor: Steve Condie

Team Members: Derek Hardin, Seohyun Kim, Vlad Kontsevoi, Jack Shi

We used a mathematical model based on distribution nodes (major cities) and natural supply sources (groundwater and surface water). We idealized each city to be a circle with equal area to a node and each state to be a circle with equal area to that state, excluding its urban centers. We used the top 60 or so most populated cities of the United States (and one city from each continental state not represented) to account for the water structure of each state. We assumed that population is equally distributed within each node and within each state, and that nodes obtain their water from the surrounding state area, which corresponds with 2000 data. We assumed that factors such as industrial or agricultural usage of water vary depending on region and are proportional to population of each region (in states with multiple regions).

We tested our model and distribution network on 2000 data and observed a relatively small margin of error. Our consideration of marginal costs was thorough in that we paid close attention to transport and to processing. We explored water conservation policies and determined that 5% conservation would be the most feasible and effective policy—while it would have a certain degree of negative cultural impact, it would save a substantial amount of money. Overall, our model was based on historic data and our own policy / node division innovation and appears to show that the future is bright for United States water.

Problem A Summary: Davis Senior High School

Advisor: Gregory Shinault

Team Members: Amanda Chen, William Liu, Saraf Nawar, Peter Wang

We first created a mathematical model to project the demand for fresh water in the United States in 2025, basing our projected needs on current trends in eight major categories of fresh water usage: public water, domestic, irrigation, livestock, aquaculture, industrial, mining, and thermoelectric-power generation. We then devised a cost-efficient and plausible national water strategy in which we can meet the 2025 projected water needs through storage, movement, desalinization, and reduction in consumption. Within our paper, part 1 addresses the projected growth in the demand for water due to population growth, energy demand, and industries, while parts 2–6 address our national water strategy and details illustrating how our strategy will create a sustainable water usage plan for the years 2010 to 2025.

For our projection model, we considered current trends in the eight major categories of the total fresh water usage as recorded by the United States Geological Survey. We created regression curves of the USGS data for each of the eight categories. We then summed up the projected 2025 fresh water usage from each of the eight categories to obtain a total fresh water usage graph that projects the total United States fresh water usage in 2025.

This parameter allowed us to devise a plan that would attempt to alleviate the projected water usage. In our plan, we addressed financial issues, physical and environmental effects. We took a multi-pronged approach, addressing various areas of water use, such as toilet flush and canal lining, as well as multiple ways of conserving the freshwater. Each plan is modeled by a sub-model, and at the end their costs and water savings are summed to create the total water saved and the total cost of our national water strategy. By minimizing waste and maximizing economic efficiency, our plan attempts to achieve a permanent sustainable national fresh water system.

In conclusion, our national water plan requires a moderate amount of funds, around $200 billion over the course of the next 15 years ($13.3 billion a year), yet it greatly lessens the burden of available freshwater in the United States. By reducing the amount of fresh water consumed every year to pre-1970s level, our national water strategy creates a plan for long term sustainable usage of the United States' fresh water resources.

Problem A Paper: Mills Godwin High School

Advisor: Todd Phillips

Team Members: Matthew Boegner, Todd Phillips, Samuel Rubin, Alex Yachanin

The U.S. is in a state of water stress, using more fresh water than can be replenished by the hydrologic cycle. This is especially true in areas that rely on groundwater from aquifers, which accounts for 99% of all usable freshwater, but only accounts for 20% of human water withdrawals.

The first strategy that must be taken to combat this problem is to measure and account for all the water resources in the U.S. Since 1950, the United States Geological Survey (USGS) has issued a series of reports estimating water withdrawals by state, the amount of water allocated by each sector, and the sources from which the water was withdrawn.

Secondly, to allow for more effective management, usage of these resources in relation to demands of all aspects of society must be known. The USGS reports eight main categories of water usage. The largest is thermoelectric power production, followed by irrigation, public supply, industry, aquaculture, mining, domestic, and livestock. Water use has changed with the evolution of technology, population, and economic conditions. Since 1950, water use for thermoelectric power has increased by about 500%, and irrigation has increased by about 50%. Thus, the largest reductions in water consumption should come from these two.

Water withdrawals peaked around 1980, and have since leveled off despite an increasing population. The USGS has underscored the importance of knowing the limitations of the drinking water supply. Continuing technological improvements in irrigation and energy production are of utmost importance, but in order to reduce water consumption even further, advances must be made in public and domestic use.

In addition to improving management practices, the looming water shortage has necessitated investigation into expanding sources from which the U.S. obtains water. Fresh water development would occur through new treatment technologies, preventing water pollution, new approaches to water storage, and creating social and economic tools to optimize spending and encourage acceptance of new management techniques.

We seek to incorporate this knowledge and these strategies to solve the nation's water constraints in the most cost-effective means possible. This happens through efficiency in management, conservation, policies, and innovative technologies. We seek to continue existing programs that serve to stabilize water consumption, expand desalination and similar approaches to gaining new fresh water sources, and incorporate new methods to minimize environmental, economic, physical, and cultural impact.

GENERAL ASSUMPTIONS FOR ALL MODELS:

1. U.S. population will reach about 353 million by 2025.
2. Total water consumption is from eight categories: public supply, domestic, irrigation, livestock, aquaculture, industrial, mining, and aquaculture.
3. Current water management methods are insufficient for long-term sustainability.
4. New technologies for water management, desalination, transport, storage, etc., will become cheaper over time.
5. Using water stored in icecaps and glaciers is not economically feasible. The best option for expanding fresh water resources, therefore, lies in saline water.
6. Groundwater sources are non-renewable on a short-term basis. Groundwater provides 31% of the water used in U.S. agriculture and is, on average, being depleted 25% in excess of recharge rates.

DETERMINING CURRENT WATER WITHDRAWAL:

Our first approach was to research the current situation.

The USGS reports that the total freshwater and saline water withdrawals are 408 billion gallons per day. Surface water withdrawals are about 67.15% of the total, groundwater withdrawals about 18.15%, and the remaining 15% consists of saline water from seawater and brackish costal water.

The USGS also reports the current distribution of water in the eight main sectors: public supply 11%, irrigation 31%, aquaculture 2%, mining 1%, domestic 11%, livestock 1%, industrial 4%, and thermoelectric power 49%.

CALCULATING/ESTIMATING WATER CONSUMPTION IN 2025:

Using USGS reports, we could see the overall trend in water usage (**Figure 1**).

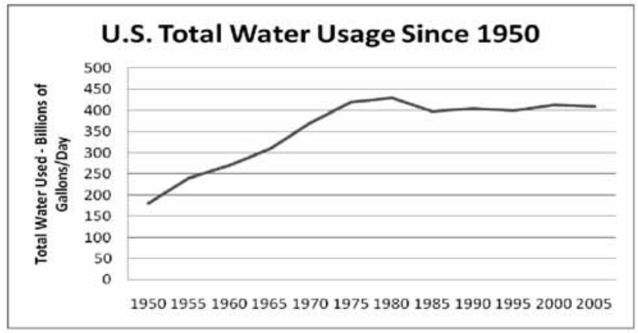


Figure 1.

Withdrawals peaked in 1980 and have been relatively constant since 1985.

Water use in the sectors has exhibited different growth. For example, irrigation and thermoelectric power water use has steadied in recent years (**Figure 2**).

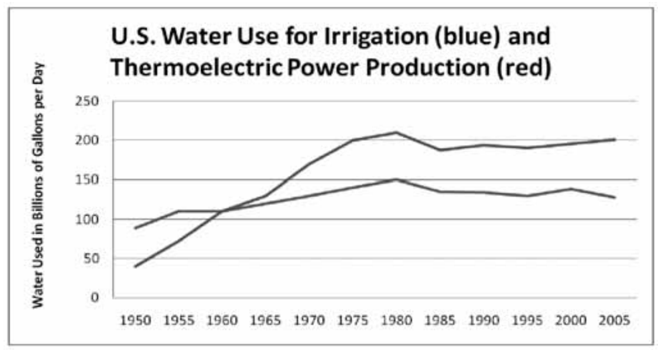


Figure 2.

Meanwhile, water use in the public sector and household domestic use has steadily increased since 1950 (**Figures 3 and 4**).

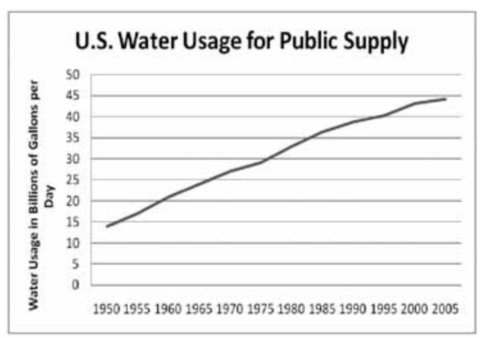


Figure 3.

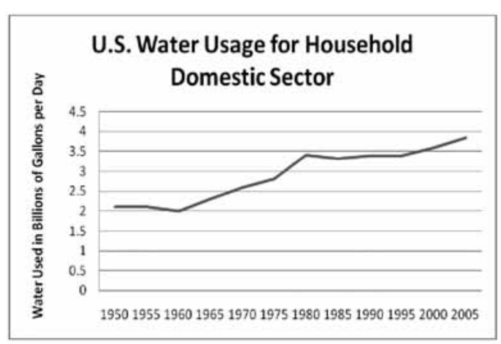


Figure 4.

From these trends, we calculate that total water usage in 2025 will be about 497.8 billion gallons per day, or 181,681 billion gallons per year.

STRATEGIES FOR OUR MODELS:

Our models seek to implement several strategies:

- Water conservation through public supply and domestic use.
- Reduce reliance on groundwater by 2025.
- Increase reliance on desalinized saline water
- Groundwater sources not replaced with desalinized water are offset by expanding surface water resources.
- We can't spend hundreds of billions of dollars on desalination projects.

OUR MODELING GOALS: REDUCING GROUNDWATER, MINIMIZING COSTS

Background:

We seek to reduce reliance on groundwater and surface water by 2025. We analyze two variables: water withdrawal and cost. We aim to model the cost of reducing groundwater and surface water consumption. Our first goal is to expand desalination technologies to replace groundwater usage.

The most obvious source of saline water is oceans, but there are also vast reserves of brackish water. Lower salt concentration means brackish water is a more economically feasible source than seawater. The economics of desalination are daunting, however. Processes that use boiling require large amounts of energy, which makes large-scale manufacturing prohibitive. New techniques, such as decreasing pressure to reduce water's boiling point, have helped, but are almost entirely limited to wealthy and energy-rich nations on the Arabian Gulf.

Another desalination method is solar distillation, but the vast area requirements make this process prohibitive. Hope lies in membrane technologies such as reverse osmosis (RO), which is now widely used in decontamination, purification, and recycling, and its potential for desalination is rising. Put simply, membranes allow passage of water while excluding passage of contaminants such as salt.

Cost remains the main problem. RO is an energy intensive method, but far less so than any thermal distillation plants. Developments in desalination technology, construction of large capacity plants, and collocation of power generation facilities and desalination plants have resulted in a dramatic decrease in the cost of desalinized water.

Assumptions Specific for Model 1:

1. Brackish water and seawater cost \$1.64 and \$2.50 per 1000 gallons to desalinize, respectively.
2. Of current desalination plants in the U.S., about 55% use brackish water and 45% use seawater. These proportions will be constant through 2025. Thus, desalinizing costs an average of (0.55)(1.64) + (0.45)(2.50) = \$2.03 per 1000 gallons.
3. The cost of extracting groundwater and surface water are \$0.50 and \$0.90 per 1000 gallons, respectively. These are constant through 2025.
4. Reductions in groundwater withdrawal result in an increase in saline water withdrawal in the proportions in assumption 2. Also, increases in water use are compensated for only with increases in saline water withdrawal.
5. All current desalination plants work to current potential through 2025.
6. New desalination plants have a lifespan that extends past 2025.
7. 2005 surface water extraction is constant through 2025.
8. All water plants can store water for use in peak times.
9. The rate of increase of groundwater usage is constant through 2025.
10. The rate of groundwater withdrawal reduction is constant.
11. The average cost of a desalination plant accounts for the low number of high-production plants and the high number of low-production plants.
12. The costs of extraction (assumptions 1 and 3) include power, operation, extraction, maintenance, etc., but not initial construction costs.
13. No new surface water or groundwater facilities are built through 2025.

EQUATION AND CALCULATIONS

Model 1:

C_T = 575.7584 – 394.052(1 + r)15

Where is total cost of water withdrawal and processing for the year 2025, and r is the rate at which groundwater resources change.

Step 1

Water withdrawal in the U.S. is 67.15% surface water, 17.85% groundwater, and 15% saline water.

We define C_T as total cost of groundwater withdrawal x, surface water withdrawal y, and saline water withdrawal z.

Based on USGS data, we can predict total water usage in 2025 as 181,681 billion gallons per year. Thus, 181,681 = $x + y + z$.

Since surface water withdrawal is constant, y = (0.6715)*(181681) = 121,998.79 billion gallons per year.

x (groundwater) decreases each year in accordance with the goals of our model. We can describe x by $x = N_o(1 + r)15$, where N_o = original groundwater withdrawal in 2025. Our exponent of 15 is the number of years (2025–2010). We assume that N_o is constant, and calculate $N_o = (0.1785)(181681) = 32{,}430.06$ billion gallons. r is negative because the rate decreases over time.

Our goal is to replace groundwater with saline water. We can therefore define z as $z = 181{,}681 - y - x$. We substitute for x and y to get $z = 59{,}682 - 32{,}430.06(1 + r)15$.

Using costs in assumptions 2 and 3, we derive total cost as $C_T = 0.5x + 0.9y + 2.03z$.

Substituting for x, y, and z, we have

$C_T = 0.5(32430.06(1 + r)15) + 0.9(121998.79) + 2.03(59682 - 32430.06(1 + r)15)$, which simplifies to $C_T = [230{,}953.37 - 49{,}617.99(1 + r)15] / 1000$.

Step 2

We expand upon this equation by noting that new desalination equipment needs to be constructed. Our new equation, therefore, is $C_T = 0.5x + 0.9y + 2.03z + C_m(m)$.

C_m is average cost of a desalination plant. We use C_m = 0.55(brackish average cost) + 0.45(saltwater average cost) = 0.55(22.95) + 0.45(326.5) = $0.15955 billion.

m is the number of desalination plants needed for 2025. Therefore m is the difference in gallons of saline water between 2025 and 2010, divided average plant output. Thus, $m = \frac{(z_{25} - z_{10})}{O}$, where O is average output (17.28 billion gallons per year), and z_n is gallons of saline water production.

With O = 17.28 billion, $z_{25} = 59{,}682 - 32{,}430.06(1 + r)^{15}$ and $z_{10} = (0.15)(148920)$, we have $m = [(59682 - 32430.06(1 + r)^{15}) - 0.15(148920)] / 17.28$, or m **= (37344 – 32430.06(1 + r)^{15})/17.28**.

We therefore rewrite the cost equation obtained in part 1 as:

$C_T = [230953.37 - 49617.99(1 + r)^{15}] / 1000 + 0.15955[(37344 - 32430.06(1 + r)^{15}] / 17.28)$

Simplification in terms of “r” and a constant value give us our final model:

$C_T = 575.7584 - 394.052(1 + r)^{15}$

Model 2:

$C_T = 1840.061 - 349.052(1 + r)^{15} - 1264.3(1 + S)^{15}$

Where C_T is total cost of water withdrawal and processing for 2025 and S is the rate at which surface water resources change.

We build upon Model 1 by acknowledging that current surface water usage y is unlikely to remain the same through 2025. From Model 1, $x = 32{,}430.06(1 + r)^{15}$. Now, we derive $y = 121{,}998.79(1 + S)^{15}$.

Recalling that $z = 181{,}681 - x - y$, we define z as

$z = 181{,}681 - 32{,}430.06(1 + r)^{15} - 121{,}998.79(1 + S)^{15}$.

Our total cost equation from Model 1, part 1, is now

$C_T = 0.5[32{,}430.06(1 + r)^{15}] + 0.9[121{,}998.79(1 + S)^{15}] + 2.03[181{,}681 - 32{,}430.06(1 + r)^{15} - 121{,}998.79(1 + S)^{15}]$, or $C_T = 368{,}812.43 - 49{,}617.992(1 + r)^{15} - 137{,}858.6313(1 + S)^{15}$.

Recalling from Model 1, part 2, we must now include $C_m(m)$.

C_m is still $0.15955 billion, and m is still $(z_{25} - z_{10}) / O$. We substitute our new value of z to get $m = [181{,}681 - 32{,}430.06(1 + r)^{15} - 121{,}998.79(1 + S)^{15} - 22{,}338] / 17.28$.

Multiplying our new values for m and C_m, we have

$C_m(m) = 1{,}471.249 - 299.4338(1 + r)^{15} - 1{,}126.442(1 + S)^{15}$.

Substituting into our original cost equation we get our final equation for Model 2:

$C_T = 1{,}840.061 - 349.052(1 + r)^{15} - 1264.3(1 + S)^{15}$

Model 3:

$C_T = 1{,}840.061 - 349.052(1 + r)^{15} - 1264.3(1 + S)^{15} - C[181{,}681 - 32{,}430.06(1 + r)^{15} - 121{,}998.79(1 + S)^{15}]$

Where C_T is total cost of water withdrawal and processing for 2025, recognizing that the cost of desalination will decrease by C by 2025.

Recalling $C_T = 0.5x + 0.9y + 2.03z + C_m(m)$, we now define the coefficient of z as $(2.03 - C)$.

Only looking at $(2.03 - C)z$, we can substitute our value for z from Model 2 to get

$(2.03 - C)[181{,}681 - 32{,}430.06(1 + r)^{15} - 121{,}998.79(1 + S)^{15}]$.

Thus, we have:

$C_T = 1{,}840.061 - 349.052(1 + r)^{15} - 1264.3(1 + S)^{15} - C[181{,}681 - 32{,}430.06(1 + r)^{15} - 121{,}998.79(1 + S)^{15}]$

Testing and Analyzing our Model:

We test our equation from Model 3 with the following goals:

• Reduce groundwater withdrawal by 10% by 2025.

• Reduce surface water withdrawal by 5% by 2025.

• Compensate for these reductions with increased desalination.

To reduce groundwater 10%, we need to define r. We set $(1 + r)^{15} = 0.9$, to get $r = 0.007$.

To reduce surface water 5%, we set $(1 + S)^{15} = 0.95$, to get $S = 0.00341$.

We include a decrease in average desalination cost of $1 per 1000 gallons, giving $C = 1$.

Results:

Analyzing first the values Cm and m, we calculate that to meet our set goals:

- $m = 828.06$. This would have to be rounded up to 829.
- C_m remains $0.15955 billion. Multiplying this by m, we get total cost of plant construction = $131.64 billion.
- From the equation in Model 3, we calculate that our plan costs $288.21 billion.

Conclusion:

It is becoming more evident that desalination remains our best hope for a sustainable fresh water future. In the U.S., a dramatic increase in proposed desalination plants such as this one are running into opposition from environmental and economic groups. We have addressed the financial concerns with a proposal that we believe is realistic and economically feasible, but the environmental concerns remain unaddressed.

We strongly advise that desalination sites be designed, operated, and planned to minimize environmental impact. Outflows of brine (waste) must avoid sensitive marine areas and incorporate adequate dilution. We propose that improvements be made in technology to use less energy, or couple desalination plants with thermoelectric plants. Also, safe disposal of residual salts and other chemicals is important. A case study in Greece has shown a zero-discharge plant is feasible through coupling desalination plants with solar salt works. Intake wells of these plants must also be carefully designed so as not to harm the ocean ecosystem from which they take water.

Desalination creates increases energy demand. Presently, a majority of U.S. energy is generated from coal-fired plants that release carbon dioxide and pollutants such as sulfur and mercury. Thus, we advise desalination plants pair with renewable energy resources such as wind, hydropower, and solar power. However, new technologies that reduce the energy needs of desalination and use novel desalination methods such as solar distillation, geothermal, and the Passarel process should be researched and developed to reduce cost. We also recommend that fewer large desalination plants be built, as opposed the many average production plants proposed in our model. We provide a list of the most strategic locations for new plants:

Proposed Desalination Plant Locations:

1. Seattle, Washington—to provide for the Pacific Northwest
2. San Jose, California—to meet demands in the arid Southwest
3. Los Angeles, California—to provide water for one of the nation's largest cities
4. San Diego, California—same as #2
5. Corpus Christi, Texas—for transportation inland for crop irrigation
6. Port Arthur, Texas—same as #5 and to provide water for Dallas and Houston
7. Gulfport, Mississippi—for agriculture, industrial, and domestic use in the Southeast
8. Panama City, Florida—to provide water in a region where saltwater intrusion has affected groundwater resources
9. Savannah, Georgia—same as #7
10. Boston, Massachusetts—to provide water for the Northeast

Current Plants:

A. Los Angeles, California

B. Houston, Texas

C. Tampa Bay, Florida

Proposed Transport and Storage Methods:

We analyzed water transportation methods and concluded that pipelines are the most cost-effective and environmentally friendly. Our main goal was to build pipelines from our main plant locations to all the necessary states inland (**Figure 5**).

Figure 5.

We calculated that half the distance between Corpus Christie and Seattle is the essential length of pipeline from each plant. We can determine the total cost of this by:

(distance)*(number of plants)*(gallon flow for each plant)*(cost per gallon per kilometer) = 1879.762km*10*(18.1681 million per year)*0.0106 = $3.62 billion per year.

The total distance used by pipes is 1879.762 km, which we found using Mapquest.

This is the cost for transporting the water, and we add to this the cost of building the pipes. This cost is (distance)*(number of plants)*(price per km) = 1879.762*10*(0.0044703) = $84.03 billion. Note that this is a one-time cost. Also, there is a substantial amount of pipeline already in place that would decrease this cost. Most pipelines are state-run, but our proposal is to build federal inter-state pipelines.

Water storage will be left to existing in-state facilities because although more water is being pumped into states, most of the storage in aquifers and other natural sources increases to compensate; we did not factor this into our costs.

Effects of Our Model:

Cultural effects include legislation that affects decreases in domestic water use and public supply. The EPA enforces federal laws such as the Marine Mammal Protection Act, the Safe Water Drinking Act, the Clean Water Act, the Endangered Species Act, and the Wetland Protection Act, and the National Environmental Protection Act. Placement of our desalination plants must not break any of these laws. Also, there are mandates that impose restrictions on desalination plants, such as regulations on the quality of drinking water and limitations on placement of output sources. Other cultural effects include humans conserving water through raised awareness.

One of the largest uses of freshwater is agriculture. The country can immediately conserve its water supply by instituting several changes to irrigation techniques. First, convert farmland that currently uses inefficient surface systems (flood, furrow, and corrugate) to much more efficient micro-irrigation, center pivot sprinkler, and moving lateral sprinkler systems. Research shows these methods can increase application efficiency to 85–95%. Moreover, the government should encourage farmers to avoid planting water-thirsty crops in desert regions and to time irrigation to avoid watering during the hottest/windiest weather to reduce water loss due to evaporation.

Physical effects mostly deal with transportation difficulties. Mountain ranges and other altitude changes cause transportation problems since more energy is required to overcome these obstacles. However, regions reap the benefits of our model through the placement of desalination plants and transportation pipelines. Water is transported mostly to regions with high agriculture rates that are currently depleting aquifers rapidly. By supplying them with fresh water, our model gives the aquifers time to restore by natural processes.

Our model serves as the best approach to outline the nation's water usage policy until 2025 because of its minimal environmental impacts and reduced economic constraints compared to recent legislation. Our final cost estimate is $110 billion for the first year, followed by $25 billion for each of the next fourteen years.

Problem B Summary: The Ellis School

Adviser: Amy Yam

Team members: Charlotte Clark, Rasha El-Jaroudi, Alana Ganz

Tsunamis are devastating to the areas they hit. They cause not only human loss, but property damage and societal harm as well. To build a model of the devastation a tsunami would cause in San Francisco, CA; Hilo, HI; New Orleans, LA; Charleston, SC; New York, NY; Boston, MA; and Seaside, OR, we considered all three types of devastation and designed a 100-point devastation scale that compares the total devastation caused to a city. Our model allows one to predict the devastation based on the city's characteristics and the magnitude of the earthquake, which determines the severity of the tsunami.

We looked at the devastation of the seven cities by comparing damaged property, threat to human life, and social impact of the destruction. Threat to human life was quantified by the thoroughness of each city's emergency response and evacuation plans, as well as the percentage of the city's population that must evacuate and the furthest distance an evacuee must walk to reach safe ground. The cost of damaged property was calculated by adding the values of all the houses and major buildings in the city's inundation zone (area of the city flooded by tsunami waves). Social impact was quantified by examining the socially important buildings, such as police stations, hospitals, and schools, in the inundation zone that would be destroyed. Each category's values were adjusted to fit on the 100-point scale, in which 0 is no damage and 100 is complete destruction of the city. Because these devastation scores assume the entire inundation area is affected by the tsunami, we also utilized a tsunami magnitude equation, which adjusts the devastation value for the severity of the tsunami based on the magnitude of the earthquake.

Overall we found that in the event of a severe tsunami, New York City would experience the most destruction, 41.292 points, followed by New Orleans, 25.862, Seaside, 25.114, San Francisco, 19.31, Charleston, 9.325, Boston, 8.264, and Hilo, 5.675. To test our model, we compared our findings to data from recent tsunamis around the world.

Problem B Summary: Maggie Walker Governor's School

Adviser: John Barnes

Team members: Susan Ballentine, William Farmer, Ashish Makadia, Milton Tyler

This year, a massive earthquake measuring 8.0 on the Richter scale caused a monstrous tsunami that hit Samoa, American Samoa, and Tonga, claiming at least 1000 lives. In 2004, another tsunami, caused by a 9.0 earthquake, killed over 200,000 throughout the countries of the Indian Ocean. Avoiding these natural disasters is a top priority among geophysicists, and thus far, they have attempted to model these incredible forces.

In this study, our goal was to compare the devastation of potential tsunamis caused by varying earthquakes on seven different cities. Our model takes into account two of the largest factors in measuring the strength of a tsunami when it hits land: the energy from the earthquake itself and the distance between the epicenter and the coast, where devastation is proportional to energy but inversely proportional to distance.

The main advantage of our model is its simplicity in comparison to others. Most of those use the topography of the surrounding ocean, geography of the bay the city lies in, and the stress released by the fault during the earthquake. None of these make for a simple equation. On the other hand, ours strikes a balance between accuracy and simplicity, and even though we aimed to optimize simplicity, ours is still very accurate.

In confirming our model, we used data from four different earthquakes: the 2009 Samoan earthquake to create the model and three others to reinforce it. These three included both Aceh's and Sri Lanka's death tolls from the 2004 Sumatra-Andaman earthquake and Flores's death toll from the 1992 Flores Sea earthquake.

To execute our model, we used five scenarios. The first scenario used data from the 2004 Sumatra-Andaman earthquake, which we related to how far inland water from the tsunami came. Therefore, we measured the death toll in each city as if the water had come into shore two kilometers. The second and fourth scenarios were computed using the largest earthquake ever recorded (9.5) and the smallest one that ever produced a deadly tsunami (5.2) respectively.

The third used a value for the average earthquake that produced a deadly tsunami, while the fifth and final found the smallest sized earthquake that would displace only one person.

To summarize our results, Hilo would be entirely demolished but Juneau would be left comparatively unharmed in scenario one due to their respective low and high land areas. For equal sized earthquakes (scenarios two through five), Juneau would be hit the hardest but Hilo would be hit the least. Oddly, the smallest earthquake-causing tsunami killed about 38 times as many people as expected (3 observed / 0.08 expected); we contributed this to some special incident in the actual case, such as evacuation.

So while our model aimed to maximize simplicity rather than exactness, it still provides a very accurate representation of tsunamis hitting various cities in the United States.

Problem B Summary: Illinois Mathematics and Science Academy

Adviser: Steven Condie
Team members: Paul Chung, Bonny Jain, Andrew Lee, Sid Narayanan

In this paper, we demonstrate a mathematical model that evaluates the economic effects of tsunamis on a number of cities in the United States. We subdivide the problem into three parts: (1) a theoretical physics-based model to determine the height and speed of a tsunami given earthquake parameters; (2) a computational simulation to determine the physical impact of a tsunami on a target city; (3) a statistics-based model to determine ultimate economic effects.

The earthquake parameters of intensity on the moment magnitude scale and epicenter-target city distance are used to calculate the tsunami parameters of height and speed. Tsunami height and speed upon reaching the shore are input into a tsunami simulation program that returns the area impacted by the tsunami. The area of impact is then used along with economic statistics (such as property values) to calculate the net economic damage caused by a tsunami.

We evaluate our model for certain earthquake parameters in seven cities: New York City, Boston, Virginia Beach, Charleston, New Orleans, San Francisco, and Hilo. Our results indicate that the potential economic costs of a tsunami range from tens of millions for smaller cities hit by small-scale tsunamis to several billion dollars when densely populated urban areas are hit by larger tsunamis. The results of our case study simulations are presented in the table below.

City	Epicenter-Cit Distance (km)	Earthquake Intensity (moment magnitude)	Net Cost of Damage
New York	3000	9	$3,608,965,061
San Francisco	500	9	$2,466,813,857
Charleston	3500	9	$174,508,083
Boston	3500	9	$6,577,758,050
Hilo	1000	9	$31,087,243
New Orleans	1000	8	$62,856,820
Virginia Beach	3000	7	$32,273,723

Problem B Paper: International School of Duesseldorf

Adviser: Philip Grant
Team members: Marko Kim, Michael Mager, Martin Shih, Konstantin Toennesmann

A. INTRODUCTION

Our investigation analyzed the mathematical background of shallow-water waves and established a link between yield energy of earthquakes caused by converging tectonic plates under the assumption that this energy is transferred into the kinetic energy of the wave produced. Also, we took each wave impulse to be an individual body of water that struck at one instant in time. However, we found that a mathematical model would only reveal a definite relationship with the devastation caused by the impact of a wave by adapting a broad and ill-defined group of variables. Thus, we regarded this as a theoretical basis. We subsequently assembled historical data and established a trend among several factors that are indicative of the scope of destruction.

These factors include population density and city area. We found the independent and dependent variables from major historical tsunamis and plotted them to find a relationship. Then we predicted what a strong tsunami would cause the cities given.

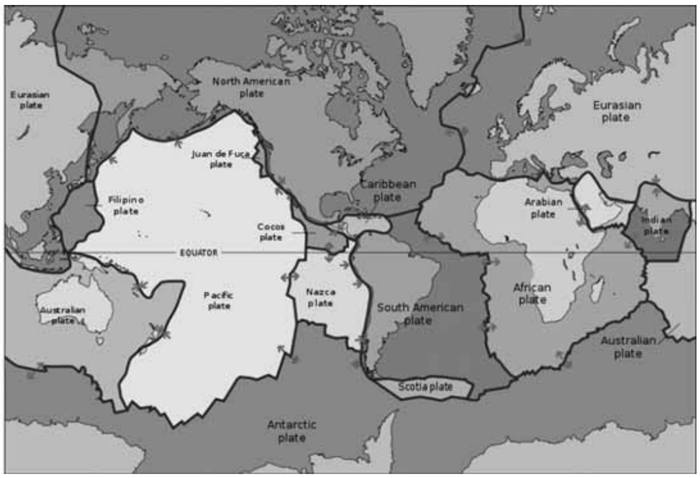
B. OUTLINE OF THE CITIES

The following is an overview of the cities we analyzed.

City	Avg. Altitude (m)	Population	Population Density (people / km^2)	GDP ($ per capita)	City Coastline (km)	City Area (km^2)
New York	0	8,274,527	10.606	52,800	40	1214.4
San Francisco	15	808,976	17,323	62,300	23	600.7
New Orleans	–3	311,853	973	38,800	40	907
Charleston	2	126,567	385	29,600	–	376.5
Hilo	18	40,000	290	38,100	–	141
Boston	43	609,023	4,860	58,000	–	232.14
Lima	43	7,605,742	2,846.1	7,600	40	2672

Our city of choice is Lima, Peru. We chose Lima because it has been exposed to significant tsunamis and thus provides a useful comparison.

C. MAIN EPICENTERS NEAR THE CITIES



This map shows tectonic plates that cause earthquakes. These earthquakes can cause tsunamis, so places where two plates meet are likely epicenters of tsunamis. Because the edge nearest to the east coast of the U.S. is relatively far away, tsunamis would have to travel quite far. Tectonic plates in the Atlantic are divergent, so they are unlikely to cause tsunamis. However, those in the Pacific are convergent, meaning that one sub-ducts another, causing tsunamis. Off the west coast of South America, for example, two plates meet, making it a fairly high-risk zone compared to the east coast of the U.S.

D. CAUSE FOR EARTHQUAKES AND TSUNAMIS

Maximum values for tsunamis are approximately:

Wave speed (open ocean)	200 ms^{-1}
Wave length	100 km
Wave period	3600 s
Run-up height	30 m
Distance of inland flooding	hundreds of meters

Water waves can be categorized as deep, intermediate, or shallow. Particularly important is that water molecules move in an elliptical cycle when a wave passes. At a certain depth the particles no longer move when a waves pass. This makes the waves of a tsunami both longitudinal and transverse.

Tsunami waves may be modeled with deep-water wave properties (The wavelength is at least half the wavelength depth; i.e., the depth of impact of the wave on a body of water is half the wavelength). This means that the speed of waves is solely dependent on the wavelength of the wave, as gravitational acceleration remains constant.

$$v = \sqrt{\frac{g\lambda}{2\pi}}$$

As they approach land, tsunami waves transform to shallow water waves; i.e., while at sea the wavelength of a tsunami is large, and it can thus transfer large amounts of energy over large distances, the wavelength of the wave shortens significantly. The tsunami wave slows down and the kinetic energy transforms to potential energy and the amplitude of the wave builds up.

$$v = \sqrt{gd}$$

In terms of plate tectonics, if one plate sub-ducts under another than the energy released will not only cause an earthquake but transfer most energy to the water.

$$v = \sqrt{gd}$$

$$E_k = \frac{1}{2}mv^2$$

$$m = Vp \qquad V = hlv \qquad m = hlvp$$

$$E_k = \frac{1}{2}hlvpv^2 = \frac{hlpv^3}{2} = \frac{hlp\left(gd\right)^{\frac{3}{2}}}{2}$$

$$E_g = \frac{g^{\frac{3}{2}}p}{2}hld^{\frac{3}{2}} \qquad E_g \propto hld^{\frac{3}{2}}$$

Therefore, the energy of the wave depends on the height of the wave, the length of the coastline, and the velocity of the wave. The density of water is constant. This means that a city with a longer coastline would suffer more from the effects of a tsunami.

Earthquake Magnitude to Wave Energy Produced:

Richter Magnitude	TNT for Seismic Energy Yield	Example
–1.5	6 ounces	Breaking a rock on a lab table
1.0	30 pounds	Large blast at a construction site
2.0	1 ton	Large quarry or mine blast
4.0	1000 tons	Small nuclear weapon
4.5	5100 tons	Average tornado (total energy)
6.0	1 million tons	Double Spring Flat, NV quake, 1994
6.5	5 million tons	Northridge, CA quake, 1994
7.0	32 million tons	Hyogo-Ken Nanbu, Japan quake, 1995
8.0	1 billion tons	San Francisco, CA quake, 1906
8.5	5 billion tons	Anchorage, AK quake, 1964
9.0	32 billion tons	Chilean quake, 1960
10.0	1 trillion tons	(San-Andreas type fault circling Earth)

Magnitude	Weight of TNT (tons)	Energy per Ton of TNT (joules)	Total Energy Produced (joules)
–1.5	0.0003125	4184000000	1307500
1	0.018575	4184000000	77717800
1.5	0.2	4184000000	836800000
2	1	4184000000	4184000000
2.5	4.6	4184000000	19246400000
3	29	4184000000	1.21336E+11
3.5	73	4184000000	3.05432E+11
4	1000	4184000000	4.184E+12
4.5	5100	4184000000	2.13384E+13
5	32000	4184000000	1.33888E+14
5.5	80000	4184000000	3.3472E+14
6	1000000	4184000000	4.184E+15
6.5	5000000	4184000000	2.092E+16
7	32000000	4184000000	1.33888E+17
7.5	160000000	4184000000	6.6944E+17
8	1000000000	4184000000	4.184E+18
8.5	5000000000	4184000000	2.092E+19
9	32000000000	4184000000	1.33888E+20
10	1E+12	4184000000	4.184E+21
12	1.6E+14	4184000000	6.6944E+23

We found an equation using best-fit line on a calculator to match the data by 99.95%: Magnitude to yield energy = 19316684.22*24.73*x*. This equation must be divided by two, since the wave of energy splits into two at the fault, sending waves in either direction.

E. ASSUMPTIONS AND LIMITATIONS

Assumptions:

- Depth of ocean from epicenter is constant until sandbank is reached.
- A wave does not lose energy until reaching a coastline; i.e., total energy is constant. This depends on the strength of the earthquake.
- All yield energy of the quake is transferred to the water.
- The cause of the earthquake and thus tsunami is identical for all situations. (This is a slipping in the mega-thrust fault; i.e., this is taken as the epicenter.)
- The source of a tsunami is a point on a fault line; i.e., the source is at one point.
- A tsunami is considered as one block of water where all waves are combined.

Limitations:

- Square Waves used for calculations.
- Coastline assumed to be linear and parallel to the tsunami.
- Inaccuracies in the measurement of coastlines.
- Used average depth of ocean in calculations.

Realization:

These calculations follow the idea that the strength of impact of the tsunami depends on the magnitude of the earthquake. However,

Instead:

We look at the:

1. Population density to death toll of a tsunami.
2. Area of city to mass of destruction (quantity of homes destroyed).

By looking at these two relationships and researching past tsunamis, we determined the effect on each city of a strong (7 and above) tsunami.

F. MODEL

New Assumptions and Limitations:

Assumptions:

- Water depth is consistent in all areas.
- Earthquakes occur the same distance away from a city in all cases.
- Sand bank leading to land is the same in gradient and distance.
- Abnormal waves formed by tsunamis are the same in shape, height, and volume and that the general nature of the wave is constant.
- Land level above water in all cities is the same.
- Population density directly affects number of people who die.
- Area of city directly affects the quantity of homes destroyed.
- The area of the city exposed to the ocean is constant.

Limitations:

- Land level above water is inconsistent in a city and can't be measured.
- Sank banks have various gradients that can affect the nature of a tsunami wave including strength of impact.
- Reflected waves and alternative movements of waves cannot be measured when a wave hits an obstacle before a city.

- The strength of a wave cannot entirely be determined directly from the earthquake magnitude due to previously mentioned limitations.

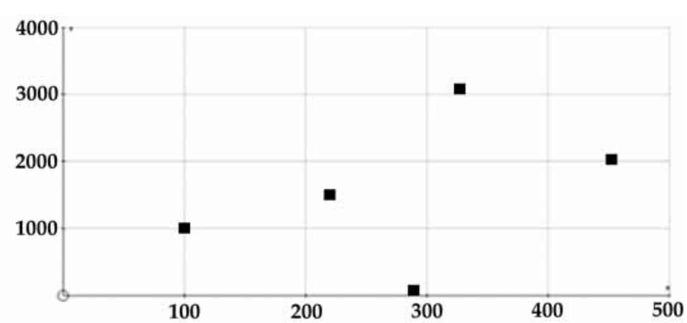
Data Collection:

We collected data by looking at previous earthquakes in Japan and one in Hawaii that had an initial earthquake of greater than magnitude 7. Since both areas are islands in the Pacific, more commonalities between the scenarios can be made in order to make a connection between the variables in the desired equations.

	Hilo, HI	Honshu, Japan	Showa, Japan	Nankaido, Japan	Sanriku, Japan
Richter Scale	9.5	7.2	8.1	8.4	8.4
Land Area (km²)	141	227962	1000	18800	140500
Population density (People per km²)	290	451.8	327	220	100
Casualties	61	2000	3068	1500	1000
Homes destroyed	541	40,000	5000	1451	7000
Year of impact	1960	1896	1933	1946	1933

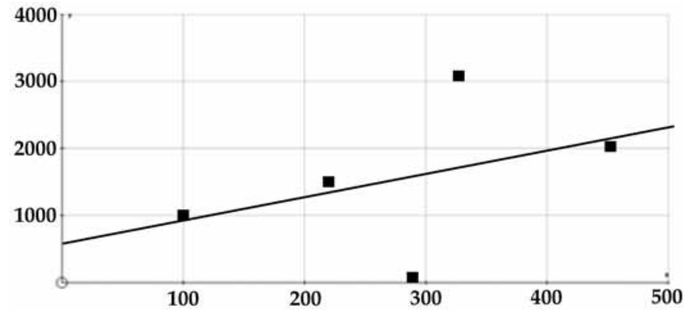
Population Density to Death Total:

	Population density (km²)	Casualties
Sanriku, Japan	100	1000
Nankaido, Japan	220	1500
Hilo, Hawaii	290	61
Showa, Japan	327	3068
Honshu, Japan	451.8	2000



The data show an upward trend: the greater the population, the greater the casualty. Hilo is an outlier that could be due to elevation.

We put the data in a TI-84 and found a best-fit line.



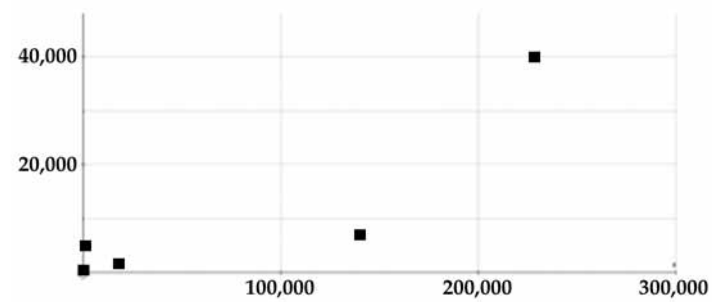
Quantity of Deaths = 561 + 3.47(Population density)
Match to Data: 40%

This equation says that as population density increases by 1, total deaths per tsunami increases by almost 3.5 people. And, per tsunami 561 people die. A flaw with this equation is that with a density of 0, people still die.

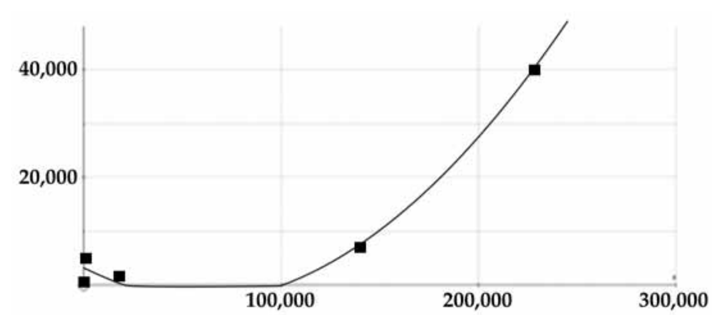
Now we must find an equation for damage to homes in relation to the area of the city.

Area of City Equation to Quantity of Homes Destroyed:

	Land Area (km²)	Homes Destroyed
Hilo, Japan	141	541
Showa, Japan	1000	5000
Nankaido, Japan	18800	1451
Sanriku, Japan	140500	7000
Honshu, Japan	227962	40,000



Using the same method, we find the best-fit line that describes the trend.



Quantity of houses destroyed = $0.0000015x^2 - 0.1821x + 3285$
Accuracy to data: 98.9%

This equation also describes an upward trend. However, the graph dips into negative values, which is inaccurate because houses cannot appear when tsunamis occur. So when a value is negative, we assume that the number of houses destroyed is 0. To solve the problem, the x-value will have 100000 added to it since that is when $x = 0$ and all values past that do not go below 0, so an obvious trend can then be used.

So which city would suffer the worst effects if a strong tsunami hit?

Data of Each City:

Fatalities = 561 + 3.47(Population density)

Home destroyed = $0.0000015(\text{area} + 100000)^2 - 0.1821(\text{area} + 100000) + 3285$

	Total Fatalities	Homes Destroyed
Hilo	1567	120.72
Charleston	1897	148.8
New York	37,364	250.06
Boston	17,425	131.57
New Orleans	3937	212.66
San Francisco	60,672	175.69
Lima	10,437	431.24

Interpreting the Results:

We can see that San Francisco is the worst hit. Hilo is least affected, both in terms of casualties and homes destroyed. However, in terms of house destruction, Lima is most affected, followed by New York. In this rating, San Francisco isn't as vulnerable as the two just named. This may make it seem like there is no one city most affected. However, because lives are more important than houses, it is sensible to say that San Francisco is hardest hit, because its potential death toll exceeds any other city. Despite Hilo having the least devastating statistics, it has to be put into perspective that it is the smallest city; therefore the fatalities might be a high percentage of its small population.

Conclusion:

Through this investigation we attempted to understand the nature of tsunamis and their effects on different cities. Although at first we tried to generate a model representing underwater earthquakes caused by tectonic plates moving and the amount of energy they release, we later turned to historic data. We initially focused on a very mathematical model, deriving the kinetic energy of waves depending on various factors, but realized that this exceeded our ability. So we changed the course of our investigation to predict the future by analyzing major past tsunamis. Although this forced us to leave out some details, it enabled us to compare the cities effectively. We defined devastation by fatalities and destruction of homes. We found that San Francisco would have the highest number of fatalities, while Lima would have the highest number of houses destroyed. The high death toll in San Francisco suggests that it would be the most significantly affected if a high-magnitude were to hit.