Team 2031 Summary Sheet

A dire financial crisis has broken out on both Wall Street and Main Street U.S.A. – but what of the one on Pennsylvania Avenue? In this solution paper, we created a simplified model for national debt, $d_t = d_{(t-1)} * (1 + s_t) * (1 + f_t) + (e_t - i_t)$. Not only does our equation factor in federal expenditures and income, but it also takes into consideration the rate of inflation as well as the interest rate levied on national debt. By using this model, we'll analyze the opposing economic policies of President-Elect Barack Obama and Senator John McCain, and propose a plan of our own that integrates the best of both worlds.

"*Blessed are the young, for they shall inherit the national debt.*"
–Herbert Hoover

RESTATEMENT OF THE PROBLEM

Exit polls during Election Day found that 62% of voters regarded the economy as their top issue for the next presidency. (CNN) Having won the election, Barack Obama now bears an immense burden – the future of the American economy, and, by extension, that of the entire world. Naturally, one of the most prominent issues he'll have to deal with is the problem of national debt. In this paper, we will develop a simplified model that mathematically explains national debt and how it changes over time according to a number of variables. We will then compare two different economic proposals using our model, analyzing how each plan would impact national debt and the nation in general. The findings of this study will be summarized and presented in a letter to President-Elect Obama.

ASSUMPTIONS

(1) That the aforementioned "national debt" refers to gross debt, the total sum of public debt and the debt held by government accounts. Specifically, we are referring to the number shown on the U.S. National Debt Clock.

(2) That interest rates on said national debt are compounded annually. As we will be looking at our variables (e.g. national debt, population growth, GDP, etc.) on a year-by-year basis, this approach eliminates the risk of potential complications and inconsistencies with the conversion of units.

(3) That government expenditures precede payments on the national debt. That is, the U.S. government will spend its money as its policies require before trying to pay off the national debt. In a real-world context, this assumption stipulates that the government would cater to all of its citizens' immediate needs before considering long-term solutions to the debt crisis. It allows us to simplify the equation by ignoring

the possibility that the government might partition funds between expenditures and debt payments.

(4) That all transactions are conducted in USD, thereby eliminating exchange rates from our calculations. For one, they are ever-fluctuating variables that are difficult to model accurately. Moreover, considering the diverse host of countries in which U.S. debt is held, we would have to create a separate model for the changing exchange rate of each currency. This is impractical and unreliable for long-term extrapolation.

(5) That the rate of population growth will change at a steady rate. In order to minimize variability and eliminate any confounding variables, we will assume that there will be no significant population-altering “wild card” events (e.g. war, natural disasters, famine/drought, etc.) for the years 2009–2017.

(6) That the plans we propose be implemented for the entire duration of the 2009–2017 period, despite President-Elect Obama’s two-term limit. The task stipulates that we analyze the effects of our plans over the entire 9-year period; thus, we must at the very least assume that the plans will stay in place for the specified duration.

(7) That our proposals only encompass fiscal and trade policies. We exclude monetary policies from our plans, as they fall under the jurisdiction of the Federal Reserve as opposed to the executive or legislative branches of government.

(8) That changes in tax policy will not affect the net GDP. A tax cut, for example, would reduce federal income but cause an increase in consumption among citizens that would compensate for the loss. As a result, there would be no significant net change in the GDP. Thus, our model for GDP will only consider the explanatory variable of time and will not take into account the tax policy adopted.

(9) That no unpredictable “wild card” events (e.g. new wars, further bailouts, depletion of fossil fuels, etc.) requiring excessive government spending or significant tax reforms will occur between 2009 and 2017. Again, this is meant to reduce variability by excluding any confounding variables. The values for both expenditures and income will change at steady rates – as specified in our proposal – over the whole 9-year period.

(10) That the current level of employment remains constant over the 2009-2017 period. We cannot forecast unemployment or job creation because they would introduce a whole host of new confounding variables. By excluding these factors completely, we can simplify and clarify the model to carry out its purpose: to explain national debt.

(11) That the current economic and credit system will continue to exist without significant fundamental changes. As our mathematical models will be based on retrospective data, we must assume that the variables will continue to follow the trends that they have up until now. To this end we will base our models solely on data from the past 8 years, since the start of the Bush administration. This is because the tax/spending policies vary too greatly from presidency to presidency. Therefore, it would be impractical to try and create models encompassing data from multiple administrations. The population would be the exception to this rule, as presidential policies would have little to no effect on population growth.

DEFINITION OF VARIABLES

To go about creating a mathematical model that explains national debt, we must start by defining the variables involved. Let

- d = national debt
- i = federal income
- e = federal expenditures
- g = gross domestic product
- s = interest rate
- n = interest value
- p = population
- t = time
- f = rate of inflation
- c = CPI (Customer Price Index)

In actuality, the national monetary values (variables d, i, e, g, n) tower into the trillions. However, to simplify our calculations, we can truncate the numbers we're dealing with. By measuring these values in billions of USD, we avoid unmanageably large numbers without significantly compromising the accuracy of the data.

As variable p will be used to calculate GDP per capita (per person), we will not apply the same abbreviation technique to it. We will measure population in its most basic unit – the number of people. In this way, there will be fewer units to convert when calculating GDP per capita (g/p), thereby reducing the room for error.

Interest rate s and rate of inflation f are both proportions that model change; therefore, they should each be lower than 1.

Time t will be measured in years since 2001, as per assumption (6). For some of these variables, we will convert t into quarters (t/4) to improve accuracy during intermediary steps. However, in the final mathematical model all values of t will be expressed in years. To preserve continuity, we will only base our models off statistics gathered during the Bush presidency. These numbers best represent the current financial situation of the nation and the ongoing trends that need to be addressed.

MODEL BUILDING DISCUSSION

With the current financial crisis, the economy is arguably more volatile than ever before. Economists and analysts have been unable to forecast and project numbers accurately. Our current financial system is based on a multitude of inextricably intertwined variables where the boundaries of explanatory and response variables become unclear. With economics, no variable is truly independent and free from confounding. Our model seeks to simplify these relationships and pick out the key variables as outlined above. By establishing logical connections between the variables, we can attempt to forecast how the national debt might change over time and how different tax/spending policies would affect that number.

As stated in the problem, “the rate at which the national debt changes depends on the difference between federal income (primarily taxes) and federal expenditures.” In terms of the variables we have defined, this value is = e – i. However, we must also take into account the interest from the previous year. Thus:

$$d_t = d_{(t-1)} * (1 + s_t) + (e_t - i_t)$$

In words, we are taking the national debt from one year prior and adding in the interest for that amount. To this we add the difference between federal income and expenditures to find the national debt for that year.

First, let us start out by determining how the rate of interest levied on national debt changes with time. We will develop a model that will allow us to forecast interest rates in the future. We can do this by comparing the annual levels of national debt with the annual amounts of interest paid. The equation would be as such:

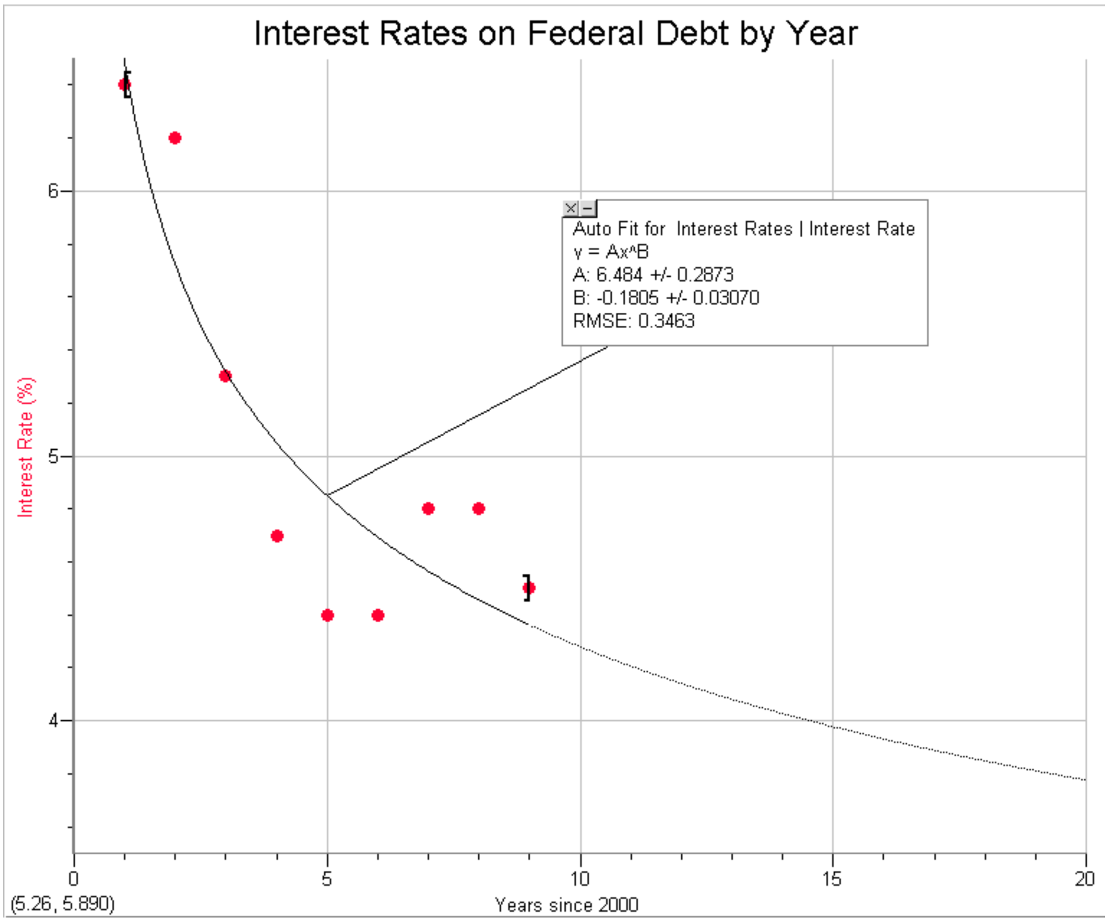
$$s_t = d_t / n_t$$

Table 1: Interest Rates (U.S. Department of the Treasury)

Year	Years since 2000	National Debt	Interest paid	Interest Rate
2000	1	5,674,178,209,886.86	$361,997,734,302.36	.064
2001	2	5,807,463,412,200.06	$359,507,635,242.41	.062
2002	3	6,228,235,965,597.16	$332,536,958,599.42	.053
2003	4	6,783,231,062,743.62	$318,148,529,151.51	.047
2004	5	7,379,052,696,330.32	$321,566,323,971.29	.044
2005	6	7,932,709,661,723.50	$352,350,252,507.90	.044
2006	7	8,506,973,899,215.23	$405,872,109,315.83	.048
2007	8	9,007,653,372,262.48	$429,977,998,108.20	.048
2008	9	10,024,724,896,912.49	$451,154,049,950.63	.045

Model 1.1: Power

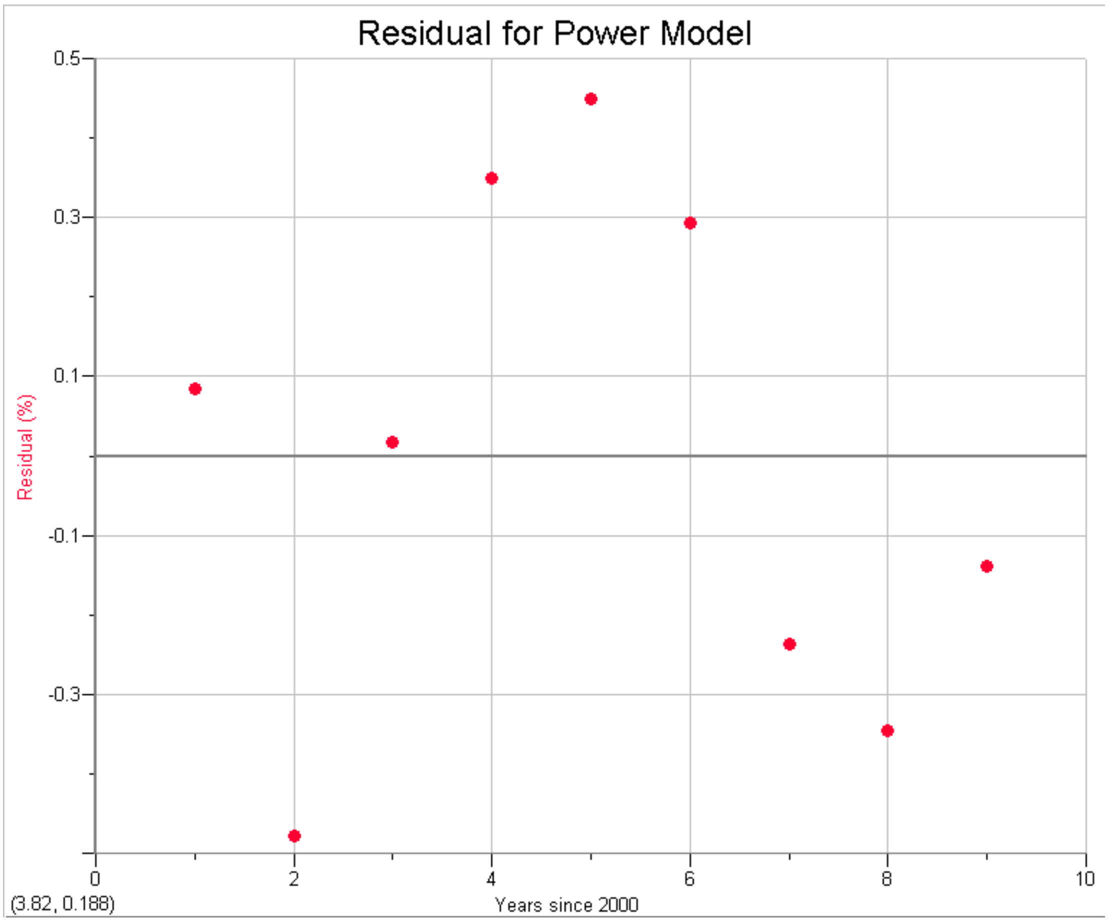
Years since 2000	Interest Rate (in %)
1	6.48403644578
2	5.72162664308
3	5.31790876172
4	5.04886296007
5	4.84958459057
6	4.69261527306
7	4.56387009409
8	4.45520457375
9	4.36150441695
10	4.27935787118
11	4.20638092923
12	4.14084541267
13	4.08146061571
14	4.02723842537
15	3.97740645052
16	3.93135007842
17	3.88857265804
18	3.84866742879



This graph shows a relative correlation between time and interest rates. Although the graph represents the general trend of this relationship, there are still significant deviations between the actual points and the calculated points.

The equation that is graphed above shows the relationship between percentage interest rate and time. To convert this percentage into decimals, we divide the equation by 100, to get:

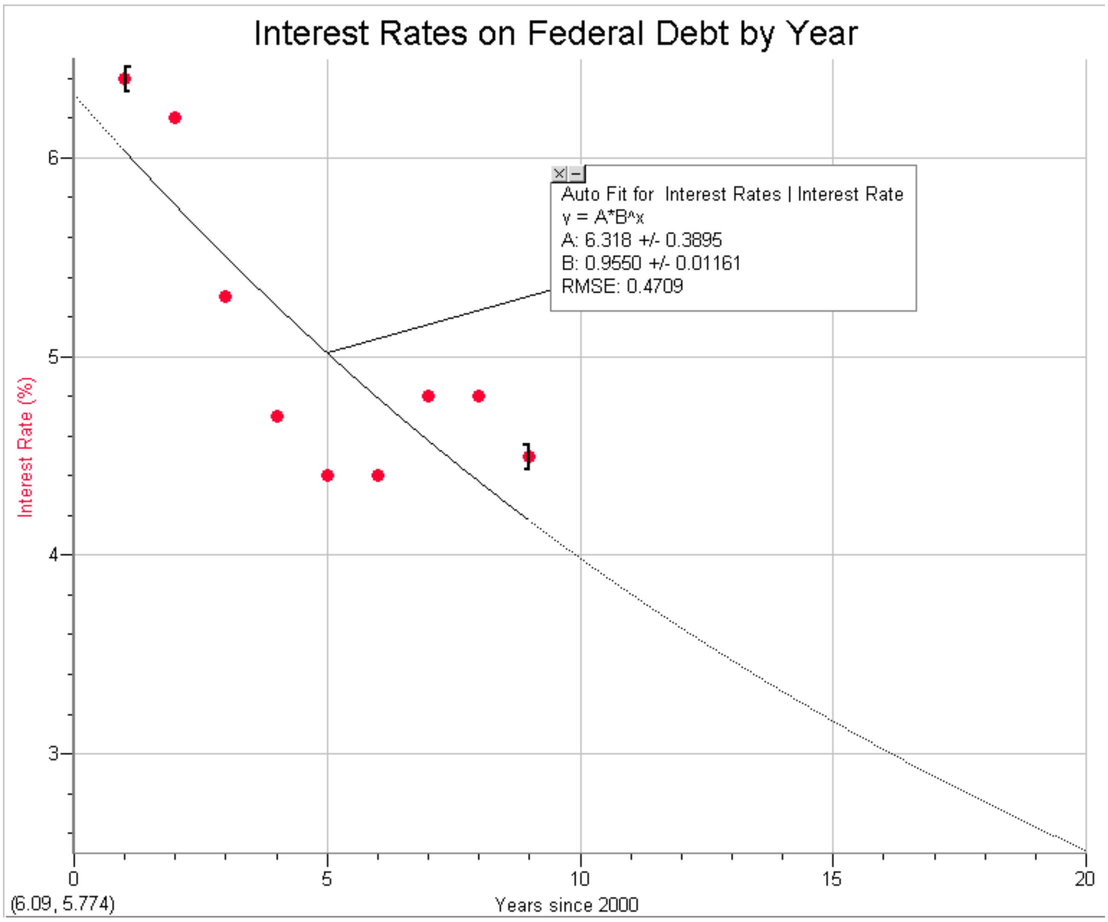
$s = .06484t^{-.1805}$



This residual plot shows that there are no patterns, but there are still large differences between the calculated and actual data points. This model, although somewhat accurate, can be improved upon.

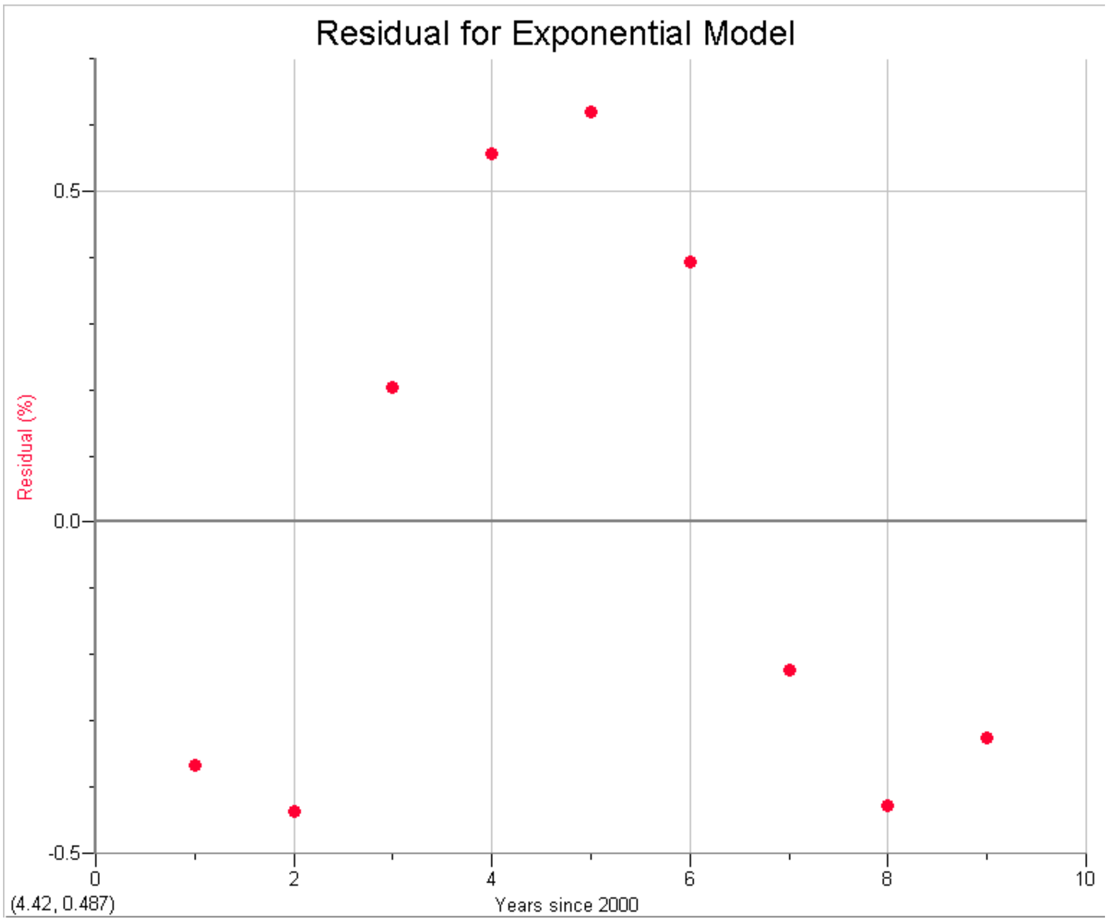
Model 1.2: Exponential

Years Since 2000	Interest Rates (in %)
1	6.03371169148
2	5.76191483097
3	5.50236143471
4	5.25449997898
5	5.01780378419
6	4.79176989577
7	4.57591801545
8	4.36978948062
9	4.17294628979
10	3.98497017183
11	3.80546169723
12	3.63403942932
13	3.47033911376
14	3.31401290458
15	3.16472862499
16	3.02216906155
17	2.88603129017
18	2.75602603236



This equation also accurately represents the general trend of the relationship, but the results are unevenly distributed along the curve and there are still significant differences between the actual data points and the calculated points. Again, this equation can be converted to a decimal proportion by dividing it by 100. This would result in the equation:

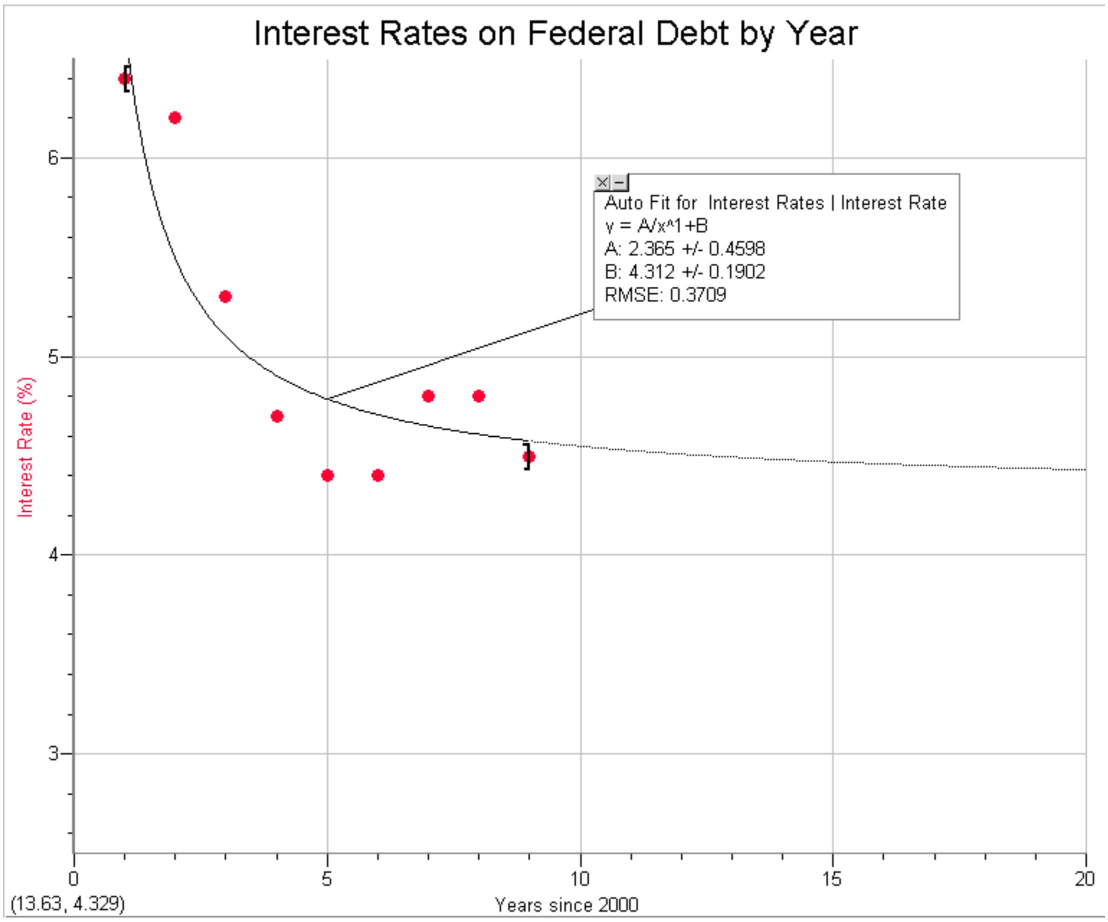
$s = .06318(.9550)^{t}$



This residual graph also shows an uneven distribution of the points and shows a slight pattern, when the residual points are plotted out. This indicates that there is a better model for this.

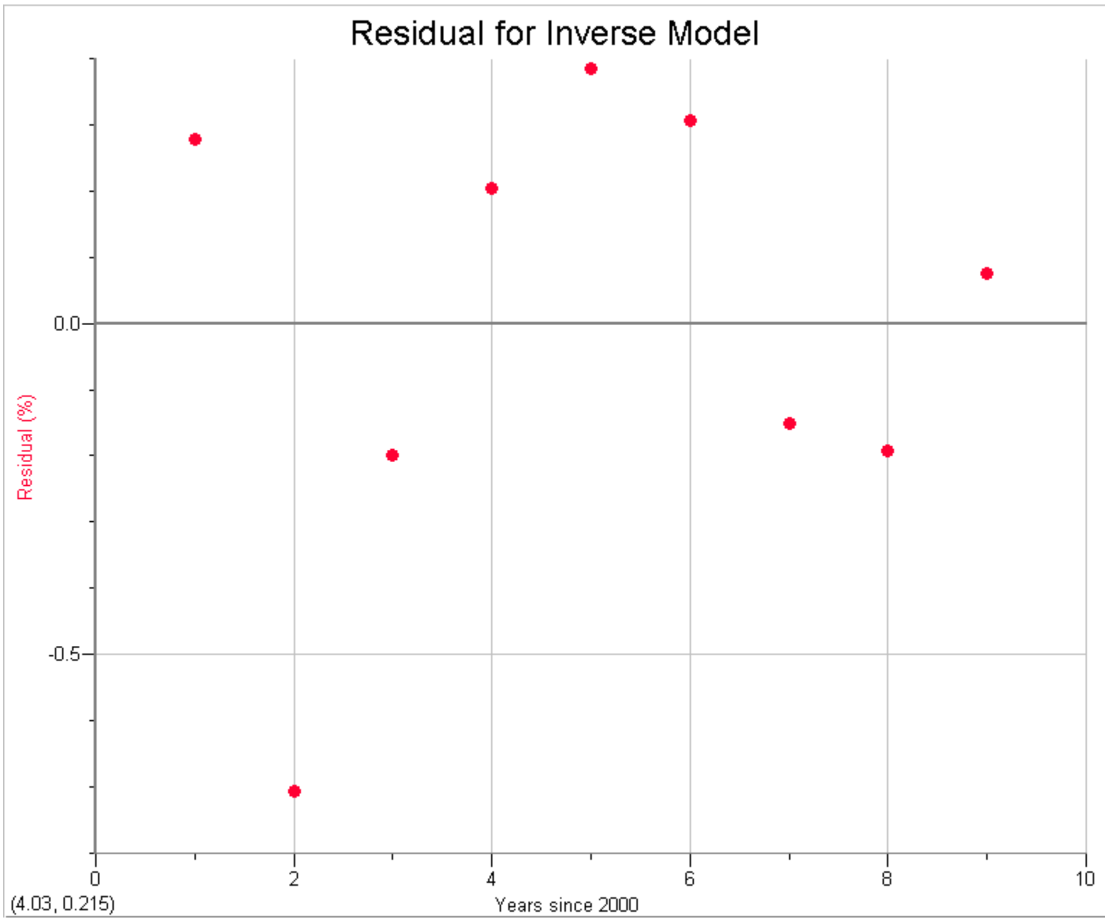
Model 1.3: Inverse

Years Since 2000	Interest Rates (in %)
1	6.67735962125
2	5.49471811235
3	5.10050427604
4	4.90339735789
5	4.785133207
6	4.70629043974
7	4.64997417741
8	4.60773698067
9	4.57488582764
10	4.54860490522
11	4.52710233233
12	4.50918352159
13	4.49402145096
14	4.48102539043
15	4.46976213796
16	4.45990679205
17	4.45121089861
18	4.44348121554



This graph shows strong correlation between the actual data points and the model equation. The equation represents the general trend of the relationship and there is an even dispersion of data points on either side of the curve. To convert s to a decimal value, we can simply divide the equation by 100, producing:

s = .02365/t + .04312



From the residual plot, there are no discernable patterns that can be observed. Again, the points are evenly distributed on either side of the x-axis. Even though there is a data point that seems to be to be an outlier, the deviation is, in actuality, very small when taken into context.

From the three equations we came up with, the one that most suitably models the growth rate of the interest rate on national debt was the inverse function. Accordingly, we will use said equation to forecast the interest rates in future years:

$$s_t = .02365/t + .04312$$

However, our model does not yet take into account the effects of inflation. The Consumer Price Index (CPI) is an index of the 12-month average price of products consumed by households that changes over time. The inflation rate f for any given year can be calculated by comparing the CPI of that year with that of the previous year, as so:

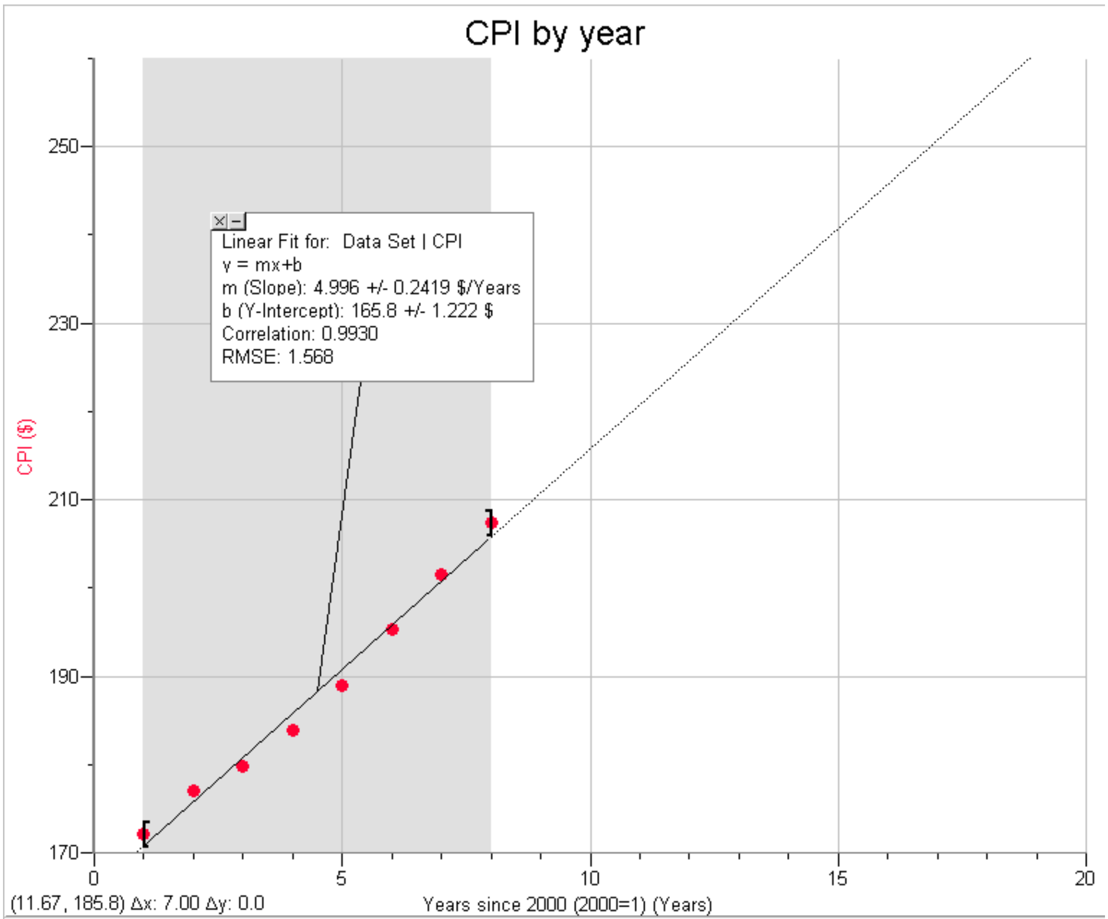
$$f = (c_t - c_{(t-1)})/c_{(t-1)}$$

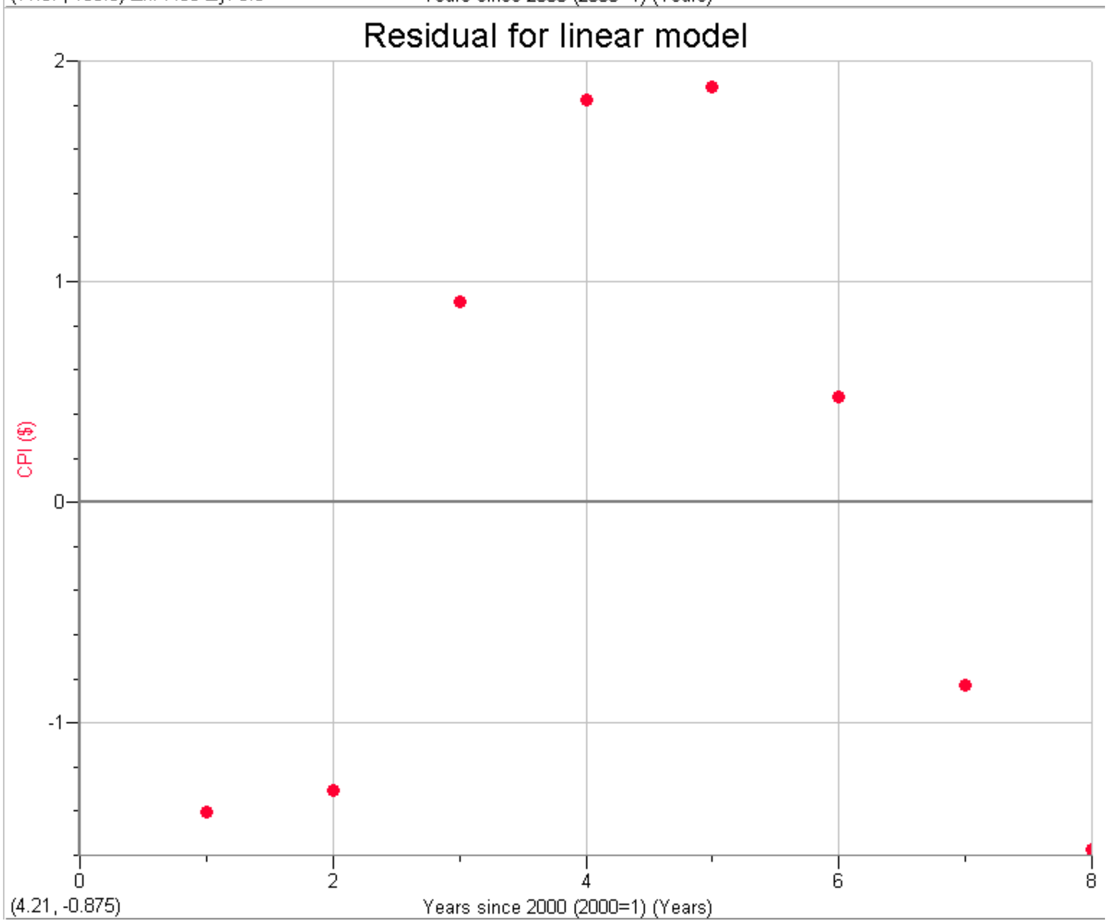
Table 2: Consumer Price Index (Financial Trend Forecaster)

Years since 2000	**Price**
1	172.2
2	177.1
3	179.88
4	183.96
5	188.9
6	195.3
7	201.6
8	207.34

Model 2.1: Linear

Years Since 2000	**Price**
1	170.798333333
2	175.79452381
3	180.790714286
4	185.786904762
5	190.783095238
6	195.779285714
7	200.77547619
8	205.771666667
9	210.767857143
10	215.764047619
11	220.760238095
12	225.756428571
13	230.752619048
14	235.748809524
15	240.745
16	245.741190476
17	250.737380952
18	255.733571429


CPI by year
Linear Fit for: Data Set | CPI
y = mx+b
m (Slope): 4.996 +/- 0.2419 $/Years
b (Y-Intercept): 165.8 +/- 1.222 $
Correlation: 0.9930
RMSE: 1.568
CPI ($)
250
230
210
190
170
0
5
10
15
20
(11.67, 185.8) Δx: 7.00 Δy: 0.0
Years since 2000 (2000=1) (Years)

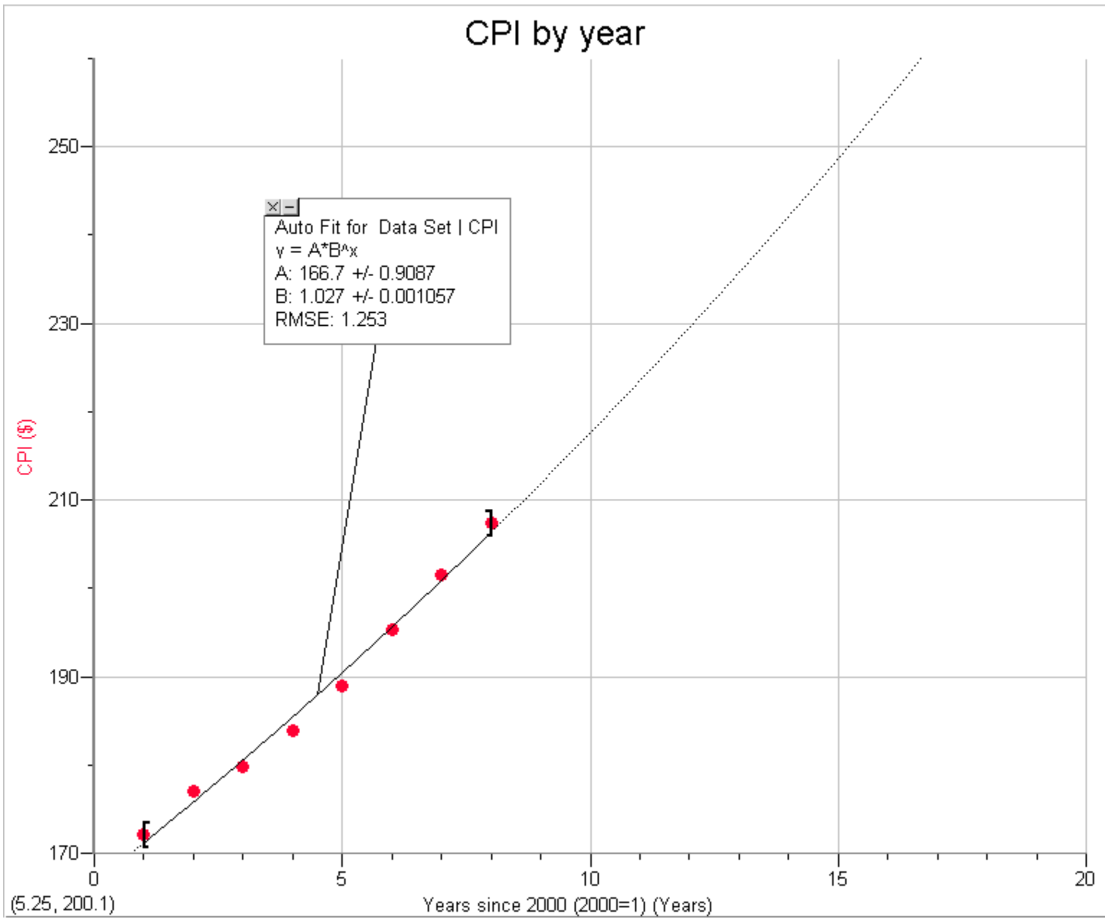

Residual for linear model
CPI ($)
2
1
0
-1
0
2
4
6
8
(4.21, -0.875)
Years since 2000 (2000=1) (Years)

$c = 4.999t + 165.779$

The linear regression relating time and CPI is strong and the line represents the actual data well. The correlation is .993, which shows a strong relationship between the two variables. The residual plot also shows a slight parabolic pattern. This suggests that there may be a better model that is available.

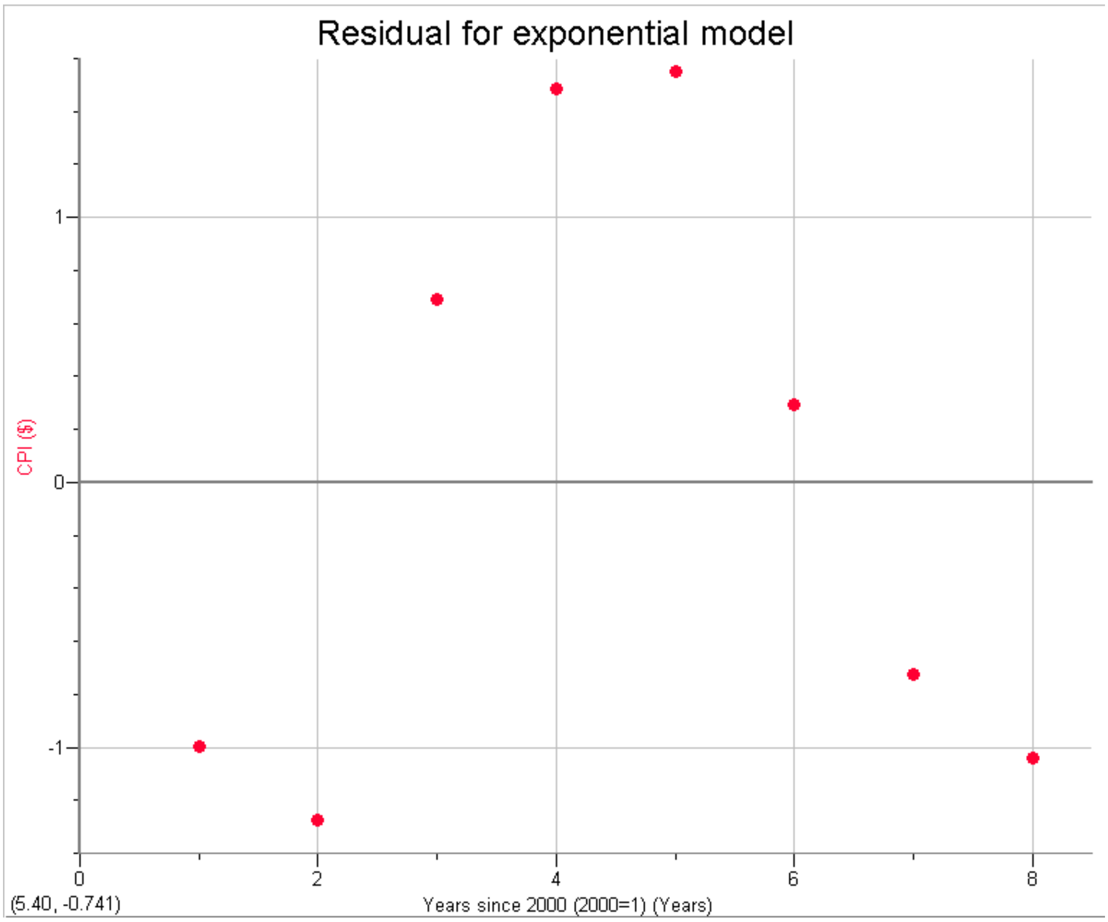
Model 2.2: Exponential

Years Since 2000	**Price**
1	171.18241921
2	175.809640008
3	180.561938908
4	185.442696891
5	190.455386323
6	195.603573437
7	200.89092086
8	206.321190226
9	211.898244849
10	217.626052471
11	223.508688087
12	229.550336843
13	235.755297011
14	242.127983053
15	248.672928755
16	255.394790457
17	262.298350364
18	271.18241921



$c = 166.832(1.027)^t$

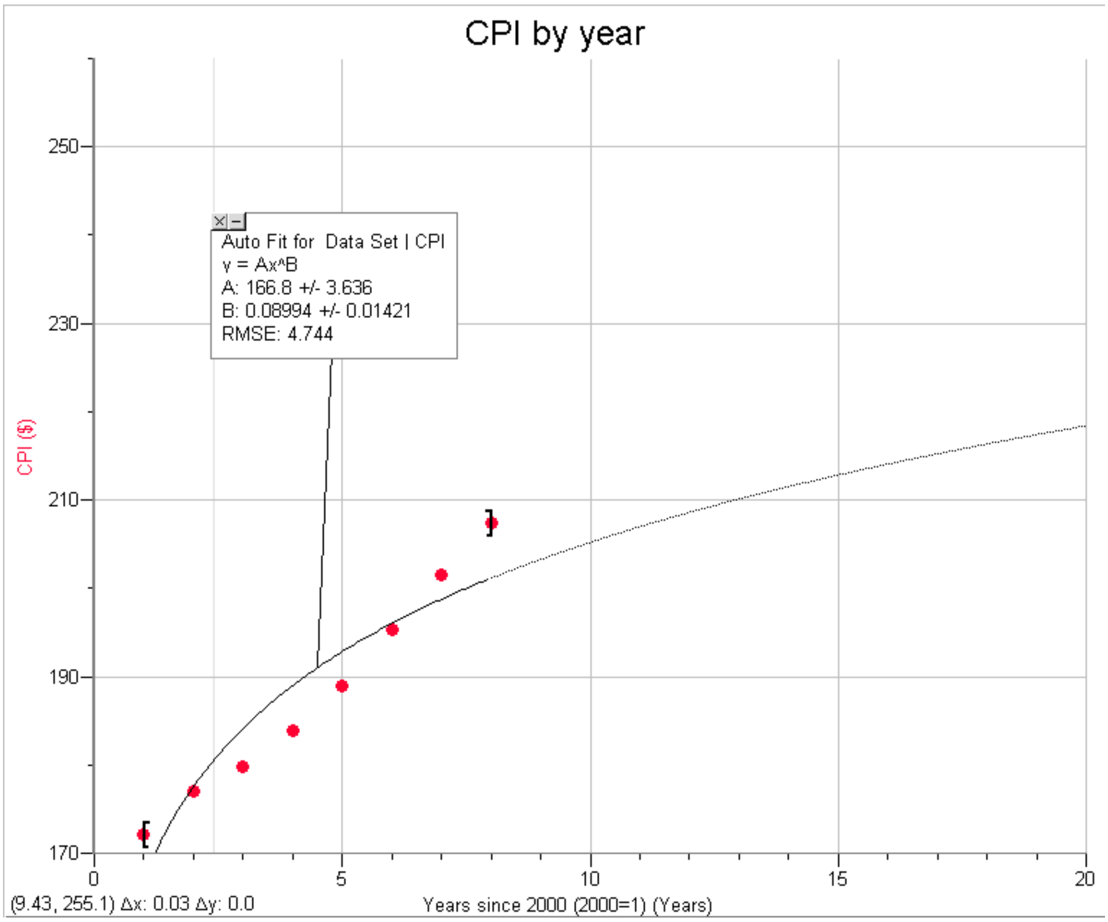
The points fit the exponential model very closely, with the same points falling above and below the line as the linear model. Both have relatively close RMSE values – it is difficult to assess which is the more appropriate model. However, we can determine a correlation coefficient for the exponential model by linearizing the data. This is done by performing a linear regression on (t, log(c)). The calculator diagnostics show that this new LSRL has a r-value of .995; thus, we can conclude that the exponential model is a better fit than the linear model.



The residual plot further reinforces the argument that the exponential model is a better fit for the data. While a parabolic pattern is still readily apparent in the residuals, it is much less smooth and consistent than in the residuals of the linear function.

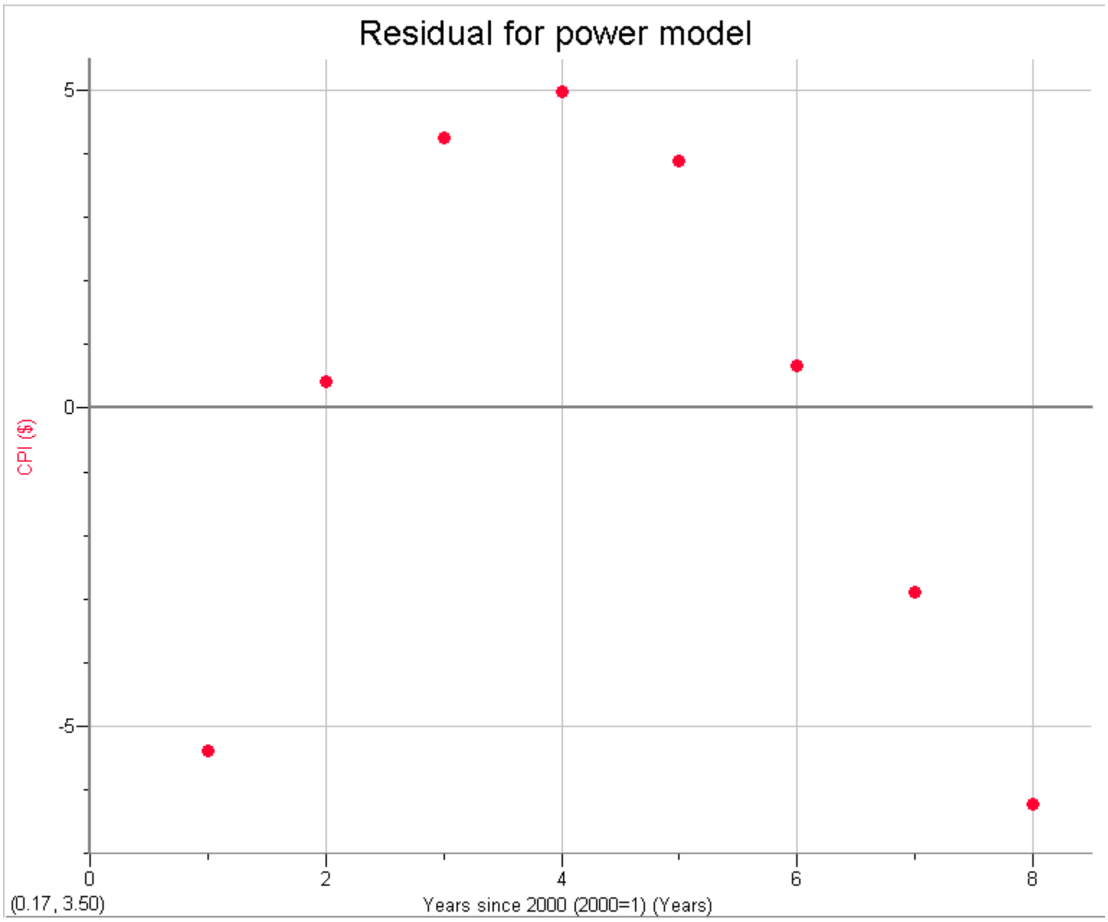
Model 2.3: Power

Years Since 2000	**Price**
1	166.830664618
2	177.562000342
3	184.156643554
4	188.983626227
5	192.814684998
6	196.002468014
7	198.738782743
8	201.139944995
9	203.28198921
10	205.217435547
11	206.984129461
12	208.610271812
13	210.117457164
14	211.522599218
15	212.839200145
16	214.078215559
17	215.248660773
18	216.358046168



$c = 167.591t^{.0864}$

The power regression used here shows very little correlation between the two points. From the graph, we can see that the rates of change for the actual data points are continually increasing, while the rate of change for the regression line is continually decreasing. The actual data points and the regression line are almost inverses of each other.



This residual graph shows a clear discernable parabolic pattern, which indicates that this regression is not suitable for the given data set.

Thus, the exponential model was the best fit for the actual value. We will use predicted CPI data from the exponential model to predict inflation for every year up to 2017, using the basic equation:

$$f_t = (CPI_t - CPI_{(t-q)})/(CPI_{(t-1)})$$

Substituting CPI $c = 166.832(1.027)^t$ into the equation defining inflation rate f, we are left with the equation:

$$f_t = (166.832(1.027)^t - 166.832(1.027)^{(t-1)})/166.832(1.027)^{(t-1)}$$

We can then use this to find predict inflation from 2009 to 2017, using our predicted data from the power model:

Years Since 2000	Price	Inflation
1	171.18241921	X

2	175.809640008	X
3	180.561938908	X
4	185.442696891	X
5	190.455386323	X
6	195.603573437	X
7	200.89092086	X
8	206.321190226	X
9	211.898244849	X
10	217.626052471	.02704
11	223.508688087	.02702
12	229.550336843	.02702
13	235.755297011	.02705
14	242.127983053	.02702
15	248.672928755	.02701
16	255.394790457	.02702
17	262.298350364	.02706
18	269.492908129	.02741

Inflation can be factored into our original debt equation by multiplying the first portion of the equation (representing the debt remaining from the previous year) by 1 + f. Ultimately, we end up with the equation

$$d_t = d_{(t-1)} * (1 + s_t) * (1 + f_t) + (e_t - i_t)$$

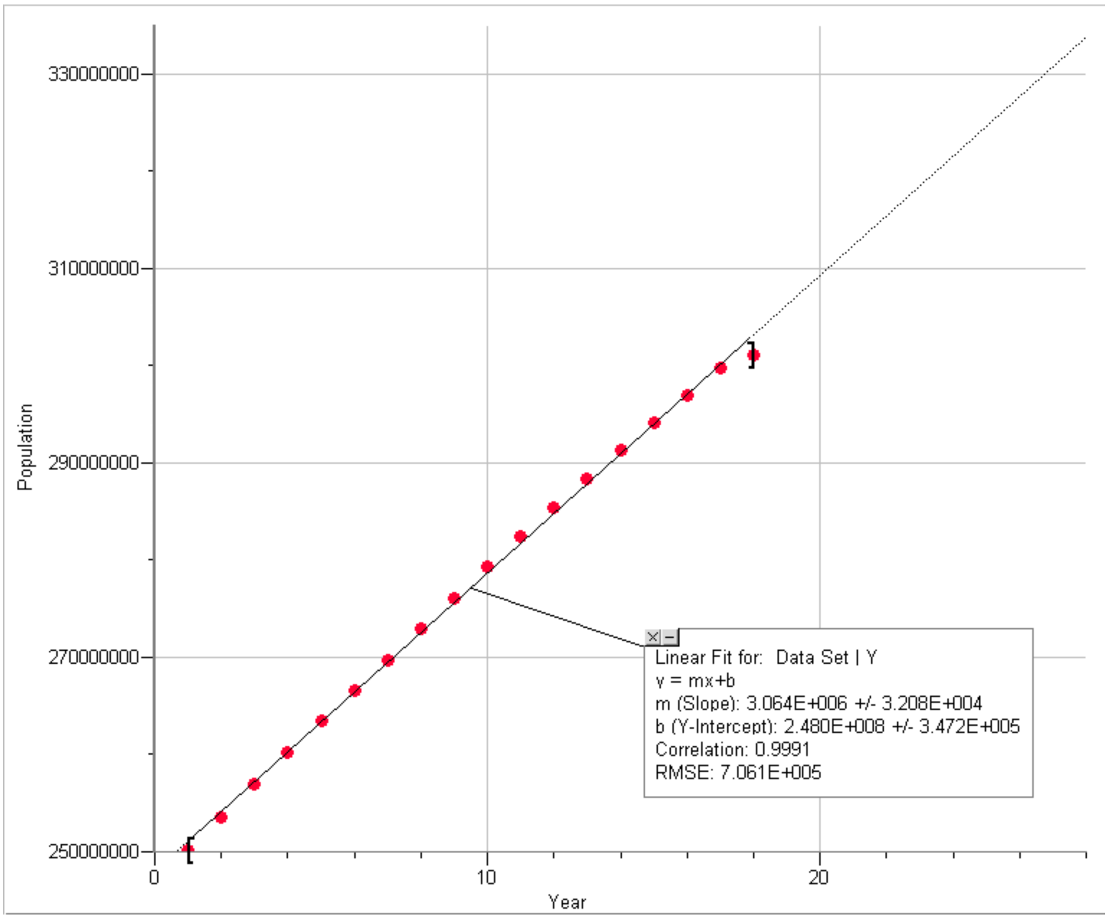
Let us now put this equation into context by looking at the national debt per capita. To do this, we must create an equation that can extrapolate the population of the U.S. in any given year. Such a model can be obtained by applying mathematical regressions on (t, p).

Table 3: U.S. Population, 1990-2007 (OECD)

Years since 1990	**Population**
1	250131894
2	253492503
3	256894189
4	260255352
5	263435673
6	266557091
7	269667391
8	272911760
9	276115288
10	279294713
11	282429565
12	285453747
13	288426877
14	291289242
15	294055604
16	296940126
17	299801097
18	301139947

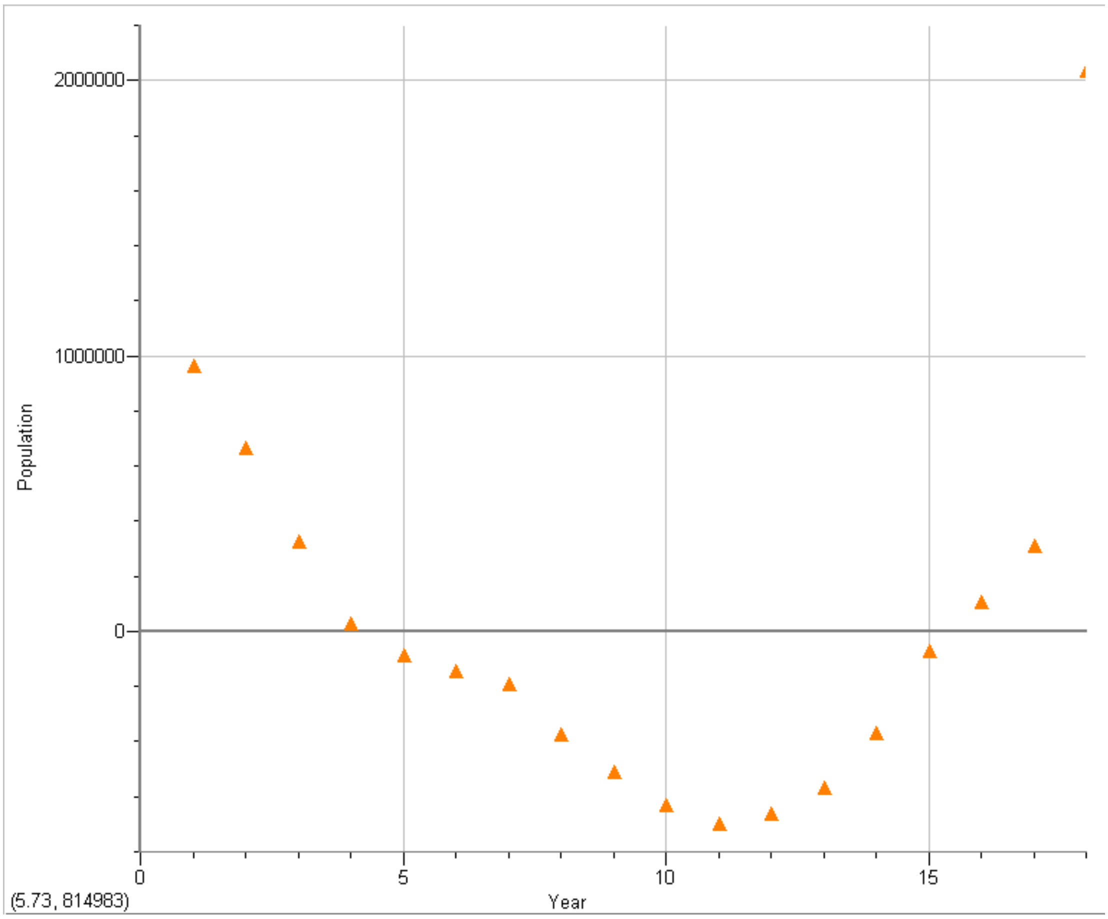
Model 3.1: Linear

Years since 1990	**Predicted population**
1	251086976
2	254150548
3	257214120
4	260277692
5	263341263
6	266404835
7	269468407
8	272531979
9	275595550
10	278659122
11	281722694
12	284786265
13	287849837
14	290913409
15	293976981
16	297040552
17	300104124
18	303167696
19	306231268
20	309294839
21	312358411
22	315421983
23	318485554
24	321549126
25	324612698
26	327676270
27	330739841
28	333803413



p = 3064643.506t + 248008762.752

The linear regression relating time and population is very strong – the points are fairly close to the line, which boasts a healthy correlation coefficient of .9991. However, the residual plot for the data paints a completely different picture…

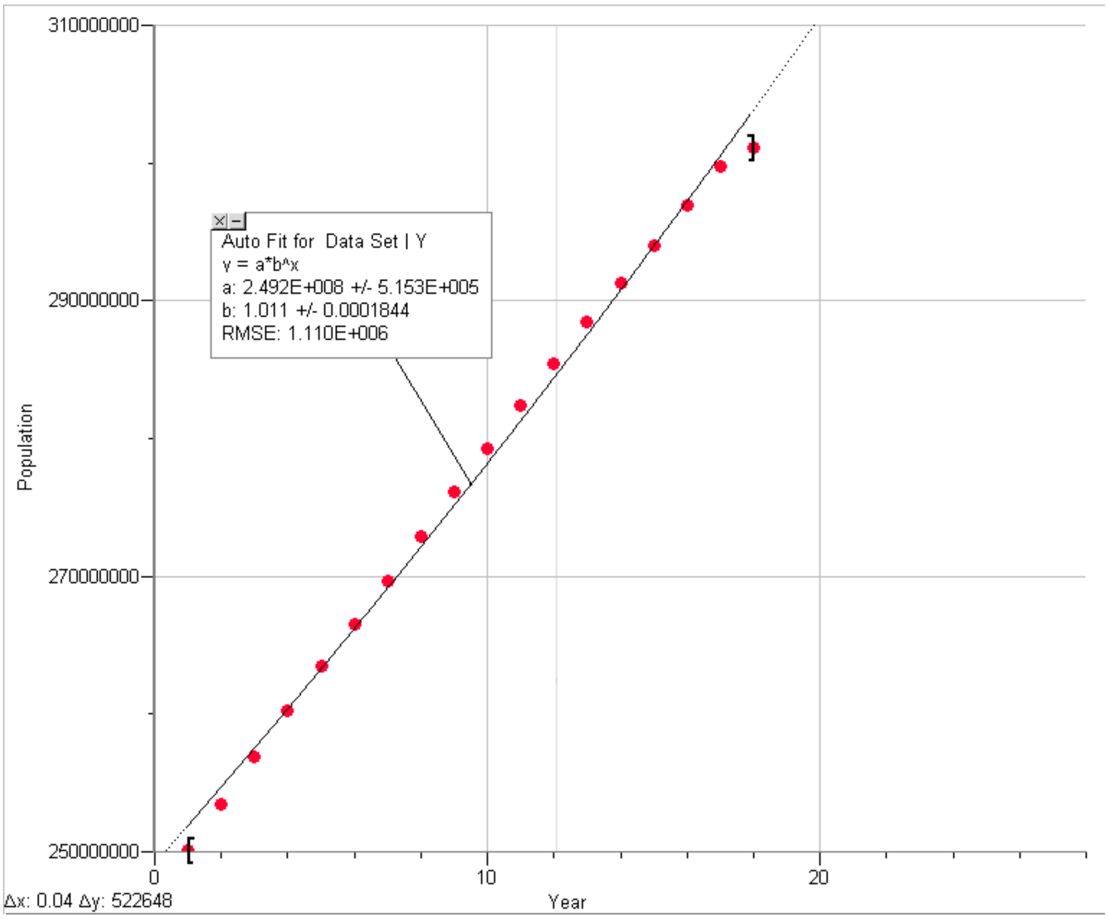


The residuals are arranged in a distinctly parabolic pattern, showing that the model may not be as suitable as it seems. The increments on the residual axis, however large they may seem, are actually quite small when considered in context – after all, population is being measured in units of 1 person each.

Model 3.2: Exponential

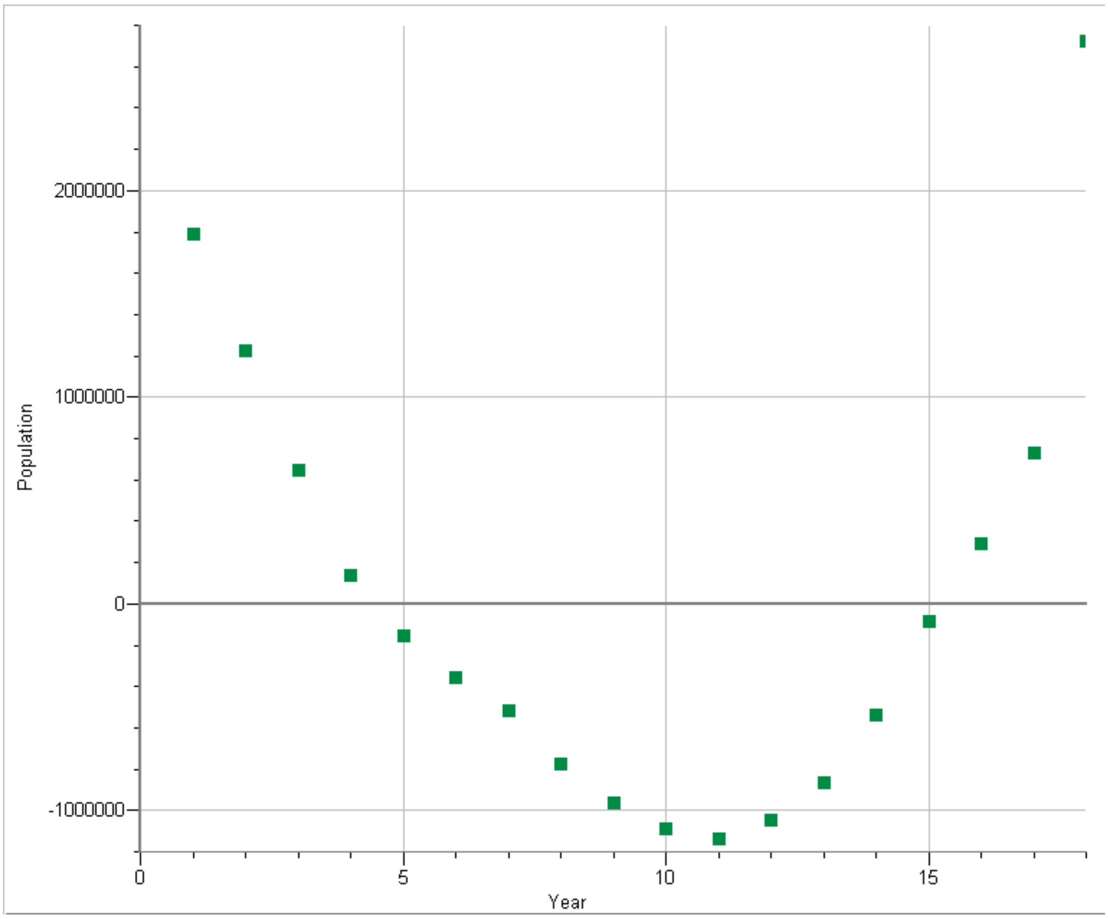
Years since 1990	**Predicted population**
1	251923135
2	254716261
3	257540354
4	260395759
5	263282822
6	266201894
7	269153331
8	272137492
9	275154738
10	278205437
11	281289960
12	284408681
13	287561981
14	290750242
15	293973851
16	297233202
17	300528690

18	303860715
19	307229684
20	310636004
21	314080092
22	317562365
23	321083246
24	324643164
25	328242552
26	331881847
27	335561492
28	339281933



$p = 248970756.678(1.011)^t$

The data points are very close to the graph of the model – the exponential relationship between time and population is very strong. Moreover, exponential growth makes more sense as a model for population, as humans do not grow at a constant rate; rather, the population grows exponentially larger as time goes on, according to the Malthusian model.

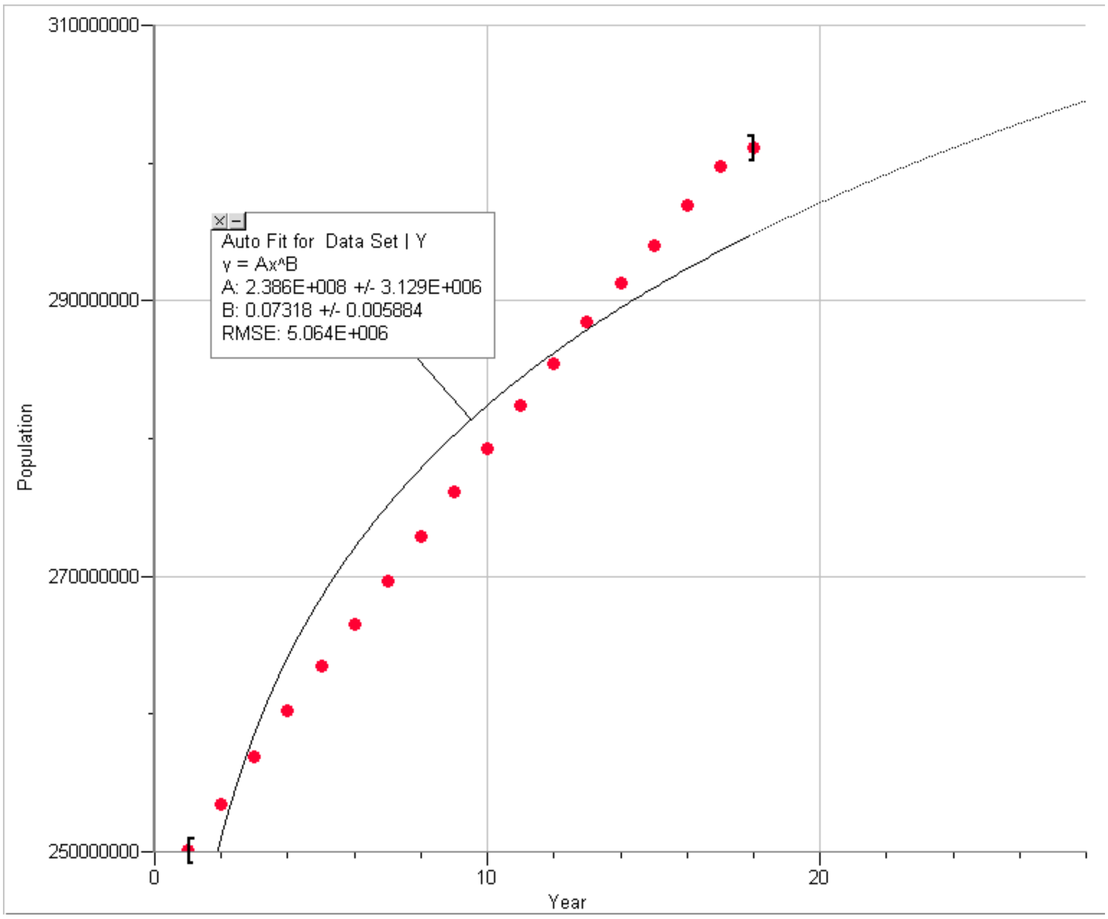


The residual plot for the exponential growth model shows a distinct parabolic pattern and features a window similar to that of the linear model. However, it has been further smoothed out and is therefore an unsuitable model for the data.

Model 3.3: Power

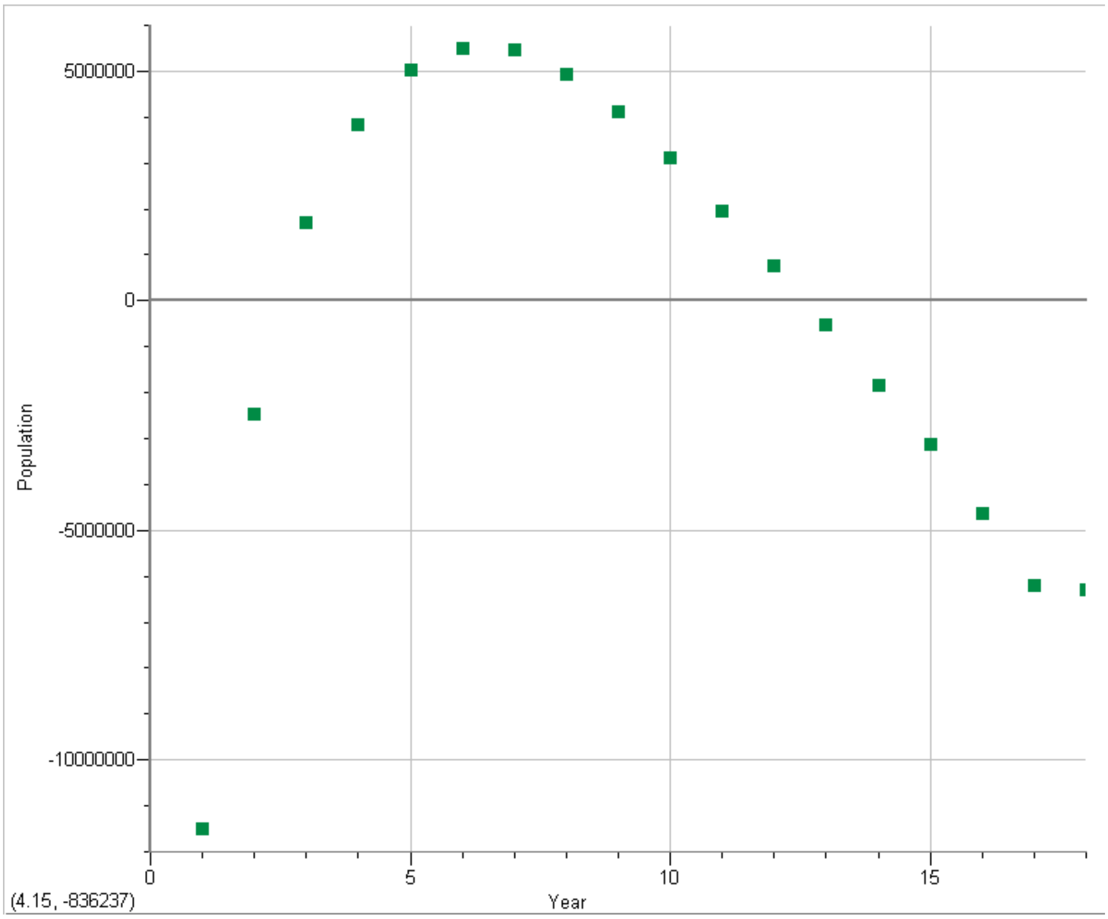
Years since 1990	Predicted population
1	238622191
2	251038855
3	258599533
4	264101617
5	268449855
6	272055714
7	275142186
8	277844098
9	280249369
10	282418596
11	284395361
12	286212085
13	287893556
14	289459161
15	290924355
16	292301667
17	293601391

18	294832096
19	296000990
20	297114197
21	298176965
22	299193823
23	300168712
24	301105080
25	302005963
26	302874046
27	303711720
28	304521117



p = 239921559.306t + .0705

The power function is evidently an inappropriate model for the relationship between time and population. The rate of change for the actual population values appears to be continually increasing whereas the rate of the change for the power model levels out over time. In fact, the two trends seem to have an inverse relationship!



The plot of the residuals adopts a pattern just as readily apparent as those of the linear and exponential models.

Out of these three models for population, all three feature residual plots that suggest that there is a better model out there. However, the exponential function makes the most sense theoretically. For that reason, we will use the second equation to model the relationship between time and population. So:

$$p_t = 248970756.678(1.011)^t$$

As such, national debt per capita can be calculated simply by dividing d_t by p_t. We will show how this relationship helps put the numbers into context when we take a look at some alternate governmental tax/spending plans.

Another way to put debt into a real-world context is to look at the ratio between national debt d and gross domestic product g. By inspecting quarterly GDP data since 2001, we can generate a mathematical model to predict future GDP values.

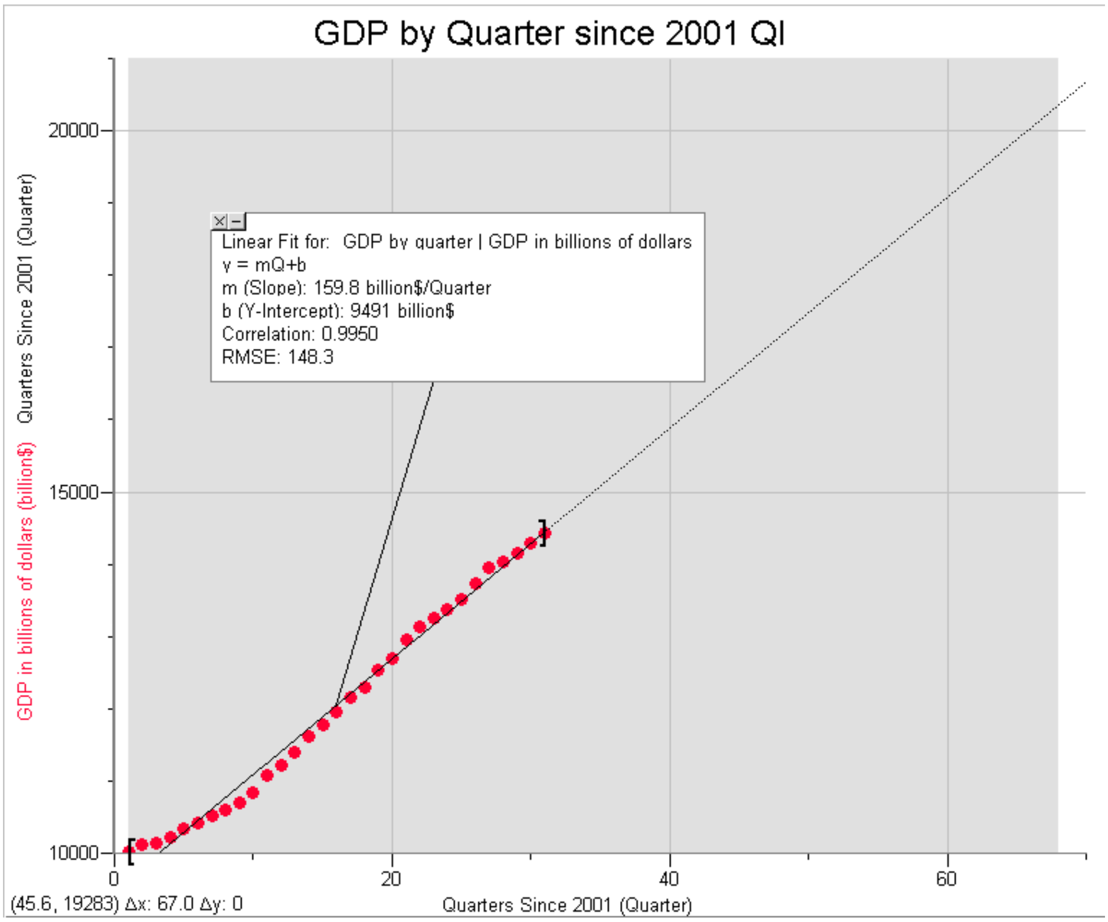
Table 4: U.S. GDP Since 2001 (U.S. Department of Commerce)

Year	Quarters since 2001	GDP (billions)
2001	1	10022
	2	10129
	3	10135
	4	10226
2002	5	10333
	6	10427
	7	10527
	8	10591
2003	9	10706
	10	10832
	11	11086
	12	11220
2004	13	11406
	14	11610
	15	11779
	16	11949
2005	17	12155
	18	12298
	19	12538
	20	12696
2006	21	12960
	22	13134
	23	13250
	24	13370
2007	25	13511
	26	13738
	27	13951
	28	14031
2008	29	14151
	30	14295
	31	14429

Model 4.1: Linear

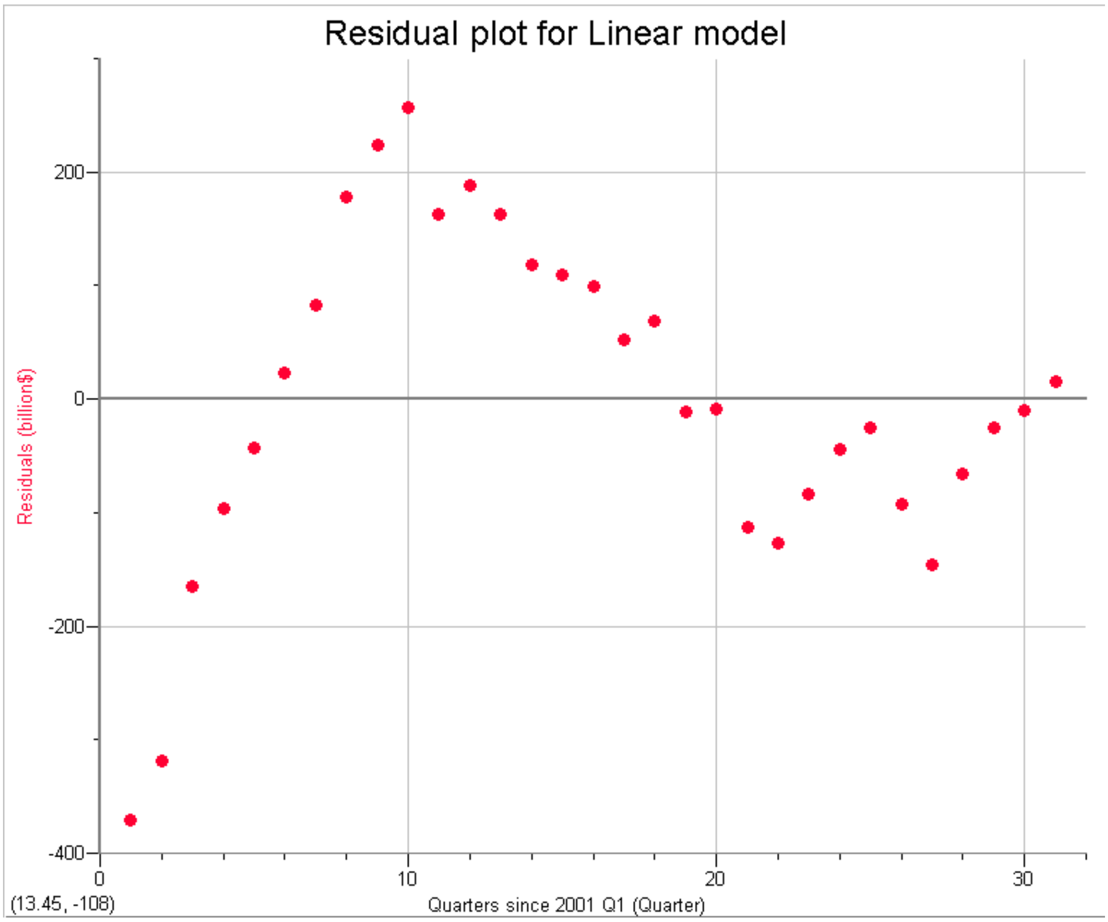
Year	Quarters since 2001	Predicted GDP (billions)
2001	1	9650.71572581
	2	9810.52822581
	3	9970.34072581
	4	10130.1532258
2002	5	10289.9657258
	6	10449.7782258
	7	10609.5907258
	8	10769.4032258
2003	9	10929.2157258
	10	11089.0282258
	11	11248.8407258
	12	11408.6532258
2004	13	11568.4657258
	14	11728.2782258
	15	11888.0907258
	16	12047.9032258
2005	17	12207.7157258
	18	12367.5282258

	19	12527.3407258
	20	12687.1532258
2006	21	12846.9657258
	22	13006.7782258
	23	13166.5907258
	24	13326.4032258
2007	25	13486.2157258
	26	13646.0282258
	27	13805.8407258
	28	13965.6532258
2008	29	14125.4657258
	30	14285.2782258
	31	14445.0907258
	32	14604.9032258
2009	33	14764.7157258
	34	14924.5282258
	35	15084.3407258
	36	15244.1532258
2010	37	15403.9657258
	38	15563.7782258
	39	15723.5907258
	40	15883.4032258
2011	41	16043.2157258
	42	16203.0282258
	43	16362.8407258
	44	16522.6532258
2012	45	16682.4657258
	46	16842.2782258
	47	17002.0907258
	48	17161.9032258
2013	49	17321.7157258
	50	17481.5282258
	51	17641.3407258
	52	17801.1532258
2014	53	17960.9657258
	54	18120.7782258
	55	18280.5907258
	56	18440.4032258
2015	57	18600.2157258
	58	18760.0282258
	59	18919.8407258
	60	19079.6532258
2016	61	19239.4657258
	62	19399.2782258
	63	19559.0907258
	64	19718.9032258
2017	65	19878.7157258
	66	20038.5282258
	67	20198.3407258
	68	20358.1532258



g = 159.8(t/4) + 9491 = 39.95t + 9491

The linear model has a very high correlation of .995 that suggests a strong linear relationship between g and t. However, we must also look at the residuals to see whether or not it is the most suitable fit for the data.

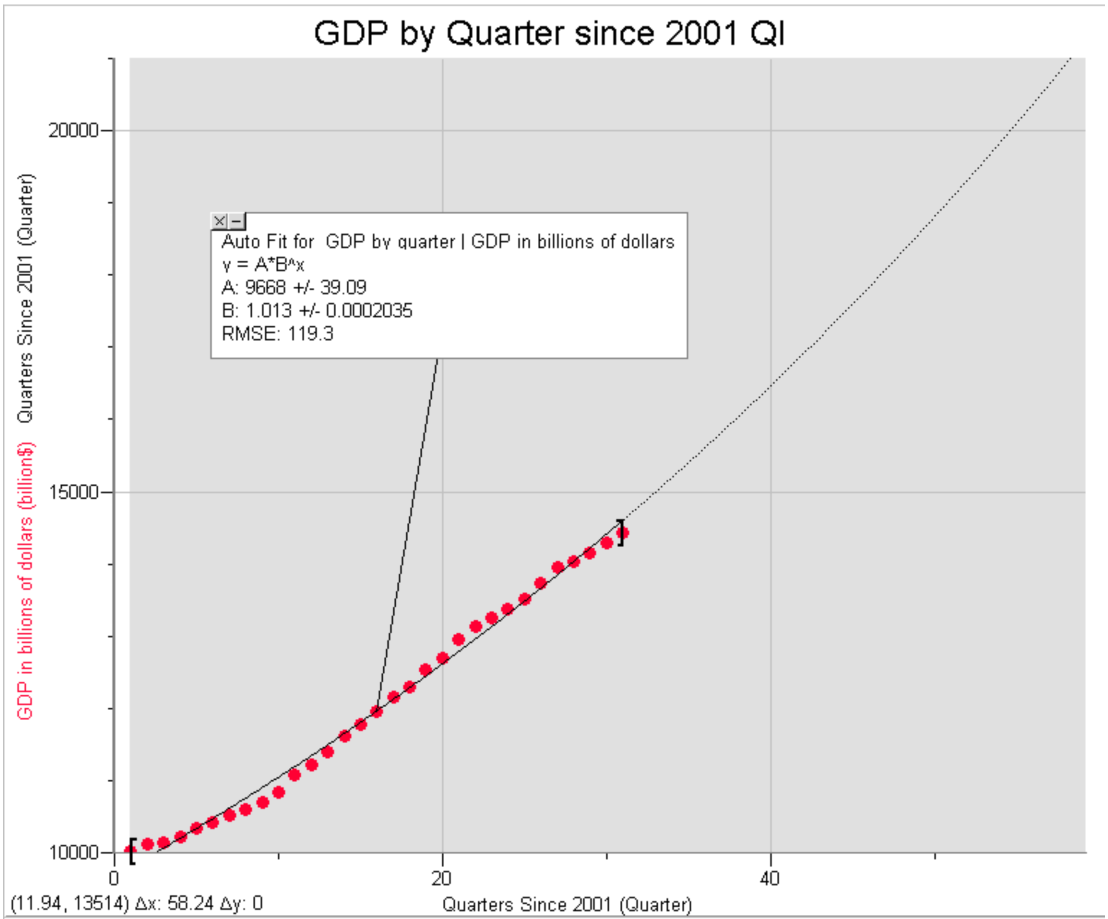


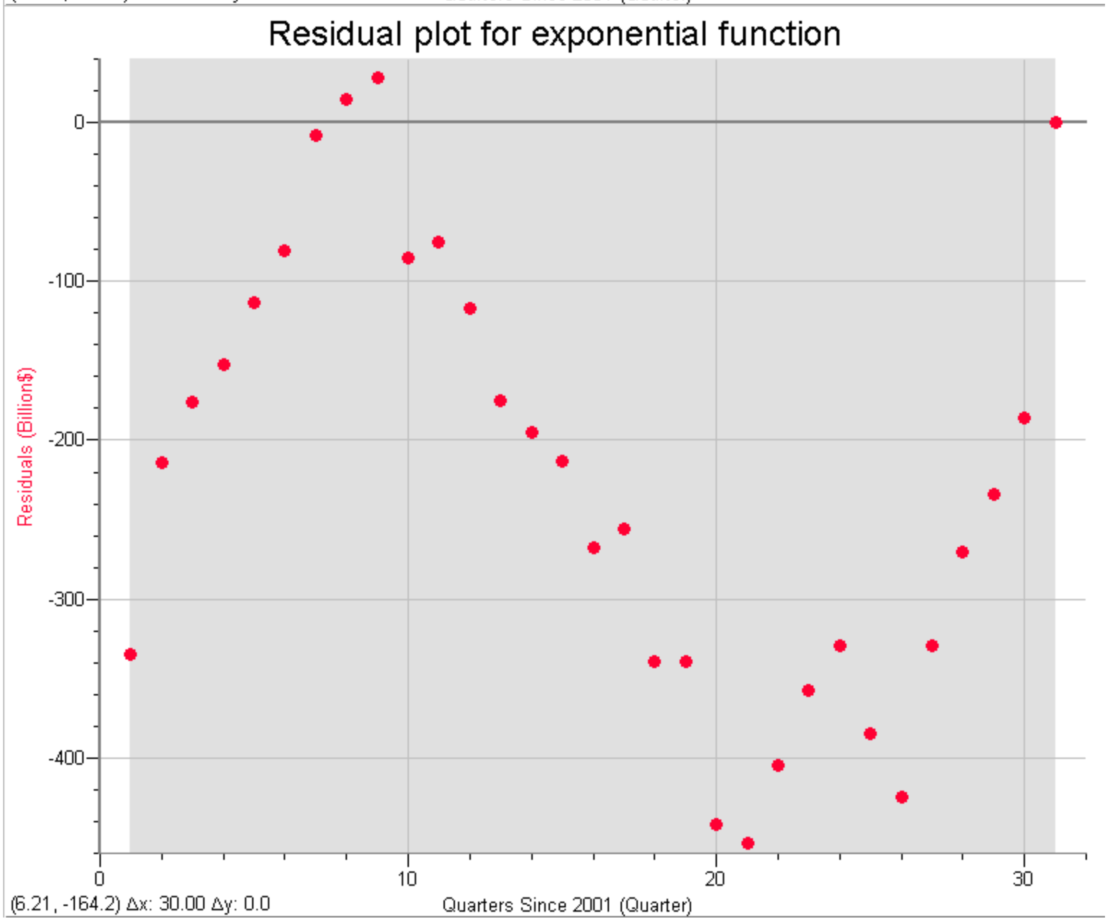
There is a distinct parabolic pattern in the residuals, suggesting that there may be a better model to fit the data. In fact, the residual axis shows an error margin in the hundreds of billions. However, we must also consider the fact that we are dealing with trillions of dollars; hence, the residuals are less significant than they appear.

<u>Model 4.2: Exponential</u>

Year	**Quarters since 2001**	**GDP (billions)**
2001	1	9797.79971973
	2	9929.08229652
	3	10062.1239534
	4	10196.9482608
2002	5	10333.5791046
	6	10472.0406909
	7	10612.3575504
	8	10754.554542
2003	9	10898.6568582
	10	11044.6900285
	11	11192.6799251
	12	11342.6527663
2004	13	11494.635122
	14	11648.6539182
	15	11804.7364415
	16	11962.910344

2005	17	12123.2036487
	18	12285.6447538
	19	12450.2624381
	20	12617.0858659
2006	21	12786.1445925
	22	12957.468569
	23	13131.088148
	24	13307.0340887
2007	25	13485.3375624
	26	13666.0301581
	27	13849.1438882
	28	14034.7111939
2008	29	14222.7649512
	30	14413.3384764
	31	14606.4655324
	32	14802.1803344
2009	33	15000.5175563
	34	15201.5123361
	35	15405.2002832
	36	15611.6174837
2010	37	15820.8005074
	38	16032.7864141
	39	16247.6127602
	40	16465.3176052
2011	41	16685.9395187
	42	16909.5175872
	43	17136.0914205
	44	17365.7011597
2012	45	17598.3874832
	46	17834.191615
	47	18073.155331
	48	18315.3209672
2013	49	18560.7314267
	50	18809.4301875
	51	19061.4613101
	52	19316.8694456
2014	53	19575.6998431
	54	19837.9983582
	55	20103.8114609
	56	20373.1862437
2015	57	20646.1704304
	58	20922.8123839
	59	21203.1611154
	60	21487.2662928
2016	61	21775.1782493
	62	22066.9479928
	63	22362.6272143
	64	22662.2682979
2017	65	22965.9243291
	66	23273.6491051
	67	23585.4971438
	68	23901.5236935


GDP by Quarter since 2001 QI
Auto Fit for GDP by quarter | GDP in billions of dollars
y = A*B^x
A: 9668 +/- 39.09
B: 1.013 +/- 0.0002035
RMSE: 119.3
GDP in billions of dollars (billion$)
Quarters Since 2001 (Quarter)
20000
15000
10000
0
20
40
(11.94, 13514) Δx: 58.24 Δy: 0
Quarters Since 2001 (Quarter)


Residual plot for exponential function
Residuals (Billion$)
0
-100
-200
-300
-400
0
10
20
30
(6.21, -164.2) Δx: 30.00 Δy: 0.0
Quarters Since 2001 (Quarter)

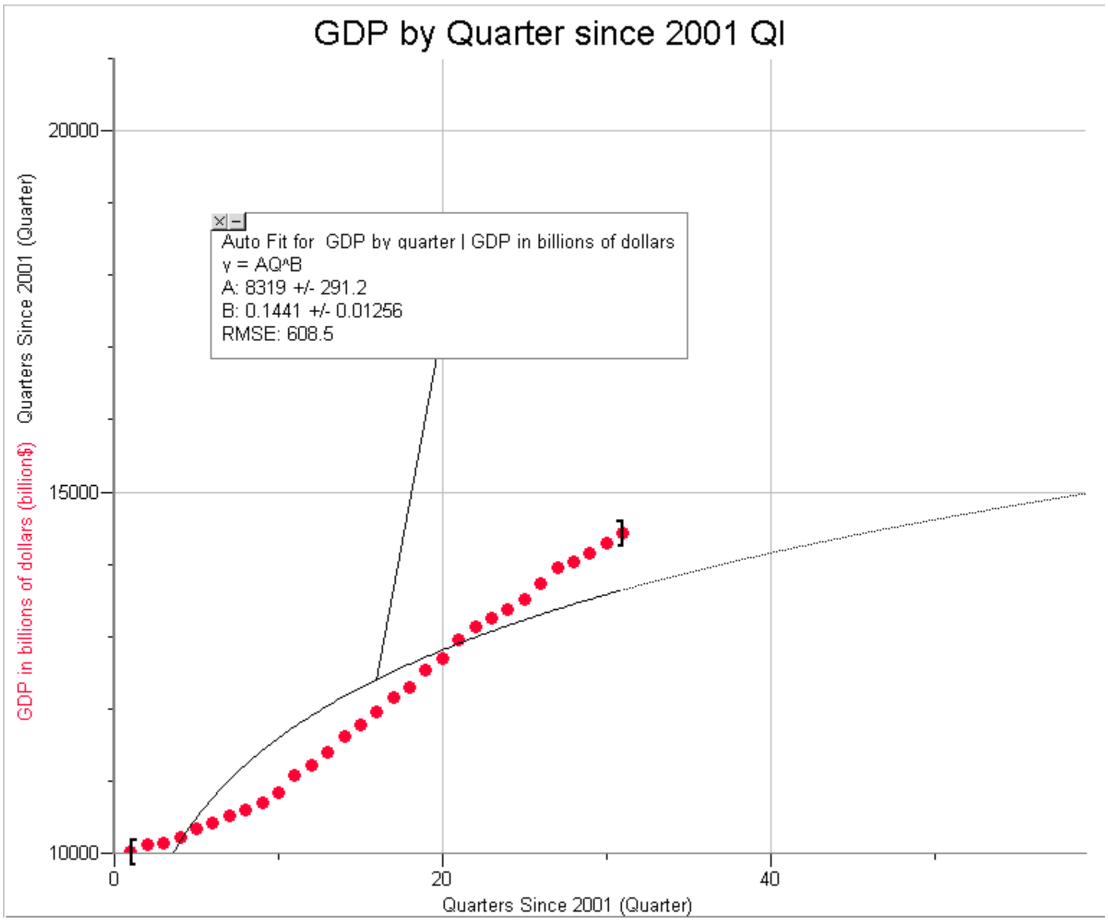
$g = 9668(1.013)^{t/4} = 9668(1.003)^{t}$

This model is clearly a better fit than the linear model, boasting a lower RMSE value and a relatively more random (somewhat cubic?) arrangement of residuals.

Model 4.3: Power

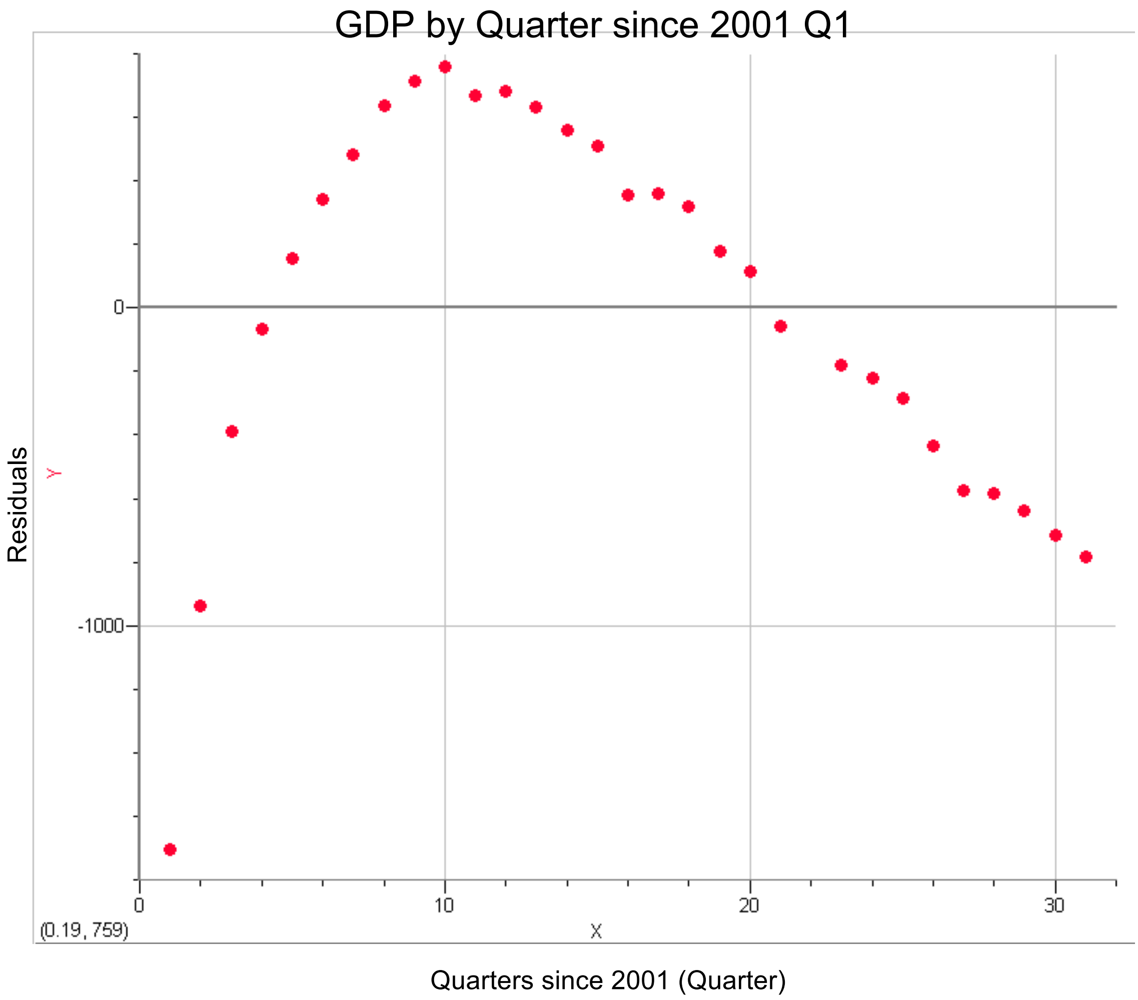
Year	**Quarters since 2001**	**GDP**
2001	1	8319.45287265
	2	9193.18750796
	3	9746.23172669
	4	10158.6844532
2002	5	10490.5903241
	6	10769.8110839
	7	11011.6790635
	8	11225.5808696
2003	9	11417.7018999
	10	11592.3445201
	11	11752.6280983
	12	11900.8899064
2004	13	12038.9286535
	14	12168.1596083
	15	12289.7173424
	16	12404.5260427
2005	17	12513.3489891
	18	12616.824217
	19	12715.4907639
	20	12809.8083445
2006	21	12900.1723425
	22	12986.9253991
	23	13070.3664904
	24	13150.7581203
2007	25	13228.332083
	26	13303.2941229
	27	13375.8277385
	28	13446.0973117
2008	29	13514.2507024
	30	13580.4214145
	31	13644.7304163
	32	13707.287679
2009	33	13768.1934854
	34	13827.5395474
	35	13885.4099667
	36	13941.8820633
2010	37	13997.0270937
	38	14050.9108757
	39	14103.5943341
	40	14155.13398
2011	41	14205.5823311
	42	14254.9882841
	43	14303.3974436
	44	14350.8524146
2012	45	14397.3930628
	46	14443.056747
	47	14487.8785278

	48	14531.8913542
2013	49	14575.1262317
	50	14617.612374
	51	14659.3773403
	52	14700.4471589
2014	53	14740.8464401
	54	14780.5984788
	55	14819.725347
	56	14858.2479796
2015	57	14896.1862517
	58	14933.5590499
	59	14970.3843375
	60	15006.6792146
2016	61	15042.4599728
	62	15077.7421463
	63	15112.5405587
	64	15146.869366
2017	65	15180.7420966
	66	15214.1716883
	67	15247.1705227
	68	15279.750457



$g = 8319(t/4)^{.1441} = 6812.622t^{.1441}$

A power function is clearly inappropriate for modeling the growth of GDP over time. Its high RMSE-value and the arrangement of the residuals only support this notion.



The residuals here show a distinct curvature that indicates the existence of a more accurate model. Of the three that we have tried, the exponential model was the best, so we will use that equation to model the growth of GDP over time:

$$g_t = 9668(1.013)^{t/4} = 9668(1.003)^t$$

We now have the necessary components of a multi-faceted mathematical model that explains national debt. At this point, we can now consider the various tax/spending plans in the algebraic context of the following equations:

$$\mathbf{d_t = d_{(t-1)} * (1 + s_t) * (1 + f_t) + (e_t - i_t)}$$

$$s_t = .02365/t + .04312$$

$$f_t = (166.832(1.027)^{(t-1)} - 166.832(1.027)^t)/166.832(1.027)^t$$

$$p_t = 248970756.678(1.011)^t$$

$$g_t = 9668(1.013)^{t/4} = 9668(1.003)^t$$

Using our model, we will go on to analyze the financial policies of Senator John McCain, President-Elect Barack Obama, and our own proposal, an optimal blend of what we consider to be the best aspects of the two plans. These pre-existing plans will ensure political feasibility and allow us to focus primarily on the national debt, not how well the plans appeal to the public. Income and expenditure data for each plan was obtained from their respective campaign websites/publications, as well as projections from independent sources. Only the most numerically significant budgets will be included in our calculations.

Table 5: Obama's Federal Expenditures (Einhorn, USBudgetWatch.org)

Policy	**Annual Cost (billions of USD / year)**
Green energy sector	15
National infrastructure reinvestment bank	6
Near-universal healthcare	50-65
Foreign aid	25
Withdrawal from Iraq (2009)	-100
Total:	103.5 increase first year
	3.5 increase subsequent years

Table 6: Obama's Federal Income (Einhorn, USBudgetWatch.org)

Policy	**Annual Revenue (billions of USD / year)**
Renewal of Bush tax cuts of households earning less than $250,000 annually (2011)	-78.2
Closing tax loopholes	+84.9
Cutting certain corporate taxes (small business)	-13.0
Eliminating senior income tax	-7.0
Universal refundable mortgage credit	-12.6
"Making Work Pay" tax credit	-71.0
New earned income tax credit	-4.7
Workplace pensions	-20.4
Child and dependent child tax credit	-2.3

College tax credit	-12.0
Production tax credit	-2.0
Alternative minimum tax	-116.7
Research and experimentation tax credit	-8.5
Reducing estate tax	-58.0
Capital gains and dividends tax reform	-16.7
TOTAL	-338.2 reduction

Obama: 2009: 103.5 increase in spending, 338.2 reduction in revenues

2010-2017: 3.5 increase in spending, 338.2 reduction in revenues

Table 7: McCain's Federal Expenditures (Einhorn, USBudgetWatch.org)

Policy	**Annual Cost (billions of USD/ year)**
Cut back Medicaid	-130
Cut back Earmarks	-18
Nuclear Power	3.7
Eliminate NEA	-0.128
TOTAL	144.428 reduction in spending

Table 8: McCain's Federal Income (Einhorn, USBudgetWatch.org)

Policy	**Annual Revenue (billions of USD / year)**
Renew Bush's Tax Cut	-173
Reduce Estate Tax	-58
Reform R&E Tax Credit	-13.3
Reform Alternative Minimum Tax (AMT)	-123.3
Double Personal Exemption for Dependence	-17.8
Reduce Federal Corporate Income Tax	-73.5
Allowing Expensing	-4.5
Close Corporate Loopholes	46.2
TOTAL	-417.2

McCain: 2009-2017: 144.428 reduction in spending, 417.2 reduction in revenues

Revenues for 2008: 2.66 trillion

Expenditure for 2008: 2.9 trillion

Under Obama Plan

Using our debt formula of:

debt (billion USD) = previous year debt * interest * inflation + (expenditure – revenues)

d_{2009} = 10024.72*1.045486*1.02704 + ((2900 + 103.5) – (2660 – 338.2))

= 11445.80 billion USD

d_{2010} = 11445.80*1.045271*1.02702 + ((2900+3.5) – (2660 – 338.2))

= 12868.93 billion USD

Expenditure for all subsequent years will be ((2900+3.5) – (2660 – 338.2)) = 581.7

d_{2011} = 12868.93*1.045092*1.02702 + 581.7

= 14394.31 billion USD

d_{2012} = 14394.31*1.044940*1.02705 + 581.7

= 16029.75 billion USD

d_{2013} = 16029.75*1.044810*1.02702 + 581.7

= 17782.28 billion USD

d_{2014} = 17782.28*1.044698*1.02701 + 581.7

= 19660.58 billion USD

d_{2015} = 19660.58*1.044599*1.02702 + 581.7

= 21674.04 billion USD

d_{2016} = 21674.04*1.044512*1.02706 + 581.7

= 23833.10 billion USD

d_{2017} = 23833.10*1.044435*1.02741 + 581.7

= 26156.12 billion USD

Under McCain Plan

Using our debt formula of:

debt (in billion USD) = previous year debt * interest * inflation + (expenditure – revenues)

d_{2009} = 10024.72*1.045486*1.02704 + ((2900 - 144.428) – (2660 -417.2))

= 11276.87 billion USD

d_{2010} = 11276.87*1.045271*1.02702 + ((2900-144.428) – (2660 – 417.2))

= 12618.65 billion USD

Expenditure for all subsequent years will be ((2900-144.428) – (2660 – 417.2))

= 512.772 billion USD

d_{2011} = 12618.65*1.045092*1.02702 + 512.772

= 14056.75 billion USD

d_{2012} = 14056.75*1.044940*1.02705 + 512.772

= 15598.56 billion USD

d_{2013} = 15598.56*1.044810*1.02702 + 512.772

= 17250.66 billion USD

d_{2014} = 17250.66*1.044698*1.02701 + 512.772

= 19021.27 billion USD

d_{2015} = 19021.27*1.044599*1.02702 + 512.772

= 20919.25 billion USD

d_{2016} = 20919.25*1.044512*1.02706 + 512.772

= 22954.45 billion USD

d_{2017} = 22954.45*1.044435*1.02741 + 512.772

= 25144.34 billion USD

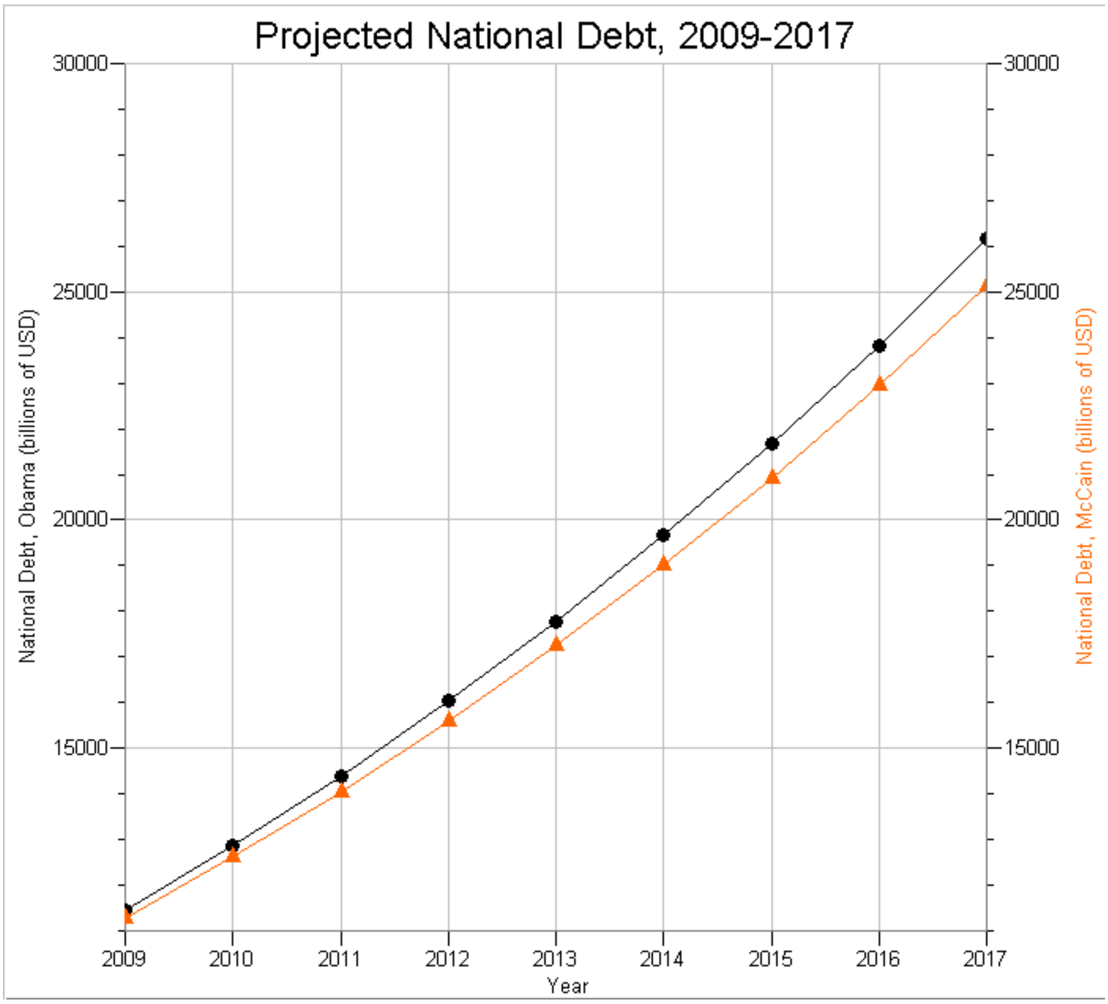
Comparison (impact on the national debt and impact on nation in general of policies)

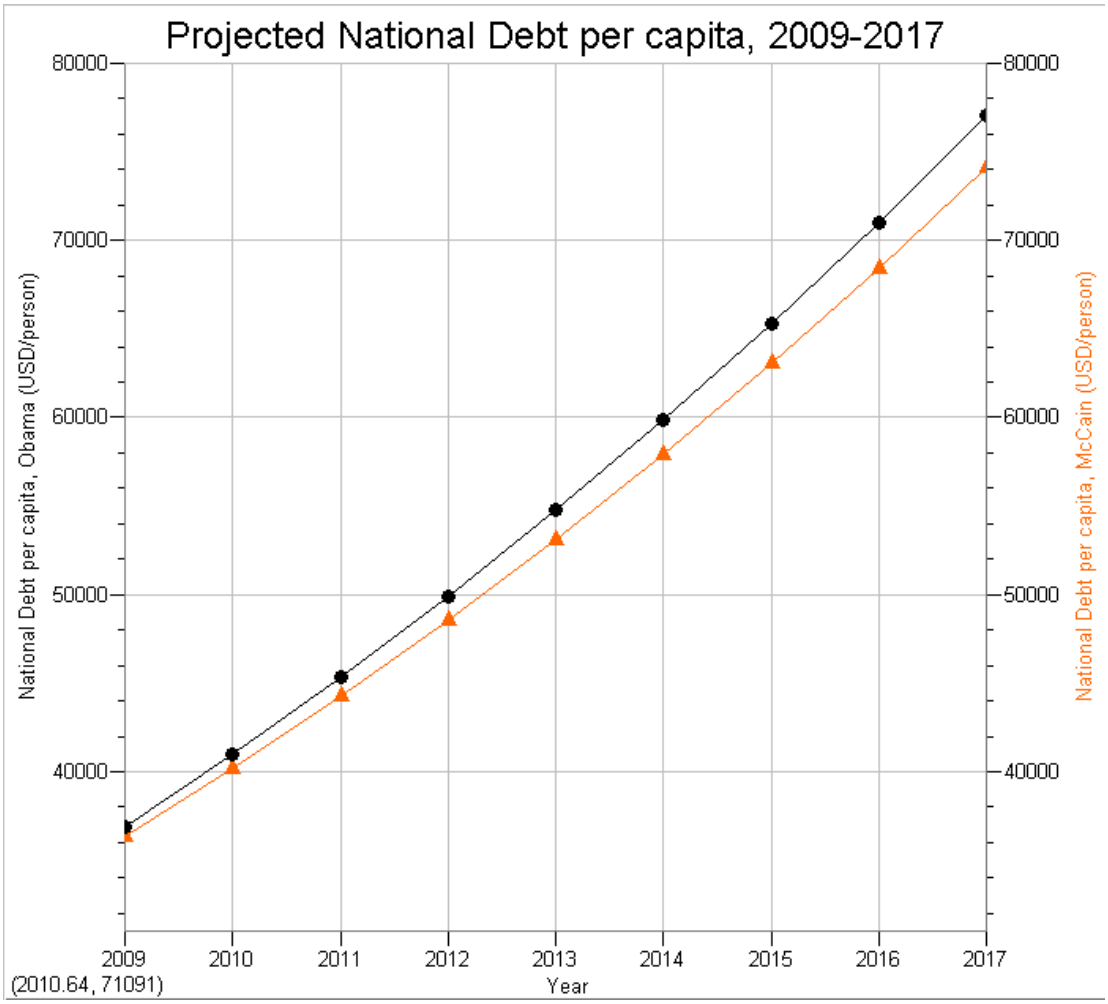
Comparing Debt/Capita

Predicted populations for 2009-2017

Year	Predicted Population
2009	310636004
2010	314080092
2011	317562365
2012	321083246
2013	324643164
2014	328242552
2015	331881847
2016	335561492
2017	339281933

	Obama		**McCain**	
Year	Debt (in billions USD)	Debt/Capita (in USD / person)	Debt (in billions USD)	Debt/Capita (in USD / person)
2009	11445.80	36846.34	11276.87	36302.52
2010	12868.93	40973.40	12618.65	40176.54
2011	14394.31	45327.51	14056.75	44264.53
2012	16029.75	49923.97	15598.56	48581.05
2013	17782.28	54774.85	17250.66	53137.30
2014	19660.58	59896.50	19021.27	57948.82
2015	21674.04	65306.49	20919.25	63032.22
2016	23833.10	71024.54	22954.45	68406.09
2017	26156.12	77080.79	25144.34	74110.46


Projected National Debt, 2009-2017
National Debt, Obama (billions of USD)
National Debt, McCain (billions of USD)
Year
30000
25000
20000
15000
2009
2010
2011
2012
2013
2014
2015
2016
2017



These diagrams clearly demonstrate that John McCain's spending plan is less strenuous on both the American government and the American people. However, both plans continuously increase national debt, with no end in sight.

Compare GDP Ratio

To better understand the effects of national debt on the U.S. economy, we will compare it to the GDP.

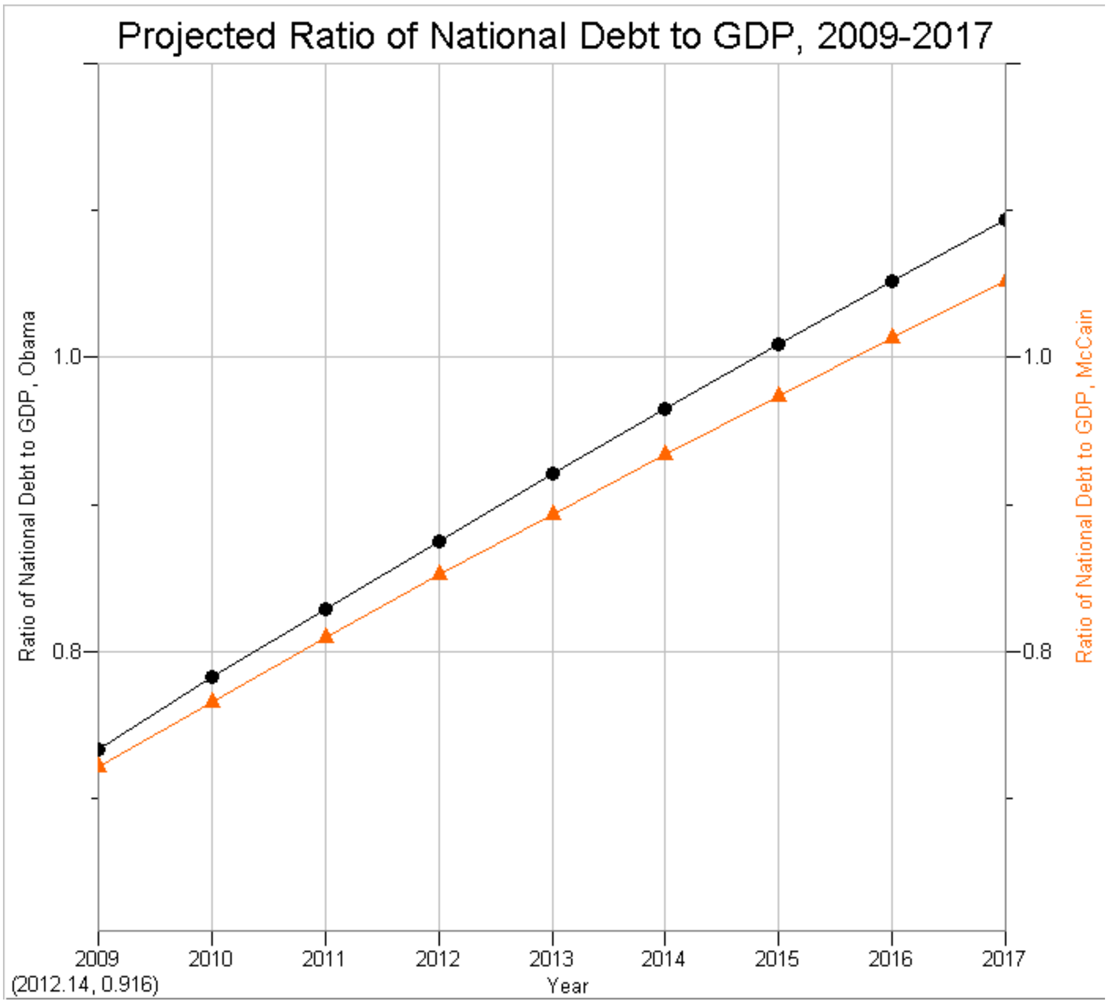
Predicted GDP for 2009-2017

2009	15000.5175563
	15201.5123361
	15405.2002832
	15611.6174837
2010	15820.8005074
	16032.7864141
	16247.6127602
	16465.31.76052

2011	16685.9395187
	16909.5175872
	17136.0914205
	17365.7011597
2012	17598.3874832
	17834.191615
	18073.155331
	18315.3209672
2013	18560.7314267
	18809.4301875
	19061.4613101
	19316.8694456
2014	19575.6998431
	19837.9983582
	20103.8114609
	20373.1862437
2015	20646.1704304
	20922.8123839
	21203.1611154
	21487.2662928
2016	21775.1782493
	22066.9479928
	22362.6272143
	22662.2682979
2017	22965.9243291
	23273.6491051
	23585.4971438
	23901.5236935

Using the GDP predicted for each year from Q4

	Obama		**McCain**	
Year	Debt (in billions USD)	Debt/GDP	Debt (in billions USD)	Debt/GDP
2009	11445.80	.733	11276.87	.722
2010	12868.93	.783	12618.65	.766
2011	14394.31	.829	14056.75	.809
2012	16029.75	.875	15598.56	.852
2013	17782.28	.921	17250.66	.893
2014	19660.58	.965	19021.27	.934
2015	21674.04	1.009	20919.25	.974
2016	23833.10	1.052	22954.45	1.013
2017	26156.12	1.094	25144.34	1.052



In essence, we are finding what percentage of the GDP that the debt is equivalent to. Ratios greater than 1 represent years in which the debt is greater than the GDP. From the table, we can see that both plans would utilize unsustainable levels of spending.

Obviously, we must opt for a more centrist approach to our tax/spending proposal – we must include elements from both Obama's and McCain's original plans. Moreover, to alleviate national debt, we would promote lower government intervention and adopt the Libertarian advocacy of minimal spending and the self-reliance of citizens. Our proposal seeks to slow the exponential increase in national debt, but we cannot neglect the immediate issues facing citizens nor compromise President-Elect Obama's original campaign promises.

PROPOSED SOLUTION – OUR ALTERNATE PLAN

Expenditures

Policy	Cost (billions of USD)
Ending the Iraq War	-100.0
Eliminating earmarks	-18.0
Universal healthcare	+69.0
Fixing up infrastructure	+47.31
Green energy	+15.0
Total	**+42.9**

Most of these policies existed on the policy packages that candidates for both parties offered. Thus, any effort to implement these policies will be with bipartisan support. These expenditures are justifiable, as currently more than 65% of the American public opposes the US war in Iraq, while more than half favor withdrawal. Meanwhile, 64% of Americans believe that the government should guarantee health care to all citizens. “Ending government inefficiency” and “governing ethics” also turned up on the top ten issues that Americans cared most about in the 2008 elections. Global warming and energy independence is also another major concern amongst the American voters. (Polling Report, Inc) Investments in infrastructure and green energy are take time to reach fruition – the benefits will only become more apparent later down the line. These are short-term trade-offs that must be made in order to see gains later.

Income

Policy	Cost (in billions of USD)
Closing tax loopholes	+84.9
Capital gain and dividend tax reform	-16.7
College tax credit	-12.0
Total	**+56.2**

Obviously, in order to finance these policies and reduce the debt, a net gain in federal income is in order. Again, one of the major concerns of American voters is “governing ethics”. Tax loopholes are seen as unfair advantages, where the rich and big corporations can get away with not paying their taxes. Capital gain reform will help encourage entrepreneurship and overall investment in the market, which will in turn help stimulate economic growth. Meanwhile, a college tax credit will help aspiring students receive the education that they deserve. Better education will give higher paying jobs to Americans and will therefore be a good long-term investment.

d_{2009} = 10024.72*1.045486*1.02704 + ((2900 + 142.9) – (2660 +56.2))

= 11090.80 billion USD

d_{2010} = 11090.80*1.045271*1.02702 + ((2900+42.9) – (2660 + 56.2))

= 12134.92 billion USD

Expenditure for all subsequent years will be ((2900+3.5) – (2660 + 56.2)) = 226.7

d_{2011} = 12134.92*1.045092*1.02702 + 226.7

= 13625.36 billion USD

d_{2012} = 13625.36*1.044940*1.02705 + 226.7

= 14849.51 billion USD

d_{2013} = 14849.51*1.044810*1.02702 + 226.7

= 16160.83 billion USD

d_{2014} = 16160.83*1.044698*1.02701 + 226.7

= 17565.90 billion USD

d_{2015} = 17565.90*1.044599*1.02702 + 226.7

= 19071.82 billion USD

d_{2016} = 19071.82*1.044512*1.02706 + 226.7

= 20686.50 billion USD

d_{2017} = 20686.50*1.044435*1.02741 + 226.7

= 22424.62 billion USD

Year	**Debt (in billions USD)**	**Debt/Capita (in USD/Person)**
2009	11090.80	35703.52
2010	12134.92	38636.39
2011	13625.36	42906.09
2012	14849.51	46248.16
2013	16160.83	49780.29
2014	17565.90	53515.00
2015	19071.82	57465.69
2016	20686.50	61647.42
2017	22424.62	66094.35

Year	Debt (in billions USD)	GDP/Debt
2009	11090.80	.710
2010	12134.92	.737
2011	13625.36	.785
2012	14849.51	.811
2013	16160.83	.837
2014	17565.90	.862
2015	19071.82	.888
2016	20686.50	.913
2017	22424.62	.938

SENSITIVITY ANALYSIS

The assumptions made are strong enough to create a simplified model of national debt, but collectively might seem a bit too strong or restricting. However, the assumptions made are important, as there are far too many complexities and variables involved in the calculation of national debt to be taken into account separately. In an attempt to group together the many variables to form a simplified model, assumptions such as steady/predictable rate of population growth, focus on USD as a main currency, and a stable condition of economy (with no 'wild card' events) are essential to form a picture of the components that make up and influence national debt, while eliminating unnecessary deviations that cannot be predicted or easily accounted for. Although it is acknowledged that many things can affect national debt, as everything in the real world is always constantly changing, we decided to assume everything by a constant rate of change to ensure a workable model, rather than include a big amount of variables that can be affected by a lot of things. In these, change in interest, inflation and ratios of debt to GDP have also been taken into account, while without it, the results from the model could fluctuate greatly while varying in the approach to include additional components to the equation. While some of the assumptions may seem unrealistic, it cuts down random variables and the results are not as far off either. Overall, although the assumptions might seem a bit too strong or limiting, it is realistic at the same time, because 'wild card' events and sudden drops in trends do not happen on a daily basis, making the model good enough for “average” estimations, ruling out unpredictable circumstances. So in an attempt to address the problem of debt, we believe that our solution/model addresses the situation adequately, and will survive the test of real-world data, as can be tested with real world recommended policies.

CONCLUSIONS

The objective of this problem was to create a model to help us understand national debt and to forcast it from 2009-2017. Our model incorporated the interest rate, inflation, change in expenditures and revnues change based on proposals. We also projected population and GDP to see how national debt would compare per capita and in ratio to GDP. Using this model, we were able to compare the two plans proposed by President-elect Obama and Senator McCain during the 2008 Presidential Campaign. According to our model, Senator McCain's and President-elect Obama's plans would continue to see increase in the national debt and will have surpassed 25 trillion by 2017. By this time, the debt per capita will be over $70,000 per person and the debt will be higher than the GDP.

After comparing these two plans, we ourselves came up with a proposal that incorporated elements from both these plans and is less detrimental to the national debt. According to our model, in order to prevent national debt from rising further, more must be done than just merely balancing the budget, in order to account for interest rates and inflation.

STRENGTHS/WEAKNESSES

However, our model is not without flaws. One of the strengths of this model for this problem is that it provides us with a general yet nuanced representation of how national debt changes, when taking inflation, interest rates, federal expenditures and incomes into account, between 2009-2017. The fact that many confounding variables were eliminated simplifies the problem and allows us to see this relationship clearly, without compromising the integrity of this model. If these variables were not taken out, we would've been left with a potentially unnecessarily complicated and lengthy solution that would not affect the outcome produced as much for it to be of any significance. As a result, this model can be more easily understood, related to and analyzed.

Some of the few weaknesses in the problem have already been mentioned above; most prominently, the model we use has been highly simplified. If given more time, more research could've have been done to investigate how other variables could've affected national debt, such as unemployment, trade deficits and aging populations. More research could have also been conducted to analyze the influence that individual variables have on each other, such as the relationship between inflation and GDP. The interaction between these variables will also influence the overall model.

If this model were to be more extensive, calculus concepts that we have not yet been exposed to could be used to determine the instantaneous rates of change and to analyze the change in GDP/inflation/population more accurately.

This model could also be improved if we were to take more detail account of the spending and incomes of the federal government. Currently, we've taken into account the budgetary items of greatest value that were subject to change. A more detailed model would be more inclusive of all the items in the budget. But we believed that the items that we've selected to model for our plans are the only significant budgetary changes there are if these plans were put into place.

But overall, we as a team believe that the model that we have chosen accurately and represents the situation. It adequately – as the problem charges – explains the national debt and makes forecasts based on different assumptions.

Dear President-Elect Obama,

Our team writes to you just less than a week after your stunning victory in the so-called “election of a lifetime” – but you and we both realize that America’s problems are far from solved. More than anyone, you should understand the financial crisis that faces our country – and our entire world – today. On the campaign trail, you and Senator McCain constantly mentioned the economic difficulties that plague both Wall Street and Main Street – but what of those on Pennsylvania Avenue?

In October of this year, the National Debt Clock ran short of digits to record the country’s federal debt of $10.2 trillion – a sad testament to the fiscal irresponsibility of presidencies past. Now that number has risen to a whopping $10.6 trillion, $6 billion of which was accumulated in the time that it has taken our team to compile this report. In this letter, we will present our proposal on altering current tax and expenditure policies to alleviate national debt.

Using a system of mathematical functions to model the American economy, we made various forecasts for the numbers that will shape the world for the next decade—such as interest rates, inflation, GDP and population. Using accurate regressions to predict these four variables, we predicted data points for the years 2009 to 2017. Using these data, we were able to determine the national debt for the years 2009 to 2017 using tax credit, tax cuts, and spending policies proposed by both you and Senator McCain during the campaign. Using our population and GDP predictions, we were able to calculate GDP to national debt ratio and national debt per capita as well. Our predictions show that the tax policies of both you and Senator McCain are unsustainable.

We would like to propose to you a tax and spending proposal that we believe would fulfill your promises to the nation while also helping alleviate the national debt. Under our proposal, you will continue to pull out of Iraq, establish full universal healthcare, establish a green energy sector, and improve infrastructure, and you will be able to eliminate earmarks. However, we propose that you give less tax credits than you have promised, and continue to close all tax loopholes. We propose that you only continue with capital gains and dividends tax reform, and continue college tax credit, but do not continue with other tax credits you have proposed and allow the whole Bush tax cut to expire. The policies we propose are largely in line with your Senatorial voting record, and are supported by the majority of your constituents.

The tax and spending policies we have outlined above are not perfect. They are not an instant remedy to all our nation's problems. But we will get there – yes, we can begin to reverse the economic quagmire that the country has found itself in. However long it takes, we *will* heal this nation. The crisis of our national debt is an issue that transcends all party lines. It's just as Congressman Kucinich said in his speech to the DNC: "this is not a call for you [the American people] to take a new direction from right to left; this is a call for you [them] to go from down to up." Mr. President, you are the one to lead our nation in the answer to that call.

Sincerely,

The Members of HiMCM Team 2031

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