HIGH SCHOOL MATHEMATICAL CONTEST IN MODELING

OUTSTANDING PAPERS

HiMCM

November 2004

The contest offers students the opportunity to compete in a team setting using applied mathematics in the solving of real-world problems.

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Editor’s Comments

This is our seventh HiMCM Special Issue. Since space does not permit printing all of the seven National Outstanding papers, this special section includes the summaries from five papers and abridged versions of two. We emphasize that the selection of these two does not imply that they are superior to the other Outstanding papers. We also wish to emphasize that the papers were not written with publication in mind. Given the 36 hours that the teams had to work on the problems and prepare their papers, it is remarkable how much was accomplished and how well written many of the papers are.

Contest Director's Article

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The High School Mathematical Contest in Modeling (HiMCM) completed its seventh year in excellent fashion. The mathematical modeling abilities of students, faculty advisors, and contest judges is evident in the professional submissions and work being done. The contest is moving ahead, consistent with our positive experiences from previous HiMCM contests.

This year the contest consisted of 228 teams from 48 institutions. There were 39 repeat institutions and 9 new institutions. These institutions were from twenty-six states, the Hong Kong International School, and a Department of Defense school. This was the second year that the contest charged a $45 registration fee for the first team and $25 for each additional team from the same institution.

The teams accomplished the vision of the contest founders by providing unique and creative mathematical solutions to complex open-ended real-world problems. This year the students had a choice of two problems.

Problem A: Motel Cleaning

Motels and hotels hire people to clean the rooms after each evening's use. Develop a mathematical model for the cleaning schedule and use of cleaning resources. Your model should include consideration of such things as stayovers, costs, number of rooms, number of rooms per floor, etc. Draft a letter to the manger of a major motel or hotel complex that recommends your model to help them in the management of their operation.

Problem B: Art Gallery Security System

An art gallery is holding a special exhibition of small watercolors. The exhibition will be held in a rectangular room that is 22 meters long and 20 meters wide with entrance and exit doors each 2 meters wide as shown in the right-hand column. Two security cameras are fixed in corners of the room, with the resulting video being watched by an attendant from a remote control room. The security cameras give at any instant a "scan beam" of 30°. They rotate backwards and forwards over the field of vision, taking 20 seconds to complete one cycle.

For the exhibition, 50 watercolors are to be shown. Each painting occupies approximately 1 meter of wall space, and must be separated from adjacent paintings by 1 meter of empty wall space and hang 2 meters away from connecting walls. For security reasons, paintings must be at least 2 meters from the entrances. The gallery also needs to add additional interior wall space in the form of portable walls. The portable walls are available in 5-meter sections. Watercolors are to be placed on both sides of these walls. To ensure adequate room for both patrons who are walking through and those stopped to view, parallel walls must be at least 5 meters apart throughout the gallery. To facilitate viewing, adjoining walls should not intersect in an acute angle.

The diagrams below illustrate the configurations of the gallery room for the last two exhibits. The present exhibitor has expressed some concern over the security of his exhibit and has asked the management to analyze the security system and rearrange the portable walls to optimize the security of the exhibit.

Define a way to measure (quantify) the security of the exhibit for different wall configurations. Use this measure to determine which of the two previous exhibitions was the more secure. Finally, determine an optimum portable wall configuration for the watercolor exhibit based on your measure of security.

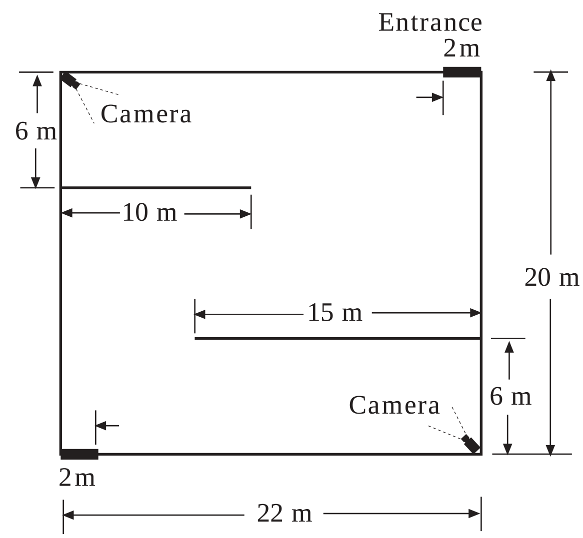

Figure 1: Exhibit Configuration: November 3–25, 2003

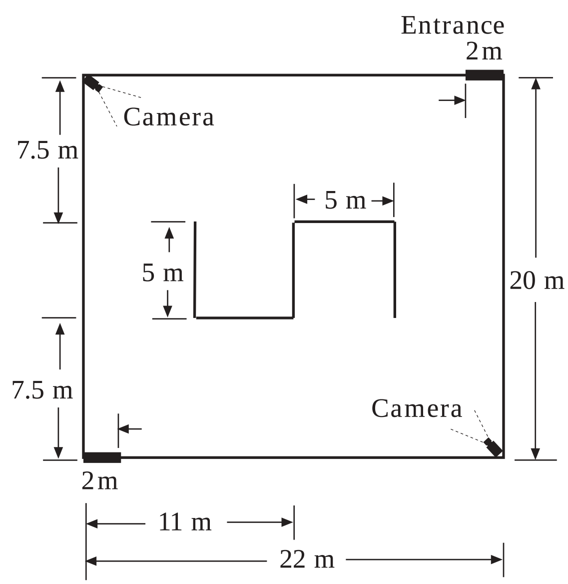

Figure 2: Exhibit Configuration: March 4–29, 2001

Commendation: All students and advisors are congratulated for their varied and creative mathematical efforts. The thirty-six continuous hours to work provided (in our opinion) an improvement in the quality of the papers. Teams are commended for the overall quality of work.

66 of the 228 teams submitted solutions to Problem A, and 162 submitted solutions to Problem B. 36% of the team members were female, showing that this competition is for both male and female students. Teams again proved to the judges that they had "fun" with the problems, demonstrating research initiative and creativity in their solutions. This year's effort was indeed a success!

Judging: We ran three regional sites in December, 2004. Each site judged both problems. Papers judged at regional sites were not from their respective region. Papers were judged as Outstanding, Meritorious, Honorable Mention, and Successful Participant. All finalist papers for the Regional Outstanding award advanced to the National Judging. For example, if eight papers were discussed for a Regional Outstanding award but only four were selected as Outstanding, all eight papers advanced to national judging. The national judging chose the "*best of the best*." The national judges commended the regional judges for their efforts and found the results very consistent.

JUDGING RESULTS:

National & Regional Combined Results

Problem	**National Outstanding* (3%)**	Regional Outstanding (6%)	Meritorious (28%)	Honorable Mention (46%)	Successful Participant (16%)	Total
A	**1**	7	18	28	12	**66**
B	**6**	8	46	78	24	**162**
Total	**7**	15	64	106	36	**228**

GENERAL JUDGING COMMENTS:

The judge's commentaries provide detailed comments on solutions to each problem. As contest director and head judge, I want to speak generally about the solutions. Papers need to be coherent, concise, and clear. Students need to restate the problem in their own words so judges can determine the paper's focus. Papers that explain model development, assumptions, and solutions and then support the findings mathematically generally do quite well. Modeling assumptions need to be listed and justified, but only those that come to bear on the team's solution. Laundry lists of assumptions that are never used in model development deter from a paper's quality. The model needs to be clearly developed and all variables need to be defined. Thinking outside of the "box" is considered important by judges. This varies from problem to problem but usually includes model extensions or sensitivity analysis of the solution to the teams' inputs. A clear conclusion and answers to specific scenario questions are key components. The strengths and weakness section is where the team can reflect on the solution. Attention to detail and proofreading the paper prior to final submission are also important because the judges look for clarity and style.

THE FUTURE:

HiMCM attempts to give the under-represented an opportunity to compete and achieve success in mathematics and appears well on its way in meeting this important mission.

We continue to strive to improve the contest, and we want the contest to grow. Any school / team can enter; there are no restrictions on the number of schools.

These are exciting times for our high school students. Mathematics continues to be more than learning skills and operations; it is a language that involves our daily lives. The ability to apply the mathematical principles that one learns is key to future success. The ability to recognize problems, formulate mathematical models, solve, and communicate and reflect on one's work is key to success. The ability to use technology aggressively to discover, experiment, analyze, resolve, and communicate results is also important to students' futures. Students learn confidence by tackling ill-defined problems and working together to generate solutions. Applying mathematics is a team sport.

Advisors need only be motivators and facilitators. They should encourage students to be creative and imaginative and to become good problem solvers. Let me encourage all high school mathematics teachers to get involved, to join the effort to make mathematics relevant to students and open doors to their future success.

Mathematical modeling is an art and a science. Through mathematical modeling students learn to think critically, communicate efficiently, and be confident, competent problem solvers for the new century.

CONTEST DATES:

Mark your calendars. The next HiMCM will be held in November 2005. Registration opens in September 2005. Teams can register via the Worldwide Web at www.comap.com.

HiMCM Judges' Commentary

Problem A: Motel Cleaning Problem

This is the purest modeling problem of the two. It is the most open-ended. No numerical information is provided. Students had to consider such variables as type of hotel / motel, location, occupancy rate, number of rooms, cleaning costs, and number of maids. Team brainstorming prior to putting pencil to paper could have been advantageous. The ill-posed nature of this problem forced students to use mathematical modeling as a process. Students needed to define what *they* meant by "develop a cleaning schedule." The judges expected to see schedules developed in accordance with some priority measure for room availability and / or occupancy. Few teams considered any priority other than all rooms had to be cleaned by check-in time. The teams should have defined and interpreted the concept of a schedule, as well as provided (even in conceptual format) a generic approach to scheduling cleaning for any size hotel / motel. Some teams did not solve the "scheduling" problem because they misinterpreted the problem to be only a minimum cost problem.

Most letters to motel managers were poor. These letters were to be written for motel / hotel managers and not for mathematicians. They needed to be in English and quickly get to the point. They were supposed to convince companies to use a mathematical model to help them schedule the cleaning of their hotels / motels. Many letters read like technical reports.

The judges were impressed with the creativity, quality of the analysis, and the writing by most of the teams. Teams used the Internet to find information such as costs of cleaning supplies, average time to clean a hotel room, and salaries of employees. It was not imperative for teams to pick a city or town for the hotel, but location affects costs. Often doing this allowed successful teams to move from the general ill-defined problem to an attackable, more specific one.

Most judges felt that models lacked realism, especially assumptions about identical room size and that all checkouts and check-ins were at one time. Judges expected the teams to develop sub-models that connected costs, sizes and types of rooms, occupancy information, and cleaning. Judges expected to see sensitivity analysis to show how the model was affected by "key" variables.

One of the items that discriminated the better papers was calculation of schedules based upon different criteria and comparison of results. A final conclusion came after considering all the possible outcomes.

Evidence of verifying or testing models to see if the results made "common sense" was a discriminator. It is noted that most answers given by teams made sense.

Problem B: Art Gallery Security System

This problem attracted three times as many teams as Problem A did. The judges felt that the numerical information and gallery schematics made the problem more attractable and allowed students to more easily enter the problem.

The problem statement did not define "security," and the teams' use of their own definition was a major factor used by judges to discriminate among papers. Security seemed to involve theft of pictures, and few teams considered damage / vandalism in their definition or measure. Judges felt that the variable *time* should have been a factor in the security measure. Few teams considered gallery set-ups to maximize the number of pictures. It appeared that the goal of 50 paintings was the only value teams considered.

The judges commented that most papers were poorly organized. This context is a very rich information environment, so teams needed to think things out logically (develop a plan) before writing. Brainstorming could have been useful here. Teams are reminded that assumptions can be developed or considered later in the modeling process, and it is fine to explain them as you develop your model. The better papers offered a diversity of security definitions and thus solutions. But many assumptions were not relevant to the model and therefore detracted from the quality of the paper. Often statistical techniques were used when they were inappropriate because the data had no variability (i.e., were deterministic).

It was critical for teams to define "security" in order to compare the two given gallery arrangements and to find a better arrangement. All teams developed a useful definition of security that was easy to model mathematically. Teams that considered aesthetic qualities were commended.

Creativity was also valuable here. One team was commended for their hexagonal design.

COMMENT ABOUT COMPUTER-GENERATED SOLUTIONS:

In both problems, many papers used computer code. Computer code used to implement mathematical expressions can be a good mathematical modeling tool. However, judges expected to see an algorithm or flow chart in the paper from which the code was developed. Successful teams provided some explanation or guide to their algorithm(s)—a step-by-step procedure for the judges to follow. The code attached to papers is usually only read for the teams reaching the final rounds. The results of any simulation or computer code should be explained and sensitivity analysis preformed.

For example, consider a flip of a fair coin. Here is the algorithm:

INPUT: Random number, number of trials

OUTPUT: Heads or tails

Step 1: Initialize all counters

Step 2: Generate a random number between 0 and 1.

Step 3: Choose an interval for heads, like [0.0.5]. If the random number falls in this interval, the flip is a head. Otherwise the flip is a tail.

Step 4: Record the result as a head or a tail.

Step 5: Count the number of trials and increment: Count = Count + 1.

An algorithm such as this is expected in the body of the paper with the code as an appendix.

COMMENTS ABOUT GRAPHS:

Judges found graphs that were not labeled or explained. Many graphs did not appear to provide information used by the teams. All graphs need a verbal explanation of what the team expects the reader (judge) to gain (or see) from the graph. Legends, labels, and points of interest need to be clearly visible and understandable, even if handwritten.

GENERAL COMMENT FROM JUDGES:

Executive Summaries:

These are one of the weakest parts of papers. Summaries should be completed after the solution is found. They should contain results and not details. They should include the "bottom line" and the key ideas used in obtaining the solution. They should include the particular questions addressed and their answers. Teams should consider a brief three-paragraph approach: restatement of the problem in their own words, a short description of their method and solution (without giving mathematical expressions), and a conclusion that provides the numerical answers in context.

Restatement of the Problem:

Problem restatements are important in order for teams to move from the general case to the specific case. They allow teams to refine their thinking to give their model uniqueness and a creative touch.

Assumptions/Justifications:

Teams should list only those assumptions that are vital to building and simplifying their model. Assumptions should not be a reiteration of facts given in the problem statement. Every assumption should have a justification. Judges do not want to see “smoke screens” of information by teams hoping that some items listed are what judges want to see. Variables chosen need to be listed with notation and well defined.

Model:

Teams need to show a clear link between the assumptions they listed and the building of the model.

Model Testing:

Model testing is not the same as testing arithmetic. Teams need to compare results or attempt to verify (even with common sense) their results.

Teams that use a computer simulation must provide a clear step-by-step algorithm. Lots of trials and related analysis are required when using a simulation. Sensitivity analysis is also expected in order to show how sensitive a simulation is to the model’s key parameters.

Conclusions:

This section deals with more than just results. Conclusions might also include speculations, extensions, and generalizations. This is where all scenario-specific questions are answered. Teams should ask themselves what other questions would be interesting if they had more time and then tell the judges about their ideas.

Strengths and Weaknesses:

Teams should be open and honest here. What could the team have done better?

References:

Teams may use references, but they must reference the source. Teams are reminded that only inanimate resources may be used. Teams cannot call upon a real estate agent, banker, hotel / motel manager, or any other real person to obtain information related to the problem.

Adherence to Rules:

Teams are reminded that detailed rules and regulations are posted on the HiMCM portion of the COMAP Website. Teams are also reminded that the 36-hour time limit is a consecutive 36 hours.

Problem B Summary: Maggie Walker Governor’s School

Advisor: John Barnes

Team Members: Christopher Boswell, Carlton Forbes, Christopher Lin, Aileen Zhang

Given a 20×22 meter art gallery with portable walls and a rapidly approaching exhibition, the exhibitor of this gallery charged the investigative team with evaluating designs of past exhibitions while at the same time designing an optimal format for this year’s eagerly anticipated event.

In order to establish a successful model, the team paid special attention to those security factors that could be easily quantified. The most important factor was the security cameras. Neither in 2001 nor in 2003 was the gallery arranged in such a way that every painting was covered by a surveillance camera. Between placing walls in the path of a security camera’s “scan beam” and placing paintings below a camera’s field of vision, the exhibitions had an unneeded risk of theft.

The team quantified this risk by creating a point system and assigning positive security points for each painting and entrance / exit that is visible to the surveillance camera. It was concluded that the 2001 exhibit was more secure than the 2003 exhibit. The goal then became to create a design where doorway and painting coverage could be maximized, thus yielding the highest point value.

The team designed an ideal model that netted all the security points possible. In addition, there were other benefits to the final product, including the flow of traffic of museum patrons.

Problem B Summary: Brookline High School

Advisor: Grace Wang

Team Members: Chien-Yee Cheng, Tzyy-Nong Liou, Herng-Fuu Yeh, Wan-Hsuan Young

We are to design a model to evaluate how secure is the exhibit given a portable wall configuration, and then use the model to determine which of two given configurations is more secure and come out with the best configuration.

Being secure implies that the thief cannot steal the watercolors; hence if the attendant in the remote control room sees all the paintings clearly, he will know immediately if a painting has been stolen and then catch the thief. Therefore, we design our model considering the following factors: the area that the cameras cannot see (the blind spots), the number of paintings in the blind spot field, and the time that the cameras spend scanning the blind spot area. The larger the blind field, the greater the insecurity or risk; also, risk is greater when there are several watercolors in this blind field. Finally, the longer time that the cameras spend in these blind spots, the greater the risk because the thief has more time to move around. Here’s our equation:

$$\text{Risk} = 10(A_{\text{abs}})(W_{\text{abs}})\left\lceil t_{\frac{1}{2}\text{abs}} \right\rceil + (A_{\text{rel}})(W_{\text{rel}})\left\lceil t_{\frac{1}{2}\text{rel}} \right\rceil.$$

A_{abs}, A_{rel} means "area of the absolute / relative blind field."

W_{abs}, W_{rel} means "number of watercolors inside the absolute / relative blind field."

$\left\lceil t_{\frac{1}{2}\text{abs}} \right\rceil$, $\left\lceil t_{\frac{1}{2}\text{rel}} \right\rceil$ means "ceiling function of the time the cameras scan the absolute / relative blind field per half cycle."

There are two kinds of blind spots—the absolute blind spot, where both cameras are unable to see, and the relative blind spot, where one camera can see but one cannot. The absolute blind spot makes sense to be much more insecure since the attendant can never check that area, and thus we single it out from the relative blind spot and multiply it by 10 to emphasize its crucial effect on insecurity. Also we use the time per half cycle for easy calculation, and take the ceiling function of it because there will be a time that $t_{\frac{1}{2}\text{abs}}$ or $t_{\frac{1}{2}\text{rel}}$ is smaller than 1 second, and multiplying the area to this $t_{\frac{1}{2}\text{abs}}$ or $t_{\frac{1}{2}\text{rel}}$ actually decreases risk, which doesn't make sense. Therefore, we use the ceiling function and then multiply only integers to A_{abs}, A_{rel} (besides, the other numbers that we multiply the areas by, W_{abs}, W_{rel}, are always integers, too). The best configuration should have the least quantity of risk.

According to our model, the 2003 exhibit has a risk of 162,843 and the 2001 exhibit has a risk of 24,395. Hence the bottom configuration is securer. The optimum configuration depicted in our paper has a risk of 26.6.

Problem B Summary: Maggie Walker Governor's School

Advisor: John Barnes

Team Members: R. W. Enoch, Brandon Herzog, Dale Swartz, Horia Todor

While large museums can afford complex security systems, many smaller galleries cannot afford to maintain security on such a scale. A cost-effective method of providing increased security is optimizing the current security measures within the context of the exhibit. Our particular scenario involved a twenty by twenty-two meter room in which fifty 1-meter paintings must be arranged on the exterior walls and on portable 5-meter walls. The only thing that we could change was the configuration of the walls and the placement of paintings on the walls. Because this only involves the movement of previously existing resources, it incurs practically no cost while allowing the gallery to be as secure as possible with limited resources.

In order to determine the security of a room on both relative and absolute scales, we needed a method to quantify security. Our solution was creation of the Cappola Security Index and the determination of danger zones. The Cappola Index was based on the factors of percentage of paintings covered by cameras, the amount of time that the cameras cover the painting, the distance of a painting to the exits, and whether the cameras can monitor the exits. These components can be used either to determine the security of an entire gallery space or of a single painting. For test cases, we used the two provided configurations, and we also designed a gallery to optimize security as quantified by the Cappola Index

As hypothesized, Gallery 2 was more secure than Gallery 1, but neither was optimal. Gallery 3, the optimal design, consisted of placing four 5-meter portable walls (the minimum required to hang 50 paintings) in a straight line centered along the diagonal between the two cameras and rotating it a small amount in order to expose the paintings on portable walls to the cameras. Our design outscored the two others because it maximized the four factors. While applying the Cappola system to a gallery provides an effective measure of its security, applying it to individual paintings identifies the safest areas of a room in which to put the most valuable paintings by determining color-coded "danger zones."

Problem B Summary: California Academy of Mathematics and Science

Advisor: Janet Lewis

Team Members: Joon-Bok Lee, Jessica Torres, Timothy Uy, Erik Van Esselstyn

We decided that the effectiveness of the exhibit's security system is determined by the area that is not covered by the cameras during their rotation (comprehensiveness) and the amount of area covered at any given time (quantity / quality). Consequently, our standard for measuring the effectiveness of security designs was 1) whether or not there were blind spots (spots not covered by either camera), 2) the total area covered by the cameras in their 20-second rotation, giving us the average area covered by the cameras per second, and 3) the area covered by both cameras sometime during the extent of their rotation (beneficial redundancy).

Using these three, we devised a way to calculate the security of a design. We determined (during the course of a full rotation) how much of the gallery area is covered by both cameras, how much of the area is covered by one camera, and how much of the area is not covered (blind spot). Using these quantities, we found the average total area covered per second by both cameras through the following equation:

> (total net area covered by the cameras) / (time taken for both cameras to sweep across once) = (single coverage area + 2*double coverage area) / (time taken for both cameras to sweep across once).

It takes the camera 20 seconds to complete one period, which is two full sweeps of the exhibit area. Since we were determining the area covered in just one full sweep, however, we used 10 seconds in all calculations of average area covered.

Using these methods, we realized that of the two given designs, the 2001 design could scan 46.4676m^2 / second, while the 2003 design could scan 31.4501 m^2 / second. Consequently, the 2001 design was by far the better of the two. The 'S' design also had much less blind area: 31.581m^2 vs. 125.499m^2.

We also tested five designs of our own, after throwing out many other proposed designs due to the existence of blind spots.

Among the five, the diagonal design-1 best met our criteria, with 100% of the gallery area covered by both cameras in a rotation. In this design, all four walls are on the diagonal in a straight line between the cameras, with the center of the four connections at the center of the gallery. Since this design gives the greatest area coverage possible, 88m^2/second, we would advise the gallery to follow this diagonal design-1.

Problem B Summary: Maggie Walker Governor's School

Advisor: John Barnes

Team Members: Eric Burcham, Ranjan Khan, Paymon Khorrami, Adam Roberts

Just recently, a famous painting known as 'The Scream" was stolen from a museum in Norway. Although there is no doubt the museum had been dubbed "secure," the picture was stolen nonetheless. In an attempt to prevent further thefts, one art gallery took a closer look at its security system. The exhibition at hand is in a room with only two security cameras. The problem arose over how to arrange the room to optimize its security.

The first step is to define factors that determine security. Such factors include the number of paintings seen by the cameras and the number of paintings that fit into the room. The numerical values are determined for such measures. These values are then scaled down to common units and weighted according to their priority to security. The final result is a score between zero and one for each room design, where zero is least secure and one is most secure. In the business world, this procedure for making choices is known as the Multi-Attribute Utility Theory (MAUT). By applying MAUT to several designs, it is possible to determine which room has the most secure design.

Based on the results from the model, the 2001 design was deemed more secure than the 2003 design. However, these did not match the security of the subsequently proposed designs. Based on the factors in the model, the best design was more than twice as secure as the 2001 design.

Problem A Paper: Davis Senior High School

Advisor: Sarah Williams

Team Members: Anna Chen, Cynthia He, Rossitza Tzankova, Serena Yeung

I. INTRODUCTION

The entertainment and hospitality industry generates billions of dollars each year towards the economies of countless nations. Profit, customer satisfaction, and efficiency are the primary factors influencing the operation of a hotel or motel.

Housekeeping is an indispensable function in a hotel and can make or break customer satisfaction. When allocating resources and finances for housekeeping, hotels must consider the predicted occupancy and the hotel's budget, as well as the hotel's layout. In this paper we present a mathematical model for management and scheduling of hotel housekeeping.

II. OUR INTERPRETATION AND RESTATEMENT OF PROBLEM

Given a motel / hotel layout and its occupancy, what is the cleaning schedule and allocation of resources that optimizes overall cleaning time and costs?

III. ASSUMPTIONS

1. For ease of communication we define "hotels" as both hotels and motels.
2. We define cleaning resources to mean human resources.
3. The names "housekeepers" and "maids" are used interchangeably.
4. The cost of cleaning materials for each room is known. Cleaning materials are resources such as detergents, washing machines, Windex®, and Lysol®.
5. Occupied rooms are cleaned once a day and may be cleaned at any time during an eight-hour period.
6. All housekeepers work at the same speed.
7. Each room is cleaned to the same standard.
8. After guests check out, the room is cleaned before the next guests check in. Hotels have definite check-in and checkout times. Hotels allot 3 hours between check-in and checkout, during which all checkout cleaning takes place.
9. Hotels do not set check-in times; however, 3 hours each day are still allotted for checkout cleaning.
10. Some guests stay in their rooms past the set checkout time, hampering checkout cleaning. These situations are called stayovers.
11. Some guests put out "Do Not Disturb" signs delaying cleaning of their rooms.
12. The hotel operates with a set budget for housekeeping staff. The consumer's cost of staying in the hotel and the hotel's profits need not be considered.
13. Each housekeeper works no more than 8 hours a day.
14. Full-time maids are paid by the month and are less expensive than part-time maids, who are paid by the hour. Part-time maids can be recruited at any time.
15. Each hotel has distinct seasons that are busy and others less busy.
16. There are three types of rooms: rooms with a king-sized bed, rooms with two double beds, and suites.
17. For a room with a king-sized bed, daily cleaning takes 20 minutes and checkout cleaning takes 30 minutes. For a room with two double beds, daily cleaning takes 30 minutes and checkout cleaning takes 45 minutes. For a suite, daily cleaning takes 45 minutes and checkout cleaning takes 55 minutes.
18. Checkout cleaning takes longer than daily cleaning because the former involves more extensive maintenance such as vacuuming or sanitizing fixtures.

19. Each maid uses one cart. Each cart holds enough cleaning materials and room supplies for one day of cleaning. Carts can hold supplies for at least 15 rooms, and each housekeeper can clean up to 15 rooms per day.
20. A11 maids carry electronic communication devices that instantly show each room's status (clean or unclean). The devices also track the location of each maid.
21. There may be guests with special requests for housekeeping assistance (e.g. extra towels or blankets).
22. There is only one exit / entrance area (e.g. an elevator) for each secluded hallway on a floor. The rooms of each secluded hallway are isolated from those of other hallways.

IV. JUSTIFICATION AND MODEL DESIGN

We chose to create a scheduling model because the daily creation of cleaning schedules is the single most important factor in the operation of a hotel's housekeeping mechanisms. Our model allows the user to input known information about hotel occupancy and layout. Based on previously determined constants regarding cleaning time, the model provides a housekeeping budget and schedule that can later be changed.

Our model consists of a two-step procedure coded in Matlab. For simplicity, we call the steps "Seasonal Model," taking into account the busy season as compared to the less busy season, and "Daily Schedule Model," providing flexible daily cleaning assignments for each maid. The former yields general budgeting and staff size recommendations and feeds into the latter, which is more dynamic.

Variables in Seasonal Budget Model:

1. *ke* = Number of king rooms needing daily cleaning.
2. *de* = Number of double rooms needing daily cleaning.
3. *se* = Number of suites needing daily cleaning.
4. *kt* = Number of king rooms needing checkout cleaning.
5. *dt* = Number of double rooms needing checkout cleaning.
6. *st* = Number of suites needing checkout cleaning.

The constants used in the Seasonal Budget Model:

1. The 8 hours of total cleaning time available each day;
2. The 3 hours of checkout cleaning time available each day;
3. Cleaning times for each room type and cleaning type (daily or checkout).

The user must input:

1. The average number of occupied king rooms, double rooms, and suites for daily cleaning (*avgke*, *avgde*, *avgse*) and checkout cleaning (*avgkt*, *avgdt*, *avgst*);
2. The cost of cleaning materials in dollars for each room type and cleaning type (*resourceske*, *resourcesde*, *resourcesse*, *resourceskt*, *resourcesdt*, *resourcesst*);
3. The monthly salary (*salary*) of housekeepers.

The Matlab code converts cleaning times to hours. Next, the code calculates total work time needed to clean all extended-occupied rooms (*totalework*) and total work time needed to checkout-clean all turnover-occupied rooms (*totaltwork*).

Because all checkout cleaning must take place within 3 hours and daily cleaning within 8 hours, we prioritize checkout cleaning. The number of maids (*maidst*) needed to checkout-clean the turnover-occupied rooms (*totaltwork*) within 3 hours is defined as *totaltwork* / 3, rounded to the nearest integer. (Intuition might tell us that we should round this number up because if there is a part of a maid's portion of work one would need a full maid to complete it, but because this is an average prediction the usual method of rounding suffices).

The code then plans for the *maidst* to spend their remaining 5 hours daily cleaning extended-occupied rooms (*totalework*). The number of maids needed to do the turnover rooms is either the same as or more than the number of maids (*maids*) needed to complete *totalework*, as well as *totaltwork*. For this reason, the code defines *maids* = *maidst*.

The budget recommended to pay the maids' salaries for one month, *budgetm*, is defined as *maids**salary*. The budget for cleaning materials for one month, *budgetm*, is defined as *resourceske**avgke* + *resourcesde**avgde* + *resourcesse**avgse* + *resourceskt**avgkt* + *resourcesst**avgst* + *resourcesdt**avgdt* + *resourcesst**avgst*.

The outputs of the Seasonal Budget Model are the number of full-time maids needed for the season and the budgets in dollars for the housekeepers' salaries and budget for cleaning materials.

The second part of the model is called the "Daily Schedule Model." This model creates dynamic cleaning schedules for individual housekeepers and provides for flexibility in the event of stayovers or special requests.

In this model, the number of housekeepers available is known (from the Seasonal Budget Model). For each floor, the Daily Schedule Model creates a somewhat circular cleaning path starting from the right of the elevator and passing through each hallway twice (because there are rooms on either side). A maid's cleaning pattern follows this path.

Variables in the Daily Schedule Model:

The rooms have any of the following identities:

1 = Unoccupied, requiring no cleaning

2 = Occupied double, requiring daily cleaning (30 minutes)

3 = Occupied double, requiring checkout cleaning (45 minutes)

4 = Occupied king, requiring daily cleaning (20 minutes)

5 = Occupied king, requiring checkout cleaning (30 minutes)

6 = Occupied suite, requiring daily cleaning (45 minutes)

7 = Occupied suite, requiring checkout cleaning (55 minutes)

The user inputs the following:

1. Number of buildings in the hotel complex
2. Number of floors in each building
3. Number of secluded hallways on each floor
4. Room types encountered traveling down the hallway from the floor's main entrance. (8 represents a return to the entrance and thus a completed path.)

After each input of a room identity, a while loop adds the cleaning times of each room to a 1-by-*n* matrix, where *n* is the total number of rooms in the hotel. The type of room (1–7) is added to a room matrix. The loop stops when 8 is inputted.

Each matrix is expanded into two matrices; one displays room numbers and the other displays their cleaning times. A pair of matrices defines and prioritizes rooms for checkout cleaning; another pair defines rooms for daily cleaning.

The code then calculates the total amount of checkout- and daily-cleaning time (prioritizing checkout cleaning) and divides this time by the number of maids available to determine the time each maid must spend checkout cleaning or daily cleaning. If the checkout-cleaning time is greater than 3 or the total-cleaning time is greater than 8, the code calculates how many additional maids are needed. At this point the user views an output of how many part-time maids are needed.

Next, the code reevaluates the time full-time maids must spend on checkout cleaning and daily cleaning, based on the number of part-time maids available.

Finally, the code generates a cleaning schedule for each maid. First, checkout rooms are assigned full-time maids. A maid's time increases by the cleaning time of the next room specified in the checkout-time matrix. As long as a maid's time total is less than the specified checkout-cleaning time, the room number is added to the schedule matrix. Once the maid's time total exceeds the specified checkout-cleaning time, the while loop for checkout cleaning stops.

Next a while loop for daily cleaning begins. This loop's time limit is *totalmaid*, which can be up to 8 hours. The code continues to add rooms to the schedule matrix. However, if a certain room is unoccupied, the schedule shows this room as a zero and thus signals the maid to skip the room. The daily cleaning while loop stops when the total time exceeds *totalmaid*.

The model generates the number of part-time maids, the total number of hours worked by each maid each day, and the cleaning schedule for each maid. Rooms needing checkout cleaning are prioritized for cleaning first.

Each maid sees room assignments on a portable device. As each room is cleaned, the maid changes its status to clean, which shows up on all maids' devices. This also allows supervisors to monitor the process.

When a housekeeper skips a stayover room, the room is marked on the portable device. This change shows up on all housekeeper devices.

After cleaning once through a path, a maid will have cleaned fewer than the assigned number of rooms because some rooms were skipped. The maid's portable device displays the next room to clean. In this way, rooms that have been skipped are cleaned.

When a guest makes a request, it is forwarded to the housekeeping manager, who adds the request to the communication database. It is displayed on the portable device of a maid near the room. That maid is responsible for responding.

Allocating maids to the closest possible room is done through a geometric model, the procedure for which we now describe. We first define some terms.

[1] Each isolated (meaning, there is no path from it to another section) section of the hotel is considered an individual building.

[2] An 'uncooperative' room is one that cannot be cleaned when scheduled.

[3] A 'reformed' room is one that was uncooperative (thus skipped) but has become available for cleaning.

[4] A maid is 'premature' at the end of a path if rooms have been skipped.

[5] A 'complete' maid has worked all of the time for which the maid is paid.

[6] Linearly equidistant means distance along the lines of hallways, not Euclidean distance.

The model aims to find the shortest distance from a reformed room to a premature maid.

In this geometric model we assume the following:

1. Each building in the hotel can be decomposed into rectangular components. A building is connected through walkways, doors, etc.
2. It is likely that a maid's schedule will include uncooperative rooms.
3. The previous assumption implies that each maid who has skipped a room will reach the scheduled ending point before the maid's working time is up.
4. The time it takes to walk past an uncooperative room is negligible.
5. The time it takes to walk across a hall is negligible.
6. The aggregate travel time it takes to follow the most efficient path is negligible, but inefficiency in scheduling reformed room cleaning significantly alters the time needed to clean the hotel.
7. A maid takes the shortest path to a room.
8. There are no more and no fewer maids working at any given time than are necessary to clean every room in 8 hours.
9. The length of a line segment is proportional to the number of rooms along the segment.

10. Rooms and room occupancy are symmetric along the hallway axis. This means that each room (with the exception of rooms at unattached vertices) has a room directly across the hall from it.
11. There is at least one maid on each floor.
12. It takes longer for a maid to change floors than to move between two points on a single floor.

Given a floor plan satisfying these assumptions, the procedure is as follows.

Line segments are drawn through each rectangular component of a building to represent the walkway in that part of the building. For the purposes of geometric analysis, we can redraw the building's floor plan using only the line-graph. The intersection of two line segments implies that there is a physical connection such as a walkway, path, or door between components.

We now define the linear equation of each line segment in the form $y_i = m_i x + b_i$ along with both its domain and range.

Each room is assigned a position (x_n, y_n) along one of these lines. The room directly across the hall is assigned the same geometric position with an irrelevant indicator differentiating the two (e.g., $(x_8, y_5)^A$ and $(x_8, y_5)^B$).

In order to have a uniform time-distance we assign time-distance value r to each type of room. We arbitrarily assign a value of 1 to the time it takes to perform king daily cleaning. Each other room has a value equal to the time it takes to clean it divided by the time it takes to perform king daily cleaning. Thus, the time distance values are $r_{kd} = 1$, $r_{kc} = 1.75$, $r_{dd} = 1.5$, $r_{dc} = 2.25$, $r_{sd} = 2.25$, and $r_{sc} = 2.75$.

Total time-distance (measured in r) is r_{total} = (# of kd) + 1.75(# of kc) + 1.5(# of dd) + 2.25(# of dc) + 2.25(# of sd) + 2.75(# of sc).

Because we assumed that no time is wasted in skipping rooms, we further assume that r is constant for all maids working equal times. Thus, if time-distance is lost (due to skipping rooms), the lost time-distance is made up at the end of the maid's path in the allotted time. By the end of the work time, each maid covers r time–distance. That is, $r_{\text{total}} = r_{\text{sequential}} + r_{\text{reformed}}$.

At any given time t there is a set of maids who become premature and also a set of reformed rooms. The position of each such maid and each such room is known. (A set may be empty. However, if either set is empty, then the other must be empty as well, and the model at that time t is irrelevant.)

For each reformed room, we find the locus of premature maids who are linearly equidistant in a given linear radius and whose remaining time-distance r is greater than the time-distance value of the room.

We add the total physical distances (measured in number of rooms) to be covered by each maid in the different combinations of maid-to-room assignments using the linear equations of each hallway. This employs the basic formula for distance along a line, taking into account the range and domain of each line so as to differentiate between hallways.

We find the maid-to-room assignments that give the lowest values and assign each maid to the appropriate room. The geometric model would be part of the scheduling program and respond to changes due to requests or stayovers.

V. SENSITIVITY AND TESTING OF THE MODEL

The results are subject to several conditions that could influence accuracy. The distribution of a tourist activity range could cause the number of full-time maids predicted by the Seasonal Budget Model to differ. For example, if a two-month tourist season contains two weeks of extremely low activity, the average level of activity is decreased significantly. The predicted number of full-time maids might then not be enough for the majority of days in a busy season.

The Daily Schedule Model is also sensitive to the fact that maids cannot be idealized—they may get sick or need to take days off. Emergencies in the hotel may also alter the accessibility of paths to be taken by maids and / or increase their workloads. In addition, malfunctions of equipment may hamper the ability of maids to receive instructions. Finally, unexpected guest arrivals / cancellations may influence the workloads in a way unaccounted for in our model.

To determine the model's error, we would input data of an existing hotel and compare the model's results with the hotel's staff and housekeeping protocol. If our results are similar or appear to surpass existing procedures in efficiency, we would consider error to be minimal and our model successful.

VI. STRENGTHS OF THE MODEL

1. Because daily housekeeping often takes place over a time interval larger than 8 hours, our model could be adapted to increase flexibility. Our model ranks high in terms of adaptability.
2. The model provides effective communication among maids and supervisors. Instantaneous updating makes the cleaning program dynamic.
3. The model allows maids to maximize the cleanliness level produced by minimizing travel distance.
4. The geometry model takes into account the fact that the closest room to any given maid may not be the best room to which to send the maid, because there are other maids in the vicinity. For this reason, the geometry model allows for the greatest aggregate efficiency.
5. The model ensures that no maid enters a room that cannot be cleaned within the maid's paid work time.

VII. WEAKNESSES OF THE MODEL

1. We grouped hotels and motels together. In reality there are meaningful housekeeping differences between motels and hotels.
2. The costs of cleaning supplies are underdeveloped.
3. We do not consider a budget for part-time staff.
4. The assumptions do not account for the varying working and movement speeds of real staff, and for their needs for breaks or other personal time.

5. In the Daily Schedule Model, each room's identity must be input by hand.

6. Our Matlab code does not allow for easy correction of input mistakes.

7. Our geometry model is not coded. For a coded version, we need software capable of image recognition. Ideally, this software could trace linear paths of the walkways and assign them equations.

8. Our geometric model is also inefficient in that it calculates distance through tedious—albeit computerized—calculation.

VIII. FUTURE INVESTIGATION

1. We would refine our matlab program to incorporate the geometry model. The final model, with the generalized time-distance optimization, would allow schedules to reflect the latest room identities and needs for housekeepers around the hotels.

2. We would modify or expand our model to incorporate further dynamic factors, including varying working or movement speeds.

3. We would expand our definition of cleaning resources to include such devices as vacuum cleaners or washing machines.

4. We might choose to expand our model into separate hotel / motel models.

Problem B Paper: Clarkstown South High School

Advisor: Mary Ann Gavioli

Team Members: Justin Coplan, Jessica Dai, Jenny Filipetti, Daniel Gendler

INTRODUCTION

When the space for exhibition of major works of art is designed, security must be taken into account. The placement of the works in relation to each other, in relation to comers and entrances, as well as in relation to security devices, must be considered in determining the best layout.

The Hood Building in downtown Manhattan will host an exhibition of fifty watercolor paintings. The O'Hara Room, a 22 by 20 meter room, is the location; two cameras in opposite comers of the room comprise its security system. A guard in a nearby room monitors the cameras, which provide a 30° field of view and complete a cycle in 20 seconds.

However, because of aesthetic and security concerns, all fifty works of art do not fit on the room's walls. In order to accommodate all artwork, portable five-foot long sections of wall can be added. Although paintings hang from both sides of these walls, their placement in the middle of the room obstructs the security camera's view of paintings on the opposite side of the room. This obstruction is the exhibition's most significant security problem.

The security company provided two room configurations. However, neither is to our liking, as one does not allow all paintings to be exhibited and the other leaves large portions of wall uncovered by security apparatus. In order to create an optimal configuration we first create a system to quantify the room's security.

Terminology

- The four walls are referenced as left, top, right, and bottom such that the entrance is at the top right corner and the exit is at the bottom left corner.
- A camera is called top left (TL) or bottom right (BR) in accordance with the corner in which it is found.

Assumptions

- The two cameras are mounted in the walls not on protruding bases.
- The camera lenses are powerful enough to see clearly across the room.
- The cameras cannot rotate to cover the two adjacent walls.
- The cameras have no special capabilities such as x-ray or infrared.
- The frames of all the works of art are the same thickness.
- There is an unlimited number of portable walls available.
- The optimal layout is such that all fifty paintings are on display.
- The time required for cameras to reverse direction of motion is negligible, and they traverse the coverage area at constant velocity.

TL and BR do not rotate a full 90°. Because the field of view of the lens of either camera is 30°, the camera can "see" 15° to either side of the point on which it is focused. Hence, TL rotates between positions that are 15° and 75° below the axis of the top wall respectively (Figure 1). Similarly, BR rotates between positions that are 15° and 75° above the axis of the bottom wall respectively.

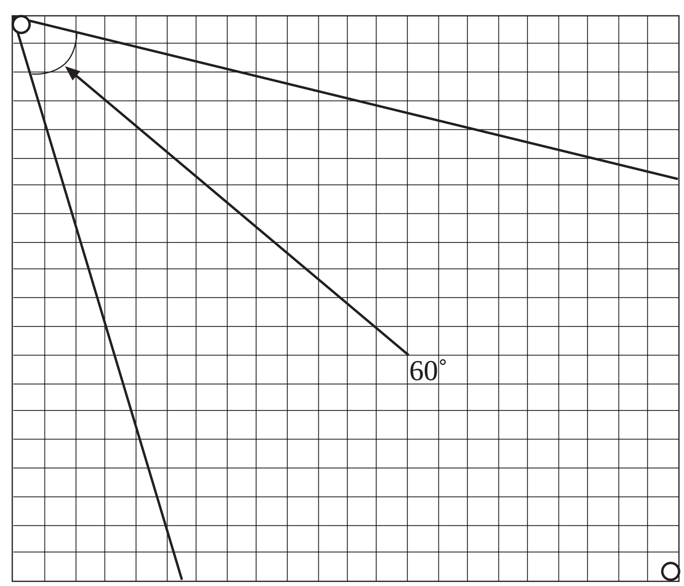

Figure 1.

METHODOLOGY

QUALITATIVE MEASURE DEVELOPMENT

To create an efficient measure we must first define an optimally secure situation. In an optimally secure situation, the room is configured so that all fifty watercolors are displayed according to the following precautions:

- The distance between a painting and an exit / entrance is at least 2 meters.
- The distance between a painting and a comer (joining of two walls) is at least 2 meters.
- The distance between any two paintings is at least 1 meter.

In addition, portable walls are configured according to the following precautions:

- Two portable walls cannot be joined by an acute angle.
- A portable wall parallel to another wall (portable or stationary) must be at least 5 meters from these parallel walls.

Lastly, a camera must cover each painting for at least 5 seconds at an interval of no more than twenty seconds.

However, we realize that this optimal situation is unlikely. Hence, we create a weighted scale to measure the security of an arrangement. The scale is:

- 2 points for a properly displayed painting that is covered by a camera for the accepted time period;
- 1 point for a properly displayed painting that is covered by a camera for less than the accepted time period.

The efficiency of a design is calculated by taking the percentage of points earned out of the maximum number of points (100 or twice the number of paintings).

NUMBER OF PAINTINGS DISPLAYED

Since no points are added for a painting not on display, the security rating decreases as the number of displayed paintings decreases. Hence, the highest security level can only be achieved if all paintings are properly displayed.

DETERMINATION OF ACCEPTABLE TIME PERIOD

Since average human visual reaction time is about 0.19 seconds, the allocation of five seconds per painting provides ample time for a busy security worker monitoring two cameras, even when multiple paintings are on the screen simultaneously. This time also coincides with the rate of camera rotation. Although a camera scans the entire room in any one sweep, it only rotates 60° to do so because any points between 0° and 15° and 75° and 90° are included in the 15° area to either side of the camera's midpoint. Thus when the camera's midpoint is at 15° with respect to its closest wall, the actual view being monitored is all area between 0° and 30°. All paintings between 0° and 15° or 75° and 90° are considered extremes. The camera scans an extreme object for less than five seconds.

GRADING AND CRITIQUE OF PREVIOUS CONFIGURATIONS

The configuration from November 2003 receives a 34 on our scale. All fifty paintings fit in this configuration; however, the configuration of the portable walls created a large visibility problem. Neither the top nor bottom walls were scanned at all. Of those paintings that did fall under the camera view, two of them did so for less than five seconds.

The configuration from March 2001 receives a 36 on our scale. Only 46 paintings are displayed; hence, the maximum possible score for this configuration is 92. Of the 46 displayed, only 21 were scanned, and only 15 of those for a satisfactory time period. The configuration of the portable walls dictates that the middle three pieces can contain only one painting per side. The exterior two segments can contain two paintings per side. Figure 2 shows the scan lines of the cameras. Note that the right angles at the joints of the portable walls create blind areas.

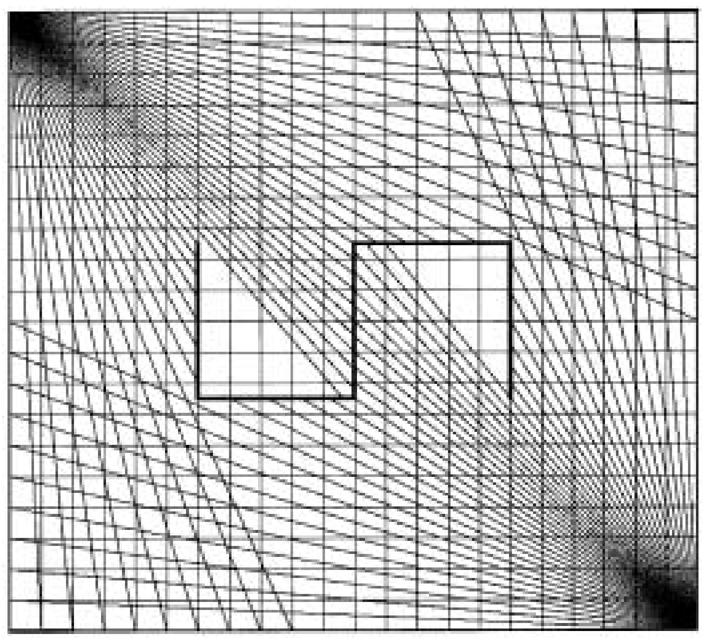

Figure 2.

DESIGNING AN OPTIMAL SOLUTION

We made it a priority to exhibit all fifty paintings. After studying the two previous exhibits, we determined that the most effective way to display all paintings was with multiple, non-joined, non-parallel sections of portable wall. Even though the two previous configurations used only five sections, we brought in an extra section. This allowed flexibility in hanging paintings, because there were more possible sites (68) than paintings. This allowed us to place paintings in those places optimally covered by the security devices.

We defined the room's axis of view as a straight line between TL and BR. When a portable wall section is placed parallel to the axis of view, it does not block any section of any stationary wall from the view of either camera. However, should the slope of a portable wall differ from the slope of the room's axis of view, the portable wall blocks parts of the stationary walls from camera view.

Figure 3(a) shows the exhibit room during pre-exhibition setup. Defining TL to be at (0, 0), with each unit equivalent to 1 meter, we see that a workman has placed a section of portable wall at position (5, 5) extending horizontally to the left. Following the ray diagram, we see that this positioning blocks most of the wall from TL's view. Figure 3(b) shows the same wall moved to position (15, 18). Now the wall blocks a much smaller portion of the bottom wall and blocks nothing of the wall on the right. Thus, the further away from a camera a wall is, the less it interferes with the camera's view.

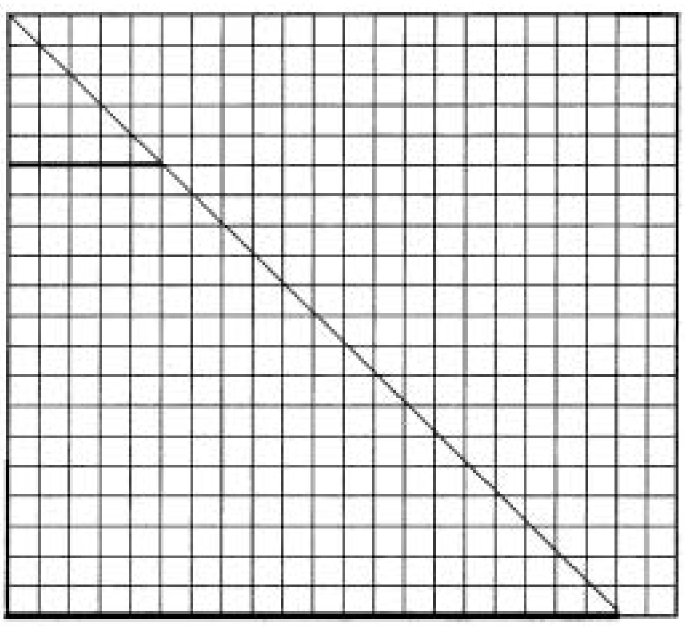

Figure 3(a).

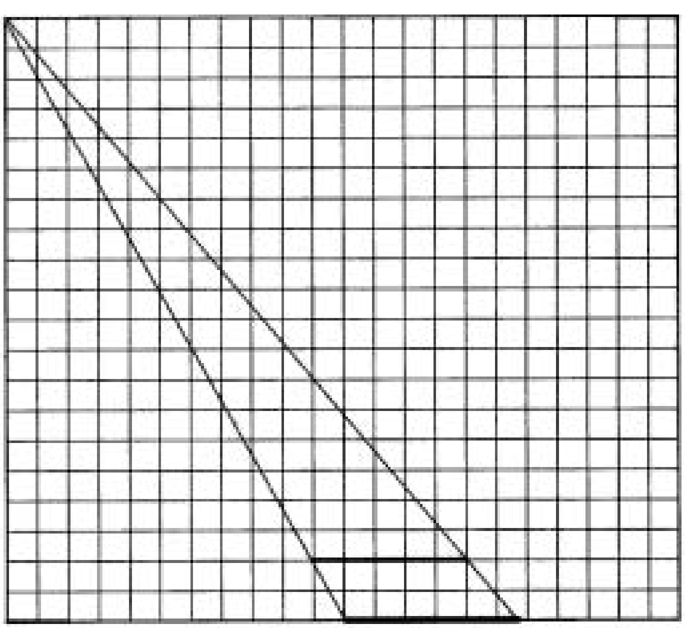

Figure 3(b).

The security specifications indicate that no painting can be placed within two meters of the entrance and exit or of any room comer. We have already determined that portable walls positioned in the center block parts of the stationary walls. Hence, another characteristic of an optimum configuration is that portable walls be positioned to block the room comers from camera view since no paintings can hang there.

The line of sight of a camera is defined as the bisector of the 30° field of view for any given angle θ, or the extension of a line that makes the angle θ with the top wall (for TL) or the bottom wall (for BR). The closer to parallel the portable walls are with respect to the line of sight of the camera for that value of the camera's angle, the less does that wall interfere with the camera's scanning of other paintings. When the line of sight is equal to the axis of sight, for example, it represents the absolute minimization of obstruction of other paintings for both cameras, because the axis of sight is common to both. However, it allows for monitoring of only two paintings per side because, assuming that the frames of all paintings are the same thickness, each camera can only see the side of the painting closest to it on either side of the wall. Thus any portable wall on the axis of sight can only display four paintings total.

In order to have all fifty paintings visible to cameras, each portable wall would best contain the maximum number of paintings. Thus no portable wall can be situated exactly on a line of sight because then it can hold four rather than six watercolors. Essentially, the closer a wall is to parallel to the line of sight, the lower the degree of obstruction but the lower the visibility of all paintings. The only point at which the middle picture on each side is completely unseen is when the wall is perfectly parallel to the line of sight. However, the effects of a lower degree of obstruction continue to apply even as the wall is not exactly parallel; that is, when a wall is close to being parallel, all paintings can still be seen and the obstruction is minimal. Thus when determining the angles at which the portable walls should be placed, we recognized that the walls should be as close to parallel to a line of sight as possible, while balancing the effects of visibility and obstruction. In determining which line of sight to use for the four exterior portable walls, we had to prioritize which effect was more important: fitting the required number of paintings or keeping all displayed paintings in view of the camera for the maximum amount of time. If the walls were placed on a more inner line of sight, all paintings would be displayed for the maximum amount of time, but the obstruction the wall caused would be greater; and thus fewer paintings would fit on the stationary walls. Based on our model for calculating security level, we determined that it was more important to fit all fifty paintings, even if not all of them are visible for the maximum amount of time.

In determining the placement of the two interior portable walls, a balance had to be struck between the amount of obstruction a wall caused and the visibility of paintings on each wall. For this scenario, it also had to be considered that the walls could not be in the same line as each other because then only four paintings total could be viewed for both walls. However, because the walls are parallel to each other, they also must be placed in accordance with the specification that parallel walls be constructed at least five meters apart from each other. Our configuration complies with this stipulation; the distance between them is approximately 8.944 meters.

Our construction fulfills the above criteria by not crowding the stationary walls, largely not descending below the 15° or above the 75° limits, minimizing the obstruction of the other paintings, and fitting and scanning all fifty paintings.

Analysis of Security of Our Configuration (Figure 4)

- Number of watercolors on display: 50
- Number of watercolors remaining in the field of vision of at least one camera for at least 5 seconds: 38
- Number of watercolors on display that remain in field of vision for less than 5 seconds: 12
- Efficiency: 88%

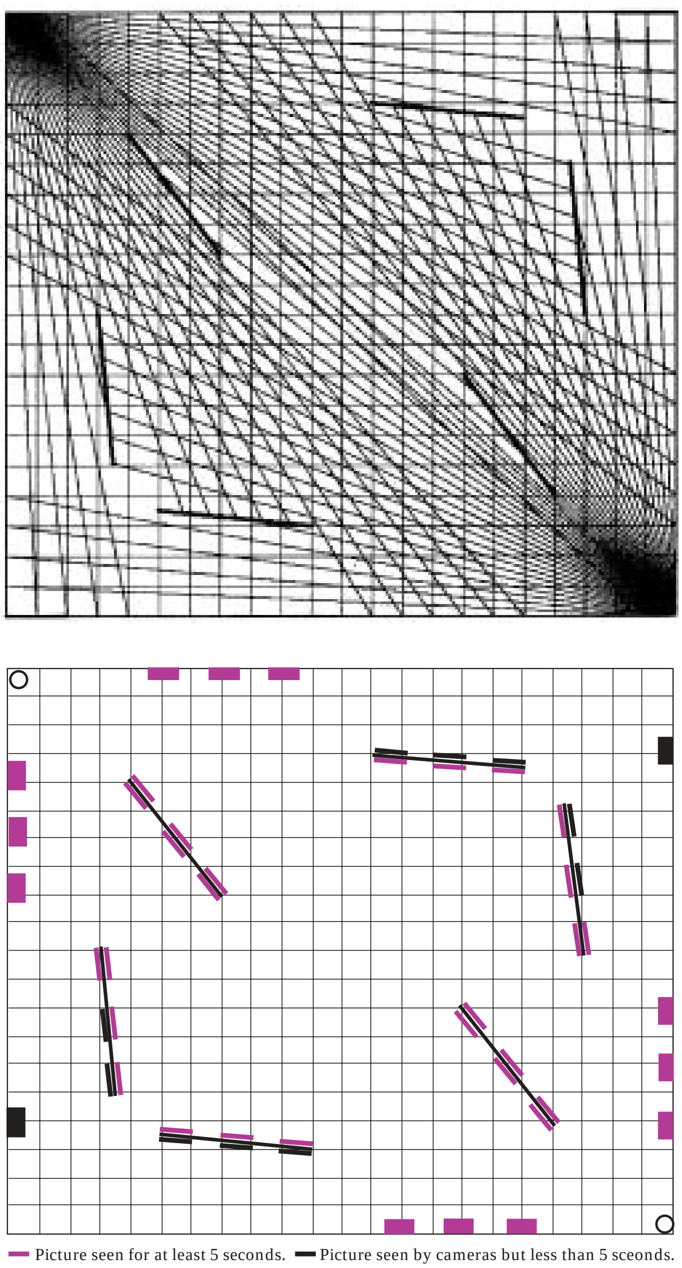

Figure 4.

TECHNICAL MODELS OF CONFIGURATIONS

We coded two Java programs to aid in our modeling. The first is a simple way to ascertain the number of paintings that can hang on a wall based on its length and the number of walls that intersect it in various manners.

The second program models a camera's lines of sight when different configurations of portable walls are entered. This is an efficient method to test designs without having to plot them manually. The application depicts a number of lines of sight, terminating a line when it intersects a wall. Any obstructed portions of the stationary walls are highlighted with the color representative of the camera from whence the terminated line of sight originated. ❑