In this paper, we demonstrate a mathematical model that evaluates the economic effects of tsunamis on a number of cities in the United States. We subdivide the problem into three parts: 1) a theoretical physics-based model to determine the height and speed of a tsunami given earthquake parameters; 2) a computational simulation to determine the physical impact of a tsunami on a target city; 3) a statistics-based model to determine ultimate economic effects.

The earthquake parameters of intensity on the moment magnitude scale and epicenter-target city distance are used to calculate the tsunami parameters of height and speed. Tsunami height and speed upon reaching the shore are input into a tsunami simulation program which returns the area impacted by the tsunami. The area of impact is then used along with economic statistics (such as property values) to calculate the net economic damage caused by a tsunami.

We evaluate our model for certain earthquake parameters in seven cities: New York City, Boston, Virginia Beach, Charleston, New Orleans, San Francisco, and Hilo. Our results indicate that the potential economic costs of a tsunami range from tens of millions for smaller cities hit by small-scale tsunamis to several billion dollars when densely populated urban areas are hit by larger tsunamis. The results of our case study simulations are presented in the table below.

City	Epicenter-City Distance (km)	Earthquake Intensity (moment magnitude)	Net Cost of Damage
New York City	3000	9	$3,608,965,061
San Francisco	500	9	$2,466,813,857
Charleston	3500	9	$174,508,083
Boston	3500	9	$6,577,758,050
ilo	1000	9	$31,087,243
New Orleans	1000	8	$62,856,820
Virginia Beach	3000	7	$32,273,723

Water, Water Everywhere…Oh Wait, it's a Tsunami!

Big Apple in Danger of Being Washed Away

Mathematicians model the effects of a tsunami on New York City.

Associated Press, 2009.

It has been a long time since the devastating 2004 Indian Ocean tsunami, and the world has become complacent about the dangers of this destructive force of nature. However, recent research by mathematicians at a well-known institution of learning has shed new light on the potential consequences of a large-scale tsunami. Imagine the following scenario.

Three thousand and five hundred miles away from our idyllic Long Island coast, lies a fissure in the ocean floor, a fault line which at any moment could shift, releasing a tremendous amount of energy into the surrounding waters. It is not unlikely that this undersea earthquake could register a 9.0 on the Richter scale, equivalent to several atomic bombs detonating simultaneously. Yet for such a powerful blast, the ocean surface will remain relatively undisturbed. Like ripples in a pond, a circular wavefront will move out from the epicenter at 500-600 miles per hour, silently and subtly, offering almost no chance of detection. Passing ships would cut through these one or two foot tall waves with ease.

However, as the waves approach the continental shelf, a mere one hundred miles away from our shores, the rising ocean floor compresses the waves, forcing them up to heights of 100 feet or more. When these monstrous columns of water hit the shore, they will transfer the entirety of their fearsome energy into a force which levels almost anything in its path. According to the new research, we could suffer casualties of over 3,314, and property losses of over $1 billion. Familiar landmarks like the Statue of Liberty, Long Island, and your favorite hot dog vendor could be gone in the blink of an eye.

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Introduction

A tsunami, sometime colloquially referred to as a tidal wave, is a large ocean wave comprising the displacement of extremely large volumes of water. A variety of factors, including volcanic eruptions, oceanic landslides, and meteorite impacts, can incite the generation of a tsunami. The most common sources of tsunamis, however, are undersea earthquakes. Tectonic earthquakes of sufficient magnitude (typically >7.0 on the Richter scale) can release tremendous amounts of energy in the form of extremely powerful ocean waves, hundreds of kilometers long, that can travel at hundreds of kilometers per hour in the deep ocean.

The word "tsunami" comes from the Japanese term for "harbor wave", as tsunamis are noticeable only near the coast. Although tsunamis are still energetic in the deep ocean (benthic zone), amplitudes are typically under one meter, making these waves virtually undetectable in the benthic zone. Upon reaching shallow water, however, tsunamis lose speed and gain amplitude without a significant loss of energy. These towering, massive, and quickly moving walls of water pose a threat to many coastal cities within a few thousand kilometers of an active fault line. Coastal cities hit by tsunamis are faced not only with the immediate destructive threat of a moving mass of water but also with lingering economic effects caused by flooding, damage to infrastructure, consequent loss of life, etc.

In this paper, we evaluate the potential economic devastation caused by earthquake-generated tsunamis in seven American cities: New York, Boston, Charleston, New Orleans, San Francisco, Hilo, and Virginia Beach. The progression of a tsunami and its damage to a city is divided into three stages: 1) the initial generation of a tsunami in the deep ocean by an earthquake and the movement of the tsunami towards the shore, 2) the breaking of the wave over the shore and its initial impact on the city, 3) immediate and long-term economic effects of the tsunami on the city. We conclude the paper with case studies in which we evaluate our model in each city.

All earthquakes considered in this paper are tsunami-generating and therefore occur along fault lines in oceanic areas. We consider earthquake parameters here to be the distance of the earthquake epicenter from the target city and earthquake intensity measure on the moment magnitude scale. In the first section of our model, we determine height, wavelength, and velocity of a tsunami generated from a given earthquake. In this section, physics-based relationships and simple wave mechanics are used to derive formulae for the quantities mentioned above.

The second section of our paper entails a computer-based simulation of the crashing of the tsunami onto the shore and initial impact on the target city. Elevation and topographical data of the various cities studied are used to generate contour maps which can then be utilized as surfaces for water to flood. Using the height and velocity of the tsunami when it hits the shoreline as parameters, we develop a model that demonstrates the distance that the wave reaches inland and the height it reaches when it floods the city.

The third section and final section of the paper assesses the damage ultimately done on the economy and infrastructure of the target city by the attacking tsunami. By considering the radius of impact of the tsunami, estimates are calculated for the economic damage in each city. The approximate monetary damage per square mile is determined, multiplied by the total affected area of the city, and then added to long-term negative effects (such as destruction of infrastructure, displacement of residents, etc.) to obtain an estimate for the total damage done by the tsunami in each city.

Seismic Tsunami Generation Model

Introduction

In this section we describe the generation of a tsunami and its journey to the shallow coastal waters along continental shoreline. We consider tsunamis generated by undersea earthquakes a significant distance off the coast. We analyze the tsunami by considering three consecutive stages of tsunami propagation. The first stage is the actual earthquake and tsunami generation, in which seismic energy from an earthquake is released in the form of a tsunami. The second stage comprises the relatively uneventful journey of the tsunami across the deep ocean, termed the benthic zone. The third and final stage consists of the shallow water journey of the tsunami and its breaking towards shore. Given parameters of earthquake intensity and epicenter distance from the target city, we aim to calculate breaking wave speed, breaking wave height, and breaking wavelength.

Initial Wave Energy

We begin by considering an earthquake on the ocean floor at depth d of intensity M_W on the moment magnitude scale, a modified version of the older and more commonly known Richter scale. The intensity M_W is defined in terms of the seismic moment of the earthquake: $M_W = \frac{2}{3}\log_{10} M_0 - 10.7$. Here, M_0 is the seismic moment in dyne centimeters, a unit equivalent to 10^{-7} N·m. The seismic moment of an earthquake is defined as $M_0 = \mu A u$, where μ is the shear modulus of the rock in GPa, A is the area of the fault rupture, and u is the displacement of the fault rupture. Since a sideways fault movement would cause no vertical water movement, for the purpose of our paper we assume that fault rupture occurs vertically.

The empirically derived Gutenberg-Richter magnitude-energy relation states that a logarithmic relation exists between the energy E_e in ergs released in an earthquake and the intensity of the earthquake. This relationship is expressed as

$$\log_{10}(E_e) = 1.5M_W + 11.4$$

Raising to the power of ten, we obtain the relation

$$E_e = 10^{11.4} \cdot 10^{1.5M_W} \approx 10^{11.4} \cdot 31.6^{M_W}\,\text{ergs}$$

We convert to joules to obtain

$$E_e = 10^{4.4} \cdot 31.6^{M_W}\,\text{J}$$

We recall that the seismic moment of an earthquake, by theoretical definition, is the total energy that could possibly be released in an earthquake. We note that that only a small portion of the seismic moment is released as mechanical energy. In light of this relationship and considering outside research, it is reasonable to assume that $\sim 5\%$ of the released energy of an undersea earthquake is absorbed by the ocean. Therefore, the mechanical energy E in Joules of the tsunami wave can be expressed as $E = 10^{3.1} \cdot 31.6^{M_W}\,\text{J} = 10^{3.1+1.5\text{M}_\text{W}}\ J$.

Wave Mechanics in the Benthic Zone

In the rest of the section, we will determine properties of the wave after the tsunami's generation by an undersea earthquake. We assume that a tsunami spreads out in a uniform circular manner from the epicenter. We also note that unlike typical waves on the ocean caused by wind, a tsunami is caused by a single mechanical event (the generating earthquake), and therefore possesses only a single wavefront. Therefore, the tsunami can be described at any time before breaking as a circular wavefront of radius vt, or more properly $\int v(t)\,dt$.

Although the majority of the tsunami's propagation time and distance occurs in the benthic zone (deep ocean), a high level of mechanical power transmission ensures us that little energy is lost in this zone. Since the mechanics in the benthic zone does not affect the wave that ultimately crashes onto shore, we do not cover the details of deep ocean tsunami mechanics here. However, it is interesting to note that in the benthic zone, tsunamis have wavelengths of hundreds of kilometers, speeds of hundreds of kilometers per hour, and barely noticeable amplitudes less than one meter.

Wave Mechanics in Shallow Water

We begin our analysis of shallow water wave mechanics by recalling that the total circumference of the tsunami wavefront at the time of collision is $2\pi r$. We denote the shore length of the target city by l_s, allowing us to use $\frac{l_s}{2\pi r}$ to denote the fraction of the total tsunami that strikes the city. Thus, $\frac{l_s}{2\pi r} \cdot E$ Joules of energy strike the target city.

We note that the wave speed in shallow water depends only on the depth of the water, which we continue to denote as $h(x)$. Since the velocity of the wave in shallow regions can be expressed as $\sqrt{gh}$ (g is assumed to be a constant 9.8 m/s^2), the kinetic energy of the wave here can be written as $\frac{mv^2}{2} = \frac{mgh}{2}$. We equate the kinetic energy of the wave derived here to the expression given above to obtain the relation

$$\frac{mgh}{2} = \frac{l_s}{2\pi r} \cdot E$$

We can simplify this relation to obtain the equation

$$m = \frac{l_s}{\pi r g h} E$$

However, we can express the mass m of the wave as $m = \rho V = \rho A l_s$, where ρ is the density of the water (assumed in this paper to be a constant 1000 kg/m^3), A is the cross-sectional area of the wave, and l_s is the length of the wave that hits the shore. Thus, we obtain the relation

$$\rho A l_s = \frac{l_s}{\pi r g h} E$$

Simplifying this expression, we obtain

$$A = \frac{E}{\pi r \rho g h}$$

Continuing our assumption of a sinusoidal tsunami waveform, we integrate under the sine curve to model the cross-sectional area of a sinusoidal waveform, shown in Figure 1. We represent the waveform as $y = a\sin\left(\frac{\pi x}{\lambda}\right)$, and we calculate the area from $x = 0$ to λ (which we refer to herein as the wavelength) to be

$$A = \int_0^\lambda a\sin\left(\frac{\pi x}{\lambda}\right) dx = -\frac{a\lambda}{\pi}\left[\cos\left(\frac{\pi x}{\lambda}\right)\right]_{x=0}^{\lambda} = \frac{2a\lambda}{\pi}$$

Thus, substituting the derived value of A into the expression shown above,

$$\frac{2a\lambda}{\pi} = \frac{E}{\pi r \rho g h}$$

We rearrange terms to obtain the relation

$$a\lambda = \frac{E}{2 r \rho g h}$$

This suggests an inverse relationship between the amplitude and wavelength of a tsunami in shallow water.

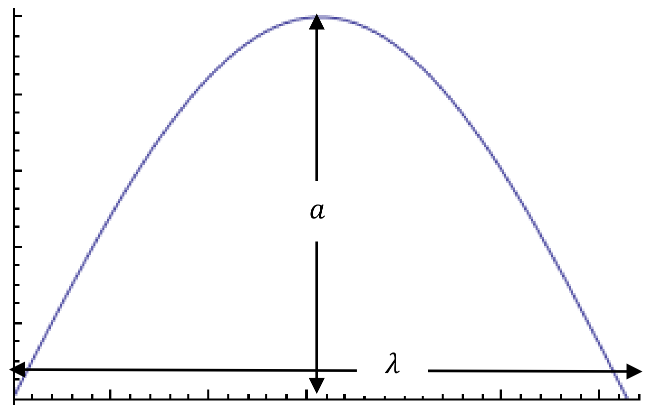


Figure 1. An ideal sinusoidal tsunami waveform of height a and wavelength λ is shown.

We now note that the velocity of a wave in water can also be expressed using the equation $v = \sqrt{\frac{g(2\lambda)}{2\pi}} = \sqrt{\frac{g\lambda}{\pi}}$. We equate the two expressions for velocity to obtain the relation $\sqrt{\frac{g\lambda}{\pi}} = \sqrt{gh}$. We now rearrange and cancel to obtain a revealing equation for shallow-water waves: $\lambda = \pi h$. We substitute this result into a previous equation to obtain the relation

$$a\pi h = \frac{E}{2r\rho gh}$$

We rearrange terms to obtain an expression for tsunami amplitude in terms of earthquake energy, distance from the epicenter, and shallow water depth.

$$a = \frac{E}{2\pi r\rho gh^2}$$

We now note that in the shallow water regions, unlike in the benthic zone, the height of the continental shelf is a non-constant but approximately linear function of distance from the shore. We note that shallow-water waves tend to break when the amplitude $a > .8h$. Thus, to solve for breaking height, we solve the inequality

$$\frac{4h}{5} < \frac{E}{2\pi r\rho gh^2}$$

Rearranging we obtain,

$$h^3 < \frac{5E}{8\pi r\rho g}, \text{or } h < \left(\frac{5E}{8\pi r\rho g}\right)^{\frac{1}{3}}$$

Thus, the tsunami wavefront begins to break when $h = \left(\frac{5E}{8\pi r\rho g}\right)^{\frac{1}{3}}$. We now use this value of h to calculate breaking values for v, a and λ.

The wavelength λ is easily calculated from h using the previously derived equation $\lambda = 2\pi h$. We now know that $\lambda = 2\pi\sqrt[3]{\frac{5E}{8\pi r\rho g}} = \left(\frac{5\pi^2 E}{r\rho g}\right)^{\frac{1}{3}}$. The breaking speed v is calculated using the equation $v = \sqrt{gh}$ to yield that $v = \sqrt{g\sqrt[3]{\frac{5E}{8\pi r\rho g}}} = \sqrt{\sqrt[3]{\frac{5g^2E}{8\pi r\rho}}} = \left(\frac{5g^2E}{8\pi r\rho}\right)^{\frac{1}{6}}$. Finally, we calculate the amplitude a to be $a = \frac{E}{4\pi r\rho g\left(\frac{5E}{8\pi r\rho g}\right)^{\frac{2}{3}}} = \frac{E^{\frac{1}{3}}(8\pi r\rho g)^{\frac{2}{3}}}{4\pi r\rho g\,(5)^{\frac{2}{3}}}$. Simplifying this expression, we obtain $a = \frac{(5E)^{\frac{1}{3}}}{5(\pi g r\rho)^{\frac{1}{3}}} = \left(\frac{5E}{125\pi\rho r g}\right)^{\frac{1}{3}} = \left(\frac{E}{25\pi\rho r g}\right)^{\frac{1}{3}}$. Dividing the expression for λ by the expression for a yields a ratio of 5π, implying that $\lambda = 5\pi a$.

Model Evaluation

Predicted wave heights, wavelengths, and wave speeds upon reaching shore are shown in Tables 1, 2, and 3 respectively. The parameters of earthquake intensity and epicenter distance from the target city are varied and the formula outputs are calculated. All predicted values appear very reasonable considering previous tsunami data.

Table 1		Earthquake Intensity (moment magnitude scale)				
Wave Height (m)		7	7.5	8	8.5	9
Distance of epicenter from target city (km)	500	4.7	8.3	14.8	26.4	46.9
	1000	3.7	6.6	11.8	21.0	37.3
	1500	3.3	5.8	10.3	18.3	32.5
	2000	3.0	5.3	9.4	16.6	29.6
	2500	2.7	4.9	8.7	15.4	27.5
	3000	2.6	4.6	8.2	14.5	25.8
	3500	2.5	4.4	7.8	13.8	24.5
	4000	2.3	4.2	7.4	13.2	23.5

Table 1. The predicted wave heights are given in meters for tsunamis generated by earthquakes of given intensity and with an epicenter a given distance from the target city.

Table 2		Earthquake Intensity (moment magnitude scale)				
Wavelength (m)		7	7.5	8	8.5	9
Distance of epicenter from target city (km)	500	168.5	224.6	299.6	399.5	532.7
	1000	150.1	200.1	266.9	355.9	474.6
	1500	140.3	187.1	249.4	332.6	443.6
	2000	133.7	178.3	237.8	317.1	422.8
	2500	128.8	171.8	229.1	305.5	407.4
	3000	125.0	166.6	222.2	296.3	395.2
	3500	121.8	162.4	216.6	288.8	385.2
	4000	119.1	158.8	211.8	282.5	376.7

Table 2. The predicted wavelengths are given in meters for tsunamis generated by earthquakes of given intensity and with an epicenter a given distance from the target city.

Table 3		Earthquake Intensity (moment magnitude scale)				
Wave Speed (m/s)		7	7.5	8	8.5	9
Distance of epicenter from target city (km)	500	10.7	14.3	19.1	25.4	33.9
	1000	9.6	12.7	17.0	22.7	30.2
	1500	8.9	11.9	15.9	21.2	28.2
	2000	8.5	11.4	15.1	20.2	26.9
	2500	8.2	10.9	14.6	19.4	25.9
	3000	8.0	10.6	14.1	18.9	25.2
	3500	7.8	10.3	13.8	18.4	24.5
	4000	7.6	10.1	13.5	18.0	24.0

Table 3. The predicted wave speeds are given in m/s for tsunamis generated by earthquakes of given intensity and with an epicenter a given distance from the target city.

Conclusion

To recapitulate, we list here the assumptions made in this section. First, we assume that the tsunami released from an earthquake spreads from the epicenter in a uniform circular fashion. Second, we assume that water is an inviscid and incompressible fluid with uniform density. Third, we assume that the acceleration due to gravity is constant. Fourth, we assume that the cross-section of a tsunami wavefront is sinusoidal in shape.

We conclude this section by providing a summary of the constants, parameters, and intermediate variables used, along with a recapitulation of derived formulae. Constants are listed in Table 4, the parameters in Table 5, the intermediate variables in Table 6, and the final formulae in Table 7. SI units are used consistently in this section.

Table 4	Symbol	Value
Density of water	ρ	1,000 kg/m^3
Acceleration due to gravity	g	9.8 m/s^2

Table 4. Constants in the seismic generation model are listed here.

Table 5	Symbol	Unit
Distance between epicenter and target city	r	Meter
Earthquake intensity (moment magnitude scale)	M_W	NA

Table 5. Parameters in the seismic generation model are listed here.

Table 6	Symbol	Formula	Unit
Tsunami energy	E	$10^{3.1+1.5M_W}$	Joule
Breaking depth	h	$\left(\frac{5E}{8\pi r\rho g}\right)^{\frac{1}{3}}$	Meter

Table 6. Intermediate variables in the seismic generation model are listed here.

Table 7	Symbol	Formula	Unit
Wavelength	λ	$\left(\frac{5\pi^2 E}{r\rho g}\right)^{\frac{1}{3}}$	Meter
Breaking wave height	a	$\left(\frac{E}{25\pi r\rho g}\right)^{\frac{1}{3}}$	Meter
Breaking wave speed	v	$\left(\frac{5g^2 E}{8\pi r\rho}\right)^{\frac{1}{6}}$	m/s

Table 7. Final outputs from the seismic generation model are listed here.

Tsunami Impact Model

Basic Assumptions

Once a tsunami wave hits the coastline, it ceases to behave as a wave. The real fluid mechanics of the tsunami at this point involve calculations far too computationally intensive to be tractable. Instead, we will model the tsunami wave as a column of water subject to the law of universal gravitation.

Imagine a cross section of the coastline when the tsunami hits; the water column at that point will have an initial height a and velocity v_0 given by the seismic tsunami generation model. By the basic laws of kinematics, it will take some time for the height of this water column to become zero, namely

$$t = k \cdot \sqrt{\frac{2a}{g}}$$

where $g = 9.8\ m/s$ is the standard acceleration due to gravity, and k is some multiplicative constant we include to account for factors that cannot be considered due to the simplistic nature of this model (for example, water will resist compression by gravity so the time t will be greater than we'd expect, suggesting $k > 1$). During this time t, the water column travels inland with speed v_0, so that the total distance traveled is

$$d = l \cdot v_0 \cdot t = (lk) \cdot v_0 \cdot \sqrt{\frac{2a}{g}}$$

where l is another empirically determined constant, included to account for the slowing of water not due to simple kinematics (buildings in the way, friction between water and the underlying terrain, et cetera). We can coalesce l and k to one constant, C, which is determined empirically:

$$d = C \cdot v_0 \cdot \sqrt{\frac{2a}{g}}$$

Tsunami Inland Propagation

Consider a coastline defined by a piecewise continuous parameterized curve $L = (x(t), y(t))$, over a topography $T(x, y)$. At some point $(u(t_0), v(t_0))$ on L, the slope of the normal vector at that point is given by

$$m = \frac{dy(t_0)}{dx(t_0)}$$

and hence the normal vector, and also the direction along which the water column will travel, is

$$\vec{N} = \langle -\frac{\partial L}{\partial y}, \frac{\partial L}{\partial x} \rangle = \langle -\frac{dy(t)}{dt}, \frac{dx(t)}{dt} \rangle$$

given a water column of height a and velocity v_0, we know the maximum traversal distance d. However, the water column may not travel the entire distance d, due to the terrain along $\vec{N}$. The height of the water column as a function of distance from its starting point is, by our water column model,

$$a(d_0) = a(0) - \left(\frac{d_0}{C \cdot v_0}\right)^2 \left(\frac{g}{2}\right)$$

It is reasonable to assume that when the height of the water column is less than the height of the terrain, the terrain will stop the water from going any further. Hence, to find when this happens for a specific $\vec{N}$, we solve for the first d_0 for which

$$a(d_0) < T(d_0 \cdot \vec{N})$$

Of course, the water cannot travel farther than d, so we compute the distance the water travels inland as

$$\min(d_0, d)$$

We repeat this process for every point along L in order to find the area which becomes flooded due to the inland propagation of the tsunami wave.

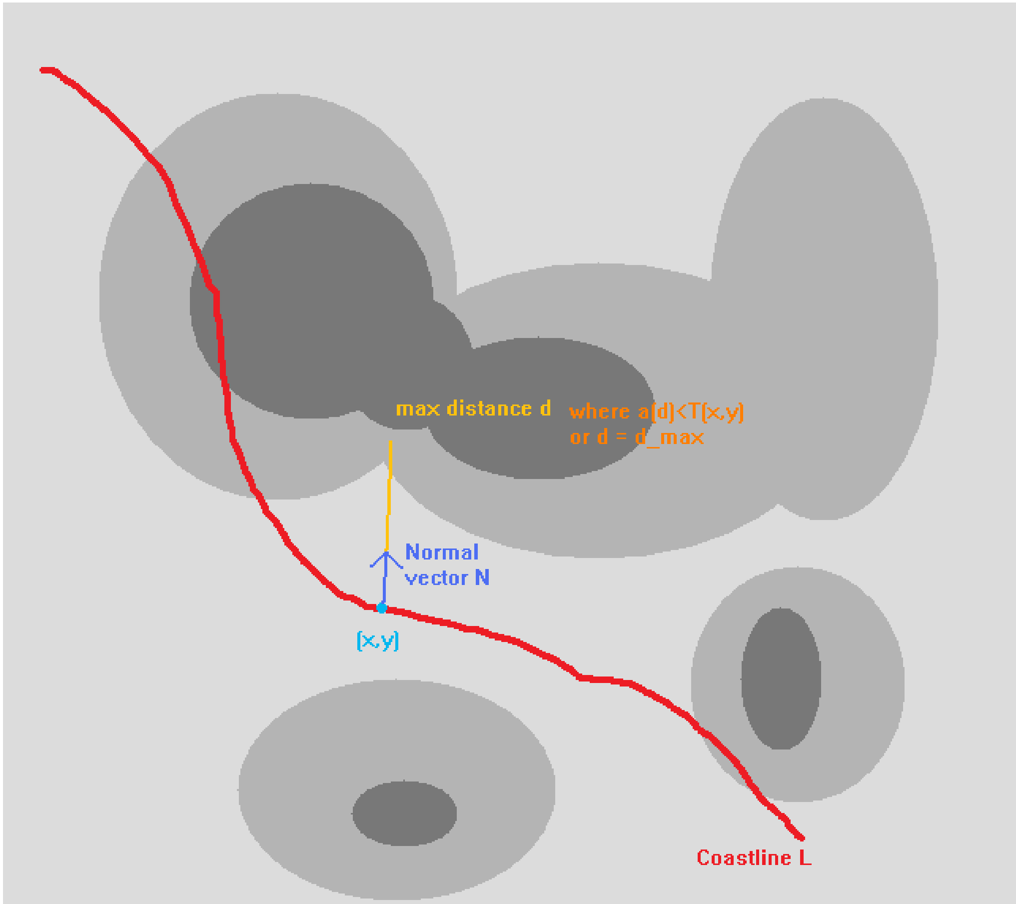


Figure 2. The coastline L and the normal vector $\vec{N}$ with underlying topography are displayed in this figure.

Economic and Human Cost of Tsunami

In this section, we calculate the property damage costs and human damage costs of a tsunami. We determine formulae to compute the economic damage done to property as a result of a tsunami and add this quantity to the economic damage resultant from human mortality. Since the impact of a tsunami is much greater at the shore than farther inland, an impact factor is calculated in order to scale the economic damage done to the target city at all distances from the coast.

Property Valuation

The single most significant effect of a tsunami is physical damage to real estate. Since property prices vary greatly across the nation, and even across cities, we use a regularly re-evaluated measure: property taxes. Levied by county and city governments, this form of revenue draws from the vast majority of residential and commercial landed assets. Thus, by analyzing the total amount of property taxes levied on a city, we can determine the total value of its real estate. Taxes are calculated by the simple formula:

$$T = V_M \cdot r$$

Where T is the tax, VM is the market value of a property, and r is the rate determined by local governments. Rearranging gives:

$$V_M = \frac{T}{r}$$

We collected the most recent data available for the seven cities. Using the above equations, we determine the following net V_M for each city:

City	Sum Property Value
New York City	$689,300,000,000
San Francisco	$103,100,000,000
Charleston	$1,356,000,000
Boston	$137,400,000,000
Hilo	$3,927,000,000
New Orleans	$4,580,000,000
Virginia Beach	$54,350,000,000

Table 8. The net property values of the seven cities considered are shown.

We then calculate the property value density, V_D, by taking these statistics and dividing by the total land area within city limits. This provides the approximate worth of one square kilometer of land, and the real estate associated with it.

City	Property Value Density ($/km²)
New York City	873,164,857
San Francisco	852,372,784
Charleston	3,754,266
Boston	1,095,265,213
Hilo	27,930,610
New Orleans	9,795,324
Virginia Beach	84,514,211

Table 9. The property value densities of the seven cities considered are shown.

For a more information on the calculation of these values for individual cities, see Appendix B.

Intensity of Impact Model

Clearly, as a tsunami flows over land, it rapidly loses energy. The devastation at the coastline will be significantly more severe than further inland. Buildings and other obstructions will significantly obstruct the progress of the water. In order to account for this, we have developed a basic intensity function to be applied to the data generated from the tsunami propagation simulator. We begin with a basic exponential decay equation:

$$I = e^{-x}, \text{where } x \text{ is the distance from the coast}$$

We would like our final intensity model to obey the following constraints:

$$x \in [0, d] \; and \; I \in [0,1]$$

$$I(0) = 1$$

$$I(d) = 0$$

where d is the maximum distance reached by the wave, as calculated by the simulation. In order to force the index to 0 when $x = d$, we add a constant:

$$I = e^{-x} - e^{-d}$$

This, however, no longer follows the second constraint ($I(0) = 1$). Multiplying by another constant gives us our desired result:

$$I = \left(\frac{1}{1 - e^{-d}}\right)(e^{-x} - e^{-d})$$

$$I = \frac{e^{-x} - e^{-d}}{1 - e^{-d}}$$

In order to apply this to the data gathered from the simulation, we must calculate the average impact factor over the domain $[0, d]$:

$$I_{avg} = \frac{1}{d - 0}\int_0^d \frac{e^{-x} - e^{-d}}{1 - e^{-d}} dx = \frac{1}{d} + \frac{1}{1 - e^{d}}$$

We note that $d = kv\sqrt{\frac{2a}{g}}$, where $k = .001$ to account for unit conversion. Substituting this expression into the one above, we obtain:

$$I_{avg} = \frac{1}{kv\sqrt{\frac{2a}{g}}} + \frac{1}{1 - e^{kv\sqrt{\frac{2a}{g}}}}$$

Finally, we multiply the average impact factor by the value density, V_D, and by the area affected, A, to calculate the total damage levied on buildings. Thus,

$$\text{Total property damage} = I_{avg} \cdot A \cdot V_D$$

The values of I_{avg} as a function of earthquake intensity and epicenter-city distance are shown in Table _. Evaluation of I_{avg} at various earthquake intensities and epicenter-city distances showed an almost constant value of $I_{avg} = .5$.

Table 10		Earthquake Intensity (moment magnitude scale)				
I_{avg}		7	7.5	8	8.5	9
Distance of epicenter from target city (km)	500	0.50	0.50	0.50	0.50	0.49
	1000	0.50	0.50	0.50	0.50	0.49
	1500	0.50	0.50	0.50	0.50	0.49
	2000	0.50	0.50	0.50	0.50	0.49
	2500	0.50	0.50	0.50	0.50	0.49
	3000	0.50	0.50	0.50	0.50	0.50
	3500	0.50	0.50	0.50	0.50	0.50
	4000	0.50	0.50	0.50	0.50	0.50

Table 10. The average impact factor (I_{avg}) for tsunamis generated by earthquakes of given intensity and with an epicenter a given distance from the target city.

Human Toll Valuation

Although the destruction of physical property due to the tsunami is devastating, we must also consider the mortality caused by the wave. While there are many factors to consider while calculating the total mortality (age dispersion, gender dispersion, location of individuals, pre-existing health problem, etc.), we decided to generalize the calculations so that the formula could be applied to all seven cities:

$$H = P_d \cdot A \cdot (.129)$$

Where H is the human death toll, P_d is the population density of the city, A is the area affected by the tsunami determined by the tsunami propagation simulator, and 0.129 is the mortality rate constant outlined by the 2006 study by Nishikiori, et al.

We collected the most recent data available for the seven cities and multiplied them by the mortality rate:

City	Population Density (in people per km²)	Mortality per km² Affected
New York City	10,606	1368.174
San Francisco	6,688.4	862.804
Charleston	384.7	49.626
Boston	4,850	625.65
Hilo	289.9	37.397
New Orleans	973	125.517
Virginia Beach	661.3	85.301

Table 11. The population densities and mortality densities of the various cities considered are shown.

We then calculate the human value density, V_H, by taking these statistics and multiplying by the cost of a human individual. Going past the semantics of what the value of a human individual is, we decided to look purely at the cost of setting public policy when determining the value of human life. According to economist Orley Ashenfelter of Princeton University, this equates to $1.54 million per person. V_H provides the approximate worth of one square kilometer of land, and the value of human life associated with it.

City	Human Value Density ($/km²)
New York City	2,106,987,960
San Francisco	1,328,718,160
Charleston	76,424,040
Boston	963,501,000
Hilo	57,591,380
New Orleans	193,296,180
Virginia Beach	131,363,540

Table 12. The human value densities of the various cities considered are shown.

We can then calculate the economic cost of human mortality in a similar way to the cost of property damage. We state essentially that

$$\text{Total human damage} = I_{avg} \cdot A \cdot V_H$$

We calculate the total economic cost by summing the total human cost and the total cost of property damage.

$$\text{Total economic damage} = I_{avg} \cdot A \cdot (V_H + V_D)$$

Case Studies

New York City

We consider an earthquake of moment magnitude intensity 9 in the Mid-Atlantic Ridge at a distance of 3000 km from New York City. The tsunami generated has a height of 25.8 m and a speed of 25.2 m/s. The area affected was simulated to be 2.422 km^2. The total economic damage was therefore calculated to be $3,608,965,061.

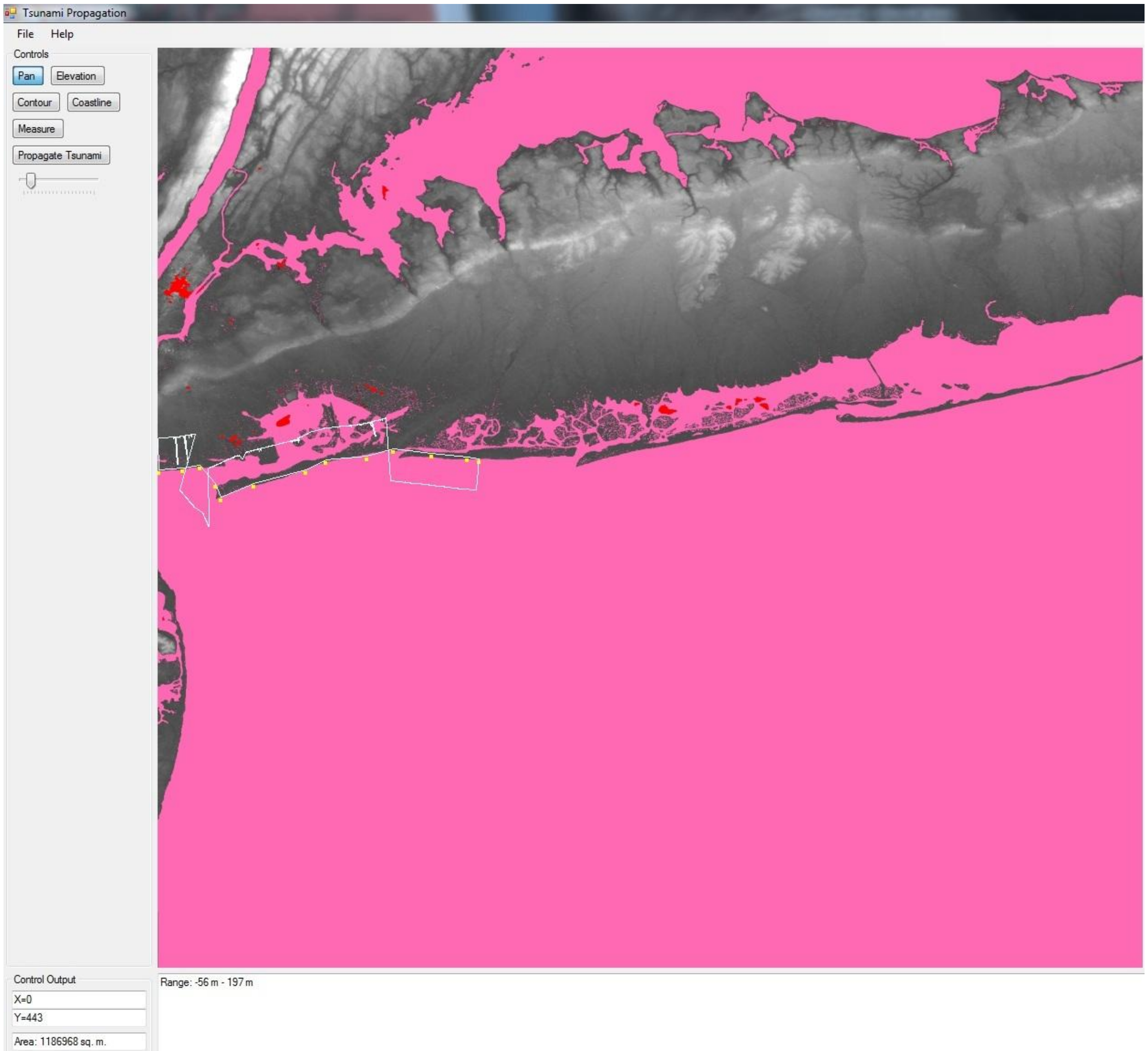


Figure 3. New York City (eastern half) is shown.

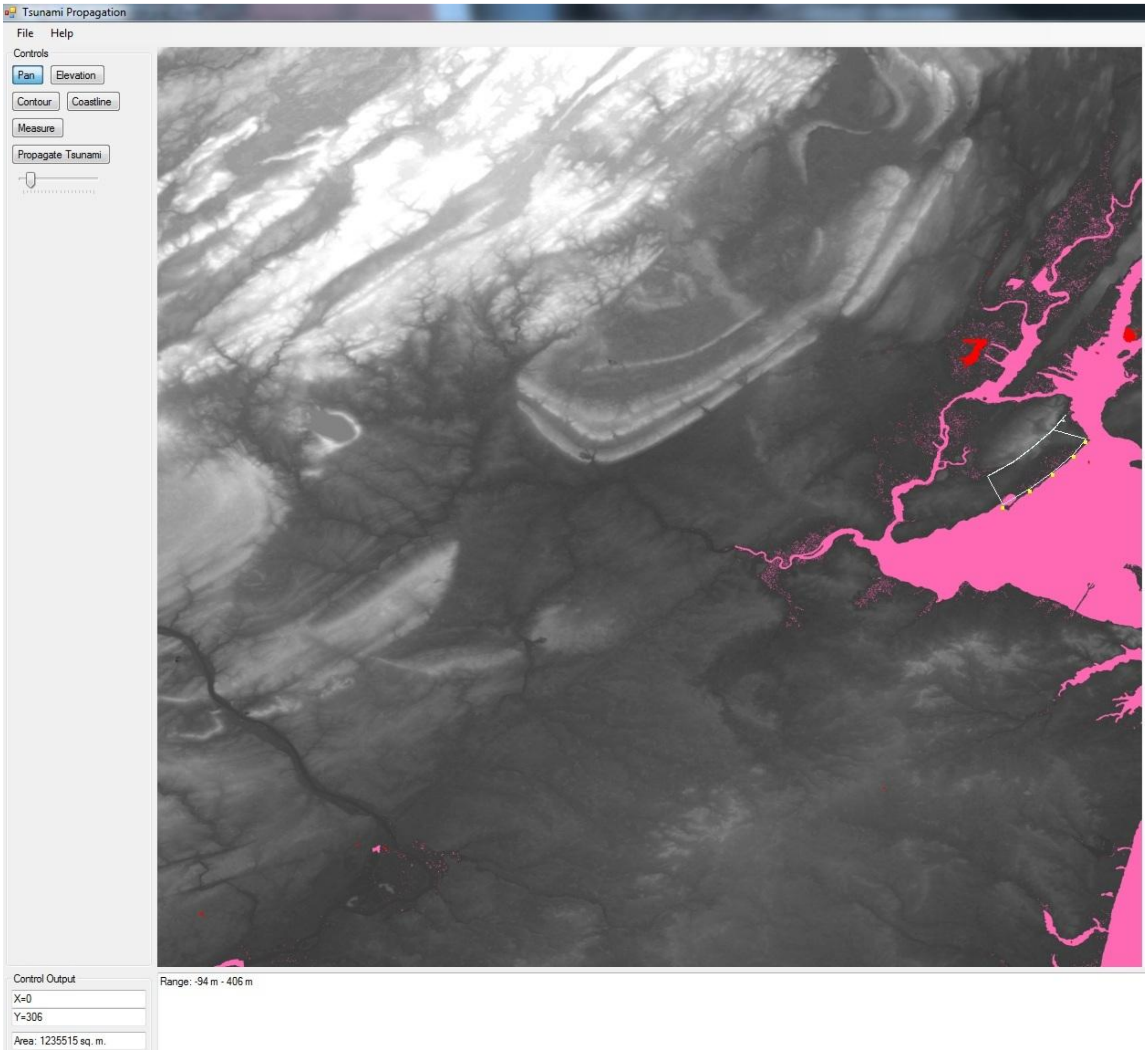


Figure 4. New York City (western half) is shown.

San Francisco

We consider an earthquake of moment magnitude intensity 9 in the Pacific Ocean at a distance of 500 km from San Francisco. The tsunami generated has a height of 46.3 m and a speed of 33.9 m/s.

The area affected was simulated to be 2.262 km^2. The total economic damage was therefore calculated to be $2,466,813,857.

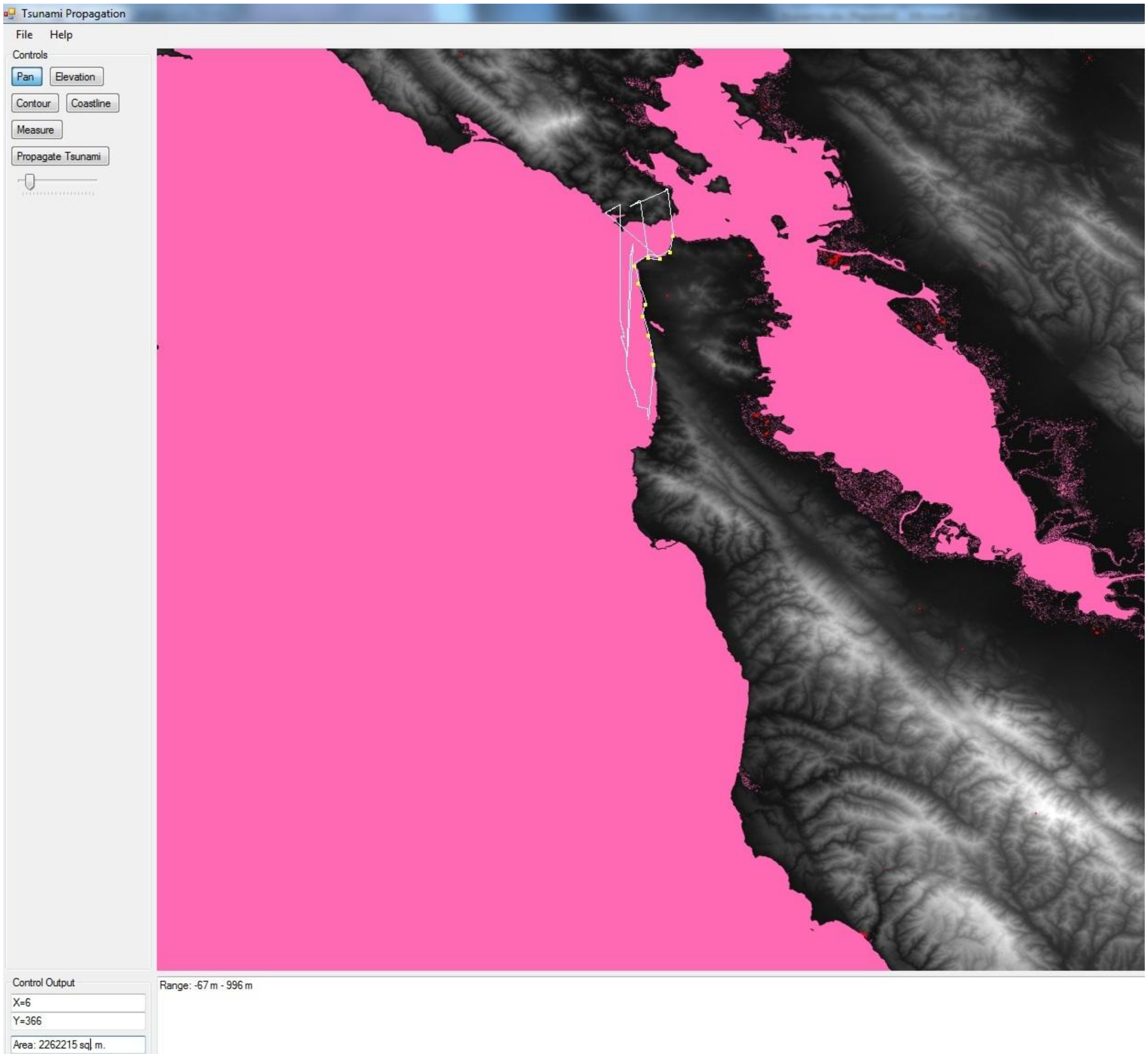


Figure 5. San Francisco is shown.

Charleston

We consider an earthquake of moment magnitude intensity 9 in the Mid-Atlantic Ridge at a distance of3500 km from Charleston. The tsunami generated has a height of 24.5 m and a speed of 24.5 m/s. The total economic damage was therefore calculated to be $174,508,083.

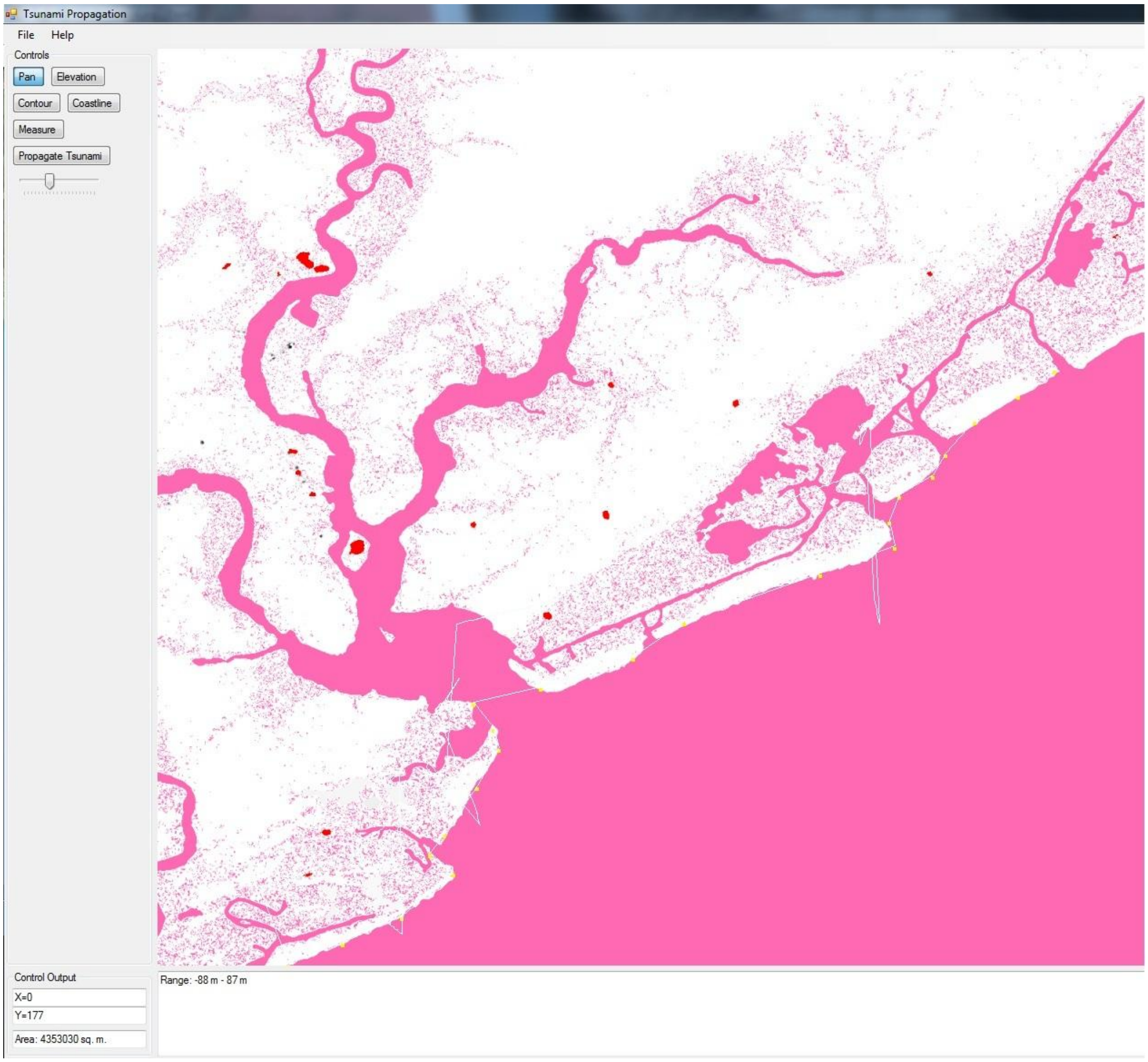


Figure 6. Charleston is shown.

Boston

We consider an earthquake of moment magnitude intensity 9 in the Mid-Atlantic Ridge at a distance of 3500 km from Boston. The tsunami generated has a height of 24.5 m and a speed of 24.5 m/s. The total economic damage was therefore calculated to be $6,577,758,050.

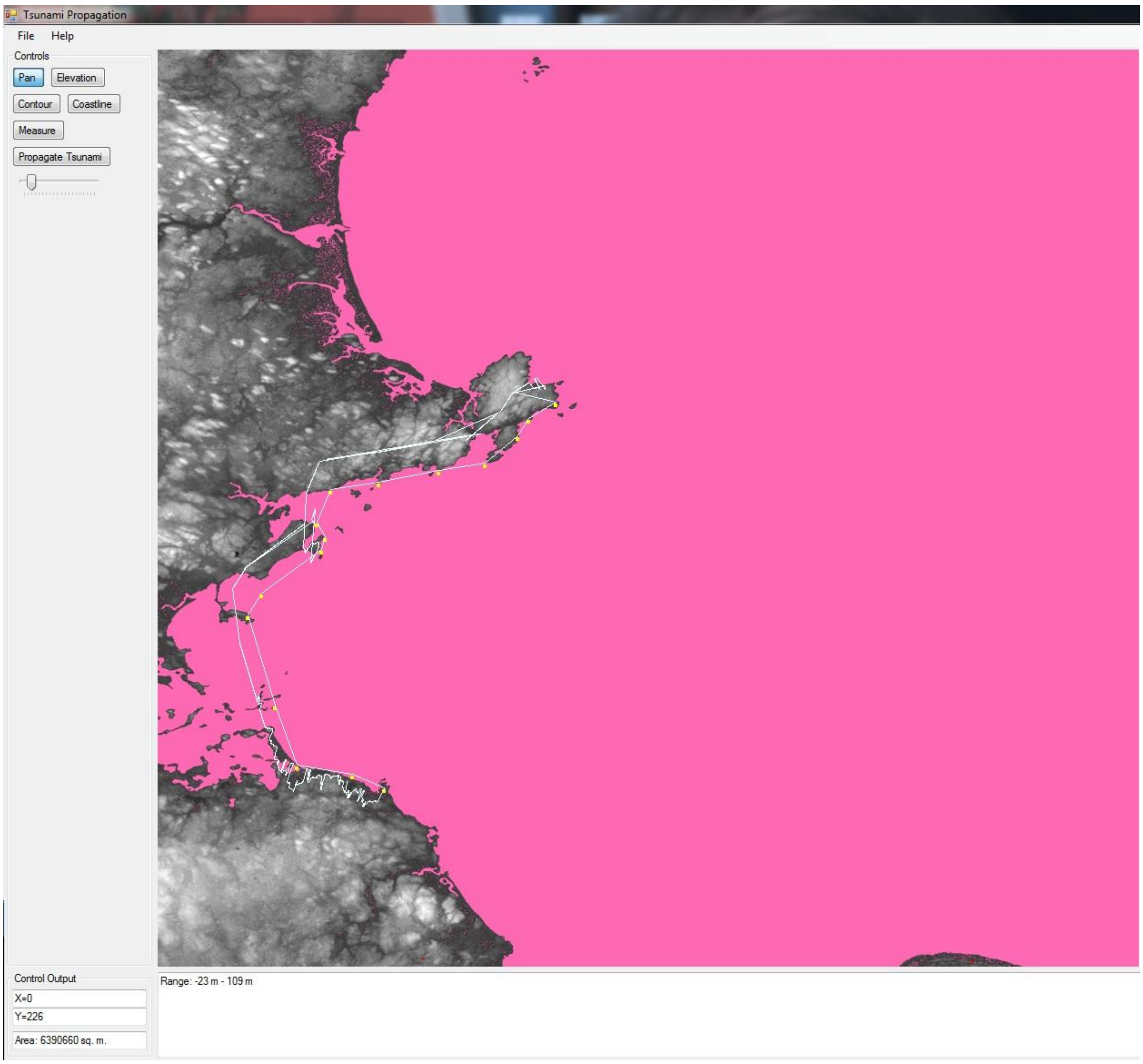


Figure 7. Boston is shown.

Hilo

We consider an earthquake of moment magnitude intensity 9 in the North Pacific at a distance of 1000 km from Hilo. The tsunami generated has a height of 46.9 m and a speed of 30.2 m/s. The total economic damage was therefore calculated to be $31,087,243.

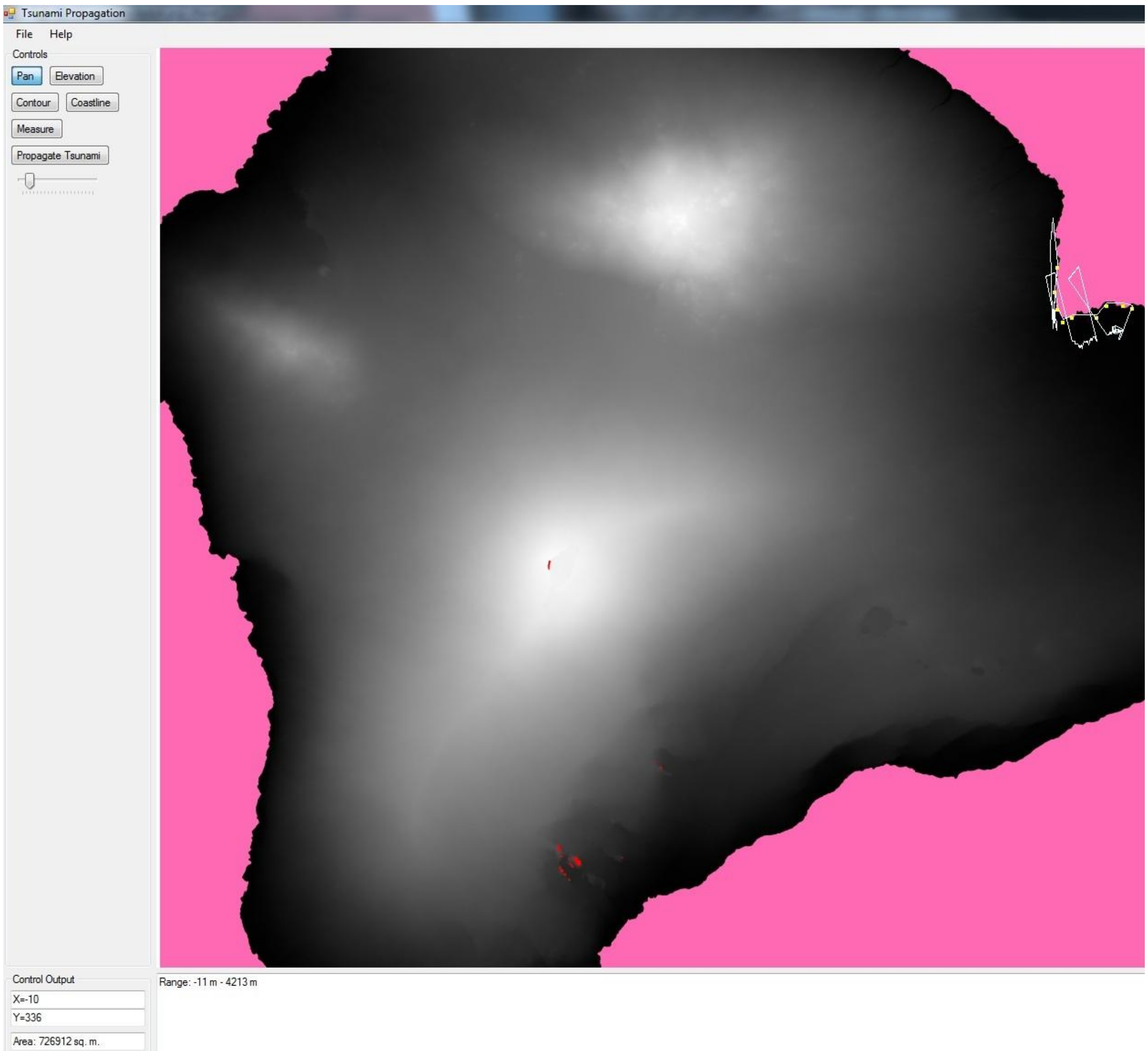


Figure 8 , Hilo is shown

New Orleans

We consider an earthquake of moment magnitude intensity 8 in the Caribbean at a distance of 1000 km from New Orleans. The tsunami generated has a height of 11.8 m and a speed of 17.0 m/s. The total economic damage was therefore calculated to be $62,856,820. It is important to note that although New Orleans is not at great direct threat from a tsunami, the possibility of breaking levees would cause havoc in regions below sea level. However, modeling breaking levees is a separate problem which we do not consider in our paper.

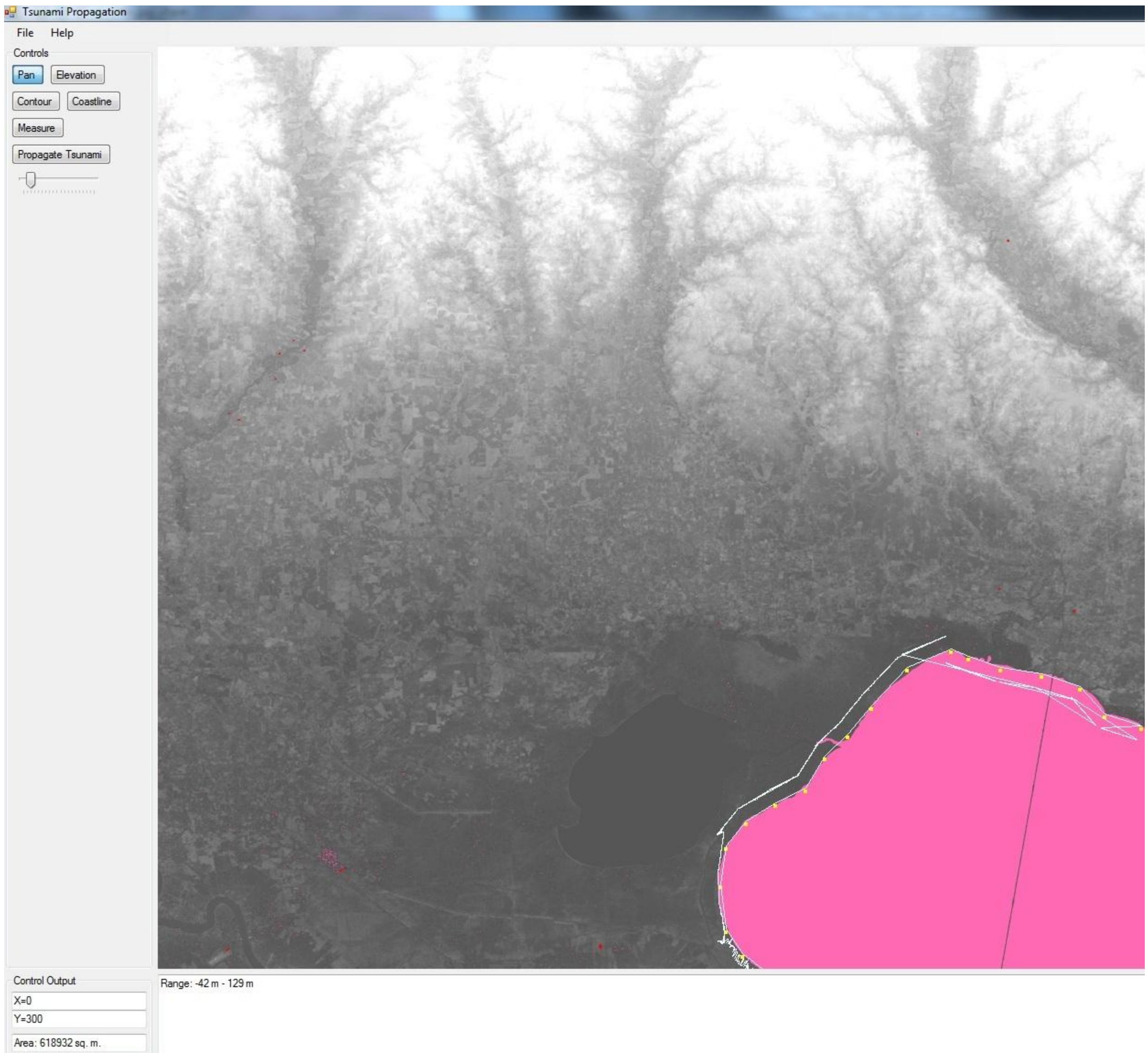


Figure 9 , New Orleans is shown

Virginia Beach

We consider an earthquake of moment magnitude intensity of 7 in the Mid-Atlantic Ridge at a distance of 3000 km from Virginia Beach. The tsunami generated has a height of 2.6 m and a speed of 8.0 m/s. The total economic damage was therefore calculated to be $32,273,723.

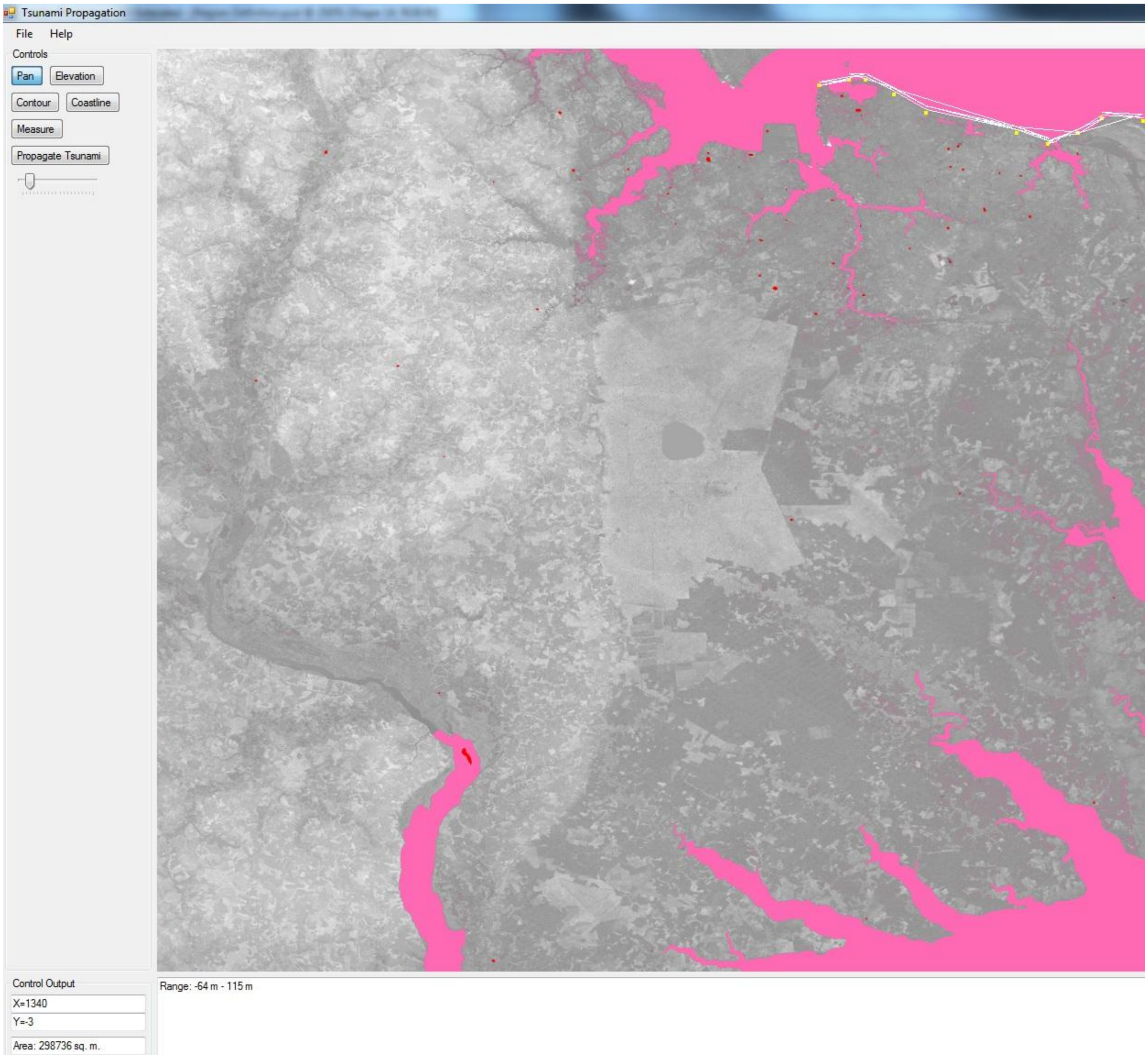


Figure 10 , Virginia Beach is shown

Conclusion

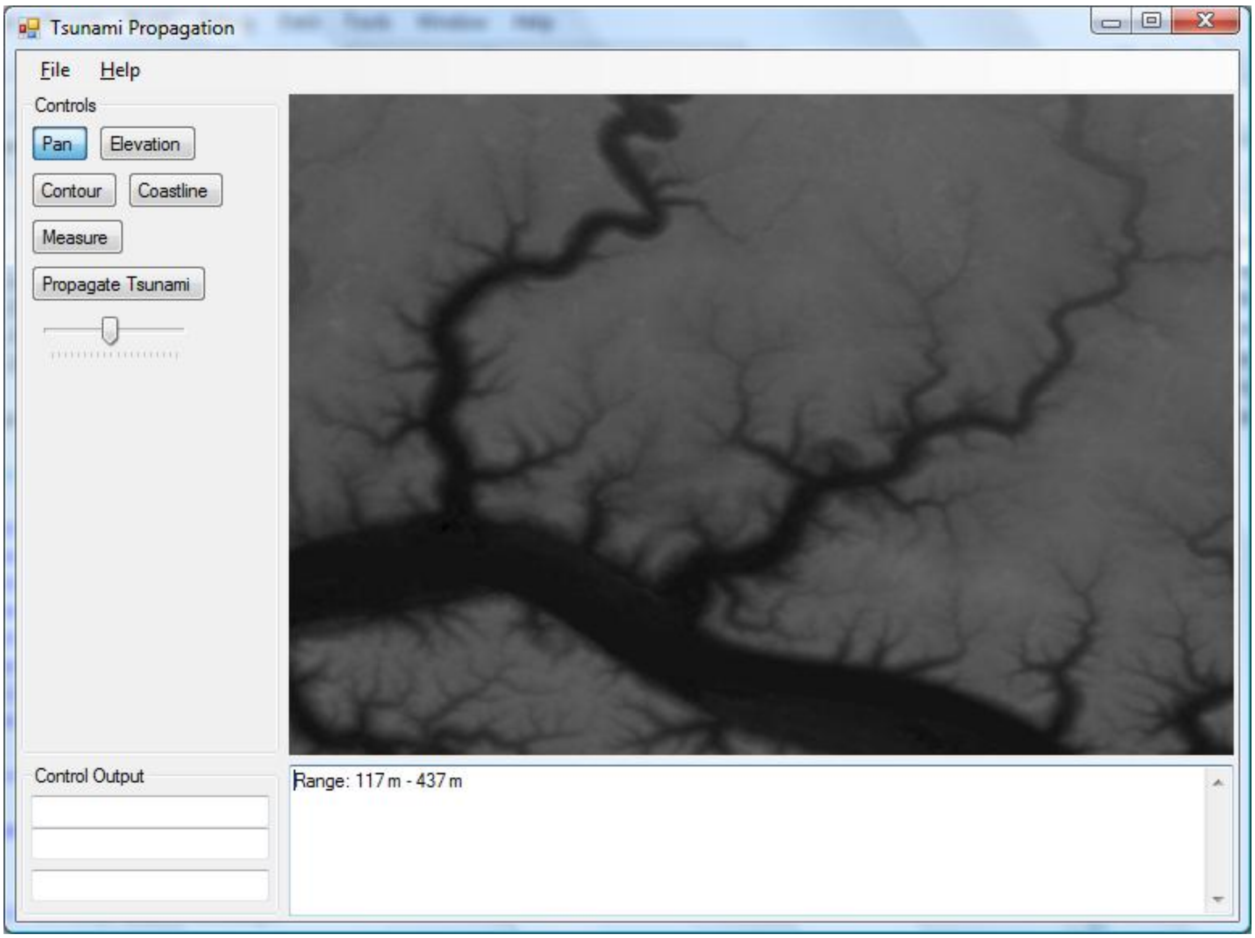
The culmination of our research has yielded a general method for calculating the devastation levied by an earthquake-generated tsunami, as long as one is given the earthquake's magnitude and distance from shore. Using equations derived from theoretical physics, we can compute the height and velocity of a wave approaching shore. These two values, when inputted into a computer simulation we wrote, give the impact radius of the tsunami. Finally, by using both empirical and theoretical models, we determine the economic and humanitarian consequences.

Appendices

Appendix A

Computer-simulated Tsunami Propagation

To facilitate the damage assessment of various cities, we created a computer program to calculate inland propagation of tsunamis. Our computer program TSUNAMI calculates the propagation of a tsunami with coastal height a and coastal velocity v_0 given a coastline defined by piecewise line segments and an underlying topography. We chose the NASA Shuttle Radar Topography Mission data sets because as of 2009 they have the highest resolution elevation data for the United States.



NSRTM Data

SRTM data is given for every degree of latitude and longitude, and each file contains 3600 rows and 3600 columns of 16-bit signed integers, expressing an elevation value of meters above sea level per every square arcsecond. To convert arcseconds to meters, we assume the Earth is a perfect sphere with radius R, and use the simple relation

$$1\ second\ of\ arc = \frac{2\pi R}{1296000}\ meters \approx 30.9219\ meters$$

Tsunami Propagation

The coastline in TSUNAMI is defined as piecewise line segments. For each segment in the path, TSUNAMI uses the Bresenham algorithm to walk along the segment to compute the maximum water column distance d for that point on the segment.

```
private void PropagateTsunami_Walk(float tHeight, float tVelocity, Point p1,
Point p2, int dir)
{
    bool steep = (Math.Abs(p2.Y - p1.Y) > Math.Abs(p2.X - p1.X));

    Point tp1 = p1;
    Point tp2 = p2;

    if (steep)
    {
        int temp = p1.X;
        p1.X = p1.Y;
        p1.Y = temp;

        temp = p2.X;
        p2.X = p2.Y;
        p2.Y = temp;
    }

    if (p1.X > p2.X)
    {
        int temp = p1.X;
        p1.X = p2.X;
        p2.X = temp;

        temp = p1.Y;
        p1.Y = p2.Y;
        p2.Y = temp;
    }

    int deltax = p2.X - p1.X;
    int deltay = Math.Abs(p2.Y - p1.Y);
    int error = deltax / 2;
    int ystep = 0;
    int y = p1.Y;

    ystep = (p1.Y < p2.Y) ? 1 : -1;

    for (int x = p1.X; x < p2.X; x++)
    {
        // plot
        float perpslope = (float)(p2.Y - p1.Y) / (float)(p2.X - p1.X);
        perpslope = -1.0f / perpslope;

        if (steep)
            PropagateTsunami_WalkPoint(tHeight, tVelocity, new Point(y, x),
perpslope, dir);
        else
            PropagateTsunami_WalkPoint(tHeight, tVelocity, new Point(x, y),
perpslope, dir);
```

```
        error = error - deltay;
        if (error < 0)
        {
            y = y + ystep;
            error = error + deltax;
        }
    }
}
```

Then, after computing the normal vector, TSUNAMI uses the Bresenham algorithm again to walk along the normal vector, comparing the water column height with the terrain height (rawData[x,y]). This allows TSUNAMI to determine the distance for which the water column stops.

```
private void PropagateTsunami_WalkPoint(float tHeight, float tVelocity, Point
p1, float m, int dir)
{
    // Maximum distance d
    float dmax = tVelocity * factorX * (float) Math.Sqrt(2 * tHeight / 9.8);
// meters
    float dx = dmax * (float) Math.Cos(Math.Atan(m)) / arcSec; // arcsec or
pixel
    float dy = dmax * (float) Math.Sin(Math.Atan(m)) / arcSec; // arcsec or
pixel

    Point p2 = new Point(p1.X + dir*(int)dx, p1.Y + dir*(int)dy);

    // Propagate
    bool steep = (Math.Abs(p2.Y - p1.Y) > Math.Abs(p2.X - p1.X));

    Point tp1 = p1;
    Point tp2 = p2;

    if (steep)
    {
        int temp = p1.X;
        p1.X = p1.Y;
        p1.Y = temp;

        temp = p2.X;
        p2.X = p2.Y;
        p2.Y = temp;
    }

    if (p1.X > p2.X)
    {
        int temp = p1.X;
        p1.X = p2.X;
        p2.X = temp;

        temp = p1.Y;
        p1.Y = p2.Y;
        p2.Y = temp;
    }
```

```
    int deltax = p2.X - p1.X;
    int deltay = Math.Abs(p2.Y - p1.Y);
    int error = deltax / 2;
    int ystep = 0;
    int y = p1.Y;

    ystep = (p1.Y < p2.Y) ? 1 : -1;

    for (int x = p1.X; x <= p2.X; x++)
    {
        // plot
        int cx, cy;

        if (steep)
        {
            cx = y;
            cy = x;
        }
        else
        {
            cx = x;
            cy = y;
        }

        // Boundary Conditions
        if (cx > 3600 || cx < 0 || cy < 0 || cy > 3600)
        {
            floodRegion.Add(new Point(cx, cy));
            return;
        }

        // Calculate height of water column here
        float dist = (float) Math.Sqrt((cx - tp1.X)*(cx - tp1.X) + (cy -
tp1.Y)*(cy - tp1.Y)) * arcSec; // meters
        float wheight = tHeight - (float)(Math.Pow((dist / (tVelocity *
factorX)), 2.0) * (9.8 / 2.0)); // meters

        // Absolute height, add the height at which the water started
        float awheight = wheight + (float)rawData[tp1.X, tp1.Y];

        // Is elevation height here greater than the absolute height?
        // If so, stop and add this (cx, cy) point to the region list
        if ((float)rawData[cx, cy] > awheight)
        {
            floodRegion.Add(new Point(cx, cy));
            return;
        }

        error = error - deltay;
        if (error < 0)
        {
            y = y + ystep;
            error = error + deltax;
        }
    }
```

```
    // fall through
    floodRegion.Add(new Point(tp2.X, tp2.Y));
}
```

To compute the area of the flooded region, TSUNAMI uses the formula for the area of a convex polygon given its vertices' coordinates (x_i, y_i):

$$A = \frac{1}{2} \cdot \left| \sum_i x_i y_{i+1} - y_i x_{i+1} \right|$$

where i is taken modulo N, for a polygon of N vertices. The polygon is taken from the points computed by the line-walking algorithm and the original coastline.

Appendix B

Different cities calculate property taxes by different methods. In addition, the information they make readily accessible to the public varies. This appendix covers the seven cities and how we computed their net worths.

New York City

Like the majority of other cities, we begin by looking at the sum taxes collected. According to the Office of Real Property Services, municipal/county taxes amount to $8,779,721,566 and school taxes amount to $8,962,251,381, with a sum of $17,741,972,947. By the equation,

$$V_A = \frac{T}{r}$$

where V_A is the assessed value of a property. With a rate of r = 13%, we determine the total assessed value to be $136,400,000,000. This, however, was not our final answer. New York City employs equalization rates in order to maintain equality between public schools in spite of economic disparities. Thus, Manhattan has an equalization rate of about 32%, whereas Brooklyn enjoys one around 10%. The ORPS website identifies the city-wide average to be 19.81%. Equalization rates are calculated by the equation:

$$E = \frac{V_A}{V_M}$$

E is the equalization rate, and as before, VM is the market value. Solving for the net V_M based on the given values, we get V_M= $689,300,000,000. Dividing by the land area (789.4 km^2), we compute V_D to be 873,164,857.

San Francisco

While the San Francisco city government does not provide tax revenue data directly, the 2009 budget does state that property taxes accounted for 22% of the city's $7,454,000,000 budget. This gives us 1,639,880,000. Using a tax rate of 1.59%, we calculate the V_M as $103,100,000,000. Dividing by the land area (121 km^2), we compute V_D to be 852,372,784.

Charleston

A Charleston City document entitled Revenue Discussion and Analysis asserts that the 2008 property tax was \$54,241,641. At a rate of 4%, this converts to \$1,356,000,000 in total property. Over 361.2 km^2, the value density is 3,754,266.

Boston

Like San Francisco, we determined property tax revenue through looking at the city fiscal plan. At 61% of the \$2,394,000,000 budget, property taxes amount to 1,460,340,000. With a rate of 1.063%, this means Boston's net real estate worth is \$137,400,000,000. Dividing by the land area (125.43 km2), we compute V_D to be 1,095,265,213.

Hilo

Hilo, unfortunately, does not provide property tax information in a readily accessible place. We remedied this by focusing solely on the net worth of all the residential real estate. This does not skew the data too much, since the majority of properties within a tsunami's reach are primarily homes. Using census data for average household value and number of households, we calculated \$3,927,000,000 as an approximation for V_M. This gives V_D as 27,930,610 over an area of 140.6 km^2.

New Orleans

Much like Charleston, New Orleans directly provides its total property tax revenue, \$77,864,995. The rate of 1.7% implies that V_M is equal to \$4,580,000,000. Dividing by the area (467.6 km^2) gives V_D = 9,795,324.

Virginia Beach

Virginia Beach also directly reveals its property tax revenue, at \$483,724,693. Dividing by the rate (0.89%) gives a V_M of \$54,350,000,000. Finally, this value is spread out over 643.1 km^2 at a V_D of 84,514,211.

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