HIGH SCHOOL MATHEMATICAL CONTEST IN MODELING

OUTSTANDING PAPERS

HiMCM

November 2007

The contest offers students the opportunity to compete in a team setting using applied mathematics in the solving of real-world problems.

Additional support provided by the National Council of Teachers of Mathematics (NCTM), the Mathematical Association of America (MAA), and the Institute for Operations Research and Management Sciences (INFORMS).

Editor's Comments

This is our tenth HiMCM Special Issue. Since space does not permit printing all of the four National Outstanding papers, this special section includes the summaries from two papers and abridged versions of two. We emphasize that the selection of these two does not imply that they are superior to the other Outstanding papers. We also wish to emphasize that the papers were not written with publication in mind. Given the 36 hours that teams have to work on the problems and prepare their papers, it is remarkable how much was accomplished and how well written many of the papers are. ❏

Contest Director's Article

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The High School Mathematical Contest in Modeling (HiMCM) completed its tenth year in excellent fashion. The mathematical and modeling ability of students and faculty advisors is very evident in the professional submissions and work being done. The contest is still moving ahead, growing with a positive first derivative, and consistent with our positive experiences from previous HiMCM contests.

This year the contest consisted of 270 teams (down 3 teams from last year) from over 52 institutions (up 2 institutions from last year). These institutions were from twenty-six states, the District of Columbia, the Hong Kong International School, China, Korea, and the United Kingdom. This year, we again charged a registration fee of $50.

The teams accomplished the vision of our founders by providing *unique* and *creative mathematical* solutions to complex open-ended real-world problems. This year the students had a choice of two problems.

Problem A

Fire is one of the leading causes of accidental deaths. It is important for everyone to take every preventative measure and precaution possible to be ready to deal with a fire emergency.

More than half of all fatal fires occur between 10 p.m. and 6 a.m. when everyone in the home is usually asleep. Smoke alarms are necessary to alert you to fires when you sleep. Will smoke alarms allow enough time to evacuate safely?

Build a mathematical model to determine the number and locations of smoke alarms to provide the maximum time for evacuation. Also include a model to determine the number and location of at-home fire extinguishers to have available. Build a mathematical model for evacuation of a family from both one and two story homes.

Prepare an advertisement for your local fire department to pass out to the community that includes the main results of your mathematical models.

Problem B

Some people rent a car when they are going on a long trip. They are convinced they save money. Even if they do not save money, they feel that the knowledge that "if the car breaks down on the trip, the problem is the rental company's" makes the rental worth it. Analyze this situation and determine under what conditions renting a car is a more appropriate option. Determine mileage limits on one's own car and a breakeven value of "ease of mind" for the driver and her family.

Commendation: All students and advisors are congratulated for their varied and creative mathematical efforts. Of the 270 teams, 157 submitted solutions to Problem A and 113 to Problem B. The thirty-six continuous hours to work on the problem provided for quality papers; teams are commended for the overall quality of their work.

Many teams had female members. There were 381 female participants on the 270 teams. There were 1011 total participants, so the females made up approximately 37.69% of the total participation, showing this competition is for both genders. Teams again proved to the judges that they had "fun" with their chosen problems, demonstrating research initiative and creativity in their solutions. This year's effort was a success!

Judging: We ran three regional sites in December 2007. Each site judged papers for problems A and B. The papers judged at each regional site may or may not have been from their respective region. Papers were judged as Outstanding, Meritorious, Honorable Mention, and Successful Participant. All finalist papers for the Regional Outstanding award were sent to the National Judging. For example, eight papers may be discussed at a Regional Final and only four selected as Regional Outstanding but all eight papers are judged for the National Outstanding. Papers receive the higher of the two awards. The national judging chooses the "best of the best" as National Outstanding. The National Judges commended the regional judges for their efforts and found the results were very consistent. We feel that this regional structure is a good one for the future as the contest grows.

Judging Results:

NATIONAL & REGIONAL COMBINED RESULTS

Problem	**National Outstanding**	Regional Outstanding Only	Meritorious	Honorable Mention	Successful Participant	Total
A	**2**	16	33	67	39	**157**
B	**2**	12	28	41	30	**113**
Total	**4**	28	61	108	69	**270**
Percents	**1.5%**	**10.37%**	**22.59%**	**40%**	**25.56%**	

General Judging Comments: The judges' commentaries provide specific comments on the solutions to each problem. As contest director and head judge, I would like to speak generally about solutions from a judge's point of view. Papers need to be coherent, concise, and clear. Students need to restate the problem in their own words so that the judges can determine the focus of the paper. Papers that explain the development of the model, assumptions, and its solutions and then support the findings mathematically generally do quite well. Modeling assumptions need to be listed and justified, but only those that come to bear on the solution (that can be part of simplifying the model). Laundry lists of assumptions that are never referred to in the context of the model development are not considered relevant and deter from the paper's quality. The mathematical model needs to be clearly developed, and all variables that are used need to be well defined. Thinking outside of the "box" is also considered important by judges. This varies from problem to problem but usually includes model extensions or sensitivity analysis of the solution to the team's inputs. Students need to attempt to validate their model even if by numerical example or intuition. A clear conclusion and answers to specific scenario questions are all key components. The strengths and weakness section is where the team can reflect on their solution and comment on the model's strengths and weaknesses. Attention to detail and proofreading the paper prior to final submission are also important since the judges look for clarity and style. Citations are also very important within the paper as well as either a reference or bibliography page at the end.

CONTEST FACTS:

Facts from the 10th Annual Contest:

- Wide range of schools/teams competed including teams from Hong Kong, China , Korea, and the UK.
- The 270 teams represented 52 institutions.
- There were 1011 student participants, 630 (62.31%) male and 381 (37.69%) female. There were 41 all-female teams.
- Schools from twenty-six states plus the District of Columbia participated in this year's contest.

THE FUTURE:

The contest, which attempts to give the under-representative an opportunity to compete and achieve success in mathematics, appears well on its way in meeting this important goal.

We continue to strive to improve the contest, and we want the contest to grow. Any school/team can enter, as there are no restrictions on the number of schools or the numbers of teams from a school. A regional judging structure is established based on the number of teams.

These are exciting times for our high school students. Mathematics continues to be more than learning skills and operations. Mathematics is a language that involves our daily lives. Applying the mathematical principles that one learns is a key to future success. The ability to recognize problems, formulate a mathematical model, use technology, and communicate and reflect on one's work are keys to success. Students gain confidence by tackling ill-defined problems and working together to generate a solution. Applying mathematics is a team sport!

Advisors need only be motivators and facilitators. They should encourage students to be creative and imaginative. It is not the technique used but the process that discovers how assumptions drive the techniques that is fundamental. Let students practice to be problem solvers. Let me encourage all high school mathematics faculty to get involved, encourage your students, make mathematics relevant, and open the doors to success.

Mathematical modeling is an art and a science. Teach your students through modeling to think critically, communicate effectively, and be confident, competent problem solvers for this new century.

CONTEST DATES:

Mark your calendars early: the next HiMCM will be held in November 2008. Registrations are due in October 2008. Teams will have a consecutive 36-hour block within the contest window to complete the problem. Teams can register via the Web at www.comap.com.

HiMCM Judges' Commentary

Problem A

We had 157 teams (over 58%) out of 270 teams attempted Problem A. Those that tried this problem fell into three categories: experimental modelers (teams that made and tested fire and smoke models and experimentally generated some flow data), geometric modelers using geometry of the residences to place alarms and extinguishers and then simulate results, and modelers using sophisticated mathematics from advanced algebra through partial differential equations to build a model. The latter were generally not successful in putting it all together. The experimental modelers created unique approaches and easily would have had Outstanding papers with better writing and better explaining. Thus, the Regional and National Outstanding were varied in their approaches.

Although the judges expected geometry to be one of the main modeling efforts, we found student teams trying to use too sophisticated mathematics. The judges felt that modeling sound from smoke alarms would be critical, and models considering decibel loss as a function of distance usually did quite well.

Some teams appeared to search the Internet and cut and paste many equations and figures into their papers. In most cases, it was clear that students did not understand the mathematics presented.

The modeling contest format was also closely adhered to in many of the papers. However, most summaries and letters were poorly written. It seemed that they could have been written before the model, analysis, and conclusions were completed. The summaries need to contain the results of the model.

The advertisement for the fire department was to showcase results to save lives, but few teams did this.

One of the items that discriminated the better papers was careful modeling of the smoke alarms sound to wake sleeping individuals. Teams modeled the fire extinguishers in a variety of ways and most were acceptable. The escape routes should have been based on some criterion such as minimum time or shortest distance, but most teams never stated their criterion explicitly.

Problem B

113 teams (41.85%) of the 270 teams chose Problem B. The problem statement was concise, but some students may have thought it too vague to get started.

The executive summaries for the most part were either absent or poorly written. Many summaries read like technical reports or were too vague to be helpful. Summaries need to contain the results of the model as well as brief explanation of the problem.

Teams examined short trips of 1 or 2 days and compared renting specific cars to personally owned cars. Teams did not consider leased cars, which would or could have a major impact on the modeling results. Most teams failed to define "ease of mind" in a quantitative manner, which was critical to any model of this problem. Many various "good" definitions and methods were used. Teams did a nice job finding "breakeven" points—either graphically, numerically, or with equations. Few teams, if any, analyzed these points or did sensitivity analysis on them.

We found many of the assumptions and research for rental cars very good. Teams that did some history of renting cars in the U.S. added some nice context to the problem.

COMMENT ABOUT COMPUTER GENERATED SOLUTIONS:

Many papers used computer code. Computer codes used to implement mathematical expressions can be a good modeling tool. However, the judges expect to see an algorithm or flow chart from which the code was developed. Successful teams provided some explanation or guide to their algorithm(s)—a step-by-step procedure for the judges to follow. Code may only be read for the papers that reach the final rounds, but not unless the code is accompanied by a good algorithm in words. The results of any simulation need to be well explained and sensitivity analysis preformed. For example, consider a flip of a fair coin. Here is an algorithm:

INPUT: Random number, number of trials

OUTPUT: Heads or tails

Step 1: Initialize all counters.

Step 2: Generate a random number between 0 and 1.

Step 3: Choose an interval for heads, like [0.0.5]. If the random number falls in this interval, the flip is a head. Otherwise the flip is a tail.

Step 4: Record the result as a head or a tail.

Step 5: Count the number of trials and increment: Count = Count + 1.

An algorithm such as this is expected in the body of the paper with the code as an appendix.

COMMENTS ABOUT GRAPHS:

Judges found many graphs that were not labeled nor explained. Many graphs did not appear to convey information used by the teams. All graphs need a verbal explanation of what the team expects the reader (judge) to gain (or see) from the graph. Legends, labels, and points of interest need to be clearly visible and understandable, even if hand written. Graphs taken from other sources should be referenced and annotated.

General Comments from Judges:

Summaries: These are still, for the most part, the weakest parts of papers. These should be written after the solution is found. They should contain results and not details. They should include the "bottom-line" and the key ideas used in obtaining the solution. They should include the particular questions addressed and their answers. Teams should consider a brief three paragraph approach: a *restatement of the problem* in their own words, a short description of *their method and solution* to the problem (without giving any mathematical expressions), and the *conclusions* providing the numerical answers in context.

Restatement of the Problem: Problem restatements are important for teams to move from the general case to the specific case. They allow teams to refine their thinking to give their model uniqueness and a creative touch.

Assumptions/Justifications: Teams should list only those assumptions that are vital to the building and simplifying of their mathematical model. Assumptions should not be a reiteration of facts given in the problem description. Assumptions are variables (issues) acting or not acting on the problem. Every assumption should have a justification. We do not want to see "smoke screens" in the hopes that some items listed are what judges are looking to see. Variables chosen need to be listed with notation and be well defined.

Model: Teams need to show a ***clear link*** between the assumptions they listed and the building of their model or models. It is required of the team to show how the model was built and why it is the model chosen. Teams should not throw out several model forms hoping to wow the judges. We prefer to see sound modeling based on good reasoning.

Model Testing: Model testing is not the same as testing arithmetic. Teams need to compare results or attempt to verify (even with common sense) their results. Teams that use a computer simulation must provide a clear step-by-step algorithm. Lots of runs and related analysis are required when using a simulation. Sensitivity analysis should be done in order to see how sensitive the simulation is to the model's key parameters.

Conclusions: This section deals with more than just results. Conclusions might also include speculations, extensions, and generalizations. This is where all scenario-specific questions should be answered. Teams should ask themselves what other questions would be interesting if they had more time and then tell the judges about their ideas.

Strengths and Weaknesses: Teams should be open and honest here. What could the team have done better?

References: Teams may use references to assist in their modeling. However, they must also ***reference the source*** of their assistance. Teams are reminded that only ***inanimate resources*** may be used. Teams cannot call upon real estate agents, bankers, hotel managers, or any other real person to obtain information related to the problem. References should be cited where used and not just listed in the back of the paper. Teams should also add a reference list or bibliography in the back of the paper.

Adherence to Rules: Teams are reminded that detailed rules and regulations are posted on the COMAP site. Teams are reminded that they may use only ***inanimate sources*** to obtain information. Teams are reminded that the ***36-hour time limit*** is a ***consecutive*** 36 hours.

MathModels.org

It is highly recommended that participants in this contest as well as future participants take a look at the new modeling Website, www.mathmodels.org, which has a wealth of information and resources.

Creativity: The judges felt the following was worth putting into this article. It is a prologue by a team (1841):

To rent or not to rent, that is the question;
Whether 'tis nobler in the mind to suffer
The wear and tear of outrageous driving
Or to rent cars against a sea of highways,
And by navigating, drive them… To rent, to drive,
And to drive, perchance to break down! Aye, there is the rub,
For in death of cars what cost may come,
When we have towed away our only car,
Must give us pause.

Another team (1662) was noted for having an excellent ad for the fire department that included results from their models. The judges were impressed by most of the ads provided by the teams.

Problem A Summary: Cheshire Academy

Advisor: Susan Eident

Team Members: Chia Jui Chen, Sen Fang, Ji Su Kwak, Jacob Tivin

In creating our models for the overall effectiveness and utility of both active and passive fire-prevention equipment, we found several characteristics that stand out in our research. One, we came to understand that it is necessary for every home to have at least one smoke alarm on each floor and should also have an additional smoke alarm on any floor that contains sleeping quarters. Two, every home should have at minimum of one fire extinguisher, with a strong preference for at least two. Each extinguisher should be in an accessible area, and every fire extinguisher should be as far as possible from every other fire extinguisher. Finally, a fire evacuation plan should be made ahead of time.

Smoke alarms have proven effective in preventing fire deaths. Smoke alarms are capable of sensing the presence of smoke in the atmosphere, and thus will always be able to warn people of a fire near them. In our mathematical models and simulations, we found that by using two smoke alarms in a one-story home, everyone in that home will be able to be notified of a fire in every part of that house. This accounts for open or closed doors, room acoustics, proximity from each smoke alarm, and likelihood that one is asleep during a critical time. Having two networked smoke alarms located in central areas, such as the great room or hallways, gives a minimum decibel level of around 75 in most every area of the one-story home, which is the minimum needed to awaken an adult from his or her sleep. In our model of a two-story house, we determined that this house needs more detectors due to the size and expanse of the house. This larger size increases the number of locations where a fire can occur, and reduces the effective spread of alert that a smoke alarm can give. Therefore, we found a function that weighs average distance with average effective decibel level that we use to compare the overall effectiveness of two models of smoke alarm number and placement. We found that on the first floor, only one smoke detector is necessary, whereas on the second floor, two smoke detectors are preferable. Logically, this arrangement makes sense because sleeping quarters need a close smoke detector in order to increase the warning effectiveness of the smoke alarm arrangement.

Fire extinguishers require a similar level of planning and preparation for maximum effectiveness. Every house should have at least one extinguisher. Fire extinguishers, unlike smoke alarms, are active fire deterrents, which allow for far greater fire protection and increased likelihood of survival in the case of a major fire. We created a simulation that determines the effectiveness of different positions and numbers of fire extinguishers. We found that having a second fire extinguisher is at minimum twice as effective as having only one. Having a third has a smaller increase in effectiveness than having two; however, having as many extinguishers as possible is our recommendation. In addition, fire extinguishers should always be placed in accessible locations such as close to hallways and exits. An extinguisher hidden from plain sight is less effective due to its constrained accessibility. Also, spreading extinguishers apart from each other is necessary in order to facilitate maximum effectiveness in response to any fire in most any location in a house.

In every household, we strongly recommend prior preparation and planning for an emergency. A fire evacuation plan should be made, and possible scenarios of fires in different locations should be analyzed. In most every case of fire, the nearest exit should be used unless it is blocked by fire or any other unanticipated factor. In the case of a two-story home, the main stairway should be used; however, if the stairway is blocked, the occupants should use a fire escape ladder, which we also strongly recommend that every family prepare beforehand.

Through all of this, the question is raised: will smoke alarms (and other measures) allow enough time to evacuate safely? We believe that with enough preparation and planning, a majority of deaths due to fire can be prevented. The answer to that question is yes; if everyone has in place at least some of our recommendations, most everyone will be able to escape to safety in time. Those precious minutes before disaster could be maximized through early warning and proper preparation.

Problem B Summary: Castilleja School

Advisor: Dave Lowell

Team Members: Rachel Vassar, Bonnie Wong, Melissa Wong

Car trips are opportunities for exploring and memory making, but also for economizing. When families go on car trips, they often consider hotel prices, meal prices, but rarely do they consider the cost of driving their car. Could renting a car possibly be more economical and less stressful? We will explore the benefits of renting a car versus driving one's own car, from both a financial and personal standpoint. For what mileage and duration of a trip is renting a car a better choice, and how does the type of car, the price of gas, or the terrain of the road affect these values? We will create a model that allows us to weigh all of these variables and reach a conclusion about when renting a car is better than driving one's own car.

We created an Excel spreadsheet to model the expenses associated with driving one's own car versus renting a car. In this model we had several variables that we could change based on type of car, type of terrain, and length of trip. These variables include type of car, gas mileage of the car, maintenance costs, tire costs, depreciation, rental car fees, and trip duration. We also made various assumptions when constructing our model. First, we used data from AAA to determine average maintenance costs, tire costs, and depreciation rates, as well as www.mpgbuddy.com to determine gas mileage ratings for various cars. Also, we calculated rental car rates using Avis rental prices and assumed that the road trip would start from San Francisco Airport and begin on June 20. We assumed that all of these values were accurate and representative of normal values for these variables.

We chose to create our model in Excel so that we could easily change variables and see their effects on the overall result. First we made a basic model, assuming an 8-day trip, mostly on highways, with a gas price of $3.231 (the current California average price for regular fuel). For a family with a midsize sedan who wanted to rent a similar midsize sedan, we found that the family would need to drive 2058 miles to "breakeven," meaning that the owned car and rented car options would cost the same. After calculating this value, we examined how renting a car *different* from the one owned would affect the "breakeven" mileage. Also, we allowed the user of the Excel model to enter the actual length of the trip to determine how much extra getting a rental car would cost, or how much money would be saved. And, since the decision to rent a car is not purely economical (some people may prefer a rented car, and this preference has some value), the user of the model can then look at the difference between the owned car and rented car costs and decide if the preference for a rented car outweighs its slightly higher cost.

Once we examined the cost of owned cars versus rented cars using our model, we performed a sensitivity analysis and tornado diagram to see how much tweaking the values of our variables would affect our results. We also considered the strengths and weaknesses of our model. Our Excel worksheet allows users to easily enter in the type of car they own, the type of car they will rent, and the duration of their trip, and from these values Excel calculates the "breakeven" mileage. This user-friendly format, along with the breadth of our variables, allows for an accurate and usable model for determining when rental cars are better for road trips. Of course, we could not include all variables that affect the operating costs of owned cars and rented cars, and we had to make various assumptions. We have successfully created a model to determine when renting a car is the best option for a road trip.

Problem A Paper: Illinois Mathematics and Science Academy

Advisor: Steven Condie

Team Members: William Hahne, David Nai, Adam Novak, Parker Schmitt

We chose to investigate the optimal locations of smoke alarms and fire extinguishers in order to warn occupants during a fire so that they could evacuate safely. Of particular importance is the fact that an alarm system must be capable of awakening sleeping occupants.

To model fire and smoke we use a variation of the NIST Consolidated Fire Growth and Smoke Transport (CFAST) model. We idealize combustion of materials in a room as combustion of an amount of pinewood determined by the room's size. We consider the people adults and assume they know what to do when alerted. We model fire and smoke as spreading outwards from an ignition point, not expecting flames to spread along gradients of good fuel, ceilings to cave in, etc. in the model's timeframe.

The factors we consider are extent of fire and temperatures in burning rooms, delay in activation of smoke alarms, distance of smoke alarms from sleeping people, and evacuation plans.

We consider distance of smoke alarms from sleeping people in order to ensure that alarms are loud enough to wake occupants. To model sound propagation, we use a variation of a model in "Guide to the Most Effective Locations for Smoke Detectors in Residential Buildings." This model accounts for the sound power of an alarm (95 dBA) and the amount of sound needed to wake a person (75 dBA). It assigns each room a hardness rating—hard, normal or soft—and calculates the room's sound level by summing all sound levels entering the room, as well as any smoke detector in the room. To each of these contributions, it applies a modifier (Appendix A). Sound traveling through a closed door receives a –10 dBA penalty; sound traveling between rooms not connected by air ducts receives a –6 dBA penalty. We modify the model to express it in discrete steps and to incorporate multiple smoke alarms; for the simulation, we accept a smoke alarm arrangement as valid if, after 10 steps (which proves sufficient to compute sound propagation completely), each bedroom of the house had a sound level of at least 75 dBA.

By testing random arrangements of a given number of alarms, we find that if bedroom doors are closed, it is impossible to wake people in every bedroom unless each has an alarm. An alarm in the hall outside a bedroom does not yield a sufficient sound level inside the room to wake a sleeping adult. As it is impossible to ensure that the door of a bedroom is open, the safest alarm arrangement is one in each bedroom.

Our integrated fire and evacuation model begins with a search for an arrangement of n alarms capable of producing at least 75 dBA in each bedroom. Alarms are randomly assigned to rooms and activated to see it they produce the required sound levels; if not, a new arrangement is tried. However, if the detectors are capable, the simulation proceeds to the next stage, in which a room is randomly lit on fire. The combustion proceeds according to our modified CFAST algorithm, which we discuss later. The fire and smoke simulation is run until an alarm activates, which occurs when room temperature is 13°K or more above the model's baseline temperature of 298°K. Temperature rise can be used in place of particulate smoke simulation to simplify the model. However, as this approach causes an alarm to activate within two to three seconds, even for very distant fires, we add another constraint to make the behavior of the simulation more realistic: an alarm does not activate unless the room it is in contains at least 0.05 kg/m^2 of heated air and smoke. With this constraint, alarm activation times increase to reasonable levels.

After an alarm activates, the final stage of the simulation begins. This stage finds the shortest available route from the master bedroom to an exit using a brute-force path-finding algorithm. This path is represented as a path between nodes (doors) along edges (routes across rooms between two doors). Although the master bedroom is used, the model can be modified for a different starting point since the problems of escaping from the master bedroom closely resemble those of escaping from another bedroom. The shortest path is defined as the path having the fewest "hops" between nodes, not necessarily the shortest actual travel distance. In practice, the distinction is meaningless because of the constraints imposed on the route by typical house geometry (i.e., small number of exits and few long distances between doors that border on the same room).

A path is not permitted to involve edges passing through rooms that are on fire or that contain excessive smoke. Excessive smoke is defined as smoke and heated air with a temperature of 100°K or more above 298°K, and with smoke concentrations of 2.0 kg/m^2 or more. In the event that such conditions block the only means of egress, the model assumes that people remain where they are and die. A real human would attempt to escape by means such as jumping out a window. However, implementing such behavior in our path-finding algorithm is too difficult.

We model smoke with a modified CFAST algorithm, using formulas derived in Appendices B through D. Our algorithm assumes that the available oxygen is turned into carbon dioxide and water vapor by fire, then carbon monoxide and soot (mostly carbon) as oxygen becomes scarce. Nitrogen and chlorine compounds in the fuel result in hydrogen chloride and hydrogen cyanide, which make the smoke highly poisonous.

All nomenclature for the smoke model is in Appendix C.

Determining smoke characteristics is a difficult task. One must account for the chemistry of combustion reactions, temperature currents, ventilation, and pressure gradients. Our model contains a system of differential equations, each representing a certain factor in smoke behavior. Some ideas are adapted from the NIST CFAST model; however, CFAST has restrictions that conflict with our goals. CFAST does not allow for spread of fires, nor does it describe smoldering fires well. Our adaptation, applicable in the first 2 minutes of a fire, accounts for spreading fire and smoke. CFAST does a fine job of diagramming flame: smoke rises and hits the ceiling first, resulting in two layers of air, a hot upper layer and a cooler lower layer (**Figure 1**).

The ceiling jet is crucial to understanding smoke. When smoke hits the ceiling it is deflected, and the smoke vector field becomes non-conservative $\bar{\mathbf{V}} \times \mathbf{F} \neq 0$. This means that there are spaces in the ceiling that smoke does not hit. It is visually obvious that corners are hit less, and since the field is non-conservative, smoke bends bend back down and increases the upper layer's volume.

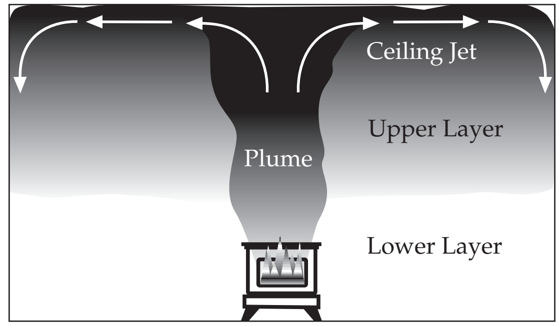

Figure 1.

Unfortunately, the CFAST differential equations are most likely unsolvable; therefore, a discrete approximation is necessary. Our model uses standard linear approximations (derived in Appendix B):

$$\forall f(t),\, f(t+\Delta t) \approx \frac{df}{dt}\Delta t + f(t)$$

In CFAST there are three differential equations (derived in Appendix C):

$$\frac{dP}{dt} = \frac{\gamma - 1}{V}\left(\dot{h}_L + \dot{h}_U\right)$$

$$\frac{dV_i}{dt} = \frac{1}{\gamma P(t+\Delta t)}\left((\gamma - 1)\dot{h}_i - V_i(t+\Delta t)\frac{dP}{dt}\right)$$

$$\frac{dT_i}{dt} = \frac{1}{c_p \rho_i V_i(t+\Delta t)}\left(\left(\dot{h}_i - c_p \dot{m}_i T_i\right) + V_i \frac{dP}{dt}\right)$$

Using linear approximations we can solve them (solutions in Appendix D):

$$P(t+\Delta t) \approx \Delta t \frac{dP}{dt} + P(t)$$

$$V_i(t+\Delta t) \approx \frac{\Delta t \dfrac{1}{\gamma P(t+\Delta t)}}{\left(1 + \dfrac{\Delta t\left(\dfrac{dP}{dt}\right)}{\gamma P(t+\Delta t)}\right)} + V_i(t)$$

$$T_i(t+\Delta t) \approx \frac{\Delta t \dfrac{1}{c_p m_i}\left(\left(\dot{h}_i\right) V_i \dfrac{dP}{dt}\right)}{\left(1 + \dfrac{\dot{m}_i}{m_i}\right)} T_i(t)$$

A major difference between our solutions and CFAST's is that ours factors out density, because vertical vents are not included in the problem. Vertical vents are easy to account for using Bernoulli's equation.

Our model has trouble with volume calculations and the density of the gas becomes excessively large. CFAST has the same problem; they claim it is because of their time slice is too big.

If it were much smaller, a supercomputer would be needed.

The ceiling jet allows room pressure to increase and the upper layer to go through the doorway, modeled as a horizontal vent. Smoke spreads based on pressure as shown in Appendix E: every second, $\frac{9.8A_d\left(P_f - P_{nf}\right)}{101325m}$ kg. However, one should do it in Δt seconds. Variable definitions for the modified CFAST algorithm are in Appendix F.

We implement our model with a C# program (Appendices G and H). The program is not intended as a general solution for end users. It includes several sections of code that we use to explore different aspects of the problem.

Our model does a reasonably good job of producing meaningful results; although without data with which to compare the model, it is difficult to say if it is accurate. However, the smoke detector placement scheme that the model recommends—namely, one smoke detector in each bedroom, and not at the corner (due to the ceiling jet)—is valid from a common sense perspective. Fire extinguisher placement is simple because one should only try to extinguish a small fire. Toxic smoke fills a house quickly (as our model shows), and a fast escape is essential. Although our model does not perfectly represent some things, such as a person's response to being trapped or, say, probability of a smoke detector breaking, it is adequate for the purposes for which it was conceived.

APPENDIX A

Flame and Smoke Simulation

Fuel Enthalpy of Combustion = 9189630.1 J/kg for fuel

Fuel Combustion Rate = 0.0236111111 kg/(m^2*sec)

Burn Fraction = 0.25. 1/4 of the floor of a room "on fire" is treated as burning.

Normal Temperature = 298°K. Room temperature is 298°K, or 25°C.

Gamma = 1.4. Gamma is a constant used in the modified CFAST model.

Cp = 1,000,000 J/(kg*K). *Cp* is a constant in the modified CFAST model.

Path Finding

A room with 2.0 kg smoke per square meter of floor and with a smoke temperature 100° or more above 298°K is considered impassable by the routing algorithm, as is a room marked as "on fire."

Smoke Alarm Modeling and Sound Transmission

A smoke level of 0.05 kg per square meter of floor, combined with a smoke temperature of 13°K above 298°K is required to activate a smoke alarm. All smoke alarms in the system activate when one alarm is tripped.

The smoke alarms are assumed to have a sound power of 95 dBa.

A sound level of 75 dBa is required in a bedroom to wake sleeping people.

The sound propagation algorithm uses **Tables 1** and **2**. All modifiers are in dBa.

Alarm sound level modifiers (Table 1) are used to determine the sound level that an alarm of a given sound power is capable of producing in a given room.

Min room size (m^2)	Max room size (m^2)	Soft room	Normal room	Hard room
2.4	2.95	0	2	4
2.95	3.65	–1	1	3
3.65	4.65	–2	0	2
4.65	5.85	–3	–1	1
5.85	7.35	–4	–2	0
7.35	9.25	–5	–3	–1
9.25	11.65	–6	–4	–2
11.65	14.75	–7	–5	–3
14.75	18.55	–8	–6	–4
18.55	23.35	–9	–78	–5
23.35	29.45	–10	–8	–6
29.45	37.05	–11	–9	–7
37.05	46.65	–12	–10	–8
46.65	58.75	–13	–11	–9
58.75	74.05	–14	–12	–10
74.05	93.25	–15	–13	–11
93.25	117.4	–16	–14	–12

Table 1.

Attenuation modifiers (Table 2) are subtracted from the contributions that adjacent rooms make to a given room's sound levels. They are based on size and hardness of the room into which sound is transferred.

Min room size (m^2)	Max room size (m^2)	Soft room	Normal room	Hard room
1.9	2.25	7	5	3
2.25	2.85	8	6	4
2.85	3.65	9	7	5
3.65	4.65	10	8	6
4.65	5.85	11	9	7
5.85	7.35	12	10	8
7.35	9.25	13	11	9
9.25	11.65	14	12	10
11.65	14.65	15	13	11
14.65	18.45	16	14	12
18.45	23.25	17	15	13
23.25	29.35	18	16	14
29.35	36.95	19	17	15
36.95	46.55	20	18	16
46.55	58.55	21	19	17
58.55	73.75	22	20	18
73.75	92.85	23	21	19
92.85	116.9	24	22	20

Table 2.

APPENDIX B

By definition:

$$\frac{df}{dt} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

We can get rid of the limit and do an approximation:

$$\frac{df}{dt} \approx \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

Of course, the increments are small.

By algebra:

$$\forall f(x), f(x+\Delta x) \approx \frac{df}{dx}\Delta x + f(x)$$

APPENDIX C

L denotes a lower layer and U denotes an upper layer; i is the general case.

$\rho_i = \frac{m_i}{v_i}$ by the definition of density.

$E_i = c_v m_i T_i$ (internal energy of the layer)

$P_i = R\rho_i T_i$

Pressure evenly distributes though, so we refer to P as total pressure.

$V_L + V_u = V$ = volume of room

Define dot notation:

$$\frac{dm_L}{dt} = \dot{m}_L$$

$$\frac{dm_U}{dt} = \dot{m}_U$$

According to the first law of thermodynamics:

$$\frac{dE_i}{dt} + P\frac{dV_i}{dt} = \dot{h}_i$$

We add the differential form of pressure and the first law of thermodynamics:

$$\frac{dP}{dt} = \frac{\gamma - 1}{V}\left(\dot{h}_L + \dot{h}_U\right)$$

Now with more substitution we get:

$$\frac{dV_i}{dt} = \frac{1}{P\gamma}\left((\gamma-1)\dot{h}_i - V_i\frac{dP}{dt}\right)$$

And with some algebra and the first law of thermodynamics we have:

$$\frac{dE_i}{dt} = \frac{1}{\gamma}\left(\dot{h}_i + V_i\frac{dP}{dt}\right)$$

To find temperature:

$$\frac{dT_i}{dt} = \frac{1}{c_p \rho_i V_i}\left(\left(\dot{h}_i - c_p \dot{m}_i T_i\right) - \frac{V_i}{\gamma - 1}\frac{dP}{dt}\right)$$

Because of the formula for density:

$$\frac{dT_i}{dt} = \frac{1}{c_p m_i}\left(\left(\dot{h}_i - c_p \dot{m}_i T_i\right) - \frac{V_i}{\gamma - 1}\frac{dP}{dt}\right)$$

APPENDIX D

We already know the time derivative of pressure:

$$\frac{dP}{dt} = \frac{\gamma - 1}{V}\left(\dot{h}_L + \dot{h}_U\right)$$

Linear approximations solve these equations:

$$P(t+\Delta t) = \Delta t\frac{dP}{dt} + P(t)$$

$$\frac{dV_i}{dt} = \frac{1}{\gamma P(t+\Delta t)}\left((\gamma-1)\dot{h}_i - V_i(t+\Delta t)\frac{dP}{dt}\right)$$

$$V_i(t+\Delta t) = \Delta t\frac{dV_i}{dt} + V_i(t)$$

By combining we get:

$$V_i(t+\Delta t) = \Delta t\frac{1}{\gamma P(t+\Delta t)}\left((\gamma-1)\dot{h}_i - V_i(t+\Delta t)\frac{dP}{dt}\right) + V_i(t)$$

$$V_i(t+\Delta t) = \frac{\Delta t\frac{1}{\gamma P(t+\Delta t)}}{1 + \frac{\Delta t\frac{dP}{dt}}{\gamma P(t+\Delta t)}} + V_i(t)$$

Now the same for temperature:

$$\frac{dT_i}{dt} = \frac{1}{c_p \rho_i V_i(t+\Delta t)}\left(\left(\dot{h}_i - c_p \dot{m}_i T_i\right) + V_i\frac{dP}{dt}\right)$$

$$T_i(t+\Delta t) = \Delta t\frac{dT_i}{dt} + T_i(t)$$

$$T_i(t+\Delta t) = \Delta t\frac{1}{c_p \rho_i V_i(t+\Delta t)}\left(\left(\dot{h}_i - c_p \dot{m}_i T_i\right) + V_i\frac{dP}{dt}\right) + T_i(t)$$

$$T_i(t+\Delta t) = \frac{\Delta t\frac{1}{c_p \rho_i V_i(t+\Delta t)}\left(\dot{h}_i + V_i\frac{dP}{dt}\right)}{1 + \frac{\dot{m}_i}{\rho_i V_i(t+\Delta t)}} + T_i(t)$$

Now eliminate density:

$$\rho = \frac{m}{v}$$

$$T_i(t+\Delta t) = \frac{\Delta t\frac{1}{c_p m_i}\left(\dot{h}_i + V_i\frac{dP}{dt}\right) + T_i(t)}{1 + \frac{\dot{m}_i}{m_i}}$$

$$\dot{m}_i = \frac{dm}{dt}$$

Although this solves continuously, discrete approximations work better in code.

$$m(t+\Delta t) = \dot{m}\Delta t + m(t)$$

APPENDIX E

A_d = area of doorway

P_f = fire room pressure

P_{nf} = non fire room pressure

$$\frac{A_d\left(P_f - P_{nf}\right)}{101325} = \frac{ma}{9.8}$$

$$\frac{9.8A_d\left(P_f - P_{nf}\right)}{101325m} = a$$

Every second $\frac{9.8A_d\left(P_f - P_{nf}\right)}{101325m}$ kg should escape, but pressure changes, so divide it into Δx sections of time.

APPENDIX F

L and U denote lower and upper layer, respectively. i denotes either layer.

A dot over a variable represents a time derivative.

m_i = mass of layer

ρ_i = density of layer

V_i = volume of layer

$V_l + V_u = V$ is total volume

$\dot{h}_i$ = enthalpy

c_p = heat capacity of air at constant pressure (J/kgK)

c_v = heat capacity of air at constant volume (J/kgK)

$\gamma = \frac{c_p}{c_v}$

E_i = energy at layer

A_d = area of doorway

$A_s = s$

P_f = fire room pressure

P_{nf} = non fire room pressure

$\dot{m}_i$ = time derivative of mass of layer

Δt = time increment

Editor's note: Appendices G (program code) and F (sample program output) are omitted due to space limitations.

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Problem B Paper: Central Academy

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We were given the task of comparing the relative merits of renting a car for a road trip versus driving one's own car. We created a model to compare the expected costs of these two options including a value of "ease of mind."

Assumptions and Justifications

1. ROUND TRIP (USA)

We assume that the family drives to the destination and back. This means our model need not account for factors such as plane fares. It is important to stay in the U. S. because rental agencies charge fees for leaving the country.

2. VACATION LENGTH IS MEASURED IN WEEKS

We assume that a vacation is one or two weeks (including driving time) because it is cheaper to rent a car by the week. To simplify, we choose not to make separate models for weekly and daily rates.

3. NO DRIVING AT DESTINATION

All driving at the destination is the same for both situations (rental or owned) and would cancel out. It is impractical for us to calculate this since it varies with the destination.

4. GOOD WEATHER AND DRIVING CONDITIONS

Bad driving conditions can change mileage and trip time, and increase probability of accident or breakdown. We used EPA mileage estimates that assume ideal driving conditions because these are the only data available.

5. COMPETENT DRIVERS WITH EXCELLENT DRIVING RECORDS

We assume competent drivers so that the probability of accident or breakdown is reasonable and does not vary with the driver. Also, drivers with an unsatisfactory driving record are charged more.

6. NO DRIVER UNDER AGE 25 OR OVER 65

Rates increase for those under 25 or over 65; many rental agencies do not rent to people over 70.

7. CAR INSURANCE IS PURCHASED FROM THE RENTAL AGENCY

If insurance is not purchased, there is no “ease of mind.” Our model assumes the main factor in “ease of mind” is risk of accident or breakdown.

8. USE HERTZ

Hertz is a major cross-country company. Also, Hertz allows significant others to drive the rental without an extra fee, so we need not model for multiple drivers.

9. NO EXTRAS

For “ease of *our* minds” our model assumes only basic insurance and no special features offered by insurance companies, which cause too much variation.

10. NO LOCATION SPECIFIC SURCHARGE

Our goal is to create a model that can be applied to any location. By ignoring location specific surcharges, which are impractical to implement, our model is applicable to any U.S. location.

11. NO INITIAL CAR PROBLEMS

Initial car troubles alter probability of breakdown in ways that are too difficult for us to account for; by negating this we standardize our model.

12. THE CAR IS CAPABLE OF DRIVING AN ADDITIONAL 5000 MILES

We assume that the car the family owns can handle the trip mileage. By setting a limit of 5000 miles, we can ignore an oil change factor.

13. GAS CONSUMED AND GAS PURCHASED ARE EQUIVALENT FOR BOTH MODELS

We assume that people pay for all gas, since rental companies charge a fee if a returned car needs gas.

14. TIME FOR REPAIRS IS THREE DAYS

This value is necessary in computing monetary loss due to accident or breakdown. We could not find an estimate for this value, so we use three days as an average from our own experience.

15. PEOPLE RENT THE SAME TYPE OF CAR THAT THEY OWN

People are comfortable driving their car. This keeps the probability of accident, cost of gas, mileage, and other model specific factors to be equal.

16. EXAMPLES USE DEFINED MODELS OF CARS

Our model can be applied to any car, but for the sake of our examples and in order to compare different types of cars, we use the Ford Focus, the Ford Taurus, the Hyundai Entourage, and the Chevy Trailblazer. All are automatic in order to limit driver influence over gas mileage and standardize gas mileage.

17. DRIVE 10 HOURS PER DAY DURING TRAVEL TIME

We use this only in application of our model, not in the model itself. We assume the family leaves around 8 A.M., stops driving by 10 P.M., and uses about 4 hours for meals, restrooms, refueling, etc.

18. 70 MILES PER HOUR AVERAGE SPEED

We use this only in application of our model. 70 mph is the speed limit on many major highways and Interstates; we assume variations average out to this.

19. EASE OF MIND IS DETERMINED BY THE RISK OF CAR TROUBLE AND ITS CONSEQUENCES

This allows us to get a quantitative value for ease of mind that we can add to our costs for driving one’s own car.

Model Explanation and Analysis

To determine a cost of driving one’s own car, we consider four factors: depreciation due to added mileage, repair costs, fuel, and vacation time lost to repairs. We then compare that cost to the cost of renting a car, for which we use three factors: rental rate, rental insurance, and fuel. We discard the depreciation value for the rental car because it is a cost to the rental company, and we ignore stress due to car trouble because the car is someone else’s concern. We discard repair cost and vacation time lost to repairs because we assume the renter buys insurance, does not pay for repairs, and has another car during repair times. We also ignore costs such as hotel bills and food as these are the same whether or not the driver rents. Since we take an economic approach, we convert all of these factors into dollars. Our final equations for cost follow.

Equation 1: Cost to Rent

$C_{rental} = Rt + \frac{d}{e}g$

where:

C is cost of the trip in dollars

R is weekly rental rate, including insurance, in dollars per week

t is vacation time in weeks

d is total trip distance in miles

e is fuel efficiency of the vehicle in miles per gallon

g is cost of gas in dollars per gallon

This cost is based on the cost of renting the car and the cost of fuel. The cost of renting is determined by the weekly rental rate, including insurance purchased for ease of mind. The fuel cost is the product of amount of gasoline and cost of gasoline; the amount of gasoline is found by dividing distance by mileage for the car model used. We used the gas price used by AAA for calculating the cost per mile driven, $2.256 per gallon.

Equation 2: Cost to Use Own Car

$C_{own} = (k + t_r IP)d$

where:

C is cost of the trip in dollars

k is operation cost constant in dollars per miles

t_r is time for repairs in days

I is median daily income in dollars per day

P is probability of breakdown or accident in incidents per mile

d is total trip distance in miles

This cost has two parts: the cost of operating one's own car, which is the product of cost per mile and distance traveled, and the expected cost of time lost to car trouble. We set the value of a vacation day equal to that of a workday, so expected cost depends on income, probability of a breakdown or accident, time lost, and distance traveled.

To calculate expected daily income we divide median family income ($44,334) by 365 days, yielding

$$I = \frac{\$44334}{1 \text{ year}} \times \frac{1 \text{ year}}{365 \text{ days}} = \frac{121.46}{1 \text{ day}}.$$

Adding the number of accidents per 100 million miles driven and the number of breakdowns per 1000 miles driven gives

$$P = \frac{0.0352 \text{ breakdowns}}{1000 \text{ miles}} + \frac{206 \text{ accidents}}{100,000,000 \text{ miles}}$$

$$= \frac{3.726 \times 10^{-5} \text{ incidents}}{1 \text{ mile}}.$$

Table of Variable Values for Four Car Models

Car type (model)	k ($/mile)	R ($/week)	e (miles/gal)	g ($/gal)	P (incidents/mile)
Small sedan (Ford Focus)	0.414	166.49	32	2.256	3.726×10^{-5}
Midsize sedan (Ford Taurus)	0.525	186.49	28	2.256	3.726×10^{-5}
Minivan (Hyundai Entourage)	0.576	300.49	25	2.256	3.726×10^{-5}
SUV (Chevy Trailblazer)	0.666	310.49	20	2.256	3.726×10^{-5}

Fuel efficiency values are for model years 2000 and 2001 because the median age of vehicles on the road is 7.7 years. Entering these values into our equations gives us functions in terms of distance traveled for a given vacation time and given car type. The point where the functions' graphs intersect is the breakeven point. For distances beyond this point, renting is better. For distances below this point, the cost of renting is higher than the cost of driving the family car (**Figure 1**).

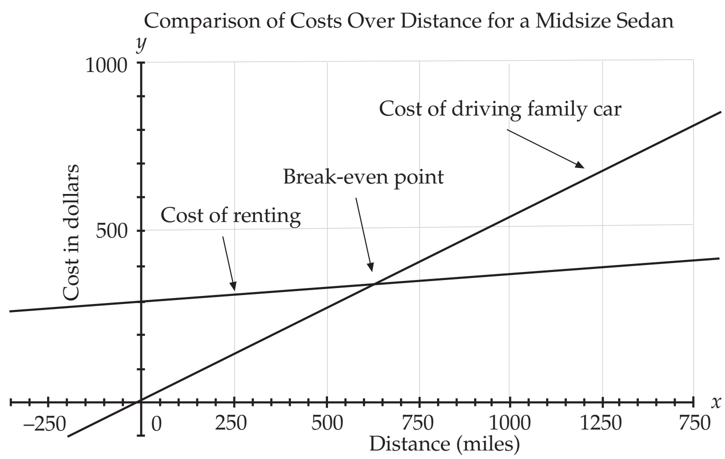

Figure 1.

One week is a good vacation length to begin analyzing travel costs.

WEEKLONG VACATION

Car type	Rental cost function	Own car cost function	Intersection (miles)
Small sedan	$C = 271.49 + 0.0705d$	$C = 0.428d$	760
Midsize sedan	$C = 291.49 + 0.0806d$	$C = 0.539d$	636
Minivan	$C = 405.49 + 0.0902d$	$C = 0.590d$	812
SUV	$C = 415.49 + 0.113d$	$C = 0.680d$	733

As these values show, whether to rent depends on car type. Rentals are better for long trips due to the increased wear on the car associated with driving long distances. Renting is a poor choice for short trips because the car is less likely to break down. The breakeven point for one-week vacation is comparable to a trip between Los Angeles to San Francisco (764 miles round trip) or Chicago and St. Louis (592 miles round trip). This is reasonable for one week since total travel time is about a day. For distances longer than about 1400 miles round trip, one week loses its effectiveness. A trip from Chicago to Denver (2106 miles round trip) requires about 1.5 travel days each way, almost half the vacation. It is therefore advisable to consider a two-week vacation.

TWO-WEEK VACATION MODEL

Car type	Rental cost function	Own car cost function	Intersection (miles)
Small sedan	$C = 542.98 + 0.0705d$	$C = 0.428d$	1521
Midsize sedan	$C = 582.98 + 0.0806d$	$C = 0.539d$	1273
Minivan	$C = 810.98 + 0.0902d$	$C = 0.590d$	1624
SUV	$C = 830.98 + 0.113d$	$C = 0.680d$	1466

These results show that vacation duration is also important when deciding whether to rent. For longer vacations the breakeven point occurs at a greater distance. In this case, the distance is comparable to a trip from Philadelphia to Chicago (1520 miles). For very long trips, such as a cross-country drive from New York City to Los Angeles (5556 miles round trip), renting is more cost-effective due to the high level of wear on the car.

BREAKEVEN POINTS

Time	Small sedan	Midsize sedan	Minivan	SUV
1 week	760	636	812	733
2 weeks	1521	1273	1624	1466

This table summarizes our results. A consumer could use it to decide whether to rent a car. For trips longer than the breakeven point it is better to rent; for trips shorter than the breakeven point it is better to not rent. The breakeven point varies with the model of car and vacation length.

STRENGTHS:

1. Our model is based on real world data. The only value we assume is time for repairs, for which we could not find a source.

2. After testing various scenarios, we found that our model provides reasonable answers to a variety of situations.

3. Our model is flexible and can be adjusted for other car models, distances traveled, and vacation lengths. One only needs to know the class of car and its gas mileage.

4. The model accounts for several components often forgotten, such as the depreciation that each mile driven causes. Since depreciation happens slowly over time, it may not be considered a reason to rent a car for a trip.

5. The equations are so straightforward and easy to use that an average consumer with no background in cars but with a few statistics can apply them.

WEAKNESSES:

1. Ease of mind considers only monetary costs and benefits, which is necessary to make the two equations comparable. There are psychological factors that cannot be modeled, although we attempt to account for them through T_rIP.

2. Time lost due to repairs in the ease of mind (C_{own}) equation is based on probability of an accident or breakdown. It is possible to take a trip longer then the breakeven point and still save money if there is no car trouble, and the opposite is also possible. We have calculated an average. It is impossible to guarantee a specific outcome.

3. Our k value includes fixed costs (not driving dependent), but overestimating the dollar per mile cost better accounts for intangible ease of mind.

4. Our model is limited to round trips of 5,000 miles. After 5,000 miles an oil change is recommended. This charge is included in the cost per mile of using one's own car (k), but we have no data to figure it into the cost of renting.

5. Our model fails to account for regional and personal variation in cost, behavior, and state of mind. A relaxed person might be better off driving his or her own car for longer than the breakeven point, whereas a worrier might place a greater value on not worrying about breakdown. Some people are harder on cars than others, which affects depreciation and breakdown probability. There is also regional variation in rental costs and gas prices.

6. Our model doesn't account for feasibility of one's own car traveling the total distance. This would be far too complicated; however, it should be considered by anyone using our model.

7. Our model does not account for city or destination driving. City traffic changes average speed and distance that can be traveled in a day. Destination driving is the same whether the car is rented or owned. Because we cannot guess the amount of destination driving, we cannot compute depreciation.

8. Our model requires gas mileage estimates, but these are based on theoretical "perfect" conditions that are not met in the real world. These values, which come from the EPA, have been questioned, but they are the only ones available.

9. Vacation time lost to accidents or breakdowns is not factored into the rental car equations. Rental companies would presumably be quick in these situations, but we have no data for how quick they are.

10. Our model does not specifically include taxes or extra fees incurred with rentals, but flat rate fees could easily be included.

POSSIBLE EXTENSIONS

There are other variables that can be added to our model to increase detail and accuracy. Adding them would allow consumers to make more informed decisions. These include AAA membership, whether the family's car is under warranty, special rental rates, and additional subgroups of trips.

AAA MEMBERSHIP

There are approximately 50 million AAA members in the United States, about 26% of registered drivers. AAA membership can offer rental discounts and reduce stress over potential car problems through services such as free roadside assistance and towing or by helping obtain car service.

WARRANTY

If the family car is under warranty, they might be better off taking it on longer trips because the car manufacturer would cover any trouble that arose.

SPECIAL RENTAL RATES

Car rental companies often offer discounts that are not included in our model. These rates could make renting viable for shorter distances.

ADDITIONAL TRIP SUBGROUPS

Additional analysis could be included depending on the nature of the trip. For example, consumers could alter the rental rates to better reflect their home region. Gas prices could also be tailored to fit specific areas through which the trip passes. Rental car companies generally charge a fee for taking cars out of the country, which could be included if a foreign destination is planned. One final subgroup is returning the rental car at the destination and using public transportation while there, which could be effective for travel to a city with an extensive mass transit system.

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