HIGH SCHOOL MATHEMATICAL CONTEST IN MODELING

OUTSTANDING PAPERS

HiMCM

The contest offers students the opportunity to compete in a team setting using applied mathematics in the solving of real-world problems.

November 2008

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Editor’s Comments

This is our eleventh HiMCM special issue. Since space does not permit printing all four National Outstanding papers, this special section includes the summaries from two papers and abridged versions of two. We emphasize that the selection of these two does not imply that they are superior to the other Outstanding papers. We also wish to emphasize that the papers were not written with publication in mind. Given the thirty-six hours that teams had to work on the problems and prepare their papers, it is remarkable how much was accomplished and how well-written many of the papers are. For the first time this year, teams were required to submit electronic versions of their papers. These electronic submissions from all National and Regional Outstanding teams are on the 2008 HiMCM CD-ROM, which is available from COMAP. ❑

Contest Director's Article

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The High School Mathematical Contest in Modeling (HiMCM) completed its eleventh year in excellent fashion. The mathematical and modeling ability of students, and faculty advisors, is very evident in the professional submissions and work being done. The contest is still moving ahead, growing with a positive first derivative, and consistent with our positive experiences from previous HiMCM contests.

This year the contest consisted of 269 teams (down one team from last year) from 52 institutions. These institutions were from twenty states (there was one new state but six other states and the District of Columbia did not compete this year), the Hong Kong International School, China, Korea, and the United Kingdom. This year, we again charged a registration fee of $75.

The teams accomplished the vision of our founders by providing *unique* and *creative mathematical* solutions to complex open-ended real-world problems. This year the students had a choice of two problems.

Problem A: National Debt and National Crisis

Mathematical modeling involves two equally important steps—building models based on real world situations and interpreting predictions made by those models back in the real world. This problem places equal emphasis on both steps.

We are at the start of the 2008 U.S. presidential elections, and one important area of debate is sure to be the national debt. As high school students, you have a particular interest in this subject since you are the people who will pay off or at least manage the national debt in the future. The rate at which the national debt changes depends on the difference between federal income (primarily taxes) and federal expenditures. Your first task is to build a model that can be used to help understand the national debt and make forecasts based on different assumptions. As usual, modeling involves a balance between so much complexity that the model may be intractable and so little complexity that it is unrealistic and useless. Your model needs, at the very least, to allow you to consider different tax policies and different expenditure policies.

As usual, raw numbers don't carry much information. Those numbers must be placed in context. For example, total national debt is less meaningful than national debt per capita. In addition, you must be careful about inflation. Many analysts look at the ratio between national debt and gross domestic product as a good indicator of the impact of the national debt. Others worry about the cost of servicing the national debt. This cost is affected by both the size of the national debt and the interest rate the government must pay to borrow money. You may want to look at the Wikipedia article:

http://en.wikipedia.org/wiki/National_debt_by_U.S._presidential_terms

for some figures involving the ratio between national debt and gross domestic product.

Tasks:

1. Build a model that can be used to help understand the national debt and make forecasts based on different assumptions. You must provide justification for the various elements of your model and you must also test the sensitivity of your model to various parameters.

2. Use your model to compare at least two alternative plans for the years 2009–2017. Your plans should be based on different tax and spending policies that are reasonable and politically feasible. Use your model to compare the impact on the national debt and then impact on the nation in general of your policies.

3. Prepare a letter to the new president advising him of your model.

Problem B: Going Green

The United States can address its national carbon footprint in two ways: by reducing carbon dioxide emissions or by increasing carbon dioxide consumption (sequestration). Assume that the total U.S. carbon dioxide emissions are capped at 2007–2008 levels indefinitely. What should the U.S. do to increase carbon dioxide consumption to achieve national carbon neutrality with minimal economic and cultural impact? Is it even possible to achieve neutrality? Model your solution to show feasibility, effectiveness, and costs. Prepare a short summary paper for the U.S. Congress to persuade them to adopt your plan.

Commendation: All students and advisors are congratulated for their varied and creative mathematical efforts. Of the 269 teams, 116 submitted solutions to Problem A and 153 to Problem B. The thirty-six continuous hours to work on the problem provided for quality papers; teams are commended for the overall quality of their work.

Many teams had female members. There were 409 female participants on the 269 teams. There were 1078 total participants, so female members made up approximately 38% of the total participation, showing this competition is for all students. (This percent is almost triple the percent of women who participate in other math competitions.) There was at least one female member on 72% of the teams, and 13% of the teams were all female (34 teams).

Teams again proved to the judges that they had "fun" with their chosen problems, demonstrating research initiative and creativity in their solutions. This year's effort was a success!

Judging: We ran three regional sites in December 2008. Each site judged papers for Problems A and B. The papers judged at each regional site may or may not have been from their respective region. Papers were judged as Outstanding, Meritorious, Honorable Mention, and Successful Participant. All finalist papers for the Regional Outstanding award were sent to the National

Judging. For example, eight papers may be discussed at a Regional Final, and only four selected as Regional Outstanding but all eight papers are judged for the National Outstanding. Papers receive the higher of the two awards. The national judging chooses the “best of the best” as National Outstanding. The National Judges commended the regional judges for their efforts and found the results were very consistent. We feel that this regional structure provides a good structure for the future as the contest grows.

Judging Results:

NATIONAL AND REGIONAL COMBINED RESULTS

Problem	National Outstanding		Regional Outstanding		Meritorious		Honorable Mention		Successful Participant		Total
	#	%	#	%	#	%	#	%	#	%	
A	**3**	2.5%	9	9%	28	25%	60	52%	16	14%	**116**
B	**1**	0.6%	9	6%	30	20%	94	61%	19	12%	**153**
Total	**4**	1.4%	18	7%	58	22%	154	57%	35	13%	**269**

National Outstanding Teams

Hong Kong International School, Tai Tam, Hong Kong
Illinois Mathematics and Science Academy, Aurora, IL
Maggie Walker Governor’s School, Richmond, VA
The Ellis School, Pittsburgh, PA

Regional Outstanding Teams

All Saints Episcopal School, Fort Worth, TX
Arkansas School for Math & Sciences, Hot Springs, AR
Central Academy, Des Moines, IA (2 Teams)
Clarkstown High School South, West Nyack, NY
Dubuque Hempstead High School, Dubuque, IA
Hanover High School, Hanover, NH (2 Teams)
Hanyoung Foreign Language High School, Seoul, Korea
Holy Ghost Prep, Bensalem, PA
Illinois Mathematics and Science Academy, Aurora, IL
Maggie Walker Governor’s School, Richmond, VA (2 Teams)
MATHS Education Research Department, Shanghai, China (2 Teams)
Middlesex School, Concord, MA
Montgomery Blair High School, Silver Spring, MD
China Normal University Secondary School No. 2, Shanghai, China

NCTM Standards: The director and the judges asked that we add this paragraph. Many of us have read the NCTM standards and clearly realize the mapping of this contest to the NCTM 9–12 mathematics standards. This contest provides a vehicle for using mathematics to build models to represent and to understand real-world behavior in a quantitative way. It enables student teams to look for patterns and think logically about mathematics and its role in our lives. Perhaps in a future *Consortium* article we will dissect a problem (paper) and map the standards into it.

General Judging Comments: The judge’s commentaries provide specific comments on the solutions to each problem. As contest director and head judge, I would like to speak generally about solutions from a judge’s point of view. Papers need to be coherent, concise, and clear. Students need to restate the problem in their own words so that the judges can determine the focus of the paper. Papers that explain the development of the model, assumptions, and its solutions, and then support the findings mathematically, generally do quite well. Modeling assumptions need to be listed and justified, but only those that come to bear on the solution (that can be part of simplifying the model). Laundry lists of assumptions that are never referred to in the context of the model development are not considered relevant and deter from a paper’s quality. The mathematical model must be clearly developed, and all variables that are used must be well defined. Thinking outside of the box is also considered important by judges. This varies from problem to problem, but usually includes model extensions or sensitivity analysis of the solution to the team’s inputs. Students must attempt to validate their model, even if by numerical example or intuition. A clear conclusion and answers to specific scenario questions are all key components. The strengths and weakness section is where the team can reflect on their solution and comment on the model’s strengths and weaknesses. Attention to detail and proofreading the paper prior to final submission are also important, since the judges look for clarity and style. Citations are very important within the paper as well as either a reference or bibliography page at the end. We encourage citations within the paper that deal directly with use of data and figures, graphs, or tables. We noticed an increased use of Wikipedia; teams should realize that, although useful, the information might not be accurate. Teams should acknowledge this.

CONTEST FACTS:

Facts from the Eleventh Annual Contest:

- Wide range of schools/teams competed including teams from Hong Kong, China, and Korea.

- The 269 teams represented 52 institutions.

- There were 1078 student participants, 669 (62%) male and 409 (38%) female. There were 34 all-female teams.

- Schools from twenty states participated in this year’s contest.

THE FUTURE:

The contest, which attempts to give the under-represented an opportunity to compete and achieve success in mathematics, appears well on its way in meeting this important goal.

We continue to strive to improve the contest, and we want the contest to grow. Any school/team can enter, as there are no restrictions on the number of schools or the numbers of teams from a school. A regional judging structure is established based on the number of teams.

These are exciting times for our high school students. Mathematics continues to be more than learning skills and operations. Mathematics is a language that involves our daily lives. Applying the mathematical principles that one learns is a key to future success. The abilities to recognize problems, formulate a mathematical model, use technology, and communicate and reflect on one’s work are keys to success.

Students gain confidence by tackling ill-defined problems and working together to generate a solution. Applying mathematics is a team sport!

Advisors need only be motivators and facilitators. They should encourage students to be creative and imaginative. It is not the technique used but the process that discovers how assumptions drive the techniques that is fundamental. Let students practice to be problem solvers. Let me encourage all high school mathematics faculty to get involved, encourage your students, make mathematics relevant, and open the doors to success.

Mathematical modeling is an art and a science. Teach your students through modeling to think critically, communicate effectively, and be confident, competent problem solvers for this new century.

CONTEST DATES:

Mark your calendars early: The next HiMCM will be held in November 2009. Registrations are due in October 2009. Teams will have a consecutive thirty-six-hour block within the contest window to complete the problem. Teams can register via the Web at www.comap.com.

HiMCM Judges' Commentary

Problem A

Out of 269 teams, 116 (43%) attempted Problem A. Most teams began by using the Internet to get data on the national debt over time. The teams then loaded the data into some "black box technology" and used linear regression and polynomial regression to fit a function to the data. Too often we saw sixth-order polynomials, which are not viable prediction models. Teams could have used other appropriate models or even broken the data into groups and fit each group with a separate function. Papers that use higher-order polynomial models usually did not achieve outstanding status.

Teams were to compare at least two plans to determine if either would improve the gap in the debt, but many teams did not do this part. Some researched well beyond the Obama and McCain plans and recommended previous administration plans. Some that were used were the Eisenhower plan, Reagan plan, and Clinton plan.

We saw papers that used mathematics from basic regression to differential equations to build a model. Thus, the Regional and National Outstanding were varied in their approaches.

Some teams appeared to have copied equations and figures from the Internet into their papers. In most cases, it was clear that these students did not understand the mathematics that they presented.

The modeling contest format was closely adhered to in many of the papers. However, many summaries and letters were poorly done—as if they could have been written before the model, analysis, and conclusions were completed. The summaries need to contain the results of the model.

The letter to the President should be concise and indicate whether all or part of the debt can be eliminated during the President's administration.

One of the items that discriminated the better papers was the distinction of spending as discretionary and obligatory, although these words did have to be used. Another clear discriminator was the use of a logical process to link assumptions to the building of the mathematical model. Judges also felt that papers of teams that created their own innovative plan were well done and showed creativity.

Data gathered from the Internet should have been obtained from sound sources, such as government sites. Data obtained from blogs or Wikipedia should be treated with suspicion.

In some papers, an issue arose with significant digits. The models contained dollar figures in scientific notation, yet numerical values were stated, at times, to sixteen decimal places. Clearly, this was not necessary.

Problem B

Of the 269 teams, 153 (57%) chose Problem B. The problem statement was concise, but clear in terms of the requirements. Most students failed to complete the required tasks. The most flagrant of this was not including a letter to the U.S. Congress, which resulted in several excellent modeling papers not obtaining National Outstanding recognition. Teams should ensure that that they complete and include all the required tasks in their submission.

The summaries, for the most part, were either absent or poorly written. Many read like technical reports or were too vague to be helpful. Summaries need to contain the results of the model as well as brief explanation of the problem. The summary should entice the reader (in our case the judge), to *read* the paper.

The letter to Congress should be a concise explanation of the results of the modeling and should include: (1) whether the carbon emission neutrality could be achieved; (2) how to obtain or close the gap; (3) a brief description or statement of the impacts; and (4) the approximate cost. The ability to summarize and present information is critical in real life and real jobs. Again, many teams failed to complete this in their submission.

The judges felt the first critical task was to compute the upper bound on the carbon emissions. With this done, teams could discuss reductions of emissions and, most importantly, sequestering carbon emissions to achieve neutrality. Student teams used algae, planting trees, injecting in water, and other interesting methods to sequester emissions. The variety of methods was interesting.

Many teams failed to consider the impacts (social, economic, cultural, etc.) of the sequestering methods, and many teams failed to consider the costs. Many calculated costs never mentioned the feasibility of those costs in terms of, "can we afford it?"

Few teams, if any, did sensitivity analysis on tradeoffs between costs and amount of emissions sequestered.

We found many of the assumptions and much of the research was very good. Teams that did some history of emissions added some nice context to the problem. We encourage teams who take data or graphics from other sources to include the reference at the point they are used in the paper, as well as on a reference page at the end. We saw the use of data from blogs and Wikipedia—these sources can be suspect, and we encourage teams to obtain data and information from reliable sources.

There was a wide variety of approaches used, from simple algebra through simulated models with Stella and AHP decision models. The latter two, however, were never explained well by the student teams that used them. It was as if these techniques were black boxes. As models, what they do and why they could be used in the scenario should be explained.

Again, in some papers, an issue arose with significant digits. The models contained dollar figures in scientific notation, yet numerical values were stated (at times) to sixteen decimal places. Clearly, this was not necessary.

COMMENT ABOUT COMPUTER GENERATED SOLUTIONS:

Many papers used computer code. Computer programs written to implement mathematical expressions can be good modeling tools. However, the judges expect to see an algorithm or flow chart from which the code was developed. Successful teams provided some explanation or guide to their algorithm(s)—a step-by-step procedure for the judges to follow. Code may only be read for the papers that reach the final rounds, but only if the code is accompanied by a good algorithm in words. The results of any simulation need to be well explained and sensitivity analysis must be performed. For example, consider a flip of a fair coin. Here is an algorithm:

INPUT: Random number, number of trials

OUTPUT: Heads or tails

Step 1: Initialize all counters.

Step 2: Generate a random number between 0 and 1.

Step 3: Choose an interval for heads, like [0, 0.5]. If the random number falls in this interval, the flip is a head. Otherwise the flip is a tail.

Step 4: Record the result as a head or a tail.

Step 5: Count the number of trials and increment: Count = Count + 1.

An algorithm such as this is expected in the body of the paper with the code as an appendix.

COMMENTS ABOUT GRAPHS:

Judges found many graphs that were not labeled nor explained. Many graphs did not appear to convey information used by the teams. All graphs need a verbal explanation of what the team expects the reader (judge) to gain (or see) from the graph. Legends, labels, and points of interest need to be clearly visible and understandable, even if hand written. Graphs taken from other sources should be referenced and annotated. Most graphs and figures were obviously taken directly from other sources and were not directly referenced.

General Comments from Judges:

Summaries: These are still, for the most part, the weakest parts of papers. These should be written after the solution is found. They should contain results and not details. They should include the “bottom line” and the key ideas used in obtaining the solution. They should include the particular questions addressed and their answers. Teams should consider a brief, three-paragraph approach: a *restatement of the problem* in their own words, a short description of *their method and solution* to the problem (without giving any mathematical expressions), and the *conclusions* that provide the numerical answers in context.

Restatement of the Problem: Problem restatements are important for teams to move from the general case to the specific case. They allow teams to refine their thinking to give their model uniqueness and a creative touch.

Assumptions/Justifications: Teams should list only those assumptions that are vital to building and simplifying their mathematical model. Assumptions should not be a reiteration of facts given in the problem statement. Assumptions are variables (issues) acting or not acting on the problem. Every assumption should have a justification. We do not want to see “smoke screens” in the hopes that some items listed are what judges want see. Variables chosen need to be listed with notation and be well defined.

Model: Teams need to show a ***clear link*** between the assumptions they listed and the building of their model or models. The teams are also required to show how the model was built and why that model was chosen. Teams should not throw out several model forms hoping to impress the judges—this does not work. We prefer to see sound modeling based on good reasoning.

Model Testing: Model testing is not the same as testing arithmetic. Teams need to compare results or attempt to verify (even with common sense) their results. Teams that use a computer simulation must provide a clear step-by-step algorithm. Lots of runs and related analysis are required when using a simulation. Sensitivity analysis should be done in order to see how sensitive the simulation is to the model’s key parameters.

Conclusions: This section deals with more than just results. Conclusions might also include speculations, extensions, and generalizations. This is where all scenario specific questions should be answered. Teams should ask themselves what other questions would be interesting if they had more time and then tell the judges about their ideas.

Strengths and Weaknesses: Teams should be open and honest here. What could the team have done better?

References: Teams may use references to assist in their modeling. However, they must also ***reference the source*** of their assistance. Teams are reminded that only ***inanimate resources*** may be used. Teams cannot call upon real estate agents, bankers, hotel managers, or any other real person to obtain information related to the problem. References should be cited where used and not just listed in the back of the paper. Teams should also have a reference list or bibliography in the back of the paper.

Adherence to Rules: Teams are reminded that detailed rules and regulations are posted on the COMAP site. Teams are reminded that they may use only ***inanimate sources*** to obtain information. Teams are reminded that the ***thirty-six-hour time limit is a consecutive thirty-six hours***.

MathModels.org

We strongly recommended that participants in this contest, as well as prospective participants, take a look at this modeling Website, www.mathmodels.org, which has a wealth of information and resources.

Problem B (Going Green) Author's Comments

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This problem was written to provide high school student teams a chance to make a significant contribution to the discussion about an ongoing national issue, using only high school mathematics. It was deliberately open-ended to allow both the creativity of the student and the widest possible set of mathematical techniques to come to bear.

As the author, I imagined that an Outstanding paper would have several characteristics. First, and most importantly, it would address all of the requirements identified in the problem statement. In particular, it would identify how much carbon dioxide needed to be removed, propose one or more approaches, show that they were feasible and effective, determine the costs, and address minimizing the impact. And, as particularly required, it would have a well-written, persuasive, short summary letter for Congress.

I imagined that a variety of approaches were possible, and that some of them were capacity constrained. I thought a "greedy algorithm" would be the easiest and likely best student approach. That is, each possible removal option had a cost per metric ton for removing carbon dioxide and an upper limit on how much the method could reasonably handle. For example, forestry approaches had a cost per metric ton removed that could be estimated, and an upper bound on how many acres could be planted and irrigated. Algae biofuel methods, as another example, had net costs and upper bounds as to acreage and water available. And so on. Most of the options that had capacity constraints, and a submission that did not address this, usually had difficulty showing that the approach was feasible.

A reasonable approach would be to find the lowest cost method and select as much as available, then the next lowest cost method, and so on, until the appropriate amount of carbon had been removed. This also had the advantage of allowing simple models to be used in a powerful way.

As I read the submissions, I was struck by how many of them failed to include the short summary paper to Congress. Such a submission was automatically excluded from consideration as an Outstanding paper. The lesson learned for subsequent teams is not a new one, but it bears repeating: Read the problem! If the problem statement asks for a letter or report to summarize your results, make sure you include one!

The paper that was chosen National Outstanding followed an excellent, straightforward approach. First, it estimated the amount of carbon dioxide that needed to be removed, above what was currently removed, to obtain neutrality. It considered four options for sequestration of carbon, and provided plausible cost estimates and capacity limits for each. The team determined a solution and provided a very well-written letter to Congress. The paper was well illustrated.

The mathematical techniques chosen were employed correctly and appropriate to the problem. The team did not try to artificially force an inappropriate technique into their paper.

This paper is an excellent example of the power of simple, well-thought-out mathematical models for analysis of complicated problems. This team, from the Ellis School in Pittsburg, Pennsylvania, is commended for their excellent work under time pressure. Their parents and teachers can be very proud of the quality of thought displayed: The students showed that they have received an excellent education.

Problem A Summary: Hong Kong International School

Advisor: William Stork

Team Members: Larry Au, Justin Hui Bon Hoa, Stephen Suen, Gillian Yi Chun Tay

A dire financial crisis has broken out on both Wall Street and Main Street U.S.A.—but what of the one on Pennsylvania Avenue? In this paper, we create a simplified model for national debt, $d_t = d_{(t-1)} * (1 + s_t) * (1 + f_t) + (e_t - i_t)$, where d is national debt, t is time, s is interest rate, f is rate of inflation, e is federal expenditures, and i is federal income. Not only does our equation factor in federal expenditures and income, but it also takes into consideration the rate of inflation as well as the interest rate levied on national debt. By using this model, we analyze the opposing economic policies of President-Elect Barack Obama and Senator John McCain, and propose a plan of our own that integrates the best of both.

Problem A Summary: Illinois Mathematics and Science Academy

Advisor: Steve Condie

Team Members: Jessica Durden, Bohao Liu, Ilya Nepomnyashchiy, Justin Troyka

Our objective is to mathematically model the behavior of the United States national public debt over the next eight years. We need to predict the value of the national debt for each year between 2009 and 2017 and be able to adjust our prediction based on different circumstances. Our model accounts for the change in national public debt based on various factors, such as government spending, previous debt, national debt interest rate, and the gross domestic product (GDP).

From our general model, we can create several extensions, allowing us to vary the parameters from which it is comprised. Different values for each parameter allow us to develop differential equations that model the change in the debt year-to-year. Since the data is based on yearly figures, we use Euler's Method to solve these equations numerically. The different extensions that we create come from varying the interest and GDP growth rates. In addition, we develop two different spending policies, which we call the "Eisenhower plan" and the "Reagan plan." The Eisenhower plan represents federal expenditures that are 18% of GDP, while the Reagan plan represents federal expenditures that are 22.4% of GDP. These two plans are demonstrative of the administrations with lowest and highest spending as a percentage of the GDP on average since World War II.

Our results show that the most efficient combination for the reduction of the national debt is low interest, high GDP growth rate, and low spending. Since it is impossible to directly control the GDP growth and interest rates, we focus on these as market circumstances upon which to build the extensions of our model, and see what effect the different spending plans have on our forecast. Our model shows that, in order for the national debt as a percentage of GDP to be lowered, the federal government must adopt a spending plan with a low expenditure during an economic situation that avoids unfavorable extrema in GDP growth and interest rates.

Problem A Paper: Maggie Walker Governor's School

Advisor: John Barnes

Team Members: Ziyi Gao, William Shimer, Wilson Weber, Adam Zedler

GENERAL MODEL VARIABLES

A fundamental equation for calculating national debt is $\Omega = E - I + D$ where Ω is national debt, $\mathbf{E}$ is federal expenditures, $\mathbf{I}$ is federal receipts (income), and $\mathbf{S}$ is non-budgeted spending.

The sub-formula for expenditure is $\mathbf{E} = \mathbf{D} + \mathbf{R_H} + \mathbf{R_P} + \mathbf{Q_E} + \boldsymbol{\beta}$. These five variables represent the major areas of federal spending according to the fiscal year 2009 budget published by the Office of Management and Budget (OMB):

1. National defense ($\mathbf{D}$) such as general military spending, excluding specific conflicts (e.g., war in Iraq);
2. Human resources ($\mathbf{R_H}$), including education, health care, Medicare, income security, social security, and veterans' benefits;
3. Physical resources ($\mathbf{R_P}$), such as energy, natural resources, transportation, and community/regional development;
4. Other ($\mathbf{Q_E}$), including international aid, general research grants, agriculture, and general government costs or allowances;
5. Interest on debt ($\boldsymbol{\beta}$), the amount spent paying off accumulated national debt.

Annual federal income is modeled by $\mathbf{I} = \mathbf{T_I} + \mathbf{T_C} + \mathbf{T_E} + \mathbf{T_W} + \mathbf{Q_I}$. Again, these five variables come directly from the fiscal year 2009 budget published by the OMB:

1. Income Taxes ($\mathbf{T_I}$) from individuals;
2. Corporate Taxes ($\mathbf{T_C}$);
3. Excise Taxes ($\mathbf{T_E}$), such as on cigarettes or gasoline;
4. Welfare Taxes ($\mathbf{T_W}$), including social security and payroll taxes;
5. Other ($\mathbf{Q_I}$), such as estate and gift taxes, customs duties, and Federal Reserve deposits.

Despite sophisticated forecasting models, the government cannot accurately predict every expense. For example, catastrophes, both natural and unnatural, are not factored into the annual budget; rather, the government passes emergency spending bills, such as the recent $700 billion rescue package. We included these non-budgeted spending items in our model as $\mathbf{S}$.

ASSUMPTIONS

We assume each plan is enacted immediately and completely. We also assume tax rates remain constant between 2009 and 2017. In addition, changes to the tax code take place instantaneously at the beginning of a fiscal year. All of our models began in 2007 since we did not have complete tax data for the fiscal year 2008. Also, our 2008 predicted figures for debt underestimate the actual debt (~$8.6 trillion vs. >$10.6 trillion) because we did not include non-budgeted expenses, which have been particularly high this year.

We assume no significant crises occur during our model time. Although we include a variable, $\mathbf{S}$, to account for the unforeseen disasters, we make this 0 because crises are too difficult to predict. Therefore, we create significant type II error. However, if calamities should occur, we could add their effects to the national debt figures of that year.

We assume the Alternative Minimum Tax (AMT) is not abolished by 2017. Moreover, the population in tax brackets affects the amount of collected taxes and the national debt. We assume the average growth rate of households for the past five years accurately projects how households will grow in 2009–2017. For all models, we assume the percentage of households in each tax bracket remains constant. For simplicity, all households are classified as married, filling jointly. We assume all federal expenditures are in one of the above five expenditure categories. In addition, we assume all federal receipts can be placed into one of the above five income categories. Furthermore, we assume future GDP, inflation, and national debt interest rates can be predicted based on previous data. Finally, we assume that funding for certain programs, such as entitlements, cannot be changed in the federal budget.

MODEL JUSTIFICATIONS AND SOURCES

We calculated Gross Domestic Product (GDP), national inflation rate, and national debt interest rate based on past data. Over the last 50 years, GDP has increased at an average annual rate of 2.70%; so we used 2.70%. We found the average inflation over 1999–2007 and applied it to the next 10 years. In every model, all of our data are adjusted for inflation. We found the national debt by year from the Treasury Department and the amount paid towards interest from the Office of Management and Budget for 2004–2007. Then, we calculated the national debt interest rate by dividing the debt paid in a given year by total national debt ($I_R = \frac{I_{T-1}}{\Psi_{T-1}}$), and applied it at a flat rate over the ten years. These two values are the same, so it appears the government pays off the debt interest at approximately the same rate as inflation.

We found data from the previous four years for all our federal income and expenditure variables, excluding debt interest, individual income tax, and corporate tax, from the OMB's 2009 Fiscal Year report. Based on these data, we modeled current government policies through 2017. For a second model, we examined a plan from the 2008 Presidential Election. Although these policies were stated vaguely in the campaign, we did find some numbers from sources with a variety of biases on major issues, including health care, energy, defense, and taxes.

Our most difficult task was determining the population spread over the six tax brackets. We found data about tax brackets for couples filling jointly and the number of households by income, but the two were incompatible. Therefore, we manipulated the data to get a percentage of households in each tax bracket, which we assumed constant over time. We used $50,000 income intervals and found a number of households in each. Then, we calculated an exponential regression function, $y = 114604e^{-2E-5x}$, where x is income and y is number of households in thousands. This function models the data well ($r^2 = 0.9874$). (See **Figure 1**.) We computed the approximate number of households in each tax bracket using the integral of the function within each bracket and divided it by the total to get a percentage breakdown of the tax brackets. Unfortunately, the exponential function slightly underestimated the last two brackets. Therefore, when we discovered the percentages did not add up to 100, we split the difference, 1.6%, equally between the top two groups.

We projected the increase in households based on changes in 2000–2007. We calculated the average increase in households between each year divided by the previous year (e.g., $H_R = \frac{H_t - H_{t-1}}{H_{t-1}}$) and averaged these seven numbers. We applied this value to 2007 and assumed households would grow at the same rate for 2009–2017. For the average tax figure, we multiplied the mean of each tax bracket by its corresponding tax rate. Finally, we multiplied the number of households for each year by the average tax rate per bracket.

We employed both the population spread described above and specific tax policies to model individual income tax over time. Although most people paid the same income taxes in the "status quo" and "change" models, the "change" plan significantly raised tax rates in the top two brackets. We got corporate tax rates by finding the average annual change for 2000–2007 ($T_C = \frac{\Sigma \Delta T_C (2000-2007)}{7}$). We found no significant differences between these two models for corporate tax rates.

MODEL DESCRIPTIONS

Model #1: "Status Quo"

We assumed the current tax and spending policies remain constant for the next nine years. This model generates a surplus for the first three years. However, by 2010, it produces a deficit, and the cumulative national debt (CND) increases every year. This model would create national debt in excess of $12.2 trillion by 2017.

Model #2: "Change"

We factored in a somewhat significant shift in both spending and tax policy. Not only do taxes on the wealthiest individuals go up, but also expenditures increase considerably in the human resource (health care, education, income security) and physical resource (energy, natural resources) subcategories. This plan creates surpluses in the first two years; however, the budget nosedives into billions of dollars of annual deficits. Again, the CND always increases, but tops $14.2 trillion in 2017.

Model #3: Our Plan

We strove for national surpluses in each fiscal year. The foundation of our policy is raising taxes by 2% across the board while simultaneously capping human resources spending at 2007 levels. Through this plan, we achieve our goal of surpluses every year and lower the CND to under $5.5 trillion in 2017.

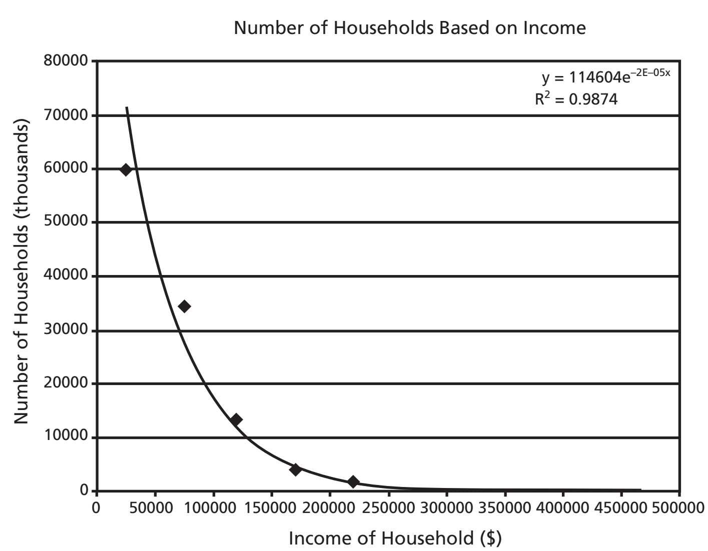

Figure 1.

ANALYZING THE MODELS

We evaluated the models in three ways. First, we examined how effectively they reduce overall national debt by calculating final CND per capita, $\frac{\Psi}{H}$. This ratio reflects the "neighborhood impact" of national debt because it directly connects the national debt to individual households. Thus, it takes a difficult-to-grasp, theoretical concept and turns it into a tangible number. Second, we divided final CND by the final projected GDP, $\frac{\Psi}{G}$, for comparison with previous Presidential administrations. Relating CND to GDP roughly replicates the national effect of each model because it predicts how much debt weighs down on the economy. A high ratio signifies a weak economy, and a low ratio occurs during growth. Finally, we ranked the three models according to our group's simple multi-attribute rating technique (SMART). This system, explained below, increases the overall value of our analysis because it judges the desirability of the models derived from individual tastes.

Currently, $\frac{\Psi}{G}$ is at the highest since 1955 during the Eisenhower administration. Under the "status quo" plan, this ratio would slightly increase for the first two years before descending again, and, by 2017, would be 67.7%. Although this is lower than the post-World War II era, it is above the ratio during the Johnson, Nixon, Ford, Carter, and Reagan administrations. Consequently, this model fits somewhere between Clinton/Bush, Sr. and Truman. In the "change" model, $\frac{\Psi}{G}$ starts at the same rate as the "status quo" model, but the end value is much larger as spending overtakes the initial income spurt. The final value for 2017 is 78.86%: much higher than the "status quo" model and approaching the 80% realm, not seen since Truman's first term.

For the "status quo" and change" models, $\frac{\Psi}{H}$ steadily becomes more negative. In the "status quo" model, the value changes from –72.18 to –91.75 (in thousands of dollars). The "change" model is similar: –72.14 escalates to –106.80. These numbers drive home the impact of government spending and tax policies on average citizens. Most American families cannot afford to pay $91,750 or $106,800 in addition to their tax burdens.

Our model is very different from the other two. We compensated for the loss of funding with optional privatization of social security. Each annual budget has a surplus. $\frac{\Psi}{G}$ decreases from the initial 60.06%, to 30.22% and passes Jimmy Carter for the lowest in history. Likewise, $\frac{\Psi}{H}$ decreases from –71.17 to –40.93.

The key difference among these models is the first two are currently on the political table while the third is hypothetical. Unfortunately, both the "status quo" and the "change" models worsen the economic situation in the near future. Our model breaks sharply with the other two. It is the only one to balance the budget every year, and it performs best in our SMART analysis.

SIMPLE MULTI-ATTRIBUTE RATING TECHNIQUE (SMART)

SMART rates the characteristics of models on a scale from 1 to 10 based upon a person's preferences; theoretically, such a system evaluates "how ideal" a given model is for an individual. We selected five variables: a balanced budget, tax ratios among the income tax brackets, defense spending, non-defense spending, and income to corporate tax ratio. SMART is especially practical because it allows politicians and citizens of differing viewpoints to see if a budget corresponds to their ideological beliefs. Thus, this model is versatile because it can be adapted to a variety of circumstances. Nevertheless, SMART does have some built-in parameters because some policy changes, like lowering corporate taxes to 2% or spending 100% of the budget on non-defense programs, are either economically or politically infeasible (or both). Most legitimate policies would fall in the middle: somewhere in the 3 to 7 range. Barring unrealistic weighting, policies outside this range might be difficult or impossible to implement without raising public discontent.

We thought balanced budget most important because it is the main objective of the given problem. A balanced budget receives a score of 10, and a deficit of 1 trillion dollars receives a 1. Anything falling in between gets a scaled score.

The tax ratio, or the tax rate of the highest bracket over the tax rate of the lowest bracket, reflects how an individual wants the tax system to work. A liberal viewpoint would want this ratio high, and a conservative stance would favor a low number. We considered a low ratio, closer to 1:1, positive. We decided that the highest feasible ratio is 5:1, and the lowest feasible ratio is 1:1. A ratio of 1:1 receives a score of 10, and a ratio of 5:1 receives a 1, with all other values scaled in between. We recognize some would disagree with us and think a high ratio better. They could simply flip the scoring system (i.e., the 10 to 5:1 and the 1 to 1:1).

The defense spending and non-defense, or domestic, spending are ratios of their respective budgets to the total discretionary expenditures (these exclude mandatory spending programs such as entitlements). We put each of these factors on a percentage scale with 100% receiving 10 points and 0% receiving 1 point.

The final variable is the ratio of individual income tax receipts to corporate tax intake. This value usually does not fluctuate much; however, we used a maximum value of 10:1 and a minimum of 1:1 and assigned points on a similar scale to the previous ratios.

Someone using our SMART system could easily change the weighting structure. First, a person assigns values to each weighted category, and those values are totaled. Each weighting is divided by the total to generate the weight ratio. Then, each variable is given a score based on an established scale. This ratio is multiplied by the score for each variable, and the totals for the variables are added. Our assigned numbers for each category are 95 (for balanced budget), 35 (for tax ratios), 50 (for defense spending), 70 (non-defense spending), and 5 (for income to corporate tax rate). The maximum possible score is 10, and under our weighting system, we got 6.414 for "status quo," 4.279 for "change," and 6.842 for our model.

SENSITIVITY ANALYSIS

Sensitivity measures how changes in each variable affect national debt. This measurement is crucial in determining what variables can be changed without significantly altering the fundamental model. Our group used variables that can be manipulated by the government and evaluated their sensitivities based upon percentage changes in the 2007 data. We also assumed that the initial variables could only be manipulated to within 20% of the 2007 data because values beyond that are not politically viable. Furthermore, sensitivity is one method of assessing the robustness of our model. To make a model applicable in many situations, we had to find how sensitive each variable is.

We calculated sensitivities using the absolute value of the slope based on national debt over the percent change in each variable. First, we used step sizes of 5% to find all the initial test values within the 80%–120% range. Thus, we were able to project the national debt for all variable changes, graphed over their percent change. The slopes of the graphs represent the final national debt's sensitivity to each variable.

Income taxes posted the most significant sensitivity because almost half of government receipts come from them. (See **Figure 2**.) Due to the large number of households taxed, changes in the income tax have a considerable ripple effect on national debt. National debt is also moderately susceptible to fluctuations in human resource expenditures. Since programs such as Medicare and Social Security take up about 62.72% of total expenditures, the human resources category has a greater sensitivity value than others, except income tax.

CONCLUSION

We applied data from the last four years to predict national debt for 2009–2017. National debt will rise from $8.429 trillion to $12.239 trillion, and the ratio of national debt to GDP will increase to 67.75% from 60.90%. Assuming these trends hold true, the United States economy will continue to weaken. Therefore, we strongly advise a reform in economic policy.

We also applied data from a popular proposal for economic change. This plan called for raising taxes on the top two federal tax brackets, as well as increasing total federal expenditures. However, since tax revenue will increase more than expenditures, national debt will still increase. If this budget were enacted, it would increase debt from $8.425 trillion to $14.247 trillion. Moreover, the ratio of national debt to GDP would increase from 60.87% to 78.86%. Thus, this proposal should not be implemented.

We created a third option. This new plan would create a surplus every year and decrease total federal debt from $8.312 trillion in 2007 to $5.460 trillion in 2017. The ratio of national debt to GDP also would decrease from 60.06% to 30.22%. These declines would have a positive effect on the economy. This proposal consists of enacting an increase of 2% in all tax brackets. Separately, we would cap human resources spending at 2007 levels while simultaneously allowing optional privatization of social security to defray the loss of revenue.

Another key aspect of our model is a weighted rating system called SMART. SMART uses a system of weighted equations based on multiplying weights, set by the user on each criterion, with the score, designated for each plan, based on how well the plan achieves each standard. The SMART rating for the model of projection based on current data is 6.414/10 while the widely supported plan received 4.279/10. Our plan achieved the highest score: 6.842.

Moreover, we analyzed sensitivity by changing one variable at a time while keeping the others constant and observing the change on the national debt in 2017. Then we graphed the national debt over the percent change to determine the sensitivity of the variable. This sensitivity allows users of the model to understand how changing each variable affects final national debt relative to the other variables.

Problem B Paper: The Ellis School

Adviser: Amy Yam

Team members: Heather Acuff, Rebecca Adelsheim, Chao Long, Xiaoyue Zhang

I. INTRODUCTION

Problem Restatement:

Due to global warming, its universal implications, and the large extent to which the United States has exacerbated this crisis, the U.S. has become increasingly conscious of its carbon footprint. It has taken actions that decrease carbon dioxide (CO_2) emissions and increase CO_2 consumption, also known as carbon sequestration. Our objective is to create a model that increases carbon sequestration to the point at which carbon neutrality is reached, assuming that the amount of emissions is indefinitely limited to that of 2007–2008. Additionally, our objective is to discover if neutrality is feasible in terms of cost, cultural impact, and effectiveness.

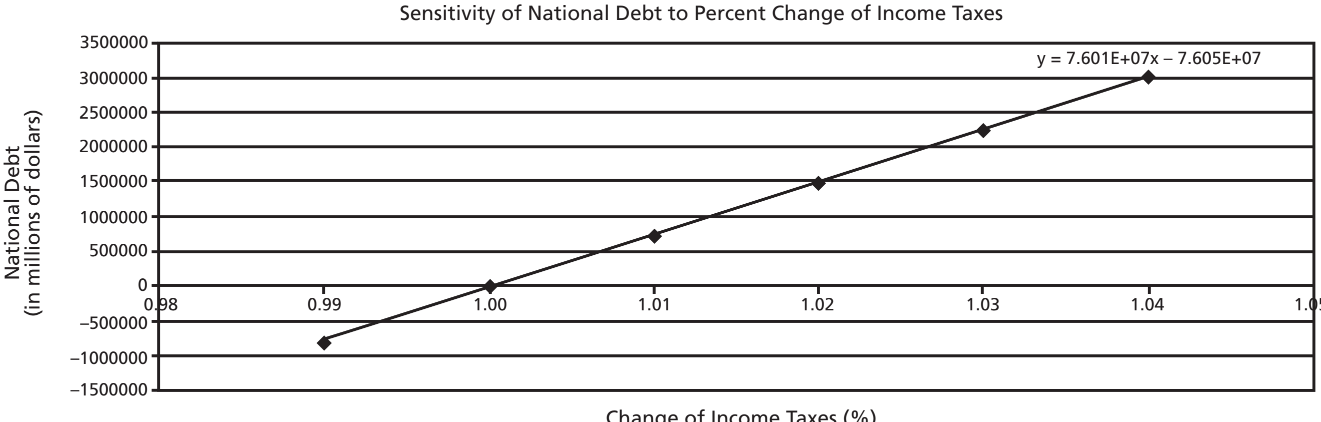

Figure 2.

Variables

Our independent variable: the method used to consume CO_2. Our dependent variable: the amount of CO_2 consumed (in million metric tons (mmt)).

II. ASSUMPTIONS

(There are other assumptions specific to parts of our model.)

1. All CO_2 produced by power plants is available for use.
2. Unless otherwise specified, consumed CO_2 is in Standard Temperature and Pressure.
3. Atmospheric CO_2 is stable, despite increased CO_2 sequestration, due to CO_2 sinks such as oceans.
4. All citizens, especially farmers and power plant owners, comply with the government's requests for change.

III. TERMINOLOGY

- Biofuel: fuel that is derived from renewable biological resources.
- Power plant emissions: CO_2 emissions that can be controlled and harnessed.
- Atmospheric emissions: other CO_2 emissions that enter the atmosphere (i.e., car exhaust, forest fires, human respiration).

IV. REASONING AND MODEL

Our model has four sub-models that implement various methods of carbon sequestration.

In 2007, humans emitted 5984 mmt of CO_2. The biosphere absorbs approximately one third, or (5984 mmt)(1/3) = 1995 mmt. Since (5984 mmt – 1995 mmt) = 3989 mmt, we develop a model that consumes 3989 mmt of CO_2.

Of the 3989 mmt, power plants emitted 2540 mmt. Thus, 1449 mmt are atmospheric emissions. It is necessary to differentiate between the two different types because some carbon sequestration methods use CO_2 from power plants, and others use CO_2 from the atmosphere.

First Sub-Model: Terrestrial Sequestration

Our first sub-model consists of two methods of carbon sequestration: planting trees and implementing land management practices that increase the amount of CO_2 that soil can absorb.

Assumption for the first method: The cost of water to sustain hardwood trees is paid by individuals, not the government.

Trees consume CO_2 via the Calvin Cycle of photosynthesis. We chose hardwood trees for several reasons. First, they grow relatively fast, thus consuming CO_2 rapidly. Second, they are relatively dense, so they can store a lot of CO_2 per cubic meter. Third, they offer a range of sizes, and can thus be planted in a variety of settings. The average hardwood tree consumes 48 lbs of CO_2 per year, and takes up an average of 60 square feet. Finally, they are low maintenance.

Next, we decided to plant trees in two settings: Brownfield lands, which are arable lands that are currently unused, and in yards of homes.

There are 4,250,000 acres of Brownfield land in the U.S. We converted acres to square feet: (4,250,000 acres)(43,560 square feet/acre) = 1.85×10^{11} ft^2. We calculated the number of hardwood trees that can be planted in Brownfield lands: (1.85×10^{11} ft^2)(1 tree/60 ft^2) = 3,085,500,000 trees. Each tree consumes an average of 0.02177 metric tons (48 lbs) of CO_2 per year, so: (3,085,500,000 trees)(0.02177 metric tons) = 67,171,335 metric tons of CO_2.

The average land area of a home is 9100 square feet. The average house is 2349 square feet, so: 9100 ft^2 – 2349 ft^2 = 6751 ft^2 = the average yard. This may seem large, but it's an average that includes homes with no yard and farms. Again, since each tree requires 60 square feet, (6751 ft^2)(1 tree/60 ft^2) = 112.5 trees can be planted per home. Since most yards would already have a few trees and a portion of yard space may be infertile, we arbitrarily halved this number: 112.5 trees/2 = 56.3 trees, which we rounded to 56 trees. Since there are 126,316,187 homes in the U.S., (126,316,187 homes)(56 trees/home) = 7,073,706,472 hardwood trees planted in yards. So (7,073,706,472 trees)(0.02177 metric tons) = 153,994,590 metric tons of CO_2 consumed.

Thus, 67,171,335 metric tons of CO_2 + 153,994,590 metric tons of CO_2 = 221,165,925 metric tons of CO_2 consumed through our first sub-model.

We found that a hardwood seed costs about \$0.20. Thus, (3,085,500,000 trees + 7,073,706,472 trees)(\$0.20/tree) = \$2,031,841,294 to plant all the trees.

The second method applies land management practices to increase soil productivity in terms of CO_2 absorption. The first agricultural option is no-till or reduced-till farming, which means not turning the soil before seeds are planted. By not disrupting the soil, previously stored CO_2 is not released into the atmosphere and more CO_2 can sink into the soil. This method sequesters carbon. It is also beneficial to farmers because it reduces soil degradation, decreases fuel and labor costs, and conserves soil nutrients.

The second agricultural option is crop rotation. Growing multiple types of crops increases carbon content in soil, and it is thus sequestered. This option also benefits farmers because it inhibits insects, disease, and weeds.

The third agricultural option is reducing soil erosion, which can be done in several ways. For example, decreasing summer fallow decreases soil erosion, and thus increases carbon storage potential. The reduction also makes soil moister, which allows for greater carbon storage. Vegetation buffers are designed to minimize soil erosion that leads to the loss of nutrients and carbon, so minimizing soil erosion decreases the amount of stored CO_2. Using a mulch tiller rather than a traditional plow reduces soil erosion by up to 40%. Planting crops perpendicular to a slope decreases soil degradation by blocking wind. Snow fences and waterways are two other methods that hinder erosion.

The fourth agricultural option uses high residue crops. While also controlling erosion, these crops (corn, grain, sorghum, and wheat) have cover that decreases soil density, which facilitates greater carbon absorption. The last agricultural option is winter cover crops, which, like residue crops, protect soil by minimizing weathering.

With these agricultural options, U.S. soils have the capacity to store 380 mmt more than current soils can. The cost to the farmers is negligible.

Second Sub-Model: The Concept Green Freedom

This sub-model assumes that normal transportation motors can use synthetic gasoline.

Green Freedom (GF) is a new concept that consumes atmospheric CO_2 and converts it into methanol and gasoline. Although GF has not yet been implemented, its viability has been verified through research and Los Alamos National Laboratory review. The process has two parts: methanol production and conversion of methanol into gasoline. Conversion uses the Mobil methanol-to-gasoline (MTG) technique. GF plants are more cost-efficient than previous methods of CO_2 conversion because the process uses breakthrough technology that extracts CO_2 from the atmosphere more efficiently. 95% of the CO_2 that passes through a plant is captured, as opposed to 80% captured with old technologies. The energy needed is obtained from nuclear power generated within the system. The energy production system accounts for over 50% of the cost of a GF plant.

We calculated the amount of atmospheric CO_2 consumed by the system per year. Since one GF plant produces 18,000 barrels (bbl) of synthetic gasoline per day, the amount produced in a year is: (18,000 barrels of synthetic gasoline/day)(365 days/year)(159 L/barrel)(0.75 kg/L)(1000 g/kg) = 7.835×10^{11} grams. Convert grams into moles: (7.835×10^{11} g/year) x (1 mol/114.23 g) = 6,858,728,005 moles. It takes 8 moles of CO_2 to produce 1 mole of gasoline (C_8H_{18} is the average molecular formula) since the gasoline's only source of the 8 carbons it needs per molecule is CO_2, so (6,858,728,005 mol C_8H_{18}) x (8 mol CO_2/1 mol C_8H_{18}) = 5.487×10^{10} mol atmospheric CO_2 consumed and converted into synthetic gasoline per year. Finally, convert moles into metric tons (mt): (5.487×10^{10} mol CO_2)(44 g/mol CO_2)(1 kg/1000 g)(1 mt/1000 kg) = 2,414,272 mt atmospheric CO_2 consumed per year by one GF plant. There are 847,400,000 metric tons of CO_2 left in the atmosphere after terrestrial sequestration methods are implemented. Thus, 847,400,000 mt of CO_2/2,414,272 mt CO_2 = 351 GF plants to achieve atmospheric carbon neutrality.

To find profit from selling synthetic gasoline, we had to calculate the cost of production of one gallon. It takes \$0.65 to produce one gallon of methanol, and it takes \$1.40 to produce one gallon of synthetic gasoline from methanol. Since the methanol production part of the plant can produce 5000 metric tons per day (enough to make 18,000 barrels of gasoline), the amount of methanol produced annually is (5000 mt methanol/day)(365 days/year)(1,000,000 g/mt)(1 cm^3/0.795 g)(1 L/1000 cm^3)(0.2642 gal/L) = 606,496,855 gal methanol/year. Thus, methanol production costs: (606,496,855 gal methanol/year)(\$0.65/gal methanol) = \$$3.94222 \times 10^8$/year. Gasoline conversion costs: (18,000 bbl/day)(365 days/year)(42 gal/bbl)(\$1.4/gal gasoline) = \$$3.86316 \times 10^8$/year. So we have (\$$3.94222 \times 10^8$ + \$$3.86316 \times 10^8$) = \$$7.80538 \times 10^8$ = cost to run one GF plant for one year. Thus, the cost to run a plant for one day = (\$$7.80538 \times 10^8$/year)(1 year/365 days) = \$2,138,460.274. A plant makes 18,000 bbl of gasoline a day, which is (18,000 bbl/day)(42 gal/bbl) = 756,000 gal per day. Thus, it costs (\$2,138,460.274/day)/(756,000 gal/day) = \$2.83 to produce a gallon.

Currently, gasoline on the market costs \$3.20 per gallon. Thus, the profit per gallon of synthetic gasoline = \$3.20 – \$2.83 + \$0.37.

The rate at which gas price increases per year is \$0.37, which we obtained by finding the slope of the line of best fit in **Figure 1**. We then turned the profit made from one gallon of synthetic gasoline into a function of time: $p(t)$ = \$0.37 + \$0.37t, where t is the number of years after 2008. Then, we turned this into a profit per year function: [(\$0.37 + \$0.37t)/gal](42 gal/bbl)(18,000 bbl/day)(365 days/year) = (\$0.37+\$0.37t)(\$275,940,000)/year = (\$102,097,800 + \$102,097,800t)/year. (Note that this function does not consider initial capital cost.)

We anti-differentiated $p(t)$ to get a new function $P(t)$ = \$102,097,800t + \$51,048,900t^2 + C, where C is a constant. We then

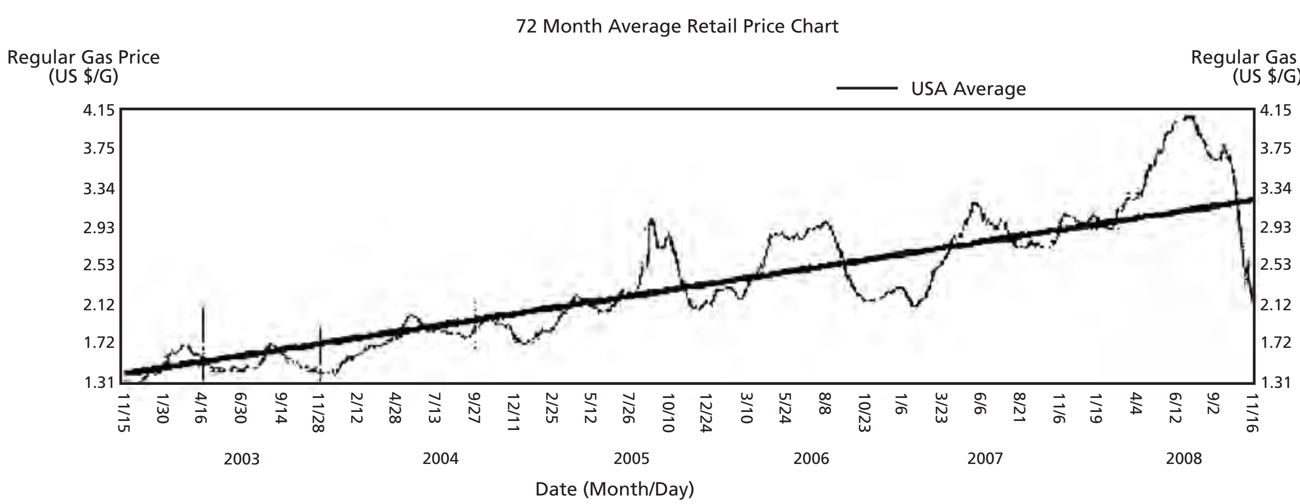

Figure 1.

set the definite integral equal to initial cost, and let x = number of years needed for one GF plant to pay back its initial costs and the costs of running the plant for one year. Thus, the total profit from t = 1 year to $t = x$ years = $\int_1^x p(t)dt = P(x) - P(1)$ = (\$102,097,800x + \$51,048,900x^2 + C) – (\$102,097,800 + \$51,048,900 – C). Since the initial investment for a plant is \$9.6 billion, we set that number equal to the total profit and solved for x: (\$9.6)(10^9) = (\$102,097,800x + \$51,048,900x^2 + C) – (\$102,097,800 + \$51,048,900 – C). We got x = 12.7 years ≈ 13 years = approximate time for a plant to get out of debt.

Thus, atmospheric emissions have been neutralized since (2,414,272 mt atmospheric CO_2 consumed per year by one GF plant + 380 mmt consumed by soil + 221,165,925 mt consumed by trees) = 603,580,197 mt of atmospheric CO_2 consumed by sub-models 1 and 2. Since 1449 mmt CO_2 is emitted into the atmosphere, our sub-models cannot neutralize atmospheric emission in the first year.

Then, we calculated the cost to operate a GF plant. The initial capital to build a sequestering system is \$9.6 billion. The amount of methanol produced per year is (5000 mt methanol/day) (365 days/year)(1,000,000 g/mt)(1 cm^3/0.795 g)(1 L/1000 cm^3) (0.2642 gallon/L) = 606,496,855 gal. Thus, methanol production costs: (606,496,855 gal methanol/year)(\$0.65/gal methanol) = \$3.94222 x 10^8/year. Gasoline conversion costs: (18,000 bbl/day)(365 days/year)(42 gal/bbl)(\$1.4/gal gasoline) = \$3.86316 x 10^8/year. Adding the two costs, we have get \$7.80538 x 10^8 for the annual cost to maintain and run one GF plant. Finally, \$7.80538 x 10^8 + \$9.6 billion = amount a GF plant needs to borrow from the government. The company will repay the government with its profits.

For our sub-model, \$9.6 x 10^{10} is invested to build ten GF plants. Profit from these plants is then invested to construct a new GF plant whenever possible. For example, the first new GF plant after can be constructed in 2012 because $\int_1^4 f(t)dt > \$9.6 \times 10^{10}$.

Third Sub-Model: Building Algae-Based Biodiesel Factories

We assume that all biodiesel created is sold at current average market price, which maximizes revenue. We also assume that excess heat from a power plant supplies the basic algae mechanisms with energy and thus the annual operating cost is negligible.

All plants consume CO_2, but release all of it when they die. However, these plants can be used to produce biofuel before they die. These biofuels are neutral in terms of CO_2 emission because the amount of CO_2 released when burning them is balanced by the CO_2 absorbed during the production of the biofuel through the biomass. This is better than petroleum, which does not absorb CO_2 despite its heavy emission.

Algae are the most efficient in producing biofuels; an acre of algae ponds can produce 15,000 gallons of biodiesel and 25,000 gallons of ethanol annually. Algae consume more CO_2 than other plants used for biodiesel production, such as corn, palm, soybeans, or rapeseeds.

There are about 1000 power plants in the U.S. that have space for algae ponds. Those power plants have, on average, 1500 unused acres. Thus, (1000 power plants)(1500 unused acres/power plant) = 1,500,000 acres. An acre of algae farm consumes approximately 202.35 mt of CO_2 per year, so (1,500,000 acres)(202.35 mt/acre) = 303,525,000 mt of CO_2 consumed per year.

Each power plant emits, on average, 2.8 billion tons of CO_2. (2.8 billion tons)(0.9072 mt/ton) = 2540 mmt available to be used for algae based biofuel plants. Since there are 1.5 million acres of new algae ponds, and it costs \$40,000 to build one hectare of algae pond, (1.5 million acres)(0.4047 hectares/acre)(\$40,000/hectare) = \$2.428 x 10^{12} = the cost to build 1.5 million acres. The government loans this money to power plants, which eventually repay the government through biofuel sales. After a power plant repays, it keeps the profits. Therefore, building algae plants consumes CO_2 and is a long-term investment for power plants and the government.

An acre of algae pond can yield 15,000 gallons of biodiesel, so (1.5 million acre)(15,000 gallons biodiesel/acre) = 2.25 x 10^{10} gallons produced per year. It costs \$0.59 to convert a gallon of algae to biodiesel, so (2.25 x 10^{10} gallons of biodiesel)(\$0.59/gallon) = \$1.3275 x 10^{10} = the cost, per year to produce biodiesel. Currently, one gallon of biodiesel can be sold for \$1.85, so (2.25 x 10^{10} gallons of biodiesel)(\$1.85/gallon) = \$4.1625 x 10^{10} = the amount earned per year from biodiesel sales. Thus, (\$4.1625 x 10^{10} – \$1.3275 x 10^{10}) = \$2.835 x 10^{10} = profit a power plant can earn. This calculation can be applied to ethanol produced by algae farms. (1.5 million acres)(25,000 gallons ethanol produced/1 acre) = 3.75 x 10^{10} gallons. (3.75 x 10^{10} gallons ethanol)(\$1.50/gallon ethanol) = \$5.625 x 10^{10} = the cost to convert algae to ethanol. (3.75 x 10^{10} gallons ethanol)(\$2.62/gallon ethanol) = \$9.825 x 10^{10} = revenue from selling ethanol. Thus, (\$9.825 x 10^{10} – \$5.625 x 10^{10}) = \$4.200 x 10^{10} = profit a power plant can earn from ethanol. Thus, total profit is \$2.835 x 10^{10} + \$4.200 x 10^{10} = \$7.035 x 10^{10} per year. Hence, \$2.428 x 10^{12}/\$7.035 x 10^{10} = 34.513 = years it takes for a power company to repay.

Fourth Sub-Model: Carbon Capture and Storage (CCS) Into Geologic Formations

Our fourth sub-model involves two types of CCS: CO_2/EOR (Enhanced Oil Recovery) and Enhanced Coalbed Methane Recovery (ECMR). Both involve injecting CO_2 into underground geological formations.

We assume that all coal present in coal seams is of the correct composition and within feasible proximity to a power plant. We also assume that all methane and oil produced is sold at current average market price, which maximizes the revenue.

For our first type of CCS, CO_2 is injected into oil reservoirs that have already had primary and secondary recovery. Primary recovery consists of directly pumping oil up (this is possible due to existent pressure), and secondary recovery consists of injecting an external fluid into a reservoir, which displaces oil, pushing it up and out of the reservoir. In these reservoirs, there is a significant amount of oil that cannot be tapped with primary and secondary methods. Once injected, CO_2 mixes with and expands the oil, increases its viscosity, and thus allows it to flow to a production well. It takes about 8000 cubic feet of CO_2 to bring up one barrel of oil. We converted this to metric tons, under the assumption that the CO_2 gas is at Standard Temperature and Pressure (STP). Since there are 28.3 liters in 1 cubic foot, (8000 ft^3 CO_2)(28.3 L/1 ft^3) = 226,400 L CO_2. The density of CO_2 gas is 1.98 g/L at STP, so (226,400 L CO_2)(1.98 g/L) = 448,272 g = 448.272 kg = 0.448 mt.

The U.S. Department of Energy says that a total of 240 billion barrels of stranded oil can be recovered through EOR. We calculated total possible CO_2 consumption: (240 billion barrels of oil)(0.448 mt CO_2/barrel of oil) = 107.5853 billion metric ton (bmt). The large scale of this number is addressed later.

Overall EOR profit = \$3.563 x 10^{12}.

The second type of CCS uses pipelines from abandoned or unmineable coal seams to sequester CO_2 and increase methane production. CO_2 is pumped from power plants into mines using equally spaced wells and displaces methane in the coal. The methane is collected and sold. The injected CO_2 is converted into carbonate within the coal and permanently stored. We assumed that all unmineable coal seams can be mined with ECMR.

There are 1000 total underground mines, 10% of which are abandoned or unmineable. Thus, there are 100 mines available for ECMR. The average consumption of CO_2 per mine is 15 bmt so (100 mines)(15 bmt CO_2 consumed/mine) = 1500 bmt of CO_2 consumed through ECMR. The large scale of this number is addressed later.

It costs \$40 to consume a ton of CO_2 through ECMR. Thus, the cost of all 1500 bmt is (1500 bmt CO_2)(\$40/ton CO_2) = \$6 x 10^{13}.

Coal can hold about twice as much CO_2 as methane, so (1500 bmt CO_2)(1 part methane/2 parts CO_2) = 750 bmt of methane. The retail price of methane is \$6.39 per 1000 ft^3 methane, so the total revenue from methane sales = (\$6.39/1000 ft^3 methane)(35.311 ft^3/m^3)(1 m^3/0.717 kg methane)(1000 kg/mt)(1,000,000,000 mt/bmt)(750 bmt) = \$2.360 x 10^{14}. Thus, total profit from ECMR = (\$2.360 x 10^{14}) – (\$6 x 10^{13})= \$1.76 x 10^{14}.

Our calculations show that ECMR is an effective long-term method of carbon sequestration because it ultimately pays for itself. However, it is difficult to incorporate into our yearly model because these numbers account for the CO_2 consumption of *all* coal seams that can be mined through ECMR. ECMR recovery is not a renewable CO_2 sequestration method because once they have been filled with CO_2, no more can ever be added. In other words, mining all coal seams in one year would consume more CO_2 than is available. Therefore this CO_2 consumption and profit would occur over a span of time.

Since our third sub-model consumes 303,525,000 mt of CO_2 annually, our fourth CCS model must consume (2540 mmt – 303,525,000 mt) = 2236.5 mmt. We found the ratio of EOR and ECMR to the total CO_2 consumption, and then applied those ratios to the necessary annual consumption. We found these ratios to ensure that the EOR and ECMR would stop consuming CO_2 at the same time.

Maximum CO_2 consumed by both EOR and ECMR is (107.64936 bmt + 1500 bmt) = 1607.646 bmt. The ratio of CO_2 consumed through EOR to the total CO_2 consumed through CCS = 107.646 bmt/1607.646 bmt = 0.06696. The ratio of CO_2 consumed through ECMR to the total CO_2 consumed through CCS = 1500 bmt/1607.646 bmt = 0.93304. We found earlier that 2236.5 mmt of power plant CO_2 must be consumed by CCS per year. So, out of that annual amount, EOR consumes (2236.5 mmt)(.0669) = 149.756 mmt/year and ECMR consumes (2236.5 mmt)(.93304) = 2086.744 mmt/year. Thus, carbon from power plants has been neutralized since 149.756 mmt + 2086.744 mmt + 303,525,000 mt = 2542 mmt ≈ amount of CO_2 emitted by factory plants (2540 mmt).

The number of years we can apply EOR is (107.646 bmt CO_2 consumed through EOR)(1000 mmt/bmt)/(149.756 mmt consumed through EOR/year) = 718.81 years. The numbers of years we can apply ECMR is the same: (1500 bmt CO_2 consumed through ECMR)(1000 mmt/bmt)/(2086.744 mmt consumed through ECMR/year) = 718.81 years. Thus, EOR and ECMR can maintain neutrality of power plant emissions for 718.81 years. It is safe to assume that by that time, new CO_2 sequestration methods and alternative fuels will have been developed.

V. ECONOMIC SUMMARY AND ANALYSIS

For all models we assume that it takes one year to get the technology in place, so these costs do not extend to subsequent years.

In summary Tables 1–5, the first sub-model (terrestrial sequestration) is split into two because they have different expenses.

Sub-Model	First Year Expenses of Programs
1. Planting Trees (Terrestrial Sequestration)	\$2,031,841,294
2. Land Management (Terrestrial Sequestration)	\$0
3. Algae-based Biodiesel Factories	\$2,428,000,000,000
4. Carbon Capture and Storage (CCS)	\$367,799,760
5. Green Freedom Technology	\$96,000,000,000
Total:	\$2,526,000,000,000

Sub-Model	Annual Cost (after the first year, not including start costs)
1. Planting Trees (Terrestrial Sequestration)	\$0
2. Land Management (Terrestrial Sequestration)	\$0
3. Algae-based Biodiesel Factories	\$0
4. Carbon Capture and Storage (CCS)	\$367,799,760
5. Green Freedom Technology	\$780,540,000 (for the first 1–3 years, this increases as investment in power plants increases)
Total:	\$1,148,339,760

Sub-Model	Annual Revenue
1. Planting Trees (Terrestrial Sequestration)	$0
2. Land Management (Terrestrial Sequestration)	$0
3. Algae-based Biodiesel Factories	$70,350,000,000 This money will go to the government until the debt has been paid off (34.5 years after establishment).
4. Carbon Capture and Storage (CCS)	$\$3.286 \times 10^{11}$
5. Green Freedom Technology	Changes because revenue is proportional to the changing prices of gasoline
Total:	$\$3.989 \times 10^{11}$

Sub-Model	Net Spending for First Year of Program (= revenue – cost)
1. Planting Trees (Terrestrial Sequestration)	–$2,031,841,294
2. Land Management (Terrestrial Sequestration)	$0
3. Algae-based Biodiesel Factories	–$2,357,650,000,000
4. Carbon Capture and Storage (CCS)	$+ \$3.282 \times 10^{11}$
5. Green Freedom Technology	$-\$1.0125 \times 10^{11}$
Total:	–$2,132,880,000,000

Sub-Model	Annual Net Spending (after first year)
1. Planting Trees (Terrestrial Sequestration)	$0
2. Land Management (Terrestrial Sequestration)	$0
3. Algae-based Biodiesel Factories	There is no annual operation fee for the plant; however, the profits are being given back to the government to repay the debt. + $70,350,000
4. Carbon Capture and Storage (CCS)	$\$3.282 \times 10^{11}$
5. Green Freedom Technology	Changes yearly with profit changes
Total:	$\$3.282 \times 10^{11}$

Current annual budget for environmental spending: $60,590,000,000

With all sub-models in action at the same time:

First year net spending = -2.1338×10^{12}

Debt: $-\$2.0722 \times 10^{12}$

This debt makes our full model unrealistic. Because the largest money sinks and most unstable numbers come from algae factories and Green Freedom Technology, we decided to eliminate them.

New model net spending: $+\$3{,}261 \times 10^{11}$

With this model we not only help the environment, but we contribute money to set aside for new initiatives. Once the surplus grows large enough, we can invest in algae factories and Green Freedom Technology.

VI. SOCIAL IMPACT STATEMENT

While these changes do not introduce a paradigm shift, they do influence the lives of a select group. For example, farmers must make changes to their crops, crop rotations, and farming practices. All Americans are asked to plant trees. However, the overall impact outweighs the slight personal burdens that may ensue. Therefore, this model is socially feasible.

VII. CONCLUSIONS

The total amount of CO_2 consumed by our all four of our sub-models, is 3,165,333,645 mt, as summarized in Table 6.

Sub-Model	CO_2 Consumed per Year (mt)
1. Planting Trees (Terrestrial Sequestration)	221,165,925 mt
2. Land Management (Terrestrial Sequestration)	380,000,000 mt
3. Algae-based Biodiesel Factories	303,525,000 mt
4. Carbon Capture and Storage (CCS)	2,236,500,000 mt
5. Green Freedom Technology	>24,142,720 mt (This will increase over time.)
Total:	3,165,333,645 mt

In our idealized model, we can reach carbon neutrality for both power plant and atmospheric emissions in 35 years.

For our economically feasible model, it is possible to reach carbon neutrality in power plant emissions by expanding the amount of CCS used in carbon sequestration per year. Thus, 2540 mmt/3989 mmt, or 63% of all CO_2 emissions are consumed by using only CCS. Without Green Freedom Technology it is impossible to reach neutrality for atmospheric emissions. With the terrestrial sub-models alone (tree planting and land management practices) 601.165/1449, or 41.49%, of atmospheric emissions are consumed. Overall, without algae and Green Freedom Technology, 78.75% of human CO_2 emissions are consumed. Eventually neutrality is reached as the new forms of technology are added.

VIII. STRENGTHS OF THE MODEL

First Sub-Model: Terrestrial Sequestration

- Minimal social impact results.
- Helps farmers by making the land healthier and more beautiful.
- Does not cost farmers additional money or energy.
- Uses unused space.

Second Sub-Model: Green Freedom Technology

- Achieves a significant amount of atmospheric CO_2.
- The system is carbon neutral.
- Allows the U.S. to become less dependent on fossil fuels.

Third Sub-Model: Building Algae-Based Biodiesel Factories

- The technology is efficient.
- Uses space that would likely not be used for anything else.
- There is no social or cultural impact.

Fourth Sub-model: CCS

- Is a very controlled because it is based on how much CO_2 we need to consume.
- Uses safe and permanent means of storage.
- Pays for itself.

Overall Model

- With sufficient loans, these four sub-models completely neutralize total CO_2 emissions.

IX. WEAKNESSES OF THE MODEL

First Sub-Model: Terrestrial Sequestration

- It is unrealistic that all trees would be planted.
- We do not consider the CO_2 emitted when the trees die.
- We do not consider the effect ecosystem when the trees are added.

Second Sub-Model: Green Freedom Technology

- New technology is introduced, and thus unforeseen problems may arise.
- Requires the Green Freedom plants and government to make long-term investments.

Third Sub-Model: Building Algae-Based Biodiesel Factories

- New technology is introduced, and thus unforeseen problems may arise.
- Requires power plants and the government to make long-term investments.

Fourth Sub-Model: CCS

- There is a limited amount of reservoirs and coal beds that can be used to store CO_2.
- It is unlikely that all unmineable areas can be used for ECMR.
- Assumes that all 240 billion barrels of oil are retrievable.
- Does not consider transportation costs of CO_2.
- Other costs are not taken into account.

Overall model

- The government may not be willing to lend the money needed to implement all sub-models.