Bicycle Club

EXECUTIVE SUMMARY:

In today's world of pollution, high fuel costs, and concerning health issues, the B-cycle Bike Sharing program attempts to give individuals a cheap, convenient, and healthy alternative for short-distance transportation. Using this program, which emphasizes using the bikes for quick trips throughout cities - think do-it-yourself taxicab - members can pick up and drop off bicycles at any automated rental station, which are scattered at various convenient locations around the regions. Several US cities, including Chicago, Des Moines, and Denver, have sponsored pilot bike share programs with notable success.

However, these systems are only in their beginning stages, and the changing populations and predicted increase in membership call for further development. Several challenges must be addressed. As the program grows, requiring more bicycles and rental stations to satisfy the population, we must determine locations for new bike rental kiosks. The optimal number of bikes to have available for use at each station, how many bikes to move to other locations, as well as a system for the timing and logistics of bike movement must also be considered. We have been asked to develop a plan for the program as it continues to grow over the next five years. In the following pages, we have detailed our creation of effective mathematical models which we have used to address the listed considerations. Using the computer programs Geometer's Sketchpad and Stella, in conjunction with extensive research, we have modeled the flow of bicycles throughout the cities based on the current locations of bike rental stations and the number of nearby attractions; with these mathematical models, we have optimized the number of bicycles at each kiosk, modeled the flow as bike usage increases over the years, and suggested locations for new kiosks.

Our model requires the use of a voronoi diagram of the region, dividing the city into portions based on proximity to bike kiosks, as well as the placement of points of interest in the divided regions. We use the percentages of area and destinations to determine how many bikes enter and leave each kiosk at any given time. This has been done using a Stella program, which models the movement of the bikes. We have used the program to optimize the initial number of bikes at each Kiosk, even as the number of bikes and the number of kiosks grows over a span of five years. Our model takes into account many important factors, but is easily adaptable to the situation of any city, regardless of layout, bike use, and location of kiosks.

We have successfully modeled the entire 5-year plan for Des Moines, Iowa, ending with 51 bikes and 7 Kiosk. We completed an initial analysis of Chicago as well. Lastly we have included how to adapt the model for Denver.

RESTATEMENT OF PROBLEM:

In every era, populations face uncertainty about the future. However, the current generation, more than any previous, faces the threats of global warming, pollution, and health concerns. One of the most effective ways to reduce our carbon emissions and increase our health is to switch from using cars to riding bicycles. Several major US cities have implemented bicycle rental programs through which program members can rent cycles from automated kiosks. In today's world of pollution, high fuel costs, and concerning health issues, the Bcycle Bike Sharing program attempts to give individuals a cheap, convenient, and healthy alternative for short-distance transportation. Using this program, members can pick up and drop off bicycles at any automated rental station, which are scattered at various convenient locations around the regions. Several US cities, including Chicago, Des Moines, and Denver, are starting bike share programs. At each location, the city inhabitants have displayed interest in the program: Chicago's pilot bike sharing program, starting with 100 bikes dispersed among six B-stations around the city, ran from July 30th through November 1st. During the first 17 days, more than 1,500 bicycles were rented and 80 temporary memberships in the pilot program were sold. The six kiosks in Chicago are located at the Museum Campus (which, with 500 rides, has been the busiest rental station), the Buckingham Fountain (with 300 rides), the John Hancock Center (200), the Chicago Park District Administrative Offices (150), the Daley Plaza (150), and the McCormick Place (57). Due to its overwhelming success, the Chicago program will restart on March 31st. As membership increases, the number of bikes will need to increase, as will the number of rental and drop off stations. These new kiosks should be added in locations where they will be most helpful, taking into account attractions such as shopping, museums, theaters, and parks, as well as transportation hubs such as train stations

The Des Moines program has an 18-Bcycle squadron and four stations, which are located at the Brenton Skating Plaza, Principal Park, at Seventh & Grand Avenue, and at Thirteenth & Grand Avenue.

The Denver program currently relies upon 1,000 bikes and almost 50 rental stations for the 554,636 residents living there.

ASSUMPTIONS AND JUSTIFICATIONS:

- **People will use the bike kiosks that are nearest to their current locations.**
  People will not need to walk or drive long distances to rent bikes; they will rent bikes from the kiosks closest to them.

- **People rent bikes at a random rate.** There is no general formula as to exactly when people rent bikes; it changes every day.

- **People will drop bikes off at the kiosks nearest to their final destinations (the final points of interest they visit).**
  People will not have to return the bikes to the same kiosks from which they were

rented. If they do return the bikes to the same kiosk, the values in our model "balance out," in essence negating each other.

- **Once a bike is rented, it will be returned by the end the day.**
  This will be one of the rules of rental. Each bike has its own built-in GPS system, so if bikes are in fact stolen, rental companies can quickly locate them.

- **Each attraction generates the same amount of interest.**
  Admittedly relatively faulty, this assumption allows us to simplistically model the situation. Nevertheless, in future improvements to our model, we can easily weight attractions in our model to accommodate more attractive points of interest.

- **The number of bikes currently in use is sufficient for the city's current bike usage needs.** ie, there is no pressing demand for more bikes, or there is no excess of unused bikes.

- **The parameters of the biking area are limited to the area defined by maps on the B-Cycle website.** People will not bike further than the boundaries the map outlines.

- **Population distribution in each city is equal.**
  For the purposes of our model, an area covering 37% of the city will contain 37% of its inhabitants.

- **In Des Moines, the lake is a popular point of interest.**
  When we added new kiosks to our Des Moines model, we assumed that many people would want to go to the lake, bike around the trails there. Thus, we weighted the lake as 2 points of interest instead of just one.

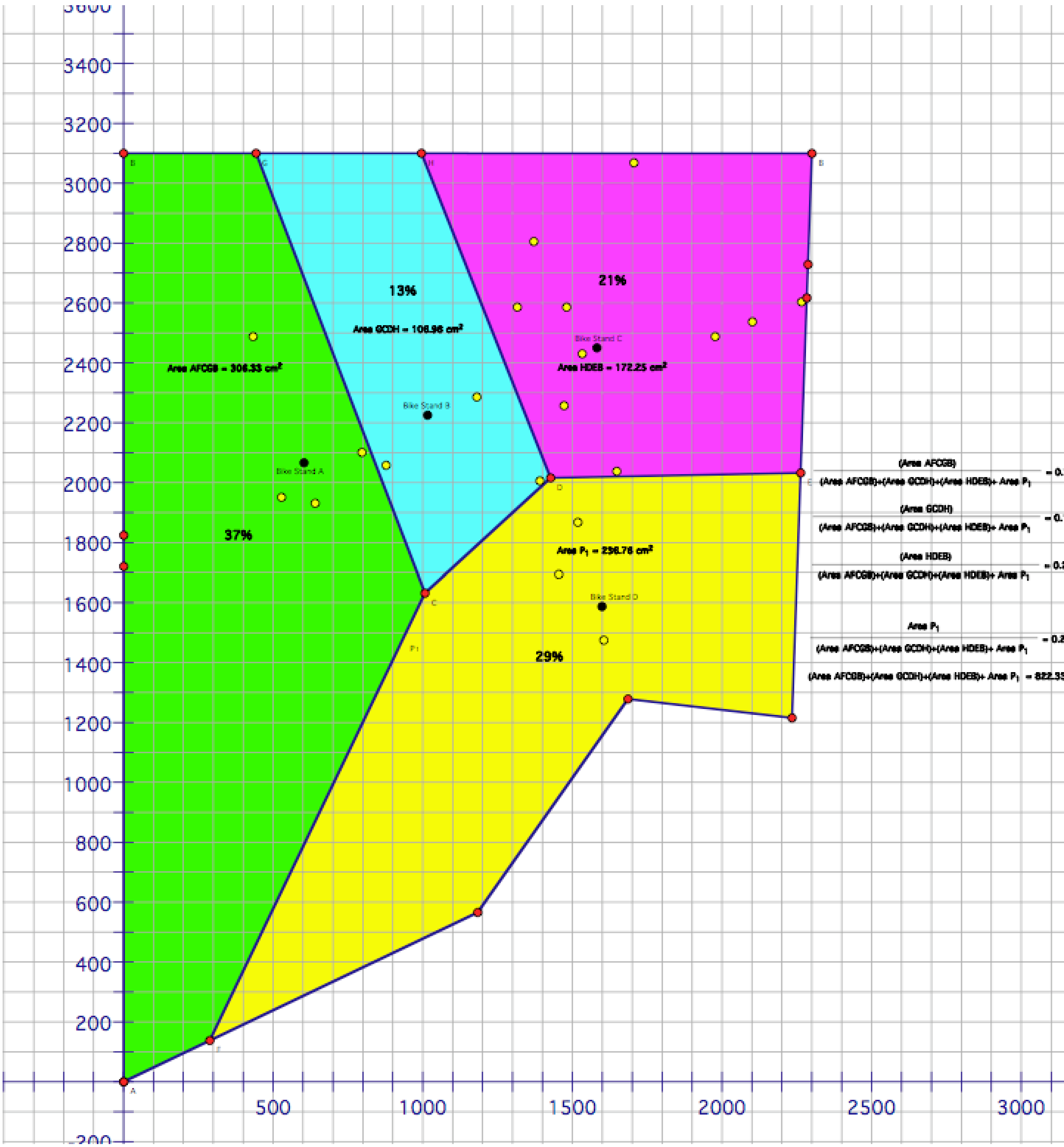
- Though the rental stations open for business at 5 AM, we assume that **B-cycle members will not begin renting and riding the bikes until 8 AM.**

MODEL AND APPLICATIONS:

Des Moines:

In the city of Des Moines, Iowa, the B-Cycle program currently has 4 kiosks. There are 18 bikes in circulation, which per our assumptions, are sufficient for the city's current biking needs. Kiosk A is located at the intersection of 13th and Grand, Kiosk B is located at 7th and Grand, Kiosk C is located at the Brenton Skating Plaza, and Kiosk D is located at principal park. With this information, we created a voronoi diagram to divide the map given on the B-Cycle website into the domains of the various kiosks. We found that the area was divided as follows:

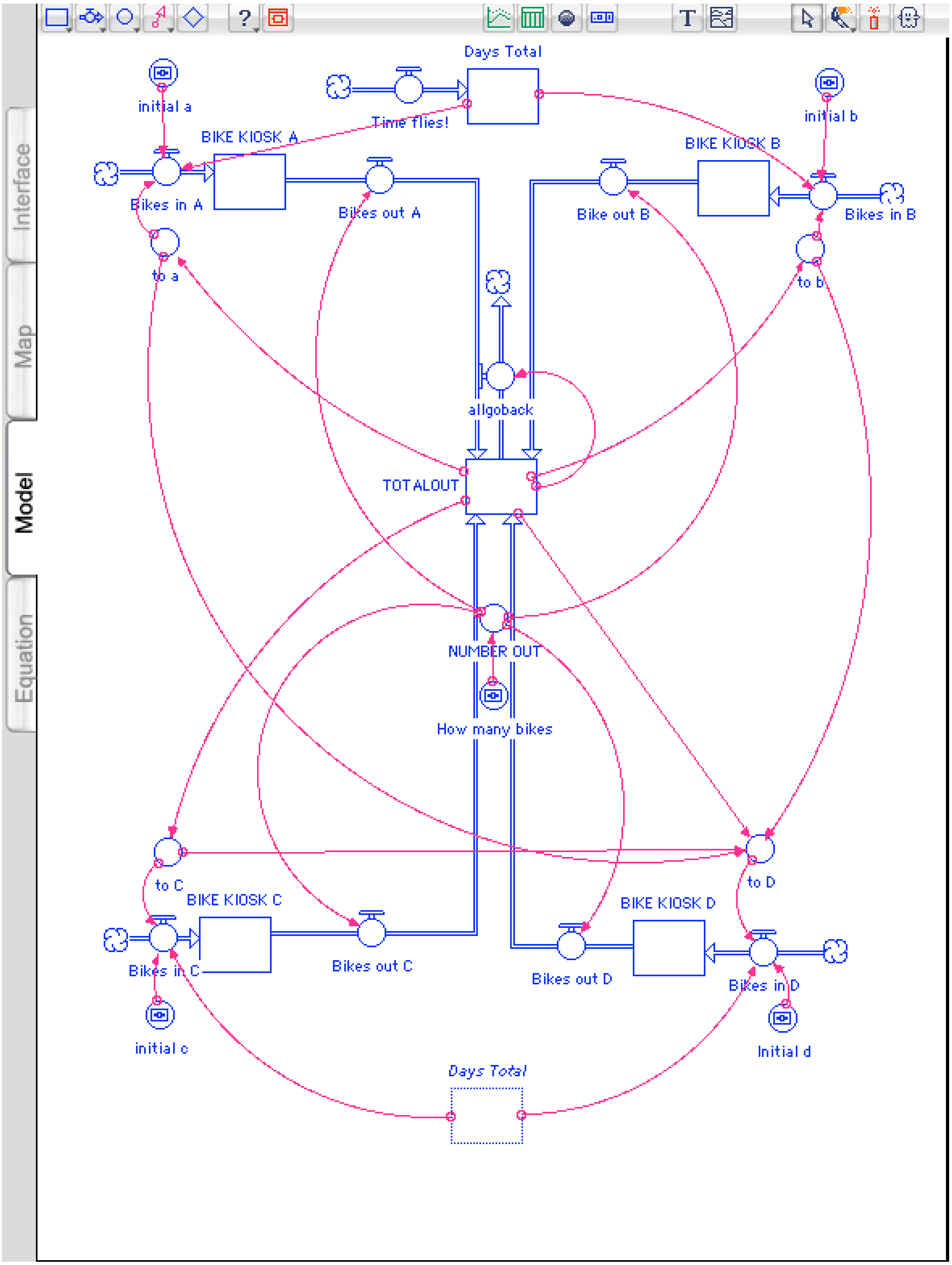
Kiosk A = 37%
Kiosk B = 13%
Kiosk C = 21%
Kiosk D = 29%



Then, to determine where the bikes are left, we calculated how many of the recommended visitation sites for bikers fell within the voronoi regions of each Kiosk. We determined what percentage of sites turned out to be in each area, and figured that those numbers would be equivalent to what proportion of bikes are returned to each Kiosk. Our logic for this is based on the assumption that once someone has visited a point of interest, they will likely leave their bike at the nearest kiosk. These percentages are as follows:

Kiosk A = 20%
Kiosk B = 15%
Kiosk C = 50%
Kiosk D = 15%

With this information, we created a model using the program, Stella.


Interface
Map
Model
Equation
Days Total
Time flies!
initial a
initial b
BIKE KIOSK A
BIKE KIOSK B
Bikes in A
Bikes out A
Bike out B
Bikes in B
to a
to b
allgoback
TOTALOUT
NUMBER OUT
How many bikes
to C
to D
BIKE KIOSK C
BIKE KIOSK D
Bikes in C
Bikes out C
Bikes out D
Bikes in D
initial c
Initial d
Days Total

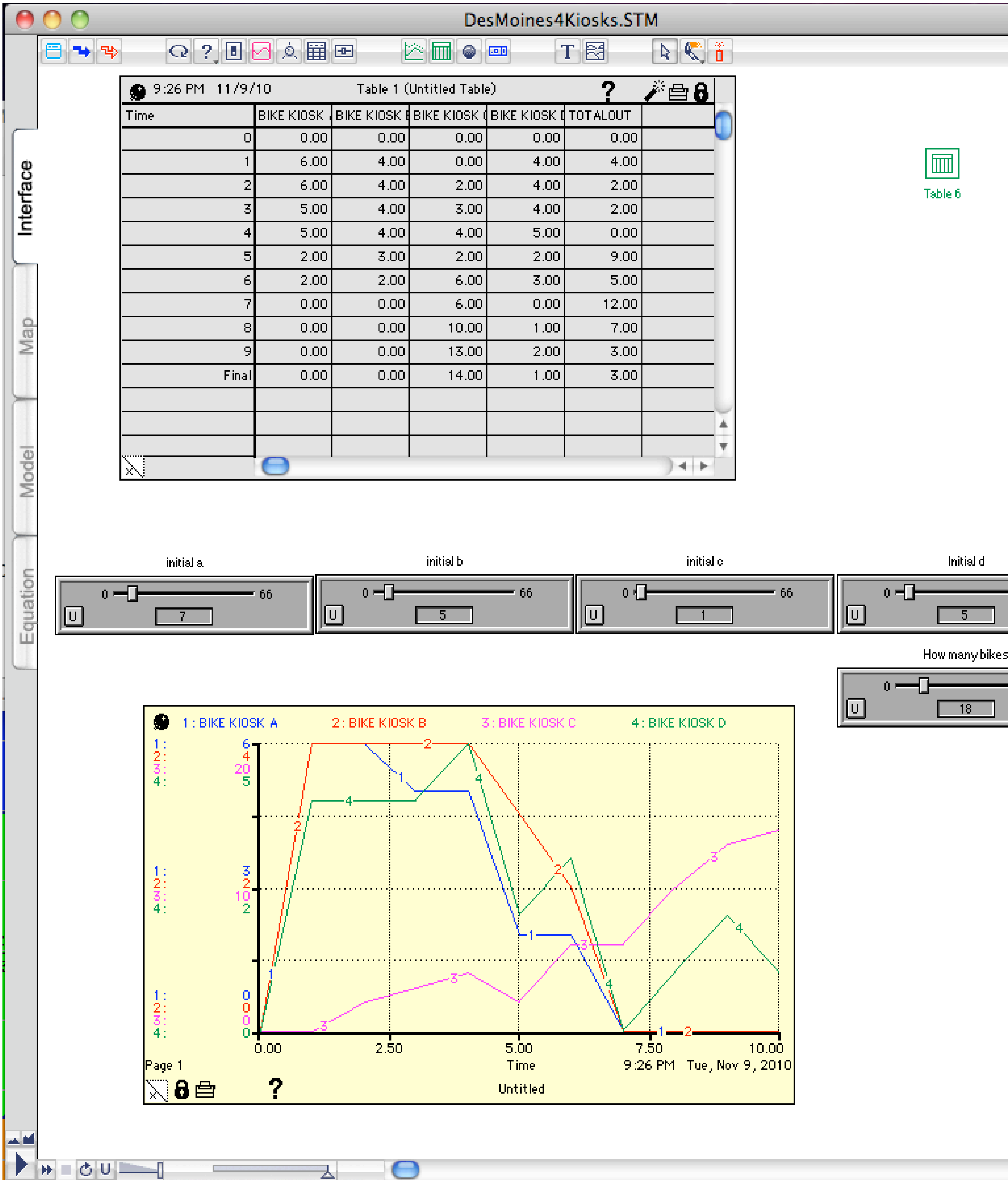
```
BIKE_KIOSK_A(t) = BIKE_KIOSK_A(t - dt) + (Bikes_in_A - Bikes_out_A) * dt
INIT BIKE_KIOSK_A = 0
  INFLOWS:
    Bikes_in_A = IF(Days_Total=0) THEN (initial_a)
    ELSE(to_a)
  OUTFLOWS:
    Bikes_out_A = ROUND(.37*NUMBER_OUT)
BIKE_KIOSK_B(t) = BIKE_KIOSK_B(t - dt) + (Bikes_in_B - Bike_out_B) * dt
INIT BIKE_KIOSK_B = 0
  INFLOWS:
    Bikes_in_B = IF(Days_Total=0) THEN (initial_b)
    ELSE(to_b)
  OUTFLOWS:
    Bike_out_B = ROUND(.13*NUMBER_OUT)
BIKE_KIOSK_C(t) = BIKE_KIOSK_C(t - dt) + (Bikes_in_C - Bikes_out_C) * dt
INIT BIKE_KIOSK_C = 0
  INFLOWS:
    Bikes_in_C = IF(Days_Total=0) THEN (initial_c)
    ELSE(to_C)
  OUTFLOWS:
    Bikes_out_C = ROUND(.21*NUMBER_OUT)
BIKE_KIOSK_D(t) = BIKE_KIOSK_D(t - dt) + (Bikes_in_D - Bikes_out_D) * dt
INIT BIKE_KIOSK_D = 0
  INFLOWS:
    Bikes_in_D = IF(Days_Total=0) THEN (initial_d)
    ELSE(to_D)
  OUTFLOWS:
    Bikes_out_D = ROUND(.29*NUMBER_OUT)
Days_Total(t) = Days_Total(t - dt) + (Time_flies!) * dt
INIT Days_Total = 0
  INFLOWS:
    Time_flies! = 1
TOTALOUT(t) = TOTALOUT(t - dt) + (Bikes_out_A + Bike_out_B + Bikes_out_D + Bikes_out_C - allgoback) *
dt
  INFLOWS:
    Bikes_out_A = ROUND(.37*NUMBER_OUT)
    Bike_out_B = ROUND(.13*NUMBER_OUT)
    Bikes_out_D = ROUND(.29*NUMBER_OUT)
    Bikes_out_C = ROUND(.21*NUMBER_OUT)
  OUTFLOWS:
    allgoback = TOTALOUT
How_many_bikes = 18
initial_a = 4
initial_b = 4
initial_c = 5
Initial_d = 5
NUMBER_OUT = ROUND(RANDOM(0,How_many_bikes))
to_a = ROUND(.2*(TOTALOUT))
to_b = ROUND(.15)*TOTALOUT
```

This model represents the flow of bikes in and out of the kiosks over time. Each station starts with a fixed number of bikes, determined by our group. The number of bikes in use at any point in time is randomly determined as a number between zero and the total number of available bikes. This simulates the fact that not all bikes will be in use at all times – essentially, it allows for variability. Anywhere from no bikes being used to all bikes being used is covered by our random number system. From where these bikes leave is determined by the percentages of area covered by the kiosk (as per our voronoi diagram). The bikes leaving Kiosk A is equal to .37*(number of bikes out). For Kiosk B, the bikes leaving is .13*(number of bikes out). For Kiosk C this is .21*(number of bikes out) and for Kiosk D this is .29*(number of bikes out). All these bikes are then returned after 1 hour. Where the bikes are returned is determined by the percentage of points of interest in the kiosk's area (see above for explanation). The number of bikes out in the previous step is split so that 20% return to Kiosk A, 15% return to Kiosk B, 50% return to Kiosk C, and 15% return to Kiosk D, at which point the number of bikes out has been depleted and the stock is regenerated randomly. Of course, people may keep bikes for more than an hour, but this simply means that one of the bikes taken out can be assumed to have never gone back in the next step . For example, if 6 bikes are out, and the next hour, 8 are out, it is possible that only 5 bikes went back and 7 new bikes came out, but one bike was never returned.

Simplified, the number of bikes leaving is random, but the number of bikes returning will always be equal to the bikes that left during the previous unit of time. We feel that this allows for a certain elegance in our model, keeping it both simple and realistic. It's impossible to know how many people will want to use a bike at any given time, something which we've sidestepped using our random number system.

YEAR 1:

We have used this model to determine what a possible outcome of the B-Cycle setup might be. We have found that most of the bikes tend to accumulate at Kiosk C, which makes sense, as this has half of the points on interest within its domain. Kiosk A, B, and D tend to run out quickly - after 3-5 hours. We have found that optimal starting values for bikes (using the B-Cycle website's value of 18 bikes in usage) are: 7 in A; 5 in B; 1 in C; 5 in D. This runs successfully for 6 hours, sometimes more, without depletion of a Kiosks bike stock. Seeing as most people won't begin using bikes until 8 AM, by 2 PM bikes will have to be shipped back to other Kiosks from Kiosk C everyday.

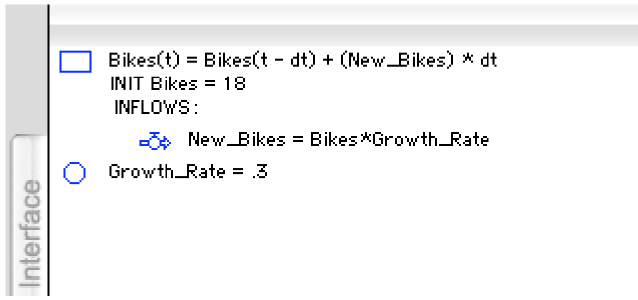


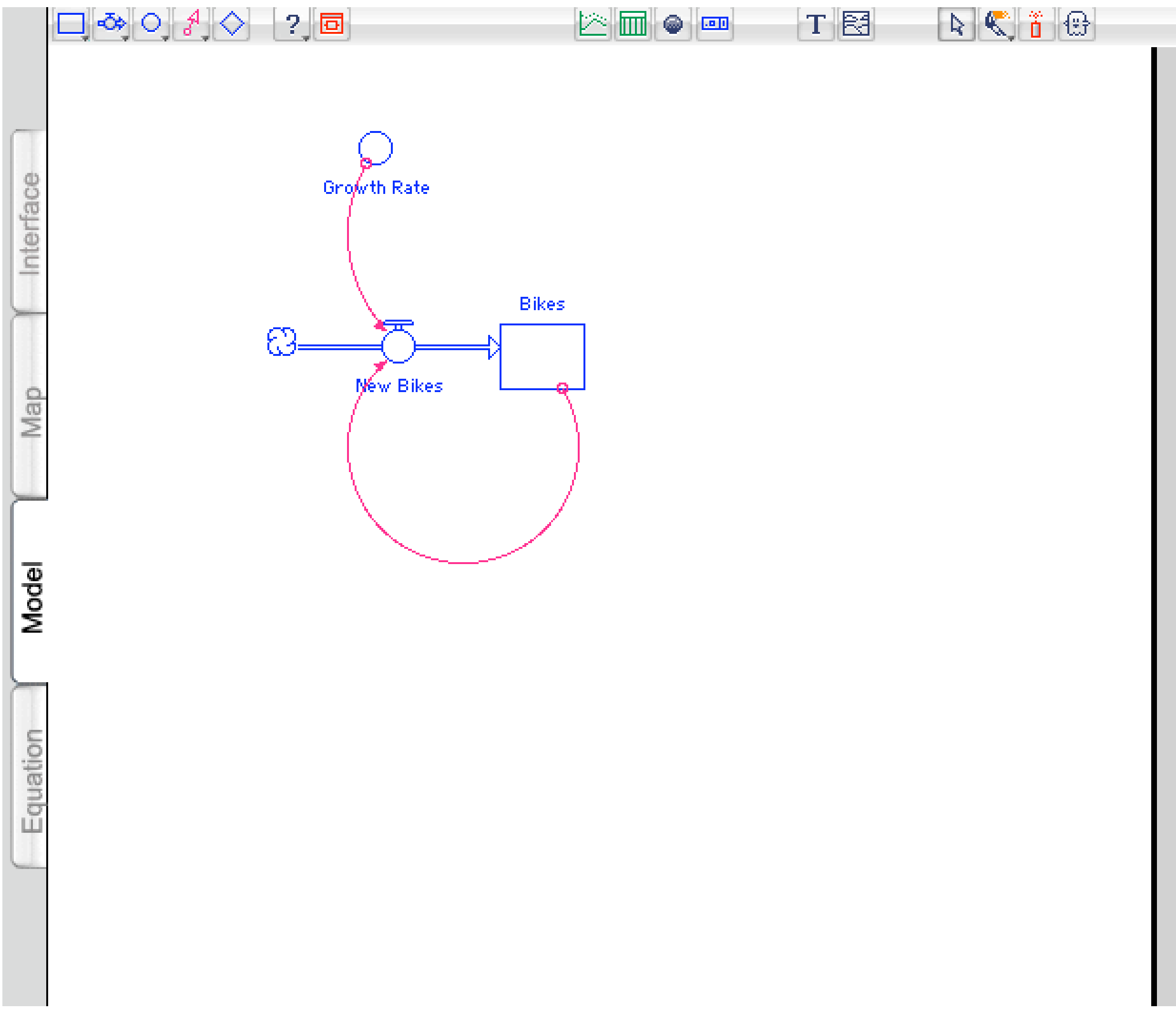
Time	BIKE KIOSK	BIKE KIOSK	BIKE KIOSK	BIKE KIOSK	TOTALOUT
0	0.00	0.00	0.00	0.00	0.00
1	6.00	4.00	0.00	4.00	4.00
2	6.00	4.00	2.00	4.00	2.00
3	5.00	4.00	3.00	4.00	2.00
4	5.00	4.00	4.00	5.00	0.00
5	2.00	3.00	2.00	2.00	9.00
6	2.00	2.00	6.00	3.00	5.00
7	0.00	0.00	6.00	0.00	12.00
8	0.00	0.00	10.00	1.00	7.00
9	0.00	0.00	13.00	2.00	3.00
Final	0.00	0.00	14.00	1.00	3.00

Something we will consider when it comes to placing new kiosks is how to divert bikes away from Kiosk C.

Growth of Bike Use

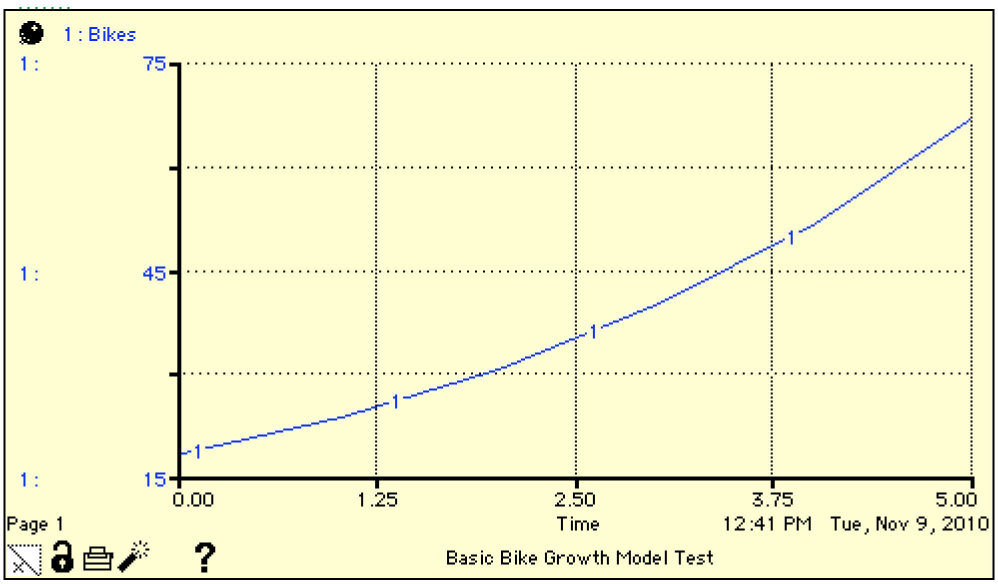
Over 5 years, the program is planned to grow by about 30%, and there will be 51 bikes total in the system after 5 years. We have modeled this growth with a different STELLA model.





12:41 PM 11/9/10 Table 1 (Basic Bike Growth Model Test)

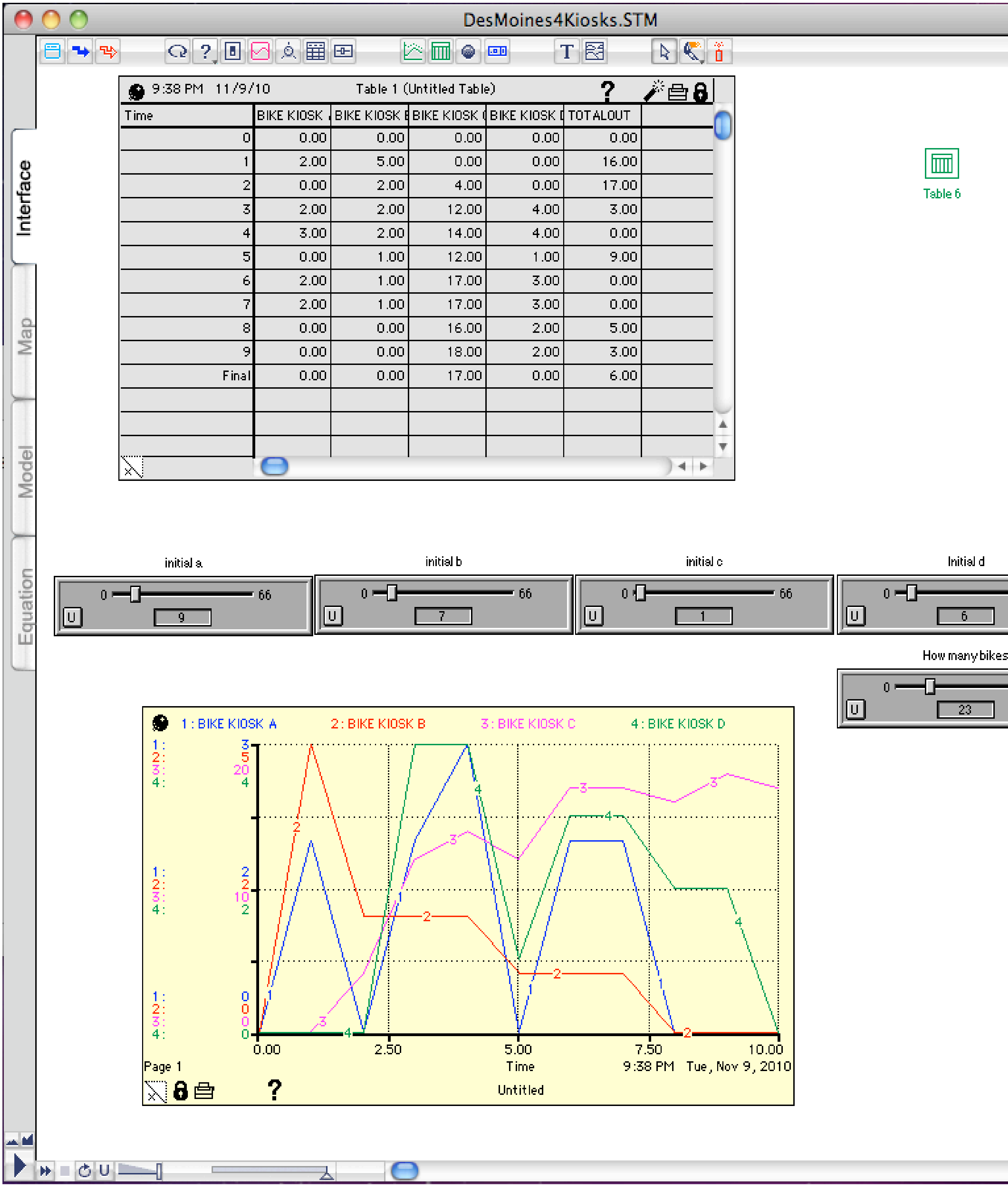
Time	Bikes
0	18.00
1	23.40
2	30.42
3	39.55
4	51.41
Final	66.83



Without doubt, as the number of bikes increases from 18 to 51 over 5 years, the city will need to set up more kiosks to accommodate the increase in bike traffic.

YEAR 2:

In the second year, the model will have grown by 30%, or 5 bikes. This means we have to re-evaluate an optimal bike placement. Using the same Stella model as Year 1, we have found new optimal values:



Time	BIKE KIOSK	BIKE KIOSK	BIKE KIOSK	BIKE KIOSK	TOTALOUT
0	0.00	0.00	0.00	0.00	0.00
1	2.00	5.00	0.00	0.00	16.00
2	0.00	2.00	4.00	0.00	17.00
3	2.00	2.00	12.00	4.00	3.00
4	3.00	2.00	14.00	4.00	0.00
5	0.00	1.00	12.00	1.00	9.00
6	2.00	1.00	17.00	3.00	0.00
7	2.00	1.00	17.00	3.00	0.00
8	0.00	0.00	16.00	2.00	5.00
9	0.00	0.00	18.00	2.00	3.00
Final	0.00	0.00	17.00	0.00	6.00

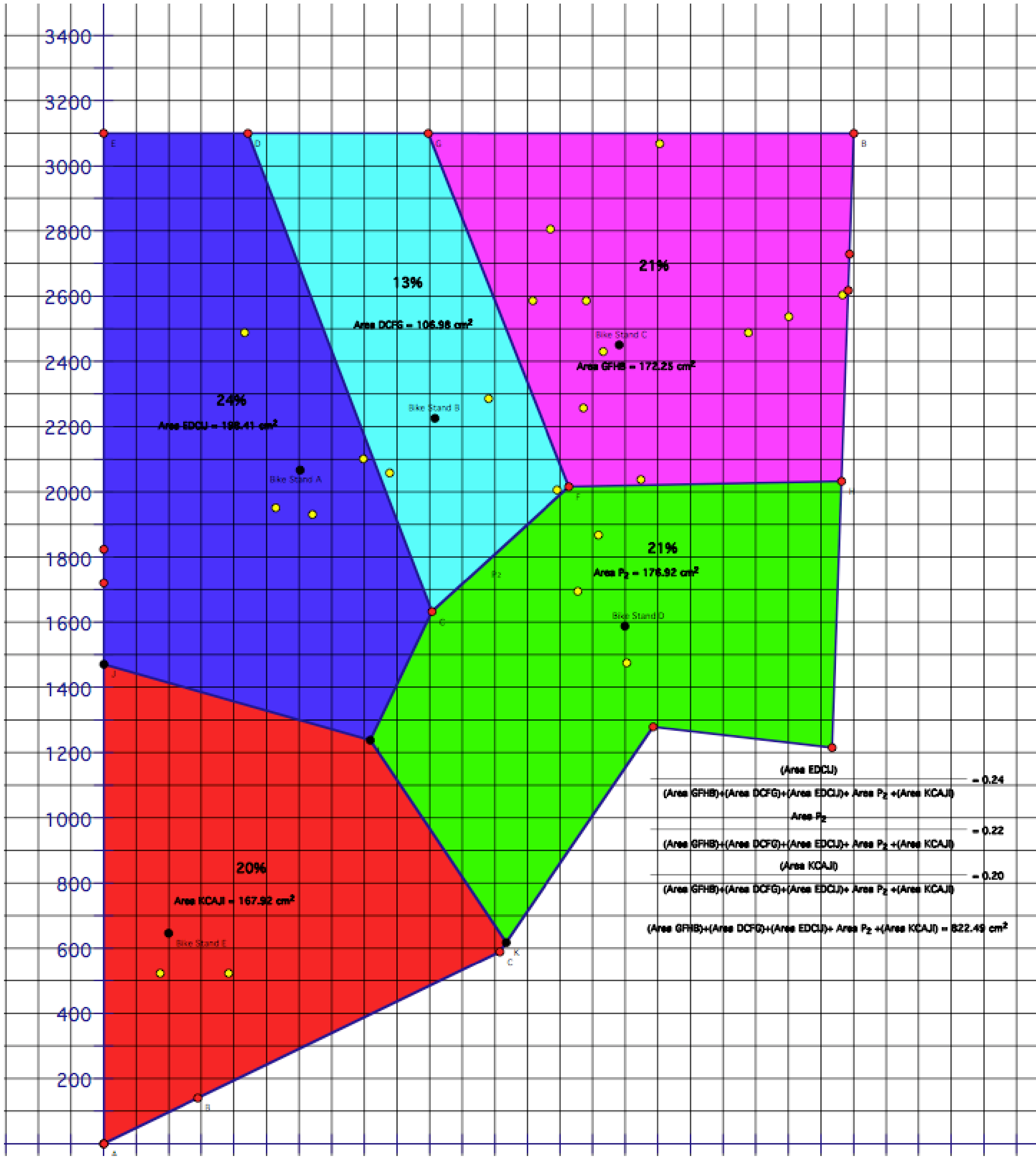
By placing 9 bikes at Kiosk A, 7 bikes at Kiosk B, 1 bike at Kiosk C, and 6 bikes at Kiosk D, we can make the program run for 7 hours without incident. Some kiosks run for even longer, but once again, after the 7 hours, by 3 PM, bikes have to be shipped away from Kiosk C to the other Kiosks.

FIRST EXPANSION (YEAR 3):

Where should be the best place for the first new kiosk? By the third year, we have reached a number of bikes that will call for a new kiosk. Looking at the voronoi diagrams we created and the map of the region, the best place for a first new bike stand will be in the lower left-hand corner of the region, where there is a large lake and many bike trails. People will rent bikes from any kiosk in the region, ride to the lake and around its trails, and return the bike to the kiosk by the lake. Since the lake is such a great point of interest, we have weighted it in our STELLA model as 2 points of interest instead of only 1.

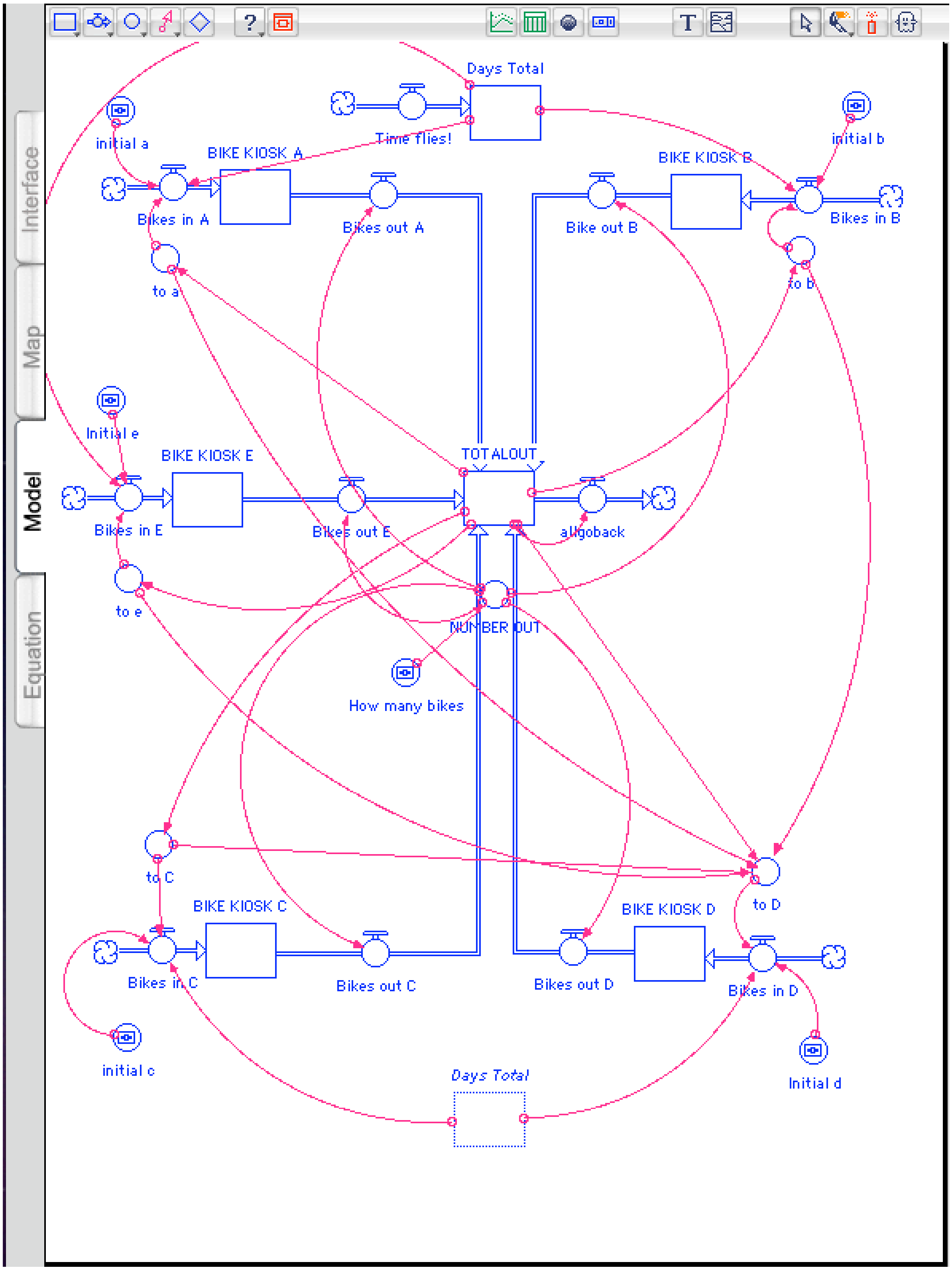
When we make a new voronoi diagram, which factors in this new kiosk, the distribution of area between each bike kiosk region is more balanced; region A no longer has so much space. The new area distribution is below, followed by the corresponding voronoi diagram.

Kiosk A = 24%
Kiosk B = 13%
Kiosk C = 21%
Kiosk D = 21%
Kiosk E =
20%



We can adjust our STELLA model to accommodate the new bike kiosk (E):

Team #2581



Interface
Map
Model
Equation
Days Total
Time flies!
initial a
BIKE KIOSK A
Bikes in A
Bikes out A
to a
initial b
BIKE KIOSK B
Bike out B
Bikes in B
to b
Initial e
BIKE KIOSK E
Bikes in E
Bikes out E
to e
TOTALOUT
allgoback
NUMBER OUT
How many bikes
to C
BIKE KIOSK C
Bikes in C
Bikes out C
initial c
to D
BIKE KIOSK D
Bikes out D
Bikes in D
Initial d
Days Total

```
BIKE_KIOSK_A(t) = BIKE_KIOSK_A(t - dt) + (Bikes_in_A - Bikes_out_A) * dt
INIT BIKE_KIOSK_A = 0
INFLOWS:
    Bikes_in_A = IF(Days_Total=0) THEN (initial_a)
    ELSE(to_a)
OUTFLOWS:
    Bikes_out_A = ROUND(.24*NUMBER_OUT)
BIKE_KIOSK_B(t) = BIKE_KIOSK_B(t - dt) + (Bikes_in_B - Bike_out_B) * dt
INIT BIKE_KIOSK_B = 0
INFLOWS:
    Bikes_in_B = IF(Days_Total=0) THEN (initial_b)
    ELSE(to_b)
OUTFLOWS:
    Bike_out_B = ROUND(.13*NUMBER_OUT)
BIKE_KIOSK_C(t) = BIKE_KIOSK_C(t - dt) + (Bikes_in_C - Bikes_out_C) * dt
INIT BIKE_KIOSK_C = 0
INFLOWS:
    Bikes_in_C = IF(Days_Total=0) THEN (initial_c)
    ELSE(to_C)
OUTFLOWS:
    Bikes_out_C = ROUND(.21*NUMBER_OUT)
BIKE_KIOSK_D(t) = BIKE_KIOSK_D(t - dt) + (Bikes_in_D - Bikes_out_D) * dt
INIT BIKE_KIOSK_D = 0
INFLOWS:
    Bikes_in_D = IF(Days_Total=0) THEN (initial_d)
    ELSE(to_D)
OUTFLOWS:
    Bikes_out_D = ROUND(.21*NUMBER_OUT)
BIKE_KIOSK_E(t) = BIKE_KIOSK_E(t - dt) + (Bikes_in_E - Bikes_out_E) * dt
INIT BIKE_KIOSK_E = 0
INFLOWS:
    Bikes_in_E = IF(Days_Total=0) THEN (initial_e)
    ELSE(to_e)
OUTFLOWS:
    Bikes_out_E = ROUND(.2*(NUMBER_OUT))
Days_Total(t) = Days_Total(t - dt) + (Time_flies!) * dt
INIT Days_Total = 0
INFLOWS:
    Time_flies! = 1
TOTALOUT(t) = TOTALOUT(t - dt) + (Bikes_out_A + Bike_out_B + Bikes_out_D + Bikes_out_C + Bikes_out_E
- allgoback) * dt
INIT TOTALOUT = 0
INFLOWS:
    Bikes_out_A = ROUND(.24*NUMBER_OUT)
    Bike_out_B = ROUND(.13*NUMBER_OUT)
    Bikes_out_D = ROUND(.21*NUMBER_OUT)
    Bikes_out_C = ROUND(.21*NUMBER_OUT)
    Bikes_out_E = ROUND(.2*(NUMBER_OUT))
OUTFLOWS:
```

What this entails is essentially changing the in and out rates of all the kiosks to accomodate the new bike kiosk (see above for new percentages). Using our model we have found new optimum values.

The STELLA interface with the optimal initial bike values at each kiosk:

DesMoines5Kiosks.STM

Interface

Map

Model

Equation

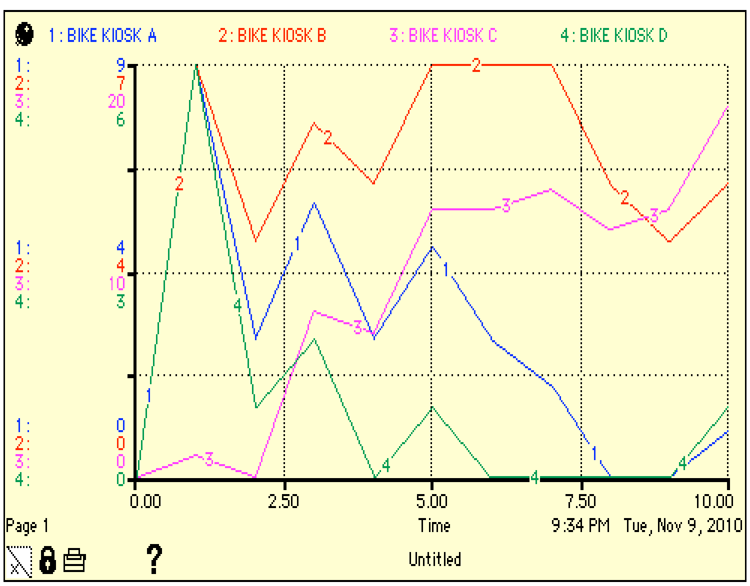
9:34 PM 11/9/10 Table 1 (Untitled Table)

Time	BIKE KIOSK A	BIKE KIOSK B	BIKE KIOSK C	BIKE KIOSK D	BIKE KIOSK E	TOTALOUT
0	0.00	0.00	0.00	0.00	0.00	0.00
1	9.00	7.00	1.00	6.00	7.00	0.00
2	3.00	4.00	0.00	1.00	2.00	20.00
3	6.00	6.00	8.00	2.00	3.00	5.00
4	3.00	5.00	7.00	0.00	0.00	15.00
5	5.00	7.00	13.00	1.00	0.00	4.00
6	3.00	7.00	13.00	0.00	0.00	7.00
7	2.00	7.00	14.00	0.00	0.00	7.00
8	0.00	5.00	12.00	0.00	0.00	13.00
9	0.00	4.00	13.00	0.00	0.00	13.00
Final	1.00	5.00	18.00	1.00	0.00	5.00

Table 6

initial a	initial b	initial c	Initial d	How many b
0 – 66: 9	0 – 66: 7	0 – 66: 1	0 – 66: 6	0: 30

Initial e: 0 – 66: 7



By starting Kiosk A at 9, B at 7, C at 1, D at 6, and E at 7, we have managed to let the model run for about 4 hours. This should be taken with a grain of salt, as people will likely rent bikes at the lake and simply return them to the same Kiosk (E). This means that a loop will be created here, and Kiosk E may not run out as quickly as it seems, and the model would run for longer than shown above.

SECOND EXPANSION (YEAR 4):

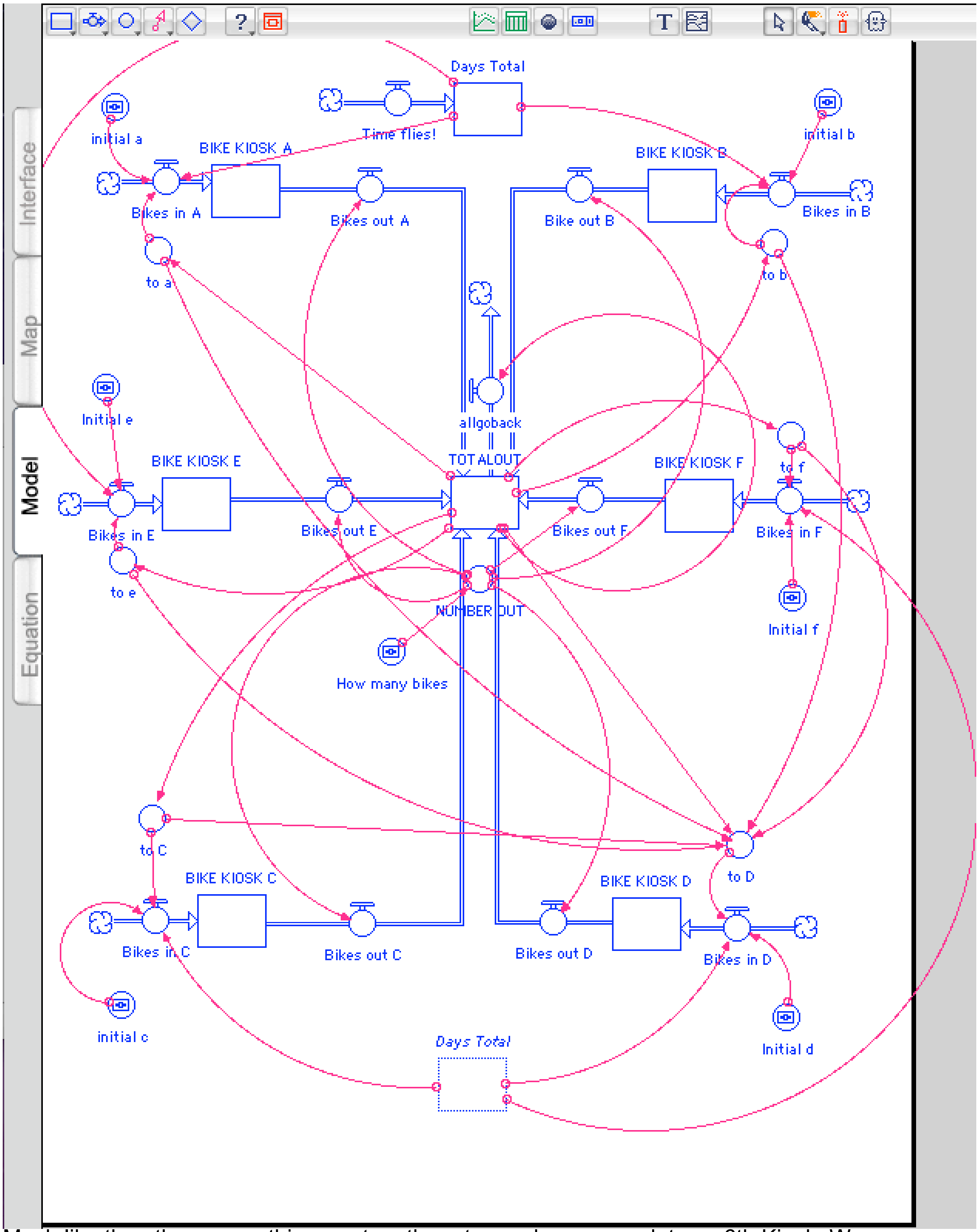
We should place a second additional kiosk to accommodate the return of all these new bikes that we have introduced into the system. When we run our model, kiosk C, in a region with many points of interest, fills up very quickly when many bikes are returned. So, we should put a second kiosk in region C where people can return and take bikes. The area distribution is as follows:

Kiosk A = 24%
Kiosk B = 13%
Kiosk C = 10%
Kiosk D = 21%
Kiosk E = 21%
Kiosk F = 11%

This evens out the land and population distributions nicely, and, more importantly, splits up the points of interest in the upper right-hand corner. The next step was to re-form our STELLA model.

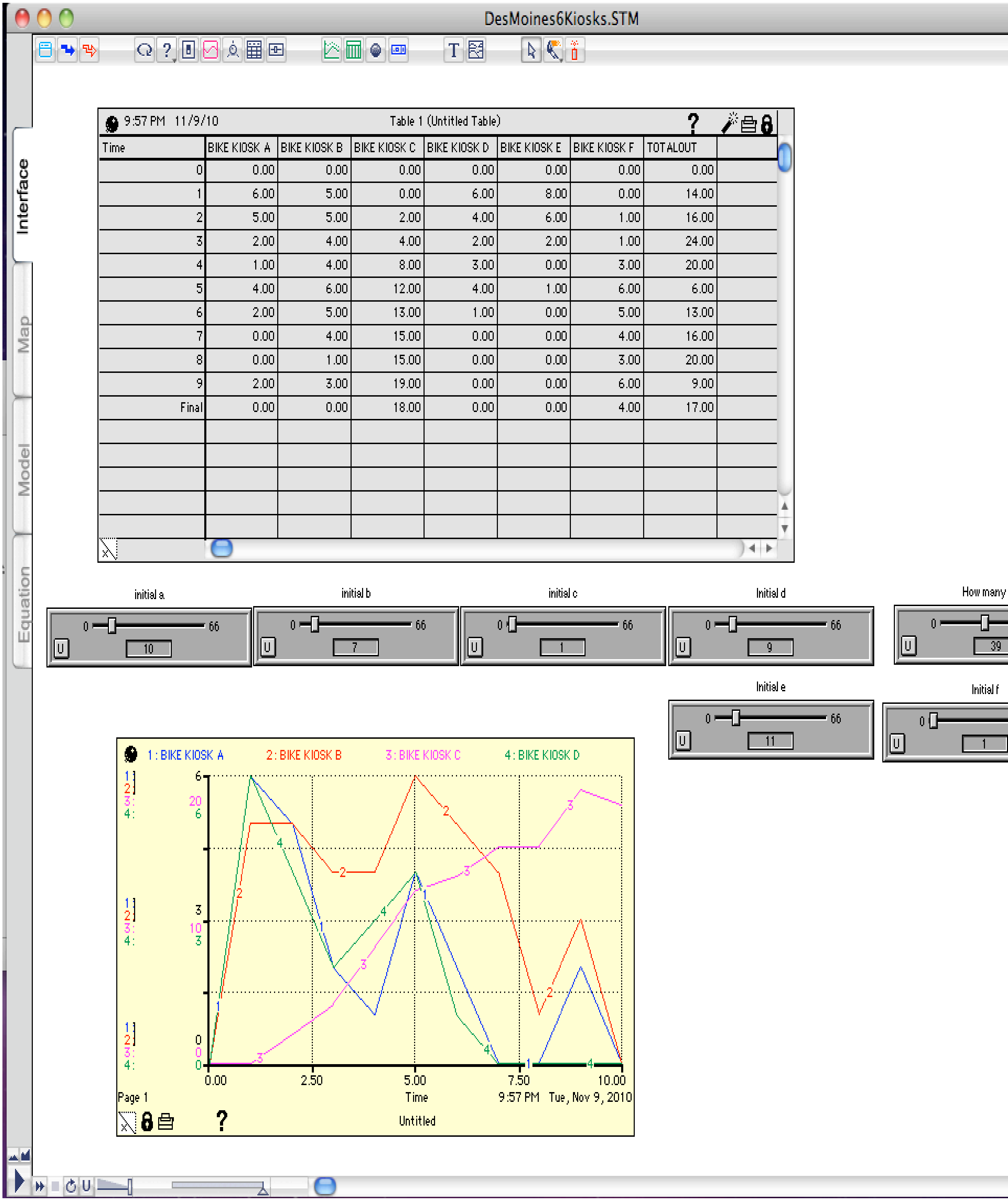
Our adjusted STELLA is as follows:

```
BIKE_KIOSK_A(t) = BIKE_KIOSK_A(t - dt) + (Bikes_in_A - Bikes_out_A) * dt
INIT BIKE_KIOSK_A = 0
INFLOWS:
    Bikes_in_A = IF(Days_Total=0) THEN (initial_a)
    ELSE(to_a)
OUTFLOWS:
    Bikes_out_A = ROUND(.24*NUMBER_OUT)
BIKE_KIOSK_B(t) = BIKE_KIOSK_B(t - dt) + (Bikes_in_B - Bike_out_B) * dt
INIT BIKE_KIOSK_B = 0
INFLOWS:
    Bikes_in_B = IF(Days_Total=0) THEN (initial_b)
    ELSE(to_b)
OUTFLOWS:
    Bike_out_B = ROUND(.13*NUMBER_OUT)
BIKE_KIOSK_C(t) = BIKE_KIOSK_C(t - dt) + (Bikes_in_C - Bikes_out_C) * dt
INIT BIKE_KIOSK_C = 0
INFLOWS:
    Bikes_in_C = IF(Days_Total=0) THEN (initial_c)
    ELSE(to_C)
OUTFLOWS:
    Bikes_out_C = ROUND(.10*NUMBER_OUT)
BIKE_KIOSK_D(t) = BIKE_KIOSK_D(t - dt) + (Bikes_in_D - Bikes_out_D) * dt
INIT BIKE_KIOSK_D = 0
INFLOWS:
    Bikes_in_D = IF(Days_Total=0) THEN (initial_d)
    ELSE(to_D)
OUTFLOWS:
    Bikes_out_D = ROUND(.21*NUMBER_OUT)
BIKE_KIOSK_E(t) = BIKE_KIOSK_E(t - dt) + (Bikes_in_E - Bikes_out_E) * dt
INIT BIKE_KIOSK_E = 0
INFLOWS:
    Bikes_in_E = IF(Days_Total=0) THEN (initial_e)
    ELSE(to_e)
OUTFLOWS:
    Bikes_out_E = ROUND(.21*(NUMBER_OUT))
BIKE_KIOSK_F(t) = BIKE_KIOSK_F(t - dt) + (Bikes_in_F - Bikes_out_F) * dt
INIT BIKE_KIOSK_F = 0
INFLOWS:
    Bikes_in_F = IF(Days_Total=0) THEN (initial_f)
    ELSE(to_f)
OUTFLOWS:
    Bikes_out_F = ROUND(.11*(NUMBER_OUT))
Days_Total(t) = Days_Total(t - dt) + (Time_flies!) * dt
INIT Days_Total = 0
INFLOWS:
    Time_flies! = 1
TOTALOUT(t) = TOTALOUT(t - dt) + (Bikes_out_A + Bike_out_B + Bikes_out_D + Bikes_out_C + Bikes_out_E
+ Bikes_out_F - allgoback) * dt
INIT TOTALOUT = 0
INFLOWS:
```



Much like the other years, this re-enters the rates and accommodates a 6th Kiosk. We proceeded to find optimal values.

The STELLA interface with the optimal initial bike values at each kiosk:



9:57 PM 11/9/10 Table 1 (Untitled Table)

Time	BIKE KIOSK A	BIKE KIOSK B	BIKE KIOSK C	BIKE KIOSK D	BIKE KIOSK E	BIKE KIOSK F	TOTALOUT
0	0.00	0.00	0.00	0.00	0.00	0.00	0.00
1	6.00	5.00	0.00	6.00	8.00	0.00	14.00
2	5.00	5.00	2.00	4.00	6.00	1.00	16.00
3	2.00	4.00	4.00	2.00	2.00	1.00	24.00
4	1.00	4.00	8.00	3.00	0.00	3.00	20.00
5	4.00	6.00	12.00	4.00	1.00	6.00	6.00
6	2.00	5.00	13.00	1.00	0.00	5.00	13.00
7	0.00	4.00	15.00	0.00	0.00	4.00	16.00
8	0.00	1.00	15.00	0.00	0.00	3.00	20.00
9	2.00	3.00	19.00	0.00	0.00	6.00	9.00
Final	0.00	0.00	18.00	0.00	0.00	4.00	17.00

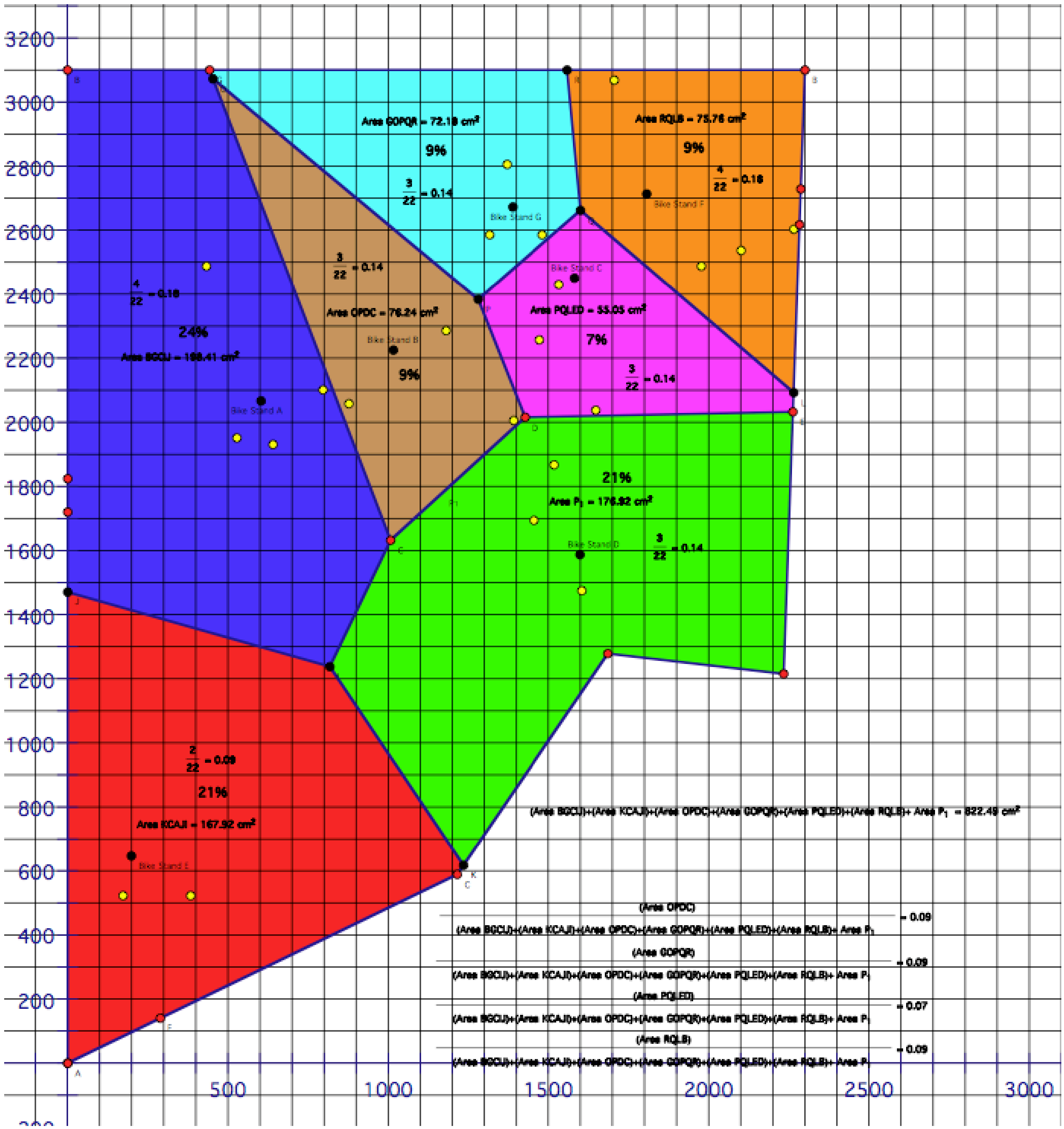
10 bikes should go to A, 7 to B, 1 to C, 9 to D, 11 to E and 1 to F. This will allow to program to run for somewhere between 5 and 6 hours on average and accommodate the growth.

THIRD EXPANSION (YEAR 5):

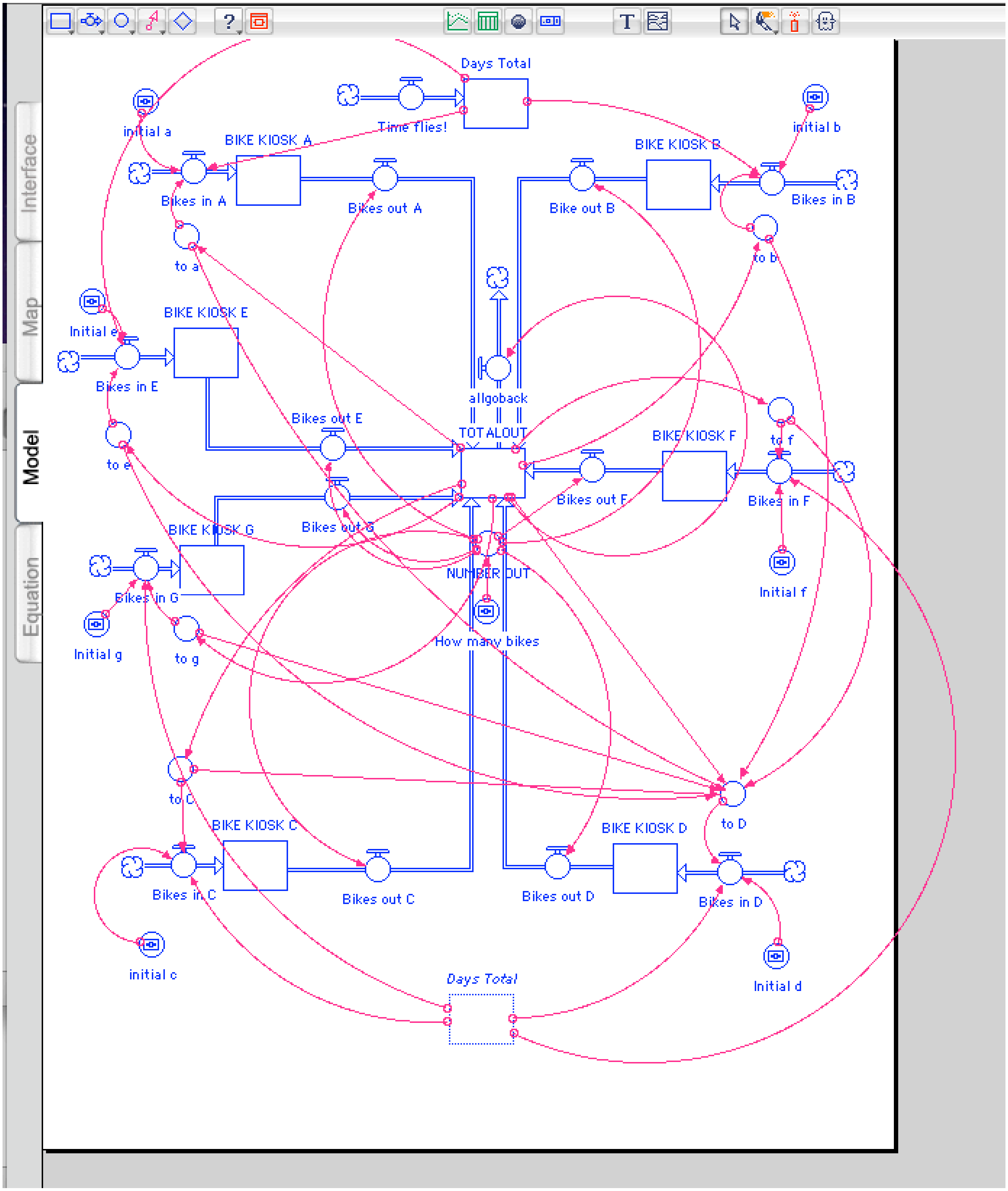
Where, now to put a third additional kiosk? With the 30% growth rate, a second kiosk will accommodate 9 more bikes after the fourth year, and there will be 51 total bikes in the system.

Running our model, we find that now region C has ends up with a *lot* of bikes. It's not excessive or uncontrollable, but it would be helpful if there was another kiosk in the area to collect bikes, too. Thus, we can split up the points of interest and add a third kiosk near regions F and C. The new area distribution is as follows:

Kiosk A = 24%
Kiosk B = 9%
Kiosk C = 7%
Kiosk D = 21%
Kiosk E = 21%
Kiosk F = 9%
Kiosk G = 9%



The corresponding STELLA model:


Interface
Map
Model
Equation
Days Total
Time flies!
initial a
initial b
BIKE KIOSK A
BIKE KIOSK B
Bikes in A
Bikes out A
Bike out B
Bikes in B
to a
to b
Initial e
BIKE KIOSK E
Bikes in E
allgoback
Bikes out E
TOTALOUT
to e
BIKE KIOSK F
to f
Bikes out F
Bikes in F
Bikes out G
BIKE KIOSK G
NUMBER OUT
Initial f
Bikes in G
How many bikes
Initial g
to g
to C
to D
BIKE KIOSK C
BIKE KIOSK D
Bikes in C
Bikes out C
Bikes out D
Bikes in D
initial c
Initial d
Days Total

```
BIKE_KIOSK_A(t) = BIKE_KIOSK_A(t - dt) + (Bikes_in_A - Bikes_out_A) * dt
INIT BIKE_KIOSK_A = 0
 INFLOWS:
    Bikes_in_A = IF(Days_Total=0) THEN (initial_a)
    ELSE(to_a)
 OUTFLOWS:
    Bikes_out_A = ROUND(.24*NUMBER_OUT)
BIKE_KIOSK_B(t) = BIKE_KIOSK_B(t - dt) + (Bikes_in_B - Bike_out_B) * dt
INIT BIKE_KIOSK_B = 0
 INFLOWS:
    Bikes_in_B = IF(Days_Total=0) THEN (initial_b)
    ELSE(to_b)
 OUTFLOWS:
    Bike_out_B = ROUND(.09*NUMBER_OUT)
BIKE_KIOSK_C(t) = BIKE_KIOSK_C(t - dt) + (Bikes_in_C - Bikes_out_C) * dt
INIT BIKE_KIOSK_C = 0
 INFLOWS:
    Bikes_in_C = IF(Days_Total=0) THEN (initial_c)
    ELSE(to_C)
 OUTFLOWS:
    Bikes_out_C = ROUND(.07*NUMBER_OUT)
BIKE_KIOSK_D(t) = BIKE_KIOSK_D(t - dt) + (Bikes_in_D - Bikes_out_D) * dt
INIT BIKE_KIOSK_D = 0
 INFLOWS:
    Bikes_in_D = IF(Days_Total=0) THEN (initial_d)
    ELSE(to_D)
 OUTFLOWS:
    Bikes_out_D = ROUND(.21*NUMBER_OUT)
BIKE_KIOSK_E(t) = BIKE_KIOSK_E(t - dt) + (Bikes_in_E - Bikes_out_E) * dt
INIT BIKE_KIOSK_E = 0
 INFLOWS:
    Bikes_in_E = IF(Days_Total=0) THEN (initial_e)
    ELSE(to_e)
 OUTFLOWS:
    Bikes_out_E = ROUND(.21*(NUMBER_OUT))
BIKE_KIOSK_F(t) = BIKE_KIOSK_F(t - dt) + (Bikes_in_F - Bikes_out_F) * dt
INIT BIKE_KIOSK_F = 0
 INFLOWS:
    Bikes_in_F = IF(Days_Total=0) THEN (initial_f)
    ELSE(to_f)
 OUTFLOWS:
    Bikes_out_F = ROUND(.09*(NUMBER_OUT))
BIKE_KIOSK_G(t) = BIKE_KIOSK_G(t - dt) + (Bikes_in_G - Bikes_out_G) * dt
INIT BIKE_KIOSK_G = 0
 INFLOWS:
    Bikes_in_G = IF(Days_Total=0) THEN (initial_g)
    ELSE(to_g)
 OUTFLOWS:
    Bikes_out_G = ROUND(.09*(NUMBER_OUT))
```

We now have 7 kiosks in out model. The rates have been adjusted as needed.

The STELLA interface with the optimal initial bike values at each kiosk:

DesMoines7Kiosks.STM

10:04 PM 11/9/10 Table 1 (Untitled Table)

Time	BIKE KIOSK A	BIKE KIOSK B	BIKE KIOSK C	BIKE KIOSK D	BIKE KIOSK E	BIKE KIOSK F	BIKE KIOSK G	TOTALOUT
0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
1	3.00	3.00	0.00	10.00	9.00	0.00	3.00	23.00
2	5.00	5.00	2.00	12.00	9.00	3.00	5.00	10.00
3	5.00	5.00	2.00	12.00	8.00	4.00	5.00	10.00
4	0.00	3.00	0.00	6.00	1.00	3.00	3.00	35.00
5	2.00	7.00	4.00	8.00	1.00	8.00	7.00	14.00
6	4.00	9.00	6.00	8.00	1.00	11.00	9.00	3.00
7	4.00	9.00	6.00	8.00	0.00	12.00	9.00	3.00
8	3.00	8.00	5.00	7.00	0.00	12.00	8.00	8.00
9	2.00	8.00	6.00	8.00	0.00	12.00	8.00	7.00
10	0.00	5.00	4.00	0.00	0.00	9.00	5.00	28.00
11	0.00	5.00	5.00	0.00	0.00	10.00	5.00	26.00
Final	1.00	8.00	8.00	0.00	0.00	14.00	8.00	12.00

initial a: 0 – 66, 8

initial b: 0 – 66, 5

initial c: 0 – 66, 2

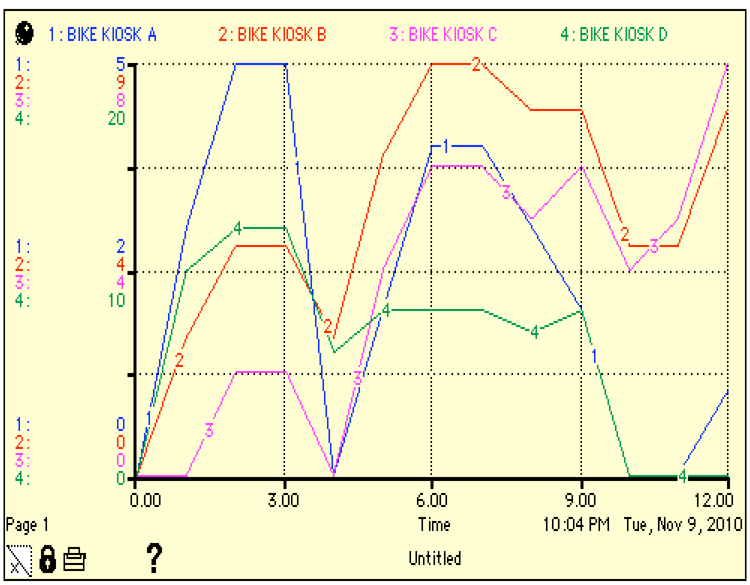
Initial d: 0 – 66, 15

How many l: 0, 51

Initial e: 0 – 66, 14

Initial f: 0, 2

Initial g: 0 – 66, 5



We'll need 8 bikes at A, 5 at B, 2 at C, 15 at D, 14 at E, 2 at F, and 5 at G. It is completely successful for 6 hours, but with the exception of Kiosk E, by the lake, it lasts for 9 hours. The more kiosks and bikes, the more complex the model becomes, but with more bikes to circulate, the model tends to last longer.

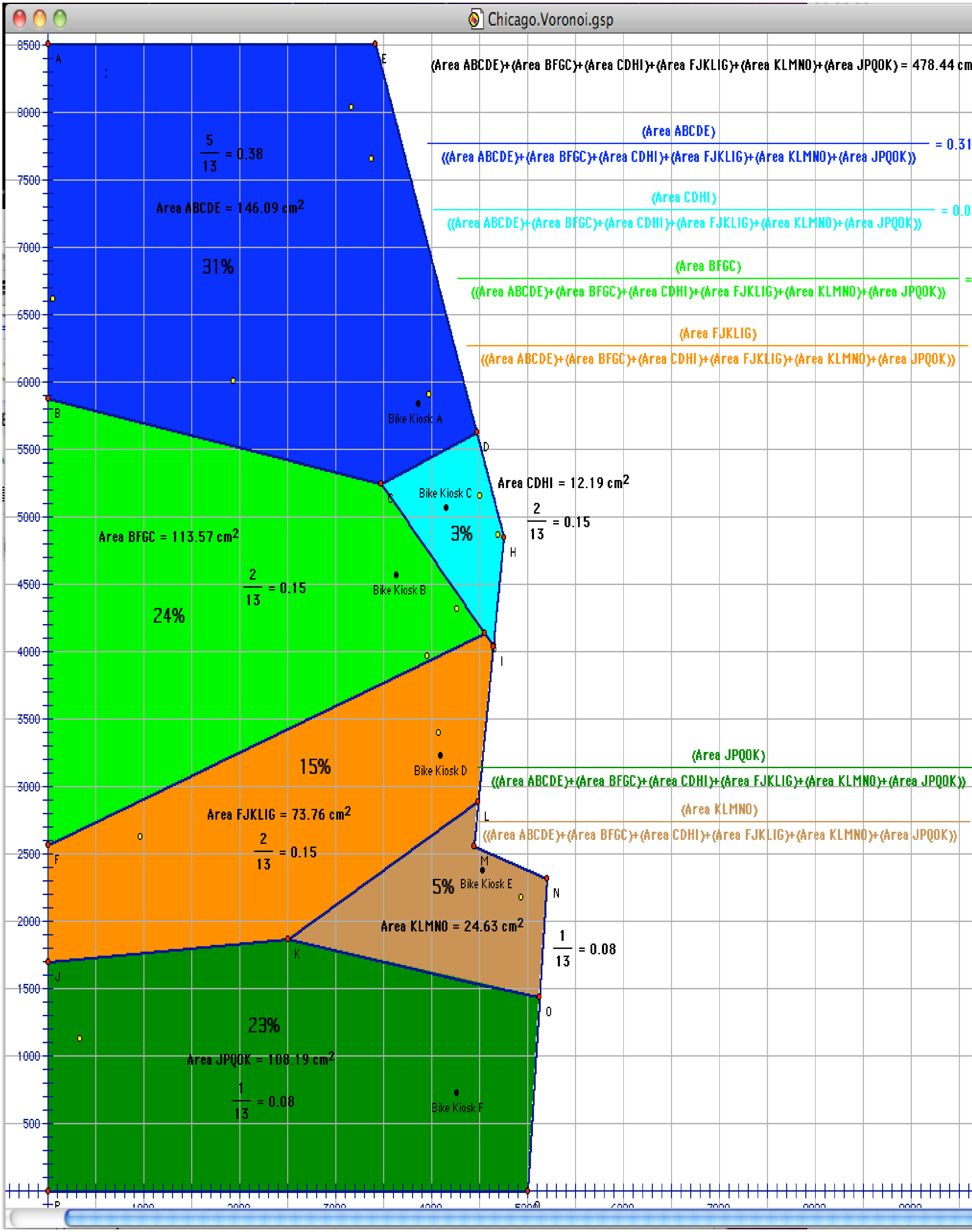
As time continues, the model can be adapted to include additional kiosks and rising bike numbers very easily, by simply adding a new kiosk to the model and adjusting the rates. This is the benefit of our model. Despite its apparent complexity, it is relatively simple to adapt.

Chicago:

Due to our severe time constraints, we have created a single, in-depth model for the city of Des Moines, Illinois, rather than three separate models for each big city. This Des Moines model, however, serves as a simple template for the other cities; it is easy to replicate and adjust it to depict the bicycle programs in the other two cities.

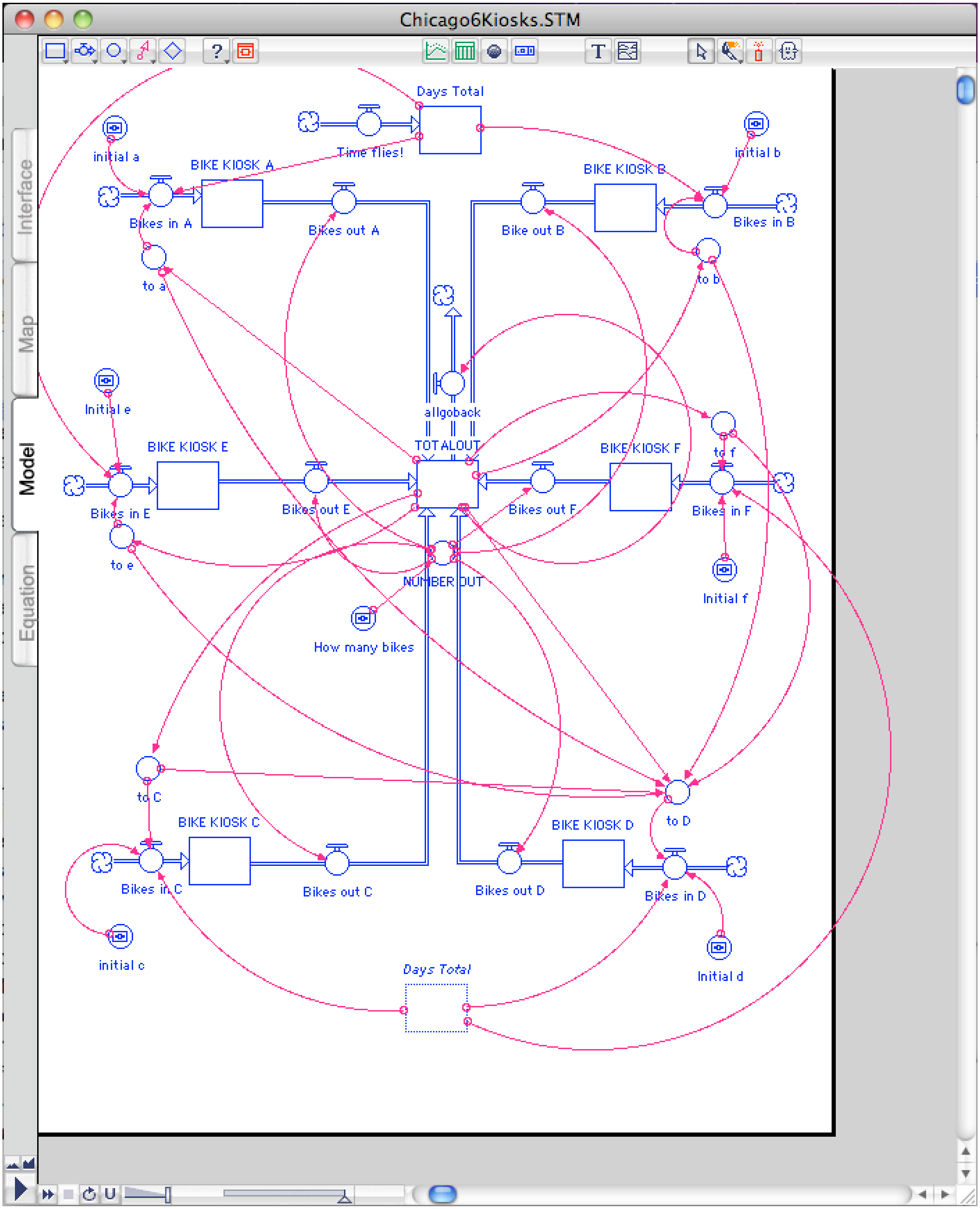
The city of Chicago was reported to have 100 bikes in circulation over 6 different B-Cycle kiosks. The great thing about our model for Des Moines is that it's incredibly easy to apply to any other city situation. The only required aspects are a voronoi diagram (to show percentage of area covered by each kiosk) and the points of interest in the city (to apply to our weighting system in our model – see Des Moines model for explanation).

Below is the voronoi diagram we created for the city of Chicago, taking into account the points of interest in the city:


Chicago.Voronoi.gsp
(Area ABCDE)+(Area BFGC)+(Area CDHI)+(Area FJKLIG)+(Area KLMNO)+(Area JPQOK) = 478.44 cm
(Area ABCDE)
((Area ABCDE)+(Area BFGC)+(Area CDHI)+(Area FJKLIG)+(Area KLMNO)+(Area JPQOK))
= 0.31
(Area CDHI)
((Area ABCDE)+(Area BFGC)+(Area CDHI)+(Area FJKLIG)+(Area KLMNO)+(Area JPQOK))
= 0.0
(Area BFGC)
((Area ABCDE)+(Area BFGC)+(Area CDHI)+(Area FJKLIG)+(Area KLMNO)+(Area JPQOK))
(Area FJKLIG)
((Area ABCDE)+(Area BFGC)+(Area CDHI)+(Area FJKLIG)+(Area KLMNO)+(Area JPQOK))
(Area JPQOK)
((Area ABCDE)+(Area BFGC)+(Area CDHI)+(Area FJKLIG)+(Area KLMNO)+(Area JPQOK))
(Area KLMNO)
((Area ABCDE)+(Area BFGC)+(Area CDHI)+(Area FJKLIG)+(Area KLMNO)+(Area JPQOK))
5/13 = 0.38
Area ABCDE = 146.09 cm2
31%
Bike Kiosk A
Area CDHI = 12.19 cm2
Bike Kiosk C
3%
2/13 = 0.15
Area BFGC = 113.57 cm2
2/13 = 0.15
24%
Bike Kiosk B
15%
Bike Kiosk D
Area FJKLIG = 73.76 cm2
2/13 = 0.15
5% Bike Kiosk E
Area KLMNO = 24.63 cm2
1/13 = 0.08
23%
Area JPQOK = 108.19 cm2
1/13 = 0.08
Bike Kiosk F
A B C D E F H I J K L M N O P
8500 8000 7500 7000 6500 6000 5500 5000 4500 4000 3500 3000 2500 2000 1500 1000 500
1000 2000 3000 4000 5000

Though our time restrictions limited us to only being able to model the growth of Des Moines, we were easily able to apply our model to the existing situation in Chicago (100 bikes placed over 6 bike kiosks). The same mathematics apply to this city, as well as any other that our model would be used for.

Below is the STELLA Model for the city of Chicago:


Chicago6Kiosks.STM
Interface
Map
Model
Equation
Days Total
Time flies!
initial a
BIKE KIOSK A
Bikes in A
to a
Bikes out A
Bike out B
BIKE KIOSK B
initial b
Bikes in B
to b
allgoback
TOTAL OUT
Initial e
BIKE KIOSK E
Bikes in E
to e
Bikes out E
Bikes out F
BIKE KIOSK F
to f
Bikes in F
Initial f
NUMBER OUT
How many bikes
to C
to D
BIKE KIOSK C
Bikes in C
Bikes out C
initial c
Bikes out D
BIKE KIOSK D
Bikes in D
Initial d
Days Total

As you can see, it looks similar to the model used for Des Moines. Next, we optimized the values of bikes placed at each kiosk so that we could keep the program going for the longest amount of time, as we did with the Des Moines model.

Below are the optimized values and graph:

Chicago6Kiosks.STM

10:38 PM 11/9/10 Table 1 (Untitled Table)

Time	BIKE KIOSK A	BIKE KIOSK B	BIKE KIOSK C	BIKE KIOSK D	BIKE KIOSK E	BIKE KIOSK F	TOTALOUT
0	0.00	0.00	0.00	0.00	0.00	0.00	0.00
1	0.00	7.00	0.00	2.00	6.00	8.00	77.00
2	21.00	13.00	11.00	8.00	11.00	8.00	28.00
3	29.00	14.00	15.00	10.00	12.00	7.00	13.00
4	7.00	0.00	14.00	0.00	9.00	0.00	70.00
5	29.00	7.00	25.00	5.00	14.00	3.00	17.00
6	25.00	2.00	27.00	0.00	13.00	0.00	33.00
7	29.00	0.00	31.00	0.00	15.00	0.00	25.00
8	37.00	2.00	35.00	1.00	17.00	0.00	8.00
9	32.00	0.00	35.00	0.00	17.00	0.00	16.00
Final	24.00	0.00	36.00	0.00	16.00	0.00	24.00

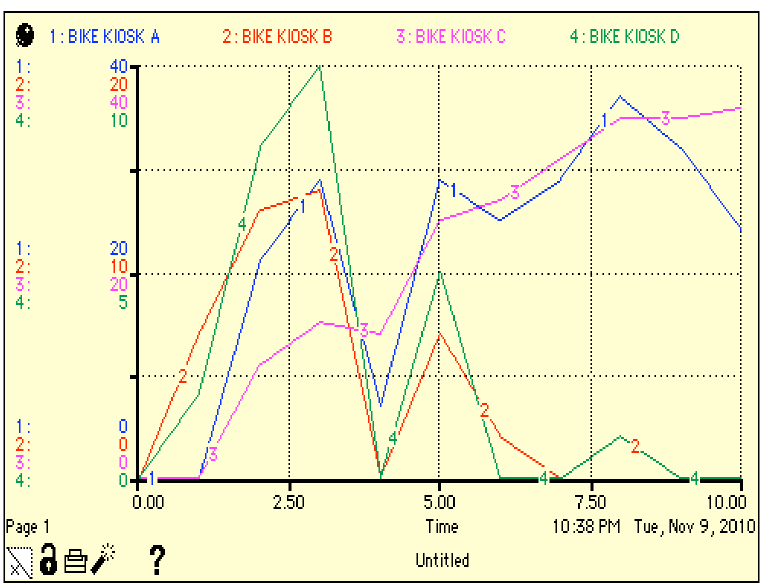
initial a: 0 – 66, 19

initial b: 0 – 66, 25

initial c: 0 – 66, 1

Initial d: 0 – 66, 20

Initial e: 0 – 66, 10



CONCLUSION:

In conclusion, we have developed a strong model for the Bike Sharing Program development in the cities of Des Moines, Chicago, and Denver. Our relatively simple model uses a voronoi diagram of the region, making portions based on proximity to bike kiosks, as well as the points of interest in the divided regions. We use the percentages of area and destinations to determine how many bikes enter and leave each kiosk every hour. Using our Stella program, which models the movement of the bikes, we have optimized the initial number of bikes at each Kiosk, even as the number of bikes and the number of kiosks grows over a span of five years. Our model takes into account many important factors, but is easily adaptable to the situation of any city, regardless of layout, bike use, and location of kiosks.

STRENGTHS AND WEAKNESSES:

- Since we were severely limited time-wise, we decided to focus our attention on an extremely in-depth model of only one of the three cities, Des Moines, with the assumption that our process for that city could be replicated (with more time) for the other two cities.
- Many of our assumptions are inaccurate, but due to our time constraints, we were forced to work with them. In our model, we assumed that the parameters of the biking area are limited to the area defined by maps on the B-Cycle website. In reality, as membership of the program increases, the regions covered by the system will expand outside the boundaries shown currently on the maps. Also, for the purposes of our model, we assumed that population distribution is equal throughout each of the cities. For example, an area covering 37% of a city will contain 37% of its inhabitants. This assumption is inaccurate because within a given city, there are areas of varying population density. Another obviously faulty assumption is that all attractions generate the same amount of interest. Clearly, not all “attractions” are equally attractive to all members of the program. To improve our model, we would need to weigh the attractions based on the member interest.
- In our model, we focused only minimally on the roads and paths themselves. Biking on a busy road with rushing traffic and exhaust is unpleasant, and bicyclists tend to avoid those routes in favor of less busy streets and pathways, regardless of directness.

Letters to Mayors:

Des Moines: Mayor Franklin Cownie

November 9, 2010
HiMCM Group #2581

Dear Mayor Cownie,

We wish to commend you on your implementation of the B-cycle Sharing program in the city of Des Moines. We are four students participating in the 2010 High School Mathematical Contest in Modeling, and we have chosen to focus on the Bcycle Sharing program development in your city, as well as the programs that have started in Denver and Chicago. After modeling the B-Cycle program in your city, we have come up with several suggestions that you may find useful to improve the bike system as the program grows.

Currently, your program includes 18 bicycles and 4 kiosk stands dispersed throughout the city. Without a doubt, this is an excellent starting point for the program. Kiosks are currently located near points of interest, and generate a large amount of bike traffic. In our assessment, we have used computer programs GeoSketchpad and STELLA to model the flow of bicycles through the system. In our model, Kiosk A is located at the intersection of 13th and Grand, Kiosk B is located at 7th and Grand, Kiosk C is located at the Brenton Skating Plaza, and Kiosk D is located at principal park.

Modeling the first year of the scenario, we found that most of the bikes tend to accumulate at Kiosk C, which makes sense, as this has *half* of the points on interest within its domain. Unfortunately, on the other hand, Kiosk A, B, and D tend to run out quickly - after 3-5 hours. After adjusting our model, we finally found that optimal starting values for bikes (using the B-Cycle website's value of 18 bikes in usage) are: 7 in A; 5 in B; 1 in C; 5 in D. This runs successfully for 6 hours, sometimes more, without depletion of a Kiosks bike stock.

In the second year, the model will have grown by 30%, or 5 bikes. This means we have to re-evaluate an optimal bike placement. Using the same Stella model as Year 1, we have found new optimal values: 9 bikes at Kiosk A, 7 bikes at Kiosk B, 1 bike at Kiosk C, and 6 bikes at Kiosk D. With this configuration, we can make the program run for 7 hours without incident. Some kiosks run for even longer, but once again, after the 7 hours, by 3 PM, bikes have to be shipped away from Kiosk C to the other Kiosks.

Still, the bike program is estimated to grow about 30% each year, and by the end of 5 years there will be 51 bikes total in the system. We would suggest three expansions, in the 3rd, 4th, and 5th years.

The best place for a first new bike stand will be in the lower left-hand corner of the region, where there is a large lake and many bike trails. People will rent bikes from any kiosk in the region, ride to the lake and around its trails, and return the bike to the kiosk by the lake. Optimal starting values are 9 bikes at Kiosk A, 7, at B, 1 at C, 6 at D, and 7 at E (the newest kiosk). Using these numbers, we managed to let the model run for about 4 hours! This should be taken with a grain of salt, however, as people will likely rent bikes at the lake and simply return them to the same Kiosk (E). This means that a loop will be created here, and Kiosk E may not run out as quickly as it seems. The model would in fact run for even longer.

We should place a second additional kiosk to accommodate the return of all these new bikes that we have introduced into the system. When we ran our model, kiosk C, in a region with many points of interest, fills up very quickly when many bikes are returned. So, we should put a second kiosk in region C where people can return and take bikes. 10 bikes should go to A, 7 to B, 1 to C, 9 to D, 11 to E and 1 to F. This will allow to program to run for somewhere between 5 and 6 hours on average and accommodate the growth.

Where, now to put a third additional kiosk? With the 30% growth rate, a second kiosk will accommodate 9 more bikes after the fourth year, and there will be 51 total bikes in the system. Running our model, we found that now region C ended up with a *lot* of bikes. It was not excessive or uncontrollable, but it would be helpful if there was another kiosk in the area to collect bikes, too. Thus, the points of interest in the upper right hand corner of the region can be divided up, and a a third kiosk added near kiosk C. We'll need 8 bikes at A, 5 at B, 2 at C, 15 at D, 14 at E, 2 at F, and 5 at G. It is completely successful for 6 hours, but with the exception of Kiosk E, by the lake, it lasts for 9 hours. The more kiosks and bikes, the more complex the model becomes, but with more bikes to circulate, the model tends to last longer.

We understand that these numbers may be confusing. Please don't hesitate to look at our full report, with details of the models and diagrams of the city for details.

We wish for the continuing success of the Bcycle Sharing Program in Des Moines, and hope you find our mathematical model useful in its development over the next five years and in the future.

Sincerely,

The Members of HiMCM Group #2581

***Letters to the other two mayors can be created in a similar format, once our Des Moines "template model" is adjusted to accommodate Chicago and Denver values.**
Chicago: Mayor Richard M. Daley

Denver Mayor John Hickenloope

SOURCES:

http://chicago.bcycle.com/

http://denver.bcycle.com/

http://desmoines.bcycle.com/

http://www.dmcityview.com/2010/09/16/arts/locker.html

http://factfinder.census.gov/servlet/ADPTable?_bm=y&-geo_id=16000US1921000&-qr_name=ACS_2008_3YR_G00_DP3YR5&-ds_name=ACS_2008_3YR_G00_&-_lang=en&-redoLog=false&-_sse=on

http://factfinder.census.gov/servlet/QTTable?_bm=n&_lang=en&qr_name=DEC_2000_SF1_U_DP1&ds_name=DEC_2000_SF1_U&geo_id=16000US0820000

http://chicago.bcycle.com/News.aspx?itemid=35

http://www.bcycle.com/tabid/75/itemid/24207616/news.aspx

http://factfinder.census.gov/servlet/QTTable?_bm=n&_lang=en&qr_name=DEC_2000_SF1_U_DP1&ds_name=DEC_2000_SF1_U&geo_id=16000US0820000

http://mayor.cityofchicago.org/mayor/en/special_pages/mayoral_greeting.html