For office use only		For office use only
T1 ________________		F1 ________________
T2 ________________		F2 ________________
T3 ________________		F3 ________________
T4 ________________		F4 ________________

2012

15th Annual High School Mathematical Contest in Modeling (HiMCM) Summary Sheet
(Please attach a copy of this page to each copy of your Solution Paper.)

Team Control Number: 3681
Problem Chosen: B

Please type a summary of your results on this page. Please remember not to include the name of your school, advisor, or team members on this page.

The average American family spent $368 per month, or $4416 per year, on gasoline in 2011. Gas has gone up by an average of 19 cents per year over the past 9 years. For our model city (New York City), our model saved 1.75% over normal consumers when driving 200 miles a week. This would translate to $77.45 savings per year for the average American family just from optimizing the timing of gas purchases. These numbers explain the immediate importance of this problem to all Americans.

Our model is a multiple regression for the change in price of gas, given past weeks change in gas, past change prices of crude oil, and the displacement of the current price of gas from the average. It uses all linear combinations between past weeks change in gas, and all linear combinations of past change prices of crude oil. The R^2 of our model for our model city, or the percentage of variance in the output variable that is explained by variance in the input variables, is .784. More importantly, the model correctly forecasted the direction of change in price of gas in 40 out of 45 weeks in 2012 after being trained on data from 2011.

We also investigated other methods of calculating the change, finding that a simple Markov process could make many effective predictions. We included this model in our analysis, and used it in our letter to the editor, as it is easy to understand and simple to implement in actual gas buying routine.

We then calculated the savings garnered using the multivariate linear model in 2012 and compared it to the simple Markov model, the model that buys gasoline whenever the tank is empty, the model that fills the tank every week, and the optimal solution, found using Dijkstra's Algorithm. Our model saved more than the simple Markov model in most cases. Variance in savings can be attributed to the severity of errors in the model's predictions.

Our model is consistent in saving money and picking the correct direction of change. It is not as useful for predicting amount of change, and does not predict wild swings with great frequency, but rapidly adapts to them. The simplified Markov model in particular would be very easy and effective to implement in real life.

A Week Ahead of the Gas Market

Team #3681

COMAP HiMCM Competition 2012
Problem B

Summary

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Contents

1 Restatement of the Problem

The price of gasoline is one of the most volatile prices on the consumer market today, and everyone who drives a car has to buy gasoline occasionally, so the question of when to buy gas is an important and difficult one. In order to evaluate when the best time to buy gas is, a model should be created that gives an answer to the question "Should I buy gas this week?" The problem presents a consumer and a car for which the model should be tailored. The car has a 16 gallon gas tank and consistently achieves 25 miles per gallon. The consumer goes to the gas station once each week. Every time the consumer is at the gas station he has to make a choice between three options: filling the gas tank of the car all the way up, filling the gas tank of the car halfway, or not buying any gas.

As a solution, we will create a model that advises the consumer on how much gas to buy every week. The model will be refined with gas price data from 2011 from numerous big cities around the country and tested with gas price data from 2012. This model will include the predicted movement in gas price in the next week and how much gas to buy in the current circumstances.

2 Information Given

1. The gas tank holds 16 gallons.

2. The car will always get 25 miles per gallon.

3. The car will travel 400 miles per tank of gas.

4. The consumer can go to the gas station once a week, as required.

5. The consumer consistently travel either exactly 100 or exactly 200 miles per week.

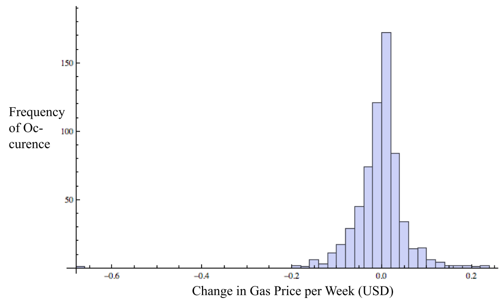


Figure 1: Change in Gas Price per Week

3 Assumptions

1. *Changes in gas price between weeks are normally distributed.*
   Inspection of Figure 1 reveals that the changes in price between weeks are normally distributed.

2. *The government reports accurate gas price data and accurate crude oil price data.*
   It is reasonable to assume that official government data is accurate as the government is required by law to be truthful in their reports.

4 Model Design

First, it is necessary to determine what data is required to predict when to buy gas. If the price of gas is going to go down, it is optimal to not purchase a full tank, whereas if the price is going to rise, it is. Because we need to determine how much gas to buy to maximize savings, the model needs to predict the action the price of gas will take in the next week.

Because the prediction only needs to be used in the short term, a variety of longer term variables were rejected. Seasonal variances due to changes in EPA regulation for the summer and increases in price of gas over a period of years due to inflation were found to be too macroscopic in scope to affect our model's predictions, verified by preliminary statistical analysis. Additionally, variance in travel due to holiday peaks, such as July Fourth, were considered, but were not found to be correlated with gas prices. Catastrophic events such as the Terrorist Attacks of 9/11 or the destruction of the gulf states due to Hurricane Katrina, while influential on gas production, are difficult to impossible to predict, as there were several cases of Class 5 hurricanes that were followed by a decrease in price of gas.

The variables that were consistently significant enough to merit inclusion in our model were "change in crude oil price in the previous week", "change in crude oil price during the week before", "displacement of price per gallon from average price", "change in the price of gas during the previous week", "change in price of gas two weeks previously", and "change in price of gas three weeks ago", as well as multiplicative combinations of some of these.

Preliminary inspection of the data showed heavy amounts of noise, but the price of gas increases from one week to the next, the next week is disproportionately likely to continue to increase, so it was predicted that change during previous week would be a large factor in determining the change in price next week. This was affirmed by our Markov model, which showed that for the majority of cases, the trend the previous week continues.It is worth noting that the 200 mile case and the 100 mile case are identical to the model at the formula stage, as the trend predicted for the next week does not in any case we tested change when attempting to predict a trend two weeks out from the same data.

Table 1: Equation to find the *Change in Price of Gas in the Next Week*

$F(x) = \sum \beta_i f_i + 3.52117 * 10^-6$ where	
β	f
0.0151589	$ChangefromAveragePriceofGas$
0.00534611	$ChangeinPriceofCrudeOil$
−0.000400537	$ChangeinPriceofCrudeOilduringthePreviousWeek$
0.843016	$ChangeinPriceofGasduringthePreviousWeek$
0.0759353	$ChangeinPriceofGasThreeWeeksAgo$
−0.245513	$ChangeinPriceofGasTwoWeeksAgo$

5 Model Testing and Analysis

Our model is a linear regression of multiple variables for the change in price of gas. The sign of the output of the function is used in conjunction with the amount of gas left in the tank to determine action taken.

The variables used in the model are change in the price of crude oil, change in price of crude oil during the previous week, displacement from average price of gas cubed, change in price of gas during the past week, change in price of gas two weeks ago, and change in price of gas three weeks ago. The only input it requires to calculate these six variables and their combinations is crude oil data and gasoline price data.

The model is coded to take input data from 2011 for a given city into the program Mathematica. It can then assess its own success using data from 2012.

5.1 Model City

The process will be demonstrated with New York City, as shown by Figures 3, 4, and 5, Table 1, and Table 2.

Miles Left in Tank	Forecast for Gas Prices for Next Week		
	Up	Stable	Down
0	Fill Tank	Fill Tank	Half Tank
100	Fill Tank	Fill Tank	No Gas
200	Fill Tank	Fill Tank	No Gas
300	Fill Tank	Fill Tank	No Gas
0	Fill Tank	Fill Tank	Half Tank
200	Fill Tank	Fill Tank	No Gas

Figure 2: When Gasoline should be Purchased

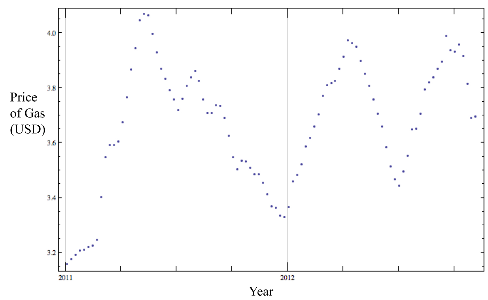


Figure 3: New York City. Price of Gas vs. Time

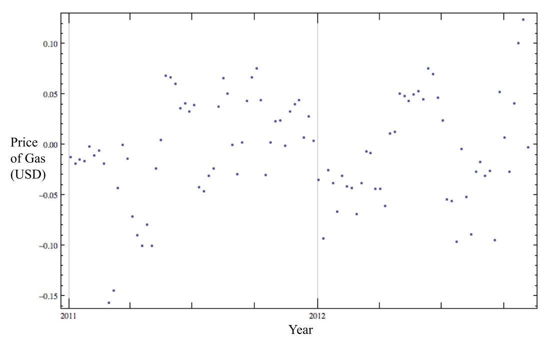


Figure 4: New York City. Change in Price of Gas vs. Time

Table 2: Statistical Values

R^2	Adjusted R^2	Standard Deviation
0.722511	0.685512	0.0278984

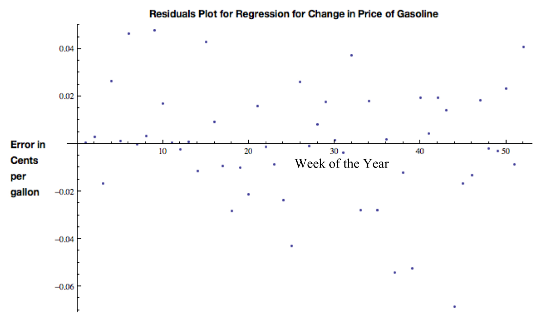


Figure 5: Residual Plot of the Regression for the Change in the Price of Gasoline

In New York City, we predicted a rise in price when the price actually dropped twice and predicted a drop in price when it raised three times. We were correct in the other 40 cases. We spent an average \$3.6636 per gallon over the course of 2012 when deciding whether to buy based on our model for 2011, driving 100 miles per week, while a simple markov model spent \$3.65723, a slight improvement. However, a consumer that bought whenever her car was out of gas would have spent \$3.74991 in that case, and a consumer that filled his car every week would have spent \$3.73764, savings of \$0.0863 and \$0.074, respectively. When driving 200 miles per week, we had less opportunity to save, saving only \$0.0656 and \$0.0602, respectively. In the 200 miles per week case, that is still a minimum savings of

$$(\frac{Dollars}{Gallons})(\frac{Gallon}{Miles})(\frac{Miles}{Week})(\frac{Weeks}{Year})$$
$$= \$50.0885 \text{ per year}$$

5.2 Other Cities

As our model was written for a general case, we also calculated models for Boston, Chicago, Denver, Houston, Los Angeles, and San Francisco.

Boston

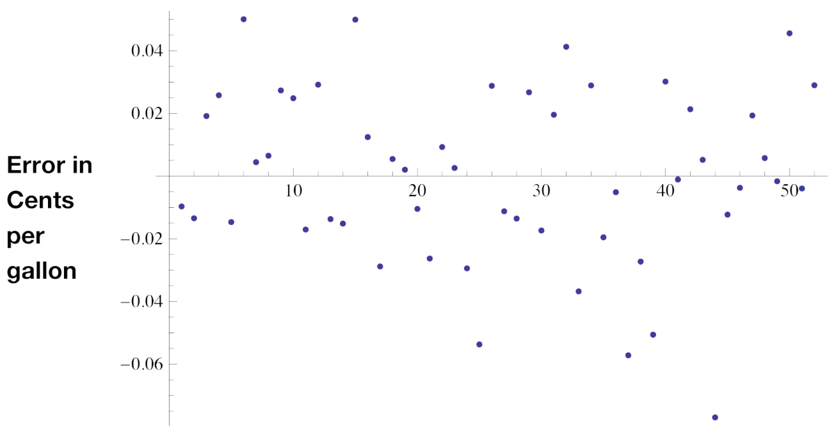
Summary for Model in 2012

Change In Price of Gas in the Next Week=

$$\begin{aligned}(&0.0015914 + 0.185289\, ChangefromAveragePriceofGas^3 + 0.0051392\, ChangeinPriceofCrudeOil + \\ &0.000561036\, ChangeinPriceofCrudeOilduringthePreviousWeek + \\ &0.841624\, ChangeinPriceofGasduringthePreviousWeek + 0.203764\, ChangeinPriceofGasThreeWeeksago - \\ &8.97711\, ChangeinPriceofGasduringthePreviousWeek\, ChangeinPriceofGasThreeWeeksago - \\ &0.160769\, ChangeinPriceofGasTwoWeeksago + \\ &3.10181\, ChangeinPriceofGasduringthePreviousWeek\, ChangeinPriceofGasTwoWeeksago - \\ &0.213953\, ChangeinPriceofGasThreeWeeksago\, ChangeinPriceofGasTwoWeeksago - \\ &67.4354\, ChangeinPriceofGasduringthePreviousWeek \\ &\quad ChangeinPriceofGasThreeWeeksago\, ChangeinPriceofGasTwoWeeksago)\end{aligned}$$

RSquared	AdjustedRSquared	Standard Deviation
0.719768	0.651419	0.027796

Residuals Plot for Regression for Change in Price of Gasoline



Overestimations	Underestimations	Correct
5	5	35

Chicago

Summary for Model in 2012

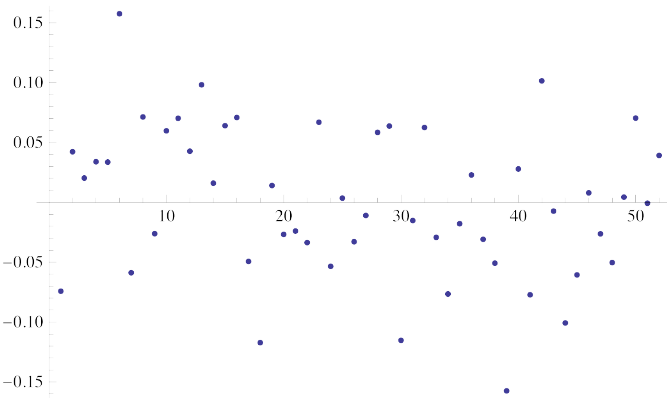
Change In Price of Gas in the Next Week=

$$\begin{aligned}(&0.00505968 + 0.125771\, ChangefromAveragePriceofGas^3 + 0.0102198\, ChangeinPriceofCrudeOil + \\ &0.00172659\, ChangeinPriceofCrudeOilduringthePreviousWeek + \\ &0.231934\, ChangeinPriceofGasduringthePreviousWeek + 0.0373265\, ChangeinPriceofGasThreeWeeksago - \\ &3.81527\, ChangeinPriceofGasduringthePreviousWeek\, ChangeinPriceofGasThreeWeeksago + \\ &0.0410316\, ChangeinPriceofGasTwoWeeksago - \\ &0.107825\, ChangeinPriceofGasduringthePreviousWeek\, ChangeinPriceofGasTwoWeeksago - \\ &0.248825\, ChangeinPriceofGasThreeWeeksago\, ChangeinPriceofGasTwoWeeksago - \\ &2.44598\, ChangeinPriceofGasduringthePreviousWeek \\ &\quad ChangeinPriceofGasThreeWeeksago\, ChangeinPriceofGasTwoWeeksago)\end{aligned}$$

RSquared	AdjustedRSquared	Standard Deviation
0.473065	0.344545	0.0629567

Residuals Plot for Regression for Change in Price of Gasoline

Error in Cents per gallon



Overestimations	Underestimations	Correct
6	10	29

Denver

Summary for Model in 2012

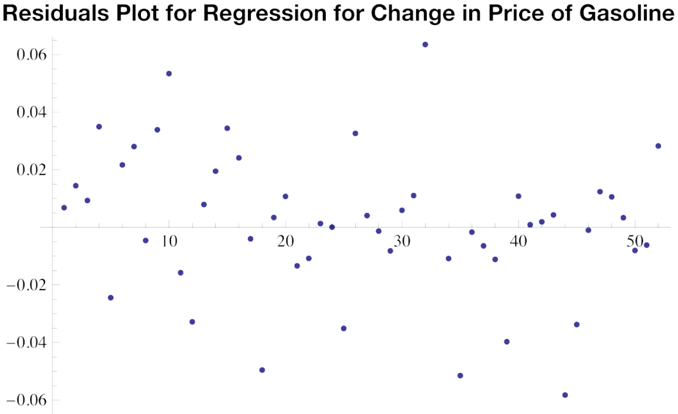
Change In Price of Gas in the Next Week=

$$\begin{aligned}&(0.00311393 + 0.106284\,\text{ChangefromAveragePriceofGas}^3 + 0.00462863\,\text{ChangeinPriceofCrudeOil} -\\ &0.00298968\,\text{ChangeinPriceofCrudeOilduringthePreviousWeek} +\\ &0.993737\,\text{ChangeinPriceofGasduringthePreviousWeek} + 0.134642\,\text{ChangeinPriceofGasThreeWeeksago} +\\ &7.36954\,\text{ChangeinPriceofGasduringthePreviousWeek}\,\text{ChangeinPriceofGasThreeWeeksago} -\\ &0.219468\,\text{ChangeinPriceofGasTwoWeeksago} -\\ &0.660892\,\text{ChangeinPriceofGasduringthePreviousWeek}\,\text{ChangeinPriceofGasTwoWeeksago} -\\ &5.36196\,\text{ChangeinPriceofGasThreeWeeksago}\,\text{ChangeinPriceofGasTwoWeeksago} -\\ &83.0337\,\text{ChangeinPriceofGasduringthePreviousWeek}\\ &\quad\text{ChangeinPriceofGasThreeWeeksago}\,\text{ChangeinPriceofGasTwoWeeksago})\end{aligned}$$

RSquared	AdjustedRSquared	Standard Deviation
0.724367	0.65714	0.0262883

Residuals Plot for Regression for Change in Price of Gasoline

Error in Cents per gallon



Overestimations	Underestimations	Correct
7	3	35

Houston

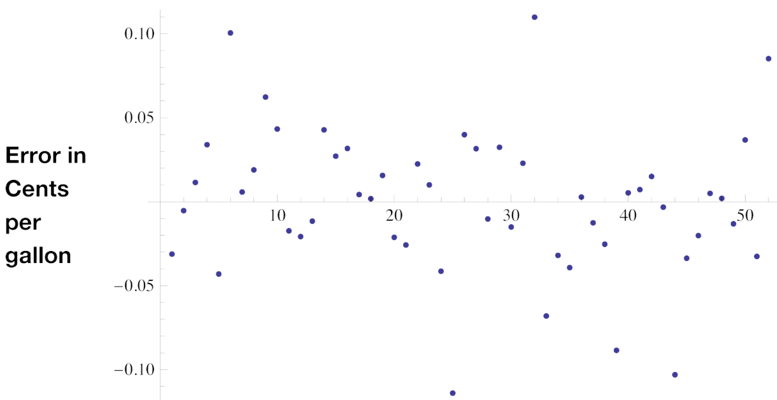
Summary for Model in 2012

Change In Price of Gas in the Next Week=

$$(0.000921406 + 0.13736\, ChangefromAveragePriceofGas^3 + 0.00873782\, ChangeinPriceofCrudeOil - 0.000859655\, ChangeinPriceofCrudeOilduringthePreviousWeek + 0.574556\, ChangeinPriceofGasduringthePreviousWeek - 0.0240324\, ChangeinPriceofGasThreeWeeksago - 2.64401\, ChangeinPriceofGasduringthePreviousWeek\, ChangeinPriceofGasThreeWeeksago - 0.00456609\, ChangeinPriceofGasTwoWeeksago + 1.56781\, ChangeinPriceofGasduringthePreviousWeek\, ChangeinPriceofGasTwoWeeksago - 1.05773\, ChangeinPriceofGasThreeWeeksago\, ChangeinPriceofGasTwoWeeksago - 16.0368\, ChangeinPriceofGasduringthePreviousWeek\, ChangeinPriceofGasThreeWeeksago\, ChangeinPriceofGasTwoWeeksago)$$

RSquared	AdjustedRSquared	Standard Deviation
0.582216	0.480318	0.0434454

Residuals Plot for Regression for Change in Price of Gasoline



Overestimations	Underestimations	Correct
1	4	40

Los Angeles

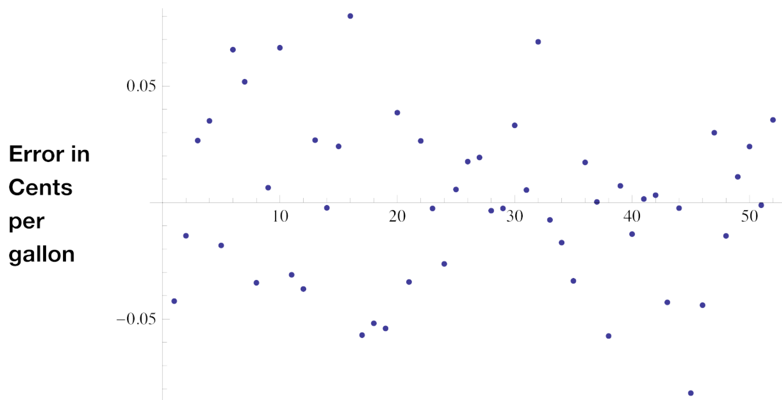
Summary for Model in 2012

Change In Price of Gas in the Next Week=

$$(0.00113018 + 0.143773\, ChangefromAveragePriceofGas^3 + 0.00557721\, ChangeinPriceofCrudeOil - 0.0026362\, ChangeinPriceofCrudeOilduringthePreviousWeek + 0.785457\, ChangeinPriceofGasduringthePreviousWeek + 0.0039152\, ChangeinPriceofGasThreeWeeksago + 0.308037\, ChangeinPriceofGasduringthePreviousWeek\, ChangeinPriceofGasThreeWeeksago - 0.0888733\, ChangeinPriceofGasTwoWeeksago - 0.177449\, ChangeinPriceofGasduringthePreviousWeek\, ChangeinPriceofGasTwoWeeksago - 1.18641\, ChangeinPriceofGasThreeWeeksago\, ChangeinPriceofGasTwoWeeksago - 14.2776\, ChangeinPriceofGasduringthePreviousWeek\, ChangeinPriceofGasThreeWeeksago\, ChangeinPriceofGasTwoWeeksago)$$

RSquared	AdjustedRSquared	Standard Deviation
0.644622	0.557945	0.0357929

Residuals Plot for Regression for Change in Price of Gasoline



Overestimations	Underestimations	Correct
3	8	34

New York City

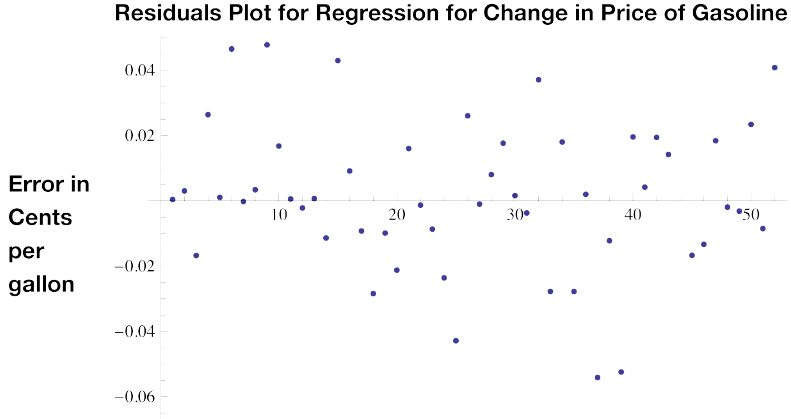
Summary for Model in 2012

Change In Price of Gas in the Next Week=

$$\begin{aligned}&(0.00240156 + 0.155081\ \mathit{ChangefromAveragePriceofGas}^3 + 0.00599815\ \mathit{ChangeinPriceofCrudeOil} -\\&\quad 0.000107701\ \mathit{ChangeinPriceofCrudeOilduringthePreviousWeek} +\\&\quad 0.825186\ \mathit{ChangeinPriceofGasduringthePreviousWeek} + 0.0336517\ \mathit{ChangeinPriceofGasThreeWeeksago} -\\&\quad 7.43102\ \mathit{ChangeinPriceofGasduringthePreviousWeek}\ \mathit{ChangeinPriceofGasThreeWeeksago} -\\&\quad 0.0658221\ \mathit{ChangeinPriceofGasTwoWeeksago} +\\&\quad 2.60236\ \mathit{ChangeinPriceofGasduringthePreviousWeek}\ \mathit{ChangeinPriceofGasTwoWeeksago} -\\&\quad 0.932797\ \mathit{ChangeinPriceofGasThreeWeeksago}\ \mathit{ChangeinPriceofGasTwoWeeksago} -\\&\quad 57.0042\ \mathit{ChangeinPriceofGasduringthePreviousWeek}\\&\qquad \mathit{ChangeinPriceofGasThreeWeeksago}\ \mathit{ChangeinPriceofGasTwoWeeksago})\end{aligned}$$

RSquared	AdjustedRSquared	Standard Deviation
0.784195	0.731559	0.024603

Residuals Plot for Regression for Change in Price of Gasoline



Overestimations	Underestimations	Correct
2	3	40

San Francisco

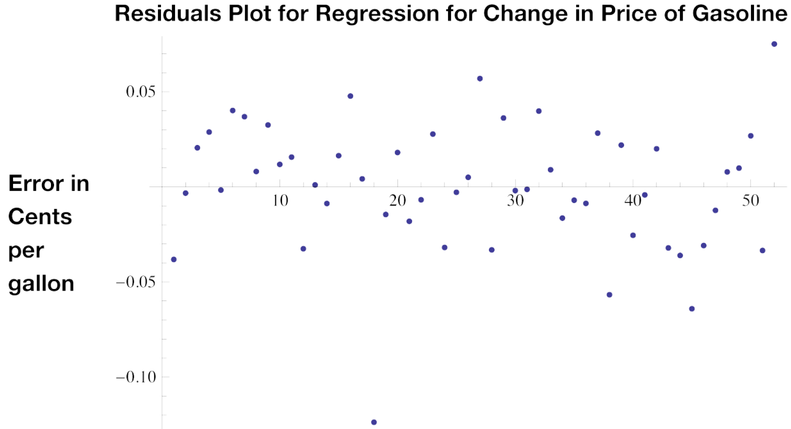
Summary for Model in 2012

Change In Price of Gas in the Next Week=

$$\left(-0.000940027 + 0.121526\,\textit{ChangefromAveragePriceofGas}^3 + 0.0051018\,\textit{ChangeinPriceofCrudeOil} - 0.00133882\,\textit{ChangeinPriceofCrudeOilduringthePreviousWeek} + 0.886847\,\textit{ChangeinPriceofGasduringthePreviousWeek} - 0.0330975\,\textit{ChangeinPriceofGasThreeWeeksago} - 2.75727\,\textit{ChangeinPriceofGasduringthePreviousWeek}\,\textit{ChangeinPriceofGasThreeWeeksago} - 0.116802\,\textit{ChangeinPriceofGasTwoWeeksago} + 2.52542\,\textit{ChangeinPriceofGasduringthePreviousWeek}\,\textit{ChangeinPriceofGasTwoWeeksago} - 0.908223\,\textit{ChangeinPriceofGasThreeWeeksago}\,\textit{ChangeinPriceofGasTwoWeeksago} - 14.3465\,\textit{ChangeinPriceofGasduringthePreviousWeek}\,\textit{ChangeinPriceofGasThreeWeeksago}\,\textit{ChangeinPriceofGasTwoWeeksago}\right)$$

RSquared	AdjustedRSquared	Standard Deviation
0.677575	0.598935	0.0335505

Residuals Plot for Regression for Change in Price of Gasoline



Overestimations	Underestimations	Correct
3	9	33

100 mi/week	Average Money Spent on Gasoline per Gallon for the 2012 Year (USD)				
Cites	Under our model	Under the Markov model	Filling only when on empty	Filling to full every week	Optimal model (Dijkstra's Algorithm)
Boston	3.58739	3.57170	3.67000	3.66153	3.54563
Chicago	3.82763	3.91729	3.94164	3.92016	3.74551
Denver	3.42832	3.45288	3.51964	3.51800	3.41200
Houston	3.43667	3.41815	3.56518	3.55422	3.43142
Los Angeles	4.09271	4.05150	4.16445	4.15087	3.98009
New York City	3.66838	3.65723	3.74991	3.73764	3.61830
San Francisco	4.05080	3.98675	4.11055	4.10349	3.93902

Figure 6: Average Money Spent per Gallon of Gasoline When Travelled 100 miles

200 mi/week	Average Money Spent on Gasoline per Gallon for the 2012 Year (USD)				
Cites	Under our model	Under the Markov model	Filling only when on empty	Filling to full every week	Optimal model (Dijkstra's Algorithm)
Boston	3.60179	3.58332	3.66777	3.66153	3.54563
Chicago	3.84495	3.93610	3.93755	3.92016	3.74551
Denver	3.44814	3.47329	3.52632	3.51800	3.41200
Houston	3.45066	3.42977	3.56336	3.55422	3.43142
Los Angeles	4.11060	4.06619	4.15382	4.15087	3.98009
New York City	3.68244	3.66787	3.74309	3.73764	3.61830
San Francisco	4.07098	4.00264	4.10473	4.10349	3.93902

Figure 7: Average Money Spent per Gallon of Gasoline When Travelled 200 miles

6 Alternative Approaches

There are many possible ways to approach this problem, of which many are not insightful to the question of whether or not to buy gas during the next week. Some other approaches we attempted were Wavelet Analysis and Markov Chain Analysis.

In Wavelet Analysis, the data is analyzed and then represented by a series of wavelets. By ignoring the wavelets that account for less than a threshold amount of the system's energy, it is possible to reduce noise vastly. This proved not useful to modeling this situation because getting rid of that noise that operates on a microscopic level renders the model useless to making predictions from one week to another.

Markov Chain Analysis is more successful. Markov Chains work by taking only the previous state of the system as input for the next state. We constructed the transition matrix using excel. For Boston,

$$A = \begin{bmatrix} 0.390243902 & 0.119918699 \\ 0.119918699 & 0.369918699 \end{bmatrix}$$

where $A_{i,j}$ indicates the probability that if it is in state i, it will transition to state j, with 1 being an increase in price and 2 being a decrease. By predicting that the highest probability in a column will come true in that case, we can make decisions about the price of gas given the price last week. For our case, this amounts to simply assuming that the trend from last week will hold during this week. A more complex matrix, using either the trend from the week before last week or the sign of the most recent change in trend, makes the same predictions, and thus the same errors. The error percentage of the model for Boston when applied to the 2012 data is 20%. We also constructed a larger Markov matrix using partitions of the data, however the same action was always taken, so no benefit was gained. Still, the very simplistic model

If the price increased during the previous week, buy as much gas as possible.
If the price declined during the previous week, buy as little gas as possible.

saved a similar amount of money per gallon over the course of the year to the much more involved Multivariate Linear Model.

7 Strengths and Weaknesses

7.1 Strengths

Despite gas price data having a massive amount of statistical noise, our model is capable of explaining the majority of the variation in the price in gasoline and predicting the change is price for the next week. The model is correct about the direction in which the prices will move 89% of the time. It only uses two data sets to calculate its decisionpast two weeks of crude oil price values and past three weeks of gas price data for the given city. The models high R^2 value indicates that a large portion of the variance in change of price in gasoline is explained by our model.

Overall, the model accomplishes its goal of saving the consumer money. As compared with other more simple models, such as filling the tank every week or only filling the tank when on empty, the model saves more than 6 cents per gallon in every city we tested. The average consumer uses one of these basic models, so the average consumer would save an average of 9 cents per gallon when driving 100 miles per week. The model is also more effective than the Markov Chain model in saving the consumer money.

This model was a good choice for the problem, using a full first order regression on the three previous weeks of change in price of gas, a cubic component of the displacement from the mean price, and a full first order regression on the two previous weeks change in price of crude oil. This effectively calculated short term differences. A model that used longer term variables would not be as reliable for one week predictions, and a model that used freak variables to account for potential spikes such as Katrina would not be possible to use, as level of devastation is immensely hard to predict.

Additionally, we provide an easy to use variant that can be easily implemented with just a post-it note on the dashboard or a smartphone notes application.

7.2 Weaknesses

The model does not accurately predict sudden changes in direction of gas due to the retrospective nature of the model. If the second-order derivative of the gas price function is near in the range used to forecast for the model, the model cannot always predict a drastic change in the slope of the cost function. Thus the model is prone to overshooting or undershooting values when the cost of gas suddenly changes. However, the model is self-correcting, and will return to the cost function after an unexpected change in slope. This weakness is minimized with increasing density of data.

Such sudden changes in slope may be caused by several factors. For example, in 2005, Hurricane Katrina severely damaged several refineries, and caused gas process to spike dramatically. Our model cannot predict such catastrophes. Another example that our model cannot predict includes terrorist activities.

One bad decision — buying when the price of gas is about to fall by a lot — can result in a much higher price per gallon than optimal. This is due partly to simple bad luck: it can just so happen that one of the few mistakes our model makes occurs when it is able to buy a full tank of gas at a very high rate.

8 Letter to the Editor

Dear Good Times Dispatch,

High gas prices have been affecting people for years. By buying fuel when the price is at its lowest, it is possible to save money on gas, however the seemingly random nature of change of gas prices makes it difficult to know when fuel prices are about to rise or drop. Attempting to save money on fuel is hardly an esoteric concern, but it is not easy to actually save money. It may seem like too much work to do the math to figure out when the optimal times to buy guess are, especially since it only shaves cents off the price, with a risk of not saving any money at all. Without going too far into the calculations, I would like you to know that it is not only very possible to make significant savings over time, but very easy. If you already take the time to drive around hunting for the lowest gas price, you won't find this difficult to add to your routine at all.

A simple trend easily observed in the data is that the change in gas price between weeks tends to not change direction more often than it does change direction. This means that if the price is going up it will most likely continue to go up and if the price is going down it will most likely continue to go down. When you drive by the gas station you normally buy from on the day you normally buy on, write down the price. If, next week, the price has gone up, it would be in your best interests to fill your tank of gas, as the price is likely to continue to increase. If the price has decreased, only buy gas if you are out. You can put a sticky note with the numbers on your dash, or record the data on a smartphone.

For a consumer driving 200 miles a week in New York City in 2012, following this process will save $62.58 if the average savings hold through the end of the year. The same model followed in Houston under the same conditions will save $113.5 over the course of the year. Would $113.5 make it worth it to you to record a few pieces of data?

Sincerely,
A Mathematical Concerned Citizen

References

[1] Chevron. "How Fuel Is Delivered." The Price of Fuel. Last modified 2011. http://www.thepriceoffuel.com/howfuelisdelivered/.

[2] CNN. "Your Monthly Gas Bill: $368." CNN Money. Last modified May 6, 2011. http://money.cnn.com/2011/05/05/news/economy/gas_prices_income_spending/index.htm.

[3] "List of Category 5 Atlantic Hurricanes." In *Wikipedia*. http://en.wikipedia.org/wiki/List_of_Category_5_Atlantic_hurricanes.

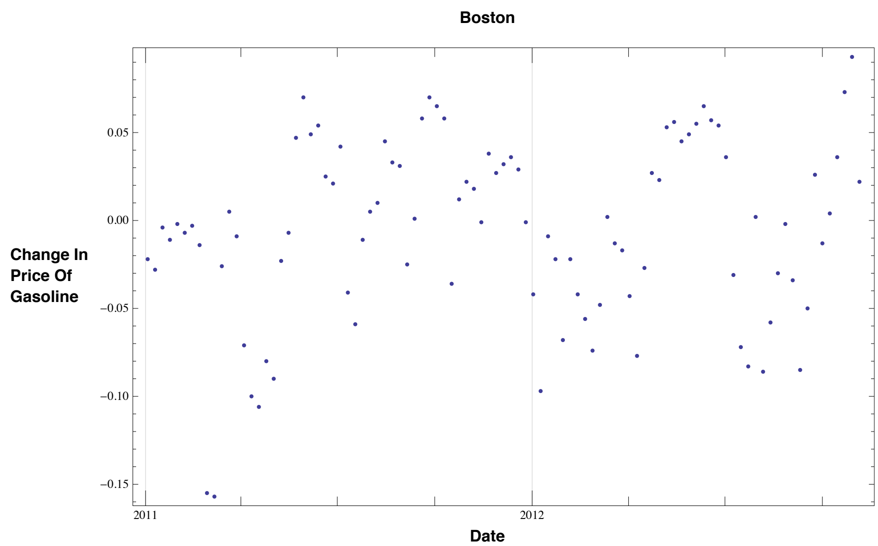
[4] U.S. Department of Energy. "Weekly Retail Gasoline and Diesel Prices." U.S. Energy Information Administration. Last modified November 5, 2012. http://www.eia.gov/dnav/pet/pet_pri_gnd_a_epmr_pte_dpgal_w.htm.

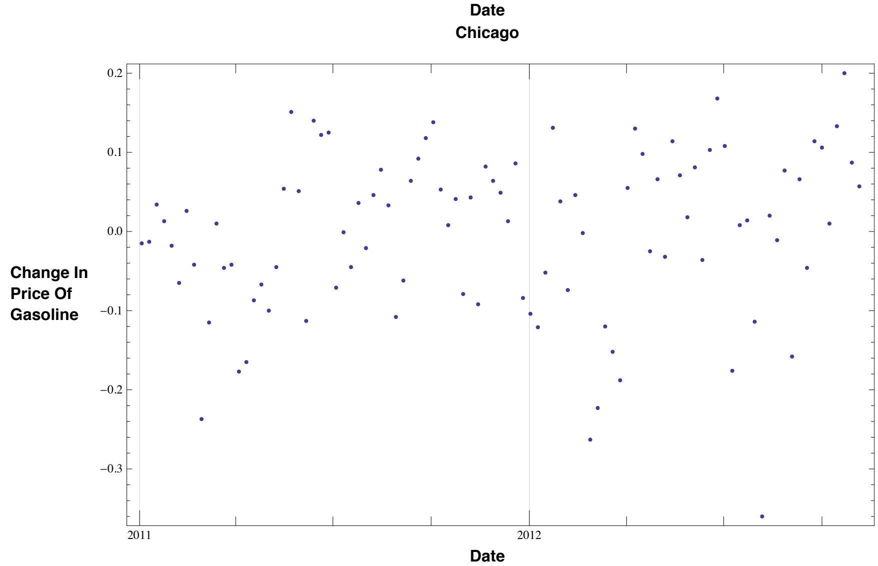
[5] "Weekly Boston, MA Regular All Formulations Retail Gasoline Prices." Table. U.S. Energy Information Administration. November 6, 2012. http://www.eia.gov/dnav/pet/hist/LeafHandler.ashx?n=PET&s=EMM_EPMR_PTE_YBOS_DPG&f=W.

A Example Data

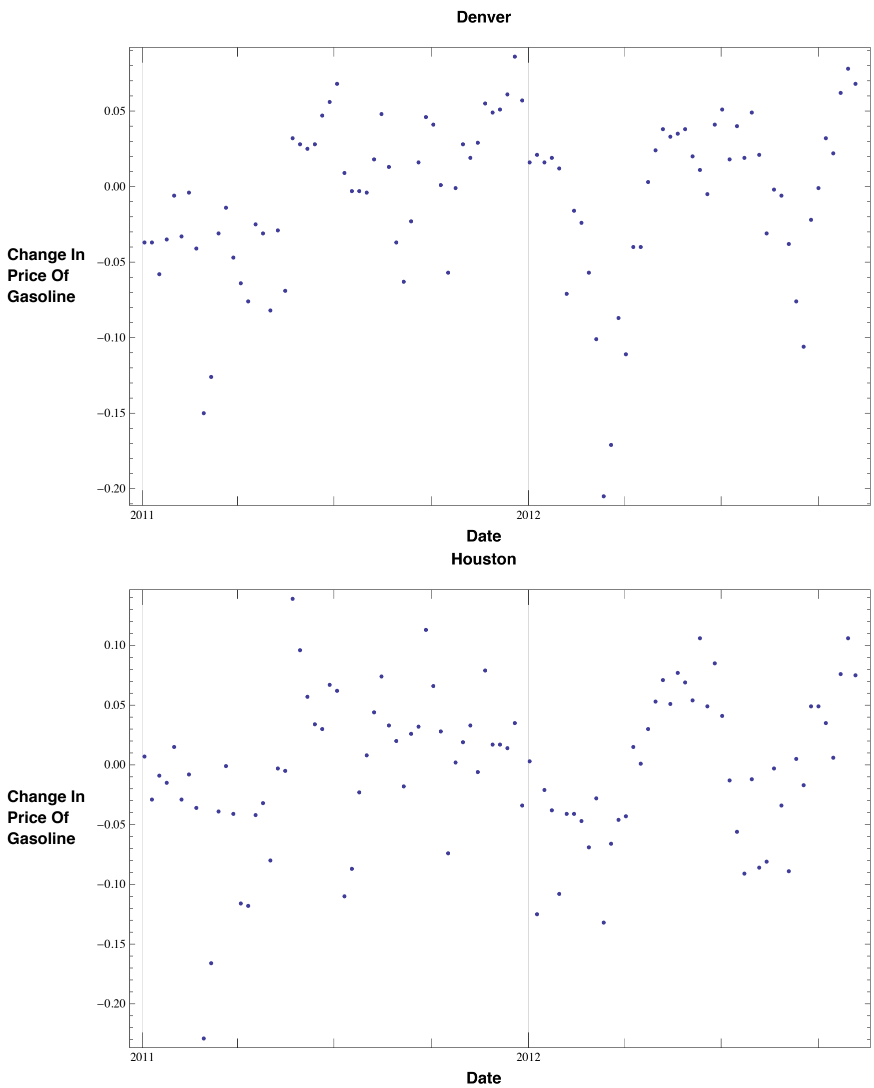
Weekly New York City Regular All Formulations Retail Gasoline Prices	
http://www.eia.gov/dnav/pet/hist/LeafHandler.ashx?n=PET&s=EMM_EPMR_PTE_Y35NY_DPG&f=W	
19:50:02 GMT-0500 (EST)	
Source: U.S. Energy Information Administration	
Week of	Weekly New York City Regular All Formulations Retail Gasoline Prices (USD)
Nov 5, 2012	3.695
Oct 29, 2012	3.692
Oct 22, 2012	3.816
Oct 15, 2012	3.917
Oct 8, 2012	3.958
Oct 1, 2012	3.931
Sep 24, 2012	3.938
Sep 17, 2012	3.99
Sep 10, 2012	3.895
Sep 3, 2012	3.869
Aug 27, 2012	3.838
Aug 20, 2012	3.821
Aug 13, 2012	3.794
Aug 6, 2012	3.705
Jul 30, 2012	3.653
Jul 23, 2012	3.649
Jul 16, 2012	3.553
Jul 9, 2012	3.497
Jul 2, 2012	3.443
Jun 25, 2012	3.467
Jun 18, 2012	3.514
Jun 11, 2012	3.584
Jun 4, 2012	3.66
May 28, 2012	3.705
May 21, 2012	3.758
May 14, 2012	3.808
May 7, 2012	3.851
Apr 30, 2012	3.899
Apr 23, 2012	3.95
Apr 16, 2012	3.963
Apr 9, 2012	3.974
Apr 2, 2012	3.913
Mar 26, 2012	3.869
Mar 19, 2012	3.825
Mar 12, 2012	3.817
Mar 5, 2012	3.81
Feb 27, 2012	3.772
Feb 20, 2012	3.703
Feb 13, 2012	3.66
Feb 6, 2012	3.619
Jan 30, 2012	3.588
Jan 23, 2012	3.522
Jan 16, 2012	3.484
Jan 9, 2012	3.459
Jan 2, 2012	3.366
Dec 26, 2011	3.331

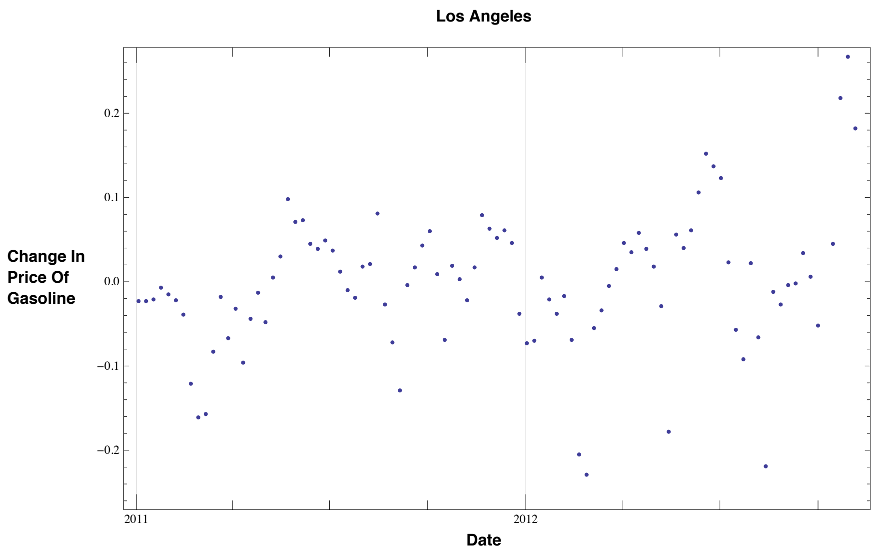
B Data Charts

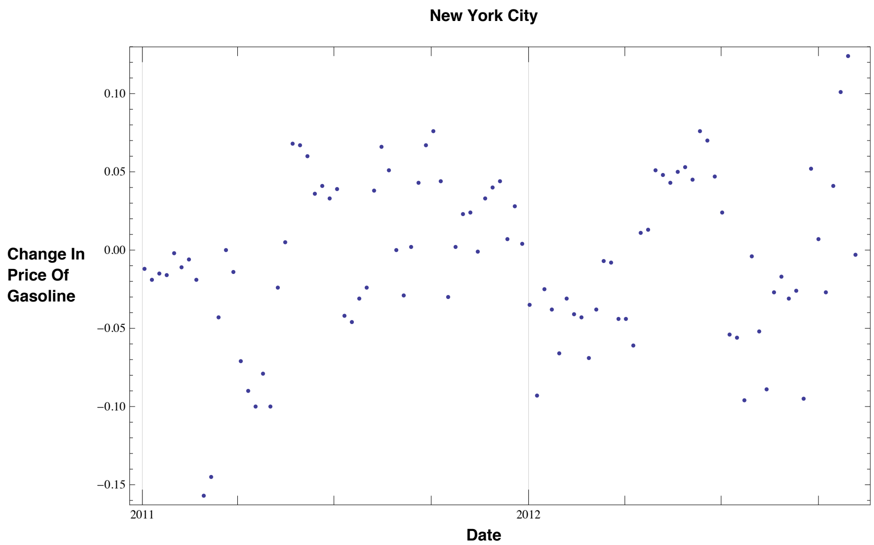

Boston
0.05
0.00
−0.05
−0.10
−0.15
Change In Price Of Gasoline
2011
2012
Date

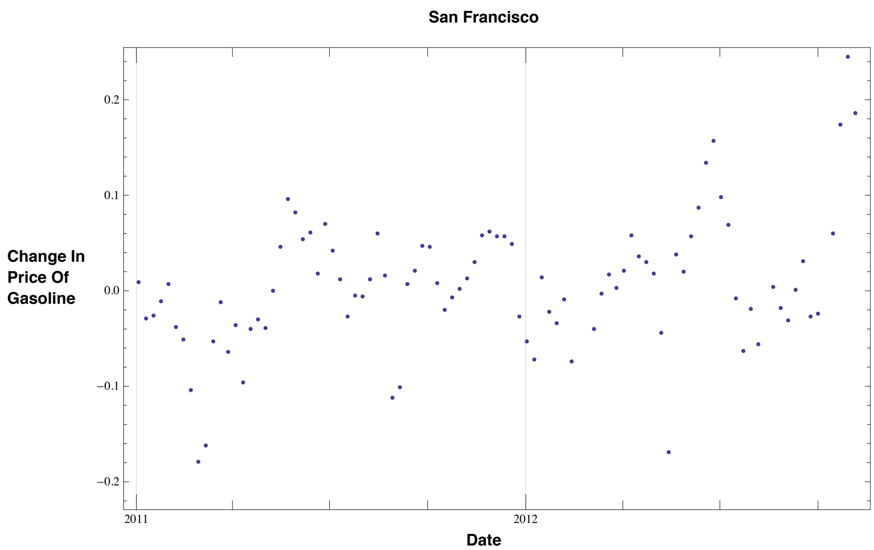

Chicago
0.2
0.1
0.0
−0.1
−0.2
−0.3
Change In Price Of Gasoline
2011
2012
Date

Out[738]=


Denver
Change In Price Of Gasoline
0.05
0.00
−0.05
−0.10
−0.15
−0.20
2011
2012
Date
Houston
Change In Price Of Gasoline
0.10
0.05
0.00
−0.05
−0.10
−0.15
−0.20
2011
2012
Date


Los Angeles
Change In Price Of Gasoline
0.2
0.1
0.0
−0.1
−0.2
2011
2012
Date


New York City
Change In Price Of Gasoline
0.10
0.05
0.00
−0.05
−0.10
−0.15
2011
2012
Date


San Francisco
Change In
Price Of
Gasoline
0.2
0.1
0.0
−0.1
−0.2
2011
2012
Date

C Mathematica Code

```
(*In our case, the data was downloaded to the desktop of a macintosh computer from http://
   www.eia.gov/dnav/pet/pet_pri_gnd_a_epmr_pte_dpgal_w.htm*)bostonGasData =
      Drop[Import["Desktop/Weekly Boston MA Regular All Formulations Retail Gasoline Prices.csv"], 5];
chicagoGasData = Drop[Import[
         "Desktop/Weekly Chicago Regular All Formulations Retail Gasoline Prices.csv"], 5];
denverGasData = Drop[Import["Desktop/Weekly Denver Regular All Formulations Retail Gasoline Prices.csv"],
      5]; houstonGasData =
   Drop[Import["Desktop/Weekly Houston All Grades All Formulations Retail Gasoline Prices.csv"], 5];
laGasData =
      Drop[Import["Desktop/Weekly Los Angeles Regular All Formulations Retail Gasoline Prices.csv"], 5];
nycGasData = Drop[Import[
            "Desktop/Weekly New York City Regular All Formulations Retail Gasoline Prices.csv"], 5];
sanfranGasData = Drop[Import[
            "Desktop/Weekly San Francisco Regular All Formulations Retail Gasoline Prices.csv"], 5];
(*We used the crude oil price data from WTI, Cushing, Oklahoma.*)
cushingCrudeData = Drop[Import["Desktop/Weekly Cushing OK WTI Spot Price FOB.csv"], 5];
```

```
(*This function takes a city's data as input and responds with a multiple regression for the change in the price
   of gas from the previous week with the variables "change in crude oil price in the previous week",
"change in crude oil price in the week before", "change in price per gallon from average price",
"change in the price of gas during the previous week", "change in price of gas two weeks previously",
and "change in price of gas three weeks ago". This uses data from 2011,
with two weeks from 2010 included for use in calculating the last two variables.*)
linearModel2011[citydata_] := LinearModelFit[Transpose[{
        Take[Drop[Differences[cushingCrudeData[[All, 2]]], 45], 52],
        Take[Drop[Differences[cushingCrudeData[[All, 2]]], 46], 52],
        Take[Drop[citydata[[All, 2]] - (Plus @@ Take[Drop[citydata[[All, 2]], 46], 52])/52, 46], 52],
        Take[Differences[Drop[citydata[[All, 2]], 46]], 52], Take[Differences[Drop[citydata[[All, 2]], 47]], 52],
        Take[Differences[Drop[citydata[[All, 2]], 48]], 52], Take[Differences[Drop[citydata[[All, 2]], 45]], 52]}],
    {ChangeinPriceofCrudeOil, ChangeinPriceofCrudeOilduringthePreviousWeek,
      ChangefromAveragePriceofGas^3, ChangeinPriceofGasduringthePreviousWeek,
      ChangeinPriceofGasTwoWeeksago, ChangeinPriceofGasThreeWeeksago,
      ChangeinPriceofGasduringthePreviousWeek*ChangeinPriceofGasThreeWeeksago,
      ChangeinPriceofGasduringthePreviousWeek*ChangeinPriceofGasTwoWeeksago,
      ChangeinPriceofGasduringthePreviousWeek*
        ChangeinPriceofGasTwoWeeksago*ChangeinPriceofGasThreeWeeksago,
      ChangeinPriceofGasTwoWeeksago*ChangeinPriceofGasThreeWeeksago},
    {ChangeinPriceofCrudeOil, ChangeinPriceofCrudeOilduringthePreviousWeek,
      ChangefromAveragePriceofGas, ChangeinPriceofGasduringthePreviousWeek,
      ChangeinPriceofGasTwoWeeksago, ChangeinPriceofGasThreeWeeksago}];
linearModel2011[citydata_, n_] := LinearModelFit[Transpose[{
        Take[Drop[Differences[cushingCrudeData[[All, 2]]], 45 + n], 52],
        Take[Drop[Differences[cushingCrudeData[[All, 2]]], 46 + n], 52],
        Take[Drop[citydata[[All, 2]] - (Plus @@ Take[Drop[citydata[[All, 2]], 46], 52])/52, 46], 52],
        Take[Differences[Drop[citydata[[All, 2]], 46 + n]], 52],
        Take[Differences[Drop[citydata[[All, 2]], 47 + n]], 52], Take[Differences[Drop[citydata[[All, 2]], 48 + n]],
          52], Take[Differences[Drop[citydata[[All, 2]], 45 + n]], 52]}],
    {ChangeinPriceofCrudeOil, ChangeinPriceofCrudeOilduringthePreviousWeek,
      ChangefromAveragePriceofGas^3, ChangeinPriceofGasduringthePreviousWeek,
      ChangeinPriceofGasTwoWeeksago, ChangeinPriceofGasThreeWeeksago,
      ChangeinPriceofGasduringthePreviousWeek*ChangeinPriceofGasThreeWeeksago,
      ChangeinPriceofGasduringthePreviousWeek*ChangeinPriceofGasTwoWeeksago,
      ChangeinPriceofGasduringthePreviousWeek*
        ChangeinPriceofGasTwoWeeksago*ChangeinPriceofGasThreeWeeksago,
      ChangeinPriceofGasTwoWeeksago*ChangeinPriceofGasThreeWeeksago},
    {ChangeinPriceofCrudeOil, ChangeinPriceofCrudeOilduringthePreviousWeek,
      ChangefromAveragePriceofGas, ChangeinPriceofGasduringthePreviousWeek,
      ChangeinPriceofGasTwoWeeksago, ChangeinPriceofGasThreeWeeksago}]
```

```
In[530]:= (*This gives summary statistics of the model for a given city,
including the model's equation, the RSquared, Adjusted RSquared, and Standard Deviation,
and a labelled Residuals Plot. Included at the bottom is the number of overestimations
  (when the model predicted that gas prices would go up and they instead declined),
underestimations, and when the model correctly predicted the direction that gas prices would
  travel. Of note is that we assumed that gas prices remaining constant could be considered
  gas prices not decreasing when considering whether to delay purchasing a full tank,
and thus should be predicted as an increase. The Summary for 2011 explains our results
  when interpolating and the Summary for 2012 explains our results when extrapolating*)
summary2011[citydata_] := (Print["Summary for Model in 2011"];
      Block[{lm = linearModel2011[citydata], m = HeavisideTheta[linearModel2011[citydata] @@@ Transpose[{
                  Take[Drop[Differences[cushingCrudeData[[All, 2]]], 45], 52],
                  Take[Drop[Differences[cushingCrudeData[[All, 2]]], 46], 52], Take[
                    Drop[nycGasData[[All, 2]] - (Plus @@ Take[Drop[citydata[[All, 2]], 46], 52])/52, 46], 52],
                  Take[Differences[Drop[citydata[[All, 2]], 46]], 52], Take[Differences[
                      Drop[citydata[[All, 2]], 47]], 52], Take[Differences[Drop[citydata[[All, 2]], 48]], 52]}]] -
            (If[# == 0, 1, HeavisideTheta[#]] & /@ Take[Drop[Differences[citydata[[All, 2]]], 45], 52])},
        Print["Change In Price of Gas in the Next Week=" Normal[lm]];
        Print[Grid[{{"RSquared", "AdjustedRSquared", "Standard Deviation"},
            {lm["RSquared"], lm["AdjustedRSquared"], StandardDeviation[lm["FitResiduals"]]}}]];
        Print[Labeled[ListPlot[lm["FitResiduals"], ImageSize → Large],
          {"Residuals Plot for Regression for Change in Price of Gasoline", "Error in
Cents
per
gallon"}, {Top, Left}]];
        Print[Grid[{{"Overestimations", "Underestimations", "Correct"},
            {Count[m, 1], Count[m, -1], Count[m, 0]}}]];]);
summary2012[citydata_] := (Print["Summary for Model in 2012"];
    Block[{lm = linearModel2011[citydata], m = HeavisideTheta[linearModel2011[citydata] @@@ Transpose[{
                Take[Differences[cushingCrudeData[[All, 2]]], 45],
                Take[Drop[Differences[cushingCrudeData[[All, 2]]], 1], 45],
                Take[Drop[nycGasData[[All, 2]] - (Plus @@ Take[Drop[citydata[[All, 2]], 1], 45])/45, 46], 45],
                Take[Differences[Drop[citydata[[All, 2]], 1]], 45], Take[Differences[Drop[citydata[[All, 2]], 2]],
                  45], Take[Differences[Drop[citydata[[All, 2]], 3]], 45]}]] -
          (If[# == 0, 1, HeavisideTheta[#]] & /@ Take[Differences[citydata[[All, 2]]], 45])},
      Print["Change In Price of Gas in the Next Week=" Normal[lm]];
      Print[Grid[{{"RSquared", "AdjustedRSquared", "Standard Deviation"},
          {lm["RSquared"], lm["AdjustedRSquared"], StandardDeviation[lm["FitResiduals"]]}}]];
      Print[Labeled[ListPlot[lm["FitResiduals"], ImageSize → Large],
        {"Residuals Plot for Regression for Change in Price of Gasoline", "Error in
Cents
per
gallon"}, {Top, Left}]]; Print[Grid[
        {{"Overestimations", "Underestimations", "Correct"}, {Count[m, 1], Count[m, -1], Count[m, 0]}}]];])

In[645]:= (*This shows the summary for extrapolation to 2012 for our model for each of the cities in alphabetic order.*)
(Print[#[[1]]]; summary2012[#[[2]]]) & /@ {{"Boston", bostonGasData},
    {"Chicago", chicagoGasData}, {"Denver", denverGasData}, {"Houston", houstonGasData},
    {"Los Angeles", laGasData}, {"New York City", nycGasData}, {"San Francisco", sanfranGasData}}

In[672]:= (*This segment of code calculates the dollars spent on gas through the course of 2012 by our algorithm,
a simpler model, someone that buys when their fuel tank is empty,
and someone that fills their tank every week. It then calculates the average cost per gallon,
```

```
and returns a list of cities and their average prices. It gives output in the form {city, model price per gallon,
    savings over markov price per gallon, savings over buy when empty price per gallon, savings over
      always buy price per gallon. It takes input in a slightly different format, so has a redone import.*)
bostonGasData = {DateList[{#[[1]], {"Month", "Day", "Year"}}], #[[2]]} & /@
      Drop[Import["Desktop/Weekly Boston MA Regular All Formulations Retail Gasoline Prices.csv"], 5];
chicagoGasData = {DateList[{#[[1]], {"Month", "Day", "Year"}}], #[[2]]} & /@
    Drop[Import["Desktop/Weekly Chicago Regular All Formulations Retail Gasoline Prices.csv"], 5];
denverGasData = {DateList[{#[[1]], {"Month", "Day", "Year"}}], #[[2]]} & /@
    Drop[Import["Desktop/Weekly Denver Regular All Formulations Retail Gasoline Prices.csv"], 5];
houstonGasData = {DateList[{#[[1]], {"Month", "Day", "Year"}}], #[[2]]} & /@
    Drop[Import["Desktop/Weekly Houston All Grades All Formulations Retail Gasoline Prices.csv"], 5];
laGasData = {DateList[{#[[1]], {"Month", "Day", "Year"}}], #[[2]]} & /@
      Drop[Import["Desktop/Weekly Los Angeles Regular All Formulations Retail Gasoline Prices.csv"], 5];
miamiGasData = {DateList[{#[[1]], {"Month", "Day", "Year"}}], #[[2]]} & /@
      Drop[Import["Desktop/Weekly Miami FL Regular All Formulations Retail Gasoline Prices.csv"], 5];
nycGasData = {DateList[{#[[1]], {"Month", "Day", "Year"}}], #[[2]]} & /@
      Drop[Import["Desktop/Weekly New York City Regular All Formulations Retail Gasoline Prices.csv"], 5];
sanfranGasData = {DateList[{#[[1]], {"Month", "Day", "Year"}}], #[[2]]} & /@ Drop[
        Import["Desktop/Weekly San Francisco Regular All Formulations Retail Gasoline Prices.csv"], 5];;
gasTankSize = 16;
mileage = 25;
totalYearPrice[cityData_, year_, milesPerWeek_] := {Total[MapIndexed[
        If[Simplify[(#2/((gasTankSize mileage)/milesPerWeek)) ∈ Integers],
                  gasTankSize #1,
                  0
          ] &,
        (#[[2]] & /@ Select[cityData, #[[1]][[1]] == year &])
      ]], gasTankSize IntegerPart[
        Length[
            (#[[2]] & /@ Select[cityData, #[[1]][[1]] == year &])
          ]/((gasTankSize mileage)/milesPerWeek)]}
totalYearPriceAlwaysFill[cityData_, year_, milesPerWeek_] :=
  {Total[(milesPerWeek/mileage ) # & /@ (#[[2]] & /@ Select[cityData, #[[1]][[1]] == year &])],
    (milesPerWeek/mileage ) Length[(#[[2]] & /@ Select[cityData, #[[1]][[1]] == year &])]}

out[citydata_] :=
    Apply[linearModel2011[citydata], Transpose[{Take[Differences[cushingCrudeData[[All, 2]]], 45],
        Take[Drop[Differences[cushingCrudeData[[All, 2]]], 1], 45],
        Take[Drop[nycGasData[[All, 2]] - Plus @@ Take[Drop[citydata[[All, 2]], 1], 45]/45, 46], 45],
        Take[Differences[Drop[citydata[[All, 2]], 1]], 45], Take[Differences[Drop[citydata[[All, 2]], 2]], 45],
        Take[Differences[Drop[citydata[[All, 2]], 3]], 45]}], {1}];

totalYearPriceModeled[cityData_, milesPerWeek_] := Block[
    {weeks = Reverse[out[cityData]], currentGas = milesPerWeek/mileage, total = 0,
      prices = Reverse[#[[2]] & /@ Select[cityData, #[[1]][[1]] == 2012 &]], totalGas = 0},
    For[week = 1, week ≤ Length[weeks], week++,
      currentGas -= (milesPerWeek/mileage);
      If[ weeks[[week]] < 0,
        total += (gasTankSize - currentGas) prices[[week]];
        totalGas += (gasTankSize - currentGas);
        currentGas = gasTankSize;, (*buy*),
```

```
      If[currentGas == 0,
          total += (gasTankSize/2) prices⟦week⟧; (*don't buy*)
          totalGas += (gasTankSize/2);
          currentGas = gasTankSize/2;];
    ];
  ];
  {total, totalGas}
]; totalYearPriceSimpleModeled[cityData_, milesPerWeek_] := Block[
  {currentGas = milesPerWeek/mileage, total = 0,
    prices = Reverse[#⟦2⟧ & /@ Select[cityData, #⟦1⟧⟦1⟧ == 2012 &]], totalGas = 0},
  For[week = 1, week ≤ Length[prices], week++,
    currentGas −= (milesPerWeek/mileage);
    If[ (prices⟦week⟧ − prices⟦week − 1⟧) ≥ 0,
      total += (gasTankSize − currentGas) prices⟦week⟧;
      totalGas += (gasTankSize − currentGas);
      currentGas = gasTankSize;, (*buy*),
      If[currentGas == 0,
          total += (gasTankSize/2) prices⟦week⟧; (*don't buy*)
          totalGas += (gasTankSize/2);
          currentGas = gasTankSize/2;];
    ];
  ];
  {total, totalGas}
]; Block[{miles = 200},
{#⟦1⟧, totalYearPriceModeled[#⟦2⟧, miles]⟦1⟧/totalYearPriceModeled[#⟦2⟧, miles]⟦2⟧,
      totalYearPriceSimpleModeled[#⟦2⟧, miles]⟦1⟧/totalYearPriceSimpleModeled[#⟦2⟧, miles]⟦2⟧ −
        totalYearPriceModeled[#⟦2⟧, miles]⟦1⟧/totalYearPriceModeled[#⟦2⟧, miles]⟦2⟧,
      totalYearPrice[#⟦2⟧, 2012, miles]⟦1⟧/totalYearPrice[#⟦2⟧, 2012, miles]⟦2⟧ −
        totalYearPriceModeled[#⟦2⟧, miles]⟦1⟧/totalYearPriceModeled[#⟦2⟧, miles]⟦2⟧,
      totalYearPriceAlwaysFill[#⟦2⟧, 2012, miles]⟦1⟧/totalYearPriceAlwaysFill[#⟦2⟧, 2012, miles]⟦2⟧ −
        totalYearPriceModeled[#⟦2⟧, miles]⟦1⟧/totalYearPriceModeled[#⟦2⟧, miles]⟦2⟧} & /@
  {{"Boston", bostonGasData}, {"Chicago", chicagoGasData}, {"Denver", denverGasData},
    {"Houston", houstonGasData}, {"Los Angeles", laGasData},
    {"New York City", nycGasData}, {"San Francisco", sanfranGasData}}]
```

```
In[719]:= (*This function returns the optimal total gas price for 2012,
given all of the data for a city, assuming that the driver drives 100 miles per week*)
optimizePrice[cityData_, 100] := optimizePrice100Miles[cityData];
optimizePrice[cityData_, 200] := optimizePrice200Miles[cityData];
optimizePrice100Miles[cityData_] := Block[
  {strippedCityData = #⟦2⟧ & /@ Select[cityData, #⟦1, 1⟧ == 2012 &]},
  GraphDistance[
    Graph[
      Flatten[
        Join[
          {1_0 ↔ 2_4, 1_0 ↔ 2_12,
            2_12 ↔ 3_12, 2_12 ↔ 3_8, 2_4 ↔ 3_12, 2_4 ↔ 3_0,
            3_12 ↔ 4_12, 3_12 ↔ 4_8, 3_8 ↔ 4_12, 3_8 ↔ 4_4, 3_0 ↔ 4_12, 3_0 ↔ 4_4,
            4_12 ↔ 5_12, 4_12 ↔ 5_8, 4_8 ↔ 5_12, 4_8 ↔ 5_4, 4_4 ↔ 5_12, 4_4 ↔ 5_0
          },
          Table[
```

```
          Join[
            {
              i_0 ⟷ (i + 1)_((gasTankSize/2)−4),
              i_0 ⟷ (i + 1)_(gasTankSize−4)
            },
            Table[
              {
                i_j ⟷ (i + 1)_(gasTankSize−4),
                i_j ⟷ (i + 1)_(j−4)
              },
              {j, 4, (gasTankSize − 4), 4}]
          ],
          {i, 5, Length[strippedCityData] − 1}
        ],
        {(Length[strippedCityData])_0 ⟷ (Length[strippedCityData] + 1),
          (Length[strippedCityData])_4 ⟷ (Length[strippedCityData] + 1),
          (Length[strippedCityData])_8 ⟷ (Length[strippedCityData] + 1),
          (Length[strippedCityData])_12 ⟷ (Length[strippedCityData] + 1)}
      ]
    ],
    EdgeWeight → Flatten[
        Join[
          {16 strippedCityData[[1]], 8 strippedCityData[[1]],
            4 strippedCityData[[2]], 0, 12 strippedCityData[[2]], 0,
            4 strippedCityData[[3]], 0,
            8 strippedCityData[[3]], 0, 16 strippedCityData[[3]], 8 strippedCityData[[3]],
            4 strippedCityData[[4]], 0, 8 strippedCityData[[4]], 0, 12 strippedCityData[[4]],
            0},
          Table[
            Join[
              {
                (gasTankSize/2) strippedCityData[[i]],
                (gasTankSize) strippedCityData[[i]]
              },
              Table[
                {
                  (gasTankSize − j) strippedCityData[[i]],
                  0
                },
                {j, 4, (gasTankSize − 4), 4}]
            ],
            {i, 5, Length[strippedCityData] − 1}
          ],
          {0, 0, 0, 0}
        ]
      ]
  ],
  1_0,
  Length[strippedCityData] + 1,
```

```
    Method → "Dijkstra"
   ]
  ]

(*This function returns the optimal total gas price for 2012,
given all of the data for a city, assuming that the driver drives 200 miles per week*)
optimizePrice200Miles[cityData_] := Block[
  {strippedCityData = #[[2]] & /@ Select[cityData, #[[1, 1]] == 2012 &]},
  GraphDistance[
   Graph[
    Flatten[
     Join[
      {1₁ ↔ 2₁, 1₁ ↔ 2₂},
      Table[{i₁ ↔ (i + 1)₁, i₁ ↔ (i + 1)₂, i₂ ↔ (i + 1)₁, i₂ ↔ (i + 2)₁}, {i, 2, Length[cityData] − 2 }],
      {(Length[strippedCityData] − 1)₁ ↔ Length[strippedCityData]₁,
       (Length[strippedCityData] − 1)₂ ↔ Length[strippedCityData]₁}
     ]
    ],
    EdgeWeight → Flatten[
      Join[
       {8 strippedCityData[[1]], 16 strippedCityData[[1]]},
       Table[{8 strippedCityData[[i]], 16 strippedCityData[[i]], 0, 8 strippedCityData[[i]]},
        {i, 2, Length[strippedCityData] − 2 }],
       {strippedCityData[[Length[strippedCityData] − 1]], 0}
      ]
     ],
    VertexLabels → "Name"
   ],
   1₁,
   Length[strippedCityData]₁,
   Method → "Dijkstra"
  ]
 ]
```