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2011

14th Annual High School Mathematical Contest in Modeling (HiMCM) Summary Sheet

Team Control Number: 3074
Problem Chosen: B

I have lost my ring! We all lose things, and the hardest part is always finding where we left them. In Problem B we were assigned the intimidating task of finding an object in a large park, and a lost jogger in an even larger park using only a penlight. Needless to say, without the mathematical and computer models contained herein, it would be like finding a needle in a haystack. However, this task is not the impossibility that it may seem at first.

When searching for a lost object, it is important to first determine the locations where there is the greatest probability of finding it. Once this has been accomplished, the next step is to develop a method for searching those locations efficiently. In both problems, we are asked to search for an object with few initial clues as to its location. However, we can make several assumptions about how the objects, and those searching for them, behave. A lost ring, for example, will not arbitrarily change its location. We therefore base our method of search for the ring off of our subject, the person who lost the ring, who is far more likely to have changed his location, and whose location changes may be predictable. To find where the ring is located, we must focus our model on where our subject would have spent the most time because he could have only dropped the ring in a place where he was present. If we are able to determine the areas where our subject spent the majority of his time, or was likely to have spent the majority of his time, those areas will be where we are most likely to find the ring. In order to determine where these areas of high probability are, we need to answer two questions: First, where must our subject have gone? And second, where could he have remained for a period of time? Because the park can only be accessed by car, we can therefore deduce that Tim must have parked his car at one of the fifteen parking lots. The areas near parking lots, therefore, are much more likely to have been places that Tim walked than other stretches of like Aikens road. In addition, Tim is likely to have stopped for a period of time at picnic tables, restrooms, or the Swimming Pond. If he stopped at these areas, the area of trail surrounding them is also likely to have been covered by our subject walking to or from the location. We have now established where the ring is most likely to be found, it is most likely found at or around one of the locations mentioned previously: parking lots, restrooms, and picnic tables.

Having determined the criteria necessary to determine whether an area is beneficial to search, being close to the attractions of the park, we can construct a probability density map that for any point in the park will determine how likely it is that the ring is located at that point. Using this map, we must construct a path that allows us to search all of the highest density, highest probability, areas as quickly as possible. We start at each high-probability location, and attempt to find the shortest route from each location that passes through all the others. We then select the shortest of these routes, which is about 3.9 miles. As Tim can walk 8 miles in the allotted 2 hours, this leaves him with 5.1 miles worth of walking to pass by as many other facilities as possible and return to the starting point. This will enable him to cover the 80\% of the road that gives him the highest probability of finding the ring.

In the second scenario, the search for Sam, our subject, presents a different problem: how to find an object that may or may not be moving. Because we cannot assume anything about Sam[n]s behavior as we search for him (we assume that he will not go in closed-off areas), we are forced to gain our information from his natural inclinations, previous behavior studies, and from the park[n]s layout.

As in the first problem, we can assume that Sam must have accessed the park by car. Therefore, there are five possible places where his run could have begun. We also assume that, as he planned to run for five miles, so at most he would have intended to turn around after 2.5 miles and then turn around. It is therefore safe to assume that at the time Sam became lost, he was within a 2.5-mile radius of one of the parking lots. He therefore is most likely to be found in an area that is within 2.5 miles of multiple parking lots. We may also assume that Sam is not lost if he is in a parking lot or at another recognizable landmark. He is therefore is most likely to be found at the edge of this 2.5-mile range, where he is farthest away from these landmark.

Sam does not, however have the same probability of being lost when starting from each of the five parking lots. There are more forks in the road in the vicinity of some parking lots than in others, giving Sam a greater probability of taking a wrong turn and becoming lost if he has started from one of those parking lots. It therefore makes sense to start our search in the area with the highest concentration of forks. In effect, Sam[*n*]s probability of being lost in a given area is determined by three factors: number of 2.5-mile radii within which the location falls, distance from landmarks, and likelihood of making a wrong turn as determined by the number of forks in the area. The third of these factors gives us our starting point; the first and second give us the order in which we should search the park making sure to hit as many forks in the road (referred to herein as decision points) because research suggests that Sam, conscious or unconscious, will be located near the decision point where he make a wrong turn. We begin at the parking lot from which Sam has the greatest probability of making a wrong turn, lot 4, and then pass through the areas within the range of the most parking lots and with the highest concentrations of wrong turns from among the areas through which we have not yet searched.

The search for a lost item in a vast area can seem like an impossible task, but with careful and methodical planning, the most likely areas in which the object could have been lost can be quickly and easily found. If these areas are searched in an efficient manner, a seemingly lost cause can be turned into a purposeful and systematic endeavor in which success is highly likely.

HighSchool Mathematical Contest in Modeling 2011
Problem B: Search and Find

Team 3074

November 5, 2011

1 Part A - The Lost Object

1.1 Question Restatement

A small object is lost at an unknown point on a trail in the given region. Our goal is to find the route of paths that when searched will produce the object as quickly and efficiently as possible. By walking along the paths where we determine the object might be located, scanning along the path and inside buildings with a pen flashlight, the object will be located if the light from the pen flashlight hits it.

1.2 Analysis of the Question

Based on the park information provided, including trail routes, parking lot, picnic table, and restroom locations, as well as the locations of other activities (like the swimming pond), we will develop a plan to most efficiently search the park for a lost object. In the example described above, a man we will call Tim has lost his favorite class ring at Hopkinton State Park and has two hours to find it with only a pen flashlight. In order to optimize the likelihood that Tim finds his class ring in the shortest amount of time, he must first look in the areas where the ring is most likely to be found, and then move to the areas where the ring is less likely to be found. It must be considered that Tim will not be able to search every trail for his ring because he is only able to maintain a walking speed of 4mph for 2 hours, giving him a range of 8 miles, while the sum of the distances in the park is 10 miles. The second consideration when designing Tim's most efficient path is that once he travels a trail section and searches it, he will find the ring if it happens to be on that trail. This means that re-traveling paths, either intentionally or un-intentionally, is inefficient because it will lead to no new discoveries. A combination of these two considerations, along with the initial observation, that a path ordered from the highest probability location to the least probability location, allows us to identify the two most important questions that our model must answer: how to differentiate, and then quantify, the differences between the high-probability locations, and the low probability locations, and then how to organize the final, optimal, path which will minimize incidence inefficient path overlap.

1.3 Assumptions

1. The hiker accessed the park by car and parked in one of the 15 parking lots
2. The hiker was equally likely to park in any one of the parking lots
3. The object was lost outside of the car
4. The object was not lost on the highway
5. Outisde the car, the probability of the object being lost at any given time varies based upon how the time is being spent.
6. Outside the car, the probability of the object being lost at any given location varies based on how much time is spent at that location.

7. The more likely Tim was to have passed through a region, the more likely that region contains his lost object

1.4 Justification of Assumptions

1. It is assumed that the hiker gained access to the park by car because the park is bounded on all sides by forest, a lake, and a highway. Because there is no reasonable way to enter the park without traveling on route 85, Tim must have entered by car. This assumption also implies that he must have parked his car in one of the 15 designated parking areas before embarking on his journey through Hopkinton.

2. Even though each parking lot is clearly designated for easy access to certain attractions - for example, one parking lot is located near the swimming pond, others are near the boat rental shop, while others, still, are near the eating areas, we have to assume that Tim was equally likely to chose one parking lot as another because the nature of Tim's visit to the park is not directly specified, and therefore his preference for parking location cannot be deduced.

3. Because the hiker arrived by car, we must assume that the missing object was not lost inside his car. If it was lost inside the car, the two hour optimized search of the park would be futile because the ring, or any other object, would have been in the car all along.

4. We can assume this because our park-goer would not have left his car while traveling on a road with a speed limit of 45(see picture 11). And per the above assumption (2), the park-goer must have been outside his car to lose his object; therefore, Tim could not have lost his object on the highway.

5. We will assume that all activities are not equally risky, that, for example, we hypothesize that using the restroom is a riskier activity than walking on a path. We define these riskier activities as those that have a higher probability of causing Tim to lose his ring/object.

6. We will assume that while Tim is not varying his type of activity, for example hiking continuously, the likelihood that he loses his ring/object during a given time interval is the same as any other equal time interval of the same activity. The probability that Tim looses his ring/object during a 5 minute meal, therefore, is 5 times as likely as that during a 1 minute meal (more of a snack).

7. By the previous assumptions, the object can only be lost in an area where Tim has travelled, therefore an area where Tim has certainly travelled is more likely to have the object than any other area of the map where we are unsure if Tim travelled or not.

1.5 Hypotheses

1. We hypothesize that because certain activities are "riskier" than others, certain paths, and certain areas of the map will be areas of "higher probability." We define areas of "higher probability" as those areas where the object was most likely to be lost, which are the same areas where the object is most likely to be found.

2. We hypothesize that these areas of higher probability are determined by the following factors: the likelihood that Tim spent time in and around the area, the amount of time that Tim spent in and around the area, and the activity occurring in and around the area. By the assumptions above, the more likely that Tim was in an area, the more likely that Time spent a lot of time in that area, and the riskier the activities associated with that area, the higher the probability of finding the object at that area.

3. We hypothesize that these areas of higher probability are located in and around the parking lots, restrooms, picnic benches, and swimming areas located in the park. We know, by assumptions 1 and 2, that Tim spent time in and around parking lots. We hypothesize that Tim, being a normal human being, also needing to eat and relieve himself, visited at least one of the 6 restrooms and at least one of the 17 picnic benches during his visit to the park. The activities at these sites are generally stationary and time consuming, which by assumptions 4 and 5 implies that the object may have been easily lost,

and if it was, it would be contained in a small location as compared to the long trails where the ring could have been lost while hiking or walking. This makes parking lots, restrooms, and picnic benches areas of "higher probability" by our definition. The concept of "probability density" will be defined later.

1.6 The Model

1.6.1 Basic Analysis and Justification of Model

Our goal in developing an effective model is to quantify the regions of higher probability so as to create a path, on which Tim should search for his lost object, that will visit the most areas of higher probability as possible in the shortest amount of time. Our entire model works off of the assumption that the more likely Tim was to have passed through a region, the more likely that region contains his lost object. However, It would be impossible to model, or even guess at where Tim went during his trip to the park or what he did there, so our model must estimate where Tim was most likely to have been during his day at the park by using what we know about Tim, namely that he is human. Being human Tim must park his car in a parking lot, as he cannot fly, eat and use the restroom. Locations where Tim can accomplish any of these three things in the park, we have designated regions of higher probability because it is likely that Tim went to at least one of each. If we allow the size of the areas of higher probability to decrease to some arbitrary, infinitesimal, size epsilon so that areas only slightly larger than points can be considered "areas" of higher probability, we can define "probability density" as the number of "areas of higher probability" contained near, or within, a defined region. Realistically, however, very few regions on the map will actually contain any areas of higher probability, so the probability density becomes a measure of how close a region is to surrounding areas of higher probability. To estimate the probability density of a region, the distance from nearest parking lot, from the nearest restroom, and from the nearest picnic bench, all areas of high probability, are averaged.

1.6.2 About Probability Density

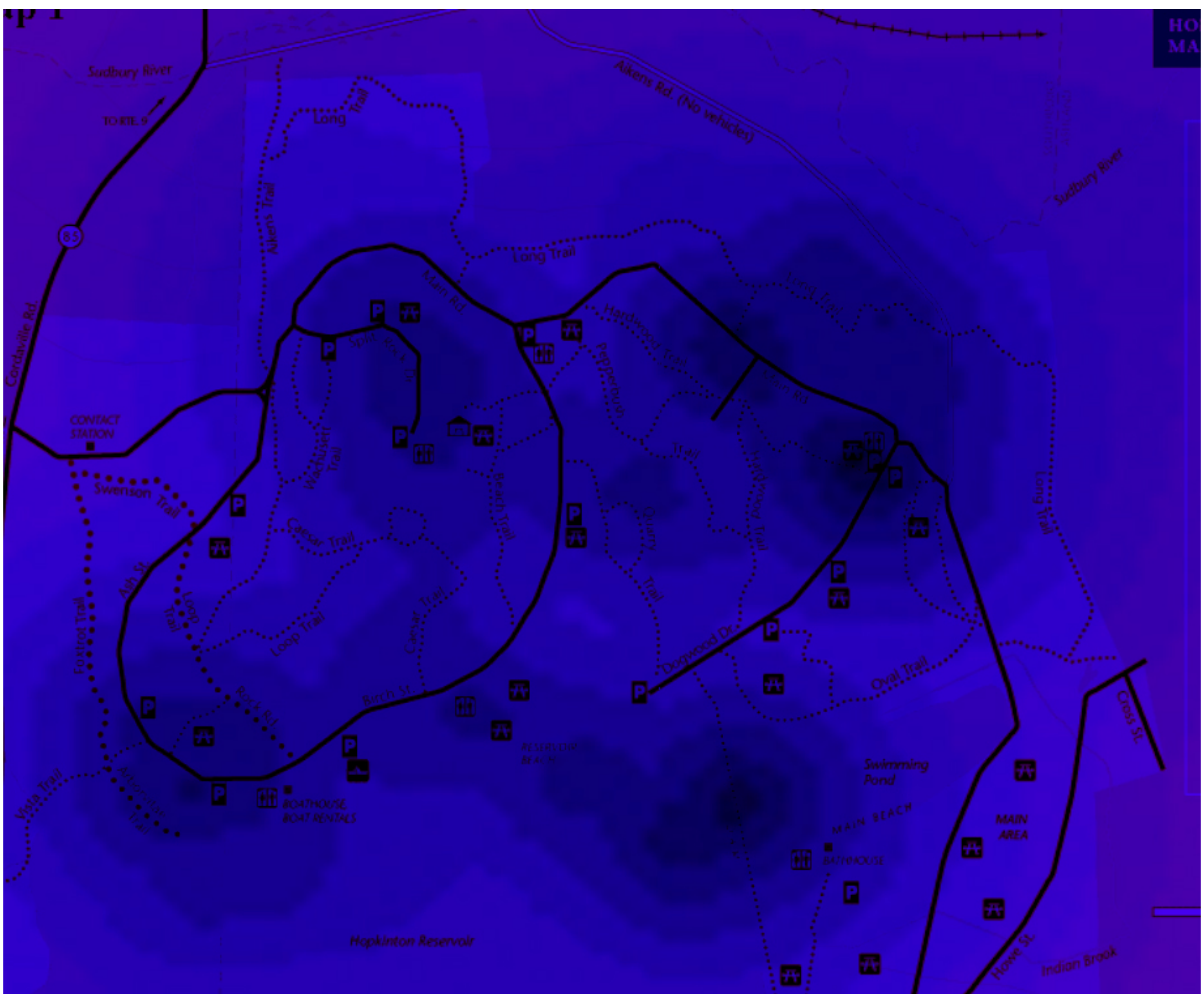
As explained above, the probability density of a region is an estimate of how close it is to the areas of higher probability around it. The definition of an area of higher probability, from Hypotheses 3, is an area where Tim was very likely to have been, and may have lost his object there. The closer a region is to an area of higher probability, the more likely that Tim passed through the region, unknowingly, on his way to or from the area of higher probability. However, this likelihood decreases the further from the area of higher probability a region is, for example, if a region is located miles away from any restroom, table, and parking lot, it is unlikely that Tim passed through that region on his way to his car, or on his way to eat lunch. If, however, a region is located near a bathroom, a parking lot, and a table, it is likely that Tim, without even realizing that he walked through the region on his way to one of the areas of higher probability, passed through this region. Areas fitting the first description will be assigned low densities, as they are not near any areas of higher probabilities. These areas will be searched last because there is nothing to indicate that Tim would have passed through that region at all during his day at the park, but they still cannot be discounted. Medium density locations will be near some types of areas of higher probability, for example, near a bathroom but not a parking lot or table. Tim has a higher likelihood of having travelled through this area than the previous because he may have passed through it on the way to a bathroom. That having been said, it is less likely that Tim passed through a medium density location than one of high density. A high density location is one that is around all three types of areas of higher probability, and Tim was very likely to have passed through this region because he could have been passing through to get to his car, a bathroom, or a picnic table. We hypothesize that this likelihood of a region near an area of higher probability being passed through by Tim decreases linearly as the distance between that region and the closest area of higher probability increases. A region that is 1/2 mile away from a bathroom is twice as likely to have been passed through on the way to or from that bathroom as a region 1 mile away. This hypothesis is the justification for the formula we derived to estimate the probability density of any region on the map.

1.6.3 Procedure, Calculations, and Results

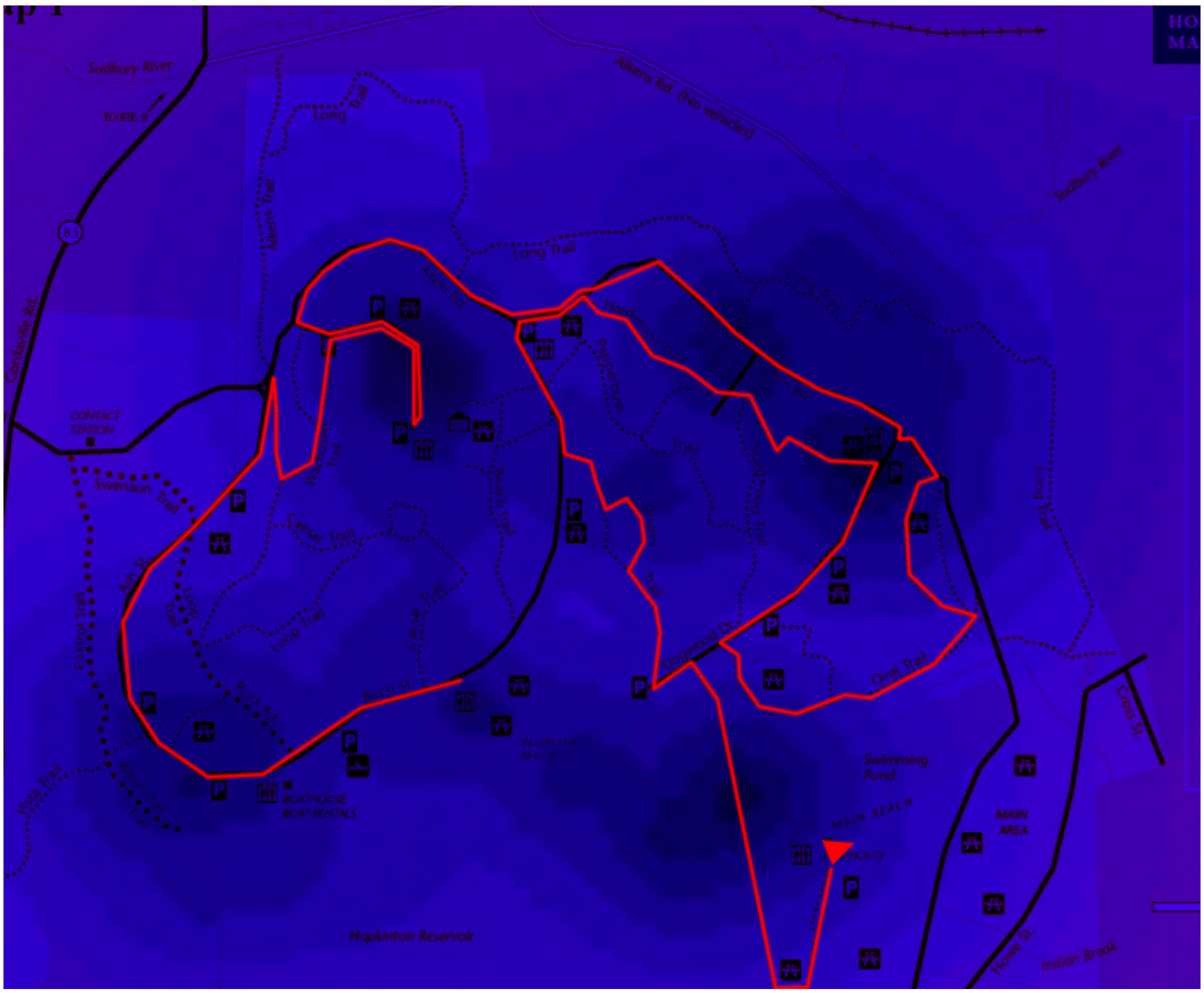
In order to map the probability density over the entire map, we first must divide it into a cartesian coordinate system where the top left corner marks the origin, movement to the right marks movement in the positive x direction, and movement down marks movement in the positive y direction. Starting at the origin, points were assigned every 1/10 inch on the printed map for a 111x86 grid. Then, using a ruler we translated each identified area of higher probability into a point on the cartesian coordinate system, and used that data in the computer program (published in its entirety as Appendix A) to determine the probability density at all points in the coordinate system. The function to find the probability density at a point (C_x, C_y) is given by:

$$D = \frac{255}{16}\sqrt{\frac{\sqrt{(P_x - C_x)^2 + (P_y - C_y)^2} + \sqrt{(T_x - C_x)^2 + (T_y - C_y)^2} + \sqrt{(R_x - C_x)^2 + (R_y - C_y)^2}}{3}} \quad (1)$$

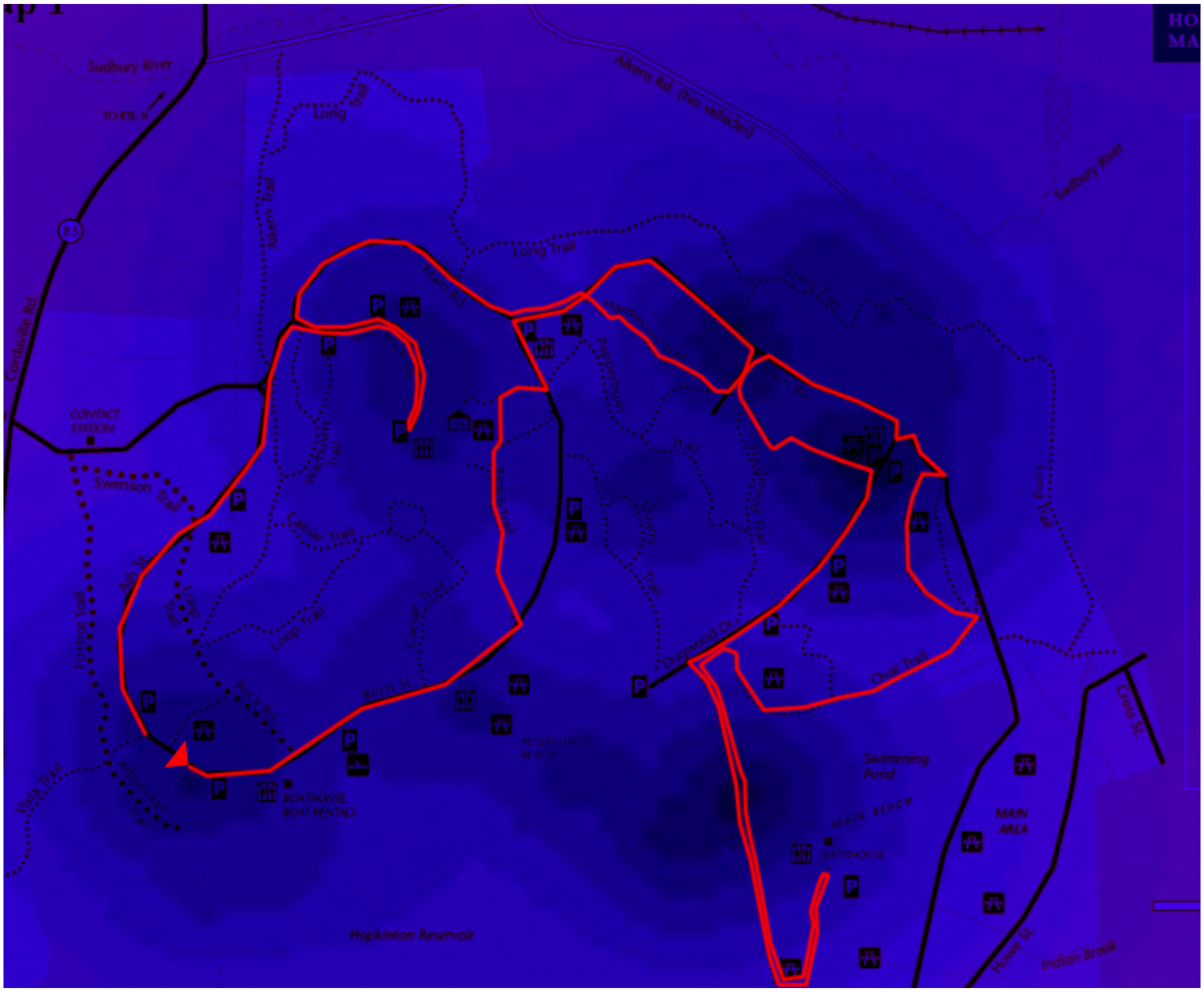
Where D is the approximate density of the region mapped to an integer from 0 to 255, P_x is the x coordinate of the nearest parking lot, and P_y is the y coordinate of the nearest parking lot. T_x and T_y represent the coordinates of the nearest tables, and R_x and R_y are the coordinate locations of the nearest restrooms. The formula uses the pythagorean theorem to calculate the arithmetic mean of the shortest distances to the three areas of higher density, and then maps it to a number between 0 and 255 by taking the square root and multiplying it by a constant. Running the program generates an image, the result of which, a density graph as shown below:



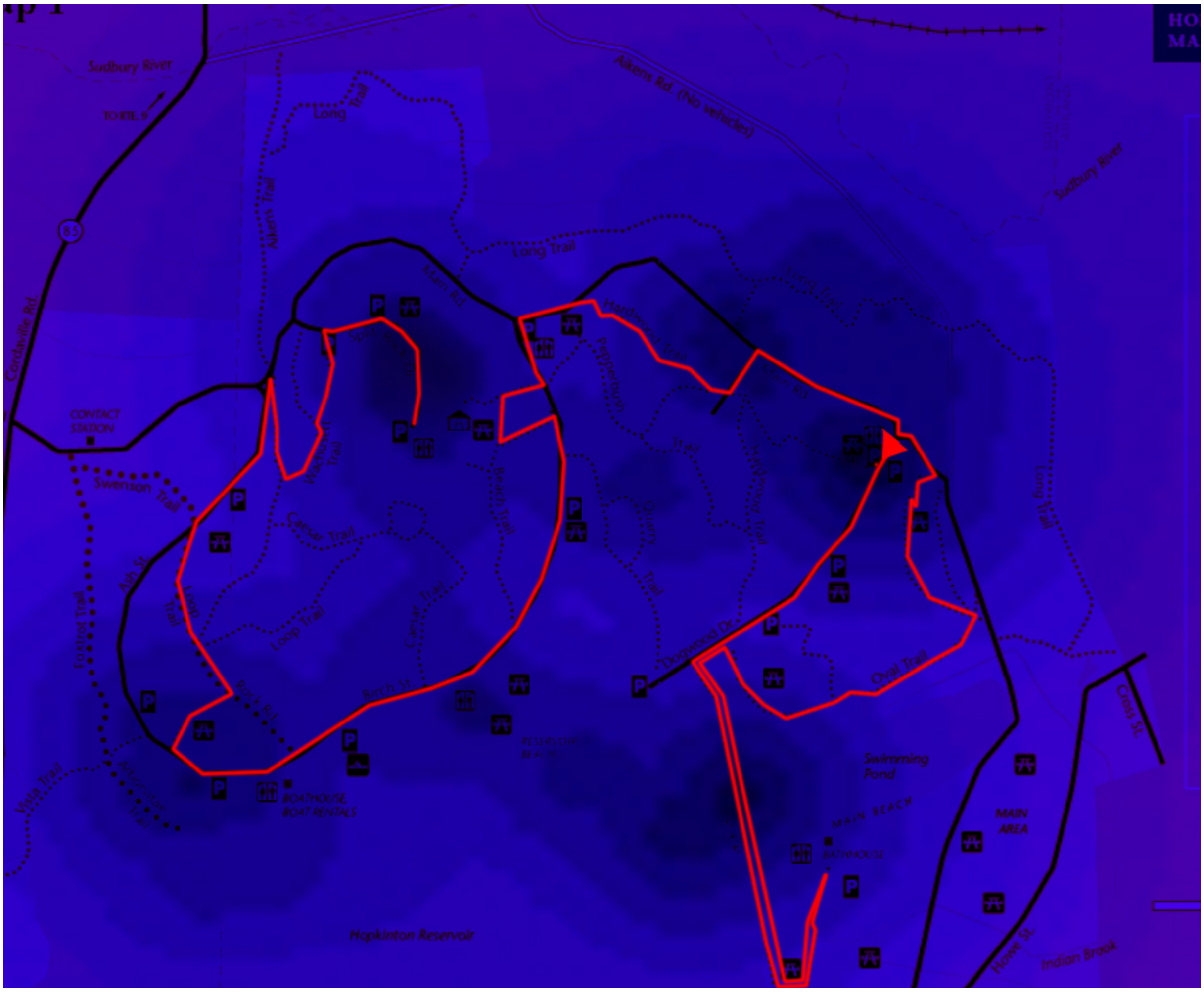
The blackest areas are those estimated to be the most "probability dense", and the bluest areas are those estimated to be the least "probability dense". As is apparent from the density map, there are four prominent areas of probability density. These are the starting points for the optimal search route. An optimal search route will travel through all four major probability density zones in the quickest time possible, and then proceed to meander its way through other semi-dense regions until it resorts to low-density regions all while minimizing path overlap which is inefficient. From the density map generated by our program which identified four areas of greatest density, we created four possible routes each one starting at one of the density locations and traveling to each of the other three in the most efficient way. The results are shown below with the chosen route highlighted in red, starting at the large red triangle, with path overlap depicted with a double line:



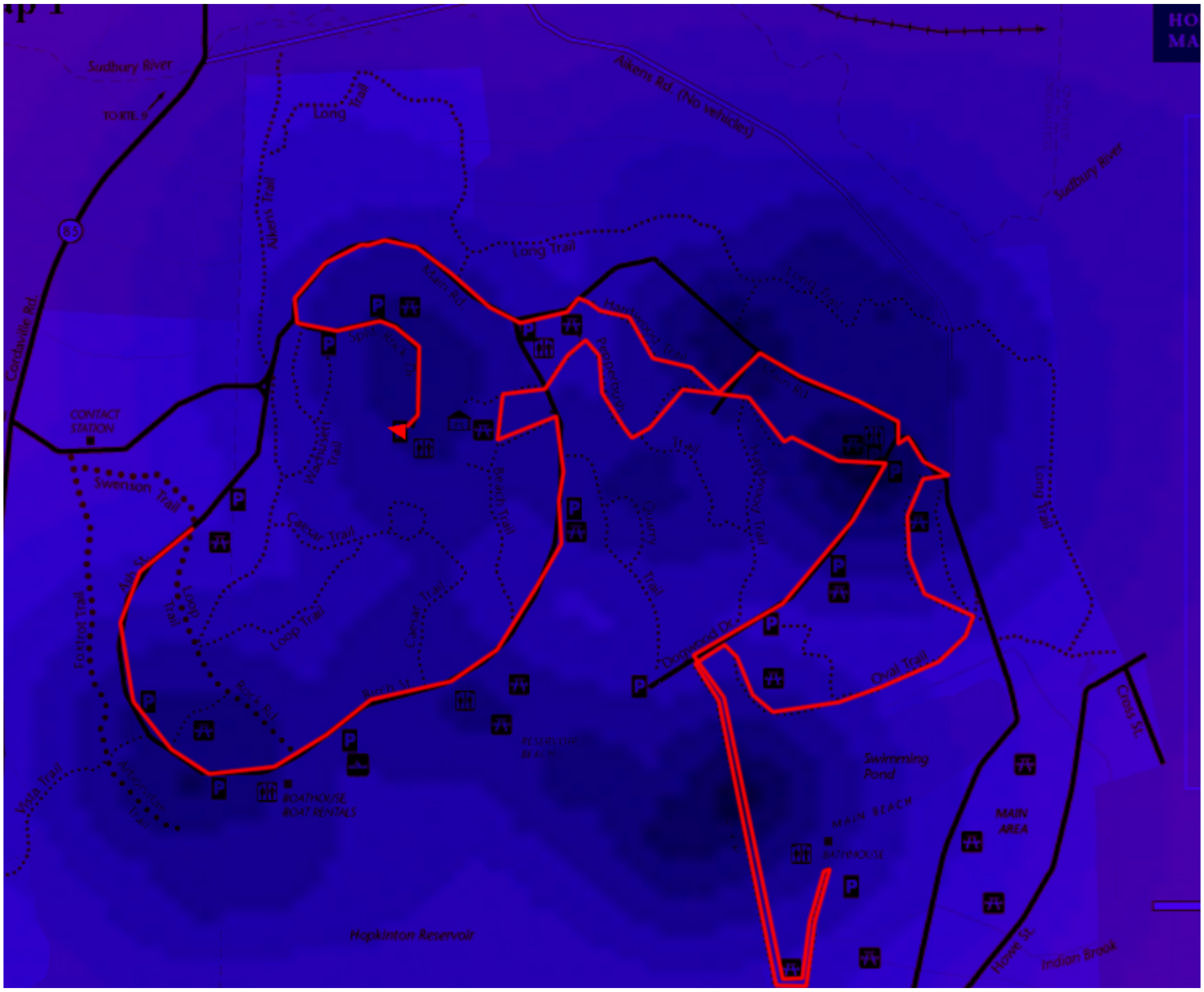
This route is 4.0 miles long



This route is 4.3miles long

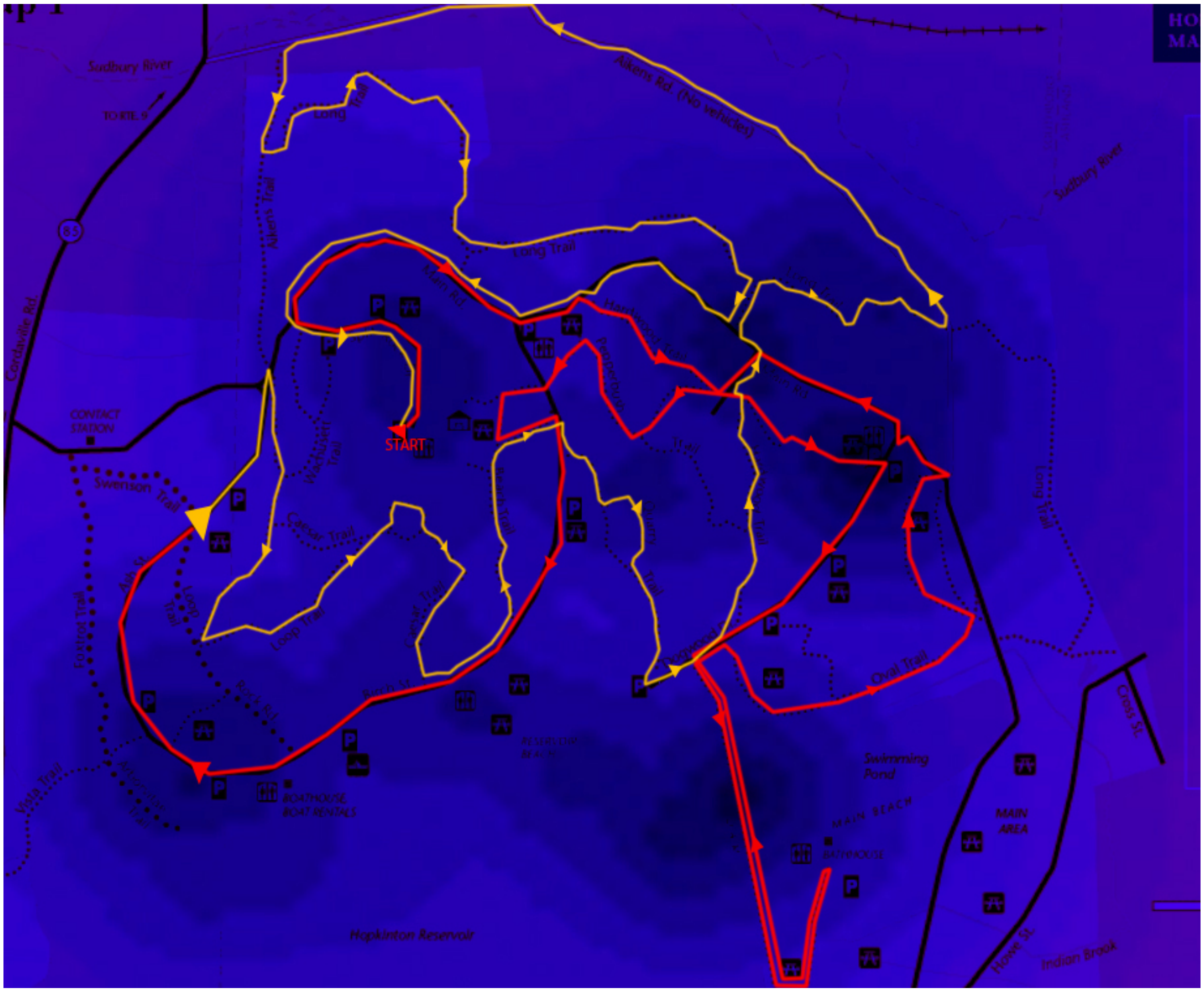


This route is 4.1 miles long



This route is 3.9 miles long

The last piece of the route travels through all of the highest density locations, and past almost all of the areas of higher probability in the shortest route possible - searching the most probable areas as quickly as possible before moving on to less probable areas accomplishes one of the main goals of the model, to find the object as quickly and efficiently as possible. From this route we move to the less dense areas, and then to the least dense areas fairly arbitrarily ending back at the starting point after having travelled exactly 8 miles, which is his range traveling at 4mph for 2 hours. The following graphic shows the search path on top of the density map where the red line is the initial route covering the high density regions and areas of higher probability as quickly as possible, and the orange line is the subsequent search through the less probability dense areas:



The route overlaid on top of the probability density map

The route overlaid over the park map without the density map

1.6.4 Probability Object will be Found

We will calculate the probability of finding the lost object using our model by considering two variables: the percentage of actual areas of higher probability visited and searched in the two hours, and the ratio of the density of the path covered, to the density of the total path. In justification of each variable, if the object is lost in one of the areas of higher probability, the parking lots tables and restrooms, the more areas of higher probability you search, the more likely you are to find the object. And the total density of the path covered takes into account the amount of trail walked and the average density of that trail. The more total density searched over the entire path, th more likely the object is located on that path. We will use these two variables in the following formula to calculate the total probability that the object will be found:

$$P_{x,y} = \frac{1}{2} \times (\frac{T_{x,y}}{T_{Total}} + \frac{A_{x,y}}{A_{Total}}) \times \frac{D_{x,y}}{D_{Total}} \tag{2}$$

Where $P_{x,y}$ is the probability of finding the object over the time interval x to y. $T_{x,y}$ is the distance of the trials covered over the time interval and T_{Total} is the total distance of al of the trails in the park. $A_{x,y}$ is the number of areas of higher probability visited over the time interval, and A_{Total} is the total number of areas of higher probability in the park. Finally $D_{x,y}$ is the average density of the path covered over the interval, and D_{Total} is the total average density of the entire path. We will use this equation to calculate the probability of finding the object during the 2 hour search

$$P_{x,y} = \frac{1}{2} \times (\frac{8}{10} + \frac{30}{40}) \times \frac{125.5}{115.6} = 84.14\% \tag{3}$$

1.7 Discussion of Model

1.7.1 Strengths

1. Our model takes into account multiple variables, probability density of the path being searched and the order of the paths to be searched, so as to optimize both the probability that the object is found at all, and minimize the time it takes to find the object.

2. Our model utilizes a computer program to provide concrete probability density data for analysis across 9546 equal regions of the map

3. Our model provides for an efficient way to structure the first half of the path around the four zones of highest probability density

1.7.2 Weaknesses

1. Our model does not provide for an efficient way to structure the last half of the path, that which will pass through the less dense areas. The path was picked relatively arbitrarily so as to stay within high density areas, while trying to cover as much of the unsearched area as possible

2. Our model is contingent on Tim using some of the areas we have identified as the higher probability areas, if he does not, and just decides to hike randomly around the park without using the bathrooms or picnic benches, our model will not produce an efficient path for finding a lost object.

2 Part B - The Lost, and Possibly Unconscious Runner

2.1 Question Restatement

A jogger went out for a 5 mile run in Fort Ord Public Lands (FOPL) and became lost, he may still be conscious, but he may be unconscious. It is our job to develop a method for most efficiently searching the lands with a penlight so as to locate the lost jogger. By searching the areas where we determine the jogger is mostly to be first, moving steadily toward those where he is least likely to be, we hope to optimize our search.

2.2 Analysis of the Question

By using the information provided on the map of the Fort Ord Public Lands including road and trail locations, stream and lake locations, private and inaccessible property locations, parking, campground, and fire station locations, and using our own research about the nature and extent of search and rescue missions, the most common human responses to becoming lost, and two dimensional random walk functions, we must determine the most optimal search path through the woods. In order to do so, as was done in the model developed for the above question A, we must first identify the locations of highest probability. Locations of highest probability in this context are defined as those areas of the forest where the jogger was most likely to become lost, and those areas where he will most likely to be found. First, however, we must consider three cases of the jogger's activity after becoming lost. The jogger, henceforth known as Sam, could have done three things after becoming lost - he is either unconscious, and therefore immobile, he has chosen, like many survival sources online suggest to do, stay in one location, and is therefore immobile, or he has decided to start walking in some direction, possibly a straight line, possibly randomly, possibly guided by some noise or light in the area. In any event, our model must be able to determine where Sam is most likely to be, unconscious or not, immobile or not. In order to create the most effective model, one that should find the jogger quickly no matter what his response to being lost was. A well-developed model in solution to the above problem will somehow determine the areas of the park that are the most probable locations of the jogger, and then will use that information to determine the most efficient search path - because there is no time limit, nearly every path can be searched, but a well-developed model will consider the order of the paths to produce the most efficient model, concerned with not only finding the jogger, but finding the jogger in the least amount of time.

2.3 Assumptions

1. The jogger became lost within the boundaries of the Fort Ord Public Lands and is still lost therein
2. The jogger accessed the Fort Ord Public Lands by car, and parked in one of the 5 parking lots
3. If the jogger had not become lost, he would have began and ended his 5 mile run at the parking lot where he parked his car
4. The jogger, even while lost, will not enter any of the private property
5. We assume that Sam is an average jogger
6. Elevation is constant across the entire park
7. The jogger never leaves the trails even when lost or unconscious
8. Once the jogger has realized that he is lost, he will slow down to a walking or stopped pace

2.4 Justification of Assumptions

1. We must be able to make this assumption in order to build a model for the effective search and rescue mission. If he did not become lost within the Public Lands, or since becoming lost has meandered outside of the given area, we will not be able to construct this model.

2. We will assume that because the Fort Ord Public Lands are completely surrounded (on all accessible sides) by roads, that the jogger would have entered the Public Lands by car. This implies that before the jogger could begin his jog, he must have parked his car in one of the 5 parking lots. These parking lots will be further discussed in the hypotheses.

3. The jogger planned to go on a 5 mile jog through the Fort Ord Public Lands which we can assume meant that he started at the parking lot, with stretching etc..., and then continued onto the trails/roads of the Public Lands for his 5 mile jog. We cannot assume that the exact length of his planned jog was exactly 5 miles, but that the jogger had found a series of paths that would provide him with an approximately 5 mile jog and end back at the parking lot where his car was located.

4. We cannot assume that the jogger knew where the private property existed, but we will assume that if he came across tall fences and "No Trespassing" signs demarcating the edges of private property, he would not enter.

5. By average jogger we assume that Sam has an average understanding of the trails

6. There is no location in the park which is significantly higher than any others

7. There is no reason for the jogger to ever leave the path even while lost, he will either be unconscious on the path, sit still and wait for rescue on the path, or wander the paths in an attempt to find his way out.

8. We will assume that the jogger cannot maintain his jogging speed forever, he will become tired, and when he realizes that he is lost, will slow down to either a stop or a walk while he tries to navigate his way out of the park

2.5 Hypotheses

1. We hypothesize that there are certain jogging paths that are of higher "risk." We define higher risk generically as a subjective measure of how likely it is for a jogger of average experience to become lost while navigating that path. We hypothesize that not all paths are of the same "risk" and that because an average jogger is more likely to become lost on a riskier trail, Sam was more likely to be traveling a riskier trail when he got lost.

2. We hypothesize that the riskiness of a jogging trail is directly proportional to the number of decision points along the trail. We define a decision point as a point along the jogging path where the jogger must chose between paths - the more decisions to make along a jogging path, the more likely the jogger is to make one wrong turn, and not realize his mistake until it is too late.

3. We hypothesize that by assumption (3), the jogger's planned course did not extend more than 2.5 miles past the starting point, the parking lot because the jogger had planned on a 5 mile run but had to make it back to his starting point. The furthers possible distance the jogger could have ever been away from his starting point, therefore, is if he travelled in a perfectly straight line away from his start location, and after 2.5 miles turned around and travelled the same path back. We then hypothesize that a circle can be drawn around each parking lot with a radius of 2.5 miles that will encompass any planned 5 mile jog the jogger could have possibly travelled because by assumption (3) the jog could have started at no other place than the parking lot.

4. We hypothesize that there exist regions of overlap on the map of the Fort Ord Public Lands where a these 2.5mile radius circles intersect, and we hypothesize that these are regions of high probability. We define these regions as regions of high probability because they are included within the score of 5 miles jogs originating from many different parking lots and therefore could be included in many more planned 5 mile jogs.

5. We hypothesize that a probability density exists over the entire map which corresponds to the probability that the jogger will be found at that location

2.6 The Model

2.6.1 Basic Analysis and Justification of Model

The two main functions of this model are to identify and quantify the areas of the highest probability over the map of the Fort Ord Public Lands and then to organize the most efficient search route that travels from the highest probability density to the lowest probability density. To create a model which is able to effectively meet these goals, we must consider and take into account the following three variables: where the jogger was most likely to have started his journey, where the jogger was most likely to have been jogging, and where the jogger was most likely to have been lost. Our model will produce a path on which a searcher would have the highest probability of finding the lost jogger. In order to determine the most efficient our model will produce a similar probability density map where the lost jogger will most probably be located.

2.6.2 Determination of the Most Probable Starting Points

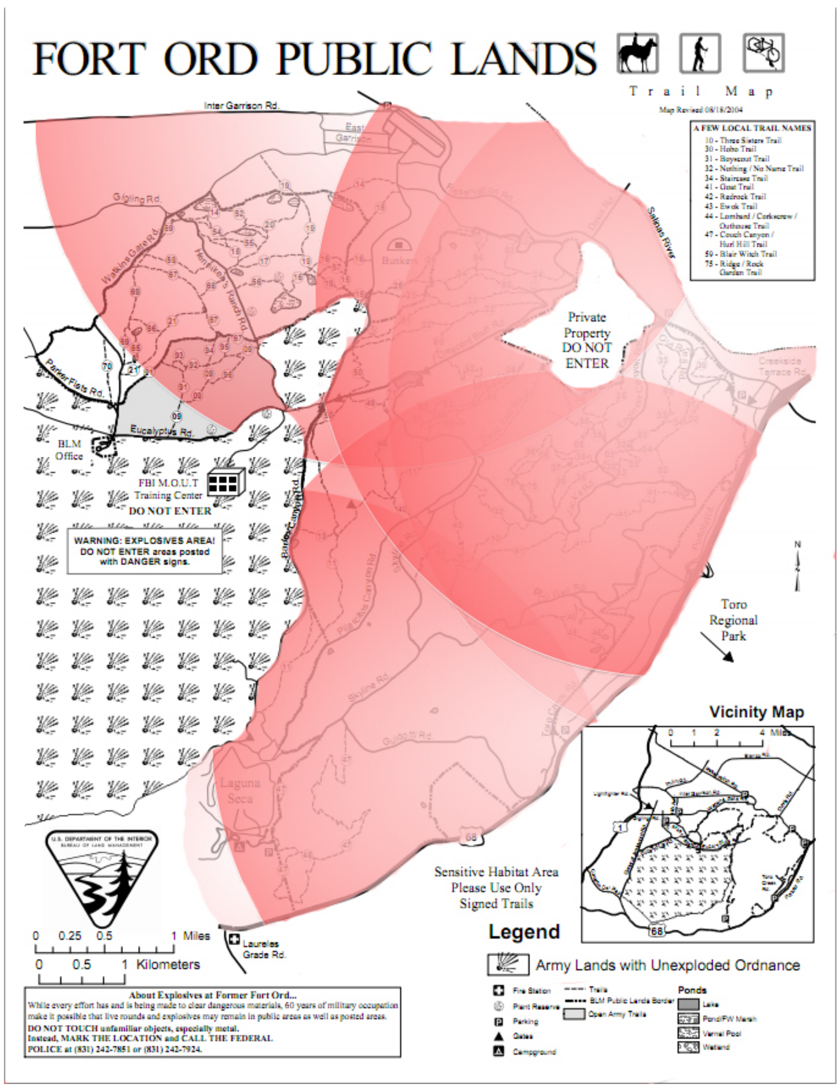
The first consideration when attempting to create a probability density map for the Fort Ord Public Lands are the 5 parking lots that serve as the 5 possible starting points for Sam's jog. We know that Sam got lost on his jog, and, therefore, by hypothesis (1) it is probable that Sam was traveling on a risky trail as opposed to an easy trail. Certain parking lots, then, have access to different types of jogging trails, and because Sam was likely to have been traveling on one of the riskier jogging paths he was likely to park in one of the parking lots with access to risky trails. By analysis of the trails surrounding each parking lot, it is possible to determine the most likely starting points for Sam - he most likely started out in the parking lot with access to the riskiest trails. In order to perform said analysis, however, it become necessary to define "risk" of trails in terms of the number of decision points along those trails - the more decisions that need to be made while jogging, the more likely a jogger will get lost. In order to asses the risk of a parking lot's surrounding trails, we must count the number of decision points in the region near the parking lot. We have defined, in hypothesis (3) that the maximum radius of a 5 mile jog originating from a parking lot is 2.5 miles, and therefore, we counted the number of decision points in a 2.5 mile radius of each parking lot. The results are below:

Parking Lot (Counterclockwise from bottom left)	Number of Decision Points Within Radius
1	37
2	88
3	91
4	113
5	95

The above analysis does not, in any way, prove that Sam started his jog at parking lot 4, it does, however, show that Sam was more likely to have started his jog at parking lot 4 or parking lot 3 over parking lot 1 because the trails he would have been jogging on near parking lot 4 would be much riskier, than those around parking lot 1. The areas where Sam was more likely to have been are factored into the probability density map which we are attempting to construct. Our model will produce a path that starts at the most probable parking lot, and surrounding region, and moves toward the least probable parking lot and its surrounding region in an effort to optimize the search effort. Here, we will start looking for the jogger somewhere int he 2.5 mile radius around parking lot 4, move to the circle around parking lot 3, then to parking lot 5, then 1, and then 2. The exact route of the path is still to be determined, and must take into account the regions of higher probability around the park area where Sam was likely to have jogged. The areas where Sam was likely to have jogged, are the same areas where Sam was most likely to have gotten lost, and may be the areas where Sam still is (depending on if he is unconscious, staying in one location, or walking around trying to find his way out).

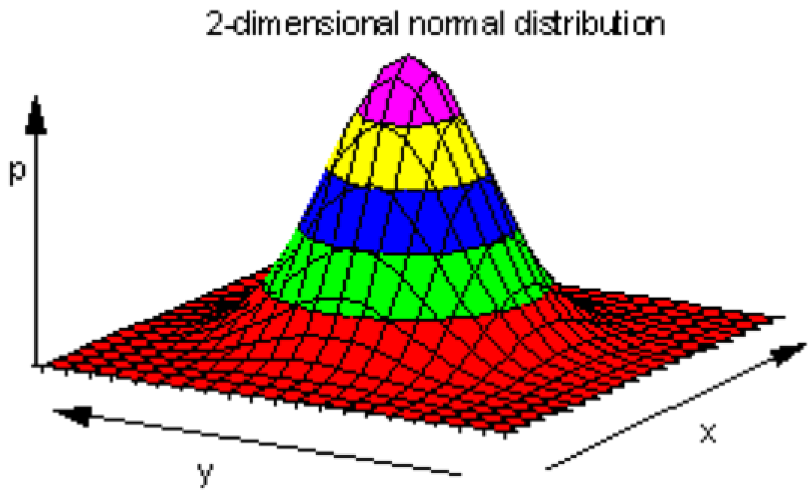
2.6.3 Determination of the Most Probable Running Trails

As established in hypothesis (3) and shown in the figure below, the 2.5 mile radius circles around all 5 possible starting points overlap. Sam's jog is most likely to have passed through points where many of these circles overlap. For example, a region outside of any of the 2.5 mile radius circles is impossible to have been part of Sam's planned jogging route, however a region inside one of the circles could have been a part of Sam's planned 5 mile jog. If that region was part of 2 circles, for example, the circles around parking lot 2 and parking lot 3, that region could have been part of a jogging path that originated at either parking lot 2 or parking lot 3. Therefore, it is twice as likely that Sam passed through that region on his jog - because twice as many jogging paths go through that region. We have named these regions where the probability of Sam's jog passing through is high, as regions of high probability, and they will be useful in creating a probability density map. Below is a graphic depicting the circles drawn around the parking lots, which depicts the probability that Sam's job took him through any point - the more white the point is, the higher possibility that Sam's jog took him through that point. The outer edges of the circle, where it is most red, is where Sam was most likely to have gotten lost - as it is suggested by one source that there is a correlation between the distance from a starting point and the probability that you will get lost - the correlation comes from the observation that if you get lost closer to your start point, it is easier to find your way back. Therefore the farther you are away from your start point, the more likely it is that you stay lost once you make a bad turn, so the darker the red, and the red has been made slightly transparent to highlight the overlap of the circles, the higher probability that Sam got lost in that region.

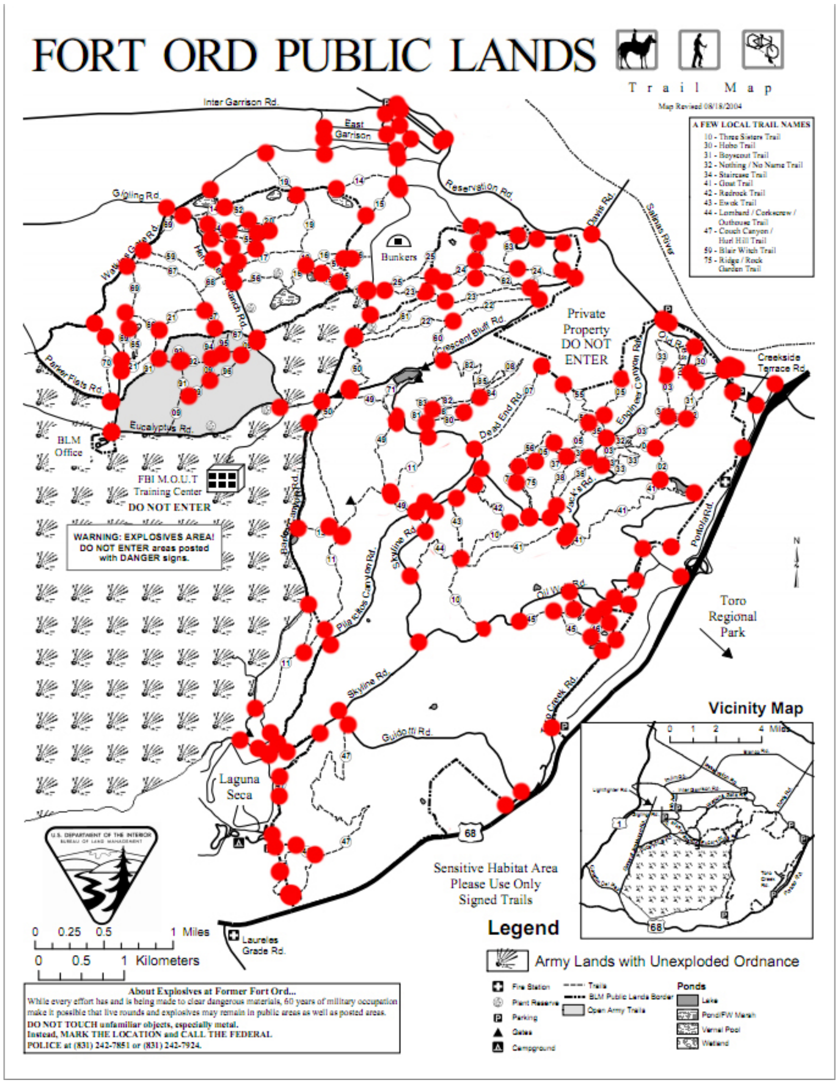


2.6.4 Determination of the Most Probable Points of Mistake

Our research suggests that the cause of Sam's loss was almost certainly making the wrong turn at a decision point, as this is the cause of nearly every incident of people getting lost in natural parks. For example, choosing to take the left path when the right path was correct would lead you down a new set of paths that you had not anticipated, and that you are not familiar with - attempting to backtrack at that point is futile, because it is unlikely that you will recognize the point where you make the mistake. It was recommended by many online sources that the best thing to do in this situation is to stay where you are, but many sources indicate that this is not, in fact, the typical human response. Knowing the locations of all of the points on the map, designated on the image below with red dots, where a mistake could have been made (we have been referring to these locations as decision points) will allow us to compute a similar probability density map as we created in Model A where the closer a point on the map is to decision points, the more likely that Sam will be located at that point. This hypothesis, that the closer points on the map are to decision points increases the likelihood that conscious or unconscious sam will be located at that point is supported by the following graphic:



This graphic depicts the distribution of end points of thousands of trials of the two dimensional random walker. The two dimensional random walker simlulates completely random walking without knowledge of where it is going. For this reason, the two dimensional walker is a good approximation of the behavior of a lost person who decides to try to find their way out of the forest and ends up walking in circles, to put it simply. What can be observed from the above graphic is that the random walker, which starts out at the center point and randomly walks around the (x, y) coordinate and stops on a coordinate (x_n, y_p). This is repeated thousands of times, and each coordinate on the (x, y) axis (n, p) is given a scalar value, graphed to the z axis, which is equal to the number of trials that landed on that point. What is apparent is that, although a random walker may think that they are walking randomly, or toward some end, they will end up with the greatest frequency, back where you started. The two dimensional random walker's application to our model is striking - that if the jogger is not unconscious and did not decide to stop walking when he realized he was lost, but instead decided to walk around trying to find his way out, he will most probably end up close to where he started. If the jogger decides to stop after getting lost, as is recommended by ever source we found on the subject, he will be located near a decision point. Therefore, if we had the time we would use a slightly edited version of the program found in Appendix B to create a new probability density map where points on the map of Fort Ord Public Lands which are closest to decision points have the highest blue-scale probability density value. Because we cannot make that map considering the time constrictions, we have to estimate where those areas of probability lie. Those areas that have been estimated to be the most "probability dense" will be targeted by the model to pass through on the search path.

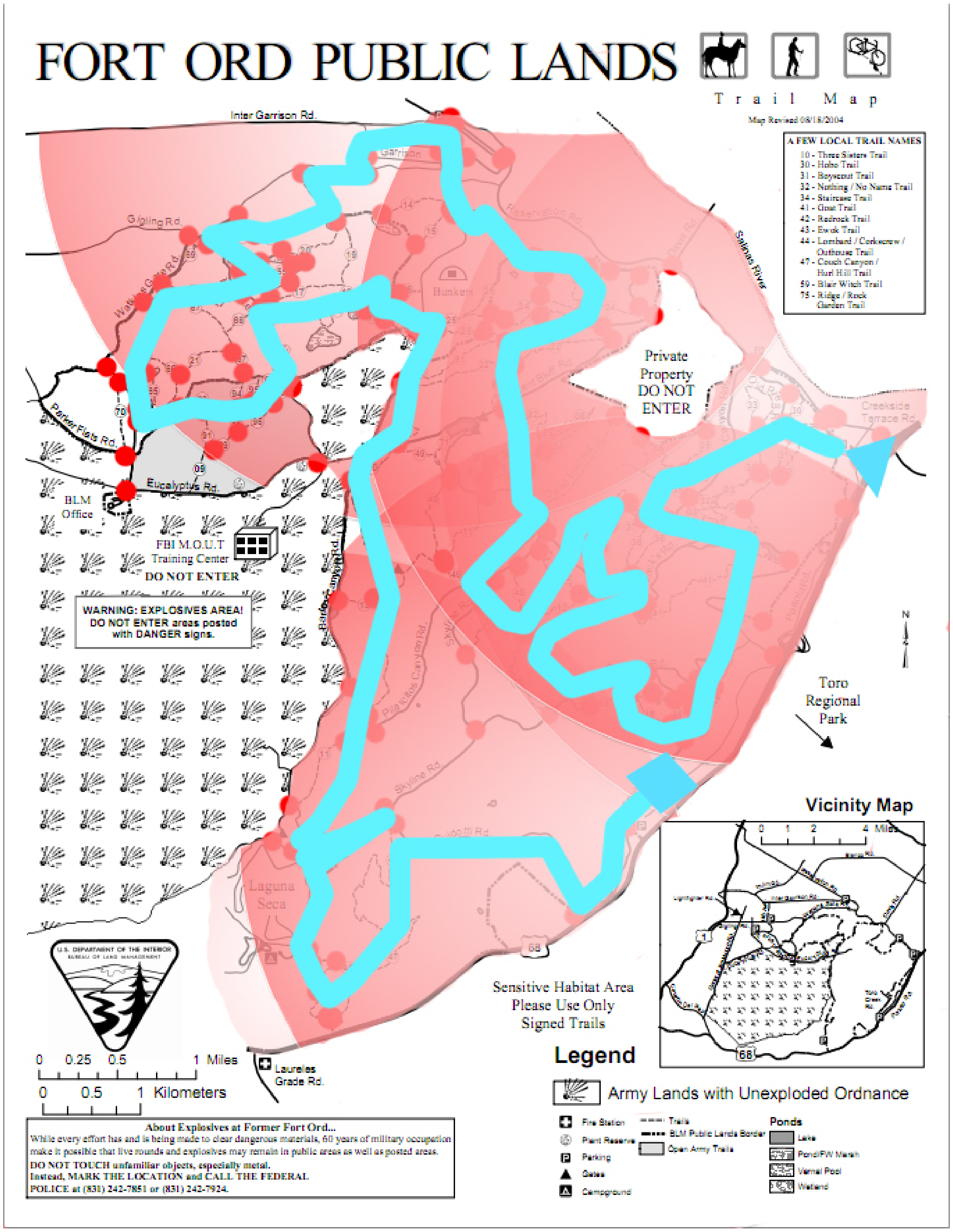


The location of every decision point in Fort Ord Public Lands given by a red dot

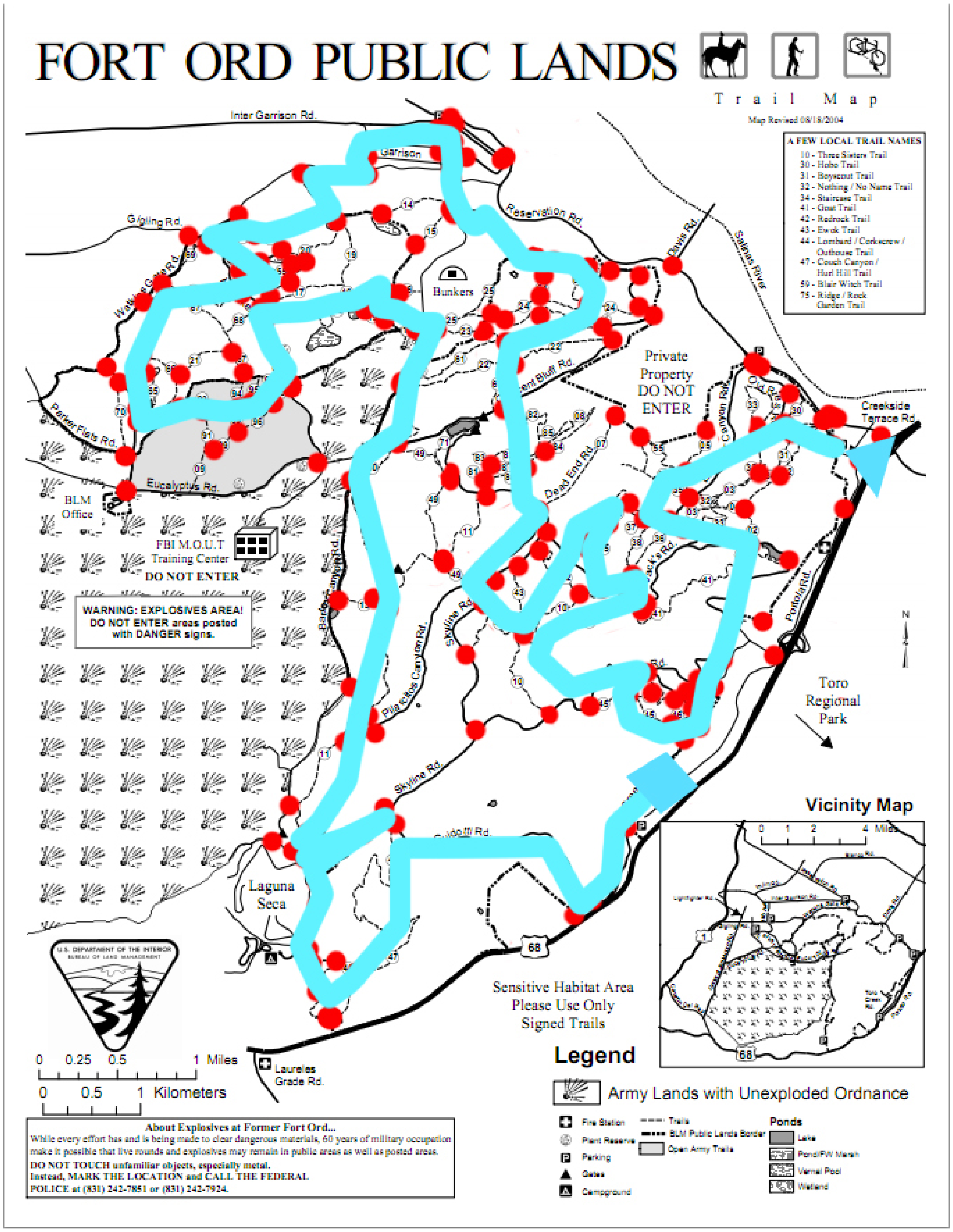
2.6.5 Finding the Most Optimal Search Path

The most optimal path contains the following - direct contact with as many decision points as possible, logical flow from high risk areas to low risk areas (based on decision point density), and contact with parking lot circle (representing the scope of that parking lot) overlap. The actual calculation of the most optimal path is separated into two procedures, first - the definition of the general path and, and second - the definition of the specific path. The general path is composed of a vague route the goes from high probability points to low probability points. The path tries not to backtrack, but its main focus is to hit as many path forks and density regions as possible. This path does not follow any existing routes or trails and skips over sections and crosses through null territory, or territory not characterized by existing trails, as the crow flies. It highlights the normal direction our specific path should go in as the specific path is then created by finding the existing trails and paths at Fort Ord Public Lands that correspond most directly to the general path.

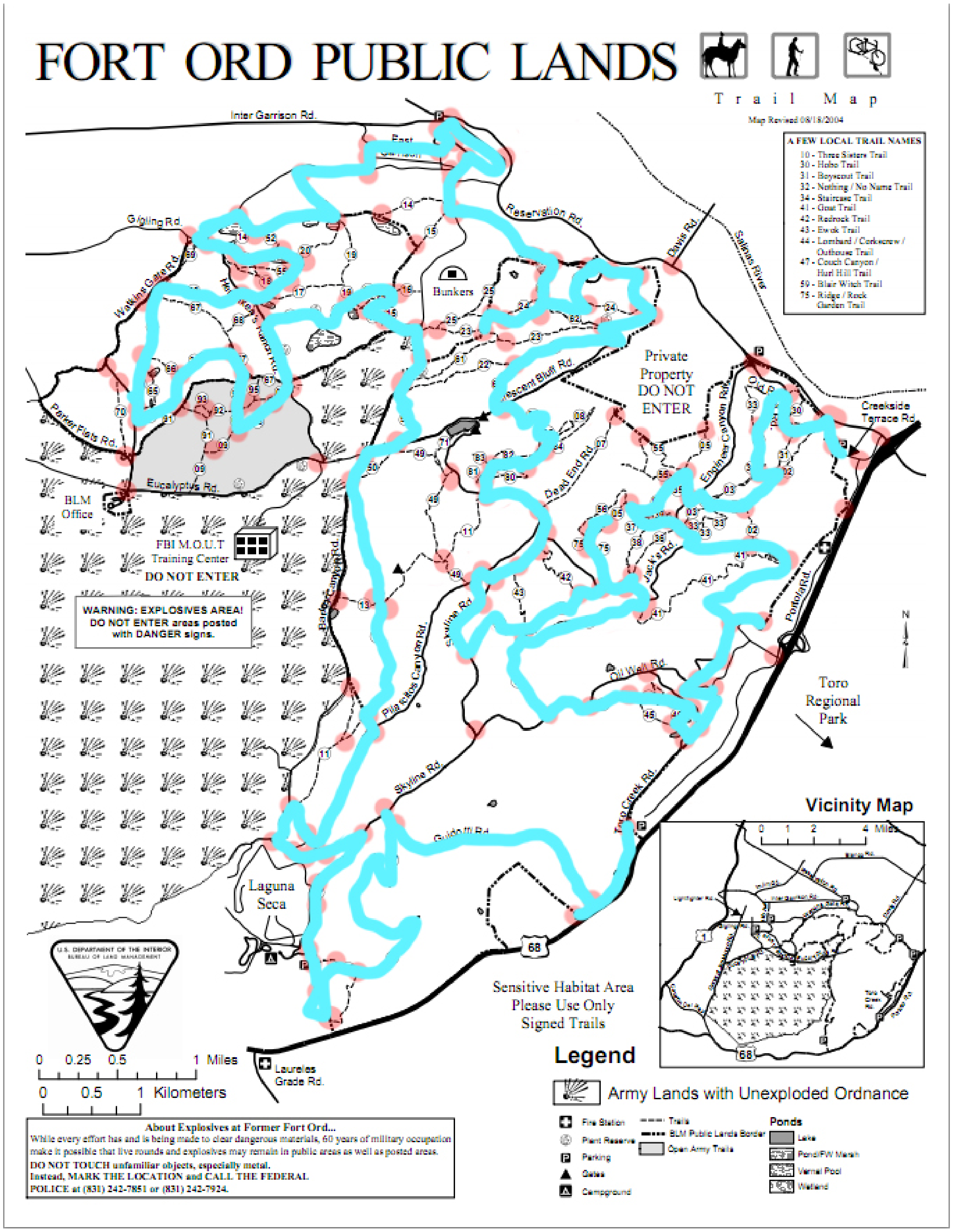
The general path was calculated as follows. The overlapping transparent circles were laid on top of all of the decision points and the path was created to follow the general trend from most risky trails to the least risky trails while covering as much red overlap on the circles as possible and intersecting with as many red dots as possible. The general path here is drawn in cyan overtop of the image just described. It starts at parking area 4, the riskiest with 111 decision points in its scope, and moves to parking area 5 by following the dark red circle overlap in the middle of the map - this area is where Sam will almost certainly have jogged through if he parked in either parking lot 1,2,3,or 4 and therefore should be searched first. The general path then continues up to the trails around parking area 5, the third riskiest area, and continues to follow the dark red areas of circle overlap to parking areas 1 and finally 2. Because it is a general "trend" line, the direct contact with decision points, a crucial component of our model, because the 2 dimensional random walker simulations and National Geographic article support our theory that Sam will be found near one a decision point. Below is the picture of the general path being drawn on top of the map of the Public Lands:



But as you can see from the image below, the general path, while it follows the greatest areas of density, does not trace either existing pathways, or intersect with the maximum amount of decision points:



The specific path is then calculated, or rather traced, from the general path so that it conforms to the pre-existing paths and trails, and intersects with as many decision points as possible. Because the general path already passes through the densest areas in the correct order, the specific path should mimic the general path as closely as possible while intersecting many decision points. Drawing out the specific path for this scenario, that path that when following will lead to the quickest discovery of the either unconscious or conscious jogger yields the following:



As you can see, the specific path closely mimics the general path, traveling through the same areas of density in the same order, but intersects with more than 80% of the decision points. If Sam were actually lost in the woods, this is the path that you would take, walking the entire way with a pen flashlight, to find him.

2.7 Discussion of Model

2.7.1 Strengths

1. The map immediately pins down the highest-probability areas.
2. It is easy for the maps user to tell likelihood of where Sam is at a glance.
3. It is easy to tell where to search first and the direction of travel.
4. Following the map will give the searcher a much greater probability of finding Sam

2.7.2 Weaknesses

1. The map does not provide an organized plan for searching the park beyond the end of the route.
2. The map does not consider features of the terrain elevation, rivers, etc.
3. The map assumes Sam is on one of the trails constantly.
4. The map does not factor in if Sam continues to move after being lost and travels onto an already searched path.

3 Appendix A:Works Cited

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4 Appendix B: Blue-Scale Density Mapping Computer Program Code

Written in JAVA. This code was used to solve Part A, finding the lost object in a park - it prints out a probability density map to then overlay over the map of the park. It requires inputting coordinate values for each of the areas of higher probability (the parking lots, the picnic tables, and the restrooms) and then is able to run by itself.

```
import java.io.*;
import java.awt.image.BufferedImage;
import javax.imageio.ImageIO;
import java.awt.Graphics;
import java.awt.Color;

public class draw {
 public static void main(String[] args) {

  int width = 111;
  int height = 86;
  int x = 0;
  int y = 0;

  ParkingLot[] P = new ParkingLot[15]; //The ParkingLot objects stored as an array
    P[0] = new ParkingLot(58,14);
    P[1] = new ParkingLot(65,20);
    P[2] = new ParkingLot(42,21);
    P[3] = new ParkingLot(61,30);
    P[4] = new ParkingLot(30, 28);
    P[5] = new ParkingLot(26,33);
    P[6] = new ParkingLot(37,34);
    P[7] = new ParkingLot(28,44);
    P[8] = new ParkingLot(43,48);
    P[9] = new ParkingLot(57,53);
    P[10] = new ParkingLot(52,64);
    P[11] = new ParkingLot(73,70);
    P[12] = new ParkingLot(48,69);
    P[13] = new ParkingLot(38,72);
    P[14] = new ParkingLot(39,73);

    Bathroom[] B = new Bathroom[6]; //The Bathroom objects stored as an array
    B[0] = new Bathroom(22,63);
    B[1] = new Bathroom(34,36);
    B[2] = new Bathroom(38,56);
    B[3] = new Bathroom(44,28);
    B[4] = new Bathroom(71,35);
    B[5] = new Bathroom(65,68);

    PicnicBenches[] PB = new PicnicBenches[17]; //The PicnicBenches objects stored as an array
    PB[0] = new PicnicBenches(17,59);
    PB[1] = new PicnicBenches(18,5);
    PB[2] = new PicnicBenches(33,25);
    PB[3] = new PicnicBenches(39,34);
    PB[4] = new PicnicBenches(41,58);
    PB[5] = new PicnicBenches(46,26);
    PB[6] = new PicnicBenches(46,43);
    PB[7] = new PicnicBenches(69,36);
```

```
  PB[8] = new PicnicBenches(74,42);
  PB[9] = new PicnicBenches(68,48);
  PB[10] = new PicnicBenches(63,54);
  PB[11] = new PicnicBenches(64,78);
  PB[12] = new PicnicBenches(70,77);
  PB[13] = new PicnicBenches(80,73);
  PB[14] = new PicnicBenches(79,68);
  PB[15] = new PicnicBenches(83,61);
  PB[16] = new PicnicBenches(38,23);

  Points[] pts = new Points[9546]; //An array of Points objects

 for(int i = 0; i < pts.length; i++) //Initializes all of the Points objects
  {
    int x2 = (int) i/86;
    int y2 = i%86;
    pts[i] = new Points(x2,y2);
  }


  for(Points px: pts) //calculates the shortest distance from every point to a parking lot
  {
    for(int i = 1; i < P.length; i++)
    {
      int nums = 0;
      if(P[i].distanceFrom(px.xCoord, px.yCoord) < px.parkingDistance)
      {
        px.parkingDistance = P[i].distanceFrom(px.xCoord, px.yCoord);
      }
    }
  }

  for(Points px: pts) //Calculates the shortest distance from every point to a bathroom
  {
    for(int i = 1; i < B.length; i++)
    {
      int nums = 0;
      if(B[i].distanceFrom(px.xCoord, px.yCoord) < px.bathroomDistance)
      {
        px.bathroomDistance = B[i].distanceFrom(px.xCoord, px.yCoord);
      }
    }
  }

  for(Points px: pts) //Calculates the shortest distance from every point to a picnic table
  {
    for(int i = 1; i < PB.length; i++)
    {
      int nums = 0;
      if(PB[i].distanceFrom(px.xCoord, px.yCoord) < px.picnicDistance)
      {
        px.picnicDistance = PB[i].distanceFrom(px.xCoord, px.yCoord);
      }
    }
  }


BufferedImage image = new BufferedImage(width, height, BufferedImage.TYPE_INT_RGB);
Graphics g = image.getGraphics();
```

```
 int p = 0;
 for(int i=1; i <= width; i++)
 {
      for(int j=1; j<=height; j++)
      {
     //   System.out.println("this worked");

           image.setRGB(x, y, pts[p].getGreyScaleNumber());
           y++;
           //System.out.println("It worked");
           if(y==86)
           {
           x++;
           y=0;



         //   System.out.println("x increased");
           }
          p++;
           //else
            //System.out.println("OMG" + y);
        //System.out.println("y:"+y+" x:"+x+" j:"+j + " i:"+i);
      }
}


  save(image);
}

public static void save(BufferedImage image) {
 try{
  ImageIO.write(image, "png", new File("image.png"));
 }
 catch(Exception e) {
  System.out.println("errore");
 }
}
}

public class Points
{
 public int xCoord;
 public int yCoord;
 public double parkingDistance = 1000;
 public double bathroomDistance = 1000;
 public double picnicDistance = 1000;
 public int grescaleNumber;

 public Points(int xCoords, int yCoords)
 {
   this.xCoord = xCoords;
   this.yCoord = yCoords;
 }

 public int getGreyScaleNumber()
 {
   int grescaleNumber = (int) ((255/60)*((parkingDistance + bathroomDistance + picnicDistance)/3));
   grescaleNumber = (int) (Math.pow(grescaleNumber, .5));
```

```
    grescaleNumber = (int) ((255/16)*(grescaleNumber));
    return grescaleNumber;
  }

}

public class ParkingLot
{
  public int xCoord;
  public int yCoord;
  public ParkingLot(int xCoords, int yCoords)
  {
    this.xCoord = xCoords;
    this.yCoord = yCoords;
  }

  public double distanceFrom(int newXCoord, int newYCoord)
  {
    int xDistance = Math.abs(newXCoord - xCoord);
    int yDistance = Math.abs(newYCoord - yCoord);

    return Math.sqrt((xDistance*xDistance) + (yDistance*yDistance));
  }
}

public class Bathroom
{
  public int xCoord;
  public int yCoord;
  public Bathroom(int xCoords, int yCoords)
  {
    this.xCoord = xCoords;
    this.yCoord = yCoords;
  }

  public double distanceFrom(int newYCoord, int newXCoord)
  {
    int xDistance = Math.abs(newXCoord - xCoord);
    int yDistance = Math.abs(newYCoord - yCoord);

    return Math.sqrt((xDistance*xDistance) + (yDistance*yDistance));
  }
}

public class PicnicBenches
{
  public int xCoord;
  public int yCoord;
  public PicnicBenches(int xCoords, int yCoords)
  {
    this.xCoord = xCoords;
    this.yCoord = yCoords;
  }

  public double distanceFrom(int newYCoord, int newXCoord)
  {
    int xDistance = Math.abs(newXCoord - xCoord);
    int yDistance = Math.abs(newYCoord - yCoord);
```

```
    return Math.sqrt((xDistance*xDistance) + (yDistance*yDistance));
  }
}
```