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2014

17th Annual High School Mathematical Contest in Modeling (HiMCM) Summary Sheet
(Please attach a copy of this page to your Solution Paper.)

Team Control Number: 4813
Problem Chosen: A

Please type a summary of your results on this page. Please remember not to include the name of your school, advisor, or team members on this page.

The goal of our model is to show the amount of time it would take people in a crowded train station to leave their trains, make their way to the stairway, and exit the station.

First, we mapped out the problem and identified constants: for some elements of the station, such as platform width and seat width, we assigned values that are similar to the average values of standard trains and that follow certain train specification and standard transportation industry guidelines. The number of people that can be on the stairway at a certain time is limited; thus, we realized that a queue would build up around the bottom of the stairs. We soon realized that the queue is a central element of the entire model because of the magnitude of the effect it has on time required to leave the station. To simulate this effect, we allowed a person waiting in the queue to take up about 4 sq ft of space and calculated a new reduced walking speed for people standing in the queue. We made the queue a rectangular shape to accurately simulate human behavior and made the width of the rectangle the platform width of 29.5 ft., which is the average platform width of a regular railway station. People enter the stairs, join the queue, and walk at three different constant rates. The rate is set by the capacity of the stairs to take up new people every second, so we were able to precisely determine the speed at which people in the queue would walk in a real-world situation.

Second, we developed a mathematical model to optimize for time. We began by obtaining the rate at which people arrive at the queue by finding the amount of time it takes the average passenger to leave the train car. Then, we calculated the distance traveled in the queue and the distance traveled on the platform. Using all of this information, we calculated the amount of time all of the passengers spent in the queue and on the platform.

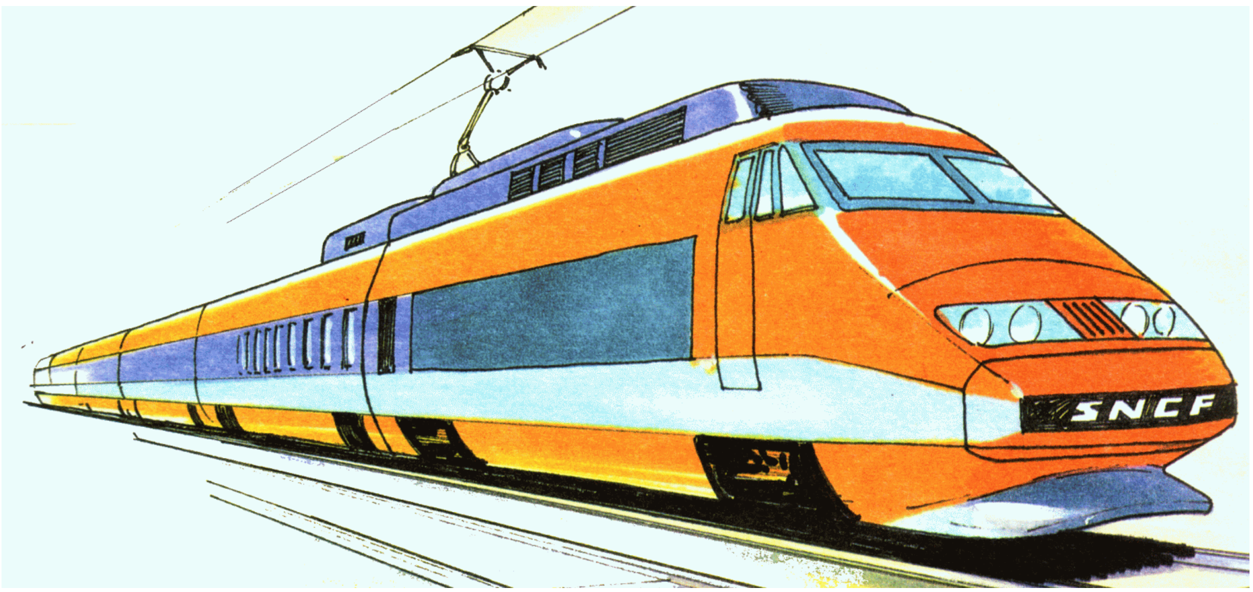
Finally, we optimized the time based on the scenario given. Using the functions we developed, we found that a queue around the bottom of the stairway would only collect when the train cars had more than 19 rows. Because the number of cars did not affect the time it takes to

exit the station as much as the number of rows in a car, choose a reasonable number of rows of cars (more than 10 to fit the constraints of the problem and use train cars of reasonable length; fewer than 20 to avoid producing a queue) for the train. We found these figures to be the same regardless of whether there are one or two trains unloading at the platform. When considering the relocation of the stairways, if there is only a single staircase, it is best to insert that staircase in the middle of the platform. With more than one staircase, it is best to pair the stairways such that the steps face opposite directions. Increases both in the number of staircases and the number of people that can be accommodated on each staircase (more precisely, on each row of the staircase, depending on the width) would reduce the amount of time needed to exit the station due to the reduction of the length of the queues by the stairs. However, having an excessive number of staircases would not do anything to optimize time once the queues are eliminated and it would only waste resources, as well as space on the platform. As long as the number and size of stairways ensure that no queue develops, the people could quickly walk out of the platform and reach street level. It is also important to note that having an infinite number of staircases is neither feasible nor desirable.

Choo-choo

Unloading Commuter Trains

HiMCM Problem A
Group #4813



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1 Introduction and General Interpretation

Congestion in train stations is a major issue for large cities. Thus, optimizing the time for a commuter to travel from their seat in the train car to the exit of the station is crucial to maintain a steady pedestrian flow and maintain efficiency within the station. In the context of this problem, a commuter rail station has a pair of tracks that each stop on one side of a central platform. For our mathematical model, we assumed that there was a two column staircase that was located at the end of the platform. As the passengers approached the stairway after exiting the train, a queue would gradually form by the staircase. The commuters exit the train by walking toward the middle of the platform and then turning to walk toward the stairs, but they are held up by the queue. The larger the queue, the longer they are held up. Our model accounts for the fact that in practice, people will not queue in a line, but rather congregate at the base of the stairs and then inch slowly forward at a rate determined by the width of the queue and the capacity of the stairs to take in new people, to ensure the most realistic interpretation of the scenario.

In this problem, we first addressed the optimization of time when one and two fully occupied trains unloaded their passengers at the station. We discussed how to change the length of a car and the number of cars to minimize the time it takes for a passenger to exit the station: by maximizing car length up to the point that a queue begins to form. Then, we analyzed current transportation research to determine the optimal location of the staircase because such a problem is more psychologically based than mathematically based. Finally, we used the mathematical model we developed to determine how the number of stairways and number of people accommodated by the stairway affects the time required to exit the station.

1.2 Assumptions

General Assumptions:

1. People are reasonable and use the exit closest to their seat.
Justification: There is no accurate way to predict how many people exit through each door if we do not assume that people exit through the door nearest to them. Further, in a crowded train, it would be very surprising if someone were to take the effort to go out of their way to use an exit that would lengthen their travel time.
2. Nothing is to be optimized other than the amount of time required to exit the station.
Justification: We were explicitly asked to minimize the amount of time required and nothing else. Optimization of other variables would likely come at the expense of time.
3. Except for Requirements 3 and 4, there is only one stairway; thus, $s = 1$.
Justification: the problem states "...there is a 'fan' of passengers from the train trying to get up <u>the stairway. The stairway</u> could accommodate two columns…" "The stairway" implies one stairway.
4. d represents the length of each car in rows (thus, a car of length $2d$ would have 2 rows of seats)
Justification: d would work as any measurement of length; we make this assumption purely for convenience and simplicity. Making d the number of rows in each car would simplify our calculations regarding the relationship between the number of passengers and the length of the car.
5. The passengers involved in the problem are the working population and physically mobile. Thus, mobility impediments that could cause congestion need not be considered.
Justification: the problem suggests that the train station is one for commuter trains. Commuter trains are designed for use by commuters, generally working-age people. Furthermore, commuters with significant mobility limitations generally cannot use a commuter train effectively because it would likely require them to walk from the station to the place of their employment.
6. The exits from the train are on the side with two seats per row.
Justification: the train company ostensibly packs as many people as possible into each car to maximize profit. The exit will inevitably displace some seats that could be used for paying passengers. The train company will prefer to displace two paying passengers rather than three; thus, they will place the exits on the sides with fewer seats.
7. A person occupies two steps of the staircase at any given moment.
Justification: climbing the stairs requires one foot on the current step and another on the next step. Once on the next step, the following step must also be free to climb.
8. People who sit directly across from the exit do not move any distance parallel to the train tracks to exit the train; in other words, they do not move "down the car," only "across it."
Justification: People will want to exit in the quickest way possible. See Assumption 1. Since they are directly across from the exit, they do not need to travel down the car. We make this assumption because this is relevant in our calculations later on.
9. Passengers walk straight forward from the exit upon exiting, turn left or right upon reaching the middle of the platform, and walk straight toward the stairs.
Justification: Because each passenger will exit at the same time as many other passengers, they will walk forward before turning toward the stairway: they will feel

"pushed" out of the train and feel compelled to move into the middle of the platform before turning toward the stairs. Although some may walk past the halfway point and others may walk less than half the width, the number of people who do each should be the same, thus making it reasonable to assume that on average, each person walks to the middle of the platform.

10. The two trains that enter the station are identical.
    Justification: If the two trains that enter into the stations are different, we would need to optimize a function with four variables, which is beyond the scope of conventional mathematics. In addition, trains that enter into the same station at the same time are likely to have similar structure and build. The problem also implies the use of uniform trains because it gives specific variables for car length and number as well as guidelines for what most trains are like - "most trains are long (perhaps 10 or more cars long)".

Numerical Assumptions:

11. People shuffle out of their seats at a constant velocity of 145 ft/min inside the car. As they move to the exit through the aisle of the train car, they walk at a constant velocity of 250 ft/min. Passengers also walk across the platform at 250 ft/min.
    Justification: Industrial transit companies assume such values when designing platforms, thus we should be able to use these constants when designing a mathematical model to optimize time. (Source: Transportation)
12. The width of the seat is 17 inches and the distance between the rows of seats is 3.67 feet.
    Justification: (Sources: Haughney and Svensson)
13. The width of the platform is 29.5 feet.
    Justification: (Source: California)
14. There are 58 stairs in a staircase.
    Justification: We found the depth of a typical train tunnel to be 394 inches and combined this with regulations regarding the acceptable height of individual stairs in the United States (about 6.75 inches). This yielded about 58 stairs in a train station staircase, a reasonable number. (Source: U.S.)
15. The aisle of the train can hold two people.
    Justification: most trains are designed this way so that people can pass each other in the aisle when the train operates on a track of standard gauge.

1.3 Variables

Given:
n = number of cars to a train; as given by the problem, $n \geq 10$
d = length of the car; in assumption 4, we set its unit to be rows for convenience (thus, d = 1 means 1 row)
p = length of platform (feet)
q = number of stairs in each staircase
s = number of stairways
k = number of people who can fit side to side on the stairway

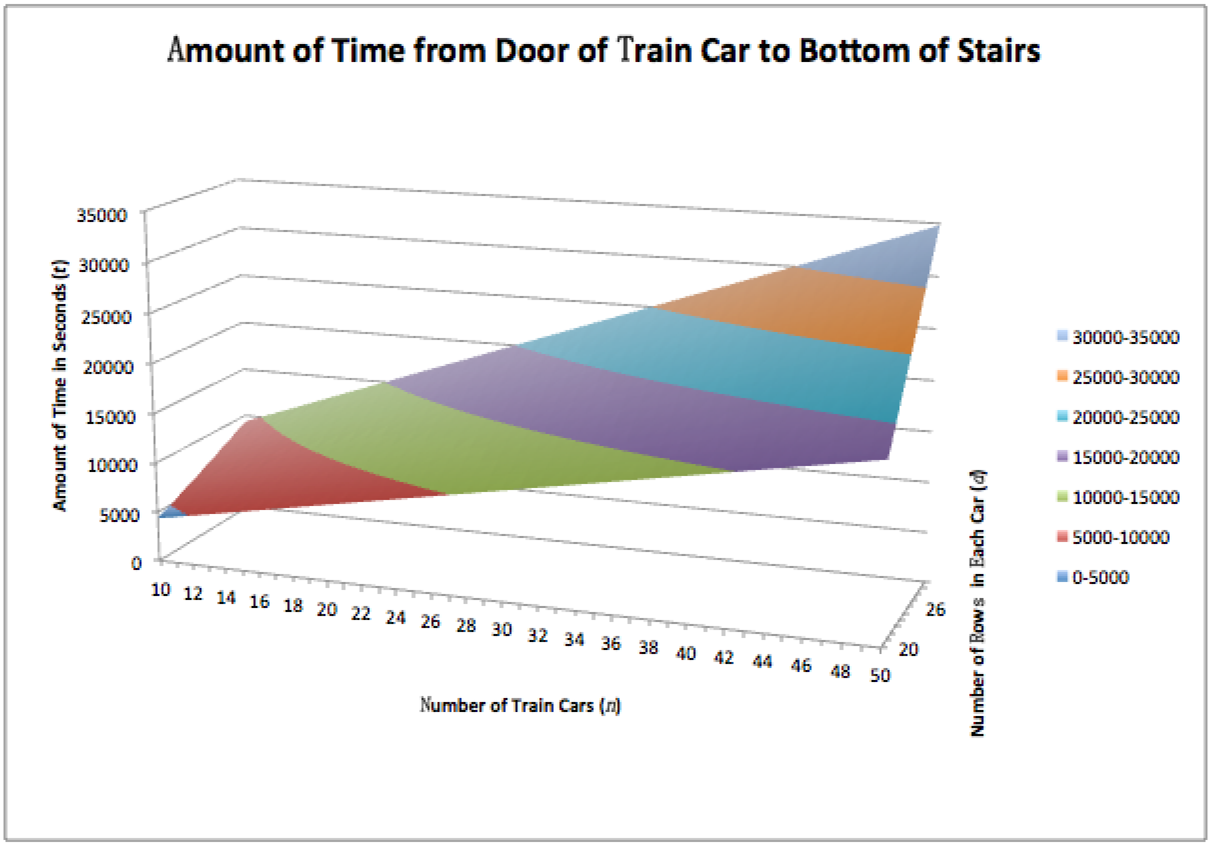
Established:
t = time taken to travel from car door to bottom stairway
l = distance traveled when walking from car seat to street level
l_q = total distance traveled by all passengers within the queue
l_p = total distance traveled by all passenger on the platform, but *not* in the queue
t_q = sum of all times passengers spend traveling in the queue
t_q = sum of all times passengers spend traveling on the platform, but *not* in the queue
B = people added to the queue for one train
B_t = people added to the queue for two trains
V = growth rate of the queue
V_t = growth rate of the queue for two trains
v_q = velocity of people traveling within the queue
v_p = velocity of people traveling on the platform, but *not* in the queue
v = general speed
x = number of people per car

2 Model

Single Train Model: t = expression below

$$\frac{158\,767. + 1.37594\, d^{6} + 0.0491012\, d^{7} + d^{3}\,(395.77 - 251.827\, n) + d\,(-728\,253. - 4.64593\, n) + d^{5}\,(58.4606 + 24.8333\, n) + d^{2}\,(-534.166 + 59.5455\, n) + d^{4}\,(743.066 + 345.238\, n)}{-6\,082.42 + 7\,608.67\, d - 18.3589\, d^{2} + 13.3202\, d^{3} + 1.\, d^{4}}$$

Surface plot for one fully occupied train:



(Figure 1)

2.1 Approach

We used the general equation:

$$l = vt$$

to set up our model for the amount of time it takes for a passenger to reach the street level of the station. We started by looking at the average distance as a function of *n* and *d*.

We chose to split the total distance traveled by each passenger from their seat in the train to the top of the stairs into three parts. Part one is the distance from their seat to the door of the train car. Part two is the distance from the train car to the bottom of the stairs. Part three is the distance from the bottom of the stairs to street level.

Part 1:

To calculate the amount of time it takes people to exit the car, we assume that they shuffle out of their seat and to the center aisle at a rate of 145 ft/min, or 1740 in/min (see Assumption 11). Furthermore, we define that a person is in the aisle once the person is no longer touching the chair on the side of the train on which s/he was sitting. From this, we assign seats certain positions, as shown below in the red numbers. Then, because we have assumed the seat width to be 17 inches, those in position 1 will have to move 17 inches to enter the aisle, those in position 2 will move 34 inches, and those in position 3 will move 51.

We must subtract one position 1 seat and one position 2 seat for each half of each car because there is an exit door in their place.

Position 1 has d-1 people per half car and thus the total time it takes all those people to move into the aisle is 17(d-1)/1740. For position 2 this value is 34(d-1)/1740. Then for position 3, instead of using d for the number of people sitting in position 3, we use d/2 because only one side of the train has position 3 seats (but we do not subtract 1 for the exit door; see Assumption 6), so the amount in people in that position take a total time to enter the aisle of 51(d/2)/1740.

Therefore, the total amount of time taken by all the people in the train to enter the aisles is

$$\frac{17(d-1)+34(d-1)+51(d/2)}{1740}$$

which simplifies to

$$\frac{76.5d-51}{1740} \text{ minutes (1)}$$

The distance between the rows is 3.67 feet, so then the total distance all the passengers travel in the aisle to reach a position where they can travel perpendicular to the train to exit is given by:

$$3.67\sum_{k=1}^{d/2}(k-1) = 3.67\left(\sum_{k=1}^{d/2}k - \sum_{k=1}^{d/2}1\right) = 3.67\left(\frac{(d/2)(d/2+1)}{2} - d/2\right)$$

Because every person will travel d/2 times the row they sit in because people travel to the door nearest them, effectively cutting the length of the car by half. Furthermore, those closer to the exit will travel less than those towards the middle of the car, which is accounted for by the summation of (k-1), which represents the people in the car who have reached the aisles.

Then, by assuming that people walk in the aisle at 250 ft/min, we arrive at a total time of

$$3.67\left(\frac{d^2-7d}{8}\right)/250 \text{ minutes} =$$
$$3.67\left(\frac{d^2-7d}{2000}\right) \text{ minutes}$$

Lastly, the total amount of time it takes all the people in a car to move from the aisle to the exit door (this is just the length of three seats: one for getting out to the the aisle from the seat plus two seats for walking by the railing) is given by:

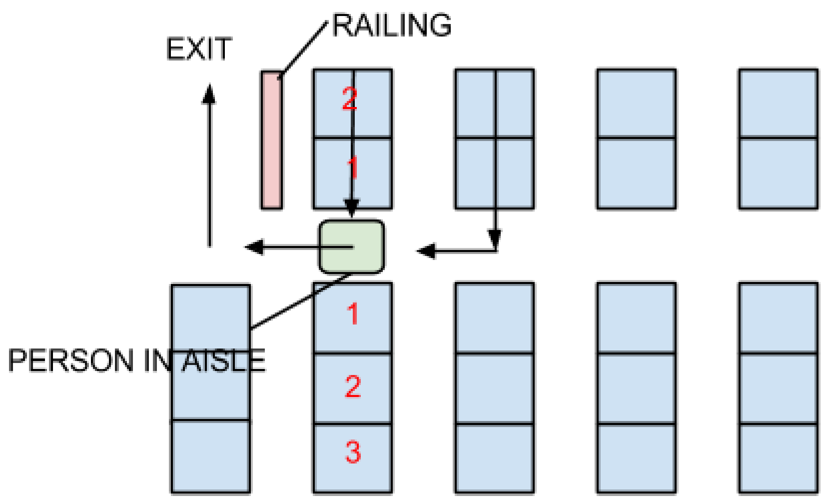
$$\frac{(5d-4)(51)}{250*12} = \frac{255d-204}{3000} \text{ minutes. (3)}$$

The total time it takes for all the people in a car to exit the train onto the platform for each half car is given by the sum of quantities (1), (2), and (3):

$$\frac{76.5d-51}{1740} + 3.67\left(\frac{d^2-7d}{2000}\right) + \frac{255d-204}{3000} \text{ minutes.}$$

Then to obtain the total time for all people to exit the entire train we have

$$2n\left(\frac{76.5d-51}{1740} + 3.67\left(\frac{d^2-7d}{2000}\right) + \frac{255d-204}{3000}\right) \text{ minutes.}$$



(Figure 2)

It is important to note that although this calculation is important in the following steps, the average amount of time it takes for a person to exit the train car is negligible in comparison to how much time it takes for them to travel down the platform and up the stairs. For instance, when plugging in n=20 and d=20, we found that it took 3.68 minutes or 220.80 seconds to travel through the car. However, the time spent on the platform and in the queue is 7647.73 seconds. Adding in the time spent on the stairs, time spent exiting the car becomes an even smaller proportion. Since the problem indicates that the time spent on the platform is extremely long, and since there is no way to optimize time spent exiting the train car except by bringing the length of the cars as close to 0 as possible (but there is a way to optimize platform time without reducing the length of the cars to 0: see our extensive discussion of the queue later on). Since it cannot be optimized and is a small proportion, we treat this amount of time as negligible in our optimization.

In order to find the number people who enter the queue per minute, we divided the total amount of people ($n(5d$-4)) by the total number of minutes that they took to exit the train (2n($\frac{76.5d-51}{1740} + 3.67\left(\frac{d^2-7d}{2000}\right) + \frac{255d-204}{3000}$) because the number of people who enter the queue per minute is the same as the number of people who exit the train per minute. Therefore, the queue gets

$$\frac{n(5d-4)}{2n(\frac{76.5d-51}{1740} + 3.67(\frac{d^2-7d}{2000}) + \frac{255d-204}{3000}} \text{ people per minute}$$

After simplification: $\frac{367d^3}{40000} + \frac{4136497d^2}{8700000} - \frac{1898321d}{2175000} + \frac{1411}{3625}$ people per minute which we called B_t.

The net growth of the queue, however, is what we obtained in the equation above subtracted by the number of people who enter the staircase per minute. This is 120 people/min because each person climbs at 2 stairs/sec and takes up 2 stairs at a time, so in each column of the stairs, one person reaches street level and one person starts climbing the stairs per second. Because we have 2 columns, 2 people go up the stairs each second, therefore 120 people up per minute. The net growth equals:

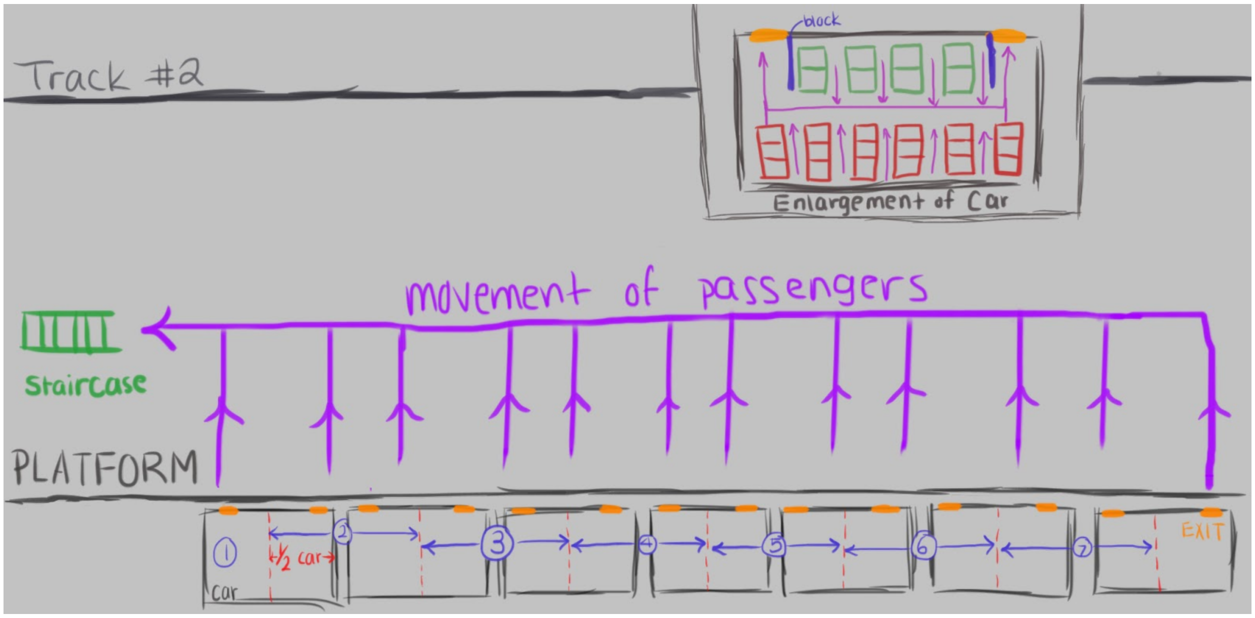
$$\frac{n(5d-4)}{2n(\frac{76.5d-51}{1740} + 3.67(\frac{d^2-7d}{2000}) + \frac{255d-204}{3000}} \text{ -120 people/min}$$

After simplification: $\frac{367d^3}{80000} + \frac{4136497d^2}{17400000} - \frac{1898321d}{4350000} - \frac{868589}{7250}$ people/min

Convert to ft/min (1 row=3.67 ft): $\frac{367(3.67d)^3}{80000} + \frac{4136497(3.67d)^2}{17400000} - \frac{1898321(3.67d)}{4350000} - \frac{868589}{7250}$ ft/min which we called V.

Part 2:

We found an expression for total distance traveled by all passengers in terms of the length of the car (d, in rows), and the number of cars (n). Then we found an expression for velocity in terms of the same variables. Using these two expressions, we were able to solve for the average amount of time taken by a passenger to reach the bottom of the stairway (by dividing by the total number of passengers, a function of the number of cars and the number of rows in each of those cars). Figure 3 is an illustration of our scenario with n = 7 and d = 6, which is explained in the paragraphs that follow.



(Figure 3)

The total distance traveled by all the people in all the cars of the train is given by:

$$\sum_{k=1}^{n}[(k-1)xd]+(1/2)xnd+(1/2)nxw$$

where x is the number of people in each car, d is the length of the car, n is the number of cars (corresponding to the blue numbers in Figure 3), and w is the width of the platform.

This is found by assuming that each passenger exits the train through the door nearest to them (Assumption 1) and that each passenger goes straight forward $w/2$ ft. once they leave the train, where w is the width of the platform, and then turn left or right and walk toward the stairs (Assumption 9). The total distance traveled by all passengers from the train car to the middle of the platform is thus

$$\frac{nw(5d-4)}{2} \text{ (distance 1)}$$

because $w/2$ is half the width of the platform and the number of passengers on the train is given by $n(5d\text{-}4)$: the number of cars times the number of people in the car, where the number of people in the car is five times the number of rows, minus the four seats displaced by the exit.

As in the picture above, we treated the two adjacent exits of adjacent train cars as one exit for $5d$-4 people (or the amount of people in one train car) because the separation between train cars is negligible. (To clarify, we did not assume people were using a single exit; rather, we treated the difference between the locations of the two exits as negligible). Therefore, if we let the first single exit be exit #1, the second and third exits be exit #2, the fourth and fifth exits be exit #3, and so on and the last pair of exits be exit #n (n = number of train cars), as shown in Figure 3, then the total horizontal distance that all passengers walk across the platform is calculated by

$$\sum_{k=1}^{n}[(k-1)*d*(5d-4)] \text{ (distance 2)}$$

At exit #1, the passengers walk 0 ft horizontally, since their exit is right next to the stairway out of the station. At exit #2, the passengers ($5d$-4 of them, to be exact), need to walk the distance d

rows to get to the stairs (see Figure 3). At exit #3, the passengers need to walk distance $2d$ rows (2 times the number of rows in each car is the length of the two cars they must walk past) to get to the stairs, and so on for each subsequent exit to exit #n from which the passengers need to walk $(n\text{-}1)d$ rows to get to the stairs.

At the last exit (the n + 1th exit), the passengers ($\frac{5d-4}{2}$ of them, because, unlike the exits in the middle of the train, the end exits take just half of a car rather than two adjacent halves of different cars; in this case, they come from the half of the last car that is closer to this last exit) walk nd rows to get to the stairway (see Figure 3). So the total distance that these last passengers walk is:

$$\frac{nd(5d-4)}{2} \text{ (distance 3)}$$

Therefore, our grand total distance (l_{total}) is the sum of the highlighted total distances 1, 2, and 3 above:

$$l_{total} = \frac{nw(5d-4)}{2} + \sum_{k=1}^{n}[(k-1)*d*(5d-4)] + \frac{nd(5d-4)}{2}$$

After simplification using summations:

$$l_{total} = \tfrac{n}{2}(5d-4)[d(n+2)+27.5]$$

A note on the queue: given the nature of human behavior, it is not realistic that humans will queue up in a straight line as they wait to climb up the stairs. Thus, in our analysis, we have assumed that a rectangular mass of people with a width of w = 29.5 ft (the width of the platform, see Assumption 13) forms at the bottom of the staircase, and the length of this mass grows as people exit the train and arrive at the base of the stairs. People will, however, travel in an essentially straight line through the queue to the staircase. Thus, when we later find the total distance traveled through the queue, this is just a straightline distance from start to end. But when we analyze how fast the queue is growing, we use the square footage each person takes up in the throng of people waiting at the bottom of the stairs in conjunction with the width of the platform to precisely quantify how quickly the back of the queue is spilling out onto the platform.

To find the growth of the queue, we looked at the rate of people stepping onto the platform from the train. Because people walk on the platform outside the queue at a constant speed, the number of people exiting the car is equal to the number of people entering the queue at any given time. To do find the queue growth, then, we divided the number of people in the train ($n(5d\text{-}4)$) by the amount of time it took for people to exit the train (See Part 1 of Approach for detailed derivation):

$$B = \frac{367d^3}{80000} + \frac{4136497d^2}{17400000} - \frac{1898321d}{4350000} + \frac{1411}{7250}$$

Because we need to find the growth rate of the queue (B represents the number of people added to the queue), we must subtract the number of people who leave the queue per minute (by entering the staircase). Knowing that 2 people exit the queue and step onto the staircase per second, 120 people leave the queue per minute. Subtracting 120 from the above expression, we get (Again, see Part 1 of Approach):

$$V = \frac{367d^3}{80000} + \frac{4136497d^2}{17400000} - \frac{1898321d}{4350000} - \frac{868589}{7250}$$

Because we need d in terms of feet, we input $3.67d$ instead of just d because each row is about 3.67 feet wide, and d has been considered to be in rows for the previous equations. Thus,

$$V = \frac{367(3.67d)^3}{80000} + \frac{4136497(3.67d)^2}{17400000} - \frac{1898321(3.67d)}{4350000} - \frac{868589}{7250}$$

Let l_q be the total distance traveled through the queue. It is the sum of the distances traveled in the queue by every person on the train. Let l_p be the total distance traveled by all the passengers on the platform. Then let t_p and t_q be the total of the times all the passengers spend traveling on the platform and in the queue, respectively. Let V be the growth rate of the queue in ft^2/min (remember that we have considered the queue to be 2-dimensional). Last, let v_p and v_q be the (constant) velocities of the people traveling on the platform and in the queue, respectively.

We know that the total distance travelled by everyone is:

$$l_{total} = l_q + l_p$$

for l_q being the total distance everyone spends in queue and l_p being the total distance everyone spends walking on the platform and not in queue.

We also know that total distance spent inside the queue (l_q) is equal to

$$l_q = v_q * t_q$$

with v_q being the velocity at which a passenger walks in queue, which is approximately .2712 ft/sec because 2 people leave the queue to go on the stairs each second, opening up 8 ft^2 of space at the front of the line (each person takes up approximately 4 ft^2 of personal space). Because the total area of the rectangular queue before a passenger in line decreases by 8 ft^2/sec, the total length of the queue before that passenger decreases by (8 ft^2/sec)/(29.7 ft)$\approx$ .2712 ft/sec, meaning that a passenger in queue moves forward by .2712 ft/sec. t_q is the total amount of time every passenger spends in queue around the stairs.

We also know that the length of the queue is equal to the speed at which it's length is changing times the given time. For each passenger the length of the queue when they arrive at the queue is the rate of change of the queue (V) times the time they spent walking on the platform. We also noticed however, that a queue does not form until the first group of passengers reach the bottom of the stairs and the time that it take that group of passengers to do so would be the time it takes them to walk halfway across the platform to reach the stairs, which is $(w/2)/v_p$ = ((29.5 ft)/2)/((250 ft/min)*((1 min)/(60 sec))) = 7.08 sec. So the actual time that we should multiply V by to find the sum of the lengths of the queue for every passenger is $(t_p - 7.08n(5d-4))$, where t_p = the sum of times every passenger spends in line because

$$l_q = (l_1 + l_2 + ... + l_k = V(t_1 - 7.08 + t_2 - 7.08 + ... + t_k - 7.08) = V(t_p - 7.08n(5d-4))$$

Thus,

$$l_q = \mathrm{V}\,(t_p - 7.08n(5d-4))$$

We also know that total distance spent walking on the platform not in queue (l_p) is equal to regular walking speed (250 ft/min = $\frac{25}{6}$ ft/sec) times the total amount of time spent walking on the platform not in queue.

$$l_p = v_p * t_p$$
$$l_p = 25/6 * t_p$$

Solving this system of four equation for two variables (t_q and t_p):

$$\mathrm{V}\,(t_p - 7.08n(5d-4)) = l_q = v_q * t_q$$
$$t_q = \mathrm{V}\,(t_p - 7.08n(5d-4)) / v_q$$
$$v_q = (8\,ft^2/s)/29.5ft \approx .2712\,ft/s$$

We were able to find t_q in terms of t_p, V, and n and d:

$$t_q = \frac{V(t_p - 7.08n(5d-4))}{.2712}$$

$$l_{total} - l_q = l_p = v_p * t_p$$
$$l_{total} - V\ (t_p - 7.08n(5d-4)) = v_p * t_p$$
$$v_p = 25/6 \text{ ft/s}$$

We found t_p in terms of l_{total}, V, n, and, d.

$$t_p = \frac{l_{total} + V(7.08n(5d-4))}{V + 25/6}$$

Now, we have a function of total time ($t_q + t_p$) in terms of n and d (because both l_{total} and V we have in terms of n and d), which is what we want.

After substituting t_p into the equation for t_q, adding the highlighted equations above, dividing by the number of passengers (n*(5d-4)) to get the average time and simplifying, we obtain:

$$\frac{158767. + 1.37594\,d^6 + 0.0491012\,d^7 + d^3\,(395.77 - 251.827\,n) + d\,(-728253. - 4.64593\,n) + d^5\,(58.4606 + 24.8333\,n) + d^2\,(-534.166 + 59.5455\,n) + d^4\,(743.066 + 345.238\,n)}{-6082.42 + 7608.67\,d - 18.3589\,d^2 + 13.3202\,d^3 + 1.\,d^4}$$

which is the equation for average amount of time each passenger spends walking from the exit of their train car to the bottom of the stairs on one fully loaded train with d rows per car and n cars.

Part 3:

We found the average stair-climbing speed of an average person to be 2 stairs/second. We also set that a person would take up 2 stairs at a time. The length of the average train station staircase is 58 steps so we decided to use that value for our staircase. Since there are 58 stairs and each person climbs the stairs at 2 stairs per second, the amount of time spent by each person climbing the stairs is 29 seconds, a constant.

2.2 Requirement 1: Optimize time traveled to reach street level for passengers on one fully occupied train

Interpretation:

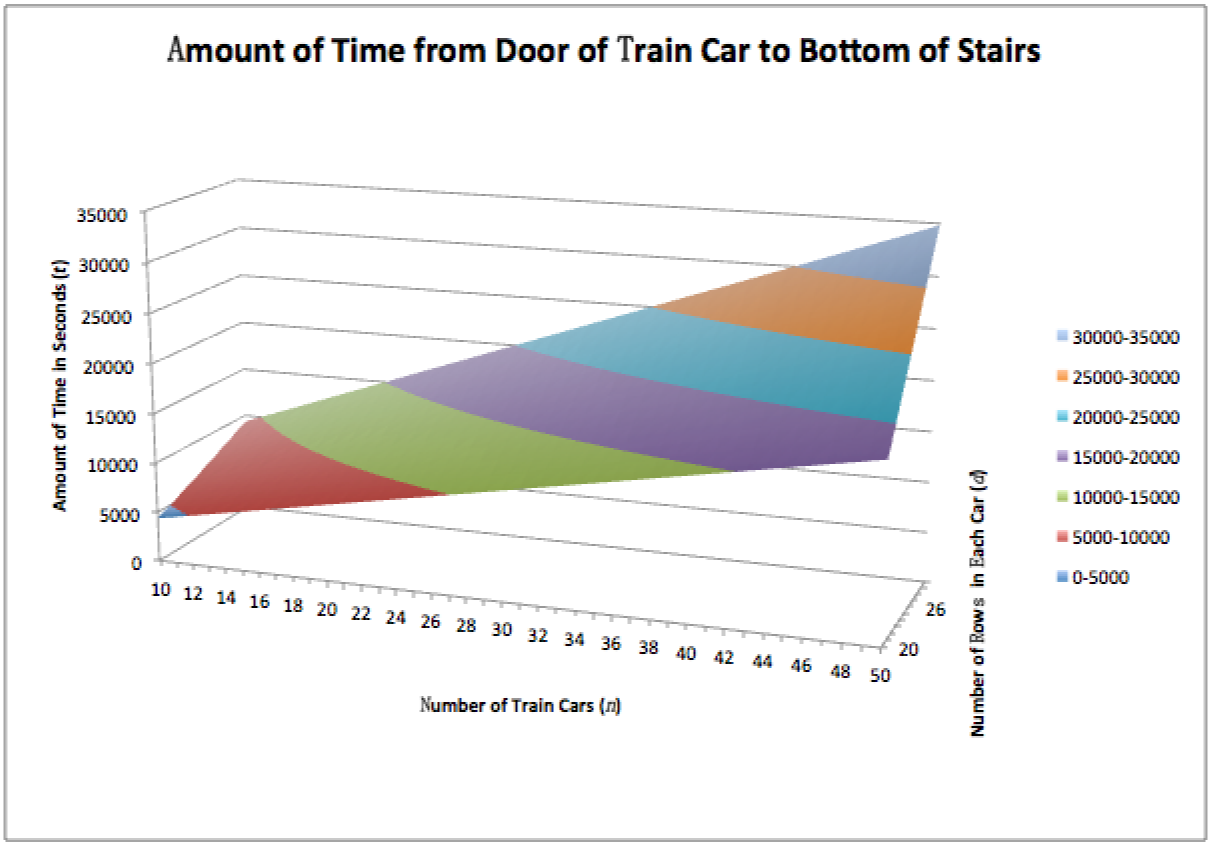
We interpreted this part of the problem as asking us to relate the time an average person takes to exit the station (start from his/her seat in the train car and finish at the top of the stairs) to n, the number of cars, and d, the number of rows per car and to find the n and d that minimize the amount of time or to suggest a general principle of selecting n and/or d that would optimize amount of time spent traveling to reach street level.

Approach:

To do this, we created a model, A, that is a function of all the variables determinant of the time required to exit the station. What determines the amount of time needed to fully exit the station?

We realized that the only period of time that matters is the amount of time it takes for a passenger to walk on the platform and arrive at the base of the stairs because first, the amount of time for each passenger to exit the train car is constant because

The method for deriving this model can be found in the Approach section. After using excel to create a surface plot (Figure 4) with the number of rows in each car (d) and the number of train cars (n) as explanatory variables and the amount of time it takes for the passenger to walk from the train car to the bottom of the stairs (t) as the response variable, we found that the time required is optimized when n and d are minimized, which is reasonable as we would expect the platform to be less congested if there are fewer people on the train.

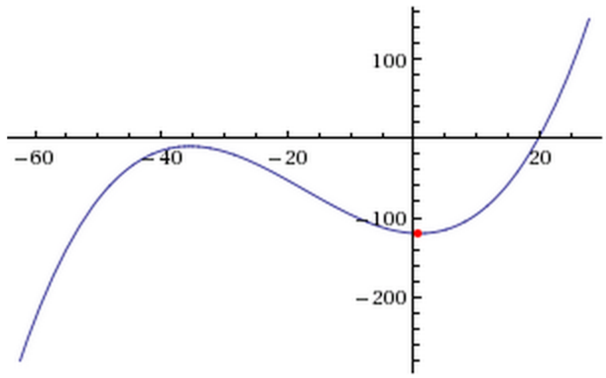


(Figure 4)

While our model confirms the intuitive idea that less people means less congestion, it does not help train manufacturers or train stations decide how to minimize wait time for passengers as it would be unrealistic to have a train with 1 car and 1 row of seats. In our model, however, is also the build up rate of the queue or fan of people at the bottom of the stairs (V).

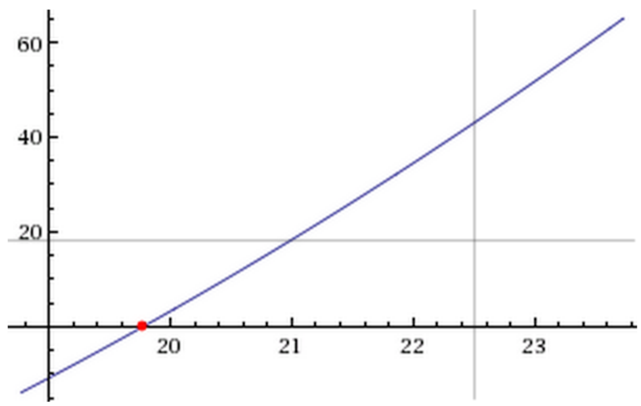
$$V = \frac{367(3.67d)^3}{80000} + \frac{4136497(3.67d)^2}{17400000} - \frac{1898321(3.67d)}{4350000} - \frac{868589}{7250}$$

V only depends on d because each passenger walks at an approximately constant speed across the platform when there is no queue (Assumption 9). Thus, the number of passengers arriving at the queue would be the same for any n as people in the farther cars would just take longer to get to the queue. Graphing and solving for zeros of $V(d)$, we found that $V(d) \geq 0 \ \forall \ d > 19.7747$, meaning that the queue will begin accumulating when the number of rows in each train car exceeds 19 or is greater or equal to 20 (Figures 4, 5, 6).



(Figure 5)

Root plot:



(Figure 6)

$$d = \frac{1}{478\,935}\Big(-8\,272\,994 + \big(778\,015\,345\,737\,730\,141\,936 - 3\,831\,480\sqrt{11\,664\,520\,877\,017\,530\,472\,275\,623\,013}\,\big)\,\hat{}\,(1/3) + 2\big(97\,251\,918\,217\,216\,267\,742 + 478\,935\sqrt{11\,664\,520\,877\,017\,530\,472\,275\,623\,013}\,\big)\,\hat{}\,(1/3)\Big) \approx 19.7746858648579$$

Therefore, we can conclude that in order to avoid the queueing/fanning phenomenon for one fully occupied train, we should have train cars with less than or equal to 19 rows. However, because the queue does not form at all for $d \leq 19$, we would recommend 19 rows to be installed in each train car to maximize the number of people transport as well as minimize the amount of time spent leaving the platform.

Table of Projected Times for One Train: $d \in (4, 30)$ and $n \in (10, 50)$ with $d > 19$ highlighted

	4	5	6	7	8	9	10	11	12
10	-59.389534	-4.1515683	75.0567693	183.432657	324.678014	501.036459	713.314574	961.001242	1242.48747
11	-55.470618	3.64213744	88.711086	205.269846	357.263446	547.067653	775.505809	1041.97643	1344.69124
12	-51.551701	11.4358432	102.365403	227.107036	389.848878	593.098848	837.697044	1122.95163	1446.89502
13	-47.632784	19.229549	116.01972	248.944225	422.434309	639.130043	899.88828	1203.92682	1549.09879
14	-43.713867	27.0232548	129.674036	270.781414	455.019741	685.161237	962.079515	1284.90201	1651.30256
15	-39.794951	34.8169606	143.328353	292.618603	487.605173	731.192432	1024.27075	1365.87721	1753.50634
16	-35.876034	42.6106663	156.98267	314.455792	520.190605	777.223627	1086.46199	1446.8524	1855.71011
17	-31.957117	50.4043721	170.636986	336.292981	552.776037	823.254821	1148.65322	1527.82759	1957.91388
18	-28.0382	58.1980779	184.291303	358.13017	585.361468	869.286016	1210.84446	1608.80278	2060.11766
19	-24.119284	65.9917837	197.94562	379.967359	617.9469	915.317211	1273.03569	1689.77798	2162.32143
20	-20.200367	73.7854895	211.599937	401.804548	650.532332	961.348406	1335.22693	1770.75317	2264.5252
21	-16.28145	81.5791952	225.254253	423.641737	683.117764	1007.3796	1397.41816	1851.72836	2366.72898
22	-12.362533	89.372901	238.90857	445.478926	715.703196	1053.4108	1459.6094	1932.70355	2468.93275
23	-8.4436167	97.1666068	252.562887	467.316115	748.288627	1099.44199	1521.80063	2013.67875	2571.13652
24	-4.5247	104.960313	266.217204	489.153304	780.874059	1145.47318	1583.99187	2094.65394	2673.3403
25	-0.6057832	112.754018	279.87152	510.990493	813.459491	1191.50438	1646.1831	2175.62913	2775.54407
26	3.31313353	120.547724	293.525837	532.827683	846.044923	1237.53557	1708.37434	2256.60432	2877.74784
27	7.23205028	128.34143	307.180154	554.664872	878.630355	1283.56677	1770.56557	2337.57952	2979.95162
28	11.150967	136.135136	320.834471	576.502061	911.215786	1329.59796	1832.75681	2418.55471	3082.15539
29	15.0698838	143.928841	334.488787	598.33925	943.801218	1375.62916	1894.94804	2499.5299	3184.35916
30	18.9888005	151.722547	348.143104	620.176439	976.38665	1421.66035	1957.13928	2580.50509	3286.56294
31	22.9077173	159.516253	361.797421	642.013628	1008.97208	1467.69155	2019.33051	2661.48029	3388.76671
32	26.826634	167.309959	375.451738	663.850817	1041.55751	1513.72274	2081.52175	2742.45548	3490.97048
33	30.7455507	175.103665	389.106054	685.688006	1074.14295	1559.75394	2143.71299	2823.43067	3593.17426
34	34.6644675	182.89737	402.760371	707.525195	1106.72838	1605.78513	2205.90422	2904.40587	3695.37803
35	38.5833842	190.691076	416.414688	729.362384	1139.31381	1651.81633	2268.09546	2985.38106	3797.5818
36	42.502301	198.484782	430.069005	751.199573	1171.89924	1697.84752	2330.28669	3066.35625	3899.78558
37	46.4212177	206.278488	443.723321	773.036762	1204.48467	1743.87872	2392.47793	3147.33144	4001.98935
38	50.3401345	214.072193	457.377638	794.873951	1237.0701	1789.90991	2454.66916	3228.30664	4104.19312
39	54.2590512	221.865899	471.031955	816.71114	1269.65554	1835.94111	2516.8604	3309.28183	4206.3969
40	58.177968	229.659605	484.686271	838.548329	1302.24097	1881.9723	2579.05163	3390.25702	4308.60067
41	62.0968847	237.453311	498.340588	860.385519	1334.8264	1928.00349	2641.24287	3471.23221	4410.80444
42	66.0158014	245.247017	511.994905	882.222708	1367.41183	1974.03469	2703.4341	3552.20741	4513.00822
43	69.9347182	253.040722	525.649222	904.059897	1399.99726	2020.06588	2765.62534	3633.1826	4615.21199
44	73.8536349	260.834428	539.303538	925.897086	1432.58269	2066.09708	2827.81657	3714.15779	4717.41576
45	77.7725517	268.628134	552.957855	947.734275	1465.16813	2112.12827	2890.00781	3795.13298	4819.61954
46	81.6914684	276.42184	566.612172	969.571464	1497.75356	2158.15947	2952.19904	3876.10818	4921.82331
47	85.6103852	284.215546	580.266489	991.408653	1530.33899	2204.19066	3014.39028	3957.08337	5024.02708
48	89.5293019	292.009251	593.920805	1013.24584	1562.92442	2250.22186	3076.58151	4038.05856	5126.23086
49	93.4482186	299.802957	607.575122	1035.08303	1595.50985	2296.25305	3138.77275	4119.03375	5228.43463
50	97.3671354	307.596663	621.229439	1056.92022	1628.09529	2342.28425	3200.96399	4200.00895	5330.6384

	13	14	15	16	17	18	19	20	21
10	1555.35072	1896.658	2263.24783	2651.96452	3059.83265	3484.17159	3922.6578	4373.34599	4834.66106
11	1680.98346	2047.63641	2441.18912	2858.18989	3295.38509	3749.8428	4219.01981	4700.78421	5193.40616
12	1806.61619	2198.61483	2619.13041	3064.41527	3530.93754	4015.514	4515.38183	5028.22242	5552.15127
13	1932.24893	2349.59324	2797.07169	3270.64064	3766.48999	4281.18521	4811.74384	5355.66064	5910.89638
14	2057.88167	2500.57166	2975.01298	3476.86601	4002.04243	4546.85642	5108.10585	5683.09885	6269.64149
15	2183.5144	2651.55007	3152.95427	3683.09139	4237.59488	4812.52762	5404.46786	6010.53706	6628.38659
16	2309.14714	2802.52849	3330.89556	3889.31676	4473.14733	5078.19883	5700.82987	6337.97528	6987.1317
17	2434.77987	2953.5069	3508.83685	4095.54213	4708.69977	5343.87004	5997.19188	6665.41349	7345.87681
18	2560.41261	3104.48532	3686.77814	4301.7675	4944.25222	5609.54124	6293.5539	6992.85171	7704.62192
19	2686.04535	3255.46373	3864.71943	4507.99288	5179.80467	5875.21245	6589.91591	7320.28992	8063.36702
20	2811.67808	3406.44215	4042.66072	4714.21825	5415.35711	6140.88366	6886.27792	7647.72814	8422.11213
21	2937.31082	3557.42056	4220.60201	4920.44362	5650.90956	6406.55487	7182.63993	7975.16635	8780.85724
22	3062.94356	3708.39898	4398.5433	5126.669	5886.462	6672.22607	7479.00194	8302.60457	9139.60235
23	3188.57629	3859.37739	4576.48459	5332.89437	6122.01445	6937.89728	7775.36395	8630.04278	9498.34745
24	3314.20903	4010.35581	4754.42588	5539.11974	6357.5669	7203.56849	8071.72597	8957.481	9857.09256
25	3439.84176	4161.33422	4932.36717	5745.34511	6593.11934	7469.23969	8368.08798	9284.91921	10215.8377
26	3565.4745	4312.31264	5110.30846	5951.57049	6828.67179	7734.9109	8664.44999	9612.35742	10574.5828
27	3691.10724	4463.29105	5288.24975	6157.79586	7064.22424	8000.58211	8960.812	9939.79564	10933.3279
28	3816.73997	4614.26947	5466.19104	6364.02123	7299.77668	8266.25331	9257.17401	10267.2339	11292.073
29	3942.37271	4765.24788	5644.13233	6570.24661	7535.32913	8531.92452	9553.53602	10594.6721	11650.8181
30	4068.00545	4916.2263	5822.07362	6776.47198	7770.88158	8797.59573	9849.89804	10922.1103	12009.5632
31	4193.63818	5067.20471	6000.01491	6982.69735	8006.43402	9063.26694	10146.26	11249.5485	12368.3083
32	4319.27092	5218.18313	6177.9562	7188.92272	8241.98647	9328.93814	10442.6221	11576.9867	12727.0534
33	4444.90365	5369.16154	6355.89749	7395.1481	8477.53892	9594.60935	10738.9841	11904.4249	13085.7985
34	4570.53639	5520.13996	6533.83878	7601.37347	8713.09136	9860.28056	11035.3461	12231.8631	13444.5436
35	4696.16913	5671.11837	6711.78007	7807.59884	8948.64381	10125.9518	11331.7081	12559.3014	13803.2887
36	4821.80186	5822.09679	6889.72136	8013.82422	9184.19626	10391.623	11628.0701	12886.7396	14162.0339
37	4947.4346	5973.0752	7067.66265	8220.04959	9419.7487	10657.2942	11924.4321	13214.1778	14520.779
38	5073.06734	6124.05362	7245.60394	8426.27496	9655.30115	10922.9654	12220.7941	13541.616	14879.5241
39	5198.70007	6275.03203	7423.54523	8632.50033	9890.8536	11188.6366	12517.1561	13869.0542	15238.2692
40	5324.33281	6426.01045	7601.48652	8838.72571	10126.406	11454.3078	12813.5182	14196.4924	15597.0143
41	5449.96555	6576.98886	7779.42781	9044.95108	10361.9585	11719.979	13109.8802	14523.9306	15955.7594
42	5575.59828	6727.96728	7957.3691	9251.17645	10597.5109	11985.6502	13406.2422	14851.3689	16314.5045
43	5701.23102	6878.94569	8135.31039	9457.40183	10833.0634	12251.3214	13702.6042	15178.8071	16673.2496
44	5826.86375	7029.92411	8313.25168	9663.6272	11068.6158	12516.9926	13998.9662	15506.2453	17031.9947
45	5952.49649	7180.90252	8491.19297	9869.85257	11304.1683	12782.6638	14295.3282	15833.6835	17390.7398
46	6078.12923	7331.88094	8669.13426	10076.0779	11539.7207	13048.335	14591.6902	16161.1217	17749.4849
47	6203.76196	7482.85935	8847.07555	10282.3033	11775.2732	13314.0062	14888.0522	16488.5599	18108.23
48	6329.3947	7633.83777	9025.01684	10488.5287	12010.8256	13579.6775	15184.4142	16815.9981	18466.9751
49	6455.02744	7784.81618	9202.95813	10694.7541	12246.3781	13845.3487	15480.7763	17143.4364	18825.7202
50	6580.66017	7935.7946	9380.89942	10900.9794	12481.9305	14111.0199	15777.1383	17470.8746	19184.4654

	22	23	24	25	26	27	28	29	30
10	5305.37138	5784.55188	6271.54297	6765.90966	7267.40323	7775.9269	8291.50601	8814.26265	9344.39462
11	5695.52875	6206.12756	6724.46614	7250.05144	7782.59221	8321.96172	8868.16543	9421.31346	9981.59773
12	6085.68611	6627.70324	7177.38932	7734.19323	8297.78119	8867.99655	9444.82486	10028.3643	10618.8008
13	6475.84347	7049.27893	7630.3125	8218.33501	8812.97017	9414.03137	10021.4843	10635.4151	11256.0039
14	6866.00083	7470.85461	8083.23567	8702.4768	9328.15915	9960.0662	10598.1437	11242.4659	11893.2071
15	7256.15819	7892.43029	8536.15885	9186.61858	9843.34813	10506.101	11174.8031	11849.5167	12530.4102
16	7646.31556	8314.00597	8989.08202	9670.76037	10358.5371	11052.1358	11751.4626	12456.5675	13167.6133
17	8036.47292	8735.58166	9442.0052	10154.9022	10873.7261	11598.1707	12328.122	13063.6183	13804.8164
18	8426.63028	9157.15734	9894.92838	10639.0439	11388.9151	12144.2055	12904.7814	13670.6691	14442.0195
19	8816.78764	9578.73302	10347.8516	11123.1857	11904.1041	12690.2403	13481.4408	14277.7199	15079.2226
20	9206.945	10000.3087	10800.7747	11607.3275	12419.293	13236.2751	14058.1003	14884.7707	15716.4257
21	9597.10237	10421.8844	11253.6979	12091.4693	12934.482	13782.31	14634.7597	15491.8215	16353.6288
22	9987.25973	10843.4601	11706.6211	12575.6111	13449.671	14328.3448	15211.4191	16098.8723	16990.8319
23	10377.4171	11265.0358	12159.5443	13059.7529	13964.86	14874.3796	15788.0785	16705.9231	17628.0351
24	10767.5745	11686.6114	12612.4674	13543.8946	14480.049	15420.4144	16364.738	17312.9739	18265.2382
25	11157.7318	12108.1871	13065.3906	14028.0364	14995.2379	15966.4493	16941.3974	17920.0247	18902.4413
26	11547.8892	12529.7628	13518.3138	14512.1782	15510.4269	16512.4841	17518.0568	18527.0755	19539.6444
27	11938.0465	12951.3385	13971.237	14996.32	16025.6159	17058.5189	18094.7162	19134.1263	20176.8475
28	12328.2039	13372.9142	14424.1601	15480.4618	16540.8049	17604.5537	18671.3757	19741.1771	20814.0506
29	12718.3613	13794.4898	14877.0833	15964.6036	17055.9939	18150.5886	19248.0351	20348.2279	21451.2537
30	13108.5186	14216.0655	15330.0065	16448.7454	17571.1828	18696.6234	19824.6945	20955.2787	22088.4568
31	13498.676	14637.6412	15782.9297	16932.8871	18086.3718	19242.6582	20401.354	21562.3295	22725.6599
32	13888.8334	15059.2169	16235.8528	17417.0289	18601.5608	19788.693	20978.0134	22169.3803	23362.863
33	14278.9907	15480.7926	16688.776	17901.1707	19116.7498	20334.7279	21554.6728	22776.4311	24000.0662
34	14669.1481	15902.3683	17141.6992	18385.3125	19631.9388	20880.7627	22131.3322	23383.4819	24637.2693
35	15059.3054	16323.9439	17594.6224	18869.4543	20147.1278	21426.7975	22707.9917	23990.5327	25274.4724
36	15449.4628	16745.5196	18047.5455	19353.5961	20662.3167	21972.8323	23284.6511	24597.5836	25911.6755
37	15839.6202	17167.0953	18500.4687	19837.7378	21177.5057	22518.8672	23861.3105	25204.6344	26548.8786
38	16229.7775	17588.671	18953.3919	20321.8796	21692.6947	23064.902	24437.9699	25811.6852	27186.0817
39	16619.9349	18010.2467	19406.3151	20806.0214	22207.8837	23610.9368	25014.6294	26418.736	27823.2848
40	17010.0922	18431.8224	19859.2382	21290.1632	22723.0727	24156.9716	25591.2888	27025.7868	28460.4879
41	17400.2496	18853.398	20312.1614	21774.305	23238.2616	24703.0065	26167.9482	27632.8376	29097.691
42	17790.407	19274.9737	20765.0846	22258.4468	23753.4506	25249.0413	26744.6076	28239.8884	29734.8942
43	18180.5643	19696.5494	21218.0078	22742.5886	24268.6396	25795.0761	27321.2671	28846.9392	30372.0973
44	18570.7217	20118.1251	21670.9309	23226.7303	24783.8286	26341.1109	27897.9265	29453.99	31009.3004
45	18960.8791	20539.7008	22123.8541	23710.8721	25299.0176	26887.1458	28474.5859	30061.0408	31646.5035
46	19351.0364	20961.2764	22576.7773	24195.0139	25814.2065	27433.1806	29051.2453	30668.0916	32283.7066
47	19741.1938	21382.8521	23029.7005	24679.1557	26329.3955	27979.2154	29627.9048	31275.1424	32920.9097
48	20131.3511	21804.4278	23482.6236	25163.2975	26844.5845	28525.2502	30204.5642	31882.1932	33558.1128
49	20521.5085	22226.0035	23935.5468	25647.4393	27359.7735	29071.2851	30781.2236	32489.244	34195.3159
50	20911.6659	22647.5792	24388.47	26131.581	27874.9625	29617.3199	31357.883	33096.2948	34832.519

(Figure 7)

2.3 Requirement 2: Optimize time traveled to reach street level for passengers of two fully occupied trains

Interpretation:

This part of the problem asks us to use the method we found in Requirement 1 for with two fully occupied and identical trains (Assumption 10), resulting in twice the amount of people as in Requirement 1 arriving in the station.

Approach:

In order to optimize time traveled to reach street level from two fully occupied trains, we applied the same equation as in requirement 1 except we double the amount by which the queue grows because the there are twice amount of passengers making their way to the staircase, assuming the two trains stationed are identical in number of rows and number of cars.

The equation for the growth rate of the queue in people per minute is given by:

$$B_t = 2\left(\frac{367d^3}{80000} + \frac{4136497d^2}{17400000} - \frac{1898321d}{4350000} + \frac{1411}{7250}\right)$$
$$B_t = \frac{367d^3}{40000} + \frac{4136497d^2}{8700000} - \frac{1898321d}{2175000} + \frac{1411}{3625}$$

To look at B_t as the growth rate of the queue, we must subtract the constant depletion rate of the queue, 120 ppl/min, from B_t. We must also convert this d into feet as well, swapping $3.67d$ for d.

$$V_t = \frac{367d^3}{40000} + \frac{4136497d^2}{8700000} - \frac{1898321d}{2175000} - \frac{433589}{3625}$$

The number of passengers also doubles with the two trains. So, the total number of commuters is:

$$2n(5d-4)$$

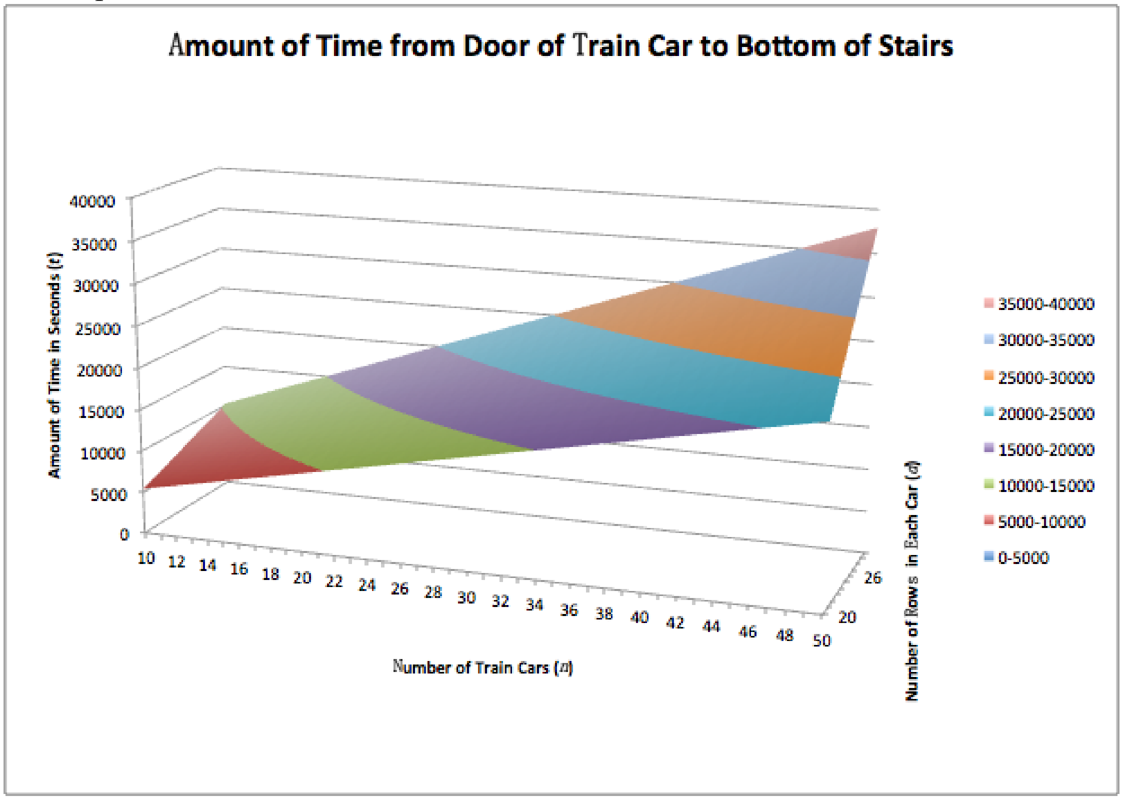
Because there are two trains, the total distance travelled to the stairway (assuming that there is no queue) is doubled as well since twice as many people walk the same distance. Thus, l_{total} is multiplied by two:

$$l_{total} = \tfrac{n}{2}(5d-4)[d(n+2)+27.5]$$

When we plug all of these changes to the original model equation for one train and simplify, we get:

$$\frac{79383.4 + 0.687968\, d^6 + 0.0245506\, d^7 + d^3\,(197.885 - 125.913\, n) + d\,(-364126. - 2.32297\, n) + d^5\,(29.2303 + 12.4166\, n) + d^2\,(-267.083 + 29.7727\, n) + d^4\,(371.533 + 172.619\, n)}{-6082.42 + 7608.67\, d - 18.3589\, d^2 + 13.3202\, d^3 + 1.\, d^4}$$

The surface plot of this function is:



(Figure 8)

Note that again the average amount of time increases and both *n* and *d* increase, so ideally, we would want *n* and *d* to be as small as possible. However, like in Requirement 1, we again find that *V* is negative up to *d* = 19 rows, because *V* for two trains is 2**V* for one train as twice as many people will join the queue per period of time since twice as many people exit their respective cars per period of time and the zero of a function does not change if that function is multiplied by a constant. Therefore, we would still recommend for each train car to have 19 rows of seats (or fewer, if necessary).

Table of Projected Times for Two Trains: $d \in (4, 30)$ and $n \in (10, 50)$ with $d > 19$ highlighted:

	4	5	6	7	8	9	10	11	12
10	48.5957439	145.914738	284.467931	467.622419	695.547682	965.758291	1273.90433	1614.63362	1982.35155
11	56.1803947	160.687733	309.697276	506.797847	752.131703	1042.97195	1374.57531	1741.11482	2136.50224
12	63.7650454	175.460728	334.926622	545.973276	808.715724	1120.18561	1475.2463	1867.59601	2290.65292
13	71.3496962	190.233723	360.155968	585.148704	865.299745	1197.39927	1575.91728	1994.0772	2444.8036
14	78.934347	205.006718	385.385314	624.324133	921.883766	1274.61293	1676.58827	2120.5584	2598.95429
15	86.5189978	219.779713	410.61466	663.499561	978.467787	1351.82659	1777.25925	2247.03959	2753.10497
16	94.1036485	234.552708	435.844005	702.674989	1035.05181	1429.04025	1877.93023	2373.52079	2907.25565
17	101.688299	249.325703	461.073351	741.850418	1091.63583	1506.2539	1978.60122	2500.00198	3061.40634
18	109.27295	264.098698	486.302697	781.025846	1148.21985	1583.46756	2079.2722	2626.48317	3215.55702
19	116.857601	278.871693	511.532043	820.201274	1204.80387	1660.68122	2179.94319	2752.96437	3369.7077
20	124.442252	293.644688	536.761389	859.376703	1261.38789	1737.89488	2280.61417	2879.44556	3523.85839
21	132.026902	308.417683	561.990734	898.552131	1317.97191	1815.10854	2381.28516	3005.92676	3678.00907
22	139.611553	323.190678	587.22008	937.72756	1374.55593	1892.3222	2481.95614	3132.40795	3832.15975
23	147.196204	337.963673	612.449426	976.902988	1431.13996	1969.53586	2582.62712	3258.88914	3986.31044
24	154.780855	352.736668	637.678772	1016.07842	1487.72398	2046.74952	2683.29811	3385.37034	4140.46112
25	162.365505	367.509663	662.908118	1055.25384	1544.308	2123.96318	2783.96909	3511.85153	4294.6118
26	169.950156	382.282658	688.137463	1094.42927	1600.89202	2201.17684	2884.64008	3638.33273	4448.76249
27	177.534807	397.055653	713.366809	1133.6047	1657.47604	2278.3905	2985.31106	3764.81392	4602.91317
28	185.119458	411.828648	738.596155	1172.78013	1714.06006	2355.60415	3085.98204	3891.29512	4757.06385
29	192.704109	426.601642	763.825501	1211.95556	1770.64408	2432.81781	3186.65303	4017.77631	4911.21454
30	200.288759	441.374637	789.054847	1251.13099	1827.2281	2510.03147	3287.32401	4144.2575	5065.36522
31	207.87341	456.147632	814.284193	1290.30641	1883.81212	2587.24513	3387.995	4270.7387	5219.51591
32	215.458061	470.920627	839.513538	1329.48184	1940.39615	2664.45879	3488.66598	4397.21989	5373.66659
33	223.042712	485.693622	864.742884	1368.65727	1996.98017	2741.67245	3589.33697	4523.70109	5527.81727
34	230.627362	500.466617	889.97223	1407.8327	2053.56419	2818.88611	3690.00795	4650.18228	5681.96796
35	238.212013	515.239612	915.201576	1447.00813	2110.14821	2896.09977	3790.67893	4776.66347	5836.11864
36	245.796664	530.012607	940.430922	1486.18356	2166.73223	2973.31343	3891.34992	4903.14467	5990.26932
37	253.381315	544.785602	965.660267	1525.35899	2223.31625	3050.52709	3992.0209	5029.62586	6144.42001
38	260.965966	559.558597	990.889613	1564.53441	2279.90027	3127.74075	4092.69189	5156.10706	6298.57069
39	268.550616	574.331592	1016.11896	1603.70984	2336.48429	3204.9544	4193.36287	5282.58825	6452.72137
40	276.135267	589.104587	1041.3483	1642.88527	2393.06831	3282.16806	4294.03386	5409.06944	6606.87206
41	283.719918	603.877582	1066.57765	1682.0607	2449.65234	3359.38172	4394.70484	5535.55064	6761.02274
42	291.304569	618.650577	1091.807	1721.23613	2506.23636	3436.59538	4495.37582	5662.03183	6915.17342
43	298.889219	633.423572	1117.03634	1760.41156	2562.82038	3513.80904	4596.04681	5788.51303	7069.32411
44	306.47387	648.196567	1142.26569	1799.58698	2619.4044	3591.0227	4696.71779	5914.99422	7223.47479
45	314.058521	662.969562	1167.49503	1838.76241	2675.98842	3668.23636	4797.38878	6041.47541	7377.62547
46	321.643172	677.742557	1192.72438	1877.93784	2732.57244	3745.45002	4898.05976	6167.95661	7531.77616
47	329.227823	692.515552	1217.95373	1917.11327	2789.15646	3822.66368	4998.73074	6294.4378	7685.92684
48	336.812473	707.288546	1243.18307	1956.2887	2845.74048	3899.87734	5099.40173	6420.919	7840.07752
49	344.397124	722.061541	1268.41242	1995.46413	2902.3245	3977.091	5200.07271	6547.40019	7994.22821
50	351.981775	736.834536	1293.64176	2034.63955	2958.90853	4054.30465	5300.7437	6673.88138	8148.37889

	13	14	15	16	17	18	19	20
10	2371.77987	2778.29498	3198.07811	3628.13115	4066.21103	4510.72456	4960.61207	5415.23647
11	2554.99309	2991.55279	3442.01922	3903.11962	4372.39942	4848.10825	5329.07378	5814.58154
12	2738.2063	3204.81061	3685.96034	4178.10809	4678.58781	5185.49193	5697.53549	6213.92661
13	2921.41951	3418.06843	3929.90145	4453.09656	4984.7762	5522.87561	6065.99719	6613.27168
14	3104.63272	3631.32624	4173.84257	4728.08503	5290.96458	5860.25929	6434.4589	7012.61675
15	3287.84593	3844.58406	4417.78368	5003.07349	5597.15297	6197.64297	6802.92061	7411.96182
16	3471.05915	4057.84187	4661.7248	5278.06196	5903.34136	6535.02666	7171.38231	7811.30689
17	3654.27236	4271.09969	4905.66591	5553.05043	6209.52974	6872.41034	7539.84402	8210.65196
18	3837.48557	4484.35751	5149.60703	5828.0389	6515.71813	7209.79402	7908.30573	8609.99703
19	4020.69878	4697.61532	5393.54814	6103.02737	6821.90652	7547.1777	8276.76743	9009.3421
20	4203.912	4910.87314	5637.48926	6378.01584	7128.0949	7884.56138	8645.22914	9408.68717
21	4387.12521	5124.13095	5881.43037	6653.00431	7434.28329	8221.94506	9013.69085	9808.03224
22	4570.33842	5337.38877	6125.37149	6927.99277	7740.47168	8559.32875	9382.15255	10207.3773
23	4753.55163	5550.64659	6369.3126	7202.98124	8046.66006	8896.71243	9750.61426	10606.7224
24	4936.76484	5763.9044	6613.25372	7477.96971	8352.84845	9234.09611	10119.076	11006.0675
25	5119.97806	5977.16222	6857.19483	7752.95818	8659.03684	9571.47979	10487.5377	11405.4125
26	5303.19127	6190.42003	7101.13595	8027.94665	8965.22522	9908.86347	10855.9994	11804.7576
27	5486.40448	6403.67785	7345.07706	8302.93512	9271.41361	10246.2472	11224.4611	12204.1027
28	5669.61769	6616.93567	7589.01818	8577.92358	9577.602	10583.6308	11592.9228	12603.4477
29	5852.8309	6830.19348	7832.95929	8852.91205	9883.79039	10921.0145	11961.3845	13002.7928
30	6036.04412	7043.4513	8076.90041	9127.90052	10189.9788	11258.3982	12329.8462	13402.1379
31	6219.25733	7256.70911	8320.84152	9402.88899	10496.1672	11595.7819	12698.3079	13801.4829
32	6402.47054	7469.96693	8564.78264	9677.87746	10802.3555	11933.1656	13066.7696	14200.828
33	6585.68375	7683.22475	8808.72375	9952.86593	11108.5439	12270.5492	13435.2313	14600.1731
34	6768.89697	7896.48256	9052.66487	10227.8544	11414.7323	12607.9329	13803.693	14999.5181
35	6952.11018	8109.74038	9296.60598	10502.8429	11720.9207	12945.3166	14172.1547	15398.8632
36	7135.32339	8322.99819	9540.5471	10777.8313	12027.1091	13282.7003	14540.6164	15798.2083
37	7318.5366	8536.25601	9784.48821	11052.8198	12333.2975	13620.084	14909.0782	16197.5534
38	7501.74981	8749.51383	10028.4293	11327.8083	12639.4859	13957.4677	15277.5399	16596.8984
39	7684.96303	8962.77164	10272.3704	11602.7967	12945.6743	14294.8513	15646.0016	16996.2435
40	7868.17624	9176.02946	10516.3116	11877.7852	13251.8626	14632.235	16014.4633	17395.5886
41	8051.38945	9389.28728	10760.2527	12152.7737	13558.051	14969.6187	16382.925	17794.9336
42	8234.60266	9602.54509	11004.1938	12427.7621	13864.2394	15307.0024	16751.3867	18194.2787
43	8417.81587	9815.80291	11248.1349	12702.7506	14170.4278	15644.3861	17119.8484	18593.6238
44	8601.02909	10029.0607	11492.076	12977.7391	14476.6162	15981.7697	17488.3101	18992.9688
45	8784.2423	10242.3185	11736.0171	13252.7275	14782.8046	16319.1534	17856.7718	19392.3139
46	8967.45551	10455.5764	11979.9582	13527.716	15088.993	16656.5371	18225.2335	19791.659
47	9150.66872	10668.8342	12223.8994	13802.7045	15395.1813	16993.9208	18593.6952	20191.0041
48	9333.88194	10882.092	12467.8405	14077.693	15701.3697	17331.3045	18962.1569	20590.3491
49	9517.09515	11095.3498	12711.7816	14352.6814	16007.5581	17668.6882	19330.6186	20989.6942
50	9700.30836	11308.6076	12955.7227	14627.6699	16313.7465	18006.0718	19699.0803	21389.0393

	21	22	23	24	25	26	27	28	29	30
10	5874.2859	6337.69272	6805.56854	7278.15358	7755.77795	8238.83259	8727.74783	9222.9777	9724.98876	10234.2522
11	6304.2694	6798.03976	7295.98926	7798.3538	8305.46663	8817.72692	9335.57645	9859.48269	10389.9266	10927.3943
12	6734.25289	7258.38681	7786.40999	8318.55401	8855.15531	9396.62125	9943.40507	10495.9877	11054.8645	11620.5364
13	7164.23638	7718.73386	8276.83071	8838.75423	9404.844	9975.51558	10551.2337	11132.4927	11719.8024	12313.6785
14	7594.21988	8179.08091	8767.25144	9358.95445	9954.53268	10554.4099	11159.0623	11768.9977	12384.7403	13006.8206
15	8024.20337	8639.42796	9257.67217	9879.15467	10504.2214	11133.3042	11766.8909	12405.5026	13049.6782	13699.9627
16	8454.18687	9099.775	9748.09289	10399.3549	11053.91	11712.1986	12374.7196	13042.0076	13714.616	14393.1048
17	8884.17036	9560.12205	10238.5136	10919.5551	11603.5987	12291.0929	12982.5482	13678.5126	14379.5539	15086.2469
18	9314.15385	10020.4691	10728.9343	11439.7553	12153.2874	12869.9872	13590.3768	14315.0176	15044.4918	15779.389
19	9744.13735	10480.8161	11219.3551	11959.9555	12702.9761	13448.8815	14198.2054	14951.5226	15709.4297	16472.5312
20	10174.1208	10941.1632	11709.7758	12480.1558	13252.6648	14027.7759	14806.034	15588.0276	16374.3676	17165.6733
21	10604.1043	11401.5102	12200.1965	13000.356	13802.3535	14606.6702	15413.8627	16224.5326	17039.3054	17858.8154
22	11034.0878	11861.8573	12690.6172	13520.5562	14352.0421	15185.5645	16021.6913	16861.0376	17704.2433	18551.9575
23	11464.0713	12322.2043	13181.038	14040.7564	14901.7308	15764.4589	16629.5199	17497.5425	18369.1812	19245.0996
24	11894.0548	12782.5514	13671.4587	14560.9566	15451.4195	16343.3532	17237.3485	18134.0475	19034.1191	19938.2417
25	12324.0383	13242.8984	14161.8794	15081.1568	16001.1082	16922.2475	17845.1772	18770.5525	19699.0569	20631.3838
26	12754.0218	13703.2455	14652.3002	15601.3571	16550.7969	17501.1418	18453.0058	19407.0575	20363.9948	21324.5259
27	13184.0053	14163.5925	15142.7209	16121.5573	17100.4856	18080.0362	19060.8344	20043.5625	21028.9327	22017.668
28	13613.9888	14623.9396	15633.1416	16641.7575	17650.1742	18658.9305	19668.663	20680.0675	21693.8706	22710.8102
29	14043.9723	15084.2866	16123.5623	17161.9577	18199.8629	19237.8248	20276.4916	21316.5725	22358.8085	23403.9523
30	14473.9558	15544.6337	16613.9831	17682.1579	18749.5516	19816.7192	20884.3203	21953.0775	23023.7463	24097.0944
31	14903.9393	16004.9807	17104.4038	18202.3582	19299.2403	20395.6135	21492.1489	22589.5825	23688.6842	24790.2365
32	15333.9228	16465.3278	17594.8245	18722.5584	19848.929	20974.5078	22099.9775	23226.0874	24353.6221	25483.3786
33	15763.9063	16925.6748	18085.2452	19242.7586	20398.6177	21553.4021	22707.8061	23862.5924	25018.56	26176.5207
34	16193.8898	17386.0219	18575.666	19762.9588	20948.3063	22132.2965	23315.6347	24499.0974	25683.4979	26869.6628
35	16623.8732	17846.3689	19066.0867	20283.159	21497.995	22711.1908	23923.4634	25135.6024	26348.4357	27562.8049
36	17053.8567	18306.716	19556.5074	20803.3592	22047.6837	23290.0851	24531.292	25772.1074	27013.3736	28255.947
37	17483.8402	18767.063	20046.9281	21323.5595	22597.3724	23868.9795	25139.1206	26408.6124	27678.3115	28949.0891
38	17913.8237	19227.4101	20537.3489	21843.7597	23147.0611	24447.8738	25746.9492	27045.1174	28343.2494	29642.2313
39	18343.8072	19687.7571	21027.7696	22363.9599	23696.7498	25026.7681	26354.7779	27681.6224	29008.1873	30335.3734
40	18773.7907	20148.1042	21518.1903	22884.1601	24246.4384	25605.6624	26962.6065	28318.1274	29673.1251	31028.5155
41	19203.7742	20608.4512	22008.611	23404.3603	24796.1271	26184.5568	27570.4351	28954.6323	30338.063	31721.6576
42	19633.7577	21068.7983	22499.0318	23924.5606	25345.8158	26763.4511	28178.2637	29591.1373	31003.0009	32414.7997
43	20063.7412	21529.1453	22989.4525	24444.7608	25895.5045	27342.3454	28786.0923	30227.6423	31667.9388	33107.9418
44	20493.7247	21989.4923	23479.8732	24964.961	26445.1932	27921.2398	29393.921	30864.1473	32332.8766	33801.0839
45	20923.7082	22449.8394	23970.2939	25485.1612	26994.8818	28500.1341	30001.7496	31500.6523	32997.8145	34494.226
46	21353.6917	22910.1864	24460.7147	26005.3614	27544.5705	29079.0284	30609.5782	32137.1573	33662.7524	35187.3681
47	21783.6752	23370.5335	24951.1354	26525.5616	28094.2592	29657.9227	31217.4068	32773.6623	34327.6903	35880.5103
48	22213.6587	23830.8805	25441.5561	27045.7619	28643.9479	30236.8171	31825.2354	33410.1673	34992.6282	36573.6524
49	22643.6422	24291.2276	25931.9768	27565.9621	29193.6366	30815.7114	32433.0641	34046.6722	35657.566	37266.7945
50	23073.6257	24751.5746	26422.3976	28086.1623	29743.3253	31394.6057	33040.8927	34683.1772	36322.5039	37959.9366

(Figure 9)

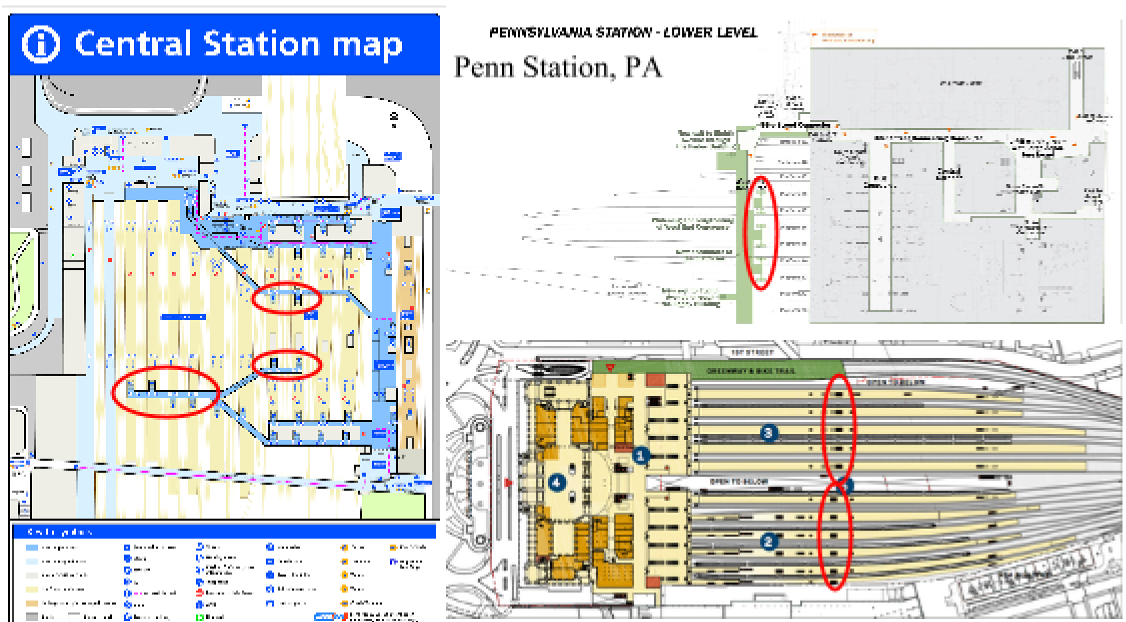
3 Requirement 3: Optimize location of stairways

<u>Interpretation</u>:

This part of the problem asks us to, if given a set number of stairways, figure out the configuration of them in the train station that would reduce the average amount of time for which it takes passengers to exit the station. We chose to interpret this question as asking us to research and find the general building guidelines and practices of major train stations and use them to compile a comprehensive report of the optimal locations of one and multiple stairways.

<u>Approach:</u>

While we assumed that the single staircase was located at the end of the platform, layouts from other central train stations show that staircases in the middle of the platform may be most optimal for controlling traffic flow. Below are images of layouts from three of the busiest train stations, Central Station (Figure 10), Penn Station (Figure 10), and Union Station (Figure 10), in the United States.



Central Station, NY

Union Station, Washington DC

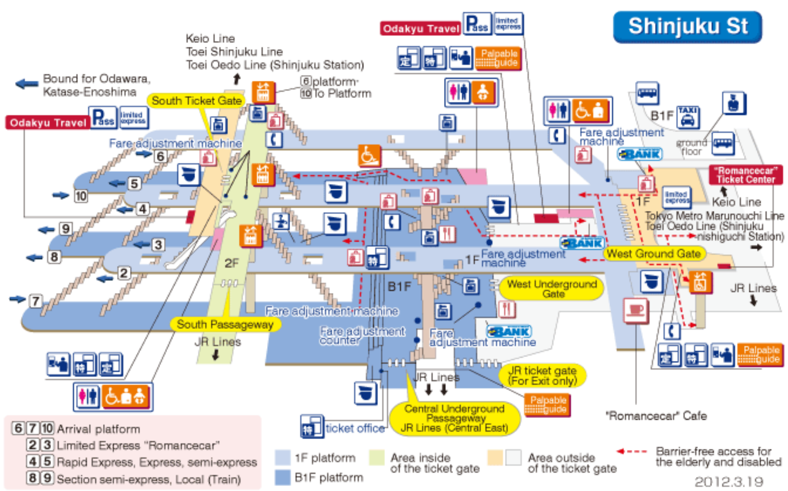
(Figure 10)

All three of the stations mentioned above include staircases located in the middle of the platform. We see this trend in Great Britain as well, in the Cardiff central station (Figure 11):

Cardiff, UK

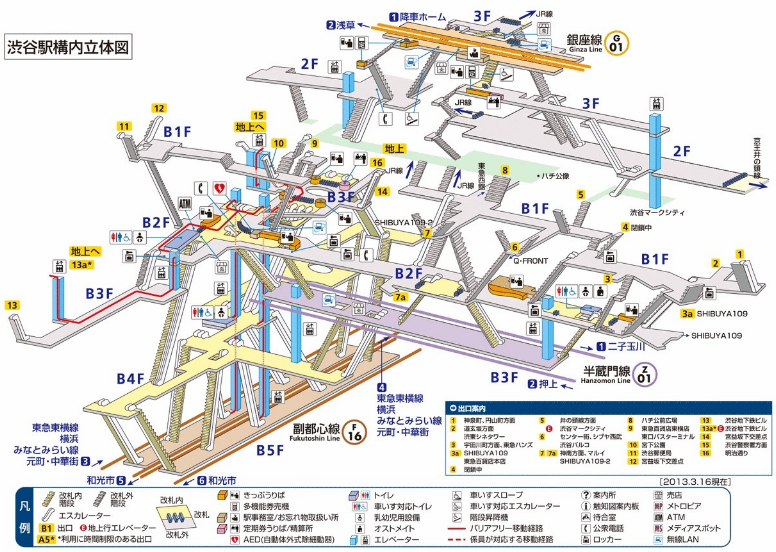
(Figure 11)

What if there was more than one staircase? Modeling the layouts from the Shinjuku and Shibuya stations in Japan (Figures 12 and 13), two of the busiest train stations in the world, we would pair staircases such that they face opposite directions. These staircases would be spread out evenly throughout the platform such that there would be enough space for passengers to wait on the platform for their respective trains but also have sufficient staircases so that traffic is minimized. Both the Shibuya and Shinjuku stations employ this model and considering each serves over one billion commuters a year, this placement of staircases should minimize traffic in the context of this problem.



Shinjuku Station, Japan

(Figure 12)



Shibuya Station, Japan

(Figure 13)

In conclusion, if we were to use a single two column staircase, we would put it in the middle of the platform to minimize the time it takes to exit the station. With multiple stairways, we would pair them such that they face opposite directions and spread them evenly across the platform. There should be enough space between the staircases to allow crowds coalesce on the platform and not have a hazardous risk.

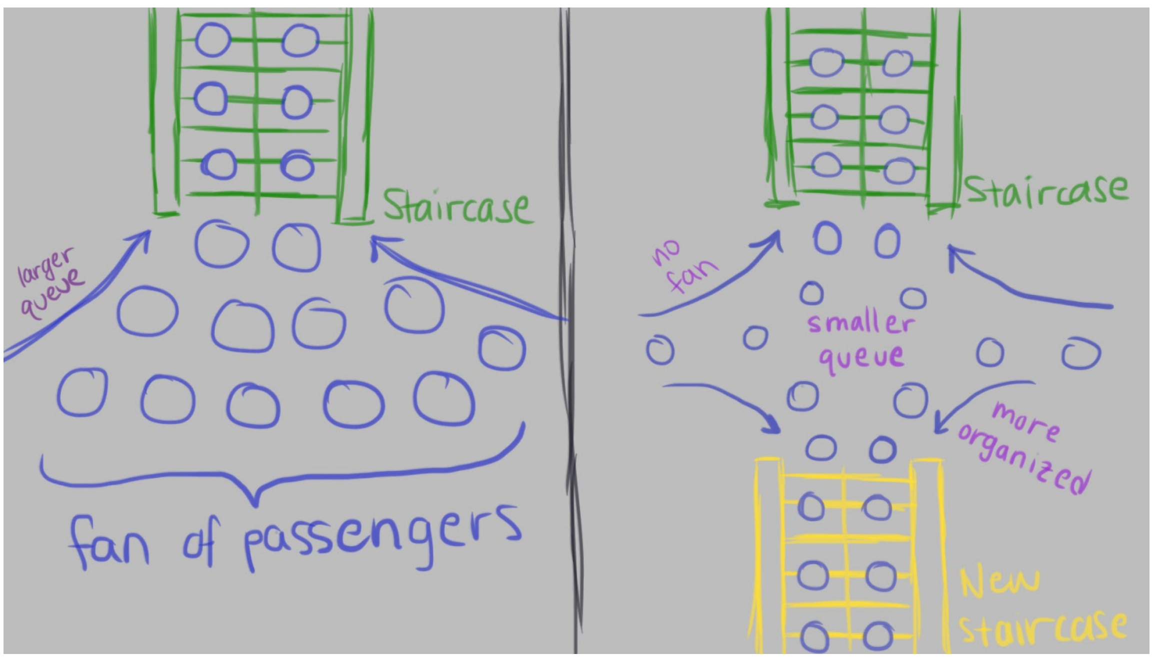
4 Requirement 4: Relationship between time and number of stairways

Interpretation:

This part asks us to find the number of stairways of a station that will minimize the average amount of time it takes a person to exit the station (again, from their seat to top of stairs). We interpreted this requirement as asking us to solve for the optimal number of stairways (s) based on n and d that minimizes time as well as concluding a relationship between time to street level and s.

Approach:

In requirements 1 and 2, we worked extensively on an analysis of the queue's effect on the time required to exit the station. The "fan" that develops at the bottom of the stairway significantly impacts people's ability to leave in a timely manner. If more stairways were made available, people would take advantage of the emptiest stairway, minimizing the crowding around the other staircases.



(Figure 14)

Increasing the number of stairways would reduce the length of the queue until there is no queue. This point is where the number of people leaving the train per second divided by the number of staircases equals 2 people per second. Because the staircases can take 2 people per second, once the rate of people entering the stairs equals the rate of people exiting the stairs, there is no queue at the bottom.

We found in Requirement 1 that the number of people exiting the train per second is:

$$\frac{367d^3}{4800000} + \frac{4136497d^2}{1044000000} - \frac{1898321d}{261000000} + \frac{1411}{435000}$$

Given that there are s staircases, the rate at which people enter each staircase is:

$$\left(\frac{367d^3}{4800000} + \frac{4136497d^2}{1044000000} - \frac{1898321d}{261000000} + \frac{1411}{435000}\right)\frac{1}{s}$$

We can set this expression equal to 2 people per second because no queue forms when the number of people leaving to go onto the stairs is equal to the number of people getting to the bottom of the stairs. Setting the expression equal to 2 and not less than 2 allows the maximum number of people to arrive in each train.

$$\left(\frac{367d^3}{4800000} + \frac{4136497d^2}{1044000000} - \frac{1898321d}{261000000} + \frac{1411}{435000}\right)\frac{1}{s} = 2$$

$$\left(\frac{367d^3}{4800000} + \frac{4136497d^2}{1044000000} - \frac{1898321d}{261000000} + \frac{1411}{435000}\right)\frac{1}{2} = s$$

Once this equality is reached, building more staircases is just costly and unhelpful. Furthermore, inserting more staircases reduces the amount of space on the platform, meaning that people would need to squeeze together when waiting for their respective trains. This would pose an injury hazard that would counteract the optimization of time. Thus, the only variable needed to optimize time is the number of people in the queue. As soon as the queue is eliminated, time is optimized.

Therefore, time to street level would decrease as s approaches
$\left(\frac{367d^3}{4800000} + \frac{4136497d^2}{1044000000} - \frac{1898321d}{261000000} + \frac{1411}{435000}\right)\frac{1}{2}$
from the negative side, but would not really change as s becomes greater than this expression as no queue would form. If we let $d = 20$, $s = 1.027$, which means that two two-column staircases would optimize amount of time needed because 1.027>1, meaning that one stairway would still cause queueing while two stairways will eliminate the queue.

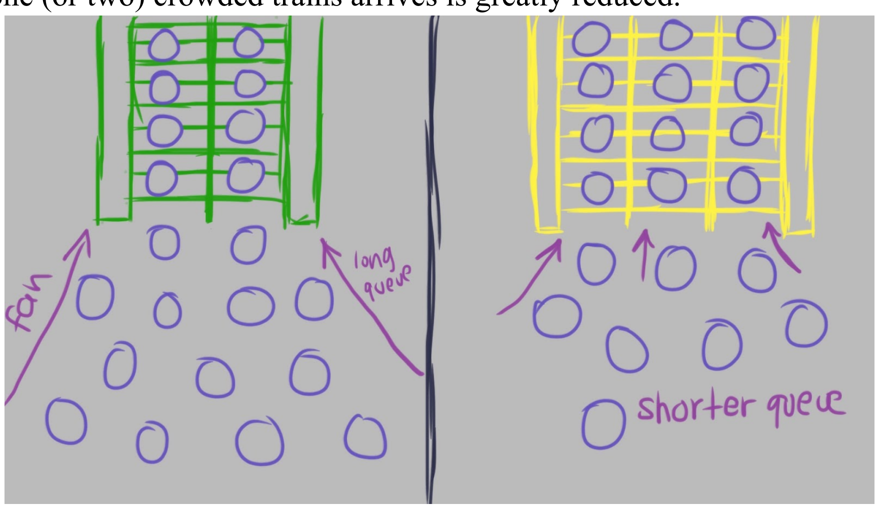
5 Requirement 5: Relationship between time and number of people accommodated by stairways

Interpretation:

We interpreted Requirement 5 to be asking for the relationship (preferably a function) between amount of time spent walking to the stairway and the number of people, *k,* the stairway can accommodate. We again chose to interpret time as the average amount of time for a passenger to walk from the exit of a train car to the bottom of the stairway because, as stated earlier in Section 2.2. We also chose to define *k* as not the total number of people that can stand on the stairway at one time but as the number of people that can stand side-by-side in one row of the stairs because the number of people that can stand on the stairs lengthwise, representing the length of the stairs, should be constant as the elevation of the train tracks and the elevation of the street level are constant. Therefore, only *k* as the number of people able to stand side-by-side in a row on the stairs is related to the amount of time passengers spend walking to street level.

Approach:

If the number of people who can fit side by side on the stairway at any one time, *k*, changes (right now, it is given that two columns of people can climb the stairs at a time; thus, there are two people on any given stair while the train is unloading), the length of the queue at the bottom of the stairway will be significantly affected. *k* is then equal to the amount of people exiting or entering the staircase every second because according to our calculations, each second one row of people exit and a new row of people enter the stairway. Queue length is a prominent determinant of the time it takes to leave the station; thus, if the length of the queue can be reduced by using a wider stairway that accommodates more people, the time it takes to exit the station when one (or two) crowded trains arrives is greatly reduced.



(Figure 15)

As with requirement 4, widening the stairways can result in a great reduction in time up to a certain point, and that point is when the stairways are wide enough such that no queue ever develops at the bottom of the stairs. We found in Requirement 1 that the amount of people leaving the train per second (B) is

$$\frac{367d^3}{4800000}+\frac{4136497d^2}{1044000000}-\frac{1898321d}{261000000}+\frac{1411}{435000}$$

which is also the number of people enter the queue per second because we assume that each passenger moves along the platform at a constant speed. Given that k, the number of people per row on the stairway, is also equal to the number of people that exit/enter the stairway every second, we know that V, the change in number of people in the queue is

V = people entering queue - people exiting queue

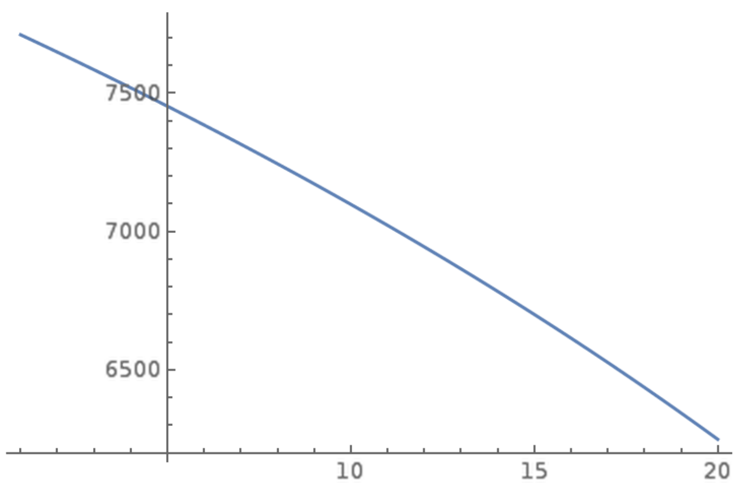
V = people entering queue - people entering stairway

$$V=\frac{367d^3}{4800000}+\frac{4136497d^2}{1044000000}-\frac{1898321d}{261000000}+\frac{1411}{435000}-k$$

We then plugged V above into our model and assigned d = 20 rows and n = 20 cars because we are looking for a relationship between time and the number of people accommodated by the stairway, so we can plug in common numbers for the number of rows and train cars in order to find time directly as a function of k and know that time as a function of k will be roughly the same shape for any ordered pair of d and n. We found t as a function of k to equal:

$$t=\frac{635591.-12744.3\,k+13.3856\,k^2}{81.7826-1.\,k}$$

Graph of t as a function of k with x-axis as number of people accommodated in one row of stairs (k) and y-axis as amount of time (t) for $k \leq 20$ because a stairway that can accommodate over 20 people per row would be unreasonably large:



(Figure 16)

As we can see, as the number of people accommodated in one row of the stairway increases (as the width of the stairway increases), the average amount of time needed to walk from the train car to the bottom of the stairway decreases nearly linearly because the queue becomes shorter as more people are able to exit per second.

6 Model Analysis

Strengths and weaknesses:

The most significant strength of our model is our analysis of the queue. We considered the queue to be critically important to time optimization and devoted much of our effort to modeling it in a robust and effective manner. First, we considered the queue to be a two-dimensional body, rather than a one-dimensional line of disciplined, orderly people, which would be much easier to calculate, yet completely unrealistic. Using realistic constants (such as platform widths recommended by major railway administrations), we were able to model the speed of individuals in the queue. We also modeled the growth of the queue as it spilled out onto the platform as a function of the number of rows in each car and cars in each train, as this affects the number of people streaming onto the platform and expanding the queue. Since people can climb stairs at a fairly rapid rate, small trains will produce no queue at the bottom of the stairs at all: the number of people leaving the train will not overwhelm the capacity of the stairs. Once the ridership of the train reaches a certain point, however, a queue will begin to develop. While a more simplistic model would eventually yield the realization that the best way to optimize time is just by shrinking the train to essentially zero to eliminate crowding and greatly reduce distance covered on the platform, we can look to a single, most preferable point. Of course, the absolute minimum would be approaching 0 cars and 0 rows, but our model allows us to find a special point: the point at which the trains can be made as large as possible before the queue begins to form. Outside of basic distance covered, crowding and congestion are the major determinants of the time required to exit the station. Modeling the queue allows us to find a point that optimizes the most relevant two variables: distance covered through the station, and time wasted moving slowly through a queue creeping toward the stairs. Our ability to find this one optimum point, a clear deliverable to the manager of the station, in a highly realistic simulation of the actual train terminal is the single most important strength of our model.

Weaknesses of our model arise mostly out of the values we have negated. For example, when we calculated the amount of time it takes for a person to exit the train car, we found that the value was around 3% of the total time taken to egress from the train station. Although three percent is a small amount, it is still a significant amount and not necessarily small enough to truly be negligible.

Sensitivity Analysis

Our model is very accurate even at large amounts of passengers. Given a train with 20 rows per car, we can fit the entire population of Bedford, Massachusetts (12595 people from 2010 US census) in a train with 131 cars. Using our model we obtain a time of 43993.369956 seconds for all the people to exit the train and leave the station. This translates to about 12.22 hours for every person to leave the station, which is a very reasonable time for a small town to get off of a train.

7 Non-technical Letter

Dear Director of Transportation,

We are high school students concerned about the excessive amount of time required to exit a train at the central train station in the city. We would like to propose a model that would minimize this time. Knowing that the layout of stations vary throughout the rail line, we designed our model with a platform that runs in between two tracks. There is one staircase at the end of the platform, and it can accommodate two people standing side by side on each stair. The platform is 29.5 feet wide. After disembarking from the train, people would form a rectangular queue at foot of the stairs. Our model describes this current situation. We then altered the model by altering the location of the staircase(s), adding additional stairways, and widening the stairs by adding more columns.

Our mathematical model describes the time needed to exit the station in terms of the number of cars on the trains and the number of rows per car. For example, using our model, when there are 10 cars per train with 10 rows each, the average person takes about 5 minutes to exit the station. This prediction is reasonable in this situation: given the nature of the station, 5 minutes seems quite normal. According to our model, increasing the number of cars does not have as drastic as an effect as changing the number of rows a car has on the time spent on the platform. We found that the optimal number of rows a car has is 19, as having less than 19 rows avoids the formation of a queue, but having more than 19 creates a queue that drastically increases the time required to leave the station.

Furthermore, stairways should be improved in both quantity and wideness to prevent the buildup of a queue at the bottom of the stairway. With one staircase, it would be best to put it in the middle of the platform. When building new more stairways, it is important to equally distribute the stairways along the length of the platform where trains will stop. The addition of staircases will decrease the amount of time required to exit the station. Because people will spread themselves out over multiple staircases, the queue length decreases, making it faster to access the staircase. The width of the staircase also decreases the time needed to exit the station. Adding more columns to the staircases indicates that more people can be on the stairs at a time, reducing the queue length.

We hope that after careful analysis of our model, you take our recommendations into consideration. We have considered a variety of important aspects of the train station with special emphasis on factors we have found to be the greatest determinants of egress time and assure you that our model is a highly robust, effective representation of the amount of time required to exit the platform. The suggestions we made, if followed, will lead to the most efficient method of unloading the train and minimal amount of time needed to exit the station.

Sincerely,
Team #4813

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