HIGH SCHOOL MATHEMATICAL CONTEST IN MODELING
OUTSTANDING PAPERS

HiMCM 2000

The contest offers students the opportunity to compete in a team setting using applied mathematics in the solving of real-world problems.

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Editor's Comments

This is the second annual HiMCM Issue of *Consortium*. Our goals in presenting it to our readers are to promote mathematical modeling in high schools and to assist students and teachers in preparing for future competitions.

As Bill Fox and Rick Jennings point out in their articles, HiMCM is growing rapidly. That is very good news, indeed. COMAP does not restrict the judges to a fixed number of outstanding papers, and the 2000 contest saw the number of national outstanding papers grow from 4 to 11. Unfortunately, printing even edited versions of all 11 papers is prohibitive. Therefore, included here are abridged versions of two of the papers—one for each of the contest problems—and the summary from each of the others. We should emphasize that selection was not based on relative merit. Rather, we picked two papers that we felt were representative and were conducive to the fairly severe abridgement required to appear here.

Our sincerest thanks and congratulations to all the students, advisors, and judges who helped to make this year's contest a success. We are looking forward to many more and hope that all of you will join us in 2001.

Contest Director's Article 2000

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The High School Mathematical Contest in Modeling (HiMCM) completed its second year in fine fashion. It is still moving along our positive first derivative and positive experience from last year's inaugural contest.

This year the contest consisted of 128 teams from among our fifty states and one team from Canada. So our contest immediately became an international contest with that Canadian entry. The teams accomplished the vision of our founders by providing unique and creative mathematical solutions to complex open-ended real-world problems. This year the students had a choice of two different problems.

Problem A: The First National Bank has just been robbed (the position of the bank on the map is marked). The clerk pressed the silent alarm to the police station. The police immediately sent out police cars to establish roadblocks at the major street junctions leading out of town. Additionally, 2 police cars were dispatched to the bank. Assume the robbers left the bank just before the police cars arrived. Develop an efficient algorithm for the police cars to sweep the area in order to force the bank robbers (who were fleeing by car) into one of the established roadblocks. Assume that no cars break down during the chase or run out of gas. Further assume that the robbers do not decide to flee via other transportation means. Map information: The bank is located at the corner of 8th Ave and Colorado Blvd and is marked with the letter B. The main exits where the two roadblocks are set up are at the intersection of Interstate 70 and Colorado Blvd, and Interstate 70 (past Riverside Drive). These are marked with a RB1 and RB2 symbol.

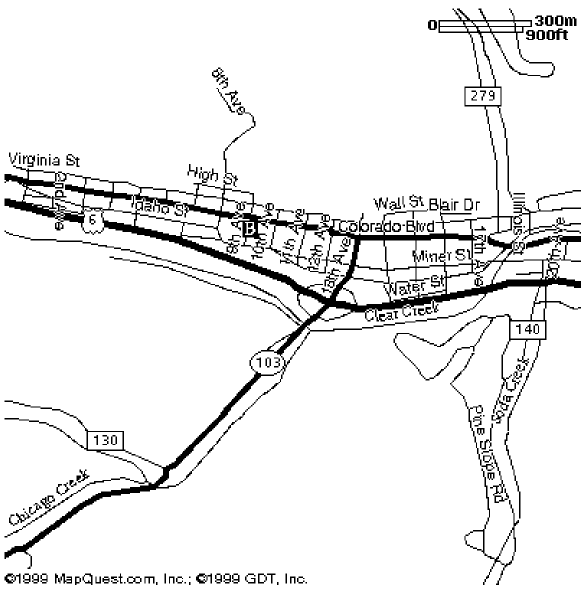
Problem B: It is almost election time, and it is time to revisit the electoral vote process. The Constitution and its amendments have provided a subjective method for awarding electoral votes to states. Additionally, a state popular vote no matter how close, awards all electoral votes to the winner of that plurality. Create a mathematical model that is different from the current electoral system. Your model might award fractional amounts of electoral votes or change the methods by which the numbers of electoral votes are awarded to the states. Carefully describe your model and test its application with the data from the 1992 election (the 1992 election data is provided). Justify why your model is better than the current model?

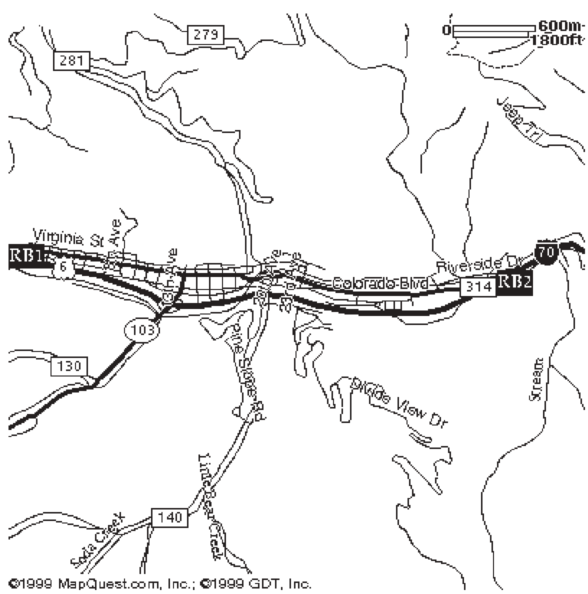
All students and their advisors are congratulated for their varied and creative mathematical efforts. The composition of the 128 teams with 74 of the 128 teams having female participation shows this competition is for both male and female students. As a matter of fact, several teams were all female, one of which earned a National Outstanding. Teams again proved to the judges that they had "fun" with their chosen problems, demonstrating research, initiative, and creativity in their solutions. The 2nd year effort was deemed a success!

We ran three regional sites in 2000. Each site judged papers from a different region as Outstanding, Meritorious, Honorable Mention, and Successful. All Outstanding, Meritorious, and a random selection of Honorable Mention papers were brought to the National Judging. The National Judging chooses the "best of the best" as National Outstanding. The National Judges commended the regional judges for their efforts and found the results were consistent. We felt this regional structure provided a good prototype for the future contest structure.

The results of the judging:

Problem	National Outstanding	Regional Outstanding	Meritorious	Honorable Mention	Successful	Total
A	4	2	15	13	8	42
B	7	1	27	33	18	86
	11	3	42	46	26	128





Facts from the 2000 contest:

- 55%, or 16 of 29, 1999 schools competed in 2000.
- 55 new schools participated in 2000.
- A wide range of schools competed including home schooling teams.
- 57.8%, or 74 of 128 teams had female participation.
- 42.2% were all male participation.
- 5 of the outstanding teams had a female participant
- 1 of the outstanding teams was an all female team.
- International HiMCM contest with a team from Waterloo Canada.

The contest, which attempts to give the under-represented an opportunity to compete and achieve success in mathematics endeavors, appears well on its way in meeting this important mission.

We continue to strive to grow. Next year any school/team will be allowed to enter the contest, as there will be no restrictions on the numbers of schools entering. A regional judging structure will be established based on the response of teams to compete in the contest.

Again, these are exciting times for our high school students. Mathematics continues to be more than learning skills and operations. Mathematics is a language that involves our daily lives. Applying the mathematical principles that one learns is the key to future success. The ability to recognize problems, formulate a mathematical model, solve, compute with technology, and communicate and reflect on one's work is key to success. The ability to use technology aggressively to discover, experiment, analyze, resolve, and communicate results is also key to success in the future. Students learn confidence by tackling ill-defined problems and working together to generate a solution. Through team building and team effort solutions are built. Applying mathematics is a team sport.

Advisors need only be a motivator and facilitator. Allow students to be creative and imaginative. It is not the technique used but the process that discovers how assumptions drive the techniques that is fundamental. Let the students practice to be problem solvers. Let me encourage all high school mathematics faculty members to get involved, encourage your students, make mathematics relevant, and open the doors to success.

Mathematical modeling is an art and a science. Teach your students through modeling to think critically, communicate efficiently, and be confident, competent problem solvers for the new century.

Judge's Commentary HiMCM 2000

Rick Jennings

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GENERAL COMMENTS ON FORMAT

The second year of the HiMCM contest found changes to the formats of both the contest and the judging of the contest—hardly a sophomore slump. Let me first discuss the judging format change.

Due to the planned increase in the number of participating teams, three regional panels were added to the process. The contest results were divided into three groups and sent out to a group of regional judging panels made up of high school teachers, as well as some university people who have had experience with the MCM (COMAP's undergraduate modeling competition). The regional panels rated their group of contest results and sent along those deemed to have a chance at national-level ratings. This process took an additional weekend, but allowed for a more careful initial reading of all the papers. Many thanks go to these regional judging panels, as well as to the national panel.

If you participated in the 1999 contest, you realize that the major change this year was the addition of a second problem. Teams were asked to select one of the problems. With the 24-hour time limit, this became another factor to weigh prior to beginning their modeling process. Although the number of regions may change, depending on the number of entrants, the board has decided, at least for the time being, to continue to offer two problems.

GENERAL COMMENTS ON THE RESULTS

The papers gave a wide range of solutions for both problems. There were, however, some common characteristics in both sets.

A common pitfall in many of the papers was in the assumptions made by the teams. In some cases, assumptions were made that reduced the problem so much that a solution became trivial. In some, assumptions were made but never used, or even contradicted in the solution. Another pitfall was the use of graphs, charts, tables, and computer programs that were either difficult to trace, because of labeling issues, or did not match either the assumptions or solutions.

The best papers were clear, concise, and well written. The summary page of these clearly outlined the modeling process used, why additional assumptions were needed and what these assumptions were, the "tools" used for the solution, as well as the solution. The assumptions made in the best papers were stated clearly, were obviously meant to allow the modeling process to proceed, gave up as little complexity as possible, and were clearly helpful in arriving at a reasonable solution. If graphs, tables, or charts were used to drive home or clarify the solution, they were clearly labeled and dealt directly with the analysis of the problem. Computer programs, if used, were well documented with remark statements identifying critical routines. The results of the program routines were printed out and labeled so the judges could identify how the program helped develop the modeling solution.

When equations or formulas were introduced to assist in the solutions, variables were defined and the formulas were justified in relation to the assumptions and reasonable extensions of the problem. The best papers also analyzed the strengths and weakness of their model—checking the sensitivity by altering the assumptions slightly and analyzing what effect it had on the results. In the case of the electoral problem they may have checked it with historical results to see how their model might have faired in past controversial elections.

THE BANK ROBBERS PROBLEM

The solutions presented for this problem had a wider variance than those for the electoral problem. The creativity of the best papers was quite interesting. Some of the papers actually found the setting of the town by searching the Internet for the street names, and researching the particular make-up of the police department. While this is not encouraged, since it does tend to limit the solution to that particular town, it was none the less interesting.

The best papers clearly defined the assumptions used to find a solution. They also used these assumptions to simplify the initial problem without trivializing it. When these papers used specific formulas or mathematical processes, they defined their variables and rationalized the use of the particular variables given the assumptions and constraints.

The best papers also did an excellent job of outlining their algorithm(s) and analyzing it in relation to not only this particular problem, but by playing "what if." The analyses of the strengths and weaknesses in the best papers were such that, given time, it was clear that the team would be able to extend their solution to other situations. Most of the solutions used some type of simplified graphs of the region, with edges and nodes defined and used within the problem. Some of the best papers focused on the timing of the search, while others focused on the search procedure. In either case, the focus was clearly defined and the results matched the focus.

THE ELECTORAL PROBLEM

The best papers clearly defined their goals for modifying the system. Most of these made an argument as to why it was desirable to change the current system and this aided them in defining their goals. In some cases the goals were to have a fair system and encourage more participation. Others chose to increase the chance that large third parties will have an equitable influence in the outcome. In the best papers, whatever was decided (as being important to adjust) was clearly used in the chosen model.

Many of these papers decided to use an electoral method similar to the current one while assigning electoral votes on a district, rather than a state-wide, basis. The best of these went to the historical data to analyze their results rather than just using the election provided. In fact, with a bit of research, the best of these looked at other elections in which the outcome of the electoral college was quite different from the results of the popular vote.

WRAP-UP

A large majority of the papers were a joy to read. From a high school teacher's perspective, it was delightful to see what high school students are capable of creating. I would encourage all to get involved in this exciting new process as it expands over the next few years. I would also like to have you consider serving as a judge if given the opportunity. Over the next few years, as the contest expands, it is probable that more teachers will be asked to assist in the judging at either the regional or national level. I know it takes time away from your already busy schedule, but it is well worth it.

Problem A Summary: Vestal High School, Vestal NY

Advisor: Jean King

Team Members: Brian King, Daniel Zaharopol

The robber is captured using a "sweep" pattern in which the police cover a large area of the city in as short a time as possible. The police cover the most sensitive areas first—those from which the robber can escape. After this, they proceed to narrow down remaining locations, attempting to force the robber into the roadblocks. Upon sighting the robber, the police take up a formation that limits the robber's mobility in such a way that he will be unable to leave the city and can move only in one direction, thus forcing him into the roadblocks.

The algorithm presented is not sufficient to capture the robber regardless of his actions. For example, by heading directly towards one of the exits not protected by a roadblock the robber can escape. Other scenarios in which the robber escapes are presented.

Problem A Summary: Waterloo Collegiate Institute, Waterloo, Ontario Canada

Advisor: Michael Burns

Team Members: Feng Cao, Shannon Murray, Jennifer Vaughan, Edward Wei

In this report, we show that two police cars are not always sufficient to force the robbers into roadblocks. Our solution guarantees a capture in certain cases, while maximizing the police's chances in others. Our model deals with two major cases: one in which the robbers turn west after leaving the bank, and the other in which they turn east. Furthermore, we redraw the map based on our interpretation of the question and the given map.

When the robbers turn west, we show that the police can always force them into a roadblock. The solution has one car following the robbers and the other car driving parallel on the street one block south. We find that having one police car directly following the robbers prevents the robbers from turning around and lets the police always know where the robbers are. There are some exceptions and special cases, which we deal with in our report: West on Virginia St., West on Colorado Blvd., West on Idaho St., and upon reaching I70.

When the robbers turn east, we demonstrate why two cars are not sufficient to completely control their path. However, we do show that the same algorithm as the westbound case can be applied to give the police the best chance of forcing the robbers into a dead end. We discuss this specifically in the scenario and then again in a more general case. The general case deals with nodes and edges. By blocking certain pathways, it limits the robbers' options. In addition, we apply this model to other situations.

To simulate our algorithm, we wrote a program that deals with the given situation of two police cars. The program illustrates our model's effectiveness.

Once our model has been described in detail, we address some of the more practical factors that had been eliminated by our original assumptions. We deal with road conditions, weather, the number of vehicles, and the preparation of both robbers and police.

Problem A Summary: Westminster Schools, Atlanta GA

Advisor: Landy Godbold

Team Members: Peter Colabuono, Jana Dopson, Michael Miller

After making several critical initial assumptions, our first step was to simplify the problem. Rather than consider an entire city of many interlocking roadways, we reduced the area to a small two by one block grid, in which the crooks were randomly hiding and not moving. Our objectives were twofold: first, to locate the criminal (we assumed that police can see one block in each direction); second, to force the perpetrator to go to one of the road-blocked exits on the interstate. We calculated the minimum number of time intervals (the amount of time it takes any driver to travel one of the uniform-length city blocks) required for police to be certain that they sight the criminals. Then, we increased the area of the grid to two by two blocks, and found the minimum time interval to locate the robbers. As we continued to increase the length of the grid, leaving the width of the grid constant at two blocks, we recorded the resulting minimum time intervals for locating the stationary robbers with 100% certainty. After evaluating the minimum time intervals for many grid lengths, we formulated a pattern for the most efficient method by which two police moving along a 2 by n grid (where n is the width of the grid in blocks) could locate a fugitive. By a similar method, we determined patterns describing the most efficient motions of police for stationary robbers on 3 by n grids (where n is again the width of the grid in blocks) and on 4 by n grids. At last, we generalized this sub-model to include variables for grid width and length, as well as controlled values for minimum time intervals to catch the robbers and the probability of locating the robbers over time. By analyzing these data, we determined that the patterns we found earlier that represented the police's most efficient path were also the most time-effective, and therefore should be used in all situations with a stationary robber.

To generalize for more realistic settings, we compounded models for simple grid configurations; that is, we viewed the entire city as a large string of simple grids. The final barrier separating our solution from being a plausible model was that the robbers were stationary. To overcome this, we first had to determine algorithms to describe police movement on simple grids, and then compound the algorithms into the more complex configuration of the actual city. As for the stationary model, the police need only follow the simpler algorithms specific to a given grid configuration for a moving fugitive.

Once the culprit is located, the police assume an L-shaped criminal herding formation to drive the perpetrator to one of the roadblocks. There are some small flaws in our model. We need to consider a few aspects of the situation more extensively, but we tried to address these issues.

Problem B Summary: Arkansas School for Math and Science, Hot Springs AR

Advisor: Bruce Turkal

Team Members: Jordan Boyd-Graber, Nicholas Hawkins, Andrew Lachowsky

This paper is intended to briefly assess the current method of apportioning a state's electoral votes to candidates and to address the problem of designing a different method of apportioning electoral votes and demonstrating the new system's superiority based on past elections.

We designed a system using a state-by-state Borda Count preference schedule of candidates with the candidates' places on the preference schedule based on the percentage of the popular vote they earned. The state's electoral votes were partitioned based on the candidates' places on the preference schedule. We present the results of the presidential elections of 1960, 1968, 1984, and 1992, compare them with the results of these elections with our system, and discuss advantages and disadvantages. Using the 1992 presidential election data with our model, the three candidates finished in the same order, but the numbers of votes differed. In 1992, Bill Clinton won 69% of the electoral votes, Bush 31%, and Perot 0%. Using our method, Clinton won 45%, Bush 39%, and Perot 17%. With our model, since Clinton did not gain a majority the election would have been decided in the House of Representatives. Electoral votes earned using our model much more closely mirror the popular vote while still emphasizing state autonomy, just one of the advantages over the current system.

Problem B Summary: Chesterfield County Math & Science High School, Midlothian VA

Advisor: Diane Leighty

Team Members: Ryan Altenbaugh, Seth Kendler, and Lindsey Mecca

To create an ideal model for the Electoral College, one must represent the popular vote as accurately as possible, and weigh the different ways to do so. Any plan that involves amending the Constitution would be difficult to achieve. Therefore, this proposal only presented the advantages and disadvantages of the current system, a direct voting system, and three other models that would not require a constitutional change.

Because of the "winner takes all' process in the current system, there is a landslide effect. On average, the winning candidate will take 17% more in percent of electoral votes than in percent of popular votes. Ideally, we found that an electoral system should reduce this to 11%. This would be to make it so that in most elections, the winner of the popular vote would get the plurality of the electoral votes, and the election would rarely go to the House of Representatives. One of our methods for comparison was how well the different models could fit that difference in percentages. We found that the best system is one in which each congressional district gets one electoral vote that goes to the candidate with the majority of the district's votes, and the other two electoral votes in a state would go to the candidate with the majority of popular votes. Under this system, voter turnout would be higher in larger states, candidates would be encouraged to campaign everywhere, and the winner of the popular vote would almost always win the electoral vote. In short, our model keeps all the benefits of the current process, promotes better campaigning, and increases voter turnout.

Problem B Summary: Goldberg Home School, Boulder CO

Advisor: David Goldberg

Team Members: Morgan Brown, Robert Brown, Benny Goldberg

We approached this problem as a thought experiment. First, we brainstormed, filling up an entire board with ideas. They were all thought provoking, and showed us new sides to the problem.

Through the process of elimination, we narrowed our possibilities. We agreed on a hypothesis, and set to testing and investigating it. We wanted the system to be more democratic, give more power to third parties, and to increase voter turnout. We believe that our model achieves all of these. In our system, the number of electoral votes per state is proportional not to total population, but rather to the number of people voting. This would increase the incentive to vote, since participation increases the state's representation in the Electoral College. To complement this change, only in a majority vote does a candidate receive all the state's electoral votes. In the case of a plurality (which in the 1992 election happened in 48 out of 51 districts), the candidates split the electoral vote proportionate to the popular vote.

By allowing an electoral vote split along the lines of the popular vote in cases of a plurality, this model is more representative than the current system. It gives third parties more of a chance because they can get electoral votes without garnering the most votes in any state. It also encourages cooperation, because third party support could be crucial. By allotting votes to states on the basis of voter turnout, it gives incentives to vote.

To show how this system works, we wrote a C++ program that determines electoral votes from the number of people voting in each state and sums them to get the final result (it also uses semi-random data entries about voting percentages).

We believe that this model fulfills our three goals of making the system more representative, giving third parties a chance, and increasing turnout.

Problem B Summary: Governor's School for Government and International Studies, Richmond VA

Advisor: Crista Hamilton

Team Members: Jonathan Charlesworth, Ben Easter, and Konstantin Lantsman

The present Electoral College system is not only archaic and inefficient, but fails to satisfy its primary duties of equally and non-subjectively representing the votes of American citizens. In addition, low voter turnout gravely hinders the country's ability to provide democratic government to all citizens. However, the merit of the present system is that it provides less populous states with electoral votes that are not based on population. The proposed system determines a maximum number of electoral votes per state by allowing one vote per 10,000 people. Increased voter turnout is rewarded by allowing a state to obtain more electoral votes as turnout increases. Thus, the maximum number of electoral votes per state is multiplied by voter turnout as a percentage to determine actual votes. With consideration to residents of small states, whose votes should count equally with those of residents of large states, each state receives a base of twenty electoral votes. No state can receive fewer.

Upon receiving electoral votes, a state must divide them among its candidates. Electors are chosen for each party in a number proportional to the votes received by the top three candidates in the popular election. The electors have binding votes.

Our model, although imperfect, accounts for the flaws in the present system. It does away with "winner takes all," a practice that is deleterious to a third party or minority candidate. In our system, the Electoral College represents voter turnout and votes cast per candidate much more precisely than does the present system. The model also accounts for the population disadvantage of smaller states, and offers them comfort in the twenty base votes. The vote allocation system preserves the popular vote while respecting states' rights and providing an incentive for an increased role of the people.

Problem B Summary: Illinois Mathematics and Science Academy, Aurora IL

Advisor: Ronald Vavrinek

Team Members: Katherine Tenhouse, Catherine Kuo, Andrea Llenos

Every four years, Americans gather at the polls to select the next president. However this public vote does not directly elect the president. Instead, an elite group of electors holds the door to the presidency, an old practice that needs review.

We found a number of problems with the current system. Among them are bias toward large states, possibility of electoral deadlock, faithless electors who vote against the voice of the people, and the ability of a third-party candidate to become president. Our model, the district division method, takes these issues into consideration and works to alleviate some of them.

The district division method eliminates the Electoral College, and so removes the possibilities of faithless electors and a deadlock. It places the election in the hands of the public. Each congressional district is given one electoral vote, which is awarded to the candidate who wins the district's popular vote. If the candidate also wins the state, he is awarded the state's two senatorial electoral votes as a bonus. After the election, all electoral votes are tallied, and the candidate with the most becomes president.

We looked at several variations of the model to deal with the possibility of a tie within a state. We decided that if candidates win the same number of districts in a state, the statewide votes must be compared to determine who wins the state's electoral vote. Otherwise, the outcome of the election is not determined by popular vote, as having a second vote might prove hard to organize and tally.

We developed a computer program to calculate the number of electoral votes for each candidate from election data. According to our model, based on the popular votes from the 1992 presidential election, Clinton wins with 315 electoral votes. Bush had

215, and Perot had 0. When compared with the actual electoral votes, we found smaller margins. We discovered several weaknesses in our model. It makes it possible for one or more splinter parties to form, which puts the two-party system at a disadvantage. The calculations can take a little longer in a state with a large number of districts. However, with fast computers and instant communication, the delay should be negligible.

We believe our model's strengths outnumber its weaknesses. It eliminates the Electoral College, reduces the bias towards large states, and gives more control to the public. It also gives a better chance to third party candidates.

Problem B Summary: Wilmington Area High School, New Wilmington PA

Advisor: Barbara T. Faires

Team Members: Erika Faires, David Stein, and Thomas Waldo

Devising a new system for the election of a president is an interesting problem because the Electoral College has such a long and celebrated, as well as controversial, history. The catch is to pacify critics of the Electoral College as well as to continue the tradition that surrounds every presidential election. Throughout the history of the United States, evolutionary steps have been taken to improve the quality of elections. The 15th, 17th, and 26th amendments gave more Americans the right to vote. Our model is another evolutionary step. We don't give all the power to the people, because we want to preserve the Electoral College, but we do force the College to vote as the people want. We allow more than one candidate to receive electoral votes in a state, but we don't allow the election to spiral into a publicity circus.

Our system specifies that the electoral votes from each state be split among the top two candidates in terms of popular votes. Ties are broken under protocol that each state is required to designate. Votes in each state are divided in this way: all the votes that were not cast for the top two candidates are eliminated. The votes that remain are summed. Then the number of votes for each candidate is divided by this sum to get a percentage. When the percentages are multiplied by the total number of electors for each state, with rounding, the number of electoral votes for each candidate is found. Also, in our system only a plurality is needed to win.

When tested against the 1992 presidential election data, Bill Clinton still wins, but not with the landslide of the current electoral system. His electoral vote of 53% (233/436) is closer than the current system's 69% (370/538). This compares favorably to the popular vote: Bill Clinton received 52% compared with President Bush's 46%, and Ross Perot's 2%.

Problem B Paper: Hunter College High School, New York NY

Advisor: David Hankin

Team Members: Seth Kleinerman, Timothy Kleiman, Ned Tyrrell

I. DEFINING OUR OBJECTIVES

Our model examined two questions. First, what are potential flaws in the type of democratic representation the Electoral College provides? And, second, how can we develop an alternative without drastically changing the existing system?

Most objections to the existing system have to do with the rigid and arbitrary way votes are assigned to states. The system is designed to ensure that a state's voice is roughly proportional to its population. However, American democracy is plagued by poor voter turnout. Our model aims to alleviate both the causes and symptoms of poor turnout. We created a system with incentive to vote by using the proportion of a state's total voters to the nation's total voters to determine the state's effective "population." States that are best at getting out the vote have the most say in the final decision.

However, we discovered that tying the influence of a state to voter turnout could encourage people to stay home! Consider the case of a candidate who is almost certain to lose a state. A clever candidate would mobilize his constituents to stay home, thereby decreasing the ratio of votes cast in that state to the total votes cast in the election, and ultimately lessening the impact of his opponent nationally. We believe this is antithetical to the principles of democracy. Therefore, we added a second criterion: that the 'voice' of each state depends on the percentage of the popular vote each candidate receives. This way, the aforementioned strategy backfires. On a more basic level, it is important that the voice given to each state is weighted according to the strength of the plurality held by the winning candidate. The function we chose for this purpose will henceforth be noted as $w(x)$.

A third concern we wanted to address was the influence of third parties. We wanted to make a "protest vote" more possible. Although it is important that a candidate who receives 5–10% of the popular vote not be in contention, we wanted to ensure that those votes are not discarded as they tend to be in the current system. Our weighting methods are designed so that a successful third-party candidate can diminish the influence of the major candidates. Our analysis examines different levels of third-party turnout, and shows the significant impact our weighting procedure can have.

With our system it is harder to predict the results of an election before all votes are tabulated. Though the media might access state results soon after polls close, it is impossible to demonstrate the effect until the ratio of voters in that state to voters nationwide is known. The practical effect would be to reduce the influence of the media. We assume that there are people who like to "vote for a winner" and will follow suit if they hear in the news that a candidate is winning by a large margin. We considered it preferable that the winner be unclear until the end.

Our system is more dynamic and flexible than the current system because it uses votes cast to determine "population." An additional benefit is that it can be calculated quickly. Since candidates do not win the entire block of electoral votes of a state, our model could track the outcome of the election as every vote is cast. This could have several interesting psychological effects. First, the model reinforces the notion that everyone's vote counts. In the current system, it may not seem worthwhile to vote for a candidate when it is likely that another candidate will win all the state's electoral votes. In our model, every opposing vote cast cuts into a candidate's weight in the national result.

Our final concern was that our system be comprehensible to the average American because democracy cannot function when the populace does not understand the voting system. It is not so important that the subtleties—for example, our choice of the *w(x)* function—be easily explained, since the method by which electoral votes are assigned to states seems equally arbitrary to voters. Anyone with a background in trigonometry would understand our weighting function.

Once we had a model, the variables were clear. The outcome of our model depends on the total number of votes cast in each state, the total number of votes cast in the country, and the percentage of popular vote each candidate receives in each state.

Our basic hypothesis was that an ideal model would not change the results of most past elections. For this reason, we tested the model on the elections of 1988, 1960, and 1948 in addition to the data given. In the present system, a candidate needs only to win a few large states by about a 1% margin to have a commanding lead in the electoral vote. In our system, the election is based primarily on the popular vote to ensure that a candidate cannot win without appealing to a significant plurality of the country.

II. ANALYSIS AND MOTIVATION OF MODEL

Our first problem with the present system involved the relative weight of each state. This bug seems to have an easy patch: we use the number of voters in each state and the total number of voters in the country to determine each state's weight.

However, this is not the most severe problem: the second flaw is that on-the-margin voting can make a big difference. We decided to completely rethink this "all-or-none" idea. It seems that distributing the benefit among all the candidates in a state would prevent most such problems. However, we cannot distribute the benefit linearly because that is a return to majority voting.

So we need a weighting system. Let us use a function $w(x)$ that maps a candidate's vote percentage in a state, x, to his weighting factor, which will be multiplied later by the state's relative weight. The range of $w(x)$ will be the interval [0, 1], and $w(0) = 0$ and $w(1) = 1$. The function should be strictly increasing on the appropriate interval—a candidate should not be punished for obtaining more votes.

What behavior do we want our function to exhibit? The initial slope should be small. It ought not to matter much whether a candidate has 1% of a state's vote or 2%. Also, a victory with 99% of the vote is not much better than one with 98% of the vote. The slope should approach zero on the ends of the interval, but maximum slope should be at the middle—the point at which the candidate reaches a majority.

A function from [0, 1] to [0, 1] that satisfies all of these criteria is $w(x) = \frac{1}{2}(1-\cos(\pi x))$. (See the graph in **Figure 1**.) To tabulate the vote for a particular candidate, sum the weighted vote won by that candidate over all fifty states. To determine the weighted vote won by a candidate in any given state, take the proportion of voters in that state to the proportion of voters in the country and multiply it by $w(x)$, where x is the number of voters who voted for that candidate divided by the number of voters in the state.

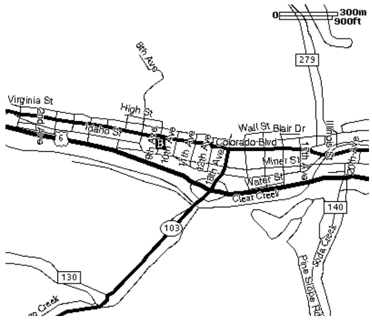

Figure 1.

III. PLAYING WITH THE FORMULA

No matter how votes are weighted, a sufficiently strong candidate will always win and a sufficiently weak candidate will always lose. What differs is how they behave in close elections. The key to understanding the behavior of our model is that a larger advantage is given for a majority of the votes in a state than for a plurality. A candidate receives the greatest number of weighted votes per votes at the point where $w(x)-x$ is greatest, which is at approximately 0.78.

Therefore, the textbook example of a candidate winning by small margins in some states and losing by a large margin in others (thereby winning the election and losing the popular vote) could not happen as easily. In one-on-one elections, our model tends to agree with the popular vote (exceptions are discussed later). The major difference is the importance of a third candidate.

The current system assigns no importance to third party candidates, unless their votes are concentrated in a particular state or group of states. A candidate with a majority of votes in just one state is of greater consequence than one with a fifth of the popular vote distributed across the country. In our system, every vote for a candidate pushes the candidate's weighted percentage further up the s-shaped graph of $w(x)$ and drags the others down the curve.

A vote for even the most obscure candidate will have some effect. A vote that prevents another candidate from obtaining a majority has the greatest effect, for that is where $w(x)$ changes most rapidly on the interval [0, 1]. However, if another candidate has a large majority, the slope of $w(x)$ is small, and so the effect of a vote for the obscure candidate is small.

In the electoral system, close three-party elections go to the candidate whose votes are concentrated in particular states. A candidate could win the popular vote and still lose the election. In our system, a candidate could only win a close three-party election with a combination of broad-based popular support and a majority in several states. A candidate might win the popular vote and lose the weighted vote without significant support in at least a few states.

For a candidate to lose the popular vote and win the weighted vote, s/he needs healthy majorities in the states won (approaching the aforementioned 78%). S/he also must not go below the point of least efficiency (22%, which can be obtained by observing the symmetry of the graph around the line $x = 0.5$). Therefore, although it is not easy for a candidate to win without a majority, it can happen when a candidate has nationwide support among a significant minority of voters and very strong support in a few states. This prevents a candidate who is unlikely to obtain pluralities in certain states from ignoring them.

IV. PLAYING WITH THE DATA

We first analyzed the 1992 data provided in the contest. The model's results corresponded closely with the popular vote. Clinton's margin of victory over Bush increased by nearly four percentage points, a change that reflects states in which he won by large margins. Our system had its most interesting effects on Perot. Although his share of the electoral vote was about eight percentage points below his share of the popular vote, his impact is infinitely greater than the zero electoral votes he received under the current system.

In order to guarantee consistency with a clear decision, we tested our model on the 1988 election. With our model, Bush clearly won. This was the kind of election we discussed earlier in which it might be profitable for Dukakis supporters to stay home in states they were going to lose in order to decrease the number of votes recorded for those states. In those states where Bush's margin was greater than 20%, we assumed that the Dukakis camp prevented its people from voting. The end result was significant: Bush's margin of victory nearly doubled. So, our model ensures that voter turnout is the best strategy for supporting a candidate.

We were curious about how our model might change recent close elections. First, we looked at the 1960 election between Kennedy and Nixon. With our model, Kennedy's margin was slightly larger than the popular vote. To test the durability of our model, we made one small change to the actual data by switching the votes of 150,000 New Yorkers from Kennedy to Nixon. Under the current system, the swing would not have changed the plurality in the state, and Kennedy still would have won. However, it would have given Nixon a plurality nationally. Our model solved this by noting that the margin of victory in New York had become very small, so the benefit Kennedy gained by winning there was small, and the adjusted votes favored Nixon. Whether this would have been good for history is, of course, a subjective question.

The final election we chose is the 1948 race between Truman, Dewey, and Thurmond. This was a close election in which Thurmond received 39, or 7.3%, of the electoral votes, which seems absurd for a candidate who received 2.4% of the popular vote. Our model tempers Thurmond's influence down to a level that reflects his popular support. Because this election had significant third-party candidates, we wanted to test some of our hypotheses pertaining to their importance. First, we assumed that all third-party voters stayed home. The data show that the candidate favored by the majority won by a slightly larger margin than he would have if third parties were present. This clearly shows that third-party support can hamper a leading candidate. Similarly, we assumed that all third-party votes went to the party that ultimately won the election. This shows that third-party support can also swing a close election—in this case, by more than seven percentage points—which should encourage mainstream candidates to court fringe voters.

The election of a president is an inherently intuitive process; for it to be truly democratic, the masses must understand the effect of their votes. The test of any election method, then, must come at the borders, where the method must select between two very closely matched candidates and the ideal solution is not blatantly clear. The electoral system fails at this task: it arbitrarily weights votes according to geographic lines with little meaning.

We must recognize that state borders are significant, in that states often share common interests and beliefs, but we must also recognize that states are made of ideologically diverse groups. Thus, the proposed model does not alter the concept that a state votes as a coherent entity, but instead removes the restriction that the state vote as a pure block.

Further, we suggest that the reliance on census data to determine Electoral College seats is outdated and unreliable. The principle expressed in this arrangement is clear; states with a larger population should get a proportionately larger representation. However, there are two failings with census data. First, it is never accurate or up to date. More importantly, it rewards states for the number of residents, not the number of people voting. The proposed model addresses both of these issues by determining the state's representation based on the number of people who vote. This prevents the situation in which a large state of people uninterested in an election overwhelms a smaller state, in which the election is very much of interest.

Problem A Paper: Severn School, Severna Park MD

Advisor: Kalla Sparrow

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ASSUMPTIONS AND EXPLANATIONS:

1. The robbers cannot pass the cops in any direction since the cops will force them off the road.
2. The robbers leave the bank in a different direction from the approaching police cars (they see them coming).
3. The robbers know the main highway might be blocked and attempt to escape by reaching one of the other roads (Route 103, Route 140, and 8th Avenue). The two unnamed roads leading north are not useable exits.
4. The robbers travel at the maximum speed (60 mph) possible in light traffic.
5. To make escape easy, the robbers choose a low-traffic time of day, but there is enough traffic to prevent extreme maneuvers.
6. The maximum speed of all the cars is the same.
7. The nearest entrance to the highway is at Route 103. The only other entrance is at Colorado Blvd (in either direction). It is impossible for the robbers to maneuver onto the highway (back towards the town) at the Colorado Blvd intersections.
8. It is sufficient either to force the robbers into a roadblock or to trap them between police cars.
9. The Interstate has a median the robbers cannot cross. They cannot avoid the roadblocks.
10. Traffic and traffic lights have the same effect on robbers and police.
11. The roads are dry and tire friction is relatively good. The maximum speed that any car can negotiate a 90-degree turn is about 40 feet per second—more for a left turn and less for a right turn. The delay that a turn causes is approximately 3 seconds.

GENERAL STRATEGY:

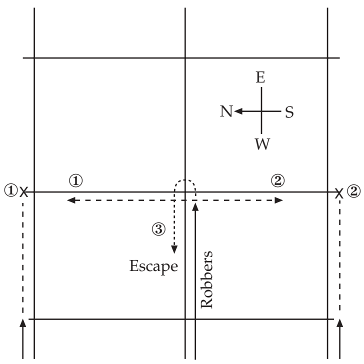
1) Cut off immediate escape routes

The first phase makes sure that the robbers cannot leave town. The easiest escape is to the north via 8th Avenue, which must be cut off by a police car. One of our assumptions in determining the later stages of the sweep strategy is that the police cars approaching the bank can approach from any direction. Indeed, only one possible approach combination prevents immediate escape: the cops must approach from the north and from the south. If no car approaches from the south, then the robbers can flee to the highway intersection at Route 103; there is no shortcut for the cops to catch up and block access to the freeway.

2) The chase pattern

The cops must follow the robbers in a manner that avoids letting them escape and forces them either into a dead end or onto the freeway, but not at the Route 103 interchange.

PATTERN A

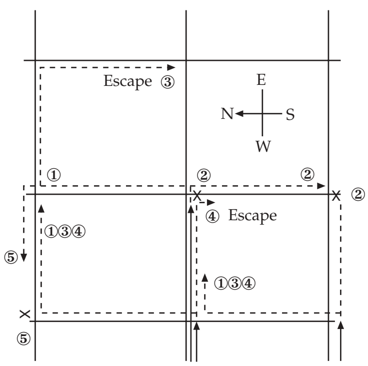


The first pattern we looked at was the most logical: the cops follow on both sides of the robbers to prevent them from making turns. On each side, a police car follows less than a block behind and one block to the side. If the robbers make a turn to the left or to the right, the police car on that side should turn onto the block. Then, before the robbers have time to turn back, the other police car will cut off that end of the block.

One problem with this approach arises from the lack of a line of sight. Another problem is that the robbers are not contained from the rear. They could simply turn around and head in the opposite direction without the police knowing.

This pattern shows that controlling sideways movement from both sides is impossible with only two cars; at least one of them must follow from the rear. Also, following from the rear resolves the line-of-sight issue since one car has the robbers in sight and can communicate with the other car.

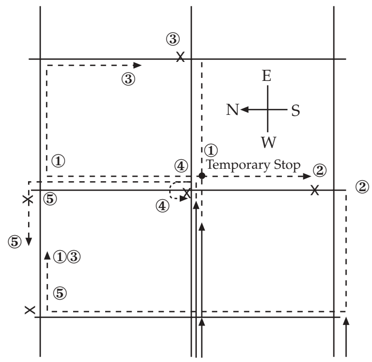
PATTERN B



This pattern contains sideways movement only on one side and has the other car follow from the rear. Since there are no northerly exits other than 8th Avenue, the cars will always follow to the rear and one block to the south. If the robbers turn to the south, then the rear car will communicate with the other car, which will turn onto the block (facing the robbers) and cut them off, while the other car blocks them from behind. If the robbers turn north and then head in their original direction again, both cars will move over one block north exactly as the robbers did. The pattern will then resume one block over. This may force the robbers into a corner rather than drive them into the roadblocks, but this is acceptable since they do not escape.

Although this improves on Pattern A, the robbers still can circumvent it. First, when the robbers turn north, the car following from the rear must be a block behind and must turn on the previous block, which temporarily breaks line of sight. At this time, the robbers may decide to turn around and head in a different direction and escape to the south. The second maneuver they may try is turning north and then away from the bank again, and then south on the next block. In this case, the car following one block to the south cannot reach the right corner in time.

PATTERN C



The algorithm we selected is very similar to the previous, except the roles of the police cars change every time the robbers turn north. The original chase positions are similar, except that the car following to the rear starts much closer to the robbers. If the robbers turn north, then the car that is following one block south must cut across 2 blocks, while the other car stops at the intersection the robbers just left in order to maintain line-of-sight. If he sees the robbers turn back towards the bank, then he follows them (Path 5 in Figure 3). The critical point here is that, while the distances the robbers and the other car must travel to reach the next intersection are almost identical, the police car must execute only one turn while the robbers must execute two. We analyzed that a turn causes a delay of about 3 seconds (see Appendix), so the police car arrives at the intersection in time to block it.

The strengths of this pattern are numerous. First, the robbers cannot get out of sight of one of the police cars long enough to execute surprise maneuvers. The only time the robbers go out of sight is on paths 1, 3, and 5. On path 5, the robbers are trapped, and on paths 1 and 3, no other surprise maneuvers can be completed by the time the other police car begins following. Once Pattern C is established, the robbers cannot turn south, and if they turn north, they will be boxed in.

3) Continue the pattern

We then applied Pattern C to the map. The final chase begins with the police cars arriving at the bank from the north and south, which forces the robbers to exit either west or east on Colorado Blvd. If the robbers exit to the west, the police cars set up a chase pattern on Virginia St and Colorado, due to the unusual grid in that area and the lack of exit streets on either side. However, in this case, the police car on Virginia must wait at the bank, as the robbers can decide to turn left on 5th Avenue. If they do, the other police car must circle around to block their passage on Idaho St. If the robbers turn right at any time, then the car waiting at the bank must circle around to Virginia to set up the pattern there. If the cop waiting at the bank is prepared to move in the right direction at the right time, then he can cut off the robbers on either side.

If the robbers continue straight on Colorado past 5th Avenue, the cop waiting at the bank must establish Pattern C quickly, but the delay complicates things. He can maintain line-of-sight at this point, so the other car should retreat one block; i.e. if the robbers turn on 4th, the cop turns on 5th. That cop cuts off escape back to the center of town, while the cop who is traveling west on Colorado can detect if the robbers attempt to change direction and maneuver back onto Colorado. Since the robbers will have made many turns, but the cop on Colorado will have made none, he will catch up, and the two cops will establish Pattern C. Once that is accomplished, the robbers will be forced into the barricades or into a dead end.

If the robbers travel east on Colorado, then the cops can establish Pattern C because there are no significant irregularities in the grid. While in the opposite direction the lack of cross streets makes it difficult for the cops to coordinate, here there are no missing cross streets until after Riverside Drive. Using Pattern C, the robbers will be forced past Riverside quickly. Once the cop following to the south (on E. Miner St) reaches Riverside, the pattern must change to accommodate the lack of cross streets. He must wait at Riverside until it is verified that the robbers have passed the last opportunity for a left turn (near 23rd Avenue). Once that happens, he must head east on Colorado, towards the intersection with the highway.

THE FINAL CHASE

The police car travelling east on Colorado will not likely beat the robbers to the crossovers between Riverside and Colorado, so he should wait at the second street after 23rd Avenue. If the robbers avoid the highway entrance, they will be coming towards him and take the one left turn in between the police cars. Then he can close off their one exit while the other police car can keep them trapped from behind. If the robbers take other paths (other than entering the highway), then the cops can work together to isolate the robbers in a small area (see paths 2 and 5 in **Figure 4**) and eventually trap them. Note that in path 4 the cop on Colorado cannot arrive quickly enough to trap the robbers. If they do not enter the highway, they then follow path 5, which carries the robbers on a long road with no turns. In path 5, one of the cops reverses direction and heads for the exit to that road, which he will get to in time; the other cop follows the robbers from behind.

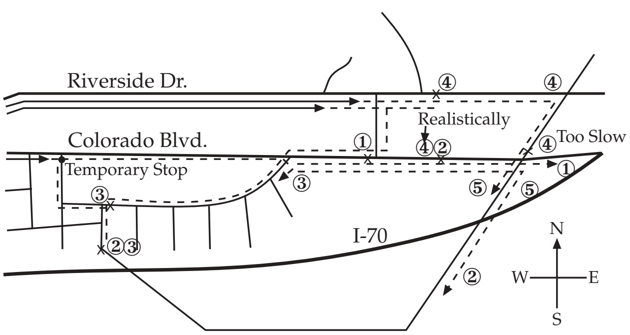


Figure 4.

ALGORITHM STRENGTHS

A chase in a non-uniform grid is intricate with only two cars. However, Pattern C (with a few special cases where there aren't enough cross streets) confines the robbers and forces them either into a dead end or onto one of the Colorado Blvd entrances and into the roadblocks. Our algorithm is sound and logical, and rides on reasonable assumptions. The cops maintain a line of sight, so there is no need for backup. Perhaps the strongest strength is that the same pattern, combined with special cases, can be adapted for almost any city where the desired exits lie only on one side.

ALGORITHM RESTRICTIONS

Because the map did not include distances, proving that the maneuvers can be executed in time is impossible, so flaws may exist in the pattern we chose. The greatest weakness is in our strategy's dependence on all exits being on one side. If there are exits to the north other than 8th Avenue, then none of the patterns we analyzed would suffice unless there is a third chase car. With only two cars, it is impossible to protect exits on both sides while maintaining protection from the suspect's rear. Also, variations in traffic and traffic lights were not considered, but were assumed to affect all vehicles similarly. We also depended on police cars arriving at the bank from the north and south.

APPENDIX

ASSUMPTIONS:

1. Gravity is 32 ft/s^2.
2. The police and robbers drive similar cars.
3. They are driving on dry roads with a static friction coefficient of 1.
4. They are driving in light traffic and do not use lanes occupied by opposing traffic.
5. Roads have four lanes and each lane is 12 feet wide including lines or barriers in the middle.
6. When turning, they try to safely maximize the radius, following a roughly circular path.
7. All acceleration is constant.
8. We used the 2000 Ford Taurus to define the average car. (http://www.mpt.org/mpt/motorwk/reviews/rtl908a.html):

   Top speed during the chase: 60 mph = 88 ft/s (Due to traffic)

   Acceleration (0 – 88 ft/s): 8.1 s

   Mass of car: 1500 kg (an estimate; we couldn't find the real value)

   Stopping distance (88 – 0 ft/s): 128ft

CALCULATIONS:

The formulas are from our physics book (Giancoli, Douglas. *Physics*. 5th edition, Prentice Hall)

1. How much time does taking a right turn waste?

Maximum speed through the corner:

A right turn has a radius of 36 feet. (See **Figure 5**.)

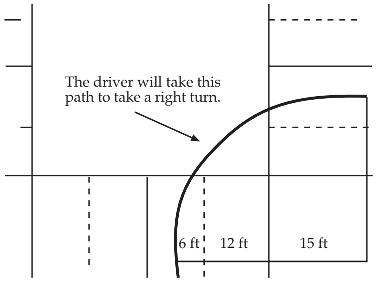


Figure 5.

The car will have to turn at a certain speed (or lower) or it will crash.

$V_{max} = \sqrt{\text{(friction coefficient)(radius)(gravity)}}$

$V_{max} = \sqrt{1 \cdot 36 \cdot 32} \approx 33.9$ ft/s

The car must stay below 33.9 ft/s

ACCELERATION:

$v = v_0 + at$

$88 = 0 + a * 8.1$

$a = 88/8.1$

$a = 10.9$ ft/s^2

DECELERATION:

$V^2 = v_0^2 + 2 * a * (x)$

$0 = (88^2) + 2 * a(128)$

$a = -(88^2)/(2 * 128)$

$a = -30.3$ ft/s^2

WHERE IT SHOULD START DECELERATING:

The car must slow from 88 ft/s to 33.9 ft/s with an acceleration of –30.3 m/s^2.

$V^2 = v_0^2 + 2 * a * x$

$33.9^2 = 88^2 + 2 * -30.3 * x$

$x = \dfrac{33.9^2 - 88^2}{2(-30.3)}$

$x = 109$ feet

The car must begin decelerating 109 feet from the intersection at full braking power.

TIME TO DECELERATE:

$v = v_0 + at$

$33.9 = 88 + -30.3t$

$t = \dfrac{33.9 - 88}{-30.3}$

$t = 1.8$s

The car will be decelerating for 1.8 seconds.

DISTANCE OF TURN AND TIME TO ACTUALLY TURN:

time = length/speed

length = $(2\pi r)/4$

length = $(2\pi * 36)/4$

length = 57 feet

speed = 33.9 ft/s

time = 57/33.9 = 1.67 seconds

The car will be turning for 1.67 seconds.

TIME TO ACCELERATE BACK TO NORMAL SPEED:

$v = v_0 + at$

$88 = 33.9 + 10.9 * t$

$t = \dfrac{88 - 33.9}{10.9} = 4.96$s

The car will return to normal speed 4.96 seconds after finishing the turn.

DISTANCE TRAVELED DURING ACCELERATION:

$x = x_0 + v_0 * t + 0.5 * at^2$

$x = 0 + 33.9(4.96) + 0.5 * 10.9 * 4.96^2 = 302$ ft

The car will travel 302 feet during the 4.96 seconds after finishing the turn.

TIME WASTED IN TURNING:

If the car didn't slow down, it would travel the distance in (total feet traveled)/(maximum speed).

T = (distance to decelerate + distance to turn + distance to accelerate)/(max speed)

$t = (109 + 57 + 302)/(88) = 5.32$ seconds

The car would usually take 5.32 seconds to travel the distance.

The car will actually take (1.8 + 1.67 + 4.96) = 8.43 seconds.

The wasted time is 8.43 – 5.32 is 3.11 seconds

2. How much time does taking a left turn waste?

A left turn has a radius of 72 feet. (See **Figure 6**.)

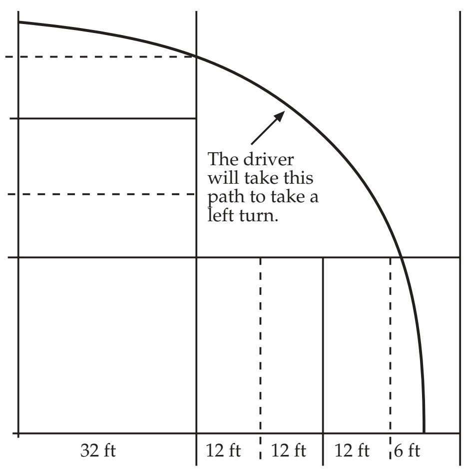


Figure 6.

Editor's Note: Similar calculations gave an estimate of 2.71 seconds for time wasted in a left turn.