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2011

14th Annual High School Mathematical Contest in Modeling (HiMCM) Summary Sheet
(Please attach a copy of this page to each copy of your Solution Paper.)

Team Control Number: 2881
Problem Chosen: B

No one enters the wilderness expecting to get lost. However, each year forest rangers search for thousands of missing or stranded people. In order to maintain public safety, it is critical to find the missing personnel in the fastest time possible.

Unfortunately, instead of calling 911, many people will operate off of their natural instinct and go searching for their friend – without help. Though some may have luck searching for their friends, many will not because the odds are very literally stacked against them. Our models show that, while certain methods of searching are somewhat effective, searching on one's own with a pen light is not the best solution, especially when searching such a large amount of land. Though the probabilities of finding the ring are not insufferably low, (as it is an inanimate object), the probabilities of finding a living being who could accidentally and repeatedly evade the searcher are horrendous. While the search methods are optimized, one person searching for a friend is never as effective as a well-equipped search party.

Introduction:

No one enters the wilderness expecting to get lost. However, each year forest rangers search for thousands of missing or stranded people (Forest Ranger Annual Report). It is critical for rescue teams to find these individuals within a few hours in order to rescue them. Due to the dire necessity of finding the lost quickly, search methods have developed to include the implementation of helicopters, rescue dogs, SARS (search and rescue squads), county sheriff departments, technicians, and local volunteers.

On October 23, 2011, Robbie Wood, an autistic eight year old child, wandered off while walking along a trail in Doswell, Virginia with his family. Mr. Wood tried to run after his son but eventually lost sight of him. Numerous rescue searches were performed with over 6,000 volunteers joining to search for the boy (Smith). After five nights, over 2,000 acres were searched. Robbie was finally found in a shallow creek bed and taken to the hospital, sustaining minor injuries (Fields).

With the many success stories, there are also numerous failures. In 2011, a hiker went missing in Yellowstone National Park. Rangers sent out search parties as night fell, hoping to find the hiker before it was too late. The search parties covered thousands of acres but could not find the hiker. A week later the hiker's dead body was found. The hiker had lacerations and had been attacked by a grizzly bear. These tragedies make it vital for rangers and search parties to find their target as quickly as possible while covering the maximum area (Fenton).

Unfortunately many of these incidences begin during the night. Therefore, without the use of artificial lighting, it would be nearly impossible to pinpoint the lost person or object. Search and rescue can be equipped with helicopters, night vision capabilities, radio/navigation equipment, and other special devices. However in this problem, the lone searcher is only equipped with a mere pen light. The question arises whether or not the searcher will even be able to see the whole trail. The answer is, with a few exceptions, yes. Because the pen light gives off 20 lumens, through a calculation of the working distance (the distance in which one can see at a certain level of luminance), the searcher could see a distance of 19.09 feet away with the luminance level of the full moon. Any ambient light would only increase this value. If he rotates the flashlight from 45 degrees to the left of his head to 45 degrees to the right of his head every second and then the opposite direction in one second, then he will periodically see the entire pathway in front of him, so for long stretches he will see the entire path. The exceptions occur when there are short paths or highly curved paths. In the first instance there will be small segments on either side of him which he misses, whereas in the second he will miss small patches on the side of the curvature. However, in either case the distance of the path would be so small that the probability of the lost object or person being there would be an insignificant enough that any effect due to curvature or length would be negligible. Thus, it is reasonable to assume that in his search he will see the entire pathway directly in front of him.

In this paper we will seek to solve two problems. In the first part of the paper, we will develop a search model in order to find a lost object, in which we present three models: one taking into account the connectivity of each path, another taking into account the distance of the path from parking spaces, and a third taking into account both. Through these models we seek to develop and improve upon a search method which would both improve our chances of finding the lost object and improve the accuracy of the calculation of the probability of finding the object. In the second part of our paper, we develop a model which should optimize the search pattern of the searcher given the possible movement of the lost jogger. However to accomplish either of these tasks, it was necessary to determine the length of each segment. To do this, we use a piece of string to follow a segment of the map and then we measured the piece of string and then used the scale to determine how long the segment actually is. Segments are any subsections of a path such that they are between two intersections of trails, as listed in the map of the park, and/or paved roads.

Assumptions for Finding Small Objects (for all Models):

1) The searcher is walking at a constant speed of 4mph.
2) The pen-flashlight used is 20 lumens.
3) If the light touching the object has a lux of X, then the object will be found.
4) The searcher has no recollection of where he has been.
5) The searcher is walking down the middle of the paths.
6) The path is nine feet wide (Virginia state law for maximum for vehicles).
7) The searcher is looking where the light is shining and he has a vision span of the radiation angle of the flashlight.

Assumptions specific for Small Object Model#1:

1. The searcher cannot walk on paved roads, except to cross them.
   a. The distance walked on a paved road is negligible
2. The searcher has a vehicle that, when being driven, maintains a constant speed of 15 miles per hour.
3. The searcher must leave the park two hours after entering.
4. The searcher always enters and exits the park through the main entrance.
5. The object is on a path.

6. The number of paths connected to the s^th^ segment's trail is directly proportional to the probability that the object is in that segment.

Small Objects Model # 1 Equation and Calculation:

$$P_{k_1} = \sum_{s=1}^{z} \left[\frac{l_s C_s}{\sum_{i=1}^{n} l_i C_i} \right]$$

k is the union of all segments that are in a given search path
P_{k_1} is the probability of finding the missing object for any given k
z is the total number of segments accounted for in the search
l_s is the length of the sth segment of K
C_s is the number of connected paths to the sth segment trail,
l_i is the length of the ith segment of the map
C_i is the number of connected paths to any

$$\frac{(l_s)(C_s)}{\sum_{i=1}^{n}(l_i)(C_i)}$$

The probability of the object being on a specific segment.

Equation Derivation:

Google has the tedious task of organizing a hierarchy of web pages. To accomplish this task, they developed a ranking system. Google correlated the number of pages linking to a website to that website's popularity. Thus, that web page would earn a higher page rank. We used a similar methodology. The more highly populated roads should be those roads with a greater number of linking trails. For instance, a trail that is connected to three other trails would be more popular than one with only one connection. So, let C_s indicate the number of paths connected to the s^{th} trail.

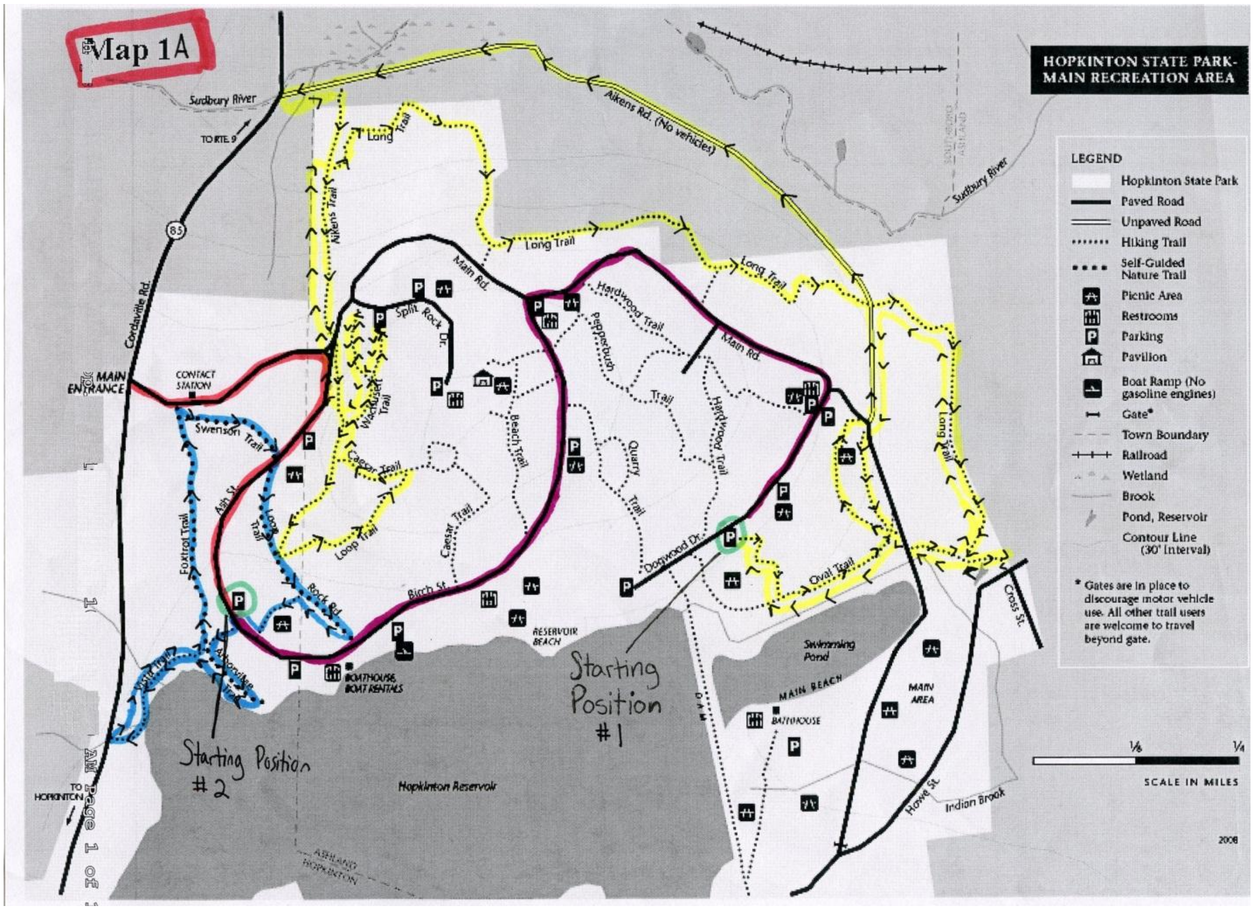
We may use this to weigh each trail *segment* with the popularity measure, C_s. Since without this weight, the probability of finding the missing object in the s^{th} segment would be precisely equal to its length, l_s , divided by the total length of all trails, we can simply multiply l_s by the popularity measure C_S to obtain the weighted measure. Furthermore, the probability of the ring being on the s^{th} segment is equal to l_s multiplied by the popularity measure C_S divided by the sum of the weighted measures of every segment in the park. If we define K to be the set of all segments in any given search path, then the probability of K containing the ring, P_k , is equal to the sum of the aforementioned probabilities for every segment in K .

Maximizing the Probability of finding object

Step 1:

$\sum_{i=1}^{n} l_i C_i$ was calculated by taking the lengths of each segment of each trail and multiplying it by the number of paths connected to that chosen trail (we did this for all 58 segments and all 17 possible trails for the given map). This calculation determined the weight of each specific segment. The “weight” was a numerical scale which determined how likely it was to find the object on a specific segment of a path. Once the weight of each segment was found the total weight of each path was found by adding the weights of all the segments in one path. The total weight, sum, (of all the paths) gave us the total weight of map 1 was **15.45**.

Step 2: Developing the best search path



After determining the sum of the weights of every segment on the map, we needed to come up with a way to search the park that allowed for the greatest probability of finding the object. Our search method allowed for visiting the paths with the highest total weight, which would increase the probability of the object being located, given the formula we developed. However, there were no parking lots within the vicinity of the two most likely trails (the two trails with the greatest weight), so the model was constructed to include other trails as well. We ended up selecting the parking lot on Dogwood Drive that directly touched the Oval Trail. This parking lot was chosen because it allowed the searcher to have access to Aikens Road and the Long Trail, which were the paths with the highest weight values. The model began with the searcher driving from the main entrance to this parking lot. The searcher would go past the Contact Station, up Main Road, and then down Dogwood Drive to the designated parking lot. This would take a total of 5.111 minutes. Once the searcher reached the parking lot (labeled "Starting Position #1" on Map 1A), he then begins his journey to find the object. This is the portion of Map 1A which is highlighted in yellow. His direction is marked by black arrows. In this one section of the model, the man searches all of Aikens Road, Long Trail, Aikens Trail, Wachusett Trail, and some of Oval Trail, Caesar Trail, and Loop Trail. He then proceeds to return to his vehicle, and then he drives it to the location on Map 1A labeled "Starting Position #2." From Starting Position #1 to Starting Position #2, the commute is about 5.556 minutes. He then follows the blue path, with the directions shown by the black arrows. This route begins going towards the Arborvitae Trail. By the end of this route, the searcher has looked through all portions of the Arborvitae Trail, the Vista Trail, the Foxtrot Trail, the Swenson Trail, Rock Road, and some of Loop Trail. Once this path is complete, the man returns to his vehicle and exits out of the park through the main entrance, along the red highlighted road. His exit takes approximately 2.64 minutes. This model results in an 82.091 percent chance of the object being found, because the routes mentioned above have a total weight of 12.683, which is 82.091 percent of the total weight. This model allows the total walking distance to be 7.044 miles. Given that he walks at a rate of four miles per hour, the walking time is 105.667 minutes. Adding this to the commute times, the total time taken for this model is 118.978 minutes, which does not exceed the 2 hour time limit.

Small Objects Model # 1 Critique:

Pros

- This model is based off of a very successful internet ranking system used for webpages, so it should provide a reasonable approximation for the popularity of the given trail paths.
- Relative to our other models, the numbers and variables are simple to calculate.

Cons

- This model does not take into account the crossing of roads.
- This model is only an approximation for the popularity of each possible trail, without actual traffic data.

Small Objects Model # 2:

Assumptions specific for Small Object Model#2:

1. The searcher cannot walk on paved roads, except to cross them.
2. The searcher has a vehicle that, when being driven, maintains a constant speed of 15 miles per hour.
3. The searcher must leave the park two hours after entering.
4. The searcher always enters and exits the park through the main entrance.
5. The object is on a path.
6. The number of intersections between the segment's nearest parking lot and the segment is inversely proportional to the probability that the object is in that segment.

Small Objects Model # 2 Equation and Calculation:

$$P_{k_2} = \sum_{s=1}^{z} \left[\frac{l_s}{S_s \sum_{i=1}^{n} \frac{l_i}{s_i}} \right]$$

P_{k_2} is the probability of finding the missing object for any given k
z is the total number of segments accounted for in the search
l_s is the length of the sth segment of k
n is total number of segments in the park
S_s is the number of intersections between the nearest parking lot of the sth segment and the sth segment.
l_i is the length to the ith segment of the map
s_i is the total number of intersections that occur between the nearest parking lot to the ith segment and the ith segment

Equation Derivation

While there are those intensive joggers who will run through an entire park, the average person will shy away from long excursions. As the sth segment is further away, the probability that one of these people would be there decreases. This distance can be effectively approximated by S_s, or the minimum number of path intersections before the sth path from the nearest parking lot. So, the probability of the sth segment containing the ring decreases as S_s increases. Thus, we weight any given path with the division of 1 by S_s.

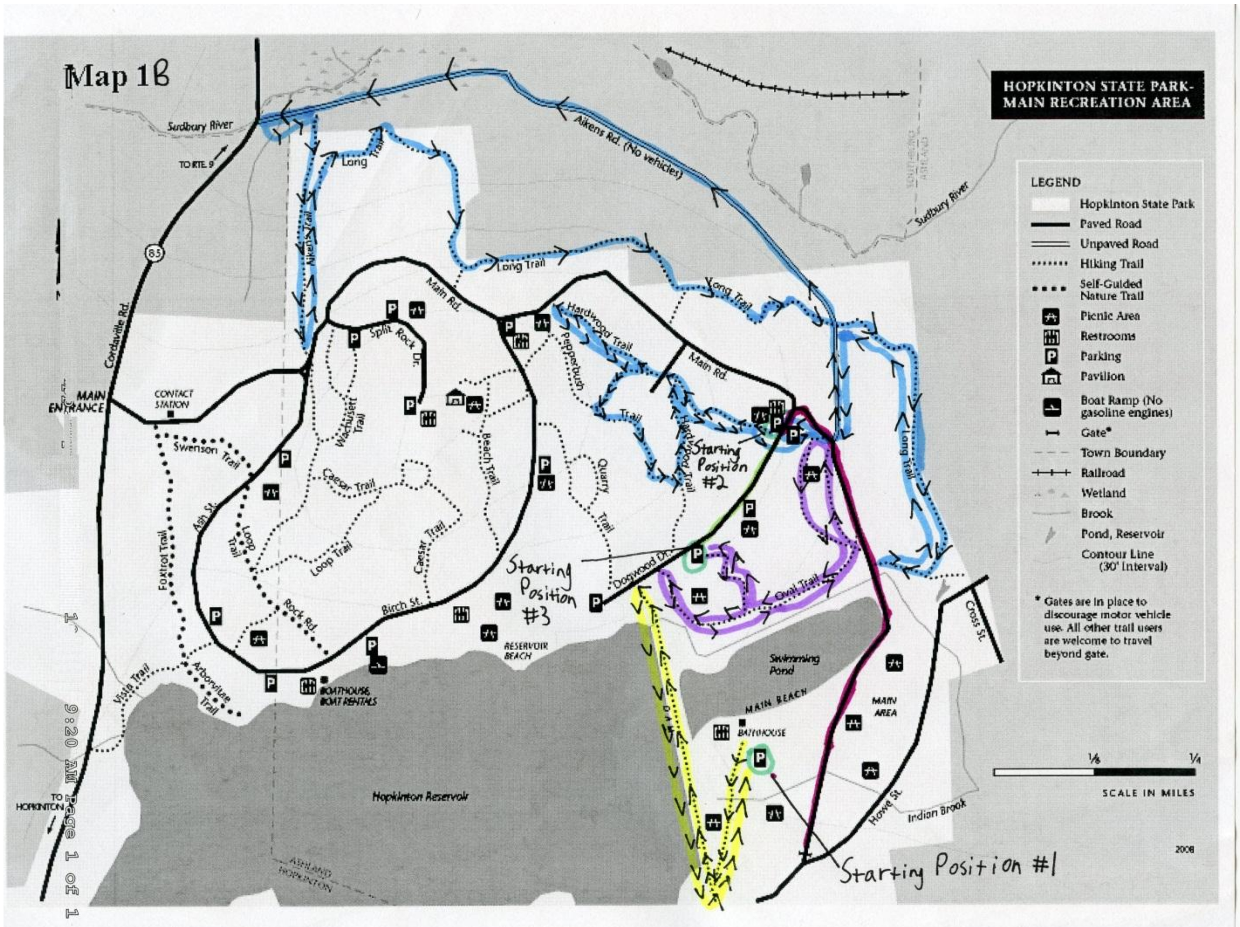
So the probability of the sth segment containing the ring is proportional to the length of the segment and inversely proportional to S_s and the sum of the weighted lengths for all possible segments. Thus, the overall probability of the ring being found on any search path K is equal to the sum of the previous probabilities for any given sub-segment of K .

Maximizing probability of finding lost object:

Step 1:

$S_s \sum_{i=1}^{n} \frac{l_i}{s_i}$ was calculated by taking the number of intersections with a paved road that a segment of a hiking trail, unpaved road, or self-guided nature trail had, and dividing it by the number of intersections it makes with a paved road. This essentially calculates this segment's distance from a parking lot. This was done for all segments in all paths used for walking. This calculation determined the weight of each specific segment. In this model, the "weight" was a numerical scale which gave the chance of finding the object on any segment chosen. Once the weight of each segment was found, the weights of all seventeen paths were found by taking the sum of the segments in each path. Adding these together, the total weight of map 1B was **5.45**.

Step 2: Developing the Best Path



After determining the sum of the weights of every segment on the map, we needed to come up with a way to search the park that allowed for the greatest probability of finding the object. Our search method allowed for visiting the trails with the highest total weight, which would increase the probability of the object being located, given the formula we developed. Given that the trails with the highest weights were located on the top half of the map and the right half of the map, a route was developed which allowed the searcher to go through these trails with the greatest weights. This method began on Starting Position #1 (refer to map 1B) and continued down the yellow highlighted path, in the direction of the black arrows. The searcher then travelled to Starting Position #2 by car, and followed the blue search path. Lastly, the searcher drove to Starting Position #3, traveling the purple path, and then exiting the park. This route took 119.374 minutes, and it allowed the searcher a 58.716% probability of finding the object.

<u>Small Objects Model # 2 Critique:</u>

Pros

- This model takes into consideration the position in which the searcher would have begun his hike, thus making it more likely that he will find the object.
- The numbers used in this model are fairly simple to calculate.
- The maximum time limit is not exceeded.
- A large portion of the map is searched, and over half of the total weight of the map is searched.

Cons

- This model does not take into account the distance or time it takes to cross the road on foot.
- This model is an approximation, and does not use the actual measurements of the distance of a segment to a parking lot.

Small Objects Model # 3 (For Future Models)

$$P_{k_3} \frac{\sum_{s=1}^{z} \frac{l_s C_s}{S_s}}{\sum_{i=1}^{n} \frac{l_i C_i}{S_i}}$$

Model 3 Derivation and Explanation:

While the first two models both provide reasonably good probabilities for finding the runner, perhaps taking into account both of their respective weight would improve the effectiveness of the model. Thus, both the number of connections to the segment's trail and the number of intersections that occur before the segment from the nearest parking spot are taken into account. The new weight, then, is equal to the division of the number of connected paths by the number of intersections occurring before the segment. This weight is multiplied by the length of the segment and divided by the number of intersections occurring before the segment. This, when divided by the sum of these values for every segment on the map gives the probability that the ring will be found in that segment according to this new model. Then, the probability of finding the ring on any given path K is equal to the sum of these individual probabilities such that each segment is a member of K. In variable from, we have that:

$$P_{k_3} \frac{\sum_{s=1}^{z} \frac{l_s C_s}{S_s}}{\sum_{i=1}^{n} \frac{l_i C_i}{S_i}}$$

When this model is implemented, though it may or may not provide a better probability of finding the object, it should provide a more accurate probability that the person searching will actually find the object. We can infer from the previous two, similar models, that the optimum path according to this third model would incorporate the similar paths of the first two. This combination should yield more accurate information about the popularity of separate paths and thus the chance that he would have dropped the ring along any one of these paths.

<u>Missing Runner Model Assumptions:</u>

1) The jogger who is missing understands that he is lost and is trying to go to one of the parking lots or he is unconscious.
2) The jogger is lost on the path and not in the woods.
3) The jogger could be anywhere in the park, according to a distribution, D.
4) The jogger will not have an initial lost position at an intersection, but rather on a certain path.
5) At the start of the search, the jogger has a 1/3 probability of being unconscious and a 2/3 probability of moving.
6) The jogger is moving at a constant velocity.
7) After two hours of searching for the jogger, we will call 911 to search.

<u>Runner Model Derivation, Explanation, and Testing:</u>

While there are many factors which could affect the ability of a runner to be lost, there are a couple of effective methods for predicting his or her location. In general, the greater the length of a trail segment, the greater the possibility of discovery or recognition of location becomes. If we simply take the length, L_s, of a given segment and divide it by the length of all trails, then we get the probability that the runner will be found on that segment *if* the runner would be lost on larger roads. However, this is the opposite of our initial assumption. So, we adjust this formula to provide greater weights to smaller roads. Given the highest initial probability of any given segment, we subtract the segment's, S's, probability. This, then, provides an unadjusted probability, P_t, that the runner will be found on one the t^{th} trail. Note that under this scenario, there is a zero percent chance that the runner will be lost on the longest trail. However, there still is the issue that these numbers do not sum to 1. This can be accomplished by implementation of the following equation:

$$a \sum_{s=1}^{n} P_s = 1 ,$$

where a is a scalar quantity by which we must multiply each P_t and n is the number of segments in the park. This final probability, a*P_t, is the probability that the initial position of the runner will be at the t^{th} segment. As there are a total of 112.09 miles of trails, it is necessary to look over a small portion of the trails. Given the assumption that we only have two hours to search, we can cover at maximum 8 miles by walking. This, however, is reduced as we drive to various drop-off points to search various trails. Thus, we want to look at as few groups of trails as possible to minimize travel time. We then introduce the idea of clusters. Clusters are groups of segments which are in the same general area and have large probability densities (P_t/mile). The probability of the runner's initial position being in one of these clusters is equal to the sum of each of the P_t's such that the t^{th} segment is in one of the clusters. This, then, would be a simple model if not for the fact that the runner has the possibility of moving. Given a perfectly straight path with no curvature attached on either end by another road, the runner, if conscious, would have a one hundred percent chance of reaching one of the roads. However, as the pathways become more convoluted and intertwined, the lower the chance that the lost runner will escape the cluster. We can approximate this notion of curvature by dividing the total length of all of the paths in the cluster, including its exterior paths (the set of paths that surround the interior portion) by the length of solely its exterior paths. In general, the greater this ratio, the greater the approximation of curvature will become, and the probability that the runner will leave will decrease. Given the original assumption that if the runner was in the middle of a completely straight road, s, he would have to travel a distance of $L_s/2$ to leave, and he would have a 100 percent chance of leaving, he would have a high chance of leaving the cluster after traveling the product of $L_s/2$ and the approximation of curvature, or:

$$\frac{L_I}{2}\left[\frac{L_I}{L_C}\right],$$

where L_I is the length of the interior of the cluster, including its circumscriptive length, and where L_C is the length of solely the circumscriptive path. Thus, if the jogger is moving at 4 miles per hour, it would take him a time of $\frac{L_I}{8}\left[\frac{L_I}{L_C}\right]$ hours to leave. Once this time is reached for any given cluster, the initial position probabilities would change. At this point, the probabilities become a function of time.

Finding Missing Runner Model Critique:

Pros

-The model limits the large amount of area to a few subsections and then restricts the subsections to an optimized search path

-It takes into account the an approximated notion of curvature in determining the amount of time it takes the runner to leave his position

Cons

-Only takes into account the notion of length and determining where the runner would have gotten lost

-It doesn't take into account the psychological decisions of a person who is lost

<u>Conclusion:</u>

Ultimately, though each of these models approximated the possible locations of the missing person or ring, the first model for the missing ring provided the best probability of finding the object. However, this does not necessarily imply that this is the best model, as the actual probability of finding the object may be, in fact, lower. In this case, though the second model has a lower probability, the result may more accurately resemble actual possibilities. In either case, the third, proposed model combines these two and should, when tested, provide a more accurate representation of probabilities.

While there was only one model for the runner, this model successfully restricted the amount of area which needed to be searched and allowed for an optimized search path. Though this produces a low probability, it is likely that this resembles reality as the total area to be searched is very large. This model illustrates the sheer improbability of finding a missing person using solely a pen light in such a large park. Further traffic analysis of each path should confirm both of these models to have a certain degree of accuracy.

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