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2010

13th Annual High School Mathematical Contest in Modeling (HiMCM)
Summary Sheet
(Please attach a copy of this page to each copy of your Solution Paper.)

Team Control Number: 2541
Problem Chosen: A

Please type a summary of your results on this page. Please remember not to include the
name of your school, advisor, or team members on this page.

Our model is 5-year plan for the development of bicycle-sharing programs in Chicago, Denver and Des Moines, aiming to popularize a new means of transport and to relieve the city center of traffic overflow.

The basis of the model is a database of roads, bike lanes, buildings, facilities and existing bike-rental stations in the three cities. All sites of buildings, facilities and existing rental stations are marked with accurate longitudes and latitudes. With this database, we define the city center for every city. According to our analysis, we should set up an initial network of bicycle-sharing program (including bicycle rental stations and bike lanes) in Des Moines and Chicago, where the existing bicycle service is far from enough. Especially, in Chicago, a metropolis covered with a developed subway system, we should place bicycle rental stations near subway stations. In Denver, a city already covered with a bicycle-sharing network, we may directly embark on improvements.

The five-year program comes in three parts. The first part is a two-year initial plan for Chicago and Des Moines, with which we build an initial network of bicycle rental stations and lanes in these two cities. When looking for ideal positions to place new stations, we divide the buildings/facilities in the city center into several types and then apply the analytic hierarchy process (AHP) to quantize the different significance of these types. Afterwards, we weigh the significance values of buildings with the distance between the buildings and the new station so as to find the ideal location of the rental station. When building bike lanes, we apply linear regression

to find a line such that the total distance between stations and this line is minimized. Then we fit the line into actual streets with appropriate linkage to existing lanes.

The second part is an additional plan to improve the initial bicycle-sharing program in the cities and thus includes all three cities. We deal with addition of new stations and lanes. We apply the Central Place Theory and find out the initial service range of every existing bicycle rental station. With the increase in demand for bicycles, the service range of each station will decrease year by year, and some “blind areas” will be left out of the coverage of the program. New stations should be built in these areas to maintain coverage of service and new lanes should be built if these new stations are far away from existing lanes.

The third part is the discussion on allocation of bicycles and regulation of bicycles according to the change in demand. We apply the Markov Chain Model to decide the different status change of bicycle stations on working days and work-free days, and distribute bicycles according to the steady state of the Markov Chain.

Finally, we put all the results together and make annual plans for each city, and prepare a letter to the mayor recommending our plan.

BIKE THE WORLD

Team # 2541

Problem A

Contents

1. Introduction

The United States is a great country – both in terms of number of highways and frequency of traffic jams. Counter-urbanization has flooded the city center with private vehicles transporting people to and from the distant residential areas – particularly during rush hours. The decreasing efficiency of driving cars, in a way, creates a good opportunity for the introduction of bicycle renting programs in downtown areas. Bikes provide flexible and swift rides inside the city. They are inexpensive and, if equipped with a carefully designed network and proper layout of rental stations and bicycle lanes, more convenient than any other means of transport. Our goal is to give a general model of planning a complete bicycle rental system in the city centers of Des Moines, Chicago and Denver. The model includes a detailed approach to determining the best locations of rental stations, an analysis of bicycle allocation, plans for bike lane and future station construction and, and a 5-year development strategy wrapped up with a short letter to the mayor. We believe one day Americans will ride to work and enjoy ham in the office, rather than jam on the road.

2. Assumptions and Basic Analysis

2-1 Assumptions

(1.) *In one year, the number of rentals and returns at each bicycle station on any working day does not change.* Since most people go to work during a fixed period on weekdays, the number of bicycles that are taken away and returned to stations does not change much from day to day. The slight difference in status change can therefore be neglected.

(2.) *In one year, the number of rentals and returns at each bicycle station on any work-free day (including weekends and public holidays) does not change.* Most people stay away from their work at weekends and on public holidays. They tend to visit museums, go to the theatre or hang out with friends. The difference in bicycle usage between work-free days is negligible.

(3.) *Everyone that takes a short bike-trip to commute to work takes round trips.* Since most people come back from work in the same way as they go, bicycles that are taken away from the station for someone to get to work will be returned at the end of the day.

(4.) *During working days, 50% of the bike renters use the bicycle to commute to work.* This value will be extremely important when we determine how to allocate bicycles at different stations. We are not able to determine the exact value of this percentage since this value may vary from place to place, so we make this assumption.

(5.) *If a bike renter does not commute to work with the bicycle, then the possibility that he/she visits any building/facility in the city center is the same.*

(6.) *The annual growth in bicycle demand at any place in Des Moines, Chicago and Denver is 30%.*

(7.) *In one year, all rental stations in the same city share the same service radius (range).* This assumption will be of vital importance when discussing where and how to add new stations to the program, and will be further discussed in the chapter three.

(8.) *The average number of bicycles at each station is ten.* Currently in Des Moines, Chicago and Denver, each rental station in use has approximately ten bicycles on average. According to our model, we will not make great changes in the number of bicycles, and we assume that the average number of bicycles at each station remains ten throughout the program.

2-2 Basic Analysis

(1.) A Brief Model Overview

We are required to develop a model that can be applied to the five-year development strategy of bike-sharing programs in Chicago, Denver and Des Moines. What we need to do is to make the program both effective and efficient. A well-built model should:

(i.) Take the existing bicycle rental stations and bicycle lanes into consideration,

(ii.) Make the bicycle rental program accessible to every citizen in the city who may need it,

(iii.) Provide a clear layout of locations of new rental stations and lanes to be built in the five-year program in chronological order, and

(iv.) Ensure stability and self-support of the program after the five-year program is completed.

(2.) Facts that we should know

Fact 1. In medium-sized American cities, driving private cars is more favored than taking public transportation (that is generally made up of bus lines) – waiting for no less than a quarter of an hour for a tortuous bus ride wastes far too much time. These cities are thus equipped with abundant parking facilities. In some of the cities, parking lots surround the city center (as in Des Moines), whereas in other cities parking lots permeate the entire downtown area (as in Denver).

Fact 2. In metropolises (like Chicago), city centers are often overcrowded with high-rises. These office blocks makes the city center more compact since each block occupies less space than buildings with ordinary heights, which results in considerable traffic pressure in the narrow streets. Fortunately, large cities are often armed with mass transit railways (like the CTA Trains in Chicago, and – even in Denver, the

RTD). These railways link residential areas to the city center more efficiently than any surface transport, and they are relatively inexpensive – but their coverage is sometimes not wide enough. Since the distance between mass transit stations are greater than that between bus stops, some of the passengers have to travel a longer distance to ride the trains. So we can install bicycle stations around these stations and at various destinations in the city, so as to make these transit stations more accessible. This would encourage more people to adopt Bike + Ride and therefore relieve the city of traffic stress.

(3.) Different features of Des Moines, Chicago and Denver

(a.) Des Moines, IA

For medium-sized cities like Des Moines (with a city center covering about 4 km^2), where private cars dominate public transportation (primarily consisting of bus routes), there is ample supply of parking spaces around the downtown area. This is evident through satellite maps obtained by Google Earth™, where we can see a sharp difference between dense blocks of buildings and extensive parking spaces surrounding them. However, while a lot of residents drive directly into the downtown area (sometimes causing slow traffic inside the very compact city), these precious parking spaces are not used to their full extent. Getting stuck in traffic jams is indeed annoying and time-consuming. It is thus desirable that, if we could take full advantage of those parking lots by setting up bicycle stations around them and inside the city center, people would tend to choose Park + Bike as their favored way when heading for the city center, if they feel what they have to pay for the bicycles is well worth the loss of time.

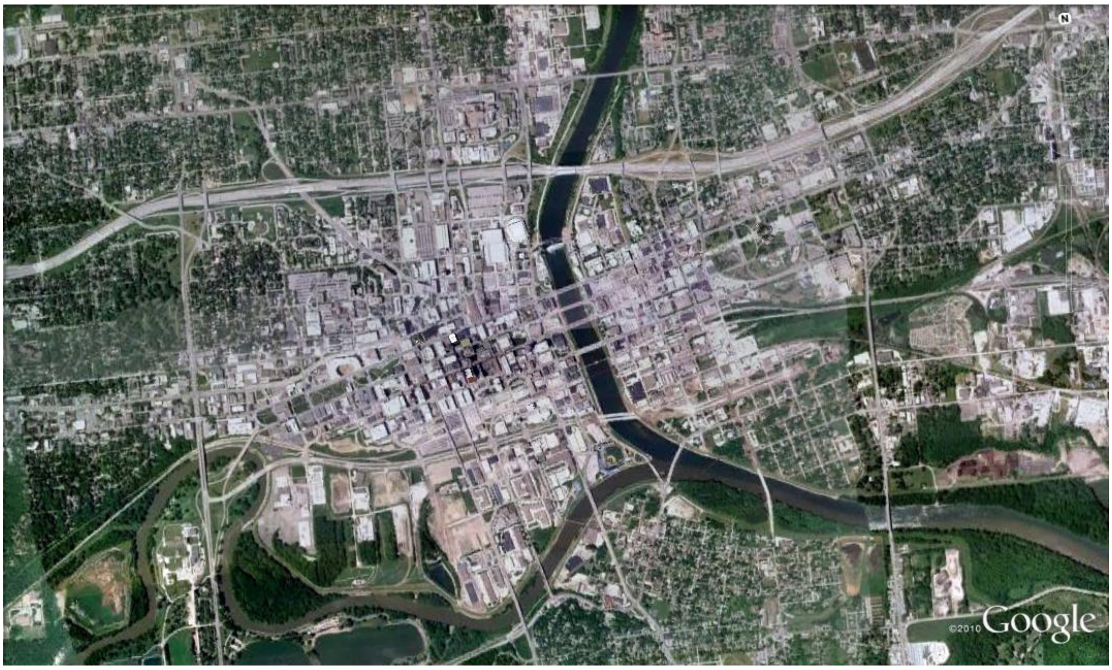


Fig. 2-2-1. Google Earth™: Downtown Des Moines

Currently, there are four bicycle rental stations in the downtown area. We will build many more new stations in this city so as to meet the needs of people.

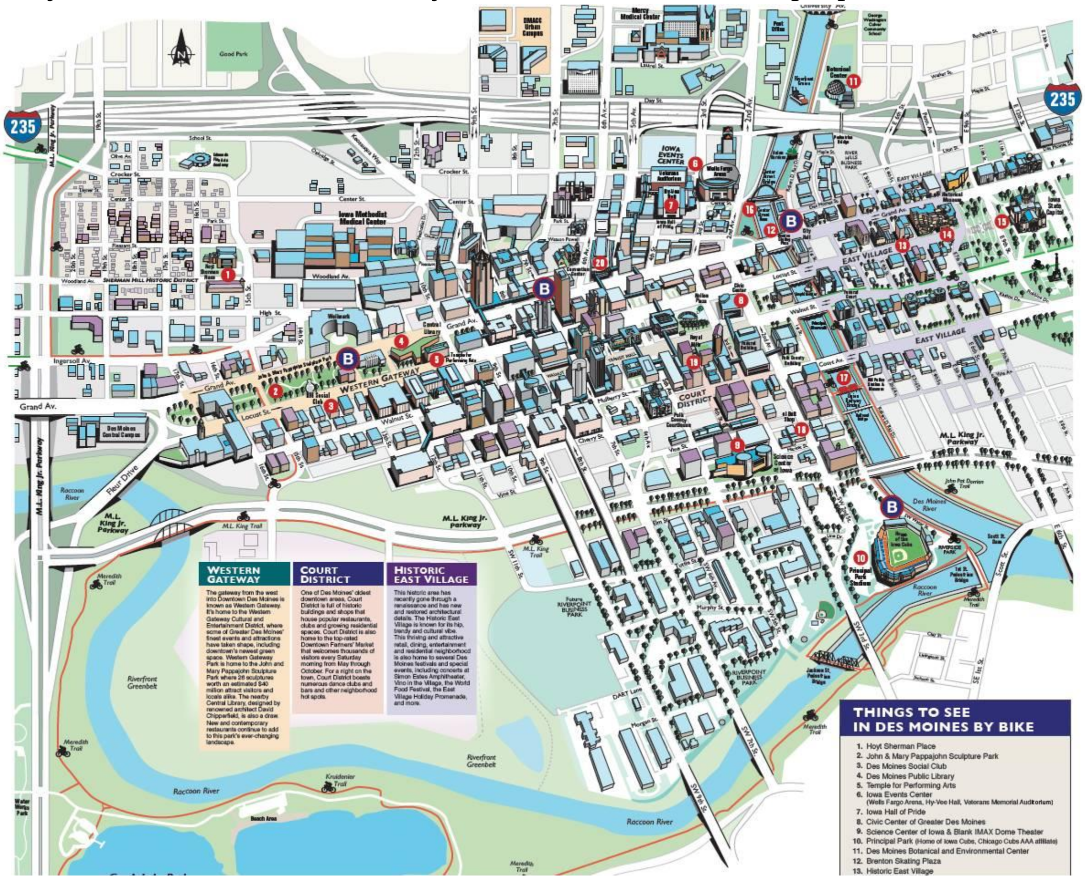


Fig.2-2-2. *B-cycle*: Current Rental Stations of Des Moines

(b.) Chicago, IL

Chicago is the third largest metro city in the United States. Various subway lines, which are crucial to the city's public transportation system, stretch throughout the city. The downtown area of Chicago is an area that is often nicknamed as "the Loop", which refers to an area rounded by several busiest subway lines in the east of the city.

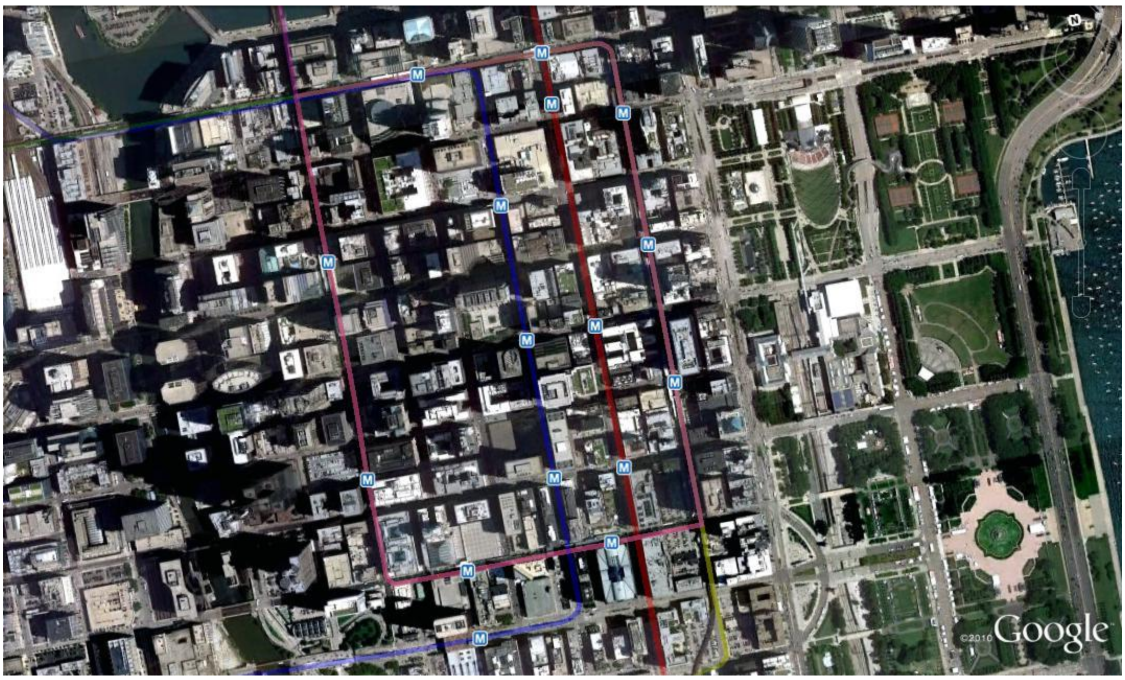


Fig. 2-2-3 *Google Earth*™: "The Loop" in Chicago (Colored Lines Stand for Subway Lines; Blue M Icons Stand for Subway Stations)

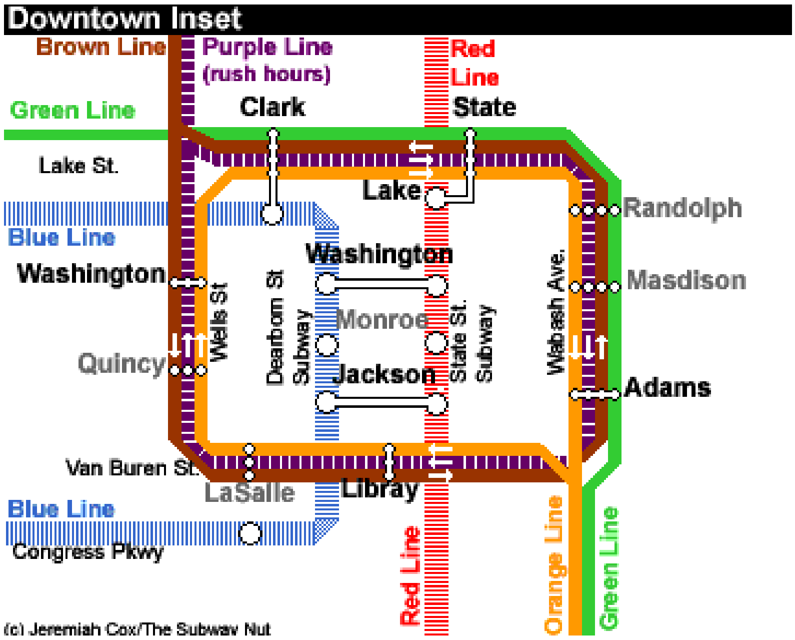


Fig. 2-2-4 *The Subway Nut*: The Loop is a Combination of the Busiest Subway Lines in Chicago

The city of Chicago stretches far beyond the Loop. Most of the residential areas are miles away from the Loop, and are connected to the center by a number of subway lines. Most of the commutations within the Loop are achieved by subway. We decide that bicycle stations should serve as a supplement to the subway system (e.g. Bike + Ride). Currently, there are five bicycle rental stations, two of which are out of use; one is far away from city center, which cuts the actual number of bicycle stations in use to two. We will analyze the transport usage and subway system of this city to see where new rental stations should be built.

(c.) Denver, CO

Denver is the largest city in the state of Colorado. More than twenty existing bicycle rental stations have already covered most of the city. The dense distribution of existing bicycle stations will have a great impact on the future plan of the bicycle rental program in the future.

Fig. 2-2-5. *Big Green Boulder*: A Partial Map of the Bicycle Rental Stations in Denver

There is a unique and well-developed network of light-rail in the city center of Denver. This undoubtedly should be taken into consideration when planning the 5-year bike-sharing program. We will take the light-rail system of Denver into consideration when placing new bicycle stations and lanes.

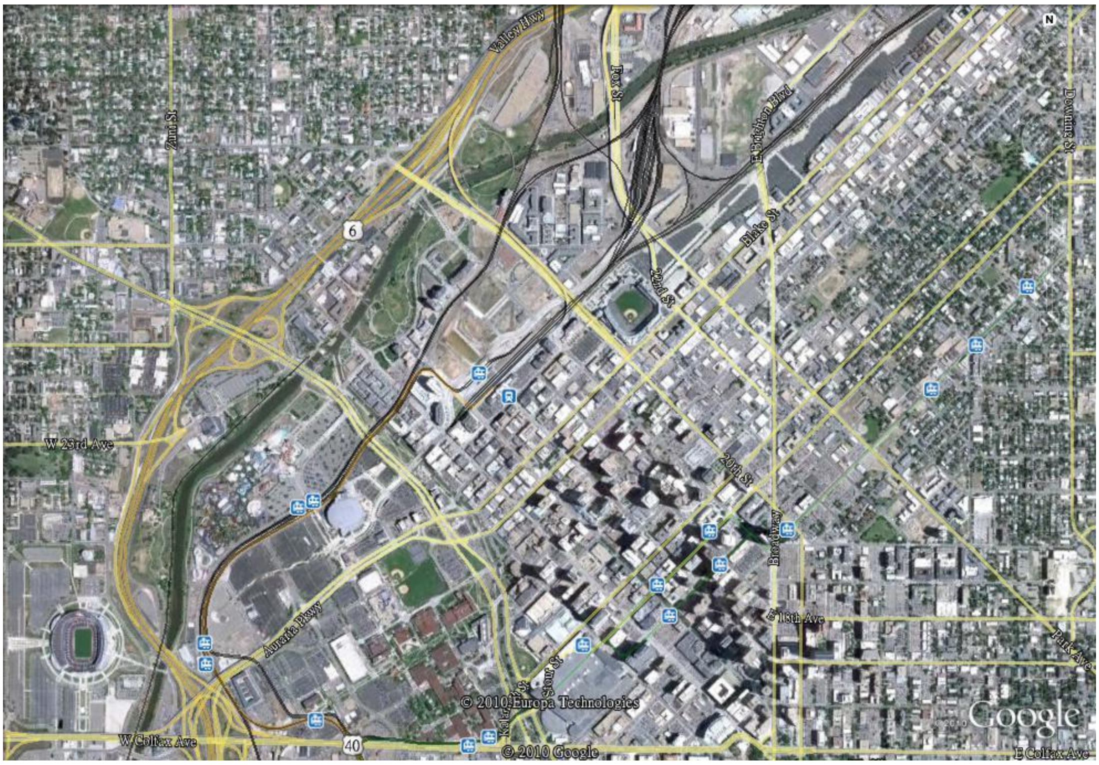


Fig. 2-2-6 *Google Earth*™: Downtown Denver (Blue Icons Stand for Light-rail Stations)

3. Methods

3-1 Basic Ideas and Concerns of Modeling

We use a flow diagram to streamline our modeling process.

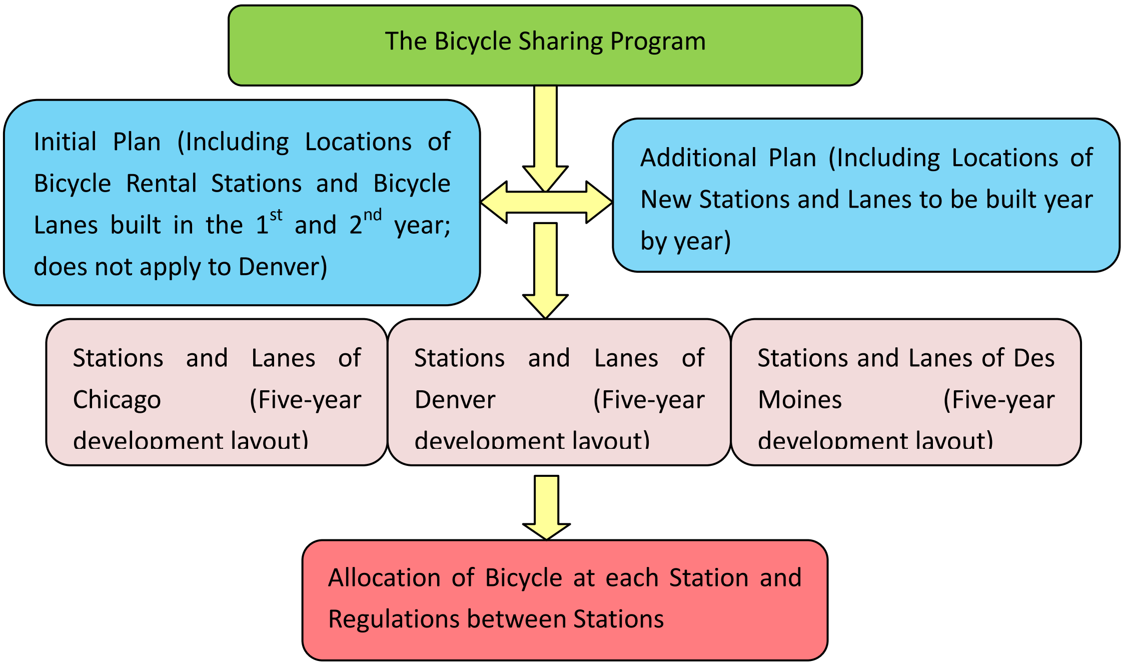


Fig. 3-1-1. Flow Diagram of Modeling

(1.) Initial Plan for Cities

The initial plan is to be implemented in the first two years of the five-year program, aiming to improve the current bicycle-sharing system in the three cities so as to meet the basic needs of the citizens.

This plan consists of two parts – (1.) building new stations, and (2.) constructing new bike lanes.

(a.) Building new Stations

Unlike those in Denver, bicycle stations in Chicago and Des Moines are far from enough (four in Des Moines, two in Chicago). Therefore, the first step of the initial plan is to construct a fundamental network of rental stations in Des Moines and Chicago.

We need to find out the ideal places for bicycle stations. This is mostly affected by different values of significance of the buildings and facilities nearby. We decide to calculate the values of significance of buildings in an area, and then find out the best point to place the rental station according to these values. It is reasonable to place the rental stations closer to buildings with larger values significance than those with smaller values.

(i.) Calculating the Significance of Buildings and Facilities

Quantizing the significance of different buildings and facilities without personal preference is implausible. In order to successfully and scientifically determine the values of significance of buildings and facilities which directly affects the location of rental stations, we decide to apply an analytic method invented by T. L. Saaty, which is a part of the famous **analytic hierarchy process (AHP)**[1].

Suppose we need to build one bicycle station in an area with n major buildings ($n \in N^*$). We assign different significance values to different types of buildings and facilities. Let s_i denote the value of significance for the i-th building (i=1, 2, … , n).

We compare the buildings in the area with each other by finding out the ratio (m_{ij}) of the significance of the i-th building to the j-th building:

$$m_{ij} = \frac{s_i}{s_j}$$

When discussing the ratio of s_i to s_j, we apply **Saaty's 9-point importance scale** to determine the value of m_{ij}. Saaty's 9-point scale features numerical grading of significance comparison.

Value of m_{ij} or s_i:s_j	What the value stands for
1	The i-th and the j-th buildings/facilities are of the same significance.
3	The i-th is relatively more significant than the j-th.
5	The i-th is clearly more significant than the j-th
7	The i-th is obviously more significant than the j-th.
9	The i-th is undoubtedly more significant than the j-th.
2, 4, 6, 8	Levels in between

Tbl. 3-1-1: Saaty's 9-point importance scale with description

When we finish the process, we can get n^2 results (m_{11} to m_{nn}) with which we build matrix M, an $\boldsymbol{n \times n}$ **pairwise comparison matrix** (here also a positive reciprocal matrix)

$$M = \begin{pmatrix} m_{11} & m_{12} & m_{13} & \cdots & m_{1n} \\ m_{21} & m_{22} & m_{23} & \cdots & m_{2n} \\ m_{31} & m_{32} & m_{33} & \cdots & m_{3n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ m_{n1} & m_{n2} & m_{n3} & \cdots & m_{nn} \end{pmatrix} = \begin{pmatrix} \frac{s_1}{s_1} & \frac{s_1}{s_2} & \frac{s_1}{s_3} & \cdots & \frac{s_1}{s_n} \\ \frac{s_2}{s_1} & \frac{s_2}{s_2} & \frac{s_2}{s_3} & \cdots & \frac{s_2}{s_n} \\ \frac{s_3}{s_1} & \frac{s_3}{s_2} & \frac{s_3}{s_3} & \cdots & \frac{s_3}{s_n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \frac{s_n}{s_1} & \frac{s_n}{s_2} & \frac{s_n}{s_3} & \cdots & \frac{s_n}{s_n} \end{pmatrix}$$

where

$$m_{ij} = m_{ji}^{-1}$$

If we were perfectly rational and reasonable when we determine these values, we should have

$$a_{ij} \cdot a_{jk} = a_{ik}$$

For example, if the value of significance of an office building is twice that of a residential site, while the value of significance of a subway station is three times that of the office building, then the ratio of $s_{\text{subway station}}$ to $s_{\text{residential site}}$ should be six. Ideally, every ratio that appears in M should follow this rule – in that case the matrix M attains **conformity**. With the **feature vector** about the **maximum positive eigenvalue** of the conformity matrix, we can determine, from those previously assigned pairwise comparison values, the significance of every building/facility in the area.

It is, however, conceivable that this would not be the usual case, since different people have different judgments for any two of these factors, and that those judgments may not be perfectly interrelated numerically. To ensure the accuracy of the final result, we have to limit the inconformity of the matrix to a certain level; therefore we have to perform a series of tests on the matrix and make adjustments thereafter. Later on in Chapter 3-2, we will provide details of these tests.

(ii.) Finding the Best Location of Rental Stations by Weighing Distance and Significance

(iii.)Let B represent a possible spot to locate the station and r_i the distance from B to the i-th building. At the best location of the bicycle station, (B^*), the parameter

$$G = \sum_{i=1}^{n} s_i r_i$$

should reach a minimum. In that case, the distance between the bicycle station and each building is weighted by the different significance of buildings/facilities. For example, in a certain area, the bicycle station should be built closer to a busy subway station or a popular tourist attraction (say, a museum), whose s_i values are relatively larger, than an ordinary bus stop or an unused office building, whose s_i values are small.

(b.) Constructing new Bike Lanes

Rome was not built in a day, just as bicycle lanes have to be constructed step by step. To make full use of existing lanes and to work out an optimized plan for bike lane construction are also among the concerns of a well-developed bicycle renting program. Some cities may already have quite a few lanes while others only have one or two lanes that run **around** the city center (like those in Des Moines and Chicago). These lanes do not help much if most of the stations are set up inside the city center (as presented later in the application of model). Therefore, constructing new lanes **inside** the city is necessary. It would be even better if these new lanes can be connected to those existing ones, thus forming a consistent network that will benefit more people.

We can tell that, at the beginning of the plan, customers have to walk a short distance with their bicycles before they reach the bicycle lane. To maximize the efficiency of those lanes, we have to carefully decide where to construct new lanes in the initial plan so that the total distance that the renters have to walk with their bicycles is minimized, i.e. we should find a number of existing streets and roads such that the extent to which all the stations diverge from these streets is the smallest.

These all lead to a simple method called **Linear Regression (with Least Squares)[2]**, which involves finding a regression (straight) line of a group of points on an xOy-plane.

Suppose we have set up a plane rectangular coordinate system on the map, and determined the locations of the n bicycle stations (B_1, B_2, … , B_n) with coordinates (x_1, y_1), (x_2, y_2), … , x_n, y_n). Let $\hat{y}_i$ represent the y-coordinates of all the points on the regression line that correspond with points (x_i, y_i), and let $Q=\sum_{i=1}^{n}(y_i-\hat{y}_i)$. The regression line is one that makes Q reach its minimum value. In this model, minimizing the value of Q means to minimize the total distance between the stations and the bicycle lanes, thus reducing the "walking distance" of renters.

Using least squares, the regression line for these points is given by

$$y=\hat{a}+\hat{b}x$$

where

$$\hat{b}=\frac{\sum_{i=1}^{n}(x_i-\bar{x})(y_i-\bar{y})}{\sum_{i=1}^{n}(x_i-\bar{x})^2}$$

$$\hat{a}=\bar{y}-\hat{b}\bar{x}$$

The three cities that we analyzed all have streets that are very close to straight lines.

We can fit the regression lines fit them into actual streets, and we will be able to figure out which of the streets, if equipped with bicycle lanes, will provide the best accessibility for renters.

There is yet another thing – minimizing the walking distance does not necessarily mean utmost convenience for all renters, since these points may not display a very strong linear relationship, and may diverge drastically from the regression line. Therefore we decided to group the points so that each group demonstrates strong linear relationship. Thus the outcome consists of more than a single bicycle lane – a good start for building up a bicycle lane network. Meanwhile, to ensure a better connection with existing networks, adjustments will be manually made to link these new lanes with older ones.

(2.) Additional Plan

Since the demand for bicycles grows year by year, the city will gradually "outgrow" the initial plan, and we need to add new bicycles and lanes to the program year by year. The additional plan aims to make the bike-sharing program in the three cities more mature, and will be implemented over a three-year period (for Denver, five, since the city does not need the two-year initial plan)

(a.) Adding new Stations

We will not bring any changes to the number of bicycles at each station. Instead, more stations will be built to shorten the distance that people need to walk to access the rental stations.

Here we adapt one part of the famous **Central Place Theory**[3][4] to suit our model. At the end of the initial plan, we have a number of bicycle stations serving customers in the city. Each station has its specific circular service area, called the **range**. The initial value of the radii of these ranges should be the smallest value such that the bicycle stations built during the initial plan can cover the whole downtown area. As the demand for bicycle rentals increases, the city embarks on the additional plan to determine the locations of new stations, and then the range of each station should decrease. The percentage of such decrease should be related to the 30% rise in bicycle demand. Suppose that each bicycle rental station offers bike-sharing service for citizens that live within a circle centered at the station with service radius R.

Fig. 3-1-2. *Google Earth*™: A Brief Demonstration of Service Radius, R (All Circles can together cover every place of this part of the city center)

According to assumption (6.), the demand for bicycles will grow by 30% at every place in the three cities annually. Since the number of bicycles at each station will remain unchanged, the number of people that each station can serve will also be similar to that of the previous years. Thus, when new stations are added to the program, there must be:

(1.) A 30% per year growth in the total number of bicycles available, and

(2.) A $\left(1-\frac{1}{1+30\%}\right)\times 100\%$ per year decrease in the range of each station.

According to assumption (7.), all the stations in the same city share the same range in a certain year. Let R(0) denote the service radius of each bicycle rental station just after the initial plan is fulfilled (for Denver, the service radius at the beginning of the five-year program). Since the service area will decrease by $\left(1-\frac{1}{1+30\%}\right)\times 100\%$ each year, the service radius, R, will decrease annually by

$$\Delta\% = \left(1-\sqrt{\frac{1}{1+30\%}}\right)\times 100\% \,.$$

Therefore, the service radius of each station after n years, which is denoted by R(n), is given by

$$\mathrm{R(n)} = \mathrm{R(0)} \times \left[\left(1-\sqrt{\frac{1}{1+30\%}}\right)\times 100\%\right]^{\mathrm{n}}$$

Since R is the smallest radius that enables all the service areas to cover the whole downtown, when R is decreased each year, some areas of the city center will no longer be within the coverage of rental stations. New bicycle stations will be built in these uncovered areas.

(b.) Adding new Lanes

When new stations are constructed, we need to see if there are any stations far away from existing bicycle lanes. If there are such stations, then we should build new lanes to connect these stations with existing lanes. If all stations are no more than 100-metres away from existing lanes, then no extra lanes will be needed.

(3.) Allocation and Regulation of Bicycles between Rental Stations

Virtually nobody could expect that the number of bicycles at each station remains the same at the end of each day. Since the total number of bicycles at the station will change, it is necessary to transport bicycles between stations in order to maintain a balanced service of bicycles. Thus, the number of bicycles to place at each station should be carefully planned so that we can minimize the need to transport bicycles between stations.

In order to estimate the number of bicycles going out of and coming into the station every day, we should predict the possible demand of bicycles at each station according to the properties and functions of different buildings/facilities nearby.

We apply the **Markov Chain Model**[5] the change in total number of bicycles at each station. Suppose there are k bicycle stations in a certain area ($k \in N^*$) numbered as 1, 2, 3, …, k. Bicycles can run freely between any two of the stations in this area. Suppose the initial number of bicycles at each station is $a_i(0)$ (i=1, 2, …, k). Thus, the initial status of all bicycle stations can be expressed as follows with the matrix A(0).

$$A(0) = (a_1(0) \quad a_2(0) \quad \ldots \quad a_k(0))$$

Assume that in one hour, the probability that a bicycle runs from station i to station j is p_{ij}. If $i=j$, then p_{ij} stands for the probability that a bicycle remains at the original place. Thus, after one hour, the numbers of bicycles at each station are

$$a_1(1) = a_1(0)p_{11} + a_2(0)p_{21} + \cdots + a_k(0)p_{k1} = \sum_{i=1}^{k} a_i(0)p_{i1}$$

$$a_2(1) = a_2(0)p_{12} + a_2(0)p_{22} + \cdots + a_k(0)p_{k2} = \sum_{i=1}^{k} a_i(0)p_{i2}$$

$$\vdots$$

$$a_k(1) = a_k(0)p_{1k} + a_2(0)p_{2k} + \cdots + a_k(0)p_{kk} = \sum_{i=1}^{k} a_i(0)p_{ik}$$

If we build a matrix P with values of p_{ij},

$$P = \begin{pmatrix} p_{11} & p_{12} & p_{13} & \cdots & p_{1k} \\ p_{21} & p_{22} & p_{23} & \cdots & p_{2k} \\ p_{31} & p_{32} & p_{33} & \cdots & p_{3k} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ p_{k1} & p_{k2} & p_{k3} & \cdots & p_{kk} \end{pmatrix}$$

Then the status of each bicycle station after an hour, if represented by A(1), can be written as

$$A(1) = (a_1(1) \quad a_2(1) \quad a_3(1) \quad \ldots \quad a_k(1)) = A(0) \times P$$

Assume that P is only affected by the different functions of buildings and does not change overtime. Then the status of bicycle stations can be regarded as a regular chain where the status after n hours, A(n), can be found by

$$\mathrm{A(n) = A(0) \times P^n}$$

According to the properties of a regular chain, we know that, given enough time, the change in number of bicycles at each station will become smaller and smaller. That means A(n) has a limit, W, which is not affected by the initial value A(0) but is only related to P:

$$\mathrm{W} = (\mathrm{a_1(final)} \quad \mathrm{a_2(final)} \quad \mathrm{a_3(final)} \quad \cdots \quad \mathrm{a_k(final)})$$

$$\mathrm{W} = \lim_{n\to\infty} \mathrm{A(n)} = \mathrm{A(0)} \times \lim_{n\to\infty} \mathrm{P^n} = \mathrm{A(0)} \times \mathrm{P^\infty}$$

According to the definition of limits, multiplying W once again by P does not affect its value:

$$\mathrm{W = W \times P}$$

W is the steady state of the Markov Chain. We can minimize the need to transport bicycles between stations if we allocate the bicycles at each station according to the results of matrix W, i.e., put a_i (final) bicycles at the *i*-th station. If bicycles are allocated this way, we can predict that there will not be a great change in the number of bicycles at each station over a period of time.

3-2 Application of the Model

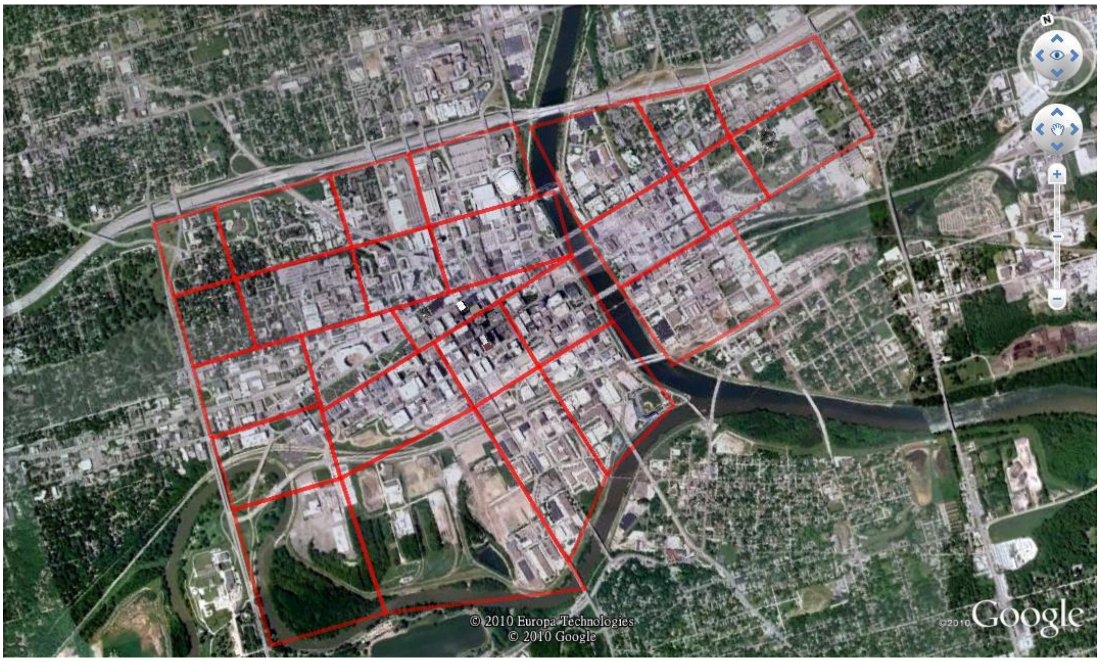
(1.) Initial Plan for Cities

As we have mentioned in the previous paragraphs, the initial plan of building rental stations and lanes basically aims to help the cities that do not have fundamentals of a bicycle sharing program. Therefore, **Denver, a city already equipped with more than 20 stations, is not included in this initial plan.** In the following paragraphs, we will focus on how to arrange initial plans in Des Moines and Chicago to enlarge the coverage of the bicycle-sharing program.

(a.) Building new Stations (in Des Moines and Chicago)

We will show how we determine the locations of stations in downtown Des Moines in the following paragraph so as to give an example of our computation. Detailed reulsts will be given in 4-1.

For convenience of discussion, we divide the city center of Des Moines into small separate sections, such that, while assuring diversity of buildings and facilities in each section, the stretch of each section is slightly beyond “walking distance”, i.e., every major building/facility in a section can easily be accessed on foot, while a bicycle will



be an ideal choice if one wishes to travel from one section to another.

Fig. 3-2-1 *Google Earth*™: Downtown Des Moines divided into sections

Therefore, we decide to place one bicycle station in each area.

Before we start to focus on certain sections, we need to analyze the significance of all types of buildings/facilities within the city center. Basically, there are five sorts of buildings/facilities in downtown Des Moines, namely, office buildings, entertainment facilities, parking lots, tourist attractions and residential sites. We first compare them pairwise to get the initial ratio of significance with the 9-point importance scale.

	Office Building	Entertainment Facility	Residential Site	Parking Lot	Tourist Attraction
Office Building	1	3	2	$\frac{1}{3}$	8
Entertainment Facility	$\frac{1}{3}$	1	$\frac{1}{2}$	$\frac{1}{7}$	5
Residential Site	$\frac{1}{2}$	2	1	$\frac{1}{5}$	7
Parking Lot	3	7	5	1	9

Tourist Attraction	$\frac{1}{8}$	$\frac{1}{5}$	$\frac{1}{7}$	$\frac{1}{9}$	1

Tbl. 3-2-1: Pairwise comparison on the 9-point scale

And then we use the method with which we find the ideal location of bicycle stations in Des Moines to place stations in Chicago. The results are marked on Fig. 3-2-1.

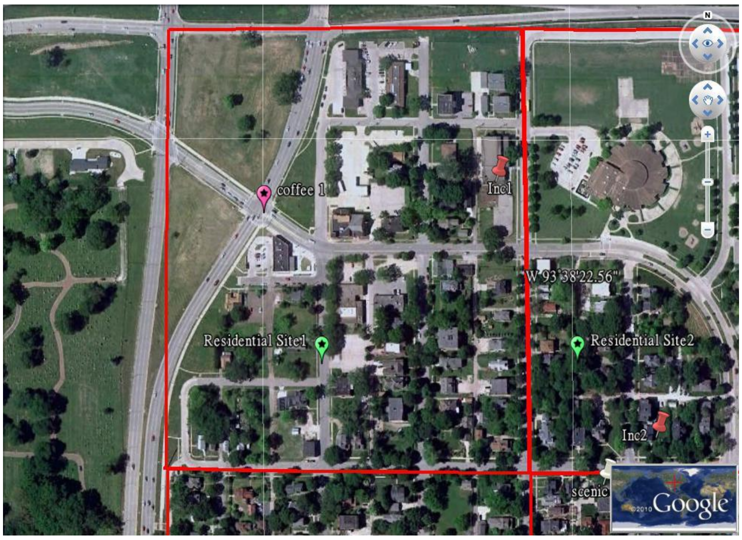


Fig. 3-2-2: *Google Earth*™: Example Section

In the section above, there are three major buildings/facilities: an office building (the red icon, which is the first building), a café (the pink icon, which is the second building) and a residential site (the green icon, which is the third building).

From the table above, we build the following pairwise comparison matrix for the three buildings in the area.

$$\mathrm{M} = \begin{pmatrix} 1 & 3 & 2 \\ \frac{1}{3} & 1 & \frac{1}{2} \\ \frac{1}{2} & 2 & 1 \end{pmatrix}$$

Obviously, this is not a conformity matrix. We will perform several tests on M to see if its inconformity is small enough to be allowed. First of all, we need to find out the maximum eigenvalue and the feature vector of M via the following transformation:

$$\mathrm{M} = \begin{pmatrix} 1 & 3 & 2 \\ \frac{1}{3} & 1 & \frac{1}{2} \\ \frac{1}{2} & 2 & 1 \end{pmatrix}$$

↓ normalization along the column

$$\mathrm{M}_1 = \begin{pmatrix} 0.545 & 0.500 & 0.571 \\ 0.182 & 0.167 & 0.143 \\ 0.273 & 0.333 & 0.286 \end{pmatrix}$$

↓ addition along the row

$$M_2 = \begin{pmatrix} 1.616 \\ 0.492 \\ 0.892 \end{pmatrix}$$

↓ normalization

$w = \begin{pmatrix} 0.539 \\ 0.164 \\ 0.297 \end{pmatrix}$, which is the feature vector of M about the maximum eigenvalue

↓ multiply M by w

$$M \times w = \begin{pmatrix} 1.625 \\ 0.4921 \\ 0.8945 \end{pmatrix}$$

And then we know that the maximum eigenvalue of M is:

$$\lambda = \frac{1}{3}\left(\frac{1.625}{0.539} + \frac{0.4921}{0.164} + \frac{0.8945}{0.297}\right) = 3.009$$

Saaty's theory states that the vector *w* can be directly viewed as the weight value that can demonstrate the values of significance of the office building, the café and the residential area. However, we need to test its conformity beforehand.

Saaty defined a **consistency index (CI)**

$$CI = \frac{\boldsymbol{\lambda} - n}{n - 1}$$

and a **random conformity index (RI)** which, together with CI, determines whether the inconformity of M is allowed. The random index of a n×n matrix is presented in Tbl. 3-2-2

n	1	2	3	4	5	6	7	8	9
RI	0	0	0.58	0.90	1.12	1.24	1.32	1.41	1.45

Tbl. 3-2-2: Random index

Obviously, the random index of M, a 3×3 matrix, should be 0.58; the CI of M is $CI = \frac{\boldsymbol{\lambda}-n}{n-1} = \frac{3.009-3}{3-1} = 4.5 \times 10^{-3}$. The ratio of CI to RI is

$$\frac{CI}{RI} = \frac{4.5 \times 10^{-3}}{0.58} = 0.08 < 0.1$$

Saaty proposes that the inconformity of M can be neglected if $\frac{CI}{RI} < 0.1$. Therefore, the inconformity of M is slight enough to be neglected. *w*, the feature vector of M, , can serve as the weight vector. Since

$$w = \begin{pmatrix} 0.539 \\ 0.164 \\ 0.297 \end{pmatrix},$$

the values of significance of the three buildings/facilities in this area are:

$$s_{\text{office building}} = s_1 = 0.539$$

$$s_{\text{café}} = s_2 = 0.164$$

$$fs_{\text{residential site}} = s_3 = 0.297$$

In finding the best point to place the new rental station, we will weigh these values with their distance to the station. First we set up a plane rectangular coordinate system xOy in the area.

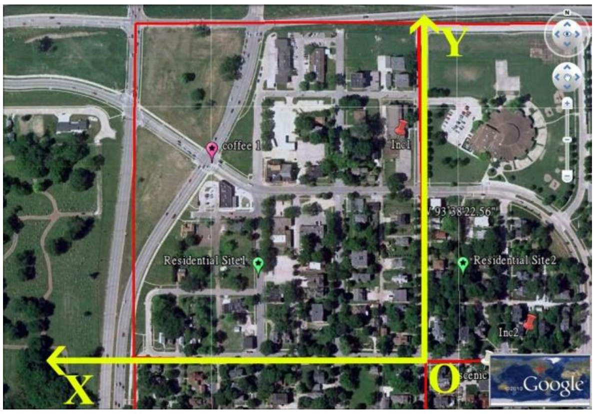


Fig. 3-2-3: *Google Earth*™: Coordinate system xOy

The longitude and latitude of the three buildings/facilities and the origin of xOy are:

O (41°35′29.75″N, 93°38′24.3″W)

Office Building (41 °35′38.55″ N, 93 °38′25.77″ W)

Café (41 °35′37.63″ N, 93 °38′35.46″ W)

Residential Site (41 °35′33.14″N, 93 °38′33.05″W)

Around 40 °N, one second (1″) change in longitude means a 30.82-meter surface distance, and one second (1″) change in latitude, a 20.2867-meter surface distance. If we transfer the longitude and latitude of these points into x-y coordinates, regarding 1-meter as 1 unit length, we have:

O (0, 0)

Office Building (29.8214, 271.216)

Café (226.3991, 242.8616)

Residential Site (177.5083, 104.4798)

In the previous chapter, we have concluded that the rental station should be placed where the parameter G, given by

$$G = \sum_{i=1}^{n} s_i r_i$$

reaches a minimum. Suppose that the ideal location of the new rental station is found at (x_0, y_0), so r_i is given by

$$r_i = \sqrt{(x_i - x_0)^2 + (y_i - y_0)^2}.$$

Therefore, the ideal point is found when

$$G = \sum_{i=1}^{n} s_i \sqrt{(x_i - x_0)^2 + (y_i - y_0)^2}$$

reaches a minimum, the ideal point is found. Eventually, the coordinate of the ideal location of the bicycle rental station is $B^*(29.9214, 271.1798)$, as shown in Fig. 3-2-4.

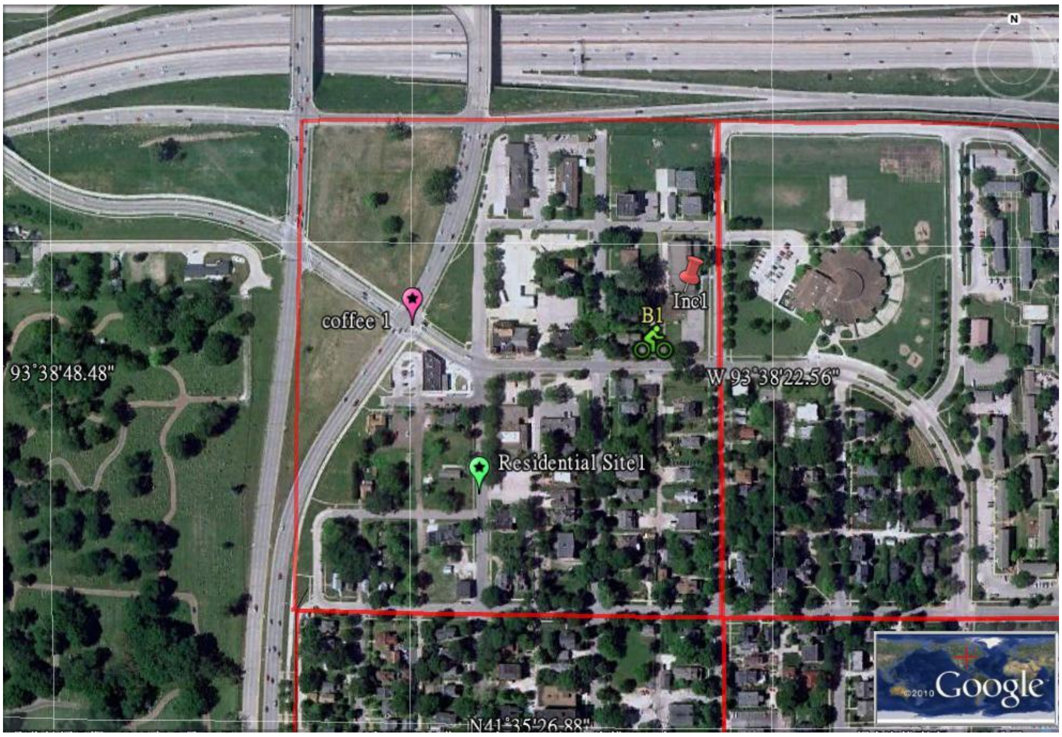


Fig. 3-2-4: *Google Earth*™: Location of new rental station in this section (B1)

With similar method, we can find the best location for the station in all the sections in downtown Des Moines.

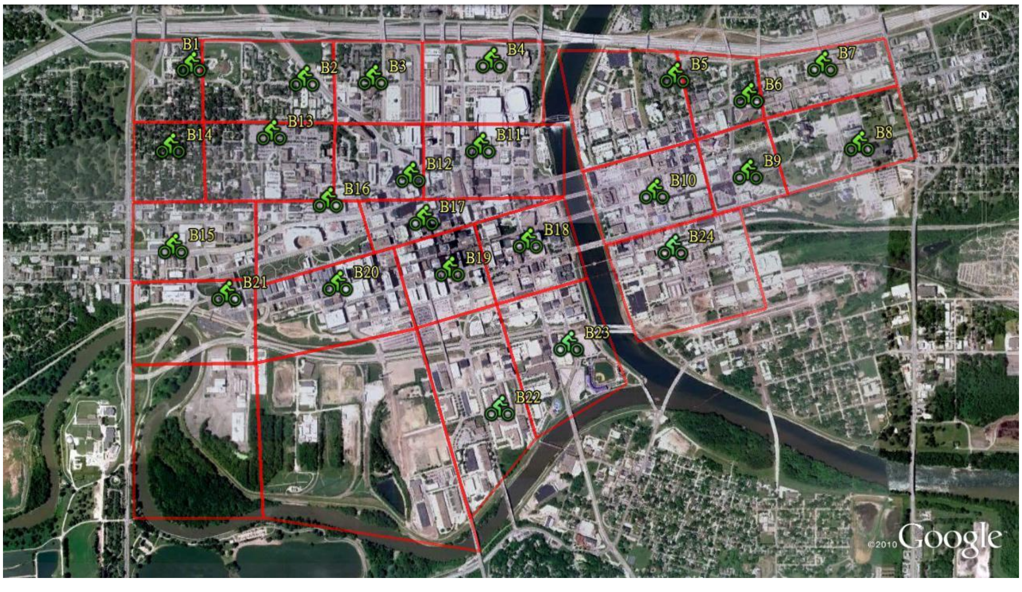


Fig. 3-2-5: *Google Earth*™: New rental stations in Des Moines (green icons)

Note that:

(1.) Two sections (on the bottom left-hand corner) are not equipped with stations because most area of these sections is covered by public parks.

(2.) As we have mentioned in the previous chapters, the locations of parking lots should be a crucial concern when we decide where to place bike stations. Since most people park their cars at public parking facilities during the day, parking lots are given the highest values of significance among all facilities. Therefore, some of the bicycle rental stations are placed right beside parking lots in downtown Des Moines.

(b.)Bicycle Lane Construction Plan

It would be too good for bike riders to have all of the streets converted into bicycle lanes – which is not possible. However, relying merely on the few existing lanes would not suffice if the city is to build around twenty rental stations. Now we will apply the linear regression model on the three cities. Detailed discussion about Des Moines serves as an example. Detailed results will be given in 4-1.

When building lanes in Des Moines, basically we group the stations so that each set of points demonstrate a relatively stronger linear relationship. Fig. 3-2-6 shows how it looks when rental stations B1 through B4 and B11 through B14 have been completed in Des Moines:

Fig. 3-2-6: *Google Earth*™: Rental stations B1 through B14

By setting up a plane rectangular coordination system and finding the coordinates of these eight points, we can calculate the formula of the regression line, $y = \hat{a} + \hat{b}x$. Fig. 3-2-7 is the map with the regression line:

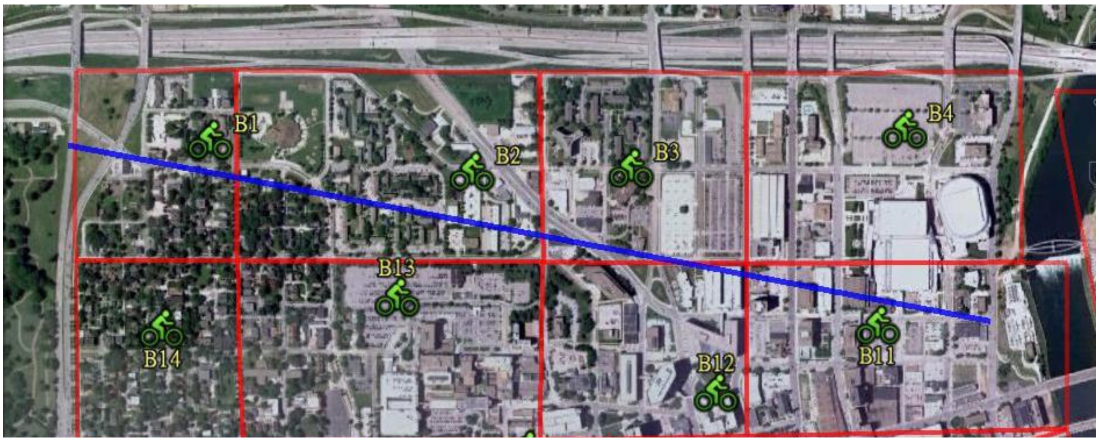


Fig. 3-2-7: *Google Earth*™: Map with regression line

Then we fit this line into actual streets to determine the final plan of the new bicycle lane.

Fig. 3-2-8: *Google Earth*™: Adjusted bicycle lane

Using the same approach, we can find regression lines and adjusted construction plan for the whole downtown area.

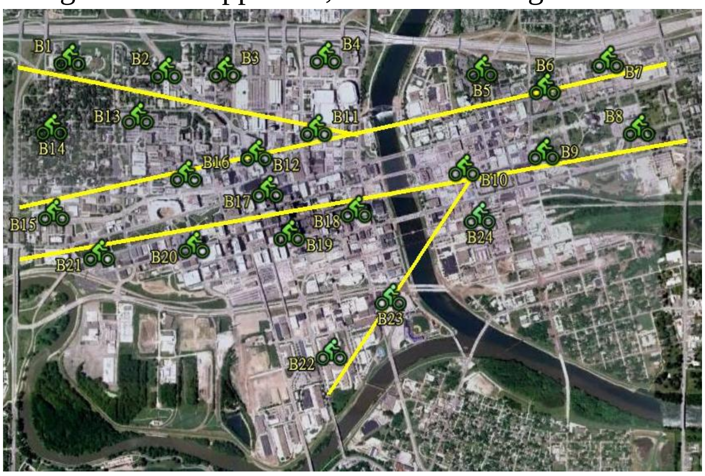


Fig. 3-2-9: *Google Earth*™: Regression lines (yellow) of the four groups of rental stations. We take care to group these stations so that each group demonstrates stronger linear relationship.

After we adjust the construction plan for the city according to the regression lines we have obtained, we will have several separate bicycle lanes stretching throughout the city center. In order to make the bicycle lanes work more efficiently, we need to link them with each other so as to form an integrated network.

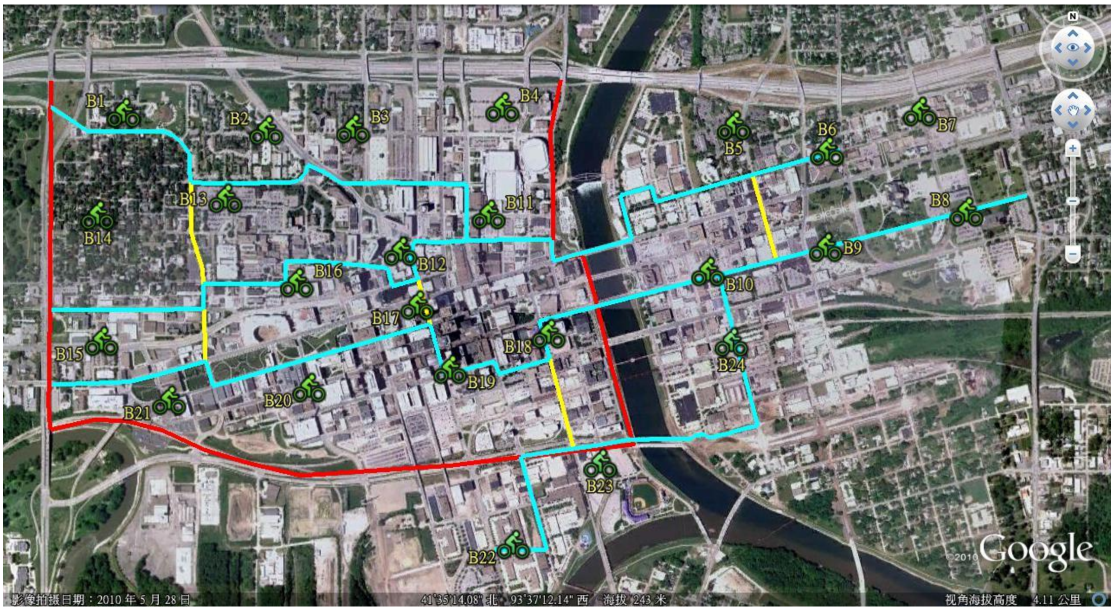


Fig. 3-2-10: *Google Earth*™: Map of all bike lanes of Des Moines, with existing lanes (red), adjusted plan for new lanes (blue and yellow). Blue lines are adaptations of the four regression lines, while yellow lines are manually added linking lanes. Please note that all new lanes are well connected with old lanes.

(2.) Additional Plan

As we have mentioned in the previous paragraphs, we will implement the additional plan to add new stations and lanes to the program so as to meet the growing demands of citizens. In Des Moines and Chicago, the additional plan will be launched in the 3rd year of the program, right after the initial plan is finished; in Denver where no initial plan is implemented, the additional plan will start at the beginning of the 1st year. In the following paragraphs, we will take the additional plan in Des Moines in the 3rd year as an example to show how we work on the problem. For other cities and other periods, we will demonstrate the final results in 4-1.

As we have mentioned, the initial plan in Des Moines will be completed within the first two years of the program. After the initial plan is finished, there will be 24 rental stations in use in the city center of Des Moines. According to our calculation[1], the initial service radius, R(0), of each station is approximately 500.88-m.

[1] We calculate the service radius with Matlab and relevant code will be given in the appendices.

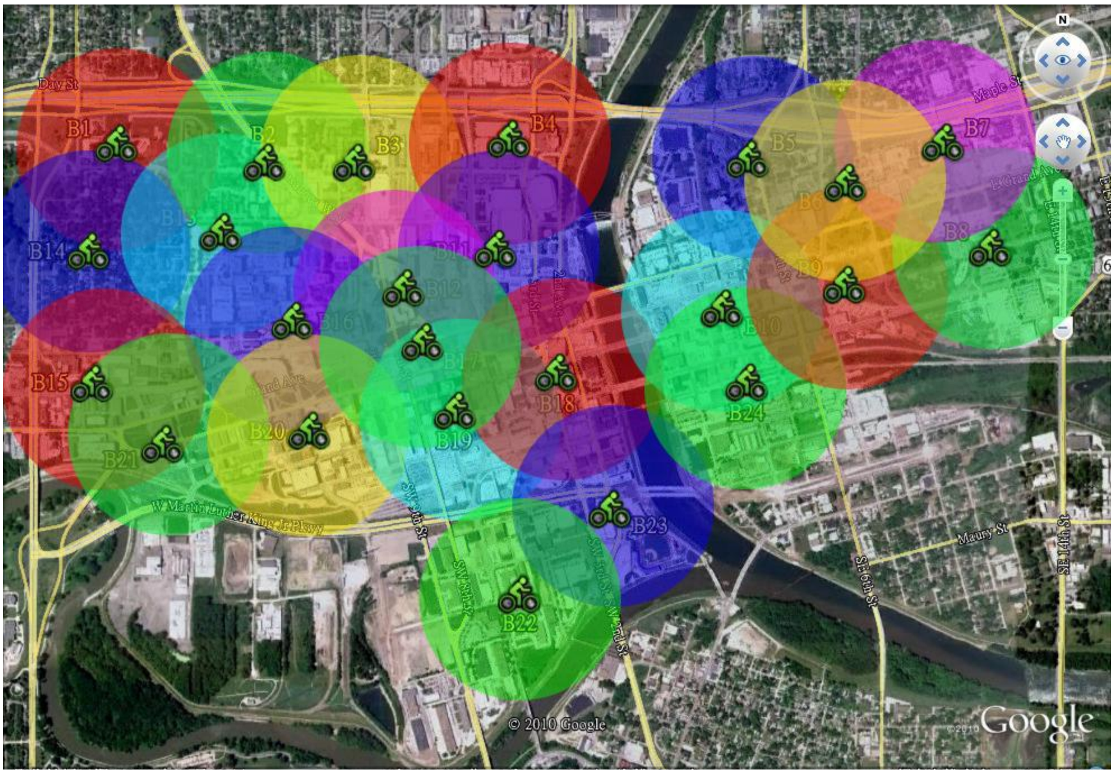


Fig. 3-2-11: *Google Earth*™: Initial range for all stations

When we implement the additional plan in Des Moines in the third year, the service radius should be decreased by $\left(1-\sqrt{\frac{1}{1+30\%}}\right)\times 100\%$, and the new radius, R(1) is given by

$$\mathrm{R}(1)=\mathrm{R}(0)\times\left[\left(1-\sqrt{\frac{1}{1+30\%}}\right)\times 100\%\right]^{1}$$

As we can see from the picture above, if the radius is reduced, then two parts of the city will be kept out of the coverage of the bicycle stations. New bicycle stations should be built in these two “blind areas” to satisfy the citizens. We place one new rental station at the center of each “blind area” to assure an all-round coverage of the program.

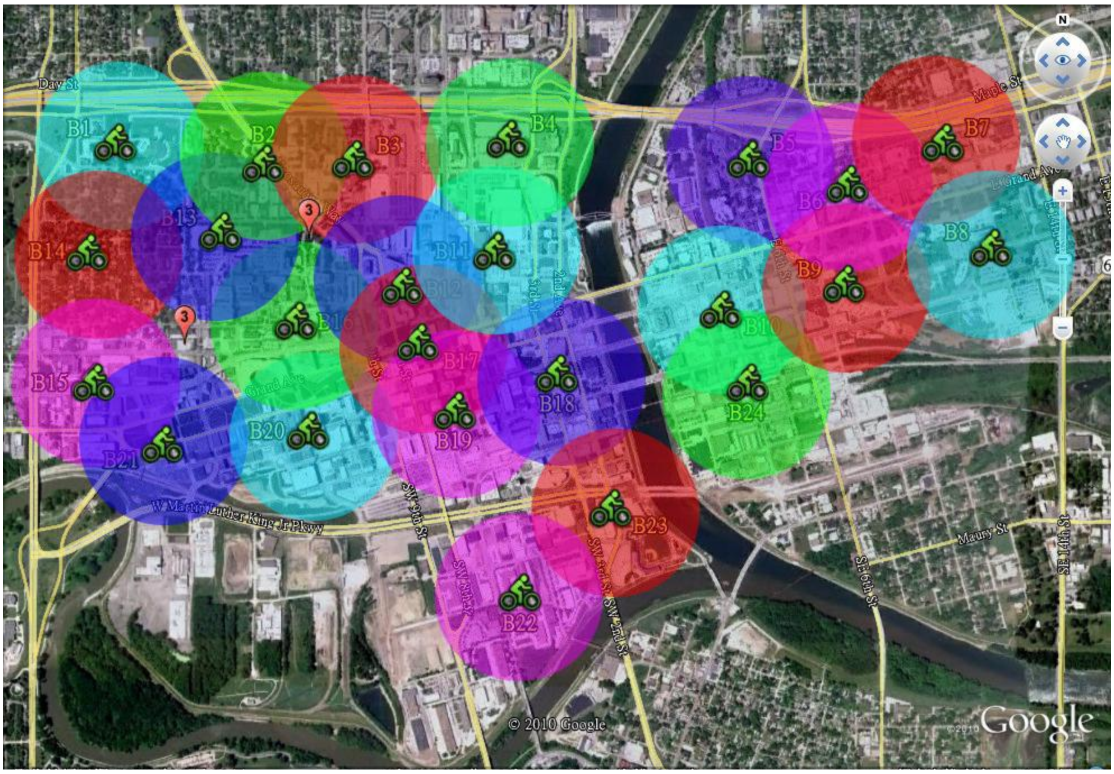


Fig. 3-2-12: *Google Earth*™: "Shrunk" ranges after one year

Locations of the new rental stations are shown in the following picture.

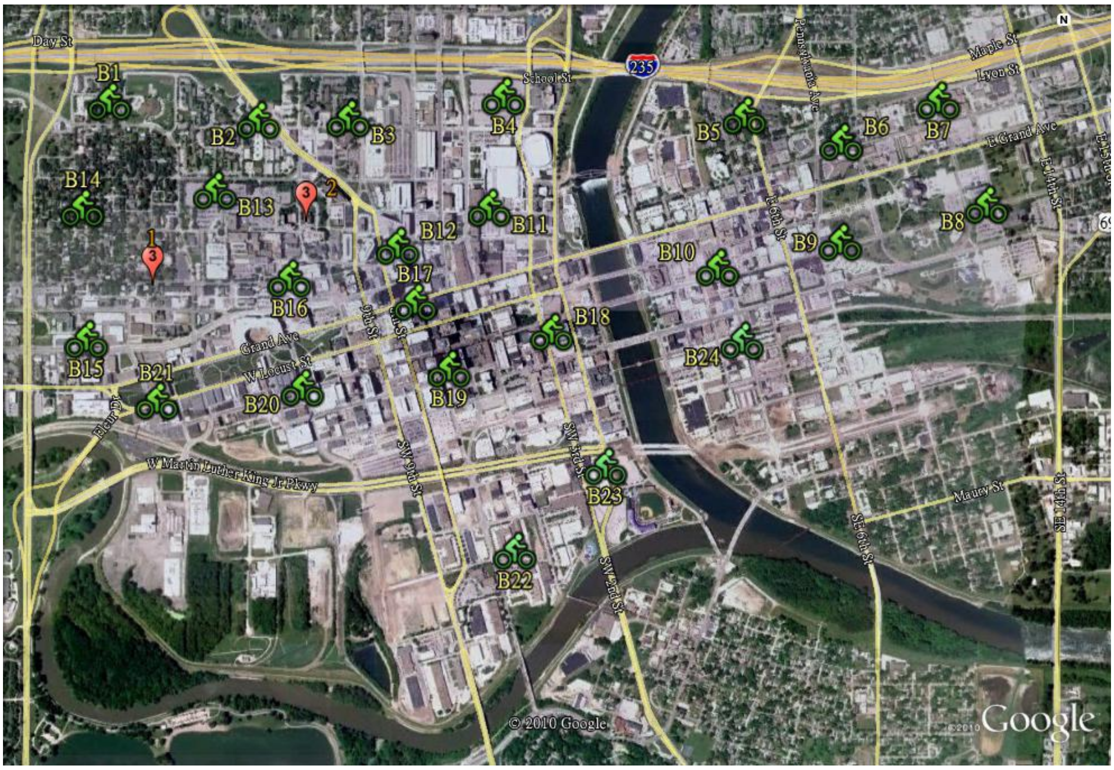


Fig. 3-2-13: *Google Earth*™: Three new rental stations

Since both new stations are within 100-metres to the nearest existing lane, there will be no need to build new bicycle lanes in the third year in Des Moines.

(3.) Allocation and Regulation of Bicycles between Rental Stations

As mentioned in 3-1, we apply the Markov Chain Model to decide how to allocate the bicycles according to different buildings and facilities nearby. We use the bicycle allocation and regulation plan in Des Moines in the second year (after the initial plan is completed) as an example to show our computations.

Just as what we have discussed in 3-1, we will allocate bicycles according to the steady state we achieve with the Markov's chain. Since the limit is only relevant to the matrix P (see 3-1), we need to determine every element in P, that is, p_{ij}.

Obviously, the value of p_{ij} on working days is different from that of work-free days. We will discuss them separately.

(i.) Working Days

According to assumption (4.), 50 percent of bike renters commute to work by bicycle. Since all these trips are considered to be round trips, all these bikes will eventually go back to their original locations at the end of the day. Therefore, we have

$$p_{ij} = 0.5 \ (i = j)$$

According to assumption (5.), the possibility that a renter visits the *i*-th bicycle station is proportional to the number of buildings/facilities near the station. With this method, we can determine the matrix P which applies to the condition on working days.

(ii.) Work-free Days

On weekends and official holidays, people visit tourist attractions or entertainment facilities rather than go to work, so there will be no need to take the working trips into consideration.

With this approach, we will be able to find out the matrix P that applies to the conditions on work-free days.

According to assumption (4.), each station has 10 bicycles on average. Since the limit is irrelevant to the values of $a_i(0)$, the original number of bicycles at the *i*-th station, we can assume that $a_i(0)$ is equal to 10 for every station. Therefore we can calculate the limit, W, for $A(n) = (a_1(n) \quad a_2(n) \quad a_3(n) \quad \dots \quad a_k(n))$.

$$W = \lim_{n\to\infty} A(n) = A(0) \times \lim_{n\to\infty} P^n = A(0) \times P^\infty$$

Now we have two different steady states for working days and work-free days, for which we make two different allocation plans. Shifts between plans should require regulation of bicycles between stations should be made on every Friday evening (or on the last evening before an official holiday) and on every Sunday evening (or on the last day of each holiday).

The final result of bike allocation and regulation of Des Moines in the 2nd year is presented in Tbl. 3-2-3.

Station Number	Bikes needed on working days	Regulations to be made on Fridays	Bikes needed on work-free days	Regulations to be made on Sundays
1	3	receive 1 bike from station 2	4	give 1 bike to station 2
2	8	give 1 bike to station 1	7	receive 1 bike from station 1
3	6	receive 1 bike from station 12	7	give 1 bike to station 12
4	5	receive 6 bikes from station 22	11	give 6 bikes to station 22
5	3	receive 4 bikes from station 22	7	give 4 bikes to station 22
6	2	does not change	2	does not change
7	6	receive 4 bikes from station 22 receive 4 bikes from station 8 receive 3 bikes from station 9	16	give 4 bikes to station 22 give 4 bikes to station 8 give 3 bikes to station 9
8	8	give 4 bikes to station 7	4	receive 4 bikes from station 7
9	5	give 3 bikes to station 7	2	receive 3 bikes from station 7
10	22	give 5 bikes to station 17 give 1 bike to station 20 give 1 bike to station 16	16	receive 5 bikes from station 17 receive 1 bike from station 20 receive 1 bike from station 16
11	15	receive 7 bikes from station 24	22	give 7 bikes to station 24
12	2	give 1 bike to station 3	1	receive 1 bike from station 3
13	8	receive 3 bikes from station 23	11	give 3 bikes to station 23
14	2	does not change	2	does not change
15	11	receive 1 bike from station 15 receive 4 bike from station 19	16	give 1 bike to station 15 give 4 bikes to station 19
16	6	receive 1 bike from station 10 receive 1 bike from station 24 receive 1 bike from station 18 receive 2 bikes from station 19	11	give 1 bike to station 10 give 1 bike to station 24 give 1 bike to station 18 give 2 bikes to station 19
17	11	receive 5 bikes from station 10	16	give 5 bikes to station 10
18	36	give 1 bike to station 16	35	receive 1 bike from station 16
19	8	give 2 bikes to station 16 give 4 bikes to station 15	2	receive 2 bikes from station 16 receive 4 bikes from station 15
20	21	receive 1 bike from station 10	22	give 1 bike to station 10
21	2	does not change	2	does not change
22	21	give 6 bikes to station 4 give 4 bikes to station 5 give 4 bikes to station 7	7	receive 6 bikes from station 4 receive 4 bikes from station 5 receive 4 bikes from station 7
23	16	give 3 bikes to station 13 give 1 bikes to station 15	12	receive 3 bikes from station 13 receive 1 bike from station 15

24	13	give 7 bikes to station 11 give 1 bike to station 16	5	receive 7 bikes from station 11 receive 1 bike from station 16

Tbl. 3-2-3: Final allocation plan

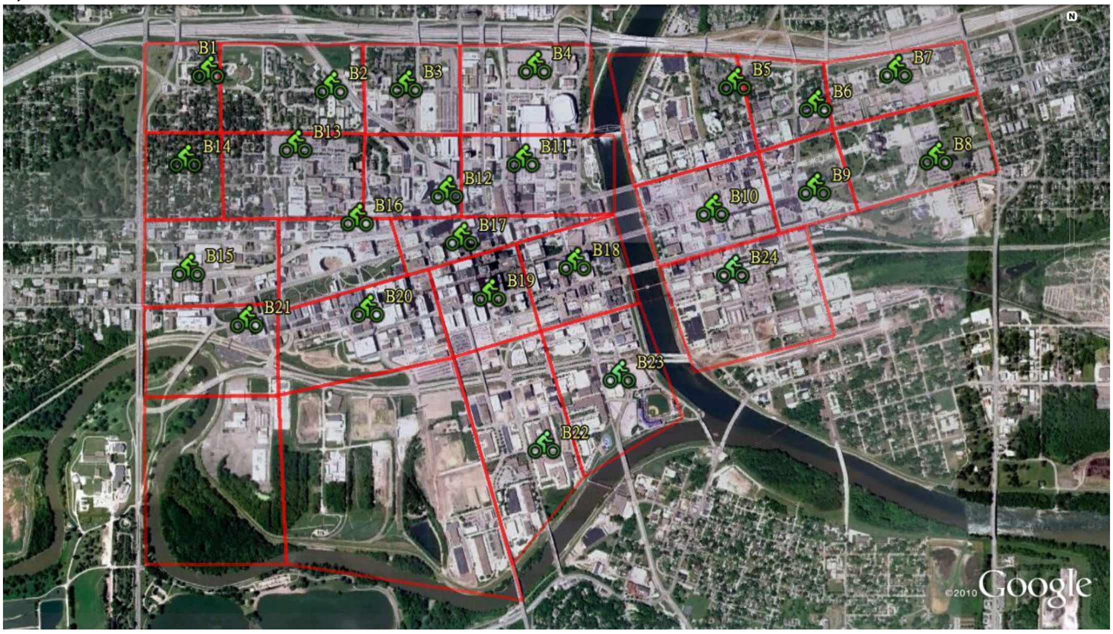
In the years to come, more and more stations will be added to the program. Please note that the allocation and regulations of bikes at new rental stations do not interfere with the plan in the previously-built stations.

4. Discussion

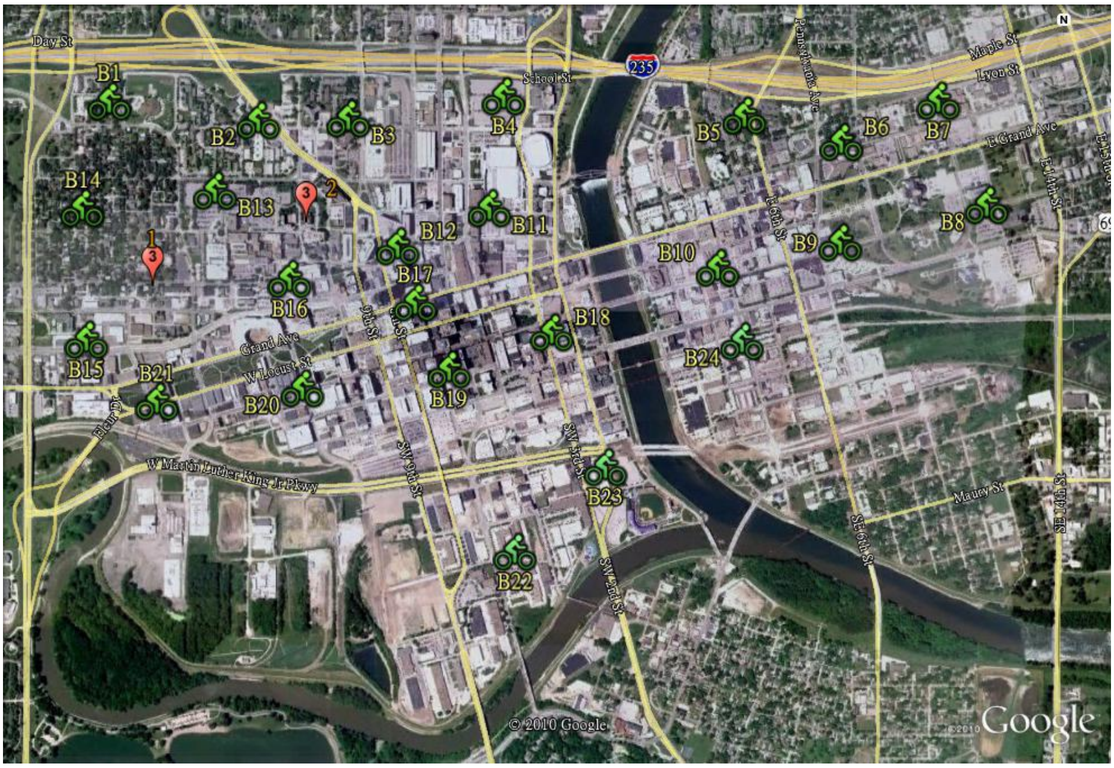
4-1 Results

(1.) Location of Bicycle Stations and Lanes

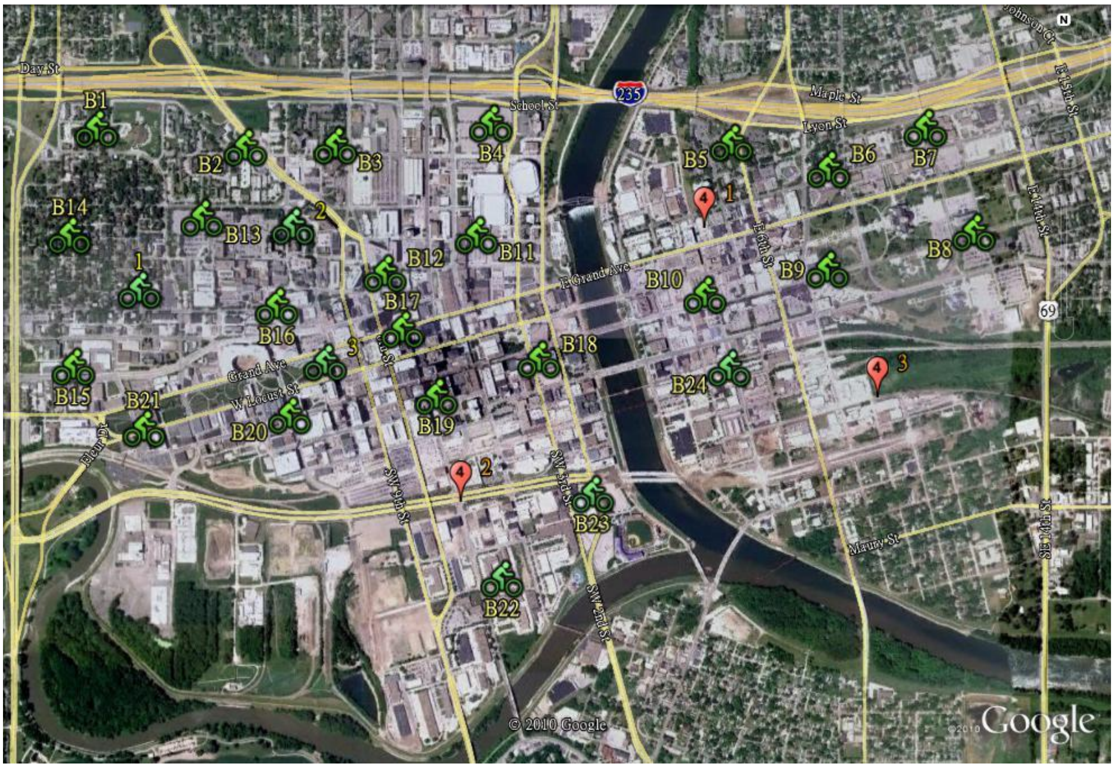
(i.) Des Moines, IA



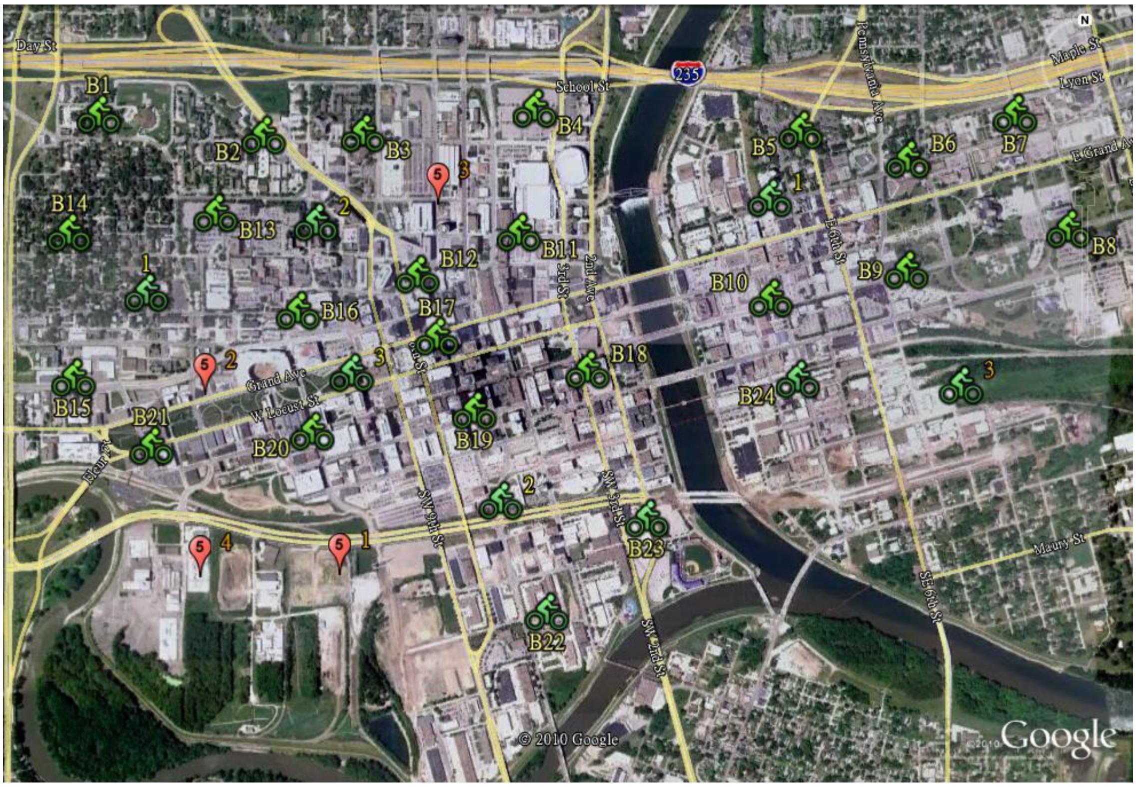
↑ Locations of rental stations in the second year (after the initial plan is completed)



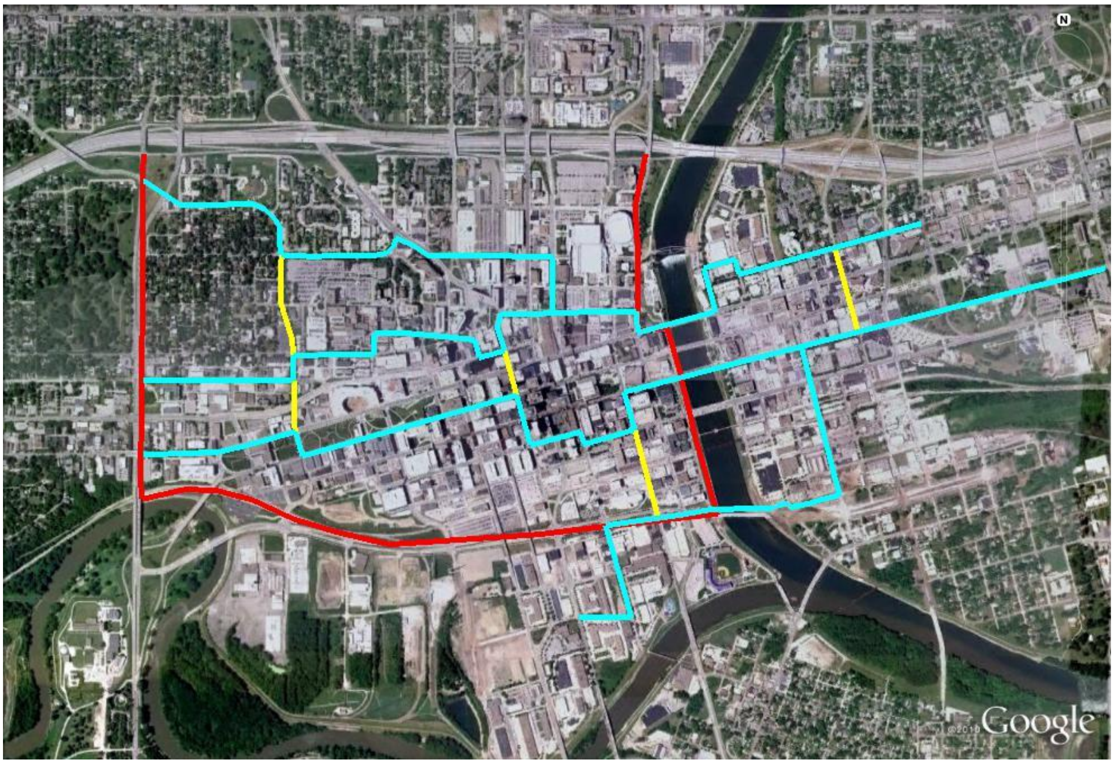
↑ Locations of rental stations in the third year (red icons marked with “3”)



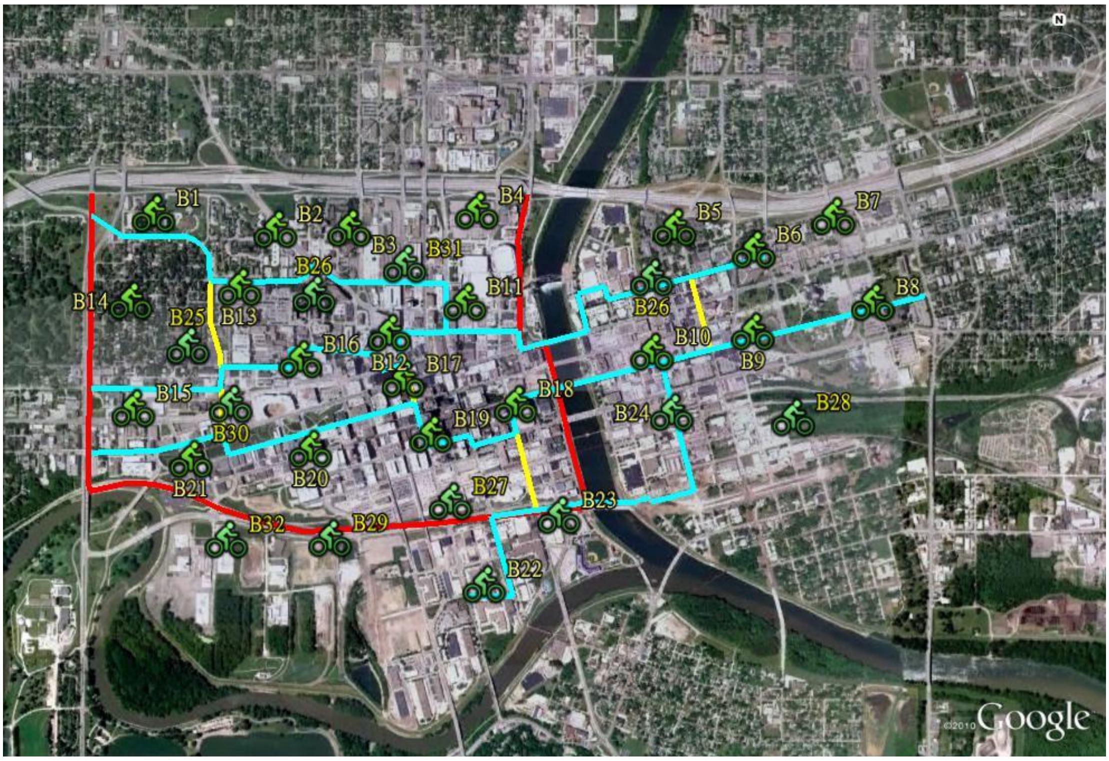
↑ Locations of rental stations in the fourth year (red icons marked with 4)



↑ Locations of rental stations in the fifth year (red icons marked with “5”)

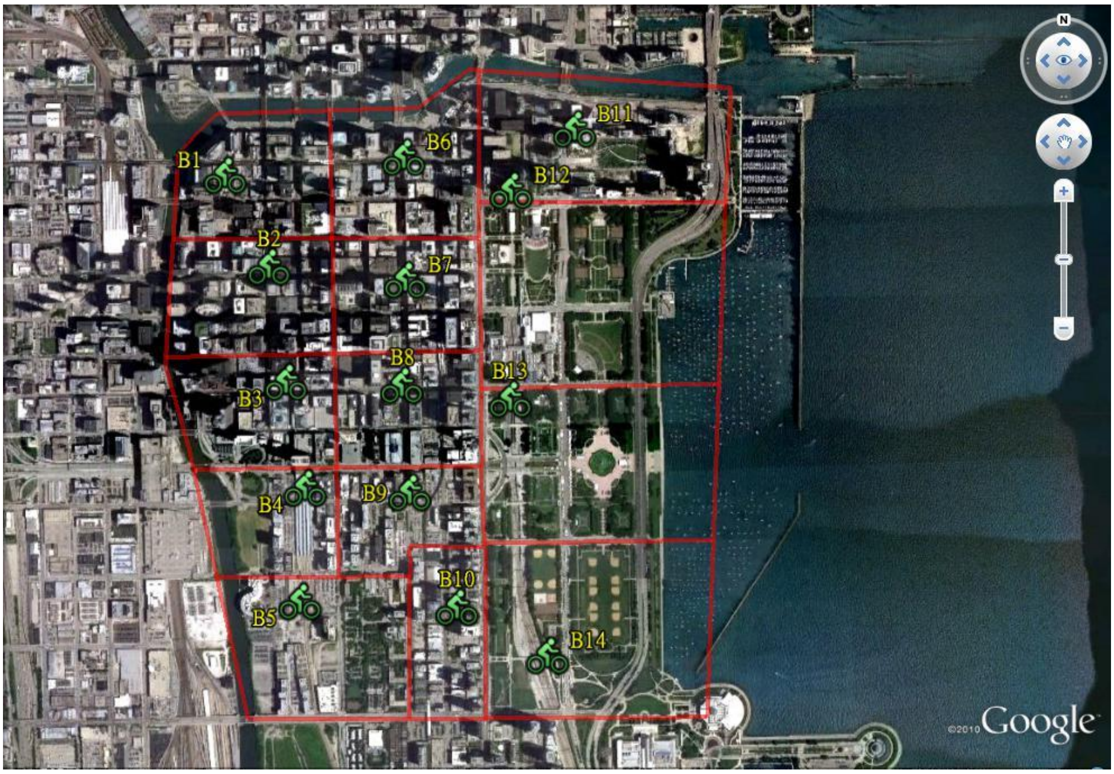


↑ Map of Lanes (Completed in the 2nd year, no changes made thereafter)

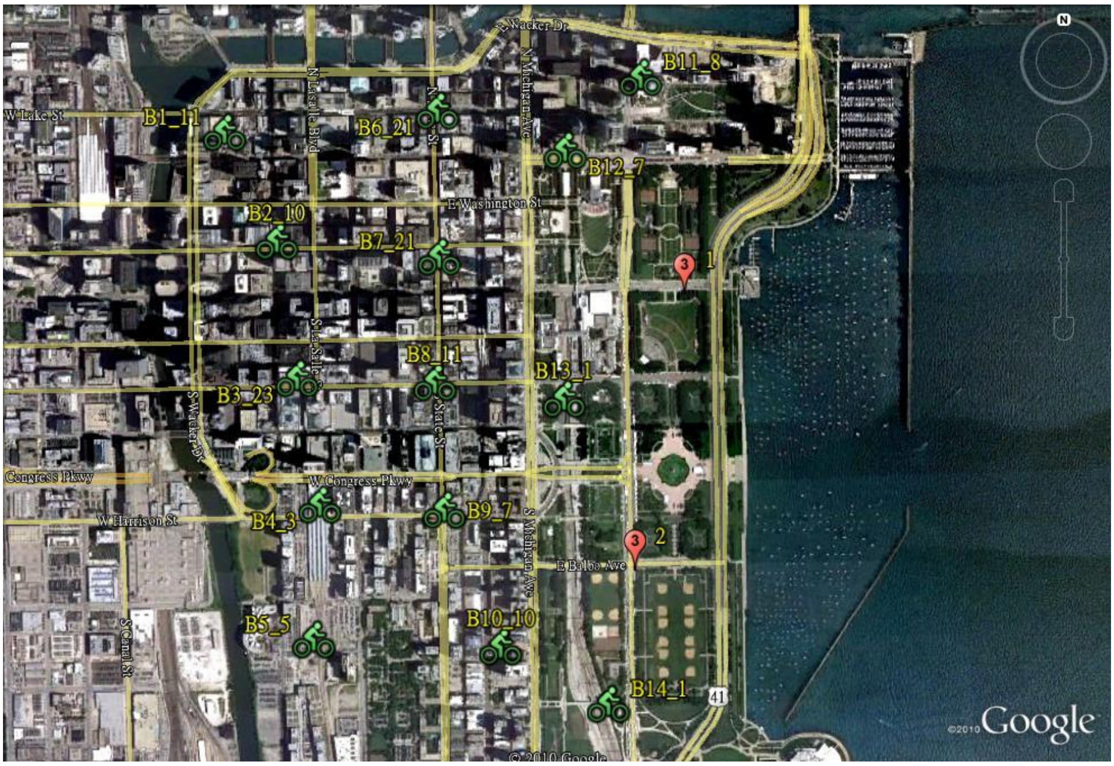


↑ Map of Des Moines after program is completed

(ii.) Chicago, IL

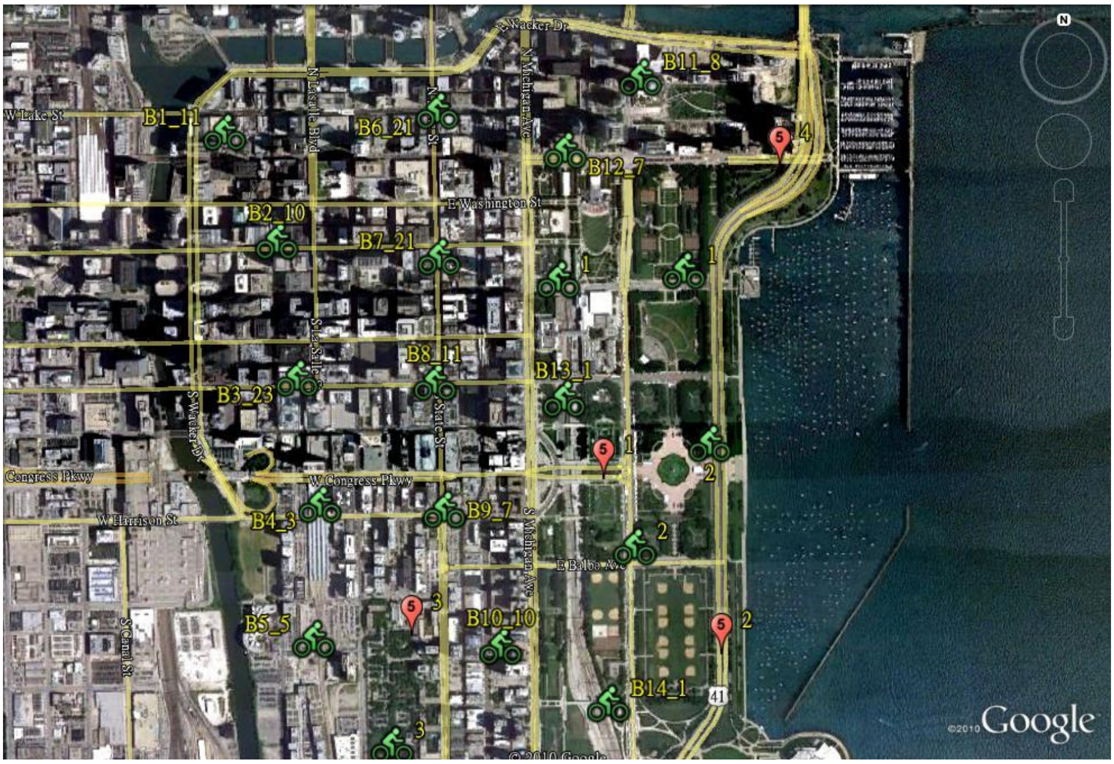


↑ Location of Stations in the second year (after initial plan is completed)

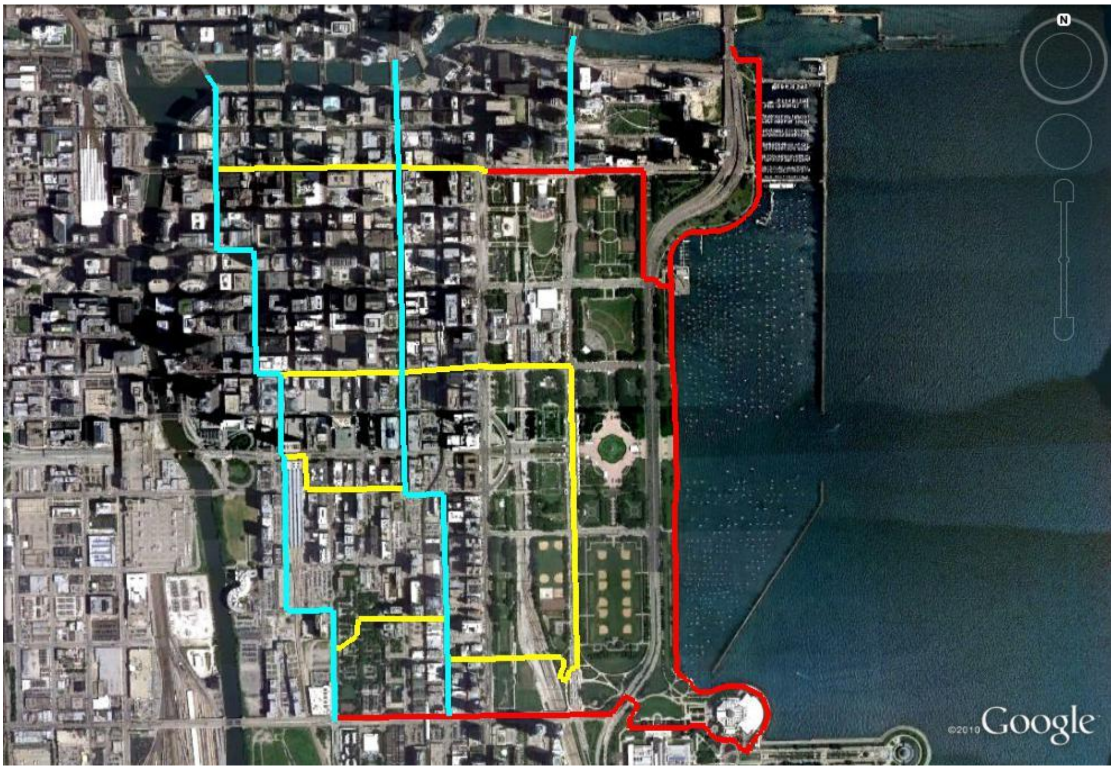


↑ New Stations added in the third year (red icons marked with “3”)

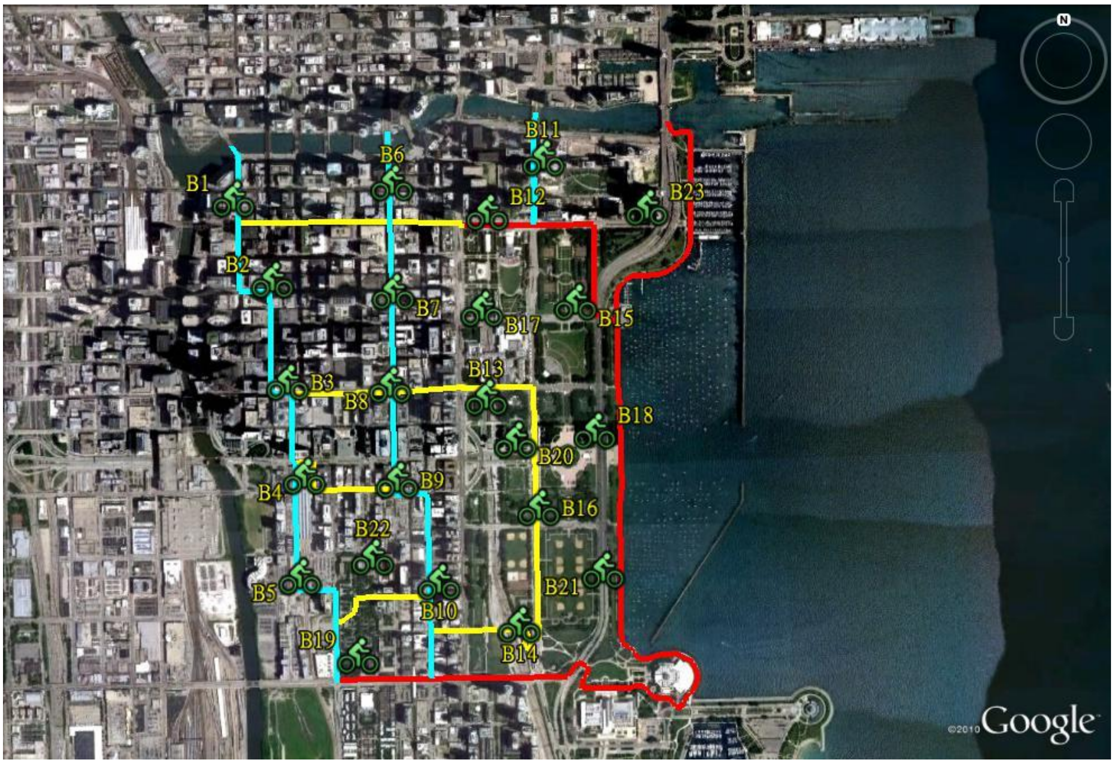
↑ New Stations added in the fourth year (red icons marked with “4”)



↑ New Stations added in the fifth year (red icons marked with “5”)

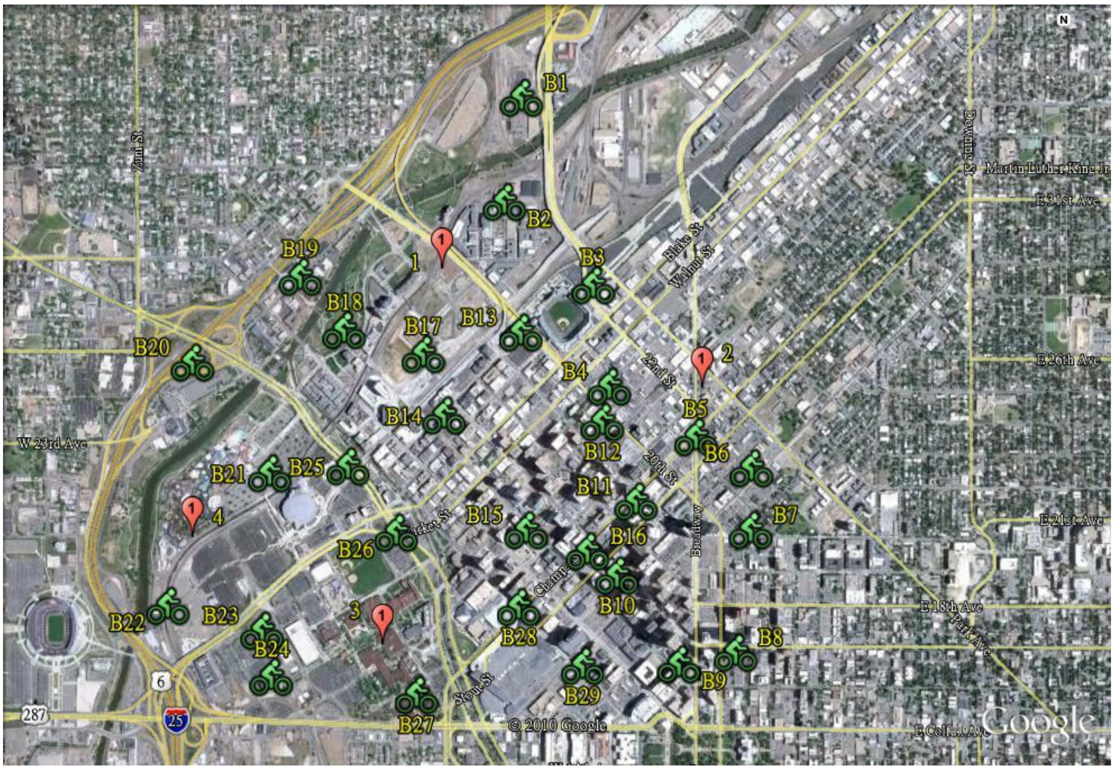


↑ Map of Bike Lanes (part of the initial plan; completed in the second year, no changes made thereafter)

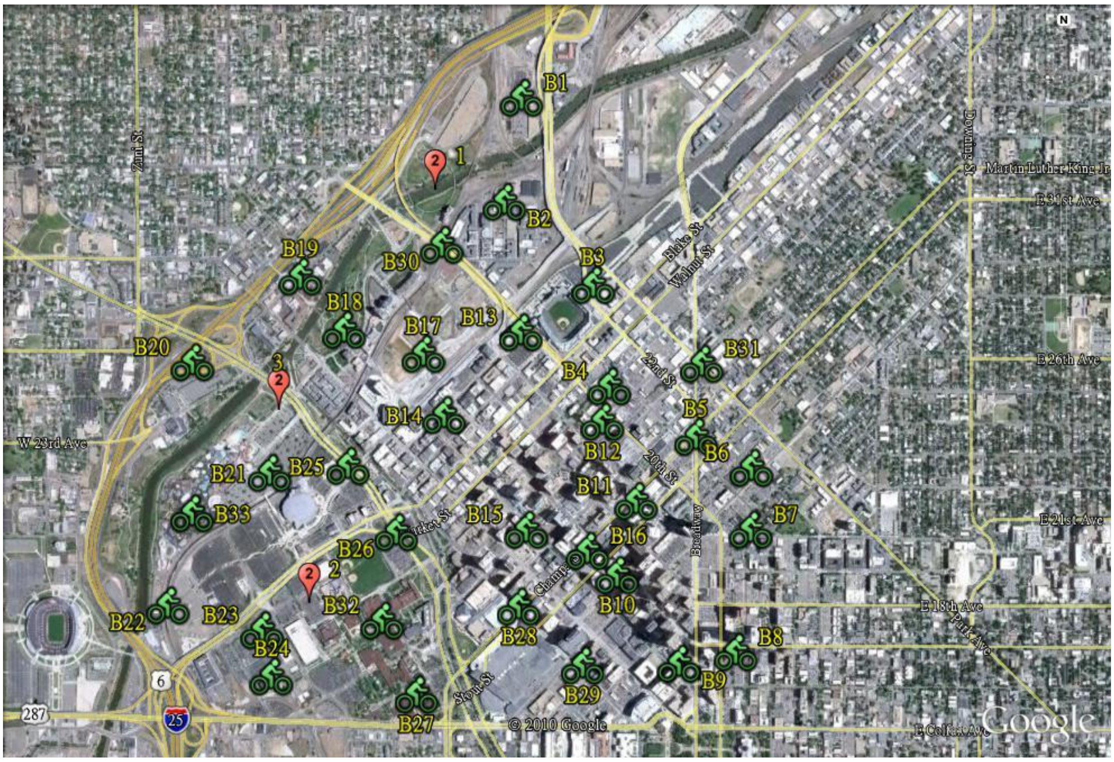


↑ Map after Program Completed

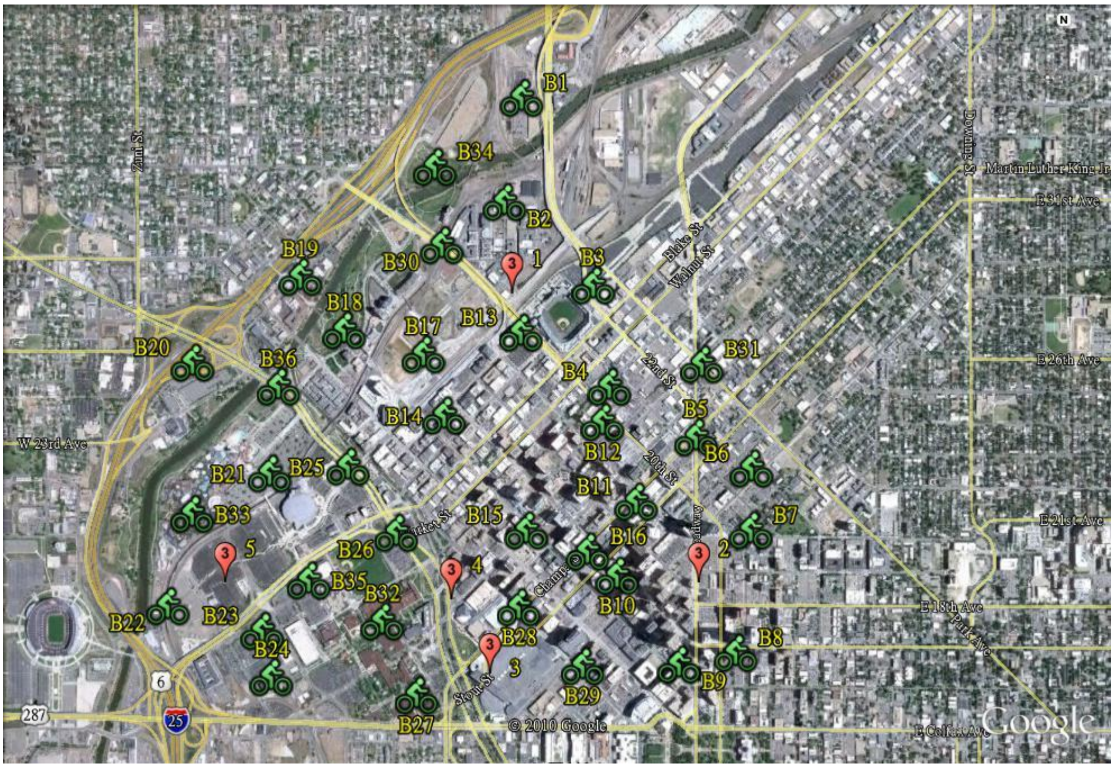
(iii.)Denver, CO



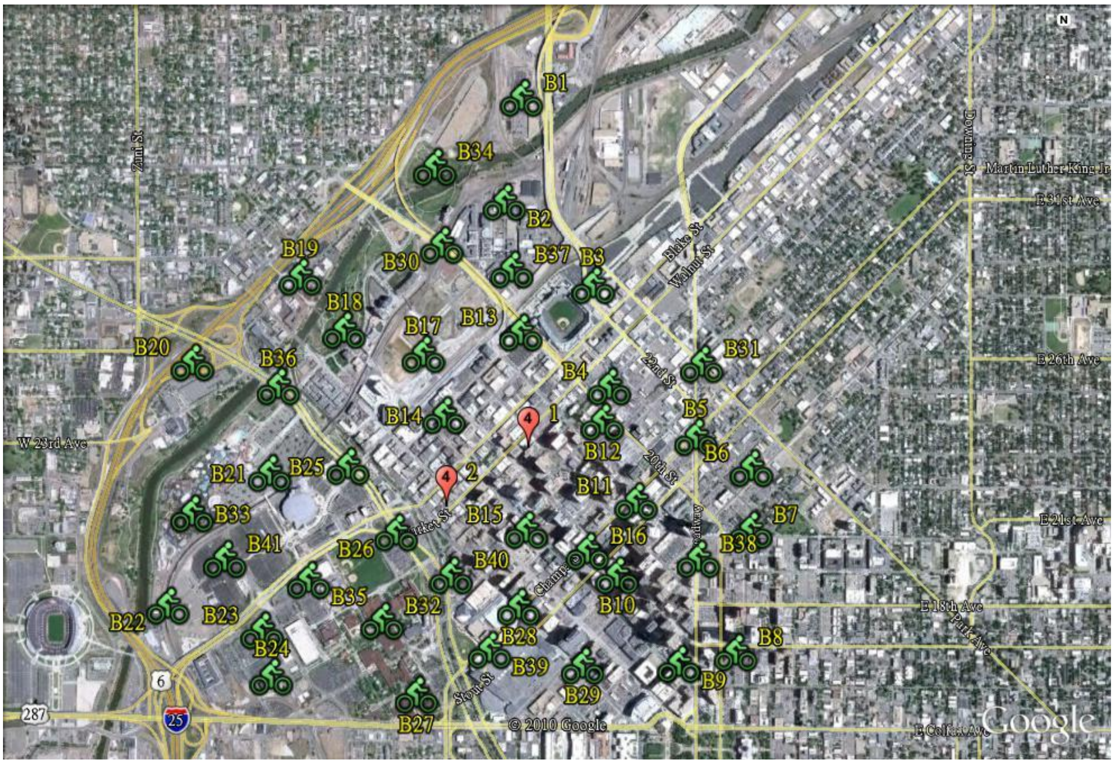
↑ Location of new Stations in the first year (red icons marked with "1")



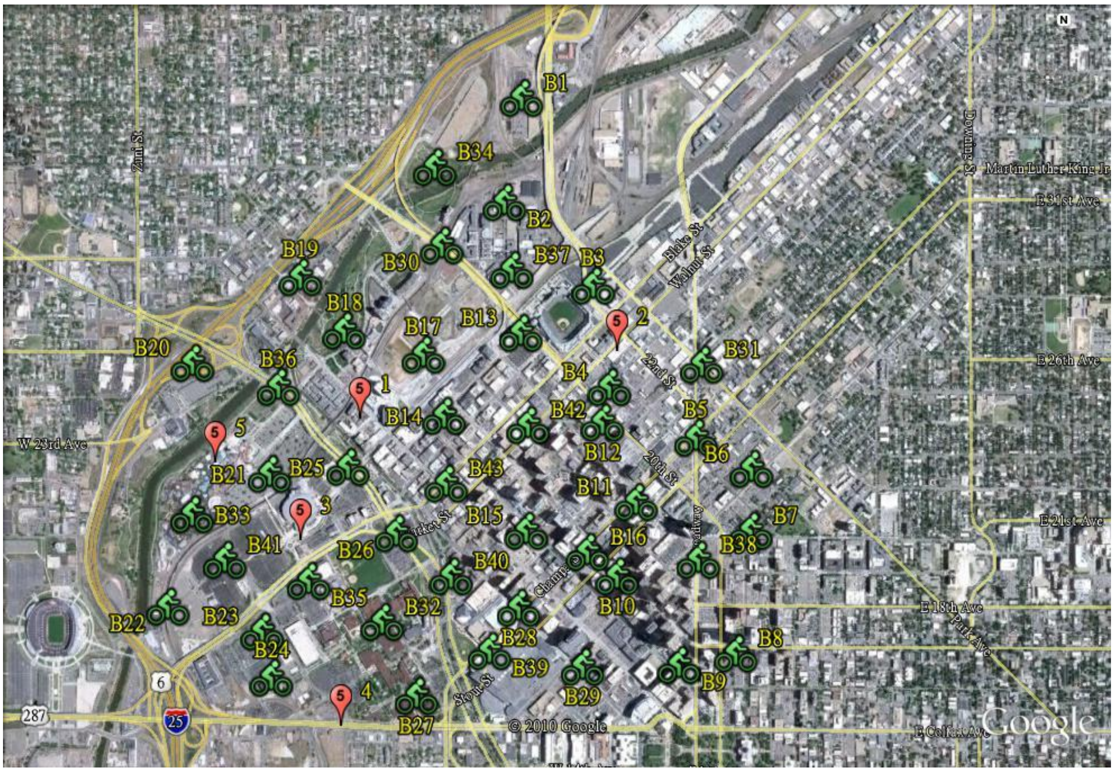
↑ Locations of new Stations in the second year (red icons marked with “2”)



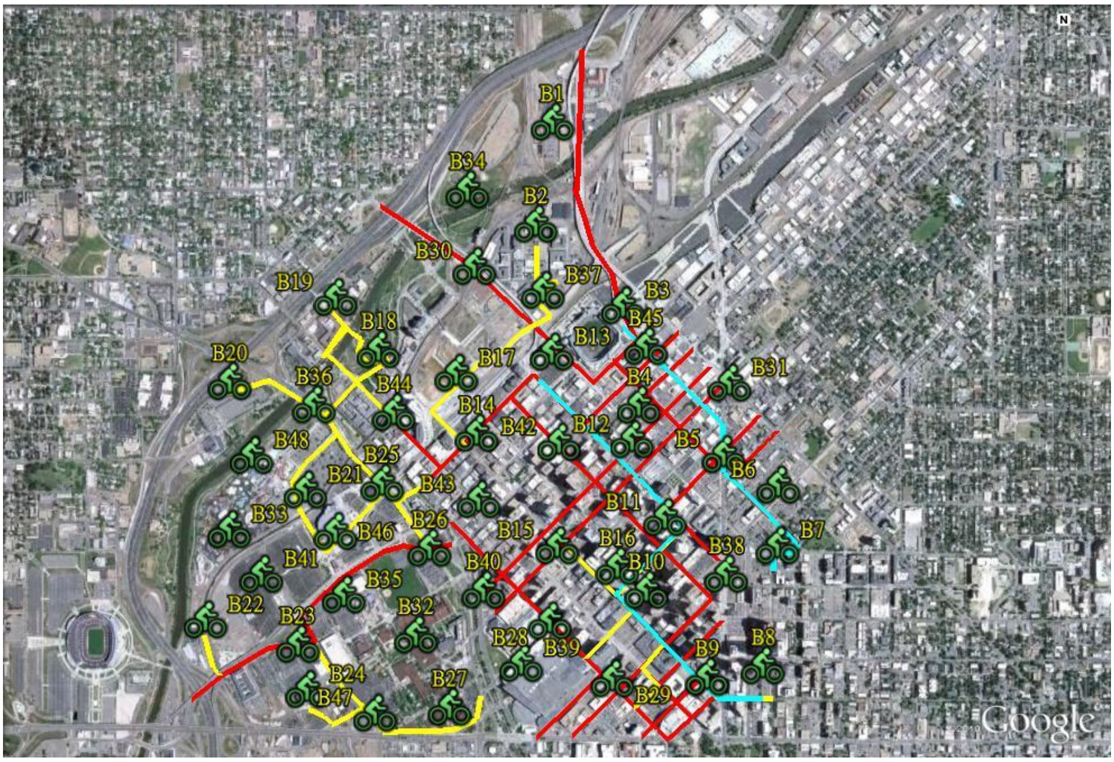
↑ Location of new Stations in the third year (red icons marked with “3”)



↑ Location of new Stations in the fourth year (red icons marked with “4”)



↑ Location of new Stations in the fifth year (red icons marked with “5”)



↑ Map after Program Completed

(2.) Allocation and Regulation of Bicycles

(i.) Des Moines, IA

Station Number	Bikes needed on working days	Regulations to be made on Fridays	Bikes needed on work-free days	Regulations to be made on Sundays
1	3	receive 1 bike from station 2	4	give 1 bike to station 2
2	8	give 1 bike to station 1	7	receive 1 bike from station 1
3	6	receive 1 bike from station 12	7	give 1 bike to station 12
4	5	receive 6 bikes from station 22	11	give 6 bikes to station 22
5	3	receive 4 bikes from station 22	7	give 4 bikes to station 22
6	2	does not change	2	does not change
7	6	receive 4 bikes from station 22 receive 4 bikes from station 8 receive 3 bikes from station 9	16	give 4 bikes to station 22 give 4 bikes to station 8 give 3 bikes to station 9
8	8	give 4 bikes to station 7	4	receive 4 bikes from station 7
9	5	give 3 bikes to station 7	2	receive 3 bikes from station 7
10	22	give 5 bikes to station 17 give 1 bike to station 20 give 1 bike to station 16	16	receive 5 bikes from station 17 receive 1 bike from station 20 receive 1 bike from station 16

11	15	receive 7 bikes from station 24	22	give 7 bikes to station 24
12	2	give 1 bike to station 3	1	receive 1 bike from station 3
13	8	receive 3 bikes from station 23	11	give 3 bikes to station 23
14	2	does not change	2	does not change
15	11	receive 1 bike from station 15 receive 4 bike from station 19	16	give 1 bike to station 15 give 4 bikes to station 19
16	6	receive 1 bike from station 10 receive 1 bike from station 24 receive 1 bike from station 18 receive 2 bikes from station 19	11	give 1 bike to station 10 give 1 bike to station 24 give 1 bike to station 18 give 2 bikes to station 19
17	11	receive 5 bikes from station 10	16	give 5 bikes to station 10
18	36	give 1 bike to station 16	35	receive 1 bike from station 16
19	8	give 2 bikes to station 16 give 4 bikes to station 15	2	receive 2 bikes from station 16 receive 4 bikes from station 15
20	21	receive 1 bike from station 10	22	give 1 bike to station 10
21	2	does not change	2	does not change
22	21	give 6 bikes to station 4 give 4 bikes to station 5 give 4 bikes to station 7	7	receive 6 bikes from station 4 receive 4 bikes from station 5 receive 4 bikes from station 7
23	16	give 3 bikes to station 13 give 1 bikes to station 15	12	receive 3 bikes from station 13 receive 1 bike from station 15
24	13	give 7 bikes to station 11 give 1 bike to station 16	5	receive 7 bikes from station 11 receive 1 bike from station 16

↑ Plan for the Stations constructed in the Initial Plan

25	12	receive 1 bike from station 26	13	give 1 bike to station 26
26	8	give 1 bike to station 25	7	receive 1 bike from station 25

↑ Plan for the Stations added in the third year

27	10	give 2 bikes to station 29	8	receive 2 bikes from station 29
28	9	does not change	9	does not change
29	11	receive 2 bikes from station 27	13	give 2 bikes to station 27

↑ Plan for the Stations added in the fourth year

30	8	receive 2 bikes from station 33	10	give 2 bikes to station 33
31	11	does not change	11	does not change
32	11	does not change	11	does not change
33	10	give 2 bikes to station 30	8	receive 2 bikes from station 30

↑ Plan for the Stations added in the fifth year

(ii.) Chicago, IL

1	10	receive 1 bike from station 6	11	give 1 bike to station 6
2	10	does not change	10	does not change
3	23	does not change	23	does not change
4	2	receive 1 bike from station 5	3	give 1 bike to station 5
5	6	give 1 bike to station 4	5	receive 1 bike from station 4
6	26	give 1 bike to station 1 give 3 bikes to station 11	22	receive 1 bike from station 1 receive 11 bikes from station 11
7	23	give 1 bike to station 8 give 1 bike to station 9	21	receive 1 bike from station 8 receive 1 bike from station 9
8	10	receive 1 bike from station 7	11	give 1 bike to station 7
9	6	receive 1 bike from station 7	7	give 1 bike to station 7
10	8	receive 1 bike from station 13 receive 1 bike from station 14	10	give 1 bike to station 13 give 1 bike to station 14
11	4	receive 1 bike from station 12 receive 3 bikes from station 6	8	give 1 bike to station 12 give 3 bikes to station 6
12	8	give 1 bike to station 11	7	receive 1 bike from station 11
13	2	give 1 bike to station 10	1	receive 1 bike from station 10
14	2	give 1 bike to station 10	1	receive 1 bike from station 10

↑ Plan for the Stations constructed in the Initial Plan

15	14	receive 2 bikes from station 16	16	give 1 bike to station 16
16	6	give 1 bike to station 15	4	receive 2 bikes from station 15

↑ Plan for the Stations added in the third year

17	12	receive 4 bikes from station 19	16	give 4 bikes to station 19
18	4	does not change	4	does not change
19	14	give 4 bikes to station 17	10	receive 4 bikes from station 17

↑ Plan for the Stations added in the fourth year

20	5	give 2 bike to station 22 give 1 bike to station 23	2	receive 2 bikes from station 22 receive 1 bike from station 23
21	3	give 1 bike to station 23	2	receive 1 bike from station 23
22	16	receive 2 bikes from station 20	18	give 2 bikes to station 20
23	16	receive 1 bike from station 20 receive 1 bike from station 21	18	give 1 bike to station 20 give 1 bike to station 21

↑ Plan for the Stations added in the fifth year

(iii.) Denver, CO

1	2	does not change	2	does not change
2	12	give 4 bikes to station 24	8	receive 4 bikes from station 24
3	16	does not change	16	does not change
4	5	does not change	5	does not change
5	12	give 3 bikes to station 10	9	receive 3 bikes from station 10
6	12	give 2 bikes to station 7 give 1 bike to station 11 give 2 bikes to station 13 give 1 bike to station 23	6	receive 2 bikes from station 7 receive 1 bike from station 11 receive 2 bikes from station 13 receive 1 bike from station 23
7	12	receive 2 bikes from station 6	14	give 2 bikes to station 6
8	12	does not change	12	does not change
9	9	give 4 bikes to station 11	5	receive 4 bikes from station 11
10	23	receive 3 bikes from station 5	26	give 3 bikes to station 5
11	9	receive 4 bikes from station 9 receive 1 bike from station 6	14	give 4 bikes to station 9 give 1 bike to station 6
12	6	receive 2 bikes from station 14	8	give 2 bikes to station 14
13	19	receive 2 bikes from station 6	21	give 2 bikes to station 6
14	19	give 2 bikes to station 12	17	receive 2 bikes from station 12
15	9	does not change	9	does not change
16	16	does not change	16	does not change
17	4	give 2 bikes to station 20	2	receive 2 bikes from station 20
18	7	give 1 bike to station 20	6	receive 1 bike from station 20
19	6	give 1 bike to station 29	5	receive 1 bike from station 29
20	6	receive 2 bikes from station 17 receive 1 bike from station 18	9	give 2 bikes to station 17 give 1 bike to station 18
21	9	receive 3 bikes from station 27	12	give 3 bikes to station 27
22	7	receive 1 bike from station 27	8	give 1 bike to station 27
23	4	receive 1 bike from station 6	5	give 1 bike to station 6
24	6	receive 4 bikes from station 2	10	give 4 bikes to station 2
25	9	give 1 bike to station 29	8	receive 1 bike from station 19
26	7	does not change	7	does not change
27	13	give 1 bike to station 22 give 3 bikes to station 21	9	receive 1 bike from station 22 receive 3 bikes from station 21
28	6	give 1 bike to station 29	5	receive 1 bike from station 29
29	13	receive 1 bike from station 28 receive 1 bike from station 25 receive 1 bike from station 19	16	give 1 bike to station 28 give 1 bike to station 25 give 1 bike to station 19

↑ Plan for Existing Stations

30	8	receive 2 bikes from station 33	10	give 2 bikes to station 33

31	11	receive 2 bikes from station 33	13	give 2 bikes to station 33
32	12	receive 2 bikes from station 33	14	give 2 bikes to station 33
33	9	give 2 bikes to station 30 give 2 bikes to station 31 give 2 bikes to station 32	3	receive 2 bikes from station 30 receive 2 bikes from station 31 receive 2 bikes from station 32

↑ Plan for the Stations added in the first year

34	9	give 1 bike to station 35	8	receive 1 bike from station 35
35	9	receive 1 bike from station 34 receive 4 bikes from station 36	14	give 1 bike to station 34 give 4 bikes to station 36
36	12	give 4 bikes to station 35	8	receive 4 bikes from station 35

Plan for the Stations added in the second year

37	8	give 3 bikes to station 40	5	receive 3 bikes from station 40
38	20	give 2 bikes to station 41	18	receive 2 bikes from station 41
39	8	give 3 bikes to station 40	5	receive 3 bikes from station 40
40	12	receive 3 bikes from station 37 receive 3 bikes from station 39	18	give 3 bikes to station 37 give 3 bikes to station 39
41	2	receive 2 bikes from station 38	4	give 2 bikes to station 38

↑ Plan for the Stations added in the third year

42	10	does not change	10	does not change
43	10	does not change	10	does not change

↑ Plan for the Stations added in the fourth year

44	6	receive 2 bikes from station 48	8	give 2 bikes to station 48
45	12	receive 1 bike from station 46	13	give 1 bike to station 46
46	14	give 1 bike to station 45	13	receive 1 bike from station 45
47	12	does not change	12	does not change
48	6	give 2 bikes to station 44	4	receive 2 bikes from station 44

↑ Plan for the Stations added in the fifth year

4-2 Assessment of the Model

Our model has the following strengths and weaknesses:

Strengths

- ☐ All of the models we applied to the problem are mature models from various fields (mathematics, statistics, human geography, etc.) and are properly adjusted and adapted.
- ☐ (The best of all strengths) we fitted our bicycle-sharing program into the large picture of the city's public transportation network (primarily through building rental stations at subway stations and near bus stops), increasing the coverage of subway stations and bus routes. We believe this will encourage more people to choose public transportation over private cars, thereby expanding the relieving effect on the traffic.
- ☐ We have taken the construction of new bicycle lanes into consideration so as to support the growing program.
- ☐ We provided detailed annual plan for each of the three cities, taking their actual situation into account.
- ☐ The application of Markov chain theory made it unnecessary to conduct large-scale bicycle transport every day, but only at weekends, thereby cutting the cost of the company.

Weaknesses

- ■ Given more time, we should consider construction cost and pricing, and try to find a balance between these two factors.
- ■ We did not provide a plan for bicycle docks at each station – it would be annoying for customers if they arrive at their destination only to find that there are not enough docks.
- ■ Our definition of "city center" may diverge from the actual bounds of the city centers, since we do not have enough data of the buildings and facilities.
- ■ Because of lack of information, we may have made mistakes in defining the category of some buildings/facilities.

5. Recommendation – Letters to Mr./Ms. Mayor

Dear Sir/ Madam,

We are a group of high school students who aim to make a 5-year plan for the development of bike-sharing programs in Chicago, Denver and Des Moines. Our plan has included the locations of new bicycle rental stations and bicycle lanes to be built within the coming five years, the allocation of bicycles at each station and the regulation of bicycles between stations. We believe that if this plan can be put into use, the bicycle-sharing program in Chicago, Denver and Des Moines will be greatly improved and will definitely be able to serve many more citizens in the city center.

When we make our plan, we have taken the existing public transport systems and other usage of transport in the city center into our consideration. We have made the bicycle-sharing program a perfect complement to the existing transport system, and we are sure that it will bring great convenience to a lot of people. The United States is known as “a nation on the wheels”, which clearly conveys the idea that cars dominate the transportation system of the country. However, the overuse of cars frequently cause traffic jams in city centers. Therefore, it is time to turn “car wheels” into “bike wheels”. We expect the bicycle –sharing program to be accepted by more and more people, and therefore relieve the stress of transportation in the city center of America.

Team #2541

Reference

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[2] Y. J. Tan et al. 《高中应用数学选讲》 *Lecture Series on Mathematics Modeling for High School Students.* Fudan Press. 2008. Page 16~19.

[3] J. R. Rubenstein. *The Cultural Landscape: An Introduction to Human Geography 8th Edition*. Pearson Education.

[4] *Central Place Theory.* Wikipedia. http://en.wikipedia.org/wiki/Central_place_theory, last modified on 25 October 2010 at 16:32

[5] D. Y. Chen et al. 《数学建模》 *Mathematics Modeling*. Science Press. 2007. Page 108~116.

Appendix: Matlab Code

1、

（1）

```
% Conformity test for the positive reciprocal matrix
A=xlsread('AHP_Denver.xlsx','0','B2:G7');%input the positive reciprocal matrix
C=xlsread('AHP_Denver.xlsx','0','B2:G7');
a=length(A);
w=0;
v=0;
t=0;
for b=1:a;
    for c=1:a;
        w=w+A(c,b);
    end
    for d=1:a;
        A(d,b)=A(d,b)/w;
    end
end
B=zeros(a,1);
for e=1:a;
    for f=1:a;
        B(e)=B(e)+A(e,f);
    end
end
for g=1:a;
    v=v+B(g);
end
for h=1:a;
    B(h)=B(h)/v;
end
D=C*B;
for i=1:a;
    t=t+D(i)/B(i);
end
m=t/a;
n=(m-a)/(a-1);
E=xlsread('AHP_Denver.xlsx','0','B15:K16');
j=E(2,a);
k=n/j;
if k<0.1;
    fprintf('OK,%f',k);
else
    fprintf('Retry, %f',k);
end
xlswrite('AHP_Denver.xlsx',B,'A21:A26');
```

（2）

```
% Finding best locations of rental stations in each section
a=30.922*cos((49/180)*pi);
A=xlsread('AHP_Chicago.xlsx','0','A21:F21');           %weight matrix
B=xlsread('AHP_Chicago.xlsx','14');    %latitude and longitude of sites
C=xlsread('AHP_Chicago.xlsx','14','A5:A6');                              %defin
e the origin of the xy plain (one for each section)
[u,w]=size(B);
D=zeros(2,w);
D(1,:)=(B(2,:)-C(1))*30.82;%turn latitude into distance measured with meters
D(2,:)=(B(3,:)-C(2))*a;%turn longitude into distance measured with meters
a1=max(D(1,:));a2=min(D(1,:));
b1=max(D(2,:));b2=min(D(2,:));
d=1000000000000;t=1;q=1;
x=0;
E=zeros(1,w);
o=0;
for j=1:w;
    k=B(1,j);
    E(j)=A(k);
```

```
end
for l=1:w;
    o=o+E(l);
end
for p=1:w;
    E(p)=E(p)/o;
end
for m=a2:0.1:a1;%calculate the ideal location of bicycle station in each section
    for n=b2:0.1:b1;
        for v=1:w;
            x=x+E(v)*(((D(2*v-1)-m)^2+(D(2*v)-n)^2)^(1/2));
        end
        if x<d;
            d=x;
            t=m;
            q=n;
        end
        x=0;
    end
end
F=zeros(1,3);
F(1)=d;
F(2)=(t/30.82)+C(1);%turn distance back into latitude
F(3)=(q/a)+C(2);%turn distance back into longitude
xlswrite('AHP_Chicago_E.xlsx',F,'A14:C14');%input the results into Excel
fprintf('OK');
```

2、

```
% Group the rental stations (preparatory step for finding regression lines)& Find
the regression lines for all of the rental stations -- new bicycle lanes
A=xlsread('AHP_Denver_E.xlsx','G1:K22');
B=xlsread('AHP_Denver_E.xlsx','M1:Q29');
C=xlsread('AHP_Denver_E.xlsx','S7:V7');
D=zeros(22,3);
E=zeros(29,3);
F=zeros(1,2);
b=1;c=1;d=1;e=1;
DA=zeros(11,2);
DB=zeros(16,2);
DC=zeros(14,2);
DD=zeros(5,2);
for a=1:29;
    if a<=22;
        D(a,1)=A(a,2)+(A(a,1)-C(1))*60-C(2);
        D(a,2)=A(a,4)+(A(a,3)-C(3))*60-C(2);
        D(a,3)=A(a,5);
    end
    E(a,1)=B(a,2)+(B(a,1)-C(1))*60-C(1);
    E(a,2)=B(a,4)+(B(a,3)-C(3))*60-C(2);
    E(a,3)=B(a,5);
    if a<=22;
        switch D(a,3)
            case 1
                DA(b,1)=D(a,1);
                DA(b,2)=D(a,2);
                b=b+1;
            case 2
                DB(c,1)=D(a,1);
                DB(c,2)=D(a,2);
                c=c+1;
            case 3
                DC(d,1)=D(a,1);
                DC(d,2)=D(a,2);
                d=d+1;
            case 4
                DD(e,1)=D(a,1);
```

```
                DD(e,2)=D(a,1);
                e=e+1;
            end
        end
        switch E(a,3)
            case 1
                DA(b,1)=E(a,1);
                DA(b,2)=E(a,2);
                b=b+1;
            case 2
                DB(c,1)=E(a,1);
                DB(c,2)=E(a,2);
                c=c+1;
            case 3
                DC(d,1)=E(a,1);
                DC(d,2)=E(a,2);
                d=d+1;
            case 4
                DD(e,1)=E(a,1);
                DD(e,2)=E(a,1);
                e=e+1;
        end
end
DAA=polyfit(DA(:,1),DA(:,2),1);
DBA=polyfit(DB(:,1),DB(:,2),1);
DCA=polyfit(DC(:,1),DC(:,2),1);
DDA=polyfit(DD(:,1),DD(:,2),1);
```

3、

（1）

```
A=xlsread('AHP_Des_Moines_LA.xlsx','0_1');%total number of buildings
a=A(26,7);%total number of buildings
b=A(26,2);%total number of office buildings
B=zeros(24,24);
for x=1:24;%Calculate the state transit matrix
   c=(a-A((x+1),7))/23;
   B(x,x)=A((x+1),7)/(A((x+1),7)+c);
   for y=1:24;
       if y~=x;
           B(x,y)=(1-B(x,x))*A((y+1),7)/(a-A((x+1),7));
       end
   end
end
```

（2）

```
format long
P=xlsread('AHP_Des_Moines_LA.xlsx','0_1','A31:X54');%state transition matrix of Markov Chain
x=100;%time of transition
[m,n]=size(P);
A=zeros(x,n);
A(1,1)=240;%original number of bicycle, 240 in all
for i=2:x;%the application of Markov Chain
    A(i,:)=A(i-1,:)*P;
end
```

4、

```
% Determining the initial range (service area) for the stations
A=xlsread('CT_Des_Moines.xlsx','G1:H24'); %Latitude of all stations
B=xlsread('CT_Des_Moines.xlsx','J1:K24'); %Longitude of all stations
C=xlsread('CT_Des_Moines.xlsx','A1:B4'); %Lati+Longitude of all four corner
[m,n]=size(A);
D=zeros(m,n);
E=zeros(2,2);
F=zeros(24,1);
G=[1000,2000];
I=xlsread('CT_Des_Moines.xlsx','A13:B14');
I(:,1)=(I(:,1)-49.16)*30.82;
I(:,2)=(I(:,2)-46.88)*(30.922*cos((49/180)*pi));
```

```
J=xlsread('CT_Des_Moines.xlsx','A22:B23');
J(:,1)=(J(:,1)-49.16)*30.82;
J(:,2)=(J(:,2)-46.88)*(30.922*cos((49/180)*pi));
K=xlsread('CT_Des_Moines.xlsx','A32:B33');
K(:,1)=(K(:,1)-49.16)*30.82;
K(:,2)=(K(:,2)-46.88)*(30.922*cos((49/180)*pi));
for a=1:24;
    D(a,1)=A(a,2)+(A(a,1)-C(1,1))*60;
    D(a,2)=B(a,2)+(B(a,1)-C(3,1))*60;
end
D(:,1)=(D(:,1)-49.16)*30.82;
D(:,2)=(D(:,2)-46.88)*(30.922*cos((49/180)*pi));
E(1,1)=C(1,2);
E(1,2)=C(3,2);
E(2,1)=C(2,2)+(C(2,1)-C(1,1))*60;
E(2,2)=C(4,2)+(C(4,1)-C(3,1))*60;
E(:,1)=(E(:,1)-49.16)*30.82;
E(:,2)=(E(:,2)-46.88)*(30.922*cos((49/180)*pi));
H=zeros(17,35);
f=1;
g=1;
x=input('Enter year');
i=0;
u=500*((1-(1/(1.3))^(1/2))^x);
for b=E(1,1):100:E(2,1);
    for c=E(1,2):100:E(2,2);
        switch 1
            case ((b>=I(1,1)) & (b<=I(2,1))) & ((c>=I(1,2)) & (c<=(I(2,2))));
                y=0;
            case ((b>=J(1,1)) & (b<=J(2,1))) & ((c>=J(1,2)) & (J<=(J(2,2))));
               y=0;
            case ((b>=K(1,1)) & (b<=K(2,1))) & ((c>=K(1,2)) & (c<=(K(2,2))));
                y=0;
            otherwise
                for d=1:24;
                    F(d,g)=(((D(d,1)-b)^2+(D(d,2)-c)^2)^(1/2));
                end
        end
        if min(F)>u;
            M(g,2*f-1)=b/30.82+49.6;
            M(g,2*f)=c/(30.922*cos((49/180)*pi))+46.88;
            i=i+1;
        end
        g=g+1;
    end
    H=min(F);
    [m,n]=size(H);
    g=1;
    L(f)=max(H);
    f=f+1;
end
a=max(L);
fprintf('%f',a)
```