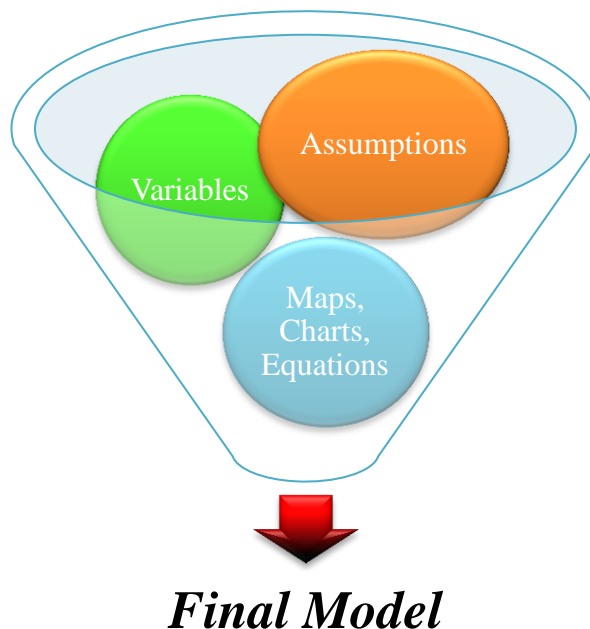


Math Modeling Team #3180 Summary Sheet

In this solution, we created innovative models that can be used to solve two common problems that were presented to us: computing the probability of finding a lost item in a small park, and developing the most effective method for searching a larger park for a lost jogger. Since the two problems had numerous similarities, we had similar solutions to both of the problems. To create these solutions, we had to first identify the variables that had to be considered in our models. Many of the variables were similar for both problems, and some of them included, but were not limited to: distance of trails, points of interests in the park, weather at the time of the event, parking station locations, the condition of the jogger, and some things even as detailed as the strength of the light of the searchers. Our model took into account all of these variables, and used maps, graphs, flow charts, and equations to come up with the optimum solution to the problem. The strengths of this model are that it accounts for a multitude of variables and the use of many visuals (such as maps, graphs, and flow charts) which help to visualize and allow for a more precise solution. Moreover, the equation that we developed for part B of the problem (the lost jogger) effectively finds the probability of finding the lost jogger while taking into account a myriad of variables and also helps to find the jogger. One weakness to our overall model is that many assumptions had to be made with limited data. Some of these assumptions include the general starting point of the jogger, ruling out areas where the jogger could not possibly be, and assuming that the lost object had to be on a trail. The way that our model will be tested is through actual values being plugged in (for the case of our equation) and real life experiences in similar situations to the given problem (lost items and lost joggers). The algorithm below shows the mixture of steps that led to our model:



2011

Math Modeling Team #3180



11/5/2011

Introduction

A wedding ring. A pocket watch. A lucky coin. A small object, precious to its owner, lost, abandoned out in the wilderness, left to fend for itself in an unfamiliar world. Imagine how the owner must feel, helpless, burning with a desire to rush headlong in and tear apart the entire area where this trinket could be concealed. His highest priority is to find the object and to find it as fast as possible. However, he knows it is impossible to cover all of the land where his precious could rest. The area: Hopkinton State Park. The task: Find the object, given only two hours to search.

Each tree looks just like the next. The trail just goes on forever, interminably, with no end. This was supposed to be only a simple 5-mile jog; how could the situation have turned so quickly? Continuing on would not help, not when there is nothing to hint at where to go. If only this jogger had planned out his path and paid attention to where he was going, then he would not have run into this problem. Instead, he is lost, alone, and lacking in any form of communication. He can only hope he does not accidentally wander into the unexploded ordnance area. His only hope is to have rescuers come and save him from this unkind fate; otherwise, he might become the latest victim of the roaming mountain lions in the area (1). The area: Fort Ord Public Lands. The task: Rescue the lost jogger.

These are the problems set for us, Math Modeling Team #3180. These problems, we solved.

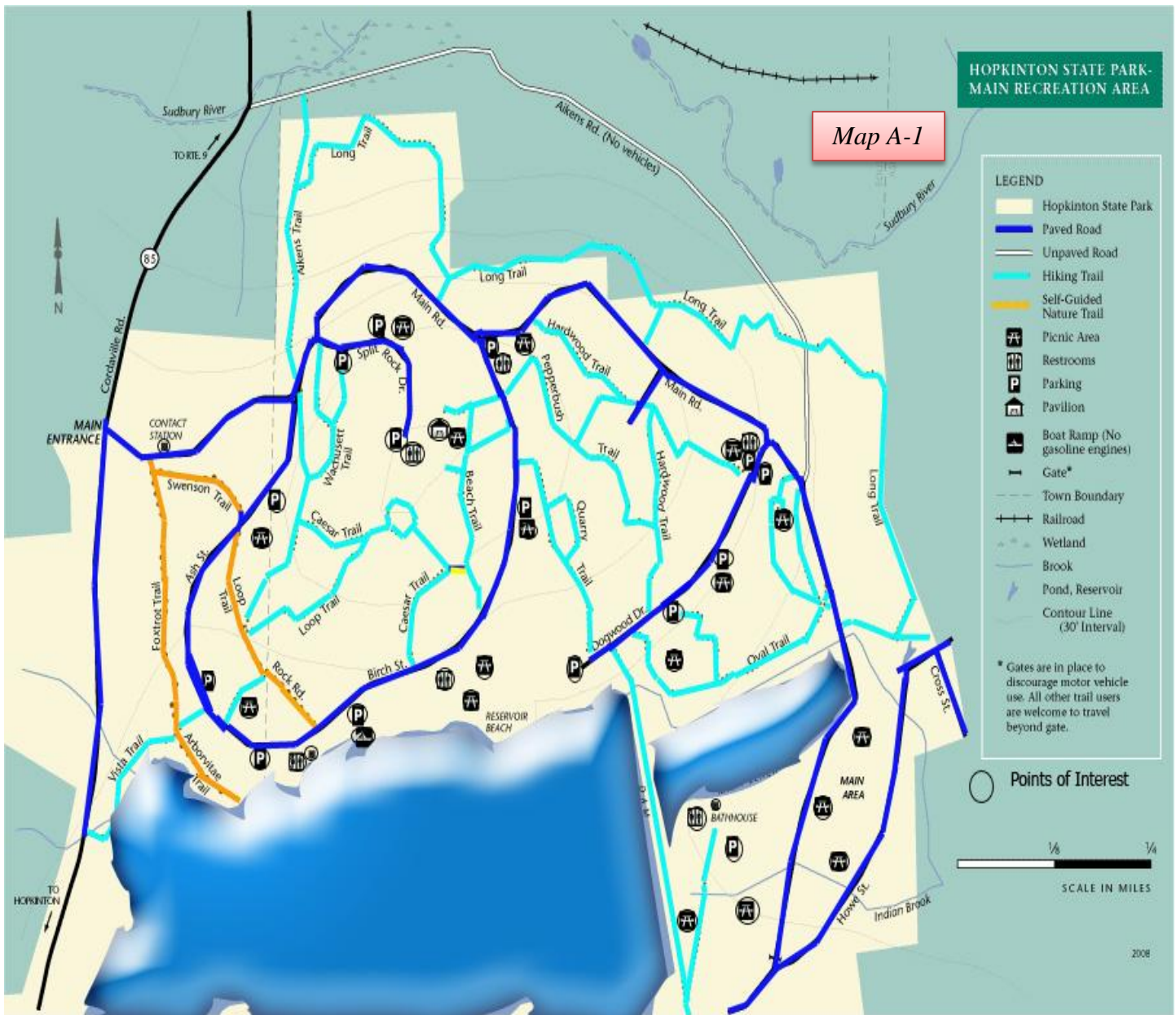
Part A- The Lost Object

In part A of the problem, we were asked to find the chance that a small object lost in a park would be found, given only two hours to search for it. At a walking pace of approximately 4 mph, about 8 total miles could be covered. In a perfect world, that means we could account for almost 97% of about 8.23 trail miles. Unfortunately, the world we live in is not perfect. We must account for backtracking, travelling on roads, and areas that are not on trails. On our sample path solution, we assumed that the object was lost on one of the trails because it is likely the object was lost while traveling and the owner did not notice its absence until later. On our solution where we divided the park into various areas, we assumed it was either on a trail or at a “Point of Interest.”

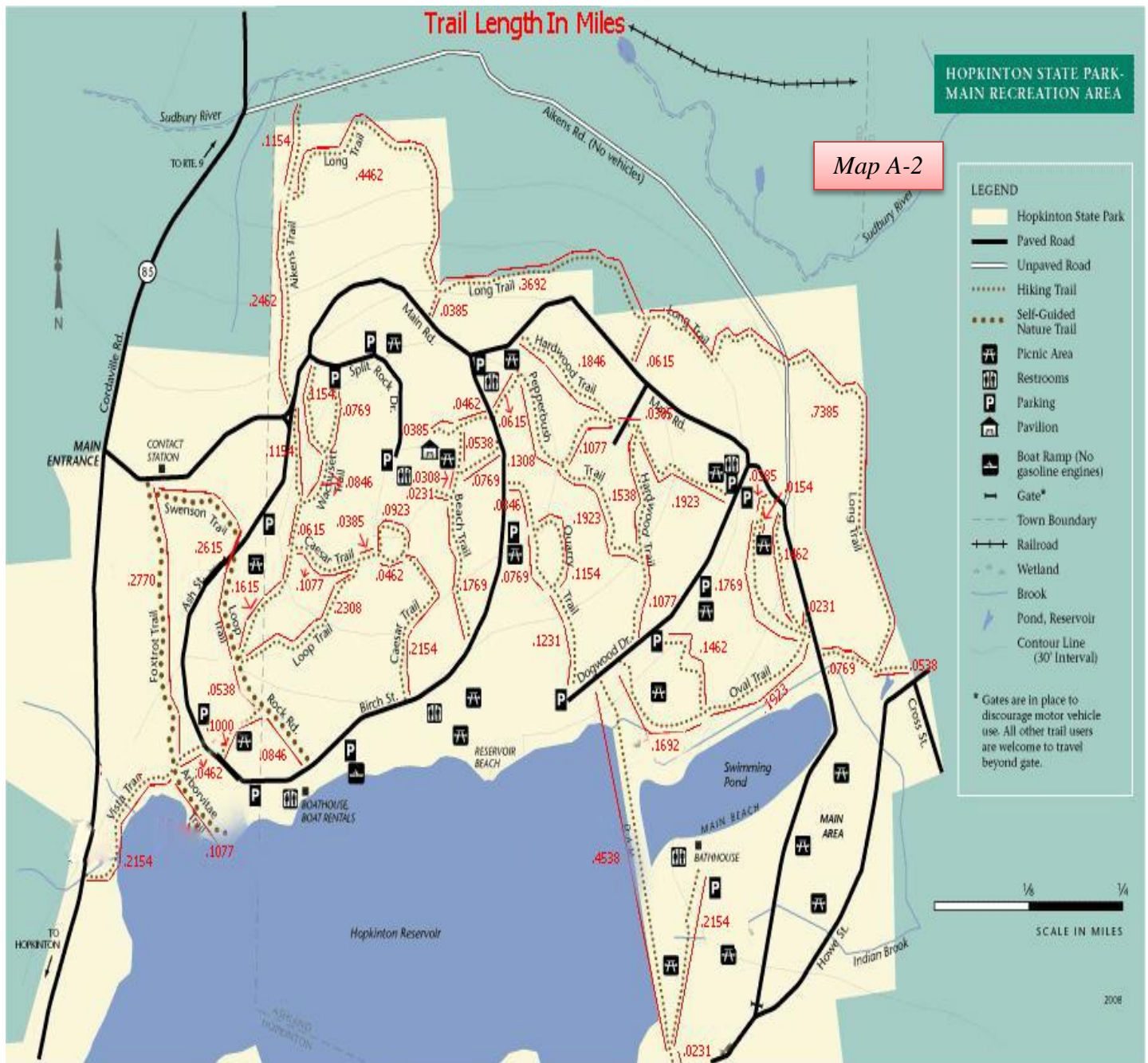
To initiate solving Problem A, we had to primarily indicate where the owner could have lost the precious object. With certainty, we deduced that when one traverses among the park, he cannot stray from the trails/paved roads or points of interests because the park mostly consists of forests (2). Furthermore, people remain on the trails for the majority of the time when they wander in the park to prevent getting lost and for convenience. We assumed that the owner of the lost object was like any average park visitor- one who stayed on the trails.

One of the first things that we realized was that without planning a path, the chance of finding the object would be significantly reduced. Through simple planning, however, the chance finding the object would be significantly increased.

The first thing we did to plan out our search was to find the lengths of each portion of the trails. This is shown in map A-2. Given these lengths, we calculated the percentage of the total trails that each section covered. We used these percentages to figure out how much of the total trail area was covered.



This is a detailed map of Hopkinton State Park. We emphasized many aspects of the park, as one can tell from the legend. Some examples of this are the highlighting of all trails and roads. This allows for easier navigation and a simpler experience in following the paths on the map. We felt that this map would be an assistance in both making our model and for others to view. The creation of this particular map, as well as every other subsequent map, took much meticulous work, but we felt it was a necessary visual aid to assist our model.

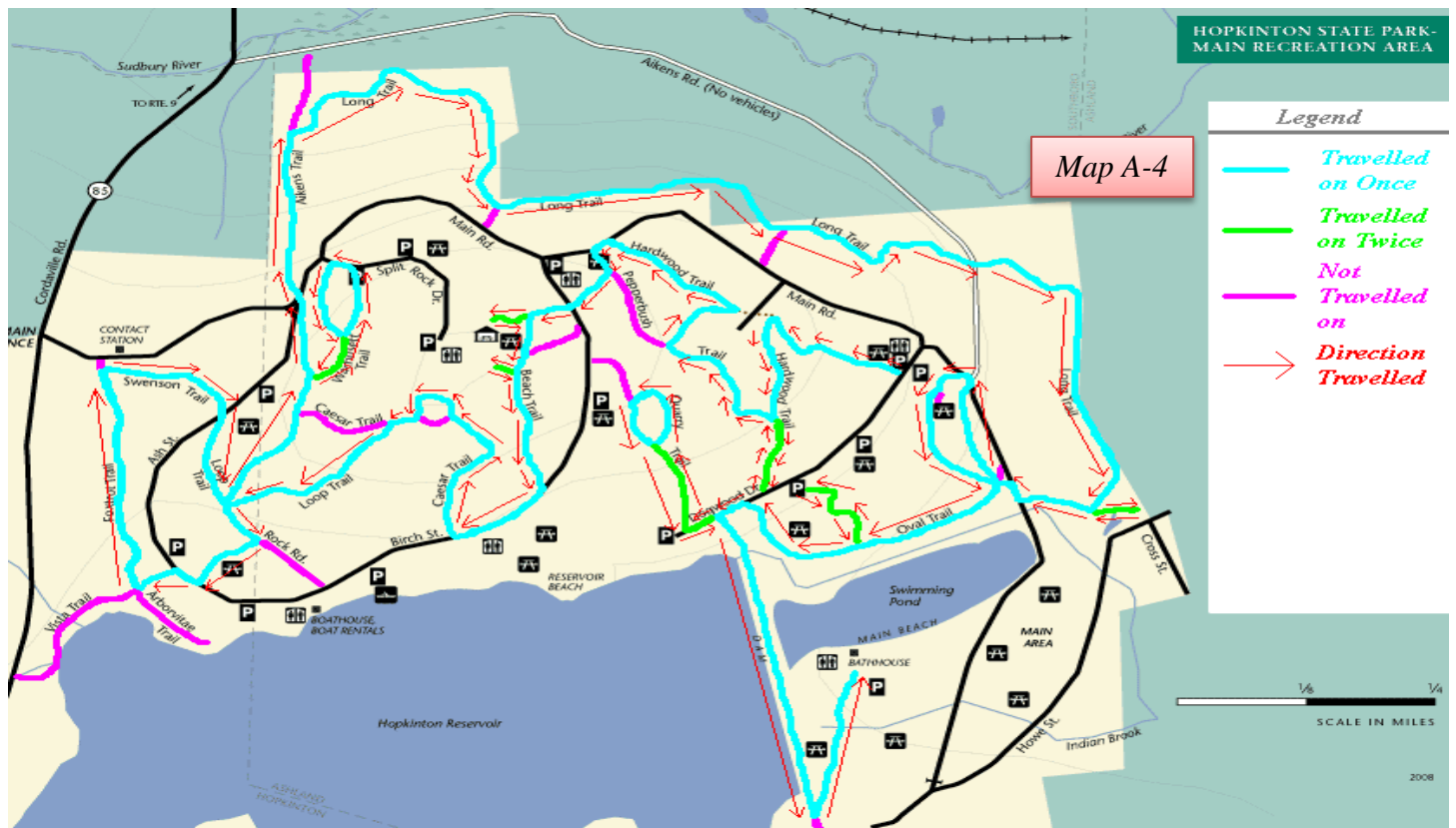


In this map, we measured the distance of the several sections for every single trail on the map. We did this by measuring the length of each section and converting them with the scale provided at the bottom of the map. This map shows one of the main factors in finding the path with the highest chance of discovering the lost object. Moreover, this map was used in many of our calculations and subsequent maps.

Math Modeling Team #3180



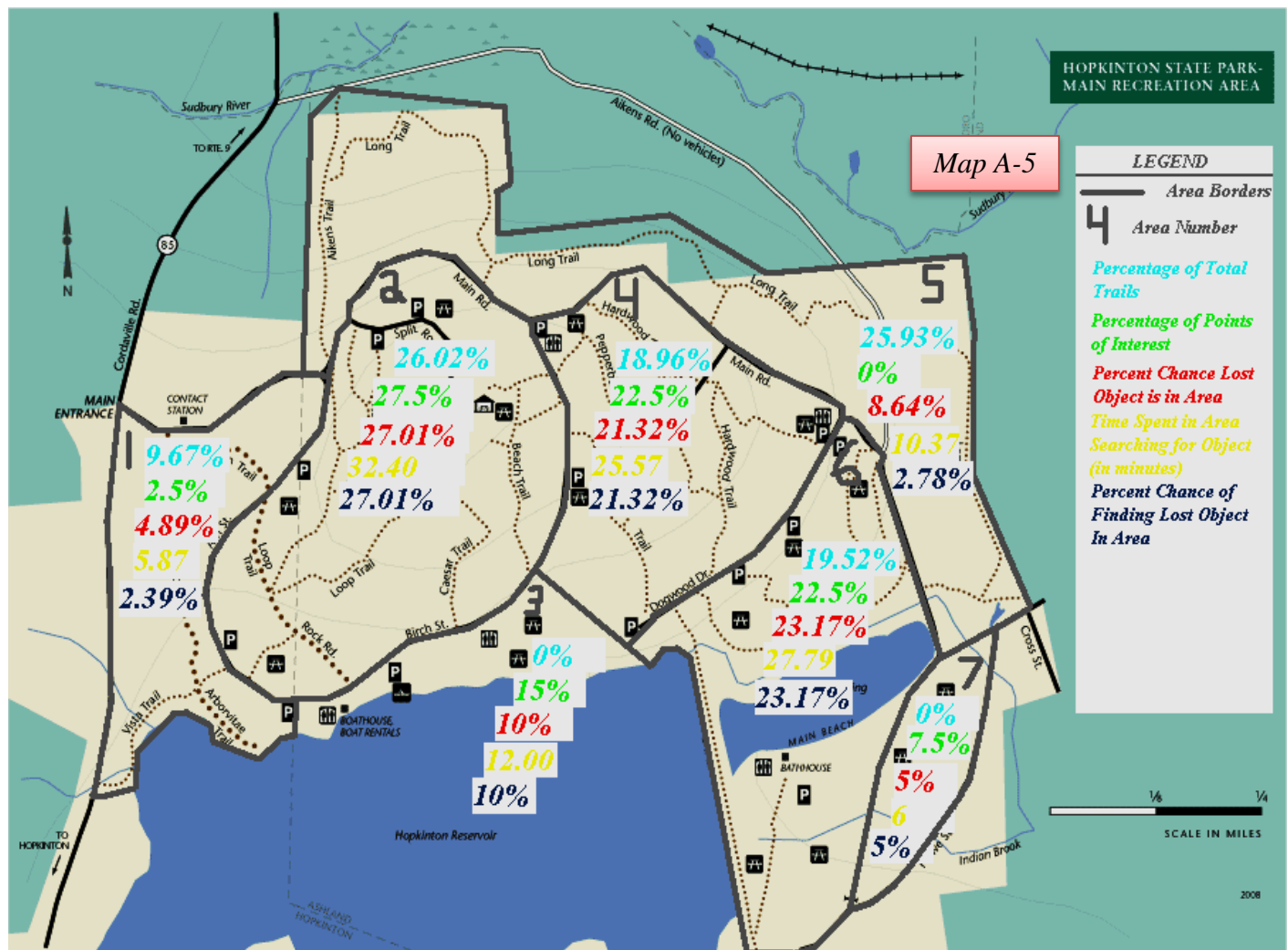
We calculated the total distance covered by all of the trails by adding up all of the values used in the previous map. Then, we divided the value of each individual piece by the total distance covered to find the percentage of total distance covered by each individual piece. We were then able to use these percentages to find the percentage of trails covered by various possible paths.



This is a map of the path that covers the most of the trails out of the 10 paths we tested. It covers slightly over 88% of the total trail distance. The blue path was travelled on once, while the green path was travelled on twice (backtracking). The purple areas were not travelled on due to a lack of time, since we were only given two hours to search. The red arrows illustrate the direction that the searcher would travel, beginning at the parking lot on the east side of the park and ending at the parking lot by the bathhouse. To find this path, we determined the total distance we would be able to travel (including backtracking) by multiplying our speed (4 miles per hour) by our time, resulting in a total of 8 miles:

$$\frac{4 \text{ miles}}{\text{hour}} * 2 \text{ hours} = 8 \text{ miles}$$

Using the second map, we were able to calculate the distance our path covered and ensure that we stayed under the 8 mile limit. Then we used the third map to determine the percentage of trails that our path covered.



First, we divided the park into 7 distinct areas, mainly using the roads as borders. Then we found the percentage of distance covered by the trails in each area relative to the total amount of distance covered by all of the trails. We did this by measuring the distances covered by each part of each trail and adding that up to find the total distance covered by the trails. Then, for each area, we added up the distance covered by the trails only in that area and divided it by the total distance. Next, we found the percentage of “Points of Interest” in each area by dividing the number of “Points of Interest” in each area by the total number of “Points of Interest.” We considered the “Points of Interest” twice as influential as the paths because visitors tend to spend twice as much of their time at a “Point of Interest” as they do on the various trails. Assigning a

weight of two to the percentage of “Points of Interest” and a weight of one to the percentage of distance covered by the trails, we calculated the chance that the person would have been in each area at the moment when he lost the object. This percentage also represents the percent chance that the lost object is in the area because if the person lost the object while in that area, the object would be in that area. We used this percentage to divide the total amount of time we had to search (2 hours) into specific amounts of time we would spend in each area (in minutes). To do this, we multiplied the number of minutes in two hours (120 minutes) by the decimal form of the percent chance that the object would be in each area. Finally, we calculated the distance on the trails that could be covered given the amount of time allotted to each area by using the approximate walking speed of four miles per hour. We then took that number and divided it by the total trail distance in that area to find how much of the trails could be covered in the given amount of time. We took that value and multiplied by the percent chance that the object would be in the specific area in order to find the percent chance that you would find the object in that area. For areas that did not include any trails, we assumed that the allotted time would be enough to search the entire relevant area; namely, the “Points of Interest.” Using this process, we concluded that the chance of finding the lost object was **91.67%**

Strengths and Weaknesses for Our Model in Part A

The strengths of our first method, using a sample route, are that it covers almost 90% of the trail areas, a favorable chance of finding the object, and it is a specific method that can be used to find the object. The weaknesses are that it does not take “Points of Interest” into consideration, it does not quite account for all of the trails, it may not actually be the most

effective path, the object may not actually be on a trail, and it is limited to only this specific park, given this specific time limit and this specific speed.

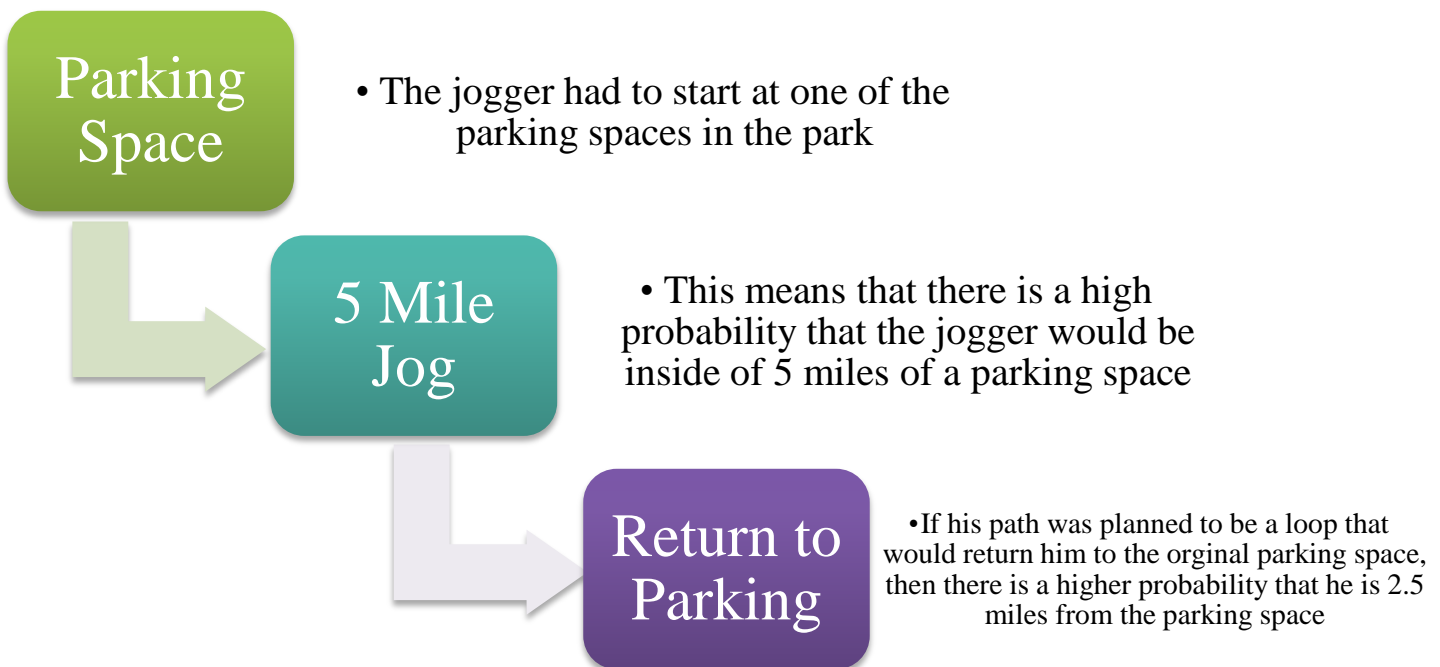
The strengths of our second method, using percentages, is that it gives a higher chance of finding the object and considers more factors, such as “Points of Interest.” The weaknesses are that we assumed that the time assigned to areas that did not have any trails would be enough to search the relevant area, which may not be true, and it does not account for travelling from one trail to another, backtracking, or if the object is not actually on a trail.

Part B- The Lost Jogger

For this part of the problem, we were given a situation in which a jogger who was going on a 5 mile run became lost. The jogger could be unconscious, as well. Our challenge was to come up with the most effective method of searching for and finding this lost jogger in his dire predicament.

The first step taken was to make logical and simple assumptions that are likely correct. By making these assumptions, we could rule out areas of the park where there is an extremely low probability of finding the jogger; this would then allow us to concentrate on the remaining areas where there is a higher probability of finding the lost jogger. We first assumed that the jogger is not in the “Explosives Area.” This area is clearly marked off, for it has numerous “Do Not Enter” signs and “Danger” signs. Thus, we can safely assume that our lost jogger is not in this area. This significantly narrows down the area that must be searched, as that area was quite large. Next, we assumed that the lost jogger was not in one of the lakes in the area: for example, we ruled out the possibility of the jogger being in the lake on Crescent Bluff Rd. or in the lake near trail #41. He could be near the lake, but not physically in it. Consequently, our model would not encourage searchers to look in the lakes. This cuts down even more area that we would have to search. Moreover, the jogger could not be in the section of the map labeled “Private Property,” as it does not allow entrance for public viewers. The jogger also had to be on one of the signed trails, as the map specifically states, “Please use only signed trails.” This means we can focus our model on searching the trails and not in the open space in between. Making all of these assumptions allows our model to focus on certain areas of higher probability of finding the lost jogger; however, it also lends to weaknesses in the case these assumptions are not correct.

Furthermore, we assumed that the jogger had to enter Fort Ord from one of the parking areas. Logically, the jogger had to park his mode of transportation and then enter from one of the lots. Therefore, the jogger must have started his 5 mile run at one of the parking stations; this means that the jogger could not have been more than 5 miles away from one of these parking stations. Continuing this even further, if we assume that the jogger planned jog 5 miles round trip (since he would need to go back to the same parking lot to get his form of transportation, be it car or bus), then we can go as far to say that the jogger will most likely be found in an area that is less than 5 miles from one of the parking stations. Using this logic, the area that must be searched is significantly reduced, and the probability of finding the lost jogger goes up considerably, because we can focus on those areas of higher probability to search in. A chart of this particular assumption is below:



The following map on the next page utilizes this assumption to find the highest probability location of finding the lost jogger.

FORT ORD PUBLIC LANDS

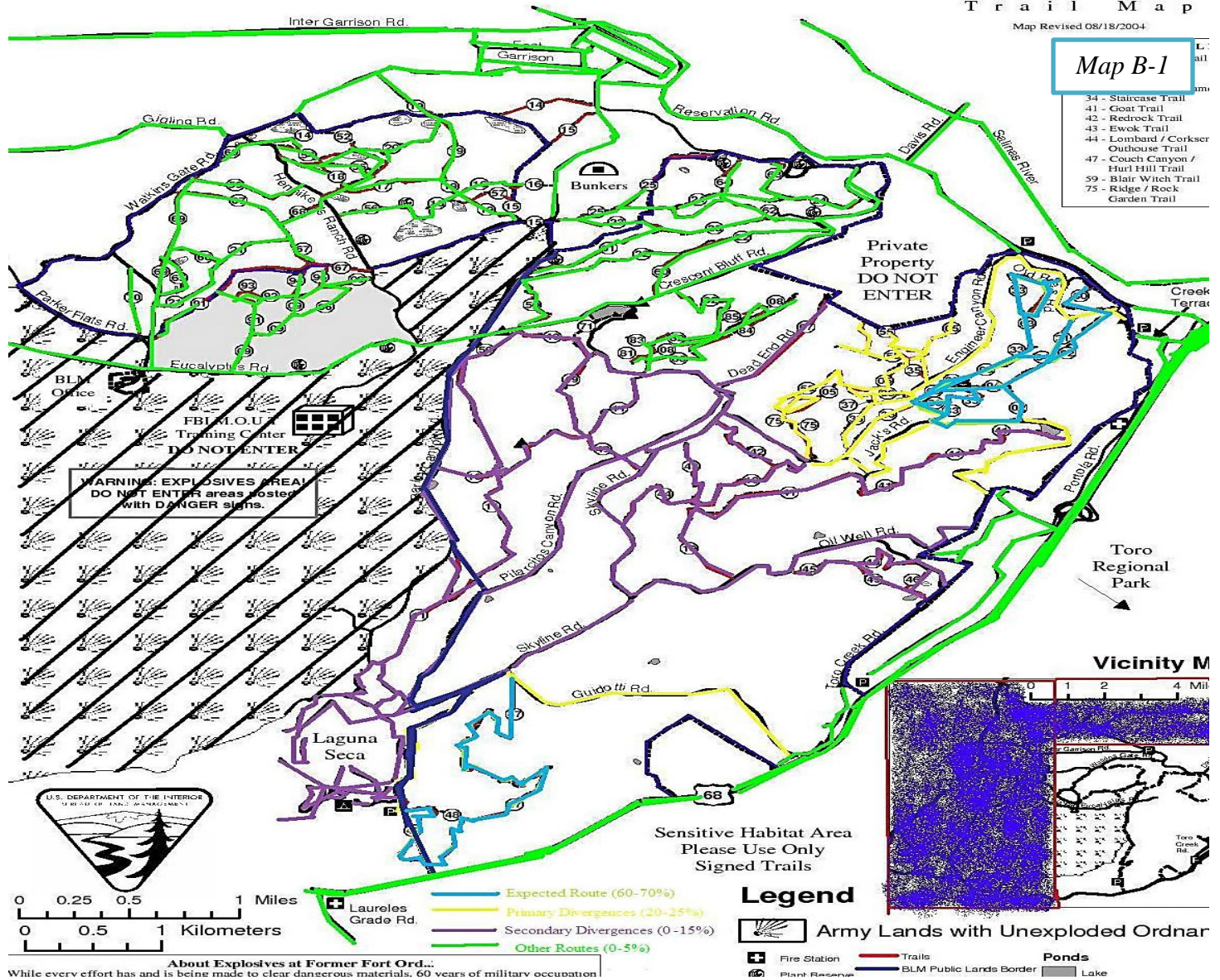


Trail Map

Map Revised 08/18/2004

Map B-1

- 34 - Staircase Trail
- 41 - Goat Trail
- 42 - Redrock Trail
- 43 - Ewok Trail
- 44 - Lombard / Corkscrew / Outhouse Trail
- 47 - Couch Canyon / Hurl Hill Trail
- 59 - Blair Witch Trail
- 75 - Ridge / Rock Garden Trail



This map represents the chance, which was estimated to the best accuracy with the information given, of finding the jogger on a certain route. Firstly, it shows the route the jogger may have intended to travel, or the expected route. These routes are relatively short due to the fact that if the jogger intended to go on a round trip, the path would have been less than 5 miles long. Next, the map shows the divergences the jogger could have made when lost. The farther away he walked, the less likely it will be to find him. The legend on the bottom shows the percentage that the jogger went on that particular road. As one can see, the higher probability of where the jogger is lies in the middle-eastern part of the map (more blue and yellow), and this correlates directly to the map below.

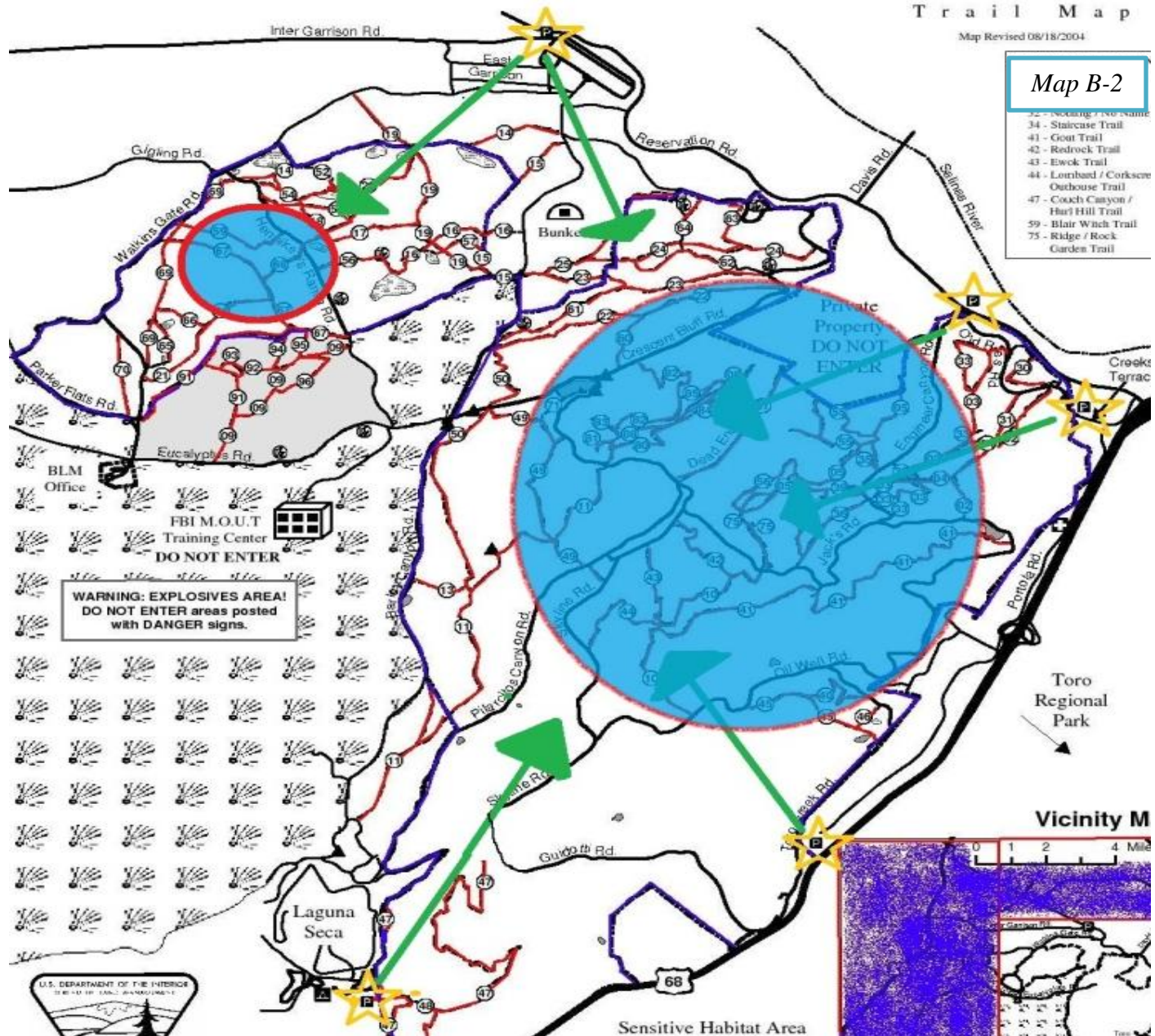
FORT ORD PUBLIC LANDS



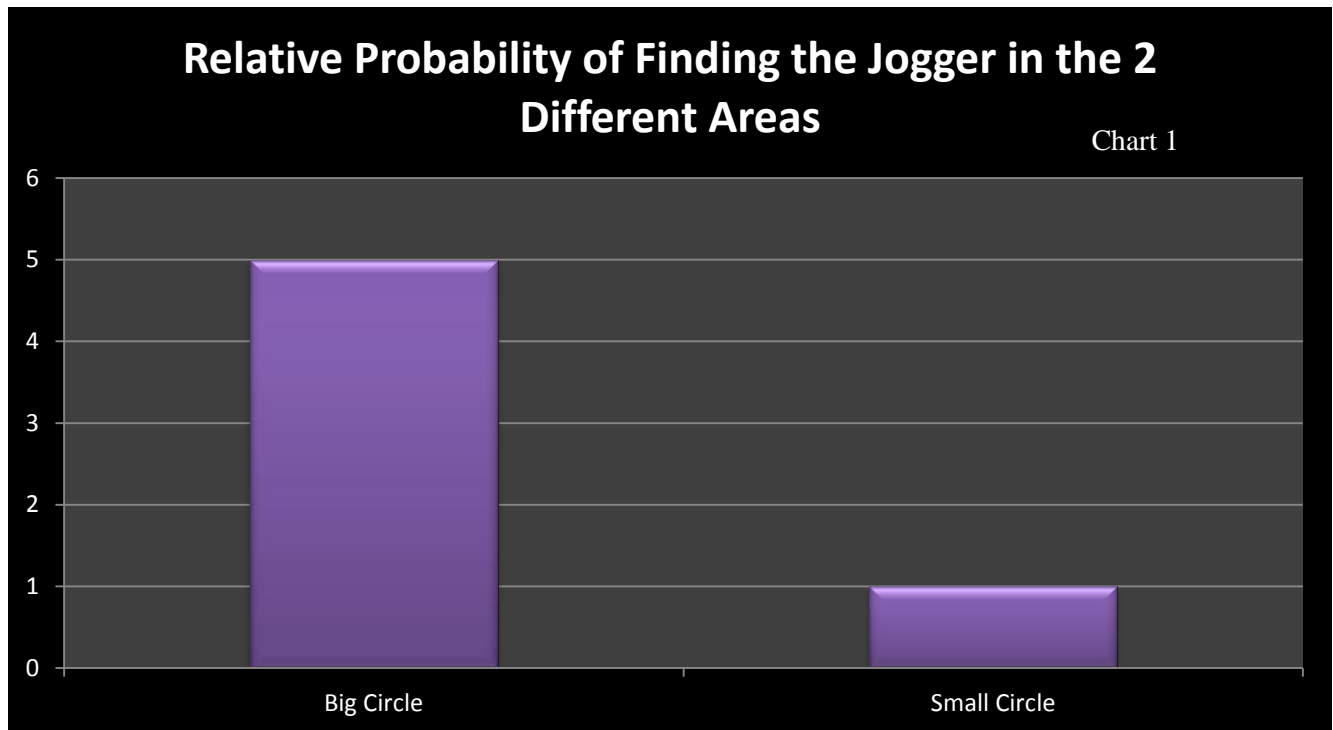
Trail Map

Map Revised 08/18/2004

Map B-2



This picture of the map shows the place of highest probability of where the jogger will be found. Since the jogger most likely started at a parking station, then he must have gone on a trail from those stations. The green arrows represent the general areas where the trails would lead the jogger from each parking station. The blue circle in the middle represents the intersections of all but one of the arrows, which means that this is the highest probability place where the jogger was located when he got lost. The other green arrow in the top left goes to a smaller area where the jogger would be if he started at that parking station. This map is specialized for the situation of if the jogger was lost in the middle of his run, either conscious or unconscious. (The arrows represent about 2.5 miles of distance, in accordance with our previous assumption) If he was lost at the end of one of his runs, and his planned route was a loop, then he would be nearer to the parking stations, as stated above. This map correlates with the map above as the Mideast is more likely and the north is unlikely.

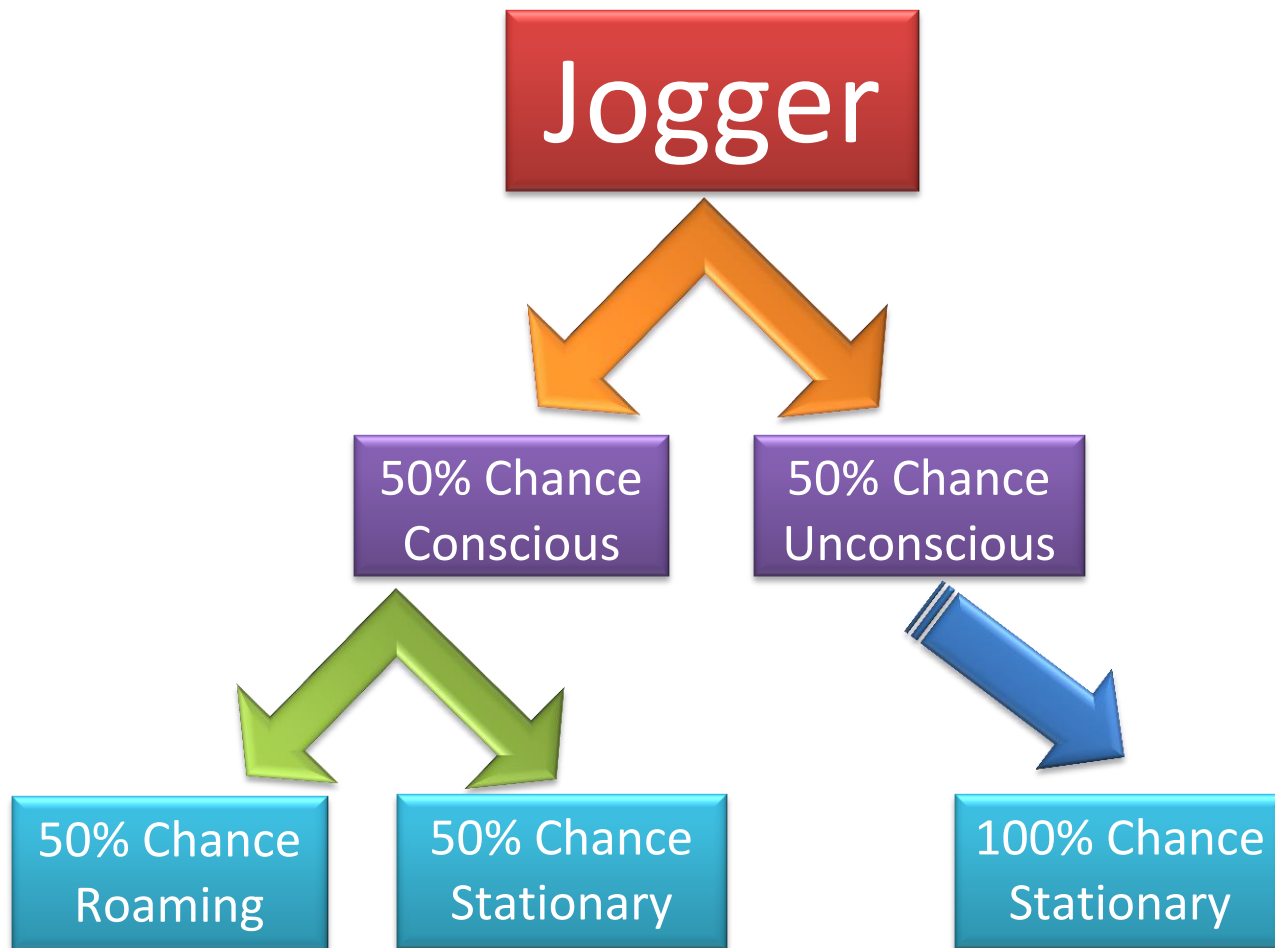


This graph shows what was represented in the map on the previous page. It simply shows that the probability of finding the jogger is 5 times as more when searching in the “Big Circle”, or the intersection of 5 out of the 6 arrows. This means that the most effective method, albeit only with the assumptions listed on the previous page, is searching in the area in the middle of the map, as there is a higher probability of finding the jogger there. This data was an integral part of the later equation we would create.

One of the other main points taken into consideration while considering a model for the situation was the condition of the jogger. For example, our model must take into account if the jogger was unconscious or not, or if he was wandering around or just staying in one place. These variables depend on each other, because if the jogger was unconscious, he obviously could not be walking around. This is important to know, because if the jogger was wandering around, then he would likely try and find his way back, and thus move out of the area of high probability he

was previously in. Since values for these situations were not given in the problem, our team came up with reasonable probabilities to represent each situation

The following algorithm helps to demonstrate the possibilities of the jogger's condition, which affects the most effective method of finding him:



With the known information, there is a 50-50 chance of the jogger being conscious and unconscious at any given moment. Each situation that the jogger could be in considerably affects our model for the jogger. For example, if the jogger was roaming, then it weakens the assumption made on the previous page (about the jogger being within 2.5 miles), as the jogger might wander into areas that are not within the 2.5 miles that we predicted. However, there is

only a 25% chance that the jogger is wandering, so it is still the best option to search within the 2.5 miles as suggested before (the middle east of the page). The other options (stationary) strengthen the assumption and the probability map shown before because if the jogger became unconscious, he is in the middle of the run and would stay in the same place that he stopped and thus not move farther ahead/move out of the predicted 2.5 miles. The probability map from before is useful for $\frac{3}{4}$ of the possible outcomes for the jogger's condition, but for the last outcome (that the jogger is wandering) another model must be created, which is the equation shown below.

The final part of our analysis was to make an equation that would take into account many factors and create a probability of finding the jogger at any given point. The equation is:

$$N = x + \frac{d+r+s}{3} + b + c + l$$

- ✚ N is the chance of finding the jogger in less than 24 hours, with the max being 100%.
- ✚ X is the value based upon the current situation of the jogger. As stated in the flow chart on the previous page, there is a 50-50 chance of the jogger being conscious or unconscious at any time. When unconscious, the jogger cannot move at all and is assigned a value of 0 as this does not affect your chance of finding the jogger positively or negatively. When conscious, there is a 50-50 chance that the jogger is roaming or inert at any given moment. When wondering around the likeliness of meeting up is determined by the number of intersections both sides meet when traveling towards each other. For example, if the jogger encounters a road with no intersections and you meet an intersection with no intersections there is a 100% chance you meet up and is assigned a

value of 1. However, if the jogger encounters a road with a two-way fork and you encounter a road with no intersections, there is a 50% chance you two will meet up and is assigned a value of $\frac{1}{2}$. Yet, this only happens in perfect conditions (theoretical probability); in reality there is the chance the two of you do not catch sight of each other and do not meet. Therefore, there must be a margin of error, estimated to be 0.6, meaning that if you both take roads with no intersections towards the same place, there is a 60% that you two will meet, and a 30% if one of you meets a two intersection. This is represented by:

$$X_{\text{number of intersections}} = \frac{1}{\text{total number of intersections}} * 0.6$$

✚ **D** is the value given by the distance from the parking lot. Assuming the jogger planned to return to the parking lot he started in, he would most likely be found <5 miles away from the parking lot he started at. Most likely found 1-2 miles or 2-3 miles away from the parking space. This is because the 4th and 5th miles are jogging back to the origin. Thus, the percentages would be 40% in the ring of a 1 and 2 mile radius, and 20% in the 3 miles radius ring. However, changes must be made to account for any errors. Once again, we assume that they may not necessarily spot each other and multiply by $\frac{3}{5}$. Thus, the percentages possible are 24% for each the first and second mile, and 12% for the 3rd mile. Outside of this ring there would still be a slight chance of finding him, around 5%.

✚ **R** is the value given depending on the route you take to find him. You should try and follow the expected route as he most likely got lost somewhere during that time. However, if he is not found on these routes follow the trails that directly diverge from the expected route, or the primary divergences, then the secondary divergences. If he is still

not found, check the other routes. Average the values from the map above and you get your chance of finding him on the route you take. Once again, multiply by $\frac{3}{5}$ and you get 39%, 13.5%, 4.5% and 1.5% as your values in respect to the different divergences.

✚ **S** is the value of whether or not you search inside the big circle in the map above. The large circle represents the area where the bulk of trails is located or lead to. Thus you have a greater chance of finding him there. As the value is 5 *times* greater (as shown in Chart 1), you have a $\frac{1}{6}$ chance of finding him outside of the circle and $5 * \frac{1}{6}$ chance finding him inside of the circle. Multiply by $\frac{3}{5}$ like before and you get 50%. The value for S will always equal either 50% or 0%

✚ **B** is the value given by the jogger's location depending on his relativity to a body of water, specifically marshes, vernal pools, and wetlands. In good weather, this does not change your chances of finding him; however during inclement weather the jogger is less likely to go to places located near bodies of water (he would not plan his route to go along these places where the weather would affect his run). Therefore, during good weather 'b' has a value of 0 and during bad weather, a value of $0\% \leq b < 12\%$. This is because when bad weather occurs, the jogger would be unlikely to go in those aforementioned areas, which means that is less area you must search in. These ranges of values were assigned by dividing the amount of wetland related area by the total area in the park (rough estimates were used for the wetland area calculation).

✚ **C** is the value of light given from circumstances. This includes from light from the pen light, darkness due to time of the night, and obstructions due to weather. Most penlights are relatively weak and may only provide a few meters worth of light; it is assigned a value of 5% to 3%. As night continues you receive less and less light from natural

sources giving you a value of -2% to 0%. Weather may also obstruct your view and gives a value of -2% to 0. To find the overall value from “C”, add up the values from each subsection.

✚ L is the constant value given from landmarks. Had the jogger reached a landmark, he would not be lost anymore. Thus, eliminating searching near landmarks would give you a value of about 5% because since the jogger will not be in that area, then you don’t need to waste your time searching near the landmarks. It gives you less land to look, and this means a higher probability of finding the jogger.

Next, we validated the model by testing values into it:

➤ X=50%, D=24%, R=13.5%, S=50%, B=8%, C=1%, L=5%

$$N = 50\% + \frac{24\% + 13.5\% + 50\%}{3} + 8\% + 1\% + 5\%$$

$$N = 93.2\%$$

With these favorable values, there is a 93.2% chance of finding the lost jogger according to our model. This validates the model; other values were also used and found to work in the model. What we found as a general trend is that if one searches in the higher probability area (the larger circle), then the chances of finding the jogger go up considerably. That variable (S) along with X is the most important variables in terms of how much they affect the overall probability of finding the jogger. Thus, they were an important part of our most effective method.

Not only can this equation give the probability of finding the jogger, but it can also help to find the most effective method of searching for the lost jogger. Since we now know, with the equation, that the most important factors in finding the lost jogger are the places in which to

search for him, we can focus our search, as mentioned before, on the places of greatest likelihood. Shown in the maps before, the places of greatest probability of finding the lost jogger is in the middle-eastern section of the map (where the large circle is in Map B-2). Thus, when searching for the lost jogger, we must make an intense search in those areas, for they will lead to the greatest probable success. However, as the equation shows, we must take into account the possibility that the jogger wandered away from that high probability area. To account for this we have come up with this plan, which is our most effective method:

1. First search the areas that have been determined as the highest probability of the jogger being there. This should lead to success, as it is extremely likely that the jogger is in this area (as shown by the maps and chart before).
2. If the jogger is not found in the designated “high probability” area, then search in the areas that have been designated as tier 2 (3 miles away from the parking stations.) This is the area where the jogger would be if he was not found in the first area.
3. If the jogger is not found in the tier 2 areas, then look in the previously disregarded areas of our model (the explosives area and other tier 3 areas). There is only a very small probability that the jogger would be in these obscure areas, though, so it is much more likely that he is found before this step. He is likely in the areas that would be searched in steps 1 and 2.

Strengths and Weaknesses of the Model for Part B

Our model for Part B of the problem has many strengths, as well as some weaknesses. The strengths for this particular model are that it is in essence 3 parts that work together to form

a more comprehensive model than each one could be on its own. To elucidate, the two maps, along with the equation, work together to form a more precise model. The two maps complement each other, as they both strongly demonstrate the area of the highest probability of where the jogger would be. Then, the equation adds in other factors to consider, which allows for a more precise model that considers the possibility that the jogger was in fact conscious (the maps go by the assumption the jogger was unconscious and that he was not wandering around). By using all three components together, our model is strong and can come up with the most effective method that is described throughout our paper.

Although our model has a myriad of strengths, it also has its share of weaknesses. For example, we had to assume some things for our model to work (such as the jogger had a planned route that was a loop of 5 miles, meaning he would end up in the same place he started). Moreover, since exact data was not given in the prompt, we had to estimate some of the values in our equation, but we did the best with what we researched and after deliberations as a team. These were some of the weaknesses in our model.

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