**Abstract**

Unmanned Aerial Vehicles (UAVs), commonly known as drones, have recently been applied in choreographed light shows and thus evoked a new boom in technology. In 2016, Intel® used a cluster of its Shooting Star™ drones that were controlled by only one pilot and a single laptop to perform an aerial light show, resulting in a twinkling digital galaxy floating above the skyline. Yet Intel is not the only company interested in aerial shows using drones since this newly formed technology has been brought under the spotlight and remained to be a heating topic.

Other related researches and finding also focused on the control and orientation of drones in application.

In response to the Mayor’s asking of an outdoor aerial light show using drones, after a careful investigation, we would like to present our mathematical model, conclusions and recommendations for the aerial light show. In brief, we address the problem of optimizing the flight paths for each drone through mapping the locus function with three-dimensional system of coordinates, aiming at finding the solution that has the shortest total flight paths as possible and satisfies the safety requirements.

Based on the historical data from all the aerial light shows using drones that have already taken place so far, we set the lower bounds to avoid crashes and the upper bounds to guarantee a visual feast within the sight for the distance between two drones referring to the basic parameters of drones. Then we use the Roberts edge detection operator to form a simplified pattern of our tridimensional design of display——a Ferris wheel, a dragon and a map of China——and pick out points that satisfy the distance bounds, calculating the coordinates of all the drones according to the coordinate system we set up. In search of the optimal solution that accomplishes both of our goals, we calculate with Euclidean distance to find the minimum flight distance for each drone, group those flight paths that share similar characteristics to match drones using clustering analysis, utilize binary integer programming to find the global optimal solution, adjust the flight paths using quadratic function to avoid crashes, write a detect function with vectors to check for colliding paths, and add Bezier curve to the rotation of our design after forming the horizontal image to make the transition more vivid. To ground this model in reality, we animate the whole display process through computer simulation and present the flight paths with mathematical functions.

Our suggested solution, which is easy to implement, includes a detailed aerial display program and flight paths for each drone. Our model showcases the improving process and the contrast among various methods on the road to finding the optimal solution. According to the sensitivity analysis, we firmly believe that our algorithm is broad and flexible enough to accommodate various local conditions, safety concerns and different parameters of drones. Since our model is based on the control and orientation of UAVs, our strategy may also contribute to other technologies related to drones including combat drone operation and orientation system.

**Key words**: Euclidean distance, the Roberts edge detection operator, binary integer programming, Bezier curve, clustering analysis

**Letter to the Mayor**

Dear Mayor,

Our team has carefully planned the light show on the night of the annual festival and succeeded in creating three possible sky displays including the pattern of a Ferris wheel, a dragon and a map of China. The whole performance will include 477 drones, and all the people in this city can enjoy this well-organized fantastic visual feast.

Taking the maximum flight time limit of drones and the overall show plan demanded into consideration, our team arrange the drone light show to last for 10 minutes. To be more specific, the actual flying time in total is 2 min and the rest of the time would be used for performance. The apron required for taking off has to cover approximately 69m×57m. all the drones are configured as a 24 drones×22 drones rectangle with 51 empty positions. As for the space needed in the midair, the flight height is within the range of 63m~224m. according to our careful calculation, the audience’s viewing position should be arranged approximately 123.48m from our light show in order to enjoy the visual feast from the best view.

The brightest shining point of our display is the third image designed. The broad territory of China fully unfolds before the audience which showcases the immensity of our motherland China, representing the uniqueness of our nationality. Another great brilliance that lies thoroughly in our model is how we improve our algorithm to achieve the goal of finding the optimal solution that has the shortest total flight distance possible with no crashes. Remarkable progress has been made through our adjustment for each drone since the total number of crashed drones is lowered down from 61 to 0. Since our model involves 477 drones, it’s clearly a great challenge to ensure that any two flying tracks do not possess intersections. Yet we manage to conquer the challenge and successfully present a structural model in response to the task.

Our model effectively achieves the goal of finding the shortest total flight distance without any crashes to meet the safety requirements. It is definitely a feasible solution and could handle large quantities of data. Admittedly, there remain several flaws in our robust and effective model. But we firmly believe that with a larger number of drones, more adjustment of the flight paths, and more factors being taken into consideration, the model can be improved to a higher and more realistic level. In addition, our model generalizes the algorithm used in the control and orientation of UAVs, flexible and broad enough to accommodate various local conditions, safety concerns and other unexpected incidents. We proudly declare that the application of our model maintains a vast potential for future development including combat drone control and orientation system.

Attached on the next page is our designed images for the aerial light show. We express our sincerest gratitude for your trust in us to organize this festive event, which we hope will develop into a worldwide carnival in which everyone enjoys and appreciates our drone light show.

Best Wishes

Team