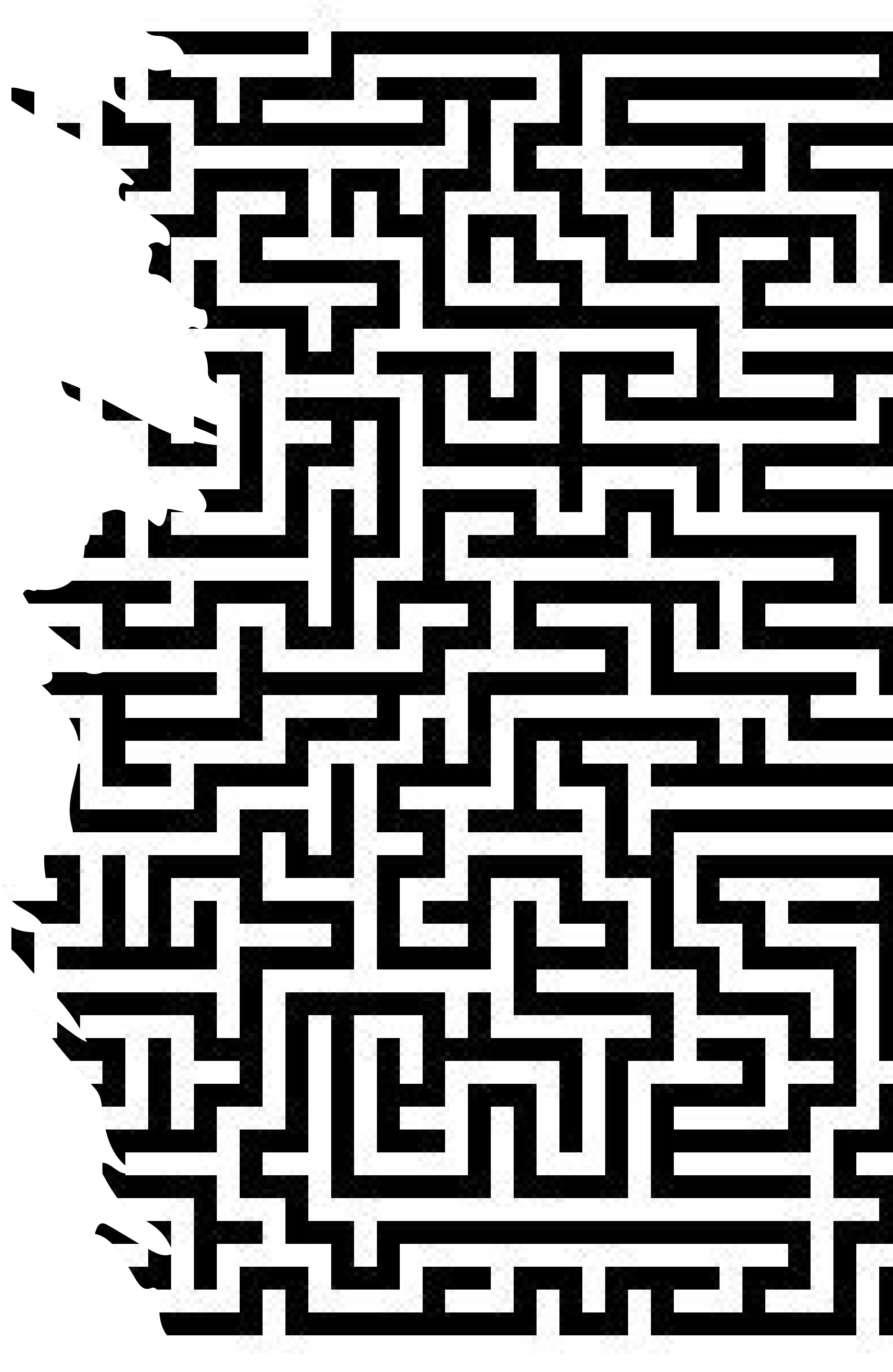
Advanced Artificial Intelligence

Solving problems by searching

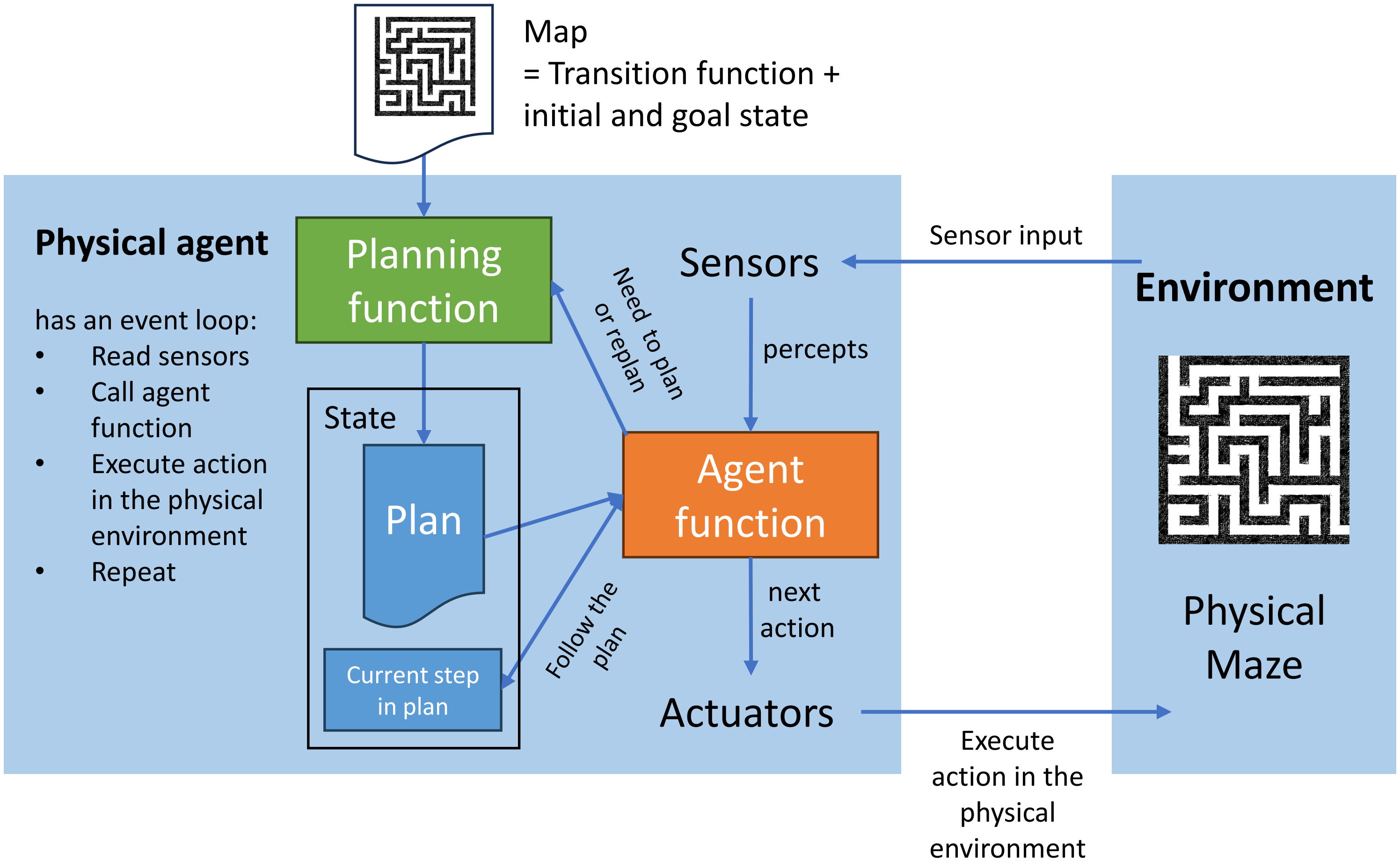
AIMA Chapter 3

Module Review

Intro and Uninformed Search



Complete Planning Agent to Solve a Maze



- The event loop calls the agent function for the next action.
- The agent function follows the plan or calls the planning function if there is no plan yet or it thinks the current plan does not work based on the percepts (replanning).

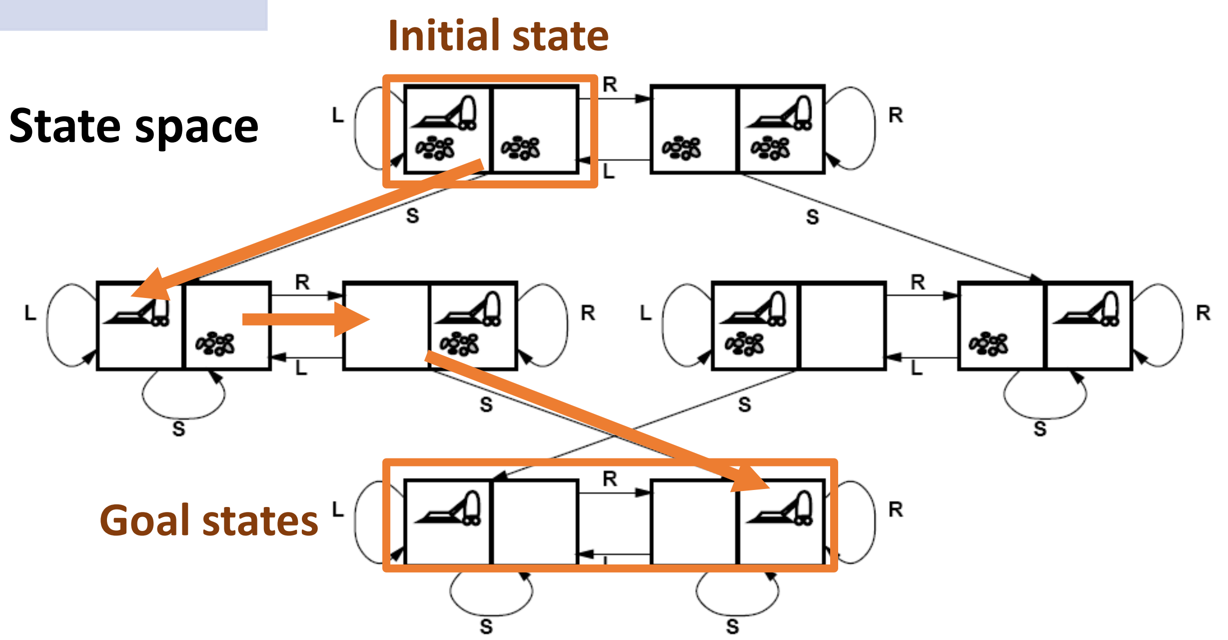
Solving Search Problems

Given a search problem definition

- Initial state
- Actions
- Transition model
- Goal state
- Path cost

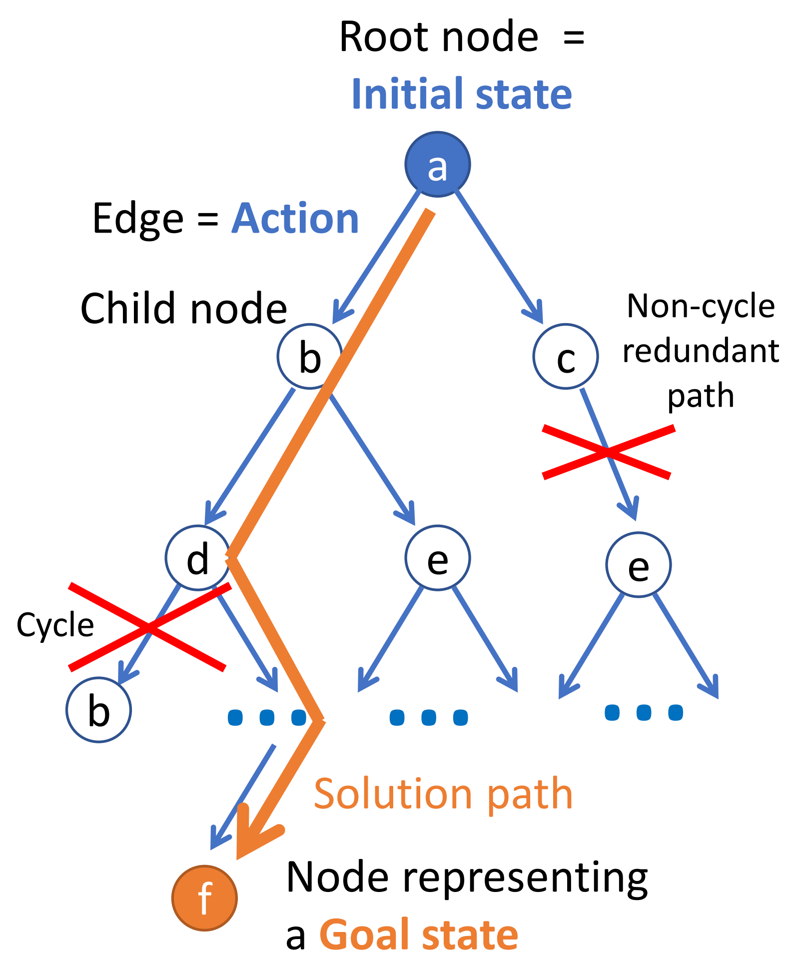
How do we find the optimal solution (sequence of actions/states)?

Construct a search tree for the state space graph!



Creating a Search Tree

- Superimpose a “what if” tree of possible actions and outcomes (states) on the state space graph.
- The **Root node** represents the initial stare.
- An action child node is reached by an **edge** representing an action. The corresponding state is defined by the transition model.
- Trees cannot have **cycles (loops).** Cycles in the search space must be broken to prevent infinite loops.
- Trees cannot have **multiple paths to the same state.** These are called redundant paths. Removing other redundant paths improves search efficiency.
- A **path** through the tree corresponds to a sequence of actions (states).
- A **solution** is a path ending in a node representing a goal state.
- **Nodes vs. states**: Each tree node represents a state of the system. If redundant path cannot be prevented then state can be represented by multiple nodes in the tree.



Differences Between Typical Tree Search and AI Search

Typical tree search

- Assumes a given tree that fits in memory.

- Trees have by construction no cycles or redundant paths.

AI tree/graph search

- The search tree is too large to fit into **memory**.
  a. **Builds parts of the tree** from the initial state using the transition function representing the graph.
  b. **Memory management** is very important.

- The search space is typically a very large and complicated graph. Memory-efficient **cycle checking** is very important to avoid infinite loops or minimize searching parts of the search space multiple times.
- Checking redundant paths often requires too much memory and we accept searching the same part multiple times.

Summary: All Search Strategies

b: maximum branching factor of the search tree
d: depth of the optimal solution
m: maximum length of any path in the state space
C*: cost of optimal solution

	Algorithm	Complete?	Optimal?	Time complexity	Space complexity
Uninformed Search	**BFS (Breadth-first search)**	Yes	If all step costs are equal	$O(b^d)$	$O(b^d)$
	Uniform-cost Search	Yes	Yes	Number of nodes with $g(n) \leq C^*$	
	DFS	In finite spaces (cycles checking)	**No**	$O(b^m)$	$\boxed{O(bm)}$
	IDS	Yes	If all step costs are equal	$O(b^d)$	$\boxed{O(bd)}$
Informed Search	**Greedy best-first Search**	In finite spaces (cycles checking)	No	Depends on heuristic Best case: $O(bd)$ Worst case: $O(b^m)$	
	A* Search	Yes	Yes	Number of nodes with $g(n) + h(n) \leq C^*$	

With a good heuristic

Breadth-First Search (BFS)

All nodes are in memory!

Expansion rule: Expand shallowest unexpanded node in the frontier (=**FIFO**).

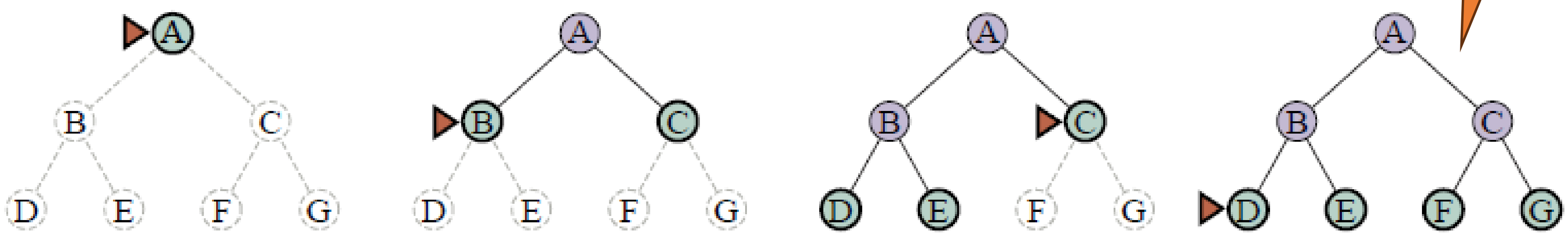


Figure 3.8 Breadth-first search on a simple binary tree. At each stage, the node to be expanded next is indicated by the triangular marker.

Data Structures

- **Frontier** data structure: holds references to the green nodes (green) and is implemented as a FIFO **queue**.
- **Reached** data structure: holds references to all visited nodes (gray and green) and is used to prevent visiting nodes more than once (cycle and redundant path checking).
- Builds a **complete tree** with links between parent and child.

This is a generalization of Breadth-first search that expands the search based on cost and not on the number of steps.

Implementation: Best-First Search Strategy

function UNIFORM-COST-SEARCH(*problem*) **returns** a solution node, or *failure*
  return BEST-FIRST-SEARCH(*problem*, PATH-COST)

function BEST-FIRST-SEARCH(*problem*, *f*) **returns** a solution node or *failure*
  node ← NODE(STATE=*problem*.INITIAL)
  frontier ← a priority queue ordered by *f*, with *node* as an element
  reached ← a lookup table, with one entry with key *problem*.INITIAL and value *node*
  while not IS-EMPTY(*frontier*) **do**
    node ← POP(*frontier*)
    if *problem*.IS-GOAL(*node*.STATE) **then return** *node*
    for each *child* **in** EXPAND(*problem*, *node*) **do**
      s ← *child*.STATE
      if *s* is not in *reached* **or** *child*.PATH-COST < *reached*[*s*].PATH-COST **then**
        reached[*s*] ← *child*
        add *child* to *frontier*
  return *failure*

The order for expanding the frontier is determined by *f(n)* = path cost from the initial state to node *n*.

This check is added to BFS! It visits a node again if it can be reached by a better (cheaper) path.

See BFS for function EXPAND.

Depth-First Search (DFS)

- **Expansion rule**: Expand deepest unexpanded node in the frontier (last added).
- **Frontier**: **stack** (LIFO)
- **No reached data structure!**

Cycle checking checks only the current path.

Redundant paths can not be identified and lead to replicated work.

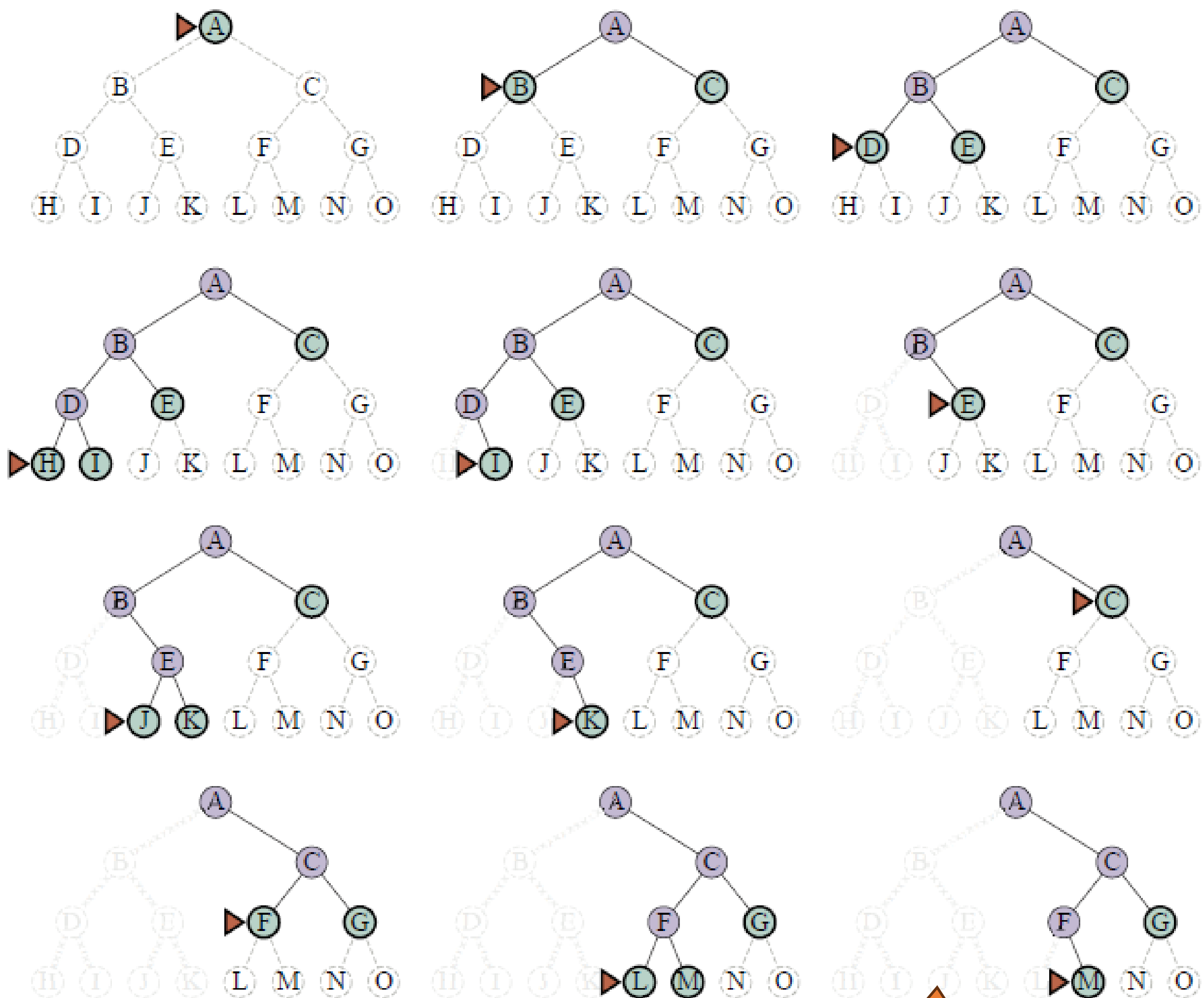
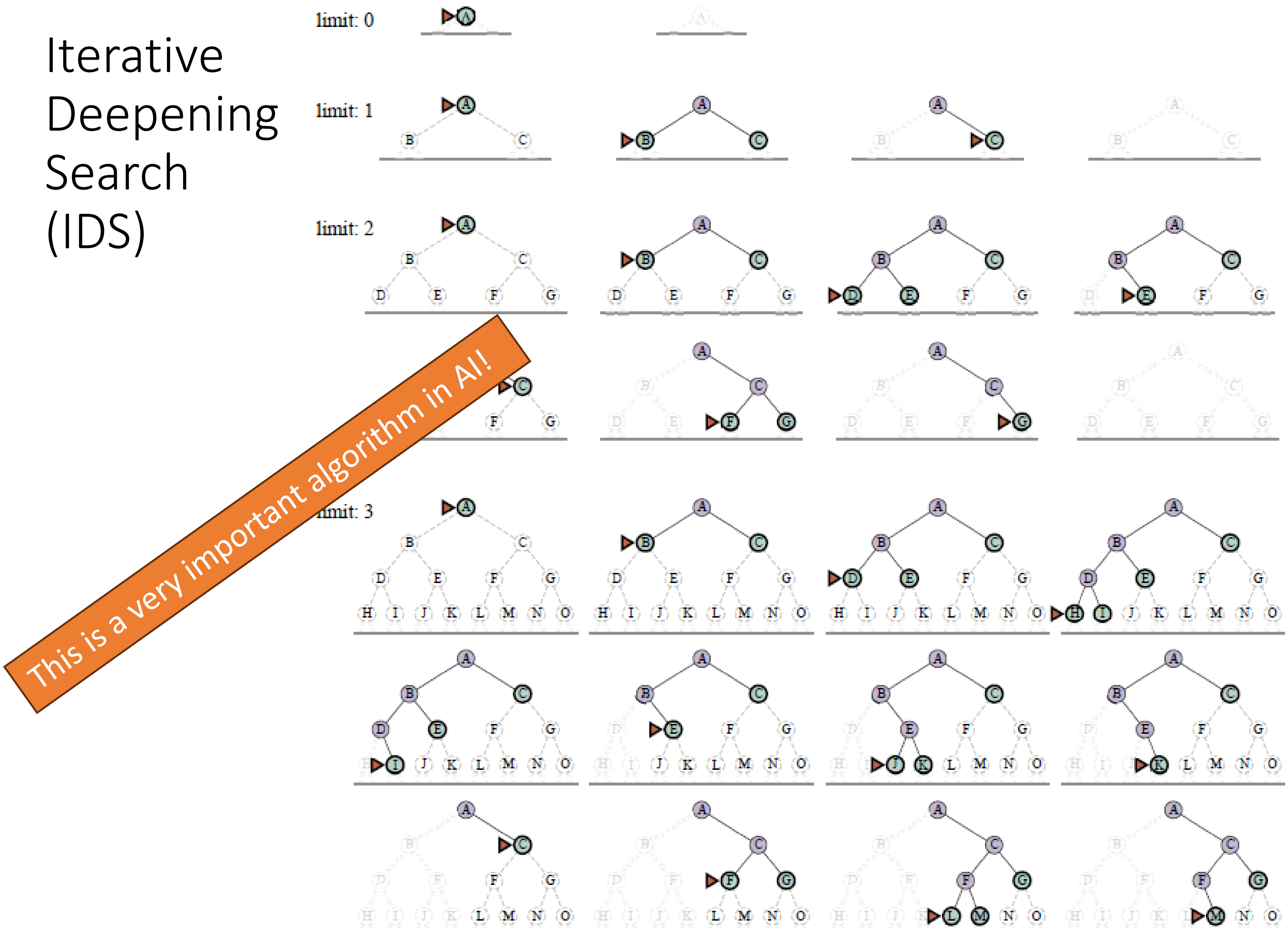


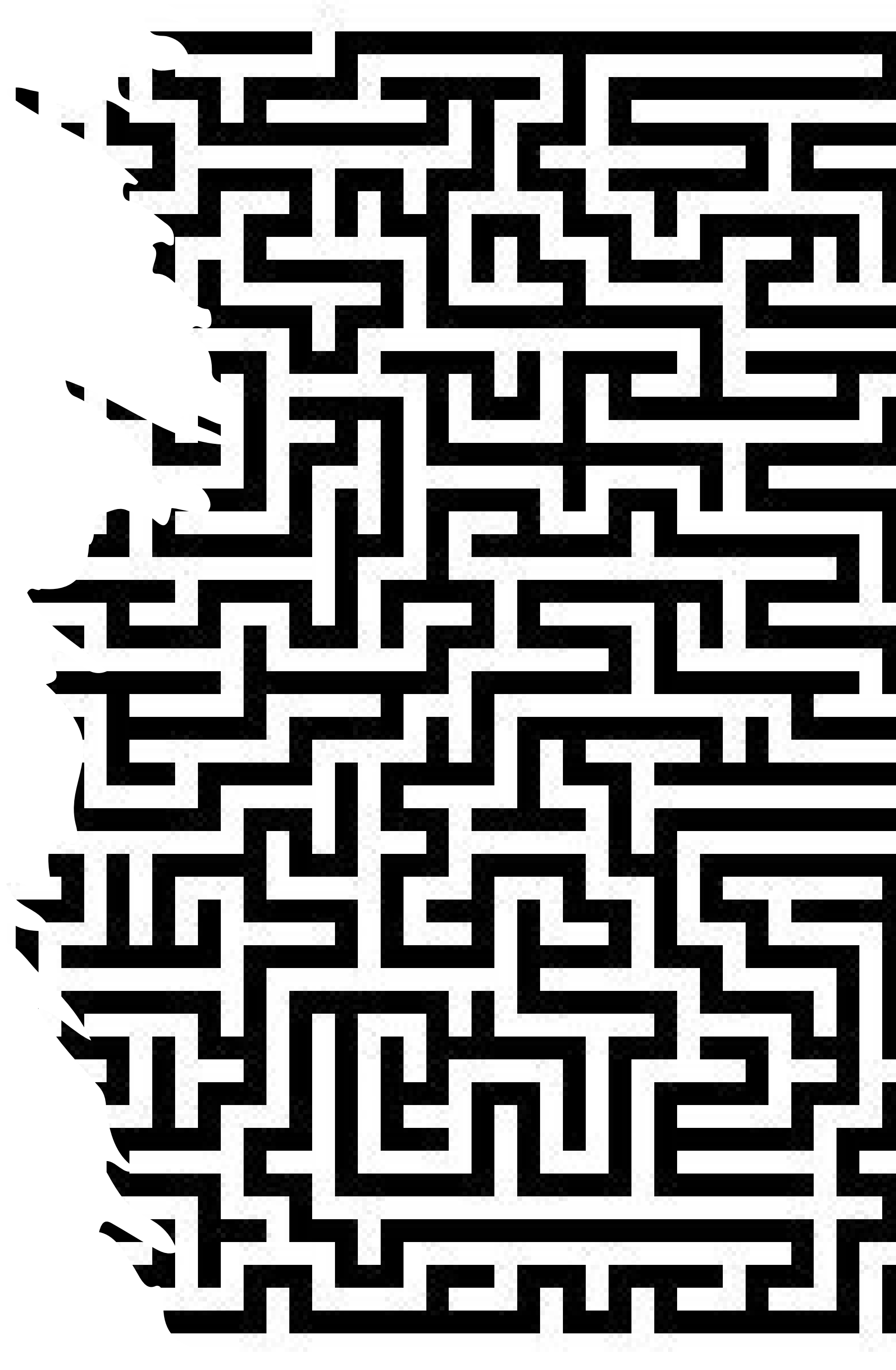
Figure 3.11 A dozen steps (left to right, top to bottom) in the progress of a depth-first search on a binary tree from start state A to goal M. The frontier is in green, with a triangle marking the node to be expanded next. Previously expanded nodes are lavender, and potential future nodes have faint dashed lines. Expanded nodes with no descendants in the frontier (very faint lines) can be discarded.

Iterative Deepening Search (IDS)



Module Review

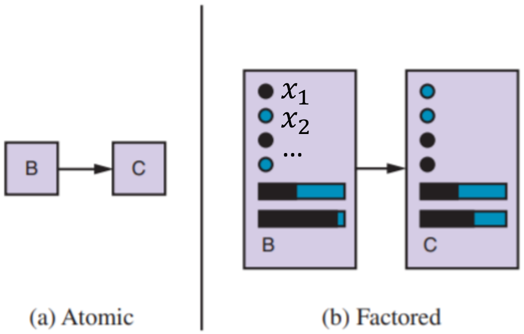
Statespace and Search Complexity



State Space

- Number of different states the agent and environment can be in.
- **Reachable states** are defined by the initial state and the transition model. Not all states may be reachable from the initial state.
- **Search tree** spans the state space. Note that a single state can be represented by several search tree nodes if we have redundant paths.
- State space size is an indication of problem size.

State representation



(a) Atomic (b) Factored

The state consists of variables called fluents that represent conditions that can change over time.

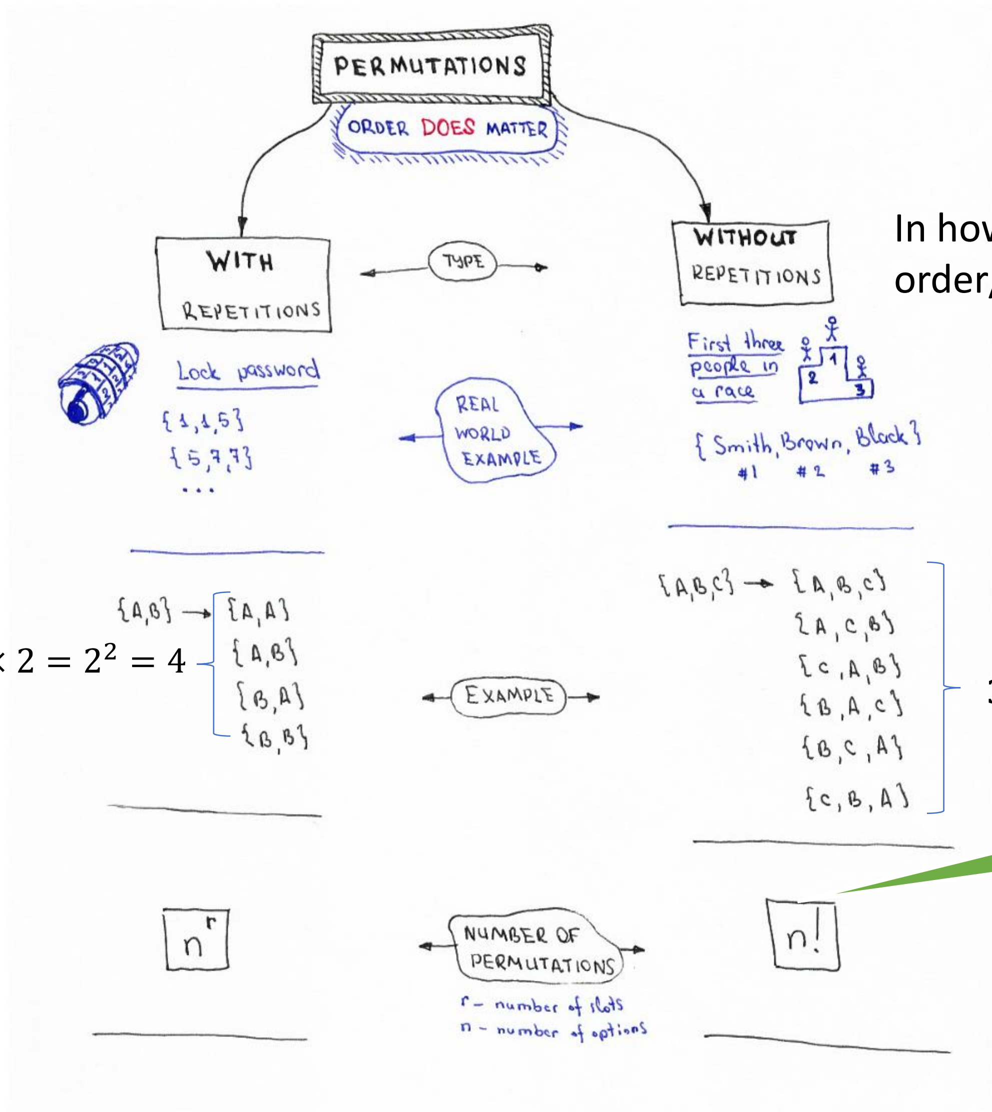
State Space Size Estimation

- Even if the used algorithm represents the state space using atomic states, we may know that internally they have a factored representation that can be used to estimate the problem size.
- The basic rule to calculate (estimate) the state space size for factored state representation with n fluents (variables) is:

$$|x_1| \times |x_2| \times \cdots \times \; |x_n|$$

where $|\cdot|$ is the number of possible values.

In how many ways can we order/arrange n objects?



$2 \times 2 = 2^2 = 4$

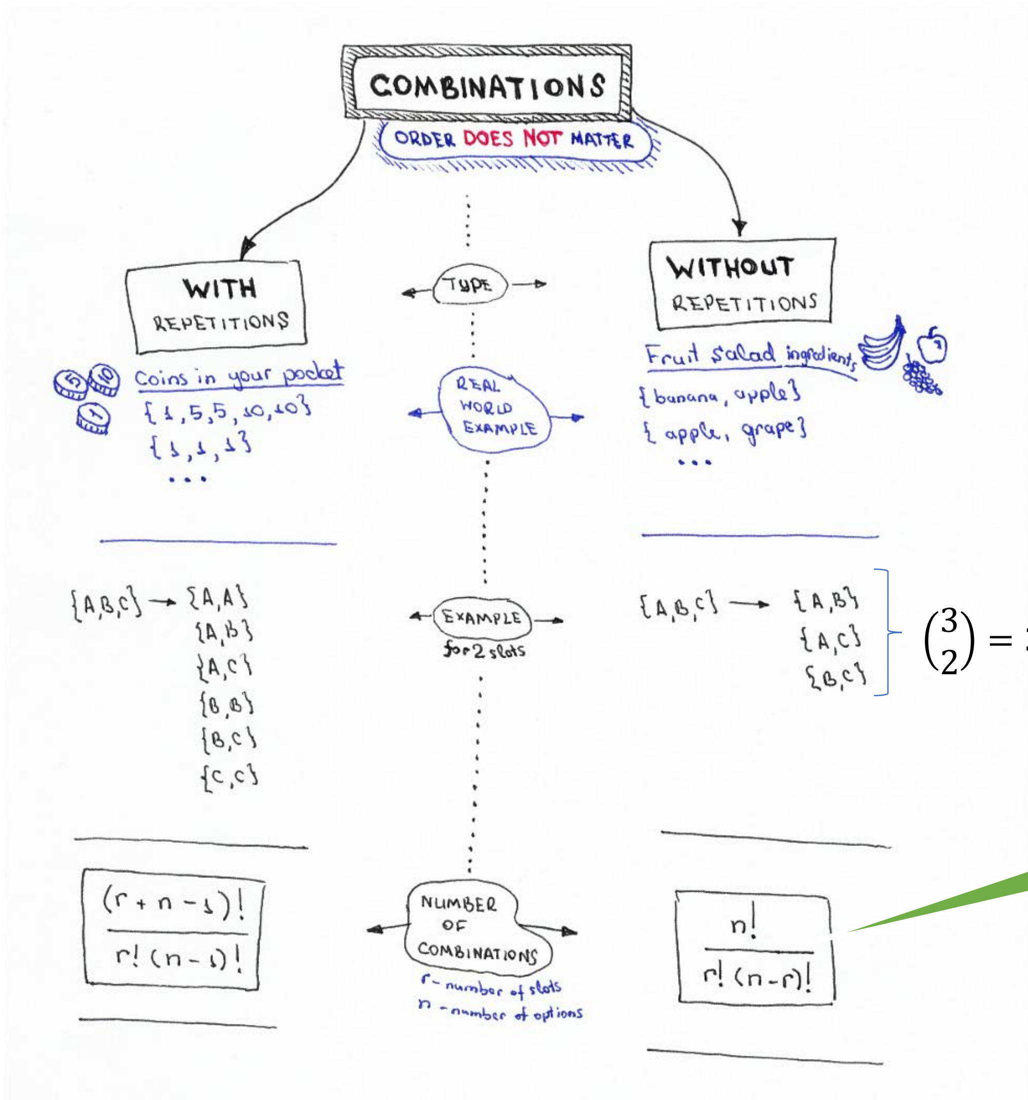
$3 \times 2 \times 1 = 6$

Factorial: $n! = n \times (n-1) \times \cdots \times 2 \times 1$

```
#Python
import math

print (math.factorial(23))
```

Source: Permutations/Combinations Cheat Sheets by Oleksii Trekhleb
https://itnext.io/permutations-combinations-algorithms-cheat-sheet-68c14879aba5



$$\binom{3}{2} = 3$$

Binomial Coefficient: $\binom{n}{r} = C(n,r) = {}_nC_r$

Read as "n choose r" because it is the number of ways can we choose r out of n objects?

Special case for $r = 2$: $\binom{n}{2} = \frac{n(n-1)}{2}$

```
#Python
import scipy.special

# the two give the same results
scipy.special.binom(10, 5)
scipy.special.comb(10, 5)
```

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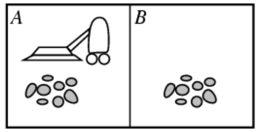
Example: What is the State Space Size?



Dirt

- **Permutation:** A and B are different rooms, order does matter!
- **With repetition:** Dirt can be in both rooms.
- There are 2 options (clean/dirty)

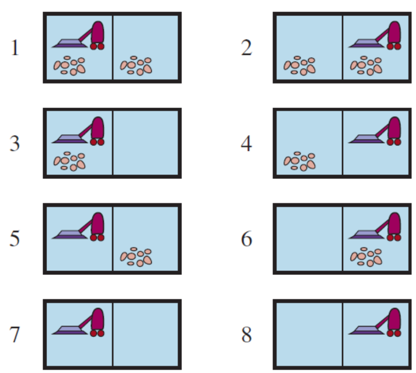
$\rightarrow 2^2$

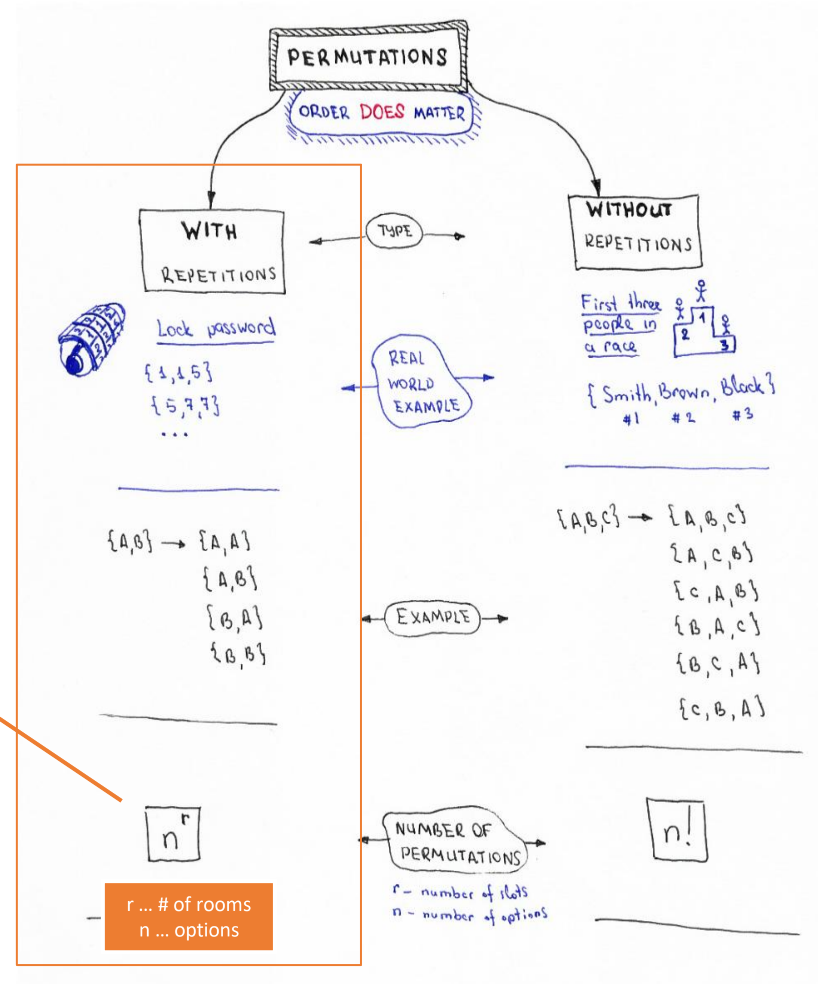
Robot location

- Can be in 1 out of 2 rooms.

$\rightarrow 2$

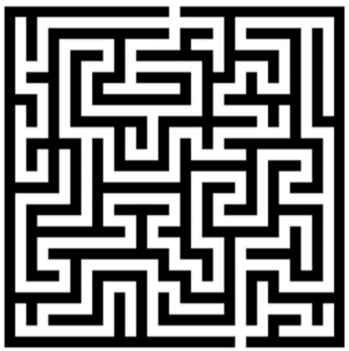
Total: $n = 2 \times 2^2 = 2^3 = 8$



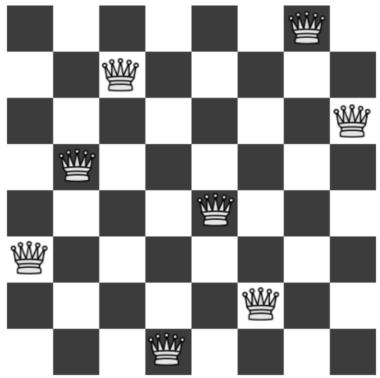


Examples: What is the State Space Size?

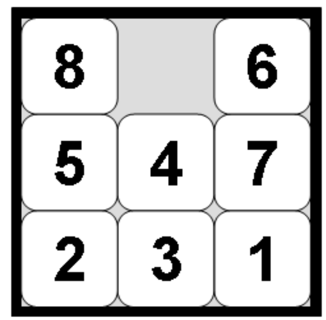
Often a rough upper limit is sufficient to determine how hard the search problem is.



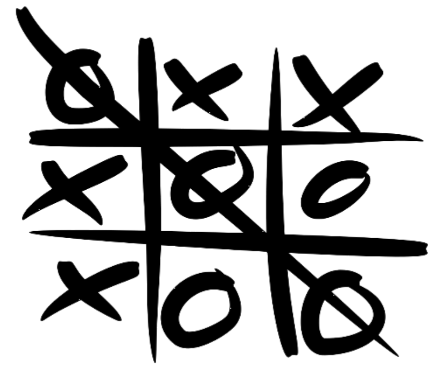
Maze



8-queens problem



8-puzzle problem



Tic-tac-toe

Examples: What is the State Space Size?

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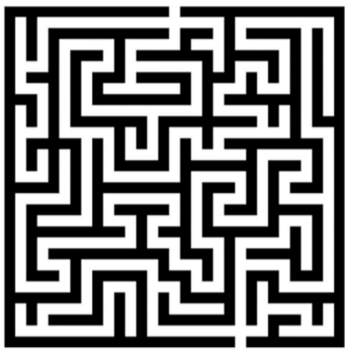
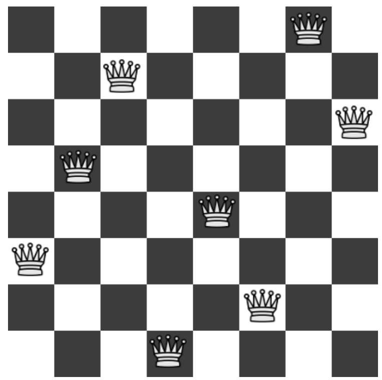


Maze

Positions the agent can be in.

n = Number of white squares.

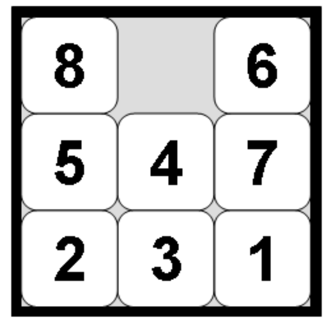


8-queens problem

All arrangements with 8 queens on the board.

$n < 2^{64} \approx 1.8 \times 10^{19}$

We can only have 8 queens:
$n = \binom{64}{8} \approx 4.4 \times 10^{9}$

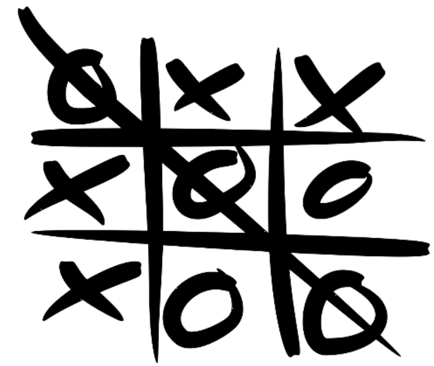


8-puzzle problem

All arrangements of 9 elements.

$n \leq 9!$

Half is unreachable:
$n = \frac{9!}{2} = 181{,}440$



Tic-tac-toe

All possible boards.

$n < 3^{9} = 19{,}683$

Many boards are not legal (e.g., all x's)

The actual number can be obtained by a depth-first traversal of the game tree.

Example: What is the Search Complexity?

- b: maximum branching factor = number of available actions?

  3

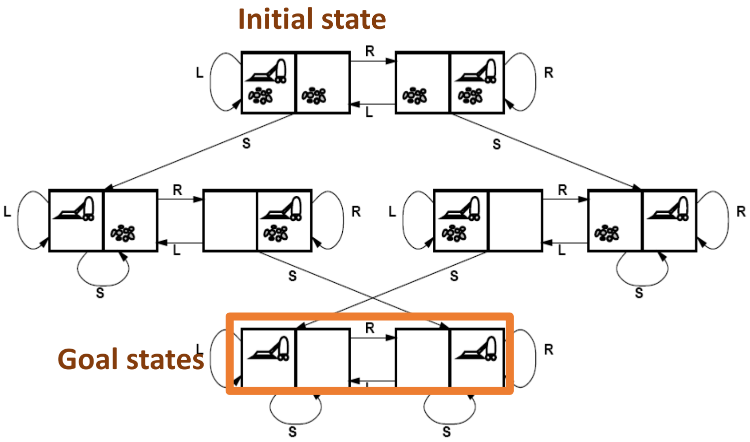
- m: the number of actions in any path? Without loops!

  4

- d: depth of the optimal solution?

  3

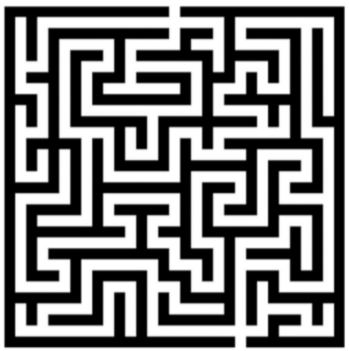
State Space with Transition Model



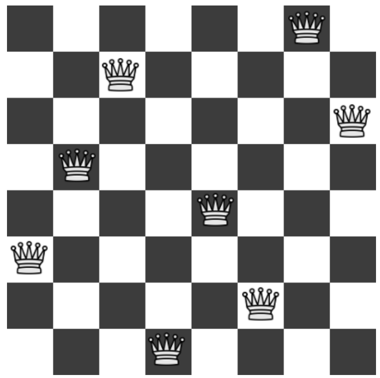
Examples: What is the Search Complexity?

b: maximum branching factor
m: max. depth of tree
d: depth of the optimal solution

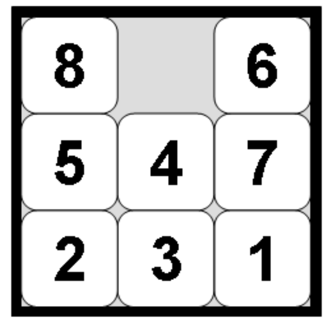
Often a rough upper limit is sufficient to determine how hard the search problem is.



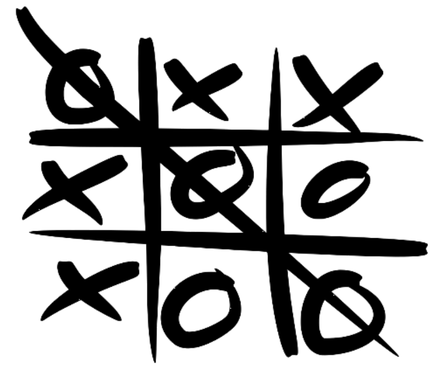
Maze



8-queens problem



8-puzzle problem



Tic-tac-toe

$b =$
$m =$
$d =$

Examples: What is the Search Complexity?

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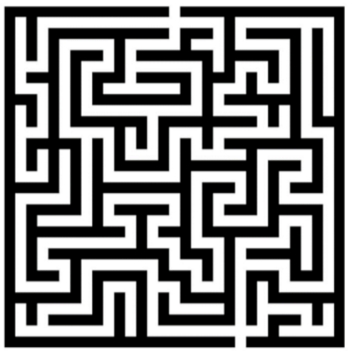
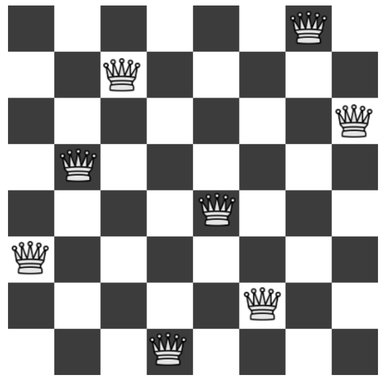


Maze

$b\ = 4$ actions

$m =$ longest path to the goal or a dead end (bounded by $x \times y$)

$d\ =$ shortest path to the goal (bounded by $x \times y$)

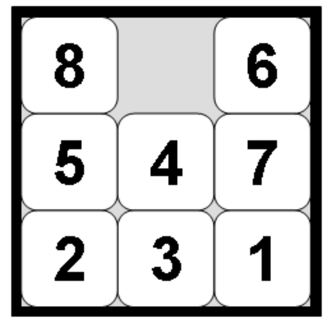


8-queens problem

$b\ =$? What are the actions? Moving one Queen: $64 - 7 = 57$

$m =$ We may have to try all: $\binom{64}{8} \approx 4.4 \times 10^9$

$d\ =$ move each queen in the right spot = 8

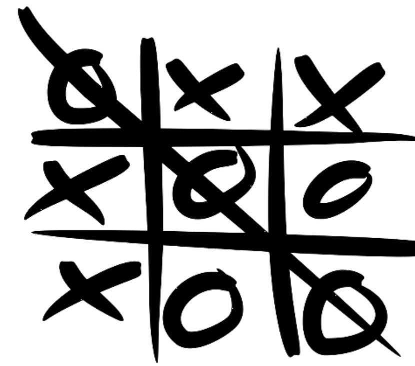


8-puzzle problem

$b\ = 4$ actions to move the empty tile.

$m =$ Try all $O(9!)$

$d\ =$???



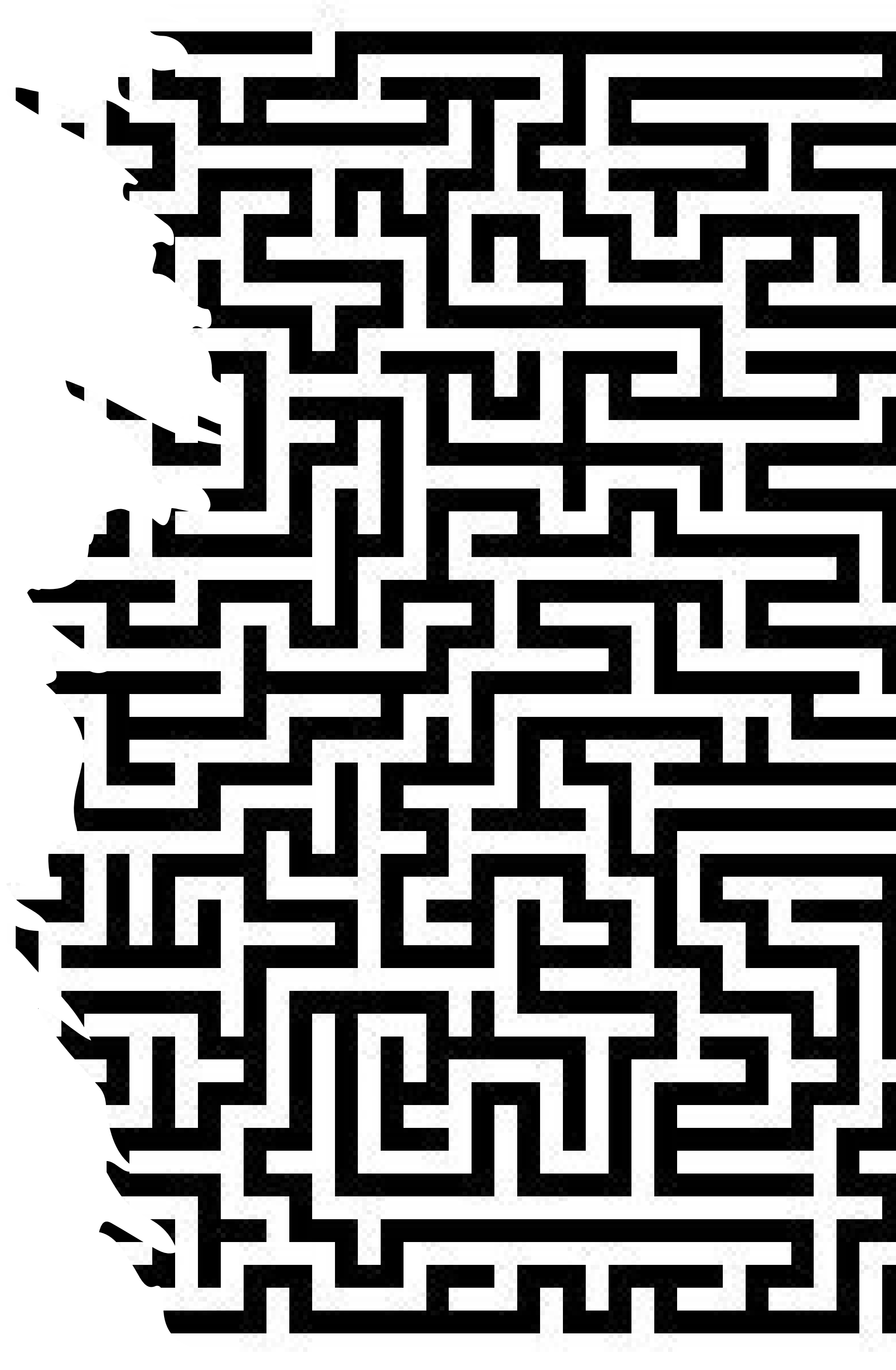
Tic-tac-toe

$b\ = 9$ actions for the first move.
$m = 9$

$d\ = 9$ (if both play optimal)

Module Review

Informed Search



Summary: All Search Strategies

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Informed Search	**Greedy best-first Search**	In finite spaces (cycles checking)	No	Depends on heuristic Best case: $O(bd)$ Worst case: $O(b^m)$	
	A* Search	Yes	Yes	With a good heuristic Number of nodes with $g(n) + h(n) \leq C^*$	

Properties of Heuristic Functions

- Evaluates a given node n.
- Provide a **good approximation** of the actual cost from node n to the goal state.
- Can be computed using additional **information that is known** to the agent or can be obtained via percepts.
- Are **fast to compute** (compared to solving the problem).

- **For A* Search**: be admissible (never overestimate the cost)

- Search algorithms will expand nodes with better heuristic values/estimated total cost first.

Implementation

Greedy Best-First search

Best-First Search

+

Expand the frontier using $f(n) = h(n)$

A* Search

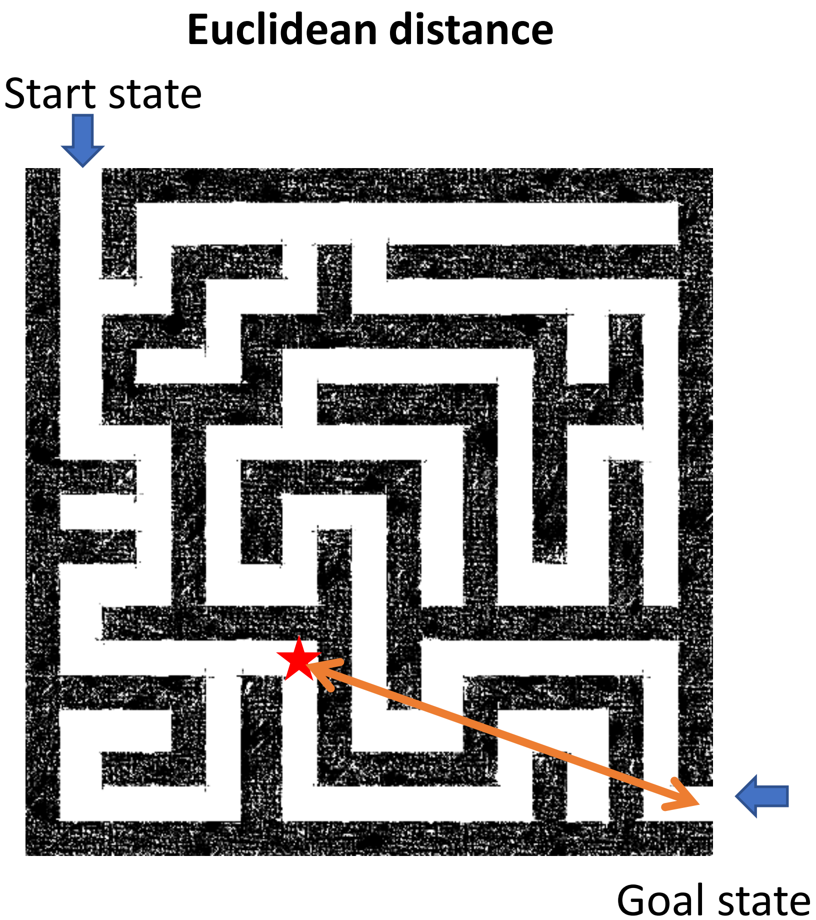
Best-First Search

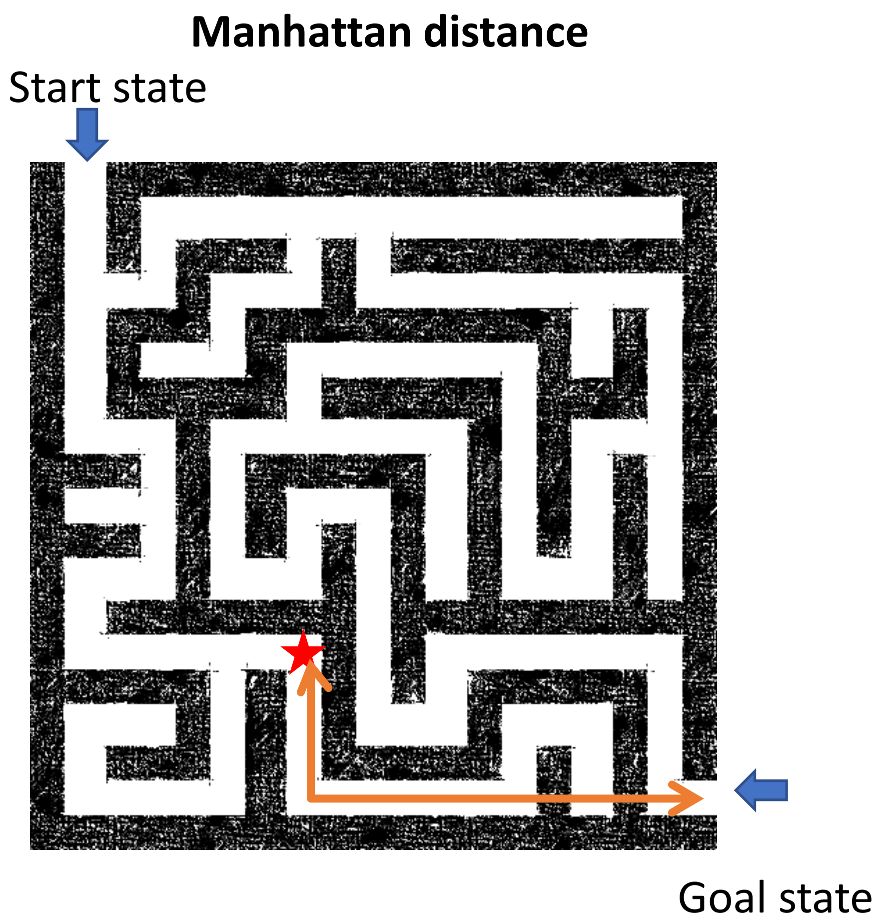
+

Expand the frontier using $f(n) = h(n) + \boldsymbol{g(n)}$

Heuristics from Relaxed Problems

What relaxations are used in these two cases?





A* Search Optimality: Admissible Heuristics

Definition: A heuristic h is **admissible** if for every node n, $h(n) \leq h^*(n)$, where $h^*(n)$ is the true cost to reach the goal state from n.

I.e., an admissible heuristic is a **lower bound** and never overestimates the true cost to reach the goal.

Example: Straight line distance never overestimates the actual road distance.

Theorem: If h is admissible, A^* is optimal.

Satisficing Search: Weighted A* Search

- Often it is sufficient to find a **"good enough" solution** if it can be found very quickly or with way less computational resources. I.e., expanding fewer nodes.

- We could use inadmissible heuristics in A* search (e.g., by multiplying $h(n)$ with a factor W) that sometimes overestimate the optimal cost to the goal slightly.
  1. It potentially reduces the number of expanded nodes significantly.
  2. **This will break the algorithm's optimality guaranty!**

$$\mathrm{f}(n) = g(n) + W \times h(n)$$

Weighted A* search: $\boldsymbol{g(n) + W \times h(n)}$ $\boldsymbol{(1 < W < \infty)}$

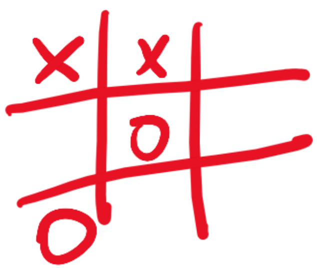
The presented algorithms are special cases:

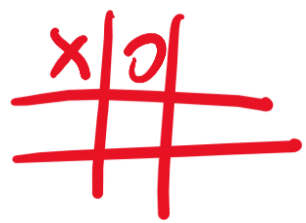
A* search:	$g(n) + h(n)$	$(W = 1)$
Uniform cost search/BFS:	$g(n)$	$(W = 0)$
Greedy best-first search:	$h(n)$	$(W = \infty)$

Case Study: Heuristic for Tic-Tac-Toe

- Define the goal states:
- What is the cost that needs to be estimated?

- What would be a heuristic value for these boards:





- How do you calculate the heuristic value?

- Is the heuristic admissible?
- Does the heuristic use a relaxation?

Assignment

Q&A