Homework 4

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Question 9:

Which of the following functions are from $\mathbb{R}$ to $\mathbb{R}$? If f is a function, give its range.

(b) $f(x) = \dfrac{1}{x^2 - 4}$

Answer:

This is not a function from $\mathbb{R}$ to $\mathbb{R}$ because for $x = 2$ or $x = -2$, there is no corresponding y.

(c) $f(x) = \sqrt{x^2}$

Answer:

This is a function from $\mathbb{R}$ to $\mathbb{R}$ because the square root is undefined only for negative numbers, and $\forall x, x^2 >= 0$. f is a function because there is exactly one y that corresponds to an x. The range is $[0, \infty)$ since $\forall x, f(x) >= 0$.

(b) Let $A = \{2, 3, 4, 5\}$. $f : A \to \mathbb{Z}$ s.t. $f(x) = x^2$

Answer:

{ 4, 9, 16, 25 }

(d) Let $f : \{0, 1\}^5 \to \mathbb{Z}$. For $x \in \{0, 1\}^5$, $f(x)$ is the number of 1's that occur in x.

Answer:

The range is { 0, 1, 2, 3, 4, 5 }. There can be at most 5 1's in a string 11111, and the lowest possible is 0 1's in a string 00000.

(h) Let $A = \{1, 2, 3\}$. $f : A \times A \to \mathbb{Z} \times \mathbb{Z}$, where $f(x, y) = (y, x)$

Answer:

Step 1: $A \times A = \{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)\}$

Step 2: Since $A = A$, $(x, y) = (y, x)$, so the range is the set in Step 1.

(i) Let $A = \{1, 2, 3\}$. $f : A \times A \to \mathbb{Z} \times \mathbb{Z}$, where $f(x, y) = (x, y + 1)$

Answer:

From (h) we have $A \times A = \{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)\}$.
Then the range is:

$$\{(1, 2), (1, 3), (1, 4), (2, 2), (2, 3), (2, 4), (3, 2), (3, 3), (3, 4)\}$$

(l) Let $A = \{1, 2, 3\}$. $f : \mathcal{P}(A) \to \mathcal{P}(A)$. For $X \subseteq A$, $f(X) = X - \{1\}$

Answer:

Step 1: $\mathcal{P}(A) = \{\varnothing, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}\}$

Step 2: All elements of the powerset are by definition subsets of A, so the range is:
$$\{\varnothing, \{2\}, \{3\}, \{2, 3\}\}$$