Linear Algebra Review

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0.1 Basics

0.1.1 Dot Product Rules

Where v and u are vectors, the dot product can be expressed as

$$v \cdot u = v'u = \|v\|\|u\| \cos\theta = \sum_{i=1}^{n} v_i u_i,$$

where $\|v\|$ is the geometric length of the vector, $\sqrt{v \cdot v}$.

1. $v \cdot u = u \cdot v$
2. $v \cdot (u + v) = v \cdot u + v \cdot v$
3. $2v \cdot u = v \cdot 2u$
4. $(v + u) \cdot (v + u) = v \cdot v + u \cdot v + v \cdot u + u \cdot u$

0.1.2 Matrix Multiplication

Two matrices can only be multiplied if they share a dimension. So $n \times k \times k \times n$is valid, but $n \times k \times n \times k$ is not valid. Multiplying matrices can be done column, row, or element-wise. For example, column-wise between a 3×3 matrix and a 3×1 vector looks like:

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = x \begin{bmatrix} a \\ b \\ g \end{bmatrix} + y \begin{bmatrix} b \\ e \\ h \end{bmatrix} + z \begin{bmatrix} c \\ f \\ i \end{bmatrix}$$

- By entry: $AB_{ij} = \text{row}A_i \cdot \text{column}B_j$
- By rows of A: i^{th}row of $AB = (i^{th}$row of $A)B$
- By columns of B: j^{th}column of $AB = A(j^{th}$column of$B)$

0.2 Row-Reduced Echelon Form