Solutions to Chapter 2 Exercises

Cangyuan Li

June 12, 2021

2.17 Let X be a random variable with $\mu = \mathbb{E}[X]$ and $\sigma^2 = var(X)$. Define

$$g(x, \mu, \sigma^2) = \begin{pmatrix} x - \mu \\ (x - \mu^2) - \sigma^2 \end{pmatrix}$$

Show that $\mathbb{E}[g(X, \mu, \sigma)] = 0$ if and only if $m = \mu$ and $s = \sigma^2$.

Answer:

Step 1: Show both set of conditions. Note that $\mu = \mathbb{E}[X]$ and $\sigma^2 = var(X)$. It is helpful to write in these terms in order to apply law of iterated expectations.

$$\begin{aligned}
\mathbb{E}[g(X, m, s)] &= \mathbb{E}\begin{pmatrix} X - \mathbb{E}[X] \\ (X - \mathbb{E}[X])^2 - var(X) \end{pmatrix} \\
&= \begin{pmatrix} \mathbb{E}[X] - \mathbb{E}[\mathbb{E}[X]] \\ (\mathbb{E}[X - \mathbb{E}[\mathbb{X}]])^2 - \mathbb{E}[var(X)] \end{pmatrix} \\
&= \begin{pmatrix} \mathbb{E}[X] - \mathbb{E}[X] \\ \mathbb{E}[var(X)] - \mathbb{E}[var(X)] \end{pmatrix} \\
&= 0
\end{aligned}$$

Step 2: For example, if $m \neq \mu$, then the law of iterated expectations would be violated.