

Math Notes

Math Notes



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Canine-Table

Github

POSIX Nexus serves as a comprehensive cross-language reference hub that explores the implementation and behavior of POSIX-compliant functionality across a diverse set of programming environments. Built atop the foundational IEEE Portable Operating System Interface (POSIX) standards, this project emphasizes compatibility, portability, and interoperability between operating systems.

Abstract

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I Algebra

I Algebraic Identities

Difference of Squares

- ⇒ **Definition** ⇒ Product of sum and difference equals difference of squares
- ⇒ **General Formula** ⇒ $a^2 - b^2 = (a - b)(a + b)$
- ⇒ **Pattern** ⇒ Always collapses into two linear factors
- ⇒ **Example** ⇒ $x^2 - 9 = (x - 3)(x + 3)$
- ⇒ **Extension** ⇒ Used in rationalizing denominators and factoring quadratics

Sum of Cubes

- ⇒ **Definition** ⇒ Sum of cubes factors into binomial and trinomial
- ⇒ **General Formula** ⇒ $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
- ⇒ **Pattern** ⇒ Binomial carries the sum, trinomial balances with alternating signs
- ⇒ **Example** ⇒ $x^3 + 8 = (x + 2)(x^2 - 2x + 4)$
- ⇒ **Extension** ⇒ Pairs with difference of cubes for full cubic factorization

Difference of Cubes

- ⇒ **Definition** ⇒ Difference of cubes factors into binomial and trinomial
- ⇒ **General Formula** ⇒ $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$
- ⇒ **Pattern** ⇒ Binomial carries the difference, trinomial balances with all positives
- ⇒ **Example** ⇒ $x^3 - 27 = (x - 3)(x^2 + 3x + 9)$
- ⇒ **Extension** ⇒ Complements sum of cubes in cubic identities



Perfect Square Trinomial

- ➔ **Definition** ⇒ Squaring a binomial produces a trinomial pattern
- ➔ **General Formula** ⇒ $(a + b)^2 = a^2 + 2ab + b^2$
- ➔ **Negative Case** ⇒ $(a - b)^2 = a^2 - 2ab + b^2$
- ➔ **Pattern** ⇒ Always: square + double product + square
- ➔ **Example** ⇒ $(x + 3)^2 = x^2 + 6x + 9$
- ➔ **Extension** ⇒ Recognizing this pattern speeds up factoring and simplification

Exponent and Radical Identities

- ➔ **Power of a Power** ⇒ $(a^m)^n = a^{mn}$
- ➔ **Negative Exponent** ⇒ $a^{-n} = \frac{1}{a^n}$
- ➔ **Fractional Exponent** ⇒ $\sqrt[n]{a^m} = a^{m/n}$
- ➔ **Product Rule** ⇒ $a^m \cdot a^n = a^{m+n}$
- ➔ **Quotient Rule** ⇒ $\frac{a^m}{a^n} = a^{m-n}$
- ➔ **Extension** ⇒ Links radicals, reciprocals, and powers into one unified system

Binomial Expansion

- ➔ **Definition** ⇒ Expansion of $(a + b)^n$ into a sum of terms
- ➔ **General Formula** ⇒ $(a + b)^n = \sum_{k=0}^n \binom{n}{k} a^{n-k} b^k$
- ➔ **Binomial Coefficient** ⇒ $\binom{n}{k} = \frac{n!}{k!(n-k)!}$
- ➔ **Pattern** ⇒ Exponents of a decrease, exponents of b increase
- ➔ **Symmetry** ⇒ Coefficients are symmetric: $\binom{n}{k} = \binom{n}{n-k}$
- ➔ **Connection** ⇒ Coefficients form Pascal's Triangle
- ➔ **Example** ⇒ $(x + 1)^4 = x^4 + 4x^3 + 6x^2 + 4x + 1$



I Nomials

Nomial Lineage

- ➊ **Monomial** ⇒ One term only, e.g. $7x^2$
- ➋ **Binomial** ⇒ Two terms, e.g. $x + 2$
- ➌ **Trinomial** ⇒ Three terms, e.g. $x^2 + 3x + 2$
- ➍ **Polynomial** ⇒ Many terms, general family
- ➎ **Technique** ⇒ Prefix indicates number of terms
- ➏ **Outcome** ⇒ Classification helps organize algebraic expressions

Monomial: ax^n (1)
 Binomial: $ax^n + bx^m$ (2)
 Trinomial: $ax^n + bx^m + cx^k$ (3)

Polynomial Forms

- ➊ **Monomial** ⇒ One term only, e.g. $7x^2$
- ➋ **Binomial** ⇒ Two unlike terms, e.g. $x + 2$
- ➌ **Trinomial** ⇒ Three terms, e.g. $x^2 + 3x + 2$
- ➍ **Polynomial** ⇒ General family with many terms
- ➎ **Technique** ⇒ Prefix indicates number of terms
- ➏ **Outcome** ⇒ Classification helps organize algebraic expressions

Like Terms

- ➊ **Definition** ⇒ Expressions with identical variable parts, e.g. $3x^2$ and $-5x^2$
- ➋ **Variable Match** ⇒ Same variables with same exponents
- ➌ **Coefficient** ⇒ Numbers in front may differ
- ➍ **Technique** ⇒ Combine by adding or subtracting coefficients
- ➏ **Outcome** ⇒ Simplifies polynomials by reducing to fewer terms



Coefficient

- ➡ **Definition** ⇒ The number in front of a variable, e.g. in $7x$ the coefficient is 7
- ➡ **Variable Match** ⇒ It scales the variable part without changing its type
- ➡ **Examples** ⇒ $3x^2$ has coefficient 3, $-5y$ has coefficient -5
- ➡ **Constants** ⇒ A constant term like 4 can be seen as coefficient 4 of x^0
- ➡ **Outcome** ⇒ Coefficients tell how strongly each variable contributes to the polynomial

I Polynomial Exponents

Definition of Negative Exponent

- ➡ **Positive Exponent** ⇒ x^n means multiply x by itself n times
- ➡ **Negative Exponent** ⇒ x^{-n} means reciprocal of x^n
- ➡ **Rule** ⇒ $x^{-n} = \frac{1}{x^n}$
- ➡ **Example** ⇒ $x^{-3} = \frac{1}{x^3}$
- ➡ **Use** ⇒ Negative exponents express division or reciprocals in algebra and calculus

$$x^3 = x \cdot x \cdot x \Rightarrow x^{-3} = \frac{1}{x^3}$$

Using Negative Exponents

- ➡ **Step 1** ⇒ Recall exponent rules: $x^a \cdot x^b = x^{a+b}$
- ➡ **Step 2** ⇒ Set $a = 3, b = -3$: $x^3 \cdot x^{-3} = x^0$
- ➡ **Step 3** ⇒ But $x^0 = 1$
- ➡ **Step 4** ⇒ So x^{-3} must equal $\frac{1}{x^3}$
- ➡ **Outcome** ⇒ Negative exponent means reciprocal of positive power



Exponent Flip Examples

$$x^5 = \frac{1}{x^{-5}} \quad x^{12} = \frac{1}{x^{-12}} \quad (4)$$

$$x^{17} = \frac{1}{x^{-17}} \quad 15x^9 = \frac{15}{x^{-9}} \quad (5)$$

$$28x^5 = \frac{28}{x^{-5}} \quad x^5 = \frac{1}{x^{-5}} \quad (6)$$

$$x^{-3} = \frac{1}{x^3} \quad x^3 = \frac{1}{x^{-3}} \quad (7)$$

$$x^{-3} = \frac{1}{x^3} \quad (8)$$

Polynomial Exponent Rules Applied

- ▶ **Power of a Power** $\Rightarrow (a^m)^n = a^{mn}$
- ▶ **Nested Powers** \Rightarrow Combine: $8 \cdot 3 = 24$
- ▶ **Outer Flip** \Rightarrow Apply (-9) : $x^{768y^{5z^3}} \rightarrow x^{-6912y^{5z^3}}$
- ▶ **Technique** \Rightarrow Multiply all exponents carefully, preserve inner structure
- ▶ **Outcome** \Rightarrow Final simplified form: $x^{-6912y^{5z^3}}$

$$\begin{aligned} (((x^{32y^{5z^3}})^8)^3)^{-9} &= (x^{32y^{5z^3}})^{8 \cdot 3} \Rightarrow x^{768y^{5z^3}} \\ &= \left(x^{768y^{5z^3}}\right)^{-9} \Rightarrow x^{-9 \cdot 768y^{5z^3}} \\ &= x^{-6912y^{5z^3}} \end{aligned}$$

Exponent Rules Applied

- ▶ **Power of a Power** $\Rightarrow (a^m)^n = a^{mn}$
- ▶ **Nested Powers** \Rightarrow Combine: $8 \cdot 3 = 24$
- ▶ **Outer Flip** \Rightarrow Apply (-9) : $x^{768y^{5z^3}} \rightarrow x^{-6912y^{5z^3}}$
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- ▶ **Outcome** \Rightarrow Final simplified form: $x^{-6912y^{5z^3}}$



$$\begin{aligned}x^2 \cdot x^4 &= x^{2+4} \Rightarrow x^6 \\x^7 \cdot x^5 &= x^{5+7} \Rightarrow x^{12} \\x^8 \cdot x^9 &= x^{8+9} \Rightarrow x^{17} \\(3x^3)(5x^6) &= (3 \cdot 5)x^{3+6} \Rightarrow 15x^9 \\(4x^2)(7x^3) &= (4 \cdot 7)x^{2+3} \Rightarrow 28x^5 \\(4xy^2)(8x^2y^3) &= (4 \cdot 8)x^{1+2}(1 \cdot 1)y^{2+3} \Rightarrow 32x^3y^5 \\(5x^2y^3)(6x^3y^4) &= (5 \cdot 6)x^{2+3}(1 \cdot 1)y^{3+4} \Rightarrow 30x^5y^7 \\(7x^3y^4)(8x^5y^7) &= (7 \cdot 8)x^{3+5}(1 \cdot 1)y^{4+7} \Rightarrow 56x^8y^{11}\end{aligned}$$

$$\begin{aligned}(x^3)^4 &= x^{3 \cdot 4} = x^{12} \\(x^4)^6 &= x^{4 \cdot 6} = x^{24} \\(x^3)^5 &= x^{3 \cdot 5} = x^{15} \\(3x^2)^4 &= 3^{1 \cdot 4}x^{2 \cdot 4} = 3^4x^8 = 81x^8 \\(2x^3)^3 &= 2^{1 \cdot 3}x^{3 \cdot 3} = 2^3x^9 = 8x^9\end{aligned}$$

$$\begin{aligned}(3x^2)^2(2x^3)^3 &= 3^{1 \cdot 2}x^{2 \cdot 2}2^{1 \cdot 3}x^3 \cdot 3 \\&= 3^2x^42^3x^9 \\&= 9 \cdot 8x^{4+9} \\&= 72x^{13}\end{aligned}$$

$$\begin{aligned}(3^2x^3y^4)^2(2^3x^2y^5)^3 &= (3^{2 \cdot 2}x^{3 \cdot 2}y^{4 \cdot 2})(2^{3 \cdot 3}x^{2 \cdot 3}y^{5 \cdot 3}) \\&= (3^4x^6y^8)(2^9x^6y^{15}) \\&= (81x^6y^8)(512x^6y^{15}) \\&= 81 \cdot 512x^6 + 6y^{8+15} \\&= 41472x^{12}y^{23}\end{aligned}$$



$$\begin{aligned}-2^3 &= -2 \cdot -2 \cdot -2 = -8 \\ (-2)^3 &= -2 \cdot -2 \cdot -2 = -8 \\ -(-2)^3 &= -2 \cdot -2 \cdot -2 = 8\end{aligned}$$

$$\begin{aligned}(-7x^2y^3)^0 &= -7^{2 \cdot 0}x^{2 \cdot 0}y^{3 \cdot 0} \\ &= -7^0x^0y^0 \\ &= 1 \cdot 1 \cdot 1 \\ &= 1\end{aligned}$$

$$\begin{aligned}3x(5x + 8) &= 15x^2 + 24x \\ 4x(x^2 - 2x + 3) &= 4x^3 - 8x^2 + 12x\end{aligned}$$

I Dividing Polynomials

$$\frac{x^8}{x^3} = x^{8-3} \Rightarrow x^5 \quad (9)$$

$$\frac{x^5}{x^2} = x^{5-2} \Rightarrow x^3 \quad (10)$$

$$\frac{x^5}{x^8} = x^{5-8} \Rightarrow x^{-3} \quad (11)$$

$$\frac{x^4}{x^7} = x^{4-7} \Rightarrow x^{-3} \quad (12)$$

(13)

$$\begin{aligned}\frac{24x^9y^5}{8x^3y^{12}} &= \frac{24}{8}x^{9-3}\frac{1}{1}y^{5-12} \\ &= 3x^6y^{-7} \\ &= \frac{3x^6}{y^7}\end{aligned}$$



$$\begin{aligned}\frac{12x^5y^{-3}z^4}{36x^8y^{-4}z^{-8}} &= \frac{\frac{12}{3}x^{5-8}}{\frac{36}{3}} \frac{1}{1}y^{-3-(-4)} \frac{1}{1}z^{4-(-8)} \\ &= \frac{x^{-3}y^1z^{12}}{3} \\ &= \frac{yz^{12}}{3x^3}\end{aligned}$$

I Multiplying Polynomials

Multiplication Symbols

- ➊ **Dot** ⇒ · is clean, algebraic, avoids confusion with x
- ➋ **Times** ⇒ × is bold, arithmetic, or cross product
- ➌ **Context** ⇒ Use · in algebra, × in arithmetic or vectors
- ➍ **Technique** ⇒ Choose based on clarity and audience
- ➎ **Outcome** ⇒ Both mean multiplication, but notation signals intent

$$x^2 \cdot x^4 = x^{2+4} \Rightarrow x^6$$

$$x^7 \cdot x^5 = x^{5+7} \Rightarrow x^{12}$$

$$x^8 \cdot x^9 = x^{8+9} \Rightarrow x^{17}$$

$$(3x^3)(5x^6) = (3 \cdot 5)x^{3+6} \Rightarrow 15x^9$$

$$(4x^2)(7x^3) = (4 \cdot 7)x^{2+3} \Rightarrow 28x^5$$



$$(4xy^2)(8x^2y^3) = (4 \cdot 8)x^{1+2}(1 \cdot 1)y^{2+3} \Rightarrow 32x^3y^5$$

$$(5x^2y^3)(6x^3y^4) = (5 \cdot 6)x^{2+3}(1 \cdot 1)y^{3+4} \Rightarrow 30x^5y^7$$

$$(7x^3y^4)(8x^5y^7) = (7 \cdot 8)x^{3+5}(1 \cdot 1)y^{4+7} \Rightarrow 56x^8y^{11}$$

I Combining Polynomials

$$x + 4 = 7 \Rightarrow x + 4 - 4 = 7 - 4 \Rightarrow x + 0 = 3$$

$$x + 9 = 15 \Rightarrow x + 9 - 9 = 15 - 9 \Rightarrow x + 0 = 6$$

$$6 + x = 13 \Rightarrow 6 + x - 6 = 13 - 6 \Rightarrow 0 + x = 7$$

$$x - 3 = 9 \Rightarrow x - 3 + 3 = 9 - 3 \Rightarrow x - 0 = 6 \Rightarrow x = 6$$

$$x - 8 = 7 \Rightarrow x - 8 + 8 = 7 + 8 \Rightarrow x - 0 = 15 \Rightarrow x = 15$$



$$3x + 5 = 11 \Rightarrow 3x + 5 - 5 = 11 - 5 \Rightarrow \frac{3x}{3} = \frac{6}{3} \Rightarrow x = 2$$

$$6.3 = -2 + x \Rightarrow 6.3 + 2 = -2 + 2 + x \Rightarrow 6.3 = 0 + x \Rightarrow x = 6.3$$

$$5 = x - 8 \Rightarrow 5 + 8 = x - 8 + 8 \Rightarrow 13 = x - 0 \Rightarrow \\ \hookrightarrow x = 13$$

$$5 - x = 12 \Rightarrow 5 - 12 - x + x = 12 - 12 + x \Rightarrow \\ \hookrightarrow -7 = 0 + x \Rightarrow x = -7$$

$$-8 = 5 - x \Rightarrow -8 + 8 = 5 + 8 - x \Rightarrow 0 + x = 13 - x + x \Rightarrow \\ \hookrightarrow x = 13$$

$$3x = 12 \Rightarrow \frac{3x}{3} = \frac{12}{3} \Rightarrow \frac{x}{1} = 4 \Rightarrow x = 4$$

$$7x = 14 \Rightarrow \frac{7x}{7} = \frac{14}{7} \Rightarrow \frac{x}{1} = 2 \Rightarrow x = 2$$

$$-6x = -30 \Rightarrow \frac{-6x}{-6} = \frac{-30}{-6} \Rightarrow \frac{-x}{-1} = 5 \Rightarrow x = 5$$



$$-8x = 48 \Rightarrow \frac{-8x}{-8} = \frac{48}{-8} \Rightarrow \frac{-x}{-1} = -6 \Rightarrow x = -6$$

$$7x = -56 \Rightarrow \frac{7x}{7} = \frac{-56}{7} \Rightarrow \frac{x}{1} = -8 \Rightarrow x = -8$$

$$-8x = -72 \Rightarrow \frac{-8x}{-8} = \frac{-72}{-8} \Rightarrow \frac{x}{1} = 9 \Rightarrow x = 9$$

$$4x + 3 = 6x - 15 \Rightarrow 4x - 4x + 3 + 15 = 6x - 4x - 15 + 15 \Rightarrow \\ \hookrightarrow \frac{18}{2} = \frac{2x}{2}; \Rightarrow x = 9$$

$$3(2x - 4) = 5(3x + 2) - 3 \Rightarrow 6x - 12 = 15x + 10 - 3 \Rightarrow \\ \hookrightarrow 6x - 6x - 12 - 7 = 15x - 6x + 7 - 7 \Rightarrow \\ \hookrightarrow \frac{-19}{9} = \frac{9x}{9} \Rightarrow x = \frac{-19}{9}$$

$$\frac{3}{4}x - \frac{2}{3} = 12 \Rightarrow (\frac{3}{4}x \cdot 4)3 - (\frac{2}{3} \cdot 3)4 = 12 \cdot 12 \\ \hookrightarrow (3x)3 - (2)4 = 144 \Rightarrow 9x - 8 + 8 = 144 + 8 \Rightarrow \\ \hookrightarrow \frac{9x}{9} = \frac{152}{9} \Rightarrow x = \frac{152}{9}$$

$$\frac{2}{3}x + 5 = 8 \Rightarrow (\frac{2}{3}x + 5 = 8)3 \Rightarrow 2x + 15 - 15 = 24 - 15 \Rightarrow \\ \hookrightarrow \frac{2x}{2} = \frac{9}{2} \Rightarrow x = \frac{9}{2}$$



$$\begin{aligned}(3x + 5) + (4x - 2) &= 7x + 3 \\(4x^2 + 3x + 9) + (5x^2 + 7x - 4) &= 9x^2 + 10x + 5 \\(5x^2 - 6x - 12) - (7x^2 + 4x - 13) &= 5x^2 - 6x - 12 - 7x^2 - 4x + 13 \\&= 5x^2 - 7x^2 - 12 + 13 - 6x - 4x \\&= -2x^2 + 1 - 10x\end{aligned}$$

I FOIL Method

$$(a + b)(c + d) = ac + ad + bc + bd \quad (14)$$

FOIL Method

- ➊ **First** ⇒ Multiply first terms: $a \cdot c$
- ➋ **Outer** ⇒ Multiply outer terms: $a \cdot d$
- ➌ **Inner** ⇒ Multiply inner terms: $b \cdot c$
- ➍ **Last** ⇒ Multiply last terms: $b \cdot d$
- ➎ **Outcome** ⇒ Sum all four products to get the expanded expression

$$\begin{aligned}(2x + 5)(4x^2 - 3x + 6) &= (2 \cdot 4)x^{1+2} + (2 \cdot -3)^{1+1} + (2 \cdot 6)x \\&\quad \hookrightarrow +(5 \cdot 4)x^2 + (5 \cdot -3)x + (5 \cdot 6) \\&= 8x^3 + (-6 + 20)^2 + (12 + -15)x + 30 \\&= 8x^3 + 14x^2 + -3x + 30\end{aligned}$$



$$\begin{aligned}
 & (3x^2 - 2x + 4)(4x^2 + 5x + -6) = \\
 & \hookrightarrow (3 \cdot 4)x^{2+2} + (3 \cdot 5)x^{2+1} + (3 \cdot -6)x^{2+0} \\
 & \hookrightarrow +(-2 \cdot 4)x^{1+2} + (-2 \cdot 5)x^{1+1} + (-2 \cdot -6)x^{1+0} \\
 & \hookrightarrow +(4 \cdot 4)x^{0+2} + (4 \cdot 5)x^{0+1} + (4 \cdot -6)x^{0+0} \\
 & = 12x^4 + (15 + -8)x^3 + (-18 + -10 + 16)x^2 + (12 + 20)x + -24 \\
 & = 12x^4 + 7x^3 + -12x^2 + 32x + -24
 \end{aligned}$$

I Factoring Polynomials

$$4a^2 + 2ab - 3a^2b + 5$$

Terms	Factors	Prime Factors
$4a^2$	$4, a^2$	$2, 2, a, a$
$2ab$	$3, a, b$	$2, a, b$
$-3a^2b$	$-3, a^2, b$	$-3, a, a, b$
5	5	5

$$xy^2 - 3x^2y^2 - 6y + z$$

Terms	Factors	Prime Factors
xy^2	$4, a^2$	$2, 2, a, a$
$-3x^2y$	$3, a, b$	$2, a, b$
$-6y$	$-6, y$	$-2, 3, y$
z	z	z

$$-5 + 2(3a^2 - 3t)$$

Terms	Factors	Prime Factors
-5	-5	-5
$2(3a^2 - 3t)$	$2 \cdot 3a^2 - 3t$	$2 \cdot 3a^2 - 3t$
$6t$	$6, t$	$3, 3, t$



$$3x^2 + 5x - 2$$

Terms	Factors	Prime Factors
$3x^2$	$3, x^2$	$3 \cdot x \cdot x$
$5x$	$5, x$	$5, x$
-2	-2	-2

Definition of Prime Factor

- ➊ **Prime** ⇒ A number greater than 1 divisible only by 1 and itself
- ➋ **Factor** ⇒ A number that divides another evenly
- ➌ **Prime Factor** ⇒ A prime number that divides another number exactly
- ➍ **Example** ⇒ $60 = 2^2 \cdot 3 \cdot 5$; prime factors are 2, 3, 5
- ➎ **Use** ⇒ Prime factors are the building blocks of integers, used in LCM, GCD, and simplification

Find LCM of 12 and 18

$$\begin{aligned}12 &= 2^2 \cdot 3 \quad \Rightarrow \\18 &= 2 \cdot 3^2 \quad \Rightarrow \\ \text{LCM} &= 2^2 \cdot 3^2 \quad \rightarrow 36\end{aligned}$$

Using Prime Factors for LCM

- ➊ **Step 1** ⇒ Prime factorize each number
- ➋ **Step 2** ⇒ Collect all distinct primes
- ➌ **Step 3** ⇒ Take the highest power of each prime
- ➍ **Step 4** ⇒ Multiply them together
- ➎ **Outcome** ⇒ $\text{LCM}(12,18) = 36$

Euclidean Modulus

```
1 define nx_pt_mod(x, y) {
2     x = nx_abs(x)
3         if (x == 0)
4             return 0
5     y = nx_abs(y)
```



```

6         if (y > 0)           return x - y * nx_pt_trunc(x / y)
7             print "<nx:impurity/>" 
8             return -1
9
10    }

```

The Greatest Common Factor

```

1 define nx_euc(x, y) {
2     auto n
3     if (x == y)           return x
4     while (x > 0 && y > 0) {
5         n = x
6         x = nx_pt_mod(y, x)
7         y = n
8     }
9     return n
10}
11

```

$$(8, 12) \mapsto \gcd(8, 12) = 4$$

$$8x + 12 \Rightarrow 4\left(\frac{8x}{2} + \frac{12}{4}\right) \Rightarrow 4(2x + 3)$$

$$(4, 2) \mapsto \gcd(4, 2) = 2$$

$$4x^2 + 2x \Rightarrow 2x\left(\frac{4x^2}{2x} + \frac{2x}{2x}\right) \Rightarrow 2x(2x + 1)$$

$$(12, 18) \mapsto \gcd(12, 18) = 6$$

$$12ab^2 + 18a^2b^3 \Rightarrow 6ab^2(2 + 3ab)$$



Definition of Perfect Square

- ⇒ **Perfect Square** ⇒ A number that can be expressed as n^2 for some integer n
- ⇒ **Integer Squared** ⇒ Formed by multiplying an integer by itself
- ⇒ **Examples** ⇒ 1, 4, 9, 16, 25, 36, ...
- ⇒ **Non-Examples** ⇒ 2, 3, 5, 6, 7, 10, ...
- ⇒ **Use** ⇒ Perfect squares appear in factoring, radicals, and Pythagorean identities

Check if 49 is a perfect square

$$\begin{aligned} 49 &= 7 \cdot 7 \Rightarrow \\ &= 7^2 \hookrightarrow \end{aligned}$$

Therefore, 49 is a perfect square.

Using Perfect Squares

- ⇒ **Step 1** ⇒ Identify the number
- ⇒ **Step 2** ⇒ Ask if it can be written as n^2
- ⇒ **Step 3** ⇒ If yes, it is a perfect square
- ⇒ **Step 4** ⇒ If no, it is not
- ⇒ **Outcome** ⇒ 49 is a perfect square since $49 = 7^2$

Difference of Squares Applied

- ⇒ **Identity** ⇒ $(a^2 - b^2) = (a - b)(a + b)$
- ⇒ **Example** ⇒ $x^2 - 9$
- ⇒ **Factorization** ⇒ Apply rule: $x^2 - 3^2 = (x - 3)(x + 3)$
- ⇒ **Technique** ⇒ Recognize perfect squares and subtract
- ⇒ **Outcome** ⇒ Final factored form: $(x - 3)(x + 3)$

$$x^2 - 9 \Rightarrow x^2 - 3^2 \Rightarrow (x - 3)(x + 3)$$



Perfect Squares Difference

- ⊕ **Identity** ⇒ $(a^2 - b^2) = (a - b)(a + b)$
- ⊕ **Example** ⇒ $x^2 - 9$
- ⊕ **Factorization** ⇒ $(x - 3)(x + 3)$
- ⊕ **Technique** ⇒ Spot the squares, apply the difference rule
- ⊕ **Outcome** ⇒ Factored polynomial form

$$(25) \mapsto \sqrt{(25)} = 5$$

$$x^2 - 25 \Rightarrow (x + 5)(x - 5)$$

$$(x - 5)(x + 5) \Rightarrow x^2 + 5x + -5x + -25 \Rightarrow$$

$$\hookrightarrow x^2 + (5x + -5x \Rightarrow 0) - 25 \Rightarrow x^2 - 25$$

$$(9) \mapsto \sqrt{(9)} = 3$$

$$x^2 - 9 \Rightarrow (x + 3)(x - 3)$$

$$(4) \mapsto \sqrt{(4)} = 2$$

$$x^2 - 4 \Rightarrow (x + 2)(x - 2)$$

$$4x^2 - 25 \Rightarrow (2x + 5)(2x - 5)$$



$$(81) \mapsto \sqrt{81} = 9$$

$$(16) \mapsto \sqrt{16} = 4$$

$$16x^2 - 25 \Rightarrow (4x + 9)(4x - 9)$$

$$25x^2 - 16y^2 \Rightarrow (5x + 4y)(5x - 4y)$$

$$81x^4 - 16y^8 \Rightarrow (9x^2 + 4y^4)(9x^2 - 4y^4) \Rightarrow (3x + 2y^2)(3x - 2y^2)$$

Factor by Grouping Applied

- ➊ **Setup** \Rightarrow Polynomial with 4 terms
- ➋ **Grouping** \Rightarrow Split into two pairs
- ➌ **Inner Factor** \Rightarrow Factor each pair separately
- ➍ **Common Binomial** \Rightarrow Extract the shared binomial
- ➎ **Outcome** \Rightarrow Final factored form

$$\begin{aligned} x^3 + 3x^2 + 2x + 6 &\Rightarrow (x^3 + 3x^2) + (2x + 6) \Rightarrow \\ &\hookrightarrow x^2(x + 3) + 2(x + 3) \Rightarrow (x^2 + 2)(x + 3) \end{aligned}$$

$$\begin{aligned} x^3 - 4x^2 + 3x - 12 &\Rightarrow x^2(x - 4) + 3(x - 4) \Rightarrow \\ &\hookrightarrow \frac{x^2(x - 4)}{x - 4} + \frac{3(x - 4)}{x - 4} \Rightarrow \\ &\hookrightarrow (x - 4)(x^2 + 3) \end{aligned}$$



$$\begin{aligned}
 2x^3 - 6x^2 + 4x - 12 &\Rightarrow 2x^2(x - 3) + 4(x - 3) \Rightarrow \\
 &\hookrightarrow \frac{2x^2(x - 3)}{x - 3} + \frac{4(x - 3)}{x - 3} \Rightarrow \\
 &\hookrightarrow (x - 3)(2x^2 + 4)
 \end{aligned}$$

$$\begin{aligned}
 3x^3 + 8x^2 - 6x - 16 &\Rightarrow x^2(3x + 8) - 2(3x + 8) \Rightarrow \\
 &\hookrightarrow \frac{x^2(3x + 8)}{3x + 8} + \frac{-2(3x + 8)}{3x + 8} \Rightarrow \\
 &\hookrightarrow (x^2 - 2)(3x + 8) \Rightarrow \\
 \hookrightarrow 3x^3 - 6x + 8x^2 - 16 &\Rightarrow 3x(x^2 - 2) + 8(x^2 - 2) \Rightarrow \\
 &\hookrightarrow \frac{3x(x^2 - 2)}{x^2 - 2} + \frac{8(x^2 - 2)}{x^2 - 2} \Rightarrow \\
 &\hookrightarrow (x^2 - 2)(3x + 8)
 \end{aligned}$$

Perfect Square Trinomial

- ▶ **Definition** ⇒ A trinomial formed by squaring a binomial
- ▶ **General Form** ⇒ $(a + b)^2 = a^2 + 2ab + b^2$
- ▶ **Expansion** ⇒ First term squared, double product of terms, last term squared
- ▶ **Example** ⇒ $(x + 3)^2 = x^2 + 6x + 9$
- ▶ **Negative Case** ⇒ $(a - b)^2 = a^2 - 2ab + b^2$
- ▶ **Pattern** ⇒ Always: square + double product + square



I Expanding Polynomials

Binomial Expansion

- ➡ **Definition** ⇒ Expansion of $(a + b)^n$ into a sum of terms
- ➡ **General Formula** ⇒ $(a + b)^n = \sum_{k=0}^n \binom{n}{k} a^{n-k} b^k$
- ➡ **Binomial Coefficient** ⇒ $\binom{n}{k} = \frac{n!}{k!(n-k)!}$
- ➡ **Pattern** ⇒ Exponents of a decrease, exponents of b increase
- ➡ **Symmetry** ⇒ Coefficients are symmetric: $\binom{n}{k} = \binom{n}{n-k}$
- ➡ **Connection** ⇒ Coefficients form Pascal's Triangle
- ➡ **Example** ⇒ $(x + 1)^4 = x^4 + 4x^3 + 6x^2 + 4x + 1$

Pascal's Triangle Connection

- ➡ **Structure** ⇒ Pascal's Triangle is built by starting with 1 at the top, each entry below is the sum of the two above
- ➡ **Coefficients** ⇒ Row n of Pascal's Triangle gives the coefficients for $(a + b)^n$
- ➡ **Symmetry** ⇒ Coefficients are symmetric: $\binom{n}{k} = \binom{n}{n-k}$
- ➡ **Example Row 4** ⇒ Coefficients 1, 4, 6, 4, 1 expand $(a + b)^4$
- ➡ **Combinatorics** ⇒ Each coefficient counts the number of ways to choose k items from n
- ➡ **Extension** ⇒ Pascal's Triangle also encodes identities: hockey-stick pattern, Fibonacci connections, binomial sums

Hockey-Stick Identity

- ➡ **Pattern** ⇒ Pick a diagonal in Pascal's Triangle, sum entries until a row
- ➡ **Result** ⇒ The sum equals the entry just below and to the right (like a hockey stick)
- ➡ **Formula** ⇒ $\binom{r}{r} + \binom{r+1}{r} + \cdots + \binom{n}{r} = \binom{n+1}{r+1}$
- ➡ **Example** ⇒ $\binom{2}{2} + \binom{3}{2} + \binom{4}{2} = \binom{5}{3}$
- ➡ **Visual** ⇒ The diagonal entries form the "shaft," the final entry is the "blade"



Fibonacci Connection

- ➡ **Pattern** ⇒ Sum shallow diagonals of Pascal's Triangle
- ➡ **Result** ⇒ The sums produce Fibonacci numbers
- ➡ **Example** ⇒ Row sums: 1, 1, 2, 3, 5, 8, 13, ...
- ➡ **Formula** ⇒ $\text{Fib}(n) = \sum \binom{n-k}{k}$
- ➡ **Visual** ⇒ Diagonal “paths” through Pascal’s Triangle trace Fibonacci lineage

Binomial Sums

- ➡ **Row Sum** ⇒ $\sum_{k=0}^n \binom{n}{k} = 2^n$
- ➡ **Alternating Sum** ⇒ $\sum_{k=0}^n (-1)^k \binom{n}{k} = 0$
- ➡ **Weighted Sum** ⇒ $\sum_{k=0}^n k \binom{n}{k} = n \cdot 2^{n-1}$
- ➡ **Combinatorial Meaning** ⇒ Row sum counts all subsets of an n -element set
- ➡ **Probability Link** ⇒ Binomial distribution probabilities sum to 1 using these identities

I Lowest Common Denominator

refer to I to see the euc function.

The Lowest Common Denominator

```

1 define nx_lcd(x, y) {
2     return x * y / nx_euc(x, y)
3 }
```

$$\frac{3}{4} + \frac{2}{5} \Rightarrow \frac{3 \cdot 5}{4 \cdot 5} + \frac{2 \cdot 4}{5 \cdot 4} \Rightarrow \frac{15}{20} + \frac{8}{20} \Rightarrow \frac{15+8}{20} \Rightarrow \frac{23}{20}$$

$$\frac{5}{6} + \frac{4}{7} \Rightarrow \frac{5 \cdot 7}{6 \cdot 7} + \frac{4 \cdot 6}{7 \cdot 6} \Rightarrow \frac{35}{42} + \frac{24}{42} \Rightarrow \frac{35-24}{42} \Rightarrow \frac{11}{42}$$

$$\frac{7}{5} \cdot \frac{4}{3} \Rightarrow \frac{7 \cdot 4}{5 \cdot 3} \Rightarrow \frac{28}{15} \Rightarrow 1\frac{13}{15}$$

$$(18, 20) \mapsto \gcd(18, 20) = 2$$

$$\frac{3}{5} \cdot \frac{6}{4} \Rightarrow \frac{3 \cdot 6}{5 \cdot 4} \Rightarrow \frac{18}{20} \Rightarrow \frac{\frac{18}{2}}{\frac{20}{2}} \Rightarrow \frac{9}{10}$$

$$(28, 63) \mapsto \gcd(28, 63) = 7$$

$$(56, 35) \mapsto \gcd(56, 35) = 7$$

$$\frac{28}{63} \cdot \frac{56}{35} \Rightarrow \frac{\frac{28}{7}}{\frac{63}{7}} \cdot \frac{\frac{56}{7}}{\frac{35}{7}} \Rightarrow \frac{4}{9} \cdot \frac{8}{5} \Rightarrow \frac{4 \cdot 8}{9 \cdot 5} \Rightarrow \frac{32}{45}$$

Definition of Keep-Change-Flip

- ➊ **Keep** ⇒ Keep the first fraction exactly as it is
- ➋ **Change** ⇒ Change the division sign to multiplication
- ➌ **Flip** ⇒ Flip the second fraction (take its reciprocal)
- ➍ **Example** ⇒ $\frac{3}{4} \div \frac{2}{5}$
- ➎ **Outcome** ⇒ $\frac{3}{4} \times \frac{5}{2} = \frac{15}{8}$

$$\frac{3}{4} \div \frac{2}{5} \Rightarrow \frac{3}{4} \times \frac{5}{2}$$



Using Keep-Change-Flip

- ⇒ **Step 1** ⇒ Write the division problem
- ⇒ **Step 2** ⇒ Keep the first fraction
- ⇒ **Step 3** ⇒ Change division to multiplication
- ⇒ **Step 4** ⇒ Flip the second fraction
- ⇒ **Step 5** ⇒ Multiply across numerators and denominators
- ⇒ **Outcome** ⇒ Simplified fraction result

I Radicals

Identity of $\sqrt[n]{a^n}$

- ⇒ **Expression** ⇒ $\sqrt[n]{a^n}$
- ⇒ **Meaning** ⇒ The n -th root of a^n
- ⇒ **Simplification** ⇒ Cancels the root and exponent, yielding a
- ⇒ **Condition** ⇒ Valid when $a \geq 0$ for real roots, or in complex domain otherwise
- ⇒ **Example** ⇒ $\sqrt[3]{2^3} = \sqrt[3]{8} = 2$
- ⇒ **Extension** ⇒ $\sqrt[n]{a^m} = a^{m/n}$

$$\sqrt[n]{a^n} = a$$

Definition of Radical

- ⇒ **Radical** ⇒ An expression that uses the root symbol $\sqrt{}$
- ⇒ **Square Root** ⇒ The most common radical, \sqrt{x} , meaning the number which squared gives x
- ⇒ **Index** ⇒ The small number above the radical, e.g. $\sqrt[3]{x}$ is the cube root
- ⇒ **Radicand** ⇒ The number or expression inside the radical sign
- ⇒ **Example** ⇒ $\sqrt{25} = 5$, $\sqrt[3]{8} = 2$
- ⇒ **Use** ⇒ Radicals are used to express roots, simplify algebraic expressions, and solve equations



$$\sqrt{50} \Rightarrow \sqrt{25 \cdot 2} \Rightarrow \sqrt{25} \cdot \sqrt{2} \Rightarrow 5\sqrt{2}$$

Using Radicals

- ➊ **Step 1** ⇒ Identify the radicand
- ➋ **Step 2** ⇒ Factor the radicand into perfect powers and leftovers
- ➌ **Step 3** ⇒ Simplify by taking the root of the perfect power
- ➍ **Step 4** ⇒ Leave the leftover inside the radical
- ➎ **Outcome** ⇒ $\sqrt{50} = 5\sqrt{2}$

Overview of nx_squares

- ➊ **Purpose** ⇒ Computes square roots using iterative scaling and subtraction
- ➋ **Input** ⇒ A single value x
- ➌ **Initialization** ⇒ Takes absolute value, sets scaling factor, prepares working variables
- ➍ **Scaling** ⇒ Doubles a seed until it exceeds the radicand
- ➎ **Iteration** ⇒ Subtracts scaled values, updates root approximation step by step
- ➏ **Condition** ⇒ Handles zero and invalid cases gracefully, returns early if needed
- ➐ **Output** ⇒ Final square root approximation, or truncated value if iteration fails

Square Roots

```
1 define nx_nr_sqrt(x) {
2     auto y, p
3     if (nx_xy_breach(x, 1) == -1)
4         return -1
5     y = x / 2
6     p = 0
7     while (y != p) {
8         p = y
9         y = (y + x / y) / 2
10    }
11 }
12 }
```



Square Roots

```

1 define nx_squares(x) {
2     auto a, s, b, y
3     a = nx_abs(x)
4     if (a == 0 || scale == 0)
5         return x
6     s = 1
7     while (s < a)
8         s = s * 2
9     b = nx_scale(1)
10    x = 0
11    y = 0
12    while (s > b) {
13        if (s <= a) {
14            a = a - s
15            x = y
16            y = nx_nr_sqrt(s)
17            if (y == -1)
18                return x
19            if (x != 0)
20                print x, ", "
21        }
22        s = s / 3
23    }
24    return y
25 }
```

Definition of Vinculum

- ➔ **Vinculum** ⇒ The horizontal bar drawn over the radicand in a radical
- ➔ **Scope** ⇒ Shows exactly which terms are included under the radical
- ➔ **Fraction Use** ⇒ Also used as the bar separating numerator and denominator in fractions
- ➔ **Repeating Decimal** ⇒ Used to mark repeating digits, e.g. $0.\overline{3}$
- ➔ **Example** ⇒ In $\sqrt{a+b}$, the vinculum extends over $a+b$
- ➔ **Use** ⇒ Ensures clarity of grouping inside radicals, fractions, and repeating decimals

Definition of Radicand

- ➔ **Radicand** ⇒ The number or expression placed under the radical sign $\sqrt{}$
- ➔ **Role** ⇒ It is the quantity from which a root is extracted
- ➔ **Example** ⇒ In $\sqrt{25}$, the radicand is 25
- ➔ **Extended** ⇒ In $\sqrt[3]{8}$, the radicand is 8, and the index is 3
- ➔ **Scope** ⇒ The vinculum (bar) shows exactly which terms belong to the radicand
- ➔ **Outcome** ⇒ Identifying the radicand clarifies what is being rooted in the expression



Square Root Expansion

- ➡ **Radical** ⇒ $\sqrt{72}$
- ➡ **Factor** ⇒ Break into perfect square and leftover: $72 = 36 \cdot 2$
- ➡ **Expand** ⇒ $\sqrt{72} = \sqrt{36 \cdot 2} = \sqrt{36} \cdot \sqrt{2}$
- ➡ **Simplify** ⇒ $\sqrt{36} = 6$
- ➡ **Outcome** ⇒ $\sqrt{72} = 6\sqrt{2}$

Square Root Truncation

- ➡ **Radical** ⇒ $\sqrt{72}$
- ➡ **Approximation** ⇒ Leave as decimal: $\sqrt{72} \approx 8.485$
- ➡ **Truncation** ⇒ Cut after two decimals: 8.48
- ➡ **Outcome** ⇒ Truncated radical value ≈ 8.48

Types of Square Roots

- ➡ **Integer Square Root** ⇒ The root of a whole number. It may simplify to a rational (e.g. $\sqrt{25} = 5$) or remain irrational (e.g. $\sqrt{2}$)
- ➡ **Fraction Square Root** ⇒ The root of a ratio $\frac{p}{q}$, defined as $\sqrt{\frac{p}{q}} = \frac{\sqrt{p}}{\sqrt{q}}$
- ➡ **Rational Square Root** ⇒ Occurs when both numerator and denominator are perfect squares, e.g. $\sqrt{\frac{9}{16}} = \frac{3}{4}$
- ➡ **Irrational Square Root** ⇒ Occurs when either numerator or denominator is not a perfect square, e.g. $\sqrt{\frac{2}{9}} = \frac{\sqrt{2}}{3}$
- ➡ **Impurity Case** ⇒ When the denominator equals 0, the expression is undefined, e.g. $\sqrt{\frac{p}{0}}$



I Brackets

Curly Braces {}

- ➊ **Definition** ⇒ Used to denote sets or grouping in mathematics, e.g. $\{1, 2, 3\}$
- ➋ **Programming** ⇒ Common in code to enclose blocks of statements
- ➌ **L^AT_EX** ⇒ Used to group arguments for commands
- ➍ **Example** ⇒ $\{x \mid x > 0\}$ means the set of all positive x
- ➎ **Outcome** ⇒ Curly braces signal structured grouping or set notation

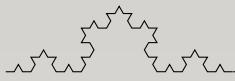
Square Brackets []

- ➊ **Definition** ⇒ Used for intervals, optional elements, or matrices
- ➋ **Interval** ⇒ $[a, b]$ means all values between a and b , inclusive
- ➌ **Matrix** ⇒ Brackets often enclose arrays of numbers
- ➍ **Example** ⇒ $[2, 5]$ includes 2 and 5
- ➎ **Outcome** ⇒ Square brackets emphasize inclusion or structured arrays

$$\begin{aligned}
 8[-6 + 8(-2 + 4)] &= 8[-6 + 8 \cdot 2] \\
 &= 8[-6 + 16] \\
 &= 8 \cdot 10 \\
 &= 80
 \end{aligned}$$

Parentheses ()

- ➊ **Definition** ⇒ Used for grouping, order of operations, or function arguments
- ➋ **Math** ⇒ $(a + b)c$ ensures addition happens before multiplication
- ➌ **Functions** ⇒ $f(x)$ shows input to a function
- ➍ **Interval** ⇒ (a, b) means values strictly between a and b
- ➎ **Outcome** ⇒ Parentheses control grouping and precedence in math and logic



$$\begin{aligned} -(-(1-2)+(5)) &= -(-(-1)+5) \\ &= -(1+5) \\ &= -(6) \\ &= -6 \end{aligned}$$

$$\begin{aligned} 3 \cdot (6 - 3 + 1) - 4^2 &= 3 \cdot 4 - 16 \\ &= 12 - 16 \\ &= -4 \end{aligned}$$

$$\begin{aligned} (10, 4) \mapsto \gcd(10, 4) &= 2 \\ \frac{2 \cdot (5-1) + 2}{4 \cdot (2-1)} &\Leftrightarrow (2 \cdot (5-1) + 2) \div (4 \cdot (2-1)) \\ &= \frac{2 \cdot 4 + 2}{4 \cdot 1} \\ &= \frac{8+2}{4} \\ &= \frac{10}{4} \\ &= \frac{\frac{10}{2}}{2} \\ &= \frac{5}{2} \end{aligned}$$

$$\begin{aligned} 5 + (1 \cdot 4 - (2 - 5) - 2) &= 4 - (-3) - 2 \\ &= 7 - 2 \\ &= 5 \end{aligned}$$



Double Vertical Bars ||

- ➔ **Definition** ⇒ Used to denote absolute value or norm
- ➔ **Absolute Value** ⇒ $|x|$ is distance of x from 0
- ➔ **Norm** ⇒ $\|v\|$ is length of vector v
- ➔ **Example** ⇒ $|-5| = 5$, $\|(3, 4)\| = 5$
- ➔ **Outcome** ⇒ Double bars measure magnitude or distance

$$4 + |3 - 9| = 4 + |-6| \quad (15)$$

$$= 4 + 6 \quad (16)$$

$$= 10 \quad (17)$$

Floor Brackets []

- ➔ **Definition** ⇒ Used for the floor function
- ➔ **Floor** ⇒ $\lfloor x \rfloor$ = greatest integer less than or equal to x
- ➔ **Example** ⇒ $\lfloor 3.7 \rfloor = 3$
- ➔ **Use** ⇒ Rounds down to nearest integer
- ➔ **Outcome** ⇒ Floor brackets capture downward rounding

Ceiling Brackets []

- ➔ **Definition** ⇒ Used for the ceiling function
- ➔ **Ceiling** ⇒ $\lceil x \rceil$ = smallest integer greater than or equal to x
- ➔ **Example** ⇒ $\lceil 3.2 \rceil = 4$
- ➔ **Use** ⇒ Rounds up to nearest integer
- ➔ **Outcome** ⇒ Ceiling brackets capture upward rounding



I Multiply

$$\left(\frac{103}{10}\right)^2 \Rightarrow \frac{103}{10} \cdot \frac{103}{10}$$

1	0	3			
×	1	0	3		
	3	0	9		
	0	0	0		
+	1	0	3	0	0
	1	0	6	0	9

$$\frac{10609}{100} \Rightarrow 106.09$$

$$\left(\frac{-235}{100}\right)^2 \Rightarrow \frac{-235}{10} \cdot \frac{-235}{10}$$

-	2	3	5		
×	-	2	3	5	
	1	0	5	1	5
	7	1	5	0	
+	4	7	0	0	0
	5	7	6	6	5

$$\frac{57665}{100} \Rightarrow 576.65$$

I Dividing

Fraction Components

- ➊ **Numerator** ⇒ Top of the fraction, counts selected parts, e.g. in $\frac{3}{4}$ the numerator is 3
- ➋ **Denominator** ⇒ Bottom of the fraction, defines total equal parts, e.g. in $\frac{3}{4}$ the denominator is 4
- ➌ **Relationship** ⇒ Fraction = Numerator ÷ Denominator
- ➍ **Technique** ⇒ Numerator changes with quantity chosen, denominator fixes the partition size
- ➎ **Outcome** ⇒ Understanding both clarifies fraction meaning and operations



$$\frac{6 - 2^3}{2} = \frac{6 - 8}{2} \quad (18)$$

$$= \frac{-2}{2} \quad (19)$$

$$= -1 \quad (20)$$

Verbose Floating-Point Multiplication

- ⇒ Step 1 ⇒ Choose two floating-point numbers, e.g. 3.25 and 2.5
- ⇒ Step 2 ⇒ Express them as fractions: $3.25 = \frac{325}{100}$, $2.5 = \frac{25}{10}$
- ⇒ Step 3 ⇒ Multiply numerators: $325 \times 25 = 8125$
- ⇒ Step 4 ⇒ Multiply denominators: $100 \times 10 = 1000$
- ⇒ Step 5 ⇒ Form product fraction: $\frac{8125}{1000}$
- ⇒ Step 6 ⇒ Simplify fraction: $\frac{8125}{1000} = 8.125$
- ⇒ Step 7 ⇒ Restore decimal form: product is 8.125

$$\begin{aligned}
 10.4 &\Rightarrow \frac{10.4}{1} \frac{10.4}{1} \cdot 10 & \Rightarrow &\Rightarrow \frac{104}{10} \\
 1.3 &\Rightarrow \frac{1.3}{1} & \Rightarrow &\Rightarrow \frac{1.3}{1} \cdot 10 \Rightarrow \frac{13}{10} \\
 1.5 &\Rightarrow \frac{1.5}{1} & \Rightarrow &\Rightarrow \frac{1.5}{1} \cdot 10 \Rightarrow \frac{15}{10} \\
 -2.35 &\Rightarrow \frac{-2.35}{1} \Leftrightarrow \frac{2.35}{-1} & \Rightarrow &\Rightarrow \frac{-2.35}{1} \cdot 100 \Rightarrow \frac{-235}{100}
 \end{aligned}$$

$$(10056, 1950) \mapsto \gcd(10056, 1950) = 26$$

$$\begin{aligned}
 \frac{10.3^2 - (-2.35)^2}{1.3(1.5)} &= \frac{106.09 - 5.522}{1.95} \\
 &= \frac{100.568}{1.95} \\
 &= \frac{\frac{100568}{26}}{\frac{1950}{26}} \\
 &= \frac{3868}{75}
 \end{aligned}$$



I Addition

I Sum and Addend

Sum

- ➡ **Definition** ⇒ The result of an addition operation
- ➡ **Formula** ⇒ $\text{Sum} = \text{Addend}_1 + \text{Addend}_2 (+ \dots)$
- ➡ **Example** ⇒ $7 + 3 = 10$, so 10 is the sum
- ➡ **Connection** ⇒ Glyph of combination, outcome of addition
- ➡ **Extension** ⇒ Used in arithmetic, algebra, and series notation

Addend

- ➡ **Definition** ⇒ A number that is added to another
- ➡ **Position** ⇒ Each term in an addition is an addend
- ➡ **Example** ⇒ In $7 + 3$, both 7 and 3 are addends
- ➡ **Connection** ⇒ Glyph of contribution, building blocks of the sum
- ➡ **Extension** ⇒ In algebra, variables can also be addends: $x + y$

Series and Summation

- ➡ **Notation** ⇒ $\sum_{i=1}^n a_i$ means addends a_1, a_2, \dots, a_n
- ➡ **Meaning** ⇒ Sum is the collective result of all addends
- ➡ **Example** ⇒ $\sum_{i=1}^4 i = 1 + 2 + 3 + 4 = 10$
- ➡ **Connection** ⇒ Summation generalizes addition to many addends
- ➡ **Extension** ⇒ Foundation of series in calculus and analysis



I Subtraction

I Difference, Minuend, Subtrahend

Difference

- ➡ **Definition** ⇒ Result of a subtraction operation
- ➡ **Formula** ⇒ Difference = Minuend – Subtrahend
- ➡ **Example** ⇒ $9 - 4 = 5$, so 5 is the difference
- ➡ **Connection** ⇒ Glyph of subtraction outcome
- ➡ **Extension** ⇒ Used in arithmetic, algebra, and measurement comparisons

Minuend

- ➡ **Definition** ⇒ The number from which another is subtracted
- ➡ **Position** ⇒ Always appears first in subtraction
- ➡ **Example** ⇒ In $9 - 4$, 9 is the minuend
- ➡ **Connection** ⇒ Glyph of the “starting quantity”
- ➡ **Extension** ⇒ Represents the whole or larger value in comparison

Subtrahend

- ➡ **Definition** ⇒ The number that is subtracted from the minuend
- ➡ **Position** ⇒ Always appears second in subtraction
- ➡ **Example** ⇒ In $9 - 4$, 4 is the subtrahend
- ➡ **Connection** ⇒ Glyph of the “taken away” quantity
- ➡ **Extension** ⇒ Represents the part removed or compared against

I Linear Algebra

I Linear Equations

$$x + 4 = 7 \Rightarrow x + 4 - 4 = 7 - 4 \Rightarrow x + 0 = 3$$

$$x + 9 = 15 \Rightarrow x + 9 - 9 = 15 - 9 \Rightarrow x + 0 = 6$$

$$6 + x = 13 \Rightarrow 6 + x - 6 = 13 - 6 \Rightarrow 0 + x = 7$$

$$x - 3 = 9 \Rightarrow x - 3 + 3 = 9 + 3 \Rightarrow x + 0 = 12 \Rightarrow x = 12$$

$$x - 8 = 7 \Rightarrow x - 8 + 8 = 7 + 8 \Rightarrow x + 0 = 15 \Rightarrow x = 15$$

$$6.3 = -2 + x \Rightarrow 6.3 + 2 = -2 + 2 + x \Rightarrow 6.3 + 0 = x \Rightarrow x = 6.3$$

$$5 = x - 8 \Rightarrow 5 + 8 = x - 8 + 8 \Rightarrow 13 = x + 0 \Rightarrow x = 13$$

$$\begin{aligned} 5 - x = 12 &\Rightarrow 5 - 12 - x + x = 12 - 12 + x \Rightarrow \\ &\Leftrightarrow -7 = 0 + x \Rightarrow x = -7 \end{aligned}$$

$$\begin{aligned} -8 = 5 - x &\Rightarrow -8 + 8 = 5 + 8 - x \Rightarrow \\ &\Leftrightarrow 0 + x = 13 - x + x \Rightarrow x = 13 \end{aligned}$$

$$3x = 12 \Rightarrow \frac{3x}{3} = \frac{12}{3} \Rightarrow 1 = 4 \Rightarrow x = 4$$



$$7x = 14 \Rightarrow \frac{7x}{7} = \frac{14}{7} \Rightarrow \frac{x}{1} = 2 \Rightarrow x = 2$$

$$-6x = -30 \Rightarrow \frac{-6x}{-6} = \frac{-30}{-6} \Rightarrow \frac{-x}{-1} = 5 \Rightarrow x = 5$$

$$-8x = 48 \Rightarrow \frac{-8x}{-8} = \frac{48}{-8} \Rightarrow \frac{-x}{-1} = -6 \Rightarrow x = -6$$

$$7x = -56 \Rightarrow \frac{7x}{7} = \frac{-56}{7} \Rightarrow \frac{x}{1} = -8 \Rightarrow x = -8$$

$$-8x = -72 \Rightarrow \frac{-8x}{-8} = \frac{-72}{-8} \Rightarrow \frac{x}{1} = 9 \Rightarrow x = 9$$

I Coordinate Plane

Domain

- ▶ **Definition** ⇒ The set of all possible input values x for a function
- ▶ **Coordinate Plane** ⇒ Represents the horizontal axis (x-axis)
- ▶ **Example** ⇒ For $f(x) = \sqrt{x}$, the domain is $x \geq 0$
- ▶ **Relation** ⇒ Domain specifies where the function is defined
- ▶ **Outcome** ⇒ Determines the allowable values you can plug into the function



Range

- ➡ **Definition** ⇒ The set of all possible output values y from a function
- ➡ **Coordinate Plane** ⇒ Represents the vertical axis (y-axis)
- ➡ **Example** ⇒ For $f(x) = \sqrt{x}$, the range is $y \geq 0$
- ➡ **Relation** ⇒ Range shows the values the function can produce
- ➡ **Outcome** ⇒ Determines the spread of results plotted on the plane

Interval Notation

- ➡ $[a,b]$ ⇒ Closed interval: includes both endpoints a and b
- ➡ (a,b) ⇒ Open interval: excludes both endpoints a and b
- ➡ $[a,b)$ ⇒ Half-open interval: includes a but excludes b
- ➡ $(a,b]$ ⇒ Half-open interval: excludes a but includes b
- ➡ $[0,x)$ ⇒ All real numbers from 0 up to but not including x
- ➡ $(0,x)$ ⇒ All real numbers greater than 0 and less than x

Relation of Ordered Pairs

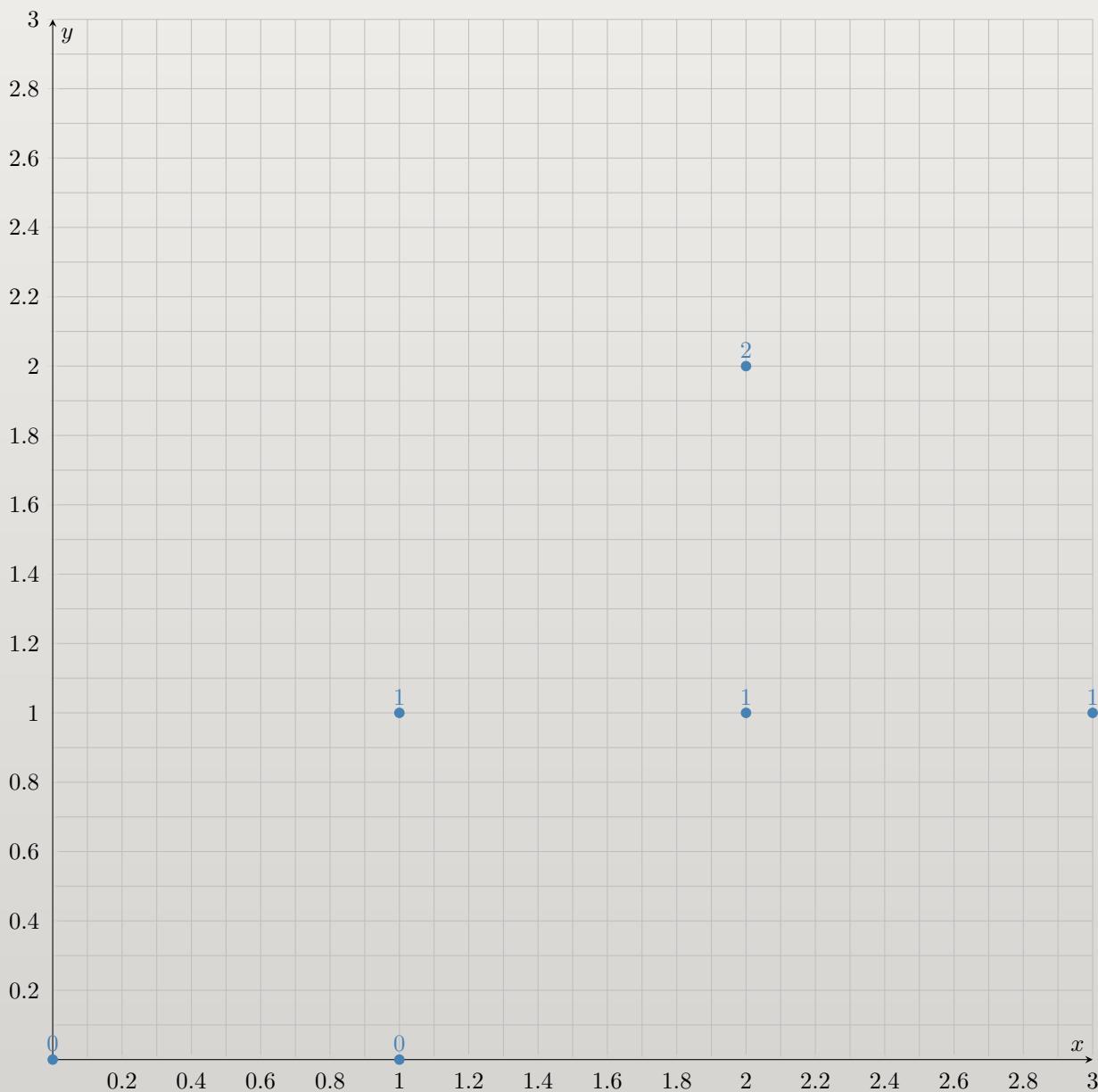
- ➡ **Ordered Pair** ⇒ A pair written as (x, y) , where x is the input and y is the output
- ➡ **Relation** ⇒ Any set of ordered pairs that connects elements from one set (domain) to another (range)
- ➡ **Domain** ⇒ The collection of all first elements x in the ordered pairs
- ➡ **Range** ⇒ The collection of all second elements y in the ordered pairs
- ➡ **Function** ⇒ A special type of relation where each x is paired with exactly one y
- ➡ **Example** ⇒ Relation: $\{(1, 2), (2, 3), (3, 4)\}$; Domain: $\{1, 2, 3\}$; Range: $\{2, 3, 4\}$



Ordered Pairs

Input (a, b)	Output Value
$(0, 0)$	5
$(1, 0)$	9
$(1, 1)$	8
$(2, 1)$	15
$(2, 2)$	17
$(3, 1)$	26

Ordered Pairs





I Distance Formula

$$\text{2D points} \quad d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad (21)$$

$$\text{3D points} \quad d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2} \quad (22)$$

Distance Formula

- ➊ **Definition** ⇒ Gives the length of the line segment between two points in the plane
- ➋ **Points** ⇒ Two points (x_1, y_1) and (x_2, y_2)
- ➌ **Formula** ⇒ $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
- ➍ **Origin** ⇒ Derived from the Pythagorean Theorem
- ➎ **Domain** ⇒ Applies to all real coordinates in 2D space
- ➏ **Extension** ⇒ Generalizes to 3D: $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

Identity of $\sqrt[n]{a^n}$

- ➊ **Expression** ⇒ $\sqrt[n]{a^n}$
- ➋ **Meaning** ⇒ The n -th root of a^n
- ➌ **Simplification** ⇒ Cancels the root and exponent, yielding a
- ➍ **Condition** ⇒ Valid when $a \geq 0$ for real roots, or in complex domain otherwise
- ➎ **Example** ⇒ $\sqrt[3]{2^3} = \sqrt[3]{8} = 2$
- ➏ **Extension** ⇒ $\sqrt[n]{a^m} = a^{m/n}$

refer to I to see the sqrt function.

Distance Formula Functions

```
1 define nx_fma_dist1(x1, x2) {  
2     return nx_nr_sqrt((x2 - x1)^2)  
3 }  
4  
5 define nx_fma_dist2(x1, x2, y1, y2) {  
6     return nx_nr_sqrt((x2 - x1)^2 + (y2 - y1)^2)  
7 }  
8  
9 define nx_fma_dist3(x1, x2, y1, y2, z1, z2) {  
10    return nx_nr_sqrt((x2 - x1)^2 + (y2 - y1)^2 + (z2 - z1)^2)
```

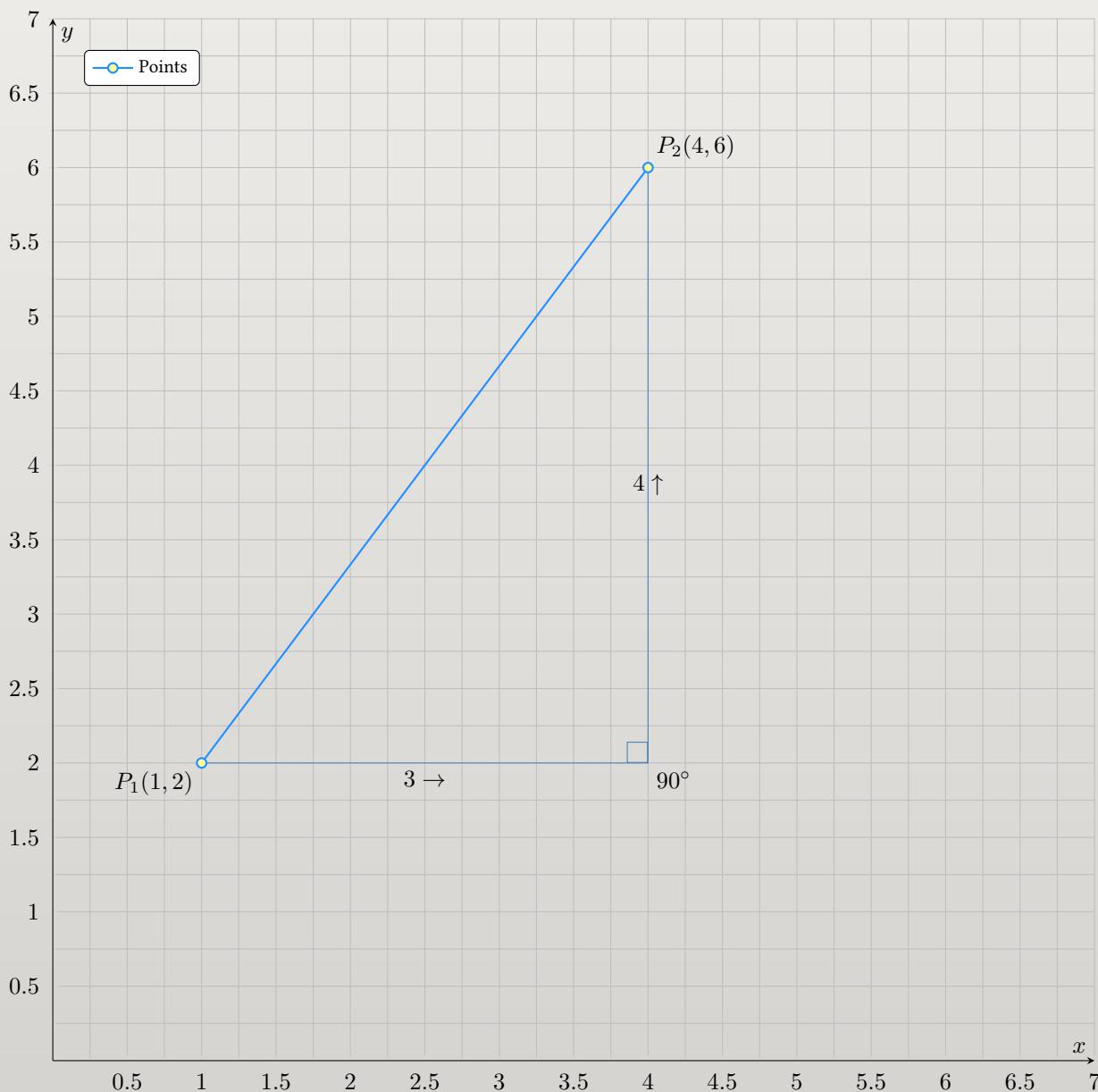


11

}

$$\begin{array}{ll}
 P_1(1, 2) & P_2(4, 6) \\
 x_1 = 1 & x_2 = 4 \\
 y_1 = 2 & y_2 = 6 \\
 d = \sqrt{(4-1)^2 + (6-2)^2} \Rightarrow \sqrt{3^2 + 4^2} \Rightarrow \sqrt{9+16} \Rightarrow \sqrt{25} \Rightarrow 5
 \end{array}$$

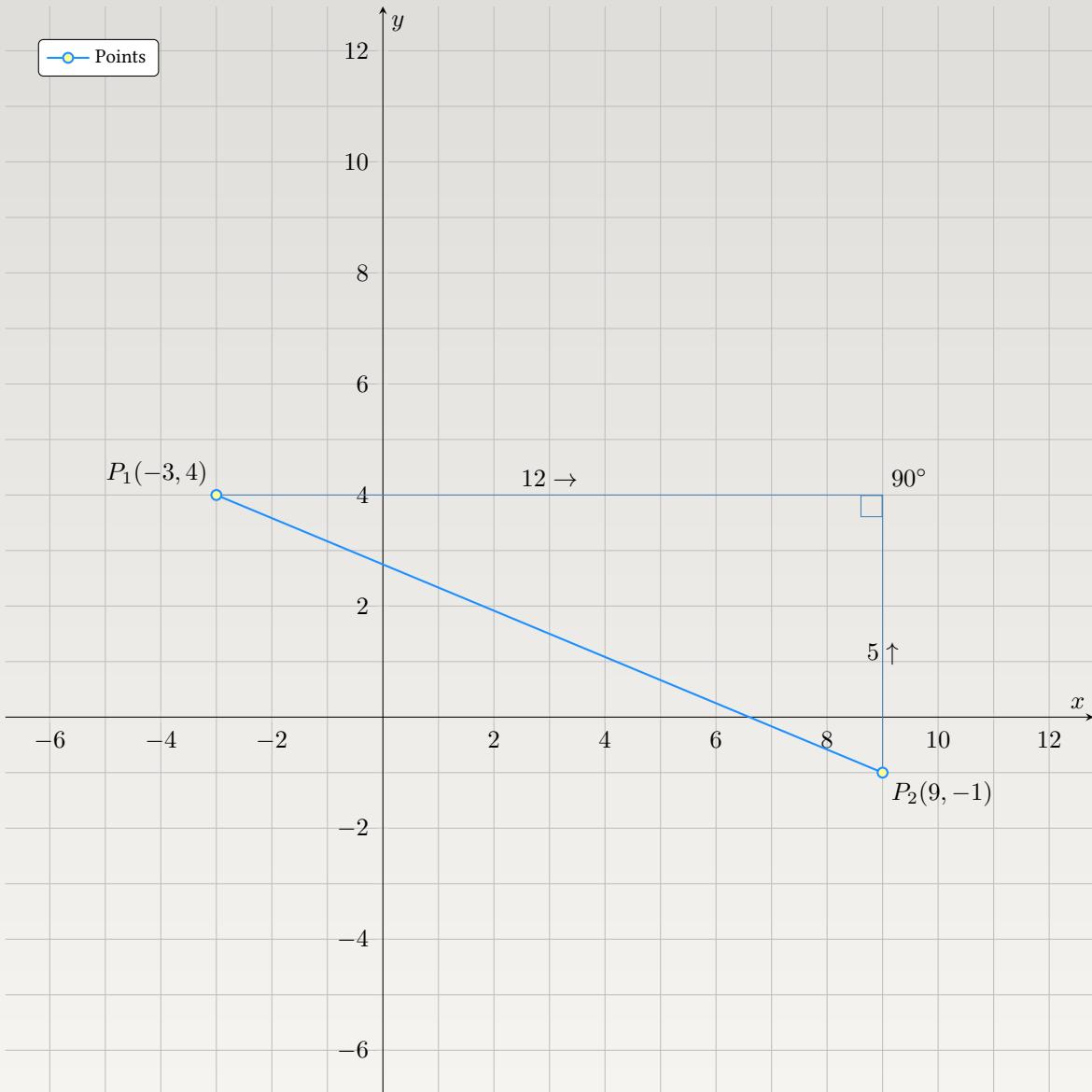
$$d = \sqrt{(4-1)^2 + (6-2)^2}$$





$$\begin{array}{ll} P_1(-3, 4) & P_2(9, -1) \\ x_1 = -3 & x_2 = 9 \\ y_1 = 4 & y_2 = -1 \end{array}$$
$$d = \sqrt{(9 - (-3))^2 + ((-1) - 4)^2} \Rightarrow \sqrt{12^2 + -5^2} \Rightarrow \sqrt{144 + 25} \Rightarrow \sqrt{169} \Rightarrow 13$$

$$d = \sqrt{(9 - (-3))^2 + ((-1) - 4)^2}$$





II Sets

$\mathbb{P} \subset \mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R} \subset \mathbb{C} \subset \mathbb{H} \subset \mathbb{R}^n$ and branches into \mathbb{F}_p

II Complex

Complex Numbers \mathbb{C}

- ⇒ **Definition** ⇒ All numbers of the form $a + bi$, where $a, b \in \mathbb{R}$ and $i = \sqrt{-1}$
- ⇒ **Examples** ⇒ $3 + 2i, -1 - i, 0 + 4i$
- ⇒ **Relation** ⇒ Contains all reals as the special case $b = 0$

II Finite Fields

Finite Fields \mathbb{F}_p

- ⇒ **Definition** ⇒ A set of integers modulo a prime p , with addition and multiplication defined mod p
- ⇒ **Examples** ⇒ $\mathbb{F}_2 = \{0, 1\}, \mathbb{F}_5 = \{0, 1, 2, 3, 4\}$
- ⇒ **Relation** ⇒ Finite fields are algebraic systems distinct from \mathbb{R} , but essential in number theory
- ⇒ **Use** ⇒ Cryptography, coding theory, error correction, and algebraic geometry

II Integers

Integers \mathbb{Z}

- ⇒ **Definition** ⇒ All whole numbers, both positive and negative, including zero
- ⇒ **Examples** ⇒ $\{-3, -2, -1, 0, 1, 2, 3, \dots\}$
- ⇒ **Relation** ⇒ Every integer can be expressed as a rational $\frac{n}{1}$, so $\mathbb{Z} \subset \mathbb{Q}$



II Natural Numbers

Natural Numbers \mathbb{N}

- ➊ **Definition** ⇒ The counting numbers, starting from 1, sometimes including 0 depending on convention
- ➋ **Examples** ⇒ $\{1, 2, 3, 4, \dots\}$ or $\{0, 1, 2, 3, \dots\}$
- ➌ **Relation** ⇒ $\mathbb{N} \subset \mathbb{Z}$, since naturals are a subset of the integers
- ➍ **Use** ⇒ Foundation for counting, arithmetic, and building larger number sets

II Primes

Primes \mathbb{P}

- ➊ **Definition** ⇒ Natural numbers greater than 1 that have no divisors other than 1 and themselves
- ➋ **Examples** ⇒ $\{2, 3, 5, 7, 11, 13, \dots\}$
- ➌ **Relation** ⇒ $\mathbb{P} \subset \mathbb{N}$, primes are a special subset of the naturals
- ➍ **Use** ⇒ Foundation of number theory, factorization, and cryptography

II Quaternions

Quaternions \mathbb{H}

- ➊ **Definition** ⇒ Numbers of the form $a + bi + cj + dk$, where $a, b, c, d \in \mathbb{R}$ and $i^2 = j^2 = k^2 = ijk = -1$
- ➋ **Examples** ⇒ $1 + 2i + 3j + 4k, -i + j$
- ➌ **Relation** ⇒ $\mathbb{C} \subset \mathbb{H}$, since complex numbers are a special case with $c = d = 0$
- ➍ **Use** ⇒ Applied in 3D rotations, computer graphics, and physics for representing orientation

II Rationals

Rationals \mathbb{Q}

- ➊ **Definition** ⇒ Numbers expressible as a fraction $\frac{p}{q}$ with integers p, q and $q \neq 0$
- ➋ **Examples** ⇒ $\frac{2}{3}, -\frac{5}{4}, 7 = \frac{7}{1}$
- ➌ **Relation** ⇒ All rationals are contained in the reals, so $\mathbb{Q} \subset \mathbb{R}$



II Reals

Reals \mathbb{R}

- ➡ **Definition** ⇒ All rationals plus irrationals (non-repeating, non-terminating decimals)
- ➡ **Examples** ⇒ $\pi, \sqrt{2}, 0.333\dots$
- ➡ **Relation** ⇒ Every real is a complex number with imaginary part 0, so $\mathbb{R} \subset \mathbb{C}$

II Vector Spaces

Vector Spaces \mathbb{R}^n

- ➡ **Definition** ⇒ Ordered tuples of real numbers, representing points or vectors in n -dimensional space
- ➡ **Examples** ⇒ $\mathbb{R}^2 = \{(x, y) \mid x, y \in \mathbb{R}\}, \mathbb{R}^3 = \{(x, y, z)\}$
- ➡ **Relation** ⇒ Built on \mathbb{R} , extending reals into higher dimensions
- ➡ **Use** ⇒ Geometry, physics, linear algebra, and computer graphics for modeling multidimensional systems

III Definitions

III Pairs

Factorial ($n!$)

Concept	Factorial ($n!$)
Definition	The product of all positive integers up to n . Defined as $n! = n \times (n - 1) \times \dots \times 1$.
Core Idea	Factorial counts permutations and combinations — it grows extremely fast.
Example	$5! = 120$.
Applications	Used in combinatorics, probability, and series expansions.
Pair	Inverse gamma function (not elementary).



Logarithm Base 2 (\log_2)

Concept	Logarithm Base 2 ($\log_2(x)$)
Definition	The inverse of the power of 2. Defined as the exponent y such that $2^y = x$.
Core Idea	$\log_2(x)$ measures how many times you multiply 2 to reach x .
Example	$\log_2(8) = 3$.
Applications	Widely used in computer science, information theory, and binary systems.
Pair	Power of 2 function (2^x).

Natural Logarithm (\ln)

Concept	Natural Logarithm ($\ln(x)$)
Definition	The inverse of the exponential function. Defined as the power to which e must be raised to equal x .
Core Idea	$\ln(x)$ undoes exponentiation with base e .
Example	$\ln(e^3) = 3$.
Applications	Used in calculus, growth/decay models, and solving exponential equations.
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Pair	Exponential function (e^x).

Tangent and Cotangent

Concept	Tangent ($\tan(x)$) and Cotangent ($\cot(x)$)
Definition	$\tan(x) = \frac{\sin(x)}{\cos(x)}$, while $\cot(x) = \frac{\cos(x)}{\sin(x)}$. They are reciprocals: $\cot(x) = \frac{1}{\tan(x)}$.
Core Idea	Tangent measures slope (rise/run). Cotangent flips that slope (run/rise).
Example	At 45° , $\tan(45^\circ) = 1$ and $\cot(45^\circ) = 1$.
Applications	Used in trigonometry, calculus, and geometry – especially for slope and angle analysis.
Pair	Reciprocal functions: $\tan(x) \leftrightarrow \cot(x)$.

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III Notation

III Delta in Mathematics

Finite Difference

- ➊ **Definition** ⇒ Delta denotes change or difference between two values
- ➋ **Formula** ⇒ $\Delta x = x_{\text{final}} - x_{\text{initial}}$
- ➌ **Usage** ⇒ Discrete counterpart to derivative: $\Delta y / \Delta x \approx dy/dx$
- ➍ **Example** ⇒ $\Delta y = f(x_2) - f(x_1)$
- ➎ **Extension** ⇒ Forms the basis of finite difference methods in numerical analysis

Discriminant

- ➊ **Definition** ⇒ In quadratic equations, Δ denotes the discriminant
- ➋ **Formula** ⇒ $\Delta = b^2 - 4ac$
- ➌ **Usage** ⇒ Determines the nature of roots of $ax^2 + bx + c = 0$
- ➍ **Example** ⇒ $\Delta > 0$: two real roots; $\Delta = 0$: one real root; $\Delta < 0$: complex roots
- ➎ **Extension** ⇒ Generalized discriminants exist for higher-degree polynomials



Triangle Symbol

- ➡ **Definition** ⇒ Δ also denotes a triangle in geometry
- ➡ **Usage** ⇒ ΔABC means triangle with vertices A, B, C
- ➡ **Connection** ⇒ Links algebraic glyph with geometric figure lineage
- ➡ **Extension** ⇒ Area of a triangle often denoted by Δ

III Theta in Mathematics

Angle Representation

- ➡ **Definition** ⇒ Greek letter θ used to denote an angle
- ➡ **Trigonometry** ⇒ Appears in sine, cosine, tangent: e.g. $\sin(\theta)$, $\cos(\theta)$, $\tan(\theta)$
- ➡ **Geometry** ⇒ Used in right triangles to relate sides and angles
- ➡ **Polar Coordinates** ⇒ Point (r, θ) defined by radius and angle
- ➡ **Example** ⇒ In a right triangle, $\cos(\theta) = \frac{\text{adjacent}}{\text{hypotenuse}}$

Other Mathematical Uses

- ➡ **Statistics** ⇒ θ often denotes parameters in probability distributions
- ➡ **Complex Numbers** ⇒ Angle of rotation in Euler's formula: $e^{i\theta} = \cos(\theta) + i \sin(\theta)$
- ➡ **Calculus** ⇒ Variable of integration in polar coordinates
- ➡ **Numerical Value** ⇒ In Greek numerals, θ has value 9
- ➡ **Connection** ⇒ Symbol of rotation, periodicity, and parameterization across math

III Epsilon in Mathematics

Infinitesimal Bound

- ➡ **Definition** ⇒ Greek letter ε used to denote a very small positive quantity
- ➡ **Limit Definition** ⇒ Appears in $\varepsilon - \delta$ proofs: for every $\varepsilon > 0$, there exists $\delta > 0$
- ➡ **Usage** ⇒ Measures closeness of a function to a limit
- ➡ **Example** ⇒ $|f(x) - L| < \varepsilon$ whenever $|x - c| < \delta$
- ➡ **Connection** ⇒ Core of rigorous calculus and real analysis



Error and Approximation

- ➡ **Definition** ⇒ ε often denotes error tolerance or margin
- ➡ **Numerical Analysis** ⇒ Used to bound approximation error
- ➡ **Example** ⇒ If $|x - x_0| < \varepsilon$, then x is within tolerance
- ➡ **Connection** ⇒ Links exact mathematics with numerical computation

Other Uses

- ➡ **Set Theory** ⇒ ε sometimes used as membership symbol, though \in is standard
- ➡ **Complexity** ⇒ ε denotes arbitrarily small constants in algorithm analysis
- ➡ **Probability** ⇒ ε used in inequalities like Chebyshev's or ε -nets
- ➡ **Connection** ⇒ Universal glyph for “smallness” across math disciplines

III Delta in Mathematics

Infinitesimal Change

- ➡ **Definition** ⇒ Greek letter δ denotes a very small positive quantity
- ➡ **Limit Proofs** ⇒ Appears in ε - δ definitions of limits
- ➡ **Formula** ⇒ For every $\varepsilon > 0$, there exists $\delta > 0$
- ➡ **Usage** ⇒ Controls how close x must be to c for $f(x)$ to be within ε of L
- ➡ **Example** ⇒ If $|x - c| < \delta$, then $|f(x) - L| < \varepsilon$
- ➡ **Connection** ⇒ δ measures input closeness, ε measures output closeness

Variation and Error

- ➡ **Definition** ⇒ δ often denotes small variation or tolerance
- ➡ **Numerical Analysis** ⇒ Used to bound input error
- ➡ **Example** ⇒ If $|x - x_0| < \delta$, then x is within tolerance of x_0
- ➡ **Connection** ⇒ Pairs with ε to formalize precision in analysis and computation



Other Mathematical Uses

- ➡ **Geometry** ⇒ δ sometimes used for small angles
- ➡ **Statistics** ⇒ δ may denote deviation or perturbation
- ➡ **Complexity** ⇒ δ used for small constants in algorithm analysis
- ➡ **Connection** ⇒ Universal glyph for “small input change” across math disciplines

III Proportional Symbol

Definition

- ➡ **Glyph** ⇒ \propto resembles the left half of ∞
- ➡ **Meaning** ⇒ Denotes proportionality between two quantities
- ➡ **Formula** ⇒ $y \propto x$ means $y = kx$ for some constant k
- ➡ **Usage** ⇒ Used in algebra, physics, and statistics to show direct proportionality
- ➡ **Example** ⇒ Gravitational force: $F \propto \frac{1}{r^2}$

Key Properties

- ➡ **Constant of Proportionality** ⇒ Always exists: $y = kx$
- ➡ **Direct Proportionality** ⇒ If one doubles, the other doubles
- ➡ **Inverse Proportionality** ⇒ Written as $y \propto \frac{1}{x}$
- ➡ **Scaling** ⇒ Proportionality preserves ratios
- ➡ **Connection** ⇒ Symbol bridges ratios, scaling laws, and functional dependence

III Infinity in Mathematics

Concept of Infinity

- ➡ **Definition** ⇒ ∞ denotes an unbounded quantity, larger than any real number
- ➡ **Calculus** ⇒ Appears in limits: $\lim_{x \rightarrow \infty} f(x)$
- ➡ **Set Theory** ⇒ Represents cardinalities of infinite sets
- ➡ **Geometry** ⇒ Used to mark points at infinity in projective geometry
- ➡ **Connection** ⇒ Symbol of endlessness, beyond finite measurement



Infinity in Calculus

- ➔ **Improper Integrals** ⇒ $\int_1^\infty \frac{1}{x^2} dx$
- ➔ **Limits** ⇒ $\lim_{x \rightarrow \infty} \frac{1}{x} = 0$
- ➔ **Series** ⇒ Infinite sums: $\sum_{n=1}^{\infty} \frac{1}{n^2}$
- ➔ **Connection** ⇒ ∞ marks the boundary of convergence and divergence
- ➔ **Example** ⇒ Harmonic series diverges: $\sum_{n=1}^{\infty} \frac{1}{n}$

Infinity in Set Theory

- ➔ **Countable Infinity** ⇒ Size of natural numbers, denoted \aleph_0
- ➔ **Uncountable Infinity** ⇒ Size of real numbers, larger than \aleph_0
- ➔ **Comparison** ⇒ Not all infinities are equal
- ➔ **Connection** ⇒ ∞ as a concept differs from cardinal numbers
- ➔ **Example** ⇒ $|\mathbb{N}| = \aleph_0$, but $|\mathbb{R}| > \aleph_0$

III Perpendicular Symbol

Definition

- ➔ **Glyph** ⇒ \perp is the mathematical symbol for perpendicularity
- ➔ **Meaning** ⇒ Two lines, segments, or planes meet at a right angle 90^{deg}
- ➔ **Notation** ⇒ $AB \perp CD$ means line AB is perpendicular to line CD
- ➔ **Geometry** ⇒ Used to denote orthogonality in Euclidean space
- ➔ **Example** ⇒ In a square, adjacent sides are \perp to each other

Linear Algebra Connection

- ➔ **Orthogonality** ⇒ \perp denotes vectors with dot product zero
- ➔ **Formula** ⇒ $\vec{u} \perp \vec{v} \iff \vec{u} \cdot \vec{v} = 0$
- ➔ **Usage** ⇒ Defines orthogonal bases and projections
- ➔ **Example** ⇒ $(1, 0) \perp (0, 1)$ in \mathbb{R}^2
- ➔ **Extension** ⇒ Orthogonality generalizes perpendicularity to higher dimensions



Other Uses

- ➡ **Logic** ⇒ \perp sometimes denotes contradiction or falsity
- ➡ **Probability** ⇒ \perp used to denote independence in some texts
- ➡ **Connection** ⇒ Symbol bridges geometry, algebra, and logic
- ➡ **Visual** ⇒ Always evokes the right-angle lineage

III Plus-Minus and Minus-Plus

Plus-Minus (\pm)

- ➡ **Definition** ⇒ Symbol \pm means “plus or minus”
- ➡ **Usage** ⇒ Represents two possible values: $a + b$ or $a - b$
- ➡ **Example** ⇒ Quadratic formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
- ➡ **Connection** ⇒ Encodes duality in solutions, symmetry in expansions
- ➡ **Extension** ⇒ Used in error bounds and approximations: $x \pm \varepsilon$

Minus-Plus (\mp)

- ➡ **Definition** ⇒ Symbol \mp means “minus or plus,” paired with \pm
- ➡ **Usage** ⇒ Ensures opposite choice when \pm is used earlier
- ➡ **Example** ⇒ If first term is $+$, second term takes $-$; if first is $-$, second takes $+$
- ➡ **Connection** ⇒ Keeps expressions consistent in paired signs
- ➡ **Extension** ⇒ Common in trigonometric identities and vector formulas

Combined Expression

- ➡ **Notation** ⇒ $a \pm b \mp c$
- ➡ **Meaning** ⇒ Two cases: $a + b - c$ or $a - b + c$
- ➡ **Pattern** ⇒ \pm and \mp always paired to flip signs consistently
- ➡ **Example** ⇒ In trig: $\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$
- ➡ **Connection** ⇒ Encodes dual solutions in compact symbolic form



III Parallel Symbol

Definition

- ➊ **Glyph** ⇒ \parallel is the mathematical symbol for parallelism
- ➋ **Meaning** ⇒ Two lines, segments, or planes never intersect and remain equidistant
- ➌ **Notation** ⇒ $AB \parallel CD$ means line AB is parallel to line CD
- ➍ **Geometry** ⇒ Used in Euclidean geometry to denote parallel lines and planes
- ➎ **Example** ⇒ In a rectangle, opposite sides are \parallel to each other

Linear Algebra Connection

- ➊ **Vectors** ⇒ \parallel denotes vectors that are scalar multiples of each other
- ➋ **Formula** ⇒ $\vec{u} \parallel \vec{v} \iff \vec{u} = k\vec{v}$
- ➌ **Usage** ⇒ Defines direction equivalence in vector spaces
- ➍ **Example** ⇒ $(2, 4) \parallel (1, 2)$ since $(2, 4) = 2(1, 2)$
- ➎ **Extension** ⇒ Parallelism generalizes beyond geometry into linear algebra and physics

Other Mathematical Uses

- ➊ **Analysis** ⇒ $\|x\|$ sometimes denotes norm of a vector
- ➋ **Logic** ⇒ \parallel used in some texts for “parallel execution” or independence
- ➌ **Connection** ⇒ Symbol bridges geometry, algebra, and analysis
- ➍ **Visual** ⇒ Always evokes equidistant, non-intersecting lineage

III Angle Symbol

Definition

- ➊ **Glyph** ⇒ \angle is the mathematical symbol for an angle
- ➋ **Meaning** ⇒ Represents the measure of rotation between two intersecting lines or rays
- ➌ **Notation** ⇒ $\angle ABC$ means the angle formed at vertex B by rays BA and BC
- ➍ **Units** ⇒ Measured in degrees ($^\circ$) or radians
- ➎ **Example** ⇒ $\angle ABC = 90^\circ$ denotes a right angle



Geometry Usage

- ⇒ **Right Angle** ⇒ $\angle = 90^\circ$
- ⇒ **Acute Angle** ⇒ $\angle < 90^\circ$
- ⇒ **Obtuse Angle** ⇒ $90^\circ < \angle < 180^\circ$
- ⇒ **Straight Angle** ⇒ $\angle = 180^\circ$
- ⇒ **Reflex Angle** ⇒ $180^\circ < \angle < 360^\circ$

Other Mathematical Uses

- ⇒ **Trigonometry** ⇒ \angle used inside sine, cosine, tangent functions
- ⇒ **Polar Coordinates** ⇒ Point (r, θ) defined by radius and angle
- ⇒ **Complex Numbers** ⇒ Angle defines argument of a complex number
- ⇒ **Vector Analysis** ⇒ Angle between vectors via dot product
- ⇒ **Connection** ⇒ Symbol bridges geometry, trigonometry, and analysis

III Similar vs Equivalent

Similar Symbol (\sim)

- ⇒ **Glyph** ⇒ \sim is the symbol for similarity
- ⇒ **Geometry** ⇒ $\triangle ABC \sim \triangle DEF$ means triangles have equal angles and proportional sides
- ⇒ **Algebra** ⇒ Sometimes used to denote asymptotic equivalence: $f(x) \sim g(x)$ as $x \rightarrow \infty$
- ⇒ **Pattern** ⇒ Similarity preserves shape but not necessarily size
- ⇒ **Example** ⇒ $\triangle ABC \sim \triangle DEF$ if $\angle A = \angle D, \angle B = \angle E, \angle C = \angle F$
- ⇒ **Connection** ⇒ Symbol of proportionality in geometry and asymptotics in analysis



Equivalent Symbol (\equiv)

- ➡ **Glyph** ⇒ \equiv is the symbol for equivalence
- ➡ **Congruence** ⇒ $\triangle ABC \equiv \triangle DEF$ means triangles are identical in shape and size
- ➡ **Number Theory** ⇒ Used for modular congruence: $a \equiv b \pmod{n}$
- ➡ **Logic** ⇒ Denotes logical equivalence: $P \equiv Q$
- ➡ **Pattern** ⇒ Equivalence preserves both shape and size, or exact relation
- ➡ **Example** ⇒ $17 \equiv 5 \pmod{12}$
- ➡ **Connection** ⇒ Symbol of exact sameness across geometry, algebra, and logic

III Approximate Symbol

Definition

- ➡ **Glyph** ⇒ \approx is the mathematical symbol for approximation
- ➡ **Meaning** ⇒ Indicates two values are close but not exactly equal
- ➡ **Notation** ⇒ $a \approx b$ means a is approximately equal to b
- ➡ **Usage** ⇒ Common in numerical analysis, applied math, and physics
- ➡ **Example** ⇒ $\pi \approx 3.1416$

Contexts of Use

- ➡ **Numerical** ⇒ Used when rounding or truncating decimals
- ➡ **Physics** ⇒ Marks measured values close to theoretical ones
- ➡ **Statistics** ⇒ Denotes approximate probabilities or estimates
- ➡ **Analysis** ⇒ Signals asymptotic closeness in limits
- ➡ **Connection** ⇒ Symbol bridges exact math with practical computation



Related Symbols

- ➔ **Equal Sign** ⇒ $=$ denotes exact equality
- ➔ **Tilde** ⇒ \sim denotes similarity or asymptotic equivalence
- ➔ **Congruence** ⇒ \equiv denotes exact equivalence or modular congruence
- ➔ **Approx** ⇒ \approx specifically signals numerical closeness
- ➔ **Extension** ⇒ Each glyph stages a different level of sameness

III Similar or Equal Symbol

Definition

- ➔ **Glyph** ⇒ \simeq is the symbol for “similar or equal”
- ➔ **Meaning** ⇒ Indicates two quantities are nearly equal and share structural similarity
- ➔ **Usage** ⇒ Common in analysis, approximation, and asymptotic notation
- ➔ **Example** ⇒ $f(x) \simeq g(x)$ means functions are close in value and form
- ➔ **Connection** ⇒ Bridges similarity (\sim) and equality ($=$)

Contexts of Use

- ➔ **Approximation** ⇒ Used when values are not exactly equal but very close
- ➔ **Asymptotics** ⇒ Signals functions behave similarly as $x \rightarrow \infty$
- ➔ **Geometry** ⇒ Sometimes used to denote figures nearly congruent
- ➔ **Physics** ⇒ Marks quantities equal within experimental tolerance
- ➔ **Extension** ⇒ \simeq is less strict than \equiv (equivalent) but stronger than \approx (approximate)



III Congruent Symbol

Definition

- ➡ **Glyph** ⇒ \cong is produced in LaTeX with \cong
- ➡ **Meaning** ⇒ Denotes congruence: “is congruent to”
- ➡ **Geometry** ⇒ $\triangle ABC \cong \triangle DEF$ means triangles are identical in shape and size
- ➡ **Pattern** ⇒ Congruence preserves both angles and side lengths
- ➡ **Example** ⇒ \triangle with sides 3,4,5 is \cong to another 3,4,5 triangle
- ➡ **Connection** ⇒ Stronger than similarity (\sim), exact match without scaling

Other Mathematical Uses

- ➡ **Number Theory** ⇒ Sometimes used interchangeably with \equiv for modular congruence
- ➡ **Analysis** ⇒ Can denote “is approximately congruent” in some texts
- ➡ **Logic** ⇒ Rarely used for structural equivalence
- ➡ **Extension** ⇒ \cong bridges geometry congruence with algebraic congruence
- ➡ **Visual** ⇒ Glyph resembles equality with a tilde, staging sameness plus shape relation

III Limit Symbol (\lim)

Definition

- ➡ **Glyph** ⇒ \lim denotes the limit of a function or sequence
- ➡ **Meaning** ⇒ Describes the value a function approaches as the input approaches some point
- ➡ **Notation** ⇒ $\lim_{x \rightarrow c} f(x)$
- ➡ **Example** ⇒ $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$
- ➡ **Connection** ⇒ Core concept in calculus, analysis, and continuity



Types of Limits

- ➊ **Finite Limit** ⇒ Function approaches a finite value as input approaches a point
- ➋ **Infinite Limit** ⇒ Function grows without bound: $\lim_{x \rightarrow 0^+} \frac{1}{x} = \infty$
- ➌ **One-Sided Limit** ⇒ $\lim_{x \rightarrow c^-} f(x)$ or $\lim_{x \rightarrow c^+} f(x)$
- ➍ **At Infinity** ⇒ $\lim_{x \rightarrow \infty} f(x)$
- ➎ **Sequence Limit** ⇒ $\lim_{n \rightarrow \infty} a_n$

Formal Definition

- ➊ **ε - δ Definition** ⇒ For every $\varepsilon > 0$, there exists $\delta > 0$
- ➋ **Condition** ⇒ If $|x - c| < \delta$, then $|f(x) - L| < \varepsilon$
- ➌ **Meaning** ⇒ $f(x)$ gets arbitrarily close to L as x approaches c
- ➍ **Connection** ⇒ Defines continuity and rigor in calculus
- ➎ **Example** ⇒ $\lim_{x \rightarrow 2} (3x + 1) = 7$

Limits

- ➊ **Usage** ⇒ Appears in limit notation: $\lim_{x \rightarrow c} f(x)$
- ➋ **Meaning** ⇒ “to” means the variable approaches a value
- ➌ **Example** ⇒ $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$
- ➍ **Connection** ⇒ Glyph of approach, not exact arrival
- ➎ **Extension** ⇒ Also used in one-sided limits: $x \rightarrow c^+$, $x \rightarrow c^-$

Mappings

- ➊ **Usage** ⇒ Appears in function notation: $f : A \rightarrow B$
- ➋ **Meaning** ⇒ “to” denotes mapping from domain to codomain
- ➌ **Example** ⇒ $f : \mathbb{R} \rightarrow \mathbb{R}$, $f(x) = x^2$
- ➍ **Connection** ⇒ Glyph of transformation, linking sets
- ➎ **Extension** ⇒ Used in category theory: arrows $A \rightarrow B$



Ranges and Intervals

- ➔ **Usage** ⇒ Appears in describing ranges: “from . . . to . . .”
- ➔ **Meaning** ⇒ Marks boundaries of intervals or sums
- ➔ **Example** ⇒ $\sum_{i=1}^n a_i$ means i runs from 1 to n
- ➔ **Connection** ⇒ Glyph of span, marking start and end
- ➔ **Extension** ⇒ Used in integrals: $\int_a^b f(x) dx$

III Summation Symbol

Definition

- ➔ **Glyph** ⇒ \sum is produced in LaTeX with \sum
- ➔ **Meaning** ⇒ Denotes summation of a sequence of terms
- ➔ **Connection** ⇒ Symbol of accumulation, discrete counterpart to integration
- ➔ **Visual** ⇒ Large sigma glyph evokes totality

Basic Usage

- ➔ **Syntax** ⇒ `\sum_{i=1}^n a_i`
- ➔ **Example** ⇒ $\sum_{i=1}^n a_i$
- ➔ **Inline** ⇒ Appears small in text mode
- ➔ **Display** ⇒ Expands with limits above/below in display math
- ➔ **Connection** ⇒ Bridges discrete series with continuous integrals

Properties

- ➔ **Linearity** ⇒ $\sum(a_i + b_i) = \sum a_i + \sum b_i$
- ➔ **Index Shift** ⇒ $\sum_{i=0}^n a_i = \sum_{j=1}^{n+1} a_{j-1}$
- ➔ **Empty Sum** ⇒ Defined as 0
- ➔ **Connection** ⇒ Properties mirror integral laws
- ➔ **Extension** ⇒ Generalizes to multiple indices and infinite series



Examples

- ➊ **Arithmetic Series** ⇒ $\sum_{i=1}^n i = \frac{n(n+1)}{2}$
- ➋ **Geometric Series** ⇒ $\sum_{i=0}^n r^i = \frac{1-r^{n+1}}{1-r}$
- ➌ **Infinite Series** ⇒ $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$
- ➍ **Connection** ⇒ Summation glyph stages discrete accumulation
- ➎ **Visual** ⇒ Limits above/below emphasize range of accumulation

III Product Symbol

Definition

- ➊ **Glyph** ⇒ \prod is produced in LaTeX with `\prod`
- ➋ **Meaning** ⇒ Denotes the product of a sequence of terms
- ➌ **Connection** ⇒ Discrete multiplicative counterpart to summation
- ➍ **Visual** ⇒ Large Pi glyph evokes multiplication lineage

Basic Usage

- ➊ **Syntax** ⇒ `\prod_{i=1}^n a_i`
- ➋ **Example** ⇒ $\prod_{i=1}^n a_i$
- ➌ **Inline** ⇒ Appears small in text mode
- ➍ **Display** ⇒ Expands with limits above/below in display math
- ➎ **Connection** ⇒ Bridges discrete multiplication with continuous exponentials

Properties

- ➊ **Empty Product** ⇒ Defined as 1
- ➋ **Factorization** ⇒ $\prod a_i b_i = (\prod a_i)(\prod b_i)$
- ➌ **Index Shift** ⇒ $\prod_{i=0}^n a_i = \prod_{j=1}^{n+1} a_{j-1}$
- ➍ **Connection** ⇒ Properties mirror exponential laws
- ➎ **Extension** ⇒ Generalizes to multiple indices and infinite products



Examples

- ➔ **Factorial** ⇒ $\prod_{i=1}^n i = n!$
- ➔ **Binomial** ⇒ $\prod_{i=1}^n (1 + x_i)$
- ➔ **Infinite Product** ⇒ $\prod_{n=1}^{\infty} \left(1 - \frac{1}{p_n^2}\right)$ converges for primes p_n
- ➔ **Connection** ⇒ Product glyph stages multiplicative accumulation
- ➔ **Visual** ⇒ Limits above/below emphasize range of multiplication

III Coproduct Symbol

Definition

- ➔ **Glyph** ⇒ \coprod is produced in L^AT_EX with `\coprod`
- ➔ **Meaning** ⇒ Denotes a categorical coproduct or disjoint union
- ➔ **Connection** ⇒ Dual to the product symbol \prod
- ➔ **Visual** ⇒ Large disjoint-union glyph evokes branching lineage

Basic Usage

- ➔ **Syntax** ⇒ `\coprod_{i \in I} A_i`
- ➔ **Example** ⇒ $\coprod_{i \in I} A_i$
- ➔ **Inline** ⇒ Appears small in text mode
- ➔ **Display** ⇒ Expands with limits above/below in display math
- ➔ **Connection** ⇒ Stages categorical disjoint union across index sets

Properties

- ➔ **Duality** ⇒ Coproduct is the categorical dual of product
- ➔ **Disjoint Union** ⇒ In set theory, \coprod denotes union with tags
- ➔ **Direct Sum** ⇒ In algebra, often used for direct sums of modules or vector spaces
- ➔ **Connection** ⇒ Mirrors \prod but accumulates by union rather than multiplication
- ➔ **Extension** ⇒ Generalizes to infinite coproducts in category theory



Examples

- ⇒ **Sets** ⇒ $\coprod_{i=1}^n A_i$ is the disjoint union of sets A_i
- ⇒ **Vector Spaces** ⇒ $\coprod_{i=1}^n V_i$ often written as $\bigoplus_{i=1}^n V_i$
- ⇒ **Categories** ⇒ Coproduct object satisfies universal property dual to product
- ⇒ **Connection** ⇒ Glyph stages branching accumulation across structures
- ⇒ **Visual** ⇒ Limits above/below emphasize index range of union

III Integral Symbol

Definition

- ⇒ **Glyph** ⇒ \int is produced in LaTeX with `\int`
- ⇒ **Meaning** ⇒ Denotes integration, the continuous counterpart to summation
- ⇒ **Connection** ⇒ Symbol of accumulation over a continuum
- ⇒ **Visual** ⇒ Elongated S glyph evokes summation lineage stretched across continuity

Basic Usage

- ⇒ **Syntax** ⇒ `\int_a^b f(x) dx`
- ⇒ **Example** ⇒ $\int_a^b f(x) dx$
- ⇒ **Indefinite** ⇒ $\int f(x) dx$ denotes antiderivative
- ⇒ **Definite** ⇒ $\int_a^b f(x) dx$ accumulates area under curve
- ⇒ **Connection** ⇒ Bridges discrete sums with continuous accumulation

Properties

- ⇒ **Linearity** ⇒ $\int (af(x) + bg(x)) dx = a \int f(x) dx + b \int g(x) dx$
- ⇒ **Additivity** ⇒ $\int_a^c f(x) dx = \int_a^b f(x) dx + \int_b^c f(x) dx$
- ⇒ **Fundamental Theorem** ⇒ $\int_a^b f(x) dx = F(b) - F(a)$ if $F' = f$
- ⇒ **Connection** ⇒ Properties mirror summation laws in continuous domain
- ⇒ **Extension** ⇒ Generalizes to multiple integrals and contour integrals



Examples

- ➡ **Area** ⇒ $\int_0^1 x \, dx = \frac{1}{2}$
- ➡ **Exponential** ⇒ $\int e^x \, dx = e^x + C$
- ➡ **Trigonometric** ⇒ $\int_0^\pi \sin x \, dx = 2$
- ➡ **Improper** ⇒ $\int_1^\infty \frac{1}{x^2} \, dx = 1$
- ➡ **Connection** ⇒ Integral glyph stages continuous accumulation across domains

III Contour Integral Symbol

Definition

- ➡ **Glyph** ⇒ \oint is produced in LaTeX with `\oint`
- ➡ **Meaning** ⇒ Denotes integration over a closed curve or contour
- ➡ **Connection** ⇒ Specialized form of \int emphasizing closed paths
- ➡ **Visual** ⇒ Integral sign with a circle evokes looping lineage

Basic Usage

- ➡ **Syntax** ⇒ `\oint_C f(z) dz`
- ➡ **Example** ⇒ $\oint_C f(z) dz$
- ➡ **Inline** ⇒ Appears small in text mode
- ➡ **Display** ⇒ Expands with limits above/below in display math
- ➡ **Connection** ⇒ Stages accumulation along closed paths in complex plane

Properties

- ➡ **Cauchy Integral** ⇒ $\oint_C \frac{f(z)}{z-a} dz = 2\pi i f(a)$
- ➡ **Residue Theorem** ⇒ $\oint_C f(z) dz = 2\pi i \sum \text{Residues}$
- ➡ **Path Independence** ⇒ Zero if integrand analytic inside contour
- ➡ **Connection** ⇒ Properties mirror fundamental theorems of complex analysis
- ➡ **Extension** ⇒ Generalizes to multiple closed integrals and homology cycles



Examples

- ⇒ **Unit Circle** ⇒ $\oint_{|z|=1} \frac{dz}{z} = 2\pi i$
- ⇒ **Residue** ⇒ $\oint_C \frac{1}{z^2+1} dz = 2\pi i \cdot \frac{1}{2}$
- ⇒ **Physics** ⇒ Closed path integrals in electromagnetism (Ampère's law)
- ⇒ **Connection** ⇒ Glyph stages looping accumulation around singularities
- ⇒ **Visual** ⇒ Circle on integral sign mythifies closure of path

III Double Integral Symbol

Definition

- ⇒ **Glyph** ⇒ \iint is produced in LaTeX with `\iint`
- ⇒ **Meaning** ⇒ Denotes integration over a two-dimensional region
- ⇒ **Connection** ⇒ Generalizes \int from 1D line segments to 2D areas
- ⇒ **Visual** ⇒ Two integral signs evoke accumulation across surfaces

Basic Usage

- ⇒ **Syntax** ⇒ `\iint_D f(x,y) dx dy`
- ⇒ **Example** ⇒ $\iint_D f(x,y) dx dy$
- ⇒ **Region** ⇒ Domain D specifies the area of integration
- ⇒ **Display** ⇒ Expands with limits above/below in display math
- ⇒ **Connection** ⇒ Stages continuous accumulation across two variables

Properties

- ⇒ **Iterated Integrals** ⇒ $\iint_D f(x,y) dx dy = \int_a^b \int_c^d f(x,y) dy dx$
- ⇒ **Fubini's Theorem** ⇒ Allows evaluation as successive 1D integrals
- ⇒ **Linearity** ⇒ $\iint (af + bg) = a \iint f + b \iint g$
- ⇒ **Connection** ⇒ Properties mirror single integrals but extend to surfaces
- ⇒ **Extension** ⇒ Generalizes to triple integrals (`\iiint`) and higher dimensions



Examples

- ➔ **Area** ⇒ $\iint_D 1 \, dx \, dy$ gives area of region D
- ➔ **Volume** ⇒ $\iint_D f(x, y) \, dx \, dy$ underlies volume via surface integration
- ➔ **Polar Coordinates** ⇒ $\iint_D f(r, \theta) r \, dr \, d\theta$
- ➔ **Physics** ⇒ Used in flux calculations across surfaces
- ➔ **Connection** ⇒ Glyph stages accumulation across 2D domains, bridging geometry and analysis

III Aleph-Null

Definition

- ➔ **Glyph** ⇒ \aleph_0 is produced in LaTeX with `\aleph_0`
- ➔ **Meaning** ⇒ Denotes the cardinality of the set of natural numbers
- ➔ **Connection** ⇒ Smallest infinite cardinal, foundation of transfinite arithmetic
- ➔ **Visual** ⇒ Hebrew letter Aleph with subscript zero evokes countable infinity

Properties

- ➔ **Countable Sets** ⇒ Integers, rationals all have size \aleph_0
- ➔ **Closure** ⇒ $\aleph_0 + n = \aleph_0$, $\aleph_0 \cdot n = \aleph_0$
- ➔ **Comparison** ⇒ Strictly smaller than continuum cardinality \mathfrak{c}
- ➔ **Connection** ⇒ Stages hierarchy of infinities in Cantorian set theory
- ➔ **Extension** ⇒ Generalizes to larger cardinals: $\aleph_1, \aleph_2, \dots$

Examples

- ➔ **Naturals** ⇒ $|\mathbb{N}| = \aleph_0$
- ➔ **Integers** ⇒ $|\mathbb{Z}| = \aleph_0$
- ➔ **Rationals** ⇒ $|\mathbb{Q}| = \aleph_0$
- ➔ **Connection** ⇒ All countable infinite sets share cardinality \aleph_0
- ➔ **Visual** ⇒ Glyph stages smallest rung of infinite ladder



III Omega Infinity

Definition

- ➊ **Glyph** ⇒ Ω_∞ staged with `\Omega_\infty \backslash infy`
- ➋ **Meaning** ⇒ Used informally for absolute infinity or infinite asymptotic bound
- ➌ **Connection** ⇒ Dual lineage: Cantorian philosophy vs algorithmic notation
- ➍ **Visual** ⇒ Greek Omega with infinity evokes boundless closure

Cantorian Context

- ➊ **Absolute Infinite** ⇒ Beyond all transfinite cardinals
- ➋ **Metaphysical** ⇒ Cantor linked Ω_∞ to divine infinity
- ➌ **Non-Cardinal** ⇒ Not part of formal set hierarchy
- ➍ **Connection** ⇒ Stages philosophical extension of infinity
- ➎ **Visual** ⇒ Glyph evokes “beyond all” lineage

Algorithmic Context

- ➊ **Definition** ⇒ Ω^∞ means bound holds infinitely often
- ➋ **Contrast** ⇒ Weaker than standard Ω which holds eventually always
- ➌ **Usage** ⇒ Appears in complexity analysis
- ➍ **Connection** ⇒ Stages weaker asymptotic guarantee
- ➎ **Extension** ⇒ Rare but useful in algorithmic lineage audits

III Intersection Symbol

Definition

- ➊ **Glyph** ⇒ \bigcap is produced in LaTeX with `\bigcap`
- ➋ **Meaning** ⇒ Denotes the intersection of a family of sets
- ➌ **Connection** ⇒ Symbol of commonality, elements shared across sets
- ➍ **Visual** ⇒ Large cap glyph evokes overlapping lineage



Basic Usage

- ➊ **Syntax** ⇒ $\bigcap_{i=1}^n A_i$
- ➋ **Example** ⇒ $\bigcap_{i=1}^n A_i$
- ➌ **Inline** ⇒ Appears small in text mode
- ➍ **Display** ⇒ Expands with limits above/below in display math
- ➎ **Connection** ⇒ Stages accumulation of shared membership across sets

Properties

- ➊ **Commutative** ⇒ Order of intersection doesn't matter
- ➋ **Associative** ⇒ Grouping doesn't affect result
- ➌ **Idempotent** ⇒ $A \cap A = A$
- ➍ **Connection** ⇒ Properties mirror logical AND
- ➎ **Extension** ⇒ Generalizes to infinite intersections

III Union Symbol

Definition

- ➊ **Glyph** ⇒ \bigcup is produced in LaTeX with `\bigcup`
- ➋ **Meaning** ⇒ Denotes the union of a family of sets
- ➌ **Connection** ⇒ Symbol of totality, elements gathered across sets
- ➍ **Visual** ⇒ Large cup glyph evokes merging lineage

Basic Usage

- ➊ **Syntax** ⇒ $\bigcup_{i=1}^n A_i$
- ➋ **Example** ⇒ $\bigcup_{i=1}^n A_i$
- ➌ **Inline** ⇒ Appears small in text mode
- ➍ **Display** ⇒ Expands with limits above/below in display math
- ➎ **Connection** ⇒ Stages accumulation of all membership across sets



Properties

- ⊕ **Commutative** ⇒ Order of union doesn't matter
- ⊕ **Associative** ⇒ Grouping doesn't affect result
- ⊕ **Idempotent** ⇒ $A \cup A = A$
- ⊕ **Connection** ⇒ Properties mirror logical OR
- ⊕ **Extension** ⇒ Generalizes to infinite unions