Math Notes

Canine-Table

Github

POSIX Nexus serves as a comprehensive cross-language reference hub that explores the implementation and behavior of POSIX-compliant functionality across a diverse set of programming environments. Built atop the foundational IEEE Portable Operating System Interface (POSIX) standards, this project emphasizes compatibility, portability, and interoperability between operating systems.

Abstract

Contents

I Algebra

I Nomials

Nomial Lineage

- **Monomial** $\Rightarrow$ One term only, e.g. $7x^2$
- **Binomial** $\Rightarrow$ Two terms, e.g. $x+2$
- **Trinomial** $\Rightarrow$ Three terms, e.g. x^2+3x+2
- **Polynomial** $\Rightarrow$ Many terms, general family
- **Technique** $\Rightarrow$ Prefix indicates number of terms
- **Outcome** $\Rightarrow$ Classification helps organize algebraic expressions

$$\text{Monomial: } ax^n \tag{1}$$
$$\text{Binomial: } ax^n + bx^m \tag{2}$$
$$\text{Trinomial: } ax^n + bx^m + cx^k \tag{3}$$

Polynomial Forms

- **Monomial** ⇒ One term only, e.g. $7x^2$
- **Binomial** ⇒ Two unlike terms, e.g. $x + 2$
- **Trinomial** ⇒ Three terms, e.g. $x^2 + 3x + 2$
- **Polynomial** ⇒ General family with many terms
- **Technique** ⇒ Prefix indicates number of terms
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Like Terms

- **Definition** ⇒ Expressions with identical variable parts, e.g. $3x^2$ and $-5x^2$
- **Variable Match** ⇒ Same variables with same exponents
- **Coefficient** ⇒ Numbers in front may differ
- **Technique** ⇒ Combine by adding or subtracting coefficients
- **Outcome** ⇒ Simplifies polynomials by reducing to fewer terms

Coefficient

- **Definition** ⇒ The number in front of a variable, e.g. in $7x$ the coefficient is 7
- **Variable Match** ⇒ It scales the variable part without changing its type
- **Examples** ⇒ $3x^2$ has coefficient 3, $-5y$ has coefficient -5
- **Constants** ⇒ A constant term like 4 can be seen as coefficient 4 of x^0
- **Outcome** ⇒ Coefficients tell how strongly each variable contributes to the polynomial

I Polynomial Exponents

Definition of Negative Exponent

- **Positive Exponent** ⇒ x^n means multiply x by itself n times
- **Negative Exponent** ⇒ x^{-n} means reciprocal of x^n
- **Rule** ⇒ $x^{-n} = \frac{1}{x^n}$
- **Example** ⇒ $x^{-3} = \frac{1}{x^3}$
- **Use** ⇒ Negative exponents express division or reciprocals in algebra and calculus

$$x^3 = x \cdot x \cdot x \;\Rightarrow\; x^{-3} = \frac{1}{x^3}$$

Using Negative Exponents

- **Step 1** $\Rightarrow$ Recall exponent rules: $x^a \cdot x^b = x^{a+b}$
- **Step 2** $\Rightarrow$ Set $a = 3$, $b = -3$: $x^3 \cdot x^{-3} = x^0$
- **Step 3** $\Rightarrow$ But $x^0 = 1$
- **Step 4** $\Rightarrow$ So x^{-3} must equal $\frac{1}{x^3}$
- **Outcome** $\Rightarrow$ Negative exponent means reciprocal of positive power

Exponent Flip Examples

$$x^5 = \frac{1}{x^{-5}} \qquad x^{12} = \frac{1}{x^{-12}} \tag{4}$$

$$x^{17} = \frac{1}{x^{-17}} \qquad 15x^9 = \frac{15}{x^{-9}} \tag{5}$$

$$28x^5 = \frac{28}{x^{-5}} \qquad x^5 = \frac{1}{x^{-5}} \tag{6}$$

$$x^{-3} = \frac{1}{x^3} \qquad x^3 = \frac{1}{x^{-3}} \tag{7}$$

$$x^{-3} = \frac{1}{x^3} \tag{8}$$

Polynomial Exponent Rules Applied

- **Power of a Power** ⇒ $(a^m)^n = a^{mn}$
- **Nested Powers** ⇒ Combine: $8 \cdot 3 = 24$
- **Outer Flip** ⇒ Apply (-9): $x^{768y^{5z^3}} \to x^{-6912y^{5z^3}}$
- **Technique** ⇒ Multiply all exponents carefully, preserve inner structure
- **Outcome** ⇒ Final simplified form: $x^{-6912y^{5z^3}}$

$$
\begin{aligned}
(((x^{32y^{5z^3}})^8)^3)^{-9} &= (x^{32y^{5z^3}})^{8\cdot 3} && \Rightarrow x^{768y^{5z^3}} \\
&= \left(x^{768y^{5z^3}}\right)^{-9} && \Rightarrow x^{-9\cdot 768y^{5z^3}} \\
&= x^{-6912y^{5z^3}}
\end{aligned}
$$

Exponent Rules Applied

- **Power of a Power** $\Rightarrow (a^m)^n = a^{mn}$
- **Nested Powers** $\Rightarrow$ Combine: $8 \cdot 3 = 24$
- **Outer Flip** $\Rightarrow$ Apply (-9): $x^{768y^{5z^3}} \to x^{-6912y^{5z^3}}$
- **Technique** $\Rightarrow$ Multiply all exponents carefully, preserve inner structure
- **Outcome** $\Rightarrow$ Final simplified form: $x^{-6912y^{5z^3}}$

$$
\begin{aligned}
x^2 \cdot x^4 &= x^{2+4} && \Rightarrow\ x^6 \\
x^7 \cdot x^5 &= x^{5+7} && \Rightarrow\ x^{12} \\
x^8 \cdot x^9 &= x^{8+9} && \Rightarrow\ x^{17} \\
(3x^3)(5x^6) &= (3 \cdot 5)x^{3+6} && \Rightarrow 15x^9 \\
(4x^2)(7x^3) &= (4 \cdot 7)x^{2+3} && \Rightarrow 28x^5 \\
(4xy^2)(8x^2y^3) &= (4 \cdot 8)x^{1+2}(1 \cdot 1)y^{2+3} && \Rightarrow\ 32x^3y^5 \\
(5x^2y^3)(6x^3y^4) &= (5 \cdot 6)x^{2+3}(1 \cdot 1)y^{3+4} && \Rightarrow\ 30x^5y^7 \\
(7x^3y^4)(8x^5y^7) &= (7 \cdot 8)x^{3+5}(1 \cdot 1)y^{4+7} && \Rightarrow\ 56x^8y^{11}
\end{aligned}
$$

$$
\begin{aligned}
(x^3)^4 &= x^{3\cdot 4} = x^{12} \\
(x^4)^6 &= x^{4\cdot 6} = x^{24} \\
(x^3)^5 &= x^{3\cdot 5} = x^{15} \\
(3x^2)^4 &= 3^{1\cdot 4}x^{2\cdot 4} = 3^4x^8 = 81x^8 \\
(2x^3)^3 &= 2^{1\cdot 3}x^{3\cdot 3} = 2^3x^9 = 8x^9
\end{aligned}
$$

$$\begin{aligned}(3x^2)^2(2x^3)^3 &= 3^{1\cdot 2}x^{2\cdot 2}2^{1\cdot 3}x3\cdot 3\\ &= 3^2x^42^3x^9\\ &= 9\cdot 8x^{4+9}\\ &= 72x^{13}\end{aligned}$$

$$\begin{aligned}(3^2x^3y^4)^2(2^3x^2y^5)^3 &= (3^{2\cdot 2}x^{3\cdot 2}y^{4\cdot 2})(2^{3\cdot 3}x^{2\cdot 3}y^{5\cdot 3})\\ &= (3^4x^6y^8)(2^9x^6y^{15})\\ &= (81x^6y^8)(512x^6y^{15})\\ &= 81\cdot 512x6+6y^{8+15}\\ &= 41472x^{12}y^{23}\end{aligned}$$

$$\begin{aligned}-2^3 &= -2\cdot -2\cdot -2 = -8\\ (-2)^3 &= -2\cdot -2\cdot -2 = -8\\ -(-2)^3 &= -2\cdot -2\cdot -2 = 8\end{aligned}$$

$$\begin{aligned}(-7x^2y^3)^0 &= -7^{2\cdot 0}x^{2\cdot 0}y^{3\cdot 0}\\ &= -7^0x^0y^0\\ &= 1\cdot 1\cdot 1\\ &= 1\end{aligned}$$

$$\begin{aligned}3x(5x+8) &= 15x^2+24x\\ 4x(x^2-2x+3) &= 4x^3-8x^2+12x\end{aligned}$$

I Dividing Polynomials

$$\frac{x^8}{x^3} = x^{8-3} \Rightarrow x^5 \tag{9}$$

$$\frac{x^5}{x^2} = x^{5-2} \Rightarrow x^3 \tag{10}$$

$$\frac{x^5}{x^8} = x^{5-8} \Rightarrow x^{-3} \tag{11}$$

$$\frac{x^4}{x^7} = x^{4-7} \Rightarrow x^{-3} \tag{12}$$

(13)

$$\begin{aligned}\frac{24x^9y^5}{8x^3y^{12}} &= \frac{24}{8}x^{9-3}\frac{1}{1}y^{5-12}\\ &= 3x^6y^{-7}\\ &= \frac{3x^6}{y^7}\end{aligned}$$

$$\begin{aligned}\frac{12x^5y^{-3}z^4}{36x^8y^{-4}z^{-8}} &= \frac{\frac{12}{3}x^{5-8}}{\frac{36}{3}}\frac{1}{1}y^{-3--4}\frac{1}{1}z^{4--8}\\ &= \frac{x^{-3}y^1z^{12}}{3}\\ &= \frac{yz^{12}}{3x^3}\end{aligned}$$

I Multiplying Polynomials

Multiplication Symbols

- **Dot** $\Rightarrow$ $\cdot$ is clean, algebraic, avoids confusion with x
- **Times** $\Rightarrow$ $\times$ is bold, arithmetic, or cross product
- **Context** $\Rightarrow$ Use $\cdot$ in algebra, $\times$ in arithmetic or vectors
- **Technique** $\Rightarrow$ Choose based on clarity and audience
- **Outcome** $\Rightarrow$ Both mean multiplication, but notation signals intent

$$x^2 \cdot x^4 = x^{2+4} \quad \Rightarrow \quad x^6$$

$$x^7 \cdot x^5 = x^{5+7} \quad \Rightarrow \quad x^{12}$$

$$x^8 \cdot x^9 = x^{8+9} \quad \Rightarrow \quad x^{17}$$

$$(3x^3)(5x^6) = (3 \cdot 5)x^{3+6} \quad \Rightarrow 15x^9$$

$$(4x^2)(7x^3) = (4 \cdot 7)x^{2+3} \quad \Rightarrow 28x^5$$

$$(4xy^2)(8x^2y^3) = (4 \cdot 8)x^{1+2}(1 \cdot 1)y^{2+3} \quad \Rightarrow \ 32x^3y^5$$

$$(5x^2y^3)(6x^3y^4) = (5 \cdot 6)x^{2+3}(1 \cdot 1)y^{3+4} \quad \Rightarrow \ 30x^5y^7$$

$$(7x^3y^4)(8x^5y^7) = (7 \cdot 8)x^{3+5}(1 \cdot 1)y^{4+7} \quad \Rightarrow \ 56x^8y^{11}$$

I Combining Polynomials

$$x + 4 = 7 \;\Rightarrow\; x + 4 - 4 = 7 - 4 \;\Rightarrow\; x + 0 = 3$$

$$x + 9 = 15 \;\Rightarrow\; x + 9 - 9 = 15 - 9 \;\Rightarrow\; x + 0 = 6$$

$$6 + x = 13 \;\Rightarrow\; 6 + x - 6 = 15 - 9 \;\Rightarrow\; 0 + x - 0 = 7$$

$$x - 3 = 9 \;\Rightarrow\; x - 3 + 3 = 9 - 3 \;\Rightarrow\; x - 0 = 6 \;\Rightarrow\; x = 6$$

$$x - 8 = 7 \;\Rightarrow\; x + 8 - 8 = 7 + 8 \;\Rightarrow\; x - 0 = 15 \;\Rightarrow\; x = 15$$

$$3x + 5 = 11 \;\Rightarrow\; 3x + 5 - 5 = 11 - 5 \;\Rightarrow\; \frac{3x}{3} = \frac{6}{3} \;\Rightarrow\; x = 2$$

$$6.3 = -2 + x \;\Rightarrow\; 6.3 + 2 = -2 + 2 + x \;\Rightarrow\; 6.3 = 0 + x \;\Rightarrow\; x = 6.3$$

$$5 = x - 8 \;\Rightarrow\; 5 + 8 = x - 8 + 8 \;\Rightarrow\; 13 = x - 0 \;\Rightarrow\; \hookrightarrow x = 13$$

$$5 - x = 12 \;\Rightarrow\; 5 - 12 - x + x = 12 - 12 + x \;\Rightarrow\; \hookrightarrow -7 = 0 + x \;\Rightarrow\; x = -7$$

$$-8 = 5 - x \;\Rightarrow\; -8 + 8 = 5 + 8 - x \;\Rightarrow\; 0 + x = 13 - x + x \;\Rightarrow\; \hookrightarrow x = 13$$

$$3x = 12 \;\Rightarrow\; \frac{3x}{3} = \frac{12}{3} \;\Rightarrow\; \frac{x}{1} = 4 \;\Rightarrow\; x = 4$$

$$7x = 14 \Rightarrow \frac{7x}{7} = \frac{12}{7} \Rightarrow \frac{x}{1} = 2 \Rightarrow x = 2$$

$$-6x = -30 \Rightarrow \frac{-6x}{-6} = \frac{-30}{-6} \Rightarrow \frac{-x}{-1} = 5 \Rightarrow x = 5$$

$$-8x = 48 \Rightarrow \frac{-8x}{-8} = \frac{48}{-8} \Rightarrow \frac{-x}{-1} = -6 \Rightarrow x = -6$$

$$7x = -56 \Rightarrow \frac{7x}{7} = \frac{-56}{7} \Rightarrow \frac{x}{1} = -8 \Rightarrow x = -8$$

$$-8x = -72 \Rightarrow \frac{-8x}{-8} = \frac{-72}{-8} \Rightarrow \frac{x}{1} = 9 \Rightarrow x = 9$$

$$4x+3=6x-15 \;\Rightarrow\; 4x-4x+3+15=6x-4x-15+15 \;\Rightarrow$$
$$\hookrightarrow \frac{18}{2}=\frac{2x}{2}; \Rightarrow\; x=9$$

$$3(2x-4)=5(3x+2)-3 \;\Rightarrow\; 6x-12=15x+10-3 \;\Rightarrow$$
$$\hookrightarrow 6x-6x-12-7=15x-6x+7-7 \;\Rightarrow$$
$$\hookrightarrow \frac{-19}{9}=\frac{9x}{9} \;\Rightarrow\; x=\frac{-19}{9}$$

$$\frac{3}{4}x-\frac{2}{3}=12 \;\Rightarrow\; (\frac{3}{4}x\cdot 4)3-(\frac{2}{3}\cdot 3)4=12\cdot 12$$
$$\hookrightarrow (3x)3-(2)4=144 \;\Rightarrow\; 9x-8+8=144+8 \;\Rightarrow$$
$$\hookrightarrow \frac{9x}{9}=\frac{152}{9} \;\Rightarrow\; x=\frac{152}{9}$$

$$\frac{2}{3}x+5=8 \;\Rightarrow\; (\frac{2}{3}x+5=8)3 \;\Rightarrow\; 2x+15-15=24-15 \;\Rightarrow$$
$$\hookrightarrow \frac{2x}{2}=\frac{9}{2} \;\Rightarrow\; x=\frac{9}{2}$$

$$
\begin{aligned}
(3x+5)+(4x-2) &= 7x+3 \\
(4x^2+3x+9)+(5x^2+7x-4) &= 9x^2+10x+5 \\
(5x^2-6x-12)-(7x^2+4x-13) &= 5x^2-6x-12-7x^2-4x+13 \\
&= 5x^2-7x^2-12+13-6x-4x \\
&= -2x^2+1-10x
\end{aligned}
$$

I FOIL Method

$$
(a+b)(c+d) = ac+ad+bc+bd \tag{14}
$$

FOIL Method

- **First** $\Rightarrow$ Multiply first terms: $a \cdot c$
- **Outer** $\Rightarrow$ Multiply outer terms: $a \cdot d$
- **Inner** $\Rightarrow$ Multiply inner terms: $b \cdot c$
- **Last** $\Rightarrow$ Multiply last terms: $b \cdot d$
- **Outcome** $\Rightarrow$ Sum all four products to get the expanded expression

$$\begin{aligned}(2x+5)(4x^2-3x+6) &= (2\cdot 4)x^{1+2} + (2\cdot -3)^{1+1} + (2\cdot 6)x \\ &\hookrightarrow +(5\cdot 4)x^2 + (5\cdot -3)x + (5\cdot 6) \\ &= 8x^3 + (-6+20)^2 + (12+-15)x + 30 \\ &= 8x^3 + 14x^2 + -3x + 30\end{aligned}$$

$$
\begin{aligned}
(3x^2 - 2x + 4)(4x^2 + 5x + -6) = \\
\hookrightarrow (3 \cdot 4)x^{2+2} + (3 \cdot 5)x^{2+1} + (3 \cdot -6)x^{2+0} \\
\hookrightarrow +(-2 \cdot 4)x^{1+2} + (-2 \cdot 5)x^{1+1} + (-2 \cdot -6)x^{1+0} \\
\hookrightarrow +(4 \cdot 4)x^{0+2} + (4 \cdot 5)x^{0+1} + (4 \cdot -6)x^{0+0} \\
= 12x^4 + (15 + -8)x^3 + (-18 + -10 + 16)x^2 + (12 + 20)x + -24 \\
= 12x^4 + 7x^3 + -12x^2 + 32x + -24
\end{aligned}
$$

I Factoring Polynomials

$4a^2 + 2ab – 3a^2b + 5$

Terms	Factors	Prime Factors
$4a^2$	$4, a^2$	$2, 2, a, a$
$2ab$	$3, a, b$	$2, a, b$
$-3a^2b$	$-3, a^2, b$	$-3, a, a, b$
5	5	5

$xy^2 - 3x^2y^2 - 6y + z$

Terms	Factors	Prime Factors
xy^2	$4, a^2$	$2, 2, a, a$
$-3x^2y$	$3, a, b$	$2, a, b$
$-6y$	$-6, y$	$-2, 3, y$
z	z	z

$-5 + 2(3a^2 - 3t)$

Terms	Factors	Prime Factors
-5	-5	-5
$2(3a^2 - 3t)$	$2.3a^2 - 3t$	$2.3a^2 - 3t$
$6t$	$6, t$	$3, 3, t$

$3x^2 + 5x - 2$

Terms	Factors	Prime Factors
$3x^2$	$3, x^2$	$3.x.x$
$5x$	$5, x$	$5, x$
-2	-2	-2

Definition of Prime Factor

- **Prime** ⇒ A number greater than 1 divisible only by 1 and itself
- **Factor** ⇒ A number that divides another evenly
- **Prime Factor** ⇒ A prime number that divides another number exactly
- **Example** ⇒ $60 = 2^2 \cdot 3 \cdot 5$; prime factors are 2, 3, 5
- **Use** ⇒ Prime factors are the building blocks of integers, used in LCM, GCD, and simplification

Find LCM of 12 and 18

$$\begin{aligned} 12 &= 2^2 \cdot 3 && \Rightarrow \\ 18 &= 2 \cdot 3^2 && \Rightarrow \\ \text{LCM} &= 2^2 \cdot 3^2 && \hookrightarrow 36 \end{aligned}$$

Using Prime Factors for LCM

- **Step 1** ⇒ Prime factorize each number
- **Step 2** ⇒ Collect all distinct primes
- **Step 3** ⇒ Take the highest power of each prime
- **Step 4** ⇒ Multiply them together
- **Outcome** ⇒ LCM(12,18) = 36

Euclidean Modulus

```
define nx_pt_mod(x, y) {
                x = nx_abs(x)
                if (x == 0)
                                return 0
                y = nx_abs(y)
                if (y > 0)
                                return x - y *
↪nx_pt_trunc(x / y)
                print "<nx:impurity/>"
                return -1
}
```

The Greatest Common Factor

```
define nx_euc(x, y) {
                auto n
                if (x == y)
```

```
                            return x
                    while (x > 0 && y > 0) {
                            n = x
                            x = nx_pt_mod(y, x)
                            y = n
                    }
                    return n
}
```

$$(8,12) \mapsto \gcd(8,12) = 4$$
$$8x + 12 \;\Rightarrow\; 4(\frac{8x}{2} + \frac{12}{4}) \;\Rightarrow\; 4(2x+3)$$

$$(4,2) \mapsto \gcd(4,2) = 2$$
$$4x^2 + 2x \;\Rightarrow\; 2x(\frac{4x^2}{2x} + \frac{2x}{2x}) \;\Rightarrow\; 2x(2x+1)$$

$$(12,18) \mapsto \gcd(12,18) = 6$$
$$12ab^2 + 18a^2b^3 \;\Rightarrow\; 6ab^2(2+3ab)$$

Definition of Perfect Square

- **Perfect Square** $\Rightarrow$ A number that can be expressed as n^2 for some integer n
- **Integer Squared** $\Rightarrow$ Formed by multiplying an integer by itself
- **Examples** $\Rightarrow$ $1, 4, 9, 16, 25, 36, \ldots$
- **Non-Examples** $\Rightarrow$ $2, 3, 5, 6, 7, 10, \ldots$
- **Use** $\Rightarrow$ Perfect squares appear in factoring, radicals, and Pythagorean identities

Check if 49 is a perfect square

$$\begin{aligned} 49 &= 7 \cdot 7 && \Rightarrow \\ &= 7^2 && \hookrightarrow \end{aligned}$$

Therefore, 49 is a perfect square.

Using Perfect Squares

- **Step 1** $\Rightarrow$ Identify the number
- **Step 2** $\Rightarrow$ Ask if it can be written as n^2
- **Step 3** $\Rightarrow$ If yes, it is a perfect square
- **Step 4** $\Rightarrow$ If no, it is not
- **Outcome** $\Rightarrow$ 49 is a perfect square since $49 = 7^2$

Difference of Squares Applied

- **Identity** $\Rightarrow$ $(a^2 - b^2) = (a - b)(a + b)$
- **Example** $\Rightarrow$ $x^2 - 9$
- **Factorization** $\Rightarrow$ Apply rule: $x^2 - 3^2 = (x - 3)(x + 3)$
- **Technique** $\Rightarrow$ Recognize perfect squares and subtract
- **Outcome** $\Rightarrow$ Final factored form: $(x - 3)(x + 3)$

$$x^2 - 9 \;\Rightarrow\; x^2 - 3^2 \;\Rightarrow\; (x - 3)(x + 3)$$

Perfect Squares Difference

- **Identity** $\Rightarrow (a^2 - b^2) = (a-b)(a+b)$
- **Example** $\Rightarrow x^2 - 9$
- **Factorization** $\Rightarrow (x-3)(x+3)$
- **Technique** $\Rightarrow$ Spot the squares, apply the difference rule
- **Outcome** $\Rightarrow$ Factored polynomial form

$$(25) \mapsto \sqrt{(25)} = 5$$
$$x^2 - 25 \;\Rightarrow\; (x+5)(x-5)$$

$$(x-5)(x+5) \Rightarrow\; x^2 + 5x + -5x + -25 \Rightarrow$$
$$\hookrightarrow\; x^2 + (5x + -5x \Rightarrow\; 0) - 25 \Rightarrow\; x^2 - 25$$

$$(9) \mapsto \sqrt{(9)} = 3$$
$$x^2 - 9 \;\Rightarrow\; (x+3)(x-3)$$

$$
\begin{aligned}
(4) &\mapsto \sqrt{(4)} = 2 \\
x^2 - 4 &\Rightarrow (x+2)(x-2)
\end{aligned}
$$

$$4x^2 - 25 \Rightarrow (2x+5)(2x-5)$$

$$
\begin{aligned}
(81) &\mapsto \sqrt{(81)} = 9 \\
(16) &\mapsto \sqrt{(16)} = 4 \\
16x^2 - 25 &\Rightarrow (4x+9)(4x-9)
\end{aligned}
$$

$$25x^2 - 16y^2 \Rightarrow (5x+4y)(5x-4y)$$

$$81x^4 - 16y^8 \Rightarrow (9x^2+4y^4)(9x^2-4y^4) \Rightarrow (3x+2y^2)(3x-2y^2)$$

Factor by Grouping Applied

- **Setup** $\Rightarrow$ Polynomial with 4 terms
- **Grouping** $\Rightarrow$ Split into two pairs
- **Inner Factor** $\Rightarrow$ Factor each pair separately
- **Common Binomial** $\Rightarrow$ Extract the shared binomial
- **Outcome** $\Rightarrow$ Final factored form

$$x^3 + 3x^2 + 2x + 6 \;\Rightarrow\; (x^3 + 3x^2) + (2x + 6) \;\Rightarrow\; \hookrightarrow x^2(x+3) + 2(x+3) \;\Rightarrow\; (x^2+2)(x+3)$$

$$x^3 - 4x^2 + 3x - 12 \;\Rightarrow\; x^2(x-4) + 3(x-4) \;\Rightarrow\; \hookrightarrow \frac{x^2(x-4)}{x-4} + \frac{3(x-4)}{x-4} \;\Rightarrow\; \hookrightarrow (x-4)(x^2+3)$$

$$
\begin{aligned}
2x^3 - 6x^2 + 4x - 12 \Rightarrow 2x^2(x-3) + 4(x-3) \Rightarrow \\
\hookrightarrow \frac{2x^2(x-3)}{x-3} + \frac{4(x-3)}{x-3} \Rightarrow \\
\hookrightarrow (x-3)(2x^2+4)
\end{aligned}
$$

$$
\begin{aligned}
3x^3 + 8x^2 - 6x - 16 \Rightarrow x^2(3x+8) - 2(3x+8) \Rightarrow \\
\hookrightarrow \frac{x^2(3x+8)}{3x+8} + \frac{-2(3x+8)}{3x+8} \Rightarrow \\
\hookrightarrow (x^2-2)(3x+8) \Rightarrow \\
\hookrightarrow 3x^3 - 6x + 8x^2 - 16 \Rightarrow 3x(x^2-2) + 8(x^2-2) \Rightarrow \\
\hookrightarrow \frac{3x(x^2-2)}{x^2-2} + \frac{8(x^2-2)}{x^2-2} \Rightarrow \\
\hookrightarrow (x^2-2)(3x+8)
\end{aligned}
$$

I Lowest Common Denominator

> refer to I to see the gcd function.

The Lowest Common Denominator

```
define nx_lcd(x, y) {
            return x * y / nx_euc(x, y)
}
```

$$\frac{3}{4}+\frac{2}{5} \Rightarrow \frac{3\cdot 5}{4\cdot 5}+\frac{2\cdot 4}{5\cdot 4} \Rightarrow \frac{15}{20}+\frac{8}{20} \Rightarrow \frac{15+8}{20} \Rightarrow \frac{23}{20}$$

$$\frac{5}{6}+\frac{4}{7} \Rightarrow \frac{5\cdot 7}{6\cdot 7}+\frac{4\cdot 6}{7\cdot 6} \Rightarrow \frac{35}{42}+\frac{24}{42} \Rightarrow \frac{35-24}{42} \Rightarrow \frac{11}{42}$$

$$\frac{7}{5}\cdot\frac{4}{3} \Rightarrow \frac{7\cdot 4}{5\cdot 3} \Rightarrow \frac{28}{15} \Rightarrow 1\frac{13}{15}$$

$$(18,20) \mapsto \gcd(18,20) = 2$$
$$\frac{3}{5} \cdot \frac{6}{4} \Rightarrow \frac{3 \cdot 6}{5 \cdot 4} \Rightarrow \frac{18}{20} \Rightarrow \frac{\frac{18}{2}}{\frac{20}{2}} \Rightarrow \frac{9}{10}$$

$$(28,63) \mapsto \gcd(28,63) = 7$$
$$(56,35) \mapsto \gcd(56,35) = 7$$
$$\frac{28}{63} \cdot \frac{56}{35} \Rightarrow \frac{\frac{28}{7}}{\frac{63}{7}} \cdot \frac{\frac{56}{7}}{\frac{35}{7}} \Rightarrow \frac{4}{9} \cdot \frac{8}{5} \Rightarrow \frac{4 \cdot 8}{9 \cdot 5} \Rightarrow \frac{32}{45}$$

Definition of Keep-Change-Flip

- **Keep** $\Rightarrow$ Keep the first fraction exactly as it is
- **Change** $\Rightarrow$ Change the division sign to multiplication
- **Flip** $\Rightarrow$ Flip the second fraction (take its reciprocal)
- **Example** $\Rightarrow$ $\frac{3}{4} \div \frac{2}{5}$
- **Outcome** $\Rightarrow$ $\frac{3}{4} \times \frac{5}{2} = \frac{15}{8}$

$$\frac{3}{4} \div \frac{2}{5} \Rightarrow \frac{3}{4} \times \frac{5}{2}$$

Using Keep-Change-Flip

- **Step 1** ⇒ Write the division problem
- **Step 2** ⇒ Keep the first fraction
- **Step 3** ⇒ Change division to multiplication
- **Step 4** ⇒ Flip the second fraction
- **Step 5** ⇒ Multiply across numerators and denominators
- **Outcome** ⇒ Simplified fraction result

I Brackets

Curly Braces {}

- **Definition** $\Rightarrow$ Used to denote sets or grouping in mathematics, e.g. $\{1, 2, 3\}$
- **Programming** $\Rightarrow$ Common in code to enclose blocks of statements
- **LaTeX** $\Rightarrow$ Used to group arguments for commands
- **Example** $\Rightarrow$ $\{x \mid x > 0\}$ means the set of all positive x
- **Outcome** $\Rightarrow$ Curly braces signal structured grouping or set notation

Square Brackets []

- **Definition** $\Rightarrow$ Used for intervals, optional elements, or matrices
- **Interval** $\Rightarrow$ $[a, b]$ means all values between a and b, inclusive
- **Matrix** $\Rightarrow$ Brackets often enclose arrays of numbers
- **Example** $\Rightarrow$ $[2, 5]$ includes 2 and 5
- **Outcome** $\Rightarrow$ Square brackets emphasize inclusion or structured arrays

$$
\begin{aligned}
8[-6+8(-2+4)] &= 8[-6+8\cdot 2] \\
&= 8[-6+16] \\
&= 8\cdot 10 \\
&= 80
\end{aligned}
$$

Parentheses ()

- **Definition** $\Rightarrow$ Used for grouping, order of operations, or function arguments
- **Math** $\Rightarrow$ $(a+b)c$ ensures addition happens before multiplication
- **Functions** $\Rightarrow$ $f(x)$ shows input to a function
- **Interval** $\Rightarrow$ (a,b) means values strictly between a and b
- **Outcome** $\Rightarrow$ Parentheses control grouping and precedence in math and logic

$$
\begin{aligned}
-(-(1-2)+(5)) &= -(-(-1)+5) \\
&= -(1+5) \\
&= -(6) \\
&= -6
\end{aligned}
$$

$$\begin{aligned} 3 \cdot (6 - 3 + 1) - 4^2 &= 3 \cdot 4 - 16 \\ &= 12 - 16 \\ &= -4 \end{aligned}$$

$$\begin{aligned} (10, 4) \mapsto \gcd(10, 4) = 2 \\ \frac{2 \cdot (5 - 1) + 2}{4 \cdot (2 - 1)} &\Leftrightarrow (2 \cdot (5 - 1) + 2) \div (4 \cdot (2 - 1)) \\ &= \frac{2 \cdot 4 + 2}{4 \cdot 1} \\ &= \frac{8 + 2}{4} \\ &= \frac{10}{4} \\ &= \frac{\frac{10}{2}}{\frac{4}{2}} \\ &= \frac{5}{2} \end{aligned}$$

$$\begin{aligned} 5 + (1 \cdot 4 - (2 - 5) - 2) &= 4 - (-3) - 2 \\ &= 7 - 2 \\ &= 5 \end{aligned}$$

Double Vertical Bars $\|$

- **Definition** $\Rightarrow$ Used to denote absolute value or norm
- **Absolute Value** $\Rightarrow$ $|x|$ is distance of x from 0
- **Norm** $\Rightarrow$ $\|v\|$ is length of vector v
- **Example** $\Rightarrow$ $|-5| = 5$, $\|(3,4)\| = 5$
- **Outcome** $\Rightarrow$ Double bars measure magnitude or distance

$$\begin{aligned} 4 + |3 - 9| &= 4 + |-6| && (15) \\ &= 4 + 6 && (16) \\ &= 10 && (17) \end{aligned}$$

Floor Brackets $\lfloor\rfloor$

- **Definition** $\Rightarrow$ Used for the floor function
- **Floor** $\Rightarrow$ $\lfloor x \rfloor$ = greatest integer less than or equal to x
- **Example** $\Rightarrow$ $\lfloor 3.7 \rfloor = 3$
- **Use** $\Rightarrow$ Rounds down to nearest integer
- **Outcome** $\Rightarrow$ Floor brackets capture downward rounding

Ceiling Brackets $\lceil\rceil$

- **Definition** ⇒ Used for the ceiling function
- **Ceiling** ⇒ $\lceil x \rceil$ = smallest integer greater than or equal to x
- **Example** ⇒ $\lceil 3.2 \rceil = 4$
- **Use** ⇒ Rounds up to nearest integer
- **Outcome** ⇒ Ceiling brackets capture upward rounding

I Dividing

Fraction Components

- **Numerator** ⇒ Top of the fraction, counts selected parts, e.g. in $\frac{3}{4}$ the numerator is 3
- **Denominator** ⇒ Bottom of the fraction, defines total equal parts, e.g. in $\frac{3}{4}$ the denominator is 4
- **Relationship** ⇒ Fraction = Numerator ÷ Denominator
- **Technique** ⇒ Numerator changes with quantity chosen, denominator fixes the partition size
- **Outcome** ⇒ Understanding both clarifies fraction meaning and operations

$$\frac{6-2^3}{2} = \frac{6-8}{2} \tag{18}$$
$$= \frac{-2}{2} \tag{19}$$
$$= -1 \tag{20}$$

Verbose Floating-Point Multiplication

- **Step 1** $\Rightarrow$ Choose two floating-point numbers, e.g. 3.25 and 2.5
- **Step 2** $\Rightarrow$ Express them as fractions: $3.25 = \frac{325}{100}$, $2.5 = \frac{25}{10}$
- **Step 3** $\Rightarrow$ Multiply numerators: $325 \times 25 = 8125$
- **Step 4** $\Rightarrow$ Multiply denominators: $100 \times 10 = 1000$
- **Step 5** $\Rightarrow$ Form product fraction: $\frac{8125}{1000}$
- **Step 6** $\Rightarrow$ Simplify fraction: $\frac{8125}{1000} = 8.125$
- **Step 7** $\Rightarrow$ Restore decimal form: product is 8.125

$$
\begin{aligned}
10.4 &\Rightarrow \frac{10.4}{1}\frac{10.4}{1}\cdot 10 && \Rightarrow\Rightarrow \frac{104}{10}\\
1.3 &\Rightarrow \frac{1.3}{1} && \Rightarrow \frac{1.3}{1}\cdot 10 \Rightarrow \frac{13}{10}\\
1.5 &\Rightarrow \frac{1.5}{1} && \Rightarrow \frac{1.5}{1}\cdot 10 \Rightarrow \frac{15}{10}\\
-2.35 &\Rightarrow \frac{-2.35}{1} \Leftrightarrow \frac{2.35}{-1} && \Rightarrow \frac{-2.35}{1}\cdot 100 \Rightarrow \frac{-235}{100}
\end{aligned}
$$

$$
\begin{aligned}
&(10056, 1950) \mapsto \sqrt{(10056, 1950)} = 26\\
&\frac{10.3^2 - (-2.35)^2}{1.3(1.5)} = \frac{106.09 - 5.522}{1.95}\\
&= \frac{100.568}{1.95}\\
&= \frac{100568}{1950}\\
&= \frac{3868}{75}
\end{aligned}
$$

I Multiply

$$\left(\frac{103}{10}\right)^2 \Rightarrow \frac{103}{10} \cdot \frac{103}{10} \Rightarrow$$

			1	0	3
		×	1	0	3
			3	0	9
		0	0	0	0
+	1	0	3	0	0
	1	0	6	0	9

$$\frac{10609}{100} \Rightarrow 106.09$$

$$\left(\frac{-235}{100}\right)^2 \Rightarrow \frac{-235}{10} \cdot \frac{-235}{10} \Rightarrow$$

		−	2	3	5
	×	−	2	3	5
	1	0	5	1	5
		7	1	5	0
+	4	7	0	0	0
	5	7	6	6	5

$$\frac{57665}{100} \Rightarrow 576.65$$