Math Notes

Canine-Table

Github

POSIX Nexus serves as a comprehensive cross-language reference hub that explores the implementation and behavior of POSIX-compliant functionality across a diverse set of programming environments. Built atop the foundational IEEE Portable Operating System Interface (POSIX) standards, this project emphasizes compatibility, portability, and interoperability between operating systems.

Abstract

Contents

I Algebra

I Nomials

Nomial Lineage

- **Monomial** ⇒ One term only, e.g. $7x^2$
- **Binomial** ⇒ Two terms, e.g. $x + 2$
- **Trinomial** ⇒ Three terms, e.g. $x^2 + 3x + 2$
- **Polynomial** ⇒ Many terms, general family
- **Technique** ⇒ Prefix indicates number of terms
- **Outcome** ⇒ Classification helps organize algebraic expressions

Monomial: ax^n (1)

Binomial: $ax^n + bx^m$ (2)

Trinomial: $ax^n + bx^m + cx^k$ (3)

Polynomial Forms

- **Monomial** ⇒ One term only, e.g. $7x^2$
- **Binomial** ⇒ Two unlike terms, e.g. $x + 2$
- **Trinomial** ⇒ Three terms, e.g. $x^2 + 3x + 2$
- **Polynomial** ⇒ General family with many terms
- **Technique** ⇒ Prefix indicates number of terms
- **Outcome** ⇒ Classification helps organize algebraic expressions

Like Terms

- **Definition** ⇒ Expressions with identical variable parts, e.g. $3x^2$ and $-5x^2$
- **Variable Match** ⇒ Same variables with same exponents
- **Coefficient** ⇒ Numbers in front may differ
- **Technique** ⇒ Combine by adding or subtracting coefficients
- **Outcome** ⇒ Simplifies polynomials by reducing to fewer terms

Coefficient

- **Definition** ⇒ The number in front of a variable, e.g. in $7x$ the coefficient is 7
- **Variable Match** ⇒ It scales the variable part without changing its type
- **Examples** ⇒ $3x^2$ has coefficient 3, $-5y$ has coefficient -5
- **Constants** ⇒ A constant term like 4 can be seen as coefficient 4 of x^0
- **Outcome** ⇒ Coefficients tell how strongly each variable contributes to the polynomial

I Polynomial Exponents

Exponent Flip Examples

$$x^5 = \frac{1}{x^{-5}} \qquad x^{12} = \frac{1}{x^{-12}} \tag{4}$$

$$x^{17} = \frac{1}{x^{-17}} \qquad 15x^9 = \frac{15}{x^{-9}} \tag{5}$$

$$28x^5 = \frac{28}{x^{-5}} \qquad x^5 = \frac{1}{x^{-5}} \tag{6}$$

$$x^{-3} = \frac{1}{x^3} \qquad x^3 = \frac{1}{x^{-3}} \tag{7}$$

$$x^{-3} = \frac{1}{x^3} \tag{8}$$

$$\begin{aligned}(((x^{32y^{5z^3}})^8)^3)^{-9} &= (x^{32y^{5z^3}})^{8\cdot 3} && \Rightarrow x^{768y^{5z^3}} \\ &= \left(x^{768y^{5z^3}}\right)^{-9} && \Rightarrow x^{-9\cdot 768y^{5z^3}} \\ &= x^{-6912y^{5z^3}}\end{aligned}$$

Polynomial Exponent Rules Applied

- **Power of a Power** $\Rightarrow (a^m)^n = a^{mn}$
- **Nested Powers** $\Rightarrow$ Combine: $8 \cdot 3 = 24$
- **Outer Flip** $\Rightarrow$ Apply (-9): $x^{768y^{5z^3}} \to x^{-6912y^{5z^3}}$
- **Technique** $\Rightarrow$ Multiply all exponents carefully, preserve inner structure
- **Outcome** $\Rightarrow$ Final simplified form: $x^{-6912y^{5z^3}}$

$$\begin{aligned}(((x^{32y^{5z^3}})^8)^3)^{-9} &= (x^{32y^{5z^3}})^{8\cdot 3} && \Rightarrow x^{768y^{5z^3}} \\ &= \left(x^{768y^{5z^3}}\right)^{-9} && \Rightarrow x^{-9\cdot 768y^{5z^3}} \\ &= x^{-6912y^{5z^3}}\end{aligned}$$

Exponent Rules Applied

- **Power of a Power** $\Rightarrow (a^m)^n = a^{mn}$
- **Nested Powers** $\Rightarrow$ Combine: $8 \cdot 3 = 24$
- **Outer Flip** $\Rightarrow$ Apply (-9): $x^{768y^{5z^3}} \to x^{-6912y^{5z^3}}$
- **Technique** $\Rightarrow$ Multiply all exponents carefully, preserve inner structure
- **Outcome** $\Rightarrow$ Final simplified form: $x^{-6912y^{5z^3}}$

$$
\begin{aligned}
x^2 \cdot x^4 = x^{2+4} &\Rightarrow x^6 \\
x^7 \cdot x^5 = x^{5+7} &\Rightarrow x^{12} \\
x^8 \cdot x^9 = x^{8+9} &\Rightarrow x^{17} \\
(3x^3)(5x^6) = (3 \cdot 5)x^{3+6} &\Rightarrow 15x^9 \\
(4x^2)(7x^3) = (4 \cdot 7)x^{2+3} &\Rightarrow 28x^5 \\
(4xy^2)(8x^2y^3) = (4 \cdot 8)x^{1+2}(1 \cdot 1)y^{2+3} &\Rightarrow 32x^3y^5 \\
(5x^2y^3)(6x^3y^4) = (5 \cdot 6)x^{2+3}(1 \cdot 1)y^{3+4} &\Rightarrow 30x^5y^7 \\
(7x^3y^4)(8x^5y^7) = (7 \cdot 8)x^{3+5}(1 \cdot 1)y^{4+7} &\Rightarrow 56x^8y^{11}
\end{aligned}
$$

$$
\begin{aligned}
(x^3)^4 &= x^{3\cdot 4} = x^{12} \\
(x^4)^6 &= x^{4\cdot 6} = x^{24} \\
(x^3)^5 &= x^{3\cdot 5} = x^{15} \\
(3x^2)^4 &= 3^{1\cdot 4}x^{2\cdot 4} = 3^4x^8 = 81x^8 \\
(2x^3)^3 &= 2^{1\cdot 3}x^{3\cdot 3} = 2^3x^9 = 8x^9
\end{aligned}
$$

$$
\begin{aligned}
(3x^2)^2(2x^3)^3 &= 3^{1\cdot 2}x^{2\cdot 2}2^{1\cdot 3}x3\cdot 3 \\
&= 3^2x^42^3x^9 \\
&= 9\cdot 8x^{4+9} \\
&= 72x^{13}
\end{aligned}
$$

$$
\begin{aligned}
(3^2x^3y^4)^2(2^3x^2y^5)^3 &= (3^{2\cdot 2}x^{3\cdot 2}y^{4\cdot 2})(2^{3\cdot 3}x^{2\cdot 3}y^{5\cdot 3}) \\
&= (3^4x^6y^8)(2^9x^6y^{15}) \\
&= (81x^6y^8)(512x^6y^{15}) \\
&= 81\cdot 512x6 + 6y^{8+15} \\
&= 41472x^{12}y^{23}
\end{aligned}
$$

$$-2^3 = -2 \cdot -2 \cdot -2 = -8$$
$$(-2)^3 = -2 \cdot -2 \cdot -2 = -8$$
$$-(-2)^3 = -2 \cdot -2 \cdot -2 = 8$$

$$\begin{aligned}(-7x^2y^3)^0 &= -7^{2\cdot 0}x^{2\cdot 0}y^{3\cdot 0}\\ &= -7^0x^0y^0\\ &= 1 \cdot 1 \cdot 1\\ &= 1\end{aligned}$$

$$3x(5x+8) = 15x^2 + 24x$$
$$4x(x^2 - 2x + 3) = 4x^3 - 8x^2 + 12x$$

I Dividing Polynomials

Fraction Components

- **Numerator** ⇒ Top of the fraction, counts selected parts, e.g. in $\frac{3}{4}$ the numerator is 3
- **Denominator** ⇒ Bottom of the fraction, defines total equal parts, e.g. in $\frac{3}{4}$ the denominator is 4
- **Relationship** ⇒ Fraction = Numerator ÷ Denominator
- **Technique** ⇒ Numerator changes with quantity chosen, denominator fixes the partition size
- **Outcome** ⇒ Understanding both clarifies fraction meaning and operations

$$\frac{x^8}{x^3} = x^{8-3} \Rightarrow x^5 \tag{9}$$

$$\frac{x^5}{x^2} = x^{5-2} \Rightarrow x^3 \tag{10}$$

$$\frac{x^5}{x^8} = x^{5-8} \Rightarrow x^{-3} \tag{11}$$

$$\frac{x^4}{x^7} = x^{4-7} \Rightarrow x^{-3} \tag{12}$$

(13)

$$\begin{aligned}\frac{24x^9y^5}{8x^3y^12} &= \frac{24}{8}x^{9-3}\frac{1}{1}y^{5-12} \\ &= 3x^6y^{-7} \\ &= \frac{3x^6}{y^7}\end{aligned}$$

$$\begin{aligned}\frac{12x^5y^{-3}z^4}{36x^8y^{-4}x^{-8}} &= \frac{\frac{12}{3}x^{5-8}}{\frac{36}{3}}\frac{1}{1}y^{-3--4}\frac{1}{1}z^{4--8} \\ &= \frac{x^{-3}y^1z^{12}}{3} \\ &= \frac{yz^{12}}{3x^3}\end{aligned}$$

I Multiplying Polynomials

Multiplication Symbols

- **Dot** $\Rightarrow$ $\cdot$ is clean, algebraic, avoids confusion with x
- **Times** $\Rightarrow$ $\times$ is bold, arithmetic, or cross product
- **Context** $\Rightarrow$ Use $\cdot$ in algebra, $\times$ in arithmetic or vectors
- **Technique** $\Rightarrow$ Choose based on clarity and audience
- **Outcome** $\Rightarrow$ Both mean multiplication, but notation signals intent

$$x^2 \cdot x^4 = x^{2+4} \quad \Rightarrow \quad x^6$$

$$x^7 \cdot x^5 = x^{5+7} \quad \Rightarrow \quad x^{12}$$

$$x^8 \cdot x^9 = x^{8+9} \quad \Rightarrow \quad x^{17}$$

$$(3x^3)(5x^6) = (3 \cdot 5)x^{3+6} \quad \Rightarrow 15x^9$$

$$(4x^2)(7x^3) = (4 \cdot 7)x^{2+3} \quad \Rightarrow 28x^5$$

$$(4xy^2)(8x^2y^3) = (4 \cdot 8)x^{1+2}(1 \cdot 1)y^{2+3} \quad \Rightarrow \ 32x^3y^5$$

$$(5x^2y^3)(6x^3y^4) = (5 \cdot 6)x^{2+3}(1 \cdot 1)y^{3+4} \quad \Rightarrow \ 30x^5y^7$$

$$(7x^3y^4)(8x^5y^7) = (7 \cdot 8)x^{3+5}(1 \cdot 1)y^{4+7} \quad \Rightarrow \ 56x^8y^{11}$$

I Combining Polynomials

$$x + 4 = 7 \Rightarrow x + 4 - 4 = 7 - 4 \Rightarrow x + 0 = 3$$

$$x + 9 = 15 \Rightarrow x + 9 - 9 = 15 - 9 \Rightarrow x + 0 = 6$$

$$6 + x = 13 \Rightarrow 6 + x - 6 = 15 - 9 \Rightarrow 0 + x - 0 = 7$$

$$x - 3 = 9 \Rightarrow x - 3 + 3 = 9 - 3 \Rightarrow x - 0 = 6 \Rightarrow x = 6$$

$$x - 8 = 7 \Rightarrow x + 8 - 8 = 7 + 8 \Rightarrow x - 0 = 15 \Rightarrow x = 15$$

$$3x + 5 = 11 \Rightarrow 3x + 5 - 5 = 11 - 5 \Rightarrow \frac{3x}{3} = \frac{6}{3} \Rightarrow x = 2$$

$$6.3 = -2 + x \;\Rightarrow\; 6.3 + 2 = -2 + 2 + x \;\Rightarrow\; 6.3 = 0 + x \;\Rightarrow\; x = 6.3$$

$$5 = x - 8 \;\Rightarrow\; 5 + 8 = x - 8 + 8 \;\Rightarrow\; 13 = x - 0 \;\Rightarrow\; \hookrightarrow x = 13$$

$$5 - x = 12 \;\Rightarrow\; 5 - 12 - x + x = 12 - 12 + x \;\Rightarrow\; \hookrightarrow -7 = 0 + x \;\Rightarrow\; x = -7$$

$$-8 = 5 - x \;\Rightarrow\; -8 + 8 = 5 + 8 - x \;\Rightarrow\; 0 + x = 13 - x + x \;\Rightarrow\; \hookrightarrow x = 13$$

$$3x = 12 \;\Rightarrow\; \frac{3x}{3} = \frac{12}{3} \;\Rightarrow\; \frac{x}{1} = 4 \;\Rightarrow\; x = 4$$

$$7x = 14 \Rightarrow \frac{7x}{7} = \frac{12}{7} \Rightarrow \frac{x}{1} = 2 \Rightarrow x = 2$$

$$-6x = -30 \Rightarrow \frac{-6x}{-6} = \frac{-30}{-6} \Rightarrow \frac{-x}{-1} = 5 \Rightarrow x = 5$$

$$-8x = 48 \Rightarrow \frac{-8x}{-8} = \frac{48}{-8} \Rightarrow \frac{-x}{-1} = -6 \Rightarrow x = -6$$

$$7x = -56 \Rightarrow \frac{7x}{7} = \frac{-56}{7} \Rightarrow \frac{x}{1} = -8 \Rightarrow x = -8$$

$$-8x = -72 \Rightarrow \frac{-8x}{-8} = \frac{-72}{-8} \Rightarrow \frac{x}{1} = 9 \Rightarrow x = 9$$

$$4x+3=6x-15 \;\Rightarrow\; 4x-4x+3+15=6x-4x-15+15 \;\Rightarrow$$
$$\hookrightarrow \frac{18}{2}=\frac{2x}{2};\Rightarrow\; x=9$$

$$3(2x-4)=5(3x+2)-3 \;\Rightarrow\; 6x-12=15x+10-3 \;\Rightarrow$$
$$\hookrightarrow 6x-6x-12-7=15x-6x+7-7 \;\Rightarrow$$
$$\hookrightarrow \frac{-19}{9}=\frac{9x}{9} \;\Rightarrow\; x=\frac{-19}{9}$$

$$\frac{3}{4}x-\frac{2}{3}=12 \;\Rightarrow\; (\frac{3}{4}x\cdot 4)3-(\frac{2}{3}\cdot 3)4=12\cdot 12$$
$$\hookrightarrow (3x)3-(2)4=144 \;\Rightarrow\; 9x-8+8=144+8 \;\Rightarrow$$
$$\hookrightarrow \frac{9x}{9}=\frac{152}{9} \;\Rightarrow\; x=\frac{152}{9}$$

$$\frac{2}{3}x+5=8 \;\Rightarrow\; (\frac{2}{3}x+5=8)3 \;\Rightarrow\; 2x+15-15=24-15 \;\Rightarrow$$
$$\hookrightarrow \frac{2x}{2}=\frac{9}{2} \;\Rightarrow\; x=\frac{9}{2}$$

$$
\begin{aligned}
(3x+5)+(4x-2) &= 7x+3 \\
(4x^2+3x+9)+(5x^2+7x-4) &= 9x^2+10x+5 \\
(5x^2-6x-12)-(7x^2+4x-13) &= 5x^2-6x-12-7x^2-4x+13 \\
&= 5x^2-7x^2-12+13-6x-4x \\
&= -2x^2+1-10x
\end{aligned}
$$

I FOIL Method

$$(a+b)(c+d) = ac+ad+bc+bd \tag{14}$$

FOIL Method

- **First** ⇒ Multiply first terms: $a \cdot c$
- **Outer** ⇒ Multiply outer terms: $a \cdot d$
- **Inner** ⇒ Multiply inner terms: $b \cdot c$
- **Last** ⇒ Multiply last terms: $b \cdot d$
- **Outcome** ⇒ Sum all four products to get the expanded expression

$$
\begin{aligned}
(2x+5)(4x^2-3x+6) &= (2\cdot 4)x^{1+2} + (2\cdot -3)^{1+1} + (2\cdot 6)x \\
&\hookrightarrow +(5\cdot 4)x^2 + (5\cdot -3)x + (5\cdot 6) \\
&= 8x^3 + (-6+20)^2 + (12+-15)x + 30 \\
&= 8x^3 + 14x^2 + -3x + 30
\end{aligned}
$$

$$
\begin{aligned}
(3x^2 - 2x + 4)(4x^2 + 5x + -6) = \\
\hookrightarrow (3 \cdot 4)x^{2+2} + (3 \cdot 5)x^{2+1} + (3 \cdot -6)x^{2+0} \\
\hookrightarrow +(-2 \cdot 4)x^{1+2} + (-2 \cdot 5)x^{1+1} + (-2 \cdot -6)x^{1+0} \\
\hookrightarrow +(4 \cdot 4)x^{0+2} + (4 \cdot 5)x^{0+1} + (4 \cdot -6)x^{0+0} \\
= 12x^4 + (15 + -8)x^3 + (-18 + -10 + 16)x^2 + (12 + 20)x + -24 \\
= 12x^4 + 7x^3 + -12x^2 + 32x + -24
\end{aligned}
$$

I Factoring Polynomials

$$x + 4 = 7 \;\Rightarrow\; x + 4 - 4 = 7 - 4 \;\Rightarrow\; x + 0 = 3$$

$$x + 9 = 15 \;\Rightarrow\; x + 9 - 9 = 15 - 9 \;\Rightarrow\; x + 0 = 6$$

$$6 + x = 13 \;\Rightarrow\; 6 + x - 6 = 15 - 9 \;\Rightarrow\; 0 + x - 0 = 7$$

$$x-3=9 \;\Rightarrow\; x-3+3=9-3 \;\Rightarrow\; x-0=6 \;\Rightarrow\; x=6$$

$$x-8=7 \;\Rightarrow\; x+8-8=7+8 \;\Rightarrow\; x-0=15 \;\Rightarrow\; x=15$$

$$3x+5=11 \;\Rightarrow\; 3x+5-5=11-5 \;\Rightarrow\; \frac{3x}{3}=\frac{6}{3} \;\Rightarrow\; x=2$$

$$6.3=-2+x \;\Rightarrow\; 6.3+2=-2+2+x \;\Rightarrow\; 6.3=0+x \;\Rightarrow\; x=6.3$$

$$5=x-8 \;\Rightarrow\; 5+8=x-8+8 \;\Rightarrow\; 13=x-0 \;\Rightarrow\; \hookrightarrow x=13$$

$$5-x=12 \;\Rightarrow\; 5-12-x+x=12-12+x \;\Rightarrow\; \hookrightarrow -7=0+x \;\Rightarrow\; x=-7$$

$$-8 = 5 - x \;\Rightarrow\; -8 + 8 = 5 + 8 - x \;\Rightarrow\; 0 + x = 13 - x + x \;\Rightarrow\; \hookrightarrow x = 13$$

$$3x = 12 \;\Rightarrow\; \frac{3x}{3} = \frac{12}{3} \;\Rightarrow\; \frac{x}{1} = 4 \;\Rightarrow\; x = 4$$

$$7x = 14 \;\Rightarrow\; \frac{7x}{7} = \frac{12}{7} \;\Rightarrow\; \frac{x}{1} = 2 \;\Rightarrow\; x = 2$$

$$-6x = -30 \;\Rightarrow\; \frac{-6x}{-6} = \frac{-30}{-6} \;\Rightarrow\; \frac{-x}{-1} = 5 \;\Rightarrow\; x = 5$$

$$-8x = 48 \;\Rightarrow\; \frac{-8x}{-8} = \frac{48}{-8} \;\Rightarrow\; \frac{-x}{-1} = -6 \;\Rightarrow\; x = -6$$

$$7x = -56 \Rightarrow \frac{7x}{7} = \frac{-56}{7} \Rightarrow \frac{x}{1} = -8 \Rightarrow x = -8$$

$$-8x = -72 \Rightarrow \frac{-8x}{-8} = \frac{-72}{-8} \Rightarrow \frac{x}{1} = 9 \Rightarrow x = 9$$

$$4x + 3 = 6x - 15 \Rightarrow 4x - 4x + 3 + 15 = 6x - 4x - 15 + 15 \Rightarrow$$
$$\hookrightarrow \frac{18}{2} = \frac{2x}{2}; \Rightarrow x = 9$$

$$3(2x - 4) = 5(3x + 2) - 3 \Rightarrow 6x - 12 = 15x + 10 - 3 \Rightarrow$$
$$\hookrightarrow 6x - 6x - 12 - 7 = 15x - 6x + 7 - 7 \Rightarrow$$
$$\hookrightarrow \frac{-19}{9} = \frac{9x}{9} \Rightarrow x = \frac{-19}{9}$$

$$\frac{3}{4}x - \frac{2}{3} = 12 \;\Rightarrow\; (\frac{3}{4}x \cdot 4)3 - (\frac{2}{3} \cdot 3)4 = 12 \cdot 12$$
$$\hookrightarrow (3x)3 - (2)4 = 144 \;\Rightarrow\; 9x - 8 + 8 = 144 + 8 \;\Rightarrow$$
$$\hookrightarrow \frac{9x}{9} = \frac{152}{9} \;\Rightarrow\; x = \frac{152}{9}$$

$$\frac{2}{3}x + 5 = 8 \;\Rightarrow\; (\frac{2}{3}x + 5 = 8)3 \;\Rightarrow\; 2x + 15 - 15 = 24 - 15 \;\Rightarrow$$
$$\hookrightarrow \frac{2x}{2} = \frac{9}{2} \;\Rightarrow\; x = \frac{9}{2}$$

II Glossary

Glossary

Glossary

Coefficient The numerical factor that multiplies a variable in a term

Like Terms Terms that have the same variable(s) raised to the same power(s)

Numerator the top number in a fraction, showing how many parts are taken

Denominator The bottom number in a fraction, showing how many equal parts make the whole

Binomial A polynomial with two terms. See equation 2.

Division Rule Subtract exponents when dividing like bases. Example: see equation 9.