Math Notes

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Canine-Table

Github

POSIX Nexus serves as a comprehensive cross-language reference hub that explores the implementation and behavior of POSIX-compliant functionality across a diverse set of programming environments. Built atop the foundational IEEE Portable Operating System Interface (POSIX) standards, this project emphasizes compatibility, portability, and interoperability between operating systems.

Abstract

Contents

I Algebra

I Nomials

Nomial Lineage

- **Monomial** ⇒ One term only, e.g. $7x^2$
- **Binomial** ⇒ Two terms, e.g. $x + 2$
- **Trinomial** ⇒ Three terms, e.g. $x^2 + 3x + 2$
- **Polynomial** ⇒ Many terms, general family
- **Technique** ⇒ Prefix indicates number of terms
- **Outcome** ⇒ Classification helps organize algebraic expressions

$$\text{Monomial: } ax^n \tag{1}$$
$$\text{Binomial: } ax^n + bx^m \tag{2}$$
$$\text{Trinomial: } ax^n + bx^m + cx^k \tag{3}$$

Polynomial Forms

- **Monomial** ⇒ One term only, e.g. $7x^2$
- **Binomial** ⇒ Two unlike terms, e.g. $x + 2$
- **Trinomial** ⇒ Three terms, e.g. $x^2 + 3x + 2$
- **Polynomial** ⇒ General family with many terms
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Like Terms

- **Definition** ⇒ Expressions with identical variable parts, e.g. $3x^2$ and $-5x^2$
- **Variable Match** ⇒ Same variables with same exponents
- **Coefficient** ⇒ Numbers in front may differ
- **Technique** ⇒ Combine by adding or subtracting coefficients
- **Outcome** ⇒ Simplifies polynomials by reducing to fewer terms

Coefficient

- **Definition** ⇒ The number in front of a variable, e.g. in $7x$ the coefficient is 7
- **Variable Match** ⇒ It scales the variable part without changing its type
- **Examples** ⇒ $3x^2$ has coefficient 3, $-5y$ has coefficient -5
- **Constants** ⇒ A constant term like 4 can be seen as coefficient 4 of x^0
- **Outcome** ⇒ Coefficients tell how strongly each variable contributes to the polynomial

I Polynomial Exponents

Definition of Negative Exponent

- **Positive Exponent** ⇒ x^n means multiply x by itself n times
- **Negative Exponent** ⇒ x^{-n} means reciprocal of x^n
- **Rule** ⇒ $x^{-n} = \frac{1}{x^n}$
- **Example** ⇒ $x^{-3} = \frac{1}{x^3}$
- **Use** ⇒ Negative exponents express division or reciprocals in algebra and calculus

$$x^3 = x \cdot x \cdot x \;\Rightarrow\; x^{-3} = \frac{1}{x^3}$$

Using Negative Exponents

- **Step 1** ⇒ Recall exponent rules: $x^a \cdot x^b = x^{a+b}$
- **Step 2** ⇒ Set $a = 3$, $b = -3$: $x^3 \cdot x^{-3} = x^0$
- **Step 3** ⇒ But $x^0 = 1$
- **Step 4** ⇒ So x^{-3} must equal $\frac{1}{x^3}$
- **Outcome** ⇒ Negative exponent means reciprocal of positive power

Exponent Flip Examples

$$x^5 = \frac{1}{x^{-5}} \qquad x^{12} = \frac{1}{x^{-12}} \tag{4}$$

$$x^{17} = \frac{1}{x^{-17}} \qquad 15x^9 = \frac{15}{x^{-9}} \tag{5}$$

$$28x^5 = \frac{28}{x^{-5}} \qquad x^5 = \frac{1}{x^{-5}} \tag{6}$$

$$x^{-3} = \frac{1}{x^3} \qquad x^3 = \frac{1}{x^{-3}} \tag{7}$$

$$x^{-3} = \frac{1}{x^3} \tag{8}$$

Polynomial Exponent Rules Applied

- **Power of a Power** $\Rightarrow$ $(a^m)^n = a^{mn}$
- **Nested Powers** $\Rightarrow$ Combine: $8 \cdot 3 = 24$
- **Outer Flip** $\Rightarrow$ Apply (-9): $x^{768y^{5z^3}} \to x^{-6912y^{5z^3}}$
- **Technique** $\Rightarrow$ Multiply all exponents carefully, preserve inner structure
- **Outcome** $\Rightarrow$ Final simplified form: $x^{-6912y^{5z^3}}$

$$\begin{aligned}(((x^{32y^{5z^3}})^8)^3)^{-9} &= (x^{32y^{5z^3}})^{8\cdot 3} && \Rightarrow x^{768y^{5z^3}} \\ &= \left(x^{768y^{5z^3}}\right)^{-9} && \Rightarrow x^{-9\cdot 768y^{5z^3}} \\ &= x^{-6912y^{5z^3}}\end{aligned}$$

Exponent Rules Applied

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$$\begin{aligned}
x^2 \cdot x^4 &= x^{2+4} &&\Rightarrow x^6\\
x^7 \cdot x^5 &= x^{5+7} &&\Rightarrow x^{12}\\
x^8 \cdot x^9 &= x^{8+9} &&\Rightarrow x^{17}\\
(3x^3)(5x^6) &= (3 \cdot 5)x^{3+6} &&\Rightarrow 15x^9\\
(4x^2)(7x^3) &= (4 \cdot 7)x^{2+3} &&\Rightarrow 28x^5\\
(4xy^2)(8x^2y^3) &= (4 \cdot 8)x^{1+2}(1 \cdot 1)y^{2+3} &&\Rightarrow 32x^3y^5\\
(5x^2y^3)(6x^3y^4) &= (5 \cdot 6)x^{2+3}(1 \cdot 1)y^{3+4} &&\Rightarrow 30x^5y^7\\
(7x^3y^4)(8x^5y^7) &= (7 \cdot 8)x^{3+5}(1 \cdot 1)y^{4+7} &&\Rightarrow 56x^8y^{11}
\end{aligned}$$

$$\begin{aligned}
(x^3)^4 &= x^{3\cdot 4} = x^{12}\\
(x^4)^6 &= x^{4\cdot 6} = x^{24}\\
(x^3)^5 &= x^{3\cdot 5} = x^{15}\\
(3x^2)^4 &= 3^{1\cdot 4}x^{2\cdot 4} = 3^4x^8 = 81x^8\\
(2x^3)^3 &= 2^{1\cdot 3}x^{3\cdot 3} = 2^3x^9 = 8x^9
\end{aligned}$$

$$\begin{aligned}
(3x^2)^2(2x^3)^3 &= 3^{1\cdot 2}x^{2\cdot 2}2^{1\cdot 3}x3 \cdot 3\\
&= 3^2x^42^3x^9\\
&= 9 \cdot 8x^{4+9}\\
&= 72x^{13}
\end{aligned}$$

$$\begin{aligned}
(3^2x^3y^4)^2(2^3x^2y^5)^3 &= (3^{2\cdot 2}x^{3\cdot 2}y^{4\cdot 2})(2^{3\cdot 3}x^{2\cdot 3}y^{5\cdot 3})\\
&= (3^4x^6y^8)(2^9x^6y^{15})\\
&= (81x^6y^8)(512x^6y^{15})\\
&= 81 \cdot 512x6 + 6y^{8+15}\\
&= 41472x^{12}y^{23}
\end{aligned}$$

$$
\begin{aligned}
-2^3 &= -2 \cdot -2 \cdot -2 = -8 \\
(-2)^3 &= -2 \cdot -2 \cdot -2 = -8 \\
-(-2)^3 &= -2 \cdot -2 \cdot -2 = 8
\end{aligned}
$$

$$
\begin{aligned}
(-7x^2y^3)^0 &= -7^{2\cdot 0}x^{2\cdot 0}y^{3\cdot 0} \\
&= -7^0x^0y^0 \\
&= 1 \cdot 1 \cdot 1 \\
&= 1
\end{aligned}
$$

$$
\begin{aligned}
3x(5x+8) &= 15x^2 + 24x \\
4x(x^2 - 2x + 3) &= 4x^3 - 8x^2 + 12x
\end{aligned}
$$

I Dividing Polynomials

$$
\begin{aligned}
\frac{x^8}{x^3} &= x^{8-3} \Rightarrow x^5 && (9) \\
\frac{x^5}{x^2} &= x^{5-2} \Rightarrow x^3 && (10) \\
\frac{x^5}{x^8} &= x^{5-8} \Rightarrow x^{-3} && (11) \\
\frac{x^4}{x^7} &= x^{4-7} \Rightarrow x^{-3} && (12) \\
& && (13)
\end{aligned}
$$

$$
\begin{aligned}
\frac{24x^9y^5}{8x^3y^{12}} &= \frac{24}{8}x^{9-3}\frac{1}{1}y^{5-12} \\
&= 3x^6y^{-7} \\
&= \frac{3x^6}{y^7}
\end{aligned}
$$

$$\frac{12x^5y^{-3}z^4}{36x^8y^{-4}z^{-8}} = \frac{\frac{12}{3}x^{5-8}}{\frac{36}{3}}\frac{1}{1}y^{-3--4}\frac{1}{1}z^{4--8}$$
$$= \frac{x^{-3}y^1z^{12}}{3}$$
$$= \frac{yz^{12}}{3x^3}$$

I Multiplying Polynomials

Multiplication Symbols

- **Dot** ⇒ $\cdot$ is clean, algebraic, avoids confusion with x
- **Times** ⇒ $\times$ is bold, arithmetic, or cross product
- **Context** ⇒ Use $\cdot$ in algebra, $\times$ in arithmetic or vectors
- **Technique** ⇒ Choose based on clarity and audience
- **Outcome** ⇒ Both mean multiplication, but notation signals intent

$$x^2 \cdot x^4 = x^{2+4} \quad \Rightarrow \; x^6$$

$$x^7 \cdot x^5 = x^{5+7} \quad \Rightarrow \; x^{12}$$

$$x^8 \cdot x^9 = x^{8+9} \quad \Rightarrow \; x^{17}$$

$$(3x^3)(5x^6) = (3 \cdot 5)x^{3+6} \quad \Rightarrow 15x^9$$

$$(4x^2)(7x^3) = (4 \cdot 7)x^{2+3} \quad \Rightarrow 28x^5$$

$$(4xy^2)(8x^2y^3) = (4 \cdot 8)x^{1+2}(1 \cdot 1)y^{2+3} \quad \Rightarrow \ 32x^3y^5$$

$$(5x^2y^3)(6x^3y^4) = (5 \cdot 6)x^{2+3}(1 \cdot 1)y^{3+4} \quad \Rightarrow \ 30x^5y^7$$

$$(7x^3y^4)(8x^5y^7) = (7 \cdot 8)x^{3+5}(1 \cdot 1)y^{4+7} \quad \Rightarrow \ 56x^8y^{11}$$

I Combining Polynomials

$$x + 4 = 7 \ \Rightarrow \ x + 4 - 4 = 7 - 4 \ \Rightarrow \ x + 0 = 3$$

$$x + 9 = 15 \ \Rightarrow \ x + 9 - 9 = 15 - 9 \ \Rightarrow \ x + 0 = 6$$

$$6 + x = 13 \ \Rightarrow \ 6 + x - 6 = 15 - 9 \ \Rightarrow \ 0 + x - 0 = 7$$

$$x - 3 = 9 \ \Rightarrow \ x - 3 + 3 = 9 - 3 \ \Rightarrow \ x - 0 = 6 \ \Rightarrow \ x = 6$$

$$x - 8 = 7 \ \Rightarrow \ x + 8 - 8 = 7 + 8 \ \Rightarrow \ x - 0 = 15 \ \Rightarrow \ x = 15$$

$$3x+5=11 \;\Rightarrow\; 3x+5-5=11-5 \;\Rightarrow\; \frac{3x}{3}=\frac{6}{3} \;\Rightarrow\; x=2$$

$$6.3=-2+x \;\Rightarrow\; 6.3+2=-2+2+x \;\Rightarrow\; 6.3=0+x \;\Rightarrow\; x=6.3$$

$$5=x-8 \;\Rightarrow\; 5+8=x-8+8 \;\Rightarrow\; 13=x-0 \;\Rightarrow$$
$$\hookrightarrow x=13$$

$$5-x=12 \;\Rightarrow\; 5-12-x+x=12-12+x \;\Rightarrow$$
$$\hookrightarrow -7=0+x \;\Rightarrow\; x=-7$$

$$-8=5-x \;\Rightarrow\; -8+8=5+8-x \;\Rightarrow\; 0+x=13-x+x \;\Rightarrow$$
$$\hookrightarrow x=13$$

$$3x=12 \;\Rightarrow\; \frac{3x}{3}=\frac{12}{3} \;\Rightarrow\; \frac{x}{1}=4 \;\Rightarrow\; x=4$$

$$7x=14 \;\Rightarrow\; \frac{7x}{7}=\frac{12}{7} \;\Rightarrow\; \frac{x}{1}=2 \;\Rightarrow\; x=2$$

$$-6x=-30 \;\Rightarrow\; \frac{-6x}{-6}=\frac{-30}{-6} \;\Rightarrow\; \frac{-x}{-1}=5 \;\Rightarrow\; x=5$$

$$-8x = 48 \;\Rightarrow\; \frac{-8x}{-8} = \frac{48}{-8} \;\Rightarrow\; \frac{-x}{-1} = -6 \;\Rightarrow\; x = -6$$

$$7x = -56 \;\Rightarrow\; \frac{7x}{7} = \frac{-56}{7} \;\Rightarrow\; \frac{x}{1} = -8 \;\Rightarrow\; x = -8$$

$$-8x = -72 \;\Rightarrow\; \frac{-8x}{-8} = \frac{-72}{-8} \;\Rightarrow\; \frac{x}{1} = 9 \;\Rightarrow\; x = 9$$

$$4x + 3 = 6x - 15 \;\Rightarrow\; 4x - 4x + 3 + 15 = 6x - 4x - 15 + 15 \;\Rightarrow$$
$$\hookrightarrow \frac{18}{2} = \frac{2x}{2}; \Rightarrow\; x = 9$$

$$3(2x - 4) = 5(3x + 2) - 3 \;\Rightarrow\; 6x - 12 = 15x + 10 - 3 \;\Rightarrow$$
$$\hookrightarrow 6x - 6x - 12 - 7 = 15x - 6x + 7 - 7 \;\Rightarrow$$
$$\hookrightarrow \frac{-19}{9} = \frac{9x}{9} \;\Rightarrow\; x = \frac{-19}{9}$$

$$\frac{3}{4}x - \frac{2}{3} = 12 \;\Rightarrow\; (\frac{3}{4}x \cdot 4)3 - (\frac{2}{3} \cdot 3)4 = 12 \cdot 12$$
$$\hookrightarrow (3x)3 - (2)4 = 144 \;\Rightarrow\; 9x - 8 + 8 = 144 + 8 \;\Rightarrow$$
$$\hookrightarrow \frac{9x}{9} = \frac{152}{9} \;\Rightarrow\; x = \frac{152}{9}$$

$$\frac{2}{3}x + 5 = 8 \;\Rightarrow\; (\frac{2}{3}x + 5 = 8)3 \;\Rightarrow\; 2x + 15 - 15 = 24 - 15 \;\Rightarrow$$
$$\hookrightarrow \frac{2x}{2} = \frac{9}{2} \;\Rightarrow\; x = \frac{9}{2}$$

$$
\begin{aligned}
(3x+5)+(4x-2) &= 7x+3 \\
(4x^2+3x+9)+(5x^2+7x-4) &= 9x^2+10x+5 \\
(5x^2-6x-12)-(7x^2+4x-13) &= 5x^2-6x-12-7x^2-4x+13 \\
&= 5x^2-7x^2-12+13-6x-4x \\
&= -2x^2+1-10x
\end{aligned}
$$

I FOIL Method

$$(a+b)(c+d) = ac+ad+bc+bd \tag{14}$$

FOIL Method

- **First** $\Rightarrow$ Multiply first terms: $a \cdot c$
- **Outer** $\Rightarrow$ Multiply outer terms: $a \cdot d$
- **Inner** $\Rightarrow$ Multiply inner terms: $b \cdot c$
- **Last** $\Rightarrow$ Multiply last terms: $b \cdot d$
- **Outcome** $\Rightarrow$ Sum all four products to get the expanded expression

$$
\begin{aligned}
(2x+5)(4x^2-3x+6) &= (2\cdot 4)x^{1+2} + (2\cdot -3)^{1+1} + (2\cdot 6)x \\
&\hookrightarrow +(5\cdot 4)x^2 + (5\cdot -3)x + (5\cdot 6) \\
&= 8x^3 + (-6+20)^2 + (12+-15)x + 30 \\
&= 8x^3 + 14x^2 + -3x + 30
\end{aligned}
$$

$$
\begin{aligned}
(3x^2-2x+4)(4x^2+5x+-6) &= \\
\hookrightarrow (3\cdot 4)x^{2+2}+(3\cdot 5)x^{2+1}+(3\cdot -6)x^{2+0} \\
\hookrightarrow +(-2\cdot 4)x^{1+2}+(-2\cdot 5)x^{1+1}+(-2\cdot -6)x^{1+0} \\
\hookrightarrow +(4\cdot 4)x^{0+2}+(4\cdot 5)x^{0+1}+(4\cdot -6)x^{0+0} \\
=12x^4+(15+-8)x^3+(-18+-10+16)x^2+(12+20)x+-24 \\
=12x^4+7x^3+-12x^2+32x+-24
\end{aligned}
$$

I Factoring Polynomials

$4a^2+2ab–3a^2b+5$

Terms	Factors	Prime Factors
$4a^2$	$4, a^2$	$2, 2, a, a$
$2ab$	$3, a, b$	$2, a, b$
$-3a^2b$	$-3, a^2, b$	$-3, a, a, b$
5	5	5

$xy^2-3x^2y^2-6y+z$

Terms	Factors	Prime Factors
xy^2	$4, a^2$	$2, 2, a, a$
$-3x^2y$	$3, a, b$	$2, a, b$
$-6y$	$-6, y$	$-2, 3, y$
z	z	z

$-5+2(3a^2-3t)$

Terms	Factors	Prime Factors
-5	-5	-5
$2(3a^2-3t)$	$2.3a^2-3t$	$2.3a^2-3t$
$6t$	$6, t$	$3, 3, t$

$3x^2 + 5x - 2$

Terms	Factors	Prime Factors
$3x^2$	$3, x^2$	$3.x.x$
$5x$	$5, x$	$5, x$
-2	-2	-2

Definition of Prime Factor

- **Prime** ⇒ A number greater than 1 divisible only by 1 and itself
- **Factor** ⇒ A number that divides another evenly
- **Prime Factor** ⇒ A prime number that divides another number exactly
- **Example** ⇒ $60 = 2^2 \cdot 3 \cdot 5$; prime factors are 2, 3, 5
- **Use** ⇒ Prime factors are the building blocks of integers, used in LCM, GCD, and simplification

Find LCM of 12 and 18

$$\begin{aligned} 12 &= 2^2 \cdot 3 && \Rightarrow \\ 18 &= 2 \cdot 3^2 && \Rightarrow \\ \text{LCM} &= 2^2 \cdot 3^2 && \hookrightarrow 36 \end{aligned}$$

Using Prime Factors for LCM

- **Step 1** ⇒ Prime factorize each number
- **Step 2** ⇒ Collect all distinct primes
- **Step 3** ⇒ Take the highest power of each prime
- **Step 4** ⇒ Multiply them together
- **Outcome** ⇒ LCM(12,18) = 36

Euclidean Modulus

```
define nx_pt_mod(x, y) {
                x = nx_abs(x)
                if (x == 0)
                                return 0
                y = nx_abs(y)
```

```
        if (y > 0)
                return x - y * nx_pt_trunc(x / y)
        print "<nx:impurity/>"
        return -1
}
```

The Greatest Common Factor

```
define nx_euc(x, y) {
        auto n
        if (x == y)
                return x
        while (x > 0 && y > 0) {
                n = x
                x = nx_pt_mod(y, x)
                y = n
        }
        return n
}
```

$$(8, 12) \mapsto \gcd(8, 12) = 4$$
$$8x + 12 \;\Rightarrow\; 4(\frac{8x}{2} + \frac{12}{4}) \;\Rightarrow\; 4(2x + 3)$$

$$(4, 2) \mapsto \gcd(4, 2) = 2$$
$$4x^2 + 2x \;\Rightarrow\; 2x(\frac{4x^2}{2x} + \frac{2x}{2x}) \;\Rightarrow\; 2x(2x + 1)$$

$$(12, 18) \mapsto \gcd(12, 18) = 6$$
$$12ab^2 + 18a^2b^3 \;\Rightarrow\; 6ab^2(2 + 3ab)$$

Definition of Perfect Square

- **Perfect Square** ⇒ A number that can be expressed as n^2 for some integer n
- **Integer Squared** ⇒ Formed by multiplying an integer by itself
- **Examples** ⇒ $1, 4, 9, 16, 25, 36, \ldots$
- **Non-Examples** ⇒ $2, 3, 5, 6, 7, 10, \ldots$
- **Use** ⇒ Perfect squares appear in factoring, radicals, and Pythagorean identities

Check if 49 is a perfect square

$$49 = 7 \cdot 7 \quad \Rightarrow$$
$$= 7^2 \quad \hookrightarrow$$

Therefore, 49 is a perfect square.

Using Perfect Squares

- **Step 1** ⇒ Identify the number
- **Step 2** ⇒ Ask if it can be written as n^2
- **Step 3** ⇒ If yes, it is a perfect square
- **Step 4** ⇒ If no, it is not
- **Outcome** ⇒ 49 is a perfect square since $49 = 7^2$

Difference of Squares Applied

- **Identity** ⇒ $(a^2 - b^2) = (a - b)(a + b)$
- **Example** ⇒ $x^2 - 9$
- **Factorization** ⇒ Apply rule: $x^2 - 3^2 = (x - 3)(x + 3)$
- **Technique** ⇒ Recognize perfect squares and subtract
- **Outcome** ⇒ Final factored form: $(x - 3)(x + 3)$

$$x^2 - 9 \;\Rightarrow\; x^2 - 3^2 \;\Rightarrow\; (x - 3)(x + 3)$$

Perfect Squares Difference

- **Identity** $\Rightarrow (a^2 - b^2) = (a - b)(a + b)$
- **Example** $\Rightarrow x^2 - 9$
- **Factorization** $\Rightarrow (x - 3)(x + 3)$
- **Technique** $\Rightarrow$ Spot the squares, apply the difference rule
- **Outcome** $\Rightarrow$ Factored polynomial form

$$(25) \mapsto \sqrt{(25)} = 5$$
$$x^2 - 25 \;\Rightarrow\; (x+5)(x-5)$$

$$(x-5)(x+5) \Rightarrow\; x^2 + 5x + -5x + -25 \Rightarrow$$
$$\hookrightarrow\; x^2 + (5x + -5x \Rightarrow\; 0) - 25 \Rightarrow\; x^2 - 25$$

$$(9) \mapsto \sqrt{(9)} = 3$$
$$x^2 - 9 \;\Rightarrow\; (x+3)(x-3)$$

$$(4) \mapsto \sqrt{(4)} = 2$$
$$x^2 - 4 \;\Rightarrow\; (x+2)(x-2)$$

$$4x^2 - 25 \;\Rightarrow\; (2x+5)(2x-5)$$

$$
\begin{aligned}
(81) &\mapsto \sqrt(81) = 9 \\
(16) &\mapsto \sqrt(16) = 4 \\
16x^2 - 25 \;\Rightarrow\; &(4x+9)(4x-9)
\end{aligned}
$$

$$25x^2 - 16y^2 \;\Rightarrow\; (5x+4y)(5x-4y)$$

$$81x^4 - 16y^8 \;\Rightarrow\; (9x^2+4y^4)(9x^2-4y^4) \;\Rightarrow\; (3x+2y^2)(3x-2y^2)$$

Factor by Grouping Applied

- **Setup** ⇒ Polynomial with 4 terms
- **Grouping** ⇒ Split into two pairs
- **Inner Factor** ⇒ Factor each pair separately
- **Common Binomial** ⇒ Extract the shared binomial
- **Outcome** ⇒ Final factored form

$$
\begin{aligned}
&x^3 + 3x^2 + 2x + 6 \;\Rightarrow\; (x^3+3x^2) + (2x+6) \;\Rightarrow \\
&\quad \hookrightarrow x^2(x+3) + 2(x+3) \;\Rightarrow\; (x^2+2)(x+3)
\end{aligned}
$$

$$
\begin{aligned}
&x^3 - 4x^2 + 3x - 12 \;\Rightarrow\; x^2(x-4) + 3(x-4) \;\Rightarrow \\
&\quad \hookrightarrow \frac{x^2(x-4)}{x-4} + \frac{3(x-4)}{x-4} \;\Rightarrow \\
&\quad \hookrightarrow (x-4)(x^2+3)
\end{aligned}
$$

$$
\begin{aligned}
2x^3 - 6x^2 + 4x - 12 \;&\Rightarrow\; 2x^2(x-3) + 4(x-3) \;\Rightarrow \\
&\hookrightarrow \frac{2x^2(x-3)}{x-3} + \frac{4(x-3)}{x-3} \;\Rightarrow \\
&\hookrightarrow (x-3)(2x^2+4)
\end{aligned}
$$

$$
\begin{aligned}
3x^3 + 8x^2 - 6x - 16 \;&\Rightarrow\; x^2(3x+8) - 2(3x+8) \;\Rightarrow \\
&\hookrightarrow \frac{x^2(3x+8)}{3x+8} + \frac{-2(3x+8)}{3x+8} \;\Rightarrow \\
&\hookrightarrow (x^2-2)(3x+8) \;\Rightarrow \\
\hookrightarrow 3x^3 - 6x + 8x^2 - 16 \;&\Rightarrow\; 3x(x^2-2) + 8(x^2-2) \;\Rightarrow \\
&\hookrightarrow \frac{3x(x^2-2)}{x^2-2} + \frac{8(x^2-2)}{x^2-2} \;\Rightarrow \\
&\hookrightarrow (x^2-2)(3x+8)
\end{aligned}
$$

I Linear Algebra

I Coordinate Plane

Domain

- **Definition** $\Rightarrow$ The set of all possible input values x for a function
- **Coordinate Plane** $\Rightarrow$ Represents the horizontal axis (x-axis)
- **Example** $\Rightarrow$ For $f(x) = \sqrt{x}$, the domain is $x \geq 0$
- **Relation** $\Rightarrow$ Domain specifies where the function is defined
- **Outcome** $\Rightarrow$ Determines the allowable values you can plug into the function

Range

- **Definition** $\Rightarrow$ The set of all possible output values y from a function
- **Coordinate Plane** $\Rightarrow$ Represents the vertical axis (y-axis)
- **Example** $\Rightarrow$ For $f(x) = \sqrt{x}$, the range is $y \geq 0$
- **Relation** $\Rightarrow$ Range shows the values the function can produce
- **Outcome** $\Rightarrow$ Determines the spread of results plotted on the plane

Interval Notation

- **[a,b]** $\Rightarrow$ Closed interval: includes both endpoints a and b
- **(a,b)** $\Rightarrow$ Open interval: excludes both endpoints a and b
- **[a,b)** $\Rightarrow$ Half-open interval: includes a but excludes b
- **(a,b]** $\Rightarrow$ Half-open interval: excludes a but includes b
- **[0,x)** $\Rightarrow$ All real numbers from 0 up to but not including x
- **(0,x)** $\Rightarrow$ All real numbers greater than 0 and less than x

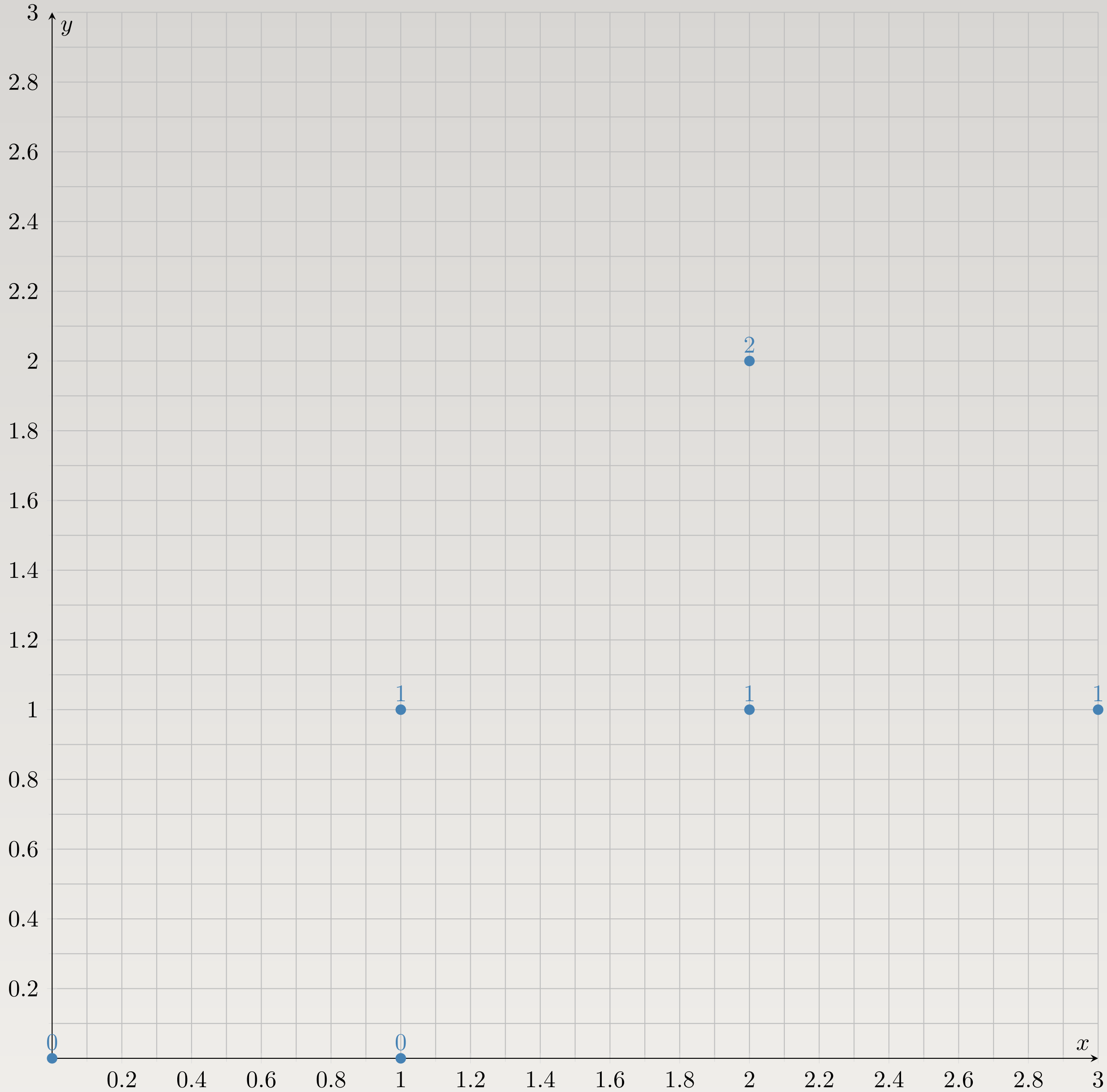
Relation of Ordered Pairs

- **Ordered Pair** $\Rightarrow$ A pair written as (x, y), where x is the input and y is the output
- **Relation** $\Rightarrow$ Any set of ordered pairs that connects elements from one set (domain) to another (range)
- **Domain** $\Rightarrow$ The collection of all first elements x in the ordered pairs
- **Range** $\Rightarrow$ The collection of all second elements y in the ordered pairs
- **Function** $\Rightarrow$ A special type of relation where each x is paired with exactly one y
- **Example** $\Rightarrow$ Relation: $\{(1, 2), (2, 3), (3, 4)\}$; Domain: $\{1, 2, 3\}$; Range: $\{2, 3, 4\}$

Ordered Pairs

Input (a, b)	**Output Value**
$(0, 0)$	5
$(1, 0)$	9
$(1, 1)$	8
$(2, 1)$	15
$(2, 2)$	17
$(3, 1)$	26

Ordered Pairs



I Distance Formula

Distance Formula

- **Definition** ⇒ Gives the length of the line segment between two points in the plane
- **Points** ⇒ Two points (x_1, y_1) and (x_2, y_2)
- **Formula** ⇒ $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
- **Origin** ⇒ Derived from the Pythagorean Theorem
- **Domain** ⇒ Applies to all real coordinates in 2D space
- **Extension** ⇒ Generalizes to 3D: $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

$$\text{2D points} \qquad d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \tag{15}$$

$$\text{2D points} \qquad d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2} \tag{16}$$