Bilkent University Department Of Electrical and Electronics Engineering

EEE212 Circuit Theory

Lab 1

Quality Factor of an Inductor

Cankut Bora Tuncer 22001770 Section 1

Purpose:

The lab aims to develop two different methods to measure the Q factor of a real inductor by getting familiar with LTSpice software.

Methodology:

The Q factor can be described as:

$$Q = 2\pi \frac{\text{stored energy in resonance frequency}}{\textit{Energy loss per cycle}}$$

In the case of an inductor:

$$Q = 2\pi \frac{\text{w}*\text{L}}{R} \qquad (1)$$

A real inductor can be represented as an ideal inductor series with a resistor. The resistance is due to copper loss and skin effect, which emerges in high frequencies. A real inductor is simulated in the software implementation, as seen in Figure 1.

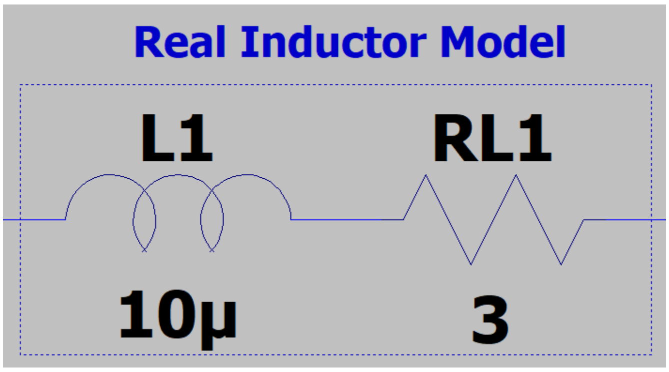


Figure 1 Real inductor model

However, both axial and toroidal inductors are used in the hardware implementation. The test frequency is selected between 1-5Mhz. The inductor values are chosen as 10uH in the simulation. For the Hardware lab, one premade inductor and one hand winded inductor with a T38-8 core with 22 turns(to achieve 10u) are used. A table of results is created for both software and hardware implementation. The overall LTSpice simulation can be seen in Figure 2. The preferred methods are as follows:

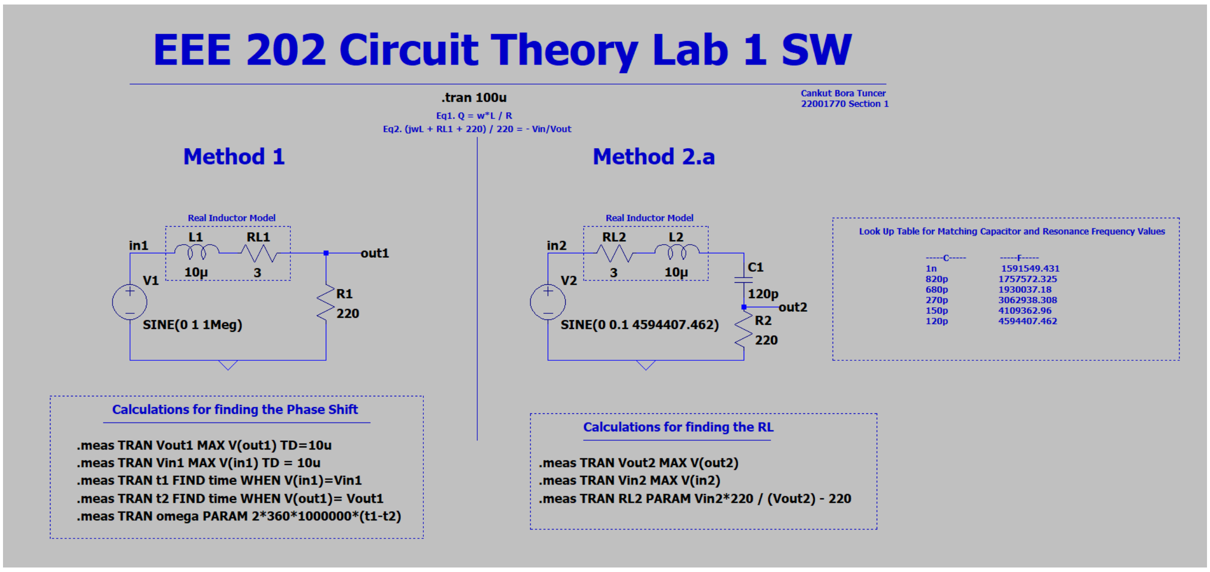
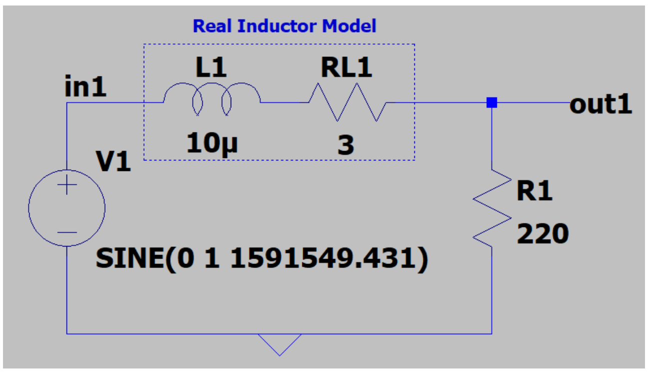


Figure 2 Overall LTSpice Simulation

Method 1: Phase Difference Method



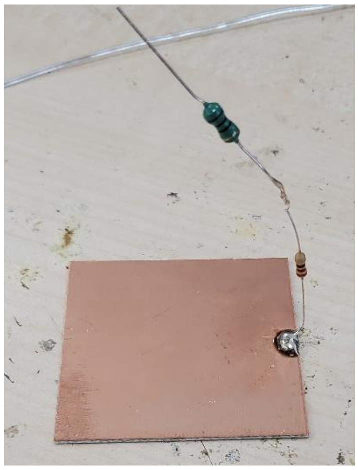


Figure 3 The Schematic and Hardware Implementation of the Method 1

Node analysis is applied at the "out1" point of Figure 3. There obtained a transfer function with two unknowns, L and R_l. With amplitude and phase equations, we got two equations with two unknowns. The inductance and internal resistance values are obtained by solving the equations at the specific frequency. Finally, the obtained results are put into (1).

KCL at out: $$\frac{V_{out}}{R1} + \frac{(V_{out} - V_{in})}{(jwL + R_l)} = 0$$

$$V_{out} * \left(\frac{1}{R1} + \frac{1}{(jwL + R_l)}\right) = \frac{V_{in}}{(jwL + R_l)}$$

Transfer Function: $$\frac{V_{out}}{V_{in}} = \left(\frac{R1}{jwL + R_l + R1}\right)$$

Amplitude Eq: $$\frac{|V_{in}|}{|V_{out}|} = \left(\frac{(wL)^2 + (R_l + R1)^2}{R1}\right)$$

$$\sqrt{\frac{\left(\frac{|V_{in}|}{|V_{out}|} * R1 - (R_l + R1)^2\right)}{w^2}}) = L \qquad (2)$$

Phase Eq: $$\theta = \Delta t * 2 * \pi * f$$

$$\theta = \arctan\left(\frac{wL}{R_l + R1}\right)$$

$$\frac{\tan(\theta)}{w} * (R_l + R1) = L \qquad (3)$$

Combining 2 and 3: $$\sqrt{\frac{\left(\frac{|V_{in}|}{|V_{out}|} * R1 - (R_l + R1)^2\right)}{w^2}}) = \frac{\tan(\theta)}{w} * (R_l + R1) \qquad (4)$$

After solving for (4,) we can find R_l and substitute into (3). From ther,e we can calculate the Q factor. The known values are Vin, Vout, R1. We find θ by measuring the time difference between peaks of Vin and Vout and inputting the result into Phase Eq.

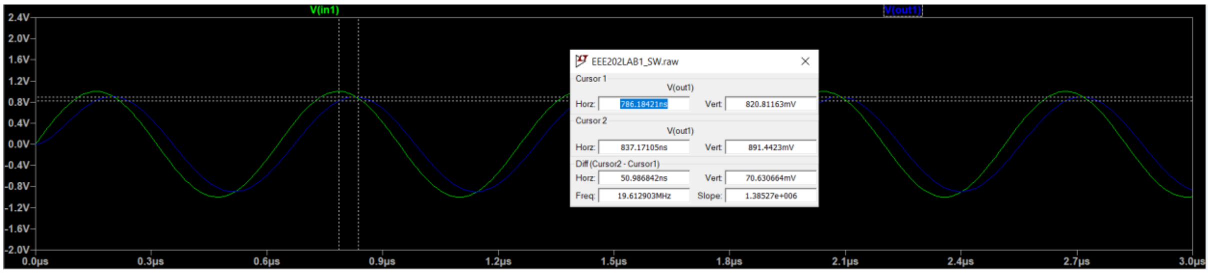


Figure 4 Example of time measurement at f=1591549Hz, Software Lab

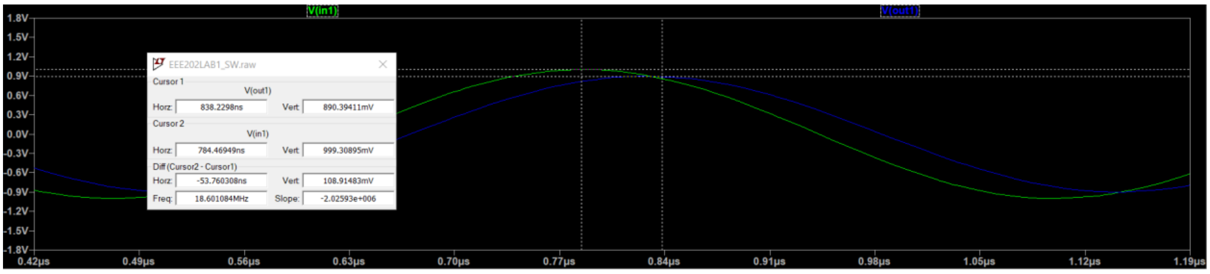
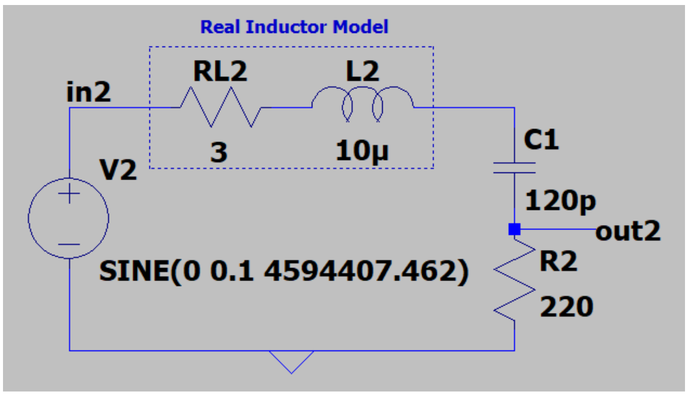


Figure 5 Voltage Measurement, Software Lab

Figure 6 Time Measurement at f=1591549, Hardware Lab

Figure 7 Voltage Measurement, Hardware Lab

Method 2: Resonance Method



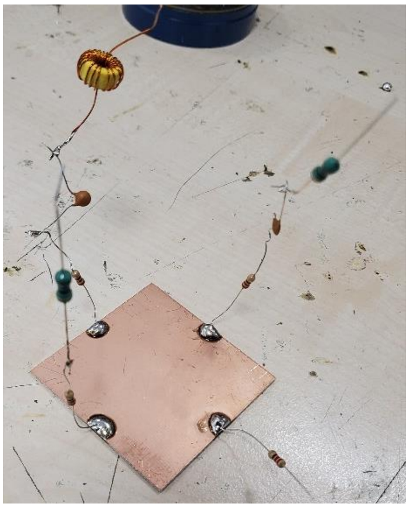


Figure 8 The Schematic and Hardware Implementation of the Method 2

The inductor is resonated with a capacitor which is accepted as ideal in the simulation. To find the resonance frequency, we will assume that the inductor is 10u and then calculate the resonance frequency. We can find the actual resonance frequency by twisting the frequency around the ideal frequency. The voltage divider formula is applied at the voltage point “out2”. The motivation is when the capacitor and inductor are resonating, they both act as a short circuit. Ideally, only the Rl and R2 are left in the circuit-. With voltage dividing, we can find the RL2. The capacitor values are selected to resonate between the 1-5Mhz frequency with the ideal value of the inductor, 10u. The calculations are as follows:

Voltage Divider at out2: $V_{out} = V_{in} * \frac{R2}{R2+R_l}$

$$R_l = \quad R2 * (\frac{V_{in}}{V_{out}} - 1) \tag{5}$$

After solving for (5), we can find R_l. Since we already know the inductor by fine adjusting the resonance frequency, the Q factor can be calculated easily.

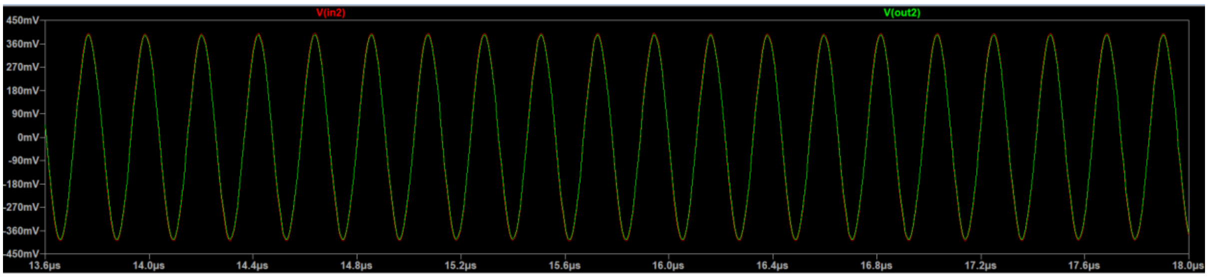


Figure 9 Vin and Vout at f=4594407Hz, Simulation Lab

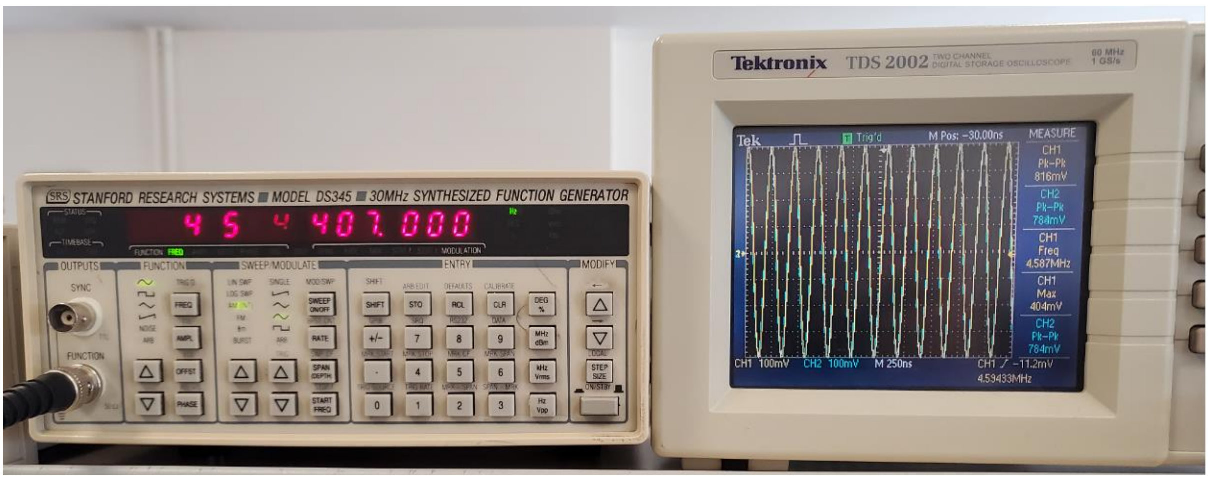


Figure 10 Vin and Vout at f=4594407Hz, Hardware Lab

Results:

The photos of the measurements can be found in the Appendix.

Software Lab:

A generic real inductor model was chosen for the software lab with 10u inductance and a 3 Ohm resistance in series. For the voltage division, it is used a 220 ohm resistor. For more precise measurements, .meas function is used. The frequencies are 1591549, 1930037, 3062938, 3062938, 4594407, with a moral of resonating the inductor with known capacitance values. The table of results is constructed for each method.

Method 1:

The lab results of method one can be seen in Figure 11.

Frequency(Hz)	Vin(mV)	Vout(mV)	Dt(ns)	**R(ohm)**	**Q(measured)**	**Q(calculated**	**L(uH)**
1591549	999.24	899.28	43.54	**3.0123**	**33.61350881**	**33.33332431**	**10.1254**
1930037	999.33	862.01	42.21	**3.4983**	**34.62443058**	**40.4226004**	**9.98836**
3062938	997.69	740.16	36.18	**3**	**64.50605609**	**64.15002346**	**10.0555**
4594407	1060	760	30.7	**3.9399**	**80.59808362**	**96.22503519**	**11.0002**

Figure 11 Method 1 Results, Software Lab

Error Q factor: %8.87917693

Method 2:

The lab results of method one can be seen in Figure 12.

Frequency(Hz)	Vin(V)	Vout(V)	**R(ohm)**	**Q**	**Q(calculated**	**L(uH)**
1591549	0.4	0.394494	**3.07056**	**32.56734046**	**33.33332431**	**10**
1930037	0.4	0.394546	**3.041166303**	**39.87542578**	**40.4226004**	**10**
3062938	0.4	0.39374	**3.097739625**	**62.12596722**	**64.15002346**	**10**
4594407	0.4	0.393212	**3.197849506**	**82.53190246**	**96.22503519**	**10**

Figure 12 Method 2 Results, Software Lab

Error Q factor: %7.273854658

Hardware Lab:

For the hardware lab, two different inductors are used. A premade axial inductor with an inductance of 9.145uH at 100kHz, and a hand winded toroidal inductor with a reading of 10.498uH at 100kHz. For the handmade inductor, the T38-8 core is used. Rather than constructing the circuit on a breadboard, I have used a PCB plate since we are working with MHz frequency waves. Using the formula A*n^2=L, with A=20nH/t^2, the turn amount is calculated as 22. The inductors are tested with an “inductance testing” machine, as seen in Figure 13. For the 2nd method, capacitor values are measured with a multimeter. Again, the frequency values are 1591549, 1930037, 3062938, 3062938, 4594407.

Figure 13 The inductance and Q factor measurements from the machine

Method 1:

The lab results of method one can be seen in Figures 14&15.

Axial Inductor						
Frequency(Hz)	Vin(mV)	Vout(mV)	Dt(ns)	**R(ohm)**	**Q(measured)**	**L(uH)**
1591549	1020	960	44	**4.512820513**	**50.5919399**	**22.8312406**
1930037	1040	940	40	**3.316582915**	**58.59014855**	**16.0239638**
3062938	1040	860	40	**5.612244898**	**5.760783578**	**1.67996448**
4594407	1080	760	36	**8.979591837**	**27.96325095**	**9.26996874**

Figure 14 Method 1 Results from axial inductor, Hardware Lab

Toroidal Inductor						
Frequency(Hz)	Vin(mV)	Vout(mV)	Dt(ns)	**R(ohm)**	**Q(measured)**	**L(uH)**
1591549	1000	920	52	**5.670103093**	**16.68087579**	**9.4582311**
1930037	980	880	52	**6.85381911**	**31.73714861**	**17.9372161**
3062938	1040	800	40	**23.5241320**	**0.4873593416**	**0.142124135**
4594407	1060	680	36	**26.3213110**	**0.3130557917**	**0.108554164**

Figure 15 Method 1 Results from toroidal inductor, Hardware Lab

Method 2:

The lab results of method one can be seen in Figures 16&17.

Axial Inductor					
Frequency(Hz)	Vin(V)	Vout(V)	**R(ohm)**	**Q**	**L(uH)**
1591549	0.398	0.390	**4.512820513**	**24.74106148**	**11.1652**
1930037	0.404	0.398	**3.316582915**	**41.41711893**	**11.3272697**
3062938	0.408	0.388	**11.34020619**	**10.92482477**	**6.4375017**
4594407	0.408	0.392	**8.979591837**	**41.28216187**	**12.8413208**

Figure 16 Method 2 Results from axial inductor, Hardware Lab

Toroidal Inductor					
Frequency(Hz)	Vin(V)	Vout(V)	**R(ohm)**	**Q**	**L(uH)**
1591549	0.398	0.388	**5.670103093**	**19.6913474**	**10.604459**
1930037	0.404	0.400	**2.2**	**58.53745924**	**10.6196706**
3062938	0.402	0.392	**5.612244898**	**31.92620618**	**9.3103467**
4594407	0.400	0.394	**3.350253807**	**103.0683762**	**11.9617249**

Figure 17 Method 2 Results from toroidal inductor, Hardware Lab

Conclusion:

This experiment aims to demonstrate two different methods to measure the quality factor of an inductor at four distinct frequencies. I have learned to design simple circuits to measure necessary values to calculate the Q factor and get familiar with the LTSpice software. For the first method, I have used the phase difference property of the inductor. For the second method, resonance is applied with a capacitor to leave out the internal resistance of the inductor.

In method 1, we deal with two unknown two nonlinear equations; thus, the calculations require more algebra or even a calculator. In terms of calculations, Method 2 is more manageable and straightforward. We adjust the frequency to make the inductor resonate with the capacitor, and thus knowing the capacitance and frequency values; we can easily calculate the inductance. And by applying simple voltage division to the resistor equivalent circuit, the internal resistance is also easily found.

Considering the Software Implementation, the results from both methods are fairly close to the expected value of 10u inductance and 3ohms of internal resistance. For the 1st and 2nd Methods, the error percentages of the Q factor values are %8.87917693 and %7.273854658, respectively. The Q factor values increase with the frequency. As a note, in the 2nd method, we assume that the capacitor is ideal. Thus, it does not affect the overall Q factor of the circuit.

In the Hardware Implementation, it is observed that the phase difference method has much more variance compared to the resonance method. The phase difference method thus is not as reliable as the resonance method. However, in the resonance method, the very critical complication is that the capacitor is not ideal in real. It contributes to the overall q factor of the circuit and thus does not give the actual Q factor of the inductor. Still, the resonance method is much more reliable for overall measurements than the phase difference method, as seen in the results. It is observed in this lab experiment that although simulation gives us an idealistic simulation of the circuit, in the actual case, the physical errors can make the measurements unpredictable. In my case, method 2 is better than the 1st method since it is easy to calculate and much reliable.

To sum up, a real inductor has parasitic resistance, and thus, the model in Figure 1 is overall appropriate.

Appendix

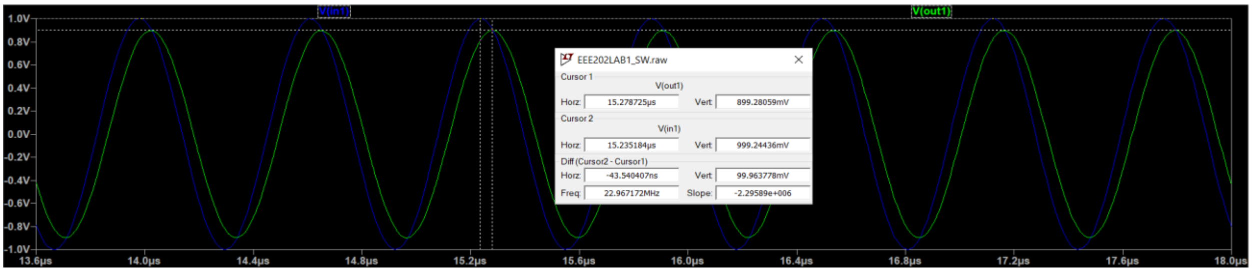


Figure 18 Method 1 at frequency = 1591549Hz

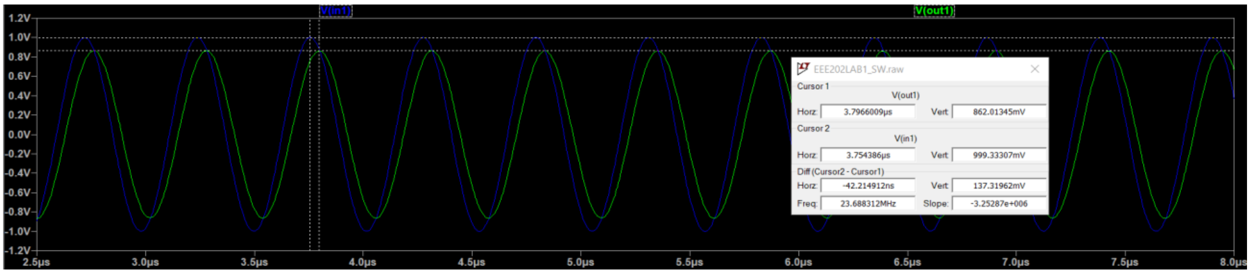


Figure 19 Method 1 at frequency =1930037Hz

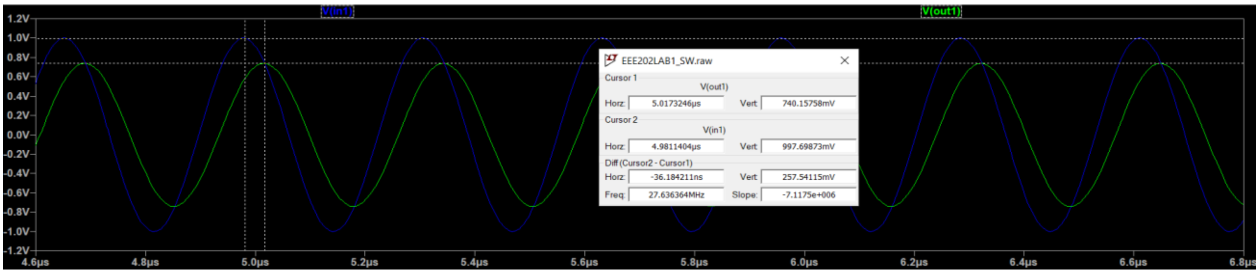


Figure 20 Method 1 f=3062938Hz

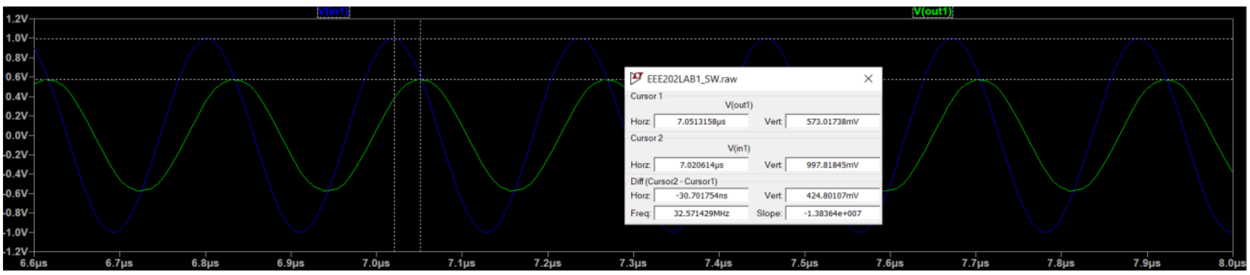
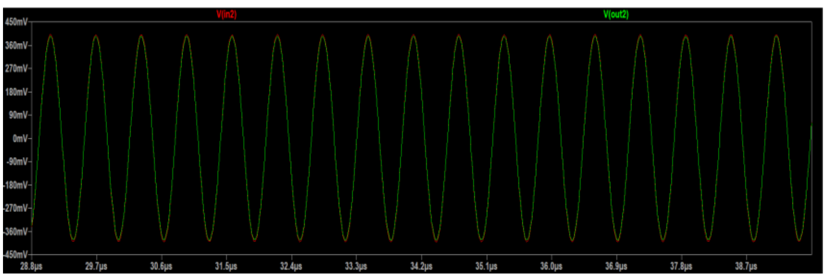
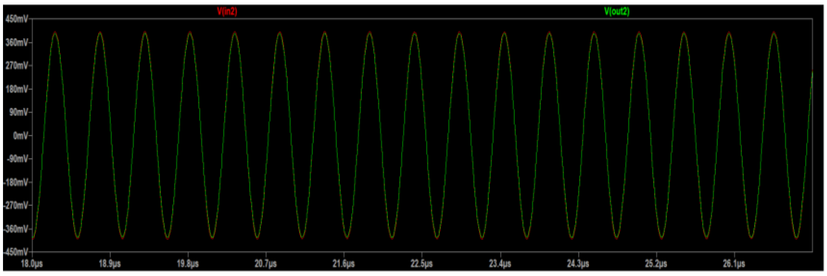


Figure 21 Method 1 f=4594407Hz



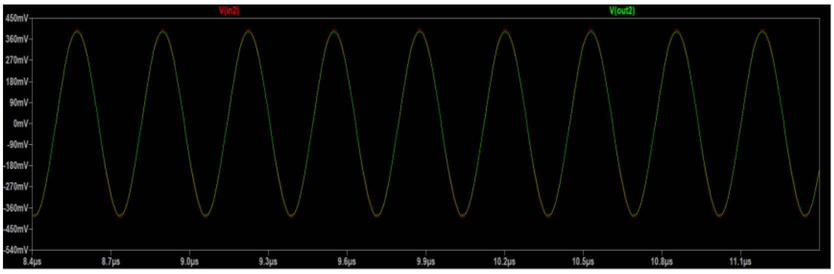

```
vout2: MAX(v(out2))=0.394494 FROM 0 TO 0.0001
vin2: MAX(v(in2))=0.4 FROM 0 TO 0.0001
```

Figure 22 Method 2 f= 1591549Hz C=1nF



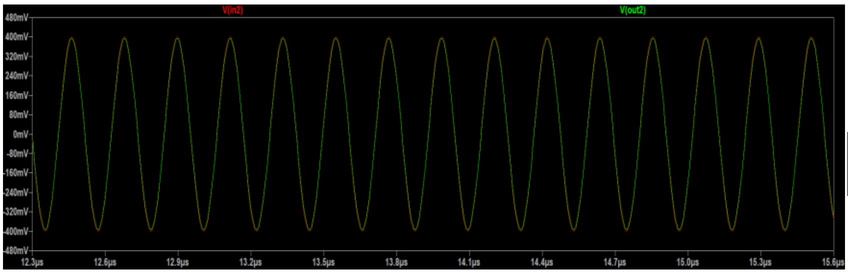
vout2: MAX(v(out2))=0.394546 FROM 0 TO 0.0001
vin2: MAX(v(in2))=0.4 FROM 0 TO 0.0001

Figure 23 Method 2 f= 1930037Hz C=680pF



vout2: MAX(v(out2))=0.39374 FROM 0 TO 0.0001
vin2: MAX(v(in2))=0.399997 FROM 0 TO 0.0001

Figure 24 Method 2 f= 3062938Hz C=270pF



vout2: MAX(v(out2))=0.393212 FROM 0 TO 0.0001
vin2: MAX(v(in2))=0.399856 FROM 0 TO 0.0001

Figure 25 Method 2 f= 4594407Hz C=120pF

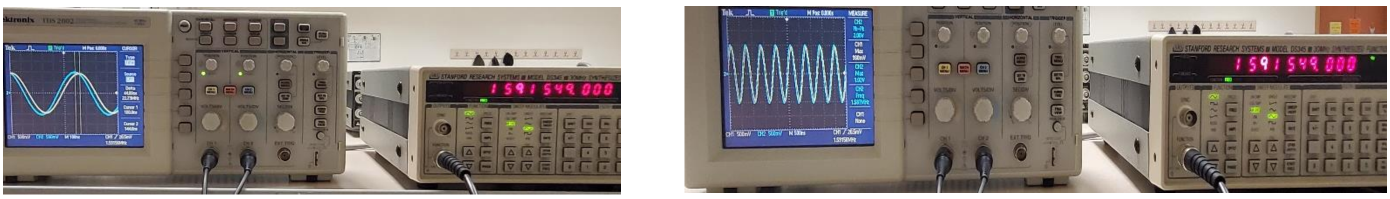


Figure 26 Method 1 at frequency = 1591549Hz for axial inductor

Figure 27 Method 1 at frequency =1930037Hz for axial inductor

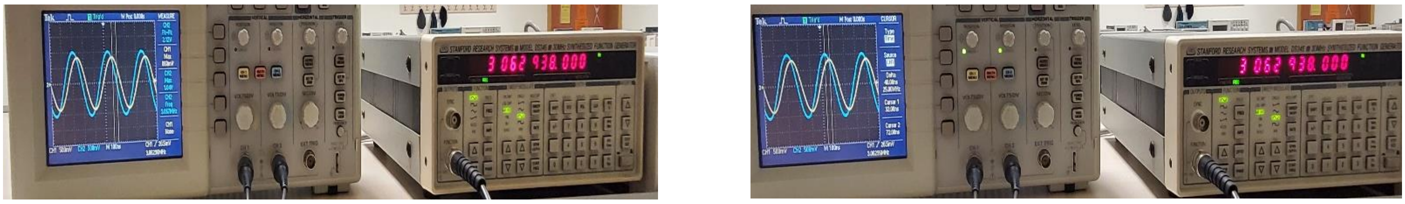


Figure 28 Method 1 at frequency =3062938Hz for axial inductor

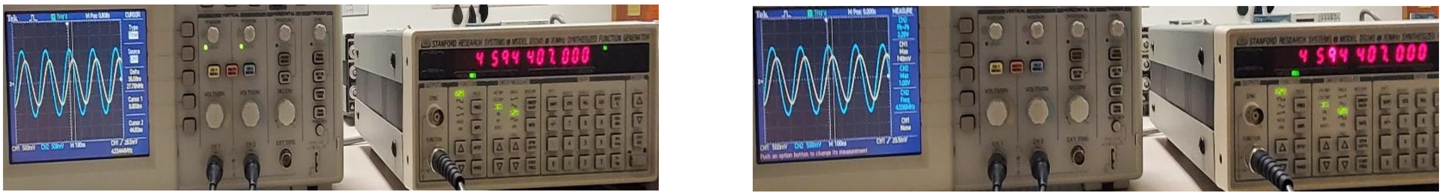


Figure 29 Method 1 at frequency =4594407Hz for axial inductor

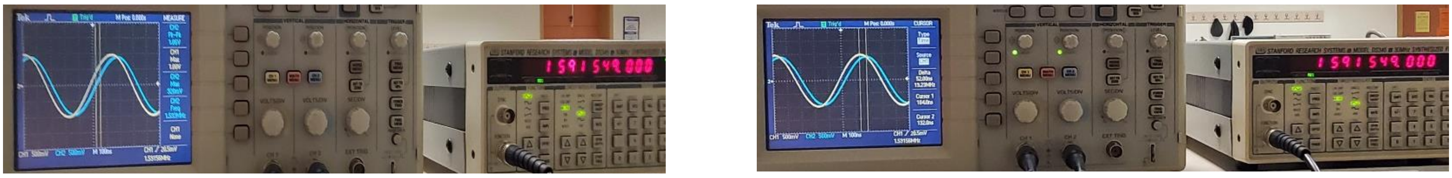


Figure 30 Method 1 at frequency = 1591549Hz for toroidal inductor

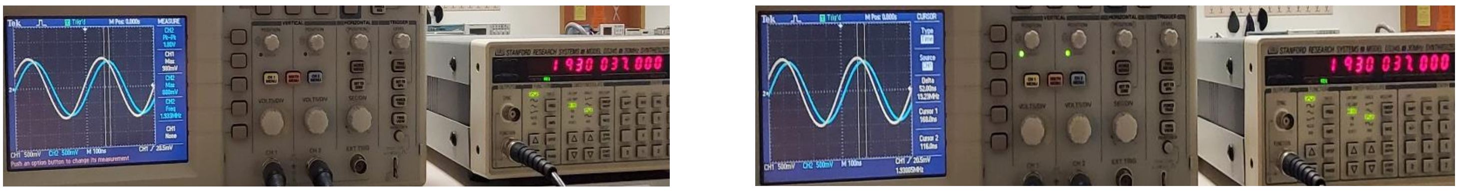


Figure 31 Method 1 at frequency = 1930037Hz for toroidal inductor

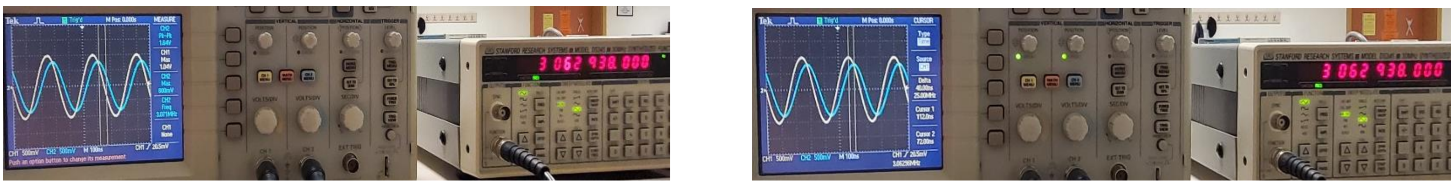


Figure 32 Method 1 at frequency =3062938Hz for toroidal inductor

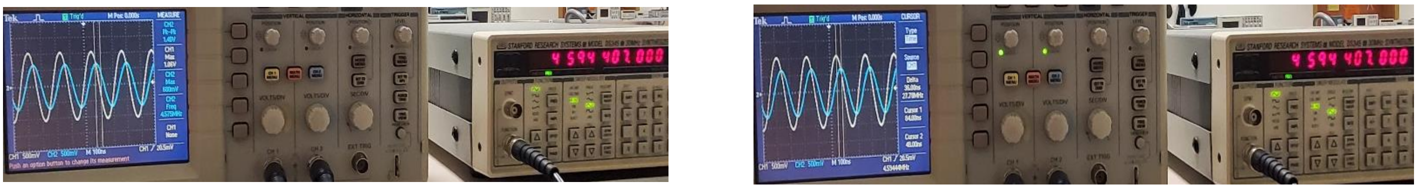


Figure 33 Method 1 at frequency =4594407Hz for toroidal inductor

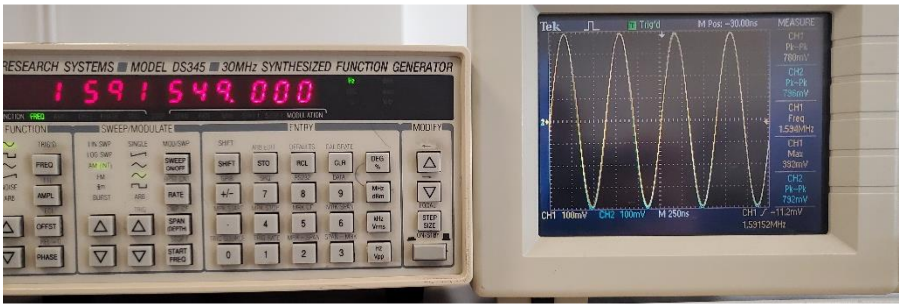


Figure 34 Method 2 at frequency =1591549Hz for axial inductor

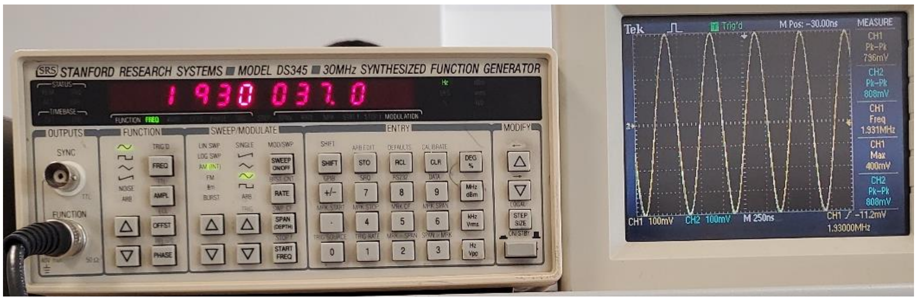


Figure 35 Method 2 at frequency =1930037Hz for axial inductor

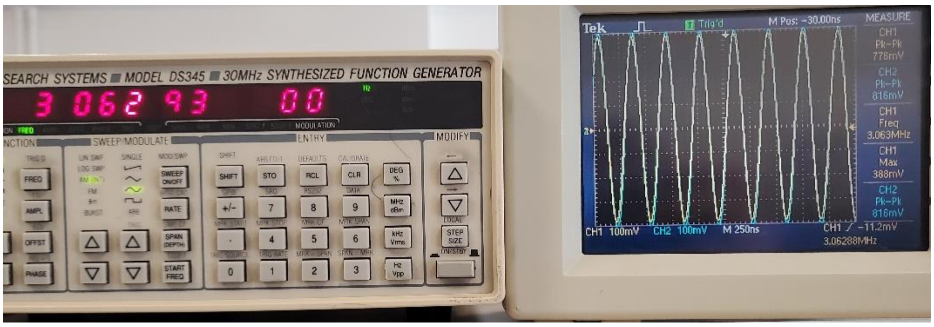


Figure 36 Method 2 at frequency =3062938Hz for axial inductor

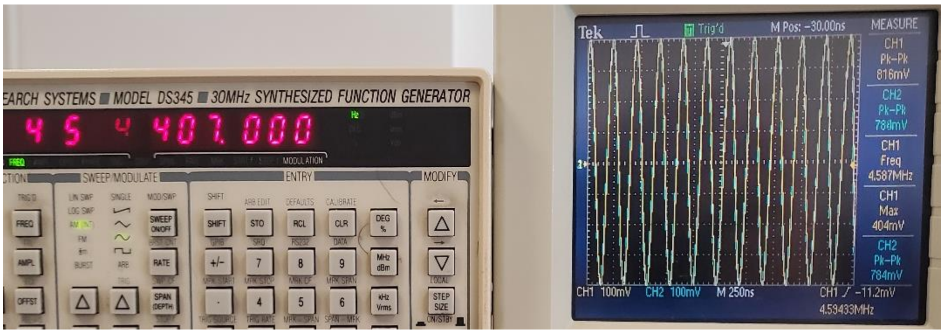


Figure 37 Method 2 at frequency =4594407Hz for axial inductor

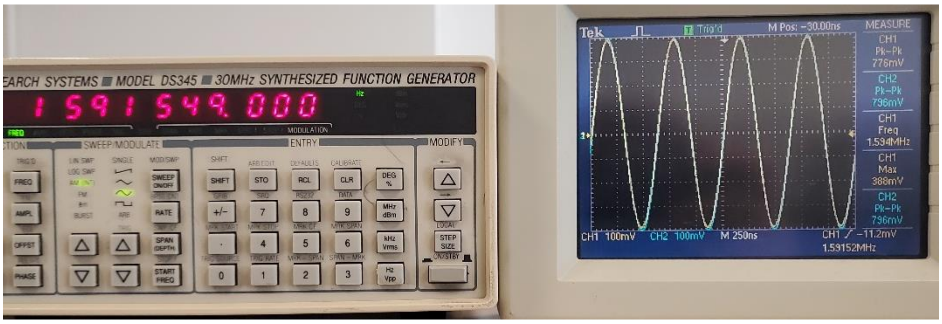


Figure 38 Method 2 at frequency =1591549Hz for toroidal inductor

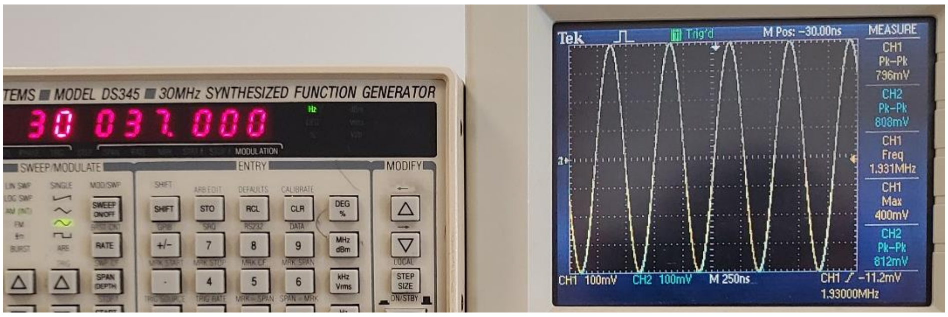


Figure 39 Method 2 at frequency =1930037Hz for toroidal inductor

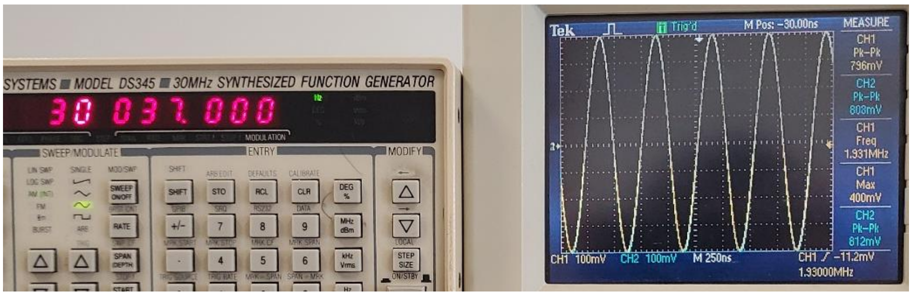


Figure 40 Method 2 at frequency =3062938Hz for toroidal inductor

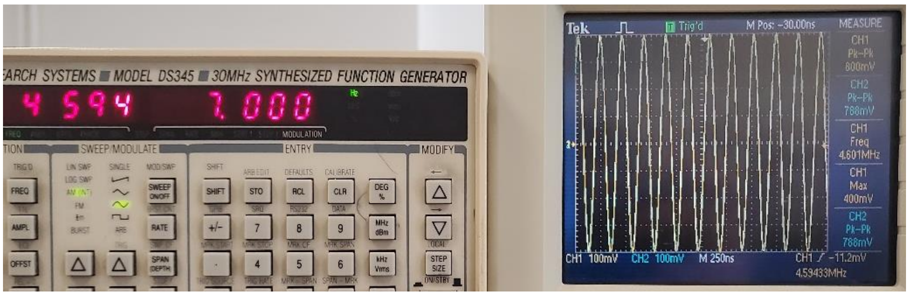


Figure 41 Method 2 at frequency =4594407Hz for axial inductor