Project 1 – 2016-11-09

FMNN05: Simulation Tools

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This is part I of the overall course project which is based on ASSIMULO/SUNDIALS in your own Python working environment and on DYMOLA/Modelica. For the later you get a student license which is valid during the course plus two more months.
In this part, we consider unconstrained ordinary differential equations and investigate the notion of order and stability by considering a spring pendulum example.
Please describe you observation in report form. It should include

- a description of your tests, i.e. what is tested and why ...
- a description of your test cases in such a way so that they are reproducible.
- a documentation of your results (tables, plots etc) obtained by DYMOLA/Modelica and your own Python/Sundials control computations.
- your own interpretation of the results.

Consider the tasks listed below as suggestions for experiments. You are free to design others as long as they lead to a statement.

This assignment has 6 tasks.

Task 1

We consider the elastic pendulum given by the following ODE

$$\dot{y}_1 = y_3 \tag{1}$$
$$\dot{y}_2 = y_4 \tag{2}$$
$$\dot{y}_3 = -y_1\lambda(y_1, y_2) \tag{3}$$
$$\dot{y}_4 = -y_2\lambda(y_1, y_2) - 1 \tag{4}$$

with $\lambda(y_1, y_2) = k\frac{\sqrt{y_1^2+y_2^2}-1}{\sqrt{y_1^2+y_2^2}}$. It corresponds to a mathematical pendulum where the bar is replaced by a linear spring with spring constant k. Also the mass-, nominal pendulum length and gravitational constant are taken to be one.

Set up the problem (rhs-function) in Python and use Assimulo's class for explicit problems provided by the course's project page to solve the problem. Use first, to see if things are correctly set up a moderately size k and make a simulation with CVODE. Plot your result.

Task 2

Implement in Assimulo BDF-4 and BDF-3 (*). You may use the previous values as predictor. As corrector iteration method use Newton's method. To begin with, you might call `fsolve` for this purpose. You find on the webpage BDF2 to give you an idea how to make such an Assimulo implementation.

Task 3

The larger k the higher the frequencies in the problem. These frequencies become excited, when you start with initial conditions, which correspond to a slightly strechted spring, i.e. $\sqrt{y_1(0)^2 + y_2(0)^2} > 1$. We study now the influence of this parameter on the choice of a method. Test the methods in Assimulo for various values of k (also really huge ones). Make first statements about the fixed order methods including those which use fixed point iteration (the BDF2 example) and the explicit Euler method. What do you observe? Can you practically verify the message from stability diagrams?

Task 4

Repeat the experiments with CVODE. Test also the influence of ATOL, RTOL, MAXORD and the choice of the discretization on the performance for the low and highly oscillating case.

Task 5

We use now these experiments to compare the results and also the performance in terms of number of function calls, numeber of steps etc with DYMOLA. For this end, set up a Dymola model and simulate in DYMOLA with integrators which you find in this package. Is there a particular method in Dymola you would recommend for this task?

Task 6

Describe your experiments briefly, but extensively enough so that they can be reproduced and make a short statement about your observations.

Lycka till!

* see also http://www.scolarpedia.org/article/Backward_differentiation_formulas