System Analysis and Decision Support Methods

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I took 5 data from my dataset. I made calculations like following;

X "Age"	y "Cancer Risk"
25	1.9
35	8.4
45	20.1
55	25.6
65	24.8

Mean

$\bar{x} = 47.06$ (average value of x)

$\bar{y} = 16.16$ (average value of y)

We are going to calculate slope;

$$\text{slope} = \sum_{i=1}^{5} \frac{(x_i - \bar{x}_i)(y_i - \bar{y}_i)}{\sum_{i=1}^{5} (x_i - x_i)^2}$$

$$= \frac{(-22.06)(-14.26) + (-12.06)(-7.76) + (-2.06)(3.94) + (7.94)(9.46) + (17.94)(8.64)}{486.64 + 145.44 + 4.24 + 63.04 + 321.84}$$

$$= \frac{314.57 + 93.58 - 8.11 + 74.95 + 155}{1021.2} = \frac{629.94}{1021.2}$$

$$= 0.616$$

So we are going to calculate intercept

$$b = \bar{y} - (\text{slope}(\bar{x}))$$

$$b = 16.16 - 0.616(47.06)$$

$$b = 16.16 - 28.98$$

$b = -12.82$ → intercept or constant

The importance of Intercept is that whenever our value of $x=0$ then the y becomes equal to intercept

The final formula ~~for~~ according to our data is;

$$\boxed{y = (0.616)x - 12.82}$$

According to your notes we cant use direct/explicit formula that is

$$a^* = \frac{\sum_{i=1}^{5} (x_i y_i)}{\sum_{i=1}^{5} (x_i)^2}$$

Now I shall be doing the analytical part of class.

$$1\begin{cases}\frac{\partial f}{\partial \beta_0} = \sum (y_i - (\beta_0 + \beta_1 x_i))^2 = \\ = \sum \frac{\partial f}{\partial \beta_0}(y_i - (\beta_0 + \beta_1 x_i)) = (-2)\sum (y_i - (\beta_0 + \beta_1 x_i))\end{cases}$$

$$2\begin{cases}\frac{\partial f}{\partial \beta_1} = \sum (y_i - (\beta_0 + \beta_1 x_i))^2 = \sum 2(y_i - (\beta_0 + \beta_1 x_i))(-x_i) \\ = (-2)\sum x_i (y_i - (\beta_0 + \beta_1) x_i))\end{cases}$$

Equating them to zero

Equation (1) $\rightarrow \cancel{(-2)} \sum (y_i - (\beta_0 + \beta_1 x_i)) = 0$

Equation (2) $\rightarrow \cancel{(-2)} \sum x_i (y_i - (\beta_0 + \beta_1 x_i)) = 0$

Solving equation (1):

$$\sum (y_i - (\beta_0 + \beta_1 x_i)) = 0$$

$$\sum y_i - n\beta_0 - \beta_1 \sum x_i = 0$$

$$n\beta_0 = \sum y_i - \beta_1 \sum x_i \Rightarrow \beta_0 = \frac{\sum y_i}{n} - \frac{\beta_1 \sum n}{n}$$

Equation (3) $\beta_0 = \bar{y} - \beta_1 \bar{x}$

Now using Equation 3 and Equation 2

$$\sum x_i (y_i - (\bar{y} - \beta_1 \bar{x}) + \beta_1 x_i) = 0$$

$$\sum x_i (y_i - \bar{y}) - \sum (\beta_1 x_i (x_i - \bar{x}) = 0$$

We calculate the slope by

$$\beta_1 = \frac{\sum x_i (y_i - \bar{y})}{\sum x_i (x_i - \bar{x})} = \frac{\sum (y_i - \bar{y})(x_i - \bar{x})}{\sum (x_i - \bar{x})^2}$$