

# System Analysis and Decision Support

## Methods

Cannur KARTUM 251585

I took 5 data from my dataset. I made calculations like following;

"X Age"	"y Cancer Risk"	Mean
25	1.9	$\bar{x} = 47.06$ (average value of x)
35	8.4	
45	20.1	$\bar{y} = 16.16$ (average value of y)
55	25.6	
65	24.8	

We are going to calculate slope;

$$\text{slope} = \frac{\sum_{i=1}^5 (x_i - \bar{x}_i)(y_i - \bar{y}_i)}{\sum_{i=1}^5 (x_i - \bar{x}_i)^2}$$

$$= \frac{(-22.06)(-14.26) + (-12.06)(-7.76) + (-2.06)(3.94) + (7.94)(9.46) + (7.94)(18.64)}{486.64 + 145.44 + 4.24 + 63.04 + 321.84}$$
$$= \frac{314.57 + 93.58 - 8.11 + 74.95 + 155}{1021.2} = \frac{629.94}{1021.2}$$
$$= \underline{\underline{0.616}}$$



So we are going to calculate intercept

$$b = \bar{y} - (\text{slope} \cdot \bar{x})$$

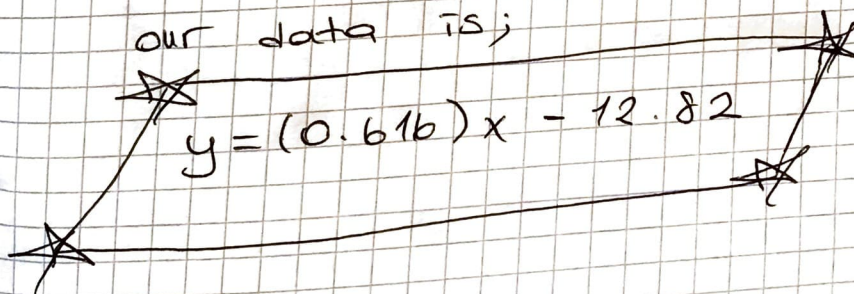
$$b = 16.16 - 0.616(47.06)$$

$$b = 16.16 - 28.98$$

$$\underline{\underline{b = -12.82}} \rightarrow \text{intercept or constant}$$

The importance of Intercept is that whenever our value of  $x=0$  then the  $y$  becomes equal to intercept

The final formula ~~for~~ according to our data is;


$$y = (0.616)x - 12.82$$

According to your notes we can't use direct/explicit formula that is

$$a^* = \frac{\sum_{i=1}^n (x_i y_i)}{\sum_{i=1}^n (x_i)^2}$$

Now I shall be doing the analytical part of class.

$$1 \left\{ \begin{aligned} \frac{\partial f}{\partial \beta_0} &= \sum (y_i - (\beta_0 + \beta_1 x_i))^2 = \cancel{\sum (y_i - (\beta_0 + \beta_1 x_i))^2} \\ &= \sum \frac{\partial f}{\partial \beta_0} (y_i - (\beta_0 + \beta_1 x_i)) = (-2) \sum (y_i - (\beta_0 + \beta_1 x_i)) \end{aligned} \right.$$

$$2 \left\{ \begin{aligned} \frac{\partial f}{\partial \beta_1} &= \sum (y_i - (\beta_0 + \beta_1 x_i))^2 = \sum 2 (y_i - (\beta_0 + \beta_1 x_i)) (-x_i) \\ &= (-2) \sum x_i (y_i - (\beta_0 + \beta_1 x_i)) \end{aligned} \right.$$

Equating them to zero

Equation (1)  $\rightarrow (-2) \sum (y_i - (\beta_0 + \beta_1 x_i)) = 0$

Equation (2)  $\rightarrow (-2) \sum x_i (y_i - (\beta_0 + \beta_1 x_i)) = 0$

Solving equation (1):

$$\sum (y_i - (\beta_0 + \beta_1 x_i)) = 0$$

$$\sum y_i - n\beta_0 - \beta_1 \sum x_i = 0$$

$$n\beta_0 = \sum y_i - \beta_1 \sum x_i \Rightarrow \beta_0 = \frac{\sum y_i}{n} - \frac{\beta_1 \sum x_i}{n}$$

Equation (3)  $\beta_0 = \bar{y} - \beta_1 \bar{x}$

Now using Equation 3 and Equation 2

$$\sum x_i (y_i - (\bar{y} - \beta_1 \bar{x}) + \beta_1 x_i) = 0$$

$$\sum x_i (y_i - \bar{y}) - \sum (\beta_1 x_i (x_i - \bar{x})) = 0$$



We calculate the slope by

$$B_1 = \frac{\sum x_i (y_i - \bar{y})}{\sum x_i (x_i - \bar{x})} = \frac{\sum (y_i - \bar{y}) (x_i - \bar{x})}{\sum (x_i - \bar{x})^2}$$