PROBLEMS OF QUANTUM FIELD THEORIES IN CURVED SPACETIMES

A MASTER THESIS

Submitted in partial fulfillment of the requirements for the award of

Master's Degree in Physics of the Universe: Cosmology, Astrophysics, Particles and Astroparticles

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September 2, 2023

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FLRW metric

$$\mathrm{d}l^2 = c^2\mathrm{d}t^2 - a^2(t)\left[\frac{\mathrm{d}r^2}{1-\kappa r^2} + r^2\mathrm{d}\Omega^2\right] \tag{3.1}$$

Weyl tensor =0 therefore the metric is conformally flat, i.e. independently of the curvature κ there must exist a coordinate system where

$$\mathrm{d}l^2 = a(t)\eta_{\mu\nu}\mathrm{d}x^\mu\mathrm{d}x^\nu = a(t)\left[c^2\mathrm{d}t^2 - \mathrm{d}\mathbf{x}^2\right] \tag{3.2}$$

the standard action describing the dynamics of a (non-minimally coupled to gravity) real scalar field is

$$s = \int \frac{1}{2}\left[\nabla_\nu\phi\,\nabla^\nu\phi - \mu^2\phi^2 - \xi R\phi^2\right]\sqrt{-g}\;\mathrm{d}^4x \tag{3.3}$$

$\sqrt{-g} = a^4$ $\chi = a\phi$

$$s = \int \frac{1}{2}\left[\partial_\nu\chi\,\partial^\nu\chi - \left(\mu^2a^2 + \xi Ra^2 - c^2\frac{a''}{a}\right)\chi^2 - \partial_t\left(c^2\chi^2\frac{a'}{a}\right)\right]\mathrm{d}^4x \tag{3.4}$$

dropping the time drivative

$$s = \int \frac{1}{2}\left[\partial_\nu\chi\,\partial^\nu\chi - \left(\mu^2a^2 + \xi Ra^2 - c^2\frac{a''}{a}\right)\chi^2\right]\mathrm{d}^4x \tag{3.5}$$

by Euler-Lagrange

$$\left[\partial_\nu\partial^\nu + \mu^2_{\text{eff}}(t)\right]\chi = 0 \tag{3.6}$$

where

$$\mu^2_{\text{eff}}(t) = \left(\mu^2 + \xi R\right)a^2 - c^2\frac{a''}{a} \tag{3.7}$$

solutions of previous equation have the form

$$\chi = a\,v(t)e^{\pm i\mathbf{k}\mathbf{x}\hbar^{-1}} \tag{3.8}$$

meaning that, the dispersion relation is

$$v''\hbar^2 + \omega^2(t)\,v = 0 \tag{3.9}$$

where $\omega(t)$ is defined as

$$\omega^2(t) = \mathbf{k}^2 + \hbar^2\mu^2_{\text{eff}}(t) = \mathbf{k}^2 + \left(m^2c^2 + \xi\hbar^2R\right)a(t) - \hbar^2c^2\frac{a''}{a} \tag{3.10}$$

now, proof that $\mathrm{Im}(vv'^*)$ is constant through time

$$\frac{\partial}{\partial t}\mathrm{Im}(vv'^*) = \frac{\partial}{\partial t}\left(\frac{vv'^* - v^*v'}{2i}\right) = \frac{vv''^* - v^*v''}{2i} = 0 \tag{3.11}$$

last step is result from dispersion relation. Since dispersion relation is scalable by a time independent function, $\mathrm{Im}(v'v^*)$ can be determined to be a chosen value, a particular useful choice is to consider it momentum independent. $\mathrm{Im}(v'v^*) = W[v, v^*]$ therefore, if its not equal to 0, they are linearly independent solutions to dispersion relation.

The most general solution to the main equation is

$$\mathcal{X} = \int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi\hbar)^3}\left[a_{\mathbf{k}}v_{\mathbf{k}}(t)e^{i\mathbf{k}\mathbf{x}\hbar^{-1}} + a^*_{\mathbf{k}}v^*_{\mathbf{k}}(t)e^{-i\mathbf{k}\mathbf{x}\hbar^{-1}}\right] \tag{3.12}$$

The field $\mathcal{X}$ and its conjugate momentum $\Pi = \partial_{ct}\mathcal{X}$ are promoted to operators on the quantum Hilbert space, with the standar canonical conmutation relations

$$\left[\hat{\mathcal{X}}(t,\mathbf{x}), \hat{\Pi}(t,\mathbf{y})\right] = i\hbar\,\delta^3(\mathbf{x}-\mathbf{y}) \tag{3.13}$$

$$\left[\hat{\mathcal{X}}(t,\mathbf{x}), \hat{\mathcal{X}}(t,\mathbf{y})\right] = \left[\hat{\Pi}(t,\mathbf{x}), \hat{\Pi}(t,\mathbf{y})\right] = 0 \tag{3.14}$$

where the operational nature of the fields arrise from the promotion of the mode amplitudes, i.e.

$$a_{\mathbf{k}} \longrightarrow \hat{a}_{\mathbf{k}} \qquad a^*_{\mathbf{k}} \longrightarrow \hat{a}^\dagger_{\mathbf{k}} \tag{3.15}$$

this operators fulfill the following conmutation relations

$$[\hat{a}_{\mathbf{k}}, \hat{a}^\dagger_{\mathbf{q}}] = \frac{(2\pi\hbar)^3\hbar c}{2\mathrm{Im}(v'v^*)}\delta^3(\mathbf{k}-\mathbf{q})\,, \qquad [\hat{a}_{\mathbf{k}}, \hat{a}_{\mathbf{q}}] = [\hat{a}^\dagger_{\mathbf{k}}, \hat{a}^\dagger_{\mathbf{q}}] = 0 \tag{3.16}$$

(note that $\hat{a}_{\mathbf{k}} \neq \hat{a}_{-\mathbf{k}}$)

To prove this, consider that

$$\left[\hat{\mathcal{X}}(\mathbf{x}),\, \hat{\Pi}(\mathbf{y})\right] = \frac{1}{c}\int \frac{\mathrm{d}^3\mathbf{k}\mathrm{d}^3\mathbf{q}}{(2\pi\hbar)^6}\Big\{[\hat{a}_{\mathbf{k}}, \hat{a}_{\mathbf{q}}]\,v_{\mathbf{k}}v'_{\mathbf{q}}e^{i(\mathbf{kx}+\mathbf{qy})\hbar^{-1}} + \left[\hat{a}^\dagger_{\mathbf{k}}, \hat{a}^\dagger_{\mathbf{q}}\right]v^*_{\mathbf{k}}v^{*\prime}_{\mathbf{q}}e^{i(\mathbf{kx}-\mathbf{qy})\hbar^{-1}} + \\ + \left[\hat{a}_{\mathbf{k}}, \hat{a}^\dagger_{\mathbf{q}}\right]v_{\mathbf{k}}v^{*\prime}_{\mathbf{q}}e^{i(\mathbf{kx}-\mathbf{qy})\hbar^{-1}} - \left[\hat{a}_{\mathbf{q}}, \hat{a}^\dagger_{\mathbf{k}}\right]v^*_{\mathbf{k}}v'_{\mathbf{q}}e^{-i(\mathbf{kx}-\mathbf{qy})\hbar^{-1}}\Big\} \tag{3.17}$$

if the operators $\hat{a}$ and $\hat{a}^\dagger$ are to be understood as creation and annihilation operators, they must fulfill

$$[\hat{a}_{\mathbf{k}}, \hat{a}^\dagger_{\mathbf{q}}] = \alpha\delta^3(\mathbf{k}-\mathbf{q})\,, \qquad [\hat{a}_{\mathbf{k}}, \hat{a}_{\mathbf{q}}] = [\hat{a}^\dagger_{\mathbf{k}}, \hat{a}^\dagger_{\mathbf{q}}] = 0 \tag{3.18}$$

where $\alpha \in \mathbb{C}$, and thus

$$\left[\hat{\mathcal{X}}(\mathbf{x}),\, \hat{\Pi}(\mathbf{y})\right] = \frac{\alpha}{c}\int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi\hbar)^6}2i\mathrm{Im}(v_{\mathbf{k}}v^{*\prime}_{\mathbf{k}})e^{i(\mathbf{kx}-\mathbf{qy})\hbar^{-1}} \tag{3.19}$$

considering $\mathrm{Im}(v'v^*)$ momentum independent, and remembering the canonical conmutation relations, one finds that

$$\alpha\mathrm{Im}(vv^{*\prime}) = \frac{1}{2}\hbar c(2\pi\hbar)^3 \tag{3.20}$$

The hamiltonian

$$\hat{\mathcal{H}}(t) = \int \frac{c}{2}\left[\hat{\Pi}^2 + \left(\nabla\hat{\mathcal{X}}\right)^2 + \mu^2_{\text{eff}}(t)\hat{\mathcal{X}}^2\right]\mathrm{d}^3\mathbf{x} \tag{3.21}$$

$$\hat{\Pi}^2 = \frac{1}{c^2}\int \frac{\mathrm{d}^3\mathbf{k}\mathrm{d}^3\mathbf{q}}{(2\pi\hbar)^6}\Big[\hat{a}_\mathbf{k}\hat{a}_\mathbf{q}v'_\mathbf{k}v'_\mathbf{q}e^{i(\mathbf{k}+\mathbf{q})\mathbf{x}\hbar^{-1}} + \hat{a}_\mathbf{k}\hat{a}^\dagger_\mathbf{q}v'_\mathbf{k}v'^*_\mathbf{q}e^{i(\mathbf{k}-\mathbf{q})\mathbf{x}\hbar^{-1}} + \\ + \hat{a}^\dagger_\mathbf{k}\hat{a}_\mathbf{q}v'^*_\mathbf{k}v'_\mathbf{q}e^{-i(\mathbf{k}-\mathbf{q})\mathbf{x}\hbar^{-1}} + \hat{a}^\dagger_\mathbf{k}\hat{a}^\dagger_\mathbf{q}v'^*_\mathbf{k}v'^*_\mathbf{q}e^{-i(\mathbf{k}+\mathbf{q})\mathbf{x}\hbar^{-1}}\Big] \quad (3.22)$$

$$\left(\nabla\hat{\chi}\right)^2 = -\frac{1}{\hbar^2}\int \frac{\mathrm{d}^3\mathbf{k}\mathrm{d}^3\mathbf{q}}{(2\pi\hbar)^6}\mathbf{kq}\Big[\hat{a}_\mathbf{k}\hat{a}_\mathbf{q}v_\mathbf{k}v_\mathbf{q}e^{i(\mathbf{k}+\mathbf{q})\mathbf{x}\hbar^{-1}} - \hat{a}_\mathbf{k}\hat{a}^\dagger_\mathbf{q}v_\mathbf{k}v^*_\mathbf{q}e^{i(\mathbf{k}-\mathbf{q})\mathbf{x}\hbar^{-1}} - \\ - \hat{a}^\dagger_\mathbf{k}\hat{a}_\mathbf{q}v^*_\mathbf{k}v_\mathbf{q}e^{-i(\mathbf{k}-\mathbf{q})\mathbf{x}\hbar^{-1}} + \hat{a}^\dagger_\mathbf{k}\hat{a}^\dagger_\mathbf{q}v^*_\mathbf{k}v^*_\mathbf{q}e^{-i(\mathbf{k}+\mathbf{q})\mathbf{x}\hbar^{-1}}\Big] \quad (3.23)$$

$$\hat{\chi}^2 = \int \frac{\mathrm{d}^3\mathbf{k}\mathrm{d}^3\mathbf{q}}{(2\pi\hbar)^6}\Big[\hat{a}_\mathbf{k}\hat{a}_\mathbf{q}v_\mathbf{k}v_\mathbf{q}e^{i(\mathbf{k}+\mathbf{q})\mathbf{x}\hbar^{-1}} + \hat{a}_\mathbf{k}\hat{a}^\dagger_\mathbf{q}v_\mathbf{k}v^*_\mathbf{q}e^{i(\mathbf{k}-\mathbf{q})\mathbf{x}\hbar^{-1}} + \\ + \hat{a}^\dagger_\mathbf{k}\hat{a}_\mathbf{q}v^*_\mathbf{k}v_\mathbf{q}e^{-i(\mathbf{k}-\mathbf{q})\mathbf{x}\hbar^{-1}} + \hat{a}^\dagger_\mathbf{k}\hat{a}^\dagger_\mathbf{q}v^*_\mathbf{k}v^*_\mathbf{q}e^{-i(\mathbf{k}+\mathbf{q})\mathbf{x}\hbar^{-1}}\Big] \quad (3.24)$$

$$\begin{aligned}\hat{\mathcal{H}} = \frac{c}{2}\int \frac{\mathrm{d}^3\mathbf{k}\mathrm{d}^3\mathbf{q}}{(2\pi\hbar)^3}\Big\{&\hat{a}_\mathbf{k}\hat{a}_\mathbf{q}\left[\frac{1}{c^2}v'_\mathbf{k}v'_\mathbf{q} - \left(\frac{1}{\hbar^2}\mathbf{kq} - \mu^2_{\text{eff}}\right)v_\mathbf{k}v_\mathbf{q}\right]\delta^3(\mathbf{k}+\mathbf{q}) + \\ &+ \hat{a}_\mathbf{k}\hat{a}^\dagger_\mathbf{q}\left[\frac{1}{c^2}v'_\mathbf{k}v'^*_\mathbf{q} + \left(\frac{1}{\hbar^2}\mathbf{kq} + \mu^2_{\text{eff}}\right)v_\mathbf{k}v^*_\mathbf{q}\right]\delta^3(\mathbf{k}-\mathbf{q}) + \\ &+ \hat{a}^\dagger_\mathbf{k}\hat{a}_\mathbf{q}\left[\frac{1}{c^2}v'^*_\mathbf{k}v'_\mathbf{q} + \left(\frac{1}{\hbar^2}\mathbf{kq} + \mu^2_{\text{eff}}\right)v^*_\mathbf{k}v_\mathbf{q}\right]\delta^3(\mathbf{k}-\mathbf{q}) + \\ &+ \hat{a}^\dagger_\mathbf{k}\hat{a}^\dagger_\mathbf{q}\left[\frac{1}{c^2}v'^*_\mathbf{k}v'^*_\mathbf{q} - \left(\frac{1}{\hbar^2}\mathbf{kq} - \mu^2_{\text{eff}}\right)v^*_\mathbf{k}v^*_\mathbf{q}\right]\delta^3(\mathbf{k}+\mathbf{q})\Big\}\end{aligned} \quad (3.25)$$

$$\begin{aligned}\hat{\mathcal{H}} = \frac{c}{2}\int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi\hbar)^3}\Big\{&\hat{a}_\mathbf{k}\hat{a}_{-\mathbf{k}}\left[\frac{1}{c^2}v'_\mathbf{k}v'_\mathbf{k} + \frac{1}{\hbar^2}\omega^2_\mathbf{k}(t)v_\mathbf{k}v_\mathbf{k}\right] + \\ &+ \hat{a}_\mathbf{k}\hat{a}^\dagger_\mathbf{k}\left[\frac{1}{c^2}v'_\mathbf{k}v'^*_\mathbf{k} + \frac{1}{\hbar^2}\omega^2_\mathbf{k}(t)v_\mathbf{k}v^*_\mathbf{k}\right] + \\ &+ \hat{a}^\dagger_\mathbf{k}\hat{a}_\mathbf{k}\left[\frac{1}{c^2}v'^*_\mathbf{k}v'_\mathbf{k} + \frac{1}{\hbar^2}\omega^2_\mathbf{k}(t)v^*_\mathbf{k}v_\mathbf{k}\right] + \\ &+ \hat{a}^\dagger_\mathbf{k}\hat{a}^\dagger_{-\mathbf{k}}\left[\frac{1}{c^2}v'^*_\mathbf{k}v'^*_\mathbf{k} + \frac{1}{\hbar^2}\omega^2_\mathbf{k}(t)v^*_\mathbf{k}v^*_\mathbf{k}\right]\Big\}\end{aligned} \quad (3.26)$$

$$\hat{\mathcal{H}} = \frac{c}{2}\int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi\hbar)^3}\left[\hat{a}_\mathbf{k}\hat{a}_{-\mathbf{k}}F_\mathbf{k} + \hat{a}^\dagger_\mathbf{k}\hat{a}^\dagger_{-\mathbf{k}}F^*_\mathbf{k} + \left(2\hat{a}^\dagger_\mathbf{k}\hat{a}_\mathbf{k} + \frac{(2\pi\hbar)^3\hbar c}{2\mathrm{Im}(v'v^*)}\delta^3(\mathbf{0})\right)E_\mathbf{k}\right] \quad (3.27)$$

where

$$F_\mathbf{k}(t) = \left(\frac{1}{\hbar c}\right)^2\left[\hbar^2 v'^2_\mathbf{k} + \omega^2_\mathbf{k}(t)\,c^2 v^2_\mathbf{k}\right] \quad (3.28)$$

$$E_\mathbf{k}(t) = \left(\frac{1}{\hbar c}\right)^2\left[\hbar^2\left|v'_\mathbf{k}\right|^2 + \omega^2_\mathbf{k}(t)\,c^2\left|v_\mathbf{k}\right|^2\right] \quad (3.29)$$

Now, the expectation value of the hamiltonian at time t_0 in the state $|_{(v)}0\rangle$

$$\langle_{(v)}0|\hat{\mathcal{H}}(t_0)|_{(v)}0\rangle = \rho(t_0)\delta^3(\mathbf{0}) = \frac{\hbar c^2\,\delta^3(\mathbf{0})}{4\mathrm{Im}(v'v^*)}\int \mathrm{d}^3\mathbf{k}\,E_\mathbf{k} \quad (3.30)$$

To minimise the energy density of de vacuum state is to fin the set of functions $v_\mathbf{k}$ that minimise $E_\mathbf{k}$. Suppose that $v_\mathbf{k}$ can be written as

$$v_\mathbf{k} = r_\mathbf{k}e^{i\alpha_\mathbf{k}} \quad (3.31)$$

since $\mathrm{Im}(vv^{'*})$ was constant through time

$$\mathrm{Im}(v_{\mathbf{k}}v_{\mathbf{k}}^{'*}) = -r_{\mathbf{k}}^2\alpha_{\mathbf{k}}' \tag{3.32}$$

this means

$$E_{\mathbf{k}} = \left(\frac{1}{\hbar c}\right)^2 \left\{\hbar^2 \left[r_{\mathbf{k}}^{'2} + \mathrm{Im}^2\left(v_{\mathbf{k}}v_{\mathbf{k}}^{'*}\right)\frac{1}{r_{\mathbf{k}}^2}\right] + \omega_{\mathbf{k}}^2\, c^2 r_{\mathbf{k}}^2\right\} \tag{3.33}$$

the minimum of this function must fulfil $r_{\mathbf{k}}'(t_0) = 0$. Now, if $\omega_{\mathbf{k}}^2(t_0)$ and $\mathrm{Im}(v_{\mathbf{k}}v_{\mathbf{k}}^{'*})$ have the same sign, the minimum of $E_{\mathbf{k}}$ happens when $r_{\mathbf{k}}(t_0) = \left[\frac{\hbar\mathrm{Im}(v_{\mathbf{k}}v_{\mathbf{k}}^{'*})}{\omega_{\mathbf{k}}(t_0)\, c}\right]^{1/2}$.

If there is a minimum, then

$$v_{\mathbf{k}}(t_0) = \left[\frac{\hbar\mathrm{Im}(v_{\mathbf{k}}v_{\mathbf{k}}^{'*})}{\omega_{\mathbf{k}}(t_0)\, c}\right]^{1/2} e^{i\alpha_{\mathbf{k}}(t_0)} \qquad v_{\mathbf{k}}'(t_0) = -c\frac{\omega_{\mathbf{k}}(t_0)}{ih}\, v_{\mathbf{k}}(t_0) \tag{3.34}$$

under these functions,

$$E_{\mathbf{k}}(t_0) = 2\frac{\mathrm{Im}(v_{\mathbf{k}}v_{\mathbf{k}}^{'*})}{\hbar c}\omega_{\mathbf{k}}(t_0) \qquad F_{\mathbf{k}}(t_0) = 0 \tag{3.35}$$

meaning

$$\hat{\mathcal{H}}(t_0) = \mathrm{Im}(vv^{'*})\frac{1}{\hbar}\int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi\hbar)^3}\left(2\hat{a}_{\mathbf{k}}^\dagger\hat{a}_{\mathbf{k}} + \frac{(2\pi\hbar)^3\hbar c}{2\mathrm{Im}(v'v^*)}\delta^3(\mathbf{0})\right)\omega_{\mathbf{k}}(t_0) \tag{3.36}$$

which is equivalent to the standard Hamiltonian for a scalar field without the presence of gravity.
Bogolyubov Transformation

$$u_{\mathbf{k}}(t) = \alpha_{\mathbf{k}}v_{\mathbf{k}}(t) + \beta_{\mathbf{k}}v_{\mathbf{k}}^*(t) \tag{3.37}$$

$\alpha_{\mathbf{k}}$, $\beta_{\mathbf{k}} \in \mathbb{C}$ (time independent)

$$\mathrm{Im}(u_{\mathbf{k}}'u_{\mathbf{k}}^*) = \mathrm{Im}(v_{\mathbf{k}}'v_{\mathbf{k}}^*)\Big(|\alpha_{\mathbf{k}}|^2 - |\beta_{\mathbf{k}}|^2\Big) \tag{3.38}$$

Changing the v functions would entail a change in the creation and annihilation, therefore if we could write the field as

$$\hat{\chi} = \int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi\hbar)^3}\left[\hat{b}_{\mathbf{k}}u_{\mathbf{k}}e^{i\mathbf{k}\mathbf{x}\hbar^{-1}} + \hat{b}_{\mathbf{k}}^\dagger u_{\mathbf{k}}^* e^{-i\mathbf{k}\mathbf{x}\hbar^{-1}}\right] \tag{3.39}$$

the field must be tha same as if it was written with de v functions and $\hat{a}$ operators, that means that

$$\hat{b}_{\mathbf{k}}u_{\mathbf{k}} + \hat{b}_{-\mathbf{k}}^\dagger u_{\mathbf{k}}^* = \hat{a}_{\mathbf{k}}v_{\mathbf{k}} + \hat{a}_{-\mathbf{k}}^\dagger v_{\mathbf{k}}^* \tag{3.40}$$

and thus, the relation between the operators would be

$$\hat{a}_{\mathbf{k}} = \alpha_{\mathbf{k}}\hat{b}_{\mathbf{k}} + \beta_{\mathbf{k}}^*\hat{b}_{-\mathbf{k}}^\dagger \qquad \hat{a}_{\mathbf{k}}^\dagger = \beta_{\mathbf{k}}\hat{b}_{-\mathbf{k}} + \alpha_{\mathbf{k}}^*\hat{b}_{\mathbf{k}}^\dagger \tag{3.41}$$

now, there are 'a' particles in the 'b' vacuum

$$\langle_{(b)}0|\hat{\mathcal{N}}_{\mathbf{k}}^{(a)}|_{(b)}0\rangle = \langle_{(b)}0|\hat{a}_{\mathbf{k}}^\dagger\hat{a}_{\mathbf{k}}|_{(b)}0\rangle = \Big|\beta_{\mathbf{k}}\Big|^2\frac{(2\pi\hbar)^3\hbar c}{2\mathrm{Im}(u'u^*)}\delta^3(\mathbf{0}) \tag{3.42}$$

therefore

$$\langle_{(t_0)}0|\hat{\mathcal{H}}(t)|_{(t_0)}0\rangle = \delta^3(\mathbf{0})\int \mathrm{d}^3\mathbf{k}\left(\frac{|\beta_{\mathbf{k}}|^2}{|\alpha_{\mathbf{k}}|^2 - |\beta_{\mathbf{k}}|^2} + \frac{1}{2}\right) c\,\omega_{\mathbf{k}}(t) \tag{3.43}$$

meaning, if $\beta_{\mathbf{k}} \neq 0$ for all $\mathbf{k}$ then, at a time $t > t_0$ the energy density will be different in relation to the original vacuum.

4 de Sitter Universe

The de Sitter Universe is a flat FLRW metric with no matter or radiation, but it does have a a positive cosmological constant Λ. Per the Friedmann equations,

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G + \Lambda c^2}{3} - \frac{\kappa c^2}{a^2} \tag{4.1}$$

the expansion parameter $a(t)$ will be equal to

$$a(t) = a_1 e^{H_\Lambda t} + a_2 e^{-H_\Lambda t} \ , \qquad H_\Lambda = \sqrt{\frac{\Lambda c^2}{3}} \tag{4.2}$$

Notas sobre unidades

- $[s] = [\hbar]$
- $[a] = [\xi] = 1$
- $[\mu] = [L]^{-1}$
- $[R] = [L]^{-2}$
- $[\phi] = [\chi] = [\hbar]^{1/2}[L]^{-1}$
- $[\Pi] = [\hbar]^{1/2}[L]^{-2}$
- $[a_{\mathbf{k}}] = [\hbar]^{1/2}[L]^{2}$

Bibliography

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