PROBLEMS OF QUANTUM FIELD THEORIES IN CURVED SPACETIMES

A MASTER THESIS

Submitted in partial fulfillment of the requirements for the award of

Master's Degree in Physics of the Universe: Cosmology, Astrophysics, Particles and Astroparticles

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August 21, 2023

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1 First Chapter

FLRW metric

$$\mathrm{d}l^2 = c^2\mathrm{d}t^2 - a^2(t)\left[\frac{\mathrm{d}r^2}{1-\kappa r^2} + r^2\mathrm{d}\Omega^2\right] \tag{1.1}$$

Weyl tensor =0 therefore the metric is conformally flat, i.e. independently of the curvature κ there must exist a coordinate system where

$$\mathrm{d}l^2 = a(t)\eta_{\mu\nu}\mathrm{d}x^\mu\mathrm{d}x^\nu = a(t)\left[c^2\mathrm{d}t^2 - \mathrm{d}\mathbf{x}^2\right] \tag{1.2}$$

the standard action describing the dynamics of a (non-minimally coupled to gravity) real scalar field is

$$s = \int \frac{1}{2}\left[\nabla_\nu\phi\,\nabla^\nu\phi - \mu^2\phi^2 - \xi R\phi^2\right]\sqrt{-g}\,\mathrm{d}^4x \tag{1.3}$$

$\sqrt{-g} = a^4$ $\chi = a\phi$

$$s = \int \frac{1}{2}\left[\partial_\nu\chi\,\partial^\nu\chi - \left(\mu^2a^2 + \xi Ra^2 - c^2\frac{a''}{a}\right)\chi^2 - \partial_t\left(c^2\chi^2\frac{a'}{a}\right)\right]\mathrm{d}^4x \tag{1.4}$$

dropping the time drivative

$$s = \int \frac{1}{2}\left[\partial_\nu\chi\,\partial^\nu\chi - \left(\mu^2a^2 + \xi Ra^2 - c^2\frac{a''}{a}\right)\chi^2\right]\mathrm{d}^4x \tag{1.5}$$

by Euler-Lagrange

$$\left[\partial_\nu\partial^\nu + \mu^2_{\text{eff}}(t)\right]\chi = 0 \tag{1.6}$$

where

$$\mu^2_{\text{eff}}(t) = \left(\mu^2 + \xi R\right)a^2 - c^2\frac{a''}{a} \tag{1.7}$$

solutions of previous equation have the form

$$\chi = a\,v(t)e^{\pm i\mathbf{k}\mathbf{x}\hbar^{-1}} \tag{1.8}$$

meaning that, the dispersion relation is

$$v''\hbar^2 + \omega^2(t)\,v = 0 \tag{1.9}$$

where $\omega(t)$ is defined as

$$\omega^2(t) = \mathbf{k}^2 + \hbar^2\mu^2_{\text{eff}}(t) = \mathbf{k}^2 + \left(m^2c^2 + \xi\hbar^2R\right)a(t) - \hbar^2c^2\frac{a''}{a} \tag{1.10}$$

now, proof that $\mathrm{Im}(vv'^*)$ is constant through time

$$\frac{\partial}{\partial t}\mathrm{Im}(vv'^*) = \frac{\partial}{\partial t}\left(\frac{vv'^* - v^*v'}{2i}\right) = \frac{vv''^* - v^*v''}{2i} = 0 \tag{1.11}$$

last step is result from dispersion relation. Since dispersion relation is scalable by a time independent function, $\mathrm{Im}(v'v^*)$ can be determined to be a chosen value, a particular useful choice is to consider it momentum independent. $\mathrm{Im}(v'v^*) = W[v, v^*]$ therefore, if its not equal to 0, they are linearly independent solutions to dispersion relation.

The most general solution to the main equation is

$$\chi = \int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi\hbar)^3}\left[a_{\mathbf{k}} v_{\mathbf{k}}(t) e^{i\mathbf{kx}\hbar^{-1}} + a^*_{\mathbf{k}} v^*_{\mathbf{k}}(t) e^{-i\mathbf{kx}\hbar^{-1}}\right] \tag{1.12}$$

The field χ and its conjugate momentum $\Pi = \partial_{ct}\chi$ are promoted to operators on the quantum Hilbert space, with the standar canonical conmutation relations

$$\left[\hat{\chi}(t,\mathbf{x}), \hat{\Pi}(t,\mathbf{y})\right] = i\hbar\,\delta^3(\mathbf{x}-\mathbf{y}) \tag{1.13}$$

$$\left[\hat{\chi}(t,\mathbf{x}), \hat{\chi}(t,\mathbf{y})\right] = \left[\hat{\Pi}(t,\mathbf{x}), \hat{\Pi}(t,\mathbf{y})\right] = 0 \tag{1.14}$$

where the operational nature of the fields arrise from the promotion of the mode amplitudes, i.e.

$$a_{\mathbf{k}} \longrightarrow \hat{a}_{\mathbf{k}} \qquad a^*_{\mathbf{k}} \longrightarrow \hat{a}^\dagger_{\mathbf{k}} \tag{1.15}$$

this operators fulfill the following conmutation relations

$$[\hat{a}_{\mathbf{k}}, \hat{a}^\dagger_{\mathbf{q}}] = \frac{(2\pi\hbar)^3\hbar c}{2\mathrm{Im}(v'v^*)}\delta^3(\mathbf{k}-\mathbf{q})\,, \qquad [\hat{a}_{\mathbf{k}}, \hat{a}_{\mathbf{q}}] = [\hat{a}^\dagger_{\mathbf{k}}, \hat{a}^\dagger_{\mathbf{q}}] = 0 \tag{1.16}$$

(note that $\hat{a}_{\mathbf{k}} \neq \hat{a}_{-\mathbf{k}}$)

To prove this, consider that

$$\begin{aligned}\left[\hat{\chi}(\mathbf{x}),\, \hat{\Pi}(\mathbf{y})\right] = \frac{1}{c}\int \frac{\mathrm{d}^3\mathbf{k}\mathrm{d}^3\mathbf{q}}{(2\pi\hbar)^6}\Big\{&[\hat{a}_{\mathbf{k}}, \hat{a}_{\mathbf{q}}]\, v_{\mathbf{k}} v'_{\mathbf{q}} e^{i(\mathbf{kx}+\mathbf{qy})\hbar^{-1}} + \left[\hat{a}^\dagger_{\mathbf{k}}, \hat{a}^\dagger_{\mathbf{q}}\right] v^*_{\mathbf{k}} v^{*\prime}_{\mathbf{q}} e^{i(\mathbf{kx}-\mathbf{qy})\hbar^{-1}} + \\ &+ \left[\hat{a}_{\mathbf{k}}, \hat{a}^\dagger_{\mathbf{q}}\right] v_{\mathbf{k}} v^{*\prime}_{\mathbf{q}} e^{i(\mathbf{kx}-\mathbf{qy})\hbar^{-1}} - \left[\hat{a}_{\mathbf{q}}, \hat{a}^\dagger_{\mathbf{k}}\right] v^*_{\mathbf{k}} v'_{\mathbf{q}} e^{-i(\mathbf{kx}-\mathbf{qy})\hbar^{-1}}\Big\}\end{aligned} \tag{1.17}$$

if the operators $\hat{a}$ and $\hat{a}^\dagger$ are to be understood as creation and annihilation operators, they must fulfill

$$[\hat{a}_{\mathbf{k}}, \hat{a}^\dagger_{\mathbf{q}}] = \alpha\delta^3(\mathbf{k}-\mathbf{q})\,, \qquad [\hat{a}_{\mathbf{k}}, \hat{a}_{\mathbf{q}}] = [\hat{a}^\dagger_{\mathbf{k}}, \hat{a}^\dagger_{\mathbf{q}}] = 0 \tag{1.18}$$

where $\alpha \in \mathbb{C}$, and thus

$$\left[\hat{\chi}(\mathbf{x}),\, \hat{\Pi}(\mathbf{y})\right] = \frac{\alpha}{c}\int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi\hbar)^6} 2i\mathrm{Im}(v_{\mathbf{k}} v^{*\prime}_{\mathbf{k}}) e^{i(\mathbf{kx}-\mathbf{qy})\hbar^{-1}} \tag{1.19}$$

considering $\mathrm{Im}(v'v^*)$ momentum independent, and remembering the canonical conmutation relations, one finds that

$$\alpha\mathrm{Im}(vv^{*\prime}) = \frac{1}{2}\hbar c(2\pi\hbar)^3 \tag{1.20}$$

The hamiltonian

$$\hat{\mathcal{H}}(t) = \int \frac{c}{2}\left[\hat{\Pi}^2 + \left(\nabla\hat{\chi}\right)^2 + \mu^2_{\text{eff}}(t)\hat{\chi}^2\right]\mathrm{d}^3\mathbf{x} \tag{1.21}$$

$$\begin{aligned}\hat{\Pi}^2 = \frac{1}{c^2}\int \frac{\mathrm{d}^3\mathbf{k}\mathrm{d}^3\mathbf{q}}{(2\pi\hbar)^6}\Big[&\hat{a}_{\mathbf{k}}\hat{a}_{\mathbf{q}} v'_{\mathbf{k}} v'_{\mathbf{q}} e^{i(\mathbf{k}+\mathbf{q})\mathbf{x}\hbar^{-1}} + \hat{a}_{\mathbf{k}}\hat{a}^\dagger_{\mathbf{q}} v'_{\mathbf{k}} v'^*_{\mathbf{q}} e^{i(\mathbf{k}-\mathbf{q})\mathbf{x}\hbar^{-1}} + \\ &+ \hat{a}^\dagger_{\mathbf{k}}\hat{a}_{\mathbf{q}} v'^*_{\mathbf{k}} v'_{\mathbf{q}} e^{-i(\mathbf{k}-\mathbf{q})\mathbf{x}\hbar^{-1}} + \hat{a}^\dagger_{\mathbf{k}}\hat{a}^\dagger_{\mathbf{q}} v'^*_{\mathbf{k}} v'^*_{\mathbf{q}} e^{-i(\mathbf{k}+\mathbf{q})\mathbf{x}\hbar^{-1}}\Big]\end{aligned} \tag{1.22}$$

$$\begin{aligned}\left(\nabla\hat{\chi}\right)^2 = -\frac{1}{\hbar^2}\int \frac{\mathrm{d}^3\mathbf{k}\mathrm{d}^3\mathbf{q}}{(2\pi\hbar)^6}\mathbf{kq}\Big[&\hat{a}_{\mathbf{k}}\hat{a}_{\mathbf{q}} v_{\mathbf{k}} v_{\mathbf{q}} e^{i(\mathbf{k}+\mathbf{q})\mathbf{x}\hbar^{-1}} - \hat{a}_{\mathbf{k}}\hat{a}^\dagger_{\mathbf{q}} v_{\mathbf{k}} v^*_{\mathbf{q}} e^{i(\mathbf{k}-\mathbf{q})\mathbf{x}\hbar^{-1}} - \\ &- \hat{a}^\dagger_{\mathbf{k}}\hat{a}_{\mathbf{q}} v^*_{\mathbf{k}} v_{\mathbf{q}} e^{-i(\mathbf{k}-\mathbf{q})\mathbf{x}\hbar^{-1}} + \hat{a}^\dagger_{\mathbf{k}}\hat{a}^\dagger_{\mathbf{q}} v^*_{\mathbf{k}} v^*_{\mathbf{q}} e^{-i(\mathbf{k}+\mathbf{q})\mathbf{x}\hbar^{-1}}\Big]\end{aligned} \tag{1.23}$$

$$\hat{\chi}^2 = \int \frac{\mathrm{d}^3\mathbf{k}\mathrm{d}^3\mathbf{q}}{(2\pi\hbar)^6} \Big[\hat{a}_{\mathbf{k}}\hat{a}_{\mathbf{q}}v_{\mathbf{k}}v_{\mathbf{q}}e^{i(\mathbf{k}+\mathbf{q})\mathbf{x}\hbar^{-1}} + \hat{a}_{\mathbf{k}}\hat{a}^\dagger_{\mathbf{q}}v_{\mathbf{k}}v^*_{\mathbf{q}}e^{i(\mathbf{k}-\mathbf{q})\mathbf{x}\hbar^{-1}} + \\ + \hat{a}^\dagger_{\mathbf{k}}\hat{a}_{\mathbf{q}}v^*_{\mathbf{k}}v_{\mathbf{q}}e^{-i(\mathbf{k}-\mathbf{q})\mathbf{x}\hbar^{-1}} + \hat{a}^\dagger_{\mathbf{k}}\hat{a}^\dagger_{\mathbf{q}}v^*_{\mathbf{k}}v^*_{\mathbf{q}}e^{-i(\mathbf{k}+\mathbf{q})\mathbf{x}\hbar^{-1}}\Big] \tag{1.24}$$

$$\begin{aligned}\hat{\mathcal{H}} = \frac{c}{2}\int \frac{\mathrm{d}^3\mathbf{k}\mathrm{d}^3\mathbf{q}}{(2\pi\hbar)^3} \Bigg\{ & \hat{a}_{\mathbf{k}}\hat{a}_{\mathbf{q}}\left[\frac{1}{c^2}v'_{\mathbf{k}}v'_{\mathbf{q}} - \left(\frac{1}{\hbar^2}\mathbf{kq} - \mu^2_{\text{eff}}\right)v_{\mathbf{k}}v_{\mathbf{q}}\right]\delta^3(\mathbf{k}+\mathbf{q}) + \\ & + \hat{a}_{\mathbf{k}}\hat{a}^\dagger_{\mathbf{q}}\left[\frac{1}{c^2}v'_{\mathbf{k}}v'^*_{\mathbf{q}} + \left(\frac{1}{\hbar^2}\mathbf{kq} + \mu^2_{\text{eff}}\right)v_{\mathbf{k}}v^*_{\mathbf{q}}\right]\delta^3(\mathbf{k}-\mathbf{q}) + \\ & + \hat{a}^\dagger_{\mathbf{k}}\hat{a}_{\mathbf{q}}\left[\frac{1}{c^2}v'^*_{\mathbf{k}}v'_{\mathbf{q}} + \left(\frac{1}{\hbar^2}\mathbf{kq} + \mu^2_{\text{eff}}\right)v^*_{\mathbf{k}}v_{\mathbf{q}}\right]\delta^3(\mathbf{k}-\mathbf{q}) + \\ & + \hat{a}^\dagger_{\mathbf{k}}\hat{a}^\dagger_{\mathbf{q}}\left[\frac{1}{c^2}v'^*_{\mathbf{k}}v'^*_{\mathbf{q}} - \left(\frac{1}{\hbar^2}\mathbf{kq} - \mu^2_{\text{eff}}\right)v^*_{\mathbf{k}}v^*_{\mathbf{q}}\right]\delta^3(\mathbf{k}+\mathbf{q})\Bigg\}\end{aligned} \tag{1.25}$$

$$\begin{aligned}\hat{\mathcal{H}} = \frac{c}{2}\int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi\hbar)^3} \Bigg\{ & \hat{a}_{\mathbf{k}}\hat{a}_{-\mathbf{k}}\left[\frac{1}{c^2}v'_{\mathbf{k}}v'_{\mathbf{k}} + \frac{1}{\hbar^2}\omega^2_{\mathbf{k}}(t)v_{\mathbf{k}}v_{\mathbf{k}}\right] + \\ & + \hat{a}_{\mathbf{k}}\hat{a}^\dagger_{\mathbf{k}}\left[\frac{1}{c^2}v'_{\mathbf{k}}v'^*_{\mathbf{k}} + \frac{1}{\hbar^2}\omega^2_{\mathbf{k}}(t)v_{\mathbf{k}}v^*_{\mathbf{k}}\right] + \\ & + \hat{a}^\dagger_{\mathbf{k}}\hat{a}_{\mathbf{k}}\left[\frac{1}{c^2}v'^*_{\mathbf{k}}v'_{\mathbf{k}} + \frac{1}{\hbar^2}\omega^2_{\mathbf{k}}(t)v^*_{\mathbf{k}}v_{\mathbf{k}}\right] + \\ & + \hat{a}^\dagger_{\mathbf{k}}\hat{a}^\dagger_{-\mathbf{k}}\left[\frac{1}{c^2}v'^*_{\mathbf{k}}v'^*_{\mathbf{k}} + \frac{1}{\hbar^2}\omega^2_{\mathbf{k}}(t)v^*_{\mathbf{k}}v^*_{\mathbf{k}}\right]\Bigg\}\end{aligned} \tag{1.26}$$

$$\hat{\mathcal{H}} = \frac{c}{2}\int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi\hbar)^3}\left[\hat{a}_{\mathbf{k}}\hat{a}_{-\mathbf{k}}F_{\mathbf{k}} + \hat{a}^\dagger_{\mathbf{k}}\hat{a}^\dagger_{-\mathbf{k}}F^*_{\mathbf{k}} + \left(2\hat{a}^\dagger_{\mathbf{k}}\hat{a}_{\mathbf{k}} + \frac{(2\pi\hbar)^3\hbar c}{2\mathrm{Im}(v'v^*)}\delta^3(\mathbf{0})\right)E_{\mathbf{k}}\right] \tag{1.27}$$

where

$$F_{\mathbf{k}}(t) = \left(\frac{1}{\hbar c}\right)^2\left[\hbar^2 v'^2_{\mathbf{k}} + \omega^2_{\mathbf{k}}(t)\,c^2 v^2_{\mathbf{k}}\right] \tag{1.28}$$

$$E_{\mathbf{k}}(t) = \left(\frac{1}{\hbar c}\right)^2\left[\hbar^2\left|v'_{\mathbf{k}}\right|^2 + \omega^2_{\mathbf{k}}(t)\,c^2\left|v_{\mathbf{k}}\right|^2\right] \tag{1.29}$$

Now, the expectation value of the hamiltonian at time t_0 in the state $|_{(v)}0\rangle$

$$\langle_{(v)}0|\hat{\mathcal{H}}(t_0)|_{(v)}0\rangle = \rho(t_0)\delta^3(\mathbf{0}) = \frac{\hbar c^2\,\delta^3(\mathbf{0})}{4\mathrm{Im}(v'v^*)}\int \mathrm{d}^3\mathbf{k}\,E_{\mathbf{k}} \tag{1.30}$$

To minimise the energy density of de vacuum state is to fin the set of functions $v_{\mathbf{k}}$ that minimise $E_{\mathbf{k}}$. Suppose that $v_{\mathbf{k}}$ can be written as

$$v_{\mathbf{k}} = r_{\mathbf{k}}e^{i\alpha_{\mathbf{k}}} \tag{1.31}$$

since $\mathrm{Im}(vv'^*)$ was constant through time

$$\mathrm{Im}(v_{\mathbf{k}}v'^*_{\mathbf{k}}) = -r^2_{\mathbf{k}}\alpha'_{\mathbf{k}} \tag{1.32}$$

this means

$$E_{\mathbf{k}} = \left(\frac{1}{\hbar c}\right)^2\left\{\hbar^2\left[r'^2_{\mathbf{k}} + \mathrm{Im}^2\left(v_{\mathbf{k}}v'^*_{\mathbf{k}}\right)\frac{1}{r^2_{\mathbf{k}}}\right] + \omega^2_{\mathbf{k}}\,c^2 r^2_{\mathbf{k}}\right\} \tag{1.33}$$

the minimum of this function must fulfil $r'_{\mathbf{k}}(t_0) = 0$. Now, if $\omega^2_{\mathbf{k}}(t_0)$ and $\mathrm{Im}(v_{\mathbf{k}}v'^*_{\mathbf{k}})$ have the same sign, the minimum of $E_{\mathbf{k}}$ happens when $r_{\mathbf{k}}(t_0) = \left[\frac{\hbar\mathrm{Im}(v_{\mathbf{k}}v'^*_{\mathbf{k}})}{\omega_{\mathbf{k}}(t_0)\,c}\right]^{1/2}$.

If there is a minimum, then

$$v_{\mathbf{k}}(t_0) = \left[\frac{\hbar \mathrm{Im}(v_{\mathbf{k}} v_{\mathbf{k}}^{\prime *})}{\omega_{\mathbf{k}}(t_0)\, c}\right]^{1/2} e^{i\alpha_{\mathbf{k}}(t_0)} \qquad v'_{\mathbf{k}}(t_0) = -c\frac{\omega_{\mathbf{k}}(t_0)}{ih}\, v_{\mathbf{k}}(t_0) \tag{1.34}$$

under these functions,

$$E_{\mathbf{k}}(t_0) = 2\frac{\mathrm{Im}(v_{\mathbf{k}} v_{\mathbf{k}}^{\prime *})}{\hbar c}\omega_{\mathbf{k}}(t_0) \qquad F_{\mathbf{k}}(t_0) = 0 \tag{1.35}$$

meaning

$$\hat{\mathcal{H}}(t_0) = \mathrm{Im}(vv^{\prime *})\frac{1}{\hbar}\int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi\hbar)^3}\left(2\hat{a}_{\mathbf{k}}^\dagger \hat{a}_{\mathbf{k}} + \frac{(2\pi\hbar)^3\hbar c}{2\mathrm{Im}(v'v^*)}\delta^3(\mathbf{0})\right)\omega_{\mathbf{k}}(t_0) \tag{1.36}$$

which is equivalent to the standard Hamiltonian for a scalar field without the presence of gravity.
Bogolyubov Transformation

$$u_{\mathbf{k}}(t) = \alpha_{\mathbf{k}} v_{\mathbf{k}}(t) + \beta_{\mathbf{k}} v_{\mathbf{k}}^*(t) \tag{1.37}$$

$\alpha_{\mathbf{k}}$, $\beta_{\mathbf{k}} \in \mathbb{C}$ (time independent)

$$\mathrm{Im}(u'_{\mathbf{k}} u_{\mathbf{k}}^*) = \mathrm{Im}(v'_{\mathbf{k}} v_{\mathbf{k}}^*)\left(|\alpha_{\mathbf{k}}|^2 - |\beta_{\mathbf{k}}|^2\right) \tag{1.38}$$

Changing the v functions would entail a change in the creation and annihilation, therefore if we could write the field as

$$\hat{\chi} = \int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi\hbar)^3}\left[\hat{b}_{\mathbf{k}} u_{\mathbf{k}} e^{i\mathbf{k}\mathbf{x}\hbar^{-1}} + \hat{b}_{\mathbf{k}}^\dagger u_{\mathbf{k}}^* e^{-i\mathbf{k}\mathbf{x}\hbar^{-1}}\right] \tag{1.39}$$

the field must be tha same as if it was written with de v functions and $\hat{a}$ operators, that means that

$$\hat{b}_{\mathbf{k}} u_{\mathbf{k}} + \hat{b}_{-\mathbf{k}}^\dagger u_{\mathbf{k}}^* = \hat{a}_{\mathbf{k}} v_{\mathbf{k}} + \hat{a}_{-\mathbf{k}}^\dagger v_{\mathbf{k}}^* \tag{1.40}$$

and thus, the relation between the operators would be

$$\hat{a}_{\mathbf{k}} = \alpha_{\mathbf{k}}\hat{b}_{\mathbf{k}} + \beta_{\mathbf{k}}^*\hat{b}_{-\mathbf{k}}^\dagger \qquad \hat{a}_{\mathbf{k}}^\dagger = \beta_{\mathbf{k}}\hat{b}_{-\mathbf{k}} + \alpha_{\mathbf{k}}^*\hat{b}_{\mathbf{k}}^\dagger \tag{1.41}$$

now, there are 'a' particles in the 'b' vacuum

$$\langle_{(b)}0|\hat{\mathcal{N}}_{\mathbf{k}}^{(a)}|_{(b)}0\rangle = \langle_{(b)}0|\hat{a}_{\mathbf{k}}^\dagger \hat{a}_{\mathbf{k}}|_{(b)}0\rangle = \left|\beta_{\mathbf{k}}\right|^2 \frac{(2\pi\hbar)^3\hbar c}{2\mathrm{Im}(u'u^*)}\delta^3(\mathbf{0}) \tag{1.42}$$

therefore

$$\langle_{(t_0)}0|\hat{\mathcal{H}}(t)|_{(t_0)}0\rangle = \delta^3(\mathbf{0})\int \mathrm{d}^3\mathbf{k}\left(\frac{|\beta_{\mathbf{k}}|^2}{|\alpha_{\mathbf{k}}|^2 - |\beta_{\mathbf{k}}|^2} + \frac{1}{2}\right) c\,\omega_{\mathbf{k}}(t) \tag{1.43}$$

meaning, if $\beta_{\mathbf{k}} \neq 0$ for all $\mathbf{k}$ then, at a time $t > t_0$ the energy density will be different in relation to the original vacuum.

Notas sobre unidades

- $[s] = [\hbar]$
- $[a] = [\xi] = 1$
- $[\mu] = [L]^{-1}$
- $[R] = [L]^{-2}$
- $[\phi] = [\chi] = [\hbar]^{1/2}[L]^{-1}$
- $[\Pi] = [\hbar]^{1/2}[L]^{-2}$
- $[a_{\mathbf{k}}] = [\hbar]^{1/2}[L]^{2}$

Bibliography

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