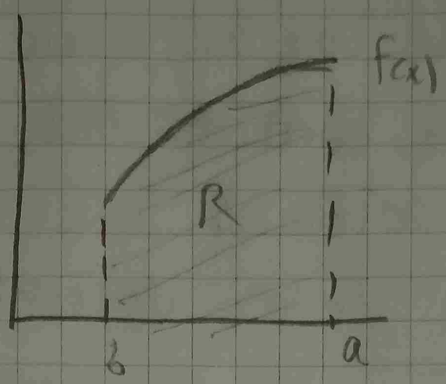
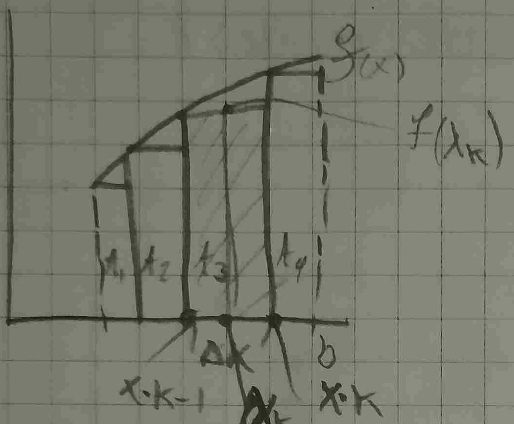
Integral Definida

$f(x)$

$[a,b]$ y positivo

continua su area esta por devajo





$y = f(x) = x+2$

$y = f(1) = 3$

$y = f(2) = 4$

$$A = (x_k - x_{k-1})\, f(\alpha_k)$$

$$A_T = A_1 + A_2 + A_3 + A_4$$

$$A = \sum_{k=1}^{n} (x_k - x_{k-1})\, f(\alpha_k)$$

$$A = \lim_{\Delta x \to 0} \sum_{k=1}^{n} (x_k - x_{k-1})\, f(\alpha_k) = \int_a^b f(x)\,dx$$

Propiedades

1 = $\int_a^b [f(x) \pm g(x)]\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$

2 = $\int_a^b c\, f(x)\,dx = c \int_a^b f(x)\,dx$; c - constante (R)

3 = $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

4 = $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$

5 = $\int_a^a f(x)\,dx = 0$

Tema: Fecha: DÍA MES AÑO / /

Reglas para resolver integrales definidas

$\int f(x)dx = (F(x)+c)' = f(x)$

$\int x^2 dx = \left(\frac{1}{3}x^3+c\right)' = \frac{1}{3}(3x^2) = x^2$

$\int_a^b f(x)dx = F(x)\Big|_a^b = F(b) - F(a)$

Metodo de Sustitucion

C.V.

$u = \operatorname{sen} x$

$du = \cos x\, dx$

$\int_0^{\pi/4} \operatorname{sen} x \cos x\, dx$

$\int_a^b f(x)dx = F(x)\Big|_a^b = F(b)-F(a)$

$\int^{\frac{\sqrt{2}}{2}} u\,du$

$x=0 \to u = \operatorname{sen} 0 = 0$

$x = \frac{\pi}{4} \to u = \operatorname{sen}\frac{\pi}{4} = \frac{\sqrt{2}}{2}$

$= \frac{1}{2}u^2\Big|_0^{\frac{\sqrt{2}}{2}} = \frac{1}{2}\left(\left(\frac{\sqrt{2}}{2}\right)^2\right) = \frac{1}{2}\left(\frac{2}{4}\right) = \frac{1}{4}$

$u = \cos x$

$du = -\operatorname{sen} x\, dx$

$-du = \operatorname{sen} x\, dx$

$x = 0 \to u = \cos 0 = 1$

$x = \frac{\pi}{4} \to u = \cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$

$-\int_1^{\sqrt{2}/2} u\,du$

$-\int_1^{\frac{\sqrt{2}}{2}} u\,du = -\frac{1}{2}u^2\Big|_1^{\frac{\sqrt{2}}{2}} = -\frac{1}{2}\left(\left(\frac{\sqrt{2}}{2}\right)^2 - 1\right) = -\frac{1}{2}\left(\frac{2}{4}-1\right) = -\frac{1}{2}\left(\frac{1}{2}-1\right) = -\frac{1}{2}\left(-\frac{1}{2}\right) = \frac{1}{4}$

$\int_0^2 (2x+4)(x^2+4x-1)^2 dx$

$u = (x^2+4x-1)$

$du = 2x+4$

$\int_{-1}^{11} u^2 du \Rightarrow \frac{1}{3}u^3 = \frac{1}{3}(11^3 - (-1)^3)$

$x = 0 \to u = -1$

$x = 2 \to u = 11$

límites
inferior
superior

Tema: Fecha:

Cuando hay raiz

$\int_1^2 x\sqrt{x-1}\,dx \longrightarrow$ $t=\sqrt{x-1}$

$t^2 = x-1$

$x = t^2+1$

$dx = 2t\,dt$

$\int (t^2+1)t\cdot 2t\,dt$

$\int (2t^4+2t^2)\,dt$

$x=1 \to t=\sqrt{1-1}=0$ lim inferior

$x=2 \to t=\sqrt{2-1}=1$ lim superior

$2\int_0^1 (t^4+t^2)\,dt$

$2\left(\frac{1}{5}t^5+\frac{1}{3}t^3\right)\Big|_0^1 = 2\left(\frac{1}{5}+\frac{1}{3}\right) = 2\left(\frac{3+5}{15}\right) = \frac{16}{15}$

metodo de integración por partes

$\int u\,dv = uv - \int v\,du$

$\int_1^2 x e^x\,dx$ $u=x$ $du=dx$

$dv=e^x$ $v=e^x$

$xe^x - \int e^x\,dx$

$xe^x - \int_1^2 e^x\,dx \Rightarrow xe^x - e^x\Big|_1^2 = 2e^2 - e^2 - (e^1 - e^1) = e^2$

$\int_1^3 x^2 \ln x\,dx$ $u=\ln x$ $du=\frac{1}{x}dx$

$dv = x^2\,dx$ $v=\frac{1}{3}x^3$

$\ln e = 1$

$uv - \int v\,du$

$\frac{1}{3}x^3\ln x - \int_1^3 \frac{1}{3}x^3\cdot\frac{1}{x}\,dx \Rightarrow \frac{1}{3}x^3\ln x - \frac{1}{3}\int_1^3 x^2\,dx = \frac{1}{3}x^3\ln x - \frac{1}{9}x^3\Big|_1^3$

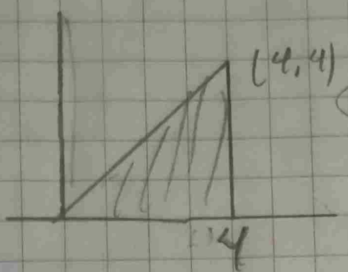
$9\ln 3 - 3 - \left(0 - \frac{1}{9}\right)$

Tema: Fecha: DÍA MES AÑO / /

Calculo de areas

ec. 1er grado = recta
2do grado = parabola

$y = x$; $x = 0$; $x = 4$



$y = x$ $x = 4$
$y = 4$

$$A = \int_0^4 x\,dx = \frac{1}{2}x^2 \Big|_0^4 = 8u^2$$

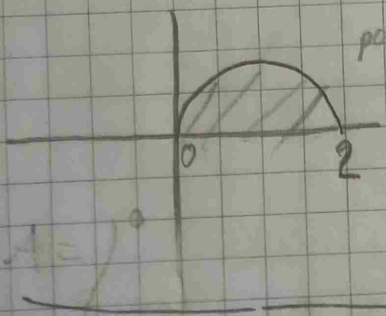
$$A = \frac{4 * 4}{2} = 8u^2$$ ← area del triangulo

$y = 2x - x^2$ eje X

parabol − ∩
\+ ∪

$y = x(2 - x) = 0$

para 0 y 2 → y = 0



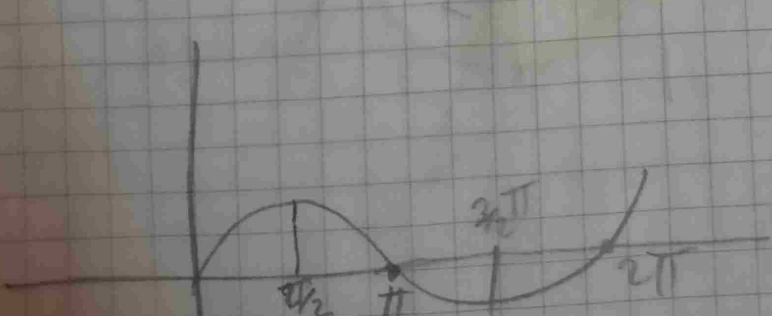
$$A = \int_0^2 (2x - x^2)\,dx = \frac{2}{2}x^2 - \frac{1}{3}x^3 \Big|_0^2$$

$$\Rightarrow 4 - \frac{8}{3} - 0 = 4 - \frac{8}{3} = \frac{4}{3}u^2$$

$$\int_0^{2\pi} \operatorname{sen} x\,dx = -\cos x \Big|_0^{2\pi}$$

$2\pi = 360°$
$\pi = 180°$
$\frac{\pi}{2} = 90°$

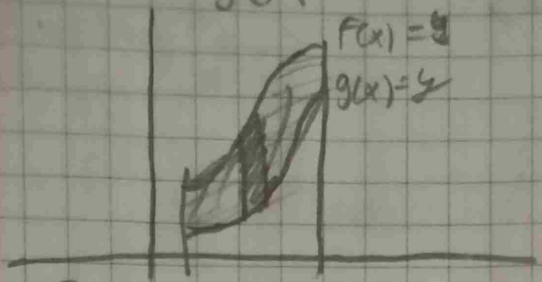
$\Rightarrow -(\cos 2\pi - \cos 0)$
$= -(1 - 1) = 0$



$$\int_0^{2\pi} \operatorname{sen} x\,dx$$

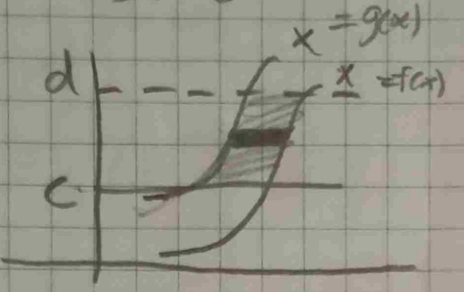
$$= \int_0^{\pi} \operatorname{sen} x\,dx - \int_{\pi}^{2\pi} \operatorname{sen} x\,dx$$

Caso 1



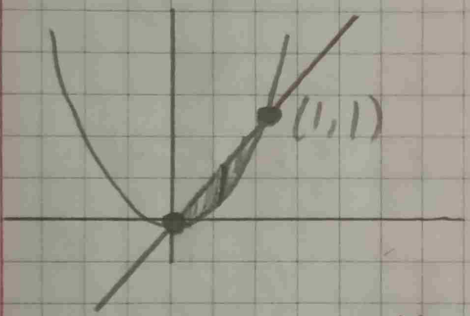
$$A = \int_a^b (f(x) - g(x))\,dx$$

Caso 2



$$A = \int_c^d (f(x) - g(x))\,dx$$

$y = x^2 ; y = x$



$x^2 = x$

$x^2 - x = 0$

$x(x-1) = 0$

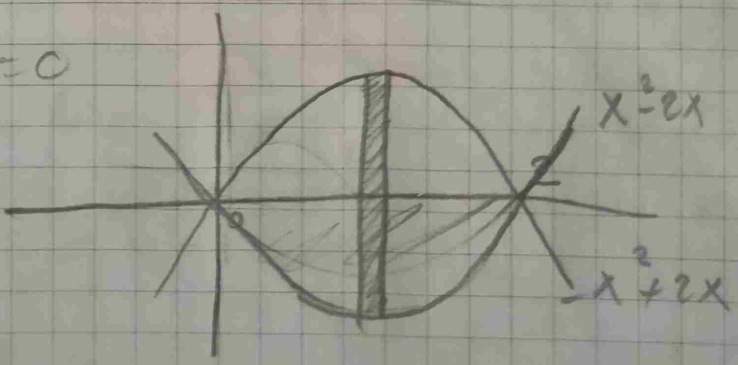
$x_1 = 0 \quad x_2 = 1$

$x = 0 ; y = 0 \qquad x = 1 ; y = 1$

$$A = \int_0^1 (x - x^2)\,dx \Rightarrow \frac{1}{2}x^2 - \frac{1}{3}x^3 = \frac{1}{2} - \frac{1}{3} = \frac{3-2}{6} = \frac{1}{6}\,u^2$$

$y = x^2 - 2x = x(x-2) = 0$

$x = -x^2 + 2x = x(2-x)$



$$A = 2\int_0^2 (-x^2 + 2x)\,dx$$

ó

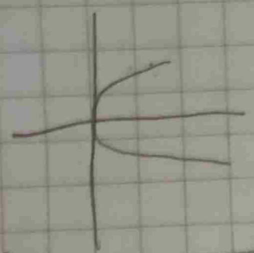
$$A = \int_0^2 (-x^2 + 2x) - (x^2 - 2x)$$

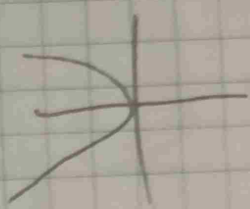
$$A = \int_0^2 (-x^2+2x) - (x^2-2x)\,dx$$

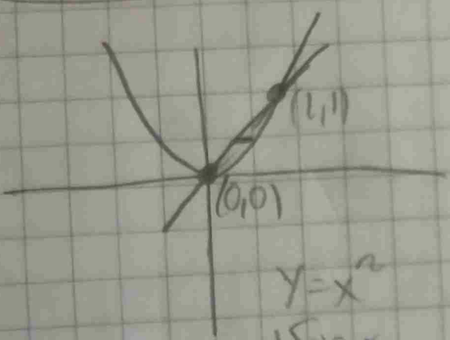
$$= \int_0^2 (-x^2+2x-x^2+2x)\,dx = \int_0^2 (-2x^2+4x)\,dx = 2\int_0^2 (-x^2+2x)\,dx$$

$$= 2\left\{ -\frac{1}{3}x^3 + x^2 \right|_0^2 \Rightarrow 2\left(-\frac{1}{3}8+4\right) = \frac{8}{3}\,u^2$$

$x = y^2$ $\qquad$ $x = -y^2$







$y = x^2$

$\sqrt{y} = x$

$y = x$

$$A = \int_0^1 (\sqrt{y} - y)\,dy$$

$$A = \int_0^1 (y^{1/2} - y)\,dy$$

$$A = \frac{2}{3}y^{3/2} - \frac{1}{2}y^2 \Big|_0^1$$

$$= \frac{2}{3} - \frac{1}{3} = \frac{4-3}{6} = \frac{1}{6}\,u^2$$

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calcular el area

$y = 2 - x^2$

$y = x$

$x = 0$

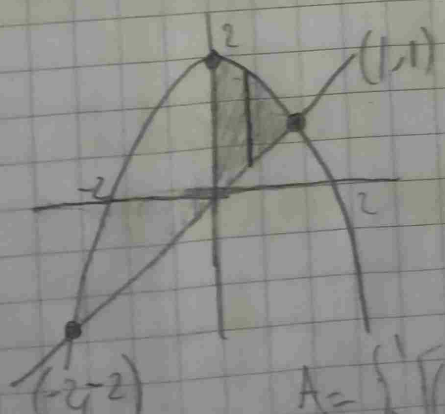
$x \geq 0$

$y = 2 - x^2 = x$

$x^2 + x - 2 = 0$

$(x+2)(x-1) = 0$

$x = -2 \,;\, x = 1$



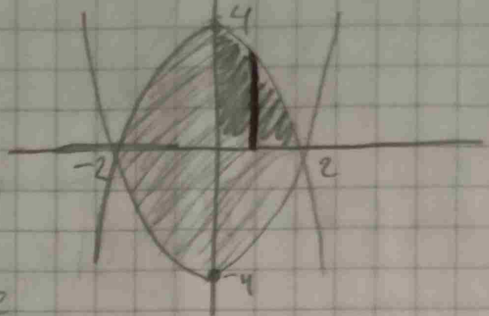
$$A = \int_0^1 \left[(2-x^2) - x\right]dx$$

Tema:

Fecha:

$y = 4 - x^2 = 0 \rightarrow x^2 = 4 \Rightarrow x \pm 2$

$y = x^2 - 4$



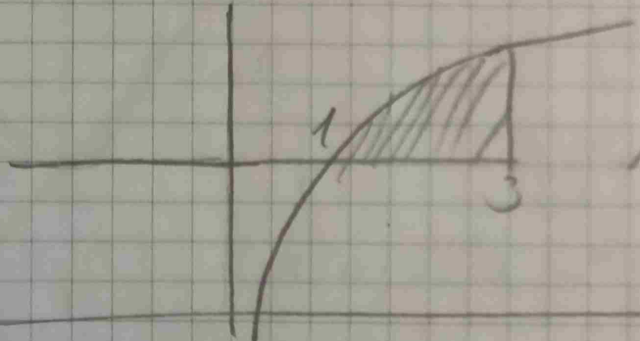
$\frac{A}{4} = \int_0^2 (4 - x^2)\,dx$

$\frac{A}{4} = 4x - \frac{1}{3}x^3 \Big|_0^2$

$\frac{A}{4} = 8 - \frac{8}{3} = \frac{24-8}{3} = \frac{16}{3}$

$\frac{A}{4} = \frac{16}{3} = \frac{64}{3}\,U^2$

$y = \ln x \;;\; x = 1 \;;\; x = 3$



$A = \int_1^3 \ln x\,dx = x\ln x - x$

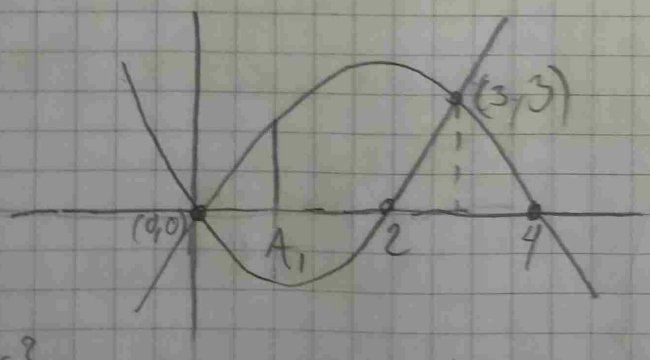
$y = x^2 - 2x \;;\; x = -x^2 + 4x$

$= x(x-2) = 0 \;;\; = x(4-x) = 0$

$x^2 - 2x = -x^2 + 4x$

$2x^2 - 6x = 0 \Rightarrow 2x(x-3) = 0$

$x = 0 \;;\; x = 3$

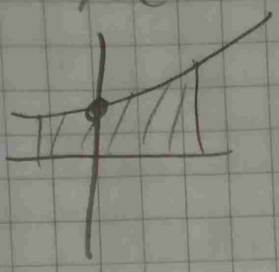


$A = 9\,U^2$

$A_1 = \int_0^2 (x^2 - 2x)\,dx = \frac{1}{3}x^3 - x^2 \Big|_0^2 = \frac{8}{3} - 4 = \frac{8-12}{3} = \frac{-4}{3}\,U$

~~Volumen de solido de revolucion~~

$y=e^x$

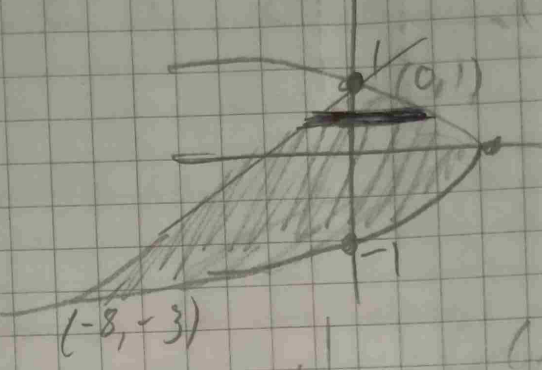


$x=-1$

$x=2$

$x=1-y^2$

~~$y^2=1 \to y=1$~~



$2y=x+2$

~~$y=\frac{1}{2}x+1$~~

$x=2y-2$

$1-y^2=2y-2$

$y^2+2y-3=0$

$(y+3)(y-1)=0$

$(-3,-3)$

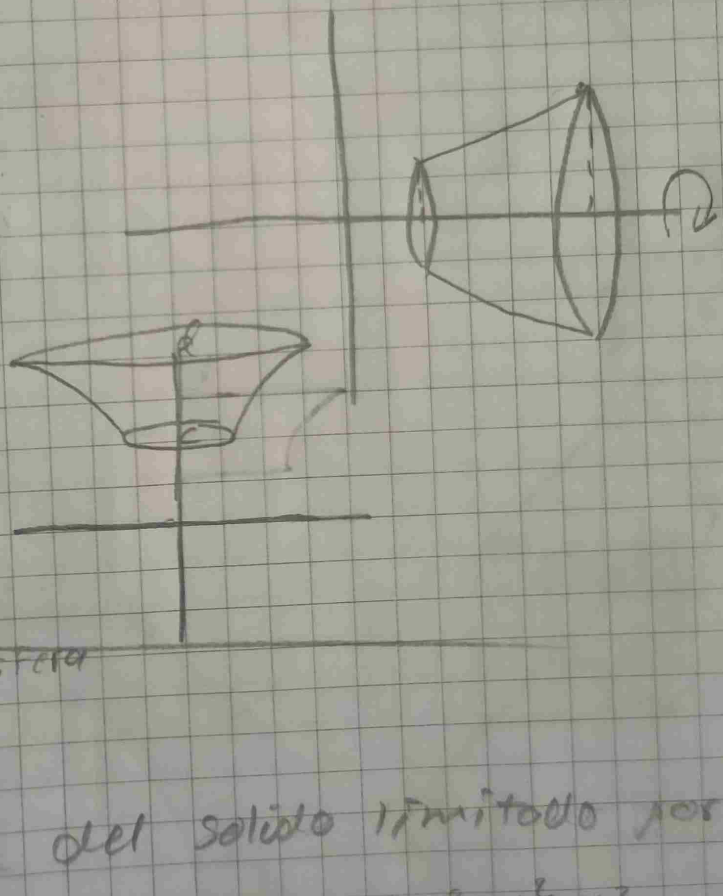
$A=\int_{-3}^{1}(1-y^2)-(2y-2)\,dy$

Volumen de solido de revolucion

$f(x)\ [a,b]$

$V=\pi\int_a^b f^2(x)\,dx$

$V=\pi\int_c^d f^2(y)\,dy$

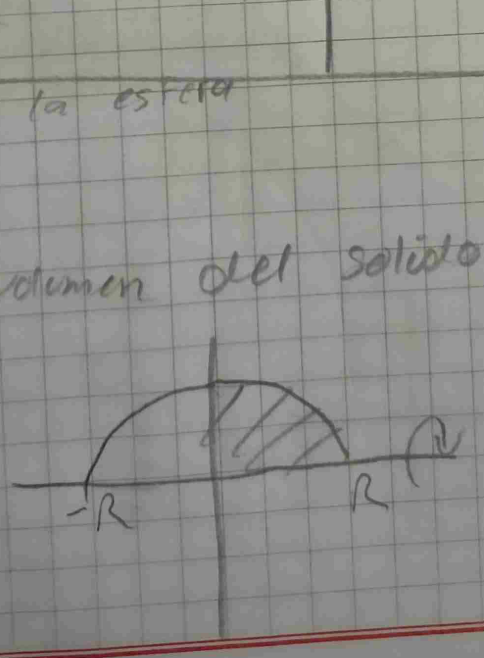


forma de la esfera

$V=\frac{4}{3}\pi R^3$

calcular el volumen del solido limitado por la curva

$x^2+y^2=R^2$



$y^2=R^2-x^2$

$y=f(x)=\sqrt{R^2-x^2}$

Tema:

$$\frac{V}{2} = \pi \int_0^R (R^2 - x^2)\,dx = \pi \left(R^2 x - \frac{1}{3} x^3 \right) \Big|_0^R$$

$$= \pi \left(R^3 - \frac{1}{3} R^3 \right) = \pi \left(\frac{3R^3}{3} - R^3 \right) = \frac{2}{3} \pi R^3$$

$$V = \frac{4}{3} \pi R^3$$

Superficies

3 ecuaciones

$F(x)=0$

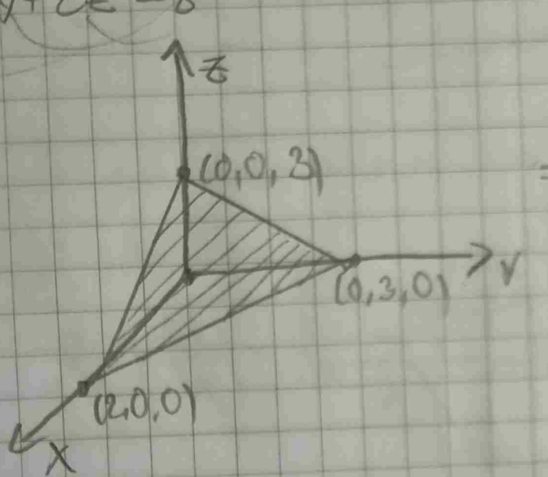
$F(x,y)=0$

$F(x,y,z)=0$ una superficie en R^3

la superficie basica

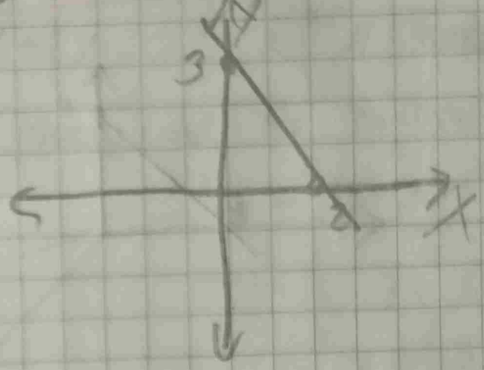
① $Ax+By+Cz+D=0$ Ecuacion de un plano

$3x+2y+2z=6$

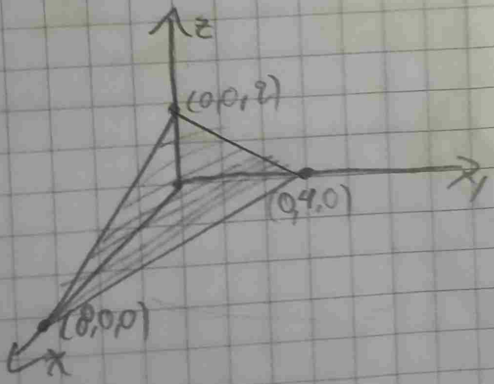


$\Rightarrow$

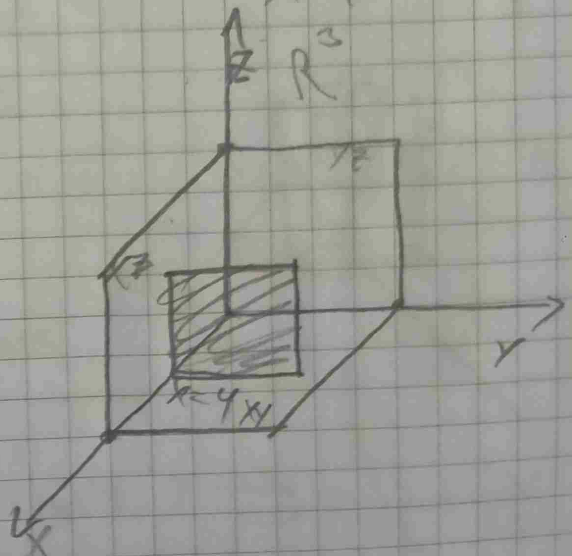
$3x+2y+2z=6$ $z=0$

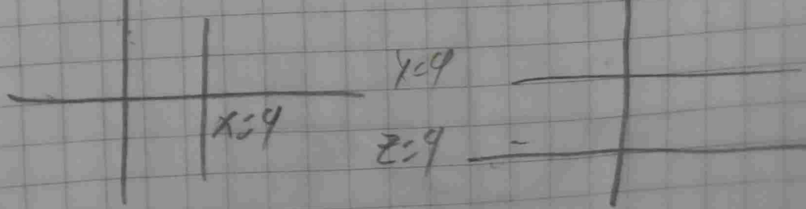


$x+2y+4z=18$



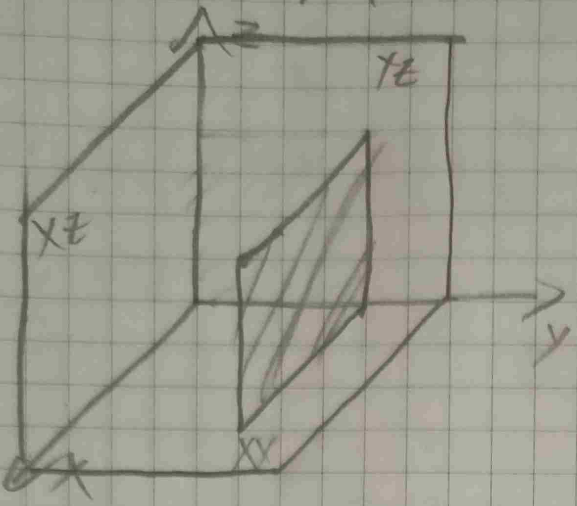
$x=4$



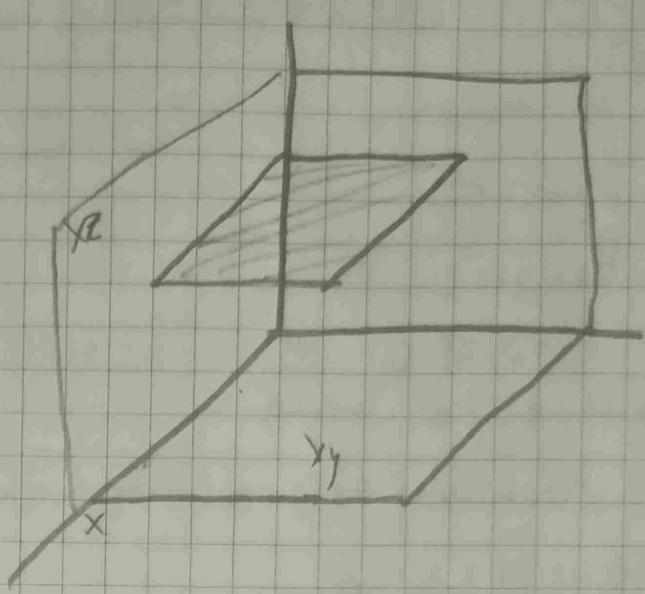


Tema: Fecha:

$y=4$



$z=4$



Superficies R^2

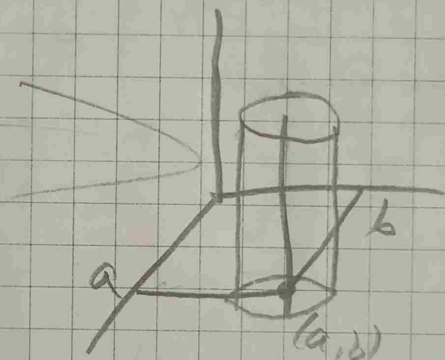
son ecuaciones de 2do grado

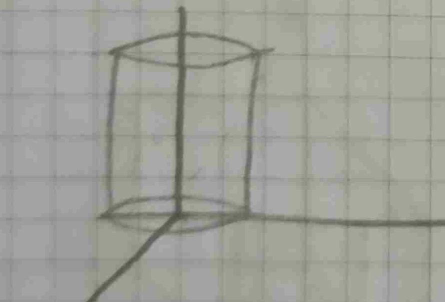
$(x-a)^2+(y-b)^2=r^2$

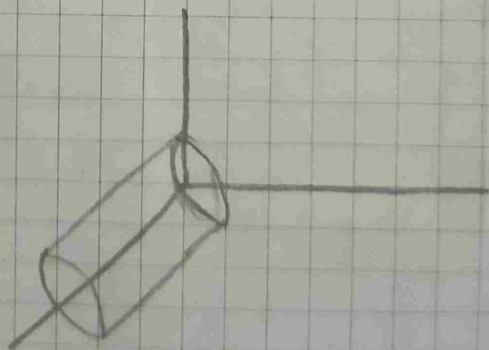
$\boxed{x^2+y^2=r^2}$

$y^2+z^2=r^2$

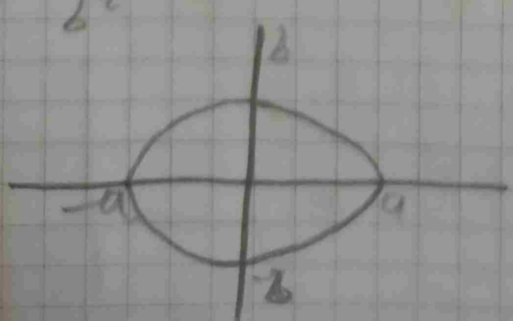
$x^2+z^2=r^2$

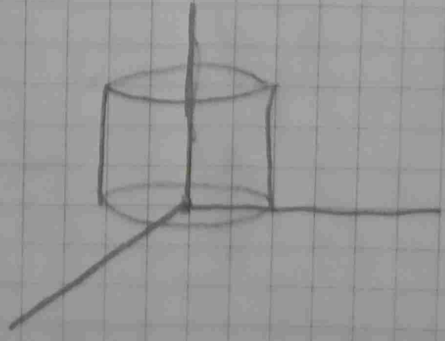






$\frac{x^2}{a^2}+\frac{y^2}{b^2}=r^2$ elipse





$X(t) = t^3 + 4t^2 - t$

$\frac{dx}{dt} = v(t) = 3t^2 + 8t - 1$

$t=1 \to v(1) = 3(1)^2 + 8(1) - 1 = 10\ m/seg$

$\frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt} = \frac{dv}{dt} =$

$= 6t + 8 = a(t)$

$a(1) = 6(1) + 8 = 14\ m/seg^2$

$\int_0^3 a(t)dt = v(t) = \int_0^3 (6t+8)dt$

$= 6\left(\frac{1}{2}t^2\right) + 8t \Big|_0^3$

$= 3t^2 + 4t \Big|_0^3 = 27 + 12 = 39\ m/seg$

$\int t\,dt = \frac{1}{2}t^2$

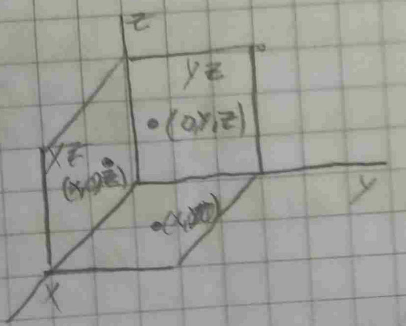
$\int (x^2+4x)dx = \left(\frac{1}{3}x^3 + 2x^2\right)\Big|^1$

Ecuacion del desplazamiento

$x(t)$

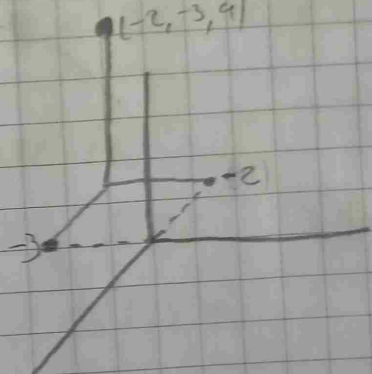
$x'(t) = \frac{dx}{dt} = v$

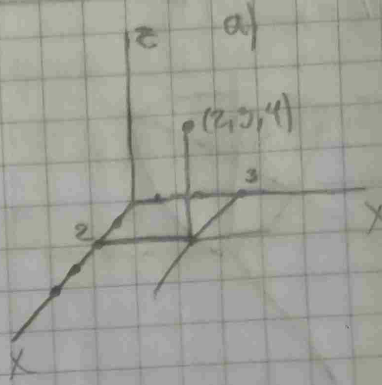
$x''(t) = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{dv}{dt} = a(t)$



$A = (2,3,4)$

$B = (-2,-3,4)$





$x^2+y^2=r^2$ cilindro

c=centro

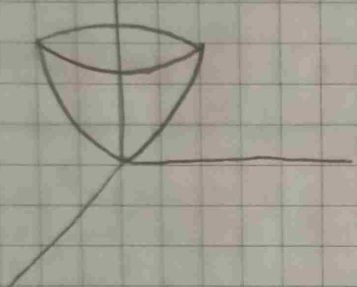
$(x-a)^2+(y-b)^2=r^2$ cilindro c(a,b)

$y^2+z^2=r^2$ cilindro

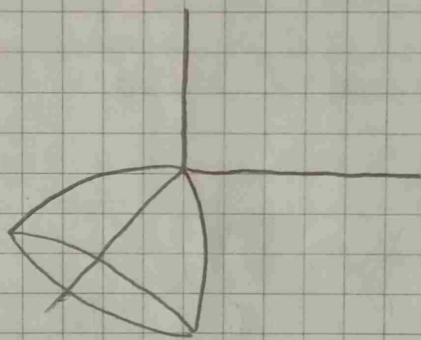
$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ cilindro eliptico

Paraboloide circular

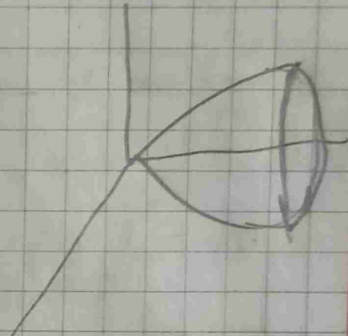
$z=x^2+y^2$ eje z $\quad y=x^2$



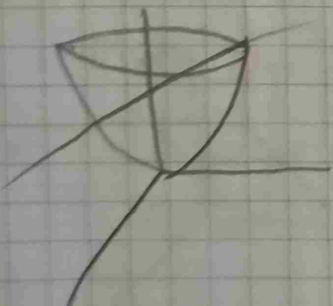
$x=y^2+z^2$



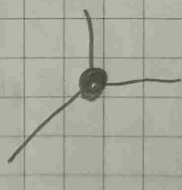
$y=x^2+z^2$



$4=x^2+y^2$



$x^2+y^2=0$



$\frac{4}{4}=\frac{x^2}{9}+\frac{y^2}{16}$

$\frac{x^2}{9}+\frac{y^2}{16}$ elipce

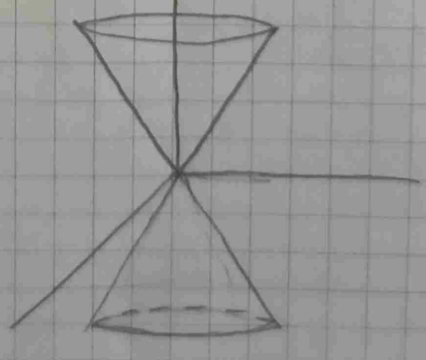
Cono circular

$x^2+y^2=z^2$

$x^2+y^2-z^2=0$ son lo mismo

$-x^2-y^2+z^2=0$

$z=4 \quad x^2+y^2=16$



Tema: Fecha: DÍA MES AÑO

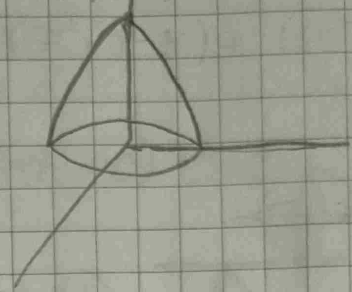
$x^2 = y^2 + z^2$ cono

$y^2 = x^2 + z^2$ cono

cono elíptico

$$\frac{z^2}{c^2} = \frac{x^2}{a^2} + \frac{y^2}{b^2}$$

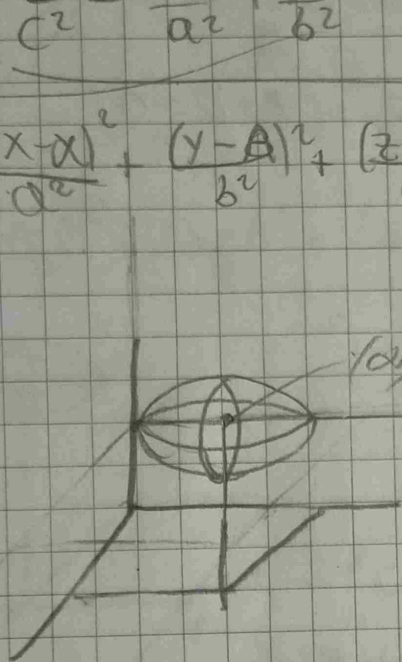
$z = 4 - x^2 - y^2$ → paraboloide



$$\frac{(x-\alpha)^2}{a^2} + \frac{(y-\beta)^2}{b^2} + \frac{(z-\gamma)^2}{c} = 1$$

$C(\alpha, \beta, \gamma)$

a, b, c



$-36x^2 + 4y^2 + 9z^2 - 72x - 24y - 72z + 180 = 0$

$-(36x^2 - 72x) + (4y^2 - 24y) + (9z^2 - 72z) + 180 = 0$

$-36(x^2 - 2x) + 4(y^2 - 6y) + 9(z^2 - 8z) + 180 = 0$

$-36(x-1)^2 + 4(y-3)^2 + 9(z-4)^2 - 36 - 36 - 144 + 180 = 0$

$36(x^2 - 2x + 1)$ $\quad 4(y^2 - 6y + 9)$ $\quad 9(z^2 - 8z + 16)$

$(a+b)^2 = a^2 + 2ab + b^2$

$(a-b)^2 = a^2 - 2ab + b^2$

$-36(x-1)^2 + 4(y-3)^2 + 9(z-4)^2 = 36$ $\quad$ %36

$-(x-1)^2 + \frac{(y-3)^2}{9} + \frac{(z-4)^2}{4} = 1$ $\quad C(1, 3, 4)$

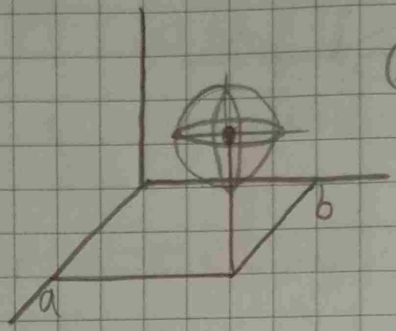
$(1, 3, 2)$

Tema:

Esfera

① $(x-a)^2+(y-b)^2+(z-c)^2=r^2$

$C(a,b,c)$
$r=$



② $x^2+y^2+z^2-2ax-2by-2cz+\underbrace{a^2+b^2+c^2-r^2}_{D}=0$

③ $x^2+y^2+z^2+Ax+By+Cz+D=0$

$A=-2a$
$B=-2b$
$C=-2c$
$D=a^2+b^2+c^2-r^2$

$x^2+y^2+z^2+4x-6y-4z-8=0$

$A=-2a$
$4=-2a \to A=-2$
$-6=-2b \to b=3$
$-4=-2c \to c=2$
$-8=4+9+4-r^2$
$-8=17-r^2$
$r^2=25$
$r=5$

$(x+2)^2+(y-3)^2+(z-2)^2=5^2$

$(x^2+4x)+(y^2-6y)+(z^2-4z)-8=0$

$(x+2)^2+(y-3)^2+(z-2)^2-4-9-4-8=0$

$(x+2)^2+(y-3)^2+(z-2)^2=25$

Ec. elipsoide

$$\frac{(x-\alpha)^2}{a^2}+\frac{(y-\beta)^2}{b^2}+\frac{(z-\gamma)^2}{c^2}=1$$

$9x^2+4y^2+z^2-18x+8y-4z-19=0$

$(9x^2-18x)+(4y^2+8y)+(z^2-4z)-19=0$

$9(x^2-2x)+4(y^2+2y)+(z^2-4z)-19=0$

$9(x-1)^2+4(y+1)^2+(z-2)^2-19-9-4-4=0$

$\Downarrow \qquad \Downarrow \qquad \Downarrow$

$9(x^2-2x+1) \qquad 4(y^2+2y+1) \qquad z^2-2z+4$

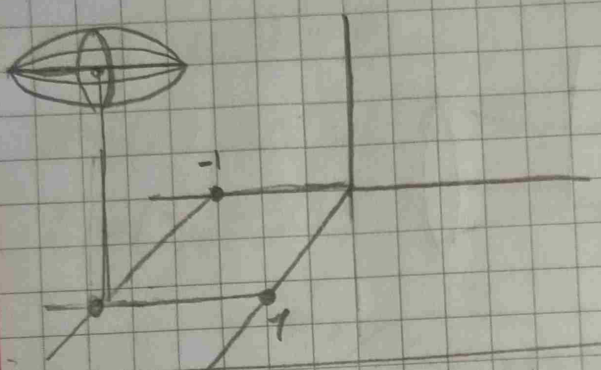
$9(x-1)^2+4(y+1)^2+(z-2)^2=36 \qquad \%36$

$\frac{(x-1)^2}{4}+\frac{(x+1)^2}{9}+\frac{(z-2)^2}{36}=1$

Elipsoide

$C(1,-1,2)$

$a=2$
$b=3$
$c=6$



$x^2+y^2+z^2-2ax-2by-2cz+a^2+b^2+c^2-r^2=0$

$A=-2a$

$B=-2b$

$C=-2c$

$D=a^2+b^2+c^3-r^2$

1er metodo

$x^2+y^2+z^2-4x+4y-6z+8=0$

$-4=-2a \rightarrow a=2$

$4=-2b \rightarrow b=-2$

$-6=-2c \rightarrow c=3$

$8=4+4+9-r^2$

$+9=+r^2 \rightarrow r=3$

$(x-2)^2+(y+2)^2+(z-3)^2=9$

$C(2,-2,3)$

$r=3$

segundo

$$(x^2-4x)+(y^2+4y)+(z^2-6z)+8=0$$

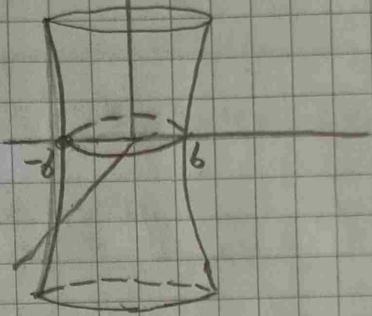
$$(x-2)^2+(y+2)^2+(z-3)^2 \cancel{-4}\cancel{-4}-9\cancel{+8}=0$$

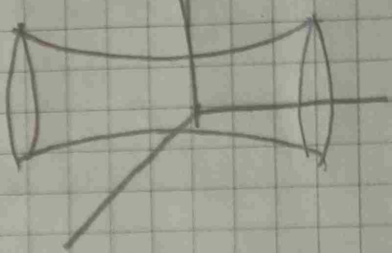
$$x^2-4x+4 \qquad (x-2)^2+(y+2)^2+(z-3)^2=9$$

Hiperboloide de una Hoja

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$$

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$$

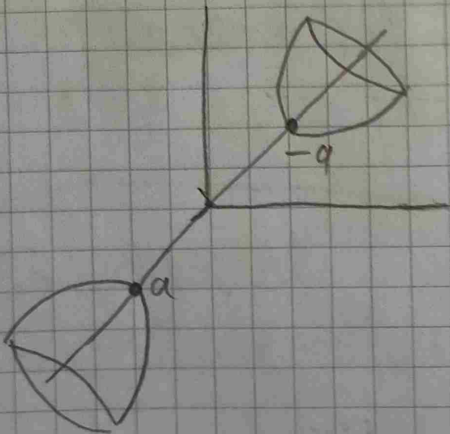


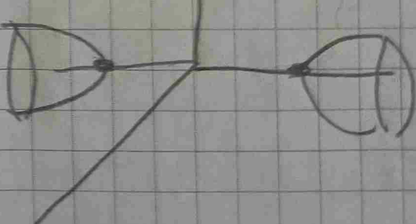


Hiperboloide de dos Hojas

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$$

$$-\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$$

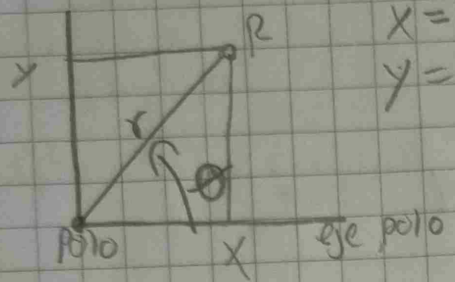




Tema: Fecha:

Cordenadas polares

$P = (r, \theta)$



$x = r\cos\theta$
$y = r\,sen\,\theta$
} polares

$r^2 = x^2 + y^2$
$\theta = tg^{-1}\left(\frac{y}{x}\right)$
} cartesianas

$tg\theta = \frac{y}{x}$

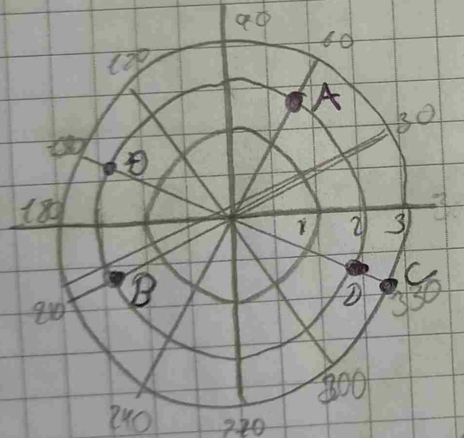
$A = (2, 60^\circ) = \left(2, \frac{\pi}{3}\right)$

$B = (-2, 30^\circ = \left(-2, \frac{\pi}{6}\right)$

$C = (3, -30)$

$D = (-2, 150^\circ)$

$E = (2, 30^\circ) = (2, 390^\circ)$



$\theta = tg^{-1}\left(\frac{2}{2}\right) = 45^\circ$

$r = \sqrt{2^2 + 2^2} = \sqrt{8}$

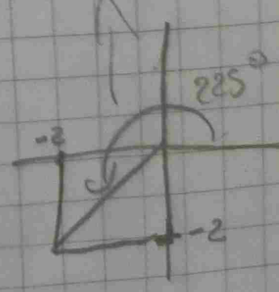
$x = r\cos\theta$
$y = r\,sen\,\theta$

$r^2 = x^2 + y^2$
$\theta = tg^{-1}\left(\frac{y}{x}\right)$

cartesianas

$A = (2, 2)$
$B = (-1, \sqrt{3})$
$C = (-2, -2)$

$A = (\sqrt{8}, 45^\circ)$
$B = 2, -60^\circ)$
$C = (\sqrt{8}, 225^\circ)$
} polares



cartesianas

$A = (3, 30^\circ) = \left(\frac{3\sqrt{3}}{2}; \frac{3}{2}\right)$

polares $B = (-2, 60^\circ) =$

$C = (2, -45^\circ) =$

$x = 3\cos 30^\circ = \left(\frac{3\sqrt{3}}{2}\right)$

$y = 3(sen\,30^\circ = \frac{3}{2}$

Tema:

Fecha:

$r = 2\cos\theta$ *r → circunferencia

$r^2 = 2r\cos\theta$

$x^2 + y^2 = 2x$

$x^2 - 2x + y^2 = 0$

$(x-1)^2 + y^2 = 1$ $C(1,0)$; $r = 1$

$r = 6\,\text{sen}\,\theta - 8\cos\theta$

$r^2 = 6r\,\text{sen}\,\theta - 8r\cos\theta$

$x^2 + y^2 = 6y - 8x$

$x^2 + y^2 - 6y + 8x = 0$

$(x^2 + 8x) + (y^2 - 6y) = 0$

$(x+4)^2 + (y-3)^2 - 16 - 9 = 0$

$(x+4)^2 + (y-3)^2 = 25$

$C(-4,3)$

$r = 5$

Tema: Fecha: DÍA MES AÑO / /

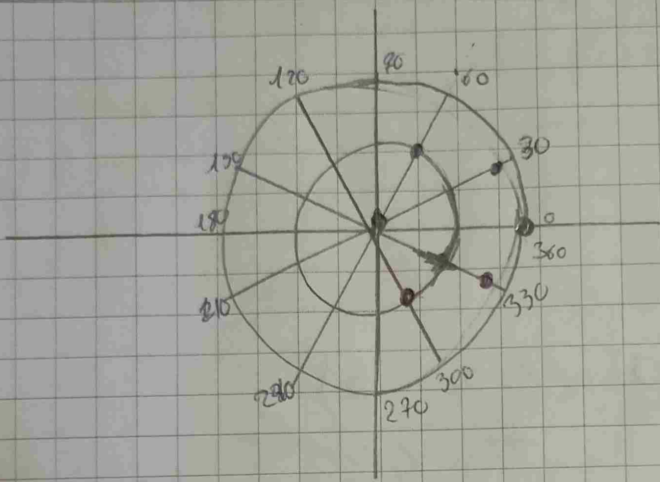
$\boxed{r = 2\cos\theta}$ $\cdot r$

$r^2 = 2r\cos\theta$

$x^2 + y^2 = 2x$

$(x^2 - 2x) + y^2 = 0$

$(x-1)^2 + y^2 = 1$ cir (1,0) r = 1



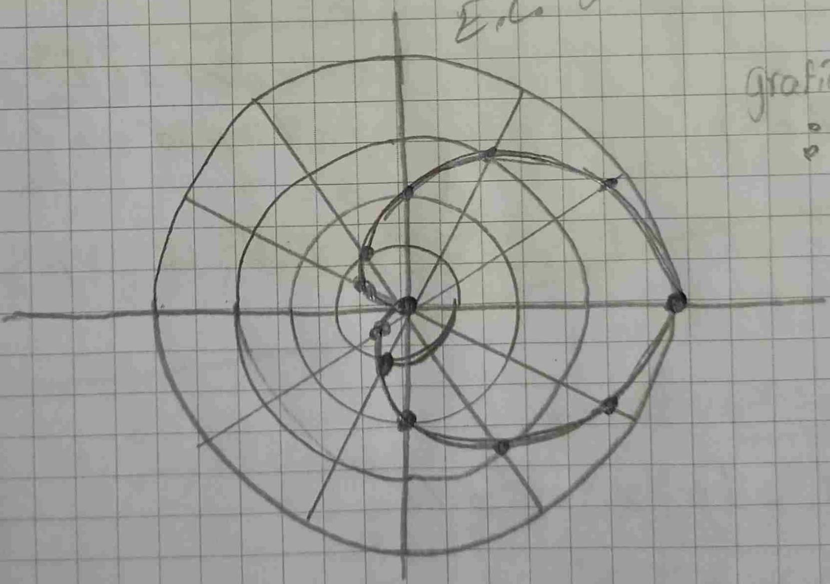
$r = 2\cos\theta$

θ	r
0	2
30	1,7
60	1
90	0
120	-1
150	-1,7
180	-2
210	-1,7
240	-1
270	0
300	1
330	1,7
360	2

$r = 2 + 2\cos\theta = 2(1+\cos\theta)$

E.C. de un cardioide

θ	r
0	4
30	3,7
60	3
90	2
120	1
150	0,3
180	0
210	0,3
240	1
270	2
300	3
330	3,7
360	4

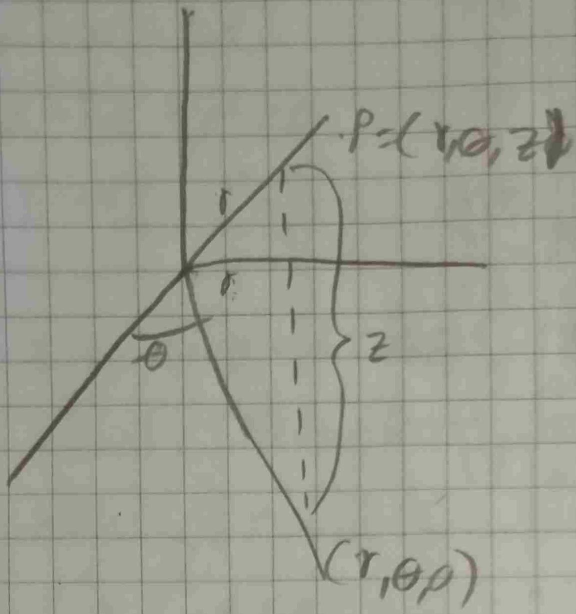


grafica llamada : Cardioide

$r = 2 + 2\,\text{sen}\,\theta$ Cardiode

Cordenadas cilíndricas

$P=(r,\theta,z)$



$A=(3,3,4)$ $\quad B=(4,\frac{\pi}{6},3)=(4,30,3)$

$\pi=180°$

$r=\sqrt{x^2+y^2}=\sqrt{9+9}=\sqrt{18}$

$\theta=tg^{-1}(1)=45°=\frac{\pi}{4}$

$z=4$

$A=(\sqrt{18},\frac{\pi}{4},4)$

$r=4$

$\theta=30$

$z=3$

$x=4\cos 30°=4\frac{\sqrt{3}}{2}=2\sqrt{3}$

$y=4\,sen\,30°=4\frac{1}{2}=2$

$z=3$

$B=(2\sqrt{3};2,3)$

$r=2$

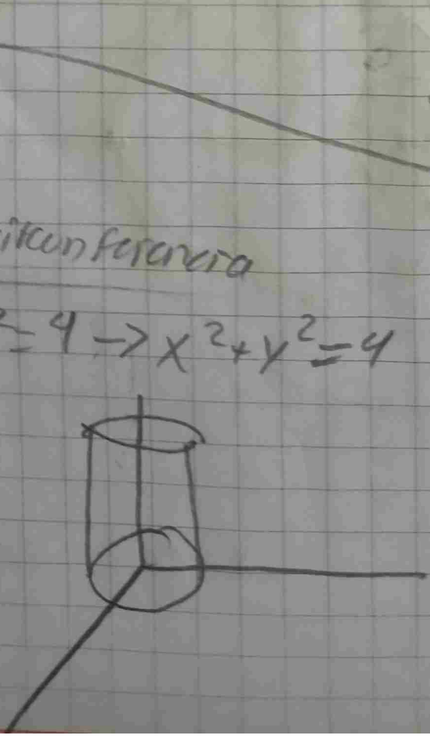
$r^2=4$

$x^2+y^2=4$ Ec circunferencia

a) $r=2 \rightarrow r^2=4 \rightarrow x^2+y^2=4$

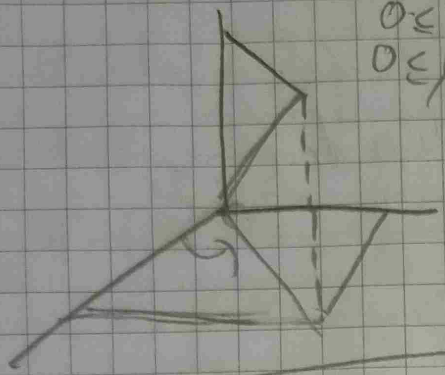
b) $z=4$

c) $\theta=\frac{\pi}{3}$



Coordenadas esfericas

$P = (\rho; \theta; \phi)$



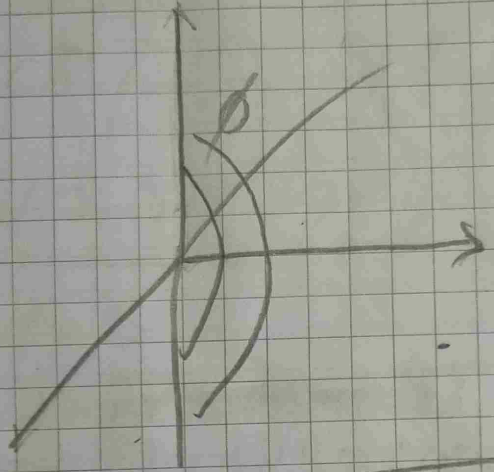
$0 \leq \theta \leq 2\pi$

$0 \leq \phi \leq \pi$

$x = \rho \operatorname{sen}\phi \cos\theta$

$y = \rho \operatorname{sen}\phi \operatorname{sen}\theta$

$z = \rho \cos\phi$



$\rho^2 = x^2 + y^2 + z^2 \rightarrow \rho = \sqrt{x^2+y^2+z^2}$

$\theta = tg^{-1}\left(\frac{y}{x}\right)$

$\phi = \cos^{-1}\left(\frac{z}{\sqrt{x^2+y^2+z^2}}\right) = \cos^{-1}\left(\frac{z}{\rho}\right)$

$A = (1, 1, \sqrt{6})$

$B = (2\sqrt{2}, \frac{3}{4}\pi, \frac{\pi}{6}) = (2\sqrt{2}, 135°, 30°)$

$\rho = \sqrt{1+1+6} = \sqrt{8}$

$\theta = tg^{-1}(1) = \frac{\pi}{4}$

$\phi = \cos^{-1}\left(\frac{\sqrt{6}}{\sqrt{3}}\right) = \frac{\pi}{6}$

$A = \left(\sqrt{8}, \frac{\pi}{4}, \frac{\pi}{6}\right)$

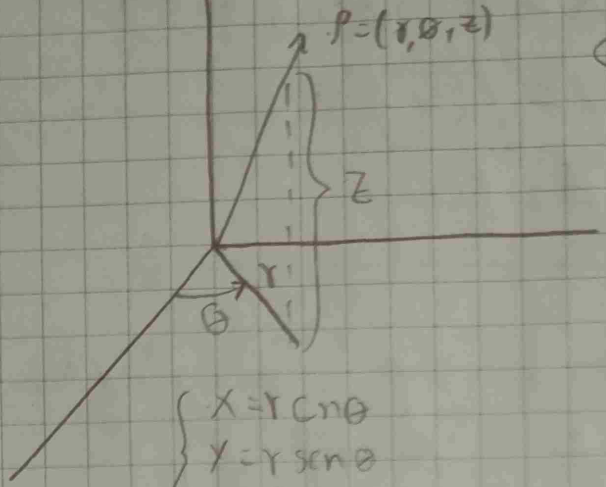
$B = (-1, -1, 2)$

$r \quad \theta \quad z$

cil $A = (2, \frac{\pi}{6}, 3) \rightarrow$ C. cart $\wedge$ C. esfericas

cart $B = (2, -2, 1) \rightarrow$ C. cil $\wedge$ C. esfe

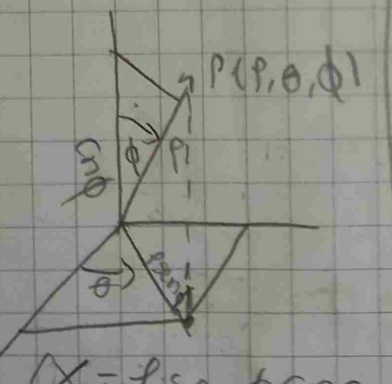
esf $C = (3, \frac{\pi}{4}, \frac{\pi}{4}) \rightarrow$ C. carte $\wedge$ C. cil



C. cilindricas

$$\begin{cases} x = r\,cn\,\theta \\ y = r\,sen\,\theta \\ z = z \end{cases}$$

$$\begin{cases} r^2 = x^2 + y^2 \\ \theta = tg^{-1}\left(\frac{y}{x}\right) \\ z = z \end{cases}$$



$$\begin{cases} x = \rho\,sen\phi\,cn\theta \\ y = \rho\,sen\phi\,sen\theta \\ z = \rho\,cn\phi \end{cases}$$

$\rho^2 = x^2 + y^2 + z^2 \quad \theta = tg^{-1}(y/x)$

$\phi\ cn^{-1}\left(\frac{z}{\rho}\right)$

A Cil ~~[illegible]~~

$x = 2\cos(30°) = 2\frac{\sqrt{3}}{2} = \sqrt{3}$

$y = 2\,sen\,30° = 2 \cdot \frac{1}{2} = 1$

$z = 3$

$A = (\sqrt{3}, 1, 3)$ cartesianas

$\rho = \sqrt{3+1+9} = \sqrt{13}$

$\theta = \frac{\pi}{6}$

$\phi = cn^{-1}\left(\frac{z}{\rho}\right) = cn^{-1}\left(\frac{3}{\sqrt{13}}\right) \cong 34°$

esfericas

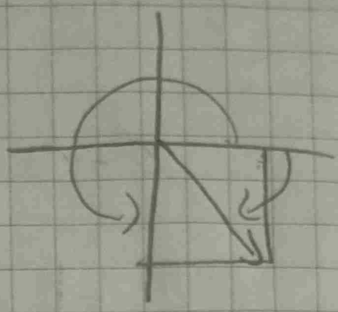
$A = (\sqrt{13}, \frac{\pi}{6}, 34°)$

(B) cil ∧ esF

$B = (2, -2, 1)$

$r = \sqrt{4+4} = \sqrt{8}$

$\theta = tg^{-1}\left(\frac{-2}{2}\right) = tg^{-1}(-1) = 315^\circ$



$z = 1$

$B = (\sqrt{8}, 315^\circ; 1)$ ← en cordenados cilindricas

$\rho = \sqrt{4+4+1^2} = 3$

$\theta = 315^\circ$

$\phi = \cos^{-1}\left(\frac{1}{3}\right) = 71^\circ$

$B = (3, 315^\circ; 71^\circ)$ en esfericas

(C) $\rho = 3$

$\theta = \frac{\pi}{4}$

$\phi = \frac{\pi}{4}$

$x = 3 \cdot \operatorname{sen}\frac{\pi}{4} \cdot \cos\frac{\pi}{4} = 3 \cdot \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{2}}{2} = \frac{3}{4}(2) = \frac{3}{2}$

$y = 3 \operatorname{sen}\frac{\pi}{4} \operatorname{sen}\frac{\pi}{4} = 3\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{2}}{2} = \frac{3}{4}(2) = \frac{3}{2}$

$z = 3 \cos\frac{\pi}{4} = \frac{3\sqrt{2}}{2}$

$C = \left(\frac{3}{2}, \frac{3}{2}, \frac{3\sqrt{2}}{2}\right)$

---

figura c. Polares	superficies c. cilindrica	c. esferi
$r = 3$	$r = 3$	$\rho = 3$
⇓	↑	↑
circunferencia $r=3$ $C(0,0)$	cilindro	esfera

$\theta = \frac{\pi}{3}$; $\phi = \frac{\pi}{4}$; $\phi \frac{\pi}{3}$ → cono circular

$$\phi = \cos^{-1}\left(\frac{z}{\sqrt{x^2+y^2+z^2}}\right)$$

$$\frac{\pi}{3} = \cos^{-1}\left(\frac{z}{\sqrt{x^2+y^2+z^2}}\right)$$

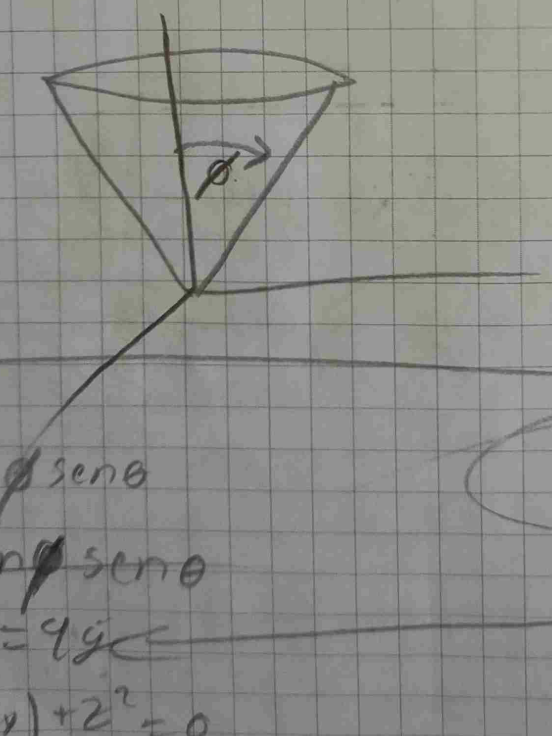
$$\cos\frac{\pi}{3} = \frac{z}{\sqrt{x^2+y^2+z^2}}$$

$$\left(\frac{1}{2}\right)^2 = \left(\frac{z}{\sqrt{x^2+y^2+z^2}}\right)^2 \rightarrow \frac{1}{4} = \frac{z^2}{x^2+y^2+z^2}$$

$$x^2+y^2+z^2 = 4z^2$$

$$x^2+y^2 = 3z^2$$

ϕ = constante = cono



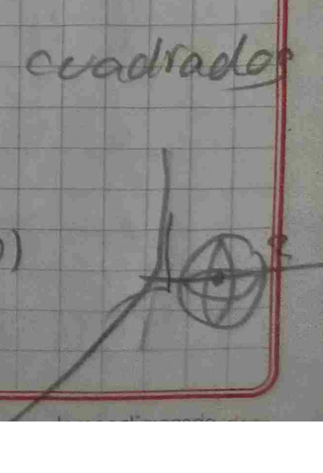
$\rho = 4 \operatorname{sen}\phi \operatorname{sen}\theta$ θ = esfera

$\rho^2 = 4\rho \operatorname{sen}\phi \operatorname{sen}\theta$

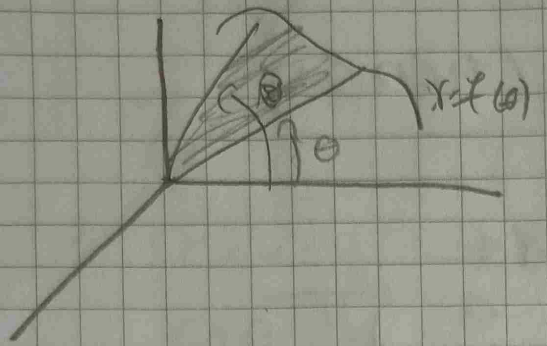
$x^2+y^2+z^2 = 4y$ ← completar cuadrados

$x^2+(y^2-4y)+z^2 = 0$

$x^2+(y-2)^2+z^2 = 4$ → esfera $C(0,2,0)$
$r = 2$

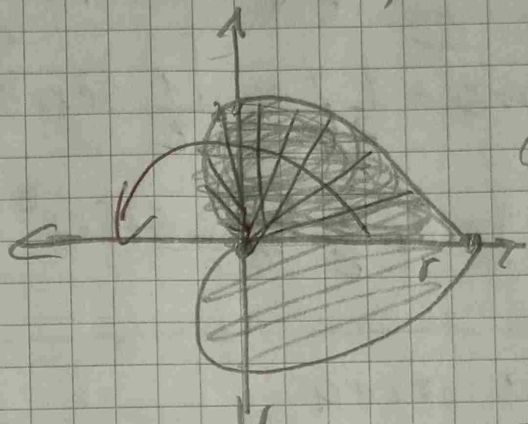


Calculo de areas



$$A = \frac{1}{2}\int_{\alpha}^{\beta} f^2(\theta)\, d\theta$$

$$r = 2(1+cn\theta)$$



cardioide

$0 \leq \theta \leq$

$$\frac{A}{\cancel{2}} = \frac{1}{\cancel{2}}\int_0^{\pi} [2(1+cn\theta)]^2\, d\theta$$

$$A = \int_0^{\pi} 4(1+2cn\theta+cn^2\theta)\, d\theta$$

$$\int sen^2x\, dx = \int \frac{1}{2}(1-cn^2x)\, dx$$

$$\int cn^2x\, dx = \int \frac{1}{2}(1+cn^2x)\, dx$$

$$sen^2x + cn^2x = 1$$

ó resta (-)

$$+\; cn^2x - sen^2x = cn2x$$

$$2cn^2x = 1+cn2x$$

$$cn^2x = \frac{1}{2}(1+cn2x)$$

$$2sen^2x = 1-cn2x$$

$$sen^2x = \frac{1}{2}(1-cn2x)$$

Tema: Fecha: DÍA MES AÑO

$\int cn^2 2x$

$$cn^2 2x = \frac{1}{2}(1 + cn\, 4x)$$

$$cn^2 3x = \frac{1}{2}(1 + cn\, 6x)$$

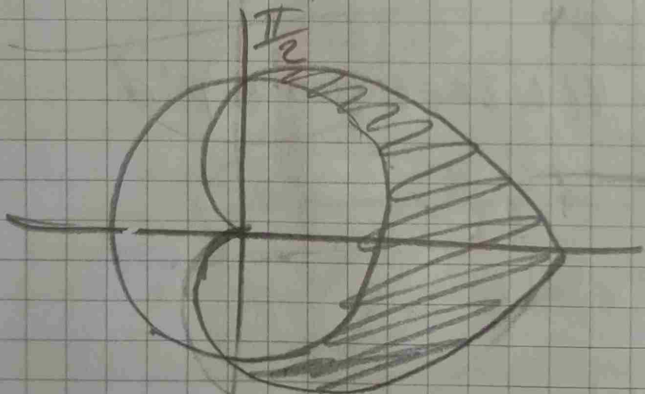
$$A = 4\int_0^{\pi} \left(1 + 2cn\theta + \frac{1}{2}(1 + cn2\theta)\right) d\theta$$

$$= 4\left(\theta + 2sen\theta + \frac{1}{2}\left(\theta + \frac{1}{2}sen2\theta\right)\right)\Big|_0^{\pi}$$

$$= 4\left(\pi + \frac{1}{2}\pi\right) = 4\left(\frac{3}{2}\pi\right) = 6\pi u^2$$

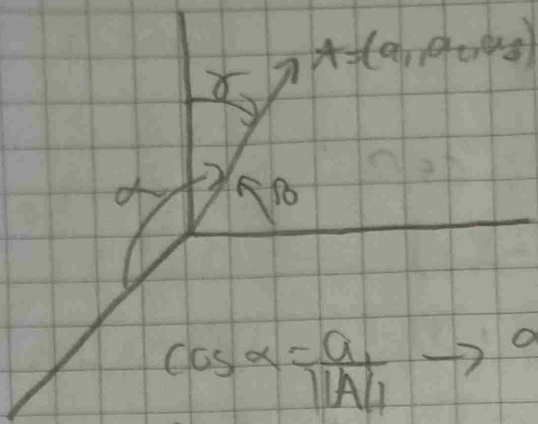
Calcular el area

$r = 2 \quad \wedge \quad r = 2(1 + cn\theta)$



$$\frac{A}{2} = \frac{1}{2}\int_0^{\frac{\pi}{2}} \left((2(1 + cn\theta))^2 - r^2\right) d\theta$$

Tema:



$$\cos\alpha = \frac{a_1}{\|A\|} \rightarrow \alpha = \cos^{-1}\left(\frac{a_1}{\|A\|}\right)$$

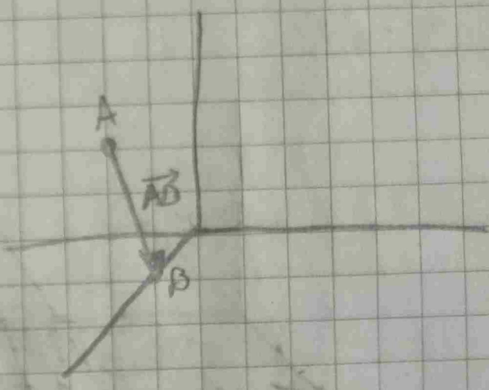
$$\cos\beta = \frac{a_2}{\|A\|}$$

$$\cos\gamma = \frac{a_3}{\|A\|}$$

$A = (2, -1, 3)$

$B = (4, 1, 2)$

Hallar el vector unitario $\overline{AB}$



$\overline{AB} = (2, 2, -1)$

$\|\overline{AB}\| = \sqrt{4+4+1} = 3$

$U = \frac{2}{3} i + \frac{2}{3} j - \frac{1}{3} k$

$\|U\| = \sqrt{\left(\frac{2}{3}\right)^2 + \left(\frac{2}{3}\right)^2 + \left(\frac{1}{3}\right)^2} = 1 = \sqrt{\frac{4}{9} + \frac{4}{9} + \frac{1}{9}}$

$A = (3, 4, -2)$

$B = (4, -1, 0)$

calcular el vector unitario "U"

$\overline{AB} = (1, -5, 2)$

$\|\overline{AB}\| = \sqrt{1+25+4} = \sqrt{30} =$

$U = \frac{1}{\sqrt{30}} i - \frac{5}{\sqrt{30}} j + \frac{2}{\sqrt{30}} k$

$A \perp B \rightarrow A \cdot B = 0$

$A = (5, -4, 0)$

$B = (4, 5, 3)$

$A \cdot B = (5, -4, 0) \cdot (4, 5, 3)$

$= 20 - 20 + 0 = 0$

$A \parallel B$

$A = cB \; ; \; c \in \mathbb{R}$

$A = (4, -2, 6)$

$B = (2, -1, 3)$

$A = 2B$

$B = \frac{1}{2} A$

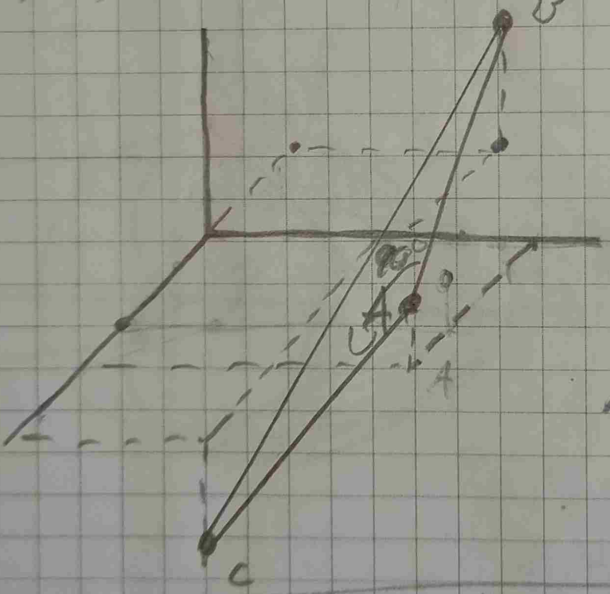
Dado los vertices de un triangulo

$A = (4, 9, 1)$

$B = (-2, 6, 3)$

$C = (6, 3, -2)$

Demostrar que el Δ es rectangulo



$\overline{AB} = (-6, -3, 2)$

$\overline{AC} = (2, -6, -3)$

$\overline{BC} = (1, -3, -5)$

$\overline{AB} \cdot \overline{AC} = -12 + 18 - 6 = 0$

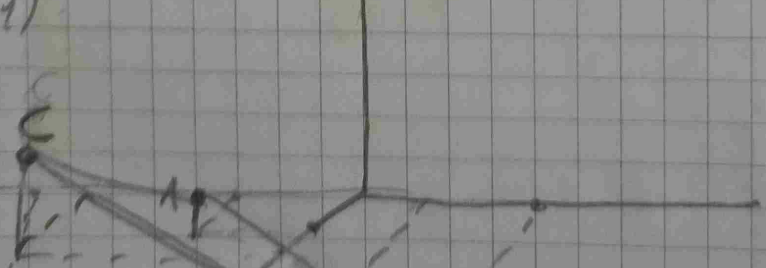
demostrar paralelogramo

$A = (2, -3, 1)$

$B = (6, 5, -1)$

$C = (3, -6, 4)$

$D = (7, 2, 2)$



$\overline{AC} = (1, -3, 3)$

$\overline{DB} = (1, -3, 3)$

$\overline{CD} = (4, 8, -2)$

$\overline{AB} = (4, 8, -2)$

$\overline{AC} \parallel \overline{DB}$

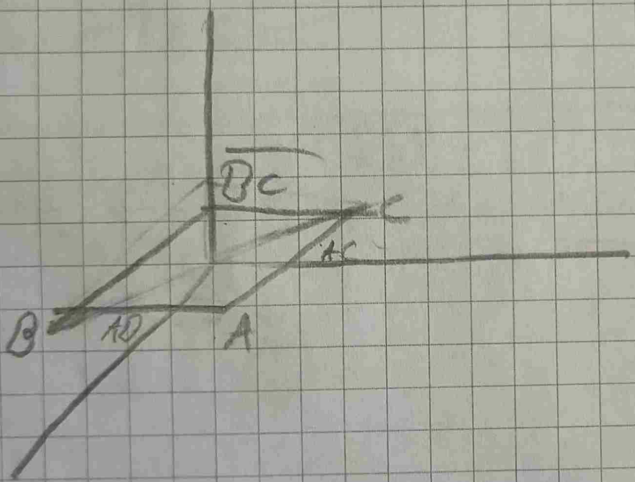
$\overline{CD} \parallel \overline{AB}$

Dado los vertices de un Δ hallar los angulos del mismo

$A = (-1, 0, -1)$

$B = (3, -2, 0)$ $\qquad \theta = \cos^{-1}\left(\frac{AB}{\|A\|\|B\|}\right)$

$C = (1, 3, 3)$



$\overline{AB} = (4, -2, 1)$

$\overline{AC} = (2, 3, 4)$

$\overline{BC} = (-2, 5, 3)$

$\|\overline{AB}\| = \sqrt{21}$

$\|\overline{AC}\| = \sqrt{29}$

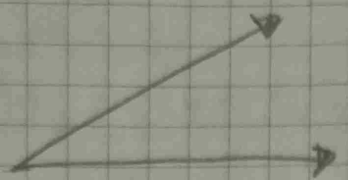
$\|\overline{BC}\| = \sqrt{38}$

$\measuredangle(A) = \cos^{-1}\left(\frac{(4, -2, 1)(2, 3, 4)}{\sqrt{21} \cdot \sqrt{29}}\right)$

$\measuredangle(C) = \cos^{-1}\left(\frac{(-2, 5, 3)(2, 3, 4)}{\sqrt{38} \cdot \sqrt{29}}\right)$

Tema: Fecha:

producto vectorial



$A = (a_1, a_2, a_3)$

$B = (b_1, b_2, b_3)$

$A \times B = N$

$A \cdot B$

$$A \times B = \begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} =$$

$$= \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} - \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} j + \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} k$$

$A = (2, -1, 3)$

$B = (1, 2, -2)$

$$A \times B = \begin{vmatrix} i & j & k \\ 2 & -1 & 3 \\ 1 & 2 & -2 \end{vmatrix} = -4i + 7j + 5k$$

Tema: vectores Fecha: DÍA MES AÑO

$A = \overset{x_0\ y_0\ z_0}{(1, 3, 2)}$ $\overline{P_0P} \cdot n = 0$ $P_0 C (x_0, y_0, z_0)$

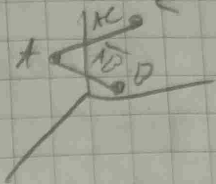
$B = (3, -2, 2)$ $P = (x, y, z)$

$C = (2, 1, 3)$ $n = (a, b, c)$

$ax + by + cz + d = 0$

$\overline{AB} = (2, -5, 0)$ $n = \underset{\text{puntos}}{\overline{AB}} \times \overline{AC}$

$\overline{AC} = (1, -2, 1)$



$$n = \begin{vmatrix} i & j & k \\ 2 & -5 & 0 \\ 1 & -2 & 1 \end{vmatrix} = -5i - 2j + k$$

$n = (\underset{a}{-5}, \underset{b}{-2}, \underset{c}{1})$ ← vector normal

2 metodos

$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$

$-5(x - 1) - 2(y - 3) + (z - 1) = 0$

$-5x - 2y + z + 9 = 0$

$A = (1, 2, -3)$ $B = (2, 3, 1)$; $C = (0, -2, -1)$

$\boxed{ax + by + cz + d = 0}$

$$\begin{cases} a + 2b - 3c = -d \\ 2a + 3b + c = -d \\ 0a - 2b - c = -d \end{cases}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -3 & -d \\ 2 & 3 & 1 & -d \\ 0 & -2 & -1 & -d \end{array}\right] \approx \left[\begin{array}{ccc|c} 1 & 2 & -1 & -d \\ 0 & -1 & 7 & d \\ 0 & -2 & -1 & -d \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & -d \\ 0 & -1 & +7 & d \\ 0 & 0 & -15 & -3d \end{array}\right]$$

$-15c = -3d$

$c = \frac{1}{5}d$

$-b + \frac{7}{5}d = d$

$-b = d - \frac{7}{5}d$

$+b = +\frac{2}{5}d$

$b = \frac{2}{5}d$

$a + 2\left(\frac{2}{5}d\right) - 3\left(\frac{1}{5}d\right) = -d$

$a + \frac{4}{5}d - \frac{3}{5}d = -d$

$a = -d - \frac{1}{5}d$

$a = -\frac{6}{5}d$

↓ ↓

Tema: Fecha:

$-\frac{6}{5}dx + \frac{2}{5}dy + \frac{1}{5}dz + d = 0$

$-6dx + 2dy + dz + 5d = 0$

$d(-6x + 2y + z + 5) = 0$

$-6x + 2y + z + 5 = 0$ Ec

$v = (-6, 2, 1)$

$N = \overline{AB} \times \overline{AC}$

$\overline{AB} = (1, 1, 4)$

$\overline{AC} = (-1, -4, 2)$

$\overline{AB} \times \overline{AC} = \begin{vmatrix} i & j & k \\ 1 & 1 & 4 \\ -1 & -4 & 2 \end{vmatrix} = 18i - 6j - 3k$

$N = (18, -6, -3)$

$a(x - x_0) + b(y - y_0) + c(z - z_0)$

$18(x-2) - 6(y-3) - 3(z-1) = 0$

$18x - 6y - 3z - 36 + 18 + 3 = 0$

$18x - 6y - 3z - 15 = 0$

$-3(-6x + 2y + z + 5) = 0$

$-6x + 2y + z + 5) = 0$

$A = c\,B$

$(2, 3, 9) = (6, 9, 12)$

$\Pi_1 \quad x - y + 2z = 12$

$\Pi_2 \quad 2x - 2y + 4z = 6 \qquad \Pi_1 /\!/ \Pi_2$

$x - 2y + z = 4$

$2x + y + 0z = 4$

$N_1 = (1, -2, 1)$

$N_2 = (2, 1, 0) \qquad N_1 N_2 = (1, -2, 1)(2, 1, 0)$

$= 2 - 2 + 0 = 0$

$\Pi_1 \to N_1$

$\Pi_2 \to N_2$

Dos planos son //

si $N_1 /\!/ N_2$

Dos planos son $\perp$

$N_1 \perp N_2$

$\theta = \cos^{-1}\left(\frac{N_1 N_2}{\|N_1\| \|N_2\|}\right)$

$N_1 = (2,1,2)$ $\quad \|N_1\| = \sqrt{4+1+4} = 3$

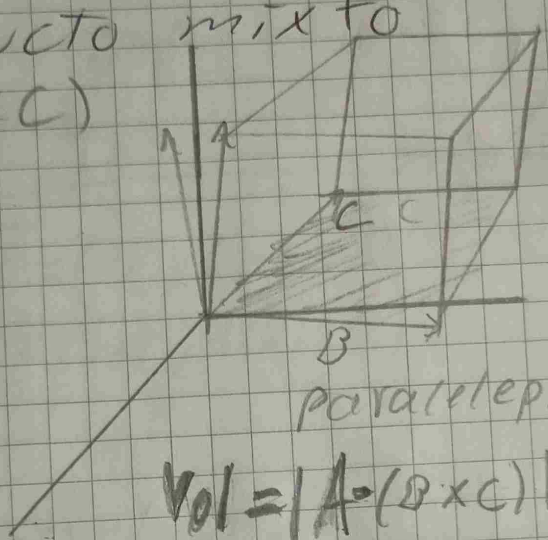
$N_2 = (3,-1,2)$ $\quad \|N_2\| = \sqrt{9+1+4} = \sqrt{14}$

$N_1 \cdot N_2 = 6-1+4 = 9$

$$\theta = \cos^{-1}\left(\frac{9}{3\sqrt{14}}\right) = \cos^{-1}\left(\frac{3}{\sqrt{14}}\right)$$

producto mixto

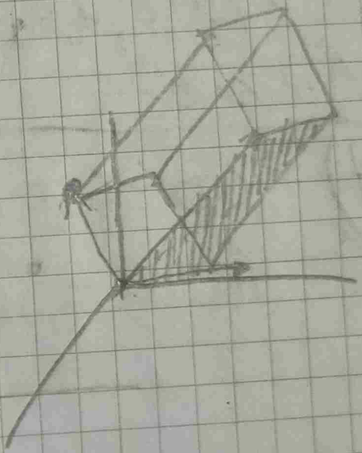
$A \cdot (B \times C)$



paralelepipedo

$$Vol = |A \cdot (B \times C)|$$

Dados los vectores



$A = (1,0,4)$

$B = (1,3,1)$

$C = (-4,2,6)$

calcular el volumen paralelepipedo

$V = \bar{A} \cdot (\bar{B} \times \bar{C})$

$$\begin{vmatrix} 1 & 0 & 4 \\ 1 & 3 & 1 \\ -4 & 2 & 6 \end{vmatrix} = \begin{vmatrix} 0 & 0 & 0 \\ 1 & 3 & -3 \\ -4 & 2 & 22 \end{vmatrix} = \begin{vmatrix} 3 & -3 \\ 2 & 22 \end{vmatrix} = 66+6 = 72$$

Tema:

1) $A=(2,-1,a+3)$

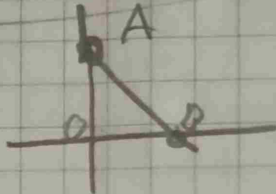
$B=(3,1-a,4)$

$C=(3,-2,-1)$

$D=(6,a-2,-1)$

calcular "a" tal $\|\overline{CD}\| = \sqrt{10}$

$\overline{AB} \perp \overline{CD}$



2)

$A=(2,x-3,2x+1)$

$B=(1,-1,-3)$

3) $A=(0,2,x)$

$B=(2,3,-1)$

$C=(y+2,1,5)$

$\overline{AB} \parallel \overline{OC}$

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1) $\overline{CD}=(3,a,0)$

$$\|\overline{CD}\| = \sqrt{9+a^2}^{\,2} = \sqrt{10}^{\,2}$$

$$= 9+a^2 = 10$$

$$a^2 = 1$$

$$a = \pm 1$$

$a=1$. $A=(2,-1,4)$

$B=(3,0,4)$

$C=(3,-2,-1)$

$D=(6,-1,-1)$

Tema: Fecha: DÍA MES AÑO / /

2) $\overline{OA} = (2, k-3, 2k+1)$

$\overline{OB} = (1, -2, -3)$

$\overline{OA} \cdot \overline{OB} = 0$

$(2, k-3, 2k+1)(1, -1, -3)$

$= 2 - k + \cancel{3} - 6k - \cancel{3} = 0$

$= -7k + 2 = 0$

$k = \frac{2}{7}$

repasar

paralelismo

perpenticuaridad

3) $\overline{AB} = (2, 1, -1-x)$

$\overline{OC} = (y+2, 1, 5)$

$\overline{AB} = \overline{OC}$

$(2, 1, -1-x) = (y+2, 1, 5)$

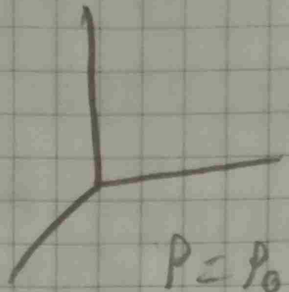
$y + \cancel{2} = \cancel{2} \rightarrow y = 0$

$1 = 1$

$-1 - x = 5$

$x = -6$

Rectas en R^3



$P_0(x_0, y_0, z_0)$; $P = (x, y, z)$

$V = (a, b, c)$

$P = P_0 + tV \quad t \in R$

$(x, y, z) = (x_0\, y_0\, z_0) + t(a, b, c)$

$$\begin{cases} x = x_0 + at \\ y = y_0 + bt \\ z = z_0 + ct \end{cases}$$ Ec. parametrica

$t = \dfrac{x - x_0}{a}$, $t = \dfrac{y - y_0}{b}$; $t = \dfrac{z - z_0}{c}$

$\dfrac{x - x_0}{a} = \dfrac{y - y_0}{b} = \dfrac{z - z_0}{c}$

Hallar las ecuaciones de la recta que pasa por el punto P_0

$P_0 = (3, -2, 4)$

$V = (4, -1, 2)$

$(x, y, z) = (3, -2, 4) + t(4, -1, 2)$

$$\begin{cases} x = 3 + 4t \\ y = -2 + t \\ z = 4 + 2t \end{cases}$$