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Analog Communication

modulation (carrier signal changes as per message signal)

- Analog modulation
  - CW modulation
    - Amplitude modulation
      - Double Side Band (DSB-SC)
      - (AM-WC)
      - (SSB-SC)
      - (VSB-SC)

      Mod-2
    - Angle modulation
      - Frequency modulation (FM)
      - phase modulation (PM)

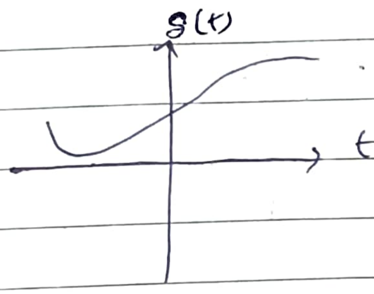
      Mod-3
  - Pulse modulation
    - PAM
    - PPM
    - PWM

    Mod 4
- Digital modulation

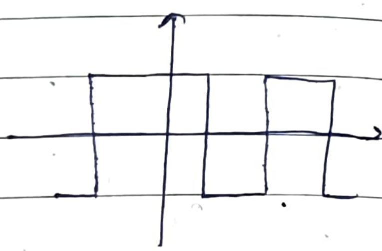
Date ___/___/______

1. Signal Analysis

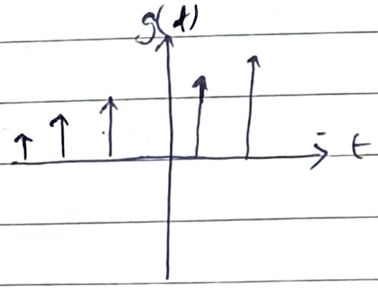
* Classification of signal
1) Continous & discrete signal
2) Analog & digital signal
3) periodic & aperiodic signal
4) Energy & power signal
5) Deterministic & Random signal
6) Even & odd signal
7) Real & complex signal



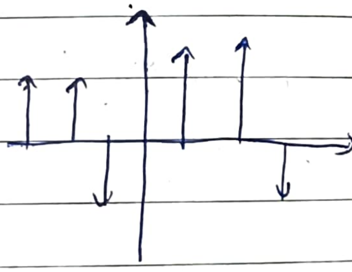
Analog, continous time



Digital, continous signal



Analog discrete



Digital, discrete signal

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Energy signal	Power signal
① $E_s = \int_{-\infty}^{\infty} \lvert s(t)\rvert^2 dt < \infty$	① $P_s = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} \lvert s(t)\rvert^2 dt$
	① $P_s = \frac{E_s}{T}$ when $T\to\infty$

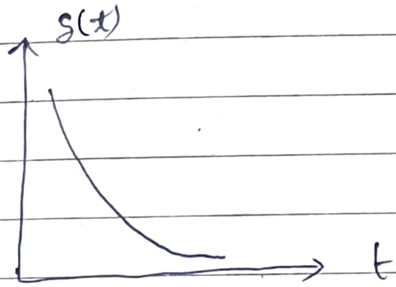
⇒ For energy signal, if $E_s < \infty$; $P_s = 0$

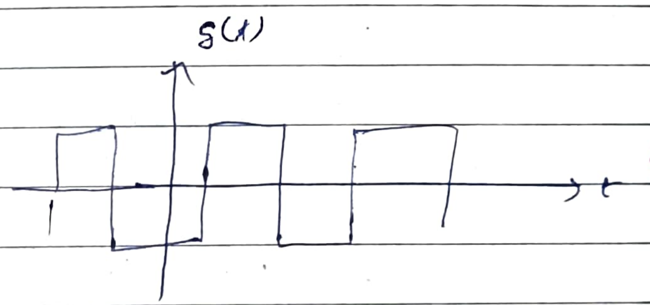
⇒ For power signal, if $P_s < \infty$; $E_s = \infty$

⇒ Signal can be ① either energy or power signal

or

② Not both

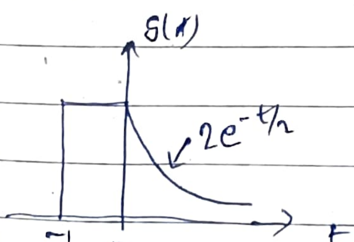




$\boxed{\text{if } s(t) = 0 \text{ at } t=\infty}$
↳ energy signal

$\boxed{\text{if } s(t) \neq 0 \text{ at } t=\infty}$
↳ power signal.

Ques) Calculate value of the signal



$\Rightarrow \int_{-1}^{0} (2)^2 dt + \int_{0}^{\infty} 2e^{-t/2} dt$

$\Rightarrow 4t\Big|_{-1}^{0} - 2\times 2\, e^{-t/2}\Big|_{0}^{\infty}$

$\Rightarrow 4 - 4(0-1)$

$= 8$

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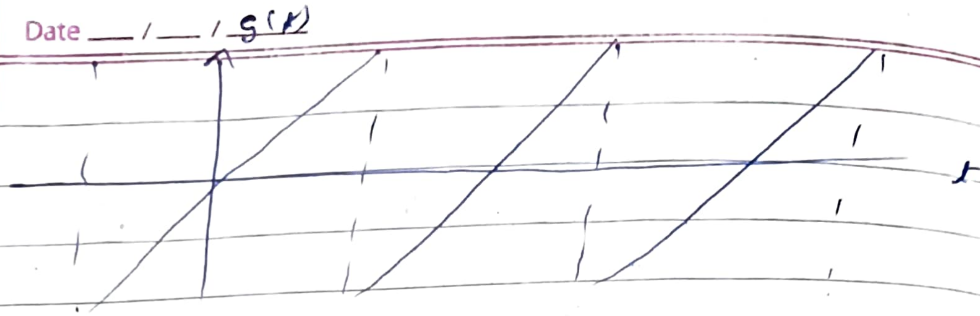
Ques)



$$P_s = \frac{1}{T}\int_{-T/2}^{T/2} t^2\,dt$$

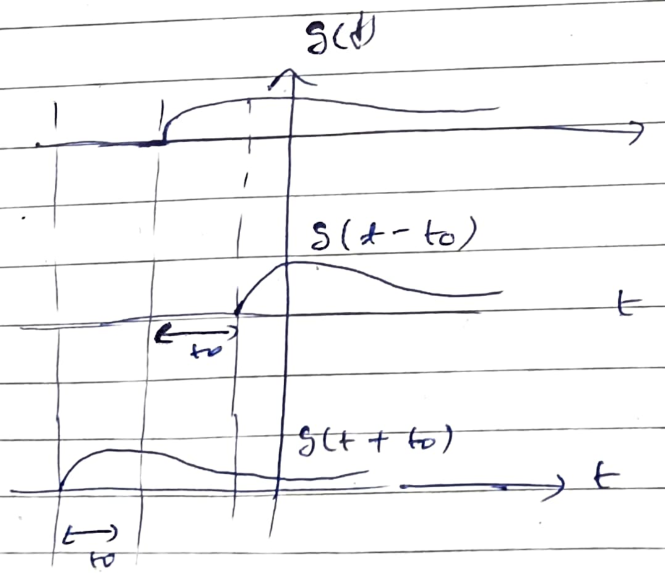
$$\Rightarrow \frac{1}{2} \times \frac{t^3}{3}\Big|_{-1}^{1}$$

$$\Rightarrow \frac{1}{6} - \left(-\frac{1}{6}\right) = \frac{1}{3}$$

* Some useful signal operations
1) Time shifting
2) Time scaling
3) Time inversion

① Time shifting

$g(t)$ shifted by $\phi(t) = g(t - t_0)$ or $g(t + t_0)$



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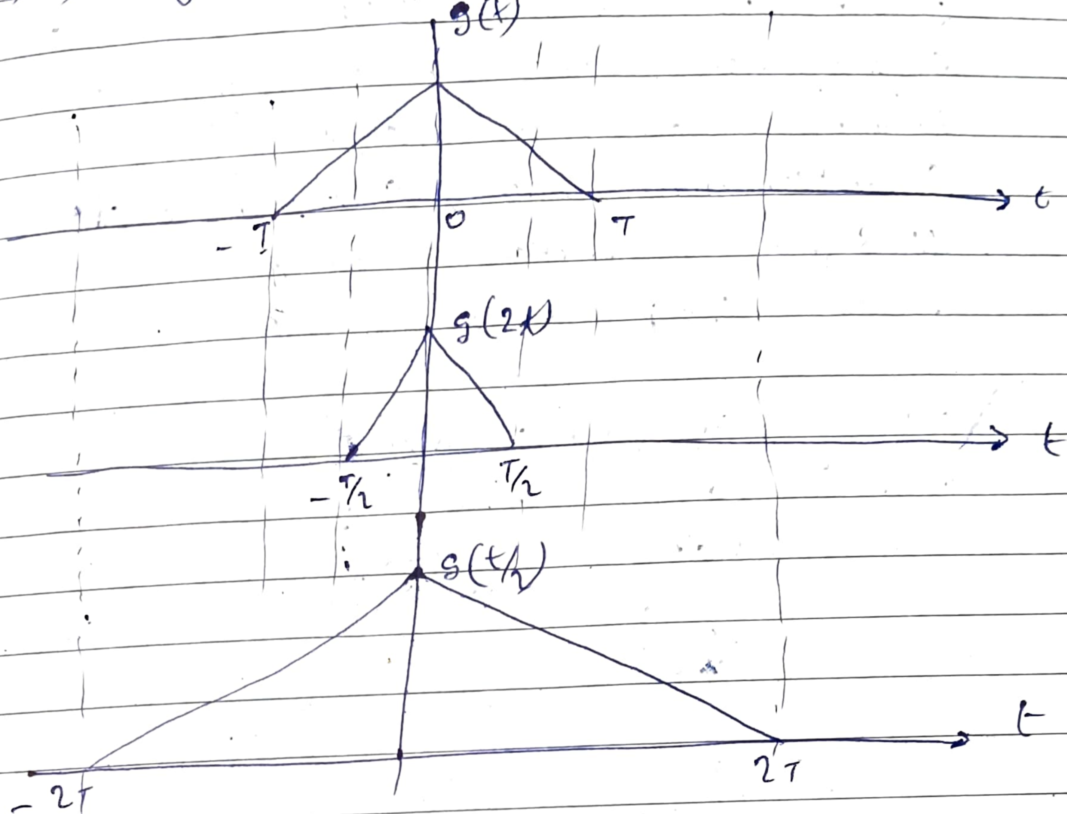
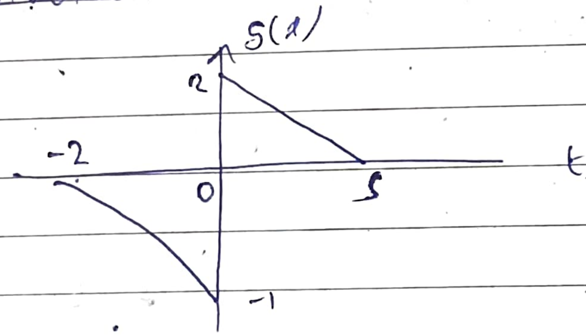
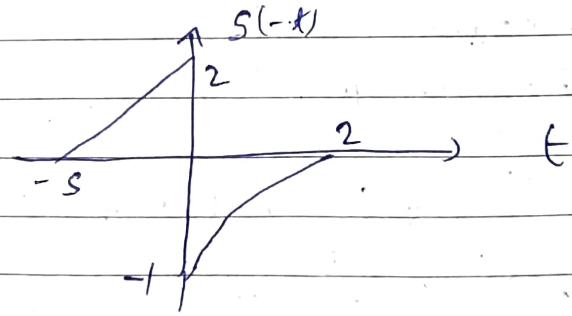
② Time scaling

$g(t)$, $g(at)$, $g(t/a)$



③ Time inversion





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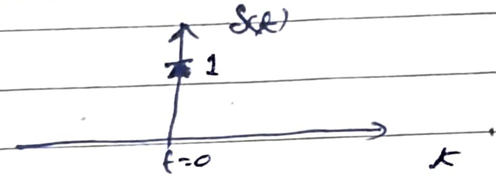
* Some useful signal or functions

1. Unit impulse function ~~f~~ $\delta(t)$
2. Unit step function $u(t)$
3. Unit Gate function or Rectangular pulse $rect(t)$
4. Unit triangle function $\Delta(t)$
5. Ienterpolation function, $Sa(t)$ or ~~Sinc~~ $sinc(t)$
6. Signum function, $Sgn(t)$
7. Unit Ramp signal $r(t)$

1. Unit impulse function $\delta(t)$

$$\delta(t) = 0 \quad , \quad t \neq 0$$
$$\int_{-\infty}^{\infty} \delta(t)\, dt = 1 \quad \text{if } t = 0$$

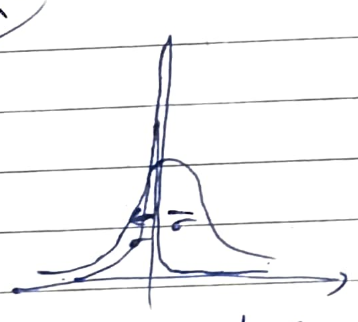


understand

Gaussian distribution function

$$f(x) = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi\sigma^2}}\, e^{-\frac{x^2}{2\sigma^2}}\, dx = 1$$

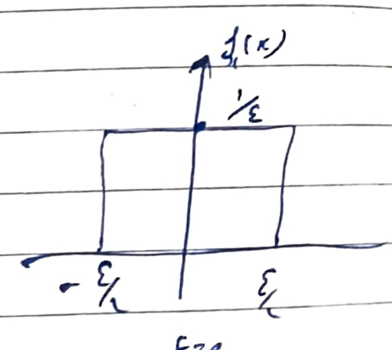
c(LC(-1



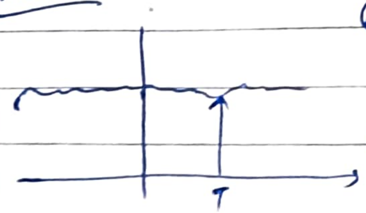
if σ decreases
it become impulse func.

or

uniform distribution function



Now



P-1 $x(t)\,\delta(t) = x(0)\,\delta t$

or

P-2 $x(t)\,\delta(t-T) = x(T)\,\delta(t-T)$

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(P-3) $\int_{-\infty}^{\infty} x(t)\,\delta(t)\,dt = \int_{-\infty}^{\infty} x(0)\,\delta(t)\,dt$

$\Rightarrow x(0)\int_{-\infty}^{\infty} \delta(t)\,dt$

$\Rightarrow x(0)$

Similarly

(P-4) $\int_{-\infty}^{\infty} x(t)\,\delta(t-T)\,dt = \int_{-\infty}^{\infty} x(T)\,\delta(t-T)\,dt$

$\Rightarrow x(T)\int_{-\infty}^{\infty} \delta(t-T)\,dt$

$\Rightarrow x(T)$

(P-5) $\int_{-\infty}^{t} \delta(\tau)\,d\tau = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases} \Rightarrow$ unit step function

Now, $\int_{-\infty}^{t} \delta(\tau)\,d\tau = u(t)$

on differentiating.

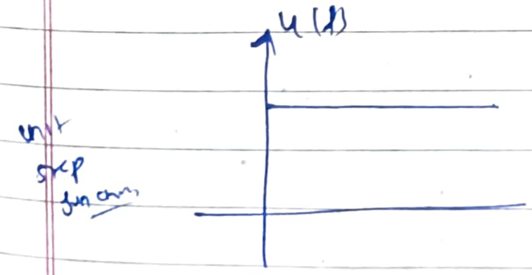
$\delta(t)\frac{dt}{dt} - \delta(-\infty)\frac{d(-\infty)}{dt} = \frac{d}{dt}u(t)$

$$\boxed{\delta(t) = \frac{d}{dt}u(t)}$$

Note:-

Integral of unit impulse function = unit step function

differential of unit step function = unit impulse function.



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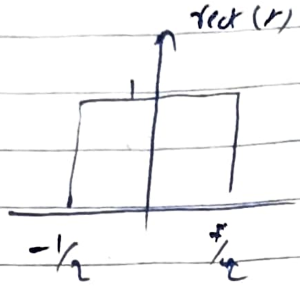
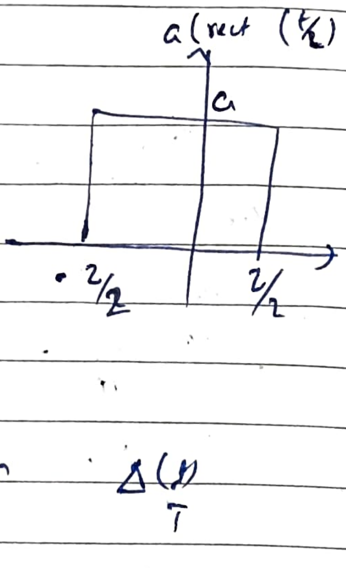
(3) Unit Gate function or rectangular function, rect (t)

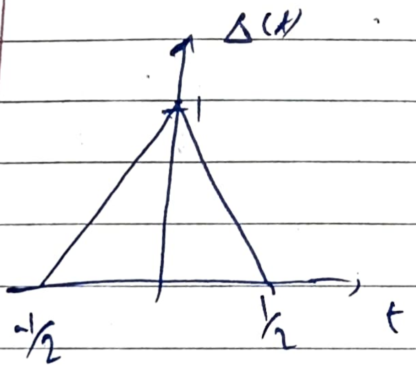


$$\text{rect}(t) = \begin{cases} 1 & |t| < 1/2 \\ 0 & |t| > 1/2 \end{cases}$$

if a rect $\left(\frac{t}{2}\right)$



(3) Unit triangular function $\Delta(t)$
T



$$\Delta t = \begin{cases} 1 - 2|t| & |t| < 1/2 \\ 0 & |t| > 1/2 \end{cases}$$

$\frac{1-0}{0-1/2}$

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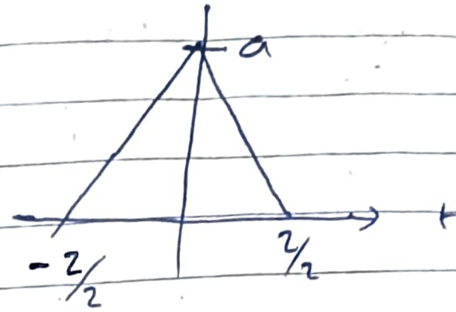
if $a\,\Delta\left(\frac{t}{2}\right)$

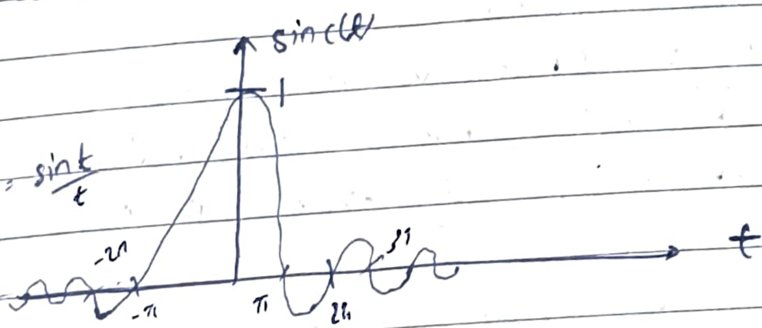


⑤ Interpolation function $Sa(t)$ or $sinc(t)$

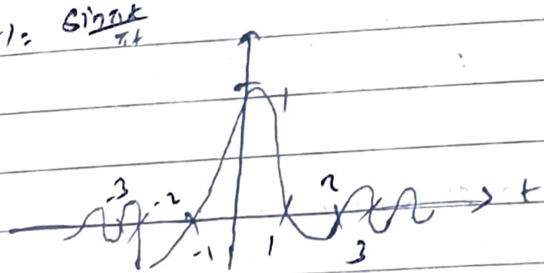
$$sinc(t) = \frac{\sin t}{t} \quad \text{or} \quad \frac{\sin \pi t}{\pi t}$$

at $t=0$, $sinc(t) = 1$ using L-Hopital limit

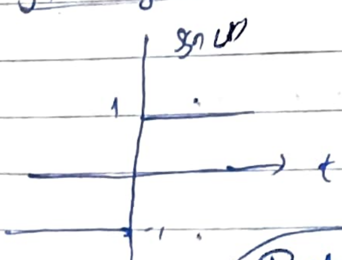
$sinc(t) = \frac{\sin t}{t}$



$sinc(t) = \frac{\sin \pi t}{\pi t}$



⑥ Signum function $sgn(t)$



$$sign(t) = \begin{cases} 1 & \text{for } t>0 \\ -1 & \text{for } t<0 \end{cases}$$

Relationship b/w sgn & unit step funct.

$sgn(t) = 2u(t) - 1$

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Trignometric fourier series → It state that any periodic signal can be represented in sum of sinosoidal signal.

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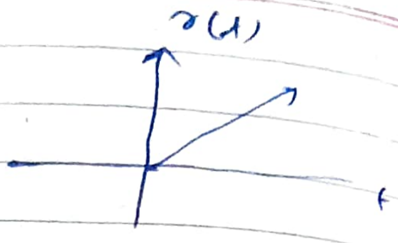
(7) Unit ramp signal : $r(t)$

$$r(t) = \begin{cases} t & \text{for } t \geq 0 \\ 0 & - \end{cases}$$



Fourier series

1. Trignometry Fourier series
2. Compact form Fourier series (Polar fourier series)
3. Complex exponential fourier series.

Dirichlet's condition

1. $x(t)$ = single valued function in T.
2. Finite no of minima and maxima of t.
3. Finite no of discontinuities in interval T.

(1) Trignometric fourier series

$$x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega_0 t + b_n \sin n\omega_0 t$$

$$a_0 = \frac{1}{T}\int_T x(t)\,dt$$

$$a_n = \frac{2}{T}\int_T x(t)\cos n\omega_0 t\,dt$$

$$b_n = \frac{2}{T}\int_T x(t)\sin n\omega_0 t\,dt.$$

if $x(t) \to$ even	if $x(t)$ = odd
a_n = ~~non~~ zero	$a_n = 0$
$b_n = 0$	$b_n \neq 0$

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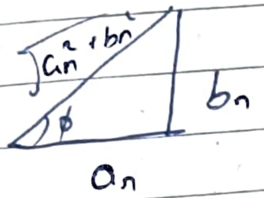
$$x(t) = a_0 + \sum_{n=1}^{\infty} \sqrt{a_n^2+b_n^2}\left[\frac{a_n}{\sqrt{a_n^2+b_n^2}} \cos n\omega t + \frac{b_n}{\sqrt{a_n^2+b_n^2}} \sin n\omega t\right]$$



$$\cos\phi = \frac{a_n}{\sqrt{a_n^2+b_n^2}}, \quad \sin\phi = \frac{-b_n}{\sqrt{a_n^2+b_n^2}}$$

$$x(t) = a_0 + \sum_{n=1}^{\infty} \sqrt{a_n^2+b_n^2}\left[\cos\phi \cos n\omega t - \sin\phi \sin n\omega t\right]$$

$$x(t) = a_0 + \sum_{n=1}^{\infty} \sqrt{a_n^2+b_n^2} \cos(n\omega_0 t + \phi)$$

$$\boxed{x(t) = D_0 + \sum_{n=1}^{\infty} D_n \cos(n\omega_0 t + \phi)}$$ Compact fourier series.

unknown, D_0, D_n, ϕ

$$D_0 = a_0 = \frac{1}{T}\int_T x(t)\,dt$$

$$D_n = \sqrt{a_n^2+b_n^2}$$

$$\phi = \tan^{-1}\left(\frac{-b_n}{a_n}\right)$$

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(3) Complex exponential fourier series

$$x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega_0 t + \sum_{n=1}^{\infty} b_n \sin n\omega_0 t$$

$$= a_0 + \sum_{n=1}^{\infty} a_n \left(\frac{e^{jn\omega_0 t} + e^{-jn\omega_0 t}}{2} \right) + b_n \left(\frac{e^{jn\omega_0 t} - e^{-jn\omega_0 t}}{2j} \right)$$

$$\Rightarrow a_0 + \sum_{n=1}^{\infty} \left[\frac{(a_n - jb_n) e^{jn\omega_0 t}}{2} + \frac{(a_n + jb_n) e^{-jn\omega_0 t}}{2} \right]$$

$$C_n = \frac{a_n - jb_n}{2} \qquad C_{-n} = \frac{a_n + jb_n}{2}$$

$\cancel{C_n = a_n +}$

$$x(t) = C_0 + \sum_{n=1}^{\infty} C_n e^{jn\omega_0 t} + \sum_{n=1}^{\infty} C_{-n} e^{-jn\omega_0 t}$$

$\Downarrow$

$n \rightarrow -n$

$$x(t) = C_0 + \sum_{n=1}^{\infty} C_n' e^{jn\omega_0 t} + \sum_{n=-\infty}^{-1} C_n e^{jn\omega_0 t}$$

$(\cancel{1}, \infty)$ $(-\infty, -1)$

So, any 0 let, when $n=0$, C_0

$$\boxed{x(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t}}$$

Now,

$$C_n = \frac{1}{2}(a_n - jb_n)$$

$$\Rightarrow \frac{1}{T} \int_T x(t) \left[\cos n\omega_0 t - j \sin n\omega_0 t \right]$$

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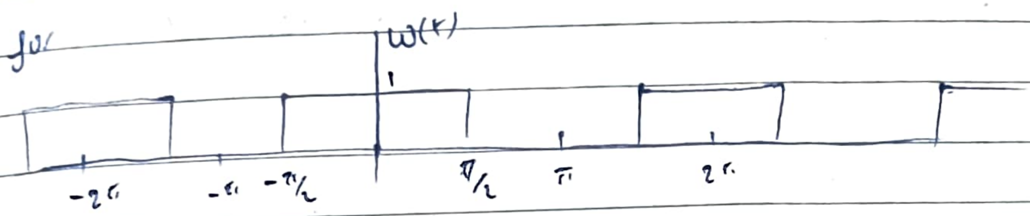
$$C_n = \frac{1}{T}\int_T x(t)\, e^{-jn\omega_0 t}\, dt$$

$|C_n| \Rightarrow$ Amplitude spectrum
$\angle C_n \Rightarrow$ Phase spectrum

Q) Calculate the trignometric fourier series for



Since even function, $b_n = 0$

$$a_0 = \frac{1}{T}\int_T x(t)\, dt$$

$$\Rightarrow \frac{1}{2\pi}\int_{-\pi/2}^{\pi/2} dt$$

$$= \frac{1}{2\pi} \times \pi \Rightarrow \frac{1}{2}$$

$$a_n = \frac{2}{T}\int_T x(t)\cos n\omega_0 t\, dt$$

$$\Rightarrow \frac{2}{2\pi}\int_{-\pi/2}^{\pi/2} \cos n\omega_0 t\, dt$$

$$\Rightarrow \frac{1}{\pi} \cdot \cancel{n\omega_0} \times \frac{1}{n\omega_0} \sin n\omega_0 t \Big|_{-\pi/2}^{\pi/2}$$

$$\Rightarrow \omega_0 = \frac{2\pi}{T_0}, \frac{2\pi}{2\pi} \Rightarrow 1$$

$$\Rightarrow \frac{1}{\pi} \times \frac{1}{n} \times \left[\sin n\frac{\pi}{2} + \sin n\frac{\pi}{2}\right]$$

$$a_n \Rightarrow \frac{2}{n\pi} \sin n\frac{\pi}{2}$$

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$$a_n = \begin{cases} 0 & n \text{ even} \\ \frac{2}{n\pi} & n = 1, 5, 9, 13 \\ \frac{-2}{n\pi} & n = 3, 7, 11 \end{cases}$$

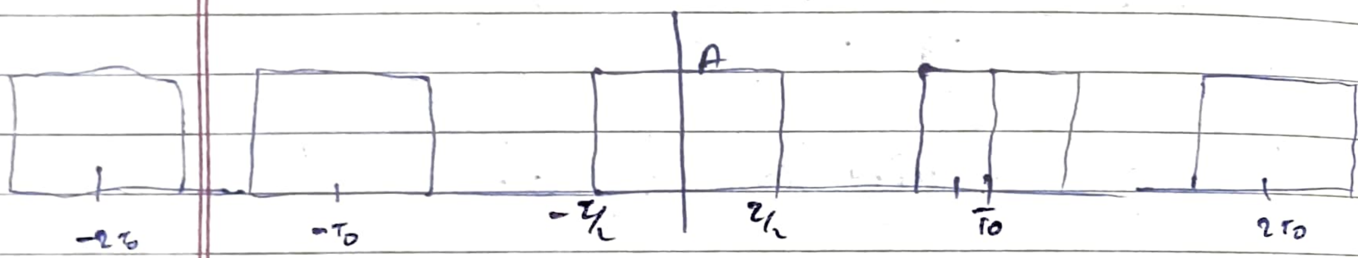
$$w(t) = \frac{1}{2} + \frac{2}{\pi}\left[\cos \omega_0 t - \frac{1}{3}\cos 3\omega_0 t + \frac{1}{5}\cos 5\omega_0 t \ldots\right.$$

Or

$$a_n = \frac{2}{n\pi} \sin \frac{n\pi}{2} = \frac{\sin \frac{n\pi}{2}}{\left(\frac{n\pi}{2}\right)} \Rightarrow \operatorname{sinc} \frac{n}{2}$$

$$x(t) = \frac{1}{2} \sum_{n=1}^{\infty} \operatorname{sinc} \frac{n}{2} \cdot \cos n\omega_0 t$$

Q Calculate exponential fourier series.



$$x(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t}$$

$$C_n = \frac{1}{T_0}\int_T x(t)\, e^{-jn\omega_0 t}\, dt$$

$$\Rightarrow \frac{1}{T_0}\int_{-\tau/2}^{\tau/2} A\, e^{-jn\omega_0 t}\, dt$$

$$\Rightarrow \frac{-A}{T_0 jn\omega_0} e^{-jn\omega_0 t}\Big|_{-\tau/2}^{\tau/2} \Rightarrow \frac{A}{T_0 jn\omega_0}\left[e^{\frac{jn\omega_0 \tau}{2}} - e^{\ldots}\right.$$

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$$W_0 = \frac{2\pi}{T_0}, \quad \frac{2\pi}{T_0}$$

$$C_n = A\left[\frac{-e^{-j\pi n \frac{z}{T_0}} + e^{j\pi n \frac{z}{T_0}}}{2j\pi n}\right]$$

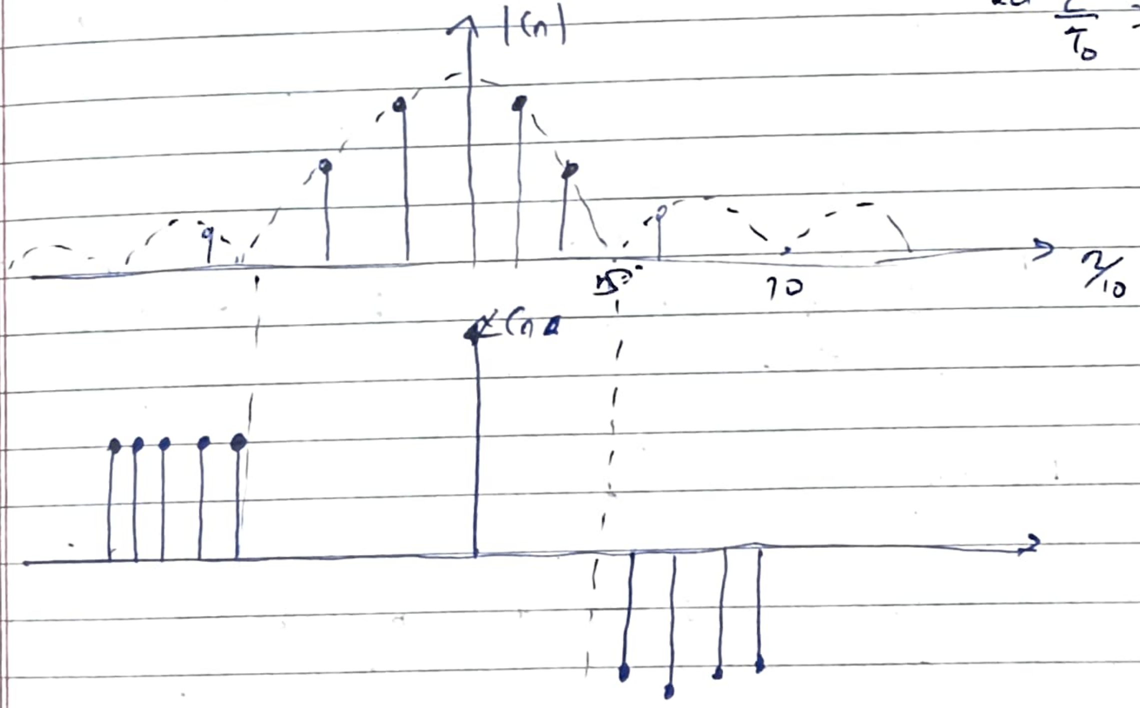
$$\Rightarrow \frac{A}{2\pi} \sin \frac{\pi n z}{T_0}$$

Writing in sinc form

$$\Rightarrow \frac{Az}{T_0} \frac{\sin \pi n z/T_0}{\pi n z/T_0} \Rightarrow \frac{Az}{T_0} \operatorname{sinc} \frac{nz}{T_0}$$

$|C_n|$ amplitude spectrum

let $\frac{z}{T_0} = 0.1$



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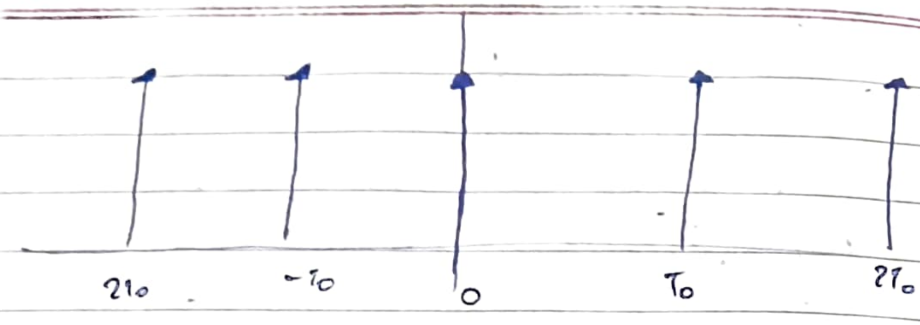
Q



* Calcute trignometric fourier

$a_0 = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} \delta(t)\, dt$

$a_0 \Rightarrow \frac{1}{T_0}$

$a_n \Rightarrow \frac{2}{T_0}\int_{-T_0/2}^{T_0/2} \delta(t) \cos n\omega_0 t \, dt$ (put $t = 0$) $\delta(t)$ ↳ $t=0$ function value

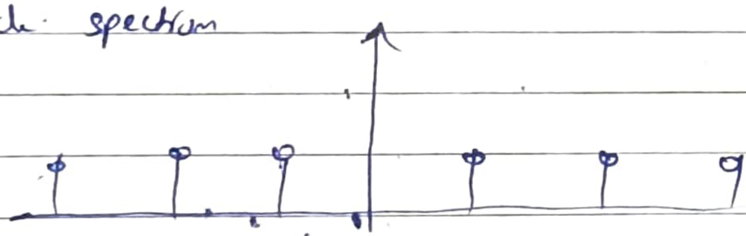
$\Rightarrow \frac{2}{T_0}$

$b_n = 0$

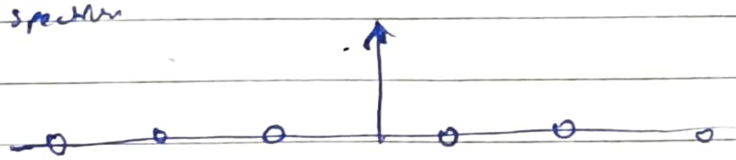
* exponential form series.

$C_n = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} \delta(t)\, e^{-jn\omega_0 t}\, dt = \frac{1}{T_0}$

Amplitude spectrum



phase spectrum



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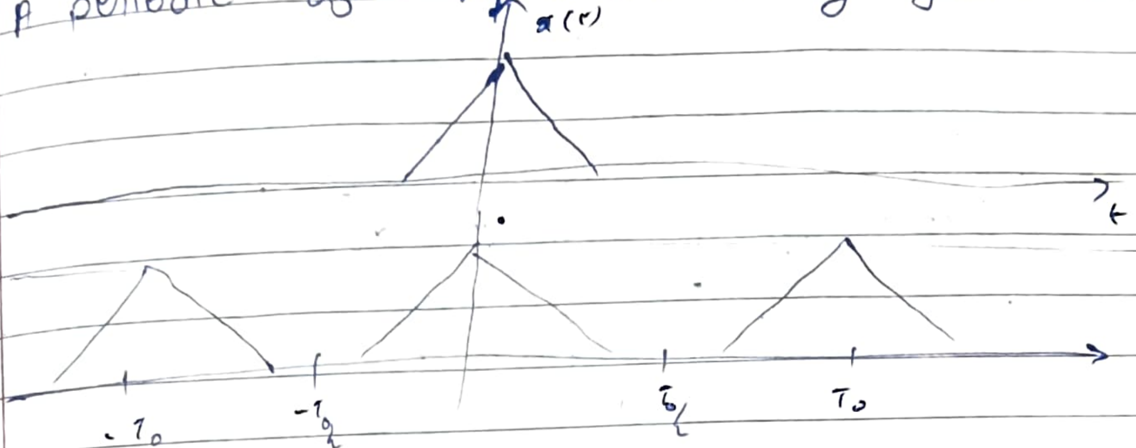
* Fourier Transform

A periodic signal representation by fourier integral



$$\boxed{\lim_{T_0 \to \infty} x_{T_0}(t) = x(t)}$$

$$x_{T_0}(t) = \sum_{n=-\infty}^{\infty} C_n e^{-jn\omega_0 t} \quad \text{--- (1)}$$

$$C_n = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} x_{T_0}(t)\, e^{-jn\omega_0 t}\, dt$$

$$\omega_0 = \frac{2\pi}{T_0}, \quad T_0 \to \infty, \quad \omega_0 \to 0$$

$$C_n = \frac{1}{T_0} \int_{-\infty}^{\infty} x(t)\, e^{-jn\omega_0 t}\, dt$$

let us assume, $x(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$

$$C_n = \frac{1}{T_0} x(n\omega_0) \qquad \text{putting in (1)}$$

$$X_{T_0}(t) = \sum_{n=-\infty}^{\infty} \frac{x(n\omega_0)}{T_0} e^{jn\omega_0 t}$$

$T_0 \to \infty$, $\omega_0 \to 0$, $\omega_0 = \Delta\omega_0$

$$x_{T_0}(t) = \sum_{n=-\infty}^{\infty} \frac{x(n\Delta\omega)\Delta\omega}{2\pi} e^{jn\Delta\omega t}$$

since $x(t) = \lim_{T \to \infty} X_{T_0}(t) = \lim_{\Delta\omega \to 0} \frac{1}{2\pi} \sum x(n\Delta\omega) e^{jn\Delta\omega}$

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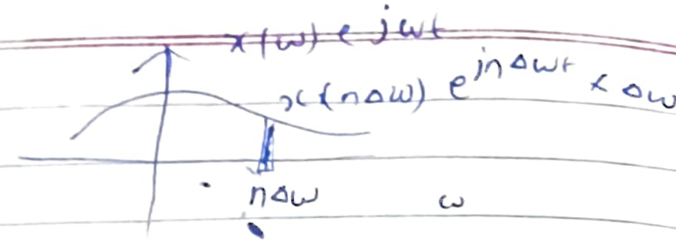
let us understand



area under curve.

$$\boxed{x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)\cdot e^{j\omega t}\,d\omega}$$

or

$x(t)$ $\quad$ $X(\omega)$ $\quad$ $\omega = 2\pi f$

$X(f)$

$$X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\,dt$$

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)\, e^{j\omega t}\,d\omega$$

$$X(f) = \int_{-\infty}^{\infty} x(t)\cdot e^{-j2\pi f t}\,dt$$

$$x(t) = \int_{-\infty}^{\infty} X(f)\, e^{j2\pi f t}\,df$$

① Finite no of maxima, minima or finite no of discontinuity
~~Integrable~~ It should be absolutely integrable.

$$\int_{-\infty}^{\infty} |x(t)|\,dt < \infty$$

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* properties of fourier transform

① linearity property

$x_1(t) \longleftrightarrow x_1(\omega)$ or $x_1(f)$

$x_2(t) \longleftrightarrow x_2(\omega)$ or $x_2(f)$

$a_1 x_1(t) + a_2 x_2(t) \longleftrightarrow a_1 X_1(\omega) + a_2 X_2(\omega)$

$a_1 x_1(t) + a_2 x_2(t) \longleftrightarrow a_1 X_1(f) + a_2 X_2(f)$

$$X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$$X'(\omega) = \int_{-\infty}^{\infty} (a_1 x_1(t) + a_2 x_2(t))\, e^{-j\omega t}\, dt$$

$$\Rightarrow a_1 \int_{-\infty}^{\infty} x_1(t)\, e^{-j\omega t}\, dt + a_2 \int_{-\infty}^{\infty} x_2(t)\, e^{-j\omega t}\, dt$$

$$X'(\omega) = a_1 X_1(\omega) + a_2 X_2(\omega)$$

② Time scaling property.

$x(t) \longleftrightarrow X(\omega)$

$x(at) \longleftrightarrow \frac{1}{|a|} X\left(\frac{\omega}{a}\right)$

$x(at) \longleftrightarrow \frac{1}{|a|} X\left(f/a\right)$

proof

$$X(\omega) = F[x[at]] = \int_{-\infty}^{\infty} x(at)\, e^{-j\omega t}\, dt$$

let $at = u$

$dt = du/a$

Case I $a > 0$

$$F[x(at)] \Rightarrow \int_{-\infty}^{\infty} x(u)\, e^{-j\omega u/a}\, \frac{du}{a} = \frac{1}{a} X\left(\frac{\omega}{a}\right)$$

Case II $a < 0$

$$F[x(at)] \Rightarrow -\int_{-\infty}^{\infty} x(u) \quad \underline{\qquad\qquad}$$

So, $F[x[at]] \Rightarrow \frac{1}{|a|} X\left(\frac{\omega}{a}\right)$

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Q) Find the fourier transform of $F[e^{-t} u(t)]$

$x(t) = e^{-t} u(t)$

$X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$

$= \int_{-\infty}^{\infty} e^{-t}\, e^{-j\omega t}\, dt$

$\Rightarrow \int_{0}^{\infty} e^{-t(1+j\omega)}\, dt$

$\Rightarrow \frac{1}{1+j\omega}$

$X(f) = \frac{1}{1+j2\pi f}$

Q) Find fourier transform of $F[e^{-at} u(t)]$

$x(t) = e^{-at} u(t)$

~~$X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$~~

$\cancel{\Rightarrow \int_{0}^{\infty} e^{-at}\, e^{-j\omega t}\, dt}$

$\cancel{\Rightarrow \frac{1}{1+ja\omega}}$

$F(x(t)) = X(\omega)$ let $a>0$

$F[x(at)] = \frac{1}{|a|} X(\omega/a)$

$= \frac{1}{a} X\left(\frac{\omega}{a}\right)$

$\Rightarrow \frac{1}{a} \times \frac{1}{1+j\omega/a}$

$F[e^{-at} u(t)] = \frac{1}{a+j\omega}$

$= u(-t)$

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Q $F[e^{at} u(-t)]$ let $a > 0$

$\Rightarrow \int_{-\infty}^{0} e^{at} e^{-j\omega t} dt$

$\Rightarrow \int_{-\infty}^{0} e^{t(a-j\omega)} dt$

$\Rightarrow \frac{1}{a-j\omega} e^{t(a-j\omega)} \Big|_{-\infty}^{0} = \frac{1}{a-j\omega}$

Q $F[e^{at} u(t)]$ let $a > 0$

$F[e^{-at} u(-t)]$ does not exist

Q $F[\delta(t)]$

$F[\delta(t)] = \int_{-\infty}^{\infty} \delta(t) e^{-j\omega t} dt = 1$

$\delta(t) \longleftrightarrow 1$

Q Find inverse fourier transform of $\delta(\omega)$

$F^{-1}[\delta(\omega)] = \frac{1}{2\pi} \int_{-\infty}^{\infty} \delta(\omega) e^{j\omega t} d\omega$

$\omega = 0$ function value $= 1$

$\Rightarrow \frac{1}{2\pi}$

$\delta(\omega) \longleftrightarrow F\left[\frac{1}{2\pi}\right]$

$\boxed{F(1) \Rightarrow 2\pi\, \delta(\omega)}$ How to find fourier transform of 1 $F(1)$