Applications of Catalan Numbers

Find a pair (n,r) in an integer array such that value of nPr is maximum

Color all boxes in line such that every M consecutive boxes are unique

Minimize the cost of partitioning an array into K groups

Count total number of even sum sequences

Find a pair (n,r) in an integer array such that value of nCr is maximum

Check whether N is a Factorion or not

Count of Multiples of A ,B or C less than or equal to N

Convert N to M with given operations using dynamic programming

Find out the correct position of the ball after shuffling

Check if two given Circles are Orthogonal or not

Sum of numbers in a range [L, R] whose count of divisors is prime

Count number of binary strings such that there is no substring of length greater than or equal to 3 with all 1's

Array containing power of 2 whose XOR and Sum of elements equals X

Count of N-digit numbers in base K with no two consecutive zeroes

Printing the Triangle Pattern using last term

▲

N

Nth number in a set of multiples of A , B or C

Reduce N to 1 with minimum number of given operations

Check if it is possible to move from (0, 0) to (X, Y) in exactly K steps

Number of words that can be made using exactly P consonants and Q vowels from the given string

Find the possible permutation of the bits of N

Find the sum of prime numbers in the Kth array

Kth number from the set of multiples of numbers A, B and C

Find the permutation of first N natural numbers such that sum of i %

P_i is maximum possible

Count number of Special Set

Sum of values of all possible non-empty subsets of the given array

Queries for the product of first N factorials

Find the next fibonacci number

Find numbers which are multiples of first array and factors of second array

Find the minimum value of X for an expression

▲

Applications of Catalan Numbers

Background :

Catalan numbers are defined using below formula:

$$C_n = (2n)!/(n+1)!n! = \prod_{k=2}^{n} \frac{n+k}{k} for_n >= 0$$

Catalan numbers can also be defined using following recursive formula.

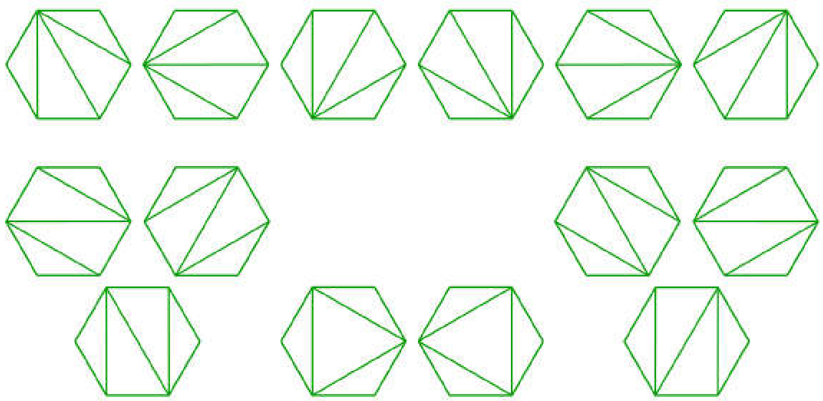
$$C_0 = 1 C_{n+1} = \sum_{i=0}^{n} C_i C_{n-i} for_n >= 0$$

The first few Catalan numbers for n = 0, 1, 2, 3, … are **1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862, …**

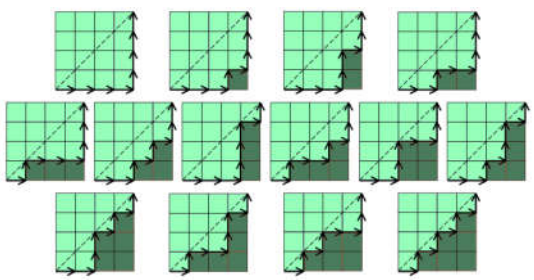
Refer this for implementation of n'th Catalan Number.

Applications :

1. Number of possible Binary Search Trees with n keys.
2. Number of expressions containing n pairs of parentheses which are correctly matched. For n = 3, possible expressions are ((())), ()(()), ()()(), (())(), (()()).
3. Number of ways a convex polygon of n+2 sides can split into triangles by connecting vertices.

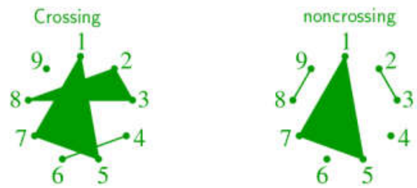


4. Number of full binary trees (A rooted binary tree is full if every vertex has either two children or no children) with n+1 leaves.
5. Number of different Unlabeled Binary Trees can be there with n nodes.
6. The number of paths with 2n steps on a rectangular grid from bottom left, i.e., (n-1, 0) to top right (0, n-1) that do not cross above the main diagonal.

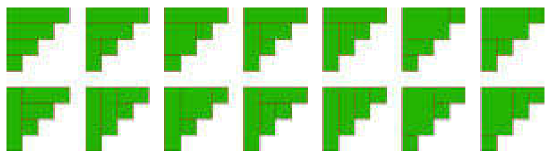


7. Number of ways to insert n pairs of parentheses in a word of n+1 letters, e.g., for n=2 there are ways: ((ab)c) or (a(bc)). For n=3 there are 5 ways, ((ab)(cd)), (((ab)c)d), ((a(bc))d), (a((bc)d)), (a(b(cd))).

8. Number of noncrossing partitions of the set {1, …, 2n} in which every block is of size 2. A partition is noncrossing if and only if in its planar diagram, the blocks are disjoint (i.e. don't cross). For example, below two are crossing and non-crossing partitions of {1, 2, 3, 4, 5, 6, 7, 8, 9}. The partition {{1, 5, 7}, {2, 3, 8}, {4, 6}, {9}} is crossing and partition {{1, 5, 7}, {2, 3}, {4}, {6}, {8, 9}} is non-crossing.



9. Number of Dyck words of length 2n. A Dyck word is a string consisting of n X's and n Y's such that no initial segment of the string has more Y's than X's. For example, the following are the Dyck words of length 6: XXXYYY XYXXYY XYXYXY XXYYXY XXYXYY.
10. Number of ways to tile a stairstep shape of height n with n rectangles. The following figure illustrates the case n = 4:



11. Number of ways to connect the points on a circle disjoint chords. This is similar to point 3 above.
12. Number of ways to form a "mountain ranges" with n upstrokes and n down-strokes that all stay above the original line.The mountain range interpretation is that the mountains will never go below the horizon.

Mountain Ranges

13. Number of stack-sortable permutations of {1, …, n}. A permutation w is called stack-sortable if S(w) = (1, …, n), where S(w) is defined recursively as follows: write w = unv where n is the largest element in w and u and v are shorter sequences, and set S(w) = S(u)S(v)n, with S being the identity for one-element sequences.
14. Number of permutations of {1, …, n} that avoid the pattern 123 (or any of the other patterns of length 3); that is, the number of permutations with no three-term increasing subsequence. For n = 3, these permutations are 132, 213, 231, 312 and 321. For n = 4, they are 1432, 2143, 2413, 2431, 3142, 3214, 3241, 3412, 3421, 4132, 4213, 4231, 4312 and 4321

Sources:

1. https://en.wikipedia.org/wiki/Catalan_number

2. http://mathworld.wolfram.com/CatalanNumber.html
3. http://www-groups.dcs.st-and.ac.uk/history/Miscellaneous/CatalanNumbers/catalan.html
4. http://www.mhhe.com/math/advmath/rosen/r5/instructor/applications/ch07.pdf
5. https://oeis.org/A000108

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Fill the missing numbers in the array of N natural numbers such that arr[i] not equal to i

Numbers less than N which are product of exactly two distinct prime numbers

Numbers within a range that can be expressed as power of two numbers

Print N lines of 4 numbers such that every pair among 4 numbers has a GCD K

Count numbers which can be constructed using two numbers

Count numbers which are divisible by all the numbers from 2 to 10

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