2005 AMC 12A

1. Two is 10% of x and 20% of y. What is $x - y$?

   (**A**) 1 (**B**) 2 (**C**) 5 (**D**) 10 (**E**) 20

2. The equations $2x + 7 = 3$ and $bx - 10 = -2$ have the same solution. What is the value of b?

   (**A**) -8 (**B**) -4 (**C**) 2 (**D**) 4 (**E**) 8

3. A rectangle with diagonal length x is twice as long as it is wide. What is the area of the rectangle?

   (**A**) $\frac{1}{4}x^2$ (**B**) $\frac{2}{5}x^2$ (**C**) $\frac{1}{2}x^2$ (**D**) x^2 (**E**) $\frac{3}{2}x^2$

4. A store normally sells windows at $100 each. This week the store is offering one free window for each purchase of four. Dave needs seven windows and Doug needs eight windows. How much will they save if they purchase the windows together rather than separately?

   (**A**) 100 (**B**) 200 (**C**) 300 (**D**) 400 (**E**) 500

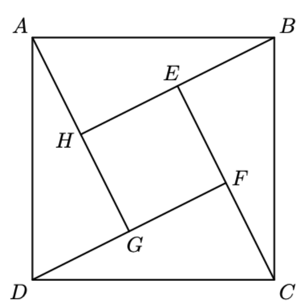
5. The average (mean) of 20 numbers is 30, and the average of 30 other numbers is 20. What is the average of all 50 numbers?

   (**A**) 23 (**B**) 24 (**C**) 25 (**D**) 26 (**E**) 27

6. Josh and Mike live 13 miles apart. Yesterday, Josh started to ride his bicycle toward Mike's house. A little later Mike started to ride his bicycle toward Josh's house. When they met, Josh had ridden for twice the length of time as Mike and at four-fifths of Mike's rate. How many miles had Mike ridden when they met?

   (**A**) 4 (**B**) 5 (**C**) 6 (**D**) 7 (**E**) 8

7. Square $EFGH$ is inside the square $ABCD$ so that each side of $EFGH$ can be extended to pass through a vertex of $ABCD$. Square $ABCD$ has side length $\sqrt{50}$ and $BE = 1$. What is the area of the inner square $EFGH$?



   (**A**) 25 (**B**) 32 (**C**) 36 (**D**) 40 (**E**) 42

8. Let A, M, and C be digits with

$$(100A + 10M + C)(A + M + C) = 2005$$

   What is A?

   (**A**) 1 (**B**) 2 (**C**) 3 (**D**) 4 (**E**) 5

9. There are two values of a for which the equation $4x^2 + ax + 8x + 9 = 0$ has only one solution for x. What is the sum of these values of a?

   (**A**) -16 (**B**) -8 (**C**) 0 (**D**) 8 (**E**) 20

10. A wooden cube n units on a side is painted red on all six faces and then cut into n^3 unit cubes. Exactly one-fourth of the total number of faces of the unit cubes are red. What is n?

    (**A**) 3 (**B**) 4 (**C**) 5 (**D**) 6 (**E**) 7

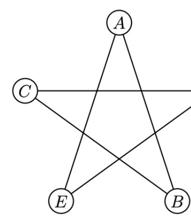
11. How many three-digit numbers satisfy the property that the middle digit is the average of the first and the last digits?

    (**A**) 41 (**B**) 42 (**C**) 43 (**D**) 44 (**E**) 45

12. A line passes through $A\ (1,1)$ and $B\ (100,1000)$. How many other points with integer coordinates are on the line and strictly between A and B?

    (**A**) 0 (**B**) 2 (**C**) 3 (**D**) 8 (**E**) 9

13. In the five-sided star shown, the letters A, B, C, D and E are replaced by the numbers 3, 5, 6, 7 and 9, although not necessarily in that order. The sums of the numbers at the ends of the line segments $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, $\overline{DE}$, and $\overline{EA}$ form an arithmetic sequence, although not necessarily in that order. What is the middle term of the arithmetic sequence?

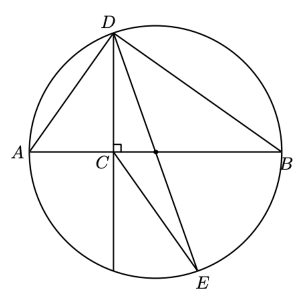


    (**A**) 9 (**B**) 10 (**C**) 11 (**D**) 12 (**E**) 13

14. On a standard die one of the dots is removed at random with each dot equally likely to be chosen. The die is then rolled. What is the probability that the top face has an odd number of dots?

    (**A**) $\frac{5}{11}$ (**B**) $\frac{10}{21}$ (**C**) $\frac{1}{2}$ (**D**) $\frac{11}{21}$ (**E**) $\frac{6}{11}$

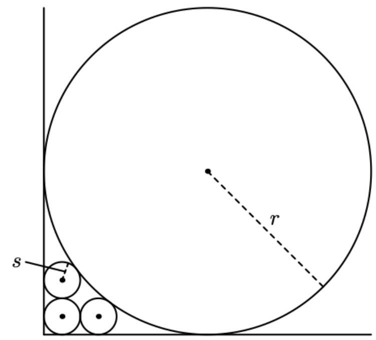
15. Let $\overline{AB}$ be a diameter of a circle and C be a point on $\overline{AB}$ with $2 \cdot AC = BC$. Let D and E be points on the circle such that $\overline{DC} \perp \overline{AB}$ and $\overline{DE}$ is a second diameter. What is the ratio of the area of $\triangle DCE$ to the area of $\triangle ABD$?



    (**A**) $\frac{1}{6}$ (**B**) $\frac{1}{4}$ (**C**) $\frac{1}{3}$ (**D**) $\frac{1}{2}$ (**E**) $\frac{2}{3}$

16. Three circles of radius s are drawn in the first quadrant of the xy-plane. The first circle is tangent to both axes, the second is tangent to the first circle and the x-axis, and the third is tangent to the first circle and the y-axis. A circle of radius $r > s$ is tangent to both axes and to the second and third circles. What is r/s?

    (**A**) 5 (**B**) 6 (**C**) 8 (**D**) 9 (**E**) 10



17. A unit cube is cut twice to form three triangular prisms, two of which are congruent, as shown in Figure 1. The cube is then cut in the same manner along the dashed lines shown in Figure 2. This creates nine pieces. What is the volume of the piece that contains vertex W?

(A) $\frac{1}{12}$ **(B)** $\frac{1}{9}$ **(C)** $\frac{1}{8}$ **(D)** $\frac{1}{6}$ **(E)** $\frac{1}{4}$

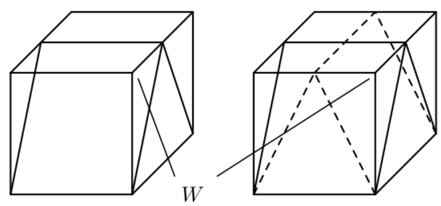


Figure 1 Figure 2

18. Call a number "prime-looking" if it is composite but not divisible by 2, 3, or 5. The three smallest prime-looking numbers are 49, 77, and 91. There are 168 prime numbers less than 1000. How many prime-looking numbers are there less than 1000?

(A) 100 **(B)** 102 **(C)** 104 **(D)** 106 **(E)** 108

19. A faulty car odometer proceeds from digit 3 to digit 5, always skipping the digit 4, regardless of position. If the odometer now reads 002005, how many miles has the car actually traveled?

(A) 1404 **(B)** 1462 **(C)** 1604 **(D)** 1605 **(E)** 1804

20. For each x in $[0, 1]$, define

$$\begin{cases} f(x) = 2x, & \textbf{if} \quad 0 \le x \le \frac{1}{2}; \\ f(x) = 2 - 2x, & \textbf{if} \quad \frac{1}{2} < x \le 1. \end{cases}$$

Let $f^{[2]}(x) = f(f(x))$, and $f^{[n+1]}(x) = f^{[n]}(f(x))$ for each integer $n \ge 2$. For how many values of x in $[0, 1]$ is $f^{[2005]}(x) = \frac{1}{2}$?

(A) 0 **(B)** 2005 **(C)** 4010 **(D)** 2005^2 **(E)** 2^{2005}

21. How many ordered triples of integers (a, b, c), with $a \ge 2$, $b \ge 1$, and $c \ge 0$, satisfy both $\log_a b = c^{2005}$ and $a + b + c = 2005$?

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

22. A rectangular box P is inscribed in a sphere of radius r. The surface area of P is 384, and the sum of the lengths of its 12 edges is 112. What is r?

(A) 8 **(B)** 10 **(C)** 12 **(D)** 14 **(E)** 16

23. Two distinct numbers a and b are chosen randomly from the set $\{2, 2^2, 2^3, \ldots, 2^{25}\}$. What is the probability that $\log_a b$ is an integer?

(A) $\frac{2}{25}$ **(B)** $\frac{31}{300}$ **(C)** $\frac{13}{100}$ **(D)** $\frac{7}{50}$ **(E)** $\frac{1}{2}$

24. Let $P(x) = (x-1)(x-2)(x-3)$. For how many polynomials $Q(x)$ does there exist a polynomial $R(x)$ of degree 3 such that $P(Q(x)) = P(x) \cdot R(x)$?

(A) 19 **(B)** 22 **(C)** 24 **(D)** 27 **(E)** 32

25. Let S be the set of all points with coordinates (x, y, z), where x, y, and z are each chosen from the set $\{0, 1, 2\}$. How many equilateral triangles have all their vertices in S?

(A) 72 **(B)** 76 **(C)** 80 **(D)** 84 **(E)** 88