Modelling with CP

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Overview

- Formulation in CP
  - Vocabulary (variables, domains)
  - Constraints

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- ReFormulation-s in CP
  - Auxiliary variables / Channeling constraints
  - Redundant constraints
  - Global constraints
  - Symmetry breaking

Motivations

Numerous CP modeling languages and platforms have been developed (OPL, ZINC, GECODE, CHOCO…) for solving combinatorial problems that arise in optimization, planning, scheduling…

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Continuous Auctions
(Lozano et al., CP10)
Portfolio Designs
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Air traffic management
(Flener et al., JATM07)
Railway management
(Chiu et al., Constraints02; Rodriguez & Kermad, Comprail98)

Constraint Programming

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  - Vocabulary (X,D)=(variables, domains)
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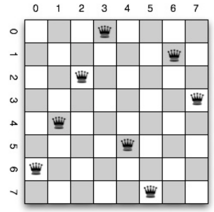

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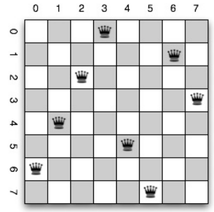

CSP:

Variables: $X = \{X_i | i \in [1,n]\}$

Domains: $X_i \in X, \ D(X_i) = \{1, 2, \ldots, n\}$

Constraints:

- $C_{lines} = \{X_i \neq X_j \mid i,j \in [1,n], i \neq j\}$
- $C_{diag1} = \{X_i \neq X_j + j - i \mid i,j \in [1,n], i \neq j\}$
- $C_{diag2} = \{X_i \neq X_j + i - j \mid i,j \in [1,n], i \neq j\}$

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CP = Model + Resolution

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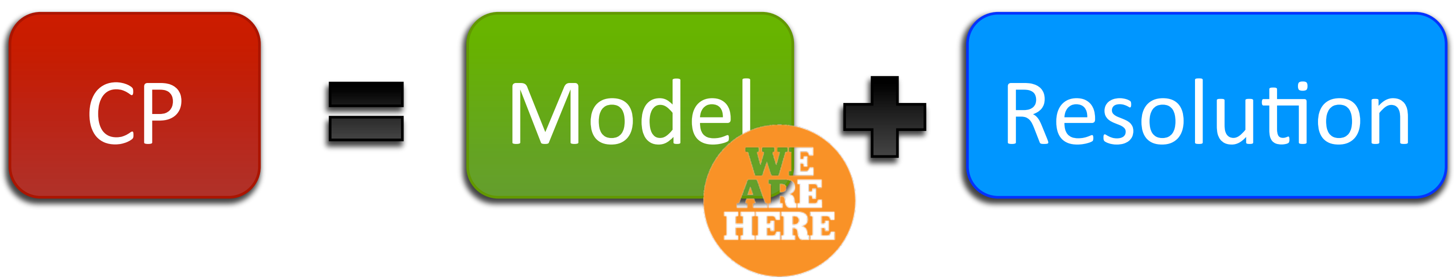

Problem = conjunction of sub-Problems

- In CP a problem can be viewed as a conjunction of sub-problems that we are able to solve
- A sub-problem can be trivial ($x \neq y$), or complex (cumulative resources of limited capacities)

    → sub-problem = constraint

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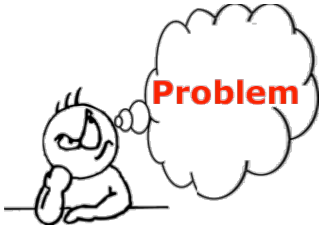
- Logical combination of constraints using OR, AND, NOT, XOR operators.

Modelling in CP



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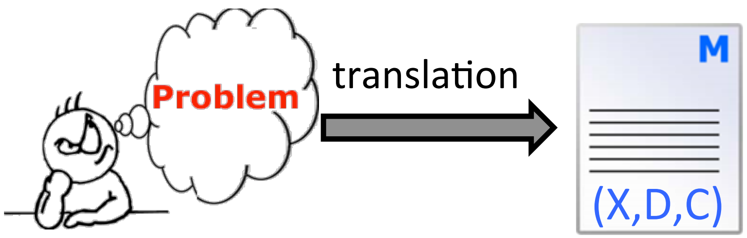

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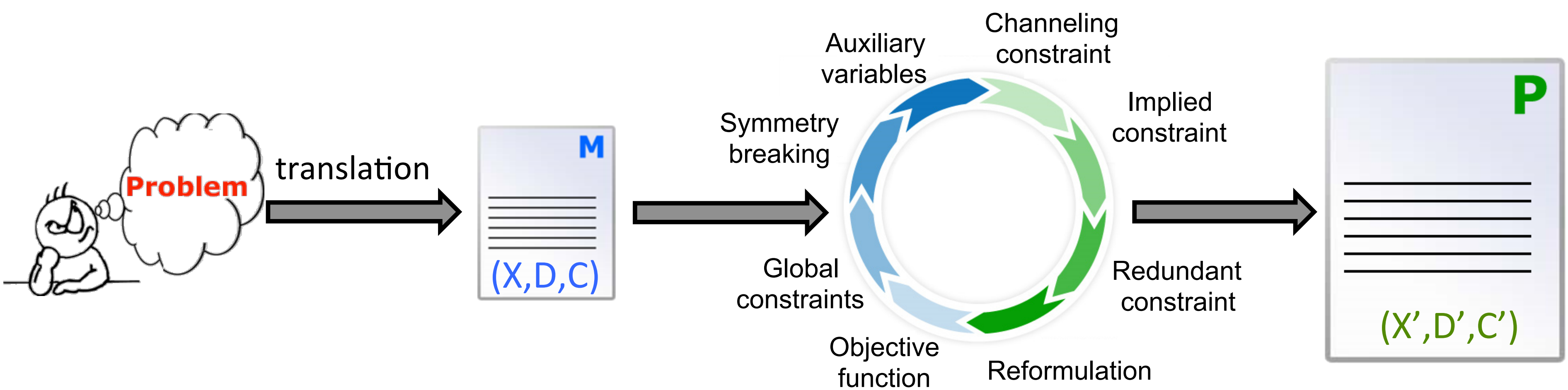
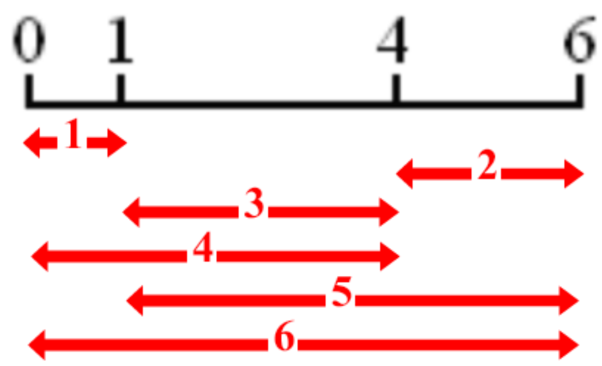


Golomb Rulers Example



Optimal Golomb Rulers (OGR-25) — *Completed 25 October 2008*[(after 3006 days)

[distributed.net, wiki]

Golomb Rulers Example

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using CP;

int m=...;

dvar int x[1..m] in 0..m*m;

minimize x[m];

subject to
{
(1)   forall (i in 1..m-1)
         x[i] < x[i+1];
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Equivalent

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dvar int x,y,z;

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AllDifferent( d );

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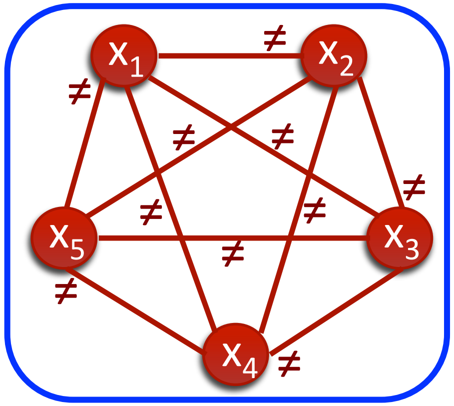

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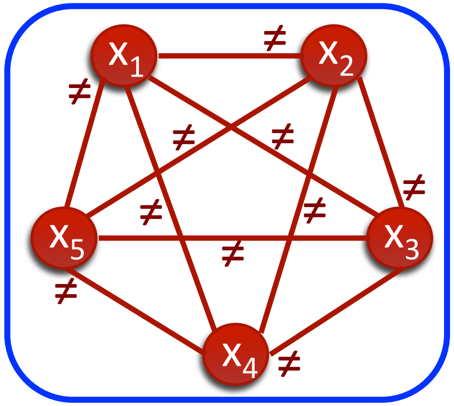

- More than 423 global constraints in the catalog (http://sofdem.github.io/gccat/)

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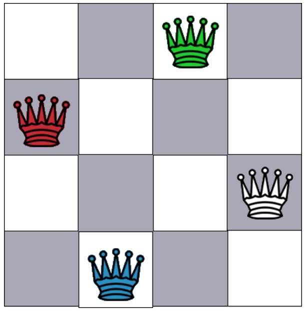
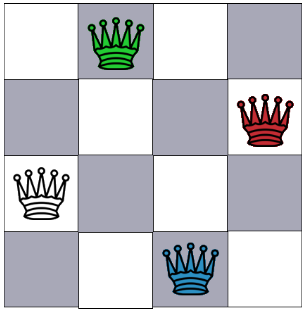


S=[2 , 4 , 1 , 3]



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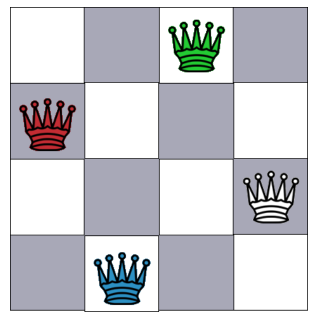

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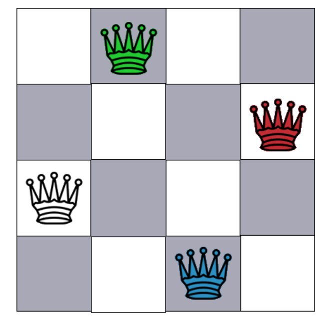

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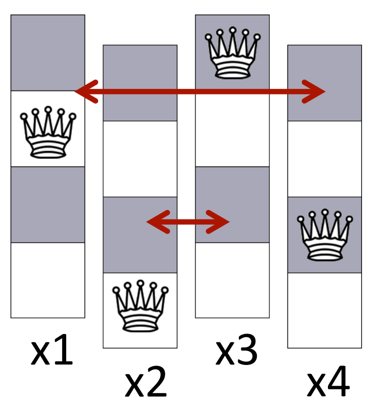
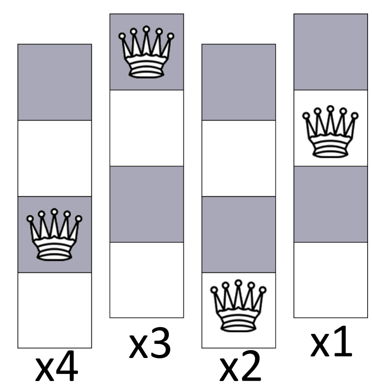
- Symmetry on variables:





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