On Common Components of DC Circuits

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This paper characterizes the properties of electrical components as measured by a variety of experiments over three circuits, and compares the observed values with those supplied by the manufacturers. Low error percentages were observed, verifying the manufacturers' reports. The function $V(t) = (1.411 \pm .001)(1 - e^{(0.096 \pm .002)t})$ was found to model the mechanisms of a capacitor. Further analysis was performed with a transient current, and the function $i(V) = ((6 \pm 1)\mathrm{E}{-10})(e^{(24.6 \pm .4)V})$ was found to fit the i-V curve observed in the diode with r^2 = 0.98. A piecewise function was also created to linearly approximate the transistor i_{ce} vs V_{ce} curves.

I Introduction

Electrical components such as capacitors, diodes, and transistors, are the basis for all modern electronics; their inventions paving the way for smaller and cheaper circuitry in radios, televisions, and computers. Using equipment, and basic circuit analysis, the values of these components can be tested, as well as compared to those supplied by the manufacturer.

II Theory

Several accepted principles and theories were used in our circuit analysis. The wires in the circuits are assumed to be ideal conductors, which would then give reasonable estimates of the current, resistance, or voltage in a circuit. As such, Ohm's law is used to find the current, voltage, or resistance across an ideal conductor. The general form is:

$$V = iR \qquad [1]$$

(*HRW 2017*)

Where:
V = The potential through the load (V)
i = The current through the load (A)
R = The resistance of the load (Ω)

The same theory can be applied to capacitors. As such, a similar equation can be derived.

$$i = C\frac{dV}{dt} \qquad [2]$$

(*HRW 2017*)

Where:
i = The current through the capacitor (A)
C = Capacitance (F)
$\frac{dV}{dT}$ = Rate of voltage change (Vs^{-1})

Kirchoff's Laws are also used to compute the current flowing into a junction, as well as the voltage drop at a particular point.

$$\sum_{k=1}^{n} V_k = 0 \quad [3]$$

(*HRW 2017*)

$$\sum_{k=1}^{n} i_k = 0 \quad [4]$$

(*HRW 2017*)

Where:
i_k = The kth current flowing to the junction
V_k = The kth voltage drop in around the loop

For the purpose of analyzing a diode, A Vi curve must be obtained. As the PN Junction diode does not follow Ohm's Law, it must be modeled by a seperate equation. This is known as Shockley's Diode Equation.

$$i_D = i_S(e^{\frac{qV_D}{nkT}} - 1) \quad [5]$$

(*HRW 2017*)

i_D = Current of diode (A)
i_S = Reverse bias saturation current (A)
q = Charge of electron (C)
V_D = Voltage of diode (V)
n = The ideality factor
k = Boltzmanns constant
T = Temperature (K)

III METHOD

The RC circuit was constructed such that the capacitor was in series with the resistor. As such, to discharge the capacitor, a switch was integrated into the circuit. After several attempts, the switches were deemed inadequate, as the capacitor would display erratic behaviour during the discharge. It was deemed that the switches were extremely sensitive to pressure, as any slight change affected the charge of the capacitor. To combat this, the wire was simply disconnected from the junction and manually inserted into the battery. A multimeter was used to detect any drop in voltage. As no difference was shown in the multimeter, if such a drop existed, it would be too insignificant to detect.

To combat the issue of the infinite resistance demonstrated by the capacitor, the potential across the resistor was taken instead.

Experiment 2 prompted the creation of a second circuit, with two resistors in series, and a resistor and PN Diode connected in parallel with the second resistor. This created the effect of a voltage divider. However, the change in voltage decreased as the power source voltage increased, meaning at 10V, only 0.6V was running through the diode. Thus, a 1:3 ratio was maintained, which created a 1:10 ratio as voltage was increased. This was acceptable, as there is little to no change in current from 0V to 0.1V. Finally, to measure the leakage current, a transistor was integrated into the circuit. The value of the current was then adjusted according to the h_{FE} value measured by a multimeter.

As three ranges of the voltage were found using three different circuits, precautions were taken to minimize the discrepancies between the data sets. The same resistors and equipment were used, each resistor was re-evaluated using the multimeter at the beginning of each lab period, and each experiment was performed in quick succession after the other.

An NPN transistor (model BC639), with an h_{FE} of (126 ± 5), was selected. To regulate the four i_b, four resistors were used with a (1.44 ± 0.01)V battery as in figure 1.

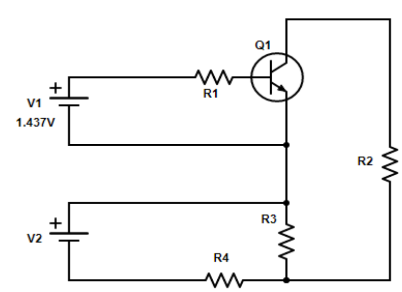

Fig. 1. Circuit diagram for the transistor circuit. V_1 may be referred to as V_{bat}. $R_1 = R_B$ and $R_2 = R_C$.

In order to regulate V_{CE} values of less than 1, a voltage divider was used similar to the setup in experiment two. Data was taken with several multimeters at regular intervals of V_{CE} ranging from 0V to 10V.

IV Analysis

The following differential equation was obtained after substituting equation [2] into equation [1], and applying equation [3], and it models the potential difference maintained by the capacitor.

$$v_s = RC\frac{dv(t)}{dt} + v(t) \quad [6]$$

Solving differential equations, and regressing the data set with r^2 = 0.996, two equations were generated.

$$V(t) = (1.45 \pm .01)(1 - e^{\frac{-t}{(9.78\pm.02)}}) \quad [7]$$

$$V(t) = (1.411 \pm .001)(1 - e^{(0.096\pm.002)t}) \quad [8]$$

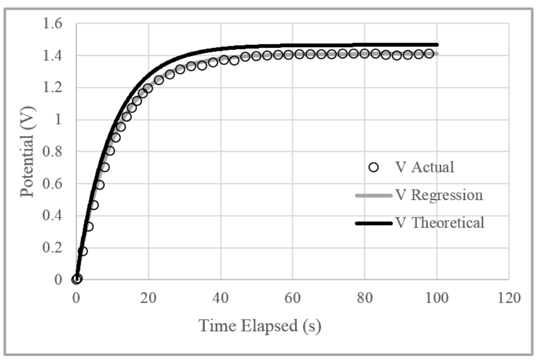

Fig. 2. The three V vs t curves for the measured potential, theoretical potential function [7], and regression curve function [8]. The curves are disjoint due to a small amount of leakage current across the capacitor.

The capacitor only charged to around 97.6% of the battery's potential. A small voltage was still measured across the resistor even as the capacitor was nearing its capacity. As a capacitor charges, its resistance increases until it can be viewed as an open circuit. However, due to the leakage current of the capacitor, some current was still flowing through the circuit, leading to the 0.03V discrepancy. This created a visible difference between the theoretical and actual values, as seen in figure 2.

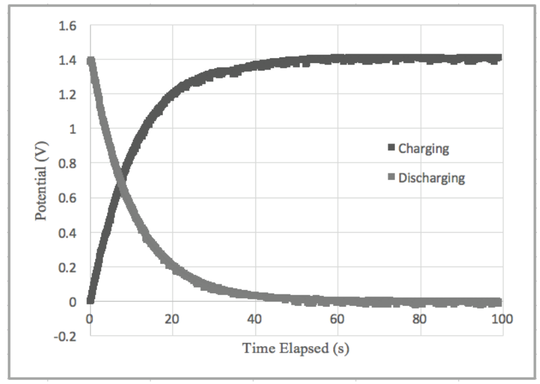

Fig. 3. The charging and discharging curves for the capacitor. The curves are symmetrical within multimeter reading error.

The charging and discharging curves were, as expected, vertically symmetric to each other. This was because the capacitor charges and discharges at the same rate, namely the absolute value of the first derivative of equation [6]. This results from the underlying symmetry of the capacitor; two identical conductive plates sandwiching a dielectric material.

From experiment 2, using equations [3] and [4] on the parallel branch, after negating the resistance of the resistor, the current and voltage through the diode can be found. A function can be generated to model the relationship, as shown in figure 4.

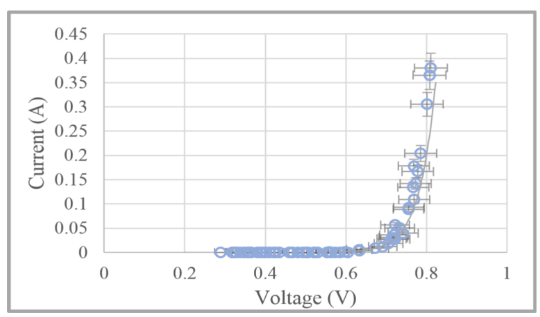

Fig. 4. i vs V graph of the PN Junction Diode only. Current measured in A. Voltage measured in V. Error bars represent the propogated uncertainty.

A exponential regression line can then be calculated to:

$$i(V) = ((6 \pm 1)\text{E}{-10})(e^{(24.6\pm.4)V}) \quad [9]$$

This equation closely resembles Shockley's Diode Equation, as presented by equation [5]. In this case, the constant term of 1 is negligible, as it represents a value in the nano decimal place. As such, in equation [9], which models ae^{bx}, a can be seen as the reverse saturation or reverse leakage current, while b can be seen as the thermal voltage multiplied by the ideality factor of the diode.

From this, we use the temperature of the physics room to analyze the thermal voltage, HRW states that a "good approximation for V_T at 23°C is 25.85mV." As such, an ideality factor of $(1.57 \pm .02)$ is obtained. This concurs with the manufacturer ideality factor of $1.62 \pm .01$, as well as its leakage current of $(6\text{E}{-}10)$A (Fairchild, 2015).

This also makes sense in the context of the whole parallel branch. With an ideality factor of 2, the linearity is expected to occur at 0.7V. With an ideality factor of 1, the linearity is expected to occur near 0.3V. With a ideality factor of 1.57, the would occur near 0.5V.

The ideality factor, which tests how ideal a diode is, is calculated through careful consideration of the materials used. As such, any small imperfection or imbalance between the P and N interfaces, the doping of the interfaces, or the depletion layer may account for this change. Given that the bulk manufacturing of the diode is not perfect, the difference in ideality factor is expected. However, the fact that the leakage current remains consistent with the observational values is somewhat surprising. Given that the reverse saturation current is erratic (most companies only give maximum leakage current), small discrepancies should cause a slight shift in leakage current as well. However, that is not the case. It is hypothesized that the composition of the materials is close enough such that, while the leakage current does deviate from the factory value, it is not enough to be analyzed by the instruments, or calculations.

A O-C residual table was also completed, with noticeable differences at certain voltages noted and listed below.

Index	Voltage (V)	$I_{observed}$ (A)	$I_{computed}$ (A)	Residual	Percent Error
1	0.32±.02	2.012E-6	1.415E-6	5.97E-7	29.6%
2	0.77±.04	0.109	0.0869	0.022	20.2%
3	0.74±.04	0.0394	0.0443	0.0049	12.3%
Mean Percent Error for All Data					3.3%

Fig. 5. Residual Tables of notable large errors. Mean value of percent errors for unshown points also displayed.

The leakage current can also be measured and reevaluated to ensure that the it stays consistent with the theoretical values.

Index	V (V)	Observed I (μA)	Adjusted I (μA)	Leakage I (μA)
1	0.11±.03	0.06±.03	0	0
2	3.12±.05	0.07±.03	0.01±.001	0.00008±.00001
3	5.12±.06	0.08±.03	0.02±.001	0.00016±.00001
4	7.11±.07	0.09±.03	0.03±.001	0.00024±.00001
5	8.56±.07	0.10±.03	0.04±.002	0.00032±.00002
6	11.11±.09	0.11±.03	0.05±.002	0.00040±.00002

Fig. 6. Leakage Current for the PN Diode. An example of the data collected, Index 6 is the highest voltage reached.

While the order of magnitude is equal to that of the theoretical and regression value, the voltage was limited to 12V, as per safety concerns. Even then, since the diode is not ideal, it would be very likely that the diode would exhibit a function type leakage current.

For the purposes of experiment 3, these two equations will be introduced:

$$i_B = \frac{i_s}{h_{FE}} e^{\frac{V_{BE}}{V_T}} \qquad [10]$$

$$i_C = i_s e^{\frac{V_{BE}}{V_T}} (1 + \frac{V_{CE}}{V_A}) \qquad [11]$$

i_B = The base current (A)
i_C = Collector current (A)
i_S = Reverse bias saturation current(A)
V_{BE} = Voltage between base/emitter node (V)
V_{CE} = Voltage between collector/emitter(V)
V_T = Thermal Voltage (V)
V_A = Early Voltage (V)
h_{FE} = Common emitter current gain

The observed values for the experiment were graphed in the graph below:

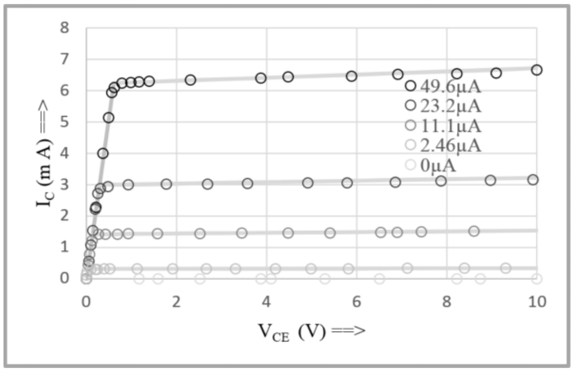

Fig. 7. Plot of i_c vs V_{CE}. There are three distinct regions: saturation, active, and cut-off. i_c rapidly increases in the saturation region until it reaches i_c,max and enters the active region. In the active region, i_c steadily increases. In the cut-off region, collector current is zero for all values of V_{CE}.

In saturation mode, the BC junction and BE junction are both forward biased. This configuration allows the current through the transistor to rapidly increase with V_{CE}. The current through the junctions can be modeled with [5]. For the sake of simplicity, the saturation region will be linearly approximated within its domain using [3] with the assumption that V_{sat} is constant.

Once i_c enters the active region, where the BE junction is forward biased and BC junction is reverse biased, the current appears to plateau. When in the active region, i_c can be approximated to be i_b*h_{FE}. However, as V_{CE} is increased, the base width gets larger, allowing for a greater flow of current. This produces a slight increase in i_c with V_{CE}.

The Early potential was found to be (-120 ± 30)V by extrapolating the active regions of the four non-zero base currents. Using [10], the saturation current (i_s) was found to be $(2.5 \pm .2)$E-9A. Using [11], a linear fit for the active region can be produced. These two fits can be combined to generate a two-part piecewise function. The max i_c will be set to $Y = (R_c)(h_{FE})(i_b)$.

$$i_c = \frac{V_{CE}}{R_c}, 0 \leq V_{CE} < Y \qquad [12]$$

$$i_c = mV_{CE} + b, Y \leq V_{CE} \leq 10 \qquad [13]$$

Index	i_B (μA)	Y (V)	m (mA V^{-1})	b (mA)
1	2.46±.04	0.0303±.0004	0.0024±.0001	.303±.004
2	11.1±.2	0.168±.002	0.0113±.0005	1.14±.01
3	23.2±.4	0.286±.005	0.024±.001	2.98±.02
4	49.6±.7	0.611±.009	0.050±.002	6.21±.05

Fig. 8. Values of Y, m and b. Y is the point at which the BJT enters active mode i.e. where i_c=$i_c$max. m and b are coefficients in the linear fit of the active region computed using the Early Effect.

In the cut-off region, the BE junction is reverse biased. Here, i_c and $i_b \approx 0$

The first part of the piecewise deviated from the theoretical by an average of 7.7% whereas the second part deviated by 4.2%. This suggests that, while not accurate on a small scale, the behavior of a transistor over a wide array of V_{CE}'s can be modeled particularly well using linear approximations.

V Sources of Error

To reduce current leakage in a capacitor, the dielectric material could be upgraded to a better material. However, it is impossible to make a capacitor without current leakage due to the existence of an electric field.

To obtain a stronger fit for the regressions, more data points would be needed from a supply in excess of 12V. Also, to reduce the signal to noise for reverse bias measurement, a second transistor could be used to amplify the current once more.

VI Conclusion

Given the study, it has been demonstrated that the given values on modern electronics are generally close to the values exhibited when the component is in practise. The charging curves of a capacitor, the characteristics of a diode, and the regional curves of the transistor are all found to concur with learned theory.

VII Sources

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