HELPING THE SPACING GUILD NAVIGATE TO ARRAKIS

CANNON TUTTLE, CURTIS EVANS, SPENCER ASHTON, TYLER SANDERS

Abstract.

1. Problem Statement and Motivation

The **Dune** Guild Naviators are tasked with delivering House Atreides from the planet Caladan to Arrakis. Usually they solve the optimal control in their mind while taking spice, but since spice production has been disrupted they've instead decided to turn to ACME students on Earth to solve the optimal control for them. They have asked the ACME students to find the best path while minimizing fuel consumption.

2. State Equations

We used Newton's second law of motion $F = ma$ to arrive at our state equations where m_p is the mass of the planet, $\mathbf{x}_s$ is the x and y position of the space ship, and $\mathbf{x}_p$ is the x and y position of the planet, and $\mathbf{u}$ is the acceleration in the x and y direction for the spaceship. TODO talk about what G is and how this equation makes sense.

$$\ddot{\mathbf{x}} = -G \sum_{p \in P} \frac{m_p}{||\mathbf{x}_s - \mathbf{x}_p||_2^3}(\mathbf{x}_s - \mathbf{x}_p) + \mathbf{u}$$

Converting $\ddot{\mathbf{x}}$ into a first order differential equation we get the following state equation.

$$\begin{pmatrix} \dot{x}_s \\ \dot{y}_s \\ \ddot{x}_s \\ \ddot{y}_s \end{pmatrix} = \begin{pmatrix} \dot{x}_s \\ \dot{y}_s \\ -G\sum_{p\in P} \frac{m_p(x_s - x_p)}{((x_s-x_p)^2+(y_s-y_p)^2)^{3/2}} + u_x \\ -G\sum_{p\in P} \frac{m_p(y_s - y_p)}{((x_s-x_p)^2+(y_s-y_p)^2)^{3/2}} + u_y \end{pmatrix}$$

We decided to minimize the control used to get the optimal fuel usage. That gives us the following cost functional and boundary conditions.

$$J[u] = \int_0^{t_f} ||\mathbf{u}||_2^2 dt$$

$$\mathbf{x}_s(0) = \text{Caladan's Position},\ \dot{\mathbf{x}}_s(0) = \text{Caladan's Velocity}$$
$$\mathbf{x}_s(t_f) = \text{Arrakis' Position},\ \dot{\mathbf{x}}_s(t_f) = \text{Arrakis' Velocity}$$

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We are now ready to use Pontryagin's maximum principle with

$$H = \mathbf{p} \cdot \mathbf{f}(\mathbf{x}) - L$$

Applying this to our state equation and Lagrangian we get the following Hamiltonian.

$$H = p_1\dot{x}_s + p_2\dot{y}_s + p_3(-G\sum_{p\in P}\frac{m_p(x_s - x_p)}{((x_s - x_p)^2 + (y_s - y_p)^2)^{3/2}} + u_x)$$

$$+p_4(-G\sum_{p\in P}\frac{m_p(y_s - y_p)}{((x_s - x_p)^2 + (y_s - y_p)^2)^{3/2}} + u_y) - u_x^2 - u_y^2$$

This give us the following co-state evolution equations by using $\mathbf{p}' = \frac{DH}{D\mathbf{x}}$

$$\dot{p}_1 = p_3G[\sum_{p\in P}\frac{m_p}{((x_s - x_p)^2 + (y_s - y_p)^2)^{3/2}} - \frac{3m_p(x_s - x_p)^2}{((x_s - x_p)^2 + (y_s - y_p)^2)^{5/2}}]$$

$$\dot{p}_2 = p_4G[\sum_{p\in P}\frac{m_p}{((x_s - x_p)^2 + (y_s - y_p)^2)^{3/2}} - \frac{3m_p(y_s - y_p)^2}{((x_s - x_p)^2 + (y_s - y_p)^2)^{5/2}}]$$

$$\dot{p}_3 = -p_1$$

$$\dot{p}_4 = -p_2.$$

$$p_1(t_f) = p_2(t_f) = 0 = p_3(tf) = p_4(t_f)$$

Now we need to find $\tilde{\mathbf{u}}$ by maximizing the Hamiltonian which gives us the following

$$\tilde{u}_x = \frac{p_3}{2} \text{ and } \tilde{u}_y = \frac{p_4}{2}.$$

We now have a system of differential equations that can be solved using TODO solve_bvp.

2.1. **Planetary Motion.** Since the ACME students aren't familiar with the planetary motion of Arrakis and Caladan they decided to look at getting from Mars to Earth with their respective masses and orbits. We have made a planet class that will give us the planets mass, position, and velocity.

References

[1] William M., Samuel L., Jeff S. University Physics Volume 1. 2021 OpenStax. https://openstax.org/details/books/university-physics-volume-1