ANGLE APPROXIMATION FROM PRESSURE MEASUREMENTS

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Abstract.

1. Problem Statement and Motivation

A BYU Acoustics Research Group project is currently using a microphone array and triangulation to calculate the direction of arrival of a source in relation to the center of the array. They've implemented some rudimentary filtering, but so far the results of the calculation are somewhat unreliable. We want to take this process and model it as a time series/Hidden Markov model in order to optimize the calculation process. We will potentially employ Kalman filtering in order to filter angle measurements. We will also explore whether cross-correlation or GCC-PHAT is the better measure of coherence between microphone signals for this purpose, and whether the hidden state should be modeled as an angle or be modeled directly as a series of coherence measurements. We will also experiment with ways to represent the observation space. This will also include trying to predict whether there is or isn't a speaker currently present.

Several of the noise reduction algorithms they have implemented depend on a highly accurate angle measurement. As those algorithms have already been implemented, we would be primarily concerned with the step of direction of arrival angle estimation and optimization, as well as optimally estimating the corresponding time delay. This would provide the research group with enhanced measurements for use in their noise processing algorithms.

This relates to the hidden markov model because we don't know what the angles are that we are looking for, but we do know how to take a measurement of the current pressure at each microphone. We will use these microphones to be our observed data to then figure out what these angles are by creating a HMM. The angles are to be calculated at discrete time steps according to the current pressure measurement. The Kalman filter might accurately represent the system and more optimally combine current and prior information about the angle in order to calculate a less noisy current angle estimate.

2. Data

The data for this project will be provided by the research team (Curtis Garner) that is currently working on it. The data includes measurements where the sound source is and is not moving, is and is not present, and measurements that do and don't include machine noise in the microphone signal.

3. Methods

We have tried several methods to compute the hidden angle measurement. We tried the following methods...

3.1. State Space Model.

First we had to set up our continuous state space model. We needed angular velocity in our state space so to do that we do a finite difference approximation where $\theta'_t \approx \frac{\theta - \theta_{t-1}}{\Delta t}$. Now that we have angular velocity we can use it to make a simple forward Euler step in our state space. With the finite difference we need to save θ_{t-1} in the state space. This yields the following setup

$$\mathbf{x}_t = \begin{pmatrix} \theta_t \\ \theta_{t-1} \\ \theta' \end{pmatrix}, \; F = \begin{pmatrix} 1 & 0 & \Delta t \\ 1 & 0 & 0 \\ \frac{1}{\Delta t} & \frac{1}{\Delta t} & 0 \end{pmatrix}, \; H = \begin{pmatrix} 1 & 0 & 0 \\ \vdots & \vdots & \vdots \\ 1 & 0 & 0 \end{pmatrix}$$

where H is of dimension 18×3. We don't have any control in our situation so our state space is

$$\mathbf{x}_t = F\mathbf{x}_{t-1} + \mathbf{w}_t,$$
$$\mathbf{z}_t = H\mathbf{x}_t + \mathbf{v}_t$$

.

3.2. Kalman Filter.

We implemented the Kalman filter from the Volume 3 Textbook [1]. To understand how angle measurements worked in the Kalman Filter we tried it with stimulated data done in the lab manuals and class. We discovered a problem with the update step in angle wraparound from the angles in the range of

$$2\pi - \epsilon \leq \theta \leq 2\pi + \epsilon$$

for some $\epsilon \geq 0$. We notices that the more noise we added the larged this ϵ got. See figure 1 to get a visual.

We discovered that the problem was occurring in the update step where

$$\tilde{\mathbf{y}}_k = \mathbf{z}_k - H\hat{\mathbf{x}}_{k|k-1}.$$

A toy example would be if we are looking at an observed angle $\mathbf{z}_i = 15°$ and a predicted angle of $(H\mathbf{x})_i = 355°$ then $\mathbf{z}_i - (H\mathbf{x})_i = 15° - 355° = -340°$ and we desire $20°$. We could simply do this by noticing that $-340 \equiv 20 \pmod{360}$, however, if we said $\mathbf{z}_i = 355°$ and $(H\mathbf{x})_i = 15°$ then $\mathbf{z}_i - (H\mathbf{x})_i = 355° - 15° = 340°$ which isn't what we want. We want the Kalman filter to think of this difference as $-20°$. The best way to do this was by implementing Algorithm 1 which will get us the right differenced needed by the correct sign so the Kalman filter can function correctly. We saw great results from this in our stimulated data. See Figure 1.

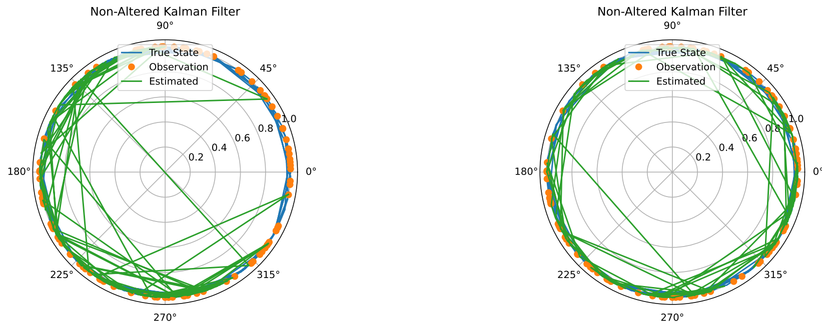

FIGURE 1. The non-altered and altered Kalman Filter

Algorithm 1 Process to Fix Wraparound

$\mathbf{v} \leftarrow \mathbf{z}_k - H\mathbf{x}_k$
if $|\mathbf{v}| \geq \pi$ **and** $\mathbf{v}_k \leq 0$ **then**
  $\mathbf{v} \leftarrow \mathbf{v} + 2\pi$
else if $|\mathbf{v}| \geq \pi$ **and** $\mathbf{v}_k \geq 0$ **then**
  $\mathbf{v} \leftarrow \mathbf{v} - 2\pi$
end if
$\tilde{\mathbf{y}} = \mathbf{v}$

3.3. **Particle Filter.** TODO graph of Von Mises distribution with explanation of how Kappa parameter correlates to certainty on particle's location.

TODO explain basis principles of Particle Filter and cite the book that Tyler referenced in implementation.

3.4. **HMM Model.**

4. Results

We had success with circular filtering techniques by using Algorithm 1 allowed us to use Kalman Filters with circular mechanics.

5. Analysis

TODO tradeoffs of Particle filter - the compute time 500x longer for 500 particles - particle filter can have a more flexible prior because it's already got particles everywhere. - particle filter better at nonlinearities bc particles can be reassigned

- Both systems have a mechanism to capture how certain they are about the current state, - and both have a way to specify how much we "belive" current measurements

6. Ethical Considerations

Some possible use cases of this project would be so that big machinery can detect an angle of danger where a person might be. This isn't a great a

7. Conclusion

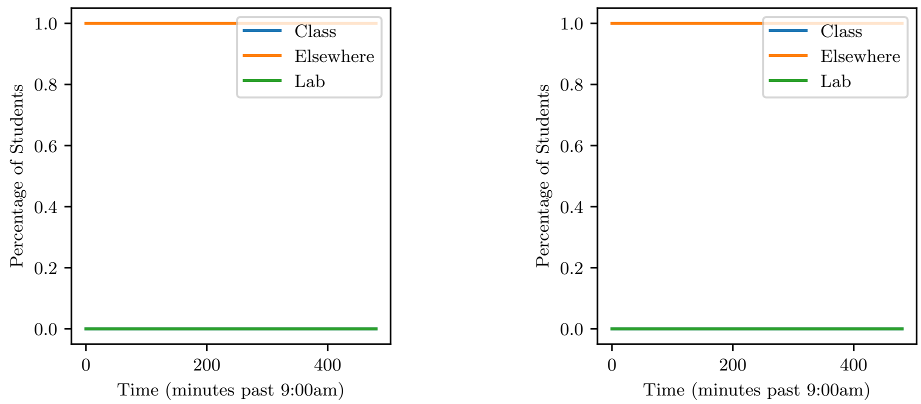

Figure 2. The constant alpha functions (left) along with the timeplot using IVP (right).

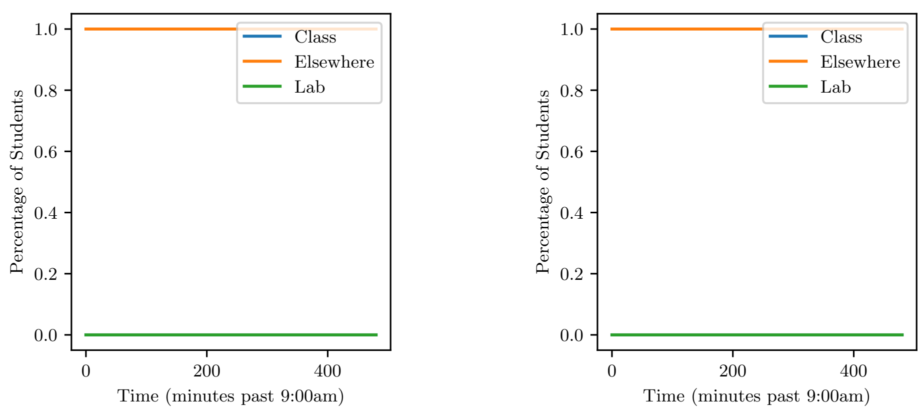

Figure 3. The discontinuous alpha functions (left) along with the timeplot using IVP (right).

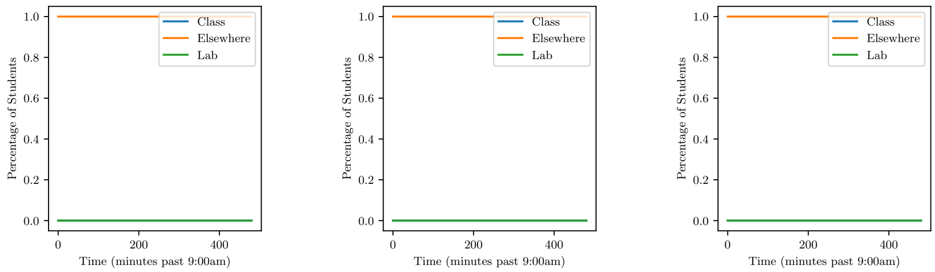

Figure 4. Simulating with $A_{interp.12}$ (left), giving the time-plots using both IVP (center) and BVP (right).

References

[1] The ACME Volume 3 Textbook

[2] Wang Xiang-Sheng and Wu Jianhong 2012. Seasonal migration dynamics: periodicity, transition delay and finite-dimensional reductionProc. R. Soc. A.468634–650. http://doi.org/10.1098/rspa.2011.0236

[3] Pierre Auger, Jean-Christophe Poggiale, Emergence of Population Growth Models: Fast Migration and Slow Growth, Journal of Theoretical Biology, Volume 182, Issue 2, 1996, Pages 99-108, ISSN 0022-5193, https://doi.org/10.1006/jtbi.1996.0145.