Helping the Spacing Guild Navigate to Arrakis

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See our Animations!

https://canyonturtle.github.io/cannon-curtis-spencer-tyler-acme/

We used optimal control to plan fuel-efficient spacecraft routes with multi-planet gravity.

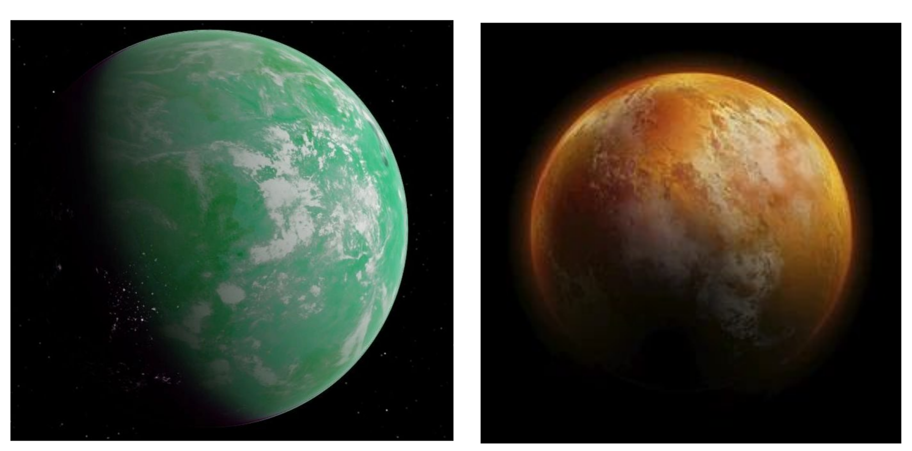
This poster showcases our thought process, model, successes, and challenges we had along the way.

Problem: Guild Navigation

The ***Dune*** Guild Navigators are tasked with delivering House Atreides from the planet Caladan to Arrakis. Usually they solve the optimal control in their mind while taking spice, but since spice production has been disrupted they've instead decided to turn to ACME students on Earth to solve the optimal control for them.

We seek to find the optimal fuel-minimizing path & controls for both fixed and variable time spans.

Figure 1: The planets Caladan and Arrakis



Initial Model

We used Newton's law of universal gravitation[1] and the ship's acceleration controls to model the ship's state equations for position, velocity, and acceleration while optimizing for fuel consumption.

Acceleration and fuel consumption are directly proportional, so our cost functional minimizes total amount of acceleration **u.**

$$J[u] = \int_0^{t_f} ||\mathbf{u}||_2^2 dt$$

Cost Functional

The only components affecting the ship's motion are its acceleration, including acceleration from gravity of each planet **p**, and the added control acceleration **u**. This defined a second order system that we turned this into the system of first order equations shown below. $\mathbf{x}_s$ and $\mathbf{x}_p$ are respectively the position of the ship and planet.

$$\ddot{\mathbf{x}} = -G \sum_{p \in P} \frac{m_p}{||\mathbf{x}_s - \mathbf{x}_p||_2^3}(\mathbf{x}_s - \mathbf{x}_p) + \mathbf{u}$$

State Update Eq.

$$\begin{pmatrix} \dot{x}_s \\ \dot{y}_s \\ \ddot{x}_s \\ \ddot{y}_s \end{pmatrix} = \begin{pmatrix} \dot{x}_s \\ \dot{y}_s \\ -G\sum_{p\in P} \frac{m_p(x_s-x_p)}{((x_s-x_p)^2+(y_s-y_p)^2)^{3/2}} + u_x \\ -G\sum_{p\in P} \frac{m_p(y_s-y_p)}{((x_s-x_p)^2+(y_s-y_p)^2)^{3/2}} + u_y \end{pmatrix}$$

First-Order State Update Eq.

Results and Analysis

Success: Using Gravity to our Advantage

Figure 2 shows the fixed-time optimal control, where the spaceship travels from Caladan to Arrakis in 6 months. Notably, the spaceship arcs alongside Kaitain and Geidi Prime in order to accelerate or decelerate using the planet's gravity, thus conserving fuel.

Success: Optimizing Over Final Time

Figure 3 shows a notably shorter path taken as compared to the fixed-time scenario from Figure 2. Optimizing over final time allowed it to take an alternate route that was shorter and used less total fuel, at the cost of it taking longer than the fixed-time path.

Issue: Sensitivity to initial guess

Figures 3, 4, and 5 show a variety of solutions calculated from different initial guesses when solving the BVP for the free-time case. Currently, these numerical solutions for the free-time case are extremely sensitive to initial guess for final time and initial guess for states/costates.

Issue: Possible numerical instability

Figure 6 shows an unstable solution when including a star in the center. We suspect this is because the mass of the sun is orders of magnitude larger than the other planets masses and other system inputs, causing the problem to be numerically unstable.

Conclusions and next steps

Our model accurately found a reasonable optimal path from Caladan to Arrakis for a fixed time and was also able to optimize over the arrival time, all while returning realistic numbers for acceleration and velocity.

As we were unable to successfully include a star in the model, our next step is modifying our solver to be more numerically stable across planetary scale differences. In order to make the model more practically useful, we will adapt it for optimization over launch time instead of arrival time. Other potential next steps include adapting the control to be thrust instead of acceleration to make acceleration dependant on the weight of the spaceship, navigating between solar systems, and including endpoint costs to avoid or incentivize landing on certain areas of the planet. We also seek to include final fuel and time scores for each trajectory and use that to perform a hyperparameter search for various initial guesses..

Figures

Figure 2: Successful Fixed Time

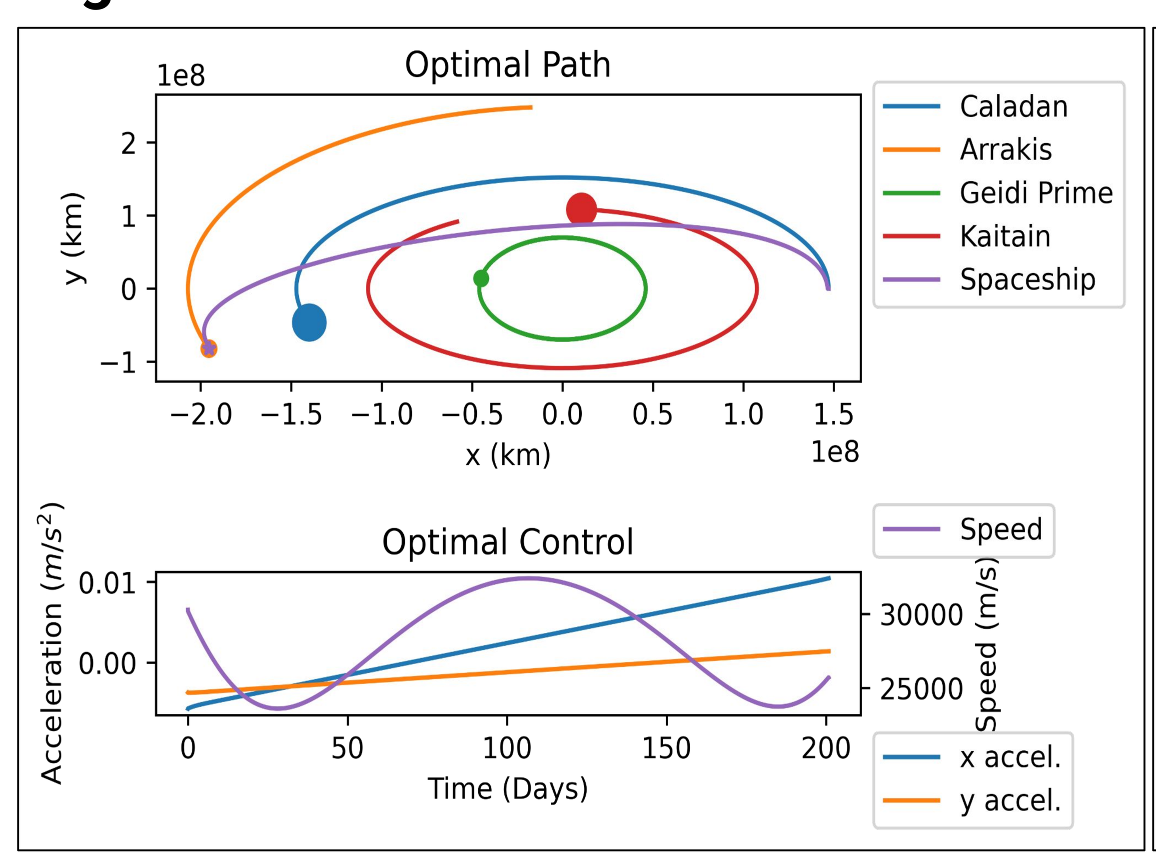


Figure 3: Free-time Initial Guess A

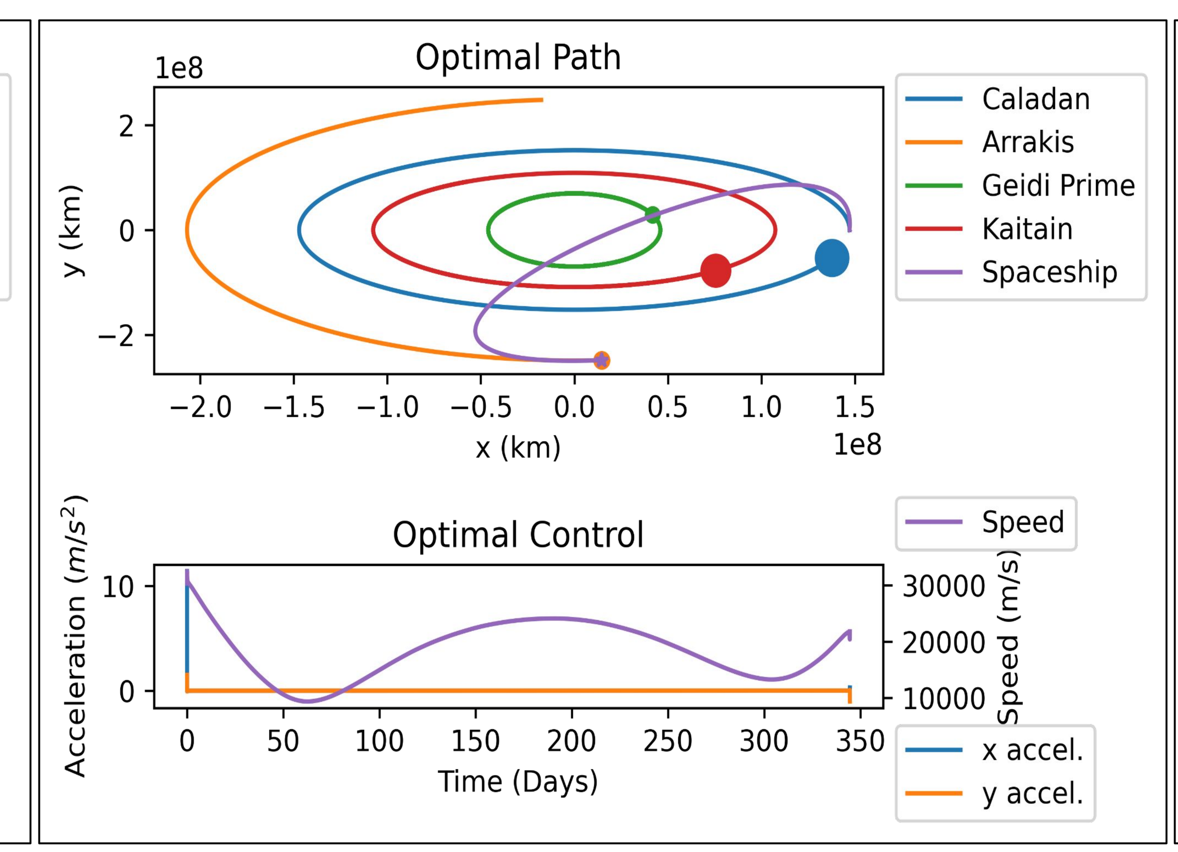


Figure 4: Free Time Initial Guess B

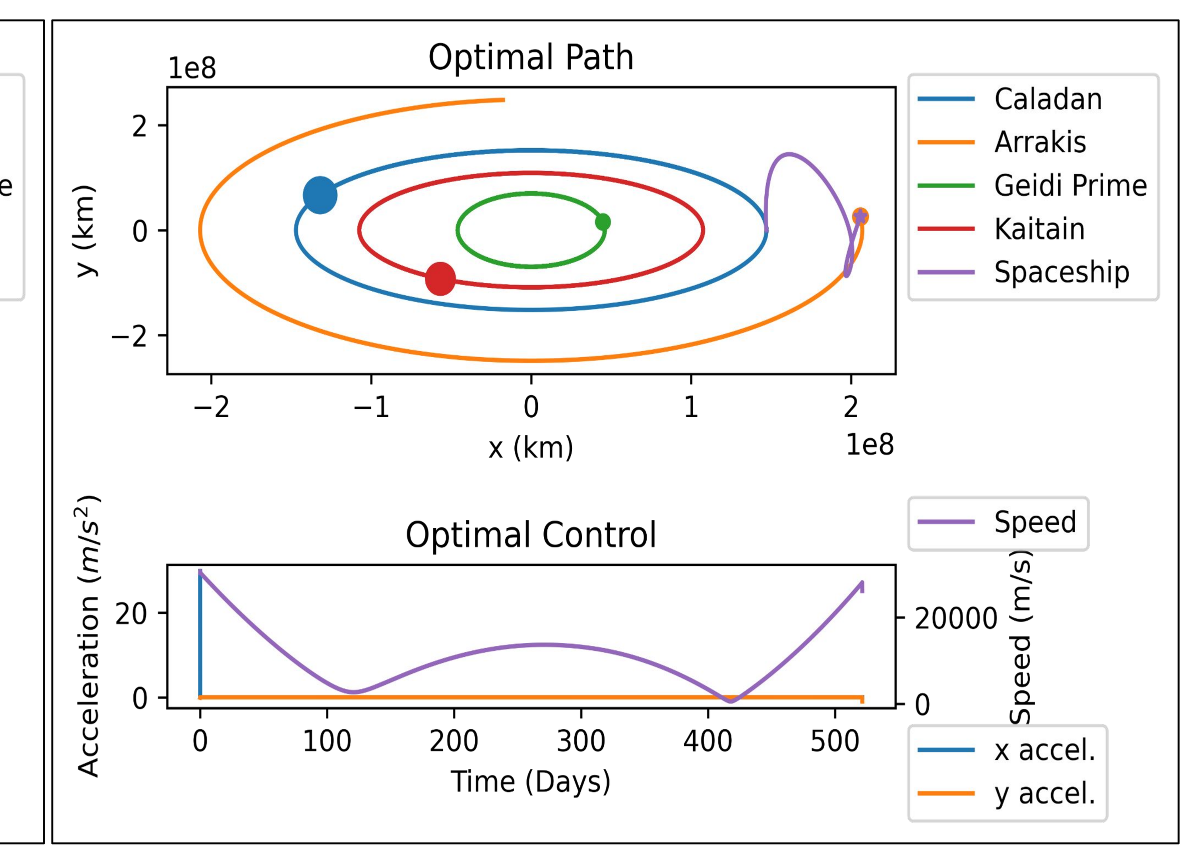


Figure 5: Free-time Initial Guess C

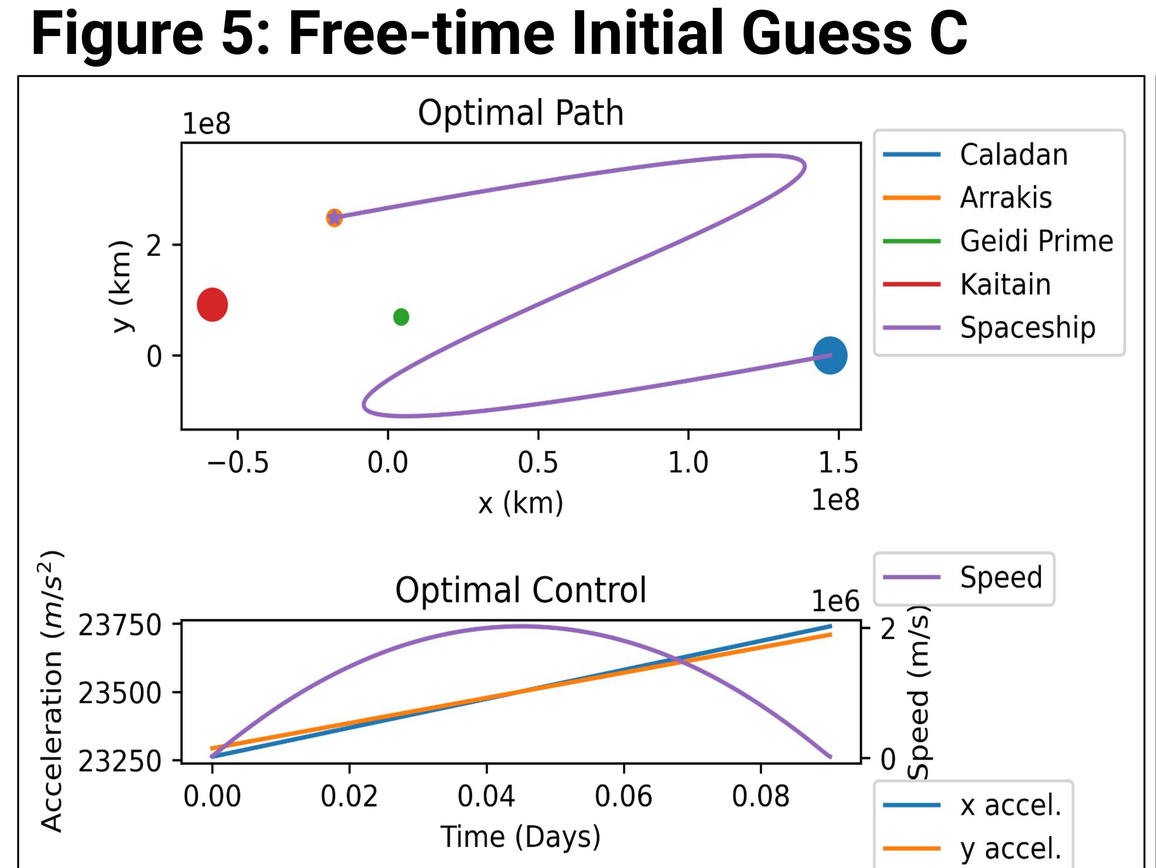


Figure 6: Fixed Time with Sun

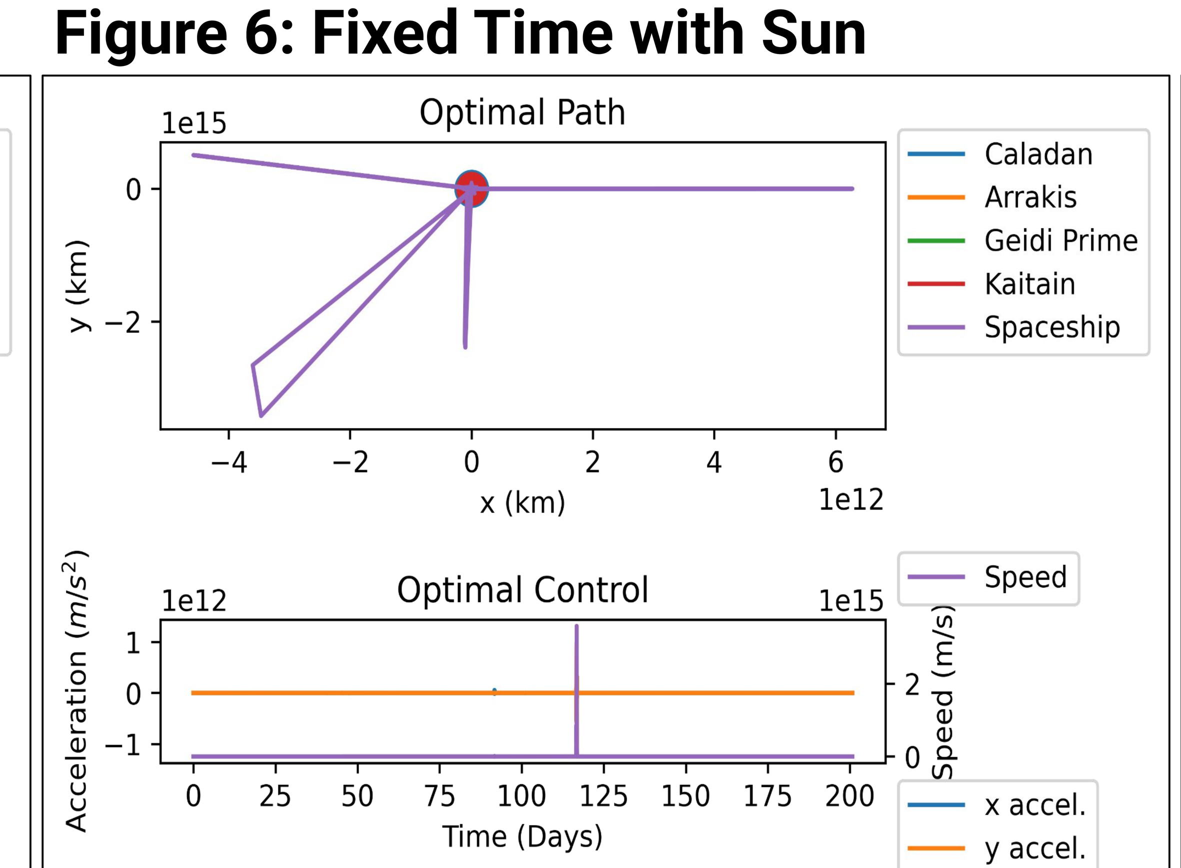
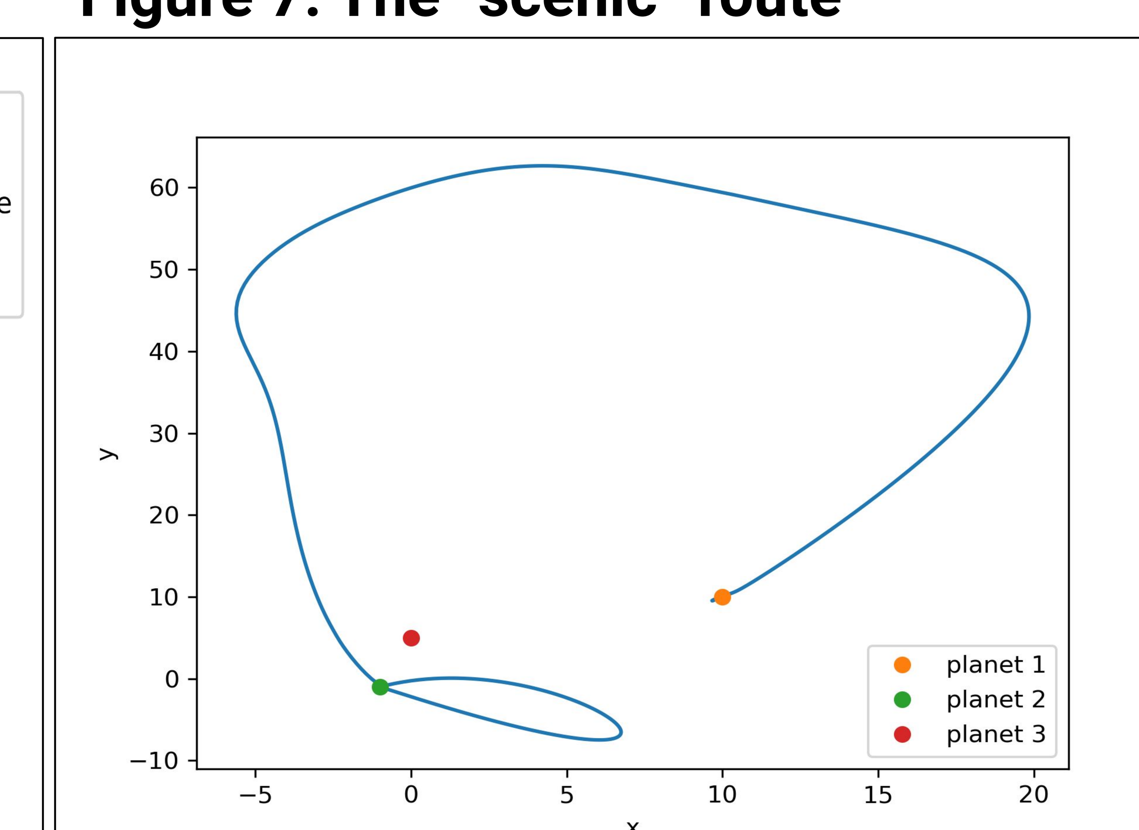


Figure 7: The "scenic" route



References

[1] William M., Samuel L., Jeff S. University Physics Volume 1. 2021 OpenStax. https://openstax.org/details/books/university-physics-volume-1