# Code Template for ACM-ICPC

Roundgod @ Nanjing University October 18, 2019

# Contents

1	1 DataStructures					
	1.1	Heap	1			
	1.2	Fenwick Tree	1			
	1.3	Mo's algorithm	2			
	1.4	Mo's algorithm on Trees				
	1.5	Mo's algorithm with Queries				
	1.6	Mo's algorithm on Trees with Queries				
	1.7	Heavy Light Decomposition				
	1.8	Long Short Decomposition				
	1.9	Lichao Segment Tree				
	-	Segment Tree Beats				
		Segment Tree Merge				
		Persistent Segment Tree				
		Persistent DSU				
		Persistent Trie				
		Monotone Stack				
		Monotone Deque				
		Splay				
		Treap				
		Link-Cut Tree				
		Union Set				
		Sparse Table				
		DSU on Tree				
	1.23	Virtual Tree				
		Young Tableaux				
	1.25	Centroid Decomposition	35			
		*				
_	~					
2		metry	36			
2	2.1	metry Geometry All-in-One	<b>36</b>			
2		metry	<b>36</b>			
	2.1 2.2	metry Geometry All-in-One	<b>36</b> 36 49			
<b>2</b> 3	2.1 2.2 <b>Gra</b>	metry Geometry All-in-One	<b>36</b> 36 49			
	2.1 2.2 <b>Gra</b> 3.1	metry Geometry All-in-One Stereometry  ph Dijkstra	<b>36</b> 36 49 <b>56</b> 56			
	2.1 2.2 <b>Gra</b> 3.1 3.2	metry Geometry All-in-One Stereometry  ph Dijkstra Floyd-Warshall	36 36 49 56 56			
	2.1 2.2 <b>Gra</b> 3.1 3.2 3.3	metry Geometry All-in-One Stereometry  ph Dijkstra Floyd-Warshall Kosaraju	36 36 49 56 56 56 57			
	2.1 2.2 <b>Gra</b> 3.1 3.2 3.3 3.4	metry Geometry All-in-One Stereometry  ph Dijkstra Floyd-Warshall Kosaraju Tarjan	36 36 49 56 56 57 57			
	2.1 2.2 <b>Gra</b> 3.1 3.2 3.3 3.4 3.5	metry Geometry All-in-One Stereometry  ph Dijkstra Floyd-Warshall Kosaraju Tarjan LCA with binary lifting	36 36 49 56 56 57 57			
	2.1 2.2 <b>Gra</b> 3.1 3.2 3.3 3.4 3.5 3.6	metry Geometry All-in-One Stereometry  ph Dijkstra Floyd-Warshall Kosaraju Tarjan LCA with binary lifting LCA with range minimum query	36 36 49 56 56 57 57 58			
	2.1 2.2 <b>Gra</b> 3.1 3.2 3.3 3.4 3.5	metry Geometry All-in-One Stereometry  ph Dijkstra Floyd-Warshall Kosaraju Tarjan LCA with binary lifting LCA with range minimum query Dinic	36 36 49 56 56 57 57 58 59 61			
	2.1 2.2 <b>Gra</b> 3.1 3.2 3.3 3.4 3.5 3.6	metry Geometry All-in-One Stereometry  ph Dijkstra Floyd-Warshall Kosaraju Tarjan LCA with binary lifting LCA with range minimum query Dinic Ear Decomposition	36 36 49 56 56 57 57 58 59 61 62			
	2.1 2.2 <b>Gra</b> 3.1 3.2 3.3 3.4 3.5 3.6 3.7	metry Geometry All-in-One Stereometry  ph Dijkstra Floyd-Warshall Kosaraju Tarjan LCA with binary lifting LCA with range minimum query Dinic	36 36 49 56 56 57 57 58 59 61 62			
	2.1 2.2 <b>Gra</b> 3.1 3.2 3.3 3.4 3.5 3.6 3.7 3.8 3.9	metry Geometry All-in-One Stereometry  ph Dijkstra Floyd-Warshall Kosaraju Tarjan LCA with binary lifting LCA with range minimum query Dinic Ear Decomposition	36 36 49 56 56 57 57 58 59 61 62 64			
	2.1 2.2 <b>Gra</b> 3.1 3.2 3.3 3.4 3.5 3.6 3.7 3.8 3.9 3.10	metry Geometry All-in-One Stereometry  ph Dijkstra Floyd-Warshall Kosaraju Tarjan LCA with binary lifting LCA with range minimum query Dinic Ear Decomposition Min-cost flow	36 36 49 56 56 57 57 58 59 61 62 64 65			
	2.1 2.2 <b>Gra</b> 3.1 3.2 3.3 3.4 3.5 3.6 3.7 3.8 3.9 3.10 3.11	metry Geometry All-in-One Stereometry  ph  Dijkstra Floyd-Warshall Kosaraju Tarjan LCA with binary lifting LCA with range minimum query Dinic Ear Decomposition Min-cost flow Bipartite Matching	36 36 49 56 56 57 57 58 59 61 62 64 65 66			
	2.1 2.2 Gra 3.1 3.2 3.3 3.4 3.5 3.6 3.7 3.8 3.9 3.10 3.11 3.12	metry Geometry All-in-One Stereometry  ph Dijkstra Floyd-Warshall Kosaraju Tarjan LCA with binary lifting LCA with range minimum query Dinic Ear Decomposition Min-cost flow Bipartite Matching Common Matching	36 36 49 56 56 57 57 58 59 61 62 64 65 66			
	2.1 2.2 Gra 3.1 3.2 3.3 3.4 3.5 3.6 3.7 3.8 3.9 3.10 3.11 3.12 3.13	metry Geometry All-in-One Stereometry  ph Dijkstra Floyd-Warshall Kosaraju Tarjan LCA with binary lifting LCA with range minimum query Dinic Ear Decomposition Min-cost flow Bipartite Matching Common Matching Kuhn-Munkres	36 36 49 56 56 57 57 58 59 61 62 64 65 66 68			
	2.1 2.2 Gra 3.1 3.2 3.3 3.4 3.5 3.6 3.7 3.8 3.9 3.10 3.11 3.12 3.13 3.14	metry Geometry All-in-One Stereometry  ph Dijkstra Floyd-Warshall Kosaraju Tarjan LCA with binary lifting LCA with range minimum query Dinic Ear Decomposition Min-cost flow Bipartite Matching Common Matching Kuhn-Munkres Linear Programming	36 36 49 56 56 57 57 58 59 61 62 64 65 66 68			
	2.1 2.2 Gra 3.1 3.2 3.3 3.4 3.5 3.6 3.7 3.8 3.9 3.10 3.11 3.12 3.13 3.14 3.15	Geometry All-in-One Stereometry  ph Dijkstra Floyd-Warshall Kosaraju Tarjan LCA with binary lifting LCA with range minimum query Dinic Ear Decomposition Min-cost flow Bipartite Matching Common Matching Kuhn-Munkres Linear Programming Dominator Tree Dynamic Connectivity	36 36 49 56 56 57 57 58 59 61 62 64 65 66 68 69 70			
	2.1 2.2 Gra 3.1 3.2 3.3 3.4 3.5 3.6 3.7 3.8 3.9 3.10 3.11 3.12 3.13 3.14 3.15 3.16	Geometry All-in-One Stereometry  ph  Dijkstra Floyd-Warshall Kosaraju Tarjan LCA with binary lifting LCA with range minimum query Dinic Ear Decomposition Min-cost flow Bipartite Matching Common Matching Kuhn-Munkres Linear Programming Dominator Tree Dynamic Connectivity Dynamic Bridge	36 36 49 56 56 57 57 58 59 61 62 64 65 66 68 69 70 72 74			
	2.1 2.2 Gra 3.1 3.2 3.3 3.4 3.5 3.6 3.7 3.8 3.10 3.11 3.12 3.13 3.14 3.15 3.16 3.17	Geometry All-in-One Stereometry  ph  Dijkstra Floyd-Warshall Kosaraju Tarjan LCA with binary lifting LCA with range minimum query Dinic Ear Decomposition Min-cost flow Bipartite Matching Common Matching Kuhn-Munkres Linear Programming Dominator Tree Dynamic Connectivity Dynamic Bridge Hopcroft-Karp	36 36 49 56 56 57 57 58 59 61 62 64 65 66 68 70 72 74 83			
	2.1 2.2 Gra 3.1 3.2 3.3 3.4 3.5 3.6 3.7 3.8 3.10 3.11 3.12 3.13 3.14 3.15 3.16 3.17 3.18	Geometry All-in-One Stereometry  ph  Dijkstra Floyd-Warshall Kosaraju Tarjan LCA with binary lifting LCA with range minimum query Dinic Ear Decomposition Min-cost flow Bipartite Matching Common Matching Kuhn-Munkres Linear Programming Dominator Tree Dynamic Connectivity Dynamic Bridge	36 36 49 56 56 57 57 58 59 61 62 64 65 66 68 70 72 74 83 84			

	3.21	Minimum Arborescence
	3.22	Minimum Diameter Spanning Tree
	3.23	Chordal Graph
	3.24	TriangleCount
		SquareCount
		•
4	Mat	
	4.1	BigNum
	4.2	Fast Multiplication
	4.3	Pohlig Hellman
	4.4	Belerkamp-Massey
	4.5	Chinese Remainder Theorem
	4.6	Matrix Determinant
	4.7	Euler Sieve
	4.8	Extended GCD
	4.9	Fast Fourier Transform
	4.10	Fast Walsh-Hadamard Transform
		Gauss-Jordan
		Linear Basis
		Linear Congruence
		Linear Recurrence
		LU Decomposition
		Matrix Operations
		Miller-Rabin primality test
		Mod-Combinatation and Mod-fact
		Fast Number-Theoretic Transform
		Pell's equation
		•
		Pollard-Rho
		Polynomial Operations
		Polynomial Summations
		Prime Counting Function
		Primitive Root
		Segmented Sieve
		Stirling number of the first kind
		Stirling number of the second kind(multiple)
		Stirling number of the second kind(single)
		Prefix Sum of Miu
	4.31	Prefix Sum of Phi
		Tonelli-Shanks
		Euclid
	4.34	Simpson
	4.35	Farey
5	Oth	
	5.1	Convex Hull Trick
	5.2	Dynamic Convex Hull Trick
	5.3	Knuth's optimization
	5.4	Multiple Backpack
	5.5	Sum Over Subsets
	5.6	Enumeration of Subsets
	5.7	Matroid Intersection
	5.8	Weighted Matroid Intersection
	5.9	Nim Multiplication
		Dynamic Dynamic Programming

Stri	$_{ m ling}$	
6.3	Hash Matching	35
6.4	Aho-Corasick Automaton	35
6.5	Suffix Array	38
6.6	SA-IS	36
6.7	Manacher	71
6.8	Suffix Automaton	71
	5.12 Stri 6.1 6.2 6.3 6.4 6.5 6.6 6.7	5.11 Simplex       16         5.12 whatday       16         String       16         6.1 Trie       16         6.2 KMP       16         6.3 Hash Matching       16         6.4 Aho-Corasick Automaton       16         6.5 Suffix Array       16         6.6 SA-IS       16         6.7 Manacher       17         6.8 Suffix Automaton       17

# 1 DataStructures

# 1.1 Heap

```
#include<bits/stdc++.h>
#define MAXN 100005
#define INF 100000000
#define MOD 100000007
#define F first
#define S second
using namespace std;
typedef long long 11;
typedef pair<int,int> P;
struct heap
{
   priority_queue<int> q1,q2;
   void push(int x) {q1.push(x);}
   void erase(int x) {q2.push(x);}
   int top()
       while(q2.size()&&q1.top()==q2.top()) q1.pop(),q2.pop();
       return q1.top();
   }
   void pop()
   {
       while(q2.size()&&q1.top()==q2.top()) q1.pop(),q2.pop();
       q1.pop();
   }
   int size()
   {
       return q1.size()-q2.size();
   }
};
int main()
   return 0;
```

#### 1.2 Fenwick Tree

```
#include<bits/stdc++.h>
#define MAXN 100000
#define MAXLOGN 20
#define INF 1000000000
using namespace std;
int bit[2*MAXN+1],n;
int sum(int i)
{
   int s=0;
   while(i>0)
   {
     s+=bit[i];
     i-=i&-i;
   }
   return s;
}
```

```
void add(int i,int x)
    while(i<=n)</pre>
    {
        bit[i] += x;
        i+=i&-i;
}
int bisearch(int v)
    int sum=0,pos=0;
    for(int i=MAXLOGN;i>=0;i--)
        if(pos+(1<<i)<=n&&sum+bit[pos+(1<<i)]<v)</pre>
            sum+=bit[pos+(1<<i)];</pre>
            pos+=(1<<i);
        }
   }
    return pos+1;
}
int main()
    return 0;
```

# 1.3 Mo's algorithm

```
#include<bits/stdc++.h>
#define MAXN 100005
#define MAXM 100005
using namespace std;
struct query
   int l,r,id;
}save[MAXM];
int cnt[MAXN],a[MAXN],out[MAXN];
int n,m,ans,block;
bool cmp(query x,query y)
   if(x.1/block!=y.1/block) return x.1/block<y.1/block;</pre>
   if(x.1/block&1) return x.r>y.r; else return x.r<y.r;</pre>
void add(int pos)
   if(cnt[a[pos]]==a[pos]) ans--;
   cnt[a[pos]]++;
   if(cnt[a[pos]] == a[pos]) ans++;
void del(int pos)
   if(cnt[a[pos]] == a[pos]) ans--;
   cnt[a[pos]]--;
   if(cnt[a[pos]] == a[pos]) ans++;
void update(int cl,int cr,int l,int r)
{
```

```
while(cr<r) add(++cr);</pre>
    while(cl>1) add(--cl);
    while(cl<1) del(cl++);</pre>
    while(cr>r) del(cr--);
}
int main()
    scanf("%d %d",&n,&m);
    block=(int)sqrt(n);
    for(int i=1;i<=n;i++)</pre>
        scanf("%d",&a[i]);
        if(a[i]>100000) a[i]=100001;
    for(int i=0;i<m;i++)</pre>
        save[i].id=i;
        scanf("%d %d",&save[i].1,&save[i].r);
    }
    sort(save,save+m,cmp);
    memset(cnt,0,sizeof(cnt));
    for(int i=save[0].1;i<=save[0].r;i++)</pre>
        if(cnt[a[i]]==a[i]) ans--;
        cnt[a[i]]++;
        if(cnt[a[i]]==a[i]) ans++;
    out[save[0].id]=ans;
    int cl=save[0].1,cr=save[0].r;
    for(int i=1;i<m;i++)</pre>
        update(cl,cr,save[i].l,save[i].r);
        out[save[i].id]=ans;
        cl=save[i].1;
        cr=save[i].r;
    }
    for(int i=0;i<m;i++)</pre>
        printf("%d\n",out[i]);
    return 0;
}
```

# 1.4 Mo's algorithm on Trees

```
#pragma GCC optimize(3)
#include<bits/stdc++.h>
#define MAXN 200005
#define MAXM 200005
#define MAXLOGN 20
#define INF 1000000000
#define MOD 1000000007
#define F first
#define S second
using namespace std;
typedef long long ll;
typedef pair<int,int> P;
int n,q,tot,st[2*MAXN],ed[2*MAXN],loc[2*MAXN],val[MAXN];
```

```
vector<int> dis;
vector<int> G[MAXN]:
int spt[MAXLOGN+1][4*MAXN];
int vs[MAXN*2],depth[MAXN*2];
int id[MAXN],pos[2*MAXN],cnt[MAXN],now,sum;
bool vis[MAXN];
vector<int> v;
void dfs(int v,int p,int d,int &k)
   st[v]=++tot; loc[tot]=v;
   id[v]=k;
   vs[k]=v;
   depth[k++]=d;
   for(auto to:G[v])
       if(to==p) continue;
       dfs(to,v,d+1,k);
       vs[k]=v;
       depth[k++]=d;
   }
   ed[v]=++tot;
   loc[tot]=v;
}
void add_edge(int u,int v)
   G[u].push_back(v);
   G[v].push_back(u);
int getMin(int x, int y)
{
   return depth[x] < depth[y]?x:y;</pre>
}
void rmq_init(int n)
   for(int i=1;i<=n;++i) spt[0][i]=i;</pre>
   for(int i=1;1<<i<n;++i)</pre>
       for(int j=1;j+(1<<i)-1<=n;++j)</pre>
           spt[i][j]=getMin(spt[i-1][j],spt[i-1][j+(1<<(i-1))]);</pre>
void init(int V)
   int k=0;
   dfs(1,0,0,k);
   rmq_init(V*2-1);
}
int query(int 1, int r)
{
   int k=31-__builtin_clz(r-l+1);
   return getMin(spt[k][1],spt[k][r-(1<<k)+1]);</pre>
}
int lca(int u,int v)
   if(u==v) return u;
   return vs[query(min(id[u],id[v]),max(id[u],id[v]))];
}
struct qry
```

```
int u,v;
   int 1,r,z,id;
}Q[MAXM];
bool cmp(qry a,qry b)
{
   return pos[a.1] == pos[b.1]?a.r < b.r:pos[a.1] < pos[b.1];</pre>
}
void deal(int x)
   if(!vis[x])
        if(!cnt[val[x]]) now++;
        cnt[val[x]]++;
        sum++;
   }
   else
   {
        cnt[val[x]]--;
        assert(cnt[val[x]]>=0);
        if(!cnt[val[x]]) now--;
       sum--;
   vis[x]^=1;
}
int ans[MAXM];
const int blocks=200;
int main()
{
   scanf("%d%d",&n,&q);
   for(int i=1;i<=n;i++)</pre>
   {
        scanf("%d",&val[i]);
       dis.push_back(val[i]);
   }
   sort(dis.begin(),dis.end());
   dis.erase(unique(dis.begin(),dis.end()),dis.end());
   for(int i=1;i<=n;i++) val[i]=lower_bound(dis.begin(),dis.end(),val[i])-dis.begin();</pre>
   for(int i=0;i<n-1;i++)</pre>
        int u,v;
        scanf("%d%d",&u,&v);
        add_edge(u,v);
   }
   init(n);
   assert(tot==2*n);
   for(int i=1;i<=tot;i++) pos[i]=i/blocks+1;</pre>
   for(int i=1;i<=q;i++)</pre>
   {
       Q[i].id=i;
       int u,v;
       scanf("%d%d",&u,&v);
       Q[i].u=u; Q[i].v=v;
        if(st[u]>st[v]) swap(u,v);
       int z=lca(u,v);
       if(z==u) Q[i].l=st[u],Q[i].r=st[v];
       else Q[i].l=ed[u],Q[i].r=st[v],Q[i].z=z;
   sort(Q+1,Q+q+1,cmp);
   int l=1,r=0;
```

```
memset(cnt,0,sizeof(cnt));
memset(vis,false,sizeof(vis));
for(int i=1;i<=q;i++)
{
    if(r<Q[i].r) {for(r++;r<=Q[i].r;r++) deal(loc[r]); r--;}
    if(r>Q[i].r) {for(;r>Q[i].r;r--) deal(loc[r]); }
    if(1<Q[i].l) {for(;1<Q[i].l;l++) deal(loc[l]); }
    if(1>Q[i].l) {for(1--;1>=Q[i].l;l--) deal(loc[l]); l++;}
    if(Q[i].z) deal(Q[i].z);
    ans[Q[i].id]=now;
    if(Q[i].z) deal(Q[i].z);
}
for(int i=1;i<=q;i++) printf("%d\n",ans[i]);
    return 0;
}</pre>
```

#### 1.5 Mo's algorithm with Queries

```
#include<bits/stdc++.h>
#define MAXN 10005
#define INF 100000000
#define MOD 100000007
#define F first
#define S second
using namespace std;
typedef long long 11;
typedef pair<int,int> P;
struct query{int 1,r,t,id;};
int n,m,c[MAXN],cnt[100*MAXN],res,cur,ans[MAXN];
query q[MAXN];
int tim[MAXN],pos[MAXN],pre[MAXN],val[MAXN];
int totq,totc,nowl,nowr;
const int blocks=462;
bool cmp(query x,query y)
   if(x.1/blocks!=y.1/blocks) return x.1<y.1;</pre>
   if(x.r/blocks!=y.r/blocks) return x.r<y.r;</pre>
   return x.t<y.t;</pre>
}
char ch[5];
void add(int p)
   if(!cnt[c[p]]) res++;
   cnt[c[p]]++;
void del(int p)
   cnt[c[p]]--;
   if(!cnt[c[p]]) res--;
void tadd(int cur)
   if(pos[cur]>=nowl&&pos[cur]<=nowr)</pre>
       cnt[c[pos[cur]]]--;
       if(!cnt[c[pos[cur]]]) res--;
```

```
pre[cur]=c[pos[cur]];
   c[pos[cur]]=val[cur];
   if(pos[cur]>=nowl&&pos[cur]<=nowr)</pre>
   {
        if(!cnt[c[pos[cur]]]) res++;
        cnt[c[pos[cur]]]++;
   }
}
void tdel(int cur)
   if(pos[cur]>=nowl&&pos[cur]<=nowr)</pre>
        cnt[c[pos[cur]]]--;
        if(!cnt[c[pos[cur]]]) res--;
   c[pos[cur]]=pre[cur];
   if(pos[cur]>=nowl&&pos[cur]<=nowr)</pre>
        if(!cnt[c[pos[cur]]]) res++;
        cnt[c[pos[cur]]]++;
   }
}
void tupd(int now)
   while(cur<totc&&tim[cur+1]<=now) tadd(++cur);</pre>
   while(cur>0&&tim[cur]>now) tdel(cur--);
void upd(int now,int 1,int r)
{
   tupd(now);
   while(nowl>1) add(--nowl);
   while(nowr<r) add(++nowr);</pre>
   while(nowl<1) del(nowl++);</pre>
   while(nowr>r) del(nowr--);
}
int main()
   scanf("%d%d",&n,&m);
   for(int i=1;i<=n;i++) scanf("%d",&c[i]);</pre>
   for(int i=1;i<=m;i++)</pre>
        scanf("%s",ch);
        if(ch[0]=='Q')
           totq++;q[totq].id=totq;q[totq].t=i;
           scanf("%d%d",&q[totq].1,&q[totq].r);
       }
       else
        {
           totc++;tim[totc]=i;
           scanf("%d%d",&pos[totc],&val[totc]);
       }
   sort(q+1,q+totq+1,cmp);
   nowl=1;nowr=0;cur=0;
   for(int i=1;i<=totq;i++)</pre>
   {
       upd(q[i].t,q[i].1,q[i].r);
        ans[q[i].id]=res;
```

```
}
for(int i=1;i<=totq;i++) printf("%d\n",ans[i]);
return 0;
}</pre>
```

### 1.6 Mo's algorithm on Trees with Queries

```
#pragma GCC optimize(3)
#include<bits/stdc++.h>
#define MAXN 100005
#define MAXM 100005
#define MAXLOGN 20
#define INF 100000000
#define MOD 100000007
#define F first
#define S second
using namespace std;
typedef long long 11;
typedef pair<int,int> P;
int n,m,q,tot,ctot,qtot,st[2*MAXN],ed[2*MAXN],loc[2*MAXN],val[MAXN];
int V[MAXN],W[MAXM],C[MAXN];
vector<int> dis;
vector<int> G[MAXN];
int spt[MAXLOGN+1][4*MAXN];
int vs[MAXN*2],depth[MAXN*2];
int id[MAXN],pos[2*MAXN],cnt[MAXN];
P change[MAXN];
ll res;
bool vis[MAXN];
vector<int> v;
void dfs(int v,int p,int d,int &k)
   st[v]=++tot; loc[tot]=v;
   id[v]=k;
   vs[k]=v;
   depth[k++]=d;
   for(auto to:G[v])
       if(to==p) continue;
       dfs(to,v,d+1,k);
       vs[k]=v;
       depth[k++]=d;
   ed[v]=++tot;
   loc[tot]=v;
void add_edge(int u,int v)
   G[u].push_back(v);
   G[v].push_back(u);
int getMin(int x, int y)
{
   return depth[x] < depth[y]?x:y;</pre>
void rmq_init(int n)
```

```
{
   for(int i=1;i<=n;++i) spt[0][i]=i;</pre>
   for(int i=1;1<<i<n;++i)</pre>
        for(int j=1;j+(1<<i)-1<=n;++j)</pre>
           spt[i][j]=getMin(spt[i-1][j],spt[i-1][j+(1<<(i-1))]);</pre>
}
void init(int V)
{
   int k=0;
   dfs(1,0,0,k);
   rmq_init(V*2-1);
}
int query(int 1, int r)
   int k=31-__builtin_clz(r-l+1);
   return getMin(spt[k][1],spt[k][r-(1<<k)+1]);</pre>
}
int lca(int u,int v)
{
   if(u==v) return u;
   return vs[query(min(id[u],id[v]),max(id[u],id[v]))];
struct qry
   int l,r,z,ti,id;
}Q[MAXM];
bool cmp(qry a,qry b)
{
   if(pos[a.1]!=pos[b.1]) return pos[a.1]<pos[b.1];</pre>
   if(pos[a.r]!=pos[b.r]) return pos[a.r]<pos[b.r];</pre>
   if(pos[a.r]&1) return a.ti>b.ti; else return a.ti<b.ti;</pre>
}
void deal(int x)
   if(!vis[x])
   {
       cnt[C[x]]++;
       res+=1LL*W[cnt[C[x]]]*V[C[x]];
   }
   else
   {
       res-=1LL*W[cnt[C[x]]]*V[C[x]];
        cnt[C[x]]--;
   }
   vis[x]^=1;
}
void modify(int ti)
   int x=change[ti].F,y=change[ti].S;
   if(vis[x])
   {
       res-=1LL*W[cnt[C[x]]]*V[C[x]];
        cnt[C[x]]--;
       cnt[y]++;
       res+=1LL*W[cnt[y]]*V[y];
   swap(C[change[ti].F],change[ti].S);
}
```

```
11 ans[MAXM];
const int blocks=2000:
int main()
{
   scanf("%d%d%d",&n,&m,&q);
   for(int i=1;i<=m;i++) scanf("%d",&V[i]);</pre>
   for(int i=1;i<=n;i++) scanf("%d",&W[i]);</pre>
   for(int i=0;i<n-1;i++)</pre>
       int u,v;
       scanf("%d%d",&u,&v);
       add_edge(u,v);
   }
   for(int i=1;i<=n;i++) scanf("%d",&C[i]);</pre>
   init(n);
   for(int i=1;i<=tot;i++) pos[i]=i/blocks+1;</pre>
   for(int i=1;i<=q;i++)</pre>
       int type,u,v;
       scanf("%d%d%d", &type, &u, &v);
       if(type==0) change[++ctot]=P(u,v);
       else
       {
           ++qtot;
           Q[qtot].id=qtot;
           Q[qtot].ti=ctot;
           if(st[u]>st[v]) swap(u,v);
           int z=lca(u,v);
           if(z==u) Q[qtot].l=st[u],Q[qtot].r=st[v];
           else Q[qtot].l=ed[u],Q[qtot].r=st[v],Q[qtot].z=z;
       }
   }
   sort(Q+1,Q+qtot+1,cmp);
   int l=1,r=0,ti=0;
   memset(cnt,0,sizeof(cnt));
   memset(vis,false,sizeof(vis));
   for(int i=1;i<=qtot;i++)</pre>
       if(r<Q[i].r) {for(r++;r<=Q[i].r;r++) deal(loc[r]); r--;}</pre>
       if(r>Q[i].r) {for(;r>Q[i].r;r--) deal(loc[r]); }
       if(1<Q[i].1) {for(;1<Q[i].1;1++) deal(loc[1]); }</pre>
       if(1>Q[i].1) {for(1--;1>=Q[i].1;1--) deal(loc[1]); 1++;}
       if(Q[i].z) deal(Q[i].z);
       while(ti<Q[i].ti) modify(++ti);</pre>
       while(ti>Q[i].ti) modify(ti--);
       ans[Q[i].id]=res;
       if(Q[i].z) deal(Q[i].z);
   for(int i=1;i<=qtot;i++) printf("%lld\n",ans[i]);</pre>
   return 0;
}
```

# 1.7 Heavy Light Decomposition

```
#include<bits/stdc++.h>
#define MAXN 400005
#define INF 1000000000
```

```
#define MOD 100000007
#define F first
#define S second
using namespace std;
typedef long long 11;
typedef pair<int,int> P;
struct node
   int l,r,maxi,sum;
};
int tot,q,n,k,a[MAXN];
int pa[MAXN],dep[MAXN],sz[MAXN],wson[MAXN],top[MAXN],spos[MAXN],tpos[MAXN];
struct segtree
   node seg[4*MAXN];
   int id[MAXN];
   void build(int k,int l,int r)
       seg[k].l=l;seg[k].r=r;
       if(l==r)
           seg[k].maxi=seg[k].sum=a[tpos[1]];
           id[1]=k;
           return;
       }
       int mid=(1+r)/2;
       build(k*2,1,mid);build(k*2+1,mid+1,r);
       seg[k].maxi=max(seg[k*2].maxi,seg[k*2+1].maxi);
       seg[k].sum=seg[k*2].sum+seg[k*2+1].sum;
   void update(int k,int x)
       k=id[k];
       seg[k].maxi=seg[k].sum=x;
       while(k>1)
       {
           k=k/2;
           seg[k].maxi=max(seg[k*2].maxi,seg[k*2+1].maxi);
           seg[k].sum=seg[k*2].sum+seg[k*2+1].sum;
   }
   int query1(int k,int l,int r)
       if(seg[k].l>r||seg[k].r<l) return -INF;</pre>
       if(seg[k].l>=l&&seg[k].r<=r) return seg[k].maxi;</pre>
       return max(query1(k*2,1,r),query1(k*2+1,1,r));
   }
   int query2(int k,int l,int r)
       if(seg[k].l>r||seg[k].r<l) return 0;</pre>
       if(seg[k].l>=l&&seg[k].r<=r) return seg[k].sum;</pre>
       return query2(k*2,1,r)+query2(k*2+1,1,r);
   int query_max(int 1,int r)
   {
       return query1(1,1,r);
   int query_sum(int 1,int r)
```

```
return query2(1,1,r);
}tree;
vector<int> G[MAXN];
void dfs1(int v,int p,int d)
   dep[v]=d;pa[v]=p;sz[v]=1;
   for(int i=0;i<(int)G[v].size();i++)</pre>
       int to=G[v][i];
       if(to==p) continue;
       dfs1(to,v,d+1);
       if(sz[to]>sz[wson[v]]) wson[v]=to;
       sz[v] += sz[to];
   }
}
void dfs2(int v,int p,int num)
   top[v]=num;
   spos[v]=++tot;
   tpos[tot]=v;
   if(wson[v]) dfs2(wson[v],v,num);
   for(int i=0;i<(int)G[v].size();i++)</pre>
       int to=G[v][i];
       if(to==p||to==wson[v]) continue;
       dfs2(to,v,to);
   }
}
void init()
   tot=0;
   memset(wson,0,sizeof(wson));//important when multiple test cases!!!
   dfs1(1,1,1);
   dfs2(1,0,1);
   tree.build(1,1,n);
}
void update(int k,int x)
   tree.update(spos[k],x);
int query_max(int u,int v)
   int res=-INF;
   while(top[u]!=top[v])
       if(dep[top[u]] < dep[top[v]]) swap(u,v);</pre>
       res=max(res,tree.query_max(spos[top[u]],spos[u]));
       u=pa[top[u]];
   }
   if(dep[u] < dep[v]) swap(u,v);</pre>
   res=max(res,tree.query_max(spos[v],spos[u]));
   return res;
int query_sum(int u,int v)
   int res=0;
   while(top[u]!=top[v])
   {
```

```
if(dep[top[u]] < dep[top[v]]) swap(u,v);</pre>
       res+=tree.query_sum(spos[top[u]],spos[u]);
       u=pa[top[u]];
   }
   if(dep[u] < dep[v]) swap(u,v);</pre>
   res+=tree.query_sum(spos[v],spos[u]);
   return res;
}
char str[10];
int x,y;
int main()
   scanf("%d",&n);
   for(int i=0;i<n-1;i++)</pre>
       int u,v;
       scanf("%d%d",&u,&v);
       G[u].push_back(v);G[v].push_back(u);
   for(int i=1;i<=n;i++) scanf("%d",&a[i]);</pre>
   init();
   scanf("%d",&q);
   while(q--)
       scanf("%s%d%d",str,&x,&y);
       if(str[1] == 'H') update(x,y);
       if(str[1]=='M') printf("%d\n",query_max(x,y));
       if(str[1] == 'S') printf("%d\n",query_sum(x,y));
   }
   return 0;
}
```

# 1.8 Long Short Decomposition

```
#pragma GCC optimize(3)
#include<bits/stdc++.h>
#define MAXN 1000005
#define INF 100000000
#define MOD 100000007
#define F first
#define S second
using namespace std;
typedef long long 11;
typedef pair<int,int> P;
int n,k,a[MAXN];
int ans[MAXN];
vector<int> G[MAXN];
struct state
   vector<int> *a;//reversed depth array
   int sz()
   {
       return a->size();
   void add(int i,int val)
       (*a)[i]+=val;
```

```
}
}s[MAXN];
state pull(state z)
   if(z.sz()==0)
   {
       state c;
       c.a=new vector<int> (1,1);
       return c;
   }
   else
   {
       state c;
       c.a=z.a;
       c.a->push_back(0);;
       c.add(c.sz()-1,1);
       return c;
   }
}
state merge(state a,state b)
   if(a.sz() < b.sz()) swap(a,b);</pre>
   int bs=b.sz();
   int as=a.sz();
   for(int i=0;i<bs;i++) a.add(as-i-1,(*(b.a))[bs-i-1]);</pre>
   return a;
void dfs(int v,int p)
   s[v].a=new vector<int>(0);
   for(auto to:G[v])
       if(to==p) continue;
       dfs(to,v);
       s[v]=merge(s[v],s[to]);
   s[v]=pull(s[v]);
}
int main()
   return 0;
}
```

# 1.9 Lichao Segment Tree

```
{
   return 1LL*p.F*x+p.S;
void insert(int &k,int l,int r,int x,int y,P p)
   if(l>y||x>r) return;
   k=++tot;
   has[k]=false;
   if(1>=x&&r<=y)</pre>
       if(!has[k])
           has[k]=true;
           ans[k]=p;
           return;
       }
       11 trl=f(ans[k],1),trr=f(ans[k],r);
       11 vl=f(p,1),vr=f(p,r);
       if(trl<=vl&&trr<=vr) return;</pre>
       if(trl>=vl&&trr>=vr) {ans[k]=p; return;}
       int mid=(1+r)/2;
       if(trl>=vl) swap(ans[k],p);
       if(f(ans[k],mid)<=f(p,mid)) insert(rson[k],mid+1,r,x,y,p);</pre>
       else swap(ans[k],p),insert(lson[k],1,mid,x,y,p);
       return;
   }
   int mid=(1+r)/2;
   insert(lson[k],1,mid,x,y,p); insert(rson[k],mid+1,r,x,y,p);
11 query(int &k,int l,int r,int x)
   if(!k) return INF;
   11 res=(!has[k]?INF:f(ans[k],x));
   if(l==r) return res;
   int mid=(1+r)/2;
   if(x<=mid) return min(res,query(lson[k],1,mid,x));</pre>
   else return min(res,query(rson[k],mid+1,r,x));
}
```

#### 1.10 Segment Tree Beats

```
#include<bits/stdc++.h>
#define MAXN 1000005
#define INF 100000000
#define MOD 100000007
#define F first
#define S second
using namespace std;
typedef long long 11;
typedef pair<int,int> P;
struct node
{
       11 l,r,sum,maxx,secx,maxnum,lazy;
}seg[4*MAXN];
11 t,n,m,a[MAXN];
void Lazy(ll k)
{
```

```
if(seg[k].l==seg[k].r||seg[k].lazy==INT_MAX) return;
       if(seg[k*2].lazy>=seg[k].lazy&&seg[k*2].maxx>seg[k].lazy)
       {
              seg[k*2].sum=(seg[k*2].maxx-seg[k].lazy)*seg[k*2].maxnum;
              seg[k*2].maxx=seg[k].lazy;
              seg[k*2].lazy=seg[k].lazy;
       }
       if(seg[k*2+1].lazy>=seg[k].lazy&&seg[k*2+1].maxx>seg[k].lazy)
              seg[k*2+1].sum=(seg[k*2+1].maxx-seg[k].lazy)*seg[k*2+1].maxnum;
              seg[k*2+1].maxx=seg[k].lazy;
              seg[k*2+1].lazy=seg[k].lazy;
       seg[k].lazy=INT_MAX;
       return;
}
void merge(ll k)
       seg[k].sum=seg[k*2].sum+seg[k*2+1].sum;
       seg[k].maxx=max(seg[k*2].maxx,seg[k*2+1].maxx);
       ll res=0, ans=-1;
       if(seg[k*2].maxx==seg[k].maxx) res+=seg[k*2].maxnum;
       if(seg[k*2+1].maxx==seg[k].maxx) res+=seg[k*2+1].maxnum;
       seg[k].maxnum=res;
       if(seg[k*2].maxx!=seg[k].maxx) ans=max(ans,seg[k*2].maxx);
       if(seg[k*2].secx!=seg[k].maxx) ans=max(ans,seg[k*2].secx);
       if(seg[k*2+1].maxx!=seg[k].maxx) ans=max(ans,seg[k*2+1].maxx);
       if(seg[k*2+1].secx!=seg[k].maxx) ans=max(ans,seg[k*2+1].secx);
       seg[k].secx=ans;
       //printf("l=%lld r=%lld maxx=%lld secx=%lld maxnum=%lld sum=%lld
           lazy=%1ld\n",seg[k].1,seg[k].r,seg[k].maxx,seg[k].secx,seg[k].maxnum,seg[k].sum,seg[k].lazy);
void build(ll k,ll l,ll r)
       seg[k].l=1;seg[k].r=r;seg[k].lazy=INT_MAX;
       if(l==r)
       {
              seg[k].maxx=seg[k].sum=a[1];
              seg[k].maxnum=1;
              seg[k].secx=-1;
              return;
       11 \text{ mid}=(1+r)/2;
       build(k*2,1,mid); build(k*2+1,mid+1,r);
       merge(k);
void update(ll k,ll l,ll r,ll x)
       if(seg[k].l>r||seg[k].r<1||seg[k].maxx<=x) return;</pre>
       if(seg[k].1>=1&&seg[k].r<=r&&seg[k].secx<x)</pre>
       {
              seg[k].sum-=(seg[k].maxx-x)*seg[k].maxnum;
              seg[k].maxx=x;
              seg[k].lazy=x;
              return;
       }
       Lazy(k);
       update(k*2,1,r,x);update(k*2+1,1,r,x);
       merge(k);
```

```
11 query1(11 k,11 1,11 r)
        if(seg[k].l>r||seg[k].r<l) return 0;</pre>
        if(seg[k].l>=l&&seg[k].r<=r) return seg[k].maxx;</pre>
       Lazy(k);
       return max(query1(k*2,1,r),query1(k*2+1,1,r));
11 query2(11 k,11 1,11 r)
        if(seg[k].l>r||seg[k].r<l) return 0;</pre>
        if(seg[k].l>=l&&seg[k].r<=r) return seg[k].sum;</pre>
       Lazy(k);
       return query2(k*2,1,r)+query2(k*2+1,1,r);
int main()
{
        scanf("%11d",&t);
       while(t--)
        {
               scanf("%lld%lld",&n,&m);
               for(ll i=1;i<=n;i++) scanf("%lld",&a[i]);</pre>
               build(1,1,n);
               for(ll i=1;i<=m;i++)</pre>
               {
                       11 type,x,y,z;
                       scanf("%lld",&type);
                       if(type==0)
                       {
                               scanf("%lld%lld",&x,&y,&z);
                              update(1,x,y,z);
                       }
                       else if(type==1)
                              scanf("%lld%lld",&x,&y);
                              printf("%lld\n",query1(1,x,y));
                       }
                       else
                       {
                               scanf("%11d%11d",&x,&y);
                              printf("%lld\n",query2(1,x,y));
                       }
               }
        }
       return 0;
}
```

# 1.11 Segment Tree Merge

```
#pragma GCC optimize(3)
#include<bits/stdc++.h>
#define MAXN 100005
#define MAXM 2000005
#define INF 1000000000
#define MOD 1000000007
#define F first
#define S second
```

```
using namespace std;
typedef long long 11;
typedef pair<int,int> P;
int n,sz,tot,root[MAXN],a[MAXN],ans[MAXN];
int cnt[MAXM],lson[MAXM],rson[MAXM];
vector<int> G[MAXN];
vector<int> v;
//ask for how many a[j] <a[i] if j is in the subtree of i for every i from 1..n
//time complexity:O(nlogn)
void pushup(int k)
   cnt[k]=cnt[lson[k]]+cnt[rson[k]];
void build(int &k,int l,int r,int p)
   k=++tot;
   if(1==r)
       cnt[k]=1;
       return;
   int mid=(1+r)/2;
   if(p<=mid) build(lson[k],1,mid,p);</pre>
   else build(rson[k],mid+1,r,p);
   pushup(k);
}
int merge(int x,int y,int 1,int r)
   if(!x) return y;
   if(!y) return x;
   if(l==r)
       cnt[x]+=cnt[y];
       return x;
   int mid=(1+r)/2;
   lson[x]=merge(lson[x],lson[y],l,mid);
   rson[x]=merge(rson[x],rson[y],mid+1,r);
   pushup(x);
   return x;
int query(int k,int l,int r,int x)
   if(k==0) return 0;
   if(r<x) return 0;</pre>
   if(l>=x) return cnt[k];
   int mid=(1+r)/2;
   return query(lson[k],1,mid,x)+query(rson[k],mid+1,r,x);
void dfs(int v,int p)
   for(auto to:G[v])
       if(to==p) continue;
       dfs(to,v);
       root[v]=merge(root[v],root[to],1,sz);
   ans[v]=query(root[v],1,sz,a[v]+1);
}
```

```
int main()
{
   scanf("%d",&n);
   for(int i=1;i<=n;i++)</pre>
       scanf("%d",&a[i]);
       v.push_back(a[i]);
   }
   sort(v.begin(),v.end());
   v.erase(unique(v.begin(),v.end()),v.end());
   for(int i=1;i<=n;i++) a[i]=lower_bound(v.begin(),v.end(),a[i])-v.begin()+1;</pre>
   sz=(int)v.size();
   for(int i=2;i<=n;i++)</pre>
       int p;scanf("%d",&p);
       G[p].push_back(i);G[i].push_back(p);
   for(int i=1;i<=n;i++) build(root[i],1,sz,a[i]);</pre>
   dfs(1,0);
   for(int i=1;i<=n;i++) printf("%d\n",ans[i]);</pre>
   return 0;
}
```

#### 1.12 Persistent Segment Tree

```
#pragma GCC optimize(3)
#include<bits/stdc++.h>
#define MAXN 100005
#define MAXM 2000005
#define INF 100000000
#define MOD 100000007
#define F first
#define S second
using namespace std;
typedef long long 11;
typedef pair<int,int> P;
int n,q,tot,cnt,a[MAXN],root[MAXN];
int lson[MAXM],rson[MAXM],mx[MAXM];
void merge(int k)
{
   mx[k]=max(mx[lson[k]],mx[rson[k]]);
void build(int &k,int 1,int r)
   k=++tot;
   if(l==r) {mx[k]=a[l]; return;}
   int mid=(1+r)/2;
   build(lson[k],1,mid);build(rson[k],mid+1,r);
   merge(k);
void insert(int &k,int last,int l,int r,int p,int v)
   k=++tot;
   mx[k]=mx[last];
   if(l==r) {mx[k]=v; return;}
   lson[k]=lson[last];rson[k]=rson[last];
   int mid=(1+r)/2;
```

```
if(p<=mid) insert(lson[k],lson[last],l,mid,p,v);</pre>
   else insert(rson[k],rson[last],mid+1,r,p,v);
   merge(k);
}
int query(int &k,int 1,int r,int x,int y)
   if(!k) return 0;
   if(1>v||r<x) return 0;</pre>
   if(l>=x&&r<=y) return mx[k];</pre>
   int mid=(1+r)/2;
   return max(query(lson[k],1,mid,x,y),query(rson[k],mid+1,r,x,y));
}
int main()
   scanf("%d%d",&n,&q);
   for(int i=1;i<=n;i++)</pre>
       scanf("%d",&a[i]);
   build(root[++cnt],1,n);
   for(int i=1;i<=q;i++)</pre>
       int type,k,x,y;
       scanf("%d%d%d%d",&type,&k,&x,&y);
       if(type==1) insert(root[++cnt],root[k],1,n,x,y);
       else printf("%d\n",query(root[k],1,n,x,y));
   }
   return 0;
```

#### 1.13 Persistent DSU

```
#include<bits/stdc++.h>
#define MAXN 100005
#define MAXM 2000005
#define INF 100000000
#define MOD 100000007
#define F first
#define S second
using namespace std;
typedef long long 11;
typedef pair<int,int> P;
int n,m,tot,root[MAXN];
int lson[MAXM],rson[MAXN],p[MAXM],rk[MAXM];
void build(int &k,int 1,int r)
{
   k=++tot;
   if(l==r) {p[k]=1; return;}
   int mid=(1+r)/2;
   build(lson[k],1,mid);build(rson[k],mid+1,r);
void insert(int &k,int last,int l,int r,int pos,int val)
   k=++tot;
   if(l==r) {p[k]=val; rk[k]=rk[last]; return;}
   lson[k]=lson[last];rson[k]=rson[last];
   int mid=(1+r)/2;
   if(pos<=mid) insert(lson[k],lson[last],l,mid,pos,val);</pre>
   else insert(rson[k],rson[last],mid+1,r,pos,val);
```

```
int query(int k,int l,int r,int pos)
   if(l==r) return k;
   int mid=(1+r)/2;
   if(pos<=mid) return query(lson[k],1,mid,pos);</pre>
   else return query(rson[k],mid+1,r,pos);
void add(int k,int l,int r,int pos)
   if(l==r) {rk[k]++; return;}
   int mid=(1+r)/2;
   if(pos<=mid) add(lson[k],1,mid,pos);</pre>
   else add(rson[k],mid+1,r,pos);
int find(int k,int x)
{
   int q=query(k,1,n,x);
   if(x==p[q]) return q;
   return find(k,p[q]);
}
int main()
{
   return 0;
```

#### 1.14 Persistent Trie

```
#include<bits/stdc++.h>
#define MAXN 100005
#define MAXM 2000005
#define INF 100000000
#define MOD 100000007
#define F first
#define S second
using namespace std;
typedef long long 11;
typedef pair<int,int> P;
int n,k,a[MAXN],tot;
int trie[MAXM][2],root[MAXN],sz[MAXM];
int newnode()
{
   ++tot;
   trie[tot][0]=trie[tot][1]=0;
   return tot;
}
void insert(int u,int v,int x)
   int now1=root[u]=newnode(),now2=root[v];
   for(int i=18;i>=0;i--)
       int id=(x>>i)&1;
       trie[now1][id]=newnode();
       trie[now1][!id]=trie[now2][!id];
       now1=trie[now1][id];now2=trie[now2][id];
       sz[now1]=sz[now2]+1;
   }
```

```
int query(int 1,int r,int x)
   int res=0;
   int now1=root[r+1],now2=root[1];
   for(int i=18;i>=0;i--)
       int id=(x>>i)&1;
       if(sz[trie[now1][!id]]-sz[trie[now2][!id]]>0)
           res+=(1<<i);
           id=!id;
       }
       now1=trie[now1][id];now2=trie[now2][id];
   return res;
}
int main()
{
   return 0;
}
```

#### 1.15 Monotone Stack

```
#include<bits/stdc++.h>
#define MAXN 100000
using namespace std;
int n;
int h[MAXN];
int L[MAXN],R[MAXN];
int st[MAXN];
void solve()
       int t=0;
       for(int i=0;i<n;i++)</pre>
               while(t>0&&h[st[t-1]]>=h[i]) t--;
               L[i]=t==0?0:(st[t-1]+1);
               st[t++]=i;
       }
       t=0;
       for(int i=n-1;i>=0;i--)
       {
               while(t>0&&h[st[t-1]]>=h[i]) t--;
               R[i]=t==0?n:st[t-1];
               st[t++]=i;
       }
       long long res=0;
       for(int i=0;i<n;i++)</pre>
       {
               res=max(res,(long long)h[i]*(R[i]-L[i]));
       }
       printf("%lld\n",res);
}
```

# 1.16 Monotone Deque

```
#include<bits/stdc++.h>
#define MAXN 100005
using namespace std;
int n,k;
int a[MAXN];
int b[MAXN];
int deq[MAXN];
void solve()
{
       int s=0,t=0;
       for(int i=0;i<n;i++)</pre>
               while(s<t&&a[deq[t-1]]>=a[i]) t--;
               deq[t++]=i;
               if(i-k+1>=0)
               {
                       b[i-k+1]=a[deq[s]];
                       if(deq[s]==i-k+1)
                       {
                               s++;
                       }
               }
       }
       for(int i=0;i<=n-k;i++)</pre>
               printf("%d%c",b[i],i==n-k?'\n':' ');
        }
}
int main()
   scanf("%d %d",&n,&k);
   for(int i=0;i<n;i++)</pre>
        scanf("%d",&a[i]);
   solve();
   return 0;
}
```

# 1.17 Splay

```
#include<bits/stdc++.h>
#define MAXN 1000005
#define INF 1000000000
#define MOD 1000000007
#define F first
#define S second
using namespace std;
typedef long long ll;
typedef pair<int,int> P;
int ch[MAXN][2],f[MAXN],size[MAXN],cnt[MAXN],key[MAXN];
int sz,root;
inline void clear(int x)
{
    ch[x][0]=ch[x][1]=f[x]=size[x]=cnt[x]=key[x]=0;
}
```

```
inline bool get(int x)
{
   return ch[f[x]][1]==x;
}
inline void pushup(int x)
   if(x)
       size[x]=cnt[x];
       if (ch[x][0]) size[x]+=size[ch[x][0]];
       if (ch[x][1]) size[x]+=size[ch[x][1]];
   }
}
inline void rotate(int x)
{
   int old=f[x],oldf=f[old],whichx=get(x);
   ch[old][whichx]=ch[x][whichx^1]; f[ch[old][whichx]]=old;
   ch[x][whichx^1]=old; f[old]=x;
   f[x]=oldf;
   if (oldf) ch[oldf][ch[oldf][1]==old]=x;
   pushup(old); pushup(x);
inline void splay(int x,int goal=0)
   for(int fa;(fa=f[x])!=goal;rotate(x))
       if(f[fa]!=goal) rotate((get(x)==get(fa))?fa:x);
   if(goal==0) root=x;
inline void insert(int x)
   if (root==0){sz++; ch[sz][0]=ch[sz][1]=f[sz]=0; root=sz; size[sz]=cnt[sz]=1; key[sz]=x;
        return;}
   int now=root,fa=0;
   while(1)
       if (x==key[now])
           cnt[now]++; pushup(now); pushup(fa); splay(now); break;
       }
       fa=now;
       now=ch[now] [key[now] < x];</pre>
       if (now==0)
           sz++;
           ch[sz][0]=ch[sz][1]=0;
           f[sz]=fa;
           size[sz]=cnt[sz]=1;
           ch[fa][key[fa]<x]=sz;</pre>
           key[sz]=x;
           pushup(fa);
           splay(sz);
           break;
   }
inline int find(int x)
   int now=root,ans=0;
   while(1)
```

```
{
       if(x<key[now]) now=ch[now][0];</pre>
       else
       {
           ans+=(ch[now][0]?size[ch[now][0]]:0);
           if (x==key[now]){splay(now); return ans+1;}
           ans+=cnt[now];
           now=ch[now][1];
       }
   }
}
inline int findx(int now,int k)
   while(now)
   {
       if(k<=size[ch[now][0]]) now=ch[now][0];</pre>
       else if(k<=size[ch[now][0]]+cnt[now]) return key[now];</pre>
       else k-=size[ch[now][0]]+cnt[now],now=ch[now][1];
   }
}
inline int pre()
   int now=ch[root][0];
   while (ch[now][1]) now=ch[now][1];
   return now;
}
inline int next()
   int now=ch[root][1];
   while (ch[now][0]) now=ch[now][0];
   return now;
}
inline void del(int x)
   int whatever=find(x);
   if (cnt[root]>1){cnt[root]--; pushup(root); return;}
   if (!ch[root][0]&&!ch[root][1]) {clear(root); root=0; return;}
   if (!ch[root][0])
   {
       int oldroot=root; root=ch[root][1]; f[root]=0; clear(oldroot); return;
   }
   else if (!ch[root][1])
   {
       int oldroot=root; root=ch[root][0]; f[root]=0; clear(oldroot); return;
   }
   int leftbig=pre(),oldroot=root;
   splay(leftbig);
   ch[root][1]=ch[oldroot][1];
   f[ch[oldroot][1]]=root;
   clear(oldroot);
   pushup(root);
}
int main()
   int n,opt,x;
   scanf("%d",&n);
   for (int i=1;i<=n;++i)</pre>
   {
       scanf("%d%d",&opt,&x);
```

```
switch(opt)
{
          case 1: insert(x); break;
          case 2: del(x); break;
          case 3: printf("%d\n",find(x)); break;
          case 4: printf("%d\n",findx(root,x)); break;
          case 5: insert(x); printf("%d\n",key[pre()]); del(x); break;
          case 6: insert(x); printf("%d\n",key[next()]); del(x); break;
}
}
```

#### 1.18 Treap

```
#include<bits/stdc++.h>
#define MAXN 50030
#define INF 100000000
using namespace std;
struct treap
   int root,treapcnt,key[MAXN],priority[MAXN],childs[MAXN][2],cnt[MAXN],size[MAXN];
   treap()
   {
       root=0:
       treapcnt=1;
       priority[0]=INF;
       size[0]=0;
   }
   void update(int x)
   {
       size[x]=size[childs[x][0]]+cnt[x]+size[childs[x][1]];
   }
   void rotate(int &x,int t)
       int y=childs[x][t];
       childs[x][t]=childs[y][1-t];
       childs[y][1-t]=x;
       update(x);
       update(y);
       x=y;
   }
   void _insert(int &x,int k)
       if(x)
           if(key[x]==k)
           {
               cnt[x]++;
           }
           else
           {
               int t=key[x]<k;</pre>
               _insert(childs[x][t],k);
```

```
if(priority[childs[x][t]]<priority[x])</pre>
               {
               rotate(x,t);
           }
       }
    }
    else
    {
       x=treapcnt++;
       key[x]=k;
       cnt[x]=1;
       priority[x]=rand();
       childs[x][0]=childs[x][1]=0;
    }
   update(x);
}
void _erase(int &x,int k)
    if(key[x]==k)
    {
       if(cnt[x]>1)
       {
           cnt[x]--;
       }
       else
           if(childs[x][0]==0&&childs[x][1]==0)
           {
               x=0;
               return;
           int t=priority[childs[x][0]]>priority[childs[x][1]];
           rotate(x,t);
           _erase(x,k);
       }
   }
    else
    {
       _erase(childs[x][key[x]<k],k);
   update(x);
}
int _getKth(int &x,int k)
    if(k<=size[childs[x][0]])</pre>
    {
       return _getKth(childs[x][0],k);
   k-=size[childs[x][0]]+cnt[x];
   if(k<=0)
       return key[x];
   return _getKth(childs[x][1],k);
}
void insert(int k)
```

```
{
    __insert(root,k);
}

void erase(int k)
{
    __erase(root,k);
}

int getKth(int k)
{
    return _getKth(root,k);
}
};

int main()
{
    return 0;
}
```

#### 1.19 Link-Cut Tree

```
#include <bits/stdc++.h>
#define MAXN 300005
#define INF 100000000
#define MOD 100000007
#define F first
#define S second
using namespace std;
typedef long long 11;
typedef pair<int,int> P;
#define lc t[x].ch[0]
#define rc t[x].ch[1]
#define pa t[x].fa
typedef long long 11;
namespace 1ct
   struct meow{int ch[2], fa, rev, sum, w;} t[2*MAXN];
   inline int wh(int x) {return t[pa].ch[1] == x;}
   inline int isr(int x) {return t[pa].ch[0] != x && t[pa].ch[1] != x;}
   inline void update(int x) {t[x].sum = t[lc].sum ^ t[rc].sum ^ t[x].w;}
   inline void rever(int x) {t[x].rev ^= 1; swap(lc, rc);}
   inline void pushdown(int x)
   {
       if(t[x].rev)
           if(lc) rever(lc);
           if(rc) rever(rc);
           t[x].rev = 0;
       }
   void pd(int x) {if(!isr(x)) pd(pa); pushdown(x);}
   inline void rotate(int x)
       int f=t[x].fa, g=t[f].fa, c=wh(x);
       if(!isr(f)) t[g].ch[wh(f)]=x;
       t[x].fa=g;
```

```
t[f].ch[c] = t[x].ch[c^1]; t[t[f].ch[c]].fa=f;
       t[x].ch[c^1] = f; t[f].fa=x;
       update(f); update(x);
   }
   inline void splay(int x)
   {
       pd(x);
       for(; !isr(x); rotate(x))
           if(!isr(pa)) rotate( wh(pa)==wh(x) ? pa : x );
   }
   inline void access(int x)
       for(int y=0; x; y=x, x=pa) splay(x), rc=y, update(x);
   }
   inline void maker(int x)
   {
       access(x); splay(x); rever(x);
   }
   inline int findr(int x)
       access(x); splay(x);
       while(lc) pushdown(x), x=lc;
       return x;
   }
   inline void link(int x, int y)
       maker(x);
       if(findr(y)!=x) t[x].fa=y;
   }
   inline void cut(int x, int y)
       maker(x);
       if(findr(y) == x&&t[x].fa == y&&t[y].ch[0] == x&&!t[y].ch[1])
           t[x].fa=t[y].ch[0]=0;
           update(y);
   }
   inline void split(int x, int y)
   {
       maker(x); access(y); splay(y);
   }
} using lct::findr;
int n, Q, op, x, y;
int main()
   scanf("%d%d",&n,&Q);
   for(int i=1; i<=n; i++) scanf("%d",&lct::t[i].w);</pre>
   for(int i=1; i<=Q; i++)</pre>
   {
       scanf("%d%d%d",&op,&x,&y);
       if(op==0) lct::split(x, y), printf("%d\n", lct::t[y].sum);
       if(op==1) lct::link(x, y);
       if(op==2) lct::cut(x, y);
       if(op==3) lct::t[x].w=y,lct::splay(x);
   }
```

{

}

#### 1.20 Union Set

```
#include<bits/stdc++.h>
#define MAXN 100000
using namespace std;
int p[MAXN],r[MAXN];
void init(int n)
   for(int i=0;i<n;i++)</pre>
       p[i]=i;
       r[i]=0;
}
int find(int x)
{
   if(p[x]==x) return x;
   else return p[x]=find(p[x]);
}
void unite(int x,int y)
   x=find(x);
   y=find(y);
   if(x==y) return;
   if(r[x]<r[y]) p[x]=y;</pre>
   else
   {
       p[y]=x;
       if(r[x]==r[y]) r[x]++;
}
bool same(int x,int y)
{
   return find(x)==find(y);
}
int main()
{
   return 0;
```

# 1.21 Sparse Table

```
#include<bits/stdc++.h>
#define MAXN 100000
using namespace std;
int N,Q;
int a[MAXN];
int st[MAXN][32];
int pre[MAXN];
void init(int n,int *arr)
{
    pre[1]=0;
    for(int i=2;i<=n;i++)</pre>
```

```
{
       pre[i]=pre[i-1];
       if ((1<<pre[i]+1)==i) ++pre[i];</pre>
   for(int i=n-1;i>=0;--i)
       st[i][0]=arr[i];
        for(int j=1;(i+(1<<j)-1)<n;++j)</pre>
           st[i][j]=min(st[i][j-1],st[i+(1<<j-1)][j-1]);
   }
}
int query(int 1,int r)
   int len=r-l+1,k=pre[len];
   return min(st[l][k],st[r-(1<<k)+1][k]);</pre>
int main()
   scanf("%d",&N);
   for(int i=0;i<N;i++)</pre>
       scanf("%d",&a[i]);
   init(N,a);
   scanf("%d",&Q);
   while(Q--)
   {
        int x,y;
        scanf("%d%d",&x,&y);
       printf("%d\n",query(x,y));
   return 0;
}
```

#### 1.22 DSU on Tree

```
#include<bits/stdc++.h>
#define MAXN 100005
#define INF 100000000
#define MOD 1000000007
#define F first
#define S second
using namespace std;
typedef long long 11;
typedef pair<int,int> P;
int n,t,c[MAXN],sz[MAXN],st[MAXN],ed[MAXN],cnt[MAXN],rev[MAXN];
vector<int> G[MAXN];
void dfs(int v,int p)
   st[v]=++t;rev[t]=v;
   sz[v]=1;
   for(int i=0;i<(int)G[v].size();i++)</pre>
       if(G[v][i]==p) continue;
       dfs(G[v][i],v);
       sz[v]+=sz[G[v][i]];
   }
   ed[v]=t;
   return;
```

```
void dfs2(int v,int p,bool keep)
   int mx=-1,wson=-1;
   for(int i=0;i<(int)G[v].size();i++)</pre>
       int to=G[v][i];
       if(to==p) continue;
       if(sz[to]>mx) {mx=sz[to]; wson=to;}
   }
   for(int i=0;i<(int)G[v].size();i++)</pre>
        int to=G[v][i];
        if(to==p||to==wson) continue;
       dfs2(to,v,0);
   if(wson!=-1) dfs2(wson,v,1);
   for(int i=0;i<(int)G[v].size();i++)</pre>
       int to=G[v][i];
       if(to==p||to==wson) continue;
       for(int j=st[to];j<=ed[to];j++)</pre>
           cnt[c[rev[j]]]++;
   }
   cnt[c[v]]++;
   //answer queries here
   if(!keep)
   {
        for(int j=st[v];j<=ed[v];j++)</pre>
           cnt[c[rev[j]]]--;
   }
}
int main()
   scanf("%d",&n);
   for(int i=1;i<=n;i++) scanf("%d",&c[i]);</pre>
   for(int i=0;i<n-1;i++)</pre>
        int u,v;
        scanf("%d%d",&u,&v);
       G[u].push_back(v);G[v].push_back(u);
   dfs(1,0);
}
```

# 1.23 Virtual Tree

```
#include<bits/stdc++.h>
#define MAXV 100005
#define INF 1000000000
#define MAXLOGV 20
using namespace std;
struct edge
{
   int to,cost;
};
vector<edge> G[MAXV];
```

```
vector<int> vt[MAXV];
int parent[MAXLOGV][MAXV];
int depth[MAXV],dfn[MAXV],dis[MAXV],st[MAXV];
int n,q,tot;
void add_edge(int from,int to)
{
   vt[from].push_back(to);
}
bool cmp(int x,int y)
   return dfn[x]<dfn[y];</pre>
void dfs(int v,int p,int d,int minx)
   dfn[v]=++tot;
   dis[v]=minx;
   parent[0][v]=p;
   depth[v]=d;
   for(int i=0;i<(int)G[v].size();i++)</pre>
       if(G[v][i].to!=p) dfs(G[v][i].to,v,d+1,min(minx,G[v][i].cost));
}
void init(int V)
   dfs(1,-1,0,INF);
   for(int k=0;k+1<MAXLOGV;k++)</pre>
       for(int v=1; v<=V; v++)</pre>
           if(parent[k][v]<0) parent[k+1][v]=-1;</pre>
           else parent[k+1][v]=parent[k][parent[k][v]];
   }
}
int lca(int u,int v)
   if(depth[u]>depth[v]) swap(u,v);
   for(int k=0;k<MAXLOGV;k++)</pre>
       if((depth[v]-depth[u])>>k&1)
           v=parent[k][v];
   if(u==v) return u;
   for(int k=MAXLOGV-1;k>=0;k--)
       if(parent[k][u]!=parent[k][v])
           u=parent[k][u];
           v=parent[k][v];
       }
   }
   return parent[0][u];
int build_vtree(vector<int> &a)
   sort(a.begin(),a.end(),cmp);
   a.erase(unique(a.begin(),a.end()),a.end());
   assert(a.size()>0);
   int t=0;
   st[t++]=a[0];
```

```
vector<int> newly;newly.clear();
   for(int i=1;i<(int)a.size();i++)</pre>
   {
       if(t==0) {st[t++]=a[i]; continue;}
       int l=lca(a[i],st[t-1]);
       while(t>1\&\&dfn[st[t-2]]>=dfn[1]) add_edge(st[t-2],st[t-1]),t--;
       if(1!=st[t-1]) {add_edge(1,st[t-1]),st[t-1]=1; newly.push_back(1);}
       st[t++]=a[i];
   while(t>1) add_edge(st[t-2],st[t-1]),t--;
   for(auto it:newly) a.push_back(it);
   return st[0];
}
int main()
{
   return 0;
}
```

#### 1.24 Young Tableaux

```
#include<bits/stdc++.h>
#define MAXN 5005
#define MAXM 305
#define INF 100000000
#define MOD 100000007
#define F first
#define S second
using namespace std;
typedef long long 11;
typedef pair<int,int> P;
int n,a[MAXN];
int t[MAXN];
int young[MAXM][MAXM];
void ins(int v)
   for(int i=1;;i++)
       if(t[i]==0||v>=young[i][t[i]])
           young[i][++t[i]]=v;
       }
       int pos=upper_bound(young[i]+1,young[i]+t[i]+1,v)-young[i];
       swap(young[i][pos],v);
   }
}
int main()
   scanf("%d",&n);
   for(int i=1;i<=n;i++)</pre>
       scanf("%d",&a[i]);
       ins(a[i]);
   }
   int x=0;
   for(int i=1;;i++)
```

```
x+=t[i];
    printf("%d\n",x);
    if(x==n) break;
}
    return 0;
}
```

#### 1.25 Centroid Decomposition

```
#include<bits/stdc++.h>
#define MAXN 100005
#define INF 100000000
#define MOD 100000007
#define F first
#define S second
using namespace std;
typedef long long 11;
typedef pair<int,int> P;
struct edge{int to,cost;};
int N,K;
vector<edge> G[MAXN];
bool centroid[MAXN];
int sz[MAXN],deep[MAXN],d[MAXN];
P getroot(int v,int p,int t)//search_centroid
   P res=P(INT_MAX,-1);
       int m=0;
   sz[v]=1;
   for(int i=0;i<(int)G[v].size();i++)</pre>
       int to=G[v][i].to;
       if(to==p||centroid[to]) continue;
       res=min(res,getroot(to,v,t));
       m=max(m,sz[to]);
       sz[v] += sz[to];
   m=max(m,t-sz[v]);
   res=min(res,P(m,v));
   return res;
}
void getdeep(int v,int p)//enumerate path
   deep[++deep[0]]=d[v];
   for(int i=0;i<(int)G[v].size();i++)</pre>
       int to=G[v][i].to;
       if(to==p||centroid[to]) continue;
       d[to]=d[v]+G[v][i].cost;
       getdeep(to,v);
   }
}
int cal(int v,int cost)
   d[v]=cost;deep[0]=0;
   getdeep(v,0);
   sort(deep+1,deep+deep[0]+1);
```

```
int l=1,r=deep[0],sum=0;
   while(l<r)</pre>
   {
        if(deep[1]+deep[r] \le K)
        {
           sum+=r-l;
           1++;
       }
       else r--;
   }
   return sum;
}
void solve(int v)
   ans+=cal(v,0);
   centroid[v]=true;
   for(int i=0;i<(int)G[v].size();i++)</pre>
        int to=G[v][i].to,cost=G[v][i].cost;
       if(centroid[to]) continue;
       ans-=cal(to,cost);
       int rt=getroot(to,v,sz[to]).S;
       solve(rt);
   }
}
void ac()
   ans=0;
   int rt=getroot(1,0,N).S;
   solve(rt);
   printf("%d\n",ans);
}
int main()
   while(scanf("%d%d",&N,&K)==2)
        if(!N&&!K) break;
       for(int i=1;i<=N;i++)</pre>
           G[i].clear();
       for(int i=0;i<N-1;i++)</pre>
           int x,y,z;
           scanf("%d%d%d",&x,&y,&z);
           G[x].push_back((edge){y,z});
           G[y].push_back((edge){x,z});
       memset(centroid,false,sizeof(centroid));
       ac();
   }
   return 0;
```

# 2 Geometry

# 2.1 Geometry All-in-One

```
#include <iostream>
#include <cstdio>
#include <cmath>
#include <algorithm>
using namespace std;
const double PI = acos(-1.0);
const double eps = 1e-10;
/*********
                          ***********/
//
           tatb
int sgn( double ta, double tb)
   if(fabs(ta-tb)<eps)return 0;</pre>
   if(ta<tb) return -1;</pre>
   return 1;
}
11
class Point
{
public:
   double x, y;
   Point(){}
   Point( double tx, double ty){ x = tx, y = ty;}
   bool operator < (const Point &_se) const</pre>
   {
       return x<_se.x || (x==_se.x && y<_se.y);</pre>
   }
   friend Point operator + (const Point &_st,const Point &_se)
       return Point(_st.x + _se.x, _st.y + _se.y);
   }
   friend Point operator - (const Point &_st,const Point &_se)
       return Point(_st.x - _se.x, _st.y - _se.y);
   }
   //
                 ( double )
   bool operator == (const Point &_off)const
       return sgn(x, _off.x) == 0 && sgn(y, _off.y) == 0;
   }
};
/******
                       *************/
double dot(const Point &po,const Point &ps,const Point &pe)
   return (ps.x - po.x) * (pe.x - po.x) + (ps.y - po.y) * (pe.y - po.y);
}
//
double xmult(const Point &po,const Point &ps,const Point &pe)
   return (ps.x - po.x) * (pe.y - po.y) - (pe.x - po.x) * (ps.y - po.y);
```

```
}
//
double getdis2(const Point &st,const Point &se)
   return (st.x - se.x) * (st.x - se.x) + (st.y - se.y) * (st.y - se.y);
}
//
double getdis(const Point &st,const Point &se)
   return sqrt((st.x - se.x) * (st.x - se.x) + (st.y - se.y) * (st.y - se.y));
}
class Line
public:
                                              [e]
   Point s, e;//
                                   [s]
   double a, b, c;//
                           ,ax+by+c=0
   double angle;//
                                  [-pi,pi]
   Line(){}
   Line( Point ts, Point te):s(ts),e(te){}//get_angle();}
   Line(double _a,double _b,double _c):a(_a),b(_b),c(_c){}
   //
   bool operator < (const Line &ta)const</pre>
   {
       return angle<ta.angle;</pre>
   }
   //
   friend double operator / (const Line &_st, const Line &_se)
       return (_st.e.x - _st.s.x) * (_se.e.y - _se.s.y) - (_st.e.y - _st.s.y) * (_se.e.x -
           _se.s.x);
   }
   //
   friend double operator *( const Line &_st, const Line &_se)
       return (_st.e.x - _st.s.x) * (_se.e.x - _se.s.x) - (_st.e.y - _st.s.y) * (_se.e.y -
           _se.s.y);
   }
   //
   //a=y2-y1,b=x1-x2,c=x2*y1-x1*y2
   bool pton()
       a = e.y - s.y;
       b = s.x - e.x;
       c = e.x * s.y - e.y * s.x;
       return true;
   }
   //
   //
   friend bool operator < (const Point &_Off, const Line &_Ori)</pre>
       return (_Ori.e.y - _Ori.s.y) * (_Off.x - _Ori.s.x)
           < (_Off.y - _Ori.s.y) * (_Ori.e.x - _Ori.s.x);
   }
   //
```

```
double get_angle( bool isVector = true)
   angle = atan2( e.y - s.y, e.x - s.x);
   if(!isVector && angle < 0)</pre>
       angle += PI;
   return angle;
}
                         1:
                                         2 s
bool has(const Point &_Off, bool isSegment = false) const
   bool ff = sgn( xmult( s, e, _Off), 0) == 0;
   if( !isSegment) return ff;
   return ff
       && sgn(_0ff.x - min(s.x, e.x), 0) >= 0 && sgn(_0ff.x - max(s.x, e.x), 0) <= 0
       && sgn(_0ff.y - min(s.y, e.y), 0) >= 0 && <math>sgn(_0ff.y - max(s.y, e.y), 0) <= 0;
}
double dis(const Point &_Off, bool isSegment = false)
   ///
   pton();
   //
   double td = (a * _0ff.x + b * _0ff.y + c) / sqrt(a * a + b * b);
   if(isSegment)
       double xp = (b * b * _Off.x - a * b * _Off.y - a * c) / (a * a + b * b);
       double yp = (-a * b * _Off.x + a * a * _Off.y - b * c) / (a * a + b * b);
       double xb = max(s.x, e.x);
       double yb = max(s.y, e.y);
       double xs = s.x + e.x - xb;
       double ys = s.y + e.y - yb;
      if(xp > xb + eps || xp < xs - eps || yp > yb + eps || yp < ys - eps)
           td = min( getdis(_Off,s), getdis(_Off,e));
   }
   return fabs(td);
}
Point mirror(const Point &_Off)
   ///
   Point ret;
   double d = a * a + b * b;
   ret.x = (b * b * _0ff.x - a * a * _0ff.x - 2 * a * b * _0ff.y - 2 * a * c) / d;
   ret.y = (a * a * _0ff.y - b * b * _0ff.y - 2 * a * b * _0ff.x - 2 * b * c) / d;
   return ret;
}
//
static Line ppline(const Point &_a,const Point &_b)
   Line ret;
   ret.s.x = (a.x + b.x) / 2;
   ret.s.y = (_a.y + _b.y) / 2;
   //
   ret.a = _b.x - _a.x;
   ret.b = _b.y - _a.y;
```

```
ret.c = (_a.y - _b.y) * ret.s.y + (_a.x - _b.x) * ret.s.x;
   //
   if(fabs(ret.a) > eps)
   {
       ret.e.y = 0.0;
       ret.e.x = - ret.c / ret.a;
       if(ret.e == ret. s)
          ret.e.y = 1e10;
          ret.e.x = - (ret.c - ret.b * ret.e.y) / ret.a;
   }
   else
       ret.e.x = 0.0;
       ret.e.y = - ret.c / ret.b;
       if(ret.e == ret. s)
          ret.e.x = 1e10;
          ret.e.y = - (ret.c - ret.a * ret.e.x) / ret.b;
   }
   return ret;
}
//
Line& moveLine( double t)
   Point of;
   of = Point( -(e.y - s.y), e.x - s.x);
   double dis = sqrt( of.x * of.x + of.y * of.y);
   of.x= of.x * t / dis, of.y = of.y * t / dis;
   s = s + of, e = e + of;
   return *this;
}
//
static bool equal(const Line &_st,const Line &_se)
   return _st.has( _se.e) && _se.has( _st.s);
}
//
static bool parallel(const Line &_st,const Line &_se)
   return sgn( _st / _se, 0) == 0;
}
//
//
static bool crossLPt(const Line &_st,const Line &_se, Point &ret)
   if(parallel(_st,_se))
       if(Line::equal(_st,_se)) return 0;
       return -1;
   }
   ret = _st.s;
   double t = ( Line(_st.s,_se.s) / _se) / ( _st / _se);
   ret.x += (_st.e.x - _st.s.x) * t;
   ret.y += (_st.e.y - _st.s.y) * t;
```

```
return 1;
   }
   //----
   11
   11
                   [_st],
                             [_se]
   friend bool crossSL( Line &_st, Line &_se)
       return sgn( xmult( _st.s, _se.s, _st.e) * xmult( _st.s, _st.e, _se.e), 0) >= 0;
                                         )
                                 eps
   static bool isCrossSS( const Line &_st, const Line &_se)
       //1.
       //2.
       return
          max(_st.s.x, _st.e.x) >= min(_se.s.x, _se.e.x) &&
          max(se.s.x, se.e.x) >= min(st.s.x, st.e.x) &&
          max(st.s.y, st.e.y) >= min(se.s.y, se.e.y) &&
          max(se.s.y, se.e.y) >= min(st.s.y, st.e.y) &&
          sgn( xmult( _se.s, _st.s, _se.e) * xmult( _se.s, _se.e, _st.s), 0) >= 0 &&
          sgn( xmult( _st.s, _se.s, _st.e) * xmult( _st.s, _st.e, _se.s), 0) >= 0;
   }
};
        graham
Point gsort;
bool gcmp( const Point &ta, const Point &tb)///
   double tmp = xmult( gsort, ta, tb);
   if( fabs( tmp) < eps)</pre>
       return getdis( gsort, ta) < getdis( gsort, tb);</pre>
   else if( tmp > 0)
       return 1;
   return 0;
}
class Polygon
public:
   const static int maxpn = 5e4+7;
   Point pt[maxpn];//
   Line dq[maxpn]; //
   int n;//
   11
   double area()
       double ans = 0.0;
       for(int i = 0; i < n; i ++)</pre>
          int nt = (i + 1) \% n;
          ans += pt[i].x * pt[nt].y - pt[nt].x * pt[i].y;
       return fabs( ans / 2.0);
   }
   //
```

```
Point gravity()
   Point ans;
    ans.x = ans.y = 0.0;
   double area = 0.0;
   for(int i = 0; i < n; i ++)</pre>
       int nt = (i + 1) \% n;
       double tp = pt[i].x * pt[nt].y - pt[nt].x * pt[i].y;
       area += tp;
       ans.x += tp * (pt[i].x + pt[nt].x);
       ans.y += tp * (pt[i].y + pt[nt].y);
    }
    ans.x /= 3 * area;
   ans.y /= 3 * area;
   return ans;
}
                                   Ε
                                             ] 0 (n)
//
bool ahas( Point &_Off)
    int ret = 0;
   double infv = 1e20;//
   Line 1 = Line( _Off, Point( -infv ,_Off.y));
   for(int i = 0; i < n; i ++)</pre>
       Line ln = Line(pt[i], pt[(i + 1) % n]);
       if(fabs(ln.s.y - ln.e.y) > eps)
       {
           Point tp = (ln.s.y > ln.e.y)? ln.s: ln.e;
           if( ( fabs( tp.y - _0ff.y) < eps && tp.x < _0ff.x + eps) || Line::isCrossSS( ln, 1))
               ret++;
       }
       else if( Line::isCrossSS( ln, l))
           ret++;
   }
   return ret&1;
}
//
                                     (logn)
                   0
bool bhas( Point & p)
   if(n < 3)
       return false;
    if( xmult( pt[0], p, pt[1]) > eps)
       return false;
    if( xmult( pt[0], p, pt[n-1]) < -eps)</pre>
       return false;
    int 1 = 2,r = n-1;
    int line = -1;
   while( 1 <= r)</pre>
       int mid = ( 1 + r) >> 1;
       if( xmult( pt[0], p, pt[mid]) >= 0)
           line = mid,r = mid - 1;
       else 1 = mid + 1;
    }
    return xmult( pt[line-1], p, pt[line]) <= eps;</pre>
}
```

```
//
Polygon split( Line &_Off)
   Polygon ret;
   Point spt[2];
   double tp = 0.0, np;
   bool flag = true;
   int i, pn = 0, spn = 0;
   for(i = 0; i < n; i ++)</pre>
       if(flag)
          pt[pn ++] = pt[i];
       else
          ret.pt[ret.n ++] = pt[i];
       np = xmult( _0ff.s, _0ff.e, pt[(i + 1) % n]);
       if(tp * np < -eps)
           flag = !flag;
          Line::crossLPt( _Off, Line(pt[i], pt[(i + 1) % n]), spt[spn++]);
       tp = (fabs(np) > eps)?np: tp;
   ret.pt[ret.n ++] = spt[0];
   ret.pt[ret.n ++] = spt[1];
   n = pn;
   return ret;
}
/**
                                                                  **/
void ConvexClosure( Point _p[], int _n)
   sort( _p, _p + _n);
   n = 0;
   for(int i = 0; i < _n; i++)</pre>
       while( n > 1 &  sgn( xmult( pt[n-2], pt[n-1], _p[i]), 0) <= 0)
       pt[n++] = _p[i];
   }
   int _key = n;
   for(int i = _n - 2; i >= 0; i--)
       while( n > key & sgn(xmult(pt[n-2], pt[n-1], p[i]), 0) <= 0)
          n--;
       pt[n++] = _p[i];
   }
   if(n>1) n--;//
}
/****
              graham
                                   ***************/
/****
                                 _n
                                         ************/
               _p
void graham( Point _p[], int _n)
{
   int cur=0;
   for(int i = 1; i < _n; i++)</pre>
```

```
p[i].x) > 0)
          cur = i;
   swap( _p[cur], _p[0]);
   n = 0, gsort = pt[n++] = _p[0];
   if( _n <= 1) return;</pre>
   sort( _p + 1, _p+_n ,gcmp);
   pt[n++] = _p[1];
   for(int i = 2; i < _n; i++)</pre>
       while(n>1 && sgn( xmult( pt[n-2], pt[n-1], _p[i]), 0) <= 0)//</pre>
       pt[n++] = _p[i];
   }
}
11
                  (
                                                       )
//
pair<Point, Point> rotating_calipers()
   int i = 1 % n;
   double ret = 0.0;
   pt[n] = pt[0];
   pair<Point,Point>ans=make_pair(pt[0],pt[0]);
   for(int j = 0; j < n; j ++)
       while( fabs( xmult( pt[i+1], pt[j], pt[j + 1])) > fabs( xmult( pt[i], pt[j], pt[j +
           1])) + eps)
          i = (i + 1) \% n;
       //pt[i] pt [j],pt[i + 1] pt [j + 1]
       if(ret < getdis2(pt[i],pt[j])) ret = getdis2(pt[i],pt[j]), ans = make_pair(pt[i],pt[j]);</pre>
       if(ret < getdis2(pt[i+1],pt[j+1])) ret = getdis(pt[i+1],pt[j+1]), ans =</pre>
           make_pair(pt[i+1],pt[j+1]);
   }
   return ans;
}
//
                  (
                                             )
//
double rotating_calipers( Polygon &_Off)
   int i = 0;
   double ret = 1e10;//inf
   pt[n] = pt[0];
   _{0ff.pt[_{0ff.n}] = _{0ff.pt[0]}}
                     pt
   while( _0ff.pt[i + 1].y > _0ff.pt[i].y)
       i = (i + 1) \% _{0ff.n};
   for(int j = 0; j < n; j ++)
   {
       double tp;
       while((tp = xmult(_Off.pt[i + 1],pt[j], pt[j + 1]) - xmult(_Off.pt[i], pt[j], pt[j +
           1])) > eps)
          i = (i + 1) \% _0ff.n;
       //(pt[i],pt[i+1]) (_Off.pt[j],_Off.pt[j + 1])
       ret = min(ret, Line(pt[j], pt[j + 1]).dis(_Off.pt[i], true));
       ret = min(ret, Line(_Off.pt[i], _Off.pt[i + 1]).dis(pt[j + 1], true));
       if(tp > -eps)//
                                          TLE
```

```
{
               ret = min(ret, Line(pt[j], pt[j + 1]).dis(_Off.pt[i + 1], true));
              ret = min(ret, Line(_Off.pt[i], _Off.pt[i + 1]).dis(pt[j], true));
           }
       }
       return ret;
   }
   11
              :0(nlog2(n))
   11
                         [1]
   //
                                          [ln];(
                                                       ]
                                                                                                0
   //
   int judege( Line &_lx, Line &_ly, Line &_lz)
   {
       Point tmp;
       Line::crossLPt(_lx,_ly,tmp);
       return sgn(xmult(_lz.s,tmp,_lz.e),0);
   }
   int halfPanelCross(Line L[], int ln)
   {
       int i, tn, bot, top;
       for(int i = 0; i < ln; i++)</pre>
           L[i].get_angle();
       sort(L, L + ln);
       //
       for(i = tn = 1; i < ln; i ++)</pre>
           if(fabs(L[i].angle - L[i - 1].angle) > eps)
              L[tn ++] = L[i];
       ln = tn, n = 0, bot = 0, top = 1;
       dq[0] = L[0], dq[1] = L[1];
       for(i = 2; i < ln; i ++)</pre>
           while(bot < top && judege(dq[top],dq[top-1],L[i]) > 0)
           while(bot < top && judege(dq[bot],dq[bot+1],L[i]) > 0)
              bot ++;
           dq[++ top] = L[i];
       }
       while(bot < top && judege(dq[top],dq[top-1],dq[bot]) > 0)
       while(bot < top && judege(dq[bot],dq[bot+1],dq[top]) > 0)
          bot ++;
       //
       //
                if(top \le bot + 1)
                   return 0;
       dq[++top] = dq[bot];
       for(i = bot; i < top; i ++)</pre>
           Line::crossLPt(dq[i],dq[i + 1],pt[n++]);
       return n;
   }
};
class Circle
public:
   Point c;//
   double r;//
```

{

```
(0 -360)
double db, de;//
//----
bool inside( Polygon &_Off)
   if(_Off.ahas(c) == false)
      return false;
   for(int i = 0; i < _0ff.n; i ++)</pre>
       Line 1 = Line(_Off.pt[i], _Off.pt[(i + 1) % _Off.n]);
       if(l.dis(c, true) < r - eps)</pre>
          return false;
   }
   return true;
}
bool has( Polygon &_Off)
   for(int i = 0; i < _0ff.n; i ++)</pre>
       if(getdis2(_0ff.pt[i],c) > r * r - eps)
          return false;
   return true;
}
//
                                          [_Off]
Circle operator-(Circle &_Off) const
   double d2 = getdis2(c,_Off.c);
   double d = getdis(c,_Off.c);
   double ans = acos((d2 + r * r - _0ff.r * _0ff.r) / (2 * d * r));
   Point py = _Off.c - c;
   double oans = atan2(py.y, py.x);
   Circle res;
   res.c = c;
   res.r = r;
   res.db = oans + ans;
   res.de = oans - ans + 2 * PI;
   return res;
}
                                          [_Off]
//
Circle operator+(Circle &_Off) const
   double d2 = getdis2(c,_Off.c);
   double d = getdis(c,_Off.c);
   double ans = acos((d2 + r * r - _0ff.r * _0ff.r) / (2 * d * r));
   Point py = _Off.c - c;
   double oans = atan2(py.y, py.x);
   Circle res;
   res.c = c;
   res.r = r;
   res.db = oans - ans;
   res.de = oans + ans;
   return res;
```

```
}
   11
   //
               [_Off](
                                                     (
                                                            s_Off
                                                                                 )
                                 ),
   pair<Line, Line> tangent( Point &_Off)
      double d = getdis(c,_Off);
      double angp = acos(r / d), ango = atan2(_Off.y - c.y, _Off.x - c.x);
      Point pl = Point(c.x + r * cos(ango + angp), c.y + r * sin(ango + angp)),
          pr = Point(c.x + r * cos(ango - angp), c.y + r * sin(ango - angp));
      return make_pair(Line(_Off, pl), Line(_Off, pr));
   }
   //
                 [_Off](
                               )
   //
   pair<Point, Point> cross(Line _Off)
      _Off.pton();
      double td = fabs(_Off.a * c.x + _Off.b * c.y + _Off.c) / sqrt(_Off.a * _Off.a + _Off.b *
          _Off.b);
      _Off.a + _Off.b * _Off.b);
      double yp = (- _Off.a * _Off.b * c.x + _Off.a * _Off.a * c.y - _Off.b * _Off.c) / (_Off.a *
          _Off.a + _Off.b * _Off.b);
      double ango = atan2(yp - c.y, xp - c.x);
      double angp = acos(td / r);
      return make_pair(Point(c.x + r * cos(ango + angp), c.y + r * sin(ango + angp)),
          Point(c.x + r * cos(ango - angp), c.y + r * sin(ango - angp));
   }
};
class triangle
public:
   Point a, b, c;//
   triangle(){}
   triangle(Point a, Point b, Point c): a(a), b(b), c(c){}
   //
   double area()
   {
      return fabs( xmult(a, b, c)) / 2.0;
   }
   //
   //
   Point circumcenter()
      double pa = a.x * a.x + a.y * a.y;
      double pb = b.x * b.x + b.y * b.y;
      double pc = c.x * c.x + c.y * c.y;
      double ta = pa * ( b.y - c.y) - pb * ( a.y - c.y) + pc * ( a.y - b.y);
      double tb = -pa * (b.x - c.x) + pb * (a.x - c.x) - pc * (a.x - b.x);
```

```
double tc = a.x * (b.y - c.y) - b.x * (a.y - c.y) + c.x * (a.y - b.y);
   return Point( ta / 2.0 / tc, tb / 2.0 / tc);
}
//
11
Point incenter()
   Line u, v;
   double m, n;
   u.s = a;
   m = atan2(b.y - a.y, b.x - a.x);
   n = atan2(c.y - a.y, c.x - a.x);
   u.e.x = u.s.x + cos((m + n) / 2);
   u.e.y = u.s.y + sin((m + n) / 2);
   v.s = b;
   m = atan2(a.y - b.y, a.x - b.x);
   n = atan2(c.y - b.y, c.x - b.x);
   v.e.x = v.s.x + cos((m + n) / 2);
   v.e.y = v.s.y + sin((m + n) / 2);
   Point ret;
   Line::crossLPt(u,v,ret);
   return ret;
}
//
//
Point perpencenter()
   Line u,v;
   u.s = c;
   u.e.x = u.s.x - a.y + b.y;
   u.e.y = u.s.y + a.x - b.x;
   v.s = b;
   v.e.x = v.s.x - a.y + c.y;
   v.e.y = v.s.y + a.x - c.x;
   Point ret;
   Line::crossLPt(u,v,ret);
   return ret;
}
//
//
//
//
Point barycenter()
   Line u,v;
   u.s.x = (a.x + b.x) / 2;
   u.s.y = (a.y + b.y) / 2;
   u.e = c;
   v.s.x = (a.x + c.x) / 2;
   v.s.y = (a.y + c.y) / 2;
   v.e = b;
   Point ret;
   Line::crossLPt(u,v,ret);
   return ret;
}
```

```
11
   Point fermentPoint()
       Point u, v;
       double step = fabs(a.x) + fabs(a.y) + fabs(b.x) + fabs(b.y) + fabs(c.x) + fabs(c.y);
       int i, j, k;
       u.x = (a.x + b.x + c.x) / 3;
       u.y = (a.y + b.y + c.y) / 3;
       while (step > eps)
           for (k = 0; k < 10; step /= 2, k ++)
              for (i = -1; i <= 1; i ++)
                  for (j =- 1; j <= 1; j ++)
                     v.x = u.x + step * i;
                      v.y = u.y + step * j;
                      if (getdis(u,a) + getdis(u,b) + getdis(u,c) > getdis(v,a) + getdis(v,b) +
                          getdis(v,c))
                         u = v;
                  }
              }
          }
       }
       return u;
   }
};
int main(void)
{
   return 0;
```

### 2.2 Stereometry

```
#pragma GCC optimize(3)
#include<bits/stdc++.h>
#define MAXN 100005
#define INF 100000000
#define MOD 100000007
#define F first
#define S second
using namespace std;
typedef long double T;
typedef long double db;
typedef long long 11;
typedef pair<int,int> P;
const T PI=acos(-1.0);
const T eps=1e-10;
int sgn( double ta, double tb)
   if(fabs(ta-tb)<eps)return 0;</pre>
   if(ta<tb) return -1;</pre>
```

```
return 1;
}
class Point
public:
   T x, y, z;
   Point(){}
   Point(T tx,T ty,T tz) {x=tx,y=ty,z=tz;}
   \label{eq:cont_p} \mbox{db dist2(Point p) } \{ \mbox{return } (x-p.x)*(x-p.x)+(y-p.y)*(y-p.y)+(z-p.z)*(z-p.z); \}
   db dist(Point p) {return sqrt(dist2(p));}
   Point operator+(Point p) {return {x+p.x,y+p.y,z+p.z};}
   Point operator-(Point p) {return {x-p.x,y-p.y,z-p.z};}
   Point operator*(T d) {return {x*d,y*d,z*d};}
   Point operator/(T d) {return {x/d,y/d,z/d};}
   bool operator==(Point p) {return tie(x,y,z)==tie(p.x,p.y,p.z);}
   bool operator!=(Point p) {return !operator==(p);}
   const bool operator<(Point &p)const {return tie(x,y,z)<tie(p.x,p.y,p.z);}</pre>
};
Point zero{0,0,0};
T operator|(Point v, Point w) {return v.x*w.x + v.y*w.y + v.z*w.z;}
T sq(Point v) {return v|v;}
db abs(Point v) {return sqrt(sq(v));}
Point unit(Point v) {return v/abs(v);}
db angle(Point v, Point w)
   db cosTheta=(v|w)/abs(v)/abs(w);
   return acos(max((db)-1.0,min((db)1.0,cosTheta)));
Point operator*(Point v,Point w) {return {v.y*w.z-v.z*w.y,v.z*w.x-v.x*w.z,v.x*w.y-v.y*w.x};}
T orient(Point p, Point q, Point r, Point s) {return (q-p)*(r-p)|(s-p);}
T orientByNormal(Point p, Point q, Point r, Point n) {return (q-p)*(r-p)|n;}
class Plane
public:
   Point n; T d;
   Plane(){}
   Plane(Point n,T d) : n(n), d(d) {}
   Plane(Point n, Point p) : n(n), d(n|p) {}
   Plane(Point p, Point q, Point r) : Plane((q-p)*(r-p), p) {}
   T side(Point p) {return (n|p)-d;}
   db dist(Point p) {return abs(side(p))/abs(n);}
   Plane translate(Point t) {return {n,d+(n|t)};}
   Plane shiftup(db dist) {return {n,d+dist*abs(n)};}
   Point proj(Point p) {return p-n*side(p)/sq(n);}
   Point refl(Point p) {return p-n*2*side(p)/sq(n);}
};
class Line
{
public:
   Point d,o;
   Line(){}
   Line(Point p,Point q):d(q-p),o(p){}
   Line(Plane p1,Plane p2)
   {
       d=p1.n*p2.n;
       o=(p2.n*p1.d-p1.n*p2.d)*d/sq(d);
   }
```

```
db dist2(Point p) {return sq(d*(p-o))/sq(d);}
   db dist(Point p) {return sqrt(dist2(p));}
   bool cmpProj(Point p,Point q) {return (d|p)<(d|q);}</pre>
   Point proj(Point p) {return o+d*(d|(p-o))/sq(d);}
   Point refl(Point p) {return proj(p)*2-p;}
   Point inter(Plane p) {return o-d*p.side(o)/(d|p.n);}
};
db dist(Line 11,Line 12)
   Point n=l1.d*l2.d;
   if(n==zero) return l1.dist(l2.o);
   return abs((12.o-11.o)|n)/abs(n);
}
Point closestOnL1(Line 11,Line 12)
   Point n2 = 12.d*(11.d*12.d);
   return 11.o+11.d*((12.o-11.o)|n2)/(11.d|n2);
}
db angle(Plane p1,Plane p2)
   return angle(p1.n,p2.n);
bool is_parallel(Plane p1,Plane p2)
{
   return p1.n*p2.n==zero;
bool is_perpendicular(Plane p1,Plane p2)
   return (p1.n|p2.n)==0;
db angle(Line 11,Line 12)
   return angle(11.d,12.d);
bool is_parallel(Line 11,Line 12)
   return 11.d*12.d==zero;
}
bool is_perpendicular(Line 11,Line 12)
   return (11.d|12.d)==0;
}
db angle(Plane p, Line 1)
   return PI/2-angle(p.n,1.d);
bool is_parallel(Plane p,Line 1)
   return (p.n|1.d)==0;
```

```
}
bool is_perpendicular(Plane p,Line 1)
   return p.n*l.d==zero;
}
Line perpthrough(Plane p,Point o) {return Line(o,o+p.n);}
Plane perpthrough(Line 1,Point o) {return Plane(1.d,o);}
Point vectorArea2(vector<Point> p)
   Point S=zero;
   for(int i=0,n=p.size();i<n;i++) S=S+p[i]*p[(i+1)%n];</pre>
   return S;
}
db area(vector<Point> p) {return abs(vectorArea2(p))/2.0;}
class Polyhedron
{
public:
       Polyhedron(){}
   vector<vector<Point> > faces;
   void clear(){faces.clear();}
   db surface_area()
       db S=0;
       for(auto f:faces) S=S+area(f);
       return S;
   }
   struct edge{int v;bool same;};
   void reorient()
       int n=faces.size();
       vector<vector<edge> > G(n);
       map<pair<Point,Point>, int> es;
       for(int u=0;u<n;u++)</pre>
           for(int i=0,m=(int)faces[u].size();i<m;i++)</pre>
               Point p=faces[u][i],q=faces[u][(i+1)%m];
               if(es.count({p,q}))
               {
                  int v=es[{p,q}];
                  G[u].push_back({v,true});G[v].push_back({u,true});
               }
               else if(es.count({q,p}))
                   int v=es[{q,p}];
                  G[u].push_back({v,false});G[v].push_back({u,false});
               else es[{p,q}]=u;
           }
       }
       vector<bool> vis(n,false),flip(n);
       flip[0]=false;
```

```
queue<int> q;q.push(0);
       while(!q.empty())
           int u=q.front();q.pop();
           for(edge e:G[u])
           {
               if(!vis[e.v])
               {
                  vis[e.v]=true;
                  flip[e.v]=flip[u]^e.same;
                  q.push(e.v);
               }
           }
       }
       for(int u=0;u<n;u++)</pre>
           if(flip[u])
              reverse(faces[u].begin(),faces[u].end());
   }
   db volume()
   {
       double ans=0.0;
       for(auto f:faces) ans+=(vectorArea2(f)|f[0]);
       return abs(ans)/6.0;
   }
};
struct fac
   int a,b,c;
   bool ok;
};
struct T3dhull
   int n;
   Point ply[MAXN];
   int trianglecnt;
   fac tri[MAXN];
   int vis[MAXN][MAXN];
   double area(Point a,Point b,Point c){return abs((b-a)*(c-a));}
   double volume(Point a, Point b, Point c, Point d){return (b-a)*(c-a)|(d-a);}
   double ptoplane(Point &p,fac &f)
   {
       Point m=ply[f.b]-ply[f.a],n=ply[f.c]-ply[f.a],t=p-ply[f.a];
       return (m*n)|t;
   }
   void deal(int p,int a,int b)
       int f=vis[a][b];
       fac add;
       if(tri[f].ok)
           if((ptoplane(ply[p],tri[f]))>eps) dfs(p,f);
           else
           {
               add.a=b,add.b=a,add.c=p,add.ok=1;
               vis[p][b]=vis[a][p]=vis[b][a]=trianglecnt;
               tri[trianglecnt++] = add;
           }
       }
```

```
void dfs(int p,int cnt)
   tri[cnt].ok=0;
   deal(p,tri[cnt].b,tri[cnt].a);
   deal(p,tri[cnt].c,tri[cnt].b);
   deal(p,tri[cnt].a,tri[cnt].c);
}
bool same(int s,int e)
   Point a=ply[tri[s].a],b=ply[tri[s].b],c=ply[tri[s].c];
   return fabs(volume(a,b,c,ply[tri[e].a]))<eps</pre>
       &&fabs(volume(a,b,c,ply[tri[e].b]))<eps
       &&fabs(volume(a,b,c,ply[tri[e].c]))<eps;
void construct()
{
   int i,j;
   trianglecnt=0;
    if(n<4) return ;</pre>
   bool tmp=true;
   for(i=1;i<n;i++) if((abs(ply[0]-ply[i]))>eps){ swap(ply[1],ply[i]); tmp=false; break;}
   if(tmp) return;
   tmp=true;
    for(i=2;i<n;i++)if((abs((ply[0]-ply[1])*(ply[1]-ply[i])))>eps){swap(ply[2],ply[i]);
        tmp=false; break;}
    if(tmp) return ;
   tmp=true;
    for(i=3;i<n;i++)</pre>
        if(fabs((ply[0]-ply[1])*(ply[1]-ply[2])|(ply[0]-ply[i]))>eps){swap(ply[3],ply[i]);
        tmp=false; break;}
         if(tmp) return;
         fac add;
         for(i=0;i<4;i++)</pre>
                add.a=(i+1)\%4,add.b=(i+2)\%4,add.c=(i+3)\%4,add.ok=1;
                if((ptoplane(ply[i],add))>0)
swap(add.b,add.c);//
vis[add.a][add.b]=vis[add.b][add.c]=vis[add.c][add.a]=trianglecnt;//
                tri[trianglecnt++] = add;
         for(i=4;i<n;i++)//</pre>
for(j=0;j<trianglecnt;j++)//</pre>
                                                     3
                                                                                                     j
                                                                                                             )
if(tri[j].ok&&(ptoplane(ply[i],tri[j]))>eps)//
                             dfs(i,j);
break;//
                                               (
                                                                                  )
                                                                                                      break
                       }
                }
         }
         int
                                        [i].ok=0
cnt=trianglecnt;//
                            tri
         trianglecnt=0;
         for(i=0;i<cnt;i++)</pre>
```

```
{
                    if(tri[i].ok)
                          tri[trianglecnt++]=tri[i];
             }
      }
      double area()//
             double ret=0;
             for(int i=0;i<trianglecnt;i++)</pre>
                    ret+=area(ply[tri[i].a],ply[tri[i].b],ply[tri[i].c]);
             return ret/2;
      }
      double volume()//
             Point p(0,0,0);
             double ret=0;
             for(int i=0;i<trianglecnt;i++)</pre>
                   ret+=volume(p,ply[tri[i].a],ply[tri[i].b],ply[tri[i].c]);
             return fabs(ret/6);
      }
      int facetri() {return trianglecnt;}//
      int facepolygon()//
      {
             int ans=0,i,j,k;
             for(i=0;i<trianglecnt;i++)</pre>
                    for(j=0,k=1;j<i;j++)</pre>
                    {
                          if(same(i,j)) {k=0;break;}
                    ans+=k;
             }
             return ans;
      }
}hull;
T point_to_segment(Point &p1,Point &p2,Point &p3)
   T 1=0.0,r=1.0,ans1,ans2;
   while(r-1>1e-14)
   {
       T dis=(r-1)/3.0;
       T lmid=l+dis,rmid=l+2.0*dis;
       Point Q=p2+((p3-p2)*lmid), R=p2+((p3-p2)*rmid);
       ans1=p1.dis2(Q);ans2=p1.dis2(R);
       if(ans1<ans2) r=rmid; else l=lmid;</pre>
   return sqrt(min(ans1,ans2));
}
T segment_dist(Point &p1, Point &p2, Point &p3, Point &p4)
   T 1=0.0,r=1.0,ans1,ans2;
   while(r-1>1e-14)
       T dis=(r-1)/3.0;
       T lmid=l+dis,rmid=l+2.0*dis;
       Point p=p1+((p2-p1)*lmid),q=p1+((p2-p1)*rmid);
       ans1=point_to_segment(p,p3,p4);ans2=point_to_segment(q,p3,p4);
```

```
if(ans1<ans2) r=rmid; else l=lmid;
}
return min(ans1,ans2);
}
int main()
{
   return 0;
}</pre>
```

# 3 Graph

### 3.1 Dijkstra

```
#include<bits/stdc++.h>
#define MAXV 1000
#define MAXE 10000
#define INF 1000000
using namespace std;
struct edge{int to,cost;};
typedef pair<int,int> P;
int V;
vector<edge> G[MAXV];
int d[MAXV];
void dijkstra(int s)
   priority_queue<P,vector<P>,greater<P> > que;
   fill(d,d+V,INF);
   d[s]=0;
   que.push(P(0,s));
   while(!que.empty())
       P p=que.top(); que.pop();
       int v=p.second;
       if(d[v]<p.first) continue;</pre>
       for(int i=0;i<(int)G[v].size();i++)</pre>
       {
           edge e=G[v][i];
           if(d[e.to]>d[v]+e.cost)
               d[e.to]=d[v]+e.cost;
               que.push(P(d[e.to],e.to));
       }
   }
}
int main()
{
   return 0;
}
```

### 3.2 Floyd-Warshall

```
#include<bits/stdc++.h>
#define MAXN 505
```

```
using namespace std;
int n,d[MAXN][MAXN];
void floyd_warshall()
{
    for(int k=1;k<=n;k++)
        for(int i=1;i<=n;i++)
        for(int j=1;j<=n;j++) d[i][j]=min(d[i][j],d[i][k]+d[k][j]);
}</pre>
```

#### 3.3 Kosaraju

```
#include<bits/stdc++.h>
#define MAXN 100005
using namespace std;
int n;
vector<int> G[MAXN];
vector<int> rG[MAXN];
vector<int> vs:
bool used[MAXN];
int cmp[MAXN];
void add_edge(int from,int to)
   G[from].push_back(to);
   rG[to].push_back(from);
void dfs(int v)
   used[v]=true;
   for(int i=0;i<(int)G[v].size();i++)</pre>
       if(!used[G[v][i]]) dfs(G[v][i]);
   vs.push_back(v);
}
void rdfs(int v,int k)
   used[v]=true;
   cmp[v]=k;
   for(int i=0;i<(int)rG[v].size();i++)</pre>
       if(!used[rG[v][i]]) rdfs(rG[v][i],k);
}
int scc()
   memset(used,0,sizeof(used));
   vs.clear();
   for(int v=1;v<=n;v++) if(!used[v]) dfs(v);</pre>
   memset(used,0,sizeof(used));
   for(int i=vs.size()-1;i>=0;i--) if(!used[vs[i]]) rdfs(vs[i],k++);
   return k;
```

# 3.4 Tarjan

```
#pragma GCC optimize(3)
#include<bits/stdc++.h>
#define MAXN 100005
```

```
#define INF 100000000
#define MOD 100000007
#define F first
#define S second
using namespace std;
typedef long long 11;
typedef pair<int,int> P;
vector<int> G[MAXN];
int n,dfn[MAXN],low[MAXN],st[MAXN];
int vis[MAXN];
int cmp[MAXN],cnt,tot,t;
void dfs(int v)
   dfn[v] = low[v] = ++tot;
   vis[v]=1;
   st[t++]=v;
   for(auto to:G[v])
       if(!vis[to])
           dfs(to);
           low[v]=min(low[v],low[to]);
       else if(vis[to]==1) low[v]=min(low[v],dfn[to]);
   if(dfn[v]==low[v])
       int u;
       do
       {
           u=st[t-1]; t--;
           cmp[u]=cnt;
           vis[u]=2;
       }while(u!=v);
       cnt++;
   }
}
int tarjan()
   t=tot=cnt=0;
   memset(vis,0,sizeof(vis));
   for(int i=1;i<=n;i++) if(!dfn[i]) dfs(i);</pre>
   return cnt;
}
int main()
{
   return 0;
}
```

# 3.5 LCA with binary lifting

```
#include<bits/stdc++.h>
#define MAXN 100005
#define MAXLOGN 20
using namespace std;
vector<int> G[MAXN];
int pa[MAXLOGN] [MAXN];
```

```
int depth[MAXN];
int n,q;
void dfs(int v,int p,int d)
   pa[0][v]=p;
   depth[v]=d;
   for(int i=0;i<(int)G[v].size();i++)</pre>
        if(G[v][i]!=p) dfs(G[v][i],v,d+1);
void init(int V)
   dfs(1,0,0);
   for(int k=0;k+1<MAXLOGN;k++)</pre>
       for(int v=1; v<=V; v++)</pre>
           if(pa[k][v]<0) pa[k+1][v]=-1;</pre>
           else pa[k+1][v]=pa[k][pa[k][v]];
        }
   }
}
int get(int v,int x)
   for(int k=0;k<MAXLOGN;k++)</pre>
        if((x>>k)&1)
           v=pa[k][v];
   return v;
int lca(int u,int v)
   if(depth[u]>depth[v]) swap(u,v);
   v=get(v,depth[v]-depth[u]);
   if(u==v) return u;
   for(int k=MAXLOGN-1;k>=0;k--)
        if(pa[k][u]!=pa[k][v])
           u=pa[k][u];
           v=pa[k][v];
        }
   }
   return pa[0][u];
}
int dis(int u,int v)
{
   return depth[u]+depth[v]-2*depth[lca(u,v)];
}
```

### 3.6 LCA with range minimum query

```
#include<bits/stdc++.h>
#define MAXN 100005
#define MAXLOGN 22
#define INF 1000000000
#define MOD 1000000007
#define F first
#define S second
```

```
using namespace std;
typedef long long 11;
typedef pair<int,int> P;
int n,q;
int st[MAXLOGN][2*MAXN];
vector<int> G[MAXN];
int vs[MAXN*2-1];
int depth[MAXN*2-1];
int id[MAXN];
void dfs(int v,int p,int d,int &k)
   id[v]=k;
   vs[k]=v;
   depth[k++]=d;
   for(int i=0;i<(int)G[v].size();i++)</pre>
       if(G[v][i]!=p)
       {
           dfs(G[v][i],v,d+1,k);
           vs[k]=v;
           depth[k++]=d;
       }
   }
}
int getMin(int x, int y)
   return depth[x] < depth[y]?x:y;</pre>
void rmq_init(int n)
   for(int i=1;i<=n;++i) st[0][i]=i;</pre>
   for(int i=1;1<<i<n;++i)</pre>
       for(int j=1;j+(1<<i)-1<=n;++j)</pre>
           st[i][j]=getMin(st[i-1][j],st[i-1][j+(1<<(i-1))]);
}
void init(int V)
   int k=0;
   dfs(1,0,0,k);
   rmq_init(V*2-1);
int query(int 1, int r)
{
   int k=31-__builtin_clz(r-l+1);
   return getMin(st[k][l],st[k][r-(1<<k)+1]);</pre>
}
int lca(int u,int v)
   if(u==v) return u;
   return vs[query(min(id[u],id[v]),max(id[u],id[v]))];
}
int dis(int u,int v)
{
   return depth[id[u]]+depth[id[v]]-2*depth[id[lca(u,v)]];
}
int main()
{
   return 0;
```

#### 3.7 Dinic

```
#include<bits/stdc++.h>
#define MAXV 3005
#define MAXE 50000
#define INF 1000000
using namespace std;
struct edge{int to,cap,rev;};
int V;
vector<edge> G[MAXV];
int level[MAXV];
int iter[MAXV];
void add_edge(int from,int to,int cap)
   G[from].push_back((edge){to,cap,(int)G[to].size()});
   G[to].push_back((edge){from,0,(int)G[from].size()-1});
}
void bfs(int s)
   memset(level,-1,sizeof(level));
   queue<int> que;
   level[s]=0;
   que.push(s);
   while(!que.empty())
       int v=que.front(); que.pop();
       for(int i=0;i<(int)G[v].size();i++)</pre>
           edge &e=G[v][i];
           if(e.cap>0&&level[e.to]<0)</pre>
               level[e.to] = level[v] + 1;
               que.push(e.to);
       }
   }
}
int dfs(int v,int t,int f)
   if(v==t) return f;
   for(int &i=iter[v];i<(int)G[v].size();i++)</pre>
       edge &e=G[v][i];
       if(level[v] < level[e.to] &&e.cap>0)
           int d=dfs(e.to,t,min(f,e.cap));
           if(d>0)
           {
               e.cap-=d;
               G[e.to][e.rev].cap+=d;
               return d;
       }
   }
```

```
return 0;
}
int max_flow(int s,int t)
{
    int flow=0;
    for(;;)
    {
        bfs(s);
        if(level[t]<0) return flow;
        memset(iter,0,sizeof(iter));
        int f;
        while((f=dfs(s,t,INF))>0)
            flow+=f;
    }
}
```

# 3.8 Ear Decomposition

```
#pragma GCC optimize(3)
#include<bits/stdc++.h>
#include<ext/pb_ds/assoc_container.hpp>
#include<ext/pb_ds/tree_policy.hpp>
#include<ext/pb_ds/priority_queue.hpp>
#define MAXN 100005
#define INF 100000000
#define MOD 100000007
#define F first
#define S second
using namespace std;
using namespace __gnu_pbds;
typedef long long 11;
typedef pair<int,int> P;
typedef tree<int,null_type,less<int>,rb_tree_tag,tree_order_statistics_node_update> ordered_set;
typedef __gnu_pbds::priority_queue<int,greater<int>,pairing_heap_tag> pq;
int n,m;
vector<int> G[MAXN];
int dep[MAXN];
vector<vector<int> > ears;
vector<int> path[MAXN];
vector<int> back[MAXN];
int low[MAXN];
int vis[MAXN];
bool f=true;
void cmin(int &a,int b)
{
   if(dep[b] < dep[a]) a=b;</pre>
}
void add_ear(int to,vector<int> &a)
   reverse(a.begin(),a.end());
   a.push_back(to);
   ears.push_back(a);
void dfs(int v,int p,int d)
   if(!f) return;
   vis[v]=1;dep[v]=d;low[v]=v;
```

```
int used=0;
   for(auto to:G[v])
       if(to==p) continue;
       if(!vis[to])
       {
           dfs(to,v,d+1);
           path[to].push_back(v);
           if(!used||dep[low[to]]<dep[low[v]])</pre>
               if(used) add_ear(low[v],path[v]);
               swap(path[v],path[to]);
               low[v] = low[to];
               used=1;
           else add_ear(low[to],path[to]);
       }
       else if(vis[to]==1) back[v].push_back(to);
   }
   vis[v]=2;
   for(int i=0;i<(int)back[v].size();i++)</pre>
       int u=back[v][i];
       if(!used||dep[u]<dep[low[v]])</pre>
           if(used) add_ear(low[v],path[v]);
           path[v].clear();path[v].push_back(v);
           low[v]=u;
           used=1;
       }
       else ears.emplace_back((vector<int>){v,u});
   }
  /* printf("%d %d %d\n",v,dep[v],low[v]);
   printf("chain %d\n",v);
   for(auto x:path[v]) printf("%d ",x);
   puts(""); */
   if(dep[low[v]]==dep[v]&&v!=1) f=false;
}
int main()
   scanf("%d%d",&n,&m);
   for(int i=0;i<m;i++)</pre>
       int u,v;
       scanf("%d%d",&u,&v);
       G[u].push_back(v);G[v].push_back(u);
   dfs(1,0,0);
   if(!f) puts("-1");
   else
       add_ear(1,path[1]);
       printf("%d\n",(int)ears.size());
       for(int i=0;i<(int)ears.size();i++)</pre>
           printf("%d ",(int)ears[i].size()-1);
           for(int j=0;j<(int)ears[i].size();j++)</pre>
               printf("%d ",ears[i][j]);
           puts("");
```

```
}
return 0;
}
```

#### 3.9 Min-cost flow

```
#include<bits/stdc++.h>
#define MAXV 1000
#define MAXE 10000
#define INF 1000000
using namespace std;
typedef pair<int,int> P;
struct edge{int to,cap,cost,rev;};
int dist[MAXV],h[MAXV],prevv[MAXV],preve[MAXV];
vector<edge> G[MAXV];
void add_edge(int from,int to,int cap,int cost)
{
   G[from].push_back((edge){to,cap,cost,G[to].size()});
   G[to].push_back((edge){from,0,-cost,G[from].size()-1});
int min_cost_flow(int s,int t,int f)
   int res=0;
   fill(h,h+V,0);
   while(f>0)
       priority_queue<P,vector<P>,greater<P> >que;
       fill(dist,dist+V,INF);
       dist[s]=0;
       que.push(P(0,s));
       while(!que.empty())
           P p=que.top(); que.pop();
           int v=p.second;
           if(dist[v]<p.first) continue;</pre>
           for(int i=0;i<G[v].size();i++)</pre>
               edge &e=G[v][i];
               if(e.cap>0&&dist[e.to]>dist[v]+e.cost+h[v]-h[e.to])
                   dist[e.to] = dist[v] + e.cost + h[v] - h[e.to];
                  prevv[e.to]=v;
                  preve[e.to]=i;
                  que.push(P(dist[e.to],e.to));
           }
       if(dist[t]==INF)
       {
           return -1;
       }
       for(int v=0;v<V;v++) h[v]+=dist[v];</pre>
       for(int v=t;v!=s;v=prevv[v])
```

```
d=min(d,G[prevv[v]][preve[v]].cap);
       }
       f-=d;
       res+=d*h[t];
       for(int v=t;v!=s;v=prevv[v])
           edge &e=G[prevv[v]][preve[v]];
           e.cap-=d;
           G[v][e.rev].cap+=d;
       }
   }
   return res;
}
int main()
{
   return 0;
}
```

# 3.10 Bipartite Matching

```
#include<cstdio>
#include<cmath>
#include<cstring>
#include<cstdlib>
#include<iostream>
#include<algorithm>
#include<queue>
#include<vector>
#define MAX_V 10000
#define MAXN 1000000
using namespace std;
int V;
vector<int> G[MAX_V];
int match[MAX_V];
bool used[MAX_V];
void add_edge(int u,int v)
{
   G[u].push_back(v);
   G[v].push_back(u);
}
bool dfs(int v)
   used[v]=true;
   for(int i=0;i<G[v].size();i++)</pre>
       int u=G[v][i],w=match[u];
       if(w<0||!used[w]&&dfs(w))</pre>
           match[v]=u;
           match[u]=v;
           return true;
       }
   }
   return false;
int bipartite_matching()
{
```

```
int res=0;
    memset(match,-1,sizeof(match));
    for(int v=0; v<V; v++)</pre>
        if(match[v]<0)</pre>
            memset(used,0,sizeof(used));
            if(dfs(v))
            {
               res++;
       }
   }
   return res;
int main()
{
    int p=sieve(1000000);
    return 0;
}
```

#### 3.11 Common Matching

```
#include<bits/stdc++.h>
#define MAXN 500
int n,m,x,y,fore,rear,cnt,ans,father[MAXN],f[MAXN],path[MAXN],tra[MAXN],que[MAXN],match[MAXN];
bool a[MAXN] [MAXN], check[MAXN], treec[MAXN], pathc[MAXN];
inline void push(int x)
   que[++rear]=x;
   check[x]=true;
   if(!treec[x])
       tra[++cnt]=x;
       treec[x]=true;
int root(int x){return f[x]?f[x]=root(f[x]):x;}
void clear()
   for(int i=1,j;i<=cnt;++i)</pre>
       j=tra[i];
       check[j]=treec[j]=false;
       father[j]=0,f[j]=0;
}
int lca(int u,int v)
   int len=0;
   for(;u;u=father[match[u]])
       u=root(u);
       path[++len]=u;
       pathc[u]=true;
```

```
}
   for(;;v=father[match[v]])
       v=root(v);
       if(pathc[v]) break;
   }
   for(int i=1;i<=len;++i)</pre>
       pathc[path[i]]=false;
   }
   return v;
}
void reset(int u,int p)
   for(int v;root(u)!=p;)
   {
       if(!check[v=match[u]]) push(v);
       if(f[u]==0) f[u]=p;
       if(f[v]==0) f[v]=p;
       u=father[v];
       if(root(u)!=p) father[u]=v;
   }
}
void flower(int u,int v)
   int p=lca(u,v);
   if(root(u)!=p) father[u]=v;
   if(root(v)!=p) father[v]=u;
   reset(u,p),reset(v,p);
}
bool find(int x)
   fore=rear=cnt=0,push(x);
   while(fore++<rear)</pre>
       int i=que[fore];
       for(int j=1;j<=n;++j)</pre>
           if(a[i][j]&&root(i)!=root(j)&&match[j]!=i)
             if(match[j]&&father[match[j]])
                flower(i,j);
             else if(father[j]==0)
                 father[j]=i;
                 tra[++cnt]=j;
                 treec[j]=true;
                 if(match[j])
                  push(match[j]);
                 else
                    for(int k=i,l=j,p;k;l=p,k=father[l])
                    {
                        p=match[k];
                        match[k]=1;
                        match[1]=k;
                    }
```

```
return true;
                 }
             }
       }
   }
   return false;
void matching()
   for(int i=1;i<=n;i++)</pre>
       if(match[i]==0)
           if(find(i)) ans++;
           clear();
       }
}
int main()
   scanf("%d%d",&n,&m);
   for(int i=1;i<=m;i++)</pre>
   {
     int x,y;
     scanf("%d%d",&x,&y);
     a[x][y]=a[y][x]=true;
   }
   matching();
   printf("%d\n",ans);
   return 0;
```

#### 3.12 Kuhn-Munkres

```
#pragma GCC optimize(3)
#include<bits/stdc++.h>
#define MAXN 1005
#define INF 100000000
#define MOD 100000007
#define F first
#define S second
using namespace std;
typedef long long 11;
typedef pair<int,int> P;
int n;
int w[MAXN][MAXN];
//minimum weight bipartite matching
11 km(int n,int m)
   vector<int> u(n+1),v(m+1),p(m+1),way(m+1);
   for(int i=1;i<=n;i++)</pre>
   {
       p[0]=i;
       int j0=0;
       vector<int> minv(m+1,INF);
       vector<char> used(m+1,false);
       do
```

```
{
            used[j0]=true;
            int i0=p[j0],delta=INF,j1;
            for(int j=1;j<=m;++j)</pre>
                if(!used[j])
                {
                    int cur=w[i0][j]-u[i0]-v[j];
                    if(cur<minv[j]) minv[j]=cur,way[j]=j0;</pre>
                    if(minv[j] < delta ) delta = minv[j], j1 = j;</pre>
                }
            for(int j=0;j<=m;++j) if(used[j]) u[p[j]]+=delta,v[j]-=delta; else minv[j]-=delta;</pre>
        }while(p[j0]!=0);
        {
            int j1=way[j0];
            p[j0]=p[j1];
            j0=j1;
        }while(j0);
   }
    ll res=0;
    for(int i=1;i<=m;i++) res+=w[p[i]][i];</pre>
    return res;
}
int main()
    return 0;
}
```

## 3.13 Linear Programming

```
#include<cstdio>
#include<cstring>
#include<algorithm>
using namespace std;
const int N = 23;
const double eps = 1e-8;
double a[N][N], ans[N];
int n, m, t, id[N << 1];</pre>
void pivot(int 1, int e)
{
   swap(id[e], id[n + 1]);
   double r = a[l][e]; a[l][e] = 1;
   for (int j = 0; j \le n; ++j)
       a[1][j] /= r;
   for (int i = 0; i <= m; ++i)</pre>
       if (i != 1) {
           r = a[i][e]; a[i][e] = 0;
           for (int j = 0; j \le n; ++j)
               a[i][j] -= r * a[1][j];
       }
}
int main()
```

```
{
   scanf("%d%d", &n, &m);
   int i, j, l, e; double k, kk;
   for (j = 1; j \le n; ++j) scanf("%lf", &a[0][j]), id[j] = j;
   for (i = 1; i <= m; ++i)</pre>
        for (j = 1; j \le n; ++j)
           scanf("%lf", &a[i][j]);
       scanf("%lf", &a[i][0]);
   }
   while (true)
       1 = e = 0; k = -eps;
       for (i = 1; i <= m; ++i)</pre>
           if (a[i][0] < k) {</pre>
               k = a[i][0];
               1 = i;
           }
       if (!1) break;
       k = -eps;
       for (j = 1; j \le n; ++j)
           if (a[l][j] < k && (!e || (rand() & 1))) {</pre>
               k = a[1][j];
               e = j;
        if (!e) {puts("Infeasible"); return 0;}
       pivot(l, e);
   }
   while (true) {
       for (j = 1; j \le n; ++j)
           if (a[0][j] > eps)
               break;
       if ((e = j) > n) break;
       k = 1e18; 1 = 0;
       for (i = 1; i <= m; ++i)</pre>
           if (a[i][e] > eps && (kk = (a[i][0] / a[i][e])) < k) {
               k = kk;
               l = i;
       if (!1) {puts("Unbounded"); return 0;}
       pivot(1, e);
   }
   printf("%.10lf\n", -a[0][0]);
   for (i = 1; i <= m; ++i) ans[id[n + i]] = a[i][0];</pre>
   for (i = 1; i <= n; ++i) printf("%.101f ", ans[i]);</pre>
   return 0;
}
```

#### 3.14 Dominator Tree

```
#include<bits/stdc++.h>
#define MAXN 100005
#define INF 1000000000
#define MOD 100000007
```

```
#define F first
#define S second
using namespace std;
typedef long long 11;
typedef pair<int,int> P;
vector<int> G[MAXN],rG[MAXN],dt[MAXN],bucket[MAXN];
int sdom[MAXN],idom[MAXN],arr[MAXN],rev[MAXN],par[MAXN],dsu[MAXN],label[MAXN];
int n,m,t;
int find(int u,int x=0)
       if(u==dsu[u]) return x?-1:u;
       int v=find(dsu[u],x+1);
       if(v<0) return u;</pre>
       if(sdom[label[dsu[u]]]<sdom[label[u]])</pre>
               label[u] = label[dsu[u]];
       dsu[u]=v;
       return x?v:label[u];
}
void unite(int u,int v)
       dsu[v]=u;
void dfs(int v)
       t++;arr[v]=t;rev[t]=v;
       label[t]=t;sdom[t]=t;dsu[t]=t;
       for(int i=0;i<(int)G[v].size();i++)</pre>
       {
               int to=G[v][i];
               if(!arr[to]) dfs(to),par[arr[to]]=arr[v];
               rG[arr[to]].push_back(arr[v]);
       }
}
void build_dominator_tree(int r)
   dfs(r);int N=t;
       for(int i=N;i>=1;i--)
               for(int j=0;j<(int)rG[i].size();j++)</pre>
                      sdom[i]=min(sdom[i],sdom[find(rG[i][j])]);
               if(i>1) bucket[sdom[i]].push_back(i);
               for(int j=0;j<(int)bucket[i].size();j++)</pre>
                      int w=bucket[i][j],v=find(w);
                      if(sdom[v]==sdom[w]) idom[w]=sdom[w];
                      else idom[w]=v;
               if(i>1) unite(par[i],i);
       }
       for(int i=2;i<=N;i++)</pre>
               if(idom[i]!=sdom[i]) idom[i]=idom[idom[i]];
               dt[rev[idom[i]]].push_back(rev[i]);
   for(int i=1;i<=N;i++) bucket[i].clear(),rG[i].clear();</pre>
}
```

# 3.15 Dynamic Connectivity

```
#pragma GCC optimize(3)
#include<bits/stdc++.h>
#include<ext/pb_ds/assoc_container.hpp>
#include<ext/pb_ds/tree_policy.hpp>
#include<ext/pb_ds/priority_queue.hpp>
#define MAXN 300005
#define INF 100000000
#define MOD 100000007
#define F first
#define S second
using namespace std;
using namespace __gnu_pbds;
typedef long long 11;
typedef pair<int,int> P;
typedef tree<int,null_type,less<int>,rb_tree_tag,tree_order_statistics_node_update> ordered_set;
typedef __gnu_pbds::priority_queue<int,greater<int>,pairing_heap_tag> pq;
int n,k,x,y;
char str[2];
vector<P> edges[4*MAXN];
bool ask[MAXN];
int p[MAXN],r[MAXN],sz[MAXN];
int ans[MAXN];
int num;
struct update
   int x,y;
   bool addrk;
};
update st[MAXN];
int t;
void init(int n)
   for(int i=1;i<=n;i++)</pre>
   {
       p[i]=i;
       r[i]=0;
   }
}
int find(int x)
  while(p[x]!=x) x=p[x];
  return x;
}
bool unite(int x,int y)
   x=find(x);
   y=find(y);
   if(x==y) return false;
   num--;
   if(r[x] < r[y])
       p[x]=y;
       st[t++]=(update){x,y,false};
   }
   else
   {
```

```
p[y]=x;
       st[t++]=(update){y,x,r[x]==r[y]};
       if(r[x]==r[y]) r[x]++;
   }
   return true;
}
void undo()
{
   assert(t);
   int x=st[t-1].x,y=st[t-1].y;
   //printf("undo %d %d %d\n",x,y,st[t-1].addrk);
   p[x]=x;p[y]=y;
   if(st[t-1].addrk) r[y]--;
   t--; num++;
bool same(int x,int y)
{
   return find(x)==find(y);
void add_edge(int k,int l,int r,int x,int y,int u,int v)
   if(x>r||1>y) return;
   if(1>=x&&r<=y)</pre>
       edges[k].push_back(P(u,v));
       return;
   }
   int mid=(1+r)/2;
   add_edge(k*2,1,mid,x,y,u,v);add_edge(k*2+1,mid+1,r,x,y,u,v);
void solve(int k,int l,int r)
   if(l>r) return;
   int cnt=0;
   for(auto e:edges[k]) if(unite(e.F,e.S)) cnt++;
   if(l==r)
   {
       if(ask[1]) ans[1]=num;
       for(int i=0;i<cnt;i++) undo();</pre>
       return;
   int mid=(1+r)/2;
   solve(k*2,l,mid); solve(k*2+1,mid+1,r);
   //printf("cnt %d %d %d\n",1,r,cnt);
   for(int i=0;i<cnt;i++) undo();</pre>
map<P,int> mp;
int main()
   freopen("connect.in","r",stdin);
   freopen("connect.out","w",stdout);
   scanf("%d%d",&n,&k);num=n;init(n);
   memset(ask,false,sizeof(ask));
   for(int i=1;i<=k;i++)</pre>
       scanf("%s",str);
       if(str[0]=='?')
       {
           ask[i]=true;
```

```
continue;
       }
       scanf("%d%d",&x,&y);
       if(x>y) swap(x,y);
       if(str[0]=='+') mp[P(x,y)]=i;
       else
       {
           add_edge(1,1,k,mp[P(x,y)],i-1,x,y);
           mp[P(x,y)]=-1;
       }
   }
   for(auto p:mp) if(p.S!=-1) add_edge(1,1,k,p.S,k,p.F.F,p.F.S);
   solve(1,1,k);
   for(int i=1;i<=k;i++) if(ask[i]) printf("%d\n",ans[i]);</pre>
   return 0;
}
```

# 3.16 Dynamic Bridge

```
/**
* Author: Sergey Kopeliovich (Burunduk30@gmail.com)
* Total Time = O(mlogm)
#include <ctime>
#include <cassert>
#include <cstdio>
#include <cstring>
#include <algorithm>
#include <map>
using namespace std;
#define forn(i, n) for (int i = 0; i < (int)(n); i++)</pre>
#define forit(i, a) for (__typeof((a).begin()) i = (a).begin(); i != (a).end(); i++)
#define mp make_pair
typedef pair <int, int> pii;
template <class T> inline void relaxMin( T &a, T b ) { a = min(a, b); }
/* Main part */
const int maxN = (int)1e5;
const int maxM = (int)1e5;
struct query
 int a, b, L, R;
 query() { }
 query( int _a, int _b, int _L, int _R ) : a(_a), b(_b), L(_L), R(_R) { }
int n, m, qn, res[maxM + 1];
query q[maxM];
```

```
const int maxMem = (int)1e7;
const int maxE = (int)1e6;
int mpos = 0, en, next[maxE], to[maxE], w[maxE];
char mem[maxMem];
template <class T> T *getMem( int n )
 char *r = mem + mpos;
 mpos += n * sizeof(T);
 assert(mpos <= maxMem);</pre>
 return (T *)r;
void addE( int a, int b, int x, int *head )
 assert(en + 2 <= maxE);</pre>
 next[en] = head[a], to[en] = b, w[en] = x, head[a] = en++;
 next[en] = head[b], to[en] = a, w[en] = x, head[b] = en++;
}
int curT, cc, used[maxN], T[maxN], minT[maxN];
int sp, ss[maxN];
int vnX, color[maxN];
int enX, ea[maxM], eb[maxM], ew[maxM];
// find components of edge-2-connectivity and bridges
void getComp( int old_sp )
 while (sp > old_sp)
   color[ss[--sp]] = vnX;
 vnX++;
void dfs( int v, int pr, int *head )
 int cnt = 0;
 ss[sp++] = v;
 used[v] = cc;
 minT[v] = T[v] = curT++;
 for (int e = head[v]; e != -1; e = next[e])
   int x = to[e];
   if (x != pr || ++cnt > 1)
     if (used[x] != cc)
       int old_sp = sp;
       dfs(x, v, head);
       if (minT[x] > T[v])
         getComp(old_sp), ew[enX] = w[e], ea[enX] = v, eb[enX++] = x;
     relaxMin(minT[v], minT[x]);
   }
 }
}
// determine important vertices in tree
```

```
int paint( int v, int *head )
{
 int num = 0;
 used[v] = cc;
 for (int e = head[v]; e != -1; e = next[e])
   if (used[to[e]] != cc)
     num += paint(to[e], head);
 color[v] |= (num >= 2);
 return color[v] || num;
}
// consolidate edges
int findEdges( int v, int start, int curLen, int *head )
 int ret = 0;
 used[v] = cc;
 if (color[v] && curLen > 0)
   ea[enX] = v, eb[enX] = start, ew[enX++] = curLen;
   start = v, curLen = 0, ret = 1;
 for (int e = head[v]; e != -1; e = next[e])
   if (used[to[e]] != cc && findEdges(to[e], start, curLen + w[e], head))
     w[e] = 0, w[e ^ 1] = 0, ret = 1;
 return ret;
// main procedure
void newGraph( int qn, query *q, int &vn, int *head, int &old_en )
 fill(head, head + vn, -1), en = old_en, vn = vnX;
 forn(i, enX)
   addE(color[ea[i]], color[eb[i]], ew[i], head);
 forn(i, qn)
   q[i].a = color[q[i].a], q[i].b = color[q[i].b];
void Solve( int L, int R, int qn2, query *q2, int vn2, int *head2, int have )
 query *q = getMem<query>(qn2);
 int old_en = en, old_mpos = mpos;
 int qn = 0, vn = vn2;
 int *head = getMem<int>(vn);
 /* Process all already obvious queries */
 memcpy(head, head2, sizeof(head2[0]) * vn);
 forn(i, qn2)
   if (q2[i].L < L && q2[i].R >= R)
     addE(q2[i].a, q2[i].b, 1, head);
   else if (q2[i].R >= L \&\& q2[i].L < R)
     q[qn++] = q2[i];
 /* Consolidate components of edge-2-connectivity */
 cc++, curT = 0;
 vnX = enX = sp = 0;
 forn(i, vn)
   if (used[i] != cc)
     dfs(i, -1, head), getComp(0);
 newGraph(qn, q, vn, head, old_en);
```

```
/* Determine all important edges */
 forn(i, vn)
   color[i] = 0;
 forn(i, qn)
   color[q[i].a] = color[q[i].b] = 1;
 cc++;
 forn(i, vn)
   if (used[i] != cc)
     paint(i, head);
 /* Reduce the graph */
 enX = 0, cc++;
 forn(i, vn)
   if (used[i] != cc && color[i])
     findEdges(i, i, 0, head);
 forn(i, vn)
   for (int e = head[i]; e != -1; e = next[e])
     if (w[e] > 0)
       have += w[e];
 vnX = 0;
 forn(i, vn)
   if (color[i])
     color[i] = vnX++;
 newGraph(qn, q, vn, head, old_en);
 /* Recursion continues... */
 mpos -= sizeof(int) * (vn2 - vn);
 if (L == R)
   res[L] = have / 2;
 else
   int M = (L + R) / 2;
   Solve(L, M, qn, q, vn, head, have);
   Solve(M + 1, R, qn, q, vn, head, have);
 en = old_en, mpos = old_mpos;
void Read()
 #define NAME "bridges3"
 assert(freopen(NAME ".in", "r", stdin));
 assert(freopen(NAME ".out", "w", stdout));
 assert(scanf("%d%d", &n, &m) == 2);
 assert(1 <= n && n <= maxN);
 assert(0 <= m && m <= maxM);
 map <pii, int> L;
 forn(i, m)
   char com[9];
   int a, b;
   assert(scanf("%s", com) == 1);
   assert(scanf("%d%d", &a, &b) == 2);
   assert(1 \le a \&\& a \le n \&\& 1 \le b \&\& b \le n \&\& a != b);
   a--, b--;
   if (a > b)
```

```
swap(a, b);
   pii p = mp(a, b);
   assert(!strcmp(com, "ADD") == !L.count(p));
   if (L.count(p))
   {
     q[qn++] = query(a, b, L[p], i);
     L.erase(p);
   else
     L[p] = i;
 forit(it, L)
   q[qn++] = query(it->first.first, it->first.second, it->second, m);
void TimeStamp( const char *s )
 fprintf(stderr, "[%05.2f] %s\n", (double)clock() / CLOCKS_PER_SEC, s);
}
int main()
{
 Read();
 TimeStamp("Data is read");
 int *head = getMem<int>(n);
 fill(head, head + n, -1);
 TimeStamp("Memory is allocated");
 Solve(0, m, qn, q, n, head, 0);
 TimeStamp("Problem is solved");
 forn(i, m)
   printf("%d\n", res[i + 1]);
 TimeStamp("Result is outputed");
 return 0;
* Author: Sergey Kopeliovich (Burunduk30@gmail.com)
* Total Time = O(mlogm)
*/
#include <ctime>
#include <cassert>
#include <cstdio>
#include <cstring>
#include <algorithm>
#include <map>
using namespace std;
#define forn(i, n) for (int i = 0; i < (int)(n); i++)</pre>
#define forit(i, a) for (__typeof((a).begin()) i = (a).begin(); i != (a).end(); i++)
#define mp make_pair
```

```
typedef pair <int, int> pii;
template <class T> inline void relaxMin( T &a, T b ) { a = min(a, b); }
/* Main part */
const int maxN = (int)1e5;
const int maxM = (int)1e5;
struct query
 int a, b, L, R;
 query() { }
 query( int _a, int _b, int _L, int _R ) : a(_a), b(_b), L(_L), R(_R) { }
int n, m, qn, res[maxM + 1];
query q[maxM];
const int maxMem = (int)1e7;
const int maxE = (int)1e6;
int mpos = 0, en, next[maxE], to[maxE], w[maxE];
char mem[maxMem];
template <class T> T *getMem( int n )
{
 char *r = mem + mpos;
 mpos += n * sizeof(T);
 assert(mpos <= maxMem);</pre>
 return (T *)r;
void addE( int a, int b, int x, int *head )
 assert(en + 2 <= maxE);</pre>
 next[en] = head[a], to[en] = b, w[en] = x, head[a] = en++;
 next[en] = head[b], to[en] = a, w[en] = x, head[b] = en++;
int curT, cc, used[maxN], T[maxN], minT[maxN];
int sp, ss[maxN];
int vnX, color[maxN];
int enX, ea[maxM], eb[maxM], ew[maxM];
// find components of edge-2-connectivity and bridges
void getComp( int old_sp )
 while (sp > old_sp)
   color[ss[--sp]] = vnX;
 vnX++;
void dfs( int v, int pr, int *head )
 int cnt = 0;
 ss[sp++] = v;
```

```
used[v] = cc;
 minT[v] = T[v] = curT++;
 for (int e = head[v]; e != -1; e = next[e])
 {
   int x = to[e];
   if (x != pr || ++cnt > 1)
     if (used[x] != cc)
       int old_sp = sp;
       dfs(x, v, head);
       if (minT[x] > T[v])
         getComp(old_sp), ew[enX] = w[e], ea[enX] = v, eb[enX++] = x;
     relaxMin(minT[v], minT[x]);
   }
 }
}
// determine important vertices in tree
int paint( int v, int *head )
 int num = 0;
 used[v] = cc;
 for (int e = head[v]; e != -1; e = next[e])
   if (used[to[e]] != cc)
     num += paint(to[e], head);
 color[v] |= (num >= 2);
 return color[v] || num;
// consolidate edges
int findEdges( int v, int start, int curLen, int *head )
{
 int ret = 0;
 used[v] = cc;
 if (color[v] && curLen > 0)
   ea[enX] = v, eb[enX] = start, ew[enX++] = curLen;
   start = v, curLen = 0, ret = 1;
 for (int e = head[v]; e != -1; e = next[e])
   if (used[to[e]] != cc && findEdges(to[e], start, curLen + w[e], head))
     w[e] = 0, w[e ^ 1] = 0, ret = 1;
 return ret;
}
// main procedure
void newGraph( int qn, query *q, int &vn, int *head, int &old_en )
 fill(head, head + vn, -1), en = old_en, vn = vnX;
 forn(i, enX)
   addE(color[ea[i]], color[eb[i]], ew[i], head);
 forn(i, qn)
   q[i].a = color[q[i].a], q[i].b = color[q[i].b];
void Solve( int L, int R, int qn2, query *q2, int vn2, int *head2, int have )
{
```

```
query *q = getMem<query>(qn2);
int old_en = en, old_mpos = mpos;
int qn = 0, vn = vn2;
int *head = getMem<int>(vn);
/* Process all already obvious queries */
memcpy(head, head2, sizeof(head2[0]) * vn);
forn(i, qn2)
 if (q2[i].L < L && q2[i].R >= R)
   addE(q2[i].a, q2[i].b, 1, head);
 else if (q2[i].R >= L \&\& q2[i].L < R)
   q[qn++] = q2[i];
/* Consolidate components of edge-2-connectivity */
cc++, curT = 0;
vnX = enX = sp = 0;
forn(i, vn)
 if (used[i] != cc)
   dfs(i, -1, head), getComp(0);
newGraph(qn, q, vn, head, old_en);
/* Determine all important edges */
forn(i, vn)
 color[i] = 0;
forn(i, qn)
 color[q[i].a] = color[q[i].b] = 1;
cc++;
forn(i, vn)
 if (used[i] != cc)
   paint(i, head);
/* Reduce the graph */
enX = 0, cc++;
forn(i, vn)
 if (used[i] != cc && color[i])
   findEdges(i, i, 0, head);
forn(i, vn)
 for (int e = head[i]; e != -1; e = next[e])
   if (w[e] > 0)
     have += w[e];
vnX = 0;
forn(i, vn)
 if (color[i])
   color[i] = vnX++;
newGraph(qn, q, vn, head, old_en);
/* Recursion continues... */
mpos -= sizeof(int) * (vn2 - vn);
if (L == R)
 res[L] = have / 2;
else
 int M = (L + R) / 2;
 Solve(L, M, qn, q, vn, head, have);
 Solve(M + 1, R, qn, q, vn, head, have);
en = old_en, mpos = old_mpos;
```

```
void Read()
{
 #define NAME "bridges3"
 assert(freopen(NAME ".in", "r", stdin));
 assert(freopen(NAME ".out", "w", stdout));
 assert(scanf("ddd", &n, &m) == 2);
 assert(1 \le n \&\& n \le maxN);
 assert(0 <= m && m <= maxM);
 map <pii, int> L;
 forn(i, m)
   char com[9];
   int a, b;
   assert(scanf("%s", com) == 1);
   assert(scanf("%d%d", &a, &b) == 2);
   assert(1 <= a && a <= n && 1 <= b && b <= n && a != b);
   a--, b--;
   if (a > b)
     swap(a, b);
   pii p = mp(a, b);
   assert(!strcmp(com, "ADD") == !L.count(p));
   if (L.count(p))
     q[qn++] = query(a, b, L[p], i);
     L.erase(p);
   }
   else
     L[p] = i;
 forit(it, L)
   q[qn++] = query(it->first.first, it->first.second, it->second, m);
void TimeStamp( const char *s )
 fprintf(stderr, "[%05.2f] %s\n", (double)clock() / CLOCKS_PER_SEC, s);
}
int main()
 Read();
 TimeStamp("Data is read");
 int *head = getMem<int>(n);
 fill(head, head + n, -1);
 TimeStamp("Memory is allocated");
 Solve(0, m, qn, q, n, head, 0);
 TimeStamp("Problem is solved");
 forn(i, m)
   printf("%d\n", res[i + 1]);
```

```
TimeStamp("Result is outputed");
  return 0;
}
```

# 3.17 Hopcroft-Karp

```
#include<bits/stdc++.h>
#define MAXN 50030
using namespace std;
int n1,n2;
vector<int> G[MAXN];
int mx[MAXN],my[MAXN];
queue<int> que;
int dx[MAXN],dy[MAXN];
bool vis[MAXN];
bool find(int u)
   for(int i=0;i<G[u].size();i++)</pre>
       if(!vis[G[u][i]]&&dy[G[u][i]]==dx[u]+1)
           vis[G[u][i]]=true;
           if(!my[G[u][i]]||find(my[G[u][i]]))
           {
               mx[u]=G[u][i];
               my[G[u][i]]=u;
               return true;
           }
       }
   }
   return false;
}
int matching()
   memset(mx,0,sizeof(mx));
   memset(my,0,sizeof(my));
   int ans=0;
   while(true)
       bool flag=false;
       while(!que.empty()) que.pop();
       memset(dx,0,sizeof(dx));
       memset(dy,0,sizeof(dy));
       for(int i=1;i<=n1;i++)</pre>
           if(!mx[i]) que.push(i);
       while(!que.empty())
           int u=que.front();
           que.pop();
           for(int i=0;i<G[u].size();i++)</pre>
               if(!dy[G[u][i]])
               {
                   dy[G[u][i]]=dx[u]+1;
                   if (my[G[u][i]])
                      dx[my[G[u][i]]]=dy[G[u][i]]+1;
                      que.push(my[G[u][i]]);
```

### 3.18 Bridge Tree

```
#pragma GCC optimize(3)
#include<bits/stdc++.h>
#define MAXN 100005
#define MAXM 100005
#define INF 100000000
#define MOD 1000000007
#define F first
#define S second
using namespace std;
typedef long long 11;
typedef pair<int,int> P;
int n,m,tot,t,bcc_cnt,mcnt;
vector<int> G[MAXN],bcc[MAXN];
int st[MAXN],dfn[MAXN],low[MAXN],bccno[MAXN];
int U[MAXM],V[MAXM];
bool isbridge[MAXM];
vector<int> tree[MAXN];
//bridge tree:
//edge-biconnected components are connected by bridges
void add_edge(int u,int v)
   U[++mcnt] = u; V[mcnt] = v;
   G[u].push_back(mcnt);G[v].push_back(mcnt);
}
int adj(int u,int e)
{
   return U[e] == u?V[e]:U[e];
void dfs1(int v,int edge)
   dfn[v]=low[v]=++tot;
   st[t++]=v;
   for(auto e:G[v])
       int to=adj(v,e);
       if(!dfn[to])
       {
           dfs1(to,e);
           low[v]=min(low[v],low[to]);
```

```
}
       else low[v]=min(low[v],dfn[to]);
   if(low[v] ==dfn[v]&&edge!=-1) isbridge[edge]=true;
}
void dfs2(int v)
{
   dfn[v]=1;
   bccno[v]=bcc_cnt;
   bcc[bcc_cnt].push_back(v);
   for(auto e:G[v])
       int to=adj(v,e);
       if(isbridge[e]) continue;
       if(!dfn[to]) dfs2(to);
   }
}
int tarjan()
   bcc_cnt=tot=0;
   memset(dfn,0,sizeof(dfn));
   memset(isbridge,false,sizeof(isbridge));
   for(int i=1;i<=n;i++) if(!dfn[i]) dfs1(i,-1);</pre>
   memset(dfn,0,sizeof(dfn));
   for(int i=1;i<=n;i++)</pre>
       if(!dfn[i])
       {
           bcc_cnt++;
           dfs2(i);
       }
   }
   return bcc_cnt;
}
void build_bridge_tree()
   tarjan();
   for(int i=1;i<=mcnt;i++)</pre>
       if(isbridge[i])
       {
           int u=bccno[U[i]],v=bccno[V[i]];
           tree[u].push_back(v); tree[v].push_back(u);
       }
   }
}
int main()
{
   return 0;
}
```

#### 3.19 Block Cut Tree

```
#pragma GCC optimize(3)
#include<bits/stdc++.h>
#define MAXN 100005
#define MAXM 100005
```

```
#define INF 100000000
#define MOD 1000000007
#define F first
#define S second
using namespace std;
typedef long long 11;
typedef pair<int,int> P;
int n,m,tot,t,bcc_cnt,mcnt;
vector<int> G[MAXN],bcc[MAXN];
int st[MAXN],dfn[MAXN],low[MAXN],bccno[MAXN];
bool art[MAXN];
vector<int> tree[MAXN];
int id[MAXN];
int N;
//block-cut tree:
//vertex-biconnected components are connected by their shared articulation point
void dfs(int v,int p,int &tot)
   dfn[v]=low[v]=++tot;
   st[t++]=v;
   for(auto to:G[v])
       if(!dfn[to])
           dfs(to,v,tot);
           low[v]=min(low[v],low[to]);
           if(low[to]>=dfn[v])
               art[v]=(dfn[v]>1||dfn[to]>2);
               bcc_cnt++;
               bcc[bcc_cnt].push_back(v); bccno[v]=bcc_cnt;
               while(bcc[bcc_cnt].back()!=v)
               {
                  bccno[st[t-1]]=bcc_cnt;
                  bcc[bcc_cnt].push_back(st[t-1]),t--;
               }
           }
       else low[v]=min(low[v],dfn[to]);
   }
}
int tarjan()
   bcc_cnt=t=0;
   memset(dfn,0,sizeof(dfn));
   memset(art,false,sizeof(art));
   for(int i=1;i<=n;i++) if(!dfn[i]) dfs(i,-1,tot=0);</pre>
   return bcc_cnt;
void build_block_cut_tree()
   tarjan();N=0;
   for(int i=1;i<=n;i++) if(art[i]) id[i]=++N;</pre>
   for(int i=1;i<=bcc_cnt;i++)</pre>
       N++;
       for(auto v:bcc[i])
           if(!art[v]) id[v]=N;
```

#### 3.20 Gomory-Hu Tree

```
#include<bits/stdc++.h>
#define MAXV 3005
#define MAXE 50000
#define INF 100000000
using namespace std;
typedef pair<int,int> P;
typedef long long 11;
struct edge{int to,cap,rev,id;};//id=1 positive edge, id=0 reverse edge
struct edge2{int to,cost;};
struct edge3{int from,to,cap;};
int V,E;
vector<edge> G[MAXV];
vector<edge2> gh[MAXV];
vector<edge3> edges;
int level[MAXV];
int iter[MAXV];
void add_edge(int from,int to,int cap)
   edges.push_back((edge3){from,to,cap});
}
void add_all()
{
   for(auto e:edges)
       G[e.from].push_back((edge){e.to,e.cap,(int)G[e.to].size(),1});
       G[e.to].push_back((edge){e.from,0,(int)G[e.from].size()-1,0});
   }
}
void clear_all()
{
   for(int i=1;i<=V;i++) G[i].clear();</pre>
}
void bfs(int s)
   memset(level,-1,sizeof(level));
   queue<int> que;
   level[s]=0;
   que.push(s);
   while(!que.empty())
       int v=que.front(); que.pop();
       for(int i=0;i<(int)G[v].size();i++)</pre>
```

```
{
           edge &e=G[v][i];
           if(e.cap>0&&level[e.to]<0)</pre>
               level[e.to] = level[v] + 1;
               que.push(e.to);
       }
   }
}
int dfs(int v,int t,int f)
   if(v==t) return f;
   for(int &i=iter[v];i<(int)G[v].size();i++)</pre>
        edge &e=G[v][i];
       if(level[v] < level[e.to] &&e.cap > 0)
           int d=dfs(e.to,t,min(f,e.cap));
           if(d>0)
           {
               e.cap-=d;
               G[e.to][e.rev].cap+=d;
               return d;
       }
   return 0;
}
int max_flow(int s,int t)
   int flow=0;
   for(;;)
       bfs(s);
       if(level[t]<0) return flow;</pre>
       memset(iter,0,sizeof(iter));
       int f;
       while((f=dfs(s,t,INF))>0)
         flow+=f;
   }
}
//0-based!!!
void build_gomory_hu_tree()
   vector<int> p(V+1,1),cap(V+1,0);
   for(int s=2;s<=V;s++)</pre>
        add_all();
        int t=p[s];
        cap[s]=max_flow(s,t);
        vector<bool> in_cut(V+1,0);
        queue<int> que({s});
        in_cut[s]=true;
       while(!que.empty())
           int v=que.front();
           que.pop();
```

```
for(auto e:G[v])
               if(e.cap>0&&!in_cut[e.to])
               {
                   in_cut[e.to]=true;
                   que.push(e.to);
               }
           }
       }
       for(int v=1; v<=V; v++)</pre>
           if(v!=s&&in_cut[v]&&p[v]==t)
               p[v]=s;
        if(in_cut[p[t]])
           p[s]=p[t];
           p[t]=s;
           swap(cap[s],cap[t]);
       }
        clear_all();
   }
   for(int v=2; v<=V; v++)</pre>
       gh[p[v]].push_back((edge2){v,cap[v]});
       gh[v].push_back((edge2){p[v],cap[v]});
   }
}
int main()
   scanf("%d%d",&V,&E);
   for(int i=0;i<E;i++)</pre>
   {
       int u,v,w;
       scanf("%d%d%d",&u,&v,&w);
       add_edge(u,v,w);
       add_edge(v,u,w);
   build_gomory_hu_tree();
   return 0;
}
```

#### 3.21 Minimum Arborescence

```
#include<bits/stdc++.h>
#define MAXN 100005
#define INF 1000000000
#define MOD 1000000007
#define F first
#define S second
using namespace std;
typedef long long ll;
typedef pair<int,int> P;
int n,k,a[MAXN];
namespace ZL
{
          const int N=100010,M=100010,inf=1e9;
          struct edge
          {
```

```
int u,v,w,use,id;
   }b[M],a[2000100];
   int n,m,ans,pre[N],id[N],vis[N],root,In[N],h[N],len,way[M];
   void init(int _n,int _root)
{
           n=_n; m=0; b[0].w=inf; root=_root; ans=0;
   }
   void add(int u,int v,int w)
{
           b[++m]=(edge)\{u,v,w,0,m\};
           a[m]=b[m];
   }
   int work()
           len=m;
       for (;;)
    {
           for (int i=1;i<=n;i++){pre[i]=0; In[i]=inf; id[i]=0; vis[i]=0; h[i]=0;}</pre>
           for (int i=1;i<=m;i++)</pre>
               if (b[i].u!=b[i].v&&b[i].w<In[b[i].v])</pre>
                   pre[b[i].v]=b[i].u; In[b[i].v]=b[i].w; h[b[i].v]=b[i].id;
           for (int i=1;i<=n;i++) if (pre[i]==0&&i!=root) return 0;</pre>
           int cnt=0; In[root]=0;
           for (int i=1;i<=n;i++)</pre>
       {
               if (i!=root) a[h[i]].use++;
               int now=i; ans+=In[i];
               while (vis[now] == 0&&now! = root)
           {
                   vis[now]=i; now=pre[now];
               }
               if (now!=root&&vis[now]==i)
           {
                   cnt++; int kk=now;
                   while (1)
               {
                       id[now] = cnt; now = pre[now];
                       if (now==kk) break;
                   }
               }
           }
           if (cnt==0) return 1;
           for (int i=1;i<=n;i++) if (id[i]==0) id[i]=++cnt;</pre>
           for (int i=1;i<=m;i++)</pre>
               int k1=In[b[i].v]; int k2=b[i].v;
               b[i].u=id[b[i].u]; b[i].v=id[b[i].v];
               if (b[i].u!=b[i].v)
           {
                   b[i].w-=k1; a[++len].u=b[i].id; a[len].v=h[k2];
                   b[i].id=len;
               }
           }
           n=cnt;
           root=id[root];
       }
       return 1;
```

### 3.22 Minimum Diameter Spanning Tree

```
#include<bits/stdc++.h>
#define MAXN 505
#define MAXM 200005
#define INF 100000000000000000LL
#define MOD 100000007
#define F first
#define S second
using namespace std;
typedef long long 11;
typedef pair<11,11> P;
typedef pair<double,11> PP;
const double eps=1e-2;
11 n,m,d[MAXN][MAXN],save[MAXN][MAXN],pre[MAXN];
double d2[MAXN];
11 u[MAXM], v[MAXM], w[MAXM];
bool used[MAXN];
vector<P> dist[MAXN];
vector<P> MDST;
void floyd_warshall()
   for(ll k=1;k<=n;k++)</pre>
       for(ll i=1;i<=n;i++)</pre>
         for(ll j=1;j<=n;j++) d[i][j]=min(d[i][j],d[i][k]+d[k][j]);</pre>
pair<P,double> absolute_center()
   11 ans=INF;
   ll uu=-1, vv=-1;
   double res=0.0;
   for(ll i=1;i<=n;i++)</pre>
        int sz=(int)dist[i].size();
       if (dist[i][sz-1].F+dist[i][sz-2].F<ans)</pre>
           ans=dist[i][sz-1].F+dist[i][sz-2].F;
           uu=vv=i; res=0.0;
   }
   for(ll i=0;i<m;i++)</pre>
       memset(used,false,sizeof(used));
       ll now=(int)dist[v[i]].size()-1;
       for(ll j=0;j<(int)dist[u[i]].size();j++)</pre>
```

```
{
           used[dist[u[i]][j].S]=true;
           while(now>0&&used[dist[v[i]][now].S]) now--;
           double pos=(dist[u[i]][j].F+dist[v[i]][now].F+w[i])/2.0-dist[u[i]][j].F;
           if(pos<eps||pos-w[i]>eps) continue;
           if(dist[u[i]][j].F+dist[v[i]][now].F+w[i]<ans)</pre>
               ans=dist[u[i]][j].F+dist[v[i]][now].F+w[i];
               uu=u[i]; vv=v[i]; res=pos;
           }
       }
   }
   printf("%lld\n",ans);
   return make_pair(P(uu,vv),res);
void minimum_diameter_spanning_tree()
{
   MDST.clear();
   auto p=absolute_center();
   fill(d2,d2+n+1,INF); memset(pre,-1,sizeof(pre));
   priority_queue<PP,vector<PP>,greater<PP> > que;
   d2[p.F.F]=p.S; d2[p.F.S]=d[p.F.F][p.F.S]-p.S;
   que.push(PP(d2[p.F.F],p.F.F)); if(p.F.F!=p.F.S) que.push(PP(d2[p.F.S],p.F.S));
   while(!que.empty())
   {
       PP p=que.top(); que.pop();
       11 v=p.S;
       if(d2[v]<p.F) continue;</pre>
       for(ll to=1;to<=n;to++)</pre>
           if(d2[to]>d2[v]+save[v][to])
           {
               d2[to]=d2[v]+save[v][to];
               pre[to]=v;
               que.push(PP(d2[to],to));
           }
       }
   }
   if(p.F.F!=p.F.S) MDST.push_back(P(p.F.F,p.F.S));
   for(ll i=1;i<=n;i++) if(pre[i]!=-1) MDST.push_back(P(pre[i],i));</pre>
int main()
   scanf("%11d%11d",&n,&m);
   for(ll i=1;i<=n;i++)</pre>
       for(ll j=1; j<=n; j++)</pre>
           d[i][j]=save[i][j]=(i==j?0:INF);
   for(ll i=0;i<m;i++)</pre>
   {
       scanf("%1ld%1ld%1ld",&u[i],&v[i],&w[i]);
       d[u[i]][v[i]]=d[v[i]][u[i]]=save[u[i]][v[i]]=save[v[i]][u[i]]=w[i];
   floyd_warshall();
   for(ll i=1;i<=n;i++)</pre>
   {
       for(ll j=1;j<=n;j++) dist[i].push_back(P(d[i][j],j));</pre>
       sort(dist[i].begin(),dist[i].end());
   }
   minimum_diameter_spanning_tree();
```

```
for(auto p:MDST) printf("%lld %lld\n",p.F,p.S);
return 0;
}
```

### 3.23 Chordal Graph

```
> File Name: ChordalGraph.cpp
   > Author: Roundgod
   > Mail: wcysai@foxmail.com
   > Created Time: 2018-10-31 15:49:59
#pragma GCC optimize(3)
#include<bits/stdc++.h>
#define MAXN 100005
#define INF 100000000
#define MOD 1000000007
#define F first
#define S second
using namespace std;
typedef long long 11;
typedef pair<int,int> P;
int n,m;
vector<int> G[MAXN];
int h[MAXN];
bool vis[MAXN];
vector<int> peo;
priority_queue<P> pque;
void MCS()
{
   memset(vis,0,sizeof(vis));
   memset(h,0,sizeof(h));
   for(int i=1;i<=n;i++) pque.push(P(0,i));</pre>
   for(int i=n;i>=1;i--)
   {
      while(1)
      {
         P p=pque.top();pque.pop();
         if(vis[p.S]) continue;
         peo.push_back(p.S); vis[p.S]=true;
         for(auto to:G[p.S])
            if(vis[to]) continue;
            h[to]++;pque.push(P(h[to],to));
         }
         break;
      }
   reverse(peo.begin(),peo.end());
}
int main()
   scanf("%d%d",&n,&m);
   for(int i=0;i<m;i++)</pre>
      int u,v;scanf("%d%d",&u,&v);
```

```
G[u].push_back(v);G[v].push_back(u);
}
MCS();
return 0;
}
```

### 3.24 TriangleCount

```
#include<bits/stdc++.h>
#define MAXN 100005
#define INF 100000000
#define MOD 1000000007
#define F first
#define S second
using namespace std;
int n,m,deg[MAXN],a[MAXN],cnt[MAXN],r[MAXN];
vector<int> G[MAXN];
vector<int> gr[MAXN];
bool cmp(int x,int y)
   return deg[x] < deg[y];</pre>
}
int main()
   memset(cnt,0,sizeof(cnt));
   scanf("%d%d",&n,&m);
   for(int i=1;i<=n;i++) a[i]=i;</pre>
   for(int i=0;i<m;i++)</pre>
       int u,v;scanf("%d%d",&u,&v);
       G[u].push_back(v);G[v].push_back(u);
       deg[v]++;deg[u]++;
   }
   sort(a,a+n+1,cmp);
   for(int i=1;i<=n;i++) r[a[i]]=i;</pre>
   for(int i=1;i<=n;i++)</pre>
       for(auto to:G[i]) if(r[to]>r[i]) gr[i].push_back(to);
   int ans=0;
   for(int i=1;i<=n;i++)</pre>
       for(auto u:gr[i]) cnt[u]++;
       for(auto u:gr[i])
           for(auto to:gr[u]) ans+=cnt[to];
       for(auto u:gr[i]) cnt[u]--;
   printf("%d\n",ans);
   return 0;
```

### 3.25 SquareCount

```
#include<bits/stdc++.h>
#define MAXN 100005
#define INF 1000000000
#define MOD 1000000007
```

```
#define F first
#define S second
using namespace std;
int n,m,deg[MAXN],a[MAXN],cnt[MAXN],r[MAXN];
vector<int> G[MAXN];
vector<int> gr[MAXN];
bool cmp(int x,int y)
{
   return deg[x] < deg[y];</pre>
}
int main()
   memset(cnt,0,sizeof(cnt));
   scanf("%d%d",&n,&m);
   for(int i=1;i<=n;i++) a[i]=i;</pre>
   for(int i=0;i<m;i++)</pre>
        int u,v;scanf("%d%d",&u,&v);
       G[u].push_back(v);G[v].push_back(u);
       deg[v]++;deg[u]++;
   sort(a,a+n+1,cmp);
   for(int i=1;i<=n;i++) r[a[i]]=i;</pre>
   for(int i=1;i<=n;i++)</pre>
        for(auto to:G[i]) if(r[to]>r[i]) gr[i].push_back(to);
   int ans=0;
   for(int i=1;i<=n;i++)</pre>
        for(auto u:G[i])
           for(auto to:gr[u])
               if(r[to]>r[i])
                   ans+=cnt[to];
                   cnt[to]++;
           }
       }
       for(auto u:G[i])
           for(auto to:gr[u])
               if(r[to]>r[i]) cnt[to]--;
        }
       printf("%d\n",ans);
   printf("%d\n",ans);
   return 0;
}
```

### 4 Math

## 4.1 BigNum

```
const int base = 1000000000;
const int base_digits = 9;
struct bigint {
   vector<int> a;
   int sign;
   /*<arpa>*/
   int size(){
       if(a.empty())return 0;
       int ans=(a.size()-1)*base_digits;
       int ca=a.back();
       while(ca)
          ans++, ca/=10;
       return ans;
   }
   bigint operator ^(const bigint &v){
       bigint ans=1,a=*this,b=v;
       while(!b.isZero()){
          if(b%2)
              ans*=a;
          a*=a,b/=2;
       }
       return ans;
   }
   string to_string(){
       stringstream ss;
       ss << *this;
       string s;
       ss >> s;
       return s;
   }
   int sumof(){
       string s = to_string();
       int ans = 0;
       for(auto c : s) ans += c - '0';
       return ans;
   /*</arpa>*/
   bigint():
       sign(1) {
   bigint(long long v) {
       *this = v;
   }
   bigint(const string &s) {
       read(s);
   }
   void operator=(const bigint &v) {
       sign = v.sign;
       a = v.a;
   }
   void operator=(long long v) {
       sign = 1;
       a.clear();
       if (v < 0)
          sign = -1, v = -v;
```

```
for (; v > 0; v = v / base)
       a.push_back(v % base);
}
bigint operator+(const bigint &v) const {
   if (sign == v.sign) {
       bigint res = v;
       for (int i = 0, carry = 0; i < (int) max(a.size(), v.a.size()) || carry; ++i) {</pre>
           if (i == (int) res.a.size())
              res.a.push_back(0);
           res.a[i] += carry + (i < (int) a.size() ? a[i] : 0);
           carry = res.a[i] >= base;
           if (carry)
              res.a[i] -= base;
       }
       return res;
   }
   return *this - (-v);
}
bigint operator-(const bigint &v) const {
   if (sign == v.sign) {
       if (abs() >= v.abs()) {
           bigint res = *this;
           for (int i = 0, carry = 0; i < (int) v.a.size() || carry; ++i) {</pre>
              res.a[i] -= carry + (i < (int) v.a.size() ? v.a[i] : 0);
              carry = res.a[i] < 0;</pre>
               if (carry)
                  res.a[i] += base;
           }
           res.trim();
           return res;
       return -(v - *this);
   return *this + (-v);
}
void operator*=(int v) {
   if (v < 0)
       sign = -sign, v = -v;
   for (int i = 0, carry = 0; i < (int) a.size() || carry; ++i) {</pre>
       if (i == (int) a.size())
           a.push_back(0);
       long long cur = a[i] * (long long) v + carry;
       carry = (int) (cur / base);
       a[i] = (int) (cur % base);
       //asm("divl %%ecx" : "=a"(carry), "=d"(a[i]) : "A"(cur), "c"(base));
   }
   trim();
}
bigint operator*(int v) const {
   bigint res = *this;
   res *= v;
   return res;
}
```

```
void operator*=(long long v) {
   if (v < 0)
       sign = -sign, v = -v;
   for (int i = 0, carry = 0; i < (int) a.size() || carry; ++i) {</pre>
       if (i == (int) a.size())
           a.push_back(0);
       long long cur = a[i] * (long long) v + carry;
       carry = (int) (cur / base);
       a[i] = (int) (cur % base);
       //asm("divl %%ecx" : "=a"(carry), "=d"(a[i]) : "A"(cur), "c"(base));
   }
   trim();
}
bigint operator*(long long v) const {
   bigint res = *this;
   res *= v;
   return res;
}
friend pair<bigint, bigint> divmod(const bigint &a1, const bigint &b1) {
   int norm = base / (b1.a.back() + 1);
   bigint a = a1.abs() * norm;
   bigint b = b1.abs() * norm;
   bigint q, r;
   q.a.resize(a.a.size());
   for (int i = a.a.size() - 1; i >= 0; i--) {
       r *= base;
       r += a.a[i];
       int s1 = r.a.size() <= b.a.size() ? 0 : r.a[b.a.size()];</pre>
       int s2 = r.a.size() <= b.a.size() - 1 ? 0 : r.a[b.a.size() - 1];</pre>
       int d = ((long long) base * s1 + s2) / b.a.back();
       r = b * d;
       while (r < 0)
          r += b, --d;
       q.a[i] = d;
   }
   q.sign = a1.sign * b1.sign;
   r.sign = a1.sign;
   q.trim();
   r.trim();
   return make_pair(q, r / norm);
}
bigint operator/(const bigint &v) const {
   return divmod(*this, v).first;
bigint operator%(const bigint &v) const {
   return divmod(*this, v).second;
void operator/=(int v) {
   if (v < 0)
       sign = -sign, v = -v;
   for (int i = (int) a.size() - 1, rem = 0; i \ge 0; --i) {
       long long cur = a[i] + rem * (long long) base;
```

```
a[i] = (int) (cur / v);
       rem = (int) (cur % v);
    }
   trim();
}
bigint operator/(int v) const {
   bigint res = *this;
   res /= v;
   return res;
}
int operator%(int v) const {
    if (v < 0)
       v = -v;
   int m = 0;
    for (int i = a.size() - 1; i >= 0; --i)
       m = (a[i] + m * (long long) base) % v;
   return m * sign;
}
void operator+=(const bigint &v) {
    *this = *this + v;
}
void operator-=(const bigint &v) {
    *this = *this - v;
}
void operator*=(const bigint &v) {
   *this = *this * v;
void operator/=(const bigint &v) {
    *this = *this / v;
}
bool operator<(const bigint &v) const {</pre>
    if (sign != v.sign)
       return sign < v.sign;</pre>
    if (a.size() != v.a.size())
       return a.size() * sign < v.a.size() * v.sign;</pre>
   for (int i = a.size() - 1; i >= 0; i--)
       if (a[i] != v.a[i])
           return a[i] * sign < v.a[i] * sign;</pre>
   return false;
}
bool operator>(const bigint &v) const {
   return v < *this;</pre>
bool operator<=(const bigint &v) const {</pre>
   return !(v < *this);</pre>
bool operator>=(const bigint &v) const {
   return !(*this < v);</pre>
bool operator==(const bigint &v) const {
   return !(*this < v) && !(v < *this);</pre>
bool operator!=(const bigint &v) const {
   return *this < v || v < *this;</pre>
```

```
}
void trim() {
   while (!a.empty() && !a.back())
       a.pop_back();
    if (a.empty())
       sign = 1;
}
bool isZero() const {
   return a.empty() || (a.size() == 1 && !a[0]);
bigint operator-() const {
   bigint res = *this;
   res.sign = -sign;
   return res;
}
bigint abs() const {
   bigint res = *this;
   res.sign *= res.sign;
   return res;
}
long longValue() const {
   long long res = 0;
   for (int i = a.size() - 1; i >= 0; i--)
       res = res * base + a[i];
   return res * sign;
}
friend bigint gcd(const bigint &a, const bigint &b) {
   return b.isZero() ? a : gcd(b, a % b);
friend bigint lcm(const bigint &a, const bigint &b) {
   return a / gcd(a, b) * b;
void read(const string &s) {
   sign = 1;
   a.clear();
   int pos = 0;
   while (pos < (int) s.size() && (s[pos] == '-' || s[pos] == '+')) {</pre>
       if (s[pos] == '-')
           sign = -sign;
       ++pos;
   for (int i = s.size() - 1; i >= pos; i -= base_digits) {
       int x = 0;
       for (int j = max(pos, i - base_digits + 1); j <= i; j++)
           x = x * 10 + s[j] - '0';
       a.push_back(x);
    }
   trim();
}
friend istream& operator>>(istream &stream, bigint &v) {
   string s;
```

```
stream >> s;
   v.read(s):
   return stream;
}
friend ostream& operator<<(ostream &stream, const bigint &v) {</pre>
    if (v.sign == -1)
       stream << '-';
   stream << (v.a.empty() ? 0 : v.a.back());</pre>
   for (int i = (int) v.a.size() - 2; i >= 0; --i)
       stream << setw(base_digits) << setfill('0') << v.a[i];</pre>
   return stream;
}
static vector<int> convert_base(const vector<int> &a, int old_digits, int new_digits) {
   vector<long long> p(max(old_digits, new_digits) + 1);
   p[0] = 1;
   for (int i = 1; i < (int) p.size(); i++)</pre>
       p[i] = p[i - 1] * 10;
   vector<int> res;
   long long cur = 0;
   int cur_digits = 0;
   for (int i = 0; i < (int) a.size(); i++) {</pre>
       cur += a[i] * p[cur_digits];
       cur_digits += old_digits;
       while (cur_digits >= new_digits) {
           res.push_back(int(cur % p[new_digits]));
           cur /= p[new_digits];
           cur_digits -= new_digits;
       }
   }
   res.push_back((int) cur);
    while (!res.empty() && !res.back())
       res.pop_back();
   return res;
}
typedef vector<long long> vll;
static vll karatsubaMultiply(const vll &a, const vll &b) {
   int n = a.size();
   vll res(n + n);
   if (n <= 32) {</pre>
       for (int i = 0; i < n; i++)</pre>
           for (int j = 0; j < n; j++)
               res[i + j] += a[i] * b[j];
       return res;
   }
    int k = n >> 1;
   vll a1(a.begin(), a.begin() + k);
   vll a2(a.begin() + k, a.end());
   vll b1(b.begin(), b.begin() + k);
   vll b2(b.begin() + k, b.end());
   vll a1b1 = karatsubaMultiply(a1, b1);
   vll a2b2 = karatsubaMultiply(a2, b2);
   for (int i = 0; i < k; i++)</pre>
```

```
a2[i] += a1[i];
       for (int i = 0; i < k; i++)</pre>
           b2[i] += b1[i];
       vll r = karatsubaMultiply(a2, b2);
       for (int i = 0; i < (int) a1b1.size(); i++)</pre>
           r[i] -= a1b1[i];
       for (int i = 0; i < (int) a2b2.size(); i++)</pre>
           r[i] = a2b2[i];
       for (int i = 0; i < (int) r.size(); i++)</pre>
           res[i + k] += r[i];
       for (int i = 0; i < (int) a1b1.size(); i++)</pre>
           res[i] += a1b1[i];
       for (int i = 0; i < (int) a2b2.size(); i++)</pre>
           res[i + n] += a2b2[i];
       return res;
   }
   bigint operator*(const bigint &v) const {
       vector<int> a6 = convert_base(this->a, base_digits, 6);
       vector<int> b6 = convert_base(v.a, base_digits, 6);
       vll a(a6.begin(), a6.end());
       vll b(b6.begin(), b6.end());
       while (a.size() < b.size())</pre>
           a.push_back(0);
       while (b.size() < a.size())</pre>
           b.push_back(0);
       while (a.size() & (a.size() - 1))
           a.push_back(0), b.push_back(0);
       vll c = karatsubaMultiply(a, b);
       bigint res;
       res.sign = sign * v.sign;
       for (int i = 0, carry = 0; i < (int) c.size(); i++) {</pre>
           long long cur = c[i] + carry;
           res.a.push_back((int) (cur % 1000000));
           carry = (int) (cur / 1000000);
       res.a = convert_base(res.a, 6, base_digits);
       res.trim();
       return res;
   }
};
```

#### 4.2 Fast Multiplication

```
#include<bits/stdc++.h>
typedef long long 11;
ll mul(ll A,ll B,ll mod)
{
    return (A*B-(ll)((long double)A*B/mod)*mod;
}
```

#### 4.3 Pohlig Hellman

```
#include<bits/stdc++.h>
#define MAXN 100005
#define INF 100000000
#define MOD 100000007
#define F first
#define S second
using namespace std;
typedef long long 11;
typedef pair<11,11> P;
int T;
ll a,b,p;
11 mul(11 A,11 B,11 mod)
   return (A*B-(11)((long double)A*B/mod)*mod+mod)%mod;
}
ll pow_mod(ll a,ll i,ll mod)
   if(i==0) return 1;
    ll s=1;
   while(i>0)
        if(i&1) s=mul(s,a);
        a=mul(a,a);
        i>>=1;
    }
    return s;
}
vector<P> fact;
11 Pohlig_Hellman(ll g,ll x,ll p,vector<P> fact)
   11 q=p-1,now=g,res=1,ker=0;
   for(int i=0;i<(int)fact.size();i++)</pre>
       for(int j=0;j<fact[i].S;j++)</pre>
           q/=fact[i].F;
           //find dlg modulo current prime, code below is for the prime 2
           if(pow_mod(x,q,p)!=1)
               x=mul(x,now,p);
              res+=ker;
           }
           now=pow_mod(now,fact[i].F,p);
           ker*=fact[i].F;
       }
   }
   return (p-1-res)%(p-1);
```

#### 4.4 Belerkamp-Massey

```
#include<bits/stdc++.h>
#define rep(i,a,n) for (int i=a;i<n;i++)
#define per(i,a,n) for (int i=n-1;i>=a;i--)
```

```
#define pb push_back
#define mp make_pair
#define all(x) (x).begin(),(x).end()
#define fi first
#define se second
#define SZ(x) ((int)(x).size())
using namespace std;
typedef vector<int> VI;
typedef long long 11;
typedef pair<int,int> PII;
const 11 mod=1000000007;
ll pow_mod(ll a,ll i)
    11 s=1;
    while(i)
    {
        if(i&1) s=s*a%mod;
        a=a*a%mod;
        i>>=1;
    }
    return s;
namespace linear_seq
    const int N=10010;
    11 res[N],base[N],_c[N],_md[N];
    vector<ll> Md;
    void mul(ll *a,ll *b,int k)
    {
       rep(i,0,k+k) _c[i]=0;
       rep(i,0,k) if(a[i]) rep(j,0,k) _c[i+j]=(_c[i+j]+a[i]*b[j])%mod;
        for(int i=k+k-1;i>=k;i--)
            if(_c[i]) rep(j,0,SZ(Md)) _c[i-k+Md[j]]=(_c[i-k+Md[j]]-_c[i]*_md[Md[j]])%mod;
       rep(i,0,k) a[i]=_c[i];
    }
    int solve(ll n,VI a,VI b)//a:coefficient b:initial value b[n+1]=a[0]*b[n]+...
       11 ans=0,pnt=0;
        int k=SZ(a);
        assert(SZ(a) == SZ(b));
       rep(i,0,k) _md[k-1-i]=-a[i];
        _md[k]=1;
       Md.clear();
       rep(i,0,k) if(_md[i]!=0) Md.push_back(i);
       rep(i,0,k) res[i]=base[i]=0;
       res[0]=1;
        while((111<<pnt)<=n) pnt++;</pre>
       for(int p=pnt;p>=0;p--)
            mul(res,res,k);
            if((n>>p)&1)
                for(int i=k-1;i>=0;i--) res[i+1]=res[i];
                \label{eq:condition} $\operatorname{rep}(j,0,SZ(Md))$ $\operatorname{res}[Md[j]]=(\operatorname{res}[Md[j]]-\operatorname{res}[k]*_md[Md[j]])$ $$mod$;
            }
        }
        rep(i,0,k) ans=(ans+res[i]*b[i])%mod;
        if(ans<0) ans+=mod;</pre>
```

```
return ans;
   }
   VI BM(VI s)
       VI C(1,1),B(1,1);
       int L=0, m=1, b=1;
       rep(n,0,SZ(s))
           11 d=0;
           rep(i,0,L+1) d=(d+(l1)C[i]*s[n-i])\mod;
           if(d==0) ++m;
           else if(2*L<=n)</pre>
               VI T=C;
               11 c=mod-d*pow_mod(b,mod-2)%mod;
               while(SZ(C)<SZ(B)+m) C.pb(0);</pre>
               rep(i,0,SZ(B)) C[i+m]=(C[i+m]+c*B[i])%mod;
               L=n+1-L; B=T; b=d; m=1;
           }
           else
           {
               11 c=mod-d*pow_mod(b,mod-2)%mod;
               while(SZ(C)<SZ(B)+m) C.pb(0);</pre>
               rep(i,0,SZ(B)) C[i+m]=(C[i+m]+c*B[i])%mod;
               ++m;
           }
       }
       return C;
   }
   int gao(VI a,ll n)
   {
       VI c=BM(a);
       c.erase(c.begin());
       rep(i,0,SZ(c)) c[i]=(mod-c[i])%mod;
       return solve(n,c,VI(a.begin(),a.begin()+SZ(c)));
   }
}
int main()
   11 n;
   while(scanf("%lld",&n)!=EOF)
       printf("\%d\n",linear_seq::gao(VI\{1,5,11,36,95,281,781,2245,6336\},n-1));
   }
   return 0;
}
```

# 4.5 Chinese Remainder Theorem

```
#include<bits/stdc++.h>
#define MAXN 105
#define INF 1000000000
#define MOD 1000000007
#define F first
#define S second
using namespace std;
typedef long long ll;
```

```
typedef pair<int,int> P;
int n,k;
int r[MAXN] [MAXN],x[MAXN];
int extgcd(int a,int b,int &x,int &y)
{
   int d=a;
   if(b!=0)
   {
       d=extgcd(b,a%b,y,x);
       y=(a/b)*x;
   }
   else
   {
       x=1;
       y=0;
   }
   return d;
}
int mod_inverse(int a,int m)
   int x,y;
   extgcd(a,m,x,y);
   return (m+x%m)%m;
}
int solve(vector<P> &v)
   int n=v.size();
   for(int i=0;i<n;i++)</pre>
       for(int j=i+1;j<n;j++)</pre>
           r[i][j]=mod_inverse(v[i].S,v[j].S);
   int ans=0;
   for(int i=0;i<n;i++)</pre>
   {
       x[i]=v[i].F;
       for(int j=0;j<i;j++)</pre>
           x[i]=r[j][i]*(x[i]-x[j]);
           x[i]=x[i]%v[i].S;
           if(x[i]<0) x[i]+=v[i].S;</pre>
       }
   }
   int base=1;
   for(int i=0;i<n;i++)</pre>
   {
        ans+=base*x[i];
       base*=v[i].S;
   }
   return ans;
}
int main()
   vector<P> v;
   v.push_back(P(4,7));
   v.push_back(P(3,13));
   printf("%d\n",solve(v));
   return 0;
}
```

### 4.6 Matrix Determinant

```
#include<bits/stdc++.h>
#define MAXN 505
using namespace std;
typedef vector<int> vec;
typedef vector<vec> mat;
int det_mod(mat A,int M)
{
        int n=A.size();
        for(int i=0;i<n;i++)</pre>
               for(int j=0;j<n;j++)</pre>
                       A[i][j]%=M;
        int ans=1;
        for(int i=0;i<n;i++)</pre>
        {
               for(int j=i+1;j<n;j++)</pre>
                       while(A[j][i]!=0)
                               int t=A[i][i]/A[j][i];
                               for(int k=0;k<n;k++)</pre>
                                       A[i][k]=A[i][k]-A[j][k]*t;
                                       swap(A[i][k],A[j][k]);
                               }
                               ans=-ans;
                       if(A[i][i]==0) return 0;
               }
               ans=ans*A[i][i];
        }
       return (ans%M+M)%M;
}
int main()
        scanf("%d",&n);
       mat A(n,vec(n));
       for(int i=0;i<n;i++)</pre>
               for(int j=0;j<n;j++)</pre>
                       scanf("%d",&A[i][j]);
       printf("%d\n",det_mod(A,3));
        return 0;
}
```

### 4.7 Euler Sieve

```
#include<bits/stdc++.h>
#define MAXN 100005
#define MOD 1000000007
#define INF 1000000000
using namespace std;
typedef long long ll;
int prime[MAXN],phi[MAXN],miu[MAXN];
bool is_prime[MAXN];
```

```
int sieve(int n)
   int p=0;
   for(int i=0;i<=n;i++) is_prime[i]=true;</pre>
   is_prime[0]=is_prime[1]=false;
   for(int i=2;i<=n;i++)</pre>
        if(is_prime[i]) prime[p++]=i;
       for(int j=0;j<p;j++)</pre>
           if(prime[j]*i>n) break;
           is_prime[prime[j]*i]=false;
           if(i%prime[j]==0) break;
       }
   }
   return p;
}
void genphi(int n)
   int p=0;
   memset(phi,0,sizeof(phi));
   phi[1]=1;
   for(int i=2;i<=n;i++)</pre>
        if(is_prime[i]) {p++; phi[i]=i-1;}
       for(int j=0;j<p;j++)</pre>
           if(prime[j]*i>n) break;
           phi[i*prime[j]]=phi[i]*(i%prime[j]?prime[j]-1:prime[j]);
           if(i%prime[j]==0) break;
       }
   }
}
void genmiu(int n)
   int p=0;
   memset(miu,0,sizeof(miu));
   miu[1]=1;
   for(int i=2;i<=n;i++)</pre>
        if(is_prime[i]) {p++; miu[i]=-1;}
       for(int j=0;j<p;j++)</pre>
           if(prime[j]*i>n) break;
           miu[i*prime[j]]=i%prime[j]?-miu[i]:0;
           if(i%prime[j]==0) break;
   }
}
int main()
   sieve(100000);
   genphi(100000);
   genmiu(100000);
   for(int i=1;i<=10;i++)</pre>
       printf("%d\n",miu[i]);
   return 0;
}
```

### 4.8 Extended GCD

```
#include<bits/stdc++.h>
using namespace std;
typedef __int64 11;
ll extgcd(ll a,ll b,ll &x,ll &y)
   11 d=a;
   if(b!=0)
       d=extgcd(b,a%b,y,x);
       y=(a/b)*x;
   }
   else
   {
       x=1;
       y=0;
   }
   return d;
}
ll a,b,x,y;
int main()
   while(scanf("%I64d%I64d",&a,&b)==2)
       if(extgcd(a,b,x,y)==1)
           while(x<0)</pre>
               x+=b;
               y-=a;
       printf("%I64d %I64d\n",x,y);
       else puts("sorry");
   return 0;
}
```

# 4.9 Fast Fourier Transform

```
#include <bits/stdc++.h>
#define MAXN 400005
#define MOD 1000000007
#define F first
#define S second
using namespace std;
typedef long long ll;
typedef pair<int,int> P;
const double PI=acos(-1.0);
namespace fft
{
    struct num
    {
        double x,y;
        num() {x=y=0;}
```

```
num(double x,double y):x(x),y(y){}
};
inline num operator+(num a,num b) {return num(a.x+b.x,a.y+b.y);}
inline num operator-(num a,num b) {return num(a.x-b.x,a.y-b.y);}
inline num operator*(num a,num b) {return num(a.x*b.x-a.y*b.y,a.x*b.y+a.y*b.x);}
inline num conj(num a) {return num(a.x,-a.y);}
int base=1;
vector<num> roots={{0,0},{1,0}};
vector<int> rev={0,1};
const double PI=acosl(-1.0);
void ensure_base(int nbase)
    if(nbase<=base) return;</pre>
   rev.resize(1<<nbase);</pre>
    for(int i=0;i<(1<<nbase);i++)</pre>
       rev[i]=(rev[i>>1]>>1)+((i&1)<<(nbase-1));
   roots.resize(1<<nbase);</pre>
    while(base<nbase)</pre>
    {
       double angle=2*PI/(1<<(base+1));</pre>
       for(int i=1<<(base-1);i<(1<<base);i++)</pre>
           roots[i<<1]=roots[i];</pre>
           double angle_i=angle*(2*i+1-(1<<base));</pre>
           roots[(i<<1)+1]=num(cos(angle_i),sin(angle_i));</pre>
       base++;
    }
}
void fft(vector<num> &a,int n=-1)
    if(n==-1) n=a.size();
    assert((n&(n-1))==0);
    int zeros=__builtin_ctz(n);
    ensure_base(zeros);
    int shift=base-zeros;
    for(int i=0;i<n;i++)</pre>
        if(i<(rev[i]>>shift))
           swap(a[i],a[rev[i]>>shift]);
   for(int k=1;k<n;k<<=1)</pre>
       for(int i=0;i<n;i+=2*k)</pre>
           for(int j=0;j<k;j++)</pre>
               num z=a[i+j+k]*roots[j+k];
               a[i+j+k]=a[i+j]-z;
               a[i+j]=a[i+j]+z;
           }
       }
    }
vector<num> fa,fb;
vector<int> multiply(vector<int> &a, vector<int> &b)
```

```
{
    int need=a.size()+b.size()-1;
   int nbase=0;
   while((1<<nbase)<need) nbase++;</pre>
    ensure_base(nbase);
    int sz=1<<nbase;</pre>
    if(sz>(int)fa.size()) fa.resize(sz);
   for(int i=0;i<sz;i++)</pre>
       int x=(i<(int)a.size()?a[i]:0);</pre>
       int y=(i<(int)b.size()?b[i]:0);</pre>
       fa[i]=num(x,y);
    }
   fft(fa,sz);
   num r(0,-0.25/sz);
   for(int i=0;i<=(sz>>1);i++)
       int j=(sz-i)&(sz-1);
       num z=(fa[j]*fa[j]-conj(fa[i]*fa[i]))*r;
       if(i!=j) fa[j]=(fa[i]*fa[i]-conj(fa[j]*fa[j]))*r;
       fa[i]=z;
   fft(fa,sz);
   vector<int> res(need);
   for(int i=0;i<need;i++) res[i]=fa[i].x+0.5;</pre>
   return res;
}
vector<int> multiply_mod(vector<int> &a,vector<int> &b,int m,int eq=0)
   int need=a.size()+b.size()-1;
   int nbase=0;
   while((1<<nbase)<need) nbase++;</pre>
    ensure_base(nbase);
   int sz=1<<nbase;</pre>
   if(sz>(int)fa.size()) fa.resize(sz);
   for(int i=0;i<(int)a.size();i++)</pre>
       int x=(a[i]%m+m)%m;
       fa[i]=num(x&((1<<15)-1),x>>15);
   fill(fa.begin()+a.size(),fa.begin()+sz,num{0,0});
    fft(fa,sz);
    if(sz>(int)fb.size()) fb.resize(sz);
    if(eq) copy(fa.begin(),fa.begin()+sz,fb.begin());
    else
       for(int i=0;i<(int)b.size();i++)</pre>
           int x=(b[i]\%m+m)\%m;
           fb[i]=num(x&((1<<15)-1),x>>15);
       fill(fb.begin()+b.size(),fb.begin()+sz,num{0,0});
       fft(fb,sz);
   double ratio=0.25/sz;
   num r2(0,-1),r3(ratio,0),r4(0,-ratio),r5(0,1);
    for(int i=0;i<=(sz>>1);i++)
    {
```

```
int j=(sz-i)&(sz-1);
           num a1=(fa[i]+conj(fa[j]));
           num a2=(fa[i]-conj(fa[j]))*r2;
           num b1=(fb[i]+conj(fb[j]))*r3;
           num b2=(fb[i]-conj(fb[j]))*r4;
           if(i!=j)
               num c1=(fa[j]+conj(fa[i]));
               num c2=(fa[j]-conj(fa[i]))*r2;
               num d1=(fb[j]+conj(fb[i]))*r3;
               num d2=(fb[j]-conj(fb[i]))*r4;
               fa[i]=c1*d1+c2*d2*r5;
               fb[i]=c1*d2+c2*d1;
           }
           fa[j]=a1*b1+a2*b2*r5;
           fb[j]=a1*b2+a2*b1;
       }
       fft(fa,sz);fft(fb,sz);
       vector<int> res(need);
       for(int i=0;i<need;i++)</pre>
           11 aa=fa[i].x+0.5;
           11 bb=fb[i].x+0.5;
           11 cc=fa[i].y+0.5;
           res[i]=(aa+((bb\m)<<15)+((cc\m)<<30))\mbox{m};
       }
       return res;
   }
   vector<int> square_mod(vector<int> &a,int m)
   {
       return multiply_mod(a,a,m,1);
   }
};
string s1,s2;
int main()
{
   cin>>s1;
   cin>>s2;
   int len1=(int)s1.size();
   vector<int> v1(len1);
   for(int i=0;i<len1;i++)</pre>
       v1[i]=(int)(s1[len1-1-i]-'0');
   int len2=(int)s2.size();
   vector<int> v2(len2);
   for(int i=0;i<len2;i++)</pre>
       v2[i]=(int)(s2[len2-1-i]-'0');
   vector<int> ans;
   ans=fft::multiply(v1,v2);
   int carry=0;
   for(int i=0;i<(int)ans.size();i++)</pre>
       carry+=ans[i];
       ans[i]=carry%10;
       carry/=10;
   while(carry>0)
       ans.push_back(carry%10);
       carry/=10;
```

```
}
while((int)ans.size()>1&&ans.back()==0) ans.pop_back();
for(int i=(int)ans.size()-1;i>=0;i--)
    printf("%d",ans[i]);
return 0;
}
```

#### 4.10 Fast Walsh-Hadamard Transform

```
#include<bits/stdc++.h>
#define MAXN 100005
#define INF 100000000
#define MOD 100000007
#define REV 50000004
#define F first
#define S second
using namespace std;
typedef long long 11;
typedef pair<int,int> P;
void FWT(int a[],int n)
   for(int d=1;d<n;d<<=1)</pre>
        for(int m=d<<1,i=0;i<n;i+=m)</pre>
           for(int j=0;j<d;j++)</pre>
           {
               int x=a[i+j],y=a[i+j+d];
               a[i+j]=(x+y)%MOD, a[i+j+d]=(x-y+MOD)%MOD;
               //and:a[i+j]=x+y;
               //or:a[i+j+d]=x+y;
           }
}
void UFWT(int a[],int n)
   for(int d=1;d<n;d<<=1)</pre>
       for(int m=d<<1,i=0;i<n;i+=m)</pre>
           for(int j=0;j<d;j++)</pre>
               int x=a[i+j],y=a[i+j+d];
               a[i+j]=1LL*(x+y)*REV%MOD,a[i+j+d]=(1LL*(x-y)*REV%MOD+MOD)%MOD;
               //and:a[i+j]=x-y;
               //or:a[i+j+d]=y-x;
void solve(int a[],int b[],int n)
   FWT(a,n);
   FWT(b,n);
   for(int i=0;i<n;i++) a[i]=1LL*a[i]*b[i]%MOD;</pre>
   UFWT(a,n);
}
int main()
{
   return 0;
```

#### 4.11 Gauss-Jordan

```
#include<bits/stdc++.h>
#define MAXN 105
using namespace std;
const double eps=1e-8;
typedef vector<double> vec;
typedef vector<vec> mat;
vec gauss_jordan(const mat& A, const vec& b)
    int n=A.size();
    mat B(n,vec(n+1));
    for(int i=0;i<n;i++)</pre>
        for(int j=0; j<n; j++)</pre>
            B[i][j]=A[i][j];
    for(int i=0;i<n;i++) B[i][n]=b[i];</pre>
    for(int i=0;i<n;i++)</pre>
        int pivot=i;
        for(int j=i;j<n;j++)</pre>
            if(abs(B[j][i])>abs(B[pivot][i])) pivot=j;
        swap(B[i],B[pivot]);
        if(abs(B[i][i]) < eps) return vec();</pre>
        for(int j=i+1;j<=n;j++) B[i][j]/=B[i][i];</pre>
        for(int j=0;j<n;j++)</pre>
            if(i!=j)
            {
                for(int k=i+1;k<=n;k++)</pre>
                    B[j][k] -= B[j][i] *B[i][k];
            }
        }
   }
    vec x(n);
    for(int i=0;i<n;i++)</pre>
        x[i]=B[i][n];
   return x;
}
int main()
    scanf("%d", &sz);
   mat A(sz,vec(sz));
    vec b(sz);
    for(int i=0;i<sz;i++)</pre>
        for(int j=0;j<sz;j++)</pre>
            A[i][j]=0;
    for(int i=0;i<sz;i++)</pre>
    {
        double x;
        int cnt=0;
        while (scanf("%lf",&x)==1)
            if(x==-1) break;
```

```
A[x-1][i]=1.0;
}

for(int i=0;i<sz;i++)
    b[i]=1.0;
vec res=gauss_jordan(A,b);
if(res==vec()) printf("No solution\n");
else
{
    for(int i=0;i<sz;i++)
        if(res[i]>0) printf("%d ",i+1);
    printf("\n");
}
return 0;
}
```

### 4.12 Linear Basis

```
#include<bits/stdc++.h>
#define MAXN 1000
using namespace std;
int a[MAXN],bas[62];
int n;
int main()
    for(int i=1;i<=n;i++)</pre>
        int x=a[i];
        for(int j=60;j>=0;j--)
           if(x&(111<<j))</pre>
           {
               if(!bas[j])
                   bas[j]=x;
                   break;
               x^=bas[j];
           }
       }
   }
}
```

# 4.13 Linear Congruence

```
#include<bits/stdc++.h>
#define MAXN 10000
using namespace std;
pair<int,int> linear_congruence(const vector<int>&A, const vector<int>&B, const vector<int>&M)
{
   int x=0,m=1;
   for(int i=0;i<A.size();i++)
   {
      int a=A[i]*m,b=B[i]-A[i]*x,d=gcd(M[i],a);
      if(b%d!=0) return make_pair(0,-1);</pre>
```

#### 4.14 Linear Recurrence

```
// Calculating kth term of linear recurrence sequence
// Complexity: init O(n^2log) query O(n^2logk)
// Requirement: const LOG const MOD
// Input(constructor): vector<int> - first n terms
                     vector<int> - transition function
//
// Output(calc(k)): int - the kth term mod MOD
// Example: In: \{1, 1\} \{2, 1\} an = 2an-1 + an-2
11
                      Out: calc(3) = 3, calc(10007) = 71480733 (MOD 1e9+7)
struct LinearRec {
       int n;
       vector<int> first, trans;
       vector<vector<int> > bin;
       vector<int> add(vector<int> &a, vector<int> &b) {
              vector < int > result(n * 2 + 1, 0);
              //You can apply constant optimization here to get a ~10x speedup
              for (int i = 0; i <= n; ++i) {</pre>
                      for (int j = 0; j <= n; ++j) {</pre>
                             if ((result[i + j] += (long long)a[i] * b[j] % MOD) >= MOD) {
                                     result[i + j] -= MOD;
                             }
                      }
              }
              for (int i = 2 * n; i > n; --i) {
                      for (int j = 0; j < n; ++j) {
                             if ((result[i - 1 - j] += (long long)result[i] * trans[j] % MOD) >=
                                     result[i - 1 - j] -= MOD;
                      }
                      result[i] = 0;
              result.erase(result.begin() + n + 1, result.end());
              return result;
       }
       LinearRec(vector<int> &first, vector<int> &trans):first(first), trans(trans) {
              n = first.size();
              vector < int > a(n + 1, 0);
              a[1] = 1;
              bin.push_back(a);
              for (int i = 1; i < LOG; ++i) {</pre>
                      bin.push_back(add(bin[i - 1], bin[i - 1]));
              }
       }
```

```
int calc(int k) {
               vector < int > a(n + 1, 0);
               a[0] = 1;
               for (int i = 0; i < LOG; ++i) {</pre>
                       if (k >> i & 1) {
                               a = add(a, bin[i]);
                       }
               }
               int ret = 0;
               for (int i = 0; i < n; ++i) {</pre>
                       if ((ret += (long long)a[i + 1] * first[i] % MOD) >= MOD) {
                               ret -= MOD;
               }
               return ret;
       }
};
```

# 4.15 LU Decomposition

```
#include<bits/stdc++.h>
#define MAXN 1000
using namespace std;
typedef vector<double> vec;
typedef vector<vec> mat;
typedef long long 11;
int n;
mat mul(mat A,mat B)
   mat C(A.size(),vec(B[0].size()));
   for(int i=0;i<A.size();i++)</pre>
        for(int k=0;k<B.size();k++)</pre>
           for(int j=0; j<B[0].size(); j++)</pre>
               C[i][j]=(C[i][j]+A[i][k]*B[k][j]);
   return C;
}
mat pow(mat A,ll n)
   mat B(A.size(),vec(A.size()));
   for(int i=0;i<A.size();i++)</pre>
       B[i][i]=1;
   while(n>0)
        if(n&1) B=mul(B,A);
       A=mul(A,A);
       n>>=1;
   }
   return B;
int main()
   scanf("%d",&n);
   mat A(n,vec(n));
   for(int i=0;i<n;i++)</pre>
        for(int j=0;j<n;j++)</pre>
           scanf("%lf",&A[i][j]);
   mat L(n,vec(n));
```

```
mat U(n,vec(n));
    for(int i=1;i<n;i++)</pre>
        for(int j=0;j<i;j++)</pre>
            U[i][j]=0;
    for(int i=0;i<n;i++)</pre>
        L[i][i]=1;
    for(int i=0;i<n;i++)</pre>
        for(int j=i+1;j<n;j++)</pre>
            L[i][j]=0;
    for(int i=0;i<n;i++)</pre>
        U[i][i]=A[i][i];
        for(int j=i+1; j<n; j++)</pre>
            L[j][i]=A[j][i]/U[i][i];
            U[i][j]=A[i][j];
        }
        for(int j=i+1; j<n; j++)</pre>
            for(int k=i+1;k<n;k++)</pre>
                 A[j][k]=A[j][k]-L[j][i]*U[i][k];
    printf("L=\n");
    for(int i=0;i<n;i++)</pre>
        for(int j=0;j<n;j++)</pre>
            printf("%6lf ",L[i][j]);
        printf("\n");
    printf("U=\n");
    for(int i=0;i<n;i++)</pre>
        for(int j=0;j<n;j++)</pre>
            printf("%6lf ",U[i][j]);
        printf("\n");
    }
}
```

# 4.16 Matrix Operations

```
#include<bits/stdc++.h>
#define MAXN 1000
using namespace std;
typedef vector<double> vec;
typedef vector<vec> mat;
typedef long long 11;
int n;
mat mul(mat A,mat B)
   mat C(A.size(),vec(B[0].size()));
   for(int i=0;i<A.size();i++)</pre>
        for(int k=0;k<B.size();k++)</pre>
           for(int j=0;j<B[0].size();j++)</pre>
               C[i][j]=(C[i][j]+A[i][k]*B[k][j]);
   return C;
}
mat pow(mat A,11 n)
{
```

```
mat B(A.size(),vec(A.size()));
    for(int i=0;i<A.size();i++)</pre>
        B[i][i]=1;
    while(n>0)
    {
        if(n&1) B=mul(B,A);
        A=mul(A,A);
        n >> = 1;
    return B;
}
int main()
    scanf("%d",&n);
    mat A(n,vec(n));
    for(int i=0;i<n;i++)</pre>
        for(int j=0;j<n;j++)</pre>
            scanf("%lf",&A[i][j]);
    mat L(n,vec(n));
    mat U(n,vec(n));
    for(int i=1;i<n;i++)</pre>
        for(int j=0;j<i;j++)</pre>
            U[i][j]=0;
    for(int i=0;i<n;i++)</pre>
        L[i][i]=1;
    for(int i=0;i<n;i++)</pre>
        for(int j=i+1; j<n; j++)</pre>
            L[i][j]=0;
    for(int i=0;i<n;i++)</pre>
        U[i][i]=A[i][i];
        for(int j=i+1; j<n; j++)</pre>
            L[j][i]=A[j][i]/U[i][i];
            U[i][j]=A[i][j];
        for(int j=i+1;j<n;j++)</pre>
            for(int k=i+1;k<n;k++)</pre>
                A[j][k]=A[j][k]-L[j][i]*U[i][k];
    printf("L=\n");
    for(int i=0;i<n;i++)</pre>
        for(int j=0;j<n;j++)</pre>
            printf("%6lf ",L[i][j]);
        printf("\n");
    printf("U=\n");
    for(int i=0;i<n;i++)</pre>
        for(int j=0; j<n; j++)</pre>
            printf("%6lf ",U[i][j]);
        printf("\n");
    }
}
```

# 4.17 Miller-Rabin primality test

```
#include<bits/stdc++.h>
using namespace std;
int pow_mod(int a,int i,int n)
   if(i==0) return 1%n;
   int temp=pow_mod(a,i>>1,n);
     temp=temp*temp%n;
   if(i&1) temp=(long long) temp*a%n;
   return temp;
}
bool test(int n,int a,int d)
   if(n==2) return true;
   if(n==a) return true;
   if((n&1)==0) return false;
   while(!(d&1)) d=d>>1;
   int t=pow_mod(a,d,n);
   while ((d!=n-1)\&\&(t!=1)\&\&(t!=n-1))
       t=(long long)t*t%n;
       d=d<<1;
   }
   return(t==n-1||(d&1)==1);
}
bool isPrime(int n)
   if(n<2) return false;</pre>
   int a[]={2,3,61};
   for(int i=0;i<=2;++i) if(!test(n,a[i],n-1)) return false;</pre>
   return true;
}
int main()
{
   return 0;
}
```

### 4.18 Mod-Combinatation and Mod-fact

```
#include<bits/stdc++.h>
#define MAXN 100000
#define MAXP 1005
using namespace std;
int gcd(int a,int b)
{
    if(b==0) return a;
    return gcd(b,a%b);
}
int extgcd(int a,int b,int &x,int &y)
{
    int d=a;
    if(b!=0)
    {
        d=extgcd(b,a%b,y,x);
        y-=(a/b)*x;
}
```

```
}
   else
   {
       x=1;
       y=0;
   }
   return d;
}
int mod_inverse(int a,int m)
{
   int x,y;
   extgcd(a,m,x,y);
   return (m+x%m)%m;
}
int fact[MAXP];
int mod_fact(int n,int p,int &e)
{
   e=0;
   if(n==0) return 1;
   int res=mod_fact(n/p,p,e);
   e+=n/p;
   if(n/p%2!=0) return res*(p-fact[n%p])%p;
   return res*fact[n%p]%p;
}
int mod_comb(int n,int k,int p)
   if(n<0||k<0||n<k) return 0;</pre>
   int e1,e2,e3;
   int a1=mod_fact(n,p,e1),a2=mod_fact(k,p,e2),a3=mod_fact(n-k,p,e3);
   if(e1>e2+e3) return 0;
   return a1*mod_inverse(a2*a3%p,p)%p;
}
int main()
   inv[1] = 1;
       for (int i = 2; i < MOD; i++)</pre>
               inv[i] = (MOD - MOD / i) * inv[ MOD % i] % MOD;
   printf("%d\n",mod_inverse(22,31));
   return 0;
}
```

### 4.19 Fast Number-Theoretic Transform

```
#include<bits/stdc++.h>
#define MAXN 100005
#define MOD 998244353
#define INF 1000000000
#define F first
#define S second
using namespace std;
typedef long long l1;
typedef pair<int,int> P;
const int g=3;
int two[31];
int dbit(int x)
{
    while(x!=(x&-x)) x+=(x&-x);
```

```
return x;
int pow_mod(int a,int i)
    if(i==0) return 1;
    int s=1;
    while(i>0)
        if(i&1) s=(1LL*s*a)%MOD;
        a=(1LL*a*a)%MOD;
        i>>=1;
    }
    return s;
}
int rev(int x,int r)
{
    int ans=0;
    for(int i=0;i<r;i++)</pre>
        if(x&(1<<i)) ans+=1<<(r-i-1);
    return ans;
void ntt(int n,int A[],int on)
    int r=0,cnt=0,t=n;
    while(t>1) {cnt++; t/=2;}
    for(;;r++) if((1<<r)==n) break;</pre>
    for(int i=0;i<n;i++)</pre>
        int tmp=rev(i,r);
        if(i<tmp) swap(A[i],A[tmp]);</pre>
   }
    for(int s=1;s<=r;s++)</pre>
        int m=1<<s;</pre>
        int wn=pow_mod(g,(MOD-1)/m);
       for(int k=0;k<n;k+=m)</pre>
           int w=1;
           for(int j=0;j<m/2;j++)</pre>
               int t,u;
                t=1LL*w*A[k+j+m/2]%MOD;
               u=A[k+j];
                A[k+j]=(u+t);
                if(A[k+j]>=MOD) A[k+j]-=MOD;
                A[k+j+m/2]=u+MOD-t;
                if(A[k+j+m/2]>=MOD) A[k+j+m/2]-=MOD;
                w=1LL*w*wn%MOD;
           }
       }
   }
    if (on==-1)
        for(int i=1;i<n/2;i++)</pre>
           swap(A[i],A[n-i]);
        for(int i=0;i<n;i++)</pre>
           A[i]=1LL*A[i]*two[cnt]%MOD;
   }
}
```

```
int A[MAXN],B[MAXN],ans[MAXN];
int main()
   int n,m;
   for(int i=1;i<=30;i++)</pre>
       two[i]=pow_mod(1<<i,MOD-2);</pre>
   string s1;
   string s2;
   while(cin>>s1>>s2)
       n=s1.size();
       m=s2.size();
       memset(A,0,sizeof(A));
       memset(B,0,sizeof(B));
       for(int i=n-1; i>=0 ; i--)
           A[i]=s1[n-i-1]-'0';
       for(int i=m-1; i>=0; i--)
           B[i]=s2[m-i-1]-'0';
       int tmp=1;
       while(tmp<max(n,m))</pre>
           tmp*=2;
       n=tmp;
       ntt(2*n,A,1);
       ntt(2*n,B,1);
       for(int i=0; i<2*n; i++)</pre>
           A[i]=1LL*A[i]*B[i]%MOD;
       ntt(2*n,A,-1);
       memset(ans,0,sizeof ans);
       for(int i=0;i<2*n;i++)</pre>
           ans[i]+=A[i];
           if(ans[i]>=10)
               ans[i+1]+=ans[i]/10;
               ans[i]%=10;
           }
       }
       int e=0;
       for(int i=2*n-1;i>=0;i--)
           if(ans[i])
           {
               e=i;
               break;
           }
       }
       for(int i=e;i>=0;i--)
           printf("%d",ans[i]);
       }
       printf("\n");
   }
   return 0;
```

# 4.20 Pell's equation

```
#include<bits/stdc++.h>
#define MAXN 10005
#define F first
#define S second
using namespace std;
typedef pair<int,int> P;
P Pell(int N)
       int p0=0,p1=1,q0=1,q1=0;
       int a0=(int)sqrt(N),a1=a0,a2=a0;
       if(a0*a0==N) return P(-1,-1);
       int g1=0,h1=1;
       while(true)
       {
              int g2=-g1+a1*h1;
              int h2=(N-g2*g2)/h1;
              a2=(g2+a0)/h2;
              int p2=a1*p1+p0;
              int q2=a1*q1+q0;
              if(p2*p2-N*q2*q2==1) return P(p2,q2);
              a1=a2;g1=g2;h1=h2;p0=p1;p1=p2;q0=q1;q1=q2;
       }
}
int main()
{
       int n;
       while (scanf("%d",&n)==1)
       {
              P p=Pell(n);
              printf("%d %d\n",p.F,p.S);
       }
       return 0;
}
```

### 4.21 Pollard-Rho

```
#include<stdio.h>
#include<string.h>
#include<stdlib.h>
#include<time.h>
#include<iostream>
#include<algorithm>
using namespace std;
// Miller_Rabin
                   <2^63
const int S=20;//
                           S
//
     (a*b)%c. a, blong
                               long
// a,b,c <2^63
long long mult_mod(long long a,long long b,long long c)
{
```

```
a%=c;
   b%=c;
   long long ret=0;
   while(b)
       if(b&1){ret+=a;ret%=c;}
       a<<=1;
       if(a>=c)a%=c;
       b>>=1;
   }
   return ret;
}
        x^n %c
long long pow_mod(long long x,long long n,long long mod)//x^n%c
   if(n==1)return x%mod;
   x\%=mod;
   long long tmp=x;
   long long ret=1;
   while(n)
       if(n&1) ret=mult_mod(ret,tmp,mod);
       tmp=mult_mod(tmp,tmp,mod);
       n>>=1;
   }
   return ret;
}
//
        ,n-1=x*2^t a^(n-1)=1 \pmod{n}
//
                        , false
          true
bool check(long long a, long long n, long long x, long long t)
   long long ret=pow_mod(a,x,n);
   long long last=ret;
   for(int i=1;i<=t;i++)</pre>
       ret=mult_mod(ret,ret,n);
       if(ret==1&&last!=1&&last!=n-1) return true;//
       last=ret;
   if(ret!=1) return true;
   return false;
}
// Miller_Rabin()
       true
                                                     )
              . (
//
      false
bool Miller_Rabin(long long n)
   if(n<2)return false;</pre>
   if(n==2)return true;
```

```
if((n&1)==0) return false;//
   long long x=n-1;
   long long t=0;
   while((x\&1)==0){x>>=1;t++;}
   for(int i=0;i<S;i++)</pre>
       long long a=rand()%(n-1)+1;//rand() stdlib .
       if(check(a,n,x,t))
          return false;//
   }
   return true;
}
//***************
//pollard_rho
//***************
long long factor[100];//
int tol;//
long long gcd(long long a,long long b)
   if(a==0)return 1;//???????
   if(a<0) return gcd(-a,b);</pre>
   while(b)
       long long t=a%b;
       a=b;
      b=t;
   }
   return a;
}
long long Pollard_rho(long long x,long long c)
   long long i=1,k=2;
   long long x0=rand()%x;
   long long y=x0;
   while(1)
   {
       i++;
       x0=(mult_mod(x0,x0,x)+c)%x;
       long long d=gcd(y-x0,x);
       if(d!=1&&d!=x) return d;
       if(y==x0) return x;
       if(i==k){y=x0;k+=k;}
}
//
void findfac(long long n)
   if(Miller_Rabin(n))//
       factor[tol++]=n;
      return;
   }
   long long p=n;
   while(p>=n)p=Pollard_rho(p,rand()%(n-1)+1);
   findfac(p);
```

# 4.22 Polynomial Operations

```
#include<bits/stdc++.h>
#define MAXN 100005
#define INF 100000000
#define MOD 998244353
#define F first
#define S second
using namespace std;
typedef long long 11;
typedef pair<int,int> P;
inline int inc(int a,int b) {a+=b; return a>=MOD?a-MOD:a;}
inline int dec(int a,int b) {a-=b; return a<0?a+MOD:a;}</pre>
int pow_mod(int a,int i,int m)
   int s=1;
   while(i)
       if(i&1) s=1LL*s*a%m;
       a=1LL*a*a%m;
       i>>=1;
   }
   return s;
11 Tonelli_Shanks(int n,ll p)
   if(p==2) return (n&1)?1:-1;
   if(pow_mod(n,p>>1,p)!=1) return -1;
   if(p&2) return pow_mod(n,(p+1)>>2,p);
   ll s=__builtin_ctzll(p^1);
   11 q=p>>s,z=2;
   for(;pow_mod(z,p>>1,p)==1;++z);
   11 c=pow_mod(z,q,p),r=pow_mod(n,(q+1)>>1,p),t=pow_mod(n,q,p),tmp;
   for(ll m=s,i;t!=1;)
       for(i=0,tmp=t;tmp!=1;++i) tmp=tmp*tmp%p;
       for(;i<--m;) c=c*c%p;</pre>
       r=r*c%p;c=c*c%p;t=t*c%p;
```

```
}
    return r;
namespace fft
    struct num
        double x,y;
       num() \{x=y=0;\}
       num(double x,double y):x(x),y(y){}
    };
    inline num operator+(num a,num b) {return num(a.x+b.x,a.y+b.y);}
    inline num operator-(num a,num b) {return num(a.x-b.x,a.y-b.y);}
    inline num operator*(num a,num b) {return num(a.x*b.x-a.y*b.y,a.x*b.y+a.y*b.x);}
    inline num conj(num a) {return num(a.x,-a.y);}
    int base=1;
    vector<num> roots={{0,0},{1,0}};
    vector<int> rev={0,1};
    const double PI=acosl(-1.0);
    void ensure_base(int nbase)
    {
        if(nbase<=base) return;</pre>
       rev.resize(1<<nbase);</pre>
        for(int i=0;i<(1<<nbase);i++)</pre>
            rev[i]=(rev[i>>1]>>1)+((i&1)<<(nbase-1));
       roots.resize(1<<nbase);</pre>
        while(base<nbase)</pre>
            double angle=2*PI/(1<<(base+1));</pre>
            for(int i=1<<(base-1);i<(1<<base);i++)</pre>
               roots[i<<1]=roots[i];</pre>
               double angle_i=angle*(2*i+1-(1<<base));</pre>
               roots[(i<<1)+1]=num(cos(angle_i),sin(angle_i));</pre>
           }
           base++;
       }
    }
    void fft(vector<num> &a,int n=-1)
        if(n==-1) n=a.size();
        assert((n&(n-1))==0);
        int zeros=__builtin_ctz(n);
        ensure_base(zeros);
        int shift=base-zeros;
        for(int i=0;i<n;i++)</pre>
            if(i<(rev[i]>>shift))
               swap(a[i],a[rev[i]>>shift]);
        for(int k=1;k<n;k<<=1)</pre>
            for(int i=0;i<n;i+=2*k)</pre>
               for(int j=0;j<k;j++)</pre>
                   num z=a[i+j+k]*roots[j+k];
                   a[i+j+k]=a[i+j]-z;
```

```
a[i+j]=a[i+j]+z;
           }
       }
   }
}
vector<num> fa,fb;
vector<int> multiply(vector<int> &a, vector<int> &b)
   int need=a.size()+b.size()-1;
   int nbase=0;
   while((1<<nbase)<need) nbase++;</pre>
    ensure_base(nbase);
    int sz=1<<nbase;</pre>
   if(sz>(int)fa.size()) fa.resize(sz);
   for(int i=0;i<sz;i++)</pre>
       int x=(i<(int)a.size()?a[i]:0);</pre>
       int y=(i<(int)b.size()?b[i]:0);</pre>
       fa[i]=num(x,y);
   fft(fa,sz);
   num r(0,-0.25/sz);
   for(int i=0;i<=(sz>>1);i++)
       int j=(sz-i)&(sz-1);
       num z=(fa[j]*fa[j]-conj(fa[i]*fa[i]))*r;
       if(i!=j) fa[j]=(fa[i]*fa[i]-conj(fa[j]*fa[j]))*r;
       fa[i]=z;
    }
   fft(fa,sz);
   vector<int> res(need);
   for(int i=0;i<need;i++) res[i]=fa[i].x+0.5;</pre>
   return res;
}
vector<int> multiply_mod(vector<int> &a,vector<int> &b,int m,int eq=0)
    int need=a.size()+b.size()-1;
   int nbase=0;
   while((1<<nbase)<need) nbase++;</pre>
   ensure_base(nbase);
    int sz=1<<nbase;</pre>
   if(sz>(int)fa.size()) fa.resize(sz);
   for(int i=0;i<(int)a.size();i++)</pre>
       int x=(a[i]%m+m)%m;
       fa[i]=num(x&((1<<15)-1),x>>15);
    }
   fill(fa.begin()+a.size(),fa.begin()+sz,num{0,0});
    fft(fa,sz);
    if(sz>(int)fb.size()) fb.resize(sz);
    if(eq) copy(fa.begin(),fa.begin()+sz,fb.begin());
    else
    {
       for(int i=0;i<(int)b.size();i++)</pre>
           int x=(b[i]\%m+m)\%m;
```

```
fb[i]=num(x&((1<<15)-1),x>>15);
           fill(fb.begin()+b.size(),fb.begin()+sz,num{0,0});
           fft(fb,sz);
       }
       double ratio=0.25/sz;
       num r2(0,-1), r3(ratio,0), r4(0,-ratio), r5(0,1);
       for(int i=0;i<=(sz>>1);i++)
           int j=(sz-i)&(sz-1);
           num a1=(fa[i]+conj(fa[j]));
           num a2=(fa[i]-conj(fa[j]))*r2;
           num b1=(fb[i]+conj(fb[j]))*r3;
           num b2=(fb[i]-conj(fb[j]))*r4;
           if(i!=j)
           {
               num c1=(fa[j]+conj(fa[i]));
               num c2=(fa[j]-conj(fa[i]))*r2;
               num d1=(fb[j]+conj(fb[i]))*r3;
              num d2=(fb[j]-conj(fb[i]))*r4;
               fa[i]=c1*d1+c2*d2*r5;
               fb[i]=c1*d2+c2*d1;
           }
           fa[j]=a1*b1+a2*b2*r5;
           fb[j]=a1*b2+a2*b1;
       }
       fft(fa,sz);fft(fb,sz);
       vector<int> res(need);
       for(int i=0;i<need;i++)</pre>
           ll aa=fa[i].x+0.5;
           11 bb=fb[i].x+0.5;
           11 cc=fa[i].y+0.5;
           res[i]=(aa+((bb\%m)<<15)+((cc\%m)<<30))\%m;
       }
       return res;
   }
   vector<int> square_mod(vector<int> &a,int m)
       return multiply_mod(a,a,m,1);
   }
};
namespace poly
   int inv(int x) {return pow_mod(x,MOD-2,MOD);}
   vector<int> fa,fb,fc,fd,fe,ff,fg,Inv;
   void get_inv(vector<int> &a,int n,vector<int> &ret)
       assert(a[0]!=0);
       if (n==1)
           ret.resize(1);
           ret[0]=inv(a[0]);
           return;
       get_inv(a,(n+1)>>1,ret);
       fa=a; fb=ret;
       fa=fft::multiply_mod(fb,fb,MOD,1);
       fa=fft::multiply_mod(fa,a,MOD);
```

```
fa.resize(n); fb.resize(n); ret.resize(n);
   for(int i=0:i<n:i++)</pre>
       ret[i]=inc(fb[i],fb[i]);
       ret[i]=dec(ret[i],fa[i]);
   }
   fa.clear(); fb.clear();
}
void get_sqrt(vector<int> &a,int n,vector<int> &ret)
   if(n==1)
       ret.resize(1);
       int x=Tonelli_Shanks(a[0],MOD);
       assert(x!=-1);
       ret[0]=x;
       return;
   }
   get_sqrt(a,(n+1)>>1,ret);
   get_inv(ret,n,fc);
   ret=fft::multiply_mod(ret,ret,MOD,1);
   ret.resize(n);
   for(int i=0;i<n;i++) fc[i]=1LL*fc[i]*((MOD+1)/2)%MOD;</pre>
   for(int i=0;i<n;i++) ret[i]=inc(ret[i],a[i]);</pre>
   ret=fft::multiply_mod(ret,fc,MOD);
   ret.resize(n);
}
void diff(vector<int> &a,int n,vector<int> &ret)
   ret.resize(n);
   for(int i=1;i<n;i++) ret[i-1]=1LL*a[i]*i%MOD;</pre>
   ret[n-1]=0;
}
void intg(vector<int> &a,int n,vector<int> &ret)
   ret.resize(n); Inv.resize(n);
   if(n>1) Inv[1]=1;
   for(int i=2;i<=n-1;i++) Inv[i]=dec(MOD,1LL*Inv[MOD%i]*(MOD/i)%MOD);</pre>
   for(int i=n-1;i>=1;i--) ret[i]=1LL*a[i-1]*Inv[i]%MOD;
   ret[0]=0;
}
void get_ln(vector<int> &a,int n,vector<int> &ret)
   assert(a[0]==1);
   diff(a,n,fc);
   get_inv(a,n,fd);
   fc=fft::multiply_mod(fc,fd,MOD);
   intg(fc,n,ret);
   ret.resize(n);
   fc.clear(); fd.clear();
void get_exp(vector<int> &a,int n,vector<int> &ret)
   if(n==1)
       ret.resize(1); ret[0]=1;
       return;
   }
   get_exp(a,(n+1)>>1,ret); ret.resize(n);
```

```
get_ln(ret,n,ff);
       for(int i=0;i<n;i++) ff[i]=dec(MOD,ff[i]);</pre>
       ff[0]+=1; if(ff[0]>=MOD) ff[0]-=MOD;
       for(int i=0;i<n;i++) ff[i]=inc(ff[i],a[i]);</pre>
       ret=fft::multiply_mod(ret,ff,MOD); ret.resize(n);
       ff.clear();
   }
   void division(vector<int> &a,vector<int> &b,vector<int> &q,vector<int> &r)
       int n=(int)a.size(),m=(int)b.size();
       if(n<m) {q.resize(1); q[0]=0; r=a; return;}</pre>
       vector<int> tmp=b; reverse(tmp.begin(),tmp.end());
       get_inv(tmp,n-m+1,tmp);
       vector<int> rev=a; reverse(rev.begin(),rev.end());
       q=fft::multiply_mod(tmp,rev,MOD); q.resize(n-m+1);
       reverse(q.begin(),q.end());
       vector<int> t=fft::multiply_mod(b,q,MOD);
       r.resize(m-1);
       for(int i=0;i<m-1;i++) r[i]=dec(a[i],t[i]);</pre>
   }
}
int n,m,k;
vector<int> f,g,a,b;
int main()
   scanf("%d",&n);
   f.resize(n);
   for(int i=0;i<n;i++) scanf("%d",&f[i]);</pre>
   vector<int> expf;
   poly::get_exp(f,n,expf);
   for(int i=0;i<n;i++) printf("%d ",expf[i]);</pre>
   return 0;
}
```

### 4.23 Polynomial Summations

```
#include<bits/stdc++.h>
#define MAXN 100005
#define INF 100000000
#define MOD 1000000007
#define F first
#define S second
using namespace std;
typedef long long 11;
typedef pair<int,int> P;
ll pow_mod(ll a,ll i)
   ll s=1;
   while(i)
       if(i&1) s=s*a%MOD;
       a=a*a%MOD;
       i>>=1;
   }
   return s;
ll gcd(ll a,ll b)
```

```
{
   if(b==0) return a:
   return gcd(b,a%b);
}
namespace polysum
   const int D=100005;
   ll a[D],f[D],g[D],p[D],p1[D],p2[D],b[D],h[D][2],C[D];
   ll calcn(int d,ll *a,ll n)
       if(n<=d) return a[n];</pre>
       p1[0]=p2[0]=1;
       for(int i=0;i<=d;i++)</pre>
          11 t=(n-i+MOD)\%MOD;
          p1[i+1]=p1[i]*t%MOD;
       }
       for(int i=0;i<=d;i++)</pre>
          11 t=(n-d+i+MOD)\%MOD;
          p2[i+1]=p2[i]*t%MOD;
       ll ans=0;
       for(int i=0;i<=d;i++)</pre>
          11 t=g[i]*g[d-i]%MOD*p1[i]%MOD*p2[d-i]%MOD*a[i]%MOD;
          if((d-i)&1) ans=(ans-t+MOD)%MOD;
          else ans=(ans+t)%MOD;
       }
       return ans;
   }
   void init(int M)
       f[0]=f[1]=g[0]=g[1]=1;
       for(int i=2;i<=M+4;i++) f[i]=f[i-1]*i%MOD;</pre>
       g[M+4] = pow_mod(f[M+4], MOD-2);
       for(int i=M+3;i>=1;i--) g[i]=g[i+1]*(i+1)%MOD;
   }
   ll polysum(ll n,ll *a,ll m) //a[0]..a[m] \sum_{i=0}^{n-1} a[i]
       a[m+1] = calcn(m,a,m+1);
       for(int i=1;i<=m+1;i++) a[i]=(a[i-1]+a[i])%MOD;</pre>
       return calcn(m+1,a,n-1);
   }
   if(R==1) return polysum(n,a,m);
       a[m+1]=calcn(m,a,m+1);
       11 r=pow_mod(R,MOD-2),p3=0,p4=0,c,ans;
       h[0][0]=0;h[0][1]=1;
       for(int i=1;i<=m+1;i++)</pre>
          h[i][0]=(h[i-1][0]+a[i-1])*r%MOD;
          h[i][1]=h[i-1][1]*r%MOD;
       for(int i=0;i<=m+1;i++)</pre>
          11 t=g[i]*g[m+1-i]%MOD;
          if(i&1) p3=((p3-h[i][0]*t)%MOD+MOD)%MOD,p4=((p4-h[i][1]*t)%MOD+MOD)%MOD;
```

# 4.24 Prime Counting Function

```
#include<bits/stdc++.h>
#define MAXN 1000005// MAXN=sqrt(upper_bound)
#define INF 100000000
#define MOD 100000007
#define F first
#define S second
using namespace std;
typedef long long 11;
typedef pair<int,int> P;
11 f[MAXN],g[MAXN],n,k; //f[i]:pi(n/i),g[i]:pi(i)
11 PrimeCount(11 n)
   ll i,j,m=0;
   for (m=1; m*m \le n; m++) f [m]=n/m-1;
   for(i=2;i<=m;i++) g[i]=i-1;</pre>
   for(i=2;i<=m;i++)</pre>
       if(g[i]==g[i-1]) continue;
       for(j=1; j<=min(m-1,n/i/i);++j)</pre>
           if(i*j<m) f[j]-=f[i*j]-g[i-1];</pre>
           else f[j]-=g[n/i/j]-g[i-1];
       for(j=m;j>=i*i;j--) g[j]-=g[j/i]-g[i-1];
   return f[1];
}
int main()
   while(scanf("%lld",&n)==1)
       printf("%lld\n",PrimeCount(n));
   return 0;
```

### 4.25 Primitive Root

```
#include<cstdio>
#include<cmath>
#include<iostream>
#include<cstdlib>
#include<cstring>
#include<algorithm>
#include<vector>
#include<queue>
#include<deque>
#include<stack>
#include<map>
#define MAXN 1005000
using namespace std;
typedef long long 11;
vector<11> a;
ll pow_mod(ll a,ll i,ll mod)
{
   if(i==0) return 1;
    ll s=1;
   while(i>0)
    {
        if(i&1) s=(s*a)%mod;
        a=(a*a)\mbox{mod};
        i>>=1;
    }
    return s;
bool g_test(ll g,ll p)
   for(ll i=0;i<a.size();i++)</pre>
       if (pow_mod(g,(p-1)/a[i],p)==1)
           return 0;
   return 1;
}
ll primitive_root(ll p)
   ll tmp=p-1;
   for(11 i=2;i<=tmp/i;i++)</pre>
       if(tmp%i==0)
           a.push_back(i);
           while(tmp%i==0)
               tmp/=i;
       }
   if(tmp!=1)
   {
       a.push_back(tmp);
   ll g=1;
   while(true)
       if(g_test(g,p))
           return g;
```

```
++g;
}
int main()
{
    ll n;
    while(scanf("%lld",&n)==1)
        printf("%lld\n",primitive_root(n));
    return 0;
}
```

# 4.26 Segmented Sieve

```
#include<bits/stdc++.h>
#define MAXL 1000005
#define MAXSQRTB 47000
#define INF 100000000
using namespace std;
typedef long long 11;
bool is_prime_small[MAXSQRTB];
bool is_prime[MAXL];
vector<ll> prime;
void segment_sieve(ll a,ll b)
   for(ll i=0;(ll)i*i<=b;i++) is_prime_small[i]=true;</pre>
   for(ll i=0;i<b-a;i++) is_prime[i]=true;</pre>
   for(11 i=2;(11)i*i<=b;i++)</pre>
        if(is_prime_small[i])
           for(ll j=2*i;(ll)j*j<=b;j+=i) is_prime_small[j]=false;</pre>
           for(ll j=max(2LL,(a+i-1)/i)*i;j<b;j+=i) is_prime[j-a]=false;</pre>
   }
   for(ll i=0;i<b-a;i++)</pre>
        if(is_prime[i]&&a+i!=1) prime.push_back(a+i);
}
```

# 4.27 Stirling number of the first kind

```
#include<bits/stdc++.h>
#define MAXN 500005
#define MOD 998244353
#define INF 1000000000
#define F first
#define S second
using namespace std;
typedef long long 11;
typedef pair<int,int> P;
const int g=3;
int tot=1;
int dbit(int x)
{
   while((x&-x)!=x) x+=x&-x;
   return x;
```

```
int two[32];
int pow_mod(int a,int i)
    if(i==0) return 1;
    int s=1;
    while(i>0)
        if(i&1) s=(1LL*s*a)%MOD;
        a=(1LL*a*a)%MOD;
        i>>=1;
    }
    return s;
}
int rev(int x,int r)
{
    int ans=0;
    for(int i=0;i<r;i++)</pre>
        if(x&(1<<i)) ans+=1<<(r-i-1);
    return ans;
void ntt(int n,int A[],int on)
    int r=0,cnt=0,t=n;
    while(t>1) {cnt++; t/=2;}
    for(;;r++) if((1<<r)==n) break;</pre>
    for(int i=0;i<n;i++)</pre>
        int tmp=rev(i,r);
        if(i<tmp) swap(A[i],A[tmp]);</pre>
   }
    for(int s=1;s<=r;s++)</pre>
        int m=1<<s;</pre>
        int wn=pow_mod(g,(MOD-1)/m);
       for(int k=0;k<n;k+=m)</pre>
           int w=1;
           for(int j=0;j<m/2;j++)</pre>
               int t,u;
               t=1LL*w*A[k+j+m/2]%MOD;
               u=A[k+j];
               A[k+j]=(u+t);
               if(A[k+j]>=MOD) A[k+j]-=MOD;
               A[k+j+m/2]=u+MOD-t;
               if(A[k+j+m/2]>=MOD) A[k+j+m/2]-=MOD;
               w=1LL*w*wn%MOD;
           }
       }
   }
    if(on==-1)
        for(int i=1;i<n/2;i++)</pre>
           swap(A[i],A[n-i]);
        for(int i=0;i<n;i++)</pre>
           A[i]=1LL*A[i]*two[cnt]%MOD;
   }
}
```

```
int A[MAXN],B[MAXN],C[10000000];
struct atom
{
    int 1,r;
};
atom solve(int l,int r)
    if (1>r){ C[++tot]=1; return (atom){tot,tot};}
    if (l==r){ C[++tot]=1; C[++tot]=1; return (atom){tot-1,tot};}
    int mid=(1+r)/2; atom k1=solve(1,mid),k2=solve(mid+1,r);
    int n=max(mid-l+1,r-mid),sz=1;
    while (sz<=(n<<1)) sz*=2;</pre>
    for (int i=0;i<sz;i++){A[i]=0; B[i]=0;}</pre>
    for (int i=k1.1;i<=k1.r;i++) A[i-k1.1]=C[i];</pre>
    for (int i=k2.1;i<=k2.r;i++) B[i-k2.1]=C[i];</pre>
    ntt(sz,A,1); ntt(sz,B,1);
    for (int i=0;i<sz;i++) A[i]=1LL*A[i]*B[i]%MOD;</pre>
    ntt(sz,A,-1);
    atom ans; ans.l=tot+1;
    for (int i=0;i<=r-l+1;i++) C[++tot]=A[i];</pre>
    ans.r=tot;
    return ans;
}
int n;
int main()
    scanf("%d",&n);
    for(int i=1;i<=30;i++)</pre>
        two[i]=pow_mod(1<<i,MOD-2);</pre>
    atom ans=solve(0,n-1);
    for(int i=ans.1;i<=ans.r;i++)</pre>
       printf("%d ",C[i]);
    return 0;
}
```

### 4.28 Stirling number of the second kind(multiple)

```
#include<bits/stdc++.h>
#define MAXN 100005
#define MOD 998244353
#define INF 100000000
#define F first
#define S second
using namespace std;
typedef long long 11;
typedef pair<int,int> P;
const int g=3;
int two[32];
int dbit(int x)
   while((x\&-x)!=x) x+=x\&-x;
   return x;
}
int pow_mod(int a,int i)
   if(i==0) return 1;
   int s=1;
```

```
while(i>0)
    {
         if(i&1) s=(1LL*s*a)%MOD;
         a=(1LL*a*a)%MOD;
         i>>=1;
    }
    return s;
}
int rev(int x,int r)
{
    int ans=0;
    for(int i=0;i<r;i++)</pre>
        if(x&(1<<i)) ans+=1<<(r-i-1);
void ntt(int n,int A[],int on)
    int r=0,cnt=0,t=n;
    while(t>1) {cnt++; t/=2;}
    for(;;r++) if((1<<r)==n) break;</pre>
    for(int i=0;i<n;i++)</pre>
        int tmp=rev(i,r);
        if(i<tmp) swap(A[i],A[tmp]);</pre>
    for(int s=1;s<=r;s++)</pre>
        int m=1<<s;</pre>
        int wn=pow_mod(g,(MOD-1)/m);
        for(int k=0;k<n;k+=m)</pre>
        {
            int w=1;
            for(int j=0; j<m/2; j++)</pre>
                int t,u;
                t=1LL*w*A[k+j+m/2]%MOD;
                u=A[k+j];
                A[k+j]=(u+t);
                if(A[k+j]>=MOD) A[k+j]-=MOD;
                A[k+j+m/2]=u+MOD-t;
                if(A[k+j+m/2]>=MOD) A[k+j+m/2]-=MOD;
                w=1LL*w*wn%MOD;
            }
        }
   }
    if(on==-1)
        for(int i=1;i<n/2;i++)</pre>
            swap(A[i],A[n-i]);
        for(int i=0;i<n;i++)</pre>
            A[i]=1LL*A[i]*two[cnt]%MOD;
   }
int fact[MAXN],inv[MAXN],A[MAXN],B[MAXN];
int main()
{
    int n;
    for(int i=1;i<=30;i++)</pre>
        two[i]=pow_mod(1<<i,MOD-2);</pre>
```

```
scanf("%d",&n);
   fact[0]=1,inv[0]=1;
   for(int i=1;i<=n;i++)</pre>
       fact[i]=1LL*fact[i-1]*i%MOD;
        inv[i]=pow_mod(fact[i],MOD-2);
   }
   int sz=dbit(n)*2;
   //printf("%d\n",sz);
   memset(A,0,sizeof(A));
   memset(B,0,sizeof(B));
   for(int i=0;i<=n;i++)</pre>
        if(i&1) A[i]=MOD-inv[i]; else A[i]=inv[i];
       B[i]=1LL*inv[i]*pow_mod(i,n)%MOD;
        //printf("%d %d\n",A[i],B[i]);
   ntt(sz,A,1);ntt(sz,B,1);
   for(int i=0;i<sz;i++)</pre>
       A[i]=1LL*A[i]*B[i]%MOD;
   ntt(sz,A,-1);
   for(int i=0;i<=n;i++)</pre>
       printf("%d ",A[i]);
   return 0;
}
```

# 4.29 Stirling number of the second kind(single)

```
#include<bits/stdc++.h>
#define MAXN 100005
#define INF 100000000
#define MOD 100000007
#define F first
#define S second
using namespace std;
typedef long long 11;
typedef pair<int,int> P;
int fact[MAXN];
int pow_mod(int a,int i)
   if(i==0) return 1;
   int s=1;
   while(i>0)
        if(i&1) s=(1LL*s*a)%MOD;
        a=(1LL*a*a)%MOD;
        i>>=1;
    }
    return s;
int inv(int x)
{
       return pow_mod(x,MOD-2);
}
int n,m;
int main()
```

### 4.30 Prefix Sum of Miu

```
#include<bits/stdc++.h>
#define MAXN 5000005
#define INF 100000000
#define MOD 100000007
#define F first
#define S second
using namespace std;
typedef long long 11;
typedef pair<int,int> P;
bool is_prime[MAXN];
int cnt,miu[MAXN],prime[MAXN];
11 n,m,f[MAXN];
map<11,11> mp;
void genmiu(int n)
   int p=0;
   for(int i=0;i<=n;i++) is_prime[i]=true;</pre>
   is_prime[0]=is_prime[1]=false;
   memset(miu,0,sizeof(miu));
   miu[1]=1;
   for(int i=2;i<=n;i++)</pre>
       if(is_prime[i]) {prime[p++]=i; miu[i]=-1;}
       for(int j=0;j<p;j++)</pre>
           if(prime[j]*i>n) break;
           is_prime[prime[j]*i]=false;
           miu[i*prime[j]]=i%prime[j]?-miu[i]:0;
           if(i%prime[j]==0) break;
       }
   }
   for(int i=1;i<=n;i++) f[i]=f[i-1]+miu[i];</pre>
11 calc(11 x)
       if(x<=5000000) return f[x];</pre>
       if(mp.find(x)!=mp.end()) return mp[x];
       ll ans=1;
       for(ll i=2,r;i<=x;i=r+1)</pre>
               r=x/(x/i);
```

```
ans-=calc(x/i)*(r-i+1);
}
return mp[x]=ans;
}
int main()
{
    genmiu(5000000);
    scanf("%1ld%1ld",&n,&m);
    printf("%1ld\n",calc(m)-calc(n-1));
    return 0;
}
```

### 4.31 Prefix Sum of Phi

```
#include<bits/stdc++.h>
#define MAXN 5000005
#define INF 100000000
#define MOD 100000007
#define F first
#define S second
using namespace std;
typedef long long 11;
typedef pair<int,int> P;
bool is_prime[MAXN];
11 cnt,phi[MAXN],prime[MAXN];
11 n,f[MAXN];
map<11,11> mp;
11 mul_mod(ll a,ll i)
       ll s=0;a%=MOD;
       while(i)
       {
               if(i&1) s=(s+a)%MOD;
               a=(a+a)\%MOD;
               i>>=1;
       }
       return s;
}
ll pow_mod(ll a,ll i)
       ll s=1;
       while(i)
       {
               if(i&1) s=mul_mod(s,a);
               a=mul_mod(a,a);
               i>>=1;
       }
       return s;
void genphi(ll n)
   11 p=0;
   memset(phi,0,sizeof(phi));
   phi[1]=1;
    for(ll i=0;i<=n;i++) is_prime[i]=true;</pre>
   is_prime[0]=is_prime[1]=false;
```

```
for(11 i=2;i<=n;i++)</pre>
        if(is_prime[i]) {prime[p++]=i; phi[i]=i-1;}
       for(11 j=0;j<p;j++)</pre>
           if(prime[j]*i>n) break;
           is_prime[prime[j]*i]=false;
           phi[i*prime[j]]=phi[i]*(i%prime[j]?prime[j]-1:prime[j]);
           if(i%prime[j]==0) break;
       }
   }
   for(ll i=1;i<=n;i++) f[i]=(f[i-1]+phi[i])%MOD;</pre>
11 calc(ll x)
{
        if(x<=5000000) return f[x];</pre>
        if(mp.find(x)!=mp.end()) return mp[x];
        11 ans=mul_mod(mul_mod(x,x+1),pow_mod(2,MOD-2));
        for(ll i=2,r;i<=x;i=r+1)</pre>
               r=x/(x/i);
               ans=(ans-calc(x/i)*((r-i+1)%MOD)%MOD+MOD)%MOD;
       }
       return mp[x]=ans;
}
int main()
       genphi(5000000);
        scanf("%11d",&n);
       printf("%lld\n",calc(n));
       return 0;
}
```

#### 4.32 Tonelli-Shanks

```
#include<bits/stdc++.h>
#define MAXN 100005
#define INF 100000000
#define MOD 100000007
#define F first
#define S second
using namespace std;
typedef long long 11;
typedef pair<int,int> P;
int n,k,a[MAXN];
ll pow_mod(ll a,ll i,ll m)
   ll s=1;
   while(i)
       if(i&1) s=s*a%m;
       a=a*a%m;
       i>>=1;
   }
   return s;
11 Tonelli_Shanks(ll n,ll p)
```

```
{
   if(p==2) return (n&1)?1:-1;
   if(pow_mod(n,p>>1,p)!=1) return -1;
   if(p&2) return pow_mod(n,(p+1)>>2,p);
   int s=__builtin_ctzll(p^1);
   11 q=p>>s,z=2;
   for(;pow_mod(z,p>>1,p)==1;++z);
   11 c=pow_mod(z,q,p),r=pow_mod(n,(q+1)>>1,p),t=pow_mod(n,q,p),tmp;
   for(int m=s,i;t!=1;)
   {
      for(i=0,tmp=t;tmp!=1;++i) tmp=tmp*tmp%p;
      for(;i<--m;) c=c*c%p;</pre>
      r=r*c%p;c=c*c%p;t=t*c%p;
   }
   return r;
}
int main()
{
   return 0;
```

#### 4.33 Euclid

```
#include<bits/stdc++.h>
#define MAXN 100005
#define INF 100000000
#define MOD 100000007
#define F first
#define S second
using namespace std;
typedef long long 11;
typedef pair<int,int> P;
11 inv2,inv6,a,b,c,l,r;//need initialize
struct E
{
   ll f,g,h;
   E(){}
   E(11 _f,11 _g,11 _h){f=_f,g=_g,h=_h;}
ll pow_mod(ll a,ll i)
   ll s=1;
   while(i)
       if(i&1) s=s*a%MOD;
       a=a*a%MOD;
       i>>=1;
   }
   return s;
// f:\sum_{i=0}^{n}\lfloor\frac{ai+b}{c}\rfloor
// g:\sum_{i=0}^{n}i\times\lfloor\frac{ai+b}{c}\rfloor
// g:\sum_{i=0}^{n}\lfloor\frac{ai+b}{c}\rfloor^{2}
E cal(ll a,ll b,ll c,ll n)
{
```

```
if(!a) return E(0,0,0);
   E x,y;
   if(a>=c||b>=c)
       x=cal(a\%c,b\%c,c,n);
       y.f = (a/c*n\%MOD*(n+1)\%MOD*inv2+b/c*(n+1)+x.f)\%MOD;
       v.g=(a/c*n%MOD*(n+1)%MOD*(n*2+1)%MOD*inv6+b/c*(n+1)%MOD*n%MOD*inv2+x.g)%MOD;
       y.h=a/c*(a/c)%MOD*n%MOD*(n+1)%MOD*(n*2+1)%MOD*inv6%MOD;
       (y.h+=b/c*(b/c)%MOD*(n+1))%=MOD;
       (y.h+=a/c*(b/c)%MOD*n%MOD*(n+1))%=MOD;
       (y.h+=2LL*(a/c)%MOD*x.g)%=MOD;
       (y.h+=2LL*(b/c)%MOD*x.f)%=MOD;
       y.f=(y.f+MOD)%MOD; y.g=(y.g+MOD)%MOD; y.h=(y.h+MOD)%MOD;
   11 m=(a*n+b)/c;
   x=cal(c,c-b-1,a,m-1);
   y.f=(n*m-x.f)%MOD;
   y.g=y.g*inv2%MOD;
   y.h = (n*m\%MOD*(m+1)-2LL*x.g-2LL*x.f-y.f)\%MOD;
   y.f=(y.f+MOD) MOD; y.g=(y.g+MOD) MOD; y.h=(y.h+MOD) MOD;
   return y;
int main()
   inv2=pow_mod(2,MOD-2);inv6=pow_mod(6,MOD-2);
   return 0;
}
```

# 4.34 Simpson

```
#include<bits/stdc++.h>
using namespace std;
double simpson(double a,double b)
{
    double c=a+(b-a)/2;
    return (F(a)+4*F(c)+F(b))*(b-a)/6;
}
double asr(double a,double b,double eps,double A)
{
    double c=a+(b-a)/2;
    double L=simpson(a,c),R=simpson(c,b);
    if(fabs(L+R-A)<=15*eps) return L+R+(L+R-A)/15.0;
    return asr(a,c,eps/2,L)+asr(c,b,eps/2,R);
}
double asr(double a,double b,double eps)
{
    return asr(a,b,eps,simpson(a,b));
}</pre>
```

## **4.35** Farey

```
#include<bits/stdc++.h>
#define MAXN 100005
#define INF 1000000000
```

```
#define MOD 1000000007
#define F first
#define S second
using namespace std;
typedef long long 11;
typedef pair<11,11> P;
ll a,b,c,d,t;
P cal(11 a,11 b,11 c,11 d)
   //printf("%lld %lld %lld %lld\n",a,b,c,d);
   ll x=a/b+1; if (x*d<c) return P(x,1);
   if(!a) return P(1,d/c+1);
   if(a<=b&&c<=d)</pre>
       P t=cal(d,c,b,a);
       swap(t.F,t.S); return t;
   x=a/b;P t=cal(a-b*x,b,c-d*x,d);
   t.F+=t.S*x;return t;
}
int main()
   while(~scanf("%lld%lld%lld",&a,&b,&c,&d))
       t=_gcd(a,b),a/=t,b/=t,t=_gcd(c,d),c/=t,d/=t;
       printf("%lld %lld %lld %lld\n",a,b,c,d);
       P p=cal(a,b,c,d);
       printf("%lld/%lld\n",p.F,p.S);
   return 0;
}
```

### 5 Others

# 5.1 Convex Hull Trick

```
#pragma GCC optimize(3)
#include<bits/stdc++.h>
#define MAXN 100005
#define INF 100000000
#define MOD 100000007
#define F first
#define S second
using namespace std;
typedef long long 11;
typedef pair<11,11> P;
11 N,tot,t,now;
P st[MAXN];
void add(ll u,ll v)
  P p=P(u,v);
  t--;
  st[t++]=p;
}
bool cmp(P x,P y)
```

```
{
    if(x.S!=y.S) return x.S<y.S;
    return x.F<y.F;
}
ll query(ll x)
{
    ll l=-1,r=t-1;
    while(r-l>1)
    {
        ll mid=(l+r)/2;
        if(st[mid].F*x+st[mid].S<=st[mid+1].F*x+st[mid+1].S) l=mid;
        else r=mid;
    }
    return st[r].F*x+st[r].S;
}
int main()
{
}</pre>
```

# 5.2 Dynamic Convex Hull Trick

```
#pragma GCC optimize(3)
#include<bits/stdc++.h>
#define MAXN 100005
#define INF 100000000
#define MOD 100000007
#define F first
#define S second
using namespace std;
typedef long long 11;
typedef pair<int,int> P;
int n,k,a[MAXN];
bool qu=0;
struct line
       long long m,b;
       mutable function<const line*()> succ;
       bool operator<(const line& rhs) const</pre>
               if (!qu) return m<rhs.m;</pre>
               const line* s=succ();
               if (!s)
               return b-s->b<rhs.m*(s->m-m);
       }
};
struct hull:public multiset<line>
       bool bad(iterator y)
               auto z=next(y);
               if (y==begin())
               {
                      if (z==end())
                      return 0;
                      return (y->m==z->m && y->b<=z->b);
              }
```

```
auto x=prev(y);
              if (z==end())
              return (y->m==x->m && y->b<=x->b);
              return 1.0*(x-b-y-b)*(z-m-y-m)=1.0*(y-b-z-b)*(y-m-x-m);
       }
       void add(long long m,long long b)
              auto it=insert({m,b});
              it->succ=[=] { return (next(it)==end())? 0:&*next(it); };
              if (bad(it))
              {
                      erase(it);
                      return;
              }
              while (next(it)!=end() && bad(next(it))) erase(next(it));
              while (it!=begin() && bad(prev(it))) erase(prev(it));
       }
       long long eval(long long x)
       {
              if (empty()) return -(1LL<<60);</pre>
              qu=1;line l=*lower_bound((line){x,0});qu=0;
              return 1.m*x+1.b;
       }
};
int main()
   return 0;
}
```

# 5.3 Knuth's optimization

```
#include<bits/stdc++.h>
#define MAXN 2005
#define INF 100000000
using namespace std;
typedef long long 11;
11 a[MAXN];
ll n,k;
11 dp[MAXN][MAXN],knuth[MAXN][MAXN];
int main()
    while(scanf("%lld %lld",&n,&k)==2)
       a[0]=0;
        for(ll i=1;i<=k;i++)</pre>
            scanf("%lld",&a[i]);
        a[k+1]=n;
        for(ll i=0;i<=k+1;i++)</pre>
            for(ll j=0;j<=k+1;j++)</pre>
                dp[i][j]=INF;
        for(ll i=0;i<=k;i++)</pre>
            dp[i][i+1]=0;
        for(11 1=3;1<=k+2;1++)</pre>
            for(ll i=0;i<=k+2-1;i++)</pre>
                if(1==3)
                {
```

### 5.4 Multiple Backpack

```
#include<bits/stdc++.h>
#define MAXN 100005
int w[MAXN],v[MAXN],m[MAXN];
int dp[MAXW+1];
int deq[MAXW+1];
int deqv[MAXW+1];
void solve()
    for(int i=0;i<n;i++)</pre>
        for(int a=0;a<w[i];a++)</pre>
           int s=0,t=0;
           for(int j=0;j*w[i]+a<=W;j++)</pre>
               int val=dp[j*w[i]+a]-j*v[i];
               while(s<t&&deqv[t-1]<=val) t--;</pre>
               deq[t]=j;
               deqv[t++]=val;
               dp[j*w[i]+a]=deqv[s]+j*v[i];
               if(deq[s]==j-m[i]) s++;
           }
       }
    }
    printf("%d\n",dp[W]);
}
```

### 5.5 Sum Over Subsets

```
#include<bits/stdc++.h>
#define MAXN 100005
#define INF 1000000000
#define MOD 1000000007
#define F first
#define S second
using namespace std;
typedef long long ll;
```

```
typedef pair<int,int> P;
int n,a[MAXN],f[MAXN];
int main()
{
        scanf("%d",&n);
        for(int i=0;i<(1<<n);i++)</pre>
                scanf("%d",&a[i]);
        for(int i=0;i<(1<<n);i++)</pre>
                f[i]=a[i];
        for(int i=0;i<n;i++)</pre>
                for(int mask=0;mask<(1<<n);mask++)</pre>
                         if(mask&(1<<i))</pre>
                                 f [mask] += f [mask^(1<<i)];
        for(int i=0;i<(1<<n);i++)</pre>
                printf("%d ",f[i]);
        puts("");
        return 0;
}
```

#### 5.6 Enumeration of Subsets

```
#include<bits/stdc++.h>
#define MAXN 100005
#define INF 100000000
#define MOD 100000007
#define F first
#define S second
using namespace std;
typedef long long 11;
typedef pair<int,int> P;
int n,k,a[MAXN];
void solve1(int sup)//all subsets
   int sub=sup;
   do
   {
       //operation here
       sub=(sub-1)⊃
   }while(sub!=sup);
}
void solve2(int n,int k) //all subsets of (1<<n) of size k</pre>
   int comb=(1<<k)-1;</pre>
   while(comb<1<<n)</pre>
       //operation here
       int x=comb&-comb,y=comb+x;
       comb=((comb\&^y)/x>>1)|y;
   }
}
int main()
{
   return 0;
```

### 5.7 Matroid Intersection

```
#include<bits/stdc++.h>
#define MAXN 65
#define MAXM 6005
#define INF 100000000
#define MOD 100000007
#define F first
#define S second
using namespace std;
typedef long long 11;
typedef pair<int,int> P;
int color[MAXM];
11 val[MAXM];
int n,m,tot,tot2;
struct LinearMatroid
   11 basis[62];
   void clear()
   {
       memset(basis,0,sizeof(basis));
   }
   void add(ll x)
       for(int j=60;j>=0;j--)
           if(!(x&(1LL<<j))) continue;</pre>
           if(!basis[j])
           {
              basis[j]=x;
              return;
           else x^=basis[j];
       }
   }
   bool test(ll x)
       for(int j=60;j>=0;j--)
           if(!(x&(1LL<<j))) continue;</pre>
           if(!basis[j]) return true; else x^=basis[j];
       return false;
   }
};
struct ColorfulMatroid
   int cnt[125];
   void clear()
   {
       memset(cnt,0,sizeof(cnt));
   }
   void add(int x)
   {
       cnt[x]++;
   bool test(int x)
```

```
return (cnt[x]==0);
   }
};
template <typename MT1, typename MT2>
struct MatroidIntersection
   int n;
   MatroidIntersection(int _n):n(_n){}
   int pre[MAXM],id[MAXM];
   bool vis[MAXM],sink[MAXM],has[MAXM];
   queue<int> que;
   void clear_all()
       memset(vis,false,sizeof(vis));
       memset(sink,false,sizeof(sink));
       memset(pre,0,sizeof(pre));
       while(que.size()) que.pop();
   }
   vector<int> getcur()
       vector<int> ret;
       for(int i=1;i<=n;i++) if(has[i]) ret.push_back(i);</pre>
       return ret;
   }
   void enqueue(int v,int p)
       vis[v]=true; pre[v]=p;
       que.push(v);
   }
   vector<int> run()
       MT1 mt1; MT2 mt2;
       memset(has,false,sizeof(has));
       while(true)
           vector<int> cur=getcur();
           int cnt=0;
           for(int i=1;i<=n;i++) if(has[i]) id[i]=cnt++;</pre>
           MT1 allmt1; MT2 allmt2; allmt1.clear(); allmt2.clear();
           vector<MT1> vmt1(cur.size()); vector<MT2> vmt2(cur.size());
           for(auto &x:vmt1) x.clear(); for(auto &x:vmt2) x.clear();
           clear_all();
           for(auto x:cur) allmt1.add(val[x]),allmt2.add(color[x]);
           for(int i=0;i<(int)cur.size();i++)</pre>
               for(int j=0;j<(int)cur.size();j++)</pre>
               {
                   if(i==j) continue;
                   vmt1[i].add(val[cur[j]]);
                  vmt2[i].add(color[cur[j]]);
           for(int i=1;i<=n;i++)</pre>
               if(has[i]) continue;
               if(allmt1.test(val[i])) {que.push(i); vis[i]=true;}
           for(int i=1;i<=n;i++)</pre>
           {
```

```
if(has[i]) continue;
               if(allmt2.test(color[i])) sink[i]=true;
           }
           int last=-1;
           while(que.size())
           {
               int v=que.front(); que.pop();
               if(sink[v]) {last=v; break;}
               for(int i=1;i<=n;i++)</pre>
                   if(vis[i]) continue;
                   if(has[i] == has[v]) continue;
                   if(has[v])
                       if(vmt1[id[v]].test(val[i])) enqueue(i,v);
                   }
                   else
                   {
                       if(vmt2[id[i]].test(color[v])) enqueue(i,v);
               }
           if(last==-1) return cur;
           while(last)
               has[last]^=1;
               last=pre[last];
       }
   }
};
//Pick Your Own Nim
//In real cases, Linear Matroid Need Optimization to Pass
int main()
{
   scanf("%d",&n);
   for(int i=0;i<n;i++)</pre>
       11 x;
       scanf("%11d",&x);
       val[++tot]=x; color[tot]=++tot2;
   }
   scanf("%d",&m);
   for(int i=0;i<m;i++)</pre>
       int k;
       scanf("%d",&k);
       tot2++;
       for(int j=0;j<k;j++)</pre>
           11 x;
           scanf("%11d",&x);
           val[++tot]=x; color[tot]=tot2;
       }
   MatroidIntersection<LinearMatroid,ColorfulMatroid> matint(tot);
   vector<int> res=matint.run();
   if(res.size()<n+m) {puts("-1"); return 0;}</pre>
   else
```

```
{
    vector<ll> ans;
    int last=n;
    for(auto x:res)
    {
        if(color[x]>last)
        {
            ans.push_back(val[x]);
            last=color[x];
        }
    }
    for(auto x:ans) printf("%lld\n",x);
}
return 0;
}
```

# 5.8 Weighted Matroid Intersection

```
#pragma GCC optimize(3)
#include<bits/stdc++.h>
#define MAXN 85
#define MAXM 205
#define INF 100000000
#define MOD 1000000007
#define F first
#define S second
using namespace std;
typedef long long 11;
typedef pair<int,int> P;
int c[MAXN],k[MAXN],color[MAXM],u[MAXM],v[MAXM],w[MAXM],cost[MAXM];
11 val[MAXM];
int T,n,m,tot,tot2;
struct LinearMatroid
   11 basis[62];
   void clear()
       memset(basis,0,sizeof(basis));
   }
   void add(ll x)
       for(int j=60;j>=0;j--)
           if(!(x&(1LL<<j))) continue;</pre>
           if(!basis[j])
               basis[j]=x;
               return;
           }
           else x^=basis[j];
       }
   }
   bool test(ll x)
       for(int j=60;j>=0;j--)
           if(!(x&(1LL<<j))) continue;</pre>
```

```
if(!basis[j]) return true; else x^=basis[j];
       return false;
   }
};
struct ColorfulMatroid
   int cnt[125];
   void clear()
   {
       memset(cnt,0,sizeof(cnt));
   }
   void add(int x)
       cnt[x]++;
   }
   bool test(int x)
   {
       return (cnt[x]==0);
   }
};
struct GraphMatroid
   vector<int> G[MAXN];
   bool vis[MAXN];
   bool exist[MAXN];
   void dfs(int v)
   {
       vis[v]=true;
       for(auto to:G[v]) if(!vis[to]) dfs(to);
   }
   bool test(vector<int> &vec)
       for(int i=1;i<=n+1;i++) G[i].clear();</pre>
       memset(vis,false,sizeof(vis));
       memset(exist,true,sizeof(exist));
       for(auto x:vec) exist[x]=false;
       for(int i=1;i<=tot;i++)</pre>
           if(exist[i])
              G[u[i]].push_back(v[i]);
               G[v[i]].push_back(u[i]);
           }
       }
       dfs(1);
       for(int i=1;i<=n+1;i++) if(!vis[i]) return false;</pre>
       return true;
   }
};
struct PartitionMatroid
   int cnt[125];
   bool test(vector<int> &vec)
       memset(cnt,0,sizeof(cnt));
       for(auto x:vec) cnt[color[x]]++;
```

```
for(int i=1;i<=m;i++) if(cnt[i]>c[i]-k[i]) return false;
       return true;
   }
};
template <typename MT1, typename MT2>
struct MatroidIntersection
   int n,S,T;
   MatroidIntersection(int _n):n(_n){}
   int pre[MAXM],id[MAXM],d[MAXM];
   bool inque[MAXM],sink[MAXM],has[MAXM];
   vector<int> g[MAXN];
   queue<int> que;
   void clear_all()
       for(int i=1;i<=n+2;i++)</pre>
           inque[i]=false;
           sink[i]=false;
           pre[i]=0;
           d[i]=-INF;
           if(has[i]) cost[i]=w[i]; else cost[i]=-w[i];
           g[i].clear();
       }
       while(que.size()) que.pop();
   }
   void add_edge(int u,int v)
   {
       g[u].push_back(v);
   }
   vector<int> getcur()
       vector<int> ret;
       for(int i=1;i<=n;i++) if(has[i]) ret.push_back(i);</pre>
       return ret;
   }
   void enqueue(int v,int p)
       pre[v]=p;
       if(!inque[v])
           inque[v]=true;
           que.push(v);
       }
   }
   pair<vector<int>,ll> run()
       ll ans=0;
       MT1 mt1; MT2 mt2;
       memset(has,false,sizeof(has));
       S=n+1; T=n+2;
       while(true)
           clear_all();
           for(int i=1;i<=n;i++)</pre>
           {
               if(!has[i])
```

```
cost[i]=w[i];
                  has[i]^=1;
                  vector<int> tmp=getcur();
                  if(mt1.test(tmp)) add_edge(S,i);
                  if(mt2.test(tmp)) add_edge(i,T);
                  has[i]^=1;
              }
               else cost[i]=-w[i];
           }
           for(int i=1;i<=n;i++)</pre>
               if(!has[i])
                  for(int j=1;j<=n;j++)</pre>
                      if(has[j])
                      {
                          has[i]^=1; has[j]^=1;
                          vector<int> tmp=getcur();
                          if(mt1.test(tmp)) add_edge(j,i);
                          if(mt2.test(tmp)) add_edge(i,j);
                          has[i]^=1; has[j]^=1;
                      }
                  }
              }
           d[S]=0; que.push(S); inque[S]=true;
           cost[S]=cost[T]=0;
           int counter=0;
           while(que.size())
           {
               counter++;
               int u=que.front(); que.pop();
               for(auto to:g[u])
                  if(d[to]<d[u]+cost[to])</pre>
                      d[to]=d[u]+cost[to];
                      enqueue(to,u);
                  }
               inque[u]=false;
           if(!pre[T]) return make_pair(getcur(),ans);
           ans+=d[T];
           int last=pre[T];
           while(last!=S)
              has[last]^=1;
               last=pre[last];
           }
       }
   }
//hdu 6636 Milk Candy
int main()
{
   scanf("%d",&T);
   while(T--)
   {
```

{

```
tot=0;
       scanf("%d%d",&n,&m);
       int sum=0;
       ll ans=0;
       for(int i=1;i<=m;i++)</pre>
           scanf("%d%d",&c[i],&k[i]);
           sum+=c[i]-k[i];
           for(int j=1;j<=c[i];j++)</pre>
               int 1,r,cost;
               scanf("%d%d%d",&1,&r,&cost);
               color[++tot]=i; u[tot]=l; v[tot]=r+1; w[tot]=cost;
               ans+=cost;
       }
       MatroidIntersection<GraphMatroid,PartitionMatroid> matint(tot);
       auto res=matint.run();
       GraphMatroid gm; PartitionMatroid pm;
       if((int)res.F.size()!=sum||!gm.test(res.F)||!pm.test(res.F)) puts("-1"); else
           printf("%lld\n",ans-res.S);
   }
   return 0;
}
```

# 5.9 Nim Multiplication

```
#include<bits/stdc++.h>
#define MAXN 100005
#define INF 100000000
#define MOD 100000007
#define F first
#define S second
using namespace std;
typedef long long 11;
typedef pair<int,int> P;
int n,sg[2][2]={0,0,0,1};
int nim_mult_pow(int x,int y)
   if(x<2)
       return sg[x][y];
   int a=0;
   for(;;a++)
       if(x>=(1<<(1<<a))&&x<(1<<(a+1))))</pre>
           break;
   int m=1<<(1<<a);</pre>
   int p=x/m,s=y/m,t=y%m;
   int d1=nim_mult_pow(p,s);
   int d2=nim_mult_pow(p,t);
   return (m*(d1^d2))^nim_mult_pow(m/2,d1);
int nim_mult(int x,int y)
       return nim_mult(y,x);
   if(x<2)
```

```
return sg[x][y];
int a=0;
for(;;a++)
    if(x>=(1<<(1<<a))&&x<(1<<(a+1))))
        break;
int m=1<<(1<<a);
int p=x/m,q=x/m,s=y/m,t=y/m;
int c1=nim_mult(p,s);
int c2=nim_mult(p,t)^nim_mult(q,s);
int c3=nim_mult(q,t);
return (m*(c1^c2))^c3^nim_mult_pow(m/2,c1);
}
int x,y;
while(scanf("%d%d",&x,&y)==2) printf("%d\n",nim_mult(x,y));
}</pre>
```

# 5.10 Dynamic Dynamic Programming

```
//luogu 4719 dynamic maximum weight vertex cover
#pragma GCC optimize(3)
#include<bits/stdc++.h>
#define MAXN 100005
#define INF 100000000
#define MOD 100000007
#define F first
#define S second
using namespace std;
typedef long long 11;
typedef pair<int,int> P;
int tot,n,q;
int dp[MAXN][2];
int ldp[MAXN][2];
    pa[MAXN],a[MAXN],dep[MAXN],sz[MAXN],wson[MAXN],top[MAXN],st[MAXN],ded[MAXN],tpos[MAXN],w[MAXN],bot[MAXN];
struct mat
{
   int v[2][2];
   mat(){v[0][0]=v[0][1]=v[1][0]=v[1][1]=-INF;}
mat mul(mat a,mat b)
   mat c;
   for(int i=0;i<2;i++)</pre>
       for(int j=0;j<2;j++)</pre>
           for(int k=0;k<2;k++)</pre>
               c.v[i][j]=max(c.v[i][j],a.v[i][k]+b.v[k][j]);
   return c;
}
mat unit;
vector<int> G[MAXN];
void dfs1(int v,int p,int d)
   dep[v]=d;pa[v]=p;sz[v]=1;
   for(int i=0;i<(int)G[v].size();i++)</pre>
   {
```

```
int to=G[v][i];
       if(to==p) continue;
       dfs1(to,v,d+1);
       if(sz[to]>sz[wson[v]]) wson[v]=to;
       sz[v] += sz[to];
   }
}
void dfs2(int v,int p,int num)
   top[v]=num; bot[num]=v;
   st[v]=++tot;
   tpos[tot]=v;
   if(wson[v]) dfs2(wson[v],v,num);
   for(int i=0;i<(int)G[v].size();i++)</pre>
   {
       int to=G[v][i];
       if(to==p||to==wson[v]) continue;
       dfs2(to,v,to);
   }
   ed[v]=tot;
}
struct segtree
   mat val[4*MAXN];
   void pushup(int k)
       val[k]=mul(val[k*2],val[k*2+1]);
   void build(int k,int l,int r)
       if(l==r)
       {
           int v=tpos[1];
           val[k].v[0][0]=val[k].v[0][1]=ldp[v][0];
           val[k].v[1][0]=ldp[v][1];
           val[k].v[1][1]=-INF;
           return;
       }
       int mid=(1+r)/2;
       build(k*2,1,mid); build(k*2+1,mid+1,r);
       pushup(k);
   void update(int k,int l,int r,int p,int v1,int v2)
       if(1==r)
       {
           val[k].v[0][0]=val[k].v[0][1]=v1;
           val[k].v[1][0]=v2;
           val[k].v[1][1]=-INF;
           return;
       }
       int mid=(1+r)/2;
       if(p<=mid) update(k*2,1,mid,p,v1,v2);</pre>
       else update(k*2+1,mid+1,r,p,v1,v2);
       pushup(k);
   }
   mat query(int k,int l,int r,int x,int y)
       if(x>r||1>y) return unit;
```

```
if(l>=x&&r<=y) return val[k];</pre>
       int mid=(1+r)/2;
       return mul(query(k*2,1,mid,x,y),query(k*2+1,mid+1,r,x,y));
}tree;
void init()
{
   memset(wson,0,sizeof(wson));//important when multiple test cases!!!
   dfs1(1,0,1);
   dfs2(1,0,1);
   tree.build(1,1,n);
}
void update(int v,int x)
   ldp[v][1]+=(x-w[v]); w[v]=x;
   while(v!=0)
   {
       int l=st[top[v]],r=st[bot[top[v]]];
       //mat tmp1(2,vec(1)),tmp2(2,vec(1));
       //tmp1[0][0]=tmp1[1][0]=tmp2[0][0]=tmp2[1][0]=0;
       mat past=tree.query(1,1,n,1,r);
       tree.update(1,1,n,st[v],ldp[v][0],ldp[v][1]);
       mat now=tree.query(1,1,n,1,r);
       v=pa[top[v]];
       ldp[v][0] += max(now.v[0][0],now.v[1][0]) - max(past.v[0][0],past.v[1][0]);
       ldp[v][1]+=now.v[0][0]-past.v[0][0];
   }
}
int main()
{
   unit.v[0][0]=unit.v[1][1]=0; unit.v[0][1]=unit.v[1][0]=-INF;
   scanf("%d%d",&n,&q);
   for(int i=1;i<=n;i++) scanf("%d",&a[i]);</pre>
   for(int i=0;i<n-1;i++)</pre>
   {
       int u,v;
       scanf("%d%d",&u,&v);
       G[u].push_back(v); G[v].push_back(u);
   }
   init();
   for(int i=1;i<=n;i++) update(i,a[i]);</pre>
   for(int i=0;i<q;i++)</pre>
   {
       int x,y;
       scanf("%d%d",&x,&y);
       update(x,y);
       int l=1,r=st[bot[1]];
       mat A=tree.query(1,1,n,l,r);
       printf("%d\n", max(A.v[0][0], A.v[1][0]));
   }
   return 0;
```

### 5.11 Simplex

#include<bits/stdc++.h>

```
#define MAXN 105
#define MAXM 105
#define INF 100000000
#define MOD 100000007
#define F first
#define S second
using namespace std;
typedef long long 11;
typedef pair<int,int> P;
typedef double db;
typedef vector<db> vec;
typedef vector<vec> mat;
const db eps=1e-8;
bool eq(db a,db b)
{
   return fabs(a-b)<eps;</pre>
}
bool ls(db a,db b)
{
   return a < b & ! eq(a,b);</pre>
}
vec simplex(mat a)
   int n=(int)a.size()-1;
   int m=(int)a[0].size()-1;
   vec left(n+1),up(m+1);
   iota(up.begin(),up.end(),0);
   iota(left.begin(),left.end(),m);
   auto pivot=[&](int x,int y)
      swap(left[x],up[y]);
      db k=a[x][y];
      a[x][y]=1;
      vector<int> vct;
      for(int j=0; j<=m; j++)</pre>
      {
         a[x][j]/=k;
         if(!eq(a[x][j],0)) vct.push_back(j);
      }
      for(int i=0;i<=n;i++)</pre>
         if(eq(a[i][y],0)||i==x) continue;
         k=a[i][y];
         a[i][y]=0;
         for(int j:vct) a[i][j]-=k*a[x][j];
      }
   };
   while(1)
      int x=-1;
      if(x==-1) break;
      int y=-1;
      assert(y!=-1);
      pivot(x,y);
   }
   while(1)
```

```
int y=-1;
    for(int j=1;j<m;j++) if(ls(0,a[0][j])&&(y==-1||a[0][j]>a[0][y])) y=j;
    if(y==-1) break;
    int x=-1;
    for(int i=1;i<=n;i++) if(ls(0,a[i][y])&&(x==-1||a[i][0]/a[i][y]<a[x][0]/a[x][y])) x=i;
    assert(x!=-1);
    pivot(x,y);
}

vector<double> ans(m+1);
for(int i=1;i<=n;i++) if(left[i]<=m) ans[left[i]]=a[i][0];
    ans[0]=-a[0][0];
    return ans;
}</pre>
```

### 5.12 whatday

```
#include<bits/stdc++.h>
using namespace std;
int whatday(int d,int m,int y)
   int ans;
   if(m==1|m==2)
       m+=12, y--;
   if((y<1752)||(y==1752&&m<9)||(y==1752&&m==9&&d<3))</pre>
       ans=(d+2*m+3*(m+1)/5+y+y/4+5)%7;
   else
       ans=(d+2*m+3*(m+1)/5+y+y/4-y/100+y/400)%7;
   return ans;
}
int main()
{
   return 0;
}
```

# 6 String

#### 6.1 Trie

```
#include<bits/stdc++.h>
#define MAXN 100005
#define INF 1000000000
#define MOD 1000000007
#define F first
#define S second
using namespace std;
typedef long long ll;
typedef pair<int,int> P;
int tot=1,n;
int trie[MAXN][26];
bool ed[MAXN];
void insert(char *s,int rt)
{
    for(int i=0;s[i];i++)
    {
```

```
int x=s[i]-'a';
       if(trie[rt][x]==0) trie[rt][x]=++tot;
       rt=trie[rt][x];
   }
   ed[rt]=true;
}
bool find(char *s,int rt)
   for(int i=0;s[i];i++)
   {
       int x=s[i]-'a';
       if(trie[rt][x]==0) return false;
       rt=trie[rt][x];
   }
   return ed[rt];
}
int main()
{
   memset(ed,false,sizeof(ed));
   return 0;
}
```

### 6.2 KMP

```
#include<bits/stdc++.h>
using namespace std;
vector<int> kmp(string a,string b) // a=pattern, b=text
   int n=a.size();
   vector<int> next(n+1,0);
   for(int i=1;i<n;++i)</pre>
       int j=i;
       while(j>0)
           j=next[j];
           if(a[j]==a[i])
               next[i+1]=j+1;
               break;
           }
       }
   }
   vector<int> p;//p=positions
   int m=b.size();
   for(int i=0,j=0;i<m;++i)</pre>
       if(j<n&&b[i]==a[j])</pre>
           j++;
       }
       else
           while(j>0)
               j=next[j];
               if(b[i]==a[j])
```

```
{
                   j++;
                   break;
               }
           }
       }
       if(j==n)
        {
           p.push_back(i-n+1);
       }
   }
   return p;
}
int main()
{
   return 0;
}
```

# 6.3 Hash Matching

```
#include<bits/stdc++.h>
#define MAXN 100005
using namespace std;
typedef unsigned long long ull;
const ull B=1000000007;
bool contain(string a,string b)
{
       int al=a.length(),bl=b.length();
       if(al>bl) return false;
       ull t=1;
       for(int i=0;i<al;i++)</pre>
               t*=B;
       ull ah=0,bh=0;
       for(int i=0;i<al;i++) ah=ah*B+a[i];</pre>
       for(int i=0;i<al;i++) bh=bh*B+b[i];</pre>
       for(int i=0;i+al<=bl;i++)</pre>
               if(ah==bh) return true;
               if(i+al<bl) bh=bh*B+b[i+al]-b[i]*t;</pre>
        }
       return false;
}
```

# 6.4 Aho-Corasick Automaton

```
#include<bits/stdc++.h>
#define MAXN 50020
using namespace std;
struct trie
{
    trie* next[26];
    trie* fail;
    bool mark;
};
trie* thead;
```

```
char str[MAXN][1001];
inline trie* newnode()
   trie* t;
   t=(trie*)malloc(sizeof(trie));
   t->fail=NULL;
   t->mark=false;
   memset(t,0,sizeof(trie));
   return t;
}
void insert(char x[])
   int i;
   trie* s=thead;
   trie* t;
   for(i=0;x[i];i++)
       if(s->next[x[i]-'a']) {s=s->next[x[i]-'a'];}
       else
       {
           t=newnode();
           s->next[x[i]-'a']=t;
           s=t;
       }
   }
   s->mark=true;
   return;
trie* g(trie* s, char x)
   if(s->next[x-'a']) return s->next[x-'a'];
   else if(s==thead) return thead;
   else return NULL;
}
void bfs()
   trie* s=thead;
   queue<trie*> que;
   for(int i=0;i<26;i++)</pre>
       if(s->next[i]){s->next[i]->fail=thead; que.push(s->next[i]);}
   while(!que.empty())
       trie* t=que.front();
       que.pop();
       for(int i=0;i<26;i++)</pre>
           if(g(t,(char)('a'+i))!=NULL)
           {
               que.push(t->next[i]);
               trie* v=t->fail;
               while(g(v,(char)('a'+i))==NULL) v=v->fail;
               t->next[i]->fail=g(v,(char)('a'+i));
   }
   return;
}
int match(char x[])
   trie* s=thead;
```

```
int cnt=0;
   for(int i=0;x[i];i++)
       while(g(s,x[i])==NULL)
           s=s->fail;
           if(s->mark) cnt++;
       s=g(s,x[i]);
       if(s->mark) cnt++;
   }
    while(s->fail!=thead)
       s=s->fail;
       if(s->mark) cnt++;
   return cnt;
}
bool find(char x[])
   trie* s=thead;
   for(int i=0;x[i];i++)
       if(s->next[x[i]-'a']==NULL) return false;
       s=s->next[x[i]-'a'];
   }
   return true;
void deltrie(trie* s)
   int i;
   for(i=0;i<26;i++)</pre>
       if(s->next[i])
       deltrie(s->next[i]);
   free(s);
   s=NULL;
}
int main()
{
   int i=0;
   thead=newnode();
   while(scanf("%s",str[i])==1)
       if(str[i][0]=='1') break;
       insert(str[i]);
       i++;
   }
   bfs();
   char p[100];
   scanf("%s",p);
   printf("%d\n",match(p));
   deltrie(thead);
   return 0;
}
```

# 6.5 Suffix Array

```
#include<bits/stdc++.h>
#define MAXN 1005
using namespace std;
int n,k;
int r[MAXN+1];
int sa[MAXN],lcp[MAXN];
int c[MAXN],t1[MAXN],t2[MAXN];
string S;
void construct_sa(string S,int *sa)
    int n=S.length()+1;
    int m=130;
    int i,*x=t1,*y=t2;
    for(i=0;i<m;i++) c[i]=0;</pre>
    for(i=0;i<n;i++) c[x[i]=S[i]]++;</pre>
    for(i=1;i<m;i++) c[i]+=c[i-1];</pre>
    for(i=n-1;i>=0;i--) sa[--c[x[i]]]=i;
    for(int k=1;k<=n;k<<=1) {</pre>
        int p=0;
        for(i=n-k;i<n;i++) y[p++]=i;</pre>
        for(i=0;i<n;i++) if(sa[i]>=k) y[p++]=sa[i]-k;
       for(i=0;i<m;i++) c[i]=0;</pre>
        for(i=0;i<n;i++) c[x[y[i]]]++;</pre>
        for(i=0;i<m;i++) c[i]+=c[i-1];</pre>
        for(i=n-1;i>=0;i--) sa[--c[x[y[i]]]]=y[i];
        swap(x,y);
       p=1; x[sa[0]]=0;
        for(i=1;i<n;i++)</pre>
           x[sa[i]]=y[sa[i]]==y[sa[i-1]] && y[sa[i]+k]==y[sa[i-1]+k]?p-1:p++;
        if(p>=n) break;
       m=p;
    }
}
void construct_lcp(string S,int *sa,int *lcp)
    int n=S.length();
    for(int i=0;i<=n;i++) r[sa[i]]=i;</pre>
    int h=0;
    lcp[0]=0;
    for(int i=0;i<n;i++)</pre>
        int j=sa[r[i]-1];
        if(h>0) h--;
        for(;j+h<n&&i+h<n;h++)</pre>
           if(S[j+h]!=S[i+h]) break;
        lcp[r[i]-1]=h;
   }
}
int main()
    cin>>S;
    n=S.size();
    construct_sa(S,sa);
    construct_lcp(S,sa,lcp);
```

```
int cnt=0;
return 0;
}
```

#### 6.6 SA-IS

```
#include<bits/stdc++.h>
#define MAXN 1000000
#define L_TYPE 0
#define S_TYPE 1
using namespace std;
inline bool is_lms_char(int *type, int x) {
   return x > 0 && type[x] == S_TYPE && type[x - 1] == L_TYPE;
inline bool equal_substring(int *S, int x, int y, int *type) {
       if (S[x] != S[y])
           return false;
       x++, y++;
   } while (!is_lms_char(type, x) && !is_lms_char(type, y));
   return S[x] == S[y];
inline void induced_sort(int *S, int *SA, int *type, int *bucket, int *lbucket,int *sbucket, int
    n, int SIGMA)
   for (int i = 0; i <= n; i++)</pre>
       if (SA[i] > 0 \&\& type[SA[i] - 1] == L_TYPE)
           SA[lbucket[S[SA[i] - 1]]++] = SA[i] - 1;
   for (int i = 1; i <= SIGMA; i++)</pre>
       sbucket[i] = bucket[i] - 1;
   for (int i = n; i >= 0; i--)
       if (SA[i] > 0 && type[SA[i] - 1] == S_TYPE)
           SA[sbucket[S[SA[i] - 1]] --] = SA[i] - 1;
}
static int *SAIS(int *S, int length, int SIGMA)
   int n = length - 1;
   int *type = new int[n + 1];
   int *position = new int[n + 1];
   int *name = new int[n + 1];
   int *SA = new int[n + 1];
   int *bucket = new int[SIGMA];
   int *lbucket = new int[SIGMA];
   int *sbucket = new int[SIGMA];
   memset(bucket, 0, sizeof(int) * (SIGMA + 1));
   for (int i = 0; i <= n; i++)</pre>
       bucket[S[i]]++;
   for (int i = 1; i <= SIGMA; i++)</pre>
       bucket[i] += bucket[i - 1];
       lbucket[i] = bucket[i - 1];
       sbucket[i] = bucket[i] - 1;
   type[n] = S_TYPE;
   for (int i = n - 1; i >= 0; i--)
```

```
if (S[i] < S[i + 1])</pre>
       type[i] = S_TYPE;
    else if (S[i] > S[i + 1])
       type[i] = L_TYPE;
    else
       type[i] = type[i + 1];
}
int cnt = 0;
for (int i = 1; i <= n; i++)</pre>
    if (type[i] == S_TYPE && type[i - 1] == L_TYPE)
       position[cnt++] = i;
fill(SA, SA + n + 1, -1);
for (int i = 0; i < cnt; i++)</pre>
    SA[sbucket[S[position[i]]]--] = position[i];
induced_sort(S, SA, type, bucket, lbucket, sbucket, n, SIGMA);
fill(name, name + n + 1, -1);
int lastx = -1, namecnt = 1;
bool flag = false;
for (int i = 1; i <= n; i++)</pre>
   int x = SA[i];
   if (is_lms_char(type, x)) {
       if (lastx >= 0 && !equal_substring(S, x, lastx, type))
           namecnt++;
       if (lastx >= 0 && namecnt == name[lastx])
           flag = true;
       name[x] = namecnt;
       lastx = x;
   }
}
name[n] = 0;
int *S1 = new int[cnt];
int pos = 0;
for (int i = 0; i <= n; i++)</pre>
   if (name[i] >= 0)
       S1[pos++] = name[i];
int *SA1;
if (!flag)
   SA1 = new int[cnt + 1];
   for (int i = 0; i < cnt; i++)</pre>
       SA1[S1[i]] = i;
}
   SA1 = SAIS(S1, cnt, namecnt);
lbucket[0] = sbucket[0] = 0;
for (int i = 1; i <= SIGMA; i++)</pre>
    lbucket[i] = bucket[i - 1];
    sbucket[i] = bucket[i] - 1;
fill(SA, SA + n + 1, -1);
for (int i = cnt - 1; i >= 0; i--)
    SA[sbucket[S[position[SA1[i]]]]--] = position[SA1[i]];
induced_sort(S, SA, type, bucket, lbucket, sbucket, n, SIGMA);
return SA;
```

```
}
int main()
{
    return 0;
}
```

#### 6.7 Manacher

```
#include<bits/stdc++.h>
#define MAXN 10000
using namespace std;
void manacher(char str[],int len[],int n)
   len[0]=1;
   for(int i=1,j=0;i<(n<<1)-1;++i)</pre>
       int p=i>>1,q=i-p,r=((j+1)>>1)+len[j]-1;
       len[i]=r<q?0:min(r-q+1,len[(j<<1)-i]);
       while(p>len[i]-1&&q+len[i]<n&&str[p-len[i]]==str[q+len[i]])</pre>
           ++len[i];
       if(q+len[i]-1>r)
           j=i;
   }
int a[MAXN];
char str[MAXN];
int main()
   scanf("%s",str);
   int x=strlen(str);
   manacher(str,a,strlen(str));
   for(int i=0;i<2*x-1;i++)</pre>
     printf("%d ",a[i]);
}
```

#### 6.8 Suffix Automaton

```
#include<bits/stdc++.h>
#define MAXN 100005
#define INF 100000000
#define MOD 100000007
#define F first
#define S second
using namespace std;
typedef long long 11;
typedef pair<int,int> P;
struct SuffixAutomaton
   vector<map<char,int>> edges;
   vector<int> link;
   vector<int> length;
   int last;
   SuffixAutomaton(string s)
       edges.push_back(map<char,int>());
```

```
link.push_back(-1);
       length.push_back(0);
       last=0;
       for(int i=0;i<s.size();i++)</pre>
       {
           edges.push_back(map<char,int>());
           length.push_back(i+1);
           link.push_back(0);
           int r=edges.size()-1;
           int p=last;
           while(p>=0 && edges[p].find(s[i])==edges[p].end())
              edges[p][s[i]]=r;
              p=link[p];
           if(p!=-1)
              int q=edges[p][s[i]];
              if(length[p]+1==length[q]) link[r]=q;
              else
              {
                  edges.push_back(edges[q]); // copy edges of q
                  length.push_back(length[p]+1);
                  link.push_back(link[q]); // copy parent of q
                  int qq=edges.size()-1;
                  // add qq as the new parent of q and r
                  link[q]=qq;
                  link[r]=qq;
                  // move short classes pointing to q to point to q'
                  while(p>=0 && edges[p][s[i]]==q)
                  {
                      edges[p][s[i]]=qq;
                      p=link[p];
              }
           }
           last=r;
       }
       vector<int> terminals;
       int p=last;
       while(p>0)
       {
           terminals.push_back(p);
           p=link[p];
       }
   }
};
int main()
{
   return 0;
}
```