Derivation of the Employability Theorem

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Abstract

This is document demonstrates the employability theorem from a set of fairly tautological axioms, which are presupposed in quantitative career choice and career development methods.

Contents

1. Premises of the Employability Theorem

1.1. Skill Set Sufficiency Axiom

1. having the required skill level of an attribute is sufficient to perform its activities at that level

2. skills are sufficient to perform activities

Idea:

$$x \iff T_x$$

Weak Version:

$$x \geq \bar{x} \iff 1 \times T_{\bar{x}}$$

Moderate Version:

$$\begin{aligned} x \geq \bar{x} &\iff 1 \times T_{\bar{x}}; \\ x < \bar{x} &\iff f_x(x, \bar{x}) \times T_{\bar{x}} \end{aligned}$$

Strong Version:

$$\begin{aligned} x \geq \bar{x} &\iff 1 \times T_{\bar{x}}; \\ x < \bar{x} &\iff (x/\bar{x}) \times T_{\bar{x}} \end{aligned}$$

1.2. Skill Set Composition Axiom

1. rational economic agents can naturally compose multiple skills to accomplish complex tasks (i.e. tasks that require multiple skills)

2. skills are composable to perform complex activities

3. $x, y \iff T_{xy}$

4. p.s.: it does not matter *how* skills are composed to perform a complex activity. the point of the axiom is that any rational person which is also sufficiently qualified can "figure out" how to "piece together", that is compose, the required skills to perform a complex activity:

$$(x, y \iff T_{xy}) \iff (x, y \iff T_{yx}).$$

$$\begin{aligned} \sigma_i^k &= \frac{a_i^k}{\sum_{i=1}^m a_i^k} \in [0,1]; \\ \boldsymbol{\sigma_k} &= (\sigma_1^k, \ldots, \sigma_m^k); \\ \boldsymbol{\sigma} &= \begin{bmatrix} \sigma_1^1 & \ldots & \sigma_m^1 \\ \vdots & \ddots & \vdots \\ \sigma_1^n & \ldots & \sigma_m^n \end{bmatrix}. \end{aligned}$$

Actually, the strong supposition is not that of complementarity, but rather of substitution. It is less realistic to suppose strong substitution than moderate complementarity. Therefore, invert the axiom's strength. Strong Composition Axiom (Maximum Complementarity, Leontiev production function?):

$$s^h_{k,q} = \min_i \left(\boldsymbol{a_k^{\sigma_q}}\right)^{1/\sigma^q_i}$$

Moderate Composition Axiom (Moderate Complementarity, Weak Substitution, Cobb-Douglas production function):

$$s^h_{k,q} = \frac{\prod_{i=1}^m \min\left(a^k_i, a^q_i\right)^{\sigma^q_i}}{\prod_{i=1}^m {a^q_i}^{\sigma^q_i}} = \prod_{i=1}^m \min(a^k_i, a^q_i)^{\sigma^q_i} \left(\prod_{i=1}^m {a^q_i}^{\sigma^q_i}\right)^{-1} \in [0,1],$$
$$\sum_{i=1}^m \sigma^q_i = 1.$$

Moderate-Low Composition Axiom (Weak Complementarity, Moderate Substitution, Cobb-Douglas production function):

$$s^h_{k,q} = \min\left(\frac{\prod_{i=1}^m {a^k_i}^{\sigma^q_i}}{\prod_{i=1}^m {a^q_i}^{\sigma^q_i}}, 1\right) \in [0,1],$$
$$\sum_{i=1}^m \sigma^q_i = 1.$$

Weak Composition Axiom (Strong Substitution, Linear production function):

$$s^h_{k,q} = \frac{\sum_{i=1}^m a^q_i \min(a^k_i, a^q_i)}{\sum_{i=1}^m {a^q_i}^2} = \sum_{i=1}^m a^q_i \min(a^k_i, a^q_i) \left(\sum_{i=1}^m {a^q_i}^2\right)^{-1} \in [0,1],$$
$$\sum_{i=1}^m \sigma^q_i = 1.$$

Weakest Composition Axiom (Strong Substitution, Linear production function):

$$s^h_{k,q} = \min\left(\frac{\sum_{i=1}^m a^q_i a^k_i}{\sum_{i=1}^m {a^q_i}^2}, 1\right) \in [0,1],$$
$$\sum_{i=1}^m \sigma^q_i = 1.$$

Probably define production and normalization separately for looks.

$$\Lambda_{k,q} = \prod_{i=1}^m {a^k_i}^{\sigma^q_i},$$

$$\sum_{i=1}^{m} \sigma_i^q = 1.$$

$$s^h_{k,q} = \tilde{\Lambda}_{k,q} = \min\left(\frac{\Lambda_{k,q}}{\Lambda_{q,q}}, 1\right) \in [0, 1]$$

Change employability notation? e.g.

1. output $h_{k,q}$
2. output vector $\boldsymbol{h_{k,q}}$
3. output matrix $\boldsymbol{h}$ or $\mathbf{h}$
4. h would stand for human-capital output
    4.1. and/or hours of equivalent labor produced
    4.2. i.e. percentage of the output a perfectly qualified q worker produces in a time period
5. it would then make more sense to call "production similarity" $s^h_{k,q} = \tilde{h}_{k,q} = h_{k,q}/h_{q,q} = \frac{h_{k,q}}{h_{q,q}}$
6. and it would be even more pedantic to call H not H, but uppercase ETA

or e.g.

1. output $Y_{k,q}$
2. output vector $\boldsymbol{Y_k}$
3. output matrix $\mathbf{Y}$

1.3. Occupational Reducibility Axiom

1. from a practical standpoint, occupations are just a collection of job activities
2. occupations can be reduced to their job activities
3. $\boldsymbol{q} \iff \boldsymbol{T_q}$

1.4. Occupational Skill-Sufficiency Corollary

1. from axioms: Reducibility, Sufficiency
2. as occupations are just a collection of job activities and skills are sufficient to perform activities, therefore occupations are just a collection of skills
3. occupations can be reduced to their skill set
4. $\boldsymbol{q} \iff \boldsymbol{T_q} \iff \boldsymbol{a_q}$

2. Demonstration of the Employability Theorem

3. Additional Corollaries

3.1. Occupational Divisibility Corollary 1 (Suboccupations)

1. from axioms: Reducibility, Sufficiency, Composition

2. as occupations are just a collection of simple and complex job activities, and skills are sufficient to perform activities, and rational agents can naturally compose skills to accomplish complex activities, therefore occupations can be "broken down" into "suboccupations", each with their own subset of the "main" occupation's activities and respective skill "subsets"

3. occupations can be divided into suboccupations, each with its own skill subset

4. $\boldsymbol{q} \iff \boldsymbol{T_q}$ and $\boldsymbol{T_q} = \{T_x, T_y, T_z, ..., T_{xy}, T_{yx}, T_{xz}, T_{xzy}\} \implies$
   $\boldsymbol{T_{q_{xyz}}} = \{T_x, T_y, T_z, ..., T_{xy}, T_{yx}, T_{xz}, T_{xzy}\}$,
   $\boldsymbol{T_{q_{xy}}} = \{T_x, T_y, ..., T_{xy}, T_{yx}\}$,
   $\boldsymbol{T_{q_{xz}}} = \{T_x, T_z, ..., T_{xz}\}$,
   $\boldsymbol{T_{q_{yz}}} = \{T_y, T_z, ...\}$,
   $\boldsymbol{T_{q_x}} = T_x$,
   $\boldsymbol{T_{q_y}} = T_y$,
   $\boldsymbol{T_{q_z}} = T_z$.

5. notice: we are not saying *how* an occupation's complex activities are composed. what the corollary states is only that an occupation, which is just a collection of activities, can *potentially* be divided into suboccupations, based on how their job activities are composed.

3.2. Occupational Divisibility Corollary 2 (Skill Subsets)

1. from corollary: Divisibility 1

2. from axioms: Reducibility, Sufficiency, Composition

3. to the extent to which an occupation is divisible, a sufficiently qualified skill set does not need to be a perfect match with an occupation's entire skill set in order to perform at least a portion of their job activities

4. partial matching: people can perform isolated subsets of an occupation's job activities

5. $\boldsymbol{q} \iff \boldsymbol{T_q}$ and $\boldsymbol{T_q} = \{T_x, T_y, T_z, ..., T_{xy}, T_{yx}, T_{xz}, T_{xzy}\} \implies$
   $\boldsymbol{T_{q_{xyz}}} = \{T_x, T_y, T_z, ..., T_{xy}, T_{yx}, T_{xz}, T_{xzy}\} \implies (x, y, z \implies \boldsymbol{T_{q_{xyz}}})$,
   $\boldsymbol{T_{q_{xy}}} = \{T_x, T_y, ..., T_{xy}, T_{yx}\} \implies (x, y \implies \boldsymbol{T_{q_{xy}}})$,
   $\boldsymbol{T_{q_{xz}}} = \{T_x, T_z, ..., T_{xz}\} \implies (x, z \implies \boldsymbol{T_{q_{xz}}})$,
   $\boldsymbol{T_{q_{yz}}} = \{T_y, T_z, ...\} \implies (y, z \implies \boldsymbol{T_{q_{yz}}})$,
   $\boldsymbol{T_{q_x}} = T_x \implies (x \implies \boldsymbol{T_{q_x}})$,
   $\boldsymbol{T_{q_y}} = T_y \implies (y \implies \boldsymbol{T_{q_y}})$,
   $\boldsymbol{T_{q_z}} = T_z \implies (z \implies \boldsymbol{T_{q_z}})$.

6. again, we are not stating *how* an occupation is divided, only that, because of skill set sufficiency, composition, and occupational reducibility, occupations are, therefore, potentially divisible into suboccupations (with skill subsets), and to the extent to which they are divisible, therefore, any sufficiently qualified person can perform the skill subset for which they are qualified

3.3. Occupational Divisibility Corollary 3 (Partial Employment)