Human Capital Flexibility

Quantifying professional competencies's amplitude of applicability

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2.1 Preliminary comparison of distributions

2.2 Preliminary equations

2.2.1 Bounds for percentage data metrics

$$\mu = \frac{1}{n}\sum_{i=1}^{n} x_i$$

If $x_i \in [0,1]$ holds for all i, then $\frac{1}{n}\sum_{i=1}^{n} x_i \in [0,1]$ as well. Likewise, the median (Q_2) and the mode (M) cannot fall away of the scale of the data from which they are calculated. Thus, when $x_i \in [0,1] \forall i$, μ, Q_2, and M are all contained within this very same interval.
In addition, the above implies that the maximum value of the population standard deviation (σ) is 0.5, while that of the sample approximation (s) is around 0.71. These results are trivial and can be verified by estimating both metrics on a vector containing only the lower and upper limits of the scale in question, thus yielding the maximum possible dispersion within it. In the case of strict zero-to-one percentage scales, such as the ones we're interested in studying, this vector is simply $(0,1)$. Therefore, the bounds of the first and second moments of the data we're modeling are known beforehand.
Finally, the third moment, or skewness (S_k), although not as neatly bound as the previous metrics, can have predictable intervals, depending on the method of estimation. For instance, Pearson's (citations) first coefficient of skewness is one such measurement, as

$$S_{kp_1} = \frac{\mu - M}{\sigma} \implies S_{kp_1} \in [-1,1].$$

Pearson's second coefficient, on the other hand, is not bound at a closed interval:

$$S_{kp_2} = \frac{3(\mu - Q_2)}{\sigma} \implies S_{kp_1} \in [-1,1].$$

However, the nonparametric skew, which is simply S_{kp_2} divided by 3, is bound at the same interval as S_{kp_1}.

At first glance, Bowley's (1901, 1912) coefficient of skewness is also a good candidate for estimating capital flexibility as a percentage, since

$$S_{kb} = \frac{Q_3 + Q_1 - 2Q_2}{Q_3 - Q_1}$$

and even further generalizations (cite Kelly's metric; Groeneveld; Meeden, 1984), all satisfy the desired constraint of being contained in a known interval.

That said, all of these metrics fail at different aspects. Firstly, as made clear in the previous exercise of visualization, capital flexibility is a function of dispersion. And both Pearson's coefficients of skewness involve dividing by σ, making it, in a sense, redundant. In addition, capital flexibility ought to be defined for whatever value of σ, even zero,

2.2.2 Initial idea of mathematical formulation

$$k_f(S_k, \sigma) = \frac{1 - S_k(1-\sigma)}{2}, S_k \in [-1,1], \sigma \in [0, 0.5]$$

$$k_f(M, \sigma) = M(1-\sigma), M \in [0,1], \sigma \in [0, 0.5]$$

2.3 Necessary adjustments

2.3.1 Required mathematical properties

$$k_f(M, \sigma) \in [0, 1]$$

$$k_f(\alpha x) = k_f(x)$$

$$k_f(M = x_u, \sigma = 0) = 1$$

$$k_f(M = 0, \sigma = 0) = 0$$

$$k_f(M = x_u, \sigma = \sigma_u) = (1 - \lambda)x_u$$

$$k_f(M = 0, \sigma = \sigma_u) = \lambda x_u$$

$$\frac{\partial k_f(M, \sigma)}{\partial M} > 0$$

$$S_k > 0 \implies \frac{\partial k_f(M, \sigma)}{\partial \sigma} > 0$$

$$S_k < 0 \implies \frac{\partial k_f(M, \sigma)}{\partial \sigma} < 0$$

$$(Alternatively)\frac{\partial k_f(S_k, \sigma)}{\partial S_k} < 0$$

$$k_f(M, \sigma = 0) = M$$

$$M_1 + M_2 = 1, \sigma_1 = \sigma_2 \implies k_f(M = M_1, \sigma = \sigma_1) + k_f(M = M_2, \sigma = \sigma_2) = 1$$

$$(implied) k_f(M_1, \sigma = c) \geq k_f(M_2, \sigma = c) \iff M_1 \geq M_2$$

2.3.2 Sketch of capital flexibility plane (side views)

2.3.3 Sketch of capital flexibility plane

2.4 Revised mathematical formulation

2.4.1 Population Capital Flexibility for percentage data

$$k_f(M, \sigma) = 0.25 + 0.75M - 0.25\sqrt{2\sigma} - 0.25(1 - M)(1 - 2\sqrt{2\sigma})$$

$$k_f(M, \sigma) = 0.25 + 0.75M - 0.25(2\sigma) - 0.25(1 - M)[1 - 2(2\sigma)]$$

2.4.2 Sample Capital Flexibility for percentage data

$$k_f(M, s) = 0.25 + 0.75M - 0.25\sqrt{1.4142136s} - 0.25(1 - M)(1 - 2\sqrt{1.4142136s})$$

[use s]

2.4.3 Visualizing Capital Flexibility planes

2.5 Intuition for mathematical formulation

2.6 Generalization for any truncated distribution

Although the present analysis is exclusively concerned with percentage data, equations (X) and (Y) can be generalized to any kind of data for which bounds are known and defined. Thus, we could say that capital flexibility or rather, in general terms, "degree of specialization" is calculated as:

$$k_f(M, \sigma_u, x_u, \lambda) = \lambda + (1-\lambda)\frac{M}{x_u} - \lambda\sqrt{\frac{\sigma}{\sigma_u}} - \lambda(1 - \frac{M}{x_u})(1 - 2\sqrt{\frac{\sigma}{\sigma_u}}), \lambda \in [0, 1]$$

$$k_f(M, \sigma_u, x_u, \lambda) = \lambda + (1-\lambda)(M/x_u) - \lambda(\sigma/\sigma_u)^{\frac{1}{2}} - \lambda[1 - (M/x_u)][1 - 2(\sigma/\sigma_u)^{\frac{1}{2}}], \lambda \in [0, 1]$$

$$k_f(M, \sigma_u, x_u, \lambda) = \lambda + (1-\lambda)(M/x_u) - \lambda(\sigma/\sigma_u) - \lambda[1 - (M/x_u)][1 - 2(\sigma/\sigma_u)], \lambda \in [0, 1]$$

$$\sigma_u = \sigma(x_l, x_u)$$

[M calculated by the Verter, Shorths, or the Meanshift methods]

[rewrite using upper and lower bounds for sigma and mode]

It is useful to write down this generalized version (Z) of the capital flexibility estimator (X), for the concepts involved in capital flexibility have a potentially wide range of applications. Indeed, we can easily remove equation (Z) out of its initial micro-economic context, and take if for what it is: essentially, a relative measure of dispersion-adjusted mode for any given variable x, truncated at a known interval $[x_l, x_u]$. After we understand it as such, then multiple applications of this simple concept come forward. Some obvious examples include [cite obvious examples]

[psychometrics, where scale lengths are previously known]

[any analysis of percentage data]

The parameter λ is the dispersion discount factor, that is the rate at which M is adjusted by σ. It seems intuitive this parameter should not allow for "overcompensation" when discounting the mode, at least in most cases. It is possible that in some applications of equation (Z) high λ values could be appropriate, but as a rule, λ should be such that

$$1 - \lambda \geq \lambda \iff \lambda \leq 1/2$$

In other words, although greater dispersion ought to increase the final flexibility estimate in right-skewed distributions, and, alternatively, ought to decrease the score when a left skew is present, the adjustment should not be so strong as to elevate the flexibility of a very low mode value to that of a very high one. For instance, at the ends of the scale, the following should hold:

$$k_f(M = x_u, \sigma = \sigma_u, \lambda) \geq k_f(M = x_l, \sigma = \sigma_u, \lambda) \iff \lambda \leq 1/2$$

This said, the dispersion discount factor does not have to be a constant, but could be determined in relation to the other variables, leading to interesting results. If we define, say, $\lambda = \sigma/2$, then as σ increases, so does the absolute value of dispersion compensation.

[plot of plane with $\lambda = \sigma/2$]

3 Testing

3.1 Data

3.2 Parameters

3.3 Results

3.4 Discussion

4 Conclusion

Ironically then, capital flexibility itself seems to have a reasonable degree of carry-over as a mathematical instrument.