The Employability Theorem

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Abstract

In this document, the Potential Employability Theorem is demonstrated from a set of fairly tautological axioms, which are presupposed in quantitative career choice and career development methods.

Keywords: Employability theorem; Career choice; Career development; Vocational choice; Occupational Information Network; O*NET.

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1. Introduction

2. The Issue of Occupational Complexity

Now, because this series of articles has been of a more applied nature, so far, we did not have a need to expound upon our most basic assumptions. This is not a problem, for these are all quite reasonable, even tautological. However, to demonstrate the Employability Theorem, it is useful to make at least one these assumptions more explicit, as a valuable intuition towards the Theorem follows directly from a simple definition. Of course, we do not need to define things this way, as this first "stepping stone" is not actually used in the demonstration itself. [But, being a reasonable and insightful assumption, which can help us understand the issue of occupational complexity and, therefore, of employability,]. To minimize digressions, the fundamental axioms behind this definition were moved to the Appendix (for these, again, are practically tautological). With this said, we proceed to our definition.

Definition 1 (Skill)**.** A professional attribute, competency, or skill, of a person k can be conceptualized as a cummulative sum of successes on binary outcome variables representing tasks of progressive difficulty which require only that skill:

$$a_i^k = \sum_{l=0}^{l_i} T_{i_l}^k, \tag{1}$$

where

$$T_{i_l}^k = \begin{cases} 1, \text{ if } k \text{ succedes in a task } T_i^l \text{ of difficulty level } l; \\ 0, \text{ otherwise.} \end{cases} \tag{2}$$

Or, more rigorously,

$$a_i^k = \sum_{l=0}^{l_i} T(l, l_i^k), \tag{3}$$

where

$$T(l, l_i^k) = T_{i_l}^k = \begin{cases} 1, \ l \leq l_i^k; \\ 0, \text{ otherwise.} \end{cases} \tag{4}$$

and $l_i^k \in [0, l_i]$ is the maximum difficulty level on which k still succeedes. Thus, we can define a person k's skill level in an attribute i as the sum of their $T_{i_l}^k$ successful trials on a $T_i = \{T_i^0, \ldots, T_i^{l_i}\}$ set of tasks of increasing difficulty.

Furthermore, as we assume scales are truncated (i.e. there is a maximum difficulty level l_i, and a trivial difficulty level, which has to be zero), we can also interpret a_i^k as the *portion* of tasks one is able to accomplish out of all difficulty levels for that skill. By normalizing l_i to 100, for example, we have:

1. $a_i^k = 0 \iff k$ cannot perform even the most basic of attribute i's tasks;

2. $a_i^k = 10 \iff k$ can perform only the bottom 10% of attribute i's tasks, but nothing more;

3. $a_i^k = 50 \iff k$ can perform the easiest half of attribute i's tasks, but not the most difficult half;

4. $a_i^k = 100 \iff k$ can perform all of attribute i's tasks.

Finally, we can define a_i^k for a continuum of task difficulty $l \in [0, 1]$:

$$a_i^k = \int_0^1 T(l, l_i^k)dl, \tag{5}$$

where $T(l, l_i^k)$ is defined as before.

Definition 2 (Skill)**.** Alternatively, we can think of a person k's professional attribute, competency, or skill, as the difficulty of the most difficult task they can accomplish, normalized by the difficulty of the most objectively difficult task of that particular skill:

$$a_i^k = \frac{l_i^k}{l_i}, \tag{6}$$

which we normalize by setting $l_i = 1$, so that

$$a_i^k = \frac{l_i^k}{1} = l_i^k, \tag{7}$$

and $l_i^k \in [0, 1]$. With this normalization, example interpretations of a_i^k are:

1. $a_i^k = 0 \iff k$ cannot perform even the most basic of attribute i's tasks;

2. $a_i^k = 0.10 \iff k$ can only perform tasks of up to 10% the difficulty of attribute i's most difficult task, but nothing more;

3. $a_i^k = 0.50 \iff k$ can perform tasks of up to half the difficulty of attribute i's most difficult task, but nothing more;

4. $a_i^k = 1 \iff k$ can perform all of attribute i's tasks.

This is, perhaps, the most natural conceptual model for understanding competencies, as, generally, it is more intuitive to think of skill as the maximum of one's capacity, rather than the portion of tasks one could potentially accomplish.

But, again, because we assume scales to be truncated, this latter interpretation actually implies and is implied by the former. For if a task is of the same difficulty as another, then they are just as difficult in relation to that skill's most difficult task (i.e. they require the same percentage of the scale's upper limit to be performed), and, likewise, are also included in the same difficulty "bracket" (i.e. they are equivalent to the same skill test in the aggregate binary outcome interpretation), and, therefore, pressupose the same a_i^k skill level.

Of course, this equivalence is quite trivial, given that

$$\int_0^1 T(l, l_i^k)dl = 1 \times \int_0^{l_i^k} dl + 0 \times \int_{l_i^k}^1 dl = l_i^k - 0 = \frac{l_i^k}{1} = a_i^k. \tag{8}$$

This means the percentage of a skill's tasks one can accomplish is also the difficulty of the most difficult task one can accomplish relative to that skill's most difficult task.

So, however one decides to interpret skill levels, the conclusion remains the same: to be skilled in an attribute is to be able to perform the activities associated with that attribute. Put simply, the capacity to act follows virtue, for virtue is, itself, the capacity to act.

Now, even though these results are basically tautological, they are still important to guide our intuition. In fact, our first insight towards the Employability Theorem, namely the Skill Sufficiency Lemma (SSL), follows directly from the definitions above.

Lemma 1 (Skill Sufficiency Lemma)**.** According to the SSL, skills are necessary and sufficient to accomplish tasks. In particular, to have a skill level of $a_i^k \in [0, 1]$ in attribute i is a necessary and sufficient condition for one to be capable of accomplishing the easier a_i^k portion of that attribute's tasks.

Proof. By definition,

$$T(l, l_i^k) = \begin{cases} 1, & l \leq l_i^k; \\ 0, & \text{otherwise.} \end{cases} \tag{9}$$

is a binary indicator of person k's ability to accomplish a task of difficulty $l \in [0, 1]$ which requires only attribute i.

With this,

$$\tilde{T}_i^k = \int_0^1 T(l, l_i^k)dl \tag{10}$$

is the percentage of tasks requiring only attribute i that k can accomplish.

But both equivalent definitions of k's skill level in attribute i, are

$$a_i^k = \int_0^1 T(l, l_i^k)dl = l_i^k, \tag{11}$$

which is precisely the $\tilde{T}_i^k$ aggregation of $T(l, l_i^k)$ in the $[0, 1]$ interval.

Therefore, having a skill level of a_i^k is a necessary and sufficient condition to be capable of accomplishing the easier a_i^k portion of that attribute's tasks:

$$a_i^k = \tilde{T}_i^k \iff \int_0^1 T(l, l_i^k)dl = \int_0^1 T(l, l_i^k)dl. \tag{12}$$

□

Definition 3 (Complex Task)**.** A task is said to be complex if it requires more than one attribute to be accomplished. More precisely, T^l_{ij} is a complex task of attributes i and j, if its binary outcome indicator is of the form

$$T(l, l^k_{ij}) = \begin{cases} 1, & l \leq l^k_{ij}; \\ 0, & \text{otherwise.} \end{cases} \tag{13}$$

Definition 3.1 (Weak Complexity)**.** dsds

Definition 3.2 (Moderate Complexity)**.** dsds

Definition 3.3 (Strong Complexity)**.** dsds

and

$$l^k_{ij} = f(l^k_i, l^k_j) \tag{14}$$

meets the following criteria:

$$\frac{\partial f(l^k_i, l^k_j)}{\partial l^k_i} \geq 0; \tag{15}$$

$$\frac{\partial f(l^k_i, l^k_j)}{\partial l^k_j} \geq 0; \tag{16}$$

$$\lim_{l^k_i \to 0} f(l^k_i, l^k_j) = 0; \tag{17}$$

$$\lim_{l^k_j \to 0} f(l^k_i, l^k_j) = 0. \tag{18}$$

Thus, a person k's capacity to perform a complex task is weakly increasing on their capacity to perform the simple tasks of its required attributes, and goes to zero when they are unskilled in at least one of these attributes.

Additionaly, this definition can be generalized for any complex task T^l_q of m attributes, requiring an entire skill set $\boldsymbol{a_q} = (a^q_1, \ldots, a^q_m)$ to be accomplished:

$$T(l, l^k_q) = \begin{cases} 1, & l \leq l^k_q; \\ 0, & \text{otherwise.} \end{cases} \tag{19}$$

where

$$l^k_q = f(l^k_1, \ldots, l^k_m) \tag{20}$$

and

$$\frac{\partial f(l^k_1, \ldots, l^k_m)}{\partial l^k_i} \geq 0 \ \forall \ i \in \{1, \ldots, m\}; \tag{21}$$

$$\lim_{l^k_i \to 0} f(l^k_1, \ldots, l^k_m) = 0 \ \forall \ i \in \{1, \ldots, m\}. \tag{22}$$

This means a task is only strictly *complex* if it cannot be reduced to any proper subset of the attributes that it requires. For example, a task of the form

$$T(l, l_{ij}^k) = \begin{cases} 1, & l \leq l_i^k + l_j^k; \\ 0, & \text{otherwise.} \end{cases} \tag{23}$$

is not, strictly speaking, a complex task, because if person k is only skilled in either attribute i or j, they can still accomplish the task. In particular, if k has precisely zero capacity in either attribute, then T_{ij}^l collapses to unidimensional, or simple, tasks T_i^l when

$$T(l, l_{ij}^k) = \begin{cases} 1, & l \leq l_i^k + 0; \\ 0, & \text{otherwise.} \end{cases} = T(l, l_i^k), \tag{24}$$

$$\tag{25}$$

or T_j^l when

$$T(l, l_{ij}^k) = \begin{cases} 1, & l \leq 0 + l_j^k; \\ 0, & \text{otherwise.} \end{cases} = T(l, l_j^k), \tag{26}$$

in which case T_{ij}^l is not *really* complex, but rather a convolution of simple tasks. Notice this has nothing to do with whether $l_{ij}^k \in [0, 1]$. In fact, if l_{ij}^k is given by, say, $l_{ij}^k = \sigma_i l_i^k + (1 - \sigma_i) l_j^k$, with $\sigma_i, l_i^k, l_j^k \in [0, 1]$, and k has zero capacity in either attribute,

$$T(l, l_{ij}^k) = \begin{cases} 1, & l \leq \sigma_i l_i^k + 0; \\ 0, & \text{otherwise.} \end{cases} = T(l, \sigma_i l_i^k) \tag{27}$$

and

$$T(l, l_{ij}^k) = \begin{cases} 1, & l \leq 0 + (1 - \sigma_i) l_j^k; \\ 0, & \text{otherwise.} \end{cases} = T(l, (1 - \sigma_i) l_j^k) \tag{28}$$

are still simple, not complex tasks, even if they are “harder versions” of T_i^l and T_j^l, respectively, as the maximum difficulty level k can accomplish of task T_{ij}^l is only limited by both attributes, not necessarily determined by them – hence why this formulation is better thought of as two separate tasks.

Of course, this does not mean a degree of substitution between attributes completely invalidates a task’s complexity. It only means a complex task must require all of its attributes at *some* level, even if its functional form allows for substitution. However, if it does not require all of its attributes, then it should be broken down into other simple and/or complex tasks. In Economic terms, this is equivalent to saying that competencies, when used to perform complex tasks, are at least weakly complementary.

Axiom 1 (Skill Composition Axiom)**.** The Skill Composition Axiom (SCA) states that skills are weakly composable to accomplish complex tasks. More precisely, let T_{ij}^l be an activity of difficulty level l that requires both attributes i and j (i.e. T_{ij}^l is a weakly complex task). With this, we define that any rational and sufficiently qualified economic agent can naturally "piece together", that is *compose*, attributes i and j to accomplish the T_{ij}^l complex task. Or, mathematically,

$$\tilde{T}_{ij}^k = \int_0^1 T(l, l_{ij}^k) dl \tag{29}$$

is the easier portion k can accomplish in a $l \in [0, 1]$ continuum of difficulty levels of complex tasks requiring attributes i and j, given

$$T(l, l_{ij}^k) = \begin{cases} 1, & l \leq l_{ij}^k; \\ 0, & \text{otherwise.} \end{cases} \tag{30}$$

and

$$l_{ij}^k = f(l_i^k, l_j^k) = f(a_i^k, a_j^k) \tag{31}$$

$$[f(a_i^k, a_j^k) \in [0, 1]], \tag{32}$$

with

$$\frac{\partial f(a_i^k, a_j^k)}{\partial a_i^k} \geq 0, \tag{33}$$

$$\frac{\partial f(a_i^k, a_j^k)}{\partial a_j^k} \geq 0. \tag{34}$$

In addition, let us assume that attributes are not perfect complements when performing a complex task, thus allowing for some substitution. actually

$$Y_{kq} = \prod_{i=1}^{m} \min((1 + a_i^k), (1 + a_i^q))^{\sigma_i^q} \tag{35}$$

violates the definition of complex task aggregation function. perhaps define task complexity with

$$\frac{\partial^2 f(\boldsymbol{l^k})}{\partial l_i^k \partial l_j^k} \geq 0 \tag{36}$$

task complexity (weak definition):

$$\frac{\partial^2 f(\boldsymbol{l^k})}{\partial l_i^k \partial l_j^k} \geq 0 \tag{37}$$

task complexity (moderate definition):

$$\lim_{l_i^k \to 0} f(l_1^k, \ldots, l_m^k) = 0 \ \forall \ i \in \{1, \ldots, m\} \tag{38}$$

task complexity (strong definition): $l_q^k = \min(l_1^k, \ldots, l_m^k)$

"[...] the whole is something besides the parts" Aristotle 980a Metaphysics, Translated by W. D. Ross

"[...] the whole is not the same as the sum of its parts" Aristotle 100a, Topics, Translated by W. D. Ross

Axiom 2 (Occupational Reducibility Axiom)**.** Occupations can be reduced to their tasks.

Axiom 3 (Occupational Complexity Axiom)**.** All of an occupation's tasks can be thought of as one indivisible task, which mobilizes their entire skill set. We call this "holistic task" an occupation's *operation*.

Lemma 2 (Occupational Composition Lemma)**.** Skill sets are composable to accomplish occupations' operations.

Proof. □

3. Labor Market Conditions and Employer Behavior

Axiom 4 (Rationality Axiom)**.** Employers are rational and will only pay for employees to work on tasks they can accomplish. Additionaly, employers will outsource parts of an occupation's operation if their employees cannot accomplish the entire operation.

Axiom 5 (Hireability Axiom)**.** Any rational employer hires employees by evaluating a hireability statistic, which quantifies potential employees' skill set similarity with an occupation, their educational attainment, and years of experience.

Lemma 3 (Occupational Essence Lemma)**.** All of an occupation's job posts are essentially the same.

Proof. □

Axiom 6 (Labor Market Completeness Axiom)**.** There is sufficient talent in the labor market to outsource difficult tasks.

4. Task Difficulty and Time Allocation

Axiom 7 (Task Duration Axiom)**.** More difficult tasks and operations require more time to complete than easier tasks and operations.

5. The Employability Theorem

5.1. Demonstration

Potential Employability Theorem. The potential employability of a person in a particular occupation is the percentage of that occupation's operation total time duration that their skill set allows them to accomplish.

Proof. □

5.2. Corollaries

Corollary 1 (Aggregate Employability Corollary)**.** A person's employability in a certain subset of the labor market is calculated by their average employability on that subset of the labor market, weighted by each occupation's employment levels.

Proof. □

Corollary 2 (Occupational Competitiveness Corollary)**.**

Proof. □

Corollary 3 (Aggregate Competitiveness Corollary)**.**

Proof. □

6. Example Implementation

6.1. Functional Specifications

6.2. Occupational Information Network Data

6.3. Results

7. Discussion

8. Conclusion

[In this paper, we demonstrated the Employability Theorem, and its corollaries. We also proposed a few functional specifications and analyzed the potential employability and competitiveness planes which they determine. In the next article, we shall implement these methods with real data from the Occupational Information Network.]

Appendix A – Basic Definitions

Appendix B – Employability and Competitiveness Statistics