R Package Copula

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Backgrounds

- Copulas have become a widely used tool for modeling multivariate dependence in a variety of fields.

- Software implementation is important in promoting the application of copulas.

- `Splus` has a collection of functions for copulas modeling in the `finmetrics` module.

  - Commercial
  - Bivariate copulas only

- Need a platform for the development of copula methods and applications.

Copulas

- A copula C is a multivariate distribution whose margins are all uniform over $(0, 1)$:

$$C(u_1, \ldots, u_p) = \Pr(U_1 \leq u1, \ldots, U_p \leq u_p).$$

- Sklar's canonical representation theorem (1959): A multivariate joint distribution can be represented by its marginal distribution and a copula:

$$F(x_1, \ldots, x_p) = C[F_1(x_1), \ldots, F_p(x_p)].$$

  The copula is unique if the margins are continuous. Otherwise, only the sub-copulas is uniquely determined on $\operatorname{Ran} F_1 \times \cdots \times \operatorname{Ran} F_p$.

- Separates the marginals and the associations.

Why R

- Quote from `http://www.r-project.org`: "R is a free software environment for statistical computing and graphics."

- Open source.

- Compiles and runs on a wide variety of UNIX platforms, Windows and MacOS.

- Cutting-edge development; hundreds of contributed packages.

- Excellent graphics.

- Easy interface with lower level compiled code (C/C++, Fortran)

- Active developer-user interaction.

Features of the Copula Package

- Classes (S4) of commonly used copula families
  - Elliptical copulas: normal, t, Clayton, Frank, and Gumbel
  - Archimedean copulas
  - Extreme value copulas (to be implemented)
- Dimension can be greater than 2.
- Methods
  - density
  - distribution
  - random number generator
- Graphics: perspective plot, contour plot.

Load the Package

The package `copula` depends on contributed packages `mvtnorm`, `scatterplot3d`, and package `sn`.

```
> library(copula)

Loading required package: mvtnorm
Loading required package: scatterplot3d
Loading required package: sn
```

Copula Objects: Elliptical

- An object of class `normalCopula` can be created by

```
> n.cop <- normalCopula(param = c(0.9, 0.5,
+       0.2), dim = 3, dispstr = "un")
```
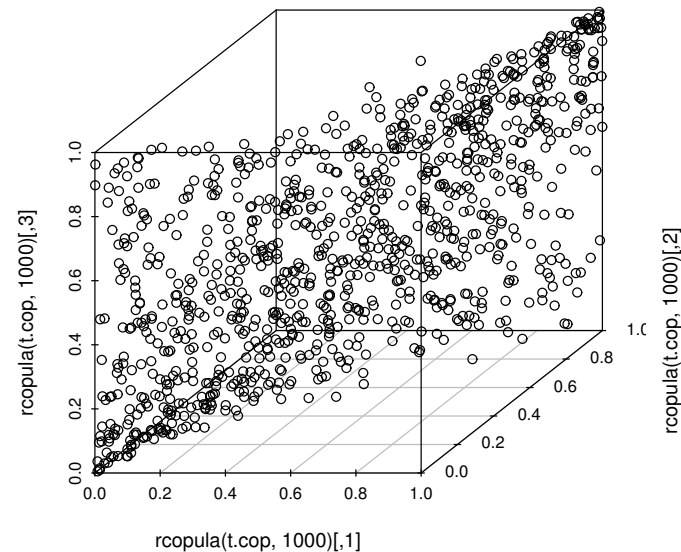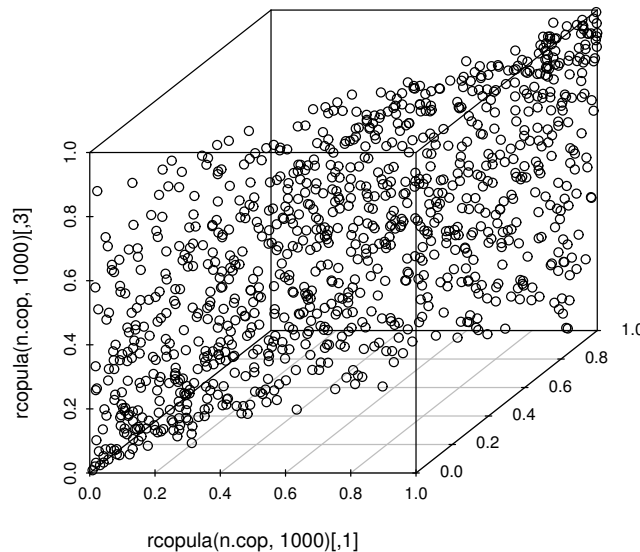
or, since `normalCopula` inherits `ellipCopula`, by

```
> n.cop <- ellipCopula(family = "normal",
+       param = c(0.9, 0.5, 0.2), dim = 3,
+       dispstr = "un")
```

- An object of class `tCopula` can be created similarly with an extra argument for the degrees of freedom, `df`.

```
> t.cop <- tCopula(param = c(0.9, 0.5, 0.2),
+       df = 5, dim = 3, dispstr = "un")
```
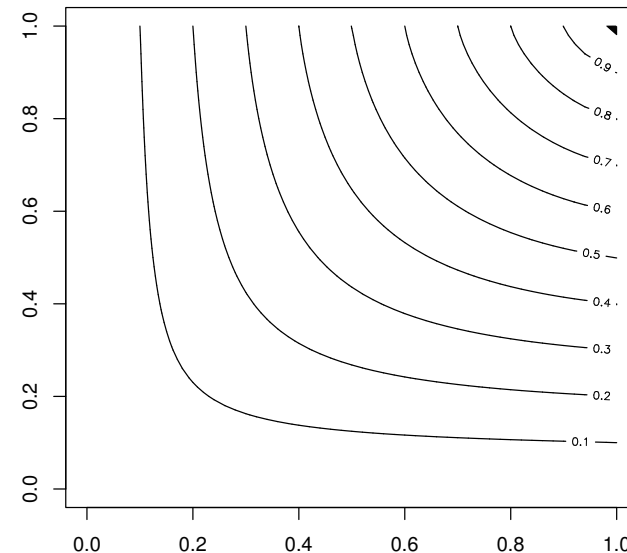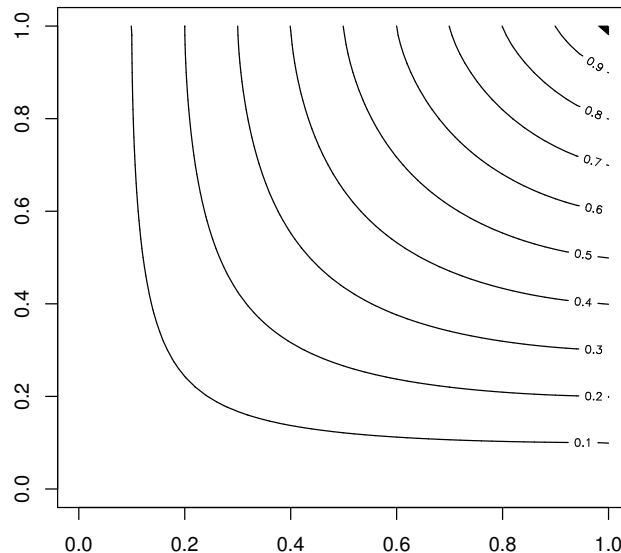
3d Scatter Plot

```
> par(mfrow = c(1, 2))
> scatterplot3d(rcopula(n.cop, 1000))
> scatterplot3d(rcopula(t.cop, 1000))
```
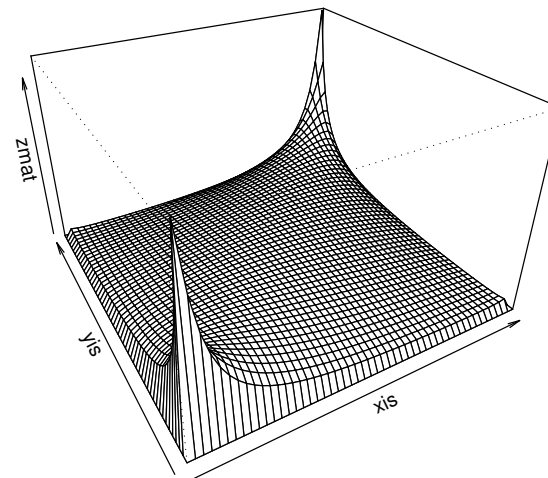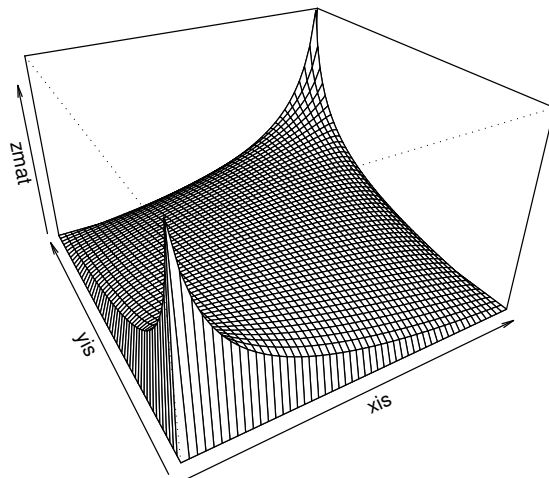
Contour Plot

```
> par(mfrow = c(1, 2))
> contour(normalCopula(0.5), pcopula)
> contour(tCopula(0.5), pcopula)
```

Perspective Plot

```
> par(mfrow = c(1, 2))
> persp(normalCopula(0.5), dcopula)
> persp(tCopula(0.5), dcopula)
```

Copula Objects: ArchmCopula

Creating Archimedean copulas:

```
> ccop <- claytonCopula(3, dim = 3)
> gcop <- gumbelCopula(10, dim = 3)
```

Expressions for the pdf and cdf of an Archimedean copula are obtained symbolically.

```
> ccop@exprdist$pdf

(1 + (u1^(-alpha) - 1 + u2^(-alpha) - 1 + u3^(-alpha) - 1))^(((((-1/alpha) -
    1) - 1) - 1) * (((((-1/alpha) - 1) - 1) * (u3^((-alpha) -
    1) * (-alpha))) * ((((-1/alpha) - 1) * (u2^((-alpha) - 1) *
    (-alpha))) * ((-1/alpha) * (u1^((-alpha) - 1) * (-alpha)))
```

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Multivariate Distribution Via Copula

```
> mv.norm <- mvdc(normalCopula(0.5), c("norm",
+     "gamma"), list(list(mean = 0, sd = 2),
+     list(rate = 2, shape = 2)))
> x <- rmvdc(mv.norm, 1000)
> x.dens <- dmvdc(mv.norm, x)
> x.cdf <- pmvdc(mv.norm, x)
> mv.gumb <- mvdc(gumbelCopula(2), c("norm",
+     "gamma"), list(list(mean = 0, sd = 2),
+     list(rate = 2, shape = 2)))
> mv.fran <- mvdc(frankCopula(5.735), c("norm",
+     "gamma"), list(list(mean = 0, sd = 2),
+     list(rate = 2, shape = 2)))
```
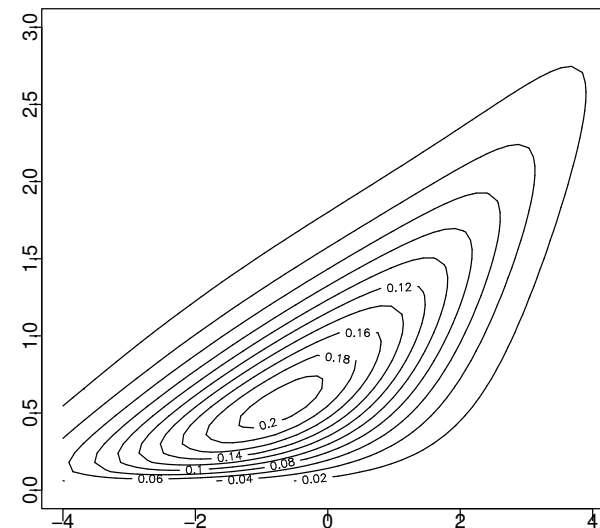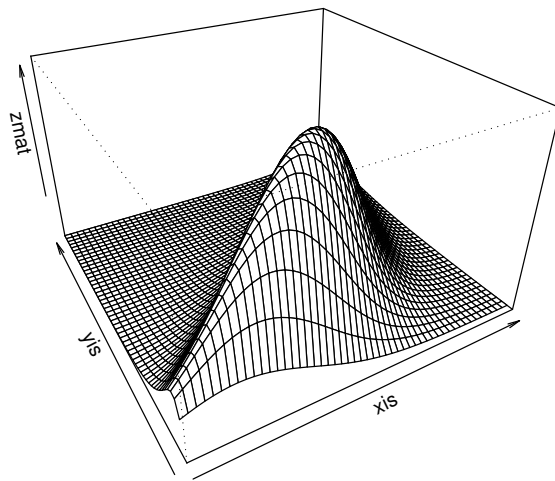
Multivariate Dependence: normalCopula

```
> par(mfrow = c(1, 2), mgp = c(0, 0, 0))
> persp(mv.norm, dmvdc, xis = seq(-4, 4,
+      len = 51), yis = seq(0, 3, len = 51))
> contour(mv.norm, dmvdc, xis = seq(-4,
+      4, len = 51), yis = seq(0, 3, len = 51))
```
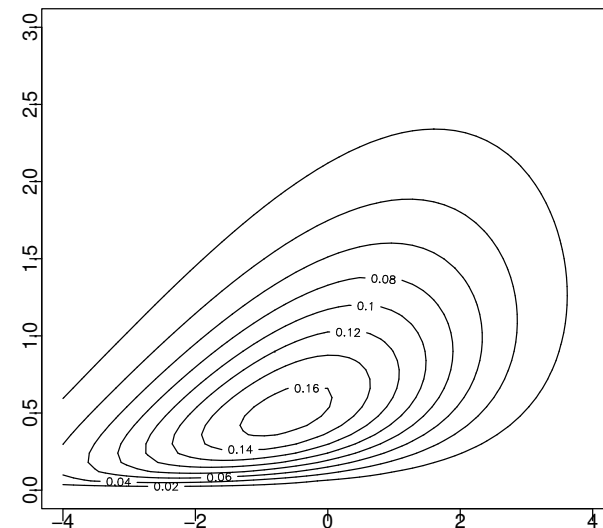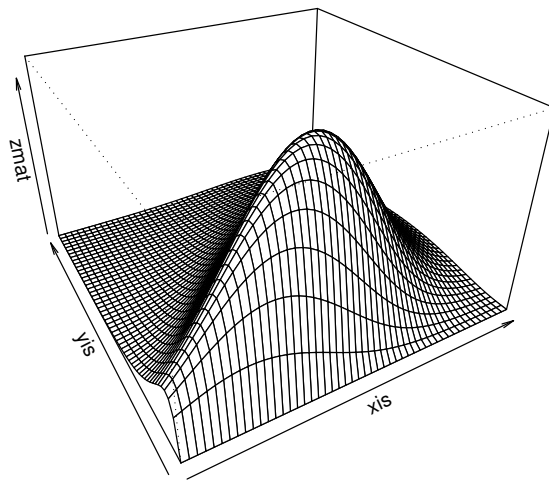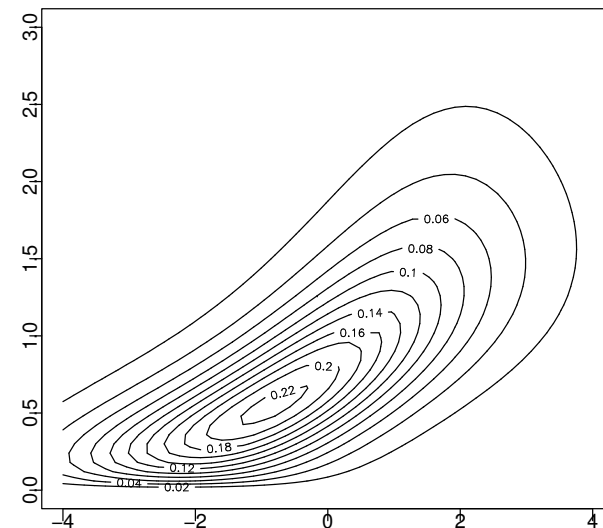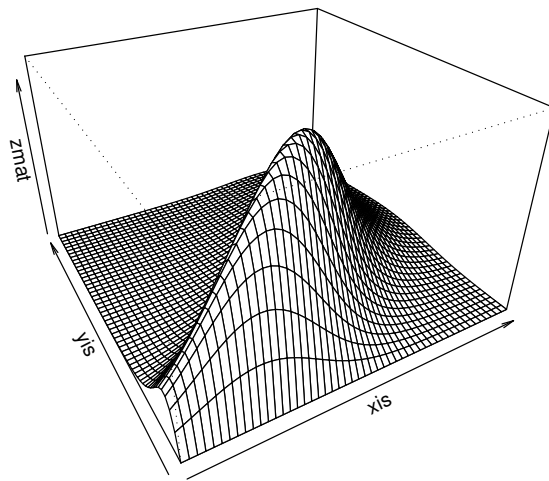
Multivariate Dependence: gumbelCopula

```
> par(mfrow = c(1, 2), mgp = c(0, 0, 0))
> persp(mv.gumb, dmvdc, xis = seq(-4, 4,
+      len = 51), yis = seq(0, 3, len = 51))
> contour(mv.gumb, dmvdc, xis = seq(-4,
+      4, len = 51), yis = seq(0, 3, len = 51))
```

Multivariate Dependence: frankCopula

```
> par(mfrow = c(1, 2), mgp = c(0, 0, 0))
> persp(mv.fran, dmvdc, xis = seq(-4, 4,
+       len = 51), yis = seq(0, 3, len = 51))
> contour(mv.fran, dmvdc, xis = seq(-4,
+       4, len = 51), yis = seq(0, 3, len = 51))
```

Future Work

- Graphical diagnosis to select copula

- Stochastic volatilities

- Dynamic copulas

- Non-symmetric dependence structure

- Extreme value copulas

- Association and tail dependence

- Potential collaborators: please contact me.

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