

# Information Security

## Asymmetric encryption

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## Objective

- ∞ Asymmetric encryption
- ∞ Modular arithmetic
- ∞ RSA
- ∞ Diffie-Hellman Key Exchange
- ∞ Message Authentication - MAC
- ∞ Cryptographic Hash Functions
- ∞ Digital Signatures

# Asymmetric encryption

- ∞ **Asymmetric encryption** is a form of cryptosystem in which *encryption* and *decryption* are performed using the different keys
  - a **public key**
  - a **private key**.
- ∞ It is also known as *public-key encryption*

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# Private – key Cryptography

- ∞ Private – key Cryptography also called **secret/ single key/ symmetric/ conventional**.
- ∞ It uses **ONLY ONE** key shared with both Recipient and Sender.
- ∞ *Private– key Cryptography looks like sealed box with message inside.*
- ∞ Private – key Cryptography's disadvantages:
  - Needs of **secure channel** to exchange keys
  - Each pair of users have to share one secret key. So the number of keys for N users should be  $N(N-1)/2$  : **so many keys!**
  - Solution: **Using Public – key Cryptography**

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# Public – key Cryptography

∞ Cryptography with **public – key/2 keys/asymmetric** uses **TWO** keys that have one owner:

- **Public - key**,
  - everyone can know and
  - use to **encrypt the message** or
  - to **check the signature** of key's owner.
- **Private – key**:
  - only owner knows and
  - use to **decrypt the message** or
  - to **create the signature**

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# Public – key Cryptography

∞ In **asymmetric cryptography**, role of sender and recipient are **not** same:

- *Person who encrypt message either check the signature*
- *that can not be decrypted or create the signature.*

∞ Mathematical basis: *One-way functions*

- *$y = f(x)$  is the one – way function if  $y = f(x)$  is easy to calculate but  $x = f^{-1}(y)$  is difficult to find*
- *$x = f^{-1}(y)$  might be easy to calculate if given additional information (key)*

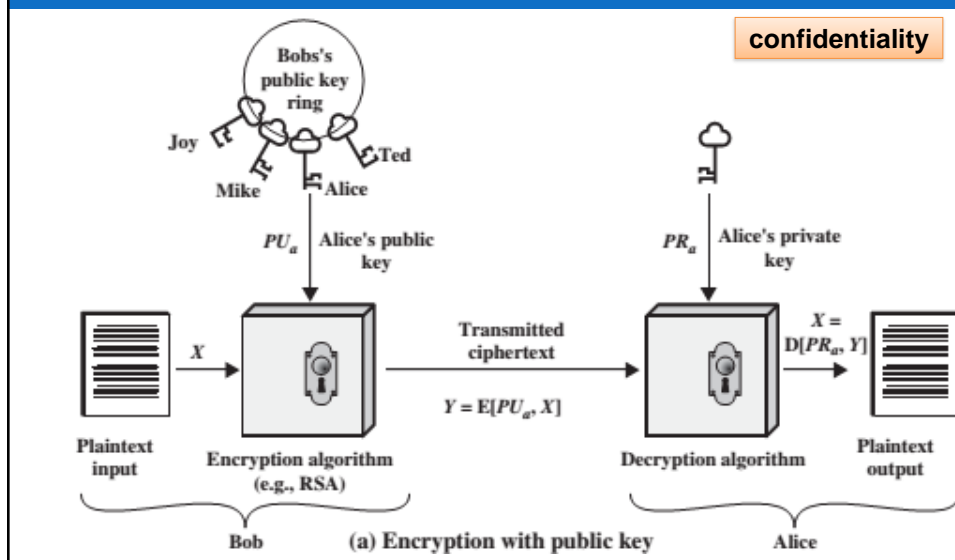
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## Public key theory

- ∞ **Public key** can be calculated from **private key** and other information of cryptography (**P problem**)
- ∞ However, if knowing the public key and the ciphertext cannot calculate the private key (**NP problem**)
- ∞ Public key needs to be distributed safely for everyone, who needs securely send message to key's owner
- ∞ Problem of public key distribution is important – that is **key distribution problem**

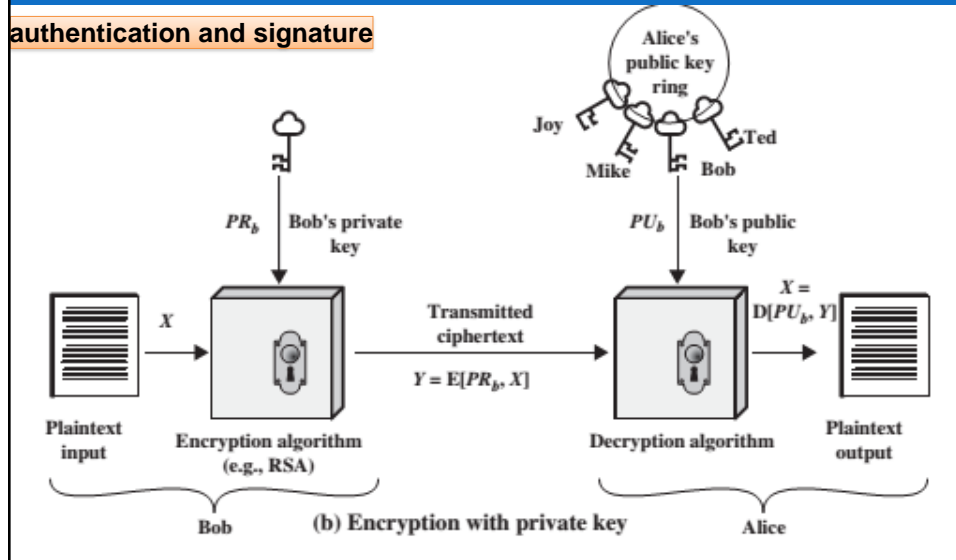
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## Encryption with public key

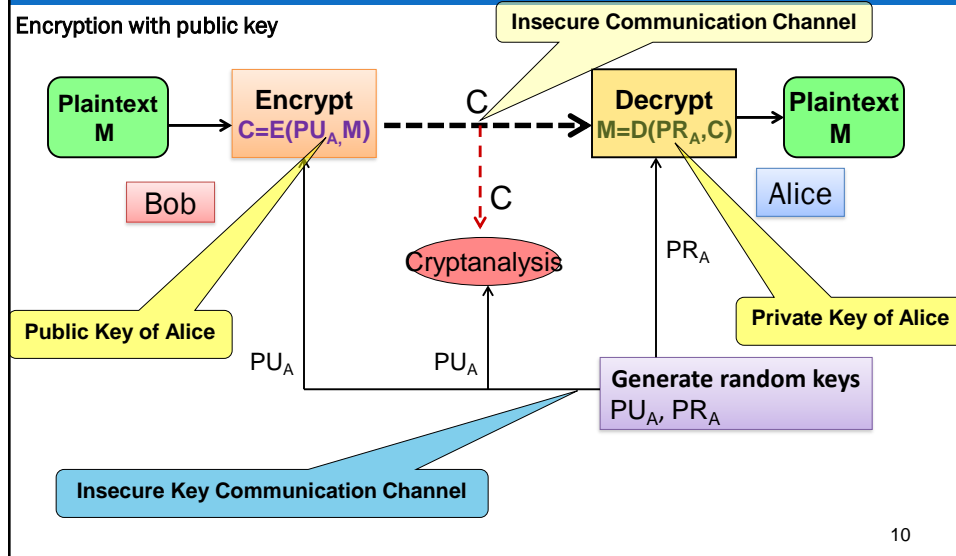


# Encryption with private key

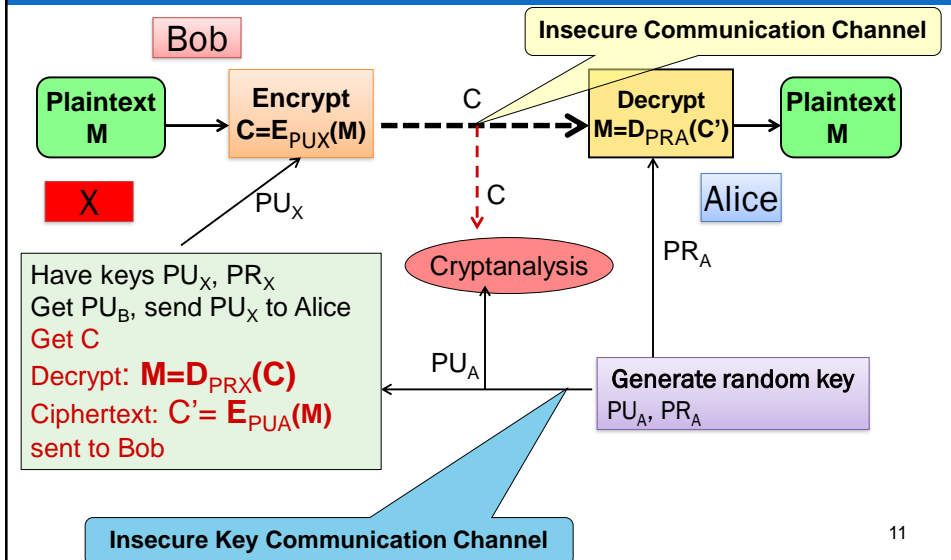
## authentication and signature



# Public – key Cryptography: Model



## Public – key Cryptography: Model



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## Public key cryptography Security

- ⌘ Security based on **the difference** between the hardness of encryption/decryption problem (**easy**) and cryptanalysis problem (**hard**)
- ⌘ Cryptanalysis using **key exhaustive key search** is always done theoretically. But in fact, the number of used keys is too large for it ( $>512$  bit)
- ⌘ To resist some other **advanced cryptanalysis methods**, need to use **the very large keys** ( $>512$  bit)
- ⌘ Therefore implementation of public key cryptography is much slower than the secret key cryptography

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# Problems of public key cryptography

## Encryption using public key:

- Using to encrypt message then sent it to key owner
- Everyone can use **public key to encrypt**
- The owner uses **private key to decrypt**

- => **Ensuring the confidentiality of message**

## Encryption using private key:

- Using to create signature for message
- Owner uses **private key** to sign message
- Everyone uses **public key to check the signature**

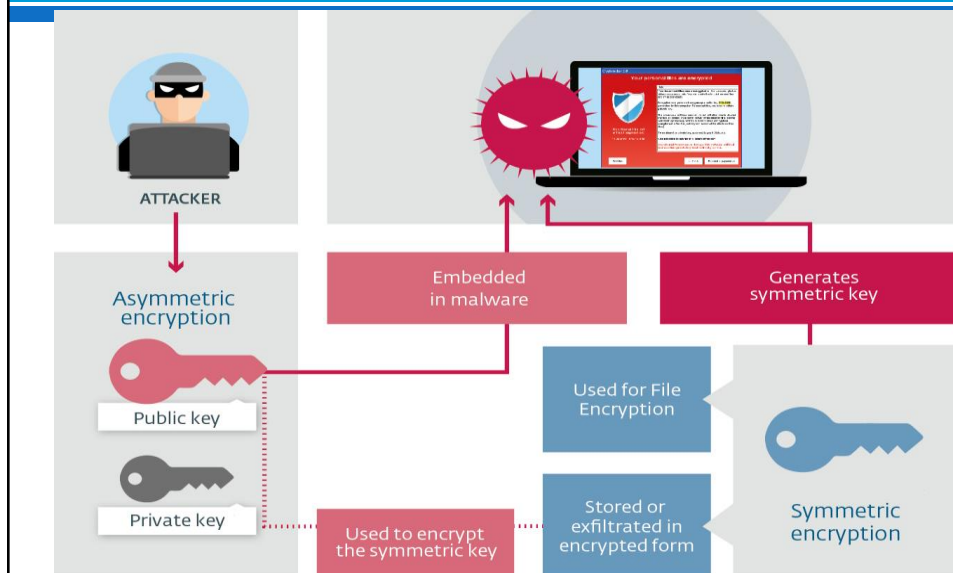
- => **Ensuring the authentication of the message**

## Public Key Distribution Scenario (PKDS):

- Methods of **public key distribution**
- Using **PK cryptography to exchange private keys**

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# Ex, Symmetric /Asymmetric encryption



# Modular Arithmetic

- ∞ Modulo, modulus
- ∞ Congruence modulo
- ∞ Properties (addition, subtraction, multiplication, exponentiation)
- ∞ Modular inverse (additive, multiplicative)
- ∞ Totient (Euler's phi) function

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## Modular, congruence



- ∞ In mathematics, **modular arithmetic** is a system of arithmetic for integers, where numbers "wrap around" upon reaching a certain value—the **modulus** (plural **moduli**).
- ∞ The modern approach to modular arithmetic was developed by Carl Friedrich Gauss
- ∞ Ex: Instead of  $13 = 1$ .
  - write  $13 \equiv 1 \pmod{12}$  and read it "13 is congruent to 1 modulo 12" or, to abbreviate, "13 is 1 modulo 12".
- ∞ Examples:
  - $12 \equiv 0 \pmod{12}$ ;  $17 \equiv 5 \pmod{12}$
  - $37 \equiv 1 \pmod{12}$ ;  $-1 \equiv 11 \pmod{12}$
- ∞ In general,  $a \equiv b \pmod{n}$  if  $a-b$  is a multiple of  $n$ .  
Equivalently,  $a \equiv b \pmod{n}$  if  $a$  and  $b$  have the same remainder when divided by  $n$  (remainder modulo  $n$ )

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# Properties

(addition, subtraction, multiplication, exponentiation)

∞ mod  $n = 6$

- $4 + 3 = 7 - 6 = 1$
- $3 - 5 = -2 + 6 = 4$
- $4 * 5 = 20 \% 6 = 2$
- $5^{13} \pmod{6}$ 
  - $5 - 6 \equiv -1, -1^{13} \equiv -1 \pmod{6}$

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## Associated with addition, multiplicative

∞ Additive

$$\begin{array}{llll} \text{If} & a \equiv b \pmod{m} & \text{and} \\ & c \equiv d \pmod{m}, & \text{then} \\ & a + c \equiv b + d \pmod{m}. \end{array}$$

∞ Multiplicative

$$\begin{array}{llll} \text{If} & a \equiv b \pmod{m} & \text{and} \\ & c \equiv d \pmod{m}, & \text{then} \\ & a \times c \equiv b \times d \pmod{m}. \end{array}$$

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## Ex, Additive

### Examples:

- $7 + 8 \equiv 3 \pmod{12}$ ;  $10 + 2 \equiv 0 \pmod{12}$   
 $13 + 2 \equiv 3 \pmod{12}$ ;  $-1 + 14 \equiv 1 \pmod{12}$

### Ex, $19 + 23 + 15 \equiv ? \pmod{12}$

- First replace each number by its remainder mod 12:  
 $7 + 11 + 3$ , then do the sum: 21
- and replace the sum by its remainder modulo 12: 9  
 $19 + 23 + 15 \equiv 9 \pmod{12}$

### If today is Sunday, what day will it be in 1000 days?

We need to find the remainder of 1000 when divided by 7

- $1000 = 700 + 280 + 20$

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## ex

### Examples multiplicative:

- $7 \times 4 \equiv 8 \pmod{10}$ ;
- $19 \times 28 \equiv 2 \pmod{10}$ :  $9 \times 8 = 72 \equiv 2 \pmod{10}$
- $-2 \times 6 \equiv 8 \pmod{10}$

### Ex, exponentiation

- $x = 3^2 \pmod{13}$ .  $x = 3^2 \pmod{13} = 9$
- $x = 3^{10} \pmod{13} = 3^4 \cdot 3^4 \cdot 3^2 \pmod{13} = 9^2 \cdot 9 \pmod{13} = (6 \cdot 13 + 3) \cdot (6 \cdot 13 + 3) \cdot 9 \pmod{13} = 3 \cdot 3 \cdot 9 \pmod{13} = 3$

### Ex, exponentiation: $17^2 \pmod{19}$

- $17^2 = 289$ , divide that by 19 and then take the remainder 4.
- However, since we know that  $17 \equiv -2 \pmod{19}$ , we can multiply this congruence equation by itself to obtain  $17^2 \equiv -2^2 \equiv 4 \pmod{19}$ .

### Ex: $x = 18^{489391312} \pmod{19}$

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## Greatest Common Divisor (GCD)

- ∞ GCD: a common problem in number theory
- ∞ GCD (a,b) of a and b is the largest number that divides evenly into both a and b
- ∞ Ex,  $\text{GCD}(60,24) = 12$
- ∞ often want no common factors (except 1) and hence numbers are relatively prime
- ∞ Ex,  $\text{GCD}(8,15) = 1$ , hence 8 & 15 are relatively prime

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## Euclidean Algorithm

- ∞ an efficient way to find the GCD(a,b):  
 $\text{GCD}(a,b) = \text{GCD}(b, a \bmod b)$
- ∞ Euclidean Algorithm to compute GCD(a,b) is:  
`EUCLID (a, b)`
  1. `A = a; B = b`
  2. `if B = 0 return A = gcd(a, b)`
  3. `R = A mod B`
  4. `A = B`
  5. `B = R`
  6. `goto 2`
- ∞ Example GCD(1970,1066)

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# Fermat's Theorem

∞  $a^{p-1} \equiv 1 \pmod{p}$

- where  $p$  is prime and  $\gcd(a,p)=1$
- also known as Fermat's Little Theorem
- Ex:  $2^{16} \equiv 1 \pmod{17}$ .
- Ex:  $2^{50} \equiv ? \pmod{17}$   
 $50 = (16 \times 3) + 2 \Rightarrow 2^{(16) \times 3} \times 2^2 = 1^3 \times 4 \equiv 4 \pmod{17}$

∞ also  $a^p \equiv a \pmod{p}$

- useful in public key and primality testing
- Ex, if  $a = 2$  and  $p = 7$ , then:  
 $2^7 = 128$ , and  $128 - 2 = 126 = 7 \times 18$  is an integer multiple of 7.

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# Euler Totient Function $\phi(n)$

∞ Euler Totient Function  $\phi(n)$

- when doing arithmetic modulo  $n$
- **complete set of residues** is:  $0..n-1$
- **reduced set of residues** is those numbers (residues) which are relatively prime to  $n$
- eg for  $n=10$ ,
  - complete set of residues is  $\{0,1,2,3,4,5,6,7,8,9\}$
  - reduced set of residues is  $\{1,3,7,9\}$
- number of elements in reduced set of residues is called the **Euler Totient Function  $\phi(n)$**

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## Euler Totient Function $\phi(n)$

- ✧ to compute  $\phi(n)$  need to count number of residues to be excluded
  - in general need prime factorization, but
    - for  $p$  ( $p$  prime):  $\phi(p) = p-1$
    - for  $p.q$  ( $p, q$  prime):  $\phi(pq) = (p-1) \times (q-1)$
  - eg.
    - $\phi(37) = 36$
    - $\phi(21) = (3-1) \times (7-1) = 2 \times 6 = 12$

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## RSA (Rivest, Shamir, Adleman)

- ✧ RSA is a well – known and widely popular public key cryptography.
- ✧ Firstly published by the authors in 1977 (MIT)
- ✧ **Its based on exponentiation on Galos' Field of the integers of modulo prime number**
  - Exponentiation has complexity  $O((\log n)^3)$  (**easy**)
- ✧ RSA security is based on hardness of the factor analysis and the discrete logarithm problem:
  - Analysis problem has complexity  $O(e^{\log n \log \log n})$  (**difficult**)
  - Similarly, discrete logarithm is very hard
- ✧ RSA has been copyrighted in North America and in some other countries.

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# RSA Algorithm

## Users create pair of public/private keys :

- Choose 2 random prime numbers  $p \neq q$  (>120 digits)
- Calculate  $N = p \times q$ ,
- Calculate  $\phi(N) = (p-1) \times (q-1)$
- Choose integer  $e$ ,  $1 < e < \phi(N)$  such as:  $\gcd(e, \phi(N)) = 1$
- Calculate  $d = e^{-1} \bmod \phi(N)$  and  $0 < d < \phi(N)$
- **Public key is the pair:  $K_u = \{e, N\}$**
- **Private key is the pair:  $K_r = \{d, N\}$**

Encryption:  $c = m^e \bmod n$ ,  $m < n$

Decryption:  $m = c^d \bmod n$

Signature:  $s = m^d \bmod n$ ,  $m < n$

Verification:  $m = s^e \bmod n$

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# RSA Example - Key Setup

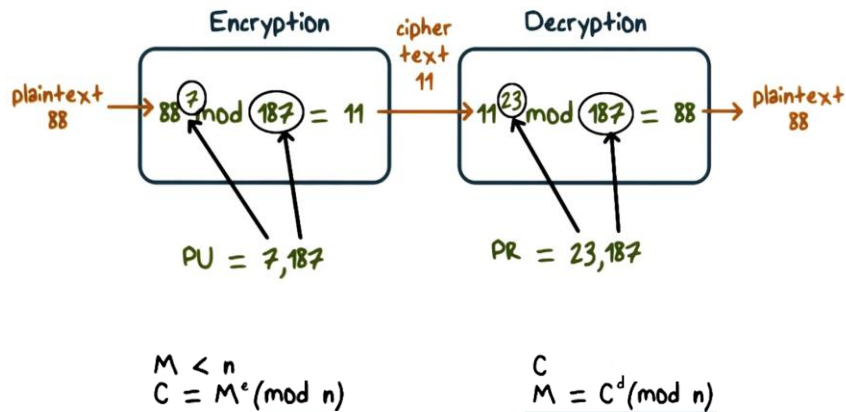


1. Select primes:  $p = 17$  &  $q = 11$
2. Calculate  $n = pq = 17 \times 11 = 187$
3. Calculate  $\phi(n) = (p-1)(q-1) = 16 \times 10 = 160$
4. Select  $e$ :  $\gcd(e, 160) = 1$ ; choose  $e = 7$
5. Determine  $d$ :  $de = 1 \bmod 160$  and  $d < 160$   
Value is  $d = 23$  since  $23 \times 7 = 161 = 1 \times 160 + 1$
1. Publish public key  $PU = \{7, 187\}$
2. Keep secret private key  $PR = \{23, 187\}$

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## Why does RSA Work?



## RSA Quiz



Fill in the text boxes:

Given  $p = 3$  and  $q = 11$

1. Compute  $n$ :  $n =$

2. Compute  $\phi(n)$ :  
 $\phi(n) =$

3. Assume  $e = 7$   
 Compute the value of  $d$ :  
 $d =$

- 4. What is the public key
- ( $e, n$ ) = (  ,  )
- 5. What is the private key
- ( $d, n$ ) = (  ,  )

## RSA Encryption Quiz

**Given:**

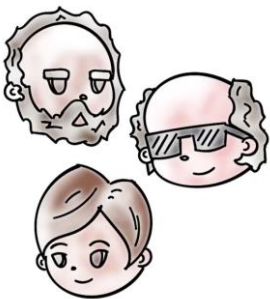
- Public key is  $(e, n) \Rightarrow (7, 33)$
- Private key is  $(d, n) \Rightarrow (3, 33)$
- Message  $m = 2$

What is the encryption of  $m$ :

What formula is used to decrypt  $m$ ?

(Use **\*\*** for denoting an exponent)

## RSA Characteristics



• **Variable key length**

• **Variable plaintext block size**

- Plaintext treated as an integer, and must be “smaller” than the key
- Ciphertext block size is the same as the key length



# RSA Security

Four possible approaches to attacking the RSA algorithm are:

1. **Brute force:** This involves trying all possible private keys.
2. **Mathematical attacks:** There are several approaches, all equivalent in effort to factoring the product of two primes.
3. **Timing attacks:** These depend on the running time of the decryption algorithm.
4. **Chosen ciphertext attacks:** This type of attack exploits properties of the RSA algorithm.

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## Why RSA is Secure?

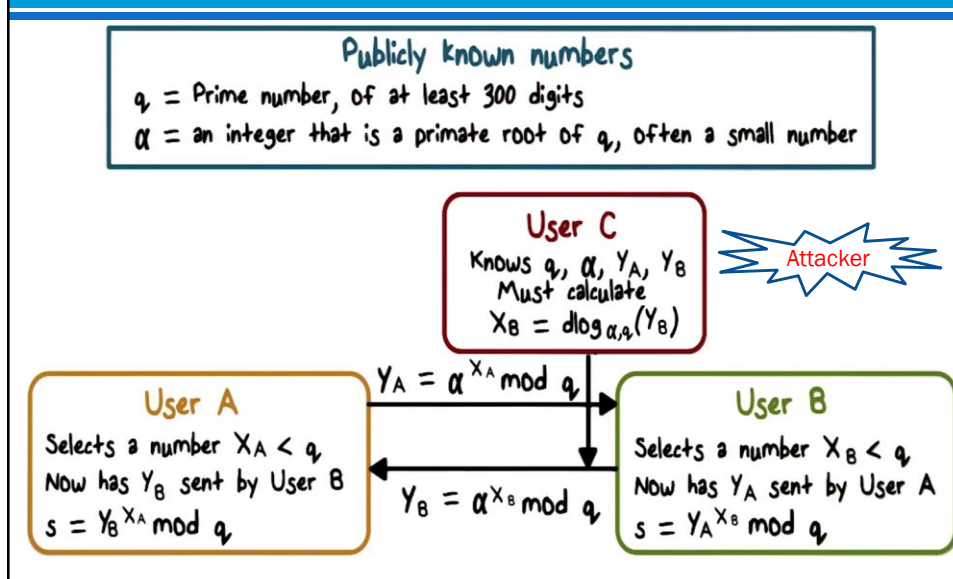


- Factoring an integer with at least 512-bit is **very hard!**
- But if you can factor big number  $n$  then given public key  $\langle e, n \rangle$ , you can find  $d$ , and hence the private key by:
  - Knowing factors  $p, q$ , such that,  $n = p \times q$
  - Then compute  $\phi(n) = (p-1)(q-1)$
  - Then find  $d$  such that  $ed \equiv 1 \pmod{\phi(n)}$

## Diffie and Hellman Key Exchange

- ❖ **First published** public-key algorithm
- ❖ By Diffie and Hellman in 1976 along with the **exposition of public key concepts**
- ❖ Used in a number of commercial products
- ❖ **Practical method to exchange a secret key** securely that can then be used for subsequent encryption of messages
- ❖ Security **relies on difficulty of computing discrete logarithms**

## Diffie and Hellman Key Exchange



## Diffie-Hellman Example

Have

- Prime number  $q = 353$
- Prime number  $\alpha = 3$

A and B each compute their public keys

- A computes  $Y_A = 3^{97} \bmod 353 = 40$
- B computes  $Y_B = 3^{233} \bmod 353 = 248$

Then exchange and compute secret key:

- For A:  $K = (Y_B)^{X_A} \bmod 353 = 248^{97} \bmod 353 = 160$
- For B:  $K = (Y_A)^{X_B} \bmod 353 = 40^{233} \bmod 353 = 160$

Attacker must solve:

- $3^x \bmod 353 = 40$  which is hard
- Desired answer is 97, then compute key as B does



## Diffie-Hellman Quiz

Alice and Bob agree to use  
prime  $q = 29$  and primitive root  $\alpha = 5$

Alice chooses secret  $a = 6$

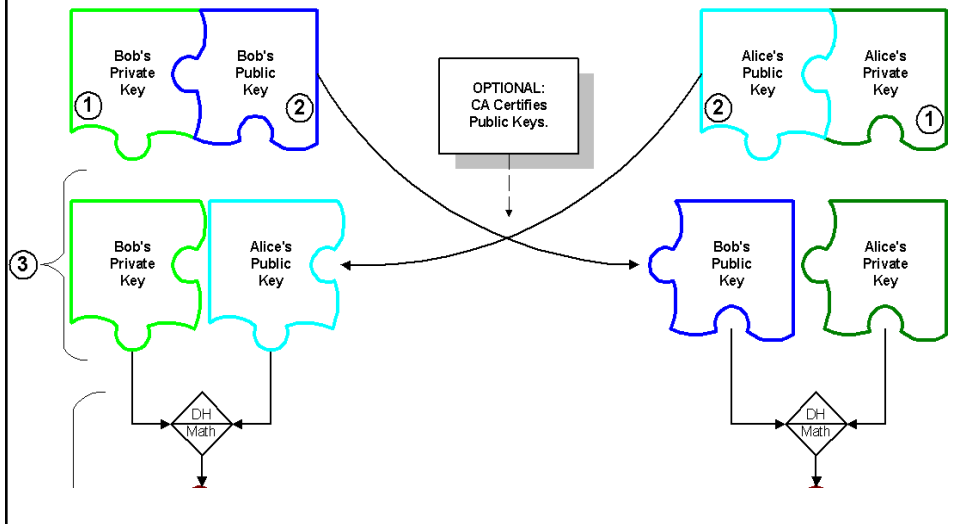
Bob chooses secret  $b = 5$

What number does Alice send Bob?

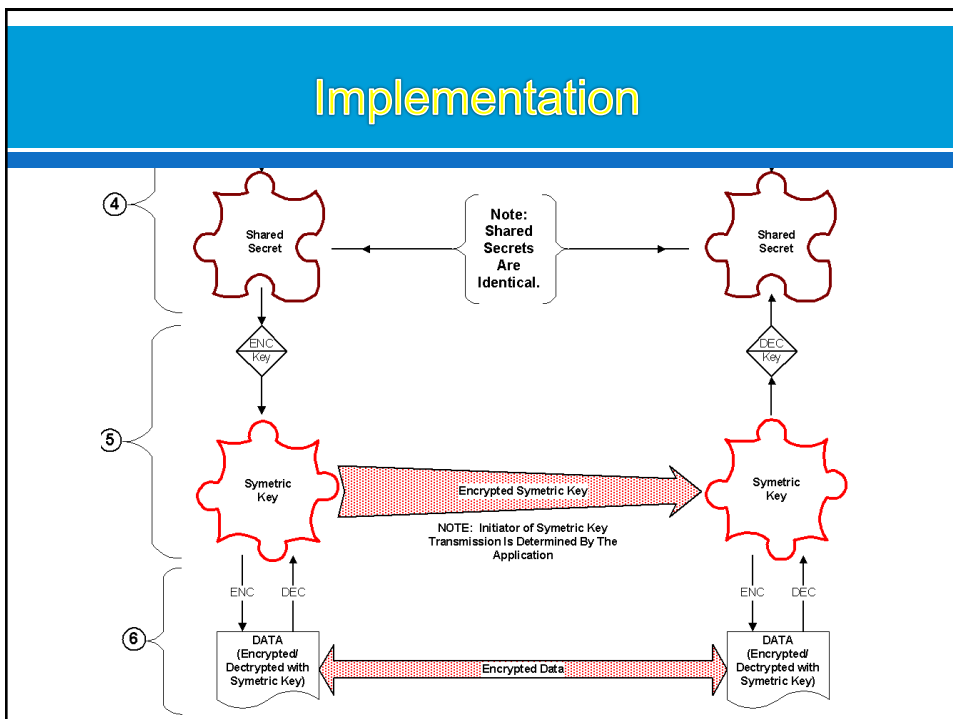
What number does Bob send Alice?

## Implementation

### Diffie-Helman Key Exchange



## Implementation



## Diffie-Hellman Security



- Shared key (the secret) itself never transmitted

- Discrete logarithm is very hard

- $Y = \alpha^x \bmod q$

- **Conjecture:** given  $Y$ ,  $\alpha$ , and  $q$ , it is extremely hard to compute the value of  $X$  because  $q$  is a very large prime (discrete logarithm)

## Applications

☞ Diffie-Hellman is currently used in many protocols, namely:

- Secure Sockets Layer (SSL)/Transport Layer Security (TLS)
- Secure Shell (SSH)
- Internet Protocol Security (IPSec)
- Public Key Infrastructure (PKI)

## Diffie-Hellman Limitations



- Expensive exponential operation
  - DoS possible
- The scheme itself **cannot be used to encrypt anything** – it is for secret key establishment
- **No authentication**, so you cannot sign anything

## Bucket Brigade Attack, Man-in-the-Middle(MIM)



$$\begin{array}{cccc}
 X_A & X_X & X_X & X_B \\
 654^{X_A} = 123^{X_X} & 255^{X_X} = 654^{X_B}
 \end{array}$$

Trudy plays Bob to Alice and Alice to Bob

## Other Public-Key Algorithms

### Digital Signal Standard:



- Makes use of SHA-1 and the Digital Signature Algorithm (DSA)
- Originally proposed in 1991, revised in 1993 due to security concerns, and another minor revision in 1996
- **Cannot be used for encryption or key exchange**
- Uses an algorithm that is designed to provide only the digital signature function

## Other Public-Key Algorithms

### Elliptic-Curve Cryptography (ECC):

- Equal security for smaller bit size than RSA
- Seen in standards such as IEEE P1363
- Confidence level in ECC is not yet as high as that in RSA
- **Based on a mathematical construct known as the elliptic curve**

## Part 3: Cryptographic data integrity algorithms

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## Contents

- Message Authentication
- Cryptographic Hash Functions**
- Digital Signatures

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# Message Authentication

## ☞ Authentication has purpose:

- Ensure **message sequentiality**
- Assure **message integrity**
- Confirm **sender's validity**

## ☞ Mechanisms for message authentication

- Message encryption (in symmetric, asymmetric)
- Hash function
- Message Authentication Code – MAC

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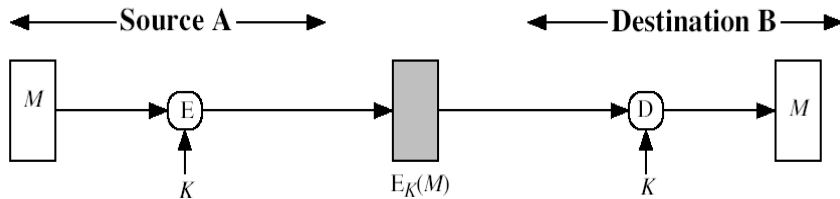
# Message encryption



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## Symmetric Cryptography

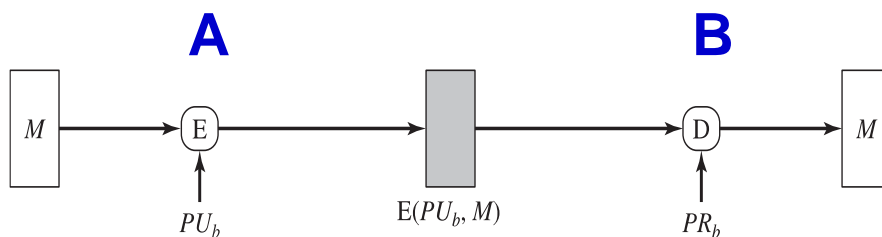


**Symmetric encryption: confidentiality and authentication**

- ∞ **Confidentiality:** Only A & B have key  $K$  to decrypt
- ∞ **Authentication:**  $M$  must only from A.  $M$  cannot be changed without detection. Need more constraints (*in case  $M$  is binary*)
- ∞ **No signature:** Recipient can fabricate message, and sender can deny message.

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## Asymmetric Cryptography

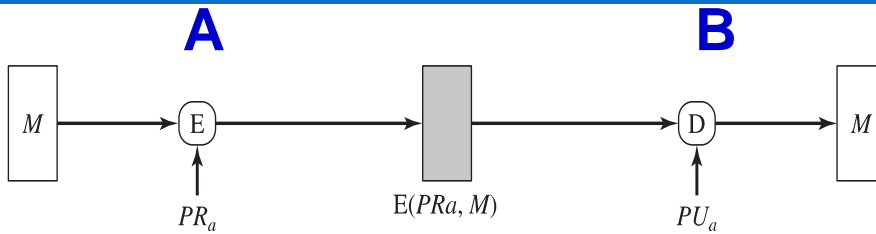


**Public-key encryption: confidentiality**

- ∞ **Confidentiality:** Only **B** has  $PR_b$  to decrypt
- ∞ **No authentication:** everyone can use  $PU_b$  to encrypt  $M$  then blame on **A**

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## Asymmetric Cryptography (2)

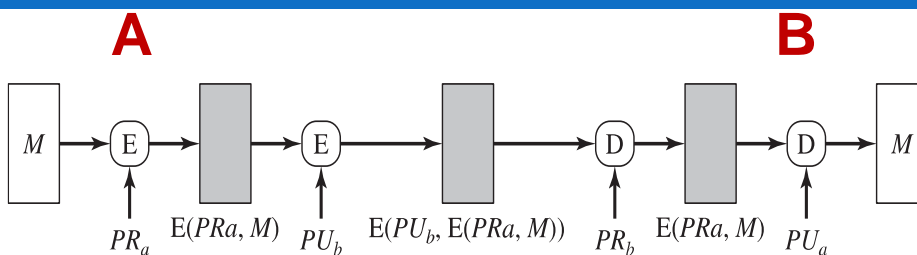


**Public-key encryption: authentication and signature**

- ⌘ Only **A** has  $PR_a$  to encrypt
- ⌘  $M$  cannot be changed
- ⌘ Need more constraints
- ⌘ Everyone can use  $PU_a$  to check signature

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## Asymmetric Cryptography (3)

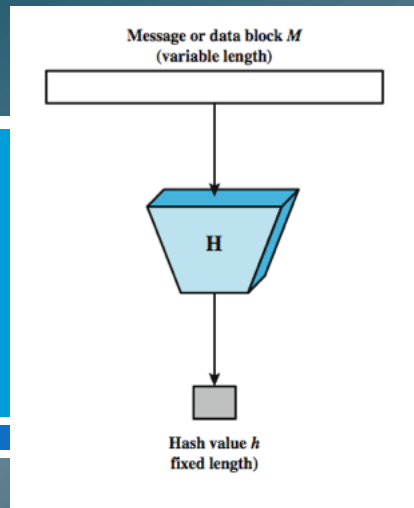


**Public-key encryption: confidentiality, authentication, and signature**

- **Confidentiality:**
  - Because of using  $PU_b$  to encrypt
- **Authentication & Signature:**
  - Because of using  $PR_a$  to encrypt

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# Hash function



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## Cryptographic Hash Functions

- ✧ What is Hash Functions
- ✧ Hash function Requirement
- ✧ Hash Functions in Message authentication
- ✧ Attacks on Hash Functions
- ✧ Secure Hash Algorithm (SHA)

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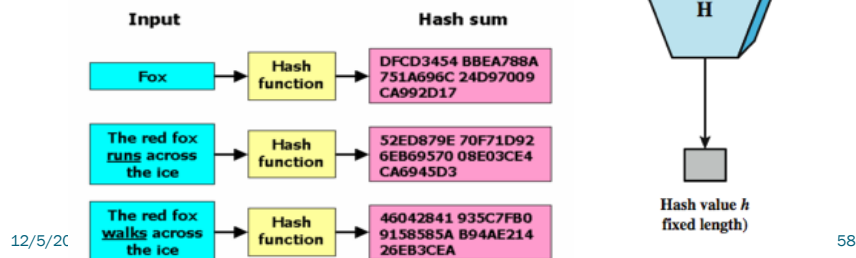
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# What is Hash Functions

- ∞ A hash function maps a *variable-length* message into a *fixed-length* hash value, or message digest

$$h = H(M)$$

- ∞ The *principal* object:
  - *data integrity*



## Hash function Requirement

- ∞ Variable input size
- ∞ Fixed output size
- ∞ Efficiency:  $H(M)$  is easily calculated with arbitrary  $M$
- ∞ For any given value  $h$ , it is difficult to find  $M$  such that  $H(M) = h$ 
  - One-way function
- ∞ For any  $M_1$ , it is very difficult to find  $M_2 \neq M_1$  such that  $H(M_2) = H(M_1)$ 
  - collision resistant: **weak**
- ∞ Very difficult to find any pair  $(M_1, M_2)$  such that  $H(M_1) = H(M_2)$ 
  - collision resistant: **Strong**

**A Strong hash function: satisfied all 6 reqs (weak: 5 reqs)**

## Hash Functions - Issues

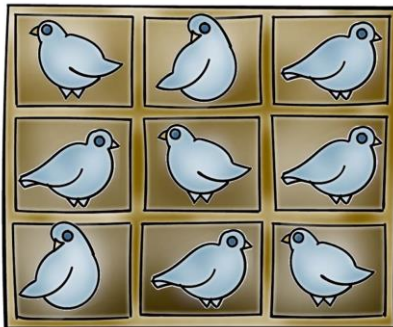


Pigeonhole  
Principle

The Birthday Paradox

## Hash Function Weaknesses

Pigeonhole Principle



$n$  = number of pigeons  
 $m$  = number of holes

$n = m$  There is one  
 pigeon per hole

$n > m$  Then at least one  
 hole must have more  
 than one pigeon

## Hash Function Weaknesses

### Hash Functions:



- There are many more 'pigeons' than 'pigeonholes'
- Many inputs will be mapped to the same output. That is, ***many input messages will have the same hash.***

**Conclusion:** The longer the length of the hash, the fewer collisions.

## Determining Hash Length

Hash Length	Possible # of hash values
1	2
64	$2^{32}$



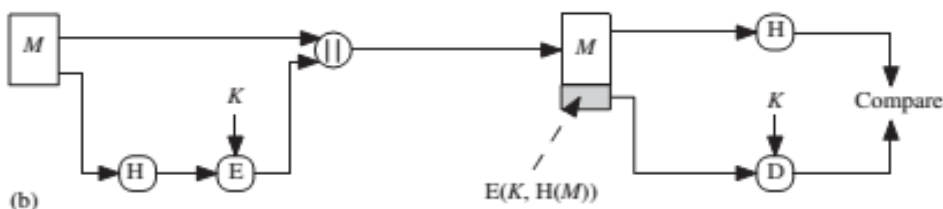
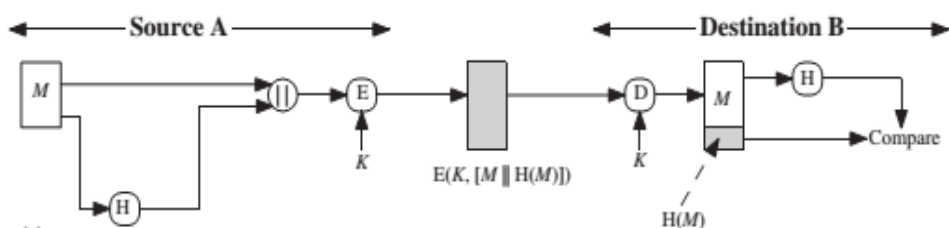
## Hash Size Quiz

Choose the correct answer:

If the length of hash is 128 bits, then how many messages does an attack need to search in order to find two that share the same hash?

- |                                    |                                    |
|------------------------------------|------------------------------------|
| <input type="checkbox"/> 128       | <input type="checkbox"/> $2^{127}$ |
| <input type="checkbox"/> $2^{128}$ | <input type="checkbox"/> $2^{64}$  |

## Exs of the Use of a Hash Function for Message Authentication

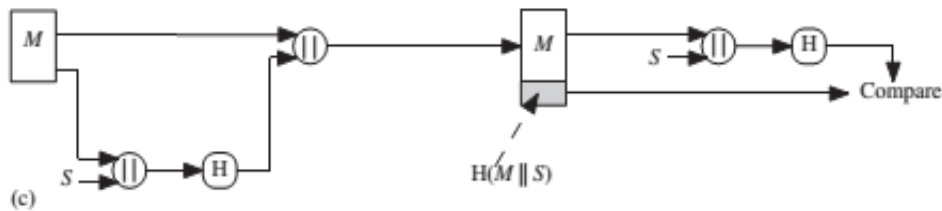


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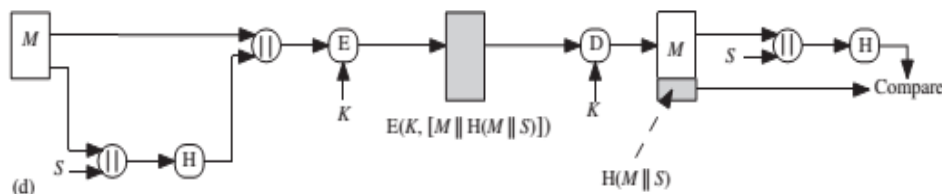
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## Exs of the Use of a Hash Function for Message Authentication



Message Auth & Integrity, no encryption



Message Auth & Conf using encryption

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## keyed hash function

- ∞ More commonly, message authentication is achieved using a **message authentication code (MAC)**, also known as a **keyed hash function**.
- ∞ Typically, MACs are used between two parties that share a secret key to authenticate information exchanged between those parties.
- ∞ A MAC function takes as input:
  - a secret key and
  - a data block
  - and produces a hash value, referred to as the MAC

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## Hash function: MD5

### MD5 creates hash value of 128-bit from message

- Calculations in 32 – bit numbers is fast and widely used with **the acceptable security** (RFC1321 standard)
- it is fast, simple and small => used in many cases even collision was found

### Calculation Process of MD5:

- Add to message 1→512 bits to get length of **448 mod 512**
- **Add one 64-bit value to the message**
- Begin with **4-word 32-bit (128-bit) block**, that is (A,B,C,D)
- In 16-word (512-bit) blocks: use **4 rounds** to calculate 16- bit numbers in the buffer and blocks. **Add outputs into inputs** to create new buffer values
- **Hash value** is the final result of (A,B,C,D)

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## SHA - Secure Hash function

SHA (Secure Hash Algorithm) originally designed by NIST & NSA in 1993, was revised in 1995 as SHA-1

US standard for use with DSA signature scheme

- standard is FIPS 180-1 1995, also Internet RFC3174
- Note that, the algorithm is SHA, the standard is SHS

based on design of MD4 with key differences

produces 160-bit hash values

recent 2005 results on security of SHA-1 have raised

concerns on its use in future applications

adds 3 additional versions of SHA: SHA-256, SHA-384, SHA-512

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## Comparison of SHA Parameters

	SHA-1	SHA-256	SHA-384	SHA-512
Message digest size	160	256	384	512
Message size	$<2^{64}$	$<2^{64}$	$<2^{128}$	$<2^{128}$
Block size	512	512	1024	1024
Word size	32	32	64	64
Number of steps	80	80	80	80
Security	80	128	192	256

Notes:

1. All sizes are measured in bits.
2. Security refers to the fact that a birthday attack on a message digest of size  $n$  produces a collision with a work factor of approximately  $2^{n/2}$ .

## Attacks on Hash function

∞ two categories of attacks on hash functions:

- **Brute-force attack:**
  - depend only on bit length of the hash value (not specific algorithm)
  - Attack to: One-way function; collision resistant - weak  
wishes to find a value  $y$  such that  $H(y)=h$ , try  $2^m-1$  values
  - Attack to: collision resistant - strong  
wishes to find 2 messages:  $x,y$ , that yield  $H(y)=H(x)$ , try  $2^{m/2}$  values
- **Cryptanalysis:**
  - based on weaknesses in a particular cryptographic algorithm.
  - require a cryptanalytic effort greater than or equal to the BF effort

# MAC – Message Authentication Code



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12/5/2018

## Message Authentication Code (MAC)

### Message Authentication Code (MAC)

- attached to message
- depends on **both message** and **private key** that only sender and recipient know
- Message length can be arbitrary, but MAC often has certain fixed length (Ex: 128 bit)
- To create MAC we can use **hash function**
  - To reduce message length
  - To keep message integrity

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# Message Authentication Code - MAC

- When A has a message to send to B, it calculates the MAC (checksum) as a function of the message **M** and the key **K**:

$$\text{MAC} = C(M, K)$$

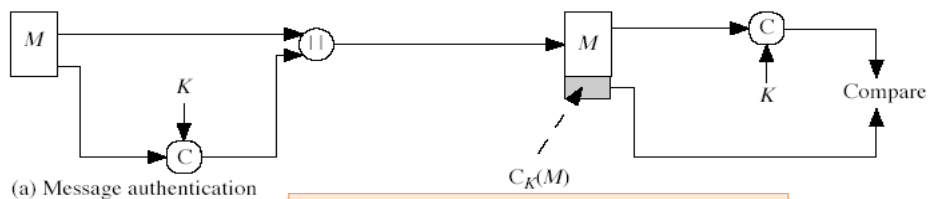
where  
 M = input message  
 K = shared secret key  
 C = MAC function

- MAC = message authentication code, is attached to M
- When B receive MAC & M, B calculates  $\text{MAC}' = C(M, K)$ ;
- If  $\text{MAC} = \text{MAC}'$  we can conclude:
  - M is not changed
  - A is the one who sent M

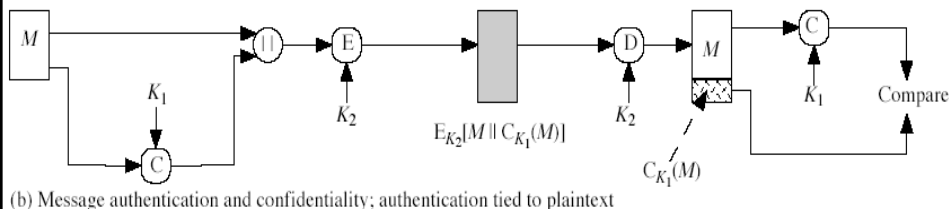
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## Basic Uses of Message Authentication Code



**Authentication:** Only A & B have key K

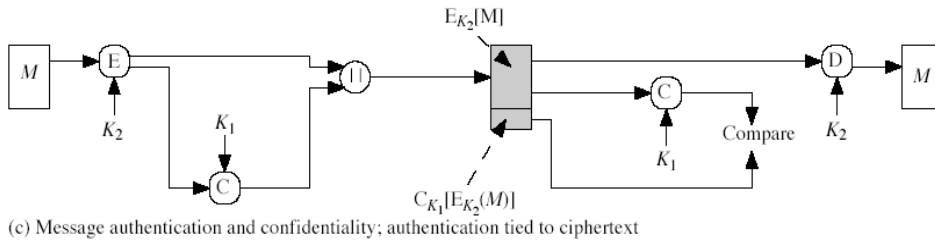


**Confidentiality:** Only A & B have key K<sub>2</sub>

**Authentication:** Only A & B have key K<sub>1</sub>

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## Basic Uses of Message Authentication Code



**Confidentiality:** Due to  $K_1$ .  
**Authentication:** Due to  $K_2$

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## Security of MAC

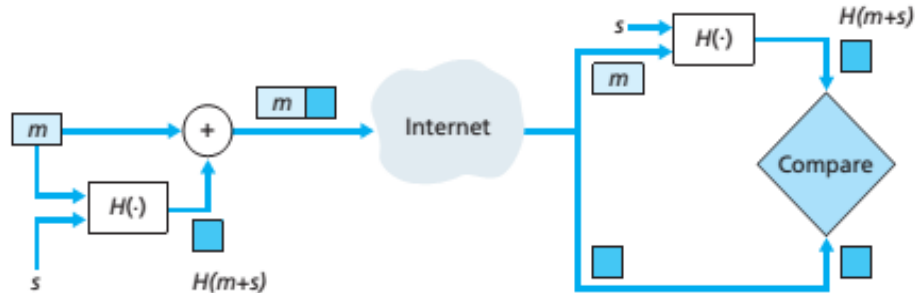
∞ two categories of attacks on MAC:

- **Brute-force attack:**
  - depends on the relative size of the key and the tag
  - more difficult undertaking than BF attack on a hash function because it requires known message-tag pairs.
- **Cryptanalysis:**
  - based on weaknesses in a particular cryptographic algorithm.
  - require a cryptanalytic effort greater than or equal to the BF effort
  - There is much more variety in the structure of MACs than in hash functions, so it is difficult to generalize about the cryptanalysis of MACs.

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## MAC based on Hash Function: HMAC



Key:

$m$  = Message

$s$  = Shared secret

Developing a MAC derived from a cryptographic hash function:

1. Cryptographic hash functions such as MD5 and SHA generally execute faster in software than symmetric block ciphers such as DES.
2. Library code for cryptographic hash functions is widely available.

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## MAC based on Hash Function: HMAC

1. Append zeros to the left end of to create a  $b$ -bit string  $K^+$  (e.g., if is of length 160 bits and , then will be appended with 44 zeroes).
2. XOR (bitwise exclusive-OR) with  $ipad$  to produce the  $b$ -bit block  $S_i$ .
3. Append  $M$  to  $S_i$
4. Apply  $H$  to the stream generated in step 3.
5. XOR with  $opad$  to produce the  $b$ -bit block  $S_0$ .
6. Append the hash result from step 4 to  $S_0$ .
7. Apply  $H$  to the stream generated in step 6 and output the result.

**HMAC:**

$$\text{HMAC}(K, M) = H[(K^+ \oplus opad) \parallel H[(K^+ \oplus ipad) \parallel M]]$$

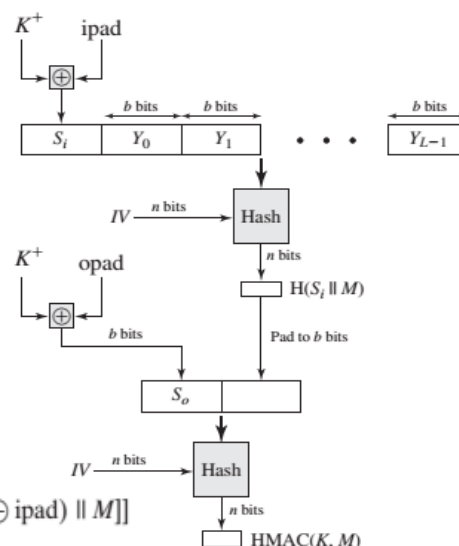


Figure 12.5 HMAC Structure

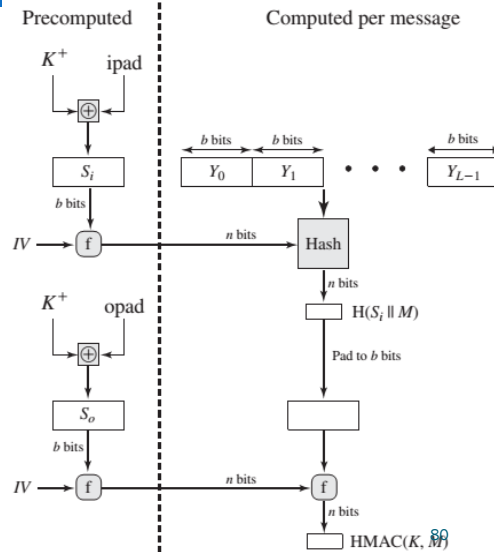
# Security of HMAC

## based on an embedded hash function

- depends on strength of the core hash function.
- the probability of successful fake with time spent and some message-tag pairs created with the same key.

## Attack:

- compute an output of the compression function
- finds collisions in the hash function

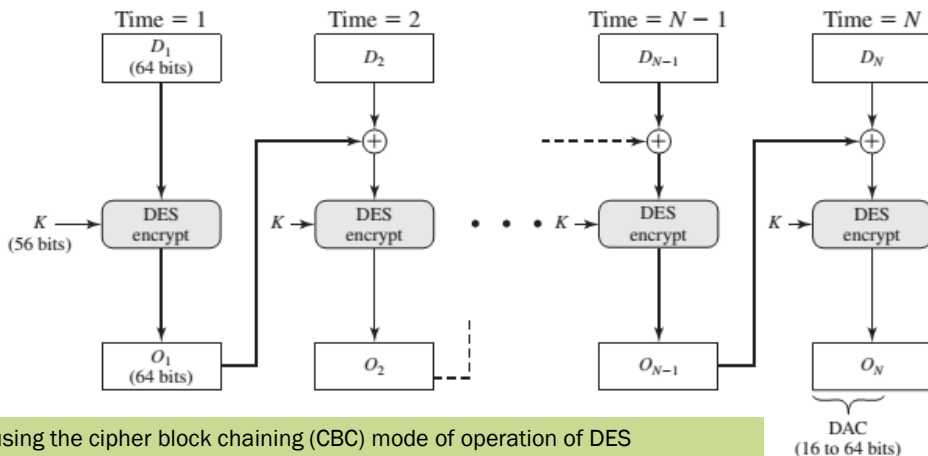


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# MACS based on block ciphers: DAA & CMAC

## Data Authentication Algorithm (DAA) (obsolete – old)

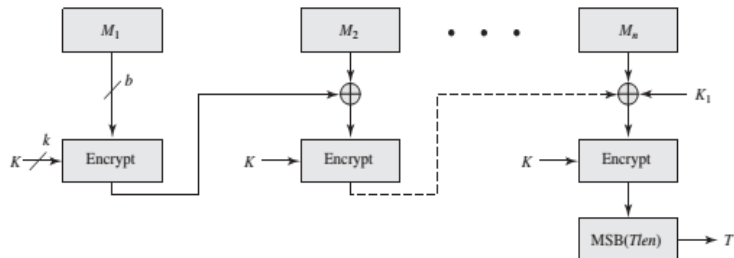
- based on DES, has been one of the most widely used MACs





# Cipher-Based Message Authentication Code (CMAC)

- ∞ operation for use with AES and triple DES:
- ∞ using three keys:
  - one key of length to be used at each step of the cipher block chaining and
  - two keys of length , where is the key length and is the cipher block length.
- ∞ This proposed construction: the two -bit keys could be derived from the encryption key, rather than being provided separately

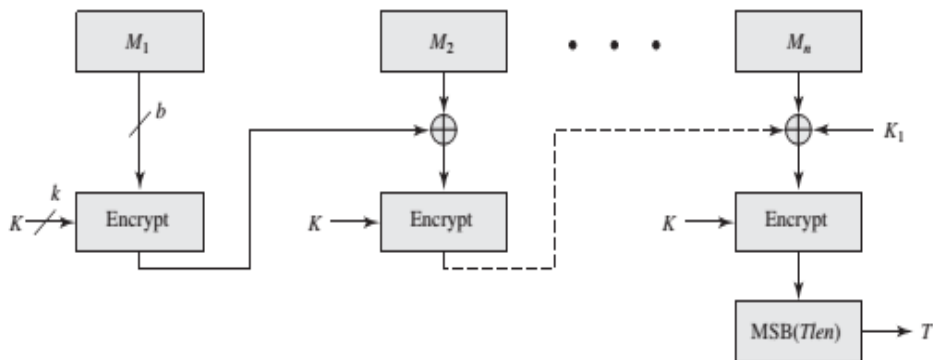


12/5/20:

(a) Message length is integer multiple of block size

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# Cipher-Based Message Authentication Code (CMAC)



(a) Message length is integer multiple of block size

$$C_n = E(K, [M_n \oplus C_{n-1} \oplus K_1])$$

$$T = \text{MSB}_{Tlen}(C_n)$$

 $T$  = message authentication code, also referred to as the tag

 $Tlen$  = bit length of  $T$ 
 $\text{MSB}_s(X)$  = the  $s$  leftmost bits of the bit string  $X$ 

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# Digital Signature



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12/5/2018

## Digital signature

### ✎ A digital signature:

- enables the creator of a message to attach a code that acts as a signature.
- is formed by taking the hash of the message and encrypting the message with the creator's private key.

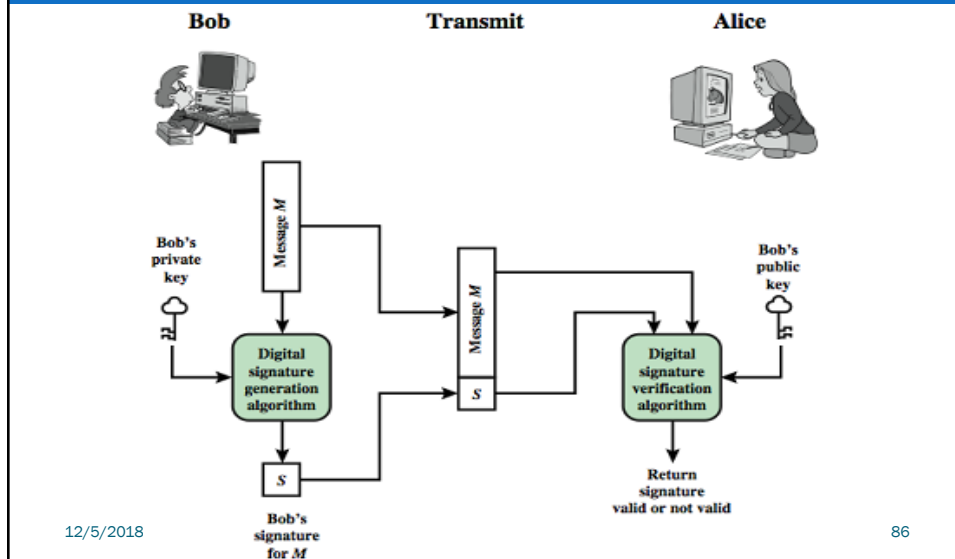
### ✎ digital signature properties:

- verify the author and time of the signature.
- authenticate the contents at the time of the signature.
- It must be verifiable by third parties, to resolve disputes.

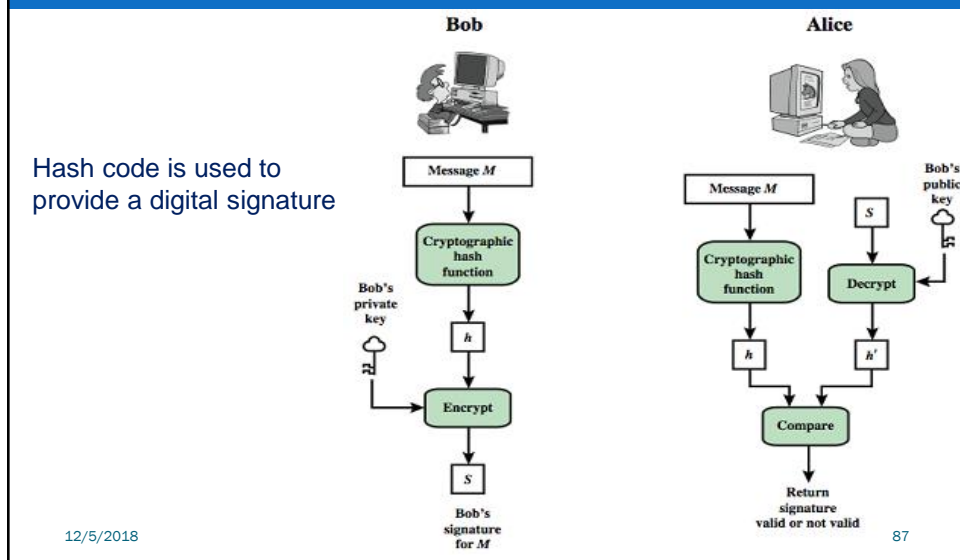
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## Generic Model of Digital Signature Process



## Simplified Depiction of Essential Elements of Digital Signature Process



## Ex of Digital Signatures

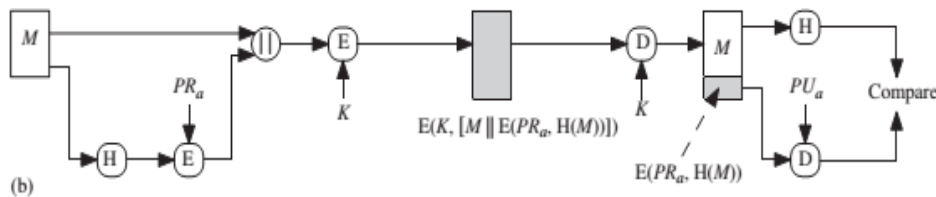


Figure 11.3 Simplified Examples of Digital Signatures

∞ hash code is used to provide a digital signature:

- $E(K, [M, E(PR_a, H(M))])$ : confidential
- This is a common technique

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## Digital Signature Standard DSS

∞ DSS: Digital Signature Standard

- US Govt approved signature scheme
- designed by NIST & NSA in early 90's
- published as FIPS-186 in 1991, revised in 1993, 1996, 2000
- Use RSA to create the digital signature process

∞ DSA: Digital Signature Algorithm

- new digital signature technique
- is a public-key technique

∞ SHA: Secure Hash Algorithm

- Is American standard in Digital Signature Algorithm DSA

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## Digital Signature using RSA

- ∞ RSA is used to create the digital signature process
- ∞ Assume we have the process RSA  $\{(e,N), (d,N)\}$
- ∞ To **sign** the message **M** we calculate:

$$S = M^d \pmod{N}.$$

- ∞ **Signature S should be attached to message M:**

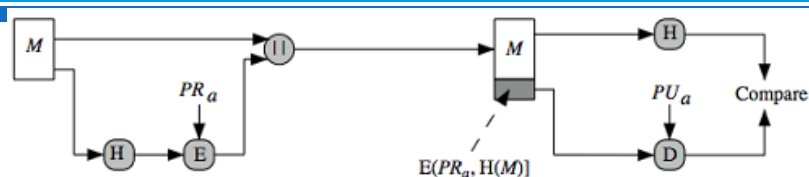
$$\{M, S\}$$

- ∞ To **check** signature we have to verify the equality of **M** and **S<sup>e</sup>**:

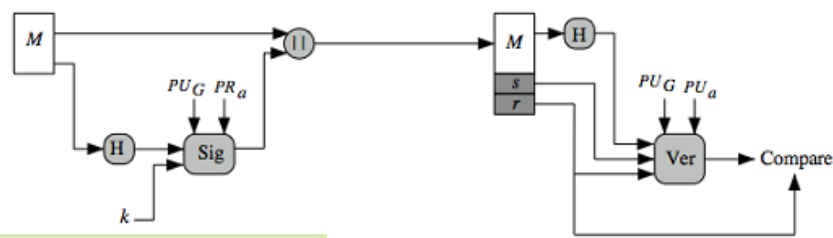
$$S^e \pmod{N} = M^{e \cdot d} \pmod{N} = M \pmod{N}$$

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## RSA vs. DSS



(a) RSA Approach



(b) DSS Approach

DSS uses an algorithm that is designed to provide only the digital signature function

it cannot be used for encryption

# Digital Signature Algorithm - DSA

The DSA is based on the difficulty of computing discrete logarithms and is based on schemes originally

## Global Public-Key Components

- $p$  prime number where  $2^{L-1} < p < 2^L$  for  $512 \leq L \leq 1024$  and  $L$  a multiple of 64; i.e., bit length of between 512 and 1024 bits in increments of 64 bits
- $q$  prime divisor of  $(p - 1)$ , where  $2^{159} < q < 2^{160}$ , i.e., bit length of 160 bits
- $g = h^{(p-1)/q} \bmod p$ , where  $h$  is any integer with  $1 < h < (p - 1)$  such that  $h^{(p-1)/q} \bmod p > 1$

## User's Private Key

$x$  random or pseudorandom integer with  $0 < x < q$

## User's Public Key

$y = g^x \bmod p$

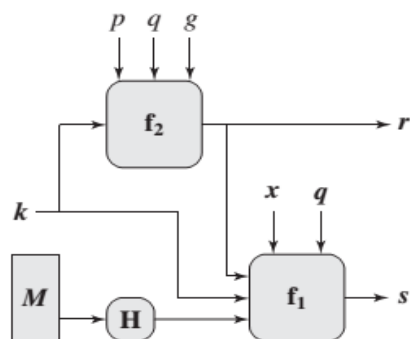
## User's Per-Message Secret Number

$k$  = random or pseudorandom integer with  $0 < k < q$

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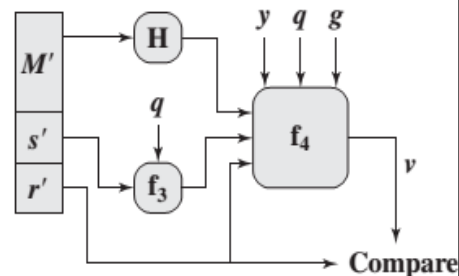
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# Digital Signature Algorithm - DSA



## Signing

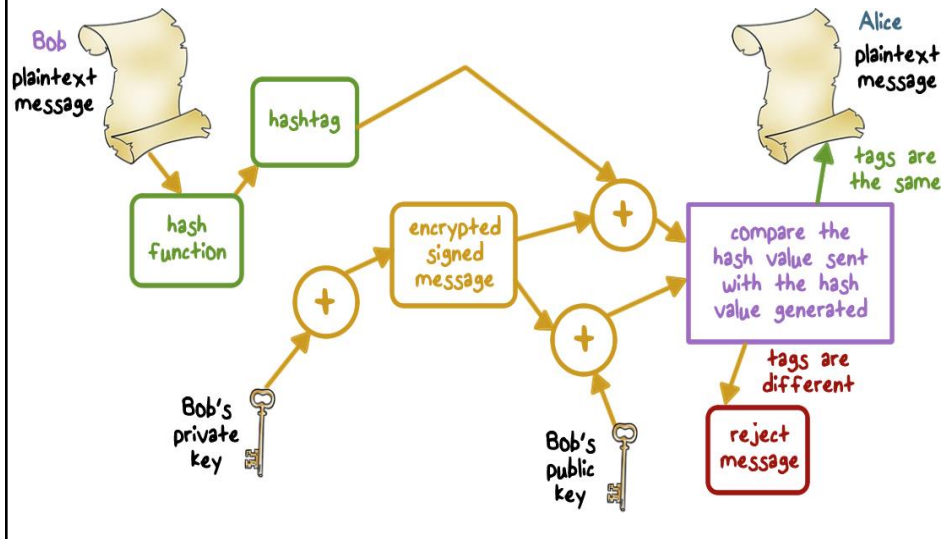
$r = (g^k \bmod p) \bmod q$   
 $s = [k^{-1} (H(M) + xr)] \bmod q$   
 Signature =  $(r, s)$



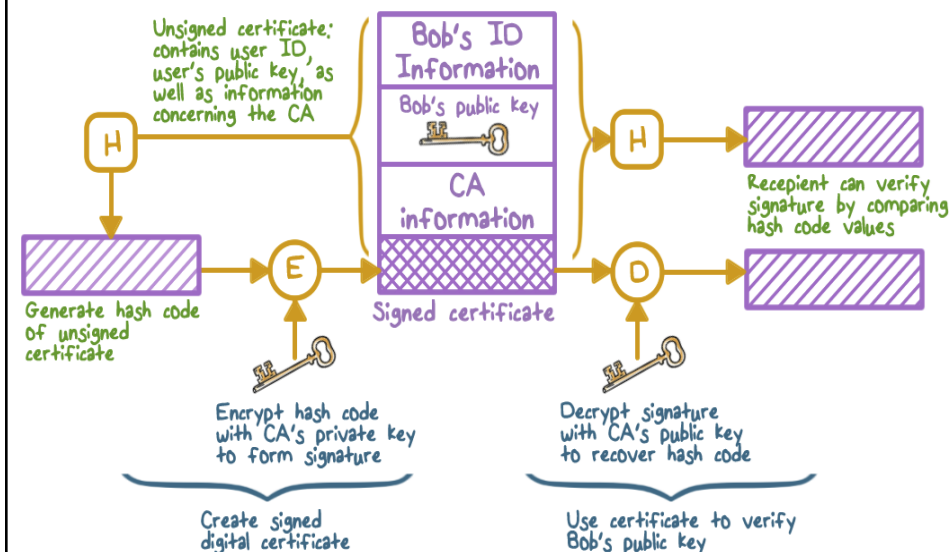
## Verifying

$w = (s')^{-1} \bmod q$   
 $u_1 = [H(M')w] \bmod q$   
 $u_2 = (r')w \bmod q$   
 $v = [(g^{u_1} y^{u_2}) \bmod p] \bmod q$   
 TEST:  $v = r'$

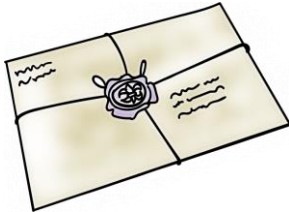
## Digital Signatures



## Digital Signatures

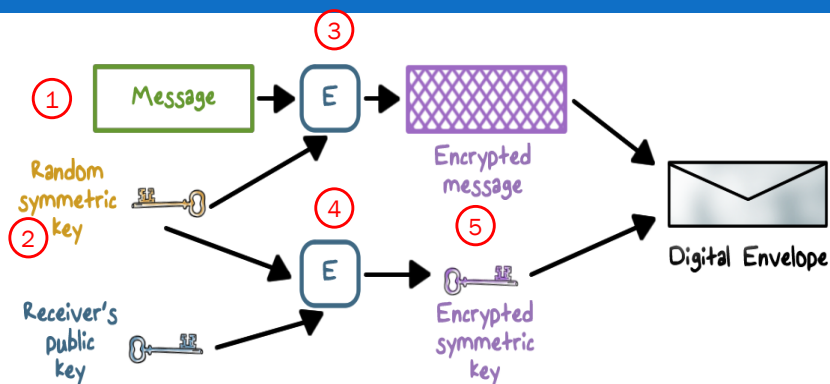


# Digital Envelopes



- Means: public-key encryption is used to protect a symmetric key
- Protects a message **without needing** to first arrange for sender and receiver to have the same secret key
- Compares to the same thing as a **wrapped envelope containing an unsigned letter**

# Digital Envelopes

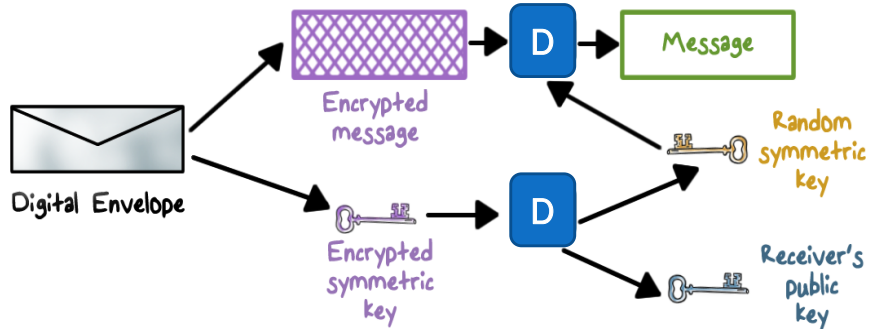


1. Prepares a message.
2. Generates a random symmetric key that will be used this one time only.
3. Encrypts that message using symmetric encryption the one-time key.
4. Encrypts the one-time key using public-key encryption with Alice's public key.
5. Attaches the encrypted one-time key to the encrypted message and sends it



# Digital Envelopes

Opening a digital envelope



Receiver is capable of decrypting the one-time key and recovering the original message. If Bob obtains Alice's public key by means of Alice's public-key certificate, then Bob is assured that it is a valid key.

# Practice openssl

- ❖ **Secure Sockets Layer (SSL)** is an application-level protocol which was developed by the Netscape Corporation for the purpose of transmitting sensitive information, such as Credit Card details, via the Internet
- ❖ **OpenSSL** is a robust, commercial-grade implementation of SSL tools, and related general-purpose library based upon SSL, developed by Eric A. Young and Tim J. Hudson
- ❖ OpenSSL is already installed on SEEDUbuntu

Q & A

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