Further Math Notes

Further Pure

Implicit Differentiation

1. Note, for some implicit differentiation such as `s03_01_07`, all of the condition in the equation must be consider, such as $x^4 + y^4 = 1$, which is useful to obtain the correct result. Be aware of those question

Series

1. CIE convergence usually related with the general term's convergence:

For a series $u_1 + u_2 + \ldots u_n$, if the general term for $\sum_{n=1}^{\infty} u_n$ converge then the single term u_n converge on a specific limit.

Note, this is not true vice versa, e.g. $u_n = \frac{1}{n}$ converge but $\sum_{n=1}^{\infty} u_n$ doesn't converge

So for question such as `s03_01_3` we can directly obtain the general term for $\sum_{n=1}^{\infty} u_n$ and deduce whether the series converge from it directly

Differentiation Equation

1. For the complementary function for characteristics having imaginary root, $y_{c.f.} = e^{-\alpha x}(A\cos\beta x + B\sin\beta x)$, this formula should be remembered precisely

Roots of Polynomial

1. For questions such as `w08_01_12` , the last bit usually involve some manipulation of the series, which could be done by doing algebraic manipulation and note the symmetry property of the summation. Usually the objective is to approach some existing conclusion for example $\sum \alpha$ or S_n, therefore we can conduct observation on this one:

   $\alpha^2(\beta^4 + \gamma^4 + \delta^4) + \beta^2(\alpha^4 + \gamma^4 + \delta^4) + \gamma^2(\alpha^4 + \beta^4 + \delta^4) + \delta^2(\alpha^4 + \beta^4 + \gamma^4)$, it is easy to realize that it is basically similar to $S_2 S_4$, but each S_4 have one term α^4 lost, so to bring it back to the polynomial, combinedly it is to obtain that the polynomial equals to $S_2 S_4 - S_6$

Integration

1. Note that for CIE both 2D and 3D centroid still exists on syllabus so it is better to remember both equation, to deal with problem such as `s08_01_1`

For 2D centroid:

$$\bar{x} = \frac{\int xydx}{\int ydx}, \bar{y} = \frac{\frac{1}{2}\int y^2 dx}{\int ydx}$$

For 3D centroid:

$$\bar{x} = \frac{\int xy^2\,dx}{\int y^2\,dx}, \bar{y} = 0$$

the derivation can be dealt with by using the **finite element method**

2. when the question ask to obtain a recurrence formula by considering some expression such as $\frac{d}{dx}Q(x)$ the algebraic result usually contains two part due to chain rule for differentiation. The recurrence formula usually comes from reintegrate the expression obtained, but sometimes like `w08_01_07` the answer is not immediately obvious.

$I_n = \int_0^1 \dfrac{1}{(1+x^4)^n}dx$, obtain $4nI_{n+1} = \frac{1}{2^n} + (4n-1)I_n$ by considering $\frac{d}{dx}(\dfrac{x}{(1+x^4)^n})$

Firstly taking the differentiation gives $\dfrac{1}{(1+x^4)^n} - \dfrac{4nx^4}{(1+x^4)^{n+1}}$, but the expression only match for one term. However, since the denominator is $(1+x^4)$, we can add and substract 1 from the right hand side part, therefore resulting $\dfrac{1}{(1+x^4)^n} - \dfrac{4n(1+x^4-1)}{(1+x^4)^{n+1}}$ and hence obtain $4nI_{n+1} = \frac{1}{2^n} + (4n-1)I_n$

Complex Number

1. For questions like `s03_01_06` asking to deduce the roots of a given polynomial from the trigonometry expression:

$\cos(6\theta) = 32\cos^6\theta - 48\cos^4\theta + 18\cos^2\theta - 1 \Rightarrow 64x^6 - 96x^4 + 36x^2 - 1 = 0$

By F.T.A. we now that the polynomial will have 6 solution, but this is not the case for the trigonometry identity which works for any θ, hence we need to limit some of the expression, by going back from polynomial we would have:

$-\frac{1}{2} = 32x^6 - 48x^4 + 18x^2 - 1$, if $x = \cos\theta$ we would have $-\frac{1}{2} = \cos 6\theta = \cos\frac{2}{3}\pi$

we would easily obtain that

$6\theta = \pm\frac{2}{3}\pi + 2\pi k, k = 0, \pm1, \pm2, \ldots$

$\theta = \pm\dfrac{\pi}{9} + \dfrac{\pi}{3}k$

$\theta = \frac{\pi}{9}, \frac{2\pi}{9}, \frac{4\pi}{9}, \frac{5\pi}{9}, \frac{7\pi}{9}, \frac{8\pi}{9}$

Note we would want all the roots having different result so be sure to eliminate the equivalent solution

2.

Vector Space

Vector Geometry

Further Mechanics

Further Statistics

P.D.F. and C.D.F.

1. For the substitution of a different variable such as `w02_02_10` , i.e. $Y = X^2$ and obtain the p.d.f. the verification for the correctness of the process could be done via directly substituting

Given that $f(x) = \int_0^\infty p(x)dx$, we would be able to obtain the p.d.f. for $g(y) = \int_0^\infty p(\sqrt(y))\frac{1}{2\sqrt{y}}dy$ by the means of substitution $y = \sqrt{x}$ (Note that the upper limit and lower limit should also been substituted) so that the result could be compared with the result from going through the definition of $G(y)$, but this is only a method for verification

Also, when asking for definition of $E(x)$ and $Var(x)$, it is probably the best by writing the original formula out and then