Assignment 5

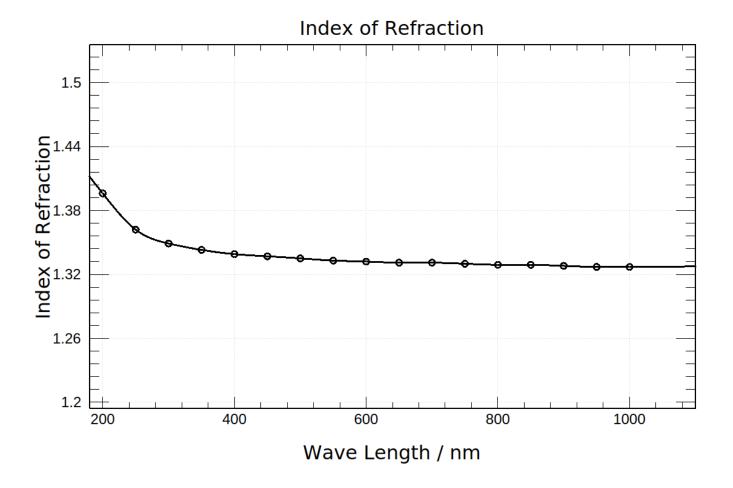
Shuyang Cao

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Chapter 4 Exercise 7

Cubic spline interpolation is used.

```
cat ./data.txt | ./dipersion
```



Chapter 5 Exercise 7

Use $M(\hat{A})$ to denote the matrix of operator \hat{A} and $\hat{F}(\hat{A})$ to denote the function of operator \hat{A} . Then, we have

$$M(\hat{F}(\hat{A})) = F(M(\hat{A}))$$

So, for \hat{H},\hat{x},\hat{D} , we have

$$\begin{array}{rcl} M(\hat{x}^2) & = & M(\hat{x})^2 \\ M(\hat{D}^2) & = & M(\hat{D})^2 \\ M(\hat{H}) & = & -\frac{1}{2}M(\hat{D})^2 + \frac{1}{2}M(\hat{x})^2 \end{array}$$

Note that the basis we use the the eigen basis of \hat{H} . Thus, the matrix of \hat{H} is diagonal. A value whose magnitude is less than 1e-12 is printed as 0.

\$./operator				
Х				
0	0.707107	0	0	0
0.707107	0	1	0	0
0	1	0	1.22474	0
0	0	1.22474	0	1.41421
0	0	0	1.41421	0
x^2				
0.5	0	0.707107	0	0
0	1.5	0	1.22474	0
0.707107	0	2.5	0	1.73205
0	1.22474	0	3.5	0
0	0	1.73205	0	4.5
D				
0	0.707107	0	0	0
-0.707107	0.707107	1	0	0
0.707107	-1	0	1.22474	0
0	0	-1.22474	0	1.41421
0	0	0	-1.41421	0
Ü	Ü	0	1.71721	0
D2				
-0.5	0	0.707107	0	0
0	-1.5	0	1.22474	0
0.707107	0	-2.5	0	1.73205
0	1.22474	0	-3.5	0
0	0	1.73205	0	-4.5
Hamiltonian	_	_	_	_
0.5	0	0	0	0
0	1.5	0	0	0
0	0	2.5	0	0
0	0	0	3.5	0
0	0	0	0	4.5

Chapter 5 Exercise 9

Effective potential is

$$V(r)=-lpharac{4\hbar c}{3r}+rac{r}{\hbar ca^2}+rac{l(l+1)\hbar^2}{2\mu r^2}$$

where $\mu=m_c/2$ is the reduced mass. So, the quantization rule is

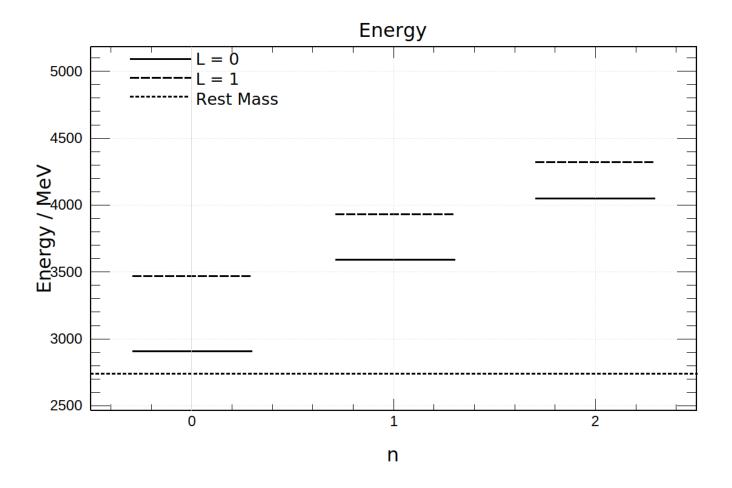
$$egin{split} \int_{r_{ ext{min}}}^{r_{ ext{max}}} \sqrt{2\mu \left(E-V(r)
ight)} dr &= \left(n+\eta
ight)\pi \hbar \ \Rightarrow \int_{r_{ ext{min}}}^{r_{ ext{max}}} \sqrt{rac{2\mu E}{\hbar^2} + rac{8lpha \mu c}{3\hbar r} - rac{2\mu r}{\hbar^3 ca^2} - rac{l(l+1)}{r^2}} dr &= \left(n+\eta
ight)\pi \end{split}$$

where $\eta=3/4$ if l=0, $\eta=1/3$ if l=1 because classically particles cannot reach the cener of the system if the angular momentum is nonzero. Choose $b=10^{-15}m$ as the length unit. The reduced position $\xi=x/a$. The quantization rule can be rewritten as

$$\int_{\xi_{\min}}^{\xi_{\max}} \sqrt{\epsilon + rac{eta}{\xi} - \gamma \xi - rac{l(l+1)}{\xi^2}} d\xi = (n+\eta) \, \pi$$

where

$$egin{cases} \epsilon = rac{E}{E_0} \ E_0 = rac{\hbar^2}{2\mu b^2} pprox 28.4219 \;\; MeV \ eta = rac{8lpha\mu cb}{3\hbar} pprox 3.51768 \ \gamma = rac{2\mu b^3}{\hbar^3 ca^2} pprox 30.1959 \end{cases}$$



Chapter 5 Exercise 12

Suppose we wait for a long enough time so that all N atoms decay.

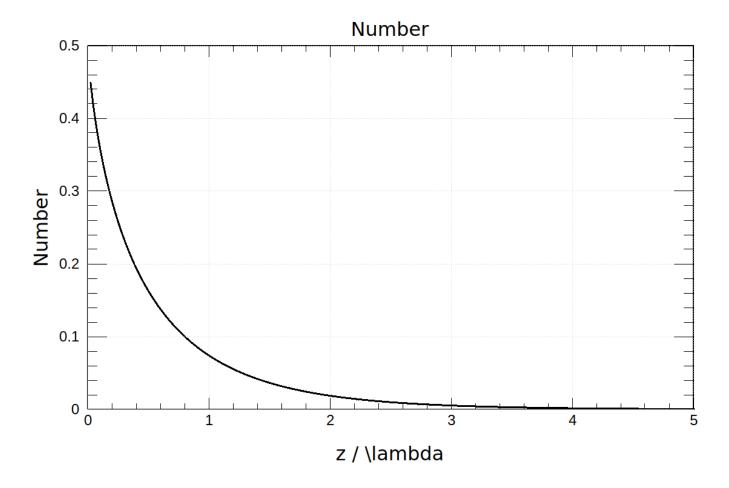
Part a

$$egin{aligned} F\left(rac{z}{\lambda}
ight) &= rac{N}{4\pi} \int_{0}^{2\pi} d\phi \int_{0}^{rac{\pi}{2}} \sin heta d heta \left(1 - \int_{0}^{rac{z}{\cos heta}} rac{1}{\lambda} e^{-rac{x}{\lambda}} dx
ight) \ &= rac{N}{2} \left(e^{-rac{z}{\lambda}} - rac{z}{\lambda} \Gamma\left(0, rac{z}{\lambda}
ight)
ight) \end{aligned}$$

where $\Gamma(a,z)$ is the upper incomplete gamma function. Check ssymptotic behaviours of $\Gamma(\frac{z}{\lambda})$

$$\lim_{z o 0} F\left(rac{z}{\lambda}
ight) = rac{N}{2} \ \lim_{z o \infty} F\left(rac{z}{\lambda}
ight) = 0$$

In the program, N is set to 1.



Part b

Ignore the limit of the speed of light and assume the gamma ray travels through the material instantly. This should be a resonable approximation in practice. The surface flux density is

$$f = \int_0^\infty \int_0^{rac{\pi}{2}} \int_0^{2\pi} \left(n r^2 \sin heta dr d heta d\phi
ight) \left(rac{1}{ au} e^{-rac{t}{ au}}
ight) \left(rac{\cos heta}{4\pi r^2}
ight) \left(1 - \int_0^r rac{1}{\lambda} e^{-rac{x}{\lambda}} dx
ight) = rac{\lambda n}{4} rac{e^{-rac{t}{ au}}}{ au}$$

Part c

Define

 $\left\{egin{array}{ll} Density \ of \ Granite &
ho=2.7g/cm^3 \ Weight \ fraction \ of \ K_2O & lpha=4\% \ Natural \ Abundance \ of \ K^{40} & eta=0.012\% \ Gamma \ ray \ production \ efficiency & \eta=11\% \ Half \ of \ K^{40} & t_0=10^9 years \ Massof K_2O & m=1.56415 imes 10^{-22} g \end{array}
ight.$

So,

$$au = t_0/\ln(2)
onumber \ n = rac{2
holpha}{m}eta$$

Surface flux density

$$egin{align} f &= \eta rac{\lambda n}{4} rac{e^{-rac{t}{ au}}}{ au} \ &pprox 0.62602 \left(rac{1}{2}
ight)^{-rac{t}{1.2 imes10^9yr}} cm^{-2}s^{-1} \end{array}$$

If we wait long enough time so that all K^{40} decay. Gamma ray surface number density is

$$N=\int_0^\infty f dt = rac{n\eta\lambda}{4}pprox 3.41782 imes 10^{16}~~cm^{-2}$$