Assignment 4

Shuyang Cao

Chapter 3 Exercise 13

equations of motion:

$$\mathbf{M} \cdot \ddot{\mathbf{x}} = -\mathbf{K} \cdot \mathbf{x}$$

where

$$\mathbf{M} = \begin{pmatrix} m & 0 & 0 & 0 & 0 \\ 0 & 3m0 & 0 & 0 & 0 \\ 0 & 0 & 2m0 & 0 & 0 \\ 0 & 0 & 0 & m & 0 \\ 0 & 0 & 0 & 0 & 2m \end{pmatrix}$$

$$\mathbf{x} = \begin{pmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}$$

$$\mathbf{K} = \begin{pmatrix} k & -k & 0 & 0 & 0 \\ -k & 3k & -2k & 0 & 0 \\ 0 & -2k & 4k & -2k & 0 \\ 0 & 0 & -2k & 3k & -k \\ 0 & 0 & 0 & -k & k \end{pmatrix}$$

Define

$$\mathbf{x} = \mathbf{a}e^{-i\omega t}$$

then

$$\mathbf{K} \cdot \mathbf{a} = \omega^2 \mathbf{M} \cdot \mathbf{a}$$

Define

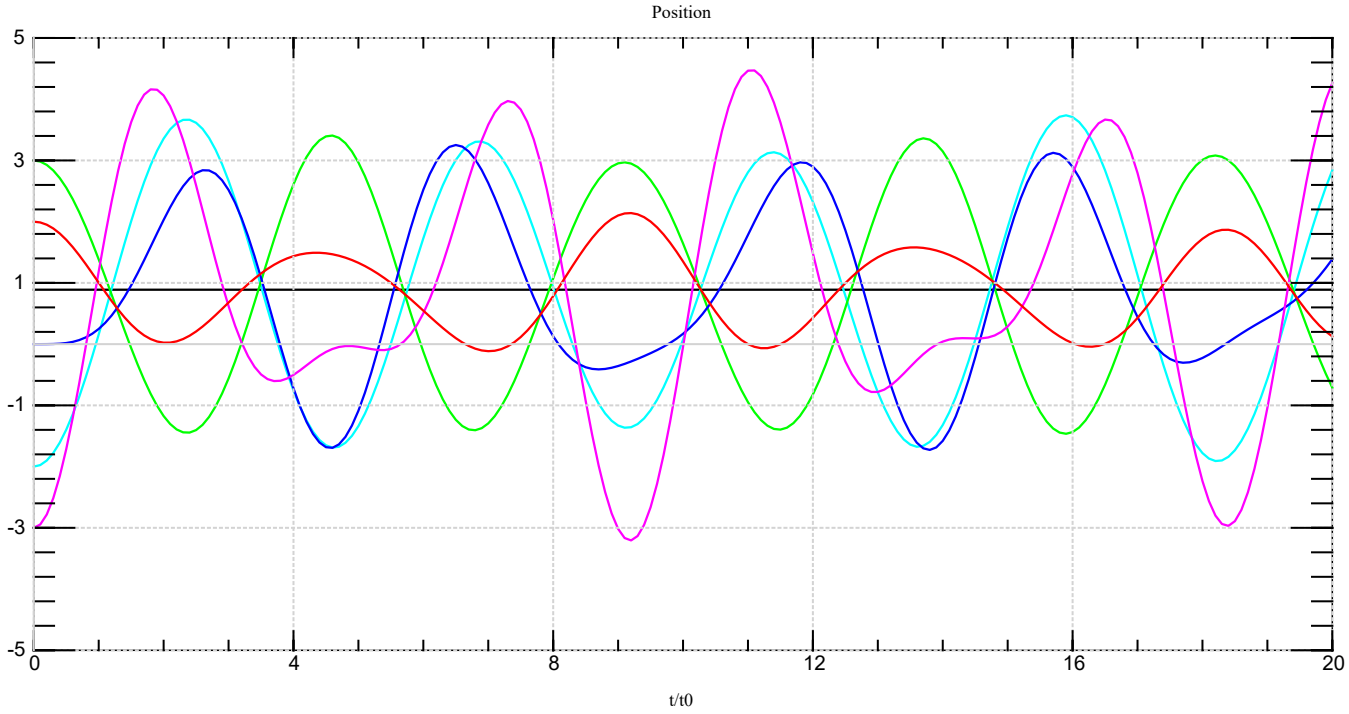
$$\Omega^2 \equiv \mathbf{M}^{-1/2} \mathbf{K} \mathbf{M}^{-1/2}$$

$$\mathbf{b} \equiv \mathbf{M}^{1/2} \mathbf{a}$$

then

$$\Omega^2 \cdot \mathbf{b} = \omega^2 \mathbf{b}$$

Positions of five balls are shown below, where the black line is the position of the center of mass of the system. A horizontal black line exhibits the conservation of total momentum of the system.



Chapter 4 Exercise 3

The skeleton code is copied directly from https://github.com/jfb3615/ACP-Misc/tree/master/SKELETONS/CH4/P1.

Coordinates of points are

```
$ cat data/points.txt
127.9 225.6
227.8 302.8
355 287
391.8 300.5
478.5 268
515.5 237
526.5 192
521.5 114.5
553.5 82.7
```

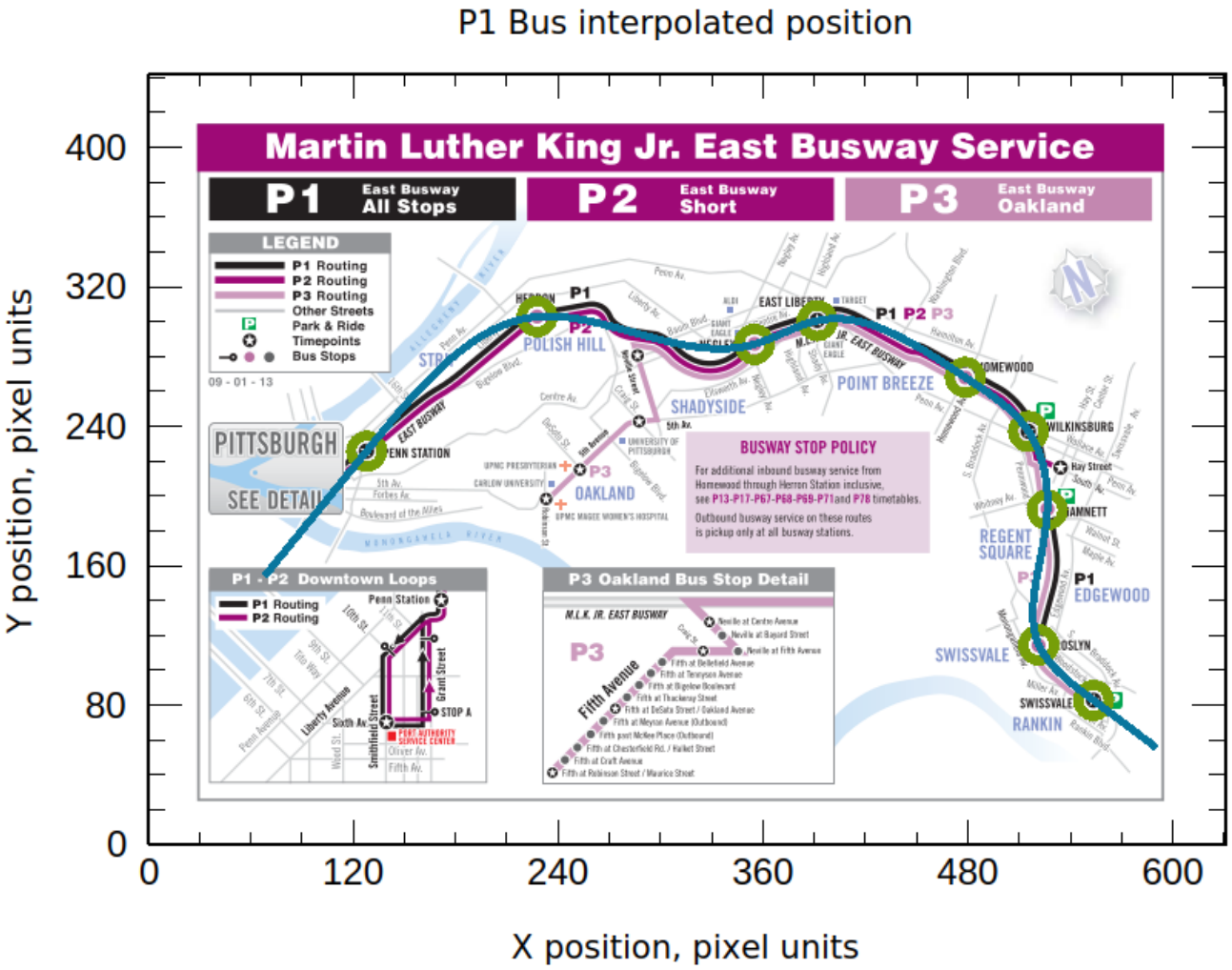
The bus route is interpolated by a curve parameterize by t.

$$\begin{cases} X(s) \\ Y(s) \end{cases}$$

where s is the distance to the port authority. s of each bus station is estimated by add up distances between adjacent bus stations from the port authority to the current bus station. By using parameterized functions, we avoid directly ploting a multi-value $Y(X)$.

As shown below, bus stations are circled by green circles and the fitted route is depicted by a blue line.

```
$ cat ../../data/points.txt | ./p1
```



Chapter 4 Exercise 4

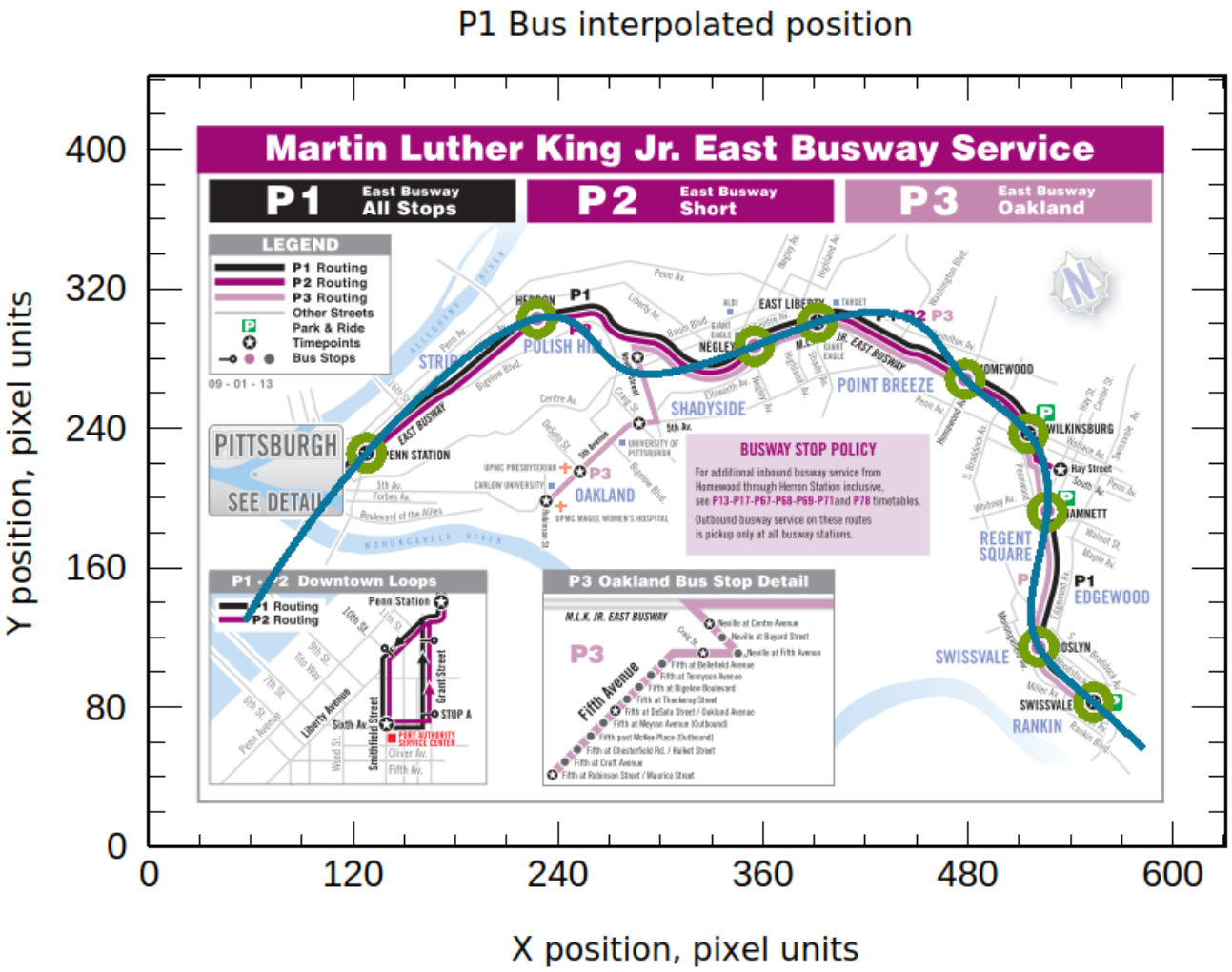
The only difference between codes in *EX3* and *EX4* is

`PlotOrbit *showFitLine(std::vector<Complex> points)` .

Cartesian coordinates of bus station are first converted to polar coordinates $\rho(\theta)$, then fitted by cubic spline interpolation. To be compatible with the interface of `PlotOrbit` , the fit function is converted back as shown below.

$$\begin{cases} X(\theta) = \rho(\theta) * \cos(\theta) \\ Y(\theta) = \rho(\theta) * \sin(\theta) \end{cases}$$

```
$ cat ../../data/points.txt | ./p1
```

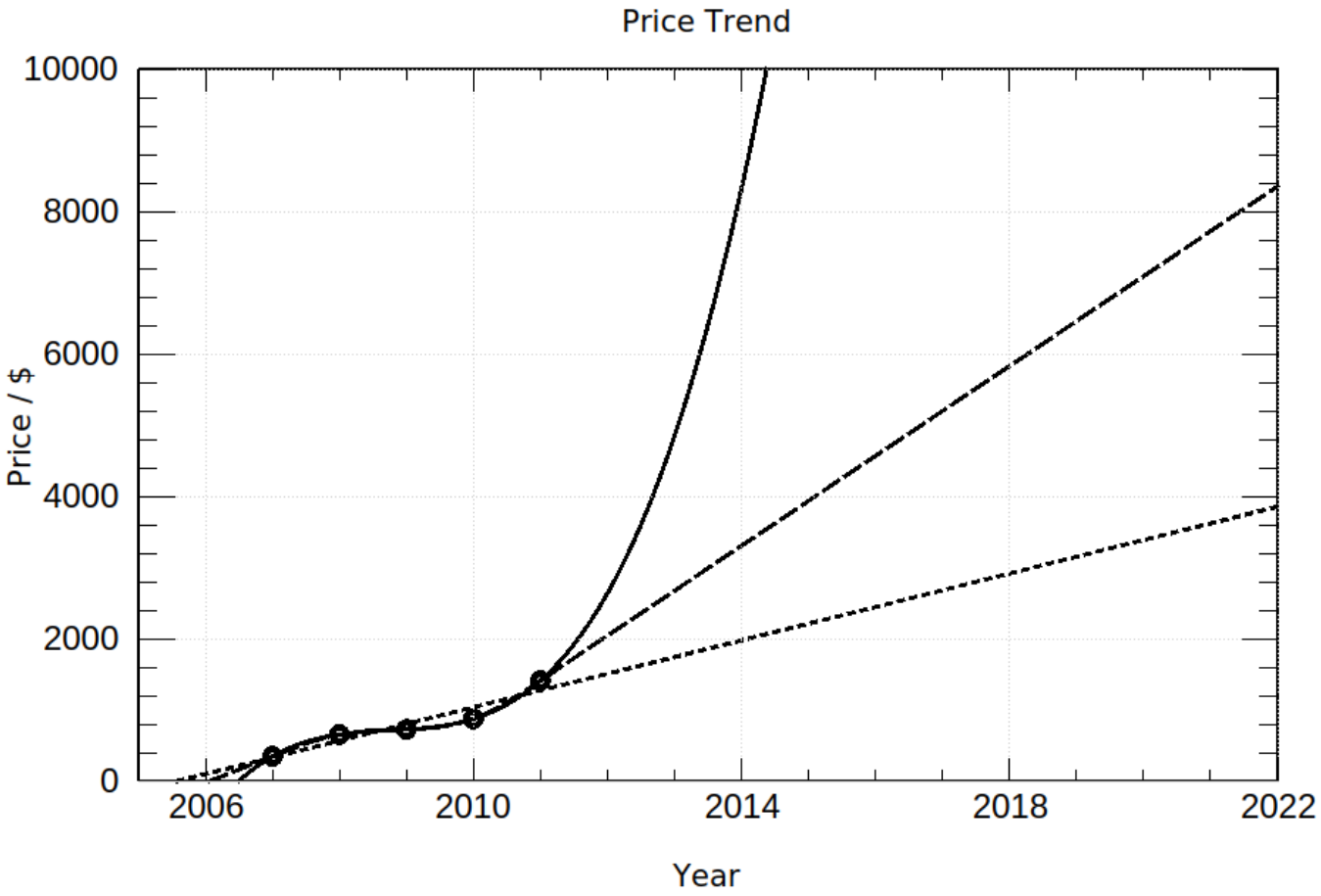


Chapter 4 Exercise 5

Predictions given by polynomical interpolation (solid line), cubic spline interpolation (dash line) and linear regression (dot line) are shown below.

Actually, the price of Château Lafite Rothschild rode a roller coaster around 2009, as shown here. So even the most conservative predicion given by linear regression is far away from the real price.

```
$ cat price.txt | ./trend
Year                2020                2021                Profit
Polynomical         70728               90380               19652
Cubic Spline        7089.32             7720.36             631.036
Linear Regression   3383.1              3617.8              234.7
```


Price Trend
10000
8000
6000
4000
2000
0
Price / $
2006
2010
2014
2018
2022
Year