HW2

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1. $9 \cdot 10^6 = 9{,}000{,}000$ seven-digit phone numbers.
2. (a) $8! = 40320$ seatings
   (b) $5! \cdot 4 \cdot 3! = 2880$ consecutive male seatings.
   (c) $4! \cdot 2^4 = 384$ consecutive couples seatings.
3. $6m, 7s, 4e \to 2$ books
   (a) $\binom{6}{2} + \binom{7}{2} + \binom{4}{2} = 42$ choices of the two books who share the same subject.
   (b) $\binom{6}{1}\binom{7}{1} + \binom{6}{1}\binom{4}{1} + \binom{7}{1}\binom{4}{1} = 94$ choices of two books who don't share the same subject.
4. $8w, 6m \to 3w, 3m$
   (a) $\binom{8}{3}[\binom{6}{3} - \binom{4}{1}] = 896$ committees where m_1, m_2 don't work together.
   (b) $\binom{6}{3}[\binom{8}{3} - \binom{6}{1}] = 1000$ committees where w_1, w_2 don't work together.
   (c) $\binom{8}{3}\binom{6}{3} - P_3^7 = 910$ committees where w_1, m_1 don't work together.[1]
5. Let $D_6 = \{1,2,3,4,5,6\}, A =$ rolling $1 \ldots 6$ in any order, and $|S| = |D_6|^6$.
   If the point $r_1 = (1,2,3,4,5,6) \in A \implies$ other points in A must be arrangements of $r_1 \implies |A| = |D_6|!$
   $\therefore P(A) = \frac{|A|}{|S|} = \frac{6!}{6^6} \approx 0.015$
6. $|S| = \binom{10}{5}, |A| = \binom{6}{5} \implies P(A) = \frac{|A|}{|S|} \approx 0.02$
7.
8. 52 cards $\to$ 5 cards; $|S| = \binom{52}{5}$
   (a) $P(3A, 2K) = \frac{\binom{4}{3}\binom{4}{2}}{\binom{52}{5}} \approx 0.000009$
   (b) $P(3R_1, 2R_2) =$
9.

[1] P_3^7 is a simplification of the number of committees where w_1, m_1 work together.