HW11

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1. $\{Y_1, Y_2, \ldots, Y_6\}$ i. i. d. $\sim$ Exponential(β)
   $\mathrm{P}\left(\min\{Y_1, Y_2, \ldots, Y_6\} \geq a\right) = \mathrm{P}\left(Y_1 \geq \alpha\right) \mathrm{P}\left(Y_2 \geq \alpha\right) \ldots \mathrm{P}\left(Y_6 \geq \alpha\right) = F_{Y_1}(a)^6 =$

$$\left(\int_a^\infty \beta^{-1} \exp\left[-\beta^{-1} y\right] \mathrm{d}y\right)^6 = \exp\left(-6a/\beta\right)$$

2. $\{Y_1, Y_2, \ldots Y_n\}$ i. i. d. with pdf $f_Y(y) = e^\theta e^{-y}, y > \theta$
   $f_{Y_{(1)}} = n\left[1 - F_Y(y)\right]^{n-1} \cdot f_Y(y) =$

$$n\left(1 - e^\theta \int_\theta^\infty e^{-y}\, \mathrm{d}y\right) e^\theta e^{-y} = n\left(1 - e^\theta \left[-e^{-y}\right]_\theta^\infty\right) e^\theta e^{-y} = n(1 - e^{2\theta}) e^\theta e^{-y}$$

3. $\{Y_1, \ldots, Y_n\}$ i. i. d. $\sim$ Beta$(2, 2)$
   (a) $F_{Y_{(n)}}(y) = F_Y(y)^n =$

$$\left(\frac{\Gamma(2+2)}{\Gamma(2)\Gamma(2)} \int y^{2-1}(1-y)^{2-1}\, \mathrm{d}y\right)^n = \left(\frac{\Gamma(4)}{\Gamma(2)^2} \int y(1-y)\, \mathrm{d}y\right)^n = \left(3y^2 - 2y^3\right)^n = y^{2n} \sum_{k=0}^{n} \binom{n}{k} (3)^k \left(-2y\right)^{n-k}$$

   (b) $f_{Y_{(n)}}(y) = \frac{\mathrm{d}}{\mathrm{d}y} F_{Y_{(n)}}(y) = 6n\left(y - y^2\right)\left(3y^2 - 2y^3\right)^{n-1}$

4. $Y = \{Y_1, Y_2, \ldots, Y_5\}$ i. i. d. $\sim$ Uniform$(0, 1)$
   $\mathrm{P}\left(1/4 < Y_{\phi_{0.5}} < 3/4\right) = \mathrm{P}\left(1/4 < Y_3 < \frac{3}{4}\right) = F_{Y(3)}(3/4) - F_{Y(3)}(1/4)$

$$\frac{5!}{(3-1)!(5-3)!} \int_{1/4}^{3/4} F_Y(y)^{3-1} \left[1 - F_Y(y)\right]^{5-3} f_Y(y)\, \mathrm{d}y = 30 \int_{1/4}^{3/4} F_Y(y)^2 \left[1 - F_Y(y)\right]^2 f_Y(y)\, \mathrm{d}y$$

All decimals are rounded to 4 places.