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An Improved Predictive Tracking Control of Discrete-Time High-Order Fully Actuated Systems

Da-Wei Zhang[1], Guo-Ping Liu[1,2]

1. Center for Control Theory and Guidance Technology, Harbin Institute of Technology, Harbin 150001, P. R. China
E-mail: zhangdawei@stu.hit.edu.cn, cldya@163.com

2. Cente for Control Science and Technology, Southern University of Science and Technology, Shenzhen 518055, P. R. China
E-mail: liugp@sustech.edu.cn

Abstract: This paper is concerned with an output tracking problem for a type of discrete-time high-order fully actuated (DT-HOFA) systems, especially improving the dynamic performance of the tracking control. An improved predictive tracking control approach is investigated to address this problem. For the improvement of dynamic performance, an attempt of this approach is to introduce an increment of output predictions into an objective function of predictive control, such that the changes of output predictions can be limited to suppress the oscillation of output responses. To ensure the full actuation feature, an incremental HOFA prediction model is developed by means of a Diophantine Equation to substitute for a first-order prediction model. Through this prediction model, the multi-step ahead output predictions and their increments are obtained to optimize the objective function involving the tracking control performance and the improvement of dynamic performance. The in-depth discussion derives a sufficient and necessary condition for the stability and tracking performance of closed-loop DT-HOFA systems. To verify the feasibility and superiority, the proposed result provides a solution to the tracking control of RLC circuit.

Key Words: DT-HOFA Systems, Improved Predictive Control, Stability and Tracking Control, Improvement of Dynamic Performance

1 Introduction

Recently, high-order fully actuated (HOFA) system theory has been proposed in [1] as an important innovation for the combination among the modeling, analysis, design and applications of control systems. Following this idea, [2] derived a fault-detectable condition of uncertain faulty HOFA systems, and then proposed an active HOFA control method to deal with the fault-tolerant control problem. Aiming at the disturbance decoupling problem, [3] designed a weak disturbance decoupling controller based on HOFA system theory to achieve the stability and the desired tracking control performance. In [4], a new control method was constructed for nonlinear impulsive HOFA systems, and then a stability condition was developed by using a matrix norm inequality. At the same time, HOFA system theory also made some process in practical applications. For instance, [5] presented a discrete-time HOFA model reference tracking control to implement the tracking control of a type of combined spacecraft with disturbances, and then applied a semi-physical experimental platform to verify the theoretical results. [6] proposed a HOFA adaptive prescribed performance controller to consider the position and attitude control of combined spacecraft under unknown disturbances and carried out the related experimental verification. In [7], a distributed learning control based on HOFA system theory was provided to realize the secondary control of DC microgrid, which illustrated the effectiveness via the experiments of dynamic current sharing performance. Meanwhile, [8–10] and the references therein have also contributed to the progress on the theoretical research and application extension of HOFA system theory.

Among so many methods, predictive control plays a significant role in the control design based on HOFA system theory. Our team has obtained a large number of results in this field (see [11–17]). Concretely, [11] firstly used a Diophantine Equation to establish an incremental HOFA prediction model, and proposed a HOFA predictive control to implement the tracking control of DT-HOFA systems. [12, 13] presented a HOFA disturbance observer to estimate both external disturbances and slowly time-variant lumped disturbances, respectively, and then combined with the HOFA predictive control to achieve the tracking control of networked HOFA systems. [14] developed a HOFA predictive sliding-mode control approach to address the random deception attacks. [15] used a packet-based HOFA predictive control to deal with the random denial-of-service attacks. In terms of application, [16, 17] expanded the HOFA predictive control for the implementation of spacecraft flying-around and used a semi-physical simulation platform to demonstrate its practicability. Additionally, other studies in [18–20] also made contributions to the theoretical and practical research on predictive control based on HOFA system theory.

Although the above results can implement the final goal of tracking control, a shortcoming is that the variation of output responses may be too drastic, which causes that it is hard for the hardware in practice to adapt to such drastic changes in amplitude. In this situation, the requirements of reliability, safety, real-time and other indices cannot be ensured. Meanwhile, the complexity of physical implementation of the controller and the mechanical losses of the devices are increased. Considering its negative effects, how to overcome this shortcoming is the key and highlight of this paper.

This study proposes an improved predictive tracking control approach to deal with this problem. In this approach, an improvement is to introduce an increment of output predictions into an objective function of predictive control, which suppresses the oscillation of output responses by limiting the changes of output predictions, such that the dynamic performance in the tracking control process can be effectively im-

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proved. Then, an incremental HOFA prediction model is established by using a Diophantine Equation to take the place of a reduced-order prediction model. Based on this prediction model, the multi-step ahead predictions and their increments are developed to achieve the optimization of tracking control and the improvement of dynamic performance. Finally, a simple necessary and sufficient condition is provided in the subsequent analysis to discuss the stability and tracking control performance of closed-loop DT-HOFA systems. Compared to the existing predictive control methods, the advantages of this paper are summarized in two aspects.

1) An IHOFA prediction model is constructed by applying a Diophantine Equation, so that the design and representation of predictive control can be realized under the framework of HOFA system theory.
2) An increment of output predictions is induced in an objective function to limit its changes, which suppresses the oscillation of output responses to improve the dynamic performance.

Notations. $\aleph_y$ and $\aleph_v$ denote the output and control prediction horizons, respectively, and $\aleph_y \geq \aleph_v$. z represents a time operator satisfying $z^\ell \vartheta(k) = \vartheta(k+\ell)$, $\ell \in \mathbb{Z}$, where $\vartheta(k)$ is an arbitrary signal. $\Delta = 1 - z^{-1}$ indicates a difference operator so that $\Delta\vartheta(k) = \vartheta(k) - \vartheta(k-1)$ denotes an increment. $\hat{\vartheta}(k+\ell|k)$ is the ℓ-th ahead prediction of $\vartheta(k)$ based on k time, and $\Delta\hat{\vartheta}(k+\ell|k) = \hat{\vartheta}(k+\ell|k) - \hat{\vartheta}(k+\ell-1|k)$ is the ℓ-th prediction increment based on k time.

2 Problem Formulation

A type of DT-HOFA systems to be investigated in this paper is provided as

$$\begin{aligned} x(k+n) &= -\sum_{\mu=0}^{n-1} A_\mu x(k+\mu) + Bu(k), \\ y(k) &= Cx(k), \end{aligned} \tag{1}$$

where $x(k)$, $u(k) \in \mathbb{R}^{\tilde{n}}$, $y(k) \in \mathbb{R}^m$ indicate the state, control and output vectors, n is the highest order of state vector. $A_\mu \in \mathbb{R}^{\tilde{n}\times\tilde{n}}$, $\mu = 0, 1, \ldots, n-1$, $B \in \mathbb{R}^{\tilde{n}\times\tilde{n}}$ and $C \in \mathbb{R}^{m\times\tilde{n}}$ represent the known coefficient matrices.

Assumption 1 ([21]). *1)* $\det(B) \neq 0$*, 2) The state vector of DT-HOFA system* (1) *is available.*

For DT-HOFA system (1), a tracking control strategy is proposed as

$$u(k) = B^{-1}\left(u_{cs}(k) + v(k)\right), u_{cs}(k) = \sum_{\mu=0}^{n-1} K_{c,\mu} x(k+\mu), \tag{2}$$

where $u_{cs}(k) \in \mathbb{R}^{\tilde{n}}$ is a stabilization control law and $K_{c,\mu} \in \mathbb{R}^{\tilde{n}\times\tilde{n}}$, $\mu = 0, 1, \cdots, n-1$, denote the associated feedback control gains to be determined. $v(k) \in \mathbb{R}^{\tilde{n}}$ indicates a tracking control law generated by an improved predictive control approach. By adopting (2), a closed-loop DT-HOFA system is implemented as

$$x(k+n) = -\sum_{\mu=0}^{n-1} A_{\mu,c} x(k+\mu) + v(k), \tag{3}$$

with $A_{\mu,c} = A_\mu - K_{c,\mu}$, $\mu = 0, 1, \ldots, n-1$. For the tracking control performance, the objective function in our previous result [11] is designed as

$$\mathcal{J}(k) = \left\|\hat{Y}(k+\aleph_y|k) - R(k+\aleph_y)\right\|_{W_1}^2 + \left\|\Delta\hat{V}(k+\aleph_v|k)\right\|_{W_2}^2, \tag{4}$$

with

$$\begin{aligned} \hat{Y}(k+\aleph_y|k) &= \begin{bmatrix} \hat{y}^{\mathrm{T}}(k+\aleph_y|k) & \cdots & \hat{y}^{\mathrm{T}}(k+1|k) \end{bmatrix}^{\mathrm{T}}, \\ \Delta\hat{V}(k+\aleph_v|k) &= \begin{bmatrix} \Delta\hat{v}^{\mathrm{T}}(k+\aleph_v|k) & \cdots & \Delta\hat{v}^{\mathrm{T}}(k|k) \end{bmatrix}^{\mathrm{T}}, \\ R(k+\aleph_y) &= \begin{bmatrix} r^{\mathrm{T}}(k+\aleph_y) & \cdots & r^{\mathrm{T}}(k+1) \end{bmatrix}^{\mathrm{T}}, \end{aligned}$$

where $\hat{y}(k+\mu|k)$, $k = 1, 2, \ldots, \aleph_y$, and $\Delta\hat{v}(k+\mu|k)$, $k = 0, 1, \ldots, \aleph_v$ are the μ-th ahead predictions of the output $y(k)$ and the tracking control increment $\Delta v(k)$ based on k time. $r(k)$ is a known reference and $R(k+\aleph_y)$ denotes the related multi-step ahead sequence of reference. W_1 and W_2 indicate two positive definite coefficient matrices. In (4), the first part aims to achieve the tracking control performance by approaching the output predictions and the reference, the second part is used to limit the changes of tracking control increment such that the physical implementation of controller is simple and the mechanical loss can be reduced.

By minimizing the $\mathcal{J}(k)$ in (4), the tracking control performance can be implemented easily. However, a shortcoming is that the variation of output response is too drastic. In practice, the hardware is difficult to adapt to such drastic changes in amplitude and frequency, so that the requirements of reliability, safety, real-time and other indices cannot be ensured. To overcome this problem, an increment of output predictions is introduced into the $\mathcal{J}(k)$ in the design of this paper, that is,

$$\mathcal{J}_{\mathrm{IPTC}}(k) = \mathcal{J}(k) + \left\|\Delta\hat{Y}(k+\aleph_y|k)\right\|_{\varepsilon W_1}^2, \tag{5}$$

where $\varepsilon > 0$ is a weighting factor, and $\Delta\hat{Y}(k+\aleph_y|k) = \hat{Y}(k+\aleph_y|k) - \hat{Y}(k+\aleph_y-1|k)$ with

$$\hat{Y}(k+\aleph_y-1|k) = \begin{bmatrix} \hat{y}^{\mathrm{T}}(k+\aleph_y-1|k) & \cdots & y^{\mathrm{T}}(k) \end{bmatrix}^{\mathrm{T}}.$$

Through inducting the increment $\left\|\Delta\hat{Y}(k+\aleph_y|k)\right\|^2$, the variation of output predictions can be effectively limited, so that the oscillation of output responses can be correspondingly suppressed. In this view, the dynamic performance in the tracking control process can be improved. Summarizing the above, the structure of the improved predictive tracking control for a type of DT-HOFA systems is shown in Fig. 1. Then, the presented work of this paper is stated as follows.

Problem 1. *For DT-NHOFA system* (1) *with Assumption 1, an improved predictive control* (2) *is designed by minimizing* (5)*, so that the stability and tracking control performance of closed-loop DT-HOFA system* (3) *can be ensured, that is, the following Conditions 1) and 2) are held.*

1) $\forall k \geq 0$, $\|y(k)\| < \infty$ *with* $\|r(k)\| < \infty$*, where* $r(k)$ *is the known reference,*
2) $\lim_{k\to\infty} \|y(k) - r(k)\| = 0$.

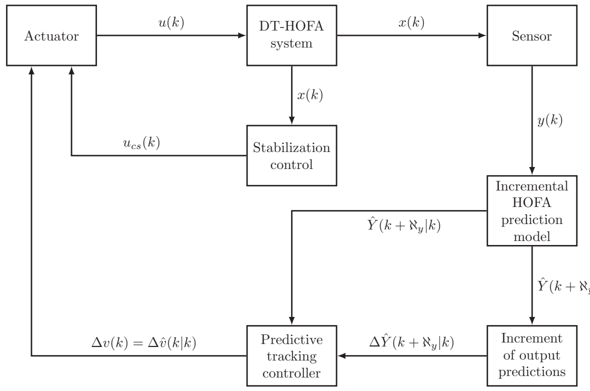

Fig. 1: The structure of improved predictive tracking control for a type of DT-HOFA systems.

3 Main Results

3.1 Design of improved predictive tracking control

According to time operator z, the closed-loop system (3) is converted as

$$\mathcal{A}(z^{-1})x(k)=\mathcal{B}(z^{-1})v(k-1), \tag{6}$$

where $\mathcal{A}(z^{-1})$ and $\mathcal{B}(z^{-1})$ indicate two polynomial coefficient matrices of system (3) and are defined as $\mathcal{A}(z^{-1})=I+\sum_{\mu=0}^{n-1}A_{\mu,c}z^{\mu-n}$ and $\mathcal{B}(z^{-1})=z^{1-n}$. From [11], a Diophantine Equation in relation to system (6) is given as

$$\mathcal{E}_\mu(z^{-1})\mathcal{A}(z^{-1})\Delta+z^{-\mu}\mathcal{F}_\mu(z^{-1})=I,$$

where $\mathcal{E}_\mu(z^{-1})=e_{\mu,0}+e_{\mu,1}z^{-1}+\cdots+e_{\mu,\mu-1}z^{-(\mu-1)}$ and $\mathcal{F}_\mu(z^{-1})=f_{\mu,0}+f_{\mu,1}z^{-1}+\cdots+f_{\mu,n}z^{-n}$ are the solutions of Diophantine Equation and are dependent on polynomial coefficient matrices $\mathcal{A}(z^{-1})$ and prediction horizon μ. By multiplying $\mathcal{E}_\mu(z^{-1})\Delta z^{\mu}$ at (6), it is yielded that

$$\mathcal{E}_\mu\mathcal{A}\Delta x(k+\mu)=\mathcal{E}_\mu\mathcal{B}\Delta v(k+\mu-1).$$

Taking the Diophantine Equation into the above, an incremental HOFA prediction model is developed as

$$x(k+\mu)=\mathcal{F}_\mu x(k)+\mathcal{G}_\mu\Delta v(k+\mu-1), \tag{7}$$

with $\mathcal{G}_\mu(z^{-1})=\mathcal{E}_\mu(z^{-1})\mathcal{B}(z^{-1})=g_{\mu,0}+g_{\mu,1}z^{-1}+\cdots+g_{\mu,n+\mu-2}z^{-(n+\mu-2)}$.

When $\mu=1,2,\ldots,\aleph_y$, the multi-step ahead predictions of $x(k)$ by adopting (7) are generated as

$$\begin{aligned}\hat{x}(k+1|k)=&\mathcal{F}_1x(k)+\mathcal{G}_1\Delta\hat{v}(k|k),\\ &\vdots\\ \hat{x}(k+\aleph_y|k)=&\mathcal{F}_{\aleph_y}x(k)+\mathcal{G}_{\aleph_y}\Delta\hat{v}(k+\aleph_y-1|k),\end{aligned}$$

and the increment of state predictions are derived as

$$\begin{aligned}\Delta\hat{x}(k+1|k)=&\hat{x}(k+1|k)-x(k)\\ =&\left(\mathcal{F}_1-I\right)x(k)+\mathcal{G}_1\Delta\hat{v}(k|k),\\ \Delta\hat{x}(k+2|k)=&\hat{x}(k+2|k)-\hat{x}(k+1|k)\\ =&\left(\mathcal{F}_2-\mathcal{F}_1\right)x(k)+\mathcal{G}_2\Delta\hat{v}(k+1|k)-\mathcal{G}_1\Delta\hat{v}(k|k),\\ &\vdots\\ \Delta\hat{x}(k+\aleph_y|k)=&\hat{x}(k+\aleph_y|k)-\hat{x}(k+\aleph_y-1|k)\\ =&\left(\mathcal{F}_{\aleph_y}-\mathcal{F}_{\aleph_y-1}\right)x(k)+\mathcal{G}_{\aleph_y}\Delta\hat{v}(k+\aleph_y-1|k)\\ &-\mathcal{G}_{\aleph_y-1}\Delta\hat{v}(k+\aleph_y-2|k).\end{aligned}$$

Meanwhile, the output prediction is yielded as

$$\hat{y}(k+\mu|k)=C\hat{x}(k+\mu|k), \tag{8}$$

and its associated increment is given as

$$\Delta\hat{y}(k+\mu|k)=C\Delta\hat{x}(k+\mu|k). \tag{9}$$

For $\mu=\aleph_v+1,\aleph_v+2,\ldots,\aleph_y$, $\hat{v}(k+\mu|k)=\hat{v}(k+\aleph_v|k)$, hence $\Delta\hat{v}(k+\mu|k)=0$. Summarizing the above shows that output prediction (8) can be rewritten in a compact form as

$$\hat{Y}(k+\aleph_y|k)=P_1x(k)+P_2\Delta\hat{V}(k+\aleph_v|k), \tag{10}$$

with

$$P_1=\begin{bmatrix}\mathcal{F}_{\aleph_y}^{\mathrm{T}} & \cdots & \mathcal{F}_1^{\mathrm{T}}\end{bmatrix}^{\mathrm{T}}C^{\mathrm{T}},$$

$$P_2=\begin{bmatrix}C\mathcal{G}_{\aleph_y} & 0 & \cdots & 0\\ \vdots & \vdots & \cdots & \vdots\\ C\mathcal{G}_{\aleph_v+1} & 0 & \cdots & 0\\ 0 & C\mathcal{G}_{\aleph_v} & \ddots & \vdots\\ \vdots & \ddots & \ddots & 0\\ 0 & \cdots & 0 & C\mathcal{G}_1\end{bmatrix},$$

and the increment of output prediction (9) is expressed as

$$\Delta\hat{Y}(k+\aleph_y|k)=P_3x(k)+P_4\Delta\hat{V}(k+\aleph_v|k), \tag{11}$$

with

$$P_3=\begin{bmatrix}\left(\mathcal{F}_{\aleph_y}-\mathcal{F}_{\aleph_y-1}\right)^{\mathrm{T}} & \cdots & \left(\mathcal{F}_1-I\right)^{\mathrm{T}}\end{bmatrix}^{\mathrm{T}}C^{\mathrm{T}},$$

$$P_4=\begin{bmatrix}C\left(\mathcal{G}_{\aleph_y}-\mathcal{G}_{\aleph_y-1}\right) & 0 & 0 & \cdots & 0\\ \vdots & \vdots & \vdots & \cdots & \vdots\\ C\left(\mathcal{G}_{\aleph_v+2}-\mathcal{G}_{\aleph_v+1}\right) & 0 & 0 & \cdots & 0\\ C\mathcal{G}_{\aleph_v+1} & -C\mathcal{G}_{\aleph_v} & 0 & \cdots & 0\\ 0 & C\mathcal{G}_{\aleph_v} & -C\mathcal{G}_{\aleph_v-1} & \ddots & \vdots\\ \vdots & \ddots & \ddots & \ddots & 0\\ \vdots & & \ddots & C\mathcal{G}_2 & -C\mathcal{G}_1\\ 0 & \cdots & \cdots & 0 & C\mathcal{G}_1\end{bmatrix}.$$

Meanwhile, a stair-like incremental limitation about predictive control is introduced as

$$\Delta\hat{v}(k+\mu|k)=\gamma\Delta\hat{v}(k+\mu-1|k)=\cdots=\gamma^{\mu}\Delta\hat{v}(k|k),$$

where $\mu=1,2,\ldots,\aleph_v$, $\gamma>0$ is a weighting factor and $\mathit{\Gamma}=\text{Blockdiag}\{\gamma^{\aleph_v}I,\ldots,\gamma I,I\}$ indicates a stair-like factor matrix. Thus, P_2 and P_4 can be transformed into $P_2\mathit{\Gamma}=P_5$ and $P_4\mathit{\Gamma}=P_6$, so that $P_2\Delta\hat{V}(k+\aleph_v|k)=P_5\Delta\hat{V}(k|k)$ and $P_4\Delta\hat{V}(k+\aleph_v|k)=P_6\Delta\hat{V}(k|k)$ with

$$\Delta\hat{V}(k|k)=\begin{bmatrix}I\\ \vdots\\ I\end{bmatrix}_{\tilde{n}(\aleph_v+1)\times\tilde{n}}\Delta\hat{v}(k|k)=\vec{\boldsymbol{I}}_{\aleph_v+1}\Delta\hat{v}(k|k).$$

In this view, output prediction (10) and its corresponding increment (11) can be represented as

$$\begin{aligned}\hat{Y}(k+\aleph_y|k) =& P_1 x(k) + P_7 \Delta\hat{v}(k|k),\\ \Delta\hat{Y}(k+\aleph_y|k) =& P_3 x(k) + P_8 \Delta\hat{V}(k+\aleph_v|k),\end{aligned} \tag{12}$$

where $P_7 = P_5 \vec{\boldsymbol{I}}_{\aleph_v+1} = P_2 \Gamma \vec{\boldsymbol{I}}_{\aleph_v+1}$ and $P_8 = P_6 \vec{\boldsymbol{I}}_{\aleph_v+1} = P_4 \Gamma \vec{\boldsymbol{I}}_{\aleph_v+1}$.

To get the optimal solution of $\Delta v(k)$, let

$$\frac{\partial}{\partial \Delta\hat{v}(k|k)} \mathcal{J}_{\mathrm{IPTC}}(k) = 0.$$

By means of (12), it is obtained that

$$\begin{aligned}P_7^{\mathrm{T}} W_1 \left(P_1 x(k) + P_7 \Delta\hat{v}(k|k) - R(k+\aleph_y)\right) + (\aleph_v+1) W_2\\ \times \Delta\hat{v}(k|k) + \varepsilon P_8^{\mathrm{T}} W_1 \left(P_3 x(k) + P_8 \Delta\hat{v}(k|k)\right) = 0,\end{aligned}$$

such that

$$M_1 \Delta\hat{v}(k|k) = M_2 x(k) + M_3 R(k+\aleph_y),$$

with

$$\begin{aligned}M_1 =& P_7^{\mathrm{T}} W_1 P_7 + (\aleph_v+1) W_2 + \varepsilon P_8^{\mathrm{T}} W_1 P_8,\\ M_2 =& - P_7^{\mathrm{T}} W_1 P_1 - \varepsilon P_8^{\mathrm{T}} W_1 P_3,\ M_3 = P_7^{\mathrm{T}} W_1.\end{aligned}$$

Then, the optimal solution of predictive control increment is selected as $\Delta v(k) = \Delta\hat{v}(k|k)$, which is expressed as

$$\Delta v(k) = M_1^{-1} M_2 x(k) + M_1^{-1} M_3 R(k+\aleph_y). \tag{13}$$

Remark 1. *The proposed tracking control strategy* (2) *is composed of $u_{cs}(k)$ and $v(k)$, where $u_{cs}(k)$ is used to stabilize the system $x(k+n) = -\sum_{\mu=0}^{n-1} A_{\mu,c} x(k+\mu)$. On the one hand, it can improve the accuracy of incremental HOFA prediction model* (7) *via a feedback correction. On the other hand, it can provide a better basis for adjusting the tracking control performance. In our previous result [11], Algorithm 1 has been provided to obtain the solutions of $K_{c,\mu}$, and its corresponding feasibility analysis has been also given. Since the focus of this paper is not on how to solve the feedback control gains $K_{c,\mu}$, the more details of Algorithm 1 in [11] are not given here, please refer to [11] if interested.*

3.2 Analysis about stability and tracking performance

Following the idea in [12], let $r(\cdot) = r$. For notations convenience, denote $\mathcal{A}_c = \mathcal{A}^{-1}(z^{-1})\mathcal{B}(z^{-1})$, such that system (6) can be transformed as

$$\Delta x(k+1) = \mathcal{A}_c \Delta v(k).$$

By taking (13) into the above system, it is derived that

$$\begin{aligned}\Delta x(k+1) =& \mathcal{A}_c \Delta v(k)\\ =& \mathcal{A}_c M_1^{-1} M_2 x(k) + \mathcal{A}_c M_1^{-1} M_3 R(k+\aleph_y)\\ =& \varPhi \Delta x(k).\end{aligned} \tag{14}$$

with $\varPhi = I + \mathcal{A}_c M_1^{-1} M_2$.

Theorem 1. *The closed-loop DT-HOFA system* (3) *achieves the stability and tracking control performance if system* (14) *implements the asymptotic stability.*

Proof. If system (14) implements the asymptotic stability, $\Delta x(k) \to 0$ as $k \to \infty$. It means that the closed-loop DT-HOFA system (3) is bounded stable, so that Condition 1) of Problem 1 is satisfied. Meanwhile, $\Delta x(k) \to 0$ as $k \to \infty$ derives that $\Delta v(k) \to 0$ as $k \to \infty$, such that Eq. (12) yields that $\hat{Y}(k+\aleph_y|k) = P_1 x(k)$ and $\Delta\hat{Y}(k+\aleph_y|k) = P_3 x(k)$. Additionally, $\Delta v(k) \to 0$ as $k \to \infty$ also implies that $M_2 x(k) + M_3 R(k+\aleph_y) = 0$, it is expressed as

$$-P_7^{\mathrm{T}} W_1 P_1 x(k) - \varepsilon P_8^{\mathrm{T}} W_1 P_3 x(k) + P_7^{\mathrm{T}} W_1 R(k+\aleph_y) = 0,$$

that is,

$$\begin{aligned}P_7^{\mathrm{T}} W_1 \left(\hat{Y}(k+\aleph_y|k) - R(k+\aleph_y)\right)\\ + \varepsilon P_8^{\mathrm{T}} W_1 \Delta\hat{Y}(k+\aleph_y|k) = 0, k \to \infty.\end{aligned}$$

A necessary condition for satisfying the above is that $\hat{Y}(k+\aleph_y) = R(k+\aleph_y)$ and $\Delta\hat{Y}(k+\aleph_y|k) = 0$ as $k \to \infty$, so $\lim_{k\to\infty} \|y(k) - r(k)\| = 0$, that is, Condition 2) of Problem 1 is held.

This completes the proof. □

Remark 2. *From* (13) *and* (14)*, it implies that W_1, W_2 and ε are implicitly embedded in the $\varPhi$ of system* (14)*, so that it is difficult to establish an explicitly analytical relationship between these parameters and stability and tracking control performance, including the regulation functionality of ε for dynamic performance improvement. Because W_1, W_2 and ε are still contained in the $\varPhi$, it is possible to further determine these parameters by characterizing the feature of $\varPhi$. On the basis of [22], the state-transformation matrix $\varPsi_{\mathrm{stm}}$ of system* (14) *is exploited to analyze the asymptotic stability, the details are provided in Lemma 1.*

Lemma 1 ([22])**.** *For a given $k_0 \in \mathbb{Z}$ and $k_\mu \in \mathbb{Z}$, $\mu = 1, 2, \ldots, \infty$, a sequence $\{k_0, k_1, \ldots, k_\infty\}$ is admissible if $\lim_{\mu\to\infty} k_\mu = \infty$ and $\exists \sigma \in \mathbb{Z}$, $\sigma > 0$ such that $\sigma_\mu \in (0, \sigma]$, $\sigma_\mu = k_{\mu+1} - k_\mu$, $\forall \mu \geq 0$. Then, system* (14) *is asymptotically stable if $\forall k_0 \in \mathbb{Z}$, there are an admissible sequence $\{k_0, k_1, \ldots, k_\infty\}$ and two scalars $\nu \in (0, 1)$ and $\hbar(k_0) \in [1, \infty)$ such that $\forall \mu \geq 0$,*

$$\begin{aligned}&\|\varPsi_{\mathrm{stm}}(k_{\mu+1}, k_\mu)\| \leq \nu,\\ &\|\varPsi_{\mathrm{stm}}(k_\mu + l, k_\mu)\| \leq \hbar(k_0),\ \ l \in \{0, 1, 2, \ldots, \sigma_\mu\},\end{aligned}$$

where $\varPsi_{\mathrm{stm}}(k, k_0)$ is the state-transformation matrix of system (14)*, that is, $\Delta x(k) = \varPsi_{\mathrm{stm}}(k, k_0) \Delta x(k_0)$.*

4 An Example

An example of RLC circuit from [13] is considered here to illustrate the feasibility and superiority of the proposed improved predictive control approach, where the circuit model is shown in Fig. 2. On the basis of Kirchhoff Law, the dy-

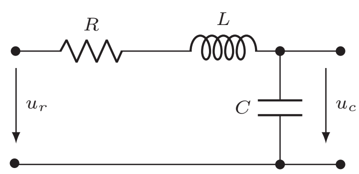

Fig. 2: RLC circuit.

namic model of RLC circuit is presented as

$$LC\ddot{x} + RC\dot{x} + x = u, \tag{15}$$

where $x = u_c(V)$, $\dot{x} = \frac{\mathrm{d}u_c}{\mathrm{d}t}(V/t)$, $\ddot{x} = \frac{\mathrm{d}^2u_c}{\mathrm{d}t^2}(V/t^2)$, $u = u_r(V)$ and $L = 0.5(H)$, $C = 1(F)$, $R = 1.5(\Omega)$. By using a forward difference operator $\dot{x} = \frac{x(k+1)-x(k)}{T}$, system (15) can be transformed into a discrete-time representation in the form of (1) as

$$x(k+2)+(3T-2)x(k+1)+(1-3T+2T^2)x(k) = 2T^2u(k), \tag{16}$$

where T is a sampling period with $T = 0.2$s.

Based on (2), a tracking control strategy is designed as

$$u(k) = \frac{K_{c,0}x(k) + K_{c,1}x(k+1) + v(k)}{2T^2}, \tag{17}$$

with $K_{c,0} = 0.18$, $K_{c,1} = -0.3$. For the predictive control part, choose $\aleph_y = 5$, $\aleph_v = 3$, $W_1 = I$, $W_2 = 10I$, $\varepsilon = 1$, $\Gamma = \mathrm{diag}\{8, 4, 2, 1\}$, the simulated results are provided in Figs. 3 and 4. From Fig. 3, it clearly shows that the control

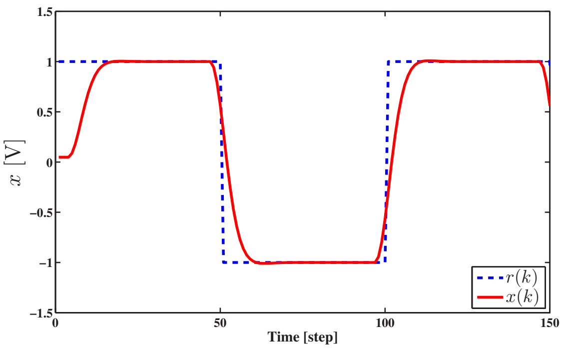

Fig. 3: Tracking control performance generated by (17).

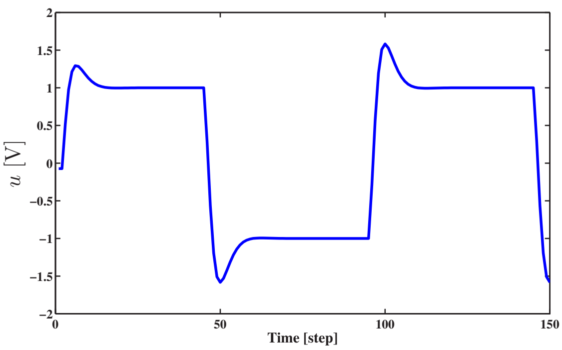

Fig. 4: Control input generated by (17).

protocol (17) can implement the tracking control objective with satisfactory performance. The control input generated by (17) is plotted in Fig. 4.

Compared to the approach in [11], the simulated results are given in Fig. 5, where the associated parameters are chosen as the same as the above without ε. Fig. 5 shows that the improved approach can reduce the overshoot and oscillation, such that a better dynamic performance can be realized. Through the comparison results, the superiority of the improved approach on adjusting the dynamic performance can be fully demonstrated, which provides a feasible idea for the control design of practical applications.

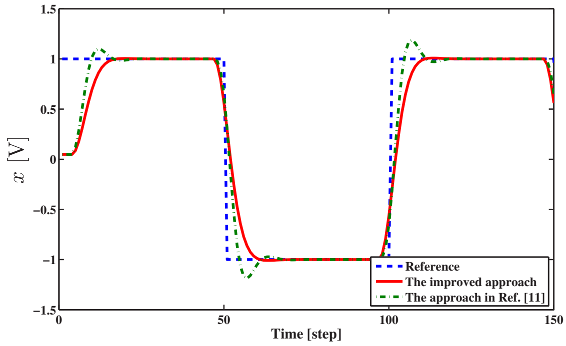

Fig. 5: Comparison of the improved approach (17) and the approach in [11].

5 Conclusions

This paper has proposed an improved predictive control to consider the output tracking control and its dynamic performance improvement for a type of DT-HOFA systems. An increment of output predictions has been introduced into an objective function for restraining the oscillation of output response. Then, a Diophantine Equation has been adopted to establish an incremental HOFA prediction model, such that multi-step ahead output predictions and their corresponding increments have been realized to optimize the objective function involving the tracking control performance and dynamic performance improvement. As a result, a necessary and sufficient condition has been presented to simply analyze the stability and tracking control performance of closed-loop DT-HOFA systems.

The future work will pay close attention to two aspects. One is to continue to investigate the analytical relationship between the design parameters W_1, W_2, ε and the stability and tracking control performance of closed-loop systems, especially the regulation mechanism of ε for dynamic performance improvement. The other will develop the improved predictive control to the control of DT-HOFA systems under some constraints, including network-induced constraints, input constraints, non-holonomic constraint and so on.

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