Proceedings of the 3rd Conference on Fully Actuated System Theory and Applications
May 10-12, 2024, Shenzhen, China

Predefined time sliding mode attitude tracking control for rigid spacecraft based on fully actuated system method

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Abstract: Based on the control theory of the fully actuated system method, this paper proposes a method of attitude-tracking control for rigid spacecraft by using a predefined time terminal sliding mode scheme. First, we establish the fully-actuated system model for the spacecraft by using the attitude dynamics and kinematics equation, then in order to improve the control efficiency, a new predefined time controller is designed by combining the fully actuated system method and the nonsingular terminal sliding mode control method. Finally, the effectiveness of the designed controller is verified by numerical simulation.

Key Words: Fully actuated system method, Attitude tracking, Predefined time stability, Terminal sliding mode

2024 3rd Conference on Fully Actuated System Theory and Applications (FASTA) | 979-8-3503-7369-1/24/$31.00 ©2024 IEEE | DOI: 10.1109/FASTA61401.2024.10595263

1 Introduction

In recent years, spacecraft attitude control has received extensive attention due to its wide application in space exploration, satellite communication, interactive docking and other space missions [1, 2]. The purpose of designing the attitude controller is to ensure that the spacecraft attitude can remain stable and still be able to complete the task of tracking the target attitude when the spacecraft has unknown external disturbances, parameter uncertainties, sensor failures, and so on.

In order to solve these control problems that may exist in aerospace missions, researchers have proposed sliding mode control methods [3, 4] backstepping control methods [5, 6] and other robust control methods for nonlinear systems, and applied these methods to spacecraft attitude control. The reference [4] designed a sliding mode attitude controller to solve the attitude tracking problem of spacecraft systems with inertial uncertainties and external disturbances. In [5], a robust controller based on the backstepping control method is proposed to solve the attitude control problem of spacecraft with external disturbance, parameter uncertainty, and input saturation. Hu proposed a finite-time observer and a linear finite-time terminal sliding mode controller to solve the attitude tracking problem [7]. In [8], an adaptive fixed-time terminal sliding mode attitude control law for rigid spacecraft.

Moreover, most of the above works are the research of asymptotic convergence, finite time convergence, and fixed time convergence, comparison with the above control method, The predefined time convergence system has the advantages of a fast convergence rate, high control accuracy, and the upper bound of convergence time is decidable. As a research hotspot in recent years, predefined time has received extensive attention. In [9], a nonsingular predefined-time controller was developed for attitude stabilization of rigid spacecraft. It is worth noting that the above research is based on the state space model. In [10], a continuous precise predefined-time attitude tracking control law was proposed for a rigid spacecraft. In [11], the time-varying technique was applied to design a predefinedtime output-feedback controller for second-order nonlinear systems.

Duan first proposed a new control method, fully actuated system method, in 2021 [12]. Compared with the state space method, the fully actuated system theory can compensate the dynamic characteristics of the system without considering the complexity of the nonlinear term of the system while keeping the physical environment unchanged. Since the fully actuated system was proposed, recent research on the control theory and application of the fully actuated system has gradually increase. Duan proposed the framework of the fully actuated system method under the conditions like full state feedback systems [13], robust control systems [14], adaptive control systems[15], adaptive robust control systems[16], discrete-time systems [17]. With the development of the fully actuated system theory, more and more researchers have applied the fully actuated system theory to practical engineering. In [18], Duan proposed the application of generalized PID control in tracking system under the theory of fully actuated system. In Reference [19], duan proposed an optimal control method in fully actuated system for spacecraft attitude stability. In [20] proposed high order fully actuated proportional integral predictive control to achieve cooperative control and compensate for fixed communication delays.In [21], a high order fully actuated predictive sliding mode control method is designed to realize the cooperative control of multi-agent systems. Although the full drive system has been studied for a period of time, there are relatively few research results on predefined time control based on fully actuated system considering external disturbances. Therefore, this paper studies a scheduled time spacecraft attitude problem based on the fully actuated system method.

The main contribution of this paper is to establish the attitude system of the spacecraft based on the fully actuated system method, and a new nonsingular terminal sliding mode controller is proposed to complete the attitude tracking control of the system.

2 Preliminaries

2.1 The fully actuated Attitude Kinematics and Dynamics model for spacecraft

The attitude model of the spacecraft is shown in Fig.1. The origin of the coordinate system is the center of the spacecraft, the z-axis is the direction from the center of the spacecraft to the center of the earth, the x-axis is consistent with

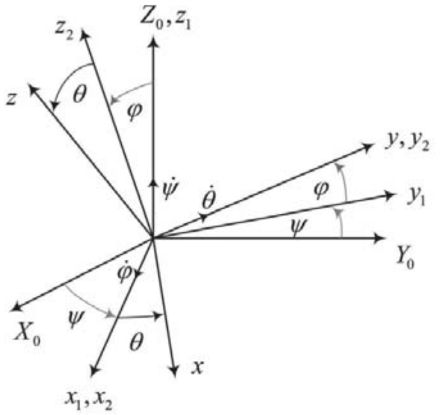

Fig. 1: Coordinate system transformation

the flight direction of the spacecraft, and the y-axis is determined by the right-handed coordinate system. It is denoted by $\sigma = [\varphi, \theta, \psi]^T$, where φ, θ, and ψ are the roll angle, pitch angle, and yaw angle. If the rotation of the coordinate system is $Z(\psi) \rightarrow Z(\varphi) \rightarrow Z(\theta)$ the spacecraft coordinate transformation matrix from ω to $\dot{\sigma}$ is defined as

$$G = \begin{bmatrix} -\sin\theta/\cos\theta & 0 & \cos\theta/\cos\varphi \\ \cos\theta & 0 & \sin\theta \\ -\sin\varphi\sin\theta/\cos\theta & 1 & -\cos\theta\sin\varphi/\cos\varphi \end{bmatrix} \tag{1}$$

As normal, the attitude kinematic and dynamic equation of the rigid spacecraft with the external disturbance is defined as

$$\dot{\sigma} = G(\sigma)\omega \tag{2}$$

$$J\dot{\omega} = -\omega^{\times} J\omega + u + d \tag{3}$$

where J is the inertia matrix, ω is the angular velocity, u is the control torque, d is the external disturbance. $\omega^{\times}$ is the skew-symmetric matrix, which is defined as

$$\omega^{\times} = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix} \tag{4}$$

From (2), there have

$$\begin{aligned} &\ddot{\sigma} = \dot{G}(\sigma)\omega + G(\sigma)\dot{\omega} \\ &= \dot{G}(\sigma)G^{-1}(\sigma)\dot{\sigma} + G(\sigma)[J^{-1}(G^{-1}(\sigma)\dot{\sigma})^{\times} \cdot J(G^{-1}(\sigma)\dot{\sigma})] \\ &+G(\sigma)J^{-1}u + G(\sigma)J^{-1}d \end{aligned} \tag{5}$$

define $f(\sigma, \dot{\sigma}) = \dot{G}(\sigma)G^{-1}(\sigma)\dot{\sigma} + G(\sigma)[J^{-1}(G^{-1}(\sigma)\dot{\sigma})^{\times} \cdot J(G^{-1}(\sigma)\dot{\sigma})]$, $B(\sigma) = G(\sigma)J^{-1}$ and $\det B(\sigma) \neq 0$.
The system satisfies the form of high order fully actuated in literature (Duan, 2021).

$$\ddot{\sigma} = f(\sigma, \dot{\sigma}) + \Delta f(\sigma, \dot{\sigma}) + B(\sigma)u \tag{6}$$

where $f(\sigma, \dot{\sigma}) = -M^{-1}(\sigma)D(\sigma, \dot{\sigma})\dot{\sigma} - M^{-1}(\sigma)\xi(\sigma, \dot{\sigma})$, $B(\sigma) = M^{-1}(\sigma)$ and $\det B(\sigma) \neq 0$

2.2 Definition and Lemma

Definition [17]: Consider the following matrix second-order system

$$x^{(n)} = f(x^{(0\sim n-1)}) + \Delta f(x^{(0\sim n-1)}) + L(x^{(0\sim n-1)})u \tag{7}$$

where x is the system state and u is the control vector, $f(x^{(0\sim n-1)})$ is a continuous vector function, $L(x^{(0\sim n-1)})$ is continuous matrix functions, and satisfies $\det L(x^{(0\sim n-1)}) \neq 0$.
Lemma 1 [17]: A stabilizing controller for the above system is defined by

$$u = u_0 + u_1 \tag{8}$$

where $u_0 = -L^{-1}[A^{(0\sim n-1)}x^{(0\sim n-1)} + \Delta f(x^{(0\sim n-1)}) + f(x^{(0\sim n-1)})]$ is the basic part, which aims to assign the linear term $A^{(0\sim n-1)}x^{(0\sim n-1)}$ to the closed-loop system, u_1 is the auxiliary controller to make the system achieve improved dynamic performance such as the predefined time convergence of the state.
Lemma 2 : consider the continuous nonlinear system $\dot{x}(t) = z(x, t)$, $z(0) = 0$ suppose there is an unbounded Lyapunov function V and $0 < \eta < 1$, such that $V > 0$ for any nonzero x, the following inequality can be satisfied

$$\dot{V} \leq -\frac{2}{t_p\eta}(2V + V^{\frac{1+\eta}{2}} + V^{\frac{3-\eta}{2}}) \tag{9}$$

Hence, the system is predefined time stability, and the settling time depends on the predefined time parameter t_p, and the settling time $t_c < t_p$.
proof: since the Lyapunov function with an initial condition $V_0 > 0$ converges to $V_f > 0$ in the time t_c, the equation can be expressed as

$$\begin{aligned} &\dot{V} \leq -\tfrac{2}{t_p\eta}(2V + V^{\frac{1+\eta}{2}} + V^{\frac{3-\eta}{2}}) \\ &t_c \leq -\tfrac{t_p\eta}{2}\int_{V_0}^{V_f} \frac{1}{2V+V^{\frac{1+\eta}{2}}+V^{\frac{3-\eta}{2}}}dV \\ &= -\tfrac{t_p\eta}{2}\int_{V_0}^{V_f} \frac{1}{V^{\frac{1+\eta}{2}}(V^{\frac{1-\eta}{2}}+2+V^{1-\eta})}dV \\ &= -\tfrac{t_p\eta}{2}\int_{V_0}^{V_f} \frac{1}{V^{\frac{1+\eta}{2}}(1+V^{\frac{1-\eta}{2}})^2}dV \\ &= -\tfrac{t_p\eta}{2}\int_{V_0}^{V_f} \frac{1}{V^{\frac{1+\eta}{2}}(1+V^{\frac{1-\eta}{2}})^2}dV \\ &= -t_p\int_{V_0}^{V_f} \frac{1}{(1+V^{\frac{1-\eta}{2}})^2}dV^{\frac{1-\eta}{2}} \\ &= t_p(1 - \frac{1}{1+V_0^{\frac{1-\eta}{2}}}) \leq t_p \end{aligned} \tag{10}$$

This completes the proof

3 The design of PTNTSM controller

Denote σ_d is the desired attitude, define the tracking error

$$\begin{cases} e_1 = \sigma - \sigma_d \\ e_2 = \dot{\sigma} - \dot{\sigma}_d \end{cases} \tag{11}$$

Assumption1: the leader spacecraft σ_d, $\dot{\sigma}_d$, $\ddot{\sigma}_d$ and the disturbance d is bounded.
A novel nonsingular terminal sliding surface proposed

$$s = \dot{e}_1 + \frac{2}{t_p\eta}(k_0e_1 + k_1sig^{\frac{\alpha}{\beta}}(e_1) + k_2sig^{2-\frac{\alpha}{\beta}}(e_1) + k_3sign(e_1)) \tag{12}$$

where s and e_1 are the sliding mode function and attitude velocity tracking error of spacecraft, respectively, and the parameter $\alpha > 0$, $\beta > 0$, $k_i > 0$ $i = 0, 1, 2, 3$ are positive constant and $\alpha < \beta$, $\eta = \frac{\alpha}{\beta}$ when $s = 0$ there have

$$\dot{e}_1 = \frac{2}{t_p\eta}(-k_0e_1 - k_1sig^{\frac{\alpha}{\beta}}(e_1) - k_2sig^{2-\frac{\alpha}{\beta}}(e_1) - k_3sign(e_1)) \tag{13}$$

construct a Lyapunov function

$$V_1 = \frac{1}{2}e_1^2 \tag{14}$$

taking the derivative of

$$\begin{aligned}
&\dot{V}_1 = e_1\dot{e}_1 = e_1(\tfrac{2}{t_p\eta}(-k_0e_1 - k_1sig^{\frac{\alpha}{\beta}}(e_1) - k_2sig^{2-\frac{\alpha}{\beta}}(e_1)\\
&-k_3sign(e_1)))\\
&= \tfrac{2}{t_p\eta}(-e_1(k_0e_1 + k_1sign(e_1)|e_1|^{\frac{\alpha}{\beta}} + k_2sign(e_1)|e_1|^{2-\frac{\alpha}{\beta}}\\
&+k_3sign(e_1)))\\
&= \tfrac{2}{t_p\eta}(-k_0e_1^2 - k_1|e_1|^{\frac{\alpha+\beta}{\beta}} - k_2|e_1|^{\frac{3\beta-\alpha}{\beta}} - k_3\,|e_1|)\\
&\le -\tfrac{2}{t_p\eta}(k_0e_1^2 - k_1|e_1|^{\frac{\alpha+\beta}{\beta}} - k_2|e_1|^{\frac{3\beta-\alpha}{\beta}})\\
&\le -\tfrac{2}{t_p\mu}(2V_1 + V_1^{\frac{1+\mu}{2}} + V_1^{\frac{3-\mu}{2}})
\end{aligned} \tag{15}$$

Then, the sliding mode reaching law can be defined as:

$$\dot{s} = l_0s + \frac{2}{t_p\eta}(l_1sig^{\frac{p}{q}}(s) + l_2sig^{2-\frac{p}{q}}(s) + l_3sign(s)) \tag{16}$$

taking the derivative of s

$$\begin{aligned}
&\dot{s} = \dot{e}_2+\\
&\tfrac{2}{t_p\eta}(k_0e_2 + k_1\tfrac{\alpha}{\beta}|e_1|^{\frac{\alpha}{\beta}-1}e_2 + k_2\tfrac{2\beta-\alpha}{\beta}|e_1|^{\frac{\alpha}{\beta}-1}e_2 + k_3\,|e_1|\,e_2)
\end{aligned} \tag{17}$$

moreover, the singularity problem will occur if $e_1 = 0$ and $\dot{e}_1 \neq 0$, when the term $\frac{\alpha}{\beta} - 1 < 0$. so, saturation function is addressed by the literature [21] to solve the singularity problem.
To limit the amplitude of the singularity term $e_1^{\frac{\alpha}{\beta}-1}$, saturation function is applied in the controller and it is defined as:

$$sat(m,n) = \begin{cases} m, n > |m| \\ nsign(m), |n| \le m \end{cases} \tag{18}$$

According to the lemma 1, for the spacecraft attitude fully actuated system model (5), the following controller is proposed in this paper.

$$u = -L^{-1}(x^{(0-1)})(A^{(0-1)}\sigma^{(0-1)} + u_s) \tag{19}$$

and the u_1 is given as

$$\begin{aligned}
&u_s = \dot{G}(\sigma)G^{-1}(\sigma)\dot{\sigma} + G(\sigma)[J^{-1}(G^{-1}(\sigma)\dot{\sigma})^{\times}\cdot J(G^{-1}(\sigma)\dot{\sigma})]\\
&-\ddot{\sigma}_d + G(\sigma)J^{-1}d + \tfrac{2}{t_p\eta}(k_0e_1 + sat(k_1\tfrac{\alpha}{\beta}sig^{\frac{\alpha}{\beta}-1}(e_1)e_2, h)\\
&+k_2\tfrac{2\beta-\alpha}{\beta}sig^{1-\frac{\alpha}{\beta}}(e_1)e_2 + k_3sign(e_1)e_2 - l_0s - l_1sig^{\frac{p}{q}}(s)\\
&-l_2sig^{2-\frac{p}{q}}(s) - l_3sign(s))
\end{aligned} \tag{20}$$

where u_0 is the linear state feedback based on the fully actuated system, u_s is a nonsingular terminal sliding mode controller for nonlinear dynamic compensation to achieve predefined time convergence control.

4 Simulation

In order to verify the effectiveness of the designed control method, considering the tracking system composed of a leader spacecraft and a follower spacecraft, the simulation parameters are defined as follows:
The inertia matrix of the spacecraft is defined as $J = [5.86, 0, 0; 0, 2.56, 0; 0, 0, 6.88]$.
The sine wave disturbance d = $0.1[\sin(0.1t), \cos(0.2t), \sin(0.3t)]^T$ of the spacecraft is randomly added.
The initial attitude and initial angular velocity of the target spacecraft are $\sigma_d = [0, 0, 0]^T$.
The initial attitude and initial angular velocity of the following spacecraft is $\sigma = [0.5825; 0. - 5225; 0.1825]^T$, $\omega = [0, 0, 0]^T$. The predefined time $t_p = 10s$, it is means the attitude tracking error will converge to a small region in 10s.
The controller parameters are given by $k_0 = l_0 = 5$,$k_1 = k_2 = l_1 = l_2 = 10$, $k_3 = l_3 = 3$,$\alpha = 7$, $\beta = 9$, $p = 5$, $q = 9$, $h = 100$, $A_0 = diag[-8, -5, -6]$, $A_1 = diag[-7, -12, -3]$ and the minimum and maximum control inputs are set to $u_{\min} = -5N/m$, $u_{\max} = 5N/m$.

Attitude tracking error (e1) and attitude tracking velocity error (e2) under the controller (20) is shown in fig.2 and fig.3, from the figures, it can be observed that all tracking errors are guaranteed to converge to small regions within the predefined time 10s. The results show that the proposed control scheme can achieve attitude tracking even if there are disturbances. The curve of sliding mode and control input are presented in fig.4 and fig.5, it is evident to see that no chattering occurs and the control inputs are continuous. The sliding mode surface converges to within 10s.

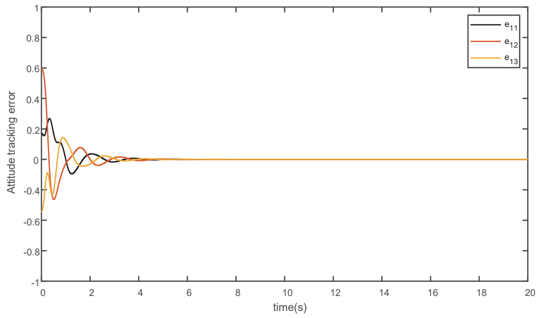

Fig. 2: Attitude tracking error under the controller (20)

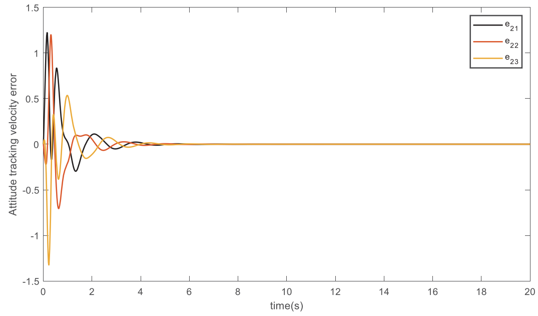

Fig. 3: Attitude tracking error velocity error under the controller (20)

In order to prove the superiority of the fully actuated system method, this paper applies the controller without the fully actuated method as a comparison, and we use u_2 as the controller.

$$u_2 = B(\sigma)u_s \tag{21}$$

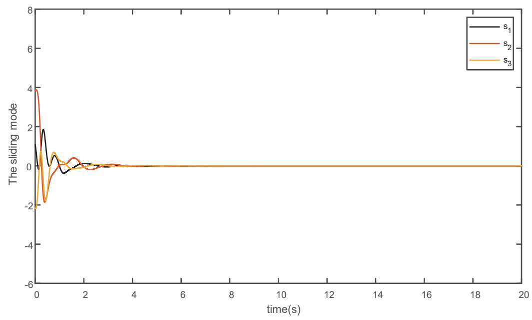

Fig. 4: Response of the sliding mode under the controller (20)

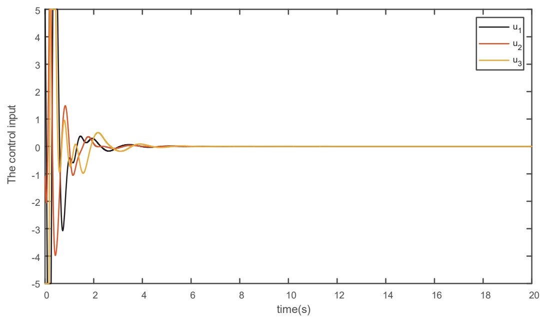

Fig. 5: Response of the controller (20)

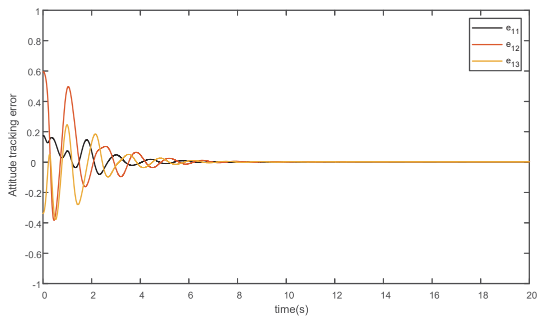

Fig. 6: Attitude tracking error under the controller (21)

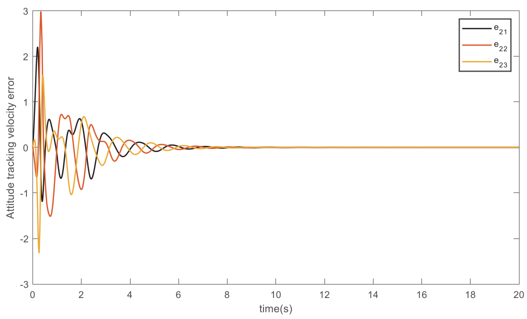

Fig. 7: Attitude tracking velocity error under the controller (21)

the simulation are shown in fig.6 and fig.7, one can see that the system can converge to a small regions in the predifined time 10s. But the control performance is not as good as the controller (20).

Therefore, the method designed in this paper is effective and can enable the spacecraft system to complete the attitude tracking task in a predefined time, which can be applied in actual working conditions because its upper bound of convergence time is decidable.

5 Conclusion

In this paper, the predefined time terminal sliding mode method and fully actuated system method are used to successfully accomplish the attitude tracking of the spacecraft in a predefined time. On this basis, the non-singular terminal sliding mode surface is used to design the control law, which ensures the predefined time convergence of the attitude tracking error in the presence of external disturbances and the effectiveness of the control scheme is verified by numerical simulation.

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