Taylor Rule Analysis

Australia, 1990-2022

BEMACS – Econometrics Assignment

By Michele Usher, Giulio Caputi, and Davide Dal Cero

What is the Taylor Rule?

The Taylor Rule is a monetary policy concept that suggests how Central Banks should adjust the interest rates in response to changes in the economy. Specifically, the Rule suggests that the interest rate set by Central Banks (i) is a linear function of inflation (π), neutral interest rate (r^*), inflation gap ($\pi - \pi^*$) and output gap ($y - y^*$)

Its mathematical expression is:

$$i_t = \pi_t + r_t^* + a_\pi(\pi_t - \pi^*_t) + a_y(y_t - y_t^*)$$

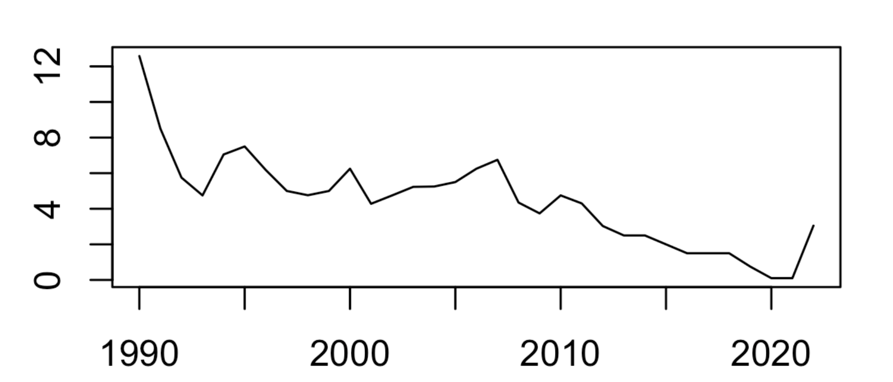
In our analysis, we are considering r^* to be a constant expressed by the intercept of our model. Moreover, given collinearity between the inflation gap and inflation, we decided to drop π_t as a regressor. Hence, our model is:

$$i_t = r^* + a_\pi(\pi_t - \pi^*_t) + a_y(y_t - y_t^*)$$

By Michele Usher, Giulio Caputi, and Davide Dal Cero

A brief historical overview

Between 1990 and 2022 (our period of interest), the Reserve Bank of Australia (RBA) implemented various monetary policies to manage inflation and stabilize the economy. The RBA used the interest rate as a tool to achieve its goals, and as such, the interest rate in Australia moved as a function of inflation gap and output gap



This is a plot of the data we used for the interest rate

In the early 1990s, Australia was coming out from an era of high inflation, and the RBA raised interest rates to combat it. Since then, the interest rate has been on a stable decline, with upwards trends only happening in the mid-1990s, after the 2008 crisis, briefly in 2010, and most recently during the Covid-19 pandemic

How we handled the data

- For our analysis we used datasets downloaded from the Reserve Bank of Australia official website
- Specifically, we used data on the interest rate (yearly, 1990-2022), the consumer price index (quarterly, 1989-2022), and the real and optimal GDP (yearly, 1990-2022). After getting the relevant data frames, we cleaned them by removing irrelevant variables/periods. You can access the data we used at https://drive.google.com/drive/folders/1qAcBaTyIBVdAXVqLp7z5rY4D9syXIX9-?usp=share_link
- In order to obtain inflation, we first turned the quarterly CPI data into yearly ones, by making an average of every 4 consecutive CPI data points. We then computed inflation as the percentage change from one year to the next one, times 100 (this is why we also included CPI data for 1989, so that inflation is between 1990 and 2022)
- Then, to obtain the inflation gap, we subtracted 2 from each data point on inflation, since over the last 30 years π^* has always been very close to 2% in Australia
- In order to obtain the output gap, we took the difference between real GDP and optimal GDP, and then we divided this difference by GDP optimal. We multiplied the result by 100

Descriptive analysis of the variables

```
> summary(output_gap)
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max. 
-11.252   3.725  31.359  35.575  70.519  93.764 
```

```
> summary(inflation_gap)
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max. 
-1.8126 -0.2463  0.4499  0.6715  1.1767  5.3330 
```

```
> summary(int_rate)
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max. 
  0.100   2.500   4.750   4.455   5.750  12.580 
```

```
> sd(output_gap)
[1] 34.64068
> sd(inflation_gap)
[1] 1.553381
> sd(int_rate)
[1] 2.586793
```

```
> cor(int_rate, inflation_gap)
[1] 0.4822343
> cor(int_rate, output_gap)
[1] 0.8110307
> cor(inflation_gap, output_gap)
[1] 0.1134564
```

The interest rate, which is our dependent variable, is moderately correlated with the inflation gap and strongly correlated with the output gap. Moreover, the correlation between the two regressors is very low, implying that they capture different economic factors

Linear regression

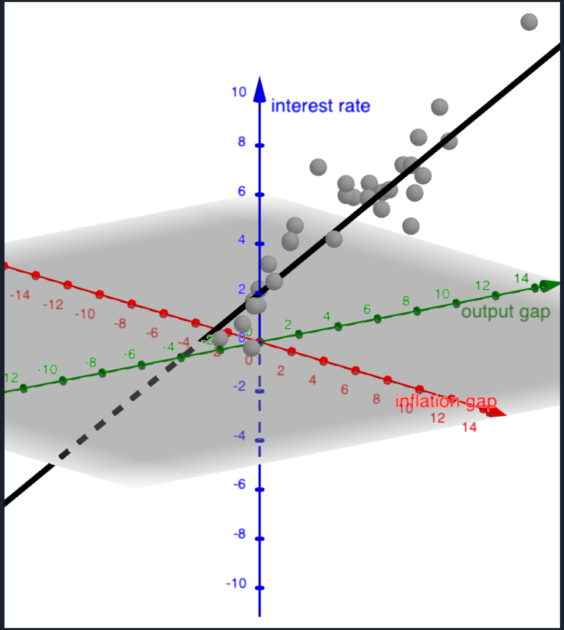
We regressed the interest rate on the inflation gap and the output gap

```
Call:
lm(formula = int_rate ~ inflation_gap + output_gap)

Residuals:
    Min      1Q  Median      3Q     Max
-2.5660 -0.5812 -0.1667  0.8435  2.9996

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)   1.977114   0.298515   6.623 2.48e-07 ***
inflation_gap 0.658290   0.132674   4.962 2.60e-05 ***
output_gap    0.057215   0.005949   9.617 1.13e-10 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 1.158 on 30 degrees of freedom
Multiple R-squared:  0.812,     Adjusted R-squared:  0.7995
F-statistic:  64.8 on 2 and 30 DF,  p-value: 1.293e-11
```



In this plot we divided the output gap by 10 for better seing the data points. This also led us to multiply the coefficient of the output gap by 10

Analysis of results

- Pr(>|t|) is less than 0.01 for all three coefficients, which indicates that both the regressors and the intercept are statistically significant to determine the short term interest rate
- The estimate for the intercept r* is ≈ 2, which is the value at which John Taylor originally set r^*
- The estimated coefficient for the inflation gap is ≈ 0.66 which is close to 0.5, the value that was estimated for it by Taylor himself
- The estimated coefficient of the output gap is 0.057, which is in line with what we expected, since it is positive
- It is important to remember that the actual values of the coefficients depend on the scale of the variables (for instance, we have multiplied both inflation and the output gap by 100, in order to analyze the results better)
- The adjusted R squared of our model is ≈ 0.8 which is high, meaning that our linear model explains a high percentage of the observed data
- Finally, the F-test reports a p-value lower than 0.1, which gives evidence for rejecting the null hypothesis that all of the coefficients are jointly equal to 0. This means that it’s likely that at least one coefficient is actually different from 0, reinforcing the result of the T-test

Tests

```
Jarque Bera Test

data:  residuals(reg)
X-squared = 0.72232, df = 2, p-value = 0.6969
```

We tested for normality of our residuals, and we failed to reject the null hypothesis that our errors follow a Gaussian distribution. So, we have no evidence to conclude that the errors significantly deviate from a Normal distribution

```
studentized Breusch-Pagan test

data:  reg
BP = 2.9932, df = 2, p-value = 0.2239
```

We then tested for homoscedasticity and, given the high p-value we obtained, we did not reject the null hypothesis of homoscedasticity in our model. Note that the Breusch-Pagan test requires the errors to be Normally-distributed, which is something we assessed with the previous test

```
RESET test

data:  reg
RESET = 6.64, df1 = 2, df2 = 28, p-value = 0.004364
```

We also checked for linearity/omition of relevant variables in the model by running a Ramsey test, and the low p-value we obtained is evidence for rejecting H_0, which means that our model could be misspecified (more on this later)

```
Durbin-Watson test

data:  reg
DW = 0.88241, p-value = 7.106e-05
```

We finally tested for autocorrelation of the residuals. Since the p-value of our Durbin-Watson test is small, we can reject H_0, according to which there is no autocorrelation in the residuals. This was expected, since our dependent variable appears autocorrelated (we have ran a Box-Pierce test on “int_rate”, and the resulting p-value was very small, plus it makes sense that the value of the interest rate this year depends on the one of last year)

A note on cointegration

Cointegration is a statistical concept used to describe a long-run relationship between two or more variables. In a cointegrated system, the variables have a long-lasting relationship, meaning that if one variable deviates from its mean, the other variable(s) will eventually adjust to bring the relationship back to its equilibrium. It is a necessary condition for theoretical meaningfulness and empirical relevance of the Taylor Rule, so we tested for the presence of cointegration between our two regressors. As can be seen below, the p-value we obtained is relatively-high, which means we cannot reject H_0 of non-stationarity, and thus we conclude that the interest rate and the output gap are likely not to be cointegrated

```
        Augmented Dickey-Fuller Test

data:  cointegration_test$residuals
Dickey-Fuller = -2.8224, Lag order = 3, p-value = 0.255
alternative hypothesis: stationary
```

Attempts of proper specification (1)

Adding powers of regressors

Since we rejected the null hypothesis of the Ramsey test, we tried adding powers of the regressors - output and inflation gap - and after some tests, we found that by adding the second and third power of the output gap, we still have statistically-significant regressors and the Ramsey test's H_0 is not rejected. Moreover, the explanatory power of our model (aka the adjusted R^2) improved by more than 9%. Note that this might be an example of overfitting, as the model may now generalize badly to new data

```
Call:
lm(formula = int_rate ~ inflation_gap + output_gap + output_gap_squared +
    output_gap_cubic)

Residuals:
     Min       1Q   Median       3Q      Max
-1.87697 -0.49755 -0.07116  0.50310  2.07827

Coefficients:
                    Estimate Std. Error t value Pr(>|t|)
(Intercept)        2.212e+00  2.306e-01   9.592 2.39e-10 ***
inflation_gap      5.884e-01  9.829e-02   5.986 1.90e-06 ***
output_gap         1.689e-01  2.237e-02   7.551 3.17e-08 ***
output_gap_squared -4.021e-03  7.359e-04  -5.464 7.82e-06 ***
output_gap_cubic   3.215e-05  6.175e-06   5.207 1.57e-05 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.8334 on 28 degrees of freedom
Multiple R-squared:  0.9092,    Adjusted R-squared:  0.8962
F-statistic: 70.07 on 4 and 28 DF,  p-value: 3.566e-14
```

```
        RESET test

data:  lm(int_rate ~ inflation_gap + output_gap + output_gap_squared +     out
put_gap_cubic)
RESET = 0.41982, df1 = 2, df2 = 26, p-value = 0.6615
```

Attempts of proper specification (2)

Adding additional regressors

To improve the fitting in our model, we added the **exchange rate** (downloaded from the RBA website). The adjusted R-squared increases by about 8% from our original model, which indicates an improvement in the explanatory power of our model. Moreover, the Ramsey test returns a p-value which is now 0.2831, which implies that we do not reject the null hypothesis that the model is properly specified

```
Call:
lm(formula = int_rate ~ inflation_gap + output_gap + exchange_rate)

Residuals:
     Min       1Q   Median       3Q      Max 
-1.59425 -0.66192 -0.04768  0.48675  2.53691 

Coefficients:
               Estimate Std. Error t value Pr(>|t|)    
(Intercept)   -3.337674   1.189764  -2.805  0.00888 ** 
inflation_gap  0.655588   0.103036   6.363 5.92e-07 ***
output_gap     0.066321   0.005034  13.173 9.09e-14 ***
exchange_rate  6.594497   1.447944   4.554 8.74e-05 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.8995 on 29 degrees of freedom
Multiple R-squared:  0.8904,    Adjusted R-squared:  0.8791 
F-statistic: 78.54 on 3 and 29 DF,  p-value: 4.98e-14
```

```
        RESET test

data:  lm(int_rate ~ inflation_gap + output_gap + exchange_rate)
RESET = 1.3227, df1 = 2, df2 = 27, p-value = 0.2831
```

Attempts of proper specification (3)

Additional additional regressors

To improve the fitting in our model, we also tried adding the **unemployment rate** as a regressor, again downloaded from the RBA official website. The adjusted R-squared increases by roughly 3% compared to our original model, but as we can see from the Ramsey test we cannot safely reject the null hypothesis that the model is misspecified

```
Call:
lm(formula = int_rate ~ inflation_gap + output_gap + unemployment_rate)

Residuals:
     Min       1Q   Median       3Q      Max 
-2.48960 -0.62899 -0.09825  0.62420  2.09868 

Coefficients:
                   Estimate Std. Error t value Pr(>|t|)    
(Intercept)        4.430953   1.015731   4.362 0.000148 ***
inflation_gap      0.456400   0.146394   3.118 0.004092 ** 
output_gap         0.076433   0.009419   8.115    6e-09 ***
unemployment_rate -0.593440   0.236458  -2.510 0.017919 *  
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 1.068 on 29 degrees of freedom
Multiple R-squared:  0.8456,	Adjusted R-squared:  0.8296 
F-statistic: 52.93 on 3 and 29 DF,  p-value: 7.03e-12
```

```
	RESET test

data:  lm(int_rate ~ inflation_gap + output_gap + unemployment_rate)
RESET = 3.0944, df1 = 2, df2 = 27, p-value = 0.06166
```

Attempts of proper specification (4)
Additional regressors and multicollinearity

An important aspect to bring to light is the little significance of the unemployment rate as additional regressor

ECONOMIC INTUITION: The trend of the unemployment rate captures that of the output gap, creating multicollinearity between the two regressors. Indeed, **Okun’s Law** describes the link between GDP and unemployment, stating that for every 1% increase in the unemployment rate, there should be roughly a 2% decrease in the economy's GDP, and vice versa. This creates multicollinearity between the output gap and the unemployment rate

STATISTICAL EVIDENCE:

```
> cor(GDP_real, unemployment_rate)
[1] -0.7449508
```

Conclusion

Overall, the Taylor Rule has gained extreme popularity since John Taylor first introduced it in 1993. Through both statistical analysis and economic considerations, we have showed that, although there surely exists a relationship among the variables of the Rule, there is a risk of both a model misspecification and of omitting relevant variables. Moreover, we have found no evidence of cointegration between the two regressors in the Taylor Rule formula, meaning that the empirical evidence of the analysis might be less relevant than originally proposed by Taylor. There is also the possibility of running spurious regressions using this formula. Our line of thought is that Central Banks are doing much more than just following this simple formula when setting the interest rate, and so the Taylor Rule might only be a relatively-vague approximation of CBs reaction function, but we should probably doubt the consistency of the model's parameters