
Inverse of a Pauli Matrix

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In general, a pauli matrix having the form:

```
aMat = a * PauliMatrix[0] + b * PauliMatrix[1] + c * PauliMatrix[2] + d * PauliMatrix[3];
```

Has an inverse:

$$\text{In[61]:= } \mathbf{cof} = \begin{pmatrix} -\frac{a}{-a^2+b^2+c^2+d^2} \\ \frac{b}{-a^2+b^2+c^2+d^2} \\ \frac{c}{-a^2+b^2+c^2+d^2} \\ \frac{d}{-a^2+b^2+c^2+d^2} \end{pmatrix};$$

```
aInverse = cof[[1]] * PauliMatrix[0] + cof[[2]] * PauliMatrix[1] +  
           cof[[3]] * PauliMatrix[2] + cof[[4]] * PauliMatrix[3];  
aInverse // MatrixForm
```

$$\text{Out[63]//MatrixForm= } \begin{pmatrix} -\frac{a}{-a^2+b^2+c^2+d^2} + \frac{d}{-a^2+b^2+c^2+d^2} & \frac{b}{-a^2+b^2+c^2+d^2} - \frac{ic}{-a^2+b^2+c^2+d^2} \\ \frac{b}{-a^2+b^2+c^2+d^2} + \frac{ic}{-a^2+b^2+c^2+d^2} & -\frac{a}{-a^2+b^2+c^2+d^2} - \frac{d}{-a^2+b^2+c^2+d^2} \end{pmatrix}$$

As confirmed by:

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In[65]:= FullSimplify[aMat.aInverse] // MatrixForm
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$$\text{Out[65]//MatrixForm= } \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$