Solution for HW2 20161013

Taper

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Abstract

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1 Describe wave function

Wave functgion is a function that is assumed to be describing the physical system. It is assumed that all physical information can be extracted from it, which is saying that a wave function is the complete description of the physical system it is complete. And we calculate any observable A by calculating its expectation value using the wave function.

As for the probability wave, I thick this is kind of a misnomer. It is the wave function that propagates in the form of waves, as can be seen by the following formal solution to the schrodinger equation:

$$\begin{aligned} i\hbar\frac{\partial}{\partial t}\Psi &= H\Psi \\ \Rightarrow \Psi(t) &= e^{-i\hbar H}\Psi(t=t_0) \end{aligned} \tag{1.0.1}$$

However, the probability is determined by the absolute value of wavefunction, so it does not exactly propagate like a wave.

2 Eigensystems

•e^x: $\frac{\mathrm{d}^2}{\mathrm{d}x^2}e^x = e^x$, so it IS an eigenfunction, with eigenvalue 1.

• $\sin(x)$: $\frac{\mathrm{d}^2}{\mathrm{d}x^2}\sin(x) = -\sin(x)$, so it IS an eigenfunction, with eigenvalue -1.

•$2\cos(x)$: $\frac{\mathrm{d}^2}{\mathrm{d}x^2}2\cos(x) = 2(-\cos(x)) = -(2\cos(x))$, so it IS an eigenfunction, with eigenvalue -1.

• $\sin^2(x)$: $\frac{\mathrm{d}^2}{\mathrm{d}x^2}\sin^2(x) = \frac{\mathrm{d}}{\mathrm{d}x}2\cos(x) = -2\sin(x)$, so it is NOT an eigenfunction.

•x^3: $\frac{\mathrm{d}^2}{\mathrm{d}x^2}x^3 = 6x$, so it is NOT an eigenfunction.

• $\sin(x)+\cos(x)$: the combination of two eigenfunction with the same eigenvalue is of course an eigenfunction, with eigenvalue -1.

3 1D infinite potential well

The time-independent Schrodinger equation inside the wall is:

$$-\frac{\hbar^2}{2m}\frac{\mathrm{d}^2}{\mathrm{d}x^2}\psi = E\psi$$

, so clearly

$$\psi = A\sin(kx) + B\cos(kx)$$

inside the wall. Here $k = \sqrt{\frac{2mE}{\hbar^2}}$.

Outside the wall the potential is infinite, not suitable for wavefunction to live. So $\psi = 0$ outside. Connecting the wave function in the boundary will clearly leads to dicretized wavelength. But the give coordinate is not convenient to determine the parameters A, B. So I shift the origin leftwards for $\frac{a}{2}$. Then the potential becomes:

$$U = \begin{cases} 0 & 0 \le x \le a \\ \infty & \text{otherwise} \end{cases}$$

Then we have $\psi(0) = 0$, hence $B = 0$. And $\psi(a) = 0$, hence

$$\sqrt{\frac{2mE}{\hbar^2}}a = n\pi \Rightarrow$$

$$E = \frac{n^2\pi^2\hbar^2}{2ma^2}$$

where $n = 1, 2, \cdots$. A is determined by normalization, but I don't have to calculate because the next exercise has already given the answer: $A = \sqrt{\frac{2}{a}}$. Now shift the coordinate back, I have

$$\psi(x) = \sqrt{\frac{2}{a}}\sin\left(k(x - \frac{a}{2})\right) = \sqrt{\frac{2}{a}}\sin\left(\frac{n\pi}{a}(x - \frac{a}{2})\right) \tag{3.0.2}$$

$$k = \sqrt{\frac{2mE}{\hbar^2}} \tag{3.0.3}$$

$$E = \frac{n^2\pi^2\hbar^2}{2ma^2} \tag{3.0.4}$$

4 Prove the orthogonality

This is simple:

$$\int_0^l \sin(\frac{n\pi x}{l})\sin(\frac{m\pi x}{l})$$

$$= \int_0^l \frac{\cos(\frac{(n-m)\pi x}{l}) - \cos(\frac{(n-m)\pi x}{l})}{2}$$

Notice that when $n \neq m$, both $\cos(\frac{(n-m)\pi x}{l})$ and $\cos(\frac{(n-m)\pi x}{l})$ oscillate with the period of l. That means the integration is taken over a whole peroid. So the result is 0, i.e.:

$$\int_0^l \sin(\frac{n\pi x}{l})\sin(\frac{m\pi x}{l}) = 0 \tag{4.0.5}$$

for $m \neq n$.