Notes of Chapter 2 of Bernevig's Book

Taper

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Abstract

Since I have already made a written one, this note is only an outline of the written script, in the hope of makeing it eaiser to read.

Contents

1 Deriving Berry Phase

From page 1 to page 2 (equation 2.7), one derives the expression for the phase $e^{i\gamma_m}$ using instaneous energy eigenstate for E_m:

$$H(\vec{R})\,|n(\vec{R})\rangle = E_m(\vec{R})\,|n(\vec{R})\rangle \tag{1.0.1}$$

$$\gamma_m = i\int_0^t \langle m\left(\vec{R}(t')\right)|\frac{\partial}{\partial t}m\left(\vec{R}(t')\right)\mathrm{d}t'\rangle \tag{1.0.2}$$

Then I writes about different ways to get the γ_m:

$$\gamma_m = i\int_{\text{curve}}\langle m|\nabla_{\vec{R}}\,m\rangle\,\mathrm{d}\vec{R} \tag{1.0.3}$$

(equation 2.8) Also, one defines

Definition 1.1 (Berry Connection, Berry Vector Potential $\vec{A}$)**.**

$$\vec{A}_n \equiv i\,\langle n|\nabla_{\vec{R}}\,n\rangle \tag{1.0.4}$$

Then I proves several facts:

Fact 1.1.

γ_n is real

Fact 1.2.

Berry connection $\vec{A}_n$ is gauge-dependent

The dependence is: If

$$|n\rangle \to |n'\rangle = e^{i\xi(\vec{R})}\,|n\rangle$$

then

$$\vec{A}_n \to \vec{A}_n - \nabla_{\vec{R}}\,\xi(\vec{R}) \tag{1.0.5}$$

Therefore we have

Fact 1.3.

γ_n is gauge-dependent, unless the path tranverses a closed loop.

Since

$$\gamma_n \to \gamma_n - \Big(\xi(\vec{R}(T)) - \xi(\vec{R}(0))\Big)$$

It is unchanged unless the integration curve is a closed loop. In which case

$$\xi(\vec{R}(T)) - \xi(\vec{R}(0)) = 2\pi m \overset{mod\ 2\pi}{=} 0$$

Example 1.1. There is a simple example on page 3 to show that the Berry phase can be actually detected.

By virtue of fact 1.1, we have

2 Anchor

References

[1] Bernevig's Topological Insulators and Superconductors

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