Calculate in the Infinite Stripe Geometry

i.e., no hopping between the 1st and Nth sites (with the corrected Hamiltonian).

```
dimensionN = 3
```

3

Now Construct the Hamiltonian Matrix

```
A = i * PauliMatrix[2] / 2 - PauliMatrix[3] / 2;
A // MatrixForm
```

$$\begin{pmatrix} -\frac{1}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

```
B[px_, m_] := 2 Sin[px] * PauliMatrix[1] / 2 -
   2 Cos[px] PauliMatrix[3] / 2 + (2 - m) PauliMatrix[3];
B[px, m] // MatrixForm
```

$$\begin{pmatrix} 2 - m - \mathrm{Cos}[px] & \mathrm{Sin}[px] \\ \mathrm{Sin}[px] & -2 + m + \mathrm{Cos}[px] \end{pmatrix}$$

```
CMatrix = -i * PauliMatrix[2] / 2 - PauliMatrix[3] / 2;
CMatrix // MatrixForm
```

$$\begin{pmatrix} -\frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

```
For[i = 1, i ≤ dimensionN, i++,
 If[i == 1,
  hamiltonianMLine[i][px_, m_] =
   ArrayFlatten[{{B[px, m], CMatrix, ConstantArray[0, {2, 2 * dimensionN - 4}]}}]
 ];
 If[i > 1 && i < dimensionN,
  hamiltonianMLine[i][px_, m_] = ArrayFlatten[{{
       ConstantArray[0, {2, 2 * (i - 2)}], A, B[px, m],
       CMatrix, ConstantArray[0, {2, 2 * dimensionN - 2 * (i + 1)}]
     }}]
 ];
 If[i == dimensionN,
  hamiltonianMLine[i][px_, m_] = ArrayFlatten[{{
       ConstantArray[0, {2, 2 * (i - 2)}], A, B[px, m]
     }}]
 ];]
hamiltonianM[px_, m_] = hamiltonianMLine[1][px, m];
For[i = 2, i ≤ dimensionN, i++,
 hamiltonianM[px_, m_] = Join[hamiltonianM[px, m], hamiltonianMLine[i][px, m]];
]
```

```
hamiltonianM[px, m] // MatrixForm
```

$$\begin{pmatrix} 2-m-\mathrm{Cos}[px] & \mathrm{Sin}[px] & -\frac{1}{2} & -\frac{1}{2} & 0 & 0 \\ \mathrm{Sin}[px] & -2+m+\mathrm{Cos}[px] & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ -\frac{1}{2} & \frac{1}{2} & 2-m-\mathrm{Cos}[px] & \mathrm{Sin}[px] & -\frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} & \mathrm{Sin}[px] & -2+m+\mathrm{Cos}[px] & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & -\frac{1}{2} & \frac{1}{2} & 2-m-\mathrm{Cos}[px] & \mathrm{Sin}[p \\ 0 & 0 & -\frac{1}{2} & \frac{1}{2} & \mathrm{Sin}[px] & -2+m+\mathrm{Cc} \end{pmatrix}$$

Solve the Eigenvalue Equation and Plot

```
eigenSystemResult[px_, m_] = Eigensystem[hamiltonianM[px, m]]
```

```
{{-1/2 √Root[
    -16 032 + 33 280 m - 29 824 m^2 + 14 848 m^3 - 4416 m^4 + 768 m^5 - 64 m^6 + 22 400 Cos[px] -
      44 992 m Cos[px] + ⋯ 11 ⋯ + 3072 m^3 Cos[2 px] - 384 m^4 Cos[2 px] +
      896 Cos[3 px] - 1472 m Cos[3 px] + 768 m^2 Cos[3 px] - 128 m^3 Cos[3 px] +
      (1776 - 2432 m + 1376 m^2 - 384 m^3 + 48 m^4 - 2048 Cos[px] + 2560 m Cos[px] -
         1152 m^2 Cos[px] + 192 m^3 Cos[px] + 368 Cos[2 px] -
         384 m Cos[2 px] + 96 m^2 Cos[2 px]) #1 +
      (-68 + 48 m - 12 m^2 + 48 Cos[px] - 24 m Cos[px]) #1^2 + #1^3 &, 1],
  1/2 √Root[⋯ 1 ⋯], ⋯ 2 ⋯,
  ⋯ 1 ⋯,
  1/2
   √Root[⋯ 1 ⋯ &, 3]}, ⋯ 1 ⋯}
```

large output | show less | show more | show all | set size limit...

The eigensystem returns a 2 x (2*dimensionN) matrix, with the eigenvalues in the first line, and the corresponding eigenvectors in the second line.

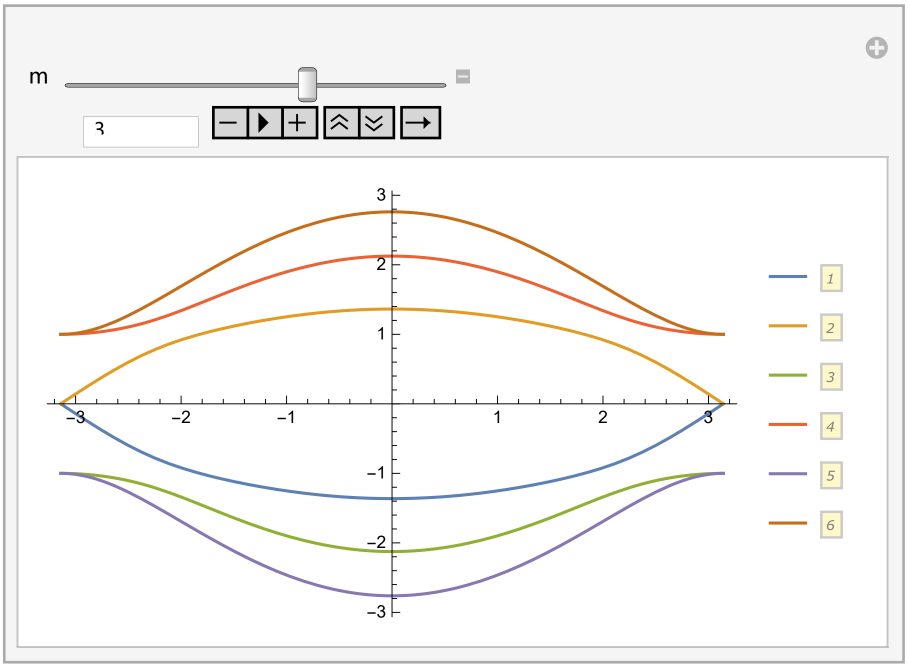
```
Dimensions[eigenSystemResult[px, m]]
```

{2, 6}

Now I seperate them and store them into respective arrays:

```
(* Experience tells me extrating things in eigenSystemResult outside
 is inefficient. Instead, define a new function of the component
 under investigation and plot using that function. Therefore,
part of the following code are commented out. *)
eigenValues[px_, m_] = ConstantArray[0, {1, 2 * dimensionN}];
(* eigenVectors[px_,m_]=ConstantArray[0,{2*dimensionN,2*dimensionN}]; *)
(* Labelled by ( ith eigenvector, jth component of that eigenvector ) *)
For[i = 0, i ≤ (2 dimensionN), i++,
 (*A primitive way that works: eigenValues[px_,m_][i]=
   eigenSystemResult[px,m][[1,i]];*)
 eigenValues[px_, m_] = ReplacePart[eigenValues[px, m],
    {1, i} → eigenSystemResult[px, m][[1, i]]];
 (* For[j=0,j≤(2 dimensionN),j++,
  eigenVectors[px_,m_]=ReplacePart[
      eigenVectors[px,m],{i,j}→ eigenSystemResult[px,m][[2,i]][[j]]
    ];
 ]*)
]

Manipulate[
 Plot[Evaluate[eigenValues[px, m]], {px, -π, π}, PlotLegends → Automatic],
 {m, -10, 10}]
```

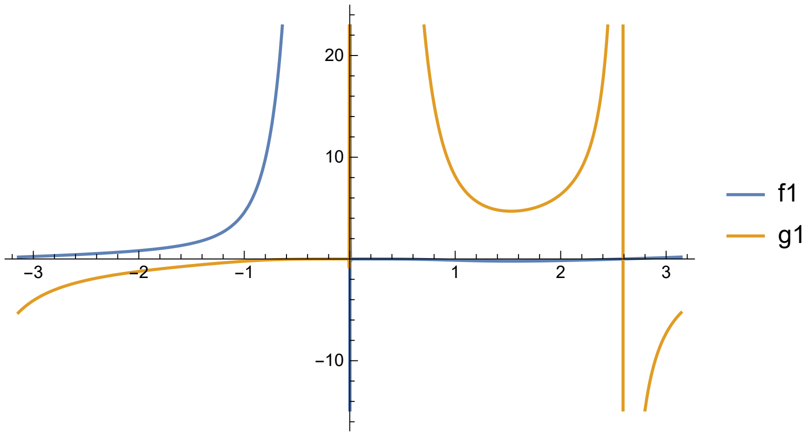


The crossing now happens at (m,px)= (1,0), (3,±π), And the band that crosses correspondes to the 1st and 2nd eigenvalues/eigenvectors

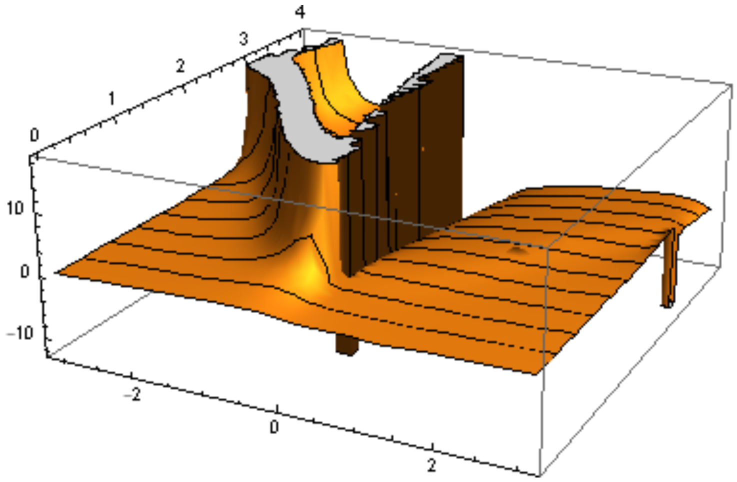
Now Analysis the Eigenvectors

In[77]:=
```
f1[px_, m_] = eigenSystemResult[px, m][[2, 1]][[1]];
f2[px_, m_] = eigenSystemResult[px, m][[2, 1]][[2]];
f3[px_, m_] = eigenSystemResult[px, m][[2, 1]][[3]];
f4[px_, m_] = eigenSystemResult[px, m][[2, 1]][[4]];
f5[px_, m_] = eigenSystemResult[px, m][[2, 1]][[5]];
f6[px_, m_] = eigenSystemResult[px, m][[2, 1]][[6]];
g1[px_, m_] = eigenSystemResult[px, m][[2, 2]][[1]];
```

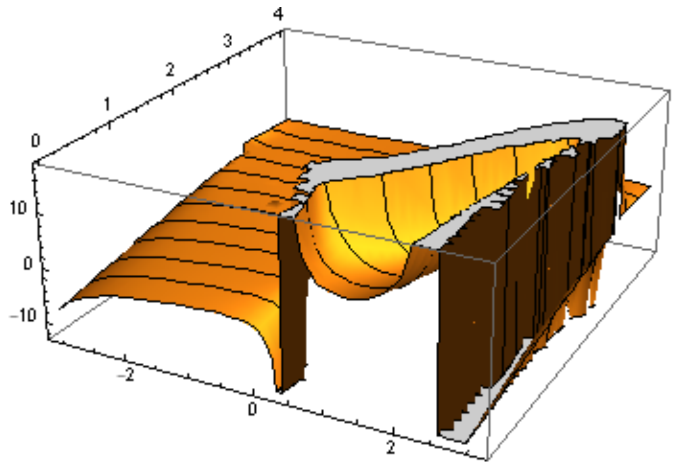
```
Plot[
 {f1[px, 1], g1[px, 1]}, {px, -π, π}, PlotLegends → {"f1", "g1"}]
```



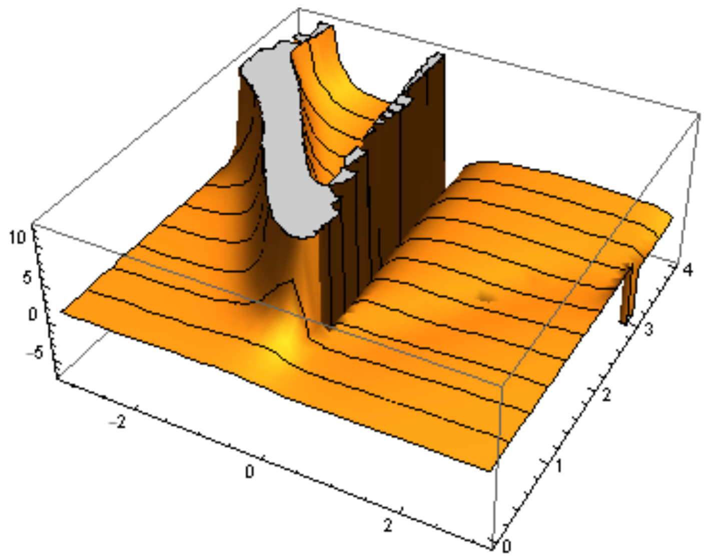

```
Plot3D[f1[px, m], {px, -Pi, Pi}, {m, 0, 4}, MeshFunctions → {#2 &},
 Mesh → 10, PerformanceGoal → "Quality", MeshStyle → {{Black, Thin}}]
```



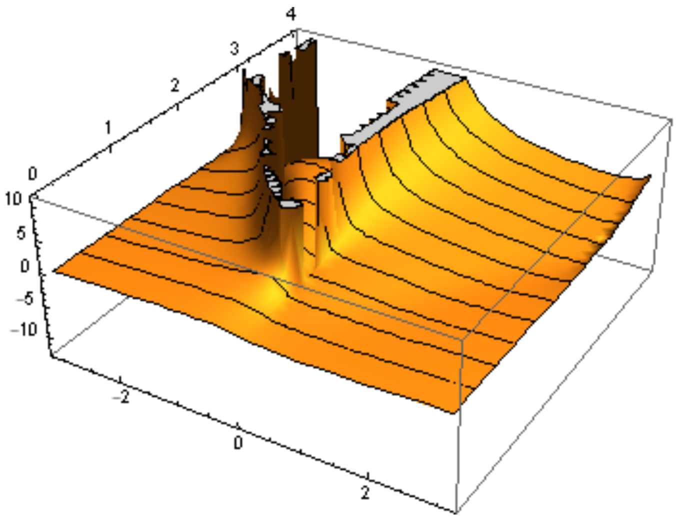

```
Plot3D[g1[px, m], {px, -Pi, Pi}, {m, 0, 4}, MeshFunctions → {#2 &},
 Mesh → 10, PerformanceGoal → "Quality", MeshStyle → {{Black, Thin}}]
```



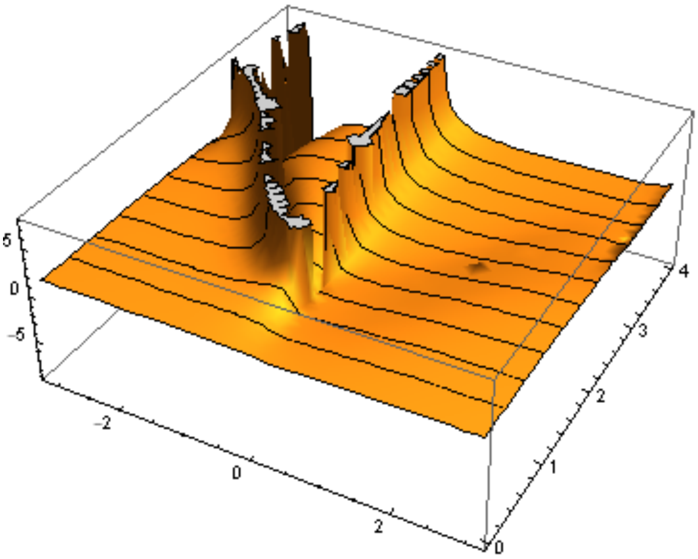
In[84]:=
```
Plot3D[f2[px, m], {px, -Pi, Pi}, {m, 0, 4}, MeshFunctions → {#2 &},
 Mesh → 10, PerformanceGoal → "Quality", MeshStyle → {{Black, Thin}}]
Plot3D[f3[px, m], {px, -Pi, Pi}, {m, 0, 4}, MeshFunctions → {#2 &},
 Mesh → 10, PerformanceGoal → "Quality", MeshStyle → {{Black, Thin}}]
Plot3D[f4[px, m], {px, -Pi, Pi}, {m, 0, 4}, MeshFunctions → {#2 &},
 Mesh → 10, PerformanceGoal → "Quality", MeshStyle → {{Black, Thin}}]
Plot3D[f5[px, m], {px, -Pi, Pi}, {m, 0, 4}, MeshFunctions → {#2 &},
  Mesh → 10, PerformanceGoal → "Quality", MeshStyle → {{Black, Thin}}]
 Plot3D[f6[px, m], {px, -Pi, Pi}, {m, 0, 4}, MeshFunctions → {#2 &},
  Mesh → 10, PerformanceGoal → "Quality", MeshStyle → {{Black, Thin}}]
```

Out[84]=


Out[85]=



Out[86]=



Out[87]=