Homework for Advanced Quantum Mechanics 9

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1. *Deadline :December 15th.*
2. Please send your homework to my email: 910493179@qq.com.
3. Contact me or discuss in QQ group if you have any question.
4. Homework written by TEX has 5 extra points as bonus.

1. Re-derive the familiar wave equation (263) in the lecture note

$$i\hbar\frac{\partial}{\partial t}\phi(\vec{x},t) = -\frac{\hbar^2}{2m}\nabla^2\phi(\vec{x},t) + V(\vec{x})\phi(\vec{x},t) \tag{1}$$

   with all details included.

2. Suppose the typist made an error and wrote a Hamiltonian as

$$H = H_{11}\left|1\right\rangle\left\langle 1\right| + H_{22}\left|2\right\rangle\left\langle 2\right| + H_{12}\left|1\right\rangle\left\langle 2\right| \tag{2}$$

   What principle is now violated?(Both H_{11} and H_{22} are real numbers.) Illustrate your point explicitly by attempting to solve the most general time-dependent problem using an illegal Hamiltonian of this kind. (You may assume $H_{11} = H_{22} = 0$ for simplicity.)

3. A spin-$\frac{1}{2}$ system is known to be in an eigenstate of $\boldsymbol{S}\cdot\vec{\boldsymbol{n}}$ with eigenvalue $\hbar/2$ where $\vec{\boldsymbol{n}}$ is a unit vector lying in the xz-plane that makes an angle γ with the positive z-axis.
   (a)Suppose S_x is measured. What is the probability of getting $\hbar/2$?
   (b)Evaluate the dispersion in S_x-that is,

$$\langle(S_x - \langle S_x\rangle)^2\rangle \tag{3}$$

   (For your own peace of mind, check your answers for the special cases $\gamma = 0$, $\pi/2$, and π.)