General Physics Formula

Taper

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Abstract

This is a collection of important formulae in Fundamentals of Physics Extended Version by Halliday and Resnick.

Contents

1 General

Table 1: Metric Prefix

tera	T	10^{12}
giga	G	10^{9}
mega	M	10^{6}
kilo	k	10^{3}
centi	c	10^{-2}
milli	m	10^{-3}
micro	μ	10^{-6}
nano	n	10^{-9}
pico	p	10^{-12}

Rotation matrix Active rotation in the counter-clockwise θ:

$$\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \tag{1.0.1}$$

2 Classical mechanics

$$L = T - V \tag{2.0.2}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial L}{\partial \dot{q}}\right) = \frac{\partial L}{\partial q} \tag{2.0.3}$$

$$\dot{q} = \frac{\partial H}{\partial p} \tag{2.0.4}$$

$$\dot{p} = -\frac{\partial H}{\partial q} \tag{2.0.5}$$

Constant Accleration These five equations in Table 2-1 describe the motion of a particle with constant acceleration:

$$v = v_0 + at \tag{2.0.6}$$

$$x - x_0 = v_0 t + \frac{1}{2}at^2 \tag{2.0.7}$$

$$x - x_0 = vt - \frac{1}{2}at^2 \tag{2.0.8}$$

$$v^2 - v_0^2 = 2a(x - x_0) \tag{2.0.9}$$

$$x - x_0 = \frac{1}{2}(v_0 + v)t \tag{2.0.10}$$

Projectile Motion Projectile motion is the motion of a particle that is launched with an initial velocity $\vec{v}_0$. During its flight, the particle's horizontal acceleration is zero and its vertical acceleration is the free-fall acceleration $-g$. (Upward is taken to be a positive direction.) If $\vec{v}_0$ is expressed as a magnitude (the speed v_0) and an angle θ_0 (measured from the horizontal), the particle's equations of motion along the horizontal x axis and vertical y axis are

$$x - x_0 = (v_0 \cos(\theta_0))t \tag{2.0.11}$$

$$y - y_0 = (v_0 \sin(\theta_0))t - \frac{1}{2}gt^2 \tag{2.0.12}$$

$$v_y = v_0 \sin\theta_0 - gt \tag{2.0.13}$$

$$v_y^2 = (v_0 \sin\theta_0)^2 - 2g(y - y_0) \tag{2.0.14}$$

The **trajectory** (path) of a particle in projectile motion is parabolic and is given by

$$y = (\tan\theta_0)x - \frac{gx^2}{2(v_0 \cos\theta_0)^2} \tag{2.0.15}$$

Uniform Circular Motion If a particle travels along a circle or circular arc of radius r at constant speed v, it is said to be in *uniform circular motion* and has an acceleration of constant magnitude

$$a = \frac{v^2}{r} \tag{2.0.16}$$

The direction of is toward the center of the circle or circular arc, and is said to be centripetal. The time for the particle to complete a circle is

$$T = \frac{2\pi r}{v} \tag{2.0.17}$$

T is called the period of revolution, or simply the period, of the motion.

This acceleration is due to a net centripetal force on the particle, with magnitude given by

$$F = \frac{mv^2}{R} \tag{2.0.18}$$

where m is the particle's mass. The vector quantities $\vec{a}$ and $\vec{F}$ are directed toward the center of curvature of the particle's path

First, Second Speed **Circular**: $v = \sqrt{gR}$.

Escape Speed $\frac{1}{2}mv^2 = \frac{GMm}{R}$, so $v = \sqrt{2GM/R}$.

Definition 2.1 (Normal force)**.** *A **normal force** is the force on a body from a surface against which the body presses.The normal force is always perpendicular to the surface.*

Drag Force When there is relative motion between air (or some other fluid) and a body, the body experiences a drag force $\vec{D}$ that opposes the relative motion and points in the direction in which the fluid flows relative to the body. The magnitude of $\vec{D}$is related to the relative speed v by an experimentally determined drag coefficient C according to

$$D = \frac{1}{2}C\rho Av^2 \tag{2.0.19}$$

where ρ is the fluid density (mass per unit volume) and A is the effective cross-sectional area of the body (the area of a cross section taken perpendicular to the relative velocity $\vec{v}$)

Power The power due to a force is the rate at which that force does work on an object. For a force $\vec{F}$ at an angle ϕ to the direction of travel of the instantaneous velocity $\vec{v}$, the instantaneous power is

$$P = Fv\cos\phi = \vec{F}\cdot\vec{v} \tag{2.0.20}$$

Collision and Impulse Applying Newton's second law in momentum form to a particle-like body involved in a collision leads to the impulse–linear momentum theorem:

$$\vec{p}_f - \vec{p}_i = \int_{t_i}^{t_f} \vec{F}\,\mathrm{d}t \tag{2.0.21}$$

where the LHS is the change in the body's linear momentum, and RHS is defined as the impulse $\vec{J}$ due to the force exerted on the body by the other body in the collision.

Elastic Collisions in One Dimension An elastic collision is a special type of collision in which the kinetic energy of a system of colliding bodies is conserved. If the system is closed and isolated, its linear momentum is also conserved. For a one-dimensional collision in which body 2 is a target and body 1 is an incoming projectile, conservation of kinetic energy and linear momentum yield the following expressions for the velocities immediately after the collision:

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} \tag{2.0.22}$$

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i} \tag{2.0.23}$$

Variable-Mass Systems In the absence of external forces a rocket accelerates at an instantaneous rate given by

$$Rv_{\mathrm{rel}} = Ma(\text{first rocket equation}) \tag{2.0.24}$$

in which M is the rocket's instantaneous mass (including unexpended fuel), R is the fuel consumption rate, and v_{rel} is the fuel's exhaust speed relative to the rocket. The term Rv_{rel} is the thrust of the rocket engine. For a rocket with constant R and v_{rel}, whose speed changes from v_i to v_f when its mass changes from M_i to M_f,

$$v_f - v_i = v_{\mathrm{rel}} \ln \frac{M_i}{M_f}(\text{second rocket equation}) \tag{2.0.25}$$

$(R \equiv |\frac{\mathrm{d}M}{\mathrm{d}t}|)$

The Kinematic Equations for Constant Angular Acceleration Constant angular acceleration (α =a constant) is an important special case of rotational motion. The appropriate kinematic equations, given in Table 10-1, are

$$\sigma = \sigma_0 + \alpha t \tag{2.0.26}$$

$$\theta - \theta_0 = \omega_0 t + \frac{1}{2}\alpha t^2 \tag{2.0.27}$$

$$\theta - \theta_0 = \omega t - \frac{1}{2}\alpha t^2 \tag{2.0.28}$$

$$\omega^2 - \omega_0^2 = 2\alpha(\theta - \theta_0) \tag{2.0.29}$$

$$\theta - \theta_0 = \frac{1}{2}(\omega_0 + \omega)t \tag{2.0.30}$$

Linear and Angular Variables Related A point in a rigid rotating body, at a perpendicular distance r from the rotation axis, moves in a circle with radius r.

The linear acceleration $\vec{a}$ of the point has both tangential and radial components.The tangential component is

$$a_t = \alpha r \tag{2.0.31}$$

where α is the magnitude of the angular acceleration (in radians per second-squared) of the body.The radial component of $\vec{a}$ is

$$a_r = \frac{v^2}{r} = \omega^2 r \tag{2.0.32}$$

If the point moves in uniform circular motion, the period T of the motion for the point and the body is

$$T = \frac{2\pi r}{v} = \frac{2\pi}{\omega} \tag{2.0.33}$$

Rotational Kinetic Energy and Rotational Inertia The kinetic energy K of a rigid body rotating about a fixed axis is given by

$$K = \frac{1}{2}I\omega^2 \tag{2.0.34}$$

in which I is the rotational inertia of the body, defined as

$$I \equiv \int r^2 \, \mathrm{d}m \tag{2.0.35}$$

The Parallel-Axis Theorem 265

$$I = I_{\text{com}} + Mh^2 \tag{2.0.36}$$

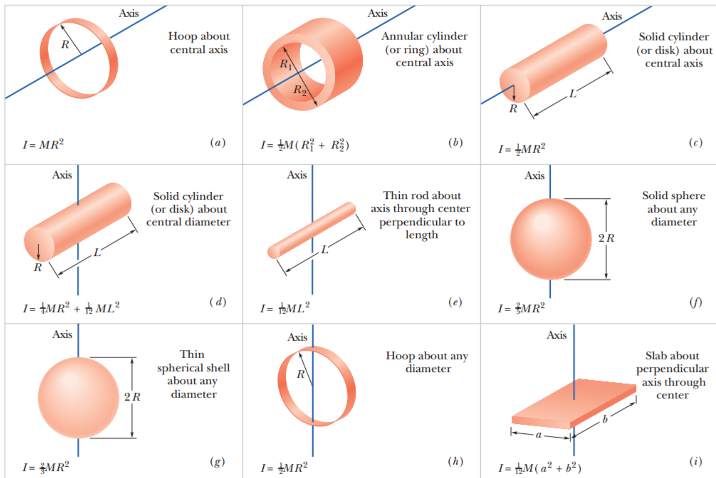


Figure 1: Some Rotational Inertias (p. 255 of [1])

Work and Rotational Kinetic Energy 265

$$W = \int_{\theta_i}^{\theta_f} \tau \, \mathrm{d}\theta \tag{2.0.37}$$

$$P = \frac{\mathrm{d}W}{\mathrm{d}t} = \tau\omega \tag{2.0.38}$$

Rolling Bodies [295]

$$v_{\mathrm{com}} = \omega R \tag{2.0.39}$$

$$K = \frac{1}{2} I_{\mathrm{com}} \omega^2 + \frac{1}{2} M v_{\mathrm{com}}^2 \tag{2.0.40}$$

$$a_{\mathrm{com}} = \alpha R \tag{2.0.41}$$

Precession of a Gyroscope

$$\Omega = \frac{Mgr}{I\omega} \tag{2.0.42}$$

Elastic Moduli [319]

$$\text{stress} = \text{modulus} \times \text{strain}$$

Young's modulus:

$$\frac{F}{A} = E \frac{\Delta L}{L} \tag{2.0.43}$$

Shear's modulus:

$$\frac{F}{A} = G \frac{\Delta x}{L} \tag{2.0.44}$$

Hydraulic bulk modulus:

$$p = B \frac{\Delta V}{V} \tag{2.0.45}$$

Gravitational Potential Energy [349]

$$U = -\frac{GMm}{r} \tag{2.0.46}$$

Escape speed

$$v = \sqrt{\frac{2GM}{R}} \tag{2.0.47}$$

Kepler's Laws

1. The law of orbits. All planets move in elliptical orbits with the Sun at one focus.
2. The law of areas. A line joining any planet to the Sun sweeps out equal areas in equal time intervals. (This statement is equivalent to conservation of angular momentum.)
3. The law of periods. The square of the period T of any planet is proportional to the cube of the semimajor axis a of its orbit. For circular orbits with radius r,

$$T^2 = \left(\frac{4\pi^2}{GM}\right) r^3 \tag{2.0.48}$$

where M is the mass of the attracting body—the Sun in the case of the solar system. For elliptical planetary orbits, the semimajor axis a is substituted for r.

Energy in Planetary Motion

$$U = -\frac{GMm}{r} \tag{2.0.49}$$

$$K = \frac{GMm}{2r} \tag{2.0.50}$$

$$E = -\frac{GMm}{2r} \text{ or} \tag{2.0.51}$$

$$E = -\frac{GMm}{2a} \tag{2.0.52}$$

3 Fluid

Flow of Ideal Fluids [377]

Definition 3.1 (Apparent weight)**.** *Omited*

Equation of continuity

$$R_v \equiv Av = \text{ a constant} \tag{3.0.53}$$

Bernoulli's Equation

$$p + \frac{1}{2}\rho v^2 + \rho gh = \text{ a constant} \tag{3.0.54}$$

4 Wave

Oscillation (**pp.403 of [1]**)

$$\omega = \frac{2\pi}{T} = 2\pi f \tag{4.0.55}$$

Linear Oscillator

$$\omega = \sqrt{\frac{k}{m}} \tag{4.0.56}$$

$$T = 2\pi\sqrt{\frac{m}{k}} \tag{4.0.57}$$

Pendulums

$$T = 2\pi\sqrt{I/\kappa} \tag{4.0.58}$$

$$T = 2\pi\sqrt{L/g} \tag{4.0.59}$$

$$T = 2\pi\sqrt{I/mgh} \tag{4.0.60}$$

Damped Harmonic Motion

$$x(t) = x_m e^{-bt/2m} \cos\left(\omega' t + \phi\right) \tag{4.0.61}$$

$$\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}} \tag{4.0.62}$$

$$E(t) \approx \frac{1}{2} k x_m^2 e^{-bt/m} \tag{4.0.63}$$

Waves (**pp.436 ch16 of [1]**)

$$y(x,t) = y_m \sin(kx - \omega t) \tag{4.0.64}$$

$$k = \frac{2\pi}{\lambda} \tag{4.0.65}$$

$$f = \frac{\omega}{2\pi} = \frac{1}{T} \tag{4.0.66}$$

$$v = \frac{\omega}{k} = \frac{\lambda}{T} = \lambda f \tag{4.0.67}$$

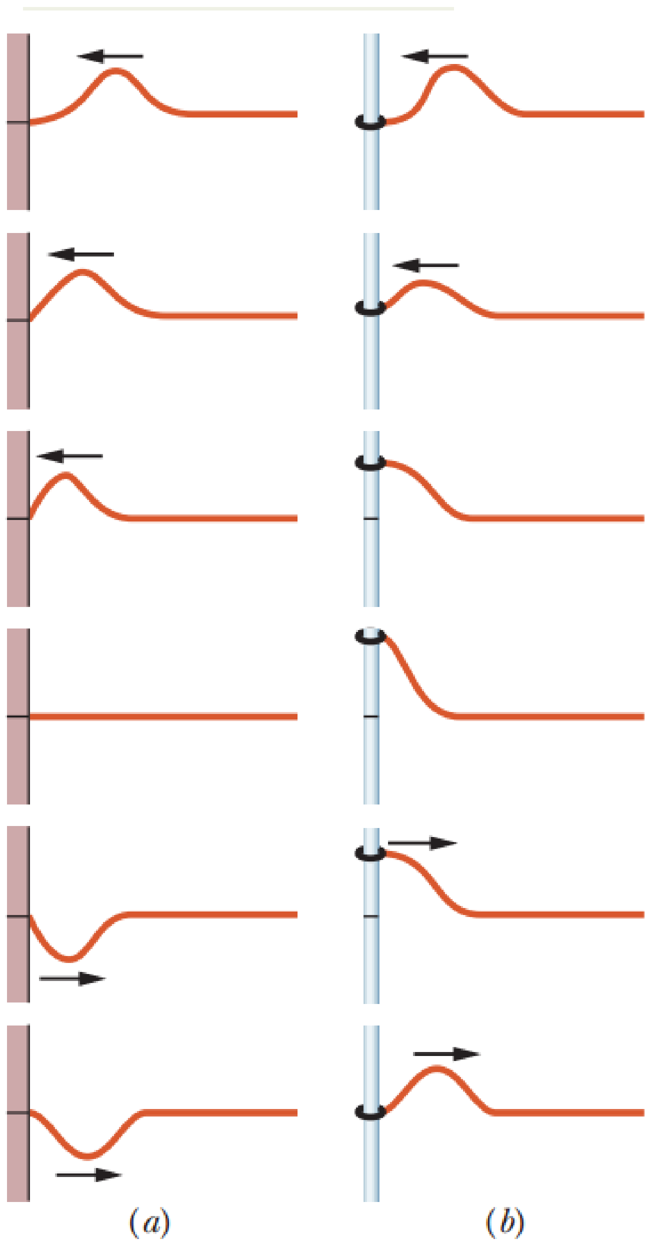


Figure 2: wave and Reflection

(a) A pulse incident from the right is reflected at the left end of the string, which is tied to a wall. Note that the reflected pulse is inverted from the incident pulse. (b) Here the left end of the string is tied to a ring that can slide without friction up and down the rod. Now the pulse is not inverted by the reflection

String

$$v = \sqrt{\frac{\tau}{\mu}} \tag{4.0.68}$$

$$P_{\text{avg}} = \frac{1}{2}\mu v \omega^2 y_m^2 \tag{4.0.69}$$

Resonance

$$n\lambda = 2L \tag{4.0.70}$$

Sound (Longitudinal) Wave (pp.466 ch17 of [1])

$$v = \sqrt{\frac{B}{\rho}} \tag{4.0.71}$$

$$s = s_m \cos(kx - \omega t) \tag{4.0.72}$$

$$\Delta p = \Delta p_m sin(kx - \omega r) \tag{4.0.73}$$

$$\Delta p_m = (v\rho\omega)s_m \tag{4.0.74}$$

Sound Intensity

$$I = \frac{P}{A} \tag{4.0.75}$$

$$I = \frac{1}{2}\rho v \omega^2 s_m^2 \tag{4.0.76}$$

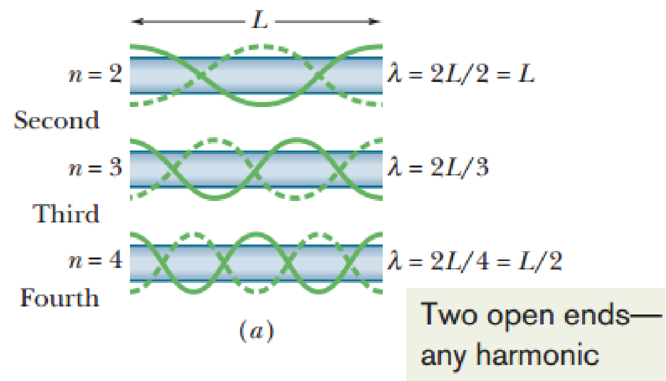
$$I = \frac{P_s}{4\pi r^2} \tag{4.0.77}$$

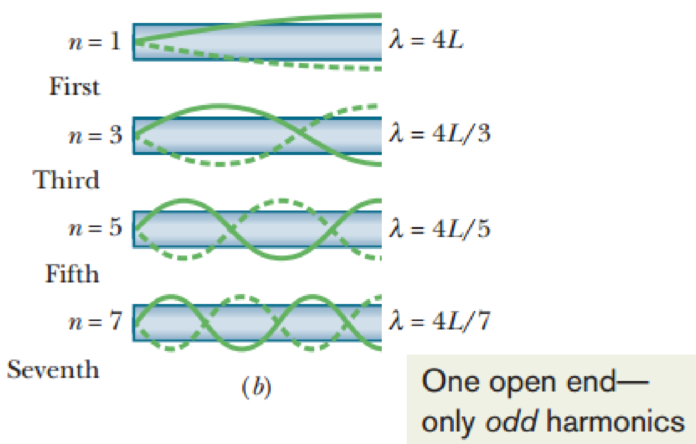
$$\tag{4.0.78}$$

Standing Wave Patterns in Pipes

$$f = \frac{v}{\lambda} = \frac{nv}{2L},\, n = 1, 2, 3, \cdots \tag{4.0.79}$$

$$f = \frac{v}{\lambda} = \frac{nv}{4L},\, n = 1, 3, 5, \cdots \tag{4.0.80}$$





Beat

$$f_{\text{beat}} = f_1 - f_2 \tag{4.0.81}$$

The Doppler Effect

$$f' = f\frac{v \pm v_D}{v \pm v_s} \tag{4.0.82}$$

The signs are chosen such that f' tends to be *greater* for motion toward and *less* for motion away. **Shock wave**

$$\sin\theta = \frac{v}{v_s}\ ,\ v_s \geq v \tag{4.0.83}$$

5 Thermal Physics

Conduction (**pp.498 ch18 of [1]**) **Thermal expansion**

$$\Delta L = L\alpha\Delta T \tag{5.0.84}$$

$$\Delta V = V\beta\Delta T \tag{5.0.85}$$

$$\beta = 3\alpha \tag{5.0.86}$$

$$Q = C\Delta T = cm\Delta T \tag{5.0.87}$$

Definition 5.1 (Convection)**.** *Convection occurs when temperature differences cause an energy transfer by motion within a fluid*

Radiation

$$P_{\text{rad}} = \sigma\varepsilon A T^4 \tag{5.0.88}$$

$$P_{\text{abs}} = \sigma\varepsilon A T_{\text{evn}}^4 \tag{5.0.89}$$

Black Body Radiation Plank's law:

$$I(\mu, T) = \frac{2h\mu^3}{c^2}\frac{1}{e^{\frac{h\mu}{kT}} - 1} \tag{5.0.90}$$

Wien's displacement law:

Wien's displacement law shows how the spectrum of black-body radiation at any temperature is related to the spectrum at any other temperature. If we know the shape of the spectrum at one temperature, we can calculate the shape at any other temperature. Spectral intensity can be expressed as a function of wavelength or of frequency.

A consequence of Wien's displacement law is that the wavelength at which the intensity per unit wavelength of the radiation produced by a black body is at a maximum, $\lambda_{\max}$, is a function only of the temperature:

$$\lambda_{\max} = \frac{b}{T} \tag{5.0.91}$$

where the constant b, known as Wien's displacement constant. **Conduction**

$$P_{\text{cond}} \equiv \frac{Q}{t} = kA\frac{T_H - T_C}{L} \tag{5.0.92}$$

$$P_{\text{cond}} = A\frac{T_H - T_C}{\sum(L_i/K_i)} \tag{5.0.93}$$

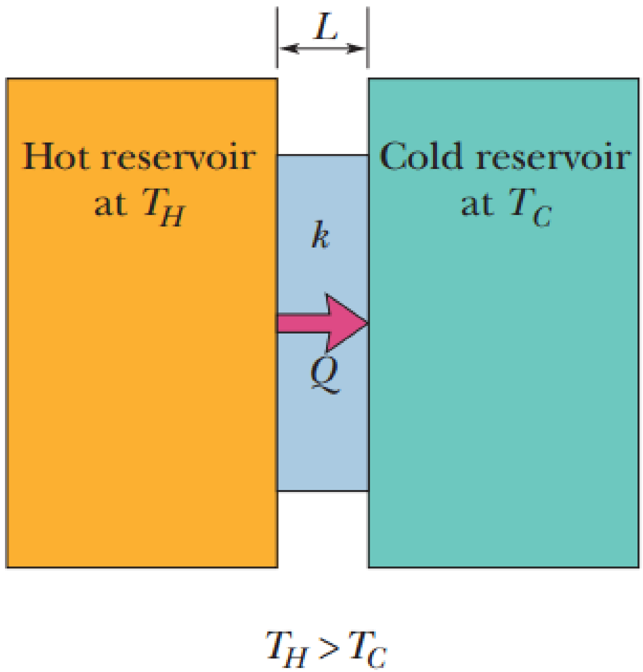


Figure 3: pp.494 of [1]

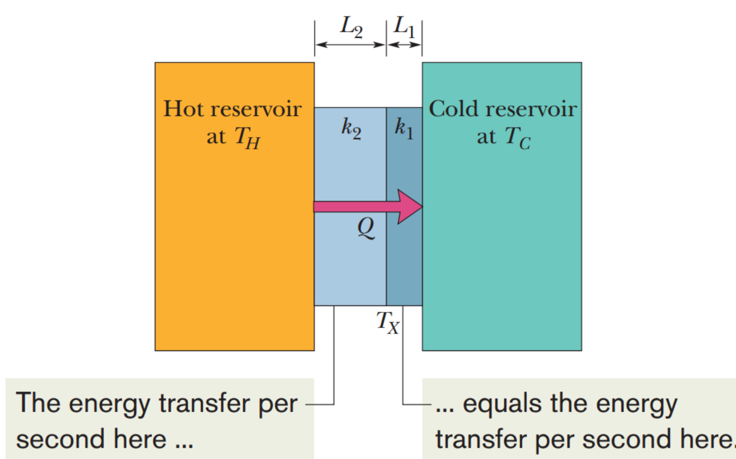


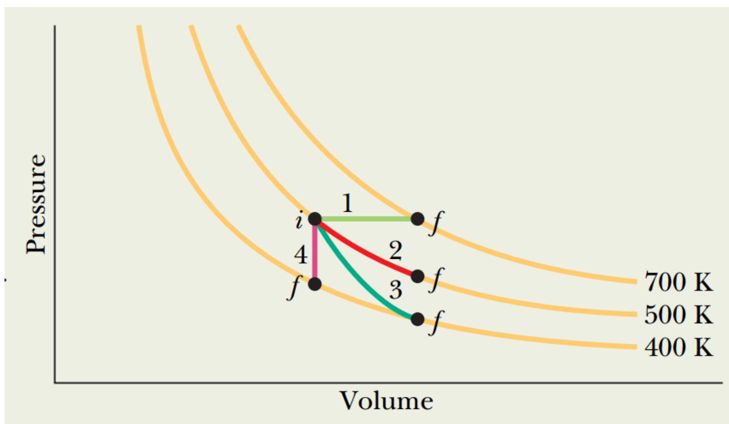
Figure 4: pp.495 of [1]

Thermal Process (**pp.529 ch19 of [1]**) Thermal process:

No.	Constant Quaility	Process Type	Special Results
1	p	Isobaric	$Q = nC_p\Delta T; W = p\Delta V$
2	T	Isothermal	$Q = W = nRT\ln(V_f/V_i); \Delta E_{\text{int}} = 0$
3	pV^γ, $TV^{\gamma-1}$	Adiabatic	$W = -\Delta E_{\text{int}}$
4	V	Isochoric	$Q = \Delta E_{\text{int}} = nC_V\Delta T; W = 0$

In particular, for Adiabatic process (let $K \equiv pV^\gamma$):

$$W = \int_i^f \frac{K}{V^\gamma}\mathrm{d}V = \frac{p_fV_f - p_iV_i}{1-\gamma} \tag{5.0.94}$$



Isothermal Process:

$$W = NkT\ln\frac{V_f}{V_i} \tag{5.0.95}$$

Temperature and Kinetic Energy

$$K_{\text{avg}} = \frac{3}{2}kT, \text{ per molecule} \tag{5.0.96}$$

$$\lambda = \frac{1}{\sqrt{2}\pi d^2 N/V} \tag{5.0.97}$$

Molar Specific Heats

$$C_V = \frac{3}{2}R \tag{5.0.98}$$

$$C_P = C_V + R \tag{5.0.99}$$

$$\Delta E_{\text{int}} = NC_V\Delta T \tag{5.0.100}$$

$$C_V = \frac{f}{2}k \tag{5.0.101}$$

$$\gamma \equiv \frac{C_P}{C_V} \tag{5.0.102}$$

Even more accurate formulae found in pp. 158 Kardar's book [5]:

$$C_V = \frac{6n-3-r}{2} k_B \tag{5.0.103}$$

$$C_p = C_V + k_B \tag{5.0.104}$$

Monatomic	He	$n = 1$	$r = 0$	$\gamma = 5/3$
Diatomic	O_2 or CO	$n = 2$	$r = 2$	$\gamma = 9/7$
Linear triatomic	O–C–O	$n = 3$	$r = 2$	$\gamma = 15/13$
Planar triatomic	H/^{O}\H	$n = 3$	$r = 3$	$\gamma = 14/12 = 7/6$
Tetra-atomic	NH_3	$n = 4$	$r = 3$	$\gamma = 20/18 = 10/9$

Figure 5: Examples of n, r, and γ.

But these are **classical results**, only valid at high temperature! (See the refered page for details).

Entropy (**pp.554 ch20 of** [**1**])

$$\mathrm{d}S = \frac{\mathrm{d}Q}{T} \tag{5.0.105}$$

$$\Delta S = nR \ln \frac{V_f}{V_i} + nC_V \ln \frac{T_f}{T_i} \tag{5.0.106}$$

$$\varepsilon = 1 - \frac{T_L}{T_H} \text{ (Carnot engine)} \tag{5.0.107}$$

$$K = \frac{T_L}{T_H - T_L} \text{ (Carnot refrigerator)} \tag{5.0.108}$$

$$\ln N! \approx N \ln N - N \tag{5.0.109}$$

6 Electrodynamics

Lorentz Force Law

$$\mathbf{F} = q\left(\mathbf{E} + \mathbf{v} \times \mathbf{B}\right) \tag{6.0.110}$$

Electric field (**pp.596 ch22 of** [**1**])

$$E = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} \tag{6.0.111}$$

$$E = \frac{1}{2\pi\varepsilon_0} \frac{p}{z^3} \tag{6.0.112}$$

$$U = -\vec{p} \cdot \vec{E} \tag{6.0.113}$$

Gauss's Law (**pp.620 ch23 of [1]**)

$$E = \frac{\sigma}{\varepsilon_0} \text{ ,charged conductor} \tag{6.0.114}$$

$$E = \frac{\sigma}{2\varepsilon_0} \text{ ,infinite sheet} \tag{6.0.115}$$

$$E = \frac{\lambda}{2\pi\varepsilon_0 r} \tag{6.0.116}$$

$$E = 0 \text{ , inside charged shell} \tag{6.0.117}$$

$$E = \left(\frac{q}{4\pi\varepsilon_0 R^3}\right) r \tag{6.0.118}$$

Electric Potential (**pp.646 ch24 of [1]**)

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r} \tag{6.0.119}$$

$$V = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2} \tag{6.0.120}$$

$$U = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r} \tag{6.0.121}$$

Capacitor (**pp.675 ch25 of [1]**)

$$C \equiv \frac{q}{V} \tag{6.0.122}$$

$$C = \frac{\varepsilon_0 A}{d} \text{ (parallel-plate)} \tag{6.0.123}$$

$$C = 2\pi\varepsilon_0 \frac{L}{\ln b/a} \text{ (cylindrical)} \tag{6.0.124}$$

$$C = 4\pi\varepsilon_0 \frac{ab}{b-a} \text{ (spherical)} \tag{6.0.125}$$

$$C = 4\pi\varepsilon_0 R \text{ (isolated spherical)} \tag{6.0.126}$$

$$C = \sum_j C_j \text{ (parallel)} \tag{6.0.127}$$

$$\frac{1}{C} = \sum_j \frac{1}{C_j} \text{ (series)} \tag{6.0.128}$$

$$U = \frac{1}{2}\frac{q^2}{C} = \frac{1}{2}CV^2 \tag{6.0.129}$$

$$u = \frac{1}{2}\varepsilon_0 E^2 \tag{6.0.130}$$

Current (**pp.698 ch26 of [1]**)

$$\vec{J} = ne\vec{v}_d \tag{6.0.131}$$
$$\rho \equiv \frac{1}{\sigma} = \frac{E}{J} \tag{6.0.132}$$
$$\vec{E} = \rho\vec{J} \tag{6.0.133}$$
$$R = \rho\frac{L}{A} \tag{6.0.134}$$
$$\rho - \rho_0 = \rho_0\alpha(T - T_0) \tag{6.0.135}$$
$$\rho = \frac{m}{e^2 n\tau} \tag{6.0.136}$$

Electro motive force (**pp.724 ch27 of [1]**)

$$\mathcal{E} = \frac{\mathrm{d}W}{\mathrm{d}q} \tag{6.0.137}$$

Cutting Magnetic field lines:

$$\mathcal{E} = BLv \tag{6.0.138}$$

Cutting Magnetic field lines in circular motion:

$$\mathcal{E} = B(\frac{1}{2}\omega R)R = \frac{1}{2}B\omega R^2 \tag{6.0.139}$$

For electric generator:

$$E_{\mathrm{max}} = nBA_{\mathrm{rea}}\omega \tag{6.0.140}$$

Circuit Rule

- **Loop Rule**. The algebraic sum of the changes in potential encountered in a complete traversal of any loop of a circuit must be zero.
- **Junction Rule**. The sum of the currents entering any junction must be equal to the sum of the currents leaving that junction

$$R = \sum_j R_j \text{ , series} \tag{6.0.141}$$
$$\frac{1}{R} = \sum_j \frac{1}{R_j} \text{ , parallel} \tag{6.0.142}$$

Magnetic Field (**pp.755 ch28 of [1]**)

$$qvB = \frac{mv^2}{r} \tag{6.0.143}$$
$$T = \frac{2\pi r}{rqB/m} = \frac{2\pi m}{qB} \tag{6.0.144}$$
$$\vec{\tau} = \vec{\mu} \times \vec{B} \tag{6.0.145}$$
$$U(\theta) = -\vec{\mu} \cdot \vec{B} \tag{6.0.146}$$

Magnetic → Electricity (**pp.781 ch29 of [1]**)

$$\mathrm{d}\vec{B} = \frac{\mu_0}{4\pi} \frac{i \, \mathrm{d}\vec{s} \times \hat{r}}{r^2} \tag{6.0.147}$$

$$B = \frac{\mu_0 i}{2\pi R} \tag{6.0.148}$$

$$B = \frac{\mu_0 i \phi}{4\pi R} \tag{6.0.149}$$

$$B = \mu_0 i N / L \tag{6.0.150}$$

$$B = \frac{\mu_i i N}{2\pi} \frac{1}{r} \tag{6.0.151}$$

$$\vec{B}(z) = \frac{\mu_0}{2\pi} \frac{\vec{\mu}}{z^3} \tag{6.0.152}$$

Faraday's Law (**pp.816 ch30 of [1]**)

$$\mathcal{E} = \oint \vec{E} \, \mathrm{d}\vec{s} \tag{6.0.153}$$

$$\tag{6.0.154}$$

Inductance

$$L \equiv \frac{N \Phi_B}{i} \tag{6.0.155}$$

$$\frac{L}{l} = \mu_n n^2 A \tag{6.0.156}$$

$$\mathcal{E}_L = -L \frac{\mathrm{d}i}{\mathrm{d}t} \tag{6.0.157}$$

$$U_B = \frac{1}{2} L i^2 \tag{6.0.158}$$

$$u_B = \frac{1}{2} \frac{B^2}{\mu_0} \tag{6.0.159}$$

LC Circuit (**pp.853 ch31 of [1]**)

$$U_E = \frac{q^2}{2C} \tag{6.0.160}$$

$$U_B = \frac{L i^2}{2} \tag{6.0.161}$$

$$\omega = \frac{1}{\sqrt{LC}} \tag{6.0.162}$$

Predicting RLC current (**pp.843 of [1]**)

$$X_C = \frac{1}{\omega_d C} \tag{6.0.163}$$

$$X_L = \omega_d L \tag{6.0.164}$$

$$V = IX \tag{6.0.165}$$

$$Z_{\text{impedance}} = \text{vector sum of all above} \tag{6.0.166}$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2} \text{ for simple RLC circuit} \tag{6.0.167}$$

Maxwell's Equations (**pp.869 of [1]**)

$$\oint \vec{E} \cdot \mathrm{d}\vec{A} = q_{\text{enc}}/\varepsilon_0 \tag{6.0.168}$$

$$\oint \vec{B} \cdot \mathrm{d}\vec{A} = 0 \tag{6.0.169}$$

$$\oint \vec{E} \cdot \mathrm{d}\vec{s} = -\frac{\mathrm{d}\Phi_B}{\mathrm{d}t} \tag{6.0.170}$$

$$\begin{aligned}\oint \vec{B} \cdot \mathrm{d}\vec{s} &= \mu_0 \varepsilon_0 \frac{\mathrm{d}\Phi_E}{\mathrm{d}t} + \mu_0 i_{\text{enc}} \\ &= \mu_0 i_d + \mu_0 i_{\text{enc}}\end{aligned} \tag{6.0.171}$$

Radiation

Electric dipole:

$$\langle P \rangle = \frac{\mu_0 p_0^2 \omega^4}{12\pi c} \tag{6.0.172}$$

where $p_0 = q_0 d$. Electric quadrupole: $\langle P \rangle \propto \omega^6$.

(The cases for Magnetic dipole is similarly $\propto \omega^4$)

Lienard's generalization of the Larmor formula

$$P = \frac{\mu_0 q^2 \gamma^6}{6\pi c} \left(a^2 - \left| \frac{\mathbf{v} \times \mathbf{a}}{c} \right|^2 \right) \tag{6.0.173}$$

$$\approx \frac{\mu_0 q^2}{6\pi c} a^2 \tag{6.0.174}$$

(**pp.448 of [3]**).

(**pp.882 ch32 of [1]**)

Spin

$$\mu_s = -\frac{e}{m} S \tag{6.0.175}$$

$$\mu_B = \frac{e\hbar}{2m} = 9.27 \times 10^{-24} \mathrm{J/T} \tag{6.0.176}$$

$$\mu_{\text{orb}} = -\frac{e}{2m} L_{\text{orb}} \tag{6.0.177}$$

Curie's law

$$M = C \frac{B_{\text{ext}}}{T} \tag{6.0.178}$$

7 Optics

Optics (pp.913 ch33 of [1])

$$\vec{S} = \frac{1}{\mu_0}\vec{E} \times \vec{B} \tag{7.0.179}$$

$$F = \frac{IA}{c} \text{ (total absorption)} \tag{7.0.180}$$

$$F = \frac{2IA}{c} \text{ (total reflection)} \tag{7.0.181}$$

$$I = I_0 \cos^2\theta \tag{7.0.182}$$

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \tag{7.0.183}$$

$$\theta_c = \sin^{-1}\frac{n_2}{n_1} \tag{7.0.184}$$

$$\theta_B = \tan^{-1}\frac{n_2}{n_1} \tag{7.0.185}$$

Optics 2 (pp.948 ch34 of [1])

$$\text{Omitted} \tag{7.0.186}$$

Light waves change phase by $\pi/2$ when they reflect from the surface of a medium with higher refractive index than that of the medium in which they are travelling.

Interference (pp.981 ch35 of [1])

$$n = \frac{c}{v_{\text{medium}}} \tag{7.0.187}$$

$$\lambda_n = \frac{\lambda}{n} \tag{7.0.188}$$

$$2L = (m + \frac{1}{2})\frac{\lambda}{n_2} \text{ (thin film, max)} \tag{7.0.189}$$

$$d\sin\theta = m\lambda \text{ (Young's, max)} \tag{7.0.190}$$

Diffraction (pp.1013 ch36 of [1])

$$a\sin\theta = m\lambda,\ m = 1, 2, \cdots \text{ (single-slit minima)} \tag{7.0.191}$$

$$\theta_R = 1.22\frac{\lambda}{d} \text{ (Rayleigh's criterion)} \tag{7.0.192}$$

$$I(\theta) = I_m(\cos^2\beta)\left(\frac{\sin\alpha}{\alpha}\right)^2 \tag{7.0.193}$$

$$\beta = (\pi d/\lambda),\ \alpha = (\pi a/\lambda)\sin\theta \tag{7.0.194}$$

$$d\sin\theta = m\lambda,\ m = 0, 1, 2\cdots \text{ (diffraction grating maxima)} \tag{7.0.195}$$

$$2d\sin\theta = m\lambda,\ m = 1, 2, \cdots \text{ (Bragg's law)} \tag{7.0.196}$$

Lenses

$$\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f} \tag{7.0.197}$$

Quantity	+	-
s_i	real	virtual
s_o	real	virtual
f	convex	concave

8 Relativity

Relativity (**pp.1048 ch37 of [1]**)

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} \tag{8.0.198}$$

$$\Delta t = \gamma \Delta t_0 \tag{8.0.199}$$

$$L = L_0/\gamma \tag{8.0.200}$$

$$x' = \gamma(x - vt) \tag{8.0.201}$$

$$t' = \gamma(t - vx/c^2) \tag{8.0.202}$$

$$u = \frac{u' + v}{1 + u'v/c^2} \tag{8.0.203}$$

$$f = f_0 \sqrt{\frac{1-\beta}{1+\beta}} \text{ (away)} \tag{8.0.204}$$

$$v = \frac{|\Delta\lambda|}{\lambda_0} c \text{ (away, for } v << c) \tag{8.0.205}$$

$$f = f_0 \sqrt{1-\beta^2} \text{ (perpendicular)} \tag{8.0.206}$$

$$\vec{p} = \gamma m \vec{v} \tag{8.0.207}$$

$$E = mc^2 + K = \gamma mc^2 \tag{8.0.208}$$

$$E^2 = (pc)^2 + (mc^2)^2 \tag{8.0.209}$$

Electrodynamics with Relativity (**pp.526-531 of [3]**)

$$E = \gamma_0 E_0 \tag{8.0.210}$$

(Only in the direction perpendicular to the motion, without B field)

$$\bar{E}_x = E_x \tag{8.0.211}$$

$$\bar{E}_y = \gamma(E_y - vB_z) \tag{8.0.212}$$

$$\bar{E}_z = \gamma(E_z + vB_y) \tag{8.0.213}$$

$$\bar{B}_x = B_x \tag{8.0.214}$$

$$\bar{B}_y = \gamma(B_y + \frac{v}{c^2} E_z) \tag{8.0.215}$$

$$\bar{B}_z = \gamma(B_z - \frac{v}{c^2} E_y) \tag{8.0.216}$$

9 Quantum Mechanics

Photon (**pp.1077 ch38 of** [**1**])

$$E = hf \tag{9.0.217}$$

$$p = \frac{hf}{c} = \frac{h}{\lambda} \tag{9.0.218}$$

$$hf = K_{\max} + \Phi_{\text{work function}} \tag{9.0.219}$$

$$\Delta\lambda = \frac{h}{mc}(1 - \cos\phi) \tag{9.0.220}$$

Infinite Square Well (**pp.32 of** [**4**])

$$k_n = \frac{n\pi}{a} \tag{9.0.221}$$

$$E_n = \frac{\hbar^2 k_n^2}{2m} \tag{9.0.222}$$

$$\psi_n = \sqrt{\frac{2}{a}}\sin(k_n x) \tag{9.0.223}$$

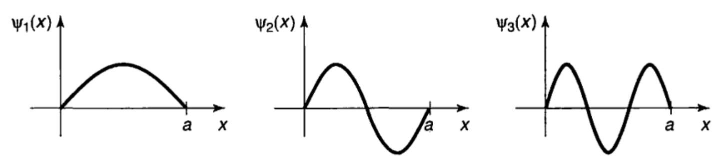


FIGURE 2.2: **The first three stationary states of the infinite square well (Equation 2.28).**

Figure 6: Wave functions

Conductions (**pp.1160 ch41 of** [**1**])

$$P(E) = \frac{1}{e^{(E-E_F)/kT} + 1} \tag{9.0.224}$$

10 Atomic Physics

$$g_J = \frac{3}{2} + \frac{1}{2}\left(\frac{S^2 - L^2}{J^2}\right) \qquad (20-12)$$

像$_1$H，$_3$Li，$_{11}$Na，$_{19}$K，$_{29}$Cu，$_{47}$Ag，$_{79}$Au 等（左下角数码代表原子序数），都是单电子体系，它们的基态状态为$^2S_{1/2}$，这里 S 表示单电子的轨道角动量 l 为 0（对多电子原子取 L 的值），因而 j 只取一个数值，$j=\frac{1}{2}$；j 的数值表示在右下角（对多电子原子取 J 的值）. 左上角表示 $2s+1$ 的数值（对多电子原子取 $2S+1$ 的值），由于单电子的 s 总是 1/2，因而 $2s+1=2$，代表双重态. 对于$^2S_{1/2}$态，可用式(20－11)算出 $g_j=2$，由于 $j=1/2$，因而 $m_j=\pm\frac{1}{2}$，于是 $m_jg_j=\pm 1$，见表 20.1 所列的第一行. 对于 P 态，相应的 $l=1$，因而 $j=1/2,3/2$，有两个原子态 $^2P_{1/2}$，$^2P_{3/2}$. 像$_{81}$Tl这样的单电子体系，它的原子基态就处于$^2P_{1/2}$. 类似地，我们有$^2D_{3/2}$，$^2D_{5/2}$. 它们对应的 g_j 因子均可由式(20－11)算出，并与 m_jg_j 一起列于表 20.1 内.

Figure 7: Electron State (from pp.162 of [2])

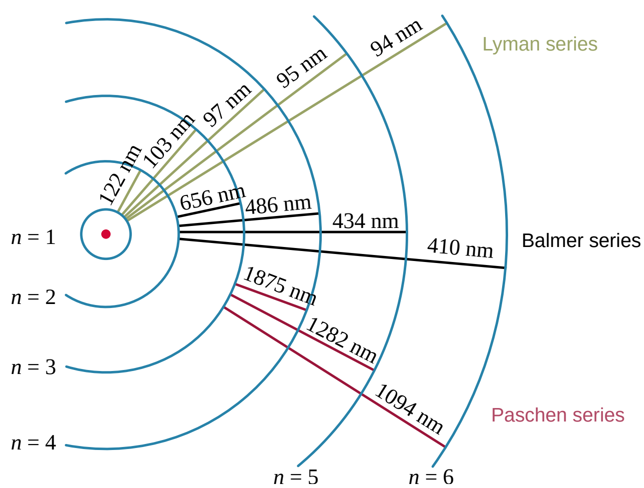


Figure 8: Hydrogen spectrum series

Table 2: Three common decay processes

Decay name	Process
α	Emitting a Helium Nucleus (two protons and two neutrons)
β	$n \rightarrow p + e^- \bar{v}_e$ or $p \rightarrow n + e^+ + v_e$
γ	Only photon (light ray) emitted

11 Standard Model

Read page 1244 of [1].

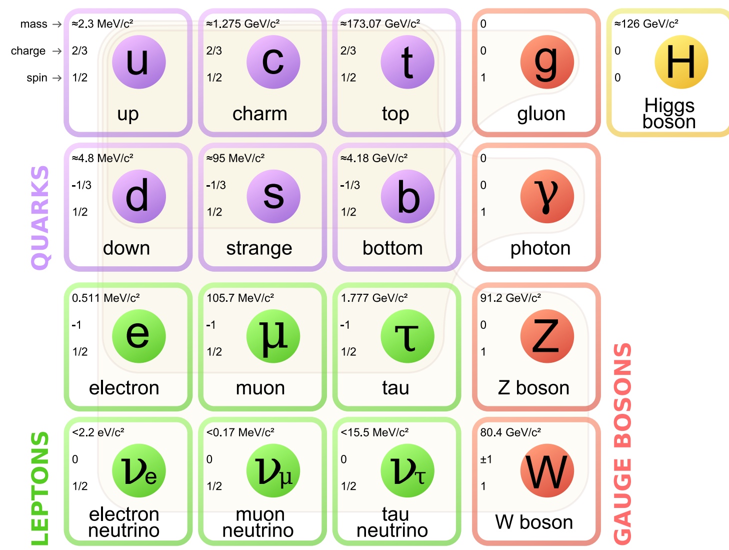


Figure 9: Standard Model of Elementary Particles (from Wikipedia)

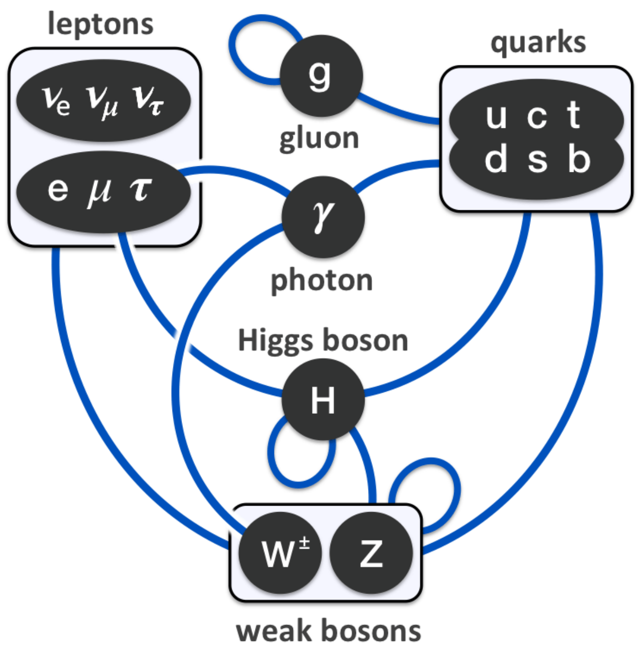


Figure 10: Elementary particle interactions in the Standard Model (from Wikipedia)

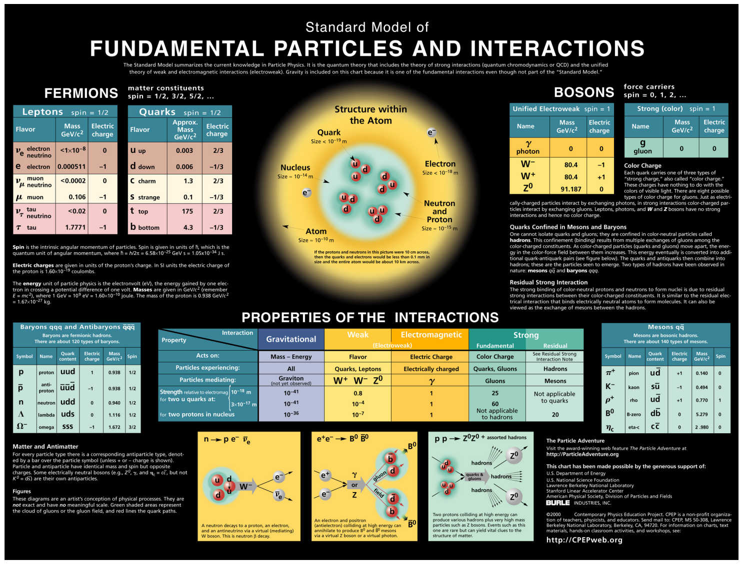


Figure 11: Comprehensive standard model particle Chart

(from Link)

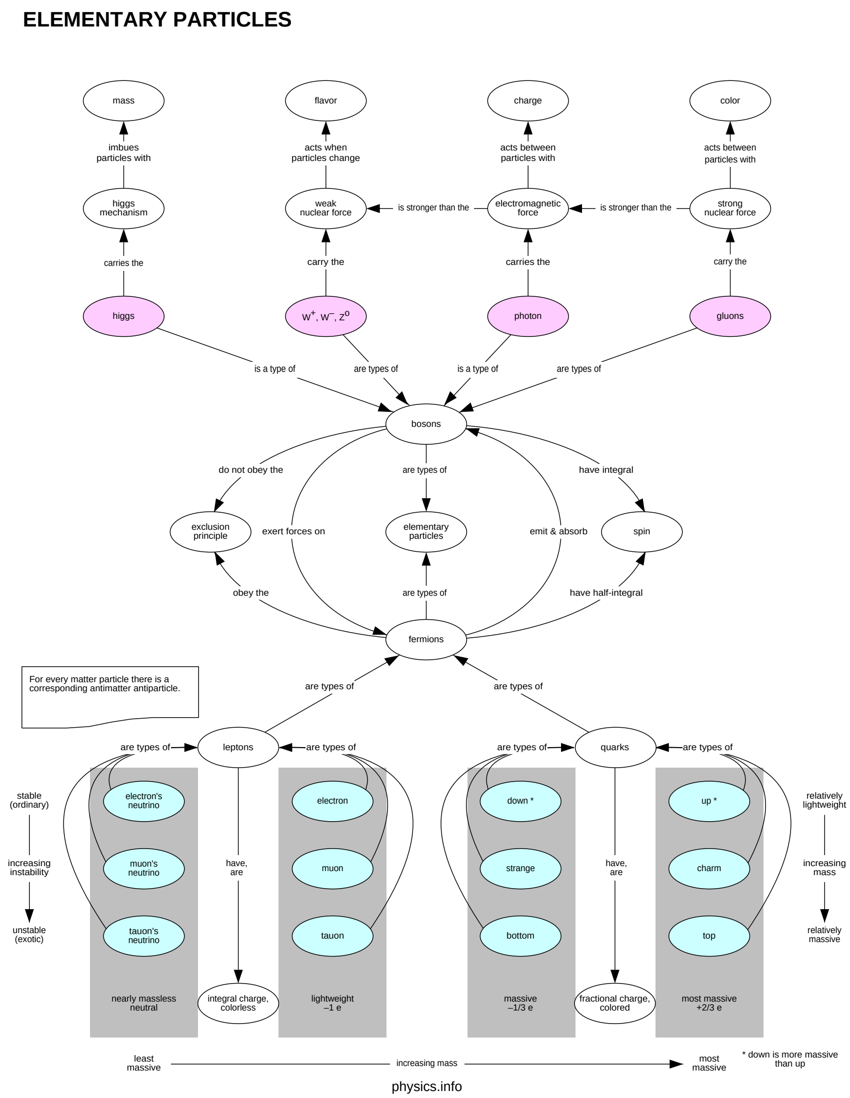


Figure 12: Elementary Particles concept Map

12 Positronium

Wiki Positronium.

Energy:

$$E_n = -\frac{6.8\text{eV}}{n^2} \tag{12.0.225}$$

13 Common Mathematical Formulae

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots \tag{13.0.226}$$

14 Anchor

References

[1] Fundamentals of Physics, Extended Edition. Halliday & Resnick.

[2] Atomic Physics. Fujia, Yang. (Chinese)

[3] Introduction to Electrodynamics, 3rd, Griffiths.

[4] Introduction to Quantum Mechanics, 2nd, Griffiths.

[5] Kardar. Statistical Physics of Particles. (2007)

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