Homework for Advanced Quantum Mechanics 7

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1. *Deadline :November 30th before class.*

2. Please send your homework to my email: 910493179@qq.com.

3. Contact me or discuss in QQ group if you have any question.

4. Homework written by TEX has 5 extra points as bonus.

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1. Prove

$$[x, p^n] = in\hbar p^{n-1}, [p, x^n] = -in\hbar x^{n-1} \tag{1}$$

and

$$[x, f(p)] = i\hbar \frac{\partial f(p)}{\partial p}, [p, g(x)] = -i\hbar \frac{\partial g(x)}{\partial x} \tag{2}$$

Hint:Repeat the application of the commutation relation $[x, p] = i\hbar$.
$[A, BC] = [A, B]C + B[A, C]$

2. Consider a three-dimensional ket space. If a certain set of orthonormal kets - say, $|1\rangle$, $|2\rangle$, $|3\rangle$ - are used as the base kets, the operators A and B are represented by

$$A = \begin{pmatrix} a & 0 & 0 \\ 0 & -a & 0 \\ 0 & 0 & -a \end{pmatrix}, B = \begin{pmatrix} b & 0 & 0 \\ 0 & 0 & ib \\ 0 & -ib & 0 \end{pmatrix} \tag{3}$$

with a and b both real.

- Obviously A exhibits a degenerate spectrum. Does B also exhibit a degenerate spectrum?
- Show that A and B commute.
- Find a new set of orthonormal kets that are simultaneous eigenkets of both A and B. Specify the eigenvalues of A and B for each of the three eigenkets.