Topological Insulators and Topological Superconductors

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Abstract

This is a draft.

Contents

1 Lectures on the Frontiers of Physics

Given by professor of physics in SUSTC

1.1 By JQ. He.

Thermal electrics

1.2 By Lang. Chen

Grow thin films.

- Rheed-Assited PLD/MBE. (Ray as an exmination).
- orbital contral of electrons -> orbitronics -> Control of Spin orbital coupling.
- Multiferroics -> multiple order parameters, and the interaction between them. E.g. $BiFeO_3$.
- Ferrotorodicity: Spontaneous Toroidal Momennt. Time and spacial symmetries simultaneous broken.
- What is a iridates $Ir_2(X)O_4$, (e.g. Sr_2RuO_4) exactly in theoretical physics?
- H_2S: 200K superconductor?
- The Double Exchange effect of oxygem -> Half-metal, phase transition.

1.3 By Alan

Photocatalysis: TiO_2. Hongkong has TiO_2 spurred on the keys.

1.4 By Li, Huang

- Computational Physics
- Surface Dynamics
- Structural factor from 2D to 3D.
- Finding Order Amid the Chaos. amorphous -> spatially resolved distributed function.
- ?: What is genetically algorithm.

Computational and theoretical studies of Surface dynamics

- Surface atoms is immersed in a very different environment compared with the bulk atoms.
- First-principle calculations
- DFT + LDA -> Conser equation
- Plane wave basis + Ultrasoft pseudopotentials to solve the Conser equation
- Continumm method ?

1.5 By Junfen, Liu

- electronic transport in mesoscopic systems:
- Spintronics
- Graphene eletronics
- Superconductors etc.

Quantum wire conductiing The conduction channel in quantum wire is quantized, with discrete value of conductance.

- λ_F Fermi wavelength
- L_m Momuntum relaxation length <– impurities.
- L_ϕ Phase relaxation length <– memory of phase, related to energy $\omega = E/\hbar$.
- L Sample length

- Ballistic transport: $L << L_m$ No scattering.
- Diffusive $L > L_m$, scattering, reduced transmission.
- Localization $L_m << L << L_\phi$ -> Prof. Haizhou Lu.
- Classical. (Omitted)

Conductance No back-scattering

$$G = \frac{I}{V} = \frac{2e^2}{h}$$

Landauer formula $G = \frac{2e^2}{h} \cdot T$, T is some coefficient accounting for the back scattering, perhaps the transmission probability. In reality, $G = \sum_{\text{Different channels}} G_i$,

We can turn the G into resisivity:

$$\text{Resistance} = \frac{h}{2e^2} + \frac{h}{2 * e^2} \frac{R}{T}$$

$R + T = 1$

Resonate Transmission (Omitted)

Spintronics Use the extra freedom of Spin.

Spin field eletroncis: Datta and Das, Appl. Phys. Lett. 56, 665(1990)

GMR: 2007 Nobel prize in Physics.

Hall Effect (Omitted) Spin Hall Effect: S. murakami, et.al. Science 301 1348(2004);

J. Sinova et.al. Phys.Rev.Lett. 92, 126603 (2004). (Omitted)

Graphene Carrier -> Relativistic Dirac fermions. **Klein Paradox**

Josephson Junction A phase difference could conduct electricity in Superconductors.

1.6 By Haizhou, Lu

Quantum Anomalous Hall Effect Requires strong magnetic field: ≈ 10 Tesla.

Anomalous Hall Effect: Without magnetic field. $R_H = R_0 B + R_A M$ where M is the magnetic susceptibility. Two-factors: SO coupling. Spin-dependent Hall Effect.

An excellent illustrations is found in [4]:

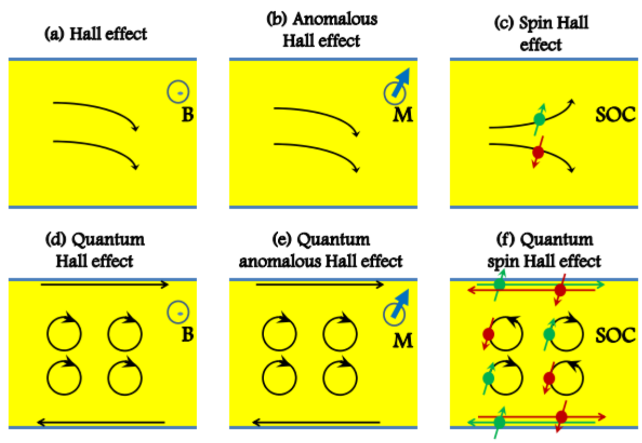

FIG. 1. (Color online) Six members in the family of Hall effect. (a) Hall effect; (b) Anomalous Hall effect; (c) Spin Hall effect; (d) Quantum Hall effect; (e) Quantum anomalous Hall effect; and (f) Quantum spin Hall effect.

Figure 1: Illustration

1.7 By Kedong Wang

Tunneling current $I \propto V e^{-2kz}$, where $k = \frac{\sqrt{2m\phi}}{\hbar}$, ϕ is the Work function. I is very sensitive to the distance z.

Work function ϕ characterize the obstruction that prevents electron from escaping the sample.

2 Problems in Bernevig's Topological ...

2.1 Chapter 2

Non-Abelian Berry Transport Derive Berry curvature to the adiabatic transport of a degenerate multiplet of states separated by a gap from the excited states. (Cautious about rotation within degenerate states).

Answer: $\gamma_{mn}(t) = i\int_0^t \langle m(R(t'))|\frac{d}{dt'}|n(R(t'))\rangle\, dt'$

Approach: Assuming that the those degenerate states are labeled by $1\cdots N$. Thus we have naturally:

$$H\phi = i\hbar\frac{\partial}{\partial t}\phi,\ \phi = \sum_n A_n\psi_n \tag{2.1.1}$$

Then we have:

$$\begin{aligned} H\sum_n A_n\psi_n &= i\hbar\frac{\partial}{\partial t}\sum_n A_n(t)\psi_n(R(t)) \\ \sum_n EA_n\psi_n &= i\hbar\sum_n\left(\frac{\partial A_n(t)}{\partial t}\psi_n(R(t)) + A_n(t)\frac{\partial\psi_n(R(t))}{\partial t}\right) \\ EA_m &= i\hbar\frac{\partial A_m(t)}{\partial t} + \sum_n A_n(t)\langle m|\frac{\partial}{\partial t}|n\rangle \end{aligned} \tag{2.1.2}$$

Put in another form:

$$\sum_n\left(\delta_m^n E - \langle m|\frac{\partial}{\partial t}|n\rangle\right)A_n = i\hbar\frac{\partial A_m(t)}{\partial t}$$

In matrix form:

$$(E-P)\mathbf{A} = i\hbar\dot{\mathbf{A}} \tag{2.1.3}$$

where:

$$E = \begin{pmatrix} \cdots & & \\ & E & \\ & & \ldots \end{pmatrix} \tag{2.1.4}$$

$$P = (P_n^m) = \left(\langle m|\frac{\partial}{\partial t}|n\rangle\right) \tag{2.1.5}$$

$$A = \begin{pmatrix} A_1(t) \\ A_2(t) \\ \cdots \end{pmatrix} \tag{2.1.6}$$

Note that $\langle n|\frac{\partial}{\partial t}|m\rangle^* \neq \langle m|\frac{\partial}{\partial t}|n\rangle$, thus P may not be Hermitian.Ergo $E-P$ is Hermitian. So it is diagonalizable.

Notice that

$$0 = \frac{\partial}{\partial t}\langle m|n\rangle = \langle\frac{\partial}{\partial t}m|n\rangle + \langle m|\frac{\partial}{\partial t}n\rangle \qquad (\text{any } m,n) \tag{2.1.7}$$

temporary mathematica code:

3 Miscellnaneous Notes

3.1 Super conductor

Mean-field approach to deal with a four operator diagonalization.

Suppose we have: D^*C^*CD, then let $\delta = CD - \langle CD\rangle = CD - avg$. Then if we assume $\langle CD\rangle \neq 0$, and $\delta \approx 0$. Then we have:

$$\delta^2 \approx 0$$

i.e.:

$$((CD)^* - avg)(CD - avg) = 0 \tag{3.1.1}$$

$$D^*C^*CD = avg * (CD + D^*C^*) - avg^2 \tag{3.1.2}$$

Hence a four operator is reduced into a few of two operators. Such method could be naturally extended to treat the operator $\sum_{i,j} D_i^* C_i^* C_j D_j$.

A copper pair has the energy of:

$$\Delta = \langle C_{k\uparrow} C_{-k\downarrow} \rangle$$

To resist the flow of current carried by Copper Pair, is equivlant to destroying a pair of Copper Pair:

$$\langle C_{k\uparrow} C_{-k\downarrow} \rangle \longrightarrow C_{k\uparrow} C_{-k\downarrow}$$

This will require an additional enegy of 2Δ.

The exact meaning of "equivalent to" is as follows:

break a copper pair $\longrightarrow$ scatter two electrons consecutively

$\longrightarrow$ create two electron-hole mixed type quasi-particle $\longrightarrow 2\Delta$

3.2 Why 0/0 is undefined?

If we suppose

$$\frac{0}{0} = \triangle$$

Consider the following derivation:

$$\frac{0}{0} \cdot 1 = \triangle \cdot 1 = \triangle \tag{3.2.1}$$

$$0 \cdot \frac{1}{0} = \triangle \tag{3.2.2}$$

$$\Rightarrow \triangle = 0 \tag{3.2.3}$$

This is already bad enough. And we are forced to define $\frac{1}{0}$. Let $\frac{1}{0} = \square$, which literally means $1 = 0 \cdot \square = 0$. This is disastrous.

Alternatively, we could let

1. Let $\frac{1}{0}$ be undefined.
2. Or let $\frac{1}{0} = \infty$.
3. Or, let $\frac{a}{b} \cdot c = a \cdot \frac{c}{b}$ be not true when $b = 0$.

The third idea is disastrous for algebraic manipulation. [1] The first idea is not good. Since defining $\frac{1}{0} = \infty$ turns out to be very useful in both mathematics and physics. Actully, in physics it is common practice to set $\frac{a}{0} = \pm\infty$ for any nonzero number a, where the sign of ∞ is determined by the sign of a. The second idea is okey. But then we are faced with a serious problem. We have to define $\triangle \equiv 0 \cdot \infty$

$\triangle \cdot 2 = \triangle$, What will be of $\triangle + 1$?

[1] Or more speicifically, it is a disaster for field theory.

3.3 Preface of BSCS

BSCS: see [2]. Parallism between theories in condensed matter physics and those in particle physics.

- Anderson-Higgs Phenomenon (Paritcle), Meissner effect (C.M.P.)
- 'inflation' in Cosmology, first order phase transition
- 'cosmic strings', magnetic field vortex lines in type II superconductors
- Hadron-meson interaction, Ginzburg-Landau theory of superfluid He^3.

Same ideas on different space-time scales, different hierachical 'layers'. Strong parallism: **strongly correlated low dimensional system**

E.g.:

The problem of formation and structure of heavy particles - hadrons and mesons. The corresponding fine structure constant $\alpha_G \approx 1$.

Approaches:

1. Exact solutions
2. Reformulate complicated interacting models in such a way that they become weekly interacting. -> Bosonization.
   Spin 1/2 anisotropic Hisenberg chain $\approx$ Model of interacting fermions. (Jordan and Wigner, 1928)

Bosonization: transformation from fermions to a scalar massless bosonic field.

3.4 System of Differential Equations

This is a small note of [3].

pp. 266.

Definition 3.1. $\mathbf{x}(\mathbf{t})$ *is a vector whose elements are* $x_i(t)$. $\frac{d}{dt}$ *acts on vector* $\mathbf{x}$ *element-wise.* $\dot{\mathbf{x}}$ *is abbrevation for* $\frac{d}{dt}\mathbf{x}$

pp. 291.

Theorem 3.1 (Existence-uniqueness theorem)**.** *There exists one, and only one, solution of the initial-value problem*

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x},\ \mathbf{x}(t_0) = \mathbf{x}^0 = \begin{pmatrix} x_1^0 \\ x_2^0 \\ \cdots \end{pmatrix} \tag{3.4.1}$$

Moreover, this solution exists for $-\infty \langle t \langle \infty$.

Remark 3.1. *By this, any non-trivial solution* $\mathbf{x}(t) \neq 0$ *at any time* t. *Also notice that the elements of* $\mathbf{A}$ *are just numbers.*

Theorem 3.2. *The dimension of the space* $\mathbf{V}$ *of all solutions of the homogeneous linear system of differential equations:*

$$\frac{d\mathbf{x}}{dt} = \mathbf{A}\mathbf{x} \tag{3.4.2}$$

is n, *i.e. the dimension of vector* $\mathbf{x}$.

3.5 ODE by Arnold

sec. 14

Definition 3.2.

$$e^A = I + A + \frac{A^2}{2!} + \frac{A^3}{3!} \tag{3.5.1}$$

or

$$e^A = \lim_{n\to\infty} (I + \frac{A}{n})^n \tag{3.5.2}$$

where I is the identity matrix.

Equivalance of the two definition will be addressed in the Theorem on pp. 165.

Important theorems:

Theorem 3.3 (pp. 158)**.** *The series e^A converges for any A uniformly on each set $X = \{A : ||A|| \le a\}$, $a \in \mathbb{R}$.*

Theorem 3.4 (pp. 160)**.**

$$e^{At} = H^t$$

where H^t is the translation operator which sends every polynomial $p(x)$ into $p(x+t)$.

Theorem 3.5 (pp. 163)**.**

$$\frac{d}{dt} e^{tA} = Ae^{tA}$$

Theorem 3.6 (Fundamental Theorem of the Theory of Linear Equations with Constant Coefficients)**.** *The solution of:*

$$\dot{\mathbf{x}} = A\mathbf{x} \tag{3.5.3}$$

with initial condition $\phi(0) = \mathbf{x}_0$ is

$$\phi(t) = e^{tA}\mathbf{x}_0 \tag{3.5.4}$$

Practically solution to

$$\dot{\mathbf{x}} = A\mathbf{x}$$

(pp. 173, Sec 17) (Assuming A is diagonalizable.)

- Find the eigenvectors $\xi_1, \cdots, \xi_n$ and eigenvalues $\lambda_1, \cdots, \lambda_n$. Use them as basis.
- Expand the initial condition in the new basis.

$$\mathbf{x}_0 = \sum_{k=1}^{n} C_k \xi_k \tag{3.5.5}$$

- Then $\phi(t) = \sum_{k=1}^{n} C_k e^{\lambda_k t} \xi_k$

3.6 Appearance of Gauge Structure in Simple Dynamical Systems

$$0 = (\eta_b, \dot{\eta_a}) = (\eta_b, \dot{U}_{ac}\psi) + (\eta_b, U_{ac}\dot{\psi}_c \tag{3.6.1}$$

4 Anchor

References

[1] Sakurai, J. J. Modern Quantum Mechanics, Addison Wesley.

[2] Bosonization and Strongly Correlated Systems. Cambridge.
Cambridge Press Link

[3] Martin Braun. Differential Equations and Their Applications. 4ed. Springer.

[4] `http://arxiv.org/abs/1508.07106v1`

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