Notes of Basic Topolgy

Taper

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Abstract

A note of Basic Topology, based on *Basic Topology* by M.A. Armstrong.

Contents

There are several parts that I will skipped for convenience. Those include chapter 1 - Introduction, chapter 2 - Continuity, chapter 3 - Compactness and Connectedness, and chapter 4 - Identification Spaces. Below is some especially confusing part that I would like to note:

1 Special Notes

sec:Special-Notes

Basic facts about maps Assuming domain $f = X$, codomain $f = Y$.

$$f(U \cup V) = f(U) \cup f(V) \tag{1.0.1}$$

$$f(U \cap V) = f(U) \cap f(V) \tag{1.0.2}$$

$$f(U^c) \supseteq f(U)^c, \text{ i.e. } f(U)^c \subseteq f(U^c) \tag{1.0.3}$$

$$f^{-1}(U \cup V) = f^{-1}(U) \cup f^{-1}(V) \tag{1.0.4}$$

$$f^{-1}(U \cap V) = f^{-1}(U) \cap f^{-1}(V) \tag{1.0.5}$$

$$f^{-1}(U^c) = [f^{-1}(U)]^c \tag{1.0.6}$$

Smallest the Largest Topolgy The set of all possible topolgies on X is partially ordered by inclusion. For a certain characteristics $\mathcal{C}$, it is possible to have the smallest or the largest one.

The **smallest topolgy** $\mathcal{T}_{\min}$ is the one such that, for any $\mathcal{T}'$ satisfying $\mathcal{C}$, $\mathcal{T}_{\min} \subseteq \mathcal{T}'$. The **largest topolgy** $\mathcal{T}_{\max}$ is the one such that, for any $\mathcal{T}'$ satisfying $\mathcal{C}$, $\mathcal{T}' \subseteq \mathcal{T}_{\max}$.

For example, assuming we have

$$f : X \to Y \tag{1.0.7}$$

where f is any function.

If X has topolgy $\mathcal{T}_X$, we ask then what kind of topolgy on Y will make f a continuous function. First, all $f^{-1}(V)$, with $V \in \mathcal{T}_Y$ should be open in X. So, the easiest choice is to make $\mathcal{T}_{Y,\min} = \{\varnothing, Y\}$, this is the smallest topolgy. Also, any set $V \in Y$ such that $f^{-1}(V) \notin \mathcal{T}_X$ should not be in $\mathcal{T}_Y$. Then the largest topolgy is $\mathcal{T}_{Y,\max} = \{V \subset Y | f^{-1}(V) \in \mathcal{T}_X\}$.

If Y has topolgy $\mathcal{T}_Y$, we also ask what kind of topolgy on X will make f a continuous function. First, all $V \in \mathcal{T}_Y$, their preimage $f^{-1}(V)$ must be in $\mathcal{T}_X$. So the smallest topolgy is $\mathcal{T}_{X,\min} = \{f^{-1}(V) | V \in \mathcal{T}_Y\}$.

2 Anchor

sec:Anchor

References

book

[1] M.A. Armstrong. Basic Topology. 2ed.

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