Homework for Advanced Quantum Mechanics 8

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1. *Deadline :December 7th before class.*
2. Please send your homework to my email: 910493179@qq.com.
3. Contact me or discuss in QQ group if you have any question.
4. Homework written by TEX has 5 extra points as bonus.

1. The Hamiltonian operator for a two-state system is given by

$$H = a(|1\rangle\langle 1| - |2\rangle\langle 2| + |1\rangle\langle 2| + |2\rangle\langle 1|) \tag{1}$$

where a is a real number, $\langle 1|2\rangle = 0$. Find the energy eigenvalues and the corresponding energy eigenkets (as linear combinations of $|1\rangle$ and $|2\rangle$).

2. Using the orthogonality of $|+\rangle$ and $|-\rangle$, prove:

$$[S_i, S_j] = i\epsilon_{ijk}\hbar S_k \tag{2}$$

$$\{S_i, S_j\} = (\frac{\hbar^2}{2})\delta_{ij} \tag{3}$$

where

$$S_x = \frac{\hbar}{2}(|+\rangle\langle -| + |-\rangle\langle +|) \tag{4}$$

$$S_y = \frac{i\hbar}{2}(-|+\rangle\langle -| + |-\rangle\langle +|) \tag{5}$$

$$S_z = \frac{\hbar}{2}(|+\rangle\langle +| - |-\rangle\langle -|) \tag{6}$$

3. (Spin-precession problem)Using the Hamiltonian

$$H = -(\frac{eB}{mc})S_z = \omega S_z \tag{7}$$

write the Heisenberg equations of motion for the time-dependent operators $S_x(t)$, $S_y(t)$, and $S_z(t)$. Solve them to obtain $S_{x,y,z}$ as functions of time. ($S_{x,y,z}(0)$ are known.)