General Physics Formula

Taper

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Abstract

This is a collection of important formulae in Fundamentals of Physics Extended Version by Halliday and Resnick.

Contents

1 Classical mechanics

Constant Accleration These five equations in Table 2-1 describe the motion of a particle with constant acceleration:

$$v = v_0 + at \tag{1.0.1}$$

$$x - x_0 = v_0 t + \frac{1}{2}at^2 \tag{1.0.2}$$

$$x - x_0 = vt - \frac{1}{2}at^2 \tag{1.0.3}$$

$$v^2 - v_0^2 = 2a(x - x_0) \tag{1.0.4}$$

$$x - x_0 = \frac{1}{2}(v_0 + v)t \tag{1.0.5}$$

Projectile Motion Projectile motion is the motion of a particle that is launched with an initial velocity $\vec{v}_0$. During its flight, the particle's horizontal acceleration is zero and its vertical acceleration is the free-fall acceleration $-g$. (Upward is taken to be a positive direction.) If $\vec{v}_0$ is expressed as a magnitude (the speed v_0) and an angle θ_0 (measured from the horizontal), the particle's equations of motion along the horizontal x axis and vertical y axis are

$$x - x_0 = (v_0 \cos(\theta_0))t \tag{1.0.6}$$

$$y - y_0 = (v_0 \sin(\theta_0))t - \frac{1}{2}gt^2 \tag{1.0.7}$$

$$v_y = v_0 \sin\theta_0 - gt \tag{1.0.8}$$

$$v_y^2 = (v_0 \sin\theta_0)^2 - 2g(y - y_0) \tag{1.0.9}$$

The **trajectory** (path) of a particle in projectile motion is parabolic and is given by

$$y = (\tan\theta_0)x - \frac{gx^2}{2(v_0\cos\theta_0)^2} \tag{1.0.10}$$

Uniform Circular Motion If a particle travels along a circle or circular arc of radius r at constant speed v, it is said to be in *uniform circular motion* and has an acceleration of constant magnitude

$$a = \frac{v^2}{r} \tag{1.0.11}$$

The direction of is toward the center of the circle or circular arc, and is said to be centripetal. The time for the particle to complete a circle is

$$T = \frac{2\pi r}{v} \tag{1.0.12}$$

T is called the period of revolution, or simply the period, of the motion.

This acceleration is due to a net centripetal force on the particle, with magnitude given by

$$F = \frac{mv^2}{R} \tag{1.0.13}$$

where m is the particle's mass. The vector quantities $\vec{a}$ and $\vec{F}$ are directed toward the center of curvature of the particle's path

Definition 1.1 (Normal force)**.** *A **normal force** is the force on a body from a surface against which the body presses.The normal force is always perpendicular to the surface.*

Drag Force When there is relative motion between air (or some other fluid) and a body, the body experiences a drag force $\vec{D}$ that opposes the relative motion and points in the direction in which the fluid flows relative to the body. The magnitude of $\vec{D}$is related to the relative speed v by an experimentally determined drag coefficient C according to

$$D = \frac{1}{2}C\rho A v^2 \tag{1.0.14}$$

where ρ is the fluid density (mass per unit volume) and A is the effective cross-sectional area of the body (the area of a cross section taken perpendicular to the relative velocity $\vec{v}$)

Power The power due to a force is the rate at which that force does work on an object. For a force $\vec{F}$ at an angle ϕ to the direction of travel of the instantaneous velocity $\vec{v}$, the instantaneous power is

$$P = Fv\cos\phi = \vec{F}\cdot\vec{v} \tag{1.0.15}$$

Collision and Impulse Applying Newton's second law in momentum form to a particle-like body involved in a collision leads to the impulse–linear momentum theorem:

$$\vec{p}_f - \vec{p}_i = \int_{t_i}^{t_f} \vec{F}\,\mathrm{d}t \tag{1.0.16}$$

where the LHS is the change in the body's linear momentum, and RHS is defined as the impulse $\vec{J}$ due to the force exerted on the body by the other body in the collision.

Elastic Collisions in One Dimension An elastic collision is a special type of collision in which the kinetic energy of a system of colliding bodies is conserved. If the system is closed and isolated, its linear momentum is also conserved. For a one-dimensional collision in which body 2 is a target and body 1 is an incoming projectile, conservation of kinetic energy and linear momentum yield the following expressions for the velocities immediately after the collision:

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} \tag{1.0.17}$$

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i} \tag{1.0.18}$$

Variable-Mass Systems In the absence of external forces a rocket accelerates at an instantaneous rate given by

$$Rv_{\mathrm{rel}} = Ma(\text{first rocket equation}) \tag{1.0.19}$$

in which M is the rocket's instantaneous mass (including unexpended fuel), R is the fuel consumption rate, and v_{rel} is the fuel's exhaust speed relative to the rocket. The term Rv_{rel} is the thrust of the rocket engine. For a rocket with constant R and v_{rel}, whose speed changes from v_i to v_f when its mass changes from M_i to M_f,

$$v_f - v_i = v_{\mathrm{rel}} \ln \frac{M_i}{M_f}(\text{second rocket equation}) \tag{1.0.20}$$

$(R \equiv |\frac{\mathrm{d}M}{\mathrm{d}t}|)$

The Kinematic Equations for Constant Angular Acceleration Constant angular acceleration ($\alpha =$a constant) is an important special case of rotational motion. The appropriate kinematic equations, given in Table 10-1, are

$$\sigma = \sigma_0 + \alpha t \tag{1.0.21}$$

$$\theta - \theta_0 = \omega_0 t + \frac{1}{2}\alpha t^2 \tag{1.0.22}$$

$$\theta - \theta_0 = \omega t - \frac{1}{2}\alpha t^2 \tag{1.0.23}$$

$$\omega^2 - \omega_0^2 = 2\alpha(\theta - \theta_0) \tag{1.0.24}$$

$$\theta - \theta_0 = \frac{1}{2}(\omega_0 + \omega)t \tag{1.0.25}$$

Linear and Angular Variables Related A point in a rigid rotating body, at a perpendicular distance r from the rotation axis, moves in a circle with radius r.

The linear acceleration $\vec{a}$ of the point has both tangential and radial components. The tangential component is

$$a_t = \alpha r \tag{1.0.26}$$

where α is the magnitude of the angular acceleration (in radians per second-squared) of the body. The radial component of $\vec{a}$ is

$$a_r = \frac{v^2}{r} = \omega^2 r \tag{1.0.27}$$

If the point moves in uniform circular motion, the period T of the motion for the point and the body is

$$T = \frac{2\pi r}{v} = \frac{2\pi}{\omega} \tag{1.0.28}$$

Rotational Kinetic Energy and Rotational Inertia The kinetic energy K of a rigid body rotating about a fixed axis is given by

$$K = \frac{1}{2} I \omega^2 \tag{1.0.29}$$

in which I is the rotational inertia of the body, defined as

$$I \equiv \int r^2 \, \mathrm{d}m \tag{1.0.30}$$

The Parallel-Axis Theorem 265

$$I = I_{\text{com}} + Mh^2 \tag{1.0.31}$$

Work and Rotational Kinetic Energy 265

$$W = \int_{\theta_i}^{\theta_f} \tau \, \mathrm{d}\theta \tag{1.0.32}$$

$$P = \frac{\mathrm{d}W}{\mathrm{d}t} = \tau \omega \tag{1.0.33}$$

Rolling Bodies [295]

$$v_{\text{com}} = \omega R \tag{1.0.34}$$

$$K = \frac{1}{2} I_{\text{com}} \omega^2 + \frac{1}{2} M v_{\text{com}}^2 \tag{1.0.35}$$

$$a_{\text{com}} = \alpha R \tag{1.0.36}$$

Precession of a Gyroscope

$$\Omega = \frac{Mgr}{I\omega} \tag{1.0.37}$$

Elastic Moduli [319]

$$\text{stress} = \text{modulus} \times \text{strain}$$

Young's modulus:

$$\frac{F}{A} = E\frac{\Delta L}{L} \tag{1.0.38}$$

Shear's modulus:

$$\frac{F}{A} = G\frac{\Delta x}{L} \tag{1.0.39}$$

Hydraulic bulk modulus:

$$p = B\frac{\Delta V}{V} \tag{1.0.40}$$

Gravitational Potential Energy [349]

$$U = -\frac{GMm}{r} \tag{1.0.41}$$

Escape speed

$$v = \sqrt{\frac{2GM}{R}} \tag{1.0.42}$$

Kepler's Laws

1. The law of orbits. All planets move in elliptical orbits with the Sun at one focus.
2. The law of areas. A line joining any planet to the Sun sweeps out equal areas in equal time intervals. (This statement is equivalent to conservation of angular momentum.)
3. The law of periods. The square of the period T of any planet is proportional to the cube of the semimajor axis a of its orbit. For circular orbits with radius r,

$$T^2 = \left(\frac{4\pi^2}{GM}\right) r^3 \tag{1.0.43}$$

where M is the mass of the attracting body—the Sun in the case of the solar system. For elliptical planetary orbits, the semimajor axis a is substituted for r.

Energy in Planetary Motion

$$U = -\frac{GMm}{r} \tag{1.0.44}$$

$$K = \frac{GMm}{2r} \tag{1.0.45}$$

$$E = -\frac{GMm}{2r} \quad \text{or} \tag{1.0.46}$$

$$E = -\frac{GMm}{2a} \tag{1.0.47}$$

Flow of Ideal Fluids [377]

Definition 1.2 (Apparent weight)**.** *Omited*

Equation of continuity

$$R_v \equiv Av = \text{ a constant} \tag{1.0.48}$$

Bernoulli's Equation

$$p + \frac{1}{2}\rho v^2 + \rho g h = \text{ a constant} \tag{1.0.49}$$

Oscillation (pp.403 of [1])

$$\omega = \frac{2\pi}{T} = 2\pi f \tag{1.0.50}$$

Linear Oscillator

$$\omega = \sqrt{\frac{k}{m}} \tag{1.0.51}$$

$$T = 2\pi\sqrt{\frac{m}{k}} \tag{1.0.52}$$

Pendulums

$$T = 2\pi\sqrt{I/\kappa} \tag{1.0.53}$$

$$T = 2\pi\sqrt{L/g} \tag{1.0.54}$$

$$T = 2\pi\sqrt{I/mgh} \tag{1.0.55}$$

Damped Harmonic Motion

$$x(t) = x_m e^{-bt/2m}\cos\left(\omega' t + \phi\right) \tag{1.0.56}$$

$$\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}} \tag{1.0.57}$$

$$E(t) \approx \frac{1}{2}kx_m^2 e^{-bt/m} \tag{1.0.58}$$

Waves (pp.436 ch16 of [1])

$$y(x,t) = y_m \sin(kx - \omega t) \tag{1.0.59}$$

$$k = \frac{2\pi}{\lambda} \tag{1.0.60}$$

$$f = \frac{\omega}{2\pi} = \frac{1}{T} \tag{1.0.61}$$

$$v = \frac{\omega}{k} = \frac{\lambda}{T} = \lambda f \tag{1.0.62}$$

String

$$v = \frac{\tau}{\mu} \tag{1.0.63}$$

$$P_{\text{avg}} = \frac{1}{2}\mu v \omega^2 y_m^2 \tag{1.0.64}$$

Resonance

$$n\lambda = 2L \tag{1.0.65}$$

Longitudinal Wave (**pp.466 ch17 of** [**1**])

$$v = \sqrt{\frac{B}{\rho}} \tag{1.0.66}$$

$$s = s_m \cos(kx - \omega t) \tag{1.0.67}$$

$$\Delta p = \Delta p_m sin(kx - \omega r) \tag{1.0.68}$$

$$\Delta p_m = (v\rho\omega)s_m \tag{1.0.69}$$

Sound Intensity

$$I = \frac{P}{A} \tag{1.0.70}$$

$$I = \frac{1}{2}\rho v \omega^2 s_m^2 \tag{1.0.71}$$

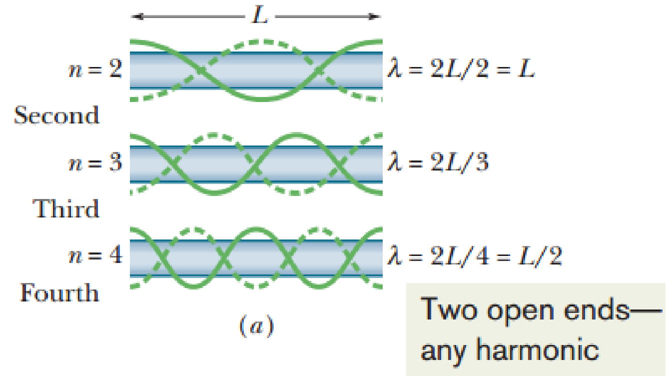
$$I = \frac{P_s}{4\pi r^2} \tag{1.0.72}$$

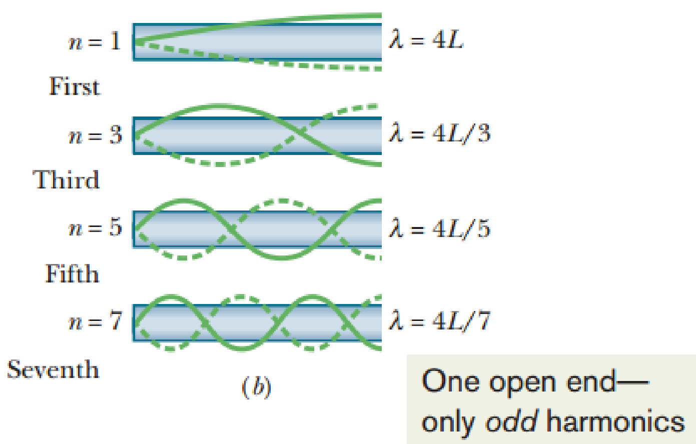
$$\tag{1.0.73}$$

Standing Wave Patterns in Pipes

$$f = \frac{v}{\lambda} = \frac{nv}{2L}, n = 1, 2, 3, \cdots \tag{1.0.74}$$

$$f = \frac{v}{\lambda} = \frac{nv}{4L}, n = 1, 3, 5, \cdots \tag{1.0.75}$$





Beat

$$f_{\text{beat}} = f_1 - f_2 \tag{1.0.76}$$

The Doppler Effect

$$f' = f\frac{v \pm v_D}{v \pm v_s} \tag{1.0.77}$$

The signs are chosen such that f' tends to be *greater* for motion toward and *less* for motion away. **Shock wave**

$$\sin\theta = \frac{v}{v_s}\ ,\ v_s \geq v \tag{1.0.78}$$

Conduction (**pp.498 ch18 of [1]**) **Thermal expansion**

$$\Delta L = L\alpha\Delta T \tag{1.0.79}$$

$$\Delta V = V\beta\Delta T \tag{1.0.80}$$

$$\beta = 3\alpha \tag{1.0.81}$$

$$Q = C\Delta T = cm\Delta T \tag{1.0.82}$$

Definition 1.3 (Convection)**.** *Convection occurs when temperature differences cause an energy transfer by motion within a fluid*

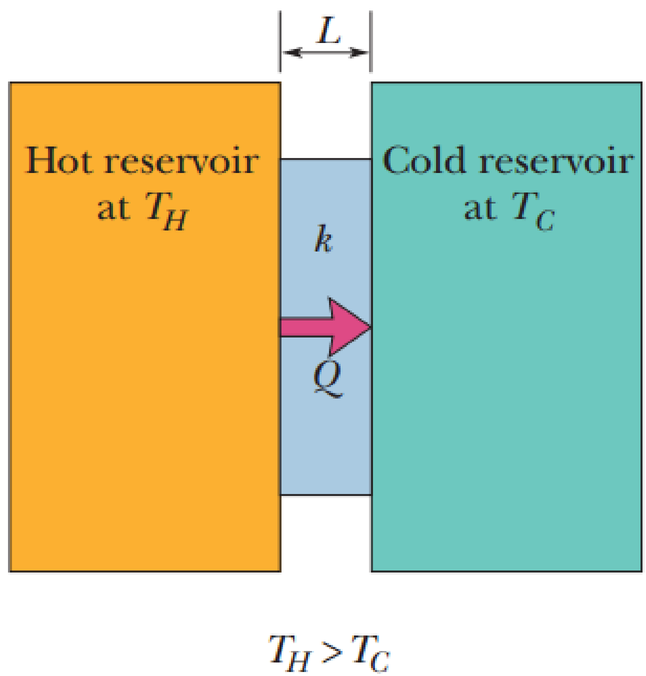
Radiation

$$P_{\text{rad}} = \sigma\varepsilon A T^4 \tag{1.0.83}$$

$$P_{\text{abs}} = \sigma\varepsilon A T_{\text{evn}}^4 \tag{1.0.84}$$

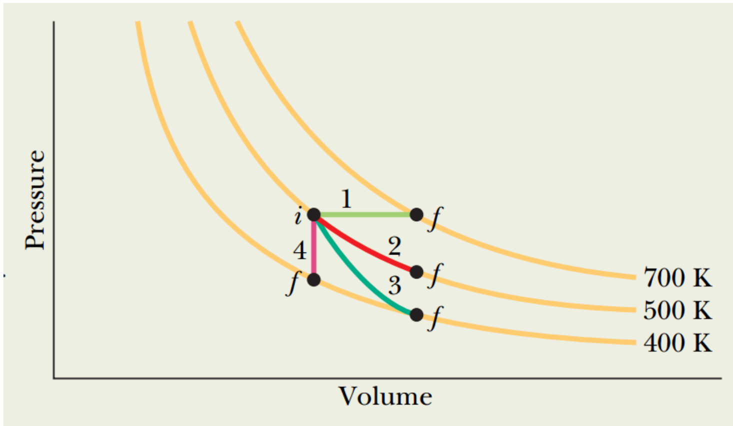
Conduction

$$P_{\text{cond}} \equiv \frac{Q}{t} = kA\frac{T_H - T_C}{L} \tag{1.0.85}$$



Thermal Process (**pp.529 ch19 of [1]**) Thermal process:

No.	Constant Quaility	Process Type	Special Results
1	p	Isobaric	$Q = nC_p\Delta T; W = p\Delta V$
2	V	Isothermal	$Q = W = nRT\ln(V_f/V_i); \Delta E_{\text{int}} = 0$
3	pV^γ, $TV^{\gamma-1}$	Adiabatic	$W = -\Delta E_{\text{int}}$
4	V	Isochoric	$Q = \Delta E_{\text{int}} = nC_V\Delta T; W = 0$



Isothermal Process:

$$W = NkT\ln\frac{V_f}{V_i} \tag{1.0.86}$$

Temperature and Kinetic Energy

$$K_{\text{avg}} = \frac{3}{2}kT, \text{ per molecule} \tag{1.0.87}$$

$$\lambda = \frac{1}{\sqrt{2}\pi d^2 N/V} \tag{1.0.88}$$

Molar Specific Heats

$$C_V = \frac{3}{2}R \tag{1.0.89}$$

$$C_P = C_V + R \tag{1.0.90}$$

$$\Delta E_{\text{int}} = NC_V\Delta T \tag{1.0.91}$$

$$C_V = \frac{f}{2}k \tag{1.0.92}$$

$$\gamma \equiv \frac{C_P}{C_V} \tag{1.0.93}$$

Entropy (**pp.554 ch20 of** [**1**])

$$\mathrm{d}S = \frac{\mathrm{d}Q}{T} \tag{1.0.94}$$

$$\Delta S = nR \ln \frac{V_f}{V_i} + nC_V \ln \frac{T_f}{T_i} \tag{1.0.95}$$

$$\varepsilon = 1 - \frac{T_L}{T_H} \text{ (Carnot engine)} \tag{1.0.96}$$

$$K = \frac{T_L}{T_H - T_L} \text{ (Carnot refrigerator)} \tag{1.0.97}$$

$$\ln N! \approx N \ln N - N \tag{1.0.98}$$

Electric field (**pp.596 ch22 of** [**1**])

$$E = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} \tag{1.0.99}$$

$$E = \frac{1}{2\pi\varepsilon_0} \frac{p}{z^3} \tag{1.0.100}$$

$$U = -\vec{p} \cdot \vec{E} \tag{1.0.101}$$

Gauss's Law (**pp.620 ch23 of** [**1**])

$$E = \frac{\sigma}{\varepsilon_0} \text{ ,charged conductor} \tag{1.0.102}$$

$$E = \frac{\sigma}{2\varepsilon_0} \text{ ,infinite sheet} \tag{1.0.103}$$

$$E = \frac{\lambda}{2\pi\varepsilon_0 r} \tag{1.0.104}$$

$$E = 0 \text{ , inside charged shell} \tag{1.0.105}$$

$$E = \left(\frac{q}{4\pi\varepsilon_0 R^3} \right) r \tag{1.0.106}$$

Electric Potential (**pp.646 ch24 of** [**1**])

$$V = \frac{1}{4\pi\varepsilon_0} \frac{q}{r} \tag{1.0.107}$$

$$V = \frac{1}{4\pi\varepsilon_0} \frac{p \cos\theta}{r^2} \tag{1.0.108}$$

$$U = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r} \tag{1.0.109}$$

Capacitor (**pp.675 ch25 of** [**1**])

$$C \equiv \frac{q}{V} \tag{1.0.110}$$

$$C = \frac{\varepsilon_0 A}{d} \text{ (parallel-plate)} \tag{1.0.111}$$

$$C = 2\pi\varepsilon_0 \frac{L}{\ln b/a} \text{ (cylindrical)} \tag{1.0.112}$$

$$C = 4\pi\varepsilon_0 \frac{ab}{b-a} \text{ (spherical)} \tag{1.0.113}$$

$$C = 4\pi\varepsilon_0 R \text{ (isolated spherical)} \tag{1.0.114}$$

$$C = \sum_j C_j \text{ (parallel)} \tag{1.0.115}$$

$$\frac{1}{C} = \sum_j \frac{1}{C_j} \text{ (series)} \tag{1.0.116}$$

$$U = \frac{1}{2}\frac{q^2}{C} = \frac{1}{2}CV^2 \tag{1.0.117}$$

$$u = \frac{1}{2}\varepsilon_0 E^2 \tag{1.0.118}$$

Current (**pp.698 ch26 of [1]**)

$$\vec{J} = ne\vec{v}_d \tag{1.0.119}$$

$$\rho \equiv \frac{1}{\sigma} = \frac{E}{J} \tag{1.0.120}$$

$$\vec{E} = \rho\vec{J} \tag{1.0.121}$$

$$R = \rho\frac{L}{A} \tag{1.0.122}$$

$$\rho - \rho_0 = \rho_0\alpha(T - T_0) \tag{1.0.123}$$

$$\rho = \frac{m}{e^2 n\tau} \tag{1.0.124}$$

Electro motive force (**pp.724 ch27 of [1]**)

$$\mathcal{E} = \frac{\mathrm{d}W}{\mathrm{d}q} \tag{1.0.125}$$

Circuit Rule

- **Loop Rule**. The algebraic sum of the changes in potential encountered in a complete traversal of any loop of a circuit must be zero.
- **Junction Rule**. The sum of the currents entering any junction must be equal to the sum of the currents leaving that junction

$$R = \sum_j R_j \text{ , series} \tag{1.0.126}$$

$$\frac{1}{R} = \sum_j \frac{1}{R_j} \text{ , parallel} \tag{1.0.127}$$

Magnetic Field (**pp.755 ch28 of [1]**)

$$qvB = \frac{mv^2}{r} \tag{1.0.128}$$

$$T = \frac{2\pi r}{rqB/m} = \frac{2\pi m}{qB} \tag{1.0.129}$$

$$\vec{\tau} = \vec{\mu} \times \vec{B} \tag{1.0.130}$$

$$U(\theta) = -\vec{\mu} \cdot \vec{B} \tag{1.0.131}$$

Magnetic -¿ Electricity (**pp.781 ch29 of [1]**)

$$\mathrm{d}\vec{B} = \frac{\mu_0}{4\pi} \frac{i\,\mathrm{d}\vec{s} \times \hat{r}}{r^2} \tag{1.0.132}$$

$$B = \frac{\mu_0 i}{2\pi R} \tag{1.0.133}$$

$$B = \frac{\mu_0 i \phi}{4\pi R} \tag{1.0.134}$$

$$B = \mu_0 i N/L \tag{1.0.135}$$

$$B = \frac{\mu_i i N}{2\pi} \frac{1}{r} \tag{1.0.136}$$

$$\vec{B}(z) = \frac{\mu_0}{2\pi} \frac{\vec{\mu}}{z^3} \tag{1.0.137}$$

Faraday's Law (**pp.816 ch30 of [1]**)

$$\mathcal{E} = \int \vec{E}\,\mathrm{d}\vec{s} \tag{1.0.138}$$

$$\tag{1.0.139}$$

Inductance

$$L \equiv \frac{N\Phi_B}{i} \tag{1.0.140}$$

$$\frac{L}{l} = \mu_n n^2 A \tag{1.0.141}$$

$$\mathcal{E}_L = -L\frac{\mathrm{d}i}{\mathrm{d}t} \tag{1.0.142}$$

$$U_B = \frac{1}{2} L i^2 \tag{1.0.143}$$

$$u_B = \frac{1}{2} \frac{B^2}{\mu_0} \tag{1.0.144}$$

2 Anchor

References

[1] Fundamentals of Physics, Extended Edition. Halliday & Resnick.

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