Solution for HW3 20161019

Taper

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Abstract

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1 Give the brief description of quantum state and operator in Hilbert space

Quantum state in Hilbert space : each quantum state correspondes to a vector of unit length in Hilbert space.

Operator in Hilbert space : each operator coorespondes to an invertible linear transformation (or a transformation of basis) in Hilbert space. In rare case, an operator may coorespondes to an invertible anti-linear transformation in Hilbert space, such as the time reversal opertor.

2 Proof

(1):

Proof.

$$\begin{aligned}\text{LHS} &= \begin{pmatrix} a_1 + b_1 \\ a_2 + b_2 \end{pmatrix}^\dagger * \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} \\ &= (a_1^* + b_1^*)c_1 + (a_2^* + b_2^*)c_2\end{aligned}$$

$$\begin{aligned}\text{RHS} &= (a_1^* c_1 + a_2^* c_2) + (b_1^* c_1 + b_2^* c_2) \\ &= (a_1^* + b_1^*)c_1 + (a_2^* + b_2^*)c_2 \\ &= \text{LHS}\end{aligned}$$

□

(2):

Proof.

$$\begin{aligned}\text{LHS} &= \begin{pmatrix} a_1 + b_1 \\ a_2 + b_2 \end{pmatrix} * \begin{pmatrix} c_1^* & c_2^* \end{pmatrix} \\ &= \begin{pmatrix} (a_1 + b_1)c_1^* & (a_1 + b_1)c_2^* \\ (a_2 + b_2)c_1^* & (a_2 + b_2)c_2^* \end{pmatrix}\end{aligned}$$

$$\begin{aligned}\text{RHS} &= \begin{pmatrix} a_1 c_1^* & a_1 c_2^* \\ a_2 c_1^* & a_2 c_2^* \end{pmatrix} + \begin{pmatrix} b_1 c_1^* & b_1 c_2^* \\ b_2 c_1^* & b_2 c_2^* \end{pmatrix} \\ &= \begin{pmatrix} (a_1 + b_1)c_1^* & (a_1 + b_1)c_2^* \\ (a_2 + b_2)c_1^* & (a_2 + b_2)c_2^* \end{pmatrix} \\ &= \text{LHS}\end{aligned}$$

$\square$

3 Check

$A^\dagger =$

$$\begin{pmatrix} 0 & 0 & i \\ 0 & 1 & 0 \\ -i & 0 & 0 \end{pmatrix}^\dagger = \overline{\begin{pmatrix} 0 & 0 & -i \\ 0 & 1 & 0 \\ i & 0 & 0 \end{pmatrix}} = \begin{pmatrix} 0 & 0 & i \\ 0 & 1 & 0 \\ -i & 0 & 0 \end{pmatrix} = A$$

$B^\dagger =$

$$\begin{pmatrix} 3 & i & 0 \\ 3 & 1 & 5 \\ 0 & -i & 2 \end{pmatrix}^\dagger = \overline{\begin{pmatrix} 3 & 3 & 0 \\ i & 1 & -i \\ 0 & 5 & 2 \end{pmatrix}} = \begin{pmatrix} 3 & 3 & 0 \\ -i & 1 & i \\ 0 & 5 & 2 \end{pmatrix} \neq B$$

So A is Hermitian whereas B is not.