$$C_m = \frac{1}{\sqrt{L_x}} \sum_{P_x} e^{iP_x m} C_{P_x} \qquad C_m^\dagger = \frac{1}{\sqrt{L_x}} \sum_{P_x} e^{-iP_x m} C_{P_x}^\dagger$$

① $\sum_{m,n} i\left[C^\dagger_{m+1,n}\, \sigma^x C_{m,n} - C^\dagger_{m,n}\, \sigma^x C_{m+1,n}\right]$

$= \sum_{m,n} i \frac{1}{L_x}\left[e^{-iP_x m} \cdot e^{-iP_x m}\, \underline{C^\dagger_{P_x,n}}\, \sigma^x\, e^{iP_x' m}\, \underline{C_{P_x' n}} - e^{-iP_x m}\, \underline{C^\dagger_{P_x,n}}\, \sigma^x\, e^{iP_x'} e^{iP_x' m}\, \underline{C_{P_x' n}}\right.$

$= i \sum_{n,P_x} e^{-iP_x}\, \cancel{} - e^{iP_x}\quad \underline{C^\dagger_{P_x n}\, \sigma^x C_{P_x n}}$

$= i \cdot -2i \sin P_x$

$= \sum_{n,P_x} 2\sin P_x\; C^\dagger_{P_x n}\, \sigma^x C_{P_x n}$

② $\sum_{m,n} i \cdot \left[C^\dagger_{m,n+1}\, \sigma^y C_{m,n} - C^\dagger_{m,n}\, \sigma^y C_{m,n+1}\right] = i \cdot \sum_{n,P_x}\left[C^\dagger_{P_x,n+1}\, \sigma^y C_{P_x,n} - C^\dagger_{P_x,n}\, \sigma^y C_{P_x,n+1}\right]$

③ $\sum_{m,n} -\left[\underline{C^\dagger_{m+1,n}\, \sigma^z C_{m,n} + C^\dagger_{m,n}\sigma^z C_{m+1,n}} + C^\dagger_{m,n+1}\, \sigma^z C_{m,n} + C^\dagger_{m,n}\, \sigma^z C_{m,n+1}\right]$

$\sum_{n,P_x} = -\left[2 \cdot \cos P_x \cdot C^\dagger_{P_x n}\, \sigma^z C_{P_x,n} + C^\dagger_{P_x,n+1}\, \sigma^z C_{P_x,n} + C^\dagger_{P_x,n}\, \sigma^z C_{P_x,n+1}\right]$

④ $\sum_{m,n} (2-m)\, C^\dagger_{m,n}\, \sigma^z C_{m,n} = (2-m) \cdot \sum_{n,P_x} C^\dagger_{P_x,n}\, \sigma^z C_{P_x,n}$

$$\phi(r) = i\vec{\alpha}\cdot\vec{p} + \alpha m \qquad \phi(r)\cdot(\psi)(\vec{r}) \quad -i(\vec{\alpha}\cdot\nabla)(\psi)(\vec{r}) + (\alpha) m (\psi)(\vec{r}).$$

$$\int d^3\vec{x}\, \chi^\dagger_\gamma(\vec{x}) \left\{ m\beta + \frac{i}{2}\left(\frac{\overleftarrow{\partial}}{\partial x_k} - \frac{\vec{\partial}}{\partial x_k}\right)\alpha_k \right\}_{\gamma\delta} \chi_\delta(\vec{x})$$

In, summary, the (P_x, n) Hamiltonian is:

$$\sum_{n,P_x} \Big\{ \underbrace{2\cdot\sin(P_x)\cdot C^\dagger_{P_x,n}\,\sigma^x C_{P_x,n}}_{A} + i\cdot\Big[\underbrace{C^\dagger_{P_x,n+1}\,\sigma^y C_{P_x,n}}_{B} - \underbrace{C^\dagger_{P_x,n}\,\sigma^y C_{P_x,n+1}}_{C}\Big]$$

$$-\Big[\underbrace{2\cdot\cos P_x\cdot C^\dagger_{P_x,n}\,\sigma^z C_{P_x,n}}_{D} + \underbrace{C^\dagger_{P_x,n+1}\,\sigma^z C_{P_x,n}}_{E} + \underbrace{C^\dagger_{P_x,n}\,\sigma^z C_{P_x,n+1}}_{F}\Big]$$

$$+ (2-m)\cdot \underbrace{C^\dagger_{P_x,n}\,\sigma^z C_{P_x,n}}_{G} \Big\}$$

~~Test wave function:~~ $\cancel{|\psi_{P_x}\rangle = \sum_{n} \psi_{n,P_x} C^\dagger_{n,P_x} |0\rangle}$

Test wave function $\quad |\psi_{P_x,u}\rangle = \sum_n \psi_{n,P_x,u}\, C^\dagger_{n,P_x,u} |0\rangle$

$$|\psi_{P_x,v}\rangle = \sum_n \psi_{n,P_x,v}\, C^\dagger_{n,P_x,v} |0\rangle$$

Quantization of Dirac Field

$\cancel{2\sin}$

$\cancel{A: \sum_n 2\sin(P_x)\cdot \psi_{n,P_x}}$

$\cancel{\mathcal{L} = \bar\psi(i\partial\!\!\!/ - m)\psi = \bar\psi(i\gamma^\mu\partial_\mu - m)\psi}$

Check:

$\cancel{a_{11} C^\dagger_{P_x n, u} + a_{12} C^\dagger}$

$\cancel{H = \int d^3x\, \psi^\dagger[-i\gamma^0\gamma\cdot\nabla + m\gamma^0]\psi}$

$a_{11} C^\dagger_{P_x n} C_{P_x n} \cdot \psi_{n P_x}$

$h_D = -i\alpha\cdot\nabla + m\beta$

$\cancel{A: \sum_{n,P_x^0} C^\dagger_{P_x^0 n}\, \sigma^x C_{P_x^0 n} |\psi_{P_x',u'}\rangle = \sum_{n,P_x} C^\dagger_{P_x n}\sigma^x C_{P_x n} \sum_{n',u'} \psi_{n',P_x'} C^\dagger_{n',P_x',u'}|0\rangle = \sum_n}$

$$A: \sum_{n',P_x'} C^\dagger_{P_x',n'}\, \sigma^x C_{P_x' n'}\, |\psi_{P_x,u}\rangle = \sum_n (C^\dagger_{P_x,n})\, \sigma^x \begin{pmatrix} \cancel{C^\dagger} 1 \\ 0 \end{pmatrix} \cdot \psi_{n,P_x,u}|0\rangle$$

$$\sum_{n',P_x'} (C^\dagger_{P_x',n'})\, \sigma^x C_{P_x',n'}\, |\psi_{P_x,v}\rangle = \sum_n (C^\dagger_{P_x,n})\, \sigma^x \begin{pmatrix} 0 \\ 1 \end{pmatrix} \psi_{n,P_x,v}|0\rangle$$

$$B: \sum_{n',P_x'} C^\dagger_{P_x',n'+1}\, \sigma^y C_{P_x',n}\, |\psi_{P_x,u}\rangle = \sum_n (C^\dagger_{P_x,\cancel{n+1}\,n})\, \sigma^y \begin{pmatrix} 1 \\ 0 \end{pmatrix} \psi_{\cancel{n}\,n-1,P_x,u}|0\rangle$$

$$- - - - |\psi_{P_x,v}\rangle = - - - - \begin{pmatrix} 0 \\ 1 \end{pmatrix} \psi_{\cancel{n}\,n-1,P_x,v}|0\rangle$$

$$C: \sum_{n',P_x'} C^\dagger_{P_x',n'}\, \sigma^y C_{P_x',n'+1}\, |\psi_{P_x,u}\rangle = \sum_n (C^\dagger_{P_x,\cancel{n-1}\,n})\, \sigma^y \begin{pmatrix} 1 \\ 0 \end{pmatrix} \psi_{\cancel{n}\,n+1,P_x,u}|0\rangle$$

$$- - - - |\psi_{P_x,v}\rangle = - - - - \begin{pmatrix} 0 \\ 1 \end{pmatrix} \psi_{\cancel{n}\,n+1,P_x,v}|0\rangle$$

$D \to A,$

$E \to B$

$F \to C$

$G \to A$

Together:

$\cancel{\sum_n \overset{A}{\cdot} \; 2 \sin P_x \; (C^\dagger_{P_x, n}}$

$\cancel{2 \cdot \sin P_x \cdot (C^\dagger_{P_x, n}) \, \sigma^x \, C_{P_x}}$

$(\psi) = \begin{pmatrix} \psi_{n P_x u} \\ \psi_{n P_x v} \end{pmatrix}$

$\psi_{n P_x}$

$$\sum_n \; (C^\dagger_{P_x n}) \underbrace{\left(\overset{Ⓐ + Ⓓ}{2 \cdot \sin P_x \sigma^x - 2\cos P_x \sigma^z} \overset{+ Ⓖ}{+ (2-m) \sigma^z} \right) (\psi)}_{B \psi_n} + \underbrace{(C^\dagger_{P_x n}) \; \overset{Ⓑ + Ⓔ}{(i \sigma^y - \sigma^z)} \begin{pmatrix} \psi_{n-1 \, P_x u} \\ \psi_{n-1 \, P_x v} \end{pmatrix}}_{A \psi_{n-1}}$$

$$+ \underbrace{(C^\dagger_{P_x n}) \; \overset{Ⓒ + Ⓕ}{(-i \sigma^y - \sigma^z)} \begin{pmatrix} \psi_{n+1 \, P_x u} \\ \psi_{n+1 \, P_x v} \end{pmatrix}}_{C \psi_{n+1}} = E \sum_n (C^\dagger_{P_x, n}) \begin{pmatrix} \psi_{P_x n u} \\ \psi_{P_x n v} \end{pmatrix} |0\rangle$$

$\cancel{A}$ $\begin{pmatrix} A & & \\ & B & \\ & & C \end{pmatrix} \begin{pmatrix} \psi_{n-1} \\ \psi_n \\ \psi_{n+1} \end{pmatrix}$

$\begin{pmatrix} \psi_{n-2} \\ \psi_{n-1} \\ \psi_n \\ \psi_{n+1} \\ \psi_{n+2} \end{pmatrix}$

$$
\begin{pmatrix}
B + A\cancel{\frac{\psi_0}{\psi_1}}\,\psi_N & C & & & A \\
A & B & C & & \\
 & A & B & C & \\
C & & & A & B + \cancel{\frac{C\psi_{N+1}}{\psi_N}}\,\psi_1
\end{pmatrix}
\begin{pmatrix} \psi_1 \\ \vdots \\ \psi_N \end{pmatrix}
= E \begin{pmatrix} \psi_1 \\ \vdots \\ \psi_N \end{pmatrix}
$$

$A = i\sigma^y - \sigma^z$

$B = 2\cdot\sin(p_x)\,\sigma^x - 2\cos p_x\,\sigma^z + (2-m)\,\sigma^z$

$C = -i\sigma^y - \sigma^z$