In[3]:= $$H := \begin{pmatrix} E_0 + \Delta & W_1 - i * W_2 \\ W_1 + i * W_2 & E_0 - \Delta \end{pmatrix};$$

In[4]:= Eigenvalues[H]

Out[4]= $$\left\{e_0 - \sqrt{\Delta^2 + W_1^2 + W_2^2},\ e_0 + \sqrt{\Delta^2 + W_1^2 + W_2^2}\right\}$$

In[18]:= $$E_- := e_0 - \sqrt{\Delta^2 + W_1^2 + W_2^2};$$

$$E_+ := e_0 + \sqrt{\Delta^2 + W_1^2 + W_2^2};$$

In[5]:= Eigenvectors[H]

Out[5]= $$\left\{\left\{-\frac{-\Delta + \sqrt{\Delta^2 + W_1^2 + W_2^2}}{W_1 + i\, W_2}, 1\right\}, \left\{-\frac{-\Delta - \sqrt{\Delta^2 + W_1^2 + W_2^2}}{W_1 + i\, W_2}, 1\right\}\right\}$$

Now express (1,0) as linear combinations of Eigenvectors:

In[8]:= $$\Lambda := \text{Transpose}\left[\left\{\left\{-\frac{-\Delta + \sqrt{\Delta^2 + W_1^2 + W_2^2}}{W_1 + i\, W_2}, 1\right\}, \left\{-\frac{-\Delta - \sqrt{\Delta^2 + W_1^2 + W_2^2}}{W_1 + i\, W_2}, 1\right\}\right\}\right]$$

In[9]:= $$a := \text{Inverse}[\Lambda].\begin{pmatrix} 1 \\ 0 \end{pmatrix} \text{ // MatrixForm}$$

In[10]:= a // MatrixForm

Out[10]//MatrixForm=

$$\begin{pmatrix} -\frac{W_1 + i\, W_2}{2\sqrt{\Delta^2 + W_1^2 + W_2^2}} \\ \frac{W_1 + i\, W_2}{2\sqrt{\Delta^2 + W_1^2 + W_2^2}} \end{pmatrix}$$

In[14]:= Λ // MatrixForm

Out[14]//MatrixForm=

$$\begin{pmatrix} -\frac{-\Delta + \sqrt{\Delta^2 + W_1^2 + W_2^2}}{W_1 + i\, W_2} & -\frac{-\Delta - \sqrt{\Delta^2 + W_1^2 + W_2^2}}{W_1 + i\, W_2} \\ 1 & 1 \end{pmatrix}$$

In[17]:= $$\text{Simplify}\left[\begin{pmatrix} -\frac{-\Delta + \sqrt{\Delta^2 + W_1^2 + W_2^2}}{W_1 + i\, W_2} & -\frac{-\Delta - \sqrt{\Delta^2 + W_1^2 + W_2^2}}{W_1 + i\, W_2} \\ 1 & 1 \end{pmatrix}.\begin{pmatrix} -\frac{W_1 + i\, W_2}{2\sqrt{\Delta^2 + W_1^2 + W_2^2}} \\ \frac{W_1 + i\, W_2}{2\sqrt{\Delta^2 + W_1^2 + W_2^2}} \end{pmatrix}\right] \text{ (* }\Lambda.a\text{ *) // MatrixForm}$$

Out[17]//MatrixForm=

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

So the time evolution is obtained by e^{-iE_j t}*a_j*|E_j> (sum over j).

In[20]:= $$\psi := \text{Exp}[-i * E_- * t] * \left(-\frac{W_1 + i\, W_2}{2\sqrt{\Delta^2 + W_1^2 + W_2^2}}\right) * \begin{pmatrix} -\frac{-\Delta + \sqrt{\Delta^2 + W_1^2 + W_2^2}}{W_1 + i\, W_2} \\ 1 \end{pmatrix} +$$

$$\text{Exp}[-i * E_+ * t] * \frac{W_1 + i\, W_2}{2\sqrt{\Delta^2 + W_1^2 + W_2^2}} * \begin{pmatrix} -\frac{-\Delta - \sqrt{\Delta^2 + W_1^2 + W_2^2}}{W_1 + i\, W_2} \\ 1 \end{pmatrix}$$

In[21]:= ψ // MatrixForm

Out[21]//MatrixForm=

$$\begin{pmatrix} -\frac{e^{-i t \left(e_0+\sqrt{\Delta^2+W_1^2+W_2^2}\right)} \left(-\Delta-\sqrt{\Delta^2+W_1^2+W_2^2}\right)}{2 \sqrt{\Delta^2+W_1^2+W_2^2}}+\frac{e^{-i t \left(e_0-\sqrt{\Delta^2+W_1^2+W_2^2}\right)} \left(-\Delta+\sqrt{\Delta^2+W_1^2+W_2^2}\right)}{2 \sqrt{\Delta^2+W_1^2+W_2^2}} \\ -\frac{e^{-i t \left(e_0-\sqrt{\Delta^2+W_1^2+W_2^2}\right)} (W_1+i W_2)}{2 \sqrt{\Delta^2+W_1^2+W_2^2}}+\frac{e^{-i t \left(e_0+\sqrt{\Delta^2+W_1^2+W_2^2}\right)} (W_1+i W_2)}{2 \sqrt{\Delta^2+W_1^2+W_2^2}} \end{pmatrix}$$

Now we get p_k and u_k

In[33]:= psiConjTranspose := $\left(-\frac{e^{i t \left(e_0+\sqrt{\Delta^2+W_1^2+W_2^2}\right)} \left(-\Delta-\sqrt{\Delta^2+W_1^2+W_2^2}\right)}{2 \sqrt{\Delta^2+W_1^2+W_2^2}}+\frac{e^{i t \left(e_0-\sqrt{\Delta^2+W_1^2+W_2^2}\right)} \left(-\Delta+\sqrt{\Delta^2+W_1^2+W_2^2}\right)}{2 \sqrt{\Delta^2+W_1^2+W_2^2}} \quad -\frac{e^{i t \left(e_0-\sqrt{\Delta^2+W_1^2+W_\right)}}}{2 \sqrt{\Delta^2+W_1^2+W}}\right.$

In[34]:= pk := psiConjTranspose.ψ

In[35]:= pk

Out[35]= $$\left\{\left\{\left(-\frac{e^{i t \left(e_0-\sqrt{\Delta^2+W_1^2+W_2^2}\right)} (W_1-i W_2)}{2 \sqrt{\Delta^2+W_1^2+W_2^2}}+\frac{e^{i t \left(e_0+\sqrt{\Delta^2+W_1^2+W_2^2}\right)} (W_1-i W_2)}{2 \sqrt{\Delta^2+W_1^2+W_2^2}}\right) \left(-\frac{e^{-i t \left(e_0-\sqrt{\Delta^2+W_1^2+W_2^2}\right)} (W_1+i W_2)}{2 \sqrt{\Delta^2+W_1^2+W_2^2}}+\frac{e^{-i t \left(e_0+\sqrt{\Delta^2+W_1^2+W_2^2}\right)} (W_1+i W_2)}{2 \sqrt{\Delta^2+W_1^2+W_2^2}}\right)+\left(-\frac{e^{-i t \left(e_0+\sqrt{\Delta^2+W_1^2+W_2^2}\right)} \left(-\Delta-\sqrt{\Delta^2+W_1^2+W_2^2}\right)}{2 \sqrt{\Delta^2+W_1^2+W_2^2}}+\frac{e^{-i t \left(e_0-\sqrt{\Delta^2+W_1^2+W_2^2}\right)} \left(-\Delta+\sqrt{\Delta^2+W_1^2+W_2^2}\right)}{2 \sqrt{\Delta^2+W_1^2+W_2^2}}\right) \left(-\frac{e^{i t \left(e_0+\sqrt{\Delta^2+W_1^2+W_2^2}\right)} \left(-\Delta-\sqrt{\Delta^2+W_1^2+W_2^2}\right)}{2 \sqrt{\Delta^2+W_1^2+W_2^2}}+\frac{e^{i t \left(e_0-\sqrt{\Delta^2+W_1^2+W_2^2}\right)} \left(-\Delta+\sqrt{\Delta^2+W_1^2+W_2^2}\right)}{2 \sqrt{\Delta^2+W_1^2+W_2^2}}\right)\right\}\right\}$$

In[29]:= uk := Arg$\left[-\frac{e^{-i t \left(e_0-\sqrt{\Delta^2+W_1^2+W_2^2}\right)} (W_1+i W_2)}{2 \sqrt{\Delta^2+W_1^2+W_2^2}}+\frac{e^{-i t \left(e_0+\sqrt{\Delta^2+W_1^2+W_2^2}\right)} (W_1+i W_2)}{2 \sqrt{\Delta^2+W_1^2+W_2^2}}\right]$