

Notes of Chapter 2 of Bernevig's Book

Taper

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Abstract

Since I have already made a written one, this note is only an outline of the written script, in the hope of making it easier to read.

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1 Deriving Berry Phase

From page 1 to page 2 (equation 2.7), one derives the expression for the phase $e^{i\gamma_m}$ using instantaneous energy eigenstate for E_m :

$$H(\vec{R}) |n(\vec{R})\rangle = E_m(\vec{R}) |n(\vec{R})\rangle \quad (1.0.1)$$

$$\gamma_m = i \int_0^t \langle m(\vec{R}(t')) | \frac{\partial}{\partial t} m(\vec{R}(t')) \rangle dt' \quad (1.0.2)$$

Then I writes about different ways to get the γ_m :

$$\gamma_m = i \int_{\text{curve}} \langle m | \nabla_{\vec{R}} m \rangle d\vec{R} \quad (1.0.3)$$

(equation 2.8) Also, one defines

Definition 1.1 (Berry Connection, Berry Vector Potential \vec{A}).

$$\vec{A}_n \equiv i \langle n | \nabla_{\vec{R}} n \rangle \quad (1.0.4)$$

Then I proves several facts:

Fact 1.1.

$$\gamma_n \text{ is real}$$

Fact 1.2.

Berry connection \vec{A}_n is gauge-dependent

The dependence is: If

$$|n\rangle \rightarrow |n'\rangle = e^{i\xi(\vec{R})} |n\rangle$$

then

$$\vec{A}_n \rightarrow \vec{A}_n - \nabla_{\vec{R}} \xi(\vec{R}) \quad (1.0.5)$$

Therefore we have

Fact 1.3.

γ_n is gauge-dependent, unless the path tranverses a closed loop.

Since

$$\gamma_n \rightarrow \gamma_n - \left(\xi(\vec{R}(T)) - \xi(\vec{R}(0)) \right)$$

It is unchanged unless the integration curve is a closed loop. In which case

$$\xi(\vec{R}(T)) - \xi(\vec{R}(0)) = 2\pi m \stackrel{\text{mod } 2\pi}{=} 0$$

Example 1.1. There is a simple example on page 3 to show that the Berry phase can be actually detected.

By virtue of fact 1.1, we have

2 Anchor

References

[1] Bernevig's Topological Insulators and Superconductors

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