Section 1.2 of Intro → Complex Geometry.

Exercises:

1.2-1.

Q: Let $(V, \langle , \rangle)$: euclidian space of dim $= 4$.

Show: $\{$ all compatible almost complex structures $\}$
$\cong$
two copies of S^2 $= \cancel{a}$ two balls.

Recap: compatible: I: $\quad I^2 = -1, \quad \langle I(v), I(w) \rangle = \langle v, w \rangle$.

Choose an orthogonal basis: $e_1 \cdots e_4$

Let $I = (\cancel{a_{ij}}\ a^i_j)$ $\qquad \boxed{I^2 = a^i_j a^j_k = -\delta^i_k}$

also $\langle , \rangle \approx \delta^i_j \quad \cancel{a}$

$$\langle I(v), I(w) \rangle = (a^i_j v^j) \cdot \delta^i_k (a^k_l w^l) = v^i \delta^i_k w^k$$

$(\forall \vec{v}, \vec{w})$

Hence $\quad a^i_j \delta^i_k a^k_l = \delta_{jl} \quad$ or $\quad \boxed{a^i_j a^i_k = \delta_{jk}}$

For example:

$$\sum_j a^1_j a^j_2 = 0 = \sum_j a^j_1 a^j_2 \Rightarrow \sum_{j=2}^4 a^1_j a^j_2 = \sum_{j=2}^4 a^j_1 a^j_2$$

This can be generalized: $\displaystyle \sum_{\substack{j=1 \\ j \neq k}}^4 a^k_j a^j_l = \sum_{\substack{j=1 \\ j \neq k}}^4 a^j_k a^j_l \quad (k \neq l)$ $\quad$ 16 sets of eq.

Also: $\displaystyle \cancel{a^i} \sum_{j=1}^4 a^i_j a^j_i = -\sum_{j=1}^4 a^j_i a^j_i$