Solution for HW1 20160929

Taper

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Abstract

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1 1.Describe and explain (or derive) the following concepts

Photoelectric effect is the effect when light is shone on metals, there will be currents induced. Also, there exists an lower limits of light frequency, which is independent of light intensity, for this to happen. This is because the energy of a single photon is determined by its frequency.

Compton effect says that the wavelength of light (especially the x-ray), after a collision and scattering with electron, changes. This shows that light also behaves like an ordinary particle when coliding with other particles.

2 Problem 2

Normalize

$$\int_{-\infty}^{\infty} e^{-\lambda*|x|}e^{-i\omega t} * e^{-\lambda*|x|}e^{i\omega t}dx = 2\int_{0}^{\infty} e^{-2\lambda x}\, dx = \frac{2}{2\lambda} = \frac{1}{\lambda} \tag{2.0.1}$$

So $A = \sqrt{\lambda}$ (up to an irrelavent phase factor).

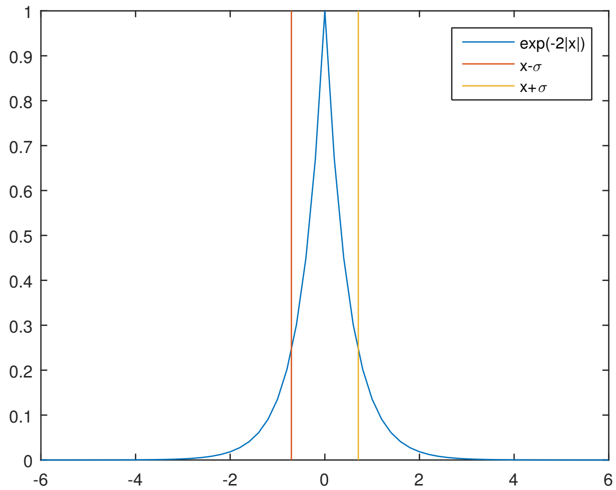
$$\langle x \rangle = \int_{-\infty}^{\infty} \lambda * e^{-\lambda*|x|}e^{-i\omega t} * x * e^{-\lambda*|x|}e^{i\omega t}dx = \lambda \int_{-\infty}^{\infty} e^{-2\lambda|x|}x\, dx = 0 \tag{2.0.2}$$

$$\langle x^2 \rangle = \int_{-\infty}^{\infty} \lambda * e^{-\lambda*|x|}e^{-\mathrm{i}\omega \mathrm{t}} * x^2 * e^{-\lambda*|x|}e^{\mathrm{i}\omega \mathrm{t}}\, dx = 2\lambda \int_{0}^{\infty} e^{-2\lambda \mathrm{x}}x^2\, dx = 2\lambda * \frac{1}{4}\frac{\partial^2}{\partial\lambda^2}\frac{1}{2\lambda} = \frac{1}{2\lambda^2} \tag{2.0.3}$$

$\sigma \quad =$

$$\sqrt{\langle x^2 \rangle - \langle x \rangle^2} = \frac{\sqrt{2}}{2}\frac{1}{\lambda} \tag{2.0.4}$$

Plot when $\lambda = 1$, $|\Psi|^2 = \exp(-2|x|)$:



5. the probability that the particle would be found outside this range $\langle \langle x \rangle - \sigma, \langle x \rangle + \sigma \rangle$:

$$\int_{\langle x \rangle - \sigma}^{\langle x \rangle + \sigma} |\Psi(x)|^2 \, \mathrm{d}x = \int_{-\sigma}^{\sigma} e^{-2\lambda|x|}\lambda \, \mathrm{d}x = 2\lambda \int_0^{\sigma} e^{-2\lambda x} \, \mathrm{d}x$$
$$= 2\lambda \frac{1}{2\lambda}(1 - e^{-2\lambda\sigma}) = 1 - e^{-2\lambda\sigma} \tag{2.0.5}$$

So the probability that the particle fall out of one sigma away from the center is: $e^{-2\lambda\sigma}$.

3 Problem 3

Derive the Compton shift formula: $\Delta\lambda = \frac{h}{mc}(1 - \cos\phi)$:

Proof. Let e for electron, h for photon. Primed for Conservation of energy:

$$hv + m_e c^2 = hv' + \sqrt{(p_e' c)^2 + m_e^2 c^4} \tag{3.0.6}$$

Conservation of momentum:

$$p_h = p_e' + p_h' \tag{3.0.7}$$

so:

$$\vec{p}_e'^2 = (\vec{p}_h - \vec{p}_h')^2 = p_h^2 + p_h'^2 - 2p_h p_h' \cos\theta \tag{3.0.8}$$

According to Einstein's relationship:

$$p_h = \frac{h}{\lambda},\, p_h' = \frac{h}{\lambda'} \tag{3.0.9}$$

Then 3.0.9 into 3.0.8:

$$p_e'^2 = h^2 \left(\frac{1}{\lambda^2} + \frac{1}{\lambda'^2} - \frac{2}{\lambda\lambda'} \cos\theta \right) \tag{3.0.10}$$

3.0.10 into 3.0.6 $\Rightarrow$

$$h(v - v') = \sqrt{h^2 \left(\frac{1}{\lambda^2} + \frac{1}{\lambda'^2} - \frac{2\cos\theta}{\lambda\lambda'} \right) c^2 + E_0^2} - E_0^2 \tag{3.0.11}$$

where we write $E_0 \equiv m_e c^2$. Squaring both sides (notice that $\lambda = \frac{c}{v}$):

$$h^2(v - v')^2 + E_0^2 + 2h(v - v')E_0 = h^2 \left(v^2 + v'^2 - 2\cos\theta v v' \right) + E_0^2 \tag{3.0.12}$$

After simplification:

$$(v - v')E_0 = h(1 - \cos\theta) v v' \tag{3.0.13}$$

Back to $\lambda = \frac{c}{v}$:

$$\frac{\frac{1}{\lambda} - \frac{1}{\lambda'}}{\frac{1}{\lambda\lambda'} c} = \frac{h}{E_0}(1 - \cos\theta) \tag{3.0.14}$$

Or:

$$\lambda' - \lambda = \Delta\lambda = \frac{h}{m_e c}(1 - \cos\theta) \tag{3.0.15}$$

$\square$