Draft

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Abstract

This is a draft.

Contents

1 Lectures on the Frontiers of Physics

Given by professor of physics in SUSTC

1.1 By JQ. He.

Thermal electrics

1.2 By Lang. Chen

Grow thin films.

- Rheed-Assited PLD/MBE. (Ray as an exmination).
- orbital contral of electrons -> orbitronics -> Control of Spin orbital coupling.
- Multiferroics -> multiple order parameters, and the interaction between them. E.g. $BiFeO_3$.
- Ferrotorodicity: Spontaneous Toroidal Momennt. Time and spacial symmetries simultaneous broken.
- What is a iridates $Ir_2(X)O_4$, (e.g. Sr_2RuO_4) exactly in theoretical physics?
- H_2S: 200K superconductor?
- The Double Exchange effect of oxygem -> Half-metal, phase transition.

1.3 By Alan

Photocatalysis: TiO_2. Hongkong has TiO_2 spurred on the keys.

1.4 By Li, Huang

- Computational Physics
- Surface Dynamics
- Structural factor from 2D to 3D.
- Finding Order Amid the Chaos. amorphous -> spatially resolved distributed function.
- ?: What is genetically algorithm.

Computational and theoretical studies of Surface dynamics

- Surface atoms is immersed in a very different environment compared with the bulk atoms.
- First-principle calculations
- DFT + LDA -> Conser equation
- Plane wave basis + Ultrasoft pseudopotentials to solve the Conser equation
- Continumm method ?

1.5 By Junfen, Liu

- electronic transport in mesoscopic systems:
- Spintronics
- Graphene eletronics
- Superconductors etc.

Quantum wire conductiing The conduction channel in quantum wire is quantized, with discrete value of conductance.

- λ_F Fermi wavelength
- L_m Momuntum relaxation length $<-$ impurities.
- L_ϕ Phase relaxation length $<-$ memory of phase, related to energy $\omega = E/\hbar$.
- L Sample length
- Ballistic transport: $L << L_m$ No scattering.
- Diffusive $L > L_m$, scattering, reduced transmission.
- Localization $L_m << L << L_\phi$ -> Prof. Haizhou Lu.
- Classical. (Omitted)

Conductance No back-scattering

$$G = \frac{I}{V} = \frac{2e^2}{h}$$

Landauer formula $G = \frac{2e^2}{h} \cdot T$, T is some coefficient accounting for the back scattering, perhaps the transmission probability. In reality, $G = \sum_{\text{Different channels}} G_i$,

We can turn the G into resisivity:

$$\text{Resistance} = \frac{h}{2e^2} + \frac{h}{2 * e^2}\frac{R}{T}$$

$R + T = 1$

Resonate Transmission (Omitted)

Spintronics Use the extra freedom of Spin.

Spin field eletroncis: Datta and Das, Appl. Phys. Lett. 56, 665(1990)

GMR: 2007 Nobel prize in Physics.

Hall Effect (Omitted) Spin Hall Effect: S. murakami, et.al. Science 301 1348(2004);

J. Sinova et.al. Phys.Rev.Lett. 92, 126603 (2004). (Omitted)

Graphene Carrier -> Relativistic Dirac fermions. **Klein Paradox**

Josephson Junction A phase difference could conduct electricity in Superconductors.

1.6 By Haizhou, Lu

Quantum Anomalous Hall Effect Requires strong magnetic field: ≈ 10 Tesla.

Anomalous Hall Effect: Without magnetic field. $R_H = R_0 B + R_A M$ where M is the magnetic susceptibility. Two-factors: SO coupling. Spin-dependent Hall Effect.

An excellent illustrations is found in [4]:

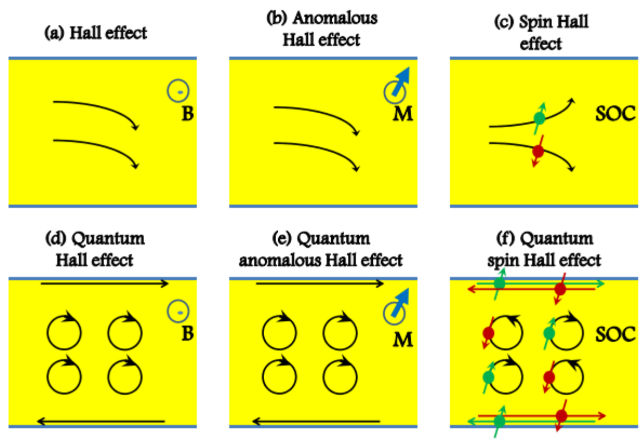

FIG. 1. (Color online) Six members in the family of Hall effect. (a) Hall effect; (b) Anomalous Hall effect; (c) Spin Hall effect; (d) Quantum Hall effect; (e) Quantum anomalous Hall effect; and (f) Quantum spin Hall effect.

Figure 1: Illustration

1.7 By Kedong Wang

Tunneling current $I \propto V e^{-2kz}$, where $k = \frac{\sqrt{2m\phi}}{\hbar}$, ϕ is the Work function. I is very sensitive to the distance z.

Work function ϕ characterize the obstruction that prevents electron from escaping the sample.

1.8 By Mingyuan Hunag

Too boring.

1.9 By Wenkang Wong

Non-clone Theorem We can easily see that there is no universal copy operators in Quantum Mechanics.

Proof. We proove it by contradiction. Let U be the copy operator. By definition, we have

$$U|\psi\rangle|0\rangle = |\psi\rangle|\psi\rangle$$

For any $|\psi\rangle$. Then, let try copying the state $|\psi\rangle = |0\rangle + |1\rangle$. We have

$$\begin{aligned}(|0\rangle + |1\rangle)(|0\rangle + |1\rangle) &= U(|0\rangle + |1\rangle)|0\rangle \\ &= U(|0\rangle|0\rangle + |1\rangle|0\rangle) \\ &= |0\rangle|0\rangle + |1\rangle|1\rangle\end{aligned}$$

This is a contradiction. □

Remark 1.1. *We assume that the copyer is universal. This might seems to be too strong. However, if we assume that the copyer only works for certain states $|\phi\rangle$, then with the knowledge of these certain states, we could in principle create an exact copy of these states. This copy ,in the sense of another instance of the same object, of original state should not be considered to be an copied version of the original state.*

2 Problems in Bernevig's Topological ...

2.1 Chapter 2

Non-Abelian Berry Transport Derive Berry curvature to the adiabatic transport of a degenerate multiplet of states separated by a gap from the excited states. (Cautious about rotation within degenerate states).

Answer: $\gamma_{mn}(t) = i\int_0^t \langle m(R(t'))|\frac{d}{dt'}|n(R(t'))\rangle\, dt'$

Approach: Assuming that the those degenerate states are labeled by $1\cdots N$. Thus we have naturally:

$$H\phi = i\hbar\frac{\partial}{\partial t}\phi,\ \phi = \sum_n A_n\psi_n \tag{2.1.1}$$

Then we have:

$$\begin{aligned}H\sum_n A_n\psi_n &= i\hbar\frac{\partial}{\partial t}\sum_n A_n(t)\psi_n(R(t)) \\ \sum_n EA_n\psi_n &= i\hbar\sum_n\left(\frac{\partial A_n(t)}{\partial t}\psi_n(R(t)) + A_n(t)\frac{\partial\psi_n(R(t))}{\partial t}\right) \\ EA_m &= i\hbar\frac{\partial A_m(t)}{\partial t} + \sum_n A_n(t)\langle m|\frac{\partial}{\partial t}|n\rangle \qquad (2.1.2)\end{aligned}$$

Put in another form:

$$\sum_n\left(\delta_m^n E - \langle m|\frac{\partial}{\partial t}|n\rangle\right)A_n = i\hbar\frac{\partial A_m(t)}{\partial t}$$

In matrix form:

$$(E-P)\mathbf{A} = i\hbar\dot{\mathbf{A}} \tag{2.1.3}$$

where:

$$E = \begin{pmatrix} \cdots & & \\ & E & \\ & & ... \end{pmatrix} \tag{2.1.4}$$

$$P = (P_n^m) = \left(\langle m | \frac{\partial}{\partial t} | n \rangle \right) \tag{2.1.5}$$

$$A = \begin{pmatrix} A_1(t) \\ A_2(t) \\ \cdots \end{pmatrix} \tag{2.1.6}$$

Note that $\langle n|\frac{\partial}{\partial t}|m\rangle^* \neq \langle m|\frac{\partial}{\partial t}|n\rangle$, thus P may not be Hermitian.Ergo $E - P$ is Hermitian. So it is diagonalizable.

Notice that

$$0 = \frac{\partial}{\partial t}\langle m|n\rangle = \langle \frac{\partial}{\partial t} m|n\rangle + \langle m|\frac{\partial}{\partial t} n\rangle \qquad \text{(any } m, n\text{)} \tag{2.1.7}$$

temporary mathematica code:

3 Miscellnaneous Notes

3.1 Super conductor

Mean-field approach to deal with a four operator diagonalization.

Suppose we have: D^*C^*CD, then let $\delta = CD - \langle CD \rangle = CD - avg$. Then if we assume $\langle CD \rangle \neq 0$, and $\delta \approx 0$. Then we have:

$$\delta^2 \approx 0$$

i.e.:

$$((CD)^* - avg)(CD - avg) = 0 \tag{3.1.1}$$

$$D^*C^*CD = avg * (CD + D^*C^*) - avg^2 \tag{3.1.2}$$

Hence a four operator is reduced into a few of two operators. Such method could be naturally extended to treat the operator $\sum_{i,j} D_i^* C_i^* C_j D_j$.

A copper pair has the energy of:

$$\Delta = \langle C_{k\uparrow} C_{-k\downarrow} \rangle$$

To resist the flow of current carried by Copper Pair, is equivlant to destroying a pair of Copper Pair:

$$\langle C_{k\uparrow} C_{-k\downarrow} \rangle \longrightarrow C_{k\uparrow} C_{-k\downarrow}$$

This will require an additional enegy of 2Δ.

The exact meaning of "equivalent to" is as follows:

$$\begin{aligned} &\text{break a copper pair} \longrightarrow \text{scatter two electrons consecutively} \\ &\longrightarrow \text{create two electron-hole mixed type quasi-particle} \longrightarrow 2\Delta \end{aligned}$$

3.2 Why 0/0 is undefined?

If we suppose

$$\frac{0}{0} = \triangle$$

Consider the following derivation:

$$\frac{0}{0} \cdot 1 = \triangle \cdot 1 = \triangle \tag{3.2.1}$$

$$0 \cdot \frac{1}{0} = \triangle \tag{3.2.2}$$

$$\Rightarrow \triangle = 0 \tag{3.2.3}$$

This is already bad enough. And we are forced to define $\frac{1}{0}$. Let $\frac{1}{0} = \square$, which literally means $1 = 0 \cdot \square = 0$. This is disastrous.

Alternatively, we could let

1. Let $\frac{1}{0}$ be undefined.
2. Or let $\frac{1}{0} = \infty$.
3. Or, let $\frac{a}{b} \cdot c = a \cdot \frac{c}{b}$ be not true when $b = 0$.

The third idea is disastrous for algebraic manipulation. [1] The first idea is not good. Since defining $\frac{1}{0} = \infty$ turns out to be very useful in both mathematics and physics. Actully, in physics it is common practice to set $\frac{a}{0} = \pm\infty$ for any nonzero number a, where the sign of ∞ is determined by the sign of a. The second idea is okey. But then we are faced with a serious problem. We have to define $\triangle \equiv 0 \cdot \infty$

$\triangle \cdot 2 = \triangle$, What will be of $\triangle + 1$?

3.3 Preface of BSCS

BSCS: see [2]. Parallism between theories in condensed matter physics and those in particle physics.

- Anderson-Higgs Phenomenon (Paritcle), Meissner effect (C.M.P.)
- 'inflation' in Cosmology, first order phase transition
- 'cosmic strings', magnetic field vortex lines in type II superconductors
- Hadron-meson interaction, Ginzburg-Landau theory of superfluid He^3.

Same ideas on different space-time scales, different hierachical 'layers'. Strong parallism: **strongly correlated low dimensional system**

E.g.:

The problem of formation and structure of heavy particles - hadrons and mesons. The corresponding fine structure constant $\alpha_G \approx 1$.

Approaches:

1. Exact solutions

[1] Or more speicifically, it is a disaster for field theory.

2. Reformulate complicated interacting models in such a way that they become weekly interacting. -> Bosonization.

   Spin 1/2 anisotropic Hisenberg chain $\approx$ Model of interacting fermions. (Jordan and Wigner, 1928)

Bosonization: transformation from fermions to a scalar massless bosonic field.

3.4 System of Differential Equations

This is a small note of [3].

pp. 266.

Definition 3.1. $\mathbf{x(t)}$ *is a vector whose elements are* $x_i(t)$. $\frac{d}{dt}$ *acts on vector* $\mathbf{x}$ *element-wise.* $\dot{\mathbf{x}}$ *is abbrevation for* $\frac{d}{dt}\mathbf{x}$

pp. 291.

Theorem 3.1 (Existence-uniqueness theorem)**.** *There exists one, and only one, solution of the initial-value problem*

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x},\ \mathbf{x}(t_0) = \mathbf{x}^0 = \begin{pmatrix} x_1^0 \\ x_2^0 \\ \cdots \end{pmatrix} \tag{3.4.1}$$

Moreover, this solution exists for $-\infty \langle t \langle \infty$.

Remark 3.1. *By this, any non-trivial solution* $\mathbf{x}(t) \neq 0$ *at any time* t. *Also notice that the elements of* $\mathbf{A}$ *are just numbers.*

Theorem 3.2. *The dimension of the space* $\mathbf{V}$ *of all solutions of the homogeneous linear system of differential equations:*

$$\frac{d\mathbf{x}}{dt} = \mathbf{A}\mathbf{x} \tag{3.4.2}$$

is n, *i.e. the dimension of vector* $\mathbf{x}$.

3.5 ODE by Arnold

sec. 14

Definition 3.2.

$$e^A = I + A + \frac{A^2}{2!} + \frac{A^3}{3!} \tag{3.5.1}$$

or

$$e^A = \lim_{n\to\infty} (I + \frac{A}{n})^n \tag{3.5.2}$$

where I *is the identity matrix.*

Equivalance of the two definition will be addressed in the Theorem on pp. 165.

Important theorems:

Theorem 3.3 (pp. 158)**.** *The series e^A converges for any A uniformly on each set $X = \{A : ||A|| \leq a\}$, $a \in \mathbb{R}$.*

Theorem 3.4 (pp. 160)**.**

$$e^{At} = H^t$$

where H^t is the translation operator which sends every polynomial $p(x)$ into $p(x+t)$.

Theorem 3.5 (pp. 163)**.**

$$\frac{d}{dt} e^{tA} = A e^{tA}$$

Theorem 3.6 (Fundamental Theorem of the Theory of Linear Equations with Constant Coefficients)**.** *The solution of:*

$$\dot{\mathbf{x}} = A\mathbf{x} \tag{3.5.3}$$

with initial condition $\phi(0) = \mathbf{x}_0$ is

$$\phi(t) = e^{tA}\mathbf{x}_0 \tag{3.5.4}$$

Practically solution to

$$\dot{\mathbf{x}} = A\mathbf{x}$$

(pp. 173, Sec 17) (Assuming A is diagonalizable.)

- Find the eigenvectors $\xi_1, \cdots, \xi_n$ and eigenvalues $\lambda_1, \cdots, \lambda_n$. Use them as basis.
- Expand the initial condition in the new basis.

$$\mathbf{x}_0 = \sum_{k=1}^{n} C_k \xi_k \tag{3.5.5}$$

- Then $\phi(t) = \sum_{k=1}^{n} C_k e^{\lambda_k t} \xi_k$

3.6 Appearance of Gauge Structure in Simple Dynamical Systems

$$0 = (\eta_b, \dot{\eta_a}) = (\eta_b, \dot{U}_{ac}\psi) + (\eta_b, U_{ac}\dot{\psi}_c \tag{3.6.1}$$

3.7 Quantum Statistical Mechanics

Definition 3.3 (Time Evolution Operator)**.** *The time evolution operator $U(t, t_0)$ is defined such that*

$$|\Psi(t)\rangle = U(t, t_0) |\Psi(t_0)\rangle \tag{3.7.1}$$

It satisfy the relationship:

$$i\hbar\partial_t U(t, t_0) = \mathcal{H} U(t, t_0) \tag{3.7.2}$$

This is obvious when substituting $U(t, t_0)$ into the Schrodinger Equations.

Quantum Macrostates Macrostates of the system depend on only a few the thermodynamic functions. We can form an ensemble of a large number $\mathcal{N}$ of microstates $\{\psi_\alpha\}$, corresponding too a given macrostates. The different microstates occur with probability p_α. When wen no longer have exact knowledge of the microstate of a system the system is said to be in a *mixed state*. The ensemble average of the quantum mechanical expectation value is given by:

$$\begin{aligned}\langle \bar{O} \rangle &= \sum_\alpha p_\alpha \langle \psi_\alpha | O | \psi_\alpha \rangle = \sum_{\alpha,m,n} p_\alpha \langle \psi_\alpha | m \rangle \langle m | O | n \rangle \langle n | \psi_\alpha \rangle \\ &= \sum_{m,n} \langle n | \rho | m \rangle \langle m | O | n \rangle = \mathrm{tr}(\rho O) \end{aligned} \tag{3.7.3}$$

where we have introduced the density matrix:

Definition 3.4 (Density Matrix)**.** *The density matrix $\rho(t)$ is defined as*

$$\langle n | \rho(t) | m \rangle \equiv \sum_\alpha p_\alpha \langle n | \psi_\alpha \rangle \langle \psi_\alpha | m \rangle \tag{3.7.4}$$

or

$$\rho(t) \equiv \sum_\alpha p_\alpha | \psi_\alpha \rangle \langle \psi_\alpha | \tag{3.7.5}$$

Density matrix is denoted by $\rho(t)$ by analogy of the notation for P.D.F, since ρ often represents density.

Density matrix satisfies several good properties:

- Normalized
- Hermiticity
- Positivity. For any Φ, $\langle \Phi | \rho | \Phi \rangle \geq 0$.

4 Anchor

References

[1] Sakurai, J. J. Modern Quantum Mechanics, Addison Wesley.

[2] Bosonization and Strongly Correlated Systems. Cambridge. Cambridge Press Link

[3] Martin Braun. Differential Equations and Their Applications. 4ed. Springer.

[4] `http://arxiv.org/abs/1508.07106v1`

[5] M. Kardar. Statistical Physics of Particles (2007)

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