Homework for Advanced Quantum Mechanics 10

Yuechi Ma

December 14, 2016

1. Deadline : December 24th.
2. Please send your homework to my email: 910493179@qq.com.
3. Contact me or discuss in QQ group if you have any question.

1. Consider a system of 2 identical particles. Define the single particle **states** as ϕ_1, ϕ_2. Write down the *number* of possible states if the particles are classical particles, bosons or fermions.(Hint. Identical fermions can not be on the same state).

2. Given density matrix of two dimension

$$\rho_A = \frac{1}{2}(1 + \boldsymbol{n}_A \cdot \boldsymbol{\sigma}), \rho_B = \frac{1}{2}(1 + \boldsymbol{n}_B \cdot \boldsymbol{\sigma})$$

   Prove

$$Tr(\rho_A \rho_B) = \frac{1}{2}(1 + \boldsymbol{n}_A \cdot \boldsymbol{n}_B)$$

3. Let $|a'\rangle$ and $|a''\rangle$ be eigenstates of a Hermitian operator A with eigenvalues a' and a'', respectively $(a' \neq a'')$. The Hamiltonian operator is given by

$$H = |a'\rangle \, \delta \, \langle a''| + |a''\rangle \, \delta \, \langle a'|$$

   where δ is just a real number.

   - Clearly, $|a'\rangle$ and $|a''\rangle$ are not eigenstates of the Hamiltonian. Write down the eigenstates of the Hamiltonian. What are their energy eigenvalues?
   - Suppose the system is known to be in state $|a'\rangle$ at $t = 0$. Write down the state vector in the Schrodinger picture for $t > 0$.
   - What is the probability for finding the system in $|a''\rangle$ for $t > 0$ if the system is known to be in state $|a'\rangle$ at $t = 0$?