Shen's View on Topological Quantum Numbers

Taper

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Abstract

A short summary of chapter 4, Topological Invariants, of [She12].

Contents

The local gauge freedom of phase gives us Berry phase, and the Hall conductance σ_{xy} can be related to this Berry phase as:

Which bands should be included in calculating the Berry phase.

$$\sigma_{xy} = \int_{\mathrm{BZ}} \frac{\mathrm{d}\mathbf{k}}{(2\pi)^2} \Omega_{k_x,k_y} \tag{0.0.1}$$

where Ω_{k_x,k_y} is the Berry curvature. The Berry curvature for each band is defined as

$$\Omega^n(\mathbf{k}) = \boldsymbol{\nabla}_{\mathbf{k}} \times \langle u_n(\mathbf{k}) | i \boldsymbol{\nabla}_{\mathbf{k}} | u_n(\mathbf{k}) \rangle \tag{0.0.2}$$

This Berry phase can also be related to the electric polarization (more exactly, the $\mathbf{P}$ in $D = \epsilon_0 \mathbf{E} + \mathbf{P}$), in a solid in which there is no electric field ($\mathbf{E} = 0$). The connection is:

$$\Delta P_\alpha = e \sum_n \int_0^T \mathrm{d}t \int_{\mathrm{BZ}} \frac{\mathrm{d}\mathbf{q}}{(2\pi)^d} \Omega^n_{q_\alpha,t} \tag{0.0.3}$$

(where d should be the dimension of the solid, though this is not mentioned in the book.)

The quantization of Hall conductance can be viewed from two perspectives. On one hand, the Berry phase is proportional to the first Chern number, which is proved mathematically to be quantized. On the other hand, there is the Laughlin's argument. The Laughlin's argument does two things. First, it relates the change of flux $\Delta\phi$ to the change of charge ΔQ by simple electrodynamics:

$$\Delta Q = \sigma_{xy} \Delta\phi \tag{0.0.4}$$

Second, it argues that the threading of one flux quantum $\hbar/e$ does not change the Hamiltonian, but instead pushes integers numbers of charge transport (just like a Thouless charge pump):

Why do we thread this flux of quantum?

$$\Delta Q = ne \tag{0.0.5}$$

Therefore, $ne = \sigma_{xy}\hbar/e$ leads to $\delta_{xy} = ne^2/\hbar$.

However the $\mathbb{Z}_2$ invariant is a complicated stuff which I do not understand.

Also, the generalization of topological insulator from two dimension to three dimension is done in section 4.9, which I do not understand. But he defines clearly the concept of strong and weak topological insulator, and mentions that a weak topological insulator is topologically equivalent to a two-dimensional topological insulator, and is not robust against disorder.

References

[She12] Shun-Qing Shen. *Topological Insulators*, volume 174 of *Springer Series in Solid-State Sciences*. Springer Berlin Heidelberg, Berlin, Heidelberg, 2012. URL: `http://link.springer.com/10.1007/978-3-642-32858-9`, `doi:10.1007/978-3-642-32858-9`.

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