# Solution for HW10

Taper

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**Abstract**

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## Problem 1

| | |
|---|---|
| Classcal | $2^2 = 4$ |
| Boson | 3 |
| Fermion | 1 |

## Problem 2

Notice two facts:

$$(\vec{\sigma} \cdot \vec{a})(\vec{\sigma} \cdot \vec{b}) = \vec{a} \cdot \vec{b} + i\vec{\sigma} \cdot (\vec{a} \times \vec{b}) \tag{0.0.1}$$

which is Eq.(3.2.39) on Sakurai's Modern Quantum Mechanics. Also

$$\mathrm{Tr}(\vec{a} \cdot \vec{\sigma}) = 0 \tag{0.0.2}$$

This is because the trace function is "almost" linear and all pauli matrices have zero trace.

Then:

$$\begin{aligned}
\mathrm{Tr}(\rho_A \rho_B) &= \frac{1}{4} \mathrm{Tr}(1 + (\vec{n}_A + \vec{n}_B) \cdot \vec{\sigma} + (\vec{n}_A \cdot \vec{\sigma})(\vec{n}_B \cdot \vec{\sigma})) \\
&= \frac{1}{4} \{\mathrm{Tr}(1 + \vec{n}_A \cdot \vec{n}_B + i\vec{\sigma} \cdot (\vec{n}_A \times \vec{n}_B))\} \\
&= \frac{1}{4} \mathrm{Tr}(1 + \vec{n}_A \cdot \vec{n}_B) \\
&= \frac{1}{2}(1 + \vec{n}_A \cdot \vec{n}_B)
\end{aligned}$$

## Problem 3

Denote $|a'\rangle$ as $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$, and $|a''\rangle$ as $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$. Then

$$H = \begin{pmatrix} 0 & \delta \\ \delta & 0 \end{pmatrix} \tag{0.0.3}$$

The eigenvectors and eigenvalues can be easily guessed:

$$|+\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \lambda_+ = \delta \tag{0.0.4}$$

$$|-\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix}, \lambda_- = -\delta \tag{0.0.5}$$

For time evolution:

Since $|a'\rangle = \frac{\sqrt{2}}{2}(|+\rangle + |-\rangle)$, under time evolution it will be

$$|a'(t)\rangle = \frac{\sqrt{2}}{2}\left(e^{-i\delta t/\hbar}|+\rangle + e^{i\delta t/\hbar}|-\rangle\right) = \frac{1}{2}\begin{pmatrix} e^{it\delta/\hbar} + e^{-it\delta/\hbar} \\ e^{-it\delta/\hbar} - e^{it\delta/\hbar} \end{pmatrix} = \begin{pmatrix} \cos(t\delta/\hbar) \\ -i\sin(t\delta/\hbar) \end{pmatrix} \tag{0.0.6}$$

For probability:

The probability is clearly: $\sin^2(t\delta/\hbar)$.

**Remark 0.1.** This is actually the famous Rabi frequency, see more at: `https://en.wikipedia.org/wiki/Rabi_cycle`.