Notes of Connecting Few-body and Many-body Pictures of Fractional Quantum Hall Physics

Taper

October 30, 2016

Abstract

I wrote this with the aim of finding some interesting topics for research. This conference has its lecture recorded and published in YouTube (link [1]). Note that, although I have the video on hand, the lecturers are speaking somehow too fast and it would be too time-consuming to replay these lectures. So the content is..... quite un-organised and bare. In addition, after typing for several hours, my fingers are getting tired. So... I could be very lazy.

Therefore, this note could be viewed just as a collection of keywords.

Contents

1 Using Optical Emission to Study Competing Phase in the Second Landau Level

By:

Antonio Levy, Aron Pinczuk, Yuliya Kuznetsova (Columbia University), Ursula Wurstbauer (Technische Uni. Munchen), Ken. W. West, Loren N. Pfeiffer (Princeton), Michael J. Manfra, Geoff C. Gardner, John D. Watson (Purdue University).

Overview Second Landau Level displays

1. FQHS
2. Ordered phases (partially). RIQHE $\overset{from}{\Leftarrow}$ Electron stalids.
3. Competition & Coexistance

   Anisotropic FQHS arise from coorelation when anisotropic $\overset{from}{\Leftarrow}$ magneti field

Advanage of Optial

1. Direct probes bulk
2. Distinguish between charge and spin modes

Sample Omitted.

RRS

light $\longrightarrow \bullet \overset{\text{quasiparticle}}{\underset{\text{quasihole}}{\rightleftarrows}} \bullet$ scattered photon

• Easy to probe Single Partical excitation and collective excitation.
... (Skipped)

2 Hyperspherical Adiabatic Approximation

By: Rachel Wooten (Purdue University)

Outline

1. QHS
2. Motivation
3. Hyperspherical adiabatic approximation (**HAA**)
4. The role of degeneracy
5. Hyperradial breathing mode
6. Results and Discussion

Table 1: Two Schemes

Conventional	$\approx$	Neutral Atom gass
2D Landau levels		2D Rotation(Ω)
ω_c		$\omega = 2\Omega$

QHS (Other schemes are also available, not treated).

Motivation

- QHS: prototype of *Strongly correlated systems*
- Nature of few-body states
- insights from collectively coordinates
- use ... ?

Few body, adiabatic hypershperical approach(AHA)

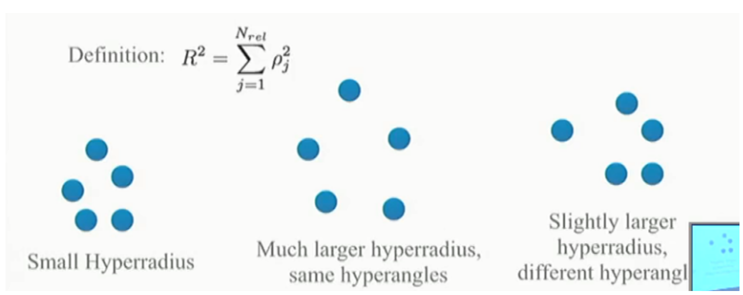
- intrinsic collective coordinates
- length scale $R \overset{\text{from}}{\Leftarrow}$ geometry
- E potential $\Leftarrow$ length scale
- introduce Grand Angular momentum K - quantum number

AHA background Joe Macek, 1968 JPB.

Scheme Sch-Ep $\overset{recast}{\Rightarrow}$ relative coordinates $\overset{diagonalize}{\Rightarrow}$ solve.

Hyperspherical Coordinates

$2N-2$ Jacobi coordinates $\Rightarrow$ $2N-3$ angular coordinates Ω + 1 length coordinates Hyperradial R.



Hyperradius

$$\rho = \left(\frac{\pi\mu \langle R^2 \rangle}{N(N-1)} \right)$$

$$\mathscr{H}_{\text{rel}} = (R, \Omega, \cdots) + \text{Colomb} V(R) = \frac{1}{R} + \text{Polarized dipoles} V(R) = \frac{1}{R^3}$$

AHA (Non-interacting case) Omitted.

AHA (Interacting case)

Treat R as an adiabatic parameter.

Interaction $\overset{\text{introduct}}{\Rightarrow}$ degeneracies

Accuracy check Omitted.

Exceptional degeneracy

3 ? (Wick Haxton)

(1995) Algebraic classification of general 4 fermion problem (Joe Ginocchio) $\Rightarrow$ has FQHE implications

Jain & Laughlin work (numerically succesful)

(No audio available for this lecture recording!, Skipped)

4 Perspectives on the half-fill Landau Level

By B.I. Halperin, N. Copper, Chong Wong, Adey Stern.

Question : What ahppens to half-filled Landau Level when there is **NO** magnetic field.

Motivation

- New research on half-filled Landau level
- Describe system by "Dirac Fermion" is simpler, generated different theories $\Rightarrow$ doubt whether they are equivalent.
- Particle-hole symmetry in explicit in "Dirac Fermion", not explicit in HLR framwork.

Physical Setup

Half-filled Landau level: in a semiconductor.

$$H_0 = \sum_i \frac{|P_i - A(r_i)|^2}{2m} + V_2(\text{interactions})$$

Units: $e = \hbar = 1$.

PH Symmetry

- $m \to 0$, e-e interactions are fixed -¿ exact PH symmetry
- For $v = 1/2$, PHS requires that $\sigma_{xy}(\sigma) = (1/2)e^2/h$

HLR Approach (Halperin, Lee, Read. PRB 1993)

- Singular gauge transformation $\Rightarrow$ Chern-Simons Gauge Field
- Transformed Hamiltonian will have fermion terms, interactions, gauge field (chern-simons interaction in it). Also the curl of a fact vector is constrained.

Antecednets

- Fractional Quantized Hall states by Jain; Lopez and Fradkin; others.
- Simultaneous work by Kalmeyer and Zhang had some features of HLR.

HLR Hypothesis

- Do mean field theory on this problem: the ground state and low energy properties of quantum hall system at $v = 1/2$. And use perturbation theory and take into account of fluctuations in gauge field and Coulomb interactions
- ...

HLR consequences

- with $1/r$ interactions, the ground state is a "marginal fermi liquid". With mass diverges as $\log(x)$ at fermi serface, due to infrared divergence.
- assume interactions that fall off slower than $1/r$.

HLR-RPA predications

- ground state at $v = 1/2$ should be compressible
- energy gaps, FQH states, occur at Jain fractions $v = p/(2p + 2)$.
- relative sizes of energy gaps close to $1/2$
- transport in absence of impurities: DC hall conductance
- Longitudianl conductivity at finite $q, \omega \to 0$:

$$\sigma_{xx}(q) = q/(8\pi k_f)$$

  in the presence impurities: formula holds for $q > l_{cf}$.

HLR-RPA problems

- most important error: ignores renormalization of fermian effective mass. gives wrong energy scale, wrong value for specific heat at $1/2$, wrong scale for FQHE gaps.
- correct effective mass is set by the e-e interaction, not by the bare mass, when m is small.

Naive renormalizaed RPA

- simply replace m by m^*.
- however: iolates Kohn's theorem for $\omega =\neq 0$.
- and many many....

Landau interaction parameters

- energy cost of a ditortion of fermi surface, independent of position.
- (several formulae)

DC conductitivity in presence of impurties

- Kivelson (1997): naive HLR leads to deviations
- ... continued of problems of HLR-RPA

Side-jump contribution ......

Commensurability oscillations: $v \neq 1/2$, with finite q and this result is symmetric w.r.t ΔB, different from HLR.

5 Composite fermi liquids in the lowest Landau level

By: T. Senthil (MIT), Chong Wang (MIT -¿ Harvord)

2d electrons in the QH regime [1].

Unquantized region

- Incompressible FQHE states: fill certain rational fractions of a Landau level
- Large degeneracy is split by e-e interactions to give a gapped ground state
- compressible metallic states: "unquantized quantum Hall effect"
- **How do interactions manage to produce a metal**

Standard theory HLR theory, idea of composite fermions

Some experimental verification of composite fermions

Unsatisfactory aspects of the theory

- HLR not suited to projecting to Lowest Landau Level (LLL)
- mean field effective mass = bare electron mass
- LLL limit: take m to zero: what happens?
- LLL theory has an extra symmetry that HLR is blind to. the PH symmetry.

PH symmetry: formal implementation ways to exam the particle-hole symmetry.

Numerical work: shows metallic ground state $v = 1/2$ preserves the C symmetry.

HLR theory: not in LLL, therefore it does not address the problem of p/h within its scope.

[1] This is the first time that I find a correct usage of the word "regime", without suspicion that it should be replaced by "region".

Related theoretical problems

- electrons at $v = 1/4, 1/6, \cdots$ No p/h. But issue of nature of a LLL theory remains
- useful to consider problem of bosons in the quantum Hall regime. Fate of such a state in LLL. For bosons, microscopically there is no p/h.

Progress old and new

- Bosons at $v = 1$: LLL theory of metallic Composite Fermi Liquid (**CFL**) state (Read 1998)
- PH symmetric theory for electronc CFL at $v = 1/2$. by Son (2016): Is the composite fermion at $v = 1/2$ a dirac particle?
  - field theoretic justification: connection to the surface of 3d topological insulators (C. Wang. TS ...)
  - Simple physical picture of the PH symmetric composite fermion (C. Wang. ...)
  - Numerical calculations (...)

Talk outline

- Understanding the p/h symmetric composite fermi liquid of electrons at $v = 1/2$ (physical picture, field theoretic derivation)
- Composite Fermi Liquid of bosons at $v = 1$:
  - review of Read's Lowest Landau Level theory
  - comparison with electrons at $v = 1/2$.
- Composite Fermi Liquids in LLL at generic v:

Old physical picture Composite Fermion in LLL LLL wave function: (Rezayi-Read 94)

$$\psi = P_{LLL}\det(e^{ik_i r_j})\prod_{i<j}(z_i - z_j)^2$$

Composite fermion = electron bound to 4π-vortex which has charge depletion $-e$. They are neutral but carry a dipole moment.

Howeer this picture misses some physics and further is not PH symmetric.

New picture of composite fermion in LLL by Wang. TS.

$$\psi = \prod_{i<j}(z_i - z_j) f(z_1, \cdots, z_N)$$

where f is symmetric.

One vortex is exactly on electron due to Pauli. Each vortex has charge $-e/2$ =¿ single vortex exactly on electron has charge $+e/2$ and the displaced vortex has chare $-e/2$.

Internal structure of composite fermion in LLL Two ends have mutual statistics of π. Solve Quantum Mechanics of just relative motion. $\cdots$

1/2-filled LL and correlated 3 dimentional Topological Insulator surface Derivation of and more insidght into PH symmetric composite fermi liquid theory.

Skipped.

6 Nematic order in fractional quantum Hall liquids

By Joseph Maciejko (University of Alberta).

FQH nematic

- A proposed novel state of matter where (topological) FQH order coexists with (conventional) nematic order
- Intrinsic topological order (blobal) is insensitive to symmetry considerations (local); Can imaagine many possible broken symmetries.
- ...

FQH nematic at $v = 7/3$ Xia et al., Nat. Phys. 2011.

FQH nematic at $v = 5/2$ Samkharadze et al., Nat. Phys. 2015.

No symmetry breaking field, but not clear if there is a Hall plateau or not

FQH nematic at $v = 5/2$ Liu et al., PRB 2013 No Hall plateu reported, but longitudinal resistances appear to vanish as $T \to 0$.

Disclaimer

- Some of these experiements may or may not be explained by a FQH nematic
- will focus on FQH nematic
- theoretical: Haldane's geometrical perspective on the FQHE.

FQH isotropic-nematic transition

- Main questions
  - Generic mechanism for a transition from isoropic FQHE to FQH nematic
  - effective field theory of the transition?
  - can we realize this transition in a microscopic model of interacting electrons?
- isotropic FQH liquid is naturally proximate to a FQH nematic

- Girvin-MacDonald-Platzman (GMP) mode in the $q = 0$ limit corresponds to gapped nematic fluctuations wich condense at a putaative isotropic-nematic QCP (JM et al., PRB 2013).

GMP mode "Intra-LL" neutral collective mode in the FQHE (by contrast with "inter-LL" Kohn/magnetoplasmon mode). G. M. P., Prl 1985.
$q = 0$ limit of GMP mode has quadrupolar symmetry

GMP mode as a quadrupole

- numerical studies $\cdots\cdots$.
- heuristic argument.

geometrical theory of the FQHE

- $q = 0$ limit of the GMP = fluctuating intrinsic "metric" $g_{ab}(r,t)$ (Haldane PRL 2011; ....)
- Metric is unimodular ($\det g = 1$): set of ellipses of fixed area, i.e., quedrupolar deformations of the circle.

....

Intrinsic metric and nematicity

- Unimodular metric equivalent to nematic order parameter Q_{ab}.

$$g = \exp Q$$

...

FQH isotropic-nematic transition $\cdots$ Skipped.

7 Rotational properties of multi-species Bose gases

By Suasanne Viefers, University of Ohio.

A good one for those not familiar with this field.

Outline

- Review of rotating bosons in the LLL quantum Hall connection
- Two- and three- species Bose gases
- Composite fermion approach: results and puzzles
- outlook

Rotating Bose condensates

- 1995: first atomic Bose condensate
- 1999: first vortex in rotating BEC (JILA, Paris)
- 2004: Abrikosov lattice in Lowest Landau Level (200 vortices)

  Increasing rotation: cloud flattens out, density decreases, weaker interaction, lowest Landau level.

  Eventually: Vortex lattice predicted to melt, so system enters quantum Hall regime
- Recent reports of small systems ($N < 10$) reaching FQH regime in novel type of optical lattice with local rotation of each site.

LLL wave functions

- Historical motivation: Quantum Hall effect
- 2-dimensional electron gas in a strong perpendicular magnetic field at low T
- Electrons residing mainly in the LLL

Construction of explicit trial wave function by various schemes (in partular) Laughlin, hierarchy/composite fermions) has proven very successful in exploring QH physics. Not exact, but o capture essential properties (topological order).

Bose condensate in the LLL Mathematical equivalence in 2D between rotation (harmonic oscillator) and perpendicular magnetic field.

In the absence of interaction: Lowest N-body state with give L is highly degenerate. The interaction lifts this degeneracy and selects the lowest ("yrast") state. (Yrast= "most dizzy"). [2]

- N-particle state: symmetric homogeneous polynomials.
- Total angular momentum = deree of polynomial

Composite fermion scheme modified and shown to work successfully for the entire yrast line, including low angular momenta.

Multi-component systems ...

Two component systems: QH regime

- Two species Bose gases in the quantum Hall regime: several theoretical studies.
- Proposed (under discussion): NASS state at $v = 4/3$.
- Fundamentl quasiholes: charge $e/3$ non-Abelian anyons
- Bosonic IQH states with topologically protected edge states.

[2] Here is one anecdote.

Two-component systems: Slow rotation

- Study 2-species Bose gas in LLL with homogeneous contact interaction.
- Recent work identified a class analytically exact many-body eigenstates for low angular momenta
- Beyond papenbrock? Study low L regime in terms of composite fermions, exploiting (pesudo) spin analogy for homogeneous interaction. Not a priori expected to work
- Choice of Slater determinants in general not unique, i.e. several CF candidates – may or may not be lineary dependent.

Simple example ...

Selection criteria for Slaters ...

Full CF diagonalization

Sample states ...

Puzzles: linear dependencies

- the number of apparent CF condidates is generally too large, after projection.
- This is true even when restricting to simple states.
- Similar problems were recently discussed in the context of higher bands for electronic FQH states. without fully succeeding to reveal the underlying mathematical structures
- ...
- Wish: identify linear dependdencies before projection (for numerical concern).

Linear dependencies (simple states): Ingredients ...

- More direct understanding of linear dependencies in the CF formalism?
- Go beyond homogeneous interaction, study vortex structures, anyonic quasiparticles in QH regime
- Future experiments in this regime?
- Spin-I
- NORDITA program "topological phases in cold atom systems" Aug 2017.

8 Artificial gauge fields with ultracold bosonic atoms in optical lattices

By Monika Aidelsburger (LMU Munich, MP institute of Quantum Optics).

Outline

- Realization of artificial magnetic fields using laser-assisted tunneling
- Experimental results
- Chern-number measurement
- Summary and prospects

Skipped. May contain a good introduction to topological Insulator concepts. But I have really limited amount of time now.

9 Generalized Topological Forces

By I.B. Spielman (JQI - Maryland).

Topology in CMP How the underlying topology gives rise to the forces.

Outline

- Geometric charge pumping
  From abelian Berry's curvature related to the 1st chern number/class.
- 2nd Chern number/class
  From non-abelian Berry's curvature.

Pumps throughout history From archimedean to NIST single electron pump.

Questions from simple 1D lattice A force "just" causes Bloch oscillations.

- Gives motion in topological/geometric pumps
- Deflects trajectories in Bloch oscillations
- Force underlying quantum Hall effect

Topology in parameter space

Monopole - topological defects He seems to want to create some concepts of force in CMP system. And I decided to skip him for the moment.

10 Anchor

References

[1] `https://www.youtube.com/playlist?list=PLCoSh1h28ieLIaD-HGi5aUQzunOTtxHTC`

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