THERMODYNAMICS AND STATISTICAL PHYSICS

Formulas and constants

Thermodynamic functions and relations

$H = E + pV \qquad F = E - TS \qquad G = E - TS + pV$

$\left(\frac{\partial E}{\partial S}\right)_V = T \qquad \left(\frac{\partial E}{\partial V}\right)_S = -p \qquad \left(\frac{\partial H}{\partial S}\right)_p = T \qquad \left(\frac{\partial H}{\partial p}\right)_S = V$

$\left(\frac{\partial F}{\partial T}\right)_V = -S \qquad \left(\frac{\partial F}{\partial V}\right)_T = -p \qquad \left(\frac{\partial G}{\partial T}\right)_p = -S \qquad \left(\frac{\partial G}{\partial p}\right)_T = V$

Maxwell relations

$\left(\frac{\partial T}{\partial V}\right)_S = -\left(\frac{\partial p}{\partial S}\right)_V \qquad \left(\frac{\partial T}{\partial p}\right)_S = \left(\frac{\partial V}{\partial S}\right)_p \qquad \left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial p}{\partial T}\right)_V \qquad \left(\frac{\partial S}{\partial p}\right)_T = -\left(\frac{\partial V}{\partial T}\right)_p$

Specific heat

$c_V = \frac{1}{\nu}\left(\frac{dQ}{dT}\right)_V \qquad c_p = \frac{1}{\nu}\left(\frac{dQ}{dT}\right)_p$

Entropy

$S = k \ln \Omega \qquad S = -k \sum_r P_r \ln P_r \qquad S = k(\ln Z + \beta \overline{E})$

Partition functions

$Z = \sum_r e^{-\beta E_r} \qquad \mathcal{Z} = \sum_r e^{-\beta E_r - \alpha N_r} \qquad \ln \mathcal{Z} = \alpha N \pm \sum_r \ln\left(1 \pm e^{-\beta\epsilon_r - \alpha}\right)$

Clausius-Clapeyron equation

$\frac{dp}{dT} = \frac{\Delta S}{\Delta V} \qquad \frac{dp}{dT} = \frac{L_{12}}{T\ \Delta V}$

Fermi energy ($\mu = -kT\alpha$)

$\mu_j = -T\left(\frac{\partial S}{\partial N_j}\right)_{E,V,N} \qquad \mu_j = \left(\frac{\partial E}{\partial N_j}\right)_{S,V,N} \qquad \mu_j = \left(\frac{\partial F}{\partial N_j}\right)_{T,V,N} \qquad \mu_j = -\left(\frac{\partial G}{\partial N_j}\right)_{T,p,N}$

Stefan-Boltzmann law

$$\mathcal{P} = a\sigma T^4 = a\frac{\pi^2 k^4}{60 c^2 \hbar^3} T^4$$

Stirlings formula

$$\ln N! = N \ln N - N + \frac{1}{2}\ln(2\pi N) + \ldots$$

Integrals

$$\int_0^\infty x^n e^{-ax} dx = \frac{n!}{a^{n+1}}$$

$$\int_0^\infty x^{2n+1} e^{-ax^2} dx = \frac{n!}{2a^{n+1}}$$

$$\int_0^\infty x^{2n} e^{-ax^2} dx = \frac{(2n-1)!!}{2(2a)^n}\sqrt{\frac{\pi}{a}}$$

The gamma function

$$\Gamma(t) = \int_0^\infty x^{t-1} e^{x} dx$$

$$\Gamma(t+1) = t\Gamma(t)$$

Value of some integrals

k	$\Gamma(k+1)$	$\int_0^\infty \frac{x^k dx}{e^x-1}$	$\int_0^\infty \frac{x^k dx}{e^x+1}$
0	1	∞	$\ln 2 = 0.6931$
$\frac{1}{2}$	$\frac{1}{2}\sqrt{\pi}$	2.315	0.6780
1	1	$\frac{1}{6}\pi^2 = 1.645$	$\frac{1}{12}\pi^2 = 0.8225$
$\frac{3}{2}$	$\frac{3}{4}\sqrt{\pi}$	1.783	1.152
2	2	2.404	1.803
3	6	$\frac{1}{15}\pi^4 = 6.491$	$\frac{7}{120}\pi^4 = 5.682$

Physical constants

$c = 2.997925 \cdot 10^8$ m/s

$e = 1.6022 \cdot 10^{-19}$ C

$h = 6.6262 \cdot 10^{-34}$ Js $= 4.1357 \cdot 10^{-15}$ eVs

$\hbar = 1.0546 \cdot 10^{-34}$ Js $= 0.65821 \cdot^{-15}$ eVs

$m_e = 0.91094 \cdot 10^{-30}$ kg $= 0.51100$ MeV/c^2

$m_p = 1.6726 \cdot 10^{-27}$ kg $= 938.27$ MeV/c^2

$m_n = 1.6605 \cdot 10^{-27}$ kg $= 931.48$ MeV/c^2

$N_A = 6.0221 \cdot 10^{23}$ mole^{-1}

$R = 8.314$ JK^{-1} mole^{-1}

$k_B = 1.38066 \cdot 10^{-23}$ J/K $= 8.61739 \cdot 10^{-5}$ eV/K

$\sigma = 5.6697 \cdot 10^{-8}$ Jm^{-2}s^{-1}K^{-4}