Solution for HW10

Taper

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Abstract

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Problem 1

Classcal	$2^2 = 4$
Boson	3
Fermion	1

Problem 2

Notice two facts:

$$(\vec{\sigma} \cdot \vec{a})(\vec{\sigma} \cdot \vec{b}) = \vec{a} \cdot \vec{b} + i\vec{\sigma} \cdot (\vec{a} \times \vec{b}) \tag{0.0.1}$$

which is Eq.(3.2.39) on Sakurai's Modern Quantum Mechanics. Also

$$\operatorname{Tr}(\vec{a} \cdot \vec{\sigma}) = 0 \tag{0.0.2}$$

This is because the trace function is linear and all pauli matrices have zero trace.

Then:

$$\begin{aligned}
\operatorname{Tr}(\rho_A \rho_B) &= \frac{1}{4} \operatorname{Tr}(1 + (\vec{n}_A + \vec{n}_B) \cdot \vec{\sigma} + (\vec{n}_A \cdot \vec{\sigma})(\vec{n}_B \cdot \vec{\sigma})) \\
&= \frac{1}{4} \{1 + \operatorname{Tr}(\vec{n}_A \cdot \vec{n}_B + i\vec{\sigma} \cdot (\vec{n}_A \times \vec{n}_B))\} \\
&= \frac{1}{4}(1 + \vec{n}_A \cdot \vec{n}_B)
\end{aligned}$$

Problem 3

Denote $|a'\rangle$ as $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$, and $|a''\rangle$ as $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$. Then

$$H = \begin{pmatrix} 0 & \delta \\ \delta & 0 \end{pmatrix} \tag{0.0.3}$$

The eigenvectors and eigenvalues can be easily guessed:

$$|+\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}, \lambda_+ = \delta \tag{0.0.4}$$

$$|-\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix}, \lambda_- = -\delta \tag{0.0.5}$$

For time evolution:

Since $|a'\rangle = \frac{\sqrt{2}}{2}(|+\rangle + |-\rangle)$, under time evolution it will be

$$|a'(t)\rangle = \frac{\sqrt{2}}{2}\left(e^{-i\delta t/\hbar}|+\rangle + e^{i\delta t/\hbar}|-\rangle\right) = \frac{1}{2}\begin{pmatrix}e^{it\delta/\hbar} + e^{-it\delta/\hbar}\\ e^{-it\delta/\hbar} - e^{it\delta/\hbar}\end{pmatrix} = \begin{pmatrix}\cos(t\delta/\hbar)\\ -i\sin(t\delta/\hbar)\end{pmatrix} \tag{0.0.6}$$

For probability:

The probability is clearly: $\sin^2(t\delta/\hbar)$.

Remark 0.1. This is actually the famous Rabi frequency, see more at: `https://en.wikipedia.org/wiki/Rabi_cycle`.