In[44]:= `pk[t_, Δ_, W1_, W2_, E0_] :=`

$$\left(-\frac{e^{i t \left(E0-\sqrt{\Delta^2+W1^2+W2^2}\right)}\,(W1 - i\, W2)}{2\sqrt{\Delta^2 + W1^2 + W2^2}} + \frac{e^{i t \left(E0+\sqrt{\Delta^2+W1^2+W2^2}\right)}\,(W1 - i\, W2)}{2\sqrt{\Delta^2 + W1^2 + W2^2}}\right)$$

$$\left(-\frac{e^{-i t \left(E0-\sqrt{\Delta^2+W1^2+W2^2}\right)}\,(W1 + i\, W2)}{2\sqrt{\Delta^2 + W1^2 + W2^2}} + \frac{e^{-i t \left(E0+\sqrt{\Delta^2+W1^2+W2^2}\right)}\,(W1 + i\, W2)}{2\sqrt{\Delta^2 + W1^2 + W2^2}}\right) +$$

$$\left(-\frac{e^{-i t \left(E0+\sqrt{\Delta^2+W1^2+W2^2}\right)}\left(-\Delta - \sqrt{\Delta^2 + W1^2 + W2^2}\right)}{2\sqrt{\Delta^2 + W1^2 + W2^2}} + \frac{e^{-i t \left(E0-\sqrt{\Delta^2+W1^2+W2^2}\right)}\left(-\Delta + \sqrt{\Delta^2 + W1^2 + W2^2}\right)}{2\sqrt{\Delta^2 + W1^2 + W2^2}}\right)$$

$$\left(-\frac{e^{i t \left(E0+\sqrt{\Delta^2+W1^2+W2^2}\right)}\left(-\Delta - \sqrt{\Delta^2 + W1^2 + W2^2}\right)}{2\sqrt{\Delta^2 + W1^2 + W2^2}} + \frac{e^{i t \left(E0-\sqrt{\Delta^2+W1^2+W2^2}\right)}\left(-\Delta + \sqrt{\Delta^2 + W1^2 + W2^2}\right)}{2\sqrt{\Delta^2 + W1^2 + W2^2}}\right)$$

In[42]:= `uk[t_, Δ_, W1_, W2_, E0_] :=`

$$\mathrm{Arg}\left[-\frac{e^{-i t \left(E0-\sqrt{\Delta^2+W1^2+W2^2}\right)}\,(W1 + i\, W2)}{2\sqrt{\Delta^2 + W1^2 + W2^2}} + \frac{e^{-i t \left(E0+\sqrt{\Delta^2+W1^2+W2^2}\right)}\,(W1 + i\, W2)}{2\sqrt{\Delta^2 + W1^2 + W2^2}}\right]$$

In[45]:= `pk[t, (Er - El) / 2, v + v1 * Cos[k], -v1 * Sin[k], (Er + El) / 2]`

Out[45]=

$$\left(-\frac{e^{-i t \left(\frac{El+Er}{2}-\sqrt{\frac{1}{4}(-El+Er)^2+(v+v1\,Cos[k])^2+v1^2\,Sin[k]^2}\right)}\left(v + v1\,Cos[k] - i\, v1\,Sin[k]\right)}{2\sqrt{\frac{1}{4}(-El + Er)^2 + (v + v1\,Cos[k])^2 + v1^2\,Sin[k]^2}} + \frac{e^{-i t \left(\frac{El+Er}{2}+\sqrt{\frac{1}{4}(-El+Er)^2+(v+v1\,Cos[k])^2+v1^2\,Sin[k]^2}\right)}\left(v + v1\,Cos[k] - i\, v1\,Sin[k]\right)}{2\sqrt{\frac{1}{4}(-El + Er)^2 + (v + v1\,Cos[k])^2 + v1^2\,Sin[k]^2}}\right)$$

$$\left(-\frac{e^{i t \left(\frac{El+Er}{2}-\sqrt{\frac{1}{4}(-El+Er)^2+(v+v1\,Cos[k])^2+v1^2\,Sin[k]^2}\right)}\left(v + v1\,Cos[k] + i\, v1\,Sin[k]\right)}{2\sqrt{\frac{1}{4}(-El + Er)^2 + (v + v1\,Cos[k])^2 + v1^2\,Sin[k]^2}} +\right.$$

$$\left.\frac{e^{i\,t\left(\frac{\text{El}+\text{Er}}{2}+\sqrt{\frac{1}{4}(-\text{El}+\text{Er})^2+(\text{v}+\text{v1}\,\text{Cos}[k])^2+\text{v1}^2\,\text{Sin}[k]^2}\right)}\left(\text{v}+\text{v1}\,\text{Cos}[k]+i\,\text{v1}\,\text{Sin}[k]\right)}{2\sqrt{\frac{1}{4}(-\text{El}+\text{Er})^2+(\text{v}+\text{v1}\,\text{Cos}[k])^2+\text{v1}^2\,\text{Sin}[k]^2}}\right)+$$

$$\left(-\left(\left(e^{-i\,t\left(\frac{\text{El}+\text{Er}}{2}+\sqrt{\frac{1}{4}(-\text{El}+\text{Er})^2+(\text{v}+\text{v1}\,\text{Cos}[k])^2+\text{v1}^2\,\text{Sin}[k]^2}\right)}\right.\right.\right.$$

$$\left.\left.\left(\frac{\text{El}-\text{Er}}{2}-\sqrt{\frac{1}{4}(-\text{El}+\text{Er})^2+(\text{v}+\text{v1}\,\text{Cos}[k])^2+\text{v1}^2\,\text{Sin}[k]^2}\right)\right)\Big/\right.$$

$$\left.\left(2\sqrt{\frac{1}{4}(-\text{El}+\text{Er})^2+(\text{v}+\text{v1}\,\text{Cos}[k])^2+\text{v1}^2\,\text{Sin}[k]^2}\right)\right)+$$

$$\left(e^{-i\,t\left(\frac{\text{El}+\text{Er}}{2}-\sqrt{\frac{1}{4}(-\text{El}+\text{Er})^2+(\text{v}+\text{v1}\,\text{Cos}[k])^2+\text{v1}^2\,\text{Sin}[k]^2}\right)}\right.$$

$$\left.\left(\frac{\text{El}-\text{Er}}{2}+\sqrt{\frac{1}{4}(-\text{El}+\text{Er})^2+(\text{v}+\text{v1}\,\text{Cos}[k])^2+\text{v1}^2\,\text{Sin}[k]^2}\right)\right)\Big/$$

$$\left(2\sqrt{\frac{1}{4}(-\text{El}+\text{Er})^2+(\text{v}+\text{v1}\,\text{Cos}[k])^2+\text{v1}^2\,\text{Sin}[k]^2}\right)\right)$$

$$\left(-\left(\left(e^{i\,t\left(\frac{\text{El}+\text{Er}}{2}+\sqrt{\frac{1}{4}(-\text{El}+\text{Er})^2+(\text{v}+\text{v1}\,\text{Cos}[k])^2+\text{v1}^2\,\text{Sin}[k]^2}\right)}\right.\right.\right.$$

$$\left.\left.\left(\frac{\text{El}-\text{Er}}{2}-\sqrt{\frac{1}{4}(-\text{El}+\text{Er})^2+(\text{v}+\text{v1}\,\text{Cos}[k])^2+\text{v1}^2\,\text{Sin}[k]^2}\right)\right)\Big/\right.$$

$$\left.\left(2\sqrt{\frac{1}{4}(-\text{El}+\text{Er})^2+(\text{v}+\text{v1}\,\text{Cos}[k])^2+\text{v1}^2\,\text{Sin}[k]^2}\right)\right)+$$

$$\left(e^{i\,t\left(\frac{\text{El}+\text{Er}}{2}-\sqrt{\frac{1}{4}(-\text{El}+\text{Er})^2+(\text{v}+\text{v1}\,\text{Cos}[k])^2+\text{v1}^2\,\text{Sin}[k]^2}\right)}\right.$$

$$\left.\left(\frac{\text{El}-\text{Er}}{2}+\sqrt{\frac{1}{4}(-\text{El}+\text{Er})^2+(\text{v}+\text{v1}\,\text{Cos}[k])^2+\text{v1}^2\,\text{Sin}[k]^2}\right)\right)\Big/$$

$$\left(2\sqrt{\frac{1}{4}(-\text{El}+\text{Er})^2+(\text{v}+\text{v1}\,\text{Cos}[k])^2+\text{v1}^2\,\text{Sin}[k]^2}\right)\right)$$

Careful examination shows that pk(k)=pk(-k).

In[46]:=
```
uk[t, (Er - El) / 2, v + v1 * Cos[k], -v1 * Sin[k], (Er + El) / 2]
```

Out[46]=
$$\mathrm{Arg}\left[-\left(\left(e^{-i\,t\left(\frac{El+Er}{2}-\sqrt{\frac{1}{4}(-El+Er)^2+(v+v1\,\mathrm{Cos}[k])^2+v1^2\,\mathrm{Sin}[k]^2}\right)}\left(v+v1\,\mathrm{Cos}[k]-i\,v1\,\mathrm{Sin}[k]\right)\right)\Big/\left(2\sqrt{\frac{1}{4}(-El+Er)^2+(v+v1\,\mathrm{Cos}[k])^2+v1^2\,\mathrm{Sin}[k]^2}\right)\right)+\left(e^{-i\,t\left(\frac{El+Er}{2}+\sqrt{\frac{1}{4}(-El+Er)^2+(v+v1\,\mathrm{Cos}[k])^2+v1^2\,\mathrm{Sin}[k]^2}\right)}\left(v+v1\,\mathrm{Cos}[k]-i\,v1\,\mathrm{Sin}[k]\right)\right)\Big/\left(2\sqrt{\frac{1}{4}(-El+Er)^2+(v+v1\,\mathrm{Cos}[k])^2+v1^2\,\mathrm{Sin}[k]^2}\right)\right]$$

I cannot see directly from above expression whether ∂_t (uk) is even in k or not.