

In[44]:= **pk**[t\_, Δ\_, w1\_, w2\_, E0\_] :=

$$\begin{aligned} & \left( -\frac{e^{\frac{i}{2} t \left( E0 - \sqrt{\Delta^2 + w1^2 + w2^2} \right)} (w1 - i w2)}{2 \sqrt{\Delta^2 + w1^2 + w2^2}} + \frac{e^{\frac{i}{2} t \left( E0 + \sqrt{\Delta^2 + w1^2 + w2^2} \right)} (w1 - i w2)}{2 \sqrt{\Delta^2 + w1^2 + w2^2}} \right) \\ & \left( -\frac{e^{\frac{-i}{2} t \left( E0 - \sqrt{\Delta^2 + w1^2 + w2^2} \right)} (w1 + i w2)}{2 \sqrt{\Delta^2 + w1^2 + w2^2}} + \frac{e^{\frac{-i}{2} t \left( E0 + \sqrt{\Delta^2 + w1^2 + w2^2} \right)} (w1 + i w2)}{2 \sqrt{\Delta^2 + w1^2 + w2^2}} \right) + \\ & \left( -\frac{e^{\frac{-i}{2} t \left( E0 + \sqrt{\Delta^2 + w1^2 + w2^2} \right)} \left( -\Delta - \sqrt{\Delta^2 + w1^2 + w2^2} \right)}{2 \sqrt{\Delta^2 + w1^2 + w2^2}} + \right. \\ & \left. \frac{e^{\frac{-i}{2} t \left( E0 - \sqrt{\Delta^2 + w1^2 + w2^2} \right)} \left( -\Delta + \sqrt{\Delta^2 + w1^2 + w2^2} \right)}{2 \sqrt{\Delta^2 + w1^2 + w2^2}} \right) \\ & \left( -\frac{e^{\frac{i}{2} t \left( E0 + \sqrt{\Delta^2 + w1^2 + w2^2} \right)} \left( -\Delta - \sqrt{\Delta^2 + w1^2 + w2^2} \right)}{2 \sqrt{\Delta^2 + w1^2 + w2^2}} + \right. \\ & \left. \frac{e^{\frac{i}{2} t \left( E0 - \sqrt{\Delta^2 + w1^2 + w2^2} \right)} \left( -\Delta + \sqrt{\Delta^2 + w1^2 + w2^2} \right)}{2 \sqrt{\Delta^2 + w1^2 + w2^2}} \right) \end{aligned}$$

In[42]:= **uk**[t\_, Δ\_, w1\_, w2\_, E0\_] :=

$$\text{Arg} \left[ -\frac{e^{\frac{-i}{2} t \left( E0 - \sqrt{\Delta^2 + w1^2 + w2^2} \right)} (w1 + i w2)}{2 \sqrt{\Delta^2 + w1^2 + w2^2}} + \frac{e^{\frac{-i}{2} t \left( E0 + \sqrt{\Delta^2 + w1^2 + w2^2} \right)} (w1 + i w2)}{2 \sqrt{\Delta^2 + w1^2 + w2^2}} \right]$$

In[45]:= **pk**[t, (Er - El)/2, v + v1 \* Cos[k], -v1 \* Sin[k], (Er + El)/2]

$$\begin{aligned} \text{Out[45]} = & \left( -\frac{e^{\frac{-i}{2} t \left( \frac{El+Er}{2} - \sqrt{\frac{1}{4} (-El+Er)^2 + (v+v1 \cos[k])^2 + v1^2 \sin[k]^2} \right)} (v + v1 \cos[k] - i v1 \sin[k])}{2 \sqrt{\frac{1}{4} (-El + Er)^2 + (v + v1 \cos[k])^2 + v1^2 \sin[k]^2}} + \right. \\ & \left. \frac{e^{\frac{-i}{2} t \left( \frac{El+Er}{2} + \sqrt{\frac{1}{4} (-El+Er)^2 + (v+v1 \cos[k])^2 + v1^2 \sin[k]^2} \right)} (v + v1 \cos[k] - i v1 \sin[k])}{2 \sqrt{\frac{1}{4} (-El + Er)^2 + (v + v1 \cos[k])^2 + v1^2 \sin[k]^2}} \right) \\ & \left( -\frac{e^{\frac{i}{2} t \left( \frac{El+Er}{2} - \sqrt{\frac{1}{4} (-El+Er)^2 + (v+v1 \cos[k])^2 + v1^2 \sin[k]^2} \right)} (v + v1 \cos[k] + i v1 \sin[k])}{2 \sqrt{\frac{1}{4} (-El + Er)^2 + (v + v1 \cos[k])^2 + v1^2 \sin[k]^2}} + \right. \end{aligned}$$

$$\begin{aligned}
 & \left. \frac{e^{i t \left( \frac{El+Er}{2} + \sqrt{\frac{1}{4} (-El+Er)^2 + (v+v1 \cos[k])^2 + v1^2 \sin[k]^2} \right)} (v + v1 \cos[k] + i v1 \sin[k])}{2 \sqrt{\frac{1}{4} (-El + Er)^2 + (v + v1 \cos[k])^2 + v1^2 \sin[k]^2}} \right) + \\
 & \left( - \left( \left( e^{-i t \left( \frac{El+Er}{2} + \sqrt{\frac{1}{4} (-El+Er)^2 + (v+v1 \cos[k])^2 + v1^2 \sin[k]^2} \right)} \right. \right. \right. \\
 & \quad \left( \frac{El - Er}{2} - \sqrt{\frac{1}{4} (-El + Er)^2 + (v + v1 \cos[k])^2 + v1^2 \sin[k]^2} \right) \Bigg) / \\
 & \quad \left( 2 \sqrt{\frac{1}{4} (-El + Er)^2 + (v + v1 \cos[k])^2 + v1^2 \sin[k]^2} \right) \Bigg) + \\
 & \left( e^{-i t \left( \frac{El+Er}{2} - \sqrt{\frac{1}{4} (-El+Er)^2 + (v+v1 \cos[k])^2 + v1^2 \sin[k]^2} \right)} \right. \\
 & \quad \left( \frac{El - Er}{2} + \sqrt{\frac{1}{4} (-El + Er)^2 + (v + v1 \cos[k])^2 + v1^2 \sin[k]^2} \right) \Bigg) / \\
 & \quad \left( 2 \sqrt{\frac{1}{4} (-El + Er)^2 + (v + v1 \cos[k])^2 + v1^2 \sin[k]^2} \right) \Bigg) \\
 & \left( - \left( \left( e^{i t \left( \frac{El+Er}{2} + \sqrt{\frac{1}{4} (-El+Er)^2 + (v+v1 \cos[k])^2 + v1^2 \sin[k]^2} \right)} \right. \right. \right. \\
 & \quad \left( \frac{El - Er}{2} - \sqrt{\frac{1}{4} (-El + Er)^2 + (v + v1 \cos[k])^2 + v1^2 \sin[k]^2} \right) \Bigg) / \\
 & \quad \left( 2 \sqrt{\frac{1}{4} (-El + Er)^2 + (v + v1 \cos[k])^2 + v1^2 \sin[k]^2} \right) \Bigg) + \\
 & \left( e^{i t \left( \frac{El+Er}{2} - \sqrt{\frac{1}{4} (-El+Er)^2 + (v+v1 \cos[k])^2 + v1^2 \sin[k]^2} \right)} \right. \\
 & \quad \left( \frac{El - Er}{2} + \sqrt{\frac{1}{4} (-El + Er)^2 + (v + v1 \cos[k])^2 + v1^2 \sin[k]^2} \right) \Bigg) / \\
 & \quad \left( 2 \sqrt{\frac{1}{4} (-El + Er)^2 + (v + v1 \cos[k])^2 + v1^2 \sin[k]^2} \right) \Bigg)
 \end{aligned}$$

Careful examination shows that  $p_k(k)=p_k(-k)$ .

$$\text{In[46]:= } \mathbf{uk} \left[ \mathbf{t}, (\mathbf{Er} - \mathbf{El}) / 2, \mathbf{v} + \mathbf{v1} * \mathbf{Cos}[\mathbf{k}], -\mathbf{v1} * \mathbf{Sin}[\mathbf{k}], (\mathbf{Er} + \mathbf{El}) / 2 \right]$$

$$\begin{aligned} \text{Out[46]= } & \text{Arg} \left[ - \left( e^{\left( -i t \left( \frac{\mathbf{El} + \mathbf{Er}}{2} - \sqrt{\frac{1}{4} (-\mathbf{El} + \mathbf{Er})^2 + (\mathbf{v} + \mathbf{v1} \mathbf{Cos}[\mathbf{k}])^2 + \mathbf{v1}^2 \mathbf{Sin}[\mathbf{k}]^2} \right)} (\mathbf{v} + \mathbf{v1} \mathbf{Cos}[\mathbf{k}] - i \mathbf{v1} \mathbf{Sin}[\mathbf{k}]) \right) \right. \\ & \left. \left( 2 \sqrt{\frac{1}{4} (-\mathbf{El} + \mathbf{Er})^2 + (\mathbf{v} + \mathbf{v1} \mathbf{Cos}[\mathbf{k}])^2 + \mathbf{v1}^2 \mathbf{Sin}[\mathbf{k}]^2} \right) \right] + \\ & \left( e^{\left( -i t \left( \frac{\mathbf{El} + \mathbf{Er}}{2} + \sqrt{\frac{1}{4} (-\mathbf{El} + \mathbf{Er})^2 + (\mathbf{v} + \mathbf{v1} \mathbf{Cos}[\mathbf{k}])^2 + \mathbf{v1}^2 \mathbf{Sin}[\mathbf{k}]^2} \right)} (\mathbf{v} + \mathbf{v1} \mathbf{Cos}[\mathbf{k}] - i \mathbf{v1} \mathbf{Sin}[\mathbf{k}]) \right) \right. \\ & \left. \left( 2 \sqrt{\frac{1}{4} (-\mathbf{El} + \mathbf{Er})^2 + (\mathbf{v} + \mathbf{v1} \mathbf{Cos}[\mathbf{k}])^2 + \mathbf{v1}^2 \mathbf{Sin}[\mathbf{k}]^2} \right) \right] \end{aligned}$$

I cannot see directly from above expression whether  $\partial_t(uk)$  is even in  $k$  or not.