

$$\begin{aligned}
 \langle \Delta m \rangle &= i \gamma \int_0^\infty dt \int \frac{dk}{2\pi} \cdot \overset{\substack{\uparrow \\ U_k(t) e^{-i\theta_k(t)}}}{\psi_k^*} \partial_k \left(U_k(t) e^{-i\theta_k(t)} \right) \\
 &= i \gamma \int_0^\infty dt \int \frac{dk}{2\pi} U_k(t) e^{-i\theta_k(t)} \cdot \left[(\partial_k U_k(t)) e^{i\theta_k(t)} + U_k(t) e^{i\theta_k(t)} \cdot i (\partial_k \theta_k(t)) \right] \\
 &\quad \downarrow \quad \swarrow \\
 &\quad \int dk (U_k \partial_k U_k) = 0
 \end{aligned}$$

$$= i \gamma \int_0^\infty dt \int \frac{dk}{2\pi} U_k^2(t) \cdot i \partial_k \theta_k(t) = -\gamma \int_0^\infty dt \int \frac{dk}{2\pi} U_k^2(t) \partial_k \theta_k(t).$$

$$= \int \frac{dk}{2\pi} \int_0^\infty (\partial_t P_k) (\partial_k \theta_k(t)).$$