Notes for General Relativity

Taper

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Abstract

(None)

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1 The Principle of Relativity

Landau's book [1] describes the speed of light, as a speed of the maximum velocity of propagation of interaction. He perceives velocity as describing the propagation of interaction.

In another aspect, I think that time does not exist. I perceive time as a agent of the environmental effect, communicating information and coordinating movement between the system under consideration and its envrionment. Therefore, a single existance is eternal by nature. Perhaps before the universe originates, time is a meaningless concept. In my view, the light is the only reliable tool to serve function of communication and coordination.

Fram either perspective, the speed of light is inherently constent. However, from my view, it is not clear why this speed should be a maximum.

1.1 Invariance of Interval

To do dynamics, we necessarily need a measure of distance. Here introduces the distance in spacetime - interval. It is:

$$\mathrm{d}s^2 = \mathrm{d}t^2 - \mathrm{d}x^2 - \mathrm{d}y^2 - \mathrm{d}z^2 \tag{1.1.1}$$

Note that from now on, I will try to use natural units as often as possible.

Such a measure should be invariant in different Lorentz frames. Landau proves it by postulating a priori that:

$$\mathrm{d}s^2 = a\,\mathrm{d}s'^2 \tag{1.1.2}$$

This is suggested by $\mathrm{d}s = 0$ is invariant (invariance of the speed of light), and $\mathrm{d}s$ and $\mathrm{d}s'$ should be infinitesimals of the same order. From this postulation, it is straightforward to argue that a should be a constant and is equal to 1.

Using this property, one can classify interval between events as being *timelike, spacelike,* and *lightlike*, by whether $ds > 0$, $ds < 0$ or $ds = 0$. A mnemonic tip is that for timelike intervals, the "time difference" is dominant, and for spacelike intervals, the spatial difference is dominant.

The following figure gives an indication of how the timelike, spacelike, lightlike classification is related to causality:

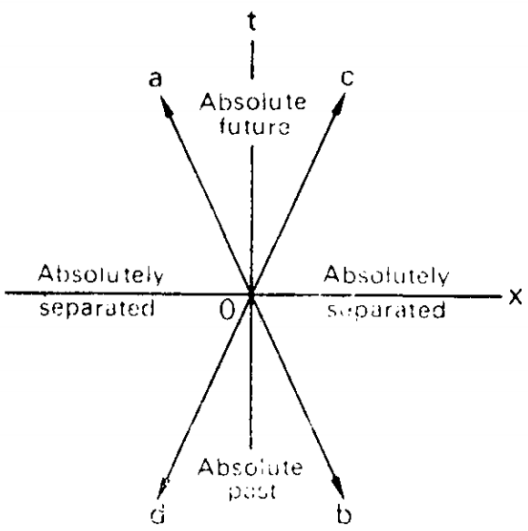

Figure 1: Spacetime regions

Only those events in the absolute future and in the absolute past regions can have a causal link with the people at O, due to the limit of speed of propagation. And the future and past combined together as the timelike region.

Another difference of timelike and spacelike regions lies in the prefix "absoluate". All events in timelike region cannot be simultaneous with O in any reference frame, since the interval must have a nonzero time component. Similarly all events in spacelike region must happen in different place with the O. An additional requirement, which comes naturally from law of causality, is that all events in the future must remain absolutely in the future in any reference frame. Similarly we have an absolute past.

Interestingly, the concept of being simultaneous with O, being before or after O in time, are relative for events in spacelike region. However,

since there can be no causal link between O and those events, this relativity does not pose a challenge to causality.

Lastly, the cone formed by all events with $ds = 0$ is called the *light cone*. Events in it is very special that it deserve to devote a separate section to discuss it, which will be done later in this note.

1.2 Proper Time

By

$$dt'^2 = dt^2 - dx^2 = dt^2 - (v\,\mathrm{d}t)^2$$

we have

$$\Delta t' = \int_{t_1}^{t_2} \mathrm{d}t\sqrt{1-v^2} \tag{1.2.1}$$

This shows the time dilation effect of a moving clock. Also, by this we can always calculate the time experienced by a clock by $t = \frac{1}{c}\int \mathrm{d}s$ (SI unit).

The Landau's book [1] explains a classical paradox about time dilation, which is omitted here.

Interestingly, we have the property that, for all lines between two events, the longest one (i.e. the path that has the longest interval), is the straight line, contrary to the classical case. To see this, we note that any two events (assumed to be causally linked) can be connected by a flying clock with uniform speed.

1.3 Light's life

The life of a proton must be miserable.

Possible sources: 1, 2. 3.

References

[1] The Classical Theory of Fields

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