Notes of Quantum Field Theory in a Nutshell

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Contents

Chapter 1

Part I: Motivation and Foundations

1.1 I:2 Path Integral Formulation of Quantum Physics

Here the path integral formulation of quantum mechanics is introduced. The intuition comes from a limiting case of the traditional double slit electron interference experiment. (**pp.7 to 10**) Then it calculates the transition probability $\langle q_F|e^{-iHT}|q_I\rangle$ by divide it into a infinite of steps:

$$\langle q_F|e^{-iHT}|q_I\rangle = \lim_{N\to\infty} \langle q_F|e^{iH\delta t}e^{iH\delta t}\cdots e^{iH\delta t}|q_I\rangle \,(\text{with } N\delta t = T) \tag{1.1.0.1}$$

For illustration, it calculates this value when $H = \frac{\hat{p}^2}{2m}$. The result is that:

$$\langle q_F|e^{-iHT}|q_I\rangle = \int Dq(t)e^{i\int_0^T dt\frac{1}{2}m\dot{q}^2}$$

where:

$$\int Dq(t) \equiv \lim_{N\to\infty}\left(\frac{-im}{2\pi\delta t}\right)^{N/2}\left(\prod_{k=1}^{N-1}\int dq_k\right) \tag{1.1.0.2}$$

It notes that when $H = \hat{p}^2/2m + V(\hat{q})$, the final result would have been:

$$\begin{aligned}\langle q_F|e^{-iHT}|q_I\rangle &= \int Dq(t)e^{i\int_0^T dt\frac{1}{2}m\dot{q}^2-V(q)} \\ &= \int Dq(t)e^{i\int_0^T dtL(\dot{q},q)}\end{aligned} \tag{1.1.0.3}$$

where L is the Lagrangian of the system.

Often, the value we need to calculate is $\langle F|e^{-iHT}|I\rangle$. Using $\int |q\rangle \langle q| \, dq = 1$, we have:

$$\langle F|e^{-iHT}|I\rangle = \int dq_F \int dq_I \, \langle F|q_F\rangle \langle q_F|e^{-iHT}|q_I\rangle \langle q_I|I\rangle \tag{1.1.0.4}$$

The value $\langle 0|e^{-iHT}|0\rangle$ is denoted Z. This part mentions that one often effect a change of coordinate $t \to -it$, called *Wick rotation*, to obtain:

$$Z = \int Dq(t) e^{-\int_0^T dt H(\dot{q},q)} \tag{1.1.0.5}$$

where H is the Hamiltonian of the system. The mathematical rigorous aspect is often ignored.

It also discuss how this formulation could explain the classical limit of quantum mechanics, i.e. classical mechanics, in a very direct manner. This is related to the saddle point approximation to the integral 1.1.0.3.

Unclear point Why is $\int dq \, |q\rangle \langle q| = 1$ while $\int \frac{p}{2\pi} |p\rangle \langle p| = 1$. What does it mean by saying "to see that the normalization is correct" (pp. 10 and 11). Why is effecting the Wick rotation is "somewhat rigorous"?

Corresponding pages in draft pp. 1 to 3.

1.1.1 Appendix 1 - Dirac Delta function and ε as infinitesimal small value

Here the Dirac Delta function is defined as the limit of another function $d_K(x)$. Since:

$$d_K(x) \equiv \int_{-K/2}^{K/2} \frac{dk}{2\pi} e^{ikx} = \frac{1}{\pi x} \sin \frac{Kx}{2} \tag{1.1.1.1}$$

$$\int_{-\infty}^{\infty} dx \; d_K(x) = 1 \tag{1.1.1.2}$$

Hence we de fine $\delta(x) = \lim_{K\to\infty} d_K(x)$. Other important formula include:

$$\delta(x) = \int_{-\infty}^{\infty} \frac{dk}{2\pi} e^{ikx} \tag{1.1.1.3}$$

$$\frac{1}{x + i\varepsilon} = \mathcal{P}\frac{1}{x} - i\pi\delta(x) \tag{1.1.1.4}$$

$$\delta(x) = \frac{1}{\pi} \frac{\varepsilon}{x^2 + \varepsilon^2} \tag{1.1.1.5}$$

Here ε is a infinitesimal value. $\mathcal{P}$ denotes the principal value integral, defined by:

$$\int dx \mathcal{P}\frac{1}{x} f(x) = \lim_{\varepsilon\to 0} \int dx \frac{x}{x^2 + \varepsilon^2} f(x) \tag{1.1.1.6}$$

1.1.2 Appendix 2 - Wick theorem in Gaussian Integral

This part introduces some very important formulae, listed below:
(It is very important that A is a real symmetric matrix.)

$$\int_{-\infty}^{+\infty} dx e^{-\frac{1}{2}ax^2+Jx} = \left(\frac{2\pi}{a}\right)^{\frac{1}{2}} e^{\frac{J^2}{2a}} \tag{1.1.2.1}$$

$$\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\cdots\int_{-\infty}^{+\infty} dx_1 dx_2 \cdots dx_N e^{-\frac{1}{2}x^T A x + J^T x} = \left(\frac{(2\pi)^N}{\det A}\right)^{\frac{1}{2}} e^{\frac{1}{2}J^T A^{-1} J} \tag{1.1.2.2}$$

$$\langle x_i x_j \cdots x_k x_l \rangle \equiv \frac{\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\cdots\int_{-\infty}^{+\infty} dx_1 dx_2 \cdots dx_N e^{-\frac{1}{2}x^T A x} x_i x_j \cdots x_k x_l}{\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\cdots\int_{-\infty}^{+\infty} dx_1 dx_2 \cdots dx_N e^{-\frac{1}{2}x^T A x}}$$

$$= \sum_{\text{Wick}} (A^{-1})_{ab} \cdots (A^{-1})_{cd} \tag{1.1.2.3}$$

For exmaple:

$$\langle x^{2n} \rangle \equiv \frac{\int_{-\infty}^{+\infty} dx e^{-\frac{1}{2}ax^2} x^{2n}}{\int_{-\infty}^{\infty} dx e^{-\frac{1}{2}ax^2}} = \frac{1}{a^n}(2n-1)!!$$

$$\langle x_i x_j x_k x_l \rangle = (A^{-1})_{ij}(A^{-1})_{kl} + (A^{-1})_{ik}(A^{-1})_{jl} + (A^{-1})_{il}(A^{-1})_{jk}$$

Corresponding pages in draft pp. 4 to 6.

Bibliography

[1] A. Zee. Quantum Field Theory in a Nutshell 2ed. PUP.

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