Notes of Preskill's Quantum Information Theory

Taper

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Abstract

This is a note taken when I read the course notes of Preskill's course on quantum computation.

Contents

1 Chapter 2 Foundations I: States and Ensembles

1.1 2.2 The Qubit

Here the concept of qubit is defined:

Definition 1.1 (Qubit)**.** *A qubit is a quantum system described by a two-dimensional Hilbert space.*

However, since measurement collapse the superposition state of quantum mechanics, there seems to be not difference between a qubit and a classical bit whose initial state is only known probabilistically. More important distinction should be and will be pointed out in the next few sections.

1.1.1 Spin-$\frac{1}{2}$

In this part, the author in fact discusses the symmetry and its role in quantum mechanis, albeit roughly.

Symmetry

Definition 1.2 (Symmetry)**.** *A symmetry is a transformation that acts on a state of a system, yet leaves all observable properties of the system unchanged.*

Hence a symmetry must leaves any $|\langle\psi|\phi\rangle|^2$ (probability) unchanged. This leads Wigner to derive a famous theorem stating that:

Theorem 1.1. *Any symmetry transformation, by adopting suitable phase conventions, can always chosen to be either unitary or antiunitary.*

He mentions that symmetry transformations formes a group. (Omitted here)

Here the author focuses on unitary ones, since "The antiunitary alternative, while important for discrete symmetries, can be excluded for continuous symmetries".

Also, he demands that symmetry "respect the dynamical evolution of the system", thus deducing:

$$[\hat{U}(R), \hat{H}] = 0 \tag{1.1.1}$$

$$[\hat{Q}, \hat{H}] = 0, \ \hat{Q} = \hat{Q}^\dagger \tag{1.1.2}$$

$$\hat{U} = \hat{I} - i\varepsilon\hat{Q} \tag{1.1.3}$$

$$\tag{1.1.4}$$

Here $\hat{U}$ is the symmetry transformation corresponding to the symmetry R. Q is an observable (since $\hat{Q}$ is hermitian) related to this symmetry by above formula, and ε as expected is an infinitesimal number. We also finds that this observable is conserved by above formula.

$\hat{Q}$ is said to be the generator of the symmetry by the following formula:

$$\hat{U}(R) = \lim_{N\to\infty}(\hat{I} + i\frac{\theta}{N}\hat{Q})^N = e^{i\theta\hat{Q}} \tag{1.1.5}$$

References

[1] Preskill's notes are available through his Caltech course website, Phy219.

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