Solution for HW4 20161026

Taper

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Abstract

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1 Explain the measurement of quantum states

A quantum system in state $|\phi\rangle$ upon measurement $\mathcal{A}$, will collpase to one of the eigenstates (for example, $|a\rangle$) of the corresponding operator A. The probability of collapsing one this eigenstate if determined by $|\langle\phi|a\rangle|^2$.

2 Solve the eigenvalues and eigenfunctions of σ_z

Physically we have a spin up eigenstate and a spin down eigenstae. So the answer can be guessed. They are just: $\alpha\begin{pmatrix}1\\0\end{pmatrix}$, with eigenvalue 1, and $\beta\begin{pmatrix}0\\1\end{pmatrix}$, with eigenvalue -1. Here α, β are arbitrary nonzero complex constants.

3 Prove some commutation relationships

Proof. With Einstein summation convention, we can write

$$L_i = \epsilon_{ijk} r_j p_k, \quad [r_j, p_k] = i\hbar\delta_{jk} \tag{3.0.1}$$

Then

$$\begin{aligned}[L_i, r_l] &= \epsilon_{ijk}[r_j p_k, r_l] = \epsilon_{ijk} r_j [p_k, r_l] = -i\hbar r_j \epsilon_{ijk}\delta_{kl} \\ &= -i\hbar r_j \epsilon_{ijl} = i\hbar\epsilon_{ilj} r_j \end{aligned} \tag{3.0.2}$$

Therefore, $[L_i, r_i] = 0$ since $\epsilon_{iij} \equiv 0$. □