$\vec{E} =$
$k\frac{Q}{|\vec{r}-vecr'|^3}(\vec{r}-$
$\vec{r'})$
$\vec{E_{z_{pc}}}(z >$
$R) =$
$k\frac{Q}{(z-R)^2}\hat{z}+$
$k\frac{Q}{(z+R)^2}\hat{z}$
$\vec{E_{z_{pc}}}(z <$
$-R) =$
$-k\frac{Q}{(z+R)^2}\hat{z}-$
$k\frac{Q}{(z-R)^2}\hat{z}$
$2\pi R$
$\lambda =$
$\frac{-2Q}{2\pi R}$
$dq =$
$\lambda *$
$Rd\theta$
$d\vec{E} =$
$k\frac{\lambda R d\theta}{|z\hat{z}-Rr\hat{h}o|^3}(z\hat{z}-$
$R\hat{\rho})$
$Here, \hat{\rho}$
$z =$
$d\vec{E}_z \cdot$
$\hat{z} =$
$k\frac{\lambda R d\theta}{(z^2+R^2)^1.5}z$
$d\vec{E_{z_{ring}}}(z >$
$R) =$
$k\frac{\lambda R d\theta}{(z^2+R^2)^2.5}z\hat{z}$
$Now, we need to add up all of the charge. I'll use a polar integral around the circumference of the ring.$
$E_{z_{ring}}(z >$
$R) =$
$k\frac{\lambda R}{(z^2+R^2)^1.5}z\int_0^2 \pi d\theta$
$E_{z_{ring}}(z >$
$R) =$
$2\pi k\frac{\lambda R}{(z^2+R^2)^1.5}z$
$Of course, \pi R\lambda =$
$-2Q$
$z_{ring}(z >$
$R) =$
$-k\frac{2Q}{(z^2+R^2)^1.5}z$
$\equiv$
$\frac{1}{4\pi\epsilon_0}$
$-\hat{z}$
$\vec{E_z} =$
$\vec{E_{z_{ring}}}+$
$\vec{E_{z_{pc}}}$
$\vec{E_z}(z >$
$R, z <$
$-R) =$
$(-k\frac{2Q}{(z^2+R^2)^1.5}z+$
$k\frac{Q}{(z-R)^2}\hat{z}+$
$k\frac{Q}{(z+R)^2})\hat{z}(\theta(z+$
$R)-$
$1)$
$\theta(z)$
$V(x_1, x_2, x_3) =$
$-\int_\infty^{x_1}\int_\infty^{x_2}\int_\infty^{x_3}\vec{E}(x_1', x_2', x_3')\cdot$
$dl\hat{dl}$
$\vec{dl}$
(∞, ∞, ∞)
(x_1, x_2, x_3)
$\int_\infty^{z>R}(k\frac{-2Q}{(z^2+R^2)^1.5}z+$
$k\frac{Q}{(z-R)^2}\hat{z}+$
$k\frac{Q}{(z+R)^2})\hat{z}\cdot$