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- $\boxed{\text{The transfer fn}}$ of a circuit ~~: dimensionless~~

$H(\omega)$ ↳ $\boxed{\dfrac{\text{output circuit quantity}}{\text{input circuit quantity}}}$ to show how the amplitude & phase of the input phasors are modified

↑ obtained from the phasor analysis

① Voltage transfer fn ⇒ "voltage gain" or "attenuation vs freq." dimensionless

② current " ⇒ "current gain" or "current attenuation vs freq." "

③ impedance ⇒ $\dfrac{\text{output phasor = voltage}}{\text{input phasor = current}}$, unit = [ohm]

often called as "transimpedance gain"

④ Admittance transfer gain ⇒ $\dfrac{\text{current}}{\text{voltage}}$ unit = [mho, siemens]

often "transconductance"

∴ the transfer fn of a circuit allows the calculation of the output signal spectrum for a given periodic input signal

$$\underset{\text{output}}{X_o(\omega)} = H(\omega)\,\underset{\text{input}}{X_i(\omega)}$$

↓ inverse F.T

$\boxed{h(t)\text{: circuit impulse response fn}}$

often $\underset{\text{output}}{x_o(t)} = \int_{-\infty}^{\infty} \underset{\text{input}}{x_i(\tau)}\, \underset{\text{impulse } \delta(t)}{h(t-\tau)}\, d\tau \xrightarrow{\text{FT}} X_o(\omega) = X_i(\omega)H(\omega)$

$= x_i(t) * h(t)$ convolution

RC series circuit $h(t-\tau) = \begin{cases} e^{-\frac{t-\tau}{RC}} & \text{for } t-\tau > 0 \quad t \text{ makes sense} \\ 0 & \text{for } t-\tau \le 0 \end{cases}$

$\boxed{\text{"Causal signals"}}$: signals which have a time origin; they are zero for $t<0$

e.g. causal signals

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Introduction to electrical circuit theory using Fourier Transforms

$\widehat{\text{circuit theory}}$ transient analysis

2 important theorems

① Differentiation theorem.

Suppose $X(\omega)$ is the Fourier transform of a signal $x(t)$.

$$F.T.\left(\frac{dx(t)}{dt}\right) = i\omega X(\omega) - x(0).$$

pf $\quad X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-i\omega t}\, dt = \int_{0}^{\infty} \underbrace{x(t)}_{u}\, \underbrace{e^{-i\omega t}\, dt}_{v'}$

if $x(t)$ is a causal signal.

integrating by parts

Assume

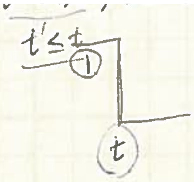
$$\boxed{\lim_{t\to\infty} \frac{x(t)}{i\omega} e^{-i\omega t} \to 0.}$$

② Integration theorem.

$$F.T.\left[\int_0^t x(t')\, dt'\right] = \frac{1}{i\omega} X(\omega) + \frac{X(\omega)}{2}\delta(\omega)$$

convolution theorem

$$\boxed{\begin{aligned} x(t) * y(t) &\xrightarrow{F.T} X(\omega)\, Y(\omega) \\ x(t)\, y(t) &\xrightarrow{F.T} X(\omega) * Y(\omega) \end{aligned}}$$



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Linear response theory (LRT)

deals with → Q: how does the system in eq. respond as its eq. state is disturbed?

→ formulation of the response fn of a many-ptl system which is stimulated by an external source

Assumption

① the external stimulation is weak enough that it can be treated as a perturbation which justifies to use the Taylor series expansion

② the perturbation expansion series are converging rapidly after the first linear term

∴ the first non-trivial linear term would be sufficient to describe the response of the system

response fn = a measurable quantity ∴ it is real-valued

e.g. in transport, response fn = a macroscopic transport coeff.

→ related to the correlation fns

∴ LRT describes a non-eq system in terms of fluctuations about its eq. ~~state~~

thus, it's essential to know the eq. system dynamics ⇒ to predict non-eq. situations

Example Suppose a system whose isolated Hamiltonian $= H_0$.

Now, ∃ a weak time-dep. disturbing field is applied at t_0

$\Rightarrow A \cdot F(t)$

$\therefore t > t_0, \quad H(t) = H_0 - A(t) F(t)$ → internal quantity conjugate to the field $F(t)$

LRT says.

$$\langle A(t) \rangle = \langle A(t) \rangle_0 + \int_{-\infty}^{t} dt' R(t,t') F(t') + O(F(t)^2)$$

$\langle A(t) \rangle_0$: average over eq. states ensembles

$R(t,t')$: linear response fn

t': the time at which the external field acts on the sys

t: the time of measurement

$\therefore t > t'$, the causality property

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Fluctuation and Dissipation Theorem

Stat Mech = a study of many-body systems to describe an effective way to treat large # of deg. of freedom.
→ quite successful to explain macroscopic thermodynamic phenomena in eq.

& it studies the non-eq. and irreversible processes, too.

F.D.T is one of numerous approaches for non-eq. cases.
→ it says that the non-eq property is quite closely related to the eq. states

ref. Bernard & Callen. "Irreversible thermodynamics of nonlinear processes and noise in driven systems." RMP 31, 1017 (1959)

ref. R. Kubo "The fluctuation-dissipation theorem"

a (general) relationship btw the response of a system disturbed by an external source and the internal fluctuations of the system w/o the disturbance

the validity of FDT lies in the "linear response regime", namely, the external disturbance is weak & the dominant response term is the linear one

- response ftn e.g. impedance, admittance
- internal fluctuations reflect the correlation ftn of physical quantities in thermal eq.

Roles of F.D.T

① predicts the fluctuation characteristics or intrinsic noise of the system from the known properties

② a basic formula to derive the known properties e.g. resistance etc from the analysis of fluctuations in the system.

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Example (prototype): Brownian Motion. to describe random behavior of objects.

↳ results in "statistical fluctuations in the thermal eq." system

Consider a 1D system. that moves in x w/ velocity $v(t)$ at $t=t$

↳ not completely isolated but only couples w/ the outside weekly

$\mathcal{F}(t)$ = the slowly varying external coupling

$F(t)$ = the interaction of the system w/ other deg. of freedom, which is rapidly fluctuating

↳ sets up 1-time scale, "correlation time" $\tau^* \cong$ the mean time btw $F(t)_{max}$

For macroscopic time τ, $\tau \gg \tau^*$.

$$\text{eq. of motion} = m\frac{dv(t)}{dt} = \mathcal{F}(t) + F(t)$$

velocity

① Integration over τ

③ Taking the ensemble average } → $m\frac{v(t+\tau)-v(t)}{\tau} \approx \int [\underbrace{\mathcal{F}(t)}_{\text{slowly varying}} + F(t)]\,dt$

$$\therefore \langle m[v(t+\tau)-v(t)]\rangle = \mathcal{F}(t)\tau + \int_t^{t+\tau}\langle F(t')\rangle dt'$$

$\langle F(t)\rangle$ is related to an energy change ΔE at T=temperature

$$\langle F(t)\rangle = \underbrace{\frac{1}{k_B T}}_{=\beta}\langle F(t)\Delta E\rangle_0$$

← ensemble average in eq. states

Note $\Delta E = -\int_t^{t'} dt''\, v(t'')F(t'') = \approx -v(t)\int_t^{t'} dt'' F(t'')$

↑ the energy change in the external world = the negative work down by the force $F(t)$

if the $v(t)$ does not vary over τ or very negligibly

$$\therefore \langle F(t')\rangle = -\beta\langle F(t)v(t)\int_t^{t'} dt'' F(t'')\rangle_0$$

$$= -\beta\overline{v(t)}\int_t^{t'} dt''\langle F(t)F(t'')\rangle_0 \quad \text{if } t''-t' \cong s$$

$-\alpha\bar{v}(t)$ frictional force

$$m\langle v(t+\tau)-v(t)\rangle = \mathcal{F}(t)\tau \boxed{-\beta\bar{v}(t)\int_t^{t+\tau}dt'\int_{t-t'}^{0} ds\underbrace{\langle F(t')F(t'+s)\rangle_0}_{\text{correlation fn}}}$$

↑ ~~fluctuations~~

Disc...