ECE 770 T14/QIC 885: Quantum Electronics & Photonics

Lecture 4

University of Waterloo, Instructor: A. H. Majedi, Winter 2013

Abstract and Objectives of Lecture 4

In the first lecture of lecture 4, I will investigate the electron propagation in an infinite and a finite periodic quantum structure using the transfer matrix method and Floquet's theorem. The general properties of periodic electron wavefunction is highlighted and the formation of energy bands is discussed in detail. As an example, we will talk about the Kronig-Penny model where an infinite series of delta-function potential barriers are considered. Using this method, the crystal momentum concept and the effective electron mass are introduced. Multiple Quantum Well as an example of a finite periodic structure will be analyzed through a combination of Floquet theorem and transfer matrix method. The WKB approximation method to find stationary states from arbitrary potential is introduced and some examples will be discussed.

In the second lecture of lecture 4, I will focus on the mathematical framework of quantum mechanics to come up with a set of postulates to formally establish a theory of quantum mechanics. Mathematical machinery of QM is based on two constructs; **state vectors** and **operators**. By recalling what we have learned from Schrodinger wave mechanics, we introduce the concept of state vector and Dirac notation. State vectors live in Hilbert space and we address briefly where is Hilbert space and present two main representations of that based on matrices and functions. Brief explanation of the generalized functions and rigged Hilbert space is presented. Then we introduce the second construct of QM framework, operators. We focus on Hermitian operators to associate them to physical observables. By introducing the eignefunctions and eigenvalues of the operators we will discuss the determinate or pure states and the two spectra associated with them, namely

discrete and continuous.

Topics of Lecture 4

2-5 Electron Propagation in a Periodic Quantum Structure

2-6 Kronig-Penny Model

2-7 Crystal Momentum and Effective Mass

2-8 Multiple Quantum Well

2-9 WKB method

3- Axiomatic Structure of QM

3-1 Anatomy of Schrodinger Wave Mechanics

3-2 Dirac Notation and State vectors

3-3 Hilbert Space

3-4 Matrix and Function Representation of Hilbert Space

3-5 Operators

3-6 Observable

3-7 Eigenvalues & Eigenvectors of an Operator

3-8 Determinate (Pure) States

References & Suggested Readings

1- D.J. Griffith, *Introduction to Quantum Mechanics*, 2^{nd} ed., PH, 2005, Chapters 2 and 8 .

2- H. Kroemer, *Quantum Mechanics: For engineering, material sciences and applied physics*, JWS, 1994, Chapters 2 and 5

3- A.F.J. Levi, *Applied Quantum Mechanics*, 2^{nd}*ed.*, Cambridge Press, 2006, Chapter 4.

4- P. Yeh, "Optical Waves in Layered Media", JW& S, 2005, Chapters 11 and 12.

5- J. Emerson, *Open Quantum Systems*, U. Waterloo, AMATH 876, Winter 2009

6- A. Peres, *Quantum Theory, Concepts and Methods*, AP, 1995.

7- L. E. Ballentine, *Quantum Mechanics: A Modern Development*, World Scientific, 2001.