Physics 760

Assignment 2

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Contents

Jackson 3rd ed. : 2.23

a

A hollow cube has conducting walls defined by six planes $x = 0$, $y = 0$, $z = 0$, and $x = a$, $y = a$, $z = a$. The walls $z = 0$ and $z = a$ are held at a constant potential V. The other four sides are at zero potential.
Find the potential $\Phi(x, y, z)$ at any point inside the cube.
The solution to a problem of this type is found by solving Laplace's equation, $\nabla^2 = 0$ everywhere in space where charges are absent. Given the rectangular symmetry of the problem I will propose a solution in Cartesian coordinates. That is, I will propose a solution of the form: $V(x, y, z) = V_x(x)V_y(y)V_z(z)$. Inserting this into Laplace's equation above yields:

$$\begin{aligned}\nabla^2 V &= V_y(y)V_z(z)\frac{\partial^2 V_x(x)}{\partial x^2} + V_x(x)V_z(z)\frac{\partial^2 V_y(y)}{\partial y^2} + V_x(x)V_y(y)\frac{\partial^2 V_z(z)}{\partial z^2} \\ &= \frac{1}{V_x(x)}\frac{\partial^2 V_x(x)}{\partial x^2} + \frac{1}{V(y)}\frac{\partial^2 V_y(y)}{\partial y^2} + \frac{1}{V_z(z)}V_x(x)V_y(y)\frac{\partial^2 V_z(z)}{\partial z^2} = 0\end{aligned}$$

This implies that each term in the sum is a constant such that the sum of all of the terms is zero for any particular x, y or z.

$$\frac{\partial^2 V_x(x)}{\partial x^2} = -\alpha^2 V_x(x) \quad , \quad \frac{\partial^2 V_y(y)}{\partial y^2} = -\beta^2 V_y(y) \quad , \quad \frac{\partial^2 V_z(z)}{\partial z^2} = -\gamma^2 V_z(z)$$

$$\begin{aligned}V_x(x) &= A\exp(-i\alpha x) + B\exp(i\alpha x) \\ V_y(y) &= C\exp(-i\beta y) + D\exp(i\beta y) \\ V_z(z) &= E\exp(\sqrt{\alpha^2+\beta^2}\, z) + F\exp(-\sqrt{\alpha^2+\beta^2}\, z)\end{aligned}$$

Solving for A and B using $V_x(x)$'s two boundary conditions:

$$V_x(a) = 0 = A + B \quad , \quad 0 = A(\exp(-i\alpha a) - \exp(i\alpha a)) = -2iA\sin(\alpha a) \quad , \quad \alpha = \frac{n\pi}{a} \quad , \quad n = 0, 1, 2, ...$$

Solving for C and D using $V_y(y)$'s two boundary conditions:

$$V_y(a) = 0 = C + D \quad , \quad 0 = C(\exp(-i\alpha a) - \exp(i\alpha a)) = -2iC\sin(\alpha a) \quad , \quad \alpha = \frac{m\pi}{a} \quad , \quad m = 0, 1, 2, ...$$

Solving for E and F using $V_z(z)$'s two boundary conditions:

$$\begin{aligned}\Phi(x,y,z) &= \sum_{\text{odd n } > \, 0}\ \sum_{odd m>0} \sin(\frac{n\pi x}{a})\sin(\frac{m\pi x}{a})(A_{nm}\exp(-\theta z) + B_{nm}\exp(\theta z)) \\ \Phi(x,y,0) = V &= \sum_{\text{odd n } > \, 0}\ \sum_{odd m>0} \sin(\frac{n\pi x}{a})\sin(\frac{m\pi x}{a})(A_{nm} + B_{nm}) \\ \Phi(x,y,a) = V &= \sum_{\text{odd n } > \, 0}\ \sum_{odd m>0} \sin(\frac{n\pi x}{a})\sin(\frac{m\pi x}{a})(A_{nm}\exp(-\theta a) + B_{nm}\exp(\theta a))\end{aligned}$$

Note above the change in variables $\sqrt{\alpha^2+\beta^2} = \sqrt{(\frac{n\pi x}{a})^2 + (\frac{m\pi y}{a})^2} = \theta$.

Using the orthogonality of sine functions I can multiply both sides by two functions $f(x) = sin(\frac{k\pi x}{a})$ and $g(y) = sin(\frac{l\pi y}{a})$ and integrate over the domain of the box to obtain the following two results. Note that I have used the following two integral identities: $\int_0^a sin(\frac{k\pi x}{a})dx = \frac{2a}{k\pi}$ for odd k (the integral is zero for even k) and $\int_0^a sin^2(\frac{k\pi x}{a})dx = \frac{a}{2}$.

$$\int_0^a \int_0^a V sin(\frac{k\pi x}{a}) sin(\frac{l\pi y}{a}) dxdy = \int_0^a sin^2(\frac{k\pi x}{a}) sin^2(\frac{l\pi y}{a})(A_{kl} + B_{kl})dxdy$$

$$\frac{16V^2}{kl\pi} = A_{kl} + B_{kl}$$

By similar analysis I can write the following below:

$$\frac{16V^2}{kl\pi} = A_{kl}\exp(-\theta a) + B_{kl}\exp(\theta a)$$

Combining these two equations into a matrix equation and inverting:

$$\begin{pmatrix} P \\ P \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ \exp(-\theta a) & \exp(\theta a) \end{pmatrix} \begin{pmatrix} A_{kl} \\ B_{kl} \end{pmatrix}$$

Above I have used the value P to absorb all the relevant constants: $P = \frac{16V}{kl\pi}$
By inverting this 2x2 matrix we obtain:

$$\begin{pmatrix} A_{kl} \\ B_{kl} \end{pmatrix} = \frac{1}{exp(\theta a) - exp(-\theta a)} \begin{pmatrix} exp(\theta a) & -1 \\ -\exp(-\theta a) & 1 \end{pmatrix} \begin{pmatrix} P \\ P \end{pmatrix}$$

Now it is clear that $A_{kl} = P\frac{(\exp(\theta a)-1)}{\exp(\theta a)-\exp(-\theta a)}$ and that $B_{kl} = P\frac{(1-\exp(-\theta a))}{\exp(\theta a)-\exp(-\theta a)}$. Exponential identities can be used to reduce these expressions: $A_{kl} = P\frac{\exp(\theta a)}{1+\exp(\theta a)}$ and $B_{kl} = P\frac{1}{1+\exp(\theta a)}$.
Thus, the potential can be written as :

$$\boxed{\Phi(x, y, z) = \sum_{oddn}\sum_{oddm} \frac{16V}{nm\pi}\frac{1}{1+\exp(\theta a)} \sin(\frac{n\pi x}{a}) \sin(\frac{m\pi y}{a})(\exp(\theta a)exp(-\theta z) + \exp(\theta z))}$$

.

b

Evaluate the potential at the center of the cube numerically, accurate to three significant figures. How many terms in the series is it necessary to keep in order to attain this accuracy? Compare your numerical result with the average value of the potential on the walls. See Problem 2.28.

Having evaluated this, I obtained $\approx \frac{V}{3}$. The value I obtained to three significant figures was $.333V$. This compares well with the average wall-potential: $V + V/6 = \frac{V}{3}$.Below, I have included a table of the contribution of each term in the double sum to the value of the potential at the center of the cube. These were obtained using Matlab code included in the appendix. The m and n values associated with the contribution of the mn^{th} term to the sum determine the ordering. I.E. The larger the “weight” of the term in the sum, the higher that value of that m and n value in the sum (they are ordered higher).

Term #	m	n	Term's Weight	Cumulative Sum
1	1	1	0.34754584866469	0.347545848664695
2	3	1	-0.00752383905851	0.340022009606181
3	1	3	-0.00752383905851	0.332498170547667
4	3	3	0.00045954463592	0.332957715183595
5	5	1	0.00021547126164	0.333173186445239
6	1	5	0.00021547126164	0.333388657706883
7	5	3	-2.27484874789757	0.333365909219404
8	3	5	-2.27484874789757	0.333343160731925
9	7	1	-6.94949638519198	0.333336211235540
10	1	7	-6.94949638519198	0.333329261739155

Thus, it seems as if I need the first 4 terms in my sub in order to reach 3 significant digits. The 5th term has a weight of $2 \cdot 10^{-4}$ which is not strong enough to alter the third significant digit.

c

Find the surface-charge density on the surface z = a.
To find the surface charge density $\sigma(x, y)$ I will take advantage of the fact that due to the discontinuity in the electric field, the change in potential with respect to the normal direction at a surface is proportional to the surface charge density. That is $\frac{\partial V}{\partial n} = \frac{\sigma}{\epsilon_0}$. In this case, the normal vector is in the z direction. So, I must take a partial derivative of my potential with respect to z and multiply it by ϵ_0 and evaluate it at $z = a$.

$$\sigma(x,y) = \epsilon_0 \sum_{odd\ n} \sum_{odd\ m} \frac{16V}{nm\pi} \frac{1}{1+\exp(\theta a)} \sin(\frac{n\pi x}{a}) \sin(\frac{m\pi y}{a}) \frac{\partial}{\partial z} \big(\exp(\theta a) exp(-\theta z) + \exp(\theta z) \big) \Big|_{z=a}$$
$$\sigma(x,y) = \epsilon_0 \sum_{odd\ n} \sum_{odd\ m} \frac{16V}{nm\pi} \frac{1}{1+\exp(\theta a)} \sin(\frac{n\pi x}{a}) \sin(\frac{m\pi y}{a}) \big(-\theta \exp(\theta a) exp(-\theta a) + \theta \exp(\theta a) \big)$$
$$\sigma(x,y) = \epsilon_0 \sum_{odd\ n} \sum_{odd\ m} \frac{16V}{nm\pi} \frac{1}{1+\exp(\theta a)} \sin(\frac{n\pi x}{a}) \sin(\frac{m\pi y}{a}) \theta \big(-1 + \exp(\theta a) \big)$$

Realizing that $\frac{e^x - 1}{e^x + 1} = \tanh(x/2)$ this last expression can be rewritten

$$\boxed{\sigma(x,y) = \epsilon_0 \sum_{odd\ n} \sum_{odd\ m} \frac{16V}{nm\pi} \sin(\frac{n\pi x}{a}) \sin(\frac{m\pi y}{a}) \tanh(\theta a/2)\theta}$$

Jackson 3rd ed. : 3.1

Two concentric spheres have radii a, b ($b > a$) and each is divided into two hemispheres by the same horizontal plane. The upper hemisphere of the inner sphere and the lower hemisphere of the outer sphere are maintained at potential V. The other hemispheres are at zero potential Determine the potential in the region $a < r < b$ as a series in Legendre polynomials. Include terms at least up to $l = 4$. Check your solution against known results in the limiting cases

$b \to \infty$, and $a \to 0$.

The boundary conditions are expressed in terms of potentials. Thus, I should use Laplace's equation and the Legendre polynomials to determine the potential everywhere between the two spheres. Due to the azimuthal symmetry of the problem I can assume that "$m = 0$" for my associated Legendre polynomials.

$$\Phi(r,\theta) = \sum_{l=0}^{\infty} \left(A_l r^l + B_l r^{-(l+1)} P_l(\cos\theta) \right)$$

Using orthogonality of the Legendre polynomials we can solve for A_l and B_l. We have two boundary conditions. At $r = a$, $\pi/2 > \phi > 0$ $\Phi = V$ and at $r = b$, $\pi/2 < \phi < \pi$, $\Phi = V$. This sets up the two equations (note that $\int\limits_{-1}^{1} P_l(\cos\theta) P_m(\cos\theta) d\cos\theta = \frac{2}{2l+1}\delta_{lm}$).

$$\frac{2k+1}{2} \int_{\theta=0}^{\theta=\pi} V(a,\theta) P_k(\cos\theta) d\cos\theta = A_k a^k + B_k a^{-(k+1)}$$
$$\frac{2k+1}{2} \int_{\theta=0}^{\theta=\pi} V(b,\theta) P_k(\cos\theta) d\cos\theta = A_k b^k + B_k b^{-(k+1)}$$

These two integrals can be substantially reduced by realizing that the potential (hence, the integrand) is zero for half of the integration.

$$V\frac{2k+1}{2} \int_{\theta=0}^{\theta=\pi/2} P_k(\cos\theta) d\cos\theta = A_k a^k + B_k a^{-(k+1)}$$
$$V\frac{2k+1}{2} \int_{\theta=\pi/2}^{\theta=\pi} P_k(\cos\theta) d\cos\theta = A_k b^k + B_k b^{-(k+1)}$$

In general, these integrals have no closed form analytic solution (truth be told, these integrals can be expressed in terms of gamma functions, but this is unnecessarily complicated and not particularly enlightening). I will, at this time, change integration variables from $\cos\theta \to x$.

$$V\frac{2k+1}{2} \int_{1}^{0} P_k(x) dx = A_k a^k + B_k a^{-(k+1)}$$
$$V\frac{2k+1}{2} \int_{0}^{-1} P_k(x) dx = A_k b^k + B_k b^{-(k+1)}$$

Now, a useful property for evaluating Legendre polynomials is the following: $P_l(x) = \dfrac{1}{2l+1}\frac{d}{dx}\left(P_{l+1}(x) + P_{l-1}(x)\right)$. One might be concerned regarding the $P_0(x)$ case. However, $P_0(x) = 1$ from $x = -1 \to 1$. Thus, the expression for the $P_0(x)$ case is given below. After, the $P_l(x)$ cases will be evaluated (where $l > 1$).

$$\frac{V}{2} \int_{1}^{0} P_0(x) dx = A_k a^k + B_k a^{-(k+1)} = -\frac{V}{2}$$
$$\frac{V}{2} \int_{0}^{-1} P_0 x dx = A_k b^k + B_k b^{-(k+1)} = -\frac{V}{2}$$

For $k > 1$ we consider the first integral (the one whose bounds are $1 \to 0$), $V\frac{(2k+1)}{2}\int_1^0 P_k(x)dx$

$$\frac{V(2k+1)}{2} \int_{1}^{0} P_k(x) dx = \frac{V}{2}\left(P_{k+1}(x) + P_{k-1}(x)\right)\Big|_1^0$$

Throwing in a couple more Legendre polynomial properties (thank you, Wikipedia!) $P_l(1) = 1 \forall l$ and $(-1)^l P_l(-x) = P_l(x) \forall l$. In plain English, the second identity states that if l is odd that P_l is odd. So, $P_l(-1) = -1$ for odd l. For even l, $P_l(-1) = 1$. Considering even $l = 2p\ p = 0, 1, 2, ...$

$$\begin{aligned}\frac{V}{2}\int_1^0 P_{2p}(x)dx &= \frac{V}{2}\int_1^0 \frac{d}{dx}\left(P_{2p+1}(x) + P_{2p-1}(x)\right)dx \\ &= \frac{V}{2}\big(P_{2p+1}(1) - P_{2p-1}(1) + P_{2p-1}(0) - P_{2p+1}(0)\big)\end{aligned}$$

According to the properties above this can be easily seen to be zero for all p. Now, considering odd $l = 2p + 1\ p = 0, 1, 2, ...$

$$\begin{aligned}\frac{V}{2}\int_1^0 P_{2p+1}(x)dx &= \frac{V}{2}\int_1^0 \frac{d}{dx}\left(P_{2p+2}(x) + P_{2p}(x)\right)dx \\ &= \frac{V}{2}\big(P_{2p+2}(0) - P_{2p}(0) + P_{2p}(1) - P_{2p+2}(1)\big)\end{aligned}$$

By the properties listed above we can reduce this to the following expression:

$$= \frac{V}{2}\big(P_{2p+2}(0) - P_{2p}(0)\big)$$

As there is no closed form solution for this expression (outside of the use of gamma functions) this integral is for all intents and purposes completed.

Performing similar analysis on the other integral yields the following:

$$\int_{-1}^0 V(2l+1)P_l(x)dx = \begin{cases} 0 & \text{l is even and } l > 0 \\ -\frac{V}{2}\big(P_{l+1}(0) - P_{l-1}(0)\big) & \text{l is odd} \\ -V & \text{l is 0} \end{cases}$$

Succinctly put, the first result is:

$$V(2l+1)\int_1^0 P_l(x)dx = \begin{cases} 0 & \text{l is even and } l > 0 \\ \frac{V}{2}\big(P_{l+1}(0) - P_{l-1}(0)\big) & \text{l is odd} \\ -\frac{V}{2} & \text{l is 0} \end{cases}$$

Given the similarity of the two expressions I will allow the expression $P_{l+1}(0) - P_{l-1}(0) = \kappa_l$. So, expressing the two equations for A_k and B_k (for odd k) in a matrix form yields:

$$\frac{V}{2}\kappa_k \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} a^k & a^{-(k+1)} \\ b^k & b^{-(k+1)} \end{pmatrix} \begin{pmatrix} A_k \\ B_k \end{pmatrix}$$

Inverting this, we obtain:

$$\boxed{\begin{pmatrix} A_k \\ B_k \end{pmatrix} = \frac{\frac{V}{2}\kappa_k}{a^k b^{-(k+1)} - b^k a^{-(k+1)}} \begin{pmatrix} b^{-(k+1)} & -a^{-(k+1)} \\ -b^k & a^k \end{pmatrix} \begin{pmatrix} 1 \\ -1 \end{pmatrix}}$$

Now, writing an expression for A_k and B_k and substituting this into the original expression for $\Phi(r, \theta)$ would be extremely cumbersome at this point. Rather than do this, we will leave this as the solution for this problem.

Considering the two cases where $a \to 0$ and $b \to \infty$. As $a \to 0$ the A_k and B_k terms go to b^{-k} and 0 respectively. Thus, for the case where the small sphere "disappears" we recover: $V(r,\theta) = \sum\limits_{l=0}^{\infty} \frac{V}{2}\kappa_l \frac{r^l}{b^l} P_l \cos\theta$ for odd k.

For the case where $b \to \infty$ the A_ks can be shown to go to zero (rather trivially) and the B_k terms go to a^{1+k}. Thus, for the case of $b \to \infty$ we recover the expression: $V(r,\theta) = \sum\limits_{l=0}^{\infty} \frac{V}{2}\kappa_l r^{-(l+1)} a^{1+l} P_l \cos\theta$
By making the appropriate substitutions for κ_l we recover the expressions for a single conducting sphere biased at two separate potentials starting at $\theta = 0$ and $\theta = \frac{\pi}{2}$ for $r < R$ (the radius of the single sphere), in the first case (as $a \to 0$) and $r > R$ in the second case (as $b \to \infty$).

Jackson 3rd ed. : Problem 3.2

A spherical surface of radius R has charge uniformly distributed over its surface with a density $\frac{Q}{4\pi R^2}$, except for a spherical cap at the north pole, defined by the cone $\theta = \alpha$.

a

Show that the potential inside the spherical surface can be expressed as

$$\frac{Q}{8\pi\epsilon_0}\sum_{l=0}^{\infty}\frac{1}{2l+1}\left(P_{l+1}(\cos\alpha) - P_{l-1}(\cos\alpha)\right)\frac{r^l}{R^{l+1}}P_l(\cos\theta)$$

where, for $l = 0$, $P_l(\cos\alpha) = -1$. What is the potential outside?
I will integrate over the charge density in the usual way to obtain the potential.

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0}\int\limits_0^{2\pi}\int\limits_0^{\pi}\int\limits_0^{\infty}\sigma\delta(|\vec{r}| - R)H(\alpha - \theta)|\vec{r} - \vec{r'}|^{-1}r'^2 d\Omega'$$

Here, we have allowed $\Omega' \equiv \sin\theta' d\theta' d\phi'$ and H(x) is the Heaviside step function in x. Now, we can use the following identity:

$$|\vec{x} - \vec{x'}|^{-1} = 4\pi\sum_{l=0}^{\infty}\frac{r^l}{R^{l+1}}P_l \cos\gamma \quad \text{where r < R and } \gamma \text{ is the angle between } \vec{r} \text{ and } \vec{r'}$$

$$V(|\vec{r}| < R) = \frac{\sigma R^2}{4\pi\epsilon_0}\sum_{l=0}^{\infty}\frac{r^l}{R^{l+1}}\int\limits_0^{2\pi}\int\limits_\alpha^{\pi}P_l(\cos\gamma)d\Omega'$$

In general, γ could be a function of both θ' and ϕ' so this could be a very hard integral. However, the following result, known as the "spherical harmonic addition theorem" allows us to express

$$P_l(\cos\gamma) = P_l(\cos\theta')P_l(\cos\theta) + 2\sum_{m=-l}^{l}\frac{(l-m)!}{(l+m)!}P_l^m(\cos\theta')P_l^m(\cos\theta)\cos(m(\phi' - \phi))$$

By noting that the charge distribution has no azimuthal asymmetry we can assume that there must be no dependence on ϕ in the final answer. This implies that $m = 0$. Substituting the reduced expression into the integral we find:

$$V(|\vec{r}| < R) = \frac{\sigma R^2}{4\pi\epsilon_0}\sum_{l=0}^{\infty}\frac{r^l}{R^{l+1}}\int\limits_0^{2\pi}\int\limits_\alpha^{\pi}P_l(\cos\theta')P_l(\cos\theta)d\Omega'$$

Now, I will seize the extremely powerful property of Legendre polynomials that I have used in the last problem:

$$P_n(x) = \frac{1}{2n+1}\frac{d}{dx}\left(P_{n+1}(x) - P_{n-1}(x)\right)$$

.

$$V(|\vec{r}| < R) = \frac{\sigma R^2}{4\pi\epsilon_0}\sum_{l=0}^{\infty}\frac{r^l}{R^{l+1}}P_l(\cos\theta)\int_0^{2\pi}\int_{\theta=\alpha}^{\theta=\pi}\frac{1}{2l+1}\frac{d}{d\cos\theta'}\left(P_{l+1}(\cos\theta') - P_{l-1}(\cos\theta')\right)d\sin\theta' d\theta' d\phi'$$

$$V(|\vec{r}| < R) = \frac{2\pi\sigma R^2}{4\pi\epsilon_0}\sum_{l=0}^{\infty}\frac{r^l}{R^{l+1}}P_l(\cos\theta)\frac{1}{2l+1}\left(P_{l+1}(\cos\theta')|_{\theta=\alpha}^{\theta=\pi} - P_{l-1}(\cos\theta')|_{\theta=\alpha}^{\theta=\pi}\right)$$

Realizing that $P_{l+1}(-1) - P_{l-1}(-1) = 0$ for all l the final expression is obtained.

$$V(|\vec{r}| < R) = \frac{2\pi\sigma R^2}{4\pi\epsilon_0}\sum_{l=0}^{\infty}\frac{1}{2l+1}\frac{r^l}{R^{l+1}}P_l(\cos\theta)\big(P_{l+1}(\alpha) - P_{l-1}(\alpha)\big)$$

$$V(|\vec{r}| < R) = \frac{2\pi Q R^2}{4\pi\epsilon_0 4\pi R^2}\sum_{l=0}^{\infty}\frac{1}{2l+1}\frac{r^l}{R^{l+1}}P_l(\cos\theta)\left(P_{l+1}(\cos\alpha) - P_{l-1}(\cos\alpha)\right)$$

$$\boxed{V(|\vec{r}| < R) = \frac{Q}{8\pi\epsilon_0}\sum_{l=0}^{\infty}\frac{1}{2l+1}\frac{r^l}{R^{l+1}}P_l(\cos\theta)\left(P_{l+1}(\cos\alpha) - P_{l-1}(\cos\alpha)\right)}$$

The field outside of the sphere would follow the same procedure except for that, now, the roles of R and r would interchange with respect to equation 1 listed above. Thus, the potential can easily be re-expressed for the case outside the sphere as : $V(|\vec{r}| > R) = \frac{Q}{8\pi\epsilon_0}\sum\limits_{l=0}^{\infty}\frac{1}{2l+1}\frac{R^l}{r^{l+1}}P_l(\cos\theta)\left(P_{l+1}(\cos\alpha) - P_{l-1}(\cos\alpha)\right)$

b

Find the magnitude and the direction of the electric field at the origin. In order to find $\vec{E} = -\nabla V$ I need to identify ∇ in spherical coordinates. According to Wikipedia, this is : $\frac{\partial f}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial f}{\partial\theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial f}{\partial\phi}\hat{\phi}$. The electric potential for this problem only depends on r and θ. This allows the derivative over ϕ to be neglected. Thus, the electric field is:

$$V = \frac{Q}{8\pi\epsilon_0}\sum_{l=0}^{\infty}\frac{1}{2l+1}\frac{\big(P_{l+1}(\cos\alpha) - P_{l-1}(\cos\alpha)\big)}{R^{l+1}}r^l P_l(\cos\theta)$$

$$-\vec{E} = \nabla\left(A\sum_{l=0}^{\infty}\gamma_l P_l(\cos\theta)\frac{\partial r^l}{\partial r}\hat{r} + A\sum_{l=0}^{\infty}\gamma_l r^l\frac{\partial p_l(\cos\theta)}{\partial\theta}\hat{\theta}\right)$$

Above, γ_l has absorbed the constants within the sum.

$$= A\sum_{l=0}^{\infty}\gamma_l l r^{l-1}P_l(\cos\theta)\hat{r} + A\sum_{l=0}^{\infty}\gamma_l\frac{1}{r}r^l\frac{\partial p_l(\cos\theta)}{\partial\theta}\hat{\theta}$$

$$= A\sum_{l=0}^{\infty}\gamma_l l r^{l-1}P_l(\cos\theta)\hat{r} + A\sum_{l=0}^{\infty}\gamma_l\frac{1}{r}r^l\frac{\partial p_l(\cos\theta)}{\partial\cos\theta}\frac{\partial\cos\theta}{\partial\theta}\hat{\theta}$$

$$= A\sum_{l=0}^{\infty}\gamma_l l r^{l-1}P_l(\cos\theta)\hat{r} - A\sum_{l=0}^{\infty}\gamma_l r^{l-1}\frac{\partial p_l(\cos\theta)}{\partial\cos\theta}\sin\theta\hat{\theta}$$

Now, I want the solution at this expression at $r = 0$. There, it really doesn't matter what value θ takes on, physically. Thus, I will select a θ that makes my life easy to work with. I will allow $\theta = 0$ such that $\cos\theta = 1$ since for all l $P_l(1) = 1$.

$$= A\sum_{l=0}^{\infty}\gamma_l l r^{l-1}P_l(\cos\theta)\hat{r} - A\sum_{l=0}^{\infty}\gamma_l r^l \frac{\partial p_l(\cos\theta)}{\partial\cos\theta}\sin\theta\hat{\theta}$$

Now, it seems that the potential is zero at $r = 0$ since r^l occurs in both sums. And the second sum is zero by the angular constraint mentioned earlier. The first sum would be zero except for the $l = 1$ term. This raises the undefined condition whereby 0^0 results in the sum. What value this should have is hotly debated in mathematical literature. However, it can be reasoned that $0^0 = 1$ since $\lim_{r\to 0} r^0 = \lim_{r\to 0} 1 = 1$. Thus, one term in the sum survives and, thus, the electric field at the origin is:

$$\vec{E}(0,0,0) = A\gamma_1\hat{r}$$

$$A = \frac{Q}{8\pi\epsilon_0} \quad , \quad \gamma_1 = \frac{P_2(\cos\alpha) - P_0(\cos\alpha)}{R^{1+1}\big(2(1)+1\big)} \quad , \quad P_2(\cos\alpha) = .5(3\cos^2\alpha - 1) \quad , \quad P_0(\cos\alpha) = 1$$

Substituting the above expressions into the expression for $\vec{E}$:

$$\begin{aligned}\vec{E}(0,0,0) &= \frac{Q}{8\pi\epsilon_0}(1.5\cos^2\alpha - 1.5)\frac{1}{3R^2}\hat{r}\\ &= \frac{Q}{16\pi\epsilon_0}\frac{\cos^2\alpha - 1}{R^2}\hat{r}\\ &= \frac{Q}{16\pi\epsilon_0}\frac{\sin^2\alpha}{R^2}\end{aligned}$$

$$\boxed{\vec{E}(0,0,0) = \frac{Q}{16\pi\epsilon_0}\frac{\sin^2\alpha}{R^2}}$$

c

Discuss the limiting forms of the potential (part a) and electric field (part b) as the spherical cap becomes A) very small, and B) so large that the area with charge on it becomes a very small cap at the south pole.

In will discuss the four cases in the following order: $\vec{E}$ field for small then large cap followed by V for small then large cap.

The first case, where the cap is small, is easily handled by considering an expansion of $\sin^2\alpha$ for small α. To first order in α this yields α^2. Thus, for small α, $\vec{E}(0,0,0) \approx \frac{Q}{16\pi\epsilon_0}\frac{\alpha^2}{R^2}$.

Similarly, when the cap is large, $\sin^2\alpha$ can be handled again. Expanding, this time about $\alpha = 2\pi$ yields $\alpha - 2\pi$ to first order in α. Thus, for large caps, the field looks approximately like $\frac{Q}{16\pi\epsilon_0}\frac{(\alpha-2\pi)^2}{R^2}$.

The potential is a little tricker to discuss. The nature of what happens inside the sphere as $\alpha \to 0$ is tucked inside of Legendre polynomials. Firs, then, I will say that $\cos\alpha$ for $\alpha \approx 0$ is $\approx 1 - \frac{\alpha^2}{2}$. Then, I will expand the Legendre polynomials to their first derivative and evaluate near $\alpha = 0$ (that is, I'll expand $P_l(x)$ about 1).

$$P_l^2(x = 1 - \frac{\alpha^2}{2})|_{x=1} = P_l(1) + \frac{d}{dx}P_l(x)|_{x=1}((1 - \frac{\alpha^2}{2}) - 1)$$

Now, considering P(1). It is 1 for all values of l greater than -1. However, we have a term in our sum whereby $l = -1$. We can thus express $P_l(1) = 1 - 2\delta_{-1,l}$. This is a nice expression for $P_l(1)$ (thanks Jean-Philippe and Jeremy Bejanin for collaborating on this homework with me.

$$P_{l+1} - P_{l-1} = 2(\delta_{0,l} - \delta_{-2,l}) - \frac{\alpha^2}{2}(P'_{l+1}(1) - P'_{l-1}(1))$$

Note, however, that l never equals -2. So, this kronecker delta function is always zero. At this time we can use our handy "Legendre derivative" property: $P_l(x) = \frac{1}{2l+1}\frac{d}{dx}(P'_{l+1}(x) - P'_{l-1}(x))$.

$$P_{l+1} - P_{l-1} = 2(\delta_{0,l}) - \frac{\alpha^2}{2}(2l+1)P_l(1)$$

This is simplified enough to substitute back into the sum.

$$V(|\vec{R}| < R \text{ for small } \alpha \text{ is}) \approx \frac{Q}{8\pi\epsilon_0}\sum_{l=0}^{\infty}\frac{1}{2l+1}\frac{r^l}{R^{l+1}}P_l(\cos\theta)\left(2(\delta_{0,l}) - \frac{\alpha^2}{2}\frac{2l+1}{2}P_l(1)\right)$$

And, finally,

$$V(|\vec{R}| < R \text{ for small } \alpha \text{ is}) \approx \frac{Q}{4\pi\epsilon_0 R} - \frac{Q\alpha^2}{4\pi\epsilon_0}\sum_{l=0}^{\infty}\frac{r^l}{R^{l+1}}P_l(\cos\theta)$$

This wouldn't change for the case where where $|\vec{r}| > R$ except for that, now, the roles of r^l and R^{l+1} would change.

Jackson 3rd ed. : 3.4a

The surface of a hollow conducting sphere of inner radius a is divided into an even number of equal segments by a set of planes; their common line of intersection is the z axis and they are distributed uniformly in the angle ϕ. (The segments are like the skin on wedges of an apple, or the earth's surface between successive meridians of longitude.) The segments are kept at fixed potentials $\pm V$, alternately.

Set up a series representation for the potential inside the sphere for the gen- eral case of 2n segments, and carry the calculation of the coefficients in the series far enough to determine exactly which coefficients are different from zero. For the nonvanishing terms, exhibit the coefficients as an integral over $\cos\theta$.

The first thing to realize is that no longer is there azimuthal symmetry in this problem. Thus, $\Phi(r, \theta, \phi) = \sum\limits_{l=0}^{\infty}\sum\limits_{m=-l}^{l}(A_{lm}r^l + B_{lm}r^{-(l+1)})Y_{lm}(\theta, \phi)$. I have been given $V(R, \phi)$. Now, I need to use orthogonality and slick mathematical tricks to get my answer into a tractable form. $V(R, \phi)$ alternates sign based on how many "slices" the sphere has in it (there are 2n wedges for n slices). Thus, the potential can be expressed in the following form if the sphere is aligned such that at $\phi = 0$ the dividing line between V and -V is along the x-axis and for $\phi \in (0, \pi/n)$ the potential is positive.

$$\Phi(\Phi, R) = \begin{cases} V & \phi \in \left(\frac{\pi 2j}{n}, \frac{\pi(2j+1)}{n}\right) \\ V & \phi \in \left(\frac{\pi(2j+1)}{n}, \frac{\pi(2j+2)}{n}\right) \end{cases} \quad \text{for } j = 0, 1, 2, ..., n-1$$

Thus, in the usual way, we will use orthogonality to obtain the coefficients in terms of the boundary conditions. Note, though, that this solution will determine Φ inside of the sphere. Thus, all B_l coefficients must drop out in order that Φ be normalizable (not divergent at $r = 0$).

$$A_{ab}R^k = \int_0^{2\pi} \int_0^{\pi} V(R, \phi) Y^*_{ab} \sin\theta d\theta d\phi$$

These spherical harmonics, Y_{km} can be expressed in terms of associated Legendre polynomials P_{lm}, some constants and a complex exponential by :

$$Y_{ab} = \sqrt{\frac{(2a+1)}{4\pi} \frac{(a-b)!}{(a+b)!}} P_{ab}(\cos\theta) \exp(ib\phi)$$

$$A_{ab}R^k = \sqrt{\frac{(2a+1)}{4\pi} \frac{(a-b)!}{(a+b)!}} \int_0^{2\pi} V(R, \phi) \exp(-ib\phi) d\phi \int_0^{\pi} P_{ab}(\cos\theta) \sin\theta d\theta d\phi \tag{1}$$

To be succinct, the first term will become a constant γ_{ab}. I will also deal with only solving the first integral for a while. This one deserves some treatment.

$$\int_0^{2\pi} V(R, \phi) \exp(-ib\phi) d\phi = V \sum_{j=0}^{n-1} \Big(\int_{\frac{\pi}{n}(2j)}^{\frac{\pi}{n}(2j+1)} \exp(-ib\phi) d\phi - \int_{\frac{\pi}{n}(2j+1)}^{\frac{\pi}{n}(2j+2)} \exp(-ib\phi) d\phi \Big)$$

$$= \frac{iV}{b} \sum_{j=0}^{n-1} \Big(\exp(-ib\frac{\pi}{n}(2j+1)) - \exp(-ib\frac{\pi}{n}(2j)) - \exp(-ib\frac{\pi}{n}(2j+2)) + \exp(-ib\frac{\pi}{n}(2j+1)) \Big)$$

$$= \frac{iV}{b} \sum_{j=0}^{n-1} \Big(2\exp(-iA(2j+1)) - \exp(-iA(2j)) - \exp(-iA(2j+2)) \qquad \text{, Allow } b\frac{\pi}{n} \equiv A$$

$$= \frac{iV}{b} \Big(2\exp(-iA) - 1 - \exp(-2iA) \Big) \sum_{j=0}^{n-1} \exp(-2iAj) \tag{2}$$

At this time, I will consider the sum over j, since I have isolated it to one term. Substituting the expression for A into the exponential yields: $\sum_{j=0}^{n-1} \exp(-2\pi i j \frac{b}{n})$. Performing this sum over n yields the following expression:

$$exp\Big(-2\pi i(b - \frac{b}{n})\Big) \frac{\exp(2\pi i b) - 1}{\exp(2\pi i \frac{b}{n}) - 1}$$

Now, b and n are integers. b is the order of the spherical harmonic in the complex exponential term. n is the number of planes that I drive through the z axis. Thus, the fraction in the above expression will return 0 for all b. It also seems that the the same fraction is undefined for whenever $b = \gamma n$ (γ being some integer). However, applying L'Hospital's rule to the above expression yields n for the entire sum. Note that in order for $b\gamma n$ to be an integer that n must divide b. Thus, the magnitude of b is lower bounded by n. Physically, this means that for this potential, no value of b (in the spherical harmonics) can exist apart from those that are n or greater or -n and smaller.

Consider now the term in front of the sum in equation 2. If we identify $\exp(-iA)$ as some quantity we will call x this expression can be rewritten as $2x - 1 - x^2$. This can easily be rewritten as $-(x-1)^2$.

$$\begin{aligned}
x - 1 &= \exp(-iA) - 1 \\
&= \exp(-iA/2)\big(\exp(iA/2) - \exp(-iA/2)\big) \\
&= -2i\exp(-iA/2)\sin(A/2) \\
&= -2i\exp(-i\pi\frac{b}{n})\sin(\frac{\pi b}{2n}) \\
-(x-1)^2 &= 4\exp(-2\pi i\frac{b}{n})sin^2(\frac{\pi b}{2n})
\end{aligned}$$

In order that this expression be nonzero, we require that $\sin(\frac{\pi b}{2n}) = \sin(\frac{\pi\gamma}{2})$ not equal zero. This expression is zero whenever γ is even. Thus, γ must be odd. So, I can write b as $(2z+1)n$ where n is the number of slices I put through the z axis and z is some integer which can take on the values $-$something, $-$something $+ 1, ..., 0, 1, 2, ...,$ something else, ...
Finally, I can rewrite equation 1 as follows:

$$A_{ab}R^k = \frac{iV}{b}\sqrt{\frac{(2a+1)}{4\pi}\frac{(a-b)!}{(a+b)!}}\int_0^\pi P_{ab}(\cos\theta)\sin\theta d\theta d\phi$$

Where b is limited to be odd and a multiple of n. So far, I have not done any work to constrain the values of a. In general, performing that integral to constrain a using θ (as was done with b using ϕ) would be difficult as this integral does not have any analytic expression. However, if certain values of Y_{ab} are odd, then we can immediately toss away those combinations of a and b. So, consider the following property of the associated Legendre polynomials. $P_{ab}(x) = (-1)^b\frac{(a-b)!}{(a+b)!}P_{ab}(x)$. In order for P_{ab} to be odd, then, it must be the case that $(a-b)! = (a+b)!$. This occurs when $b = 0$. But, this can not be any instance of b as b must be odd and a multiple of n. Thus, there are so constraints on the value of b. Finally, writing this solution with the usual Legendre parameters l and m:

$$\Phi(r,\theta,\phi) = \sum_{l=0}^{\infty}\sum_{-l\leq\text{odd}}^{l\geq\text{odd}}\sum_{\text{m}\leq-(2j+1)n}^{\text{m}\geq(2j+1)n}\Big(\frac{iV}{mR^l}\sqrt{\frac{(2l+1)}{4\pi}\frac{(l-m)!}{(l+m)!}}\int_0^\pi P_{lm}(\cos\theta)\sin\theta d\theta\Big)r^lY_{lm}(\theta,\phi)$$
$$\frac{-(l-n)}{2n} < j < \frac{(l-n)}{2n}\ ,\ j\in\mathbb{Z}$$

I could use orthonormality conditions to reduce the amount of terms in this sum even more. I could even use the fact that $\Im(V) = 0$. But, it's easier to just leave the expression in this form and apply the orthonormality conditions and the "reality condition" once n is fixed.

(3)

We now need to determine the first few cases where terms in this sum are nonzero. Consider the case when $n = 0$. Then, no m's can exist. Consider the case when $n = 1$. For this case, m can be 1 (the smallest odd multiple of n) 3, 5, etc. as long as l will allow it. So, the first few (l,m)'s that survive the sum are $(1,-1)$,$(1,1)$,$(2,-1)$,$(2,1)$,$(3,-3)$,$(3,-1)$,$(3,-1)$,$(3,3)$, etc. Thus, it seems that the general form of a surviving term goes as $(l,m) = (l,(2i+1)n)$ as long as

1. 2i+1 is less than l+1 or greater than -l-1
2. i is an integer such as to make (2i+1)n an odd multiple of n

Jackson 3rd ed. : 3.7

a

Three point charges (q, -2q, q) are located in a straight line with separation a and with the middle charge (2q) at the origin of a grounded conducting spherical shell of radius b, as indicated in the sketch.

Write down the potential of the three charges in the absence of the grounded sphere. Find the limiting form of the potential as $a \to 0$, but the product $qa^2 = Q$ remains finite. Write this latter answer in spherical coordinates.

To begin, the potential of a point charge can be expressed as $V(\vec{r}) = k\frac{q}{|\vec{r}-\vec{r'}|}$. $\vec{r}$ is the point at which the potential is to be evaluated. $\vec{r'}$ is the point at which the charges exist. Thus, by inspection, the potential due to three point charges can be expressed as :

$$V(\vec{r}) = kq\Big(\frac{-2}{|\vec{r}|} + \frac{1}{|\vec{r} + a\hat{z}|} + \frac{1}{|\vec{r} - a\hat{z}|}\Big)$$

To find the potential in the limiting case, let us first write $|\vec{r} + a\hat{z}|$ as $\sqrt{r^2 + a^2 - 2ar\cos\theta}$. Here, θ is the polar angle referenced from the $+z$ axis. Doing so results in the following:

$$V(\vec{r}) = kq\Big(\frac{-2}{r} + \frac{1}{\sqrt{r^2 + a^2 - 2ar\cos\theta}} + \frac{1}{\sqrt{r^2 + a^2 + 2ar\cos\theta}}\Big)$$
$$V(\vec{r}) = kq\Big(\frac{-2}{r} + \frac{1}{r\sqrt{1 + (a/r)^2 - 2(a/r)\cos\theta}} + \frac{1}{r\sqrt{1 + (a/r)^2 + 2(a/r)\cos\theta}}\Big)$$

Using the following expansion (to second order) $(x^2 + 2\beta x + 1) \to^2 1 + \beta x + (1.5\beta^2 - .5)r^2$

$$V(\vec{r}) = kq\Big(\frac{-2}{r} + \frac{1 - \cos\theta(a/r) + (1.5\cos^2\theta - .5)(a/r)^2}{r} + \frac{1 + \cos\theta(a/r) + (1.5\cos^2\theta - .5)(a/r)^2}{r}\Big)$$

Note that all terms except for the $(1.5\cos^2\theta - .5)$ terms die out. But, there are two of them.

$$V(\vec{r}) = kq\Big(\frac{(3\cos^2\theta - 1)(a/r)^2}{r}\Big)$$

Now, according to the problem statement $qa^2 \to Q$.

$$\boxed{V(\vec{r}) = kQ\frac{3\cos^2\theta - 1}{r^3}}$$

This is already in spherical coordinates.

b

The presence of the grounded sphere of radius b alters the potential for $r < b$. The added potential can be viewed as caused by the surface-charge density induced on the inner surface at $r = b$ or by image charges located at $r > b$. Use linear superposition to satisfy the boundary

conditions and find the potential everywhere inside the sphere for $r < a$ and $r > a$. Show that in the limit $a \to 0$,

$$\mathbf{\Phi(r}, \theta, \phi) \to \frac{\mathbf{Q}}{\mathbf{2}\pi\epsilon_\mathbf{0}\mathbf{r^3}}\left(\mathbf{1} - \frac{\mathbf{r^5}}{\mathbf{b^5}}\right)\mathbf{P_2} \cos\theta$$

To find the solution to part (b) of this problem it will be best to do as Jackson says and use "linear superposition to satisfy the boundary conditions". See, the charges set up a potential at radius $r = b$. If I can find this potential I will flip its sign and slap that potential on top of the potential generated by the point charges. Whatever, potential I have in $\Phi(r, \theta, \phi)$ at $r = b$ due to the point charges can be used along with the potential generated by the point charges, themselves to determine the potential everywhere inside and outside of the sphere. In general, because of the problem's symmetry, my solution will be of the form : $\Theta(r, \theta) = \sum\limits_{l=0}^{\infty} A_l r^l + B_l r^{-(l+1)} P_l \cos\theta$.

The nice thing about point charges is that when that axis is located on the z-axis it is easy to express the potential due to a point charge in spherical coordinates: $\Phi(r, \theta) = \sum\limits_{l=0}^{\infty} \frac{r_<^l}{r_>^{l+1}} P_l \cos\theta$ where $r_<$ is the smaller of the following two distances: the distance from the point charge to the origin and the distance away from the origin at which the potential is being considered. This equation is valid for $\vec{r} = z\hat{z}$ and $z > 0$ (i.e. the charge is located on the positive z-axis). For charges on the negative z-axis this must be slightly modified: $\Phi(r, \theta) = \sum\limits_{l=0}^{\infty} (-1)^l \frac{r_<^l}{r_>^{l+1}} P_l \cos\theta$.

Thus, the potential due to these three point charges at $\vec{r} = b\hat{z}$ is :

$$kQ \sum_{l=0}^{\infty} \Big(\frac{a^l}{b^{l+1}} + \frac{(-1)^l a^l}{b^{l+1}} \Big) P_l \cos\theta$$

Note that in the above all of the odd terms in the sum die because of the symmetry of the placement of the two point charges.

Note also that in the above I have chosen to ignore the potential due to the charge at the center of the coordinate system (it just shifts the above potential by a constant). If this is the potential at $r = b$ then if I flip the sign of the above potential and add it to the potential generated by the three point charges then I will effectively ground a sphere of radius b, centered at the origin. Thus, allow the "grounded sphere" to be constructed from both a charged sphere (with potential distribution given by the negative of the above) and the charge distribution given in the problem.

Before we set up the grounded sphere, though, let us consider what the potential inside the sphere due to the fictitious sphere looks like. For $r < b$ we can assume that because our boundary conditions (those given by the fictitious sphere) exhibit azimuthal symmetry. Thus, the potential inside the fictitious sphere looks like

$$V_{fict} = -kQ \sum_{l=0}^{\infty} \Big(\frac{a^l}{b^{l+1}} + \frac{(-1)^l a^l}{b^{l+1}} \Big) P_l \cos\theta$$

However, this potential must be related to the general solution for Laplace's equation inside of the sphere. Namely, these are the boundary conditions on the potential inside of the sphere.

$$V_{fict}(b, \theta) = \sum_{k=0}^{\infty} \Big(A_k b^k + B_k b^{-k+1} P_k \cos\theta \Big) = -kq \sum_{l=0}^{\infty} \Big(\frac{a^l + (-1)^l a^l}{b^{l+1}} \Big) P_l \cos\theta$$

If this is to hold for all r (even $r = 0$) then it must be the case that all B_ks are zero. This forces A_ks $= -2\frac{kqa^l}{b^{2l+1}}$. I can now write the potential within the enclosed fictitious sphere as:

$$V_{fict}(r,\theta) = (-2kq)\sum_{evenk=0}^{\infty} \frac{a^k}{b^{2l+1}} r^{-(k+1)} P_k \cos\theta$$

Now, I just need to superpose this with the potential due to the point charges. The result for $|\vec{r}| > a$ is:

$$\Phi(r,\theta) = 2kq\left(\sum_{evenk=0}^{\infty} a^k \left(r^{-(k+1)} - \frac{r^l}{b^{2l+1}}\right) P_k \cos\theta\right)$$

Note that as $r \to 0$ we allow the summand to go to zero for k = 0. Thus, for small a, the largest term that survives is the $k = 2$ term:

$$\Phi_2(r,\theta) = 2kq\left(a^2 (r^{-3} - \frac{r^2}{b^5}\right) P_2 \cos\theta$$

This can be seen to readily reduce to the desired expression.
Given that $k = \frac{1}{4\pi\epsilon_0}$, $Q = qa^2$:

$$\boxed{\Phi_2(r,\theta) = \frac{Q}{2\pi\epsilon_0 r^3}\left(1 - \frac{r^5}{b^5}\right) P_2 \cos\theta}$$

For $|\vec{r} < a|$ the potential inside the sphere only changes in the point charge term. The role of $r_<$ and $r_>$ switch. Thus, it is written:

$$\Phi(r < a,\theta) = 2kq\sum_{evenk=0}^{\infty}\left(\frac{r^k}{a^{k+1}} - \frac{a^k}{b^{2l+1}} r^{-(k+1)}\right) P_k \cos\theta$$

Appendix

Matlab Code for 2.23

```
format long;
V = zeros(25,3);
i = 0;
temp = zeros(25,3);
for n = 1:2:10
    for m = 1:2:10
        i = i + 1;
        theta_a = sqrt((n^2*pi^2)+(m^2*pi^2));
        exp(theta_a);
        V(i,1) = (16/(n*m*pi^2*(1+exp(theta_a))))*sin(n*pi/2)*sin(m*pi/2)*(2*
            exp(theta_a/2));
        V(i,2) = m;
        V(i,3) = n;
    end
end
[partialsums,order] = sortrows(abs(V),-1);
for i = 1:size(partialsums,1)
    temp(i,:) = V(order(i,1),:);
end
table = [cumsum(temp(:,1)) temp];

scrsz = get(groot,'ScreenSize');
f = figure('Position',[0 0 scrsz(3)/2 scrsz(4)/2]);
cnames = {'Cumulative Sum','Term''s Contribution','n','m'};
t = uitable('Parent',f,'Data',table,'ColumnNames',cnames,'Position',[5 5 200
    200]);
```