ECE 770-T14/QIC 885: Quantum Electronics & Photonics

Winter 2013, Problem Set 5, Instructor: A. H. Majedi

1- 1D Cavity and Single-Mode Quantized EM Field

Consider a single-mode quantized EM field in a one-dimensional cavity resonator of length L along the z axis. If the electric field operator is x-polarized

a) Compute the uncertainty of the electric field operator.

b) Find the commutation relation between the electric field operator and the number operator in terms of creation and annihilation operators.

c) Find the uncertainty product between number and electric field operators.

2- Kerr-Type Nonlinearity in SHO

Consider a 1D lossless cavity resonator that is uniformly filled with a material that exhibit third order optical nonlinearity. The cavity is assumed to consist of a single-mode optical field. In the presence of the nonlinearity the Hamiltonian becomes:

$$\hat{H} = \hbar\omega_o \hat{a}^\dagger \hat{a} - \frac{\hbar\kappa}{2}\hat{a}^\dagger \hat{a}^\dagger \hat{a}\hat{a}$$

where κ is a real number that represents the optical nonlinearity. The Hamiltonian describes the photons interacting with each other via the optical nonlinearity by means of ladder operators, $\hat{a}^\dagger$ and $\hat{a}$.

a) Having the commutation relationship $[\hat{a}, \hat{a}^\dagger] = 1$ and the definition of the number operator $\hat{N} = \hat{a}^\dagger \hat{a}$, prove two commutation relations: $[\hat{N}, \hat{a}^\dagger] = \hat{a}^\dagger$ and $[\hat{a}, \hat{N}] = \hat{a}$.

b) Show that $[\hat{N}, \hat{H}] = 0$.

c) Using the result of part (b), Find all the eigenstates and the corresponding eigenvalues of the Hamiltonian.

d) Write down the Heisenberg time evolution equation (Heisenberg equation of motion) for number operator and solve it to find the $\hat{N}(t)$ for $t \geq 0$ in terms of the initial number operator at $t = 0$, namely $\hat{N}$. Interpret your result.

e) Solve the Heisenberg time evolution equation for the annihilation operator, $\hat{a}(t)$, in terms of the initial annihilation operator at $t = 0$, namely $\hat{a}$.

f) Using the results obtained in part (d) and (e), write down the time-dependent creation operator, $\hat{a}^\dagger$, in terms of the initial creation operator at $t = 0$, namely $\hat{a}^\dagger$.

g) Suppose at time $t = 0$, the quantum state of the field is given by $|n\rangle$. At time $T > 0$, a single photon is somehow lost from the cavity and its frequency is measured by a spectrometer. Assuming a perfect spectrometer, what should be the result of this frequency measurement? (i.e. what frequencies could be measured by the spectrometer and with what probabilities.)

3- Schrodinger's Cat & Q Function

Consider a Schrodinger's cat state as the equal superposition of two coherent photon states, $|\psi\rangle = A(|\alpha\rangle + |-\alpha\rangle)$.

a) Find the normalization coefficient, A.

b) What is the constant A, when α is very large.

Considering α is very large,

c) Determine the photon number probability distribution.

d) What is the density operator associated with this state, i.e. $\hat{\rho}$.

e) Consider another coherent state as a probe state expressed by $|\mu\rangle$, find the so called Q function defined as, $Q(\mu) = \frac{1}{2\pi}\langle\mu|\hat{\rho}|\mu\rangle$.

4-Second Order Correlation Function & Single Photon Experiment

Consider a typical Hanbury Brown-Twiss (HBT) experiment with a 50/50 beam splitter. Consider the case when the quantized EM radiation is introduced through just one input port (this means the second input is the vacuum state).

a) Compute the second order correlation function $g^2(0)$ in terms of the expectation value of a function of input number operator.

b) If the input is photon number state $|n\rangle$ what is $g^2(0)$?

Due: Monday April 8, 2013, 11:59 p.m. via email to: ahmajedi@uwaterloo.ca