Lecture 18: Sources of decoherence in superconducting qubits

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I. INTRODUCTION

In this lecture we discuss different sources of noise relevant to decoherence of superconducting qubits.

A. Important reading list

This is the reading list for this lecture.

- van der Wal *et al* [1]
- Martinis *et al* [2]
- Yoshihara *et al* [3]

II. NOISE DUE TO AN ELECTRICAL CIRCUIT COUPLED TO A QUBIT

This type of noise contribution is similar to the situation of an atom coupled to the electromagnetic field in free space. In that situation the atom is affected by high frequency noise and decoherence is of the relaxation limited type. For superconducting qubits, the coupling to the free electromagnetic field leads to a negligible contribution to relaxation. The reason for this is the fact that the transition rate is proportional with density of electromagnetic modes (varying as ω^2 with ω the relevant frequency) times the amplitude of quantum fluctuations (varying as ω), so altogether as ω^3 [4]. Despite the fact that dipole moments of superconducting qubits are significantly higher and that thermal fluctuations may enhance the decoherence rate as well, the overall result is that the decay rate is negligible.

The qubit is however strongly coupled to electrical circuits used for control and readout. In this case the structure of electromagnetic modes is strongly perturbed. For this reason, it is more appropriate to consider an impedance model of the circuit around the qubit. In the following we consider one model of decoherence as it applies to a flux qubit coupled to its control line. Another situation explored in many recent experiments is the relaxation of a qubit in circuit QED, which leads to relaxation through he so called Purcell effect; this will be considered later.

A. Decoherence of a flux qubit due to coupling to its control circuit

The circuit considered is shown in Fig. 1. A control line with an impedance $Z(\omega)$ is used to send a current I in an on chip-line modeled by an inductance L, which results in a magnetic flux MI coupled to the qubit, with M the mutual inductance between the qubit ring and the inductor. Fluctuations of the current I, which are quantum mechanical in origin and may have as well a thermal component at large enough temperatures, result in decoherence of the qubit. We denote the noise component of the current by I_n.

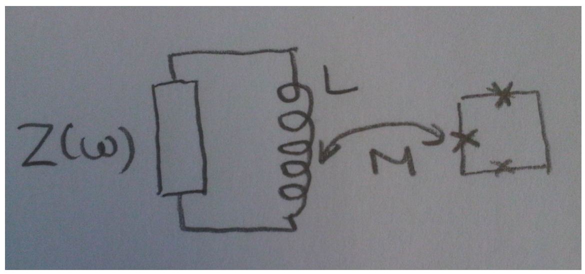

FIG. 1. Electrical circuit coupled to a persistent current qubit.

The circuit is modeled by the following Hamiltonian:

$$H = H_{qb} + \sigma_z f + H_{bath} \tag{1}$$

with f the operator corresponding to coupling to the bath and H_{qb}/H_{bath} the qubit/bath operators. The qubit Hamiltonian is given by

$$H_{qb} = -\frac{\Delta}{2}\sigma_x + \frac{\epsilon}{2}\sigma_z \tag{2}$$

with σ_x and σ_z operators having the corresponding Pauli matrix representation when represented in the flux basis. One has $\epsilon = 2I_p(\Phi_b - \Phi_0/2)$ with Φ_b the qubit bias. Δ and I_p are the usual parameters of the PCQ. The bath coupling operator is

$$f = I_p M I_n. \tag{3}$$

We next use the basis in which the qubit is diagonal formed of

$$|g\rangle = \cos\frac{\theta}{2}|acw\rangle + \sin\frac{\theta}{2}|cw\rangle, \tag{4}$$

$$|e\rangle = \sin\frac{\theta}{2}|acw\rangle - \cos\frac{\theta}{2}|cw\rangle \tag{5}$$

with $|acw\rangle$ and $|cw\rangle$ the two flux states of the qubit and the angle θ given by $\tan\theta = \frac{\Delta}{\epsilon}$. In this basis one has the following expression of the Hamiltonian:

$$H = -\frac{\hbar\omega_{qb}}{2}\tau_z + (\cos\theta\tau_z + \sin\theta\tau_x)f + H_{bath} \tag{6}$$

where the τ operators are Pauli matrices when represented in the energy eigenbasis of the qubit. The correlation function for the bath operator is given by

$$\langle f(t)f(0)\rangle = M^2 I_p^2 \langle I_n(t)I_n(0)\rangle. \tag{7}$$

In the next step we relate the fluctuations of the current through the inductance L to the fluctuations of the voltage across the impedance. Taking the Fourier transform of the correlation function we have

$$S_f(\omega) = M^2 S_{I_n}(\omega) = \frac{M^2}{\omega^2 L^2} S_{V_n}(\omega). \tag{8}$$

The voltage fluctuations can be related to the total impedance, $Z_t(\omega)$, formed by the parallel group of $Z(\omega)$ and the inductance L, through the fluctuation-dissipation theorem [5, 6]:

$$S_{V_n}(\omega) = \hbar\omega Re\left[Z_t(\omega)\right]\left[\coth\frac{\hbar\omega}{2k_BT} + 1\right]. \tag{9}$$

1. *Energy relaxation*

The relaxation rate is given by

$$\begin{aligned}\Gamma_1 &= \Gamma_{0\to1} + \Gamma_{1\to0} \\ &= \frac{1}{\hbar^2}\sin^2\theta\left[S_f(\omega_{qb}) + S_f(-\omega_{qb})\right].\end{aligned} \tag{10}$$

We assume the impedance to be a constant Z_0 and a small enough inductance so that $\omega_{qb}L \ll Z_0$. In this limit

$$\Gamma_1\nu_{qb}^{-1} = 2\sin^2\theta\left(\frac{MI_p}{\phi_0}\right)^2\frac{R_Q}{Z_0}. \tag{11}$$

Above $R_Q = h/(2e)^2 = 6.4\,\mathrm{kOhm}$ is the quantum resistance and Z_0 is usually close to the quantum impedance, so of the order of $100\,\mathrm{Ohm}$. With the adimensional coupling factor $\left(\frac{MI_p}{\phi_0}\right)$ below 0.1% the relaxation rate can easily be made large enough.

2. *Pure dephasing*

The pure dephasing rate is given by

$$\Gamma_\phi = \frac{1}{\hbar^2}\cos^2\theta S_f(0). \tag{12}$$

We assume again the impedance to be a constant Z_0. We obtain

$$\Gamma_\phi \nu_{qb}^{-1} = 2\cos^2\theta \left(\frac{MI_p}{\phi_0}\right)^2 \frac{R_Q}{Z_0}\frac{k_B T}{h\nu_{qb}}. \tag{13}$$

The pure dephasing rate is found to be smaller than the relaxation rate, by a factor $\frac{k_B T}{h\nu_{qb}}$ which is a small number. Note that exactly at the symmetry point, the dephasing rate is found to vanish. At this bias point, one has to consider the quadratic coupling of the noise, as treated in [7].

This type of analysis was first applied in [1] to calculate not only decoherence due to the control circuit, but also the effect of the readout circuit on decoherence.

III. DECOHERENCE DUE TO TWO LEVEL SYSTEMS

Note: this is complemented by the presentation in class.

The presence of two level systems in the environment of the qubit and their important role for the qubit has been identified for the first time in [8]. Two level systems are identified spectroscopically through anticrossings, which are the signature of the resonant coupling of an external object to the qubit. The position and splitting at each anticrossing are randomly distributed. Experiments have shown however that given a qubit technology, the average distribution is the same when examining different samples.

Most reports on two level systems come from phase qubit. With phase qubit, TLSs are routinely observed. The reason for this is the fact that phase qubits have junctions with large areas and the TLS are likely to be observed when positioned inside the barrier where the coupling to the electromagnetic field is the strongest (see [8, 9]). Estimates in [2] shown indeed that the number of defects is $0.05/mu\mathrm{m}^2$GHz for AlOx tunnel junctions. For junctions used in flux qubits less than one defect should be observed per junction. For the even smaller junctions used in the Cooper pair box the probability of occurence of defects becomes even smaller. For qubits using very large area Josepshon junctions (see [2]) estimates show that TLSs could be a significant source of decoherence.

Experiments in [9] probed spectroscopically the multilevel system of the coupled qubit -microscopic defect system and showed that the TLSs is indeed anharmonic. This fully removed the possibility that the observed anticrossings could be related to bosonic excitation modes of the environment.

The proposal in [10] discussed the possibility that TLSs could be used as qubits. This was motivated by the observation that in many experiments the TLSs were observed to be more coherent than the qubits themselves. In [11] a quantum memory based on a TLS for a phase qubit was demonstrated.

Finally, we note that two level systems in dielectrics are sources of losses in microwave resonators. The signature of this type of loss is the anaomalous variation of the quality factor: the quality factor increases as the driving strength and/or temperature are increased.

IV. LOW FREQUENCY FLUCTUATIONS

Note: this is complemented by the presentation in class.

In qubit devices, as well as in classical devices using mesoscopic metal structures (*eg* single electron tunneling transistors, SQUIDs) low frequency parametric fluctuations are observed. In this lecture we focus on flux noise. Charge noise may be important as well (this had a major effect on decoherence in the first experiments with charge qubits) however through proper design it can be significantly reduced (see [12, 13]). A different type of noise is observed as a fluctuation of the critical current of Josephson junctions; this noise is not a dominant source of decoherence and recent experiments have shown that the magnitude of these fluctuations may in fact be smaller than determined based on other measurements in the past[14].

A. Flux noise

Flux noise has been known and measured in SQUID devices long before the advent of quantum computing. It was typically found that the level of noise is close to 1/f from very low frequency up to the band allowed by the experimental procedure, typically kiloHertzs. The experiments in [15] report data on a very large set of DC-SQUIDs. Quite surprisingly the magnitude of the noise does not change significantly with the size of the device. The magnitude of the noise is independent of temperatures at very low temperature.

Flux noise is particularly important for coherence of flux qubits, however it may have an effect on other qubits with loops (eg the phase qubit and the transmon). Detailed measurements of flux noise in qubits have been reported in [3]. It was found that with a spin echo techniques the pure dephasing was completely canceled at the symmetry point. Away from the symmetry point the decay of coherence is Gaussian with a rate dependent on the angle of the coupling. These measurements were consistent with noise spectral densities of about $1e-6\Phi_0/\sqrt{Hz}$, with small variation from sample to sample, consistent with measurements on SQUIDs.

A more recent study was done with a persistent current qubit which had a very large $T_1 = 10\mu$s. The long relaxation time enables the application of long sequences of CPMG pulses, which allows characterizing noise up to the MHz range [16].

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