ECE770-T14/QIC 885: Quantum Electronics & Photonics

Problem Set 1, Winter 2014, Instructor: A. Hamed Majedi

Problem 1- Consider two-slit experiment with sources of monoenergetic electrons where detectors are placed along the vertical screen, e.g. x direction, to monitor the diffraction pattern of electrons. When only one slit is open, the amplitude of detected electrons can be written as $\Psi_1(x,t) = A_1 \dfrac{e^{-i(kx-\omega t)}}{1+x^2}$ and when only the other one is open, the amplitude is $\Psi_2(x,t) = A_2 \dfrac{e^{-i(kx-\omega t+\pi x)}}{1+x^2}$. A_1 and A_2 are constants to be found.

Calculate the density of the detected electrons on the screen when

a) both slits are open and a light source is used to determine which of the slits the electron went through.

b) both slits are open and no light source is used.

Plot the intensity registered on the screen as a function of x for both cases in parts a) and b).

Problem 2- Consider a simplest wave function that potentially describes a surface bound state of the form of $\Psi(z,t) = Ae^{-\alpha|z|}e^{-i\omega t}$

where A, α and ω are positive real constants.

a) Find A.

b) Find the uncertainty in z.

c) Find the momentum wave function associated with $\Psi(z, t = 0)$.

d) If $\Psi(z, t = 0)$ is the initial wave function of a free electron, construct $\Psi(z,t)$, in the form of integral.

e) Discuss the limiting cases (α very large and α very small).

Problem 3- A free object is located at $z = a$ at $t = 0$ and its wavefunction is given by $\Psi(z, t = 0) = \delta(z - a)$.

Find the wavefunction for $t > 0$, namely $\Psi(z,t)$. This solution is called the **free object propagator**.

Hint: To evaluate an integral of the form $\int_{-\infty}^{\infty} \exp\left[i\lambda z^2 + i\beta z\right] dz$, pretend that λ is $\lambda + i\epsilon$ with $\epsilon > 0$

so that the integral is convergent. Then complete the square in the exponent of the exponential and change variables. The $i\epsilon$ in your answer will allow you to decide whether to take the positive or negative square root. Finally let $i\epsilon$ goes to zero. This technique is called *regularization*.

Problem 4- Consider an object with mass m in the one-dimensional short range Dirac Delta function potential, $V(z) = V_o\delta(z)$.

a) Find the criterion for V_o for having a bound state.

b) What is the binding energy?

c) Find the value of z_o such that the probability of finding the object within $|z| < z_o$ is equal to 0.9.

Problem 5- Consider an object of mass m that is constrained to move in a circle of radius a in the $x - y$ plane.

Determine the quantized energy levels for the following cases:

a) The motion of the object is nonrelativistic with a constant linear momentum.

b) The motion of the object is relativistic, namely $E^2 = |\mathbf{p}|^2c^2 + m_o^2c^4$, where m_o is the rest mass and c is the speed of light.

Due: Wednesday Jan 29, 2014. (before starting the class)