Q1

a) $P_{avs} = \frac{|V_s|^2}{8R_s} = \frac{20^2}{8\times 100} = 0.5\ W$

$V_s = 20\angle 0\ V$
$R_s = 100\ \Omega$
$Z_0 = 50\ \Omega$
$R_L = 50\ \Omega$

b) $b_1 = S_{11} a_1 + \cancel{S_{12} a_2}$

$b_2 = S_{21} a_1 + \cancel{S_{22} a_2}$

$\frac{a_2}{b_2} = \Gamma_L = \frac{R_L - Z_0}{R_L + Z_0} = 0 \rightarrow a_2 = 0$

$V_1 = \frac{Z_{in}}{R_s + Z_{in}} V_s = V_1^+ + V_1^- = \sqrt{Z_0}(a_1 + \underbrace{b_1}_{\Gamma_{in} a_1}) = \sqrt{Z_0}\, a_1 (1 + \Gamma_{in})$ (I)

$\Gamma_{in} = \frac{Z_{in} - Z_0}{Z_{in} + Z_0} = S_{11}$ (Port 2 is matched) (II)

(I) (II) $\Rightarrow a_1 = \frac{V_s}{2\sqrt{Z_0}} \frac{Z_{in} + Z_0}{Z_{in} + R_s}$, $Z_{in} = Z_0 \frac{1 + \Gamma_{in}}{1 - \Gamma_{in}}$

substitution $\rightarrow$ $a_1 = 0.9705 - j0.0167 = 0.971\angle -1^\circ$

$b_1 = S_{11} a_1 = 0.0832 - j0.05 = 0.097\angle -31^\circ$

$b_2 = S_{21} a_1 = 0.6408 - j0.6632 = 0.922\angle -46^\circ$

Q1 c) Wrong: $\cancel{P_{ref} = \frac{|b_1|^2}{2} = 0.0047\ W}$

because a_1, b_1 are referenced to $50\ \Omega$ whereas $R_s = 100\ \Omega$

Correct: $\Gamma_{in}' = \frac{Z_{in} - R_s}{Z_{in} + R_s} = 0.26 \angle -169.5^\circ$

$\Gamma_{in}' \neq S_{11}$

$$P_{ref} = P_{avs} \, |\Gamma_{in}'|^2 = 33.6\ mW = 0.0336\ W$$

$$P_{del} = \frac{|b_2|^2 - \cancel{|a_2|^2}}{2} \overset{\to 0}{} = 0.4252\ W$$

- An alternative approach is to write $[b] = [S][a]$ in $100\ \Omega$ system. For that, you need to convert the $[S]$ from $50\ \Omega$ to $100\ \Omega$. a_i's and b_i's in that case would be referenced to $100\ \Omega$.

- Even though in this case $P_{ref} \neq \frac{|b_1|^2}{2}$ (as mentioned above), the equations for power delivered are still valid, so

$$P_{in} = \frac{|b_1|^2 - |a_1|^2}{2} = P_{avs} - P_{ref} = 0.466\ W$$

Power delivered to the 2PN

~~A Portion of this Power is reflected at the load~~

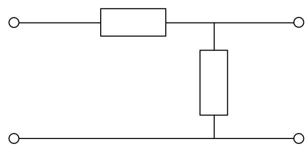
Q2.

For single OC shunt stub matching:

$d_1 = 0.348\lambda, l_1 = 0.098\lambda$, or $d_2 = 0.152\lambda, l_2 = 0.402\lambda$

For lumped LC matching:

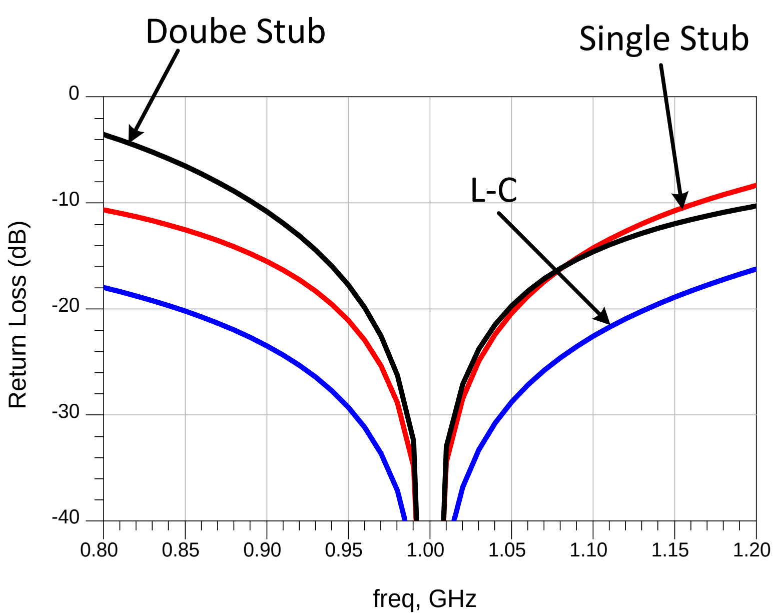
$L_s = 7.96\ nH, C_p = 1.59\ pF$, or $C_s = 3.18pF, L_p = 15.9nH$



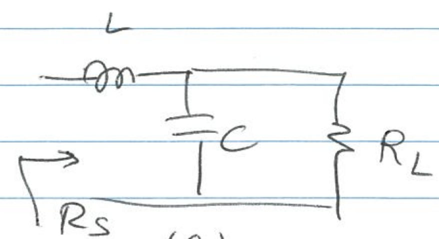
For double OC shunt stu'b matching with $d = \lambda/8$:

$l_1 = 0.194\lambda, l_2 = 0.172\lambda$, or $l_1 = 0.399\lambda, l_2 = 0.021\lambda$

According to the Figure below, LC-matching has the best bandwidth.



Q3 Using 5.3(a) & (b): $\left(R_L/R_S = N^2,\ X_L = 0,\ Z_0 = R_S\right)$

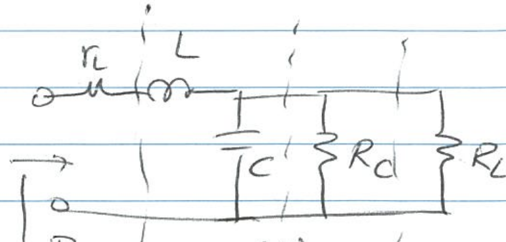


(a)

$$\omega_0 C = B = \frac{N\sqrt{R_L^2 - R_L R_S}}{R_L^2}$$

$$\omega_0 C = \frac{N}{R_L^{\cancel{2}}}\cdot \cancel{R_L}\sqrt{1 - 1/N^2}$$

$$C = \frac{N}{\omega_0 R_L}\cdot\frac{\sqrt{N^2-1}}{N} = \frac{\sqrt{N^2-1}}{R_L\,\omega_0} \quad \text{(I)}$$



(b)

$$\omega_0 L = X = \frac{R_L}{\sqrt{N^2-1}}\left(1 - \underbrace{\frac{R_S}{R_L}}_{1/N^2}\right)$$

$$\omega_0 L = \frac{R_L}{\sqrt{N^2-1}}\left(\frac{N^2-1}{N^2}\right) = R_S\sqrt{N^2-1}$$

$$\to \boxed{L = \frac{R_S\sqrt{N^2-1}}{\omega_0}} \quad \text{(II)}$$

$$I.L = \frac{P_L}{P_{in}} = \underbrace{\frac{P_L}{P_L'}}\cdot\underbrace{\frac{P_L'}{P_{in}'}}\cdot\underbrace{\frac{P_{in}'}{P_{in}}}$$

Assuming $Q \gg 1$

$$\cong \frac{R_C \| R_L}{R_L}\cdot 1 \cdot \frac{R_S}{r_L + R_S}$$

(Note that if $\begin{cases} R_C \to \infty \\ r_L \to 0 \end{cases}$ then $I.L = 1 \equiv 0\,dB$)

$$= \frac{R_C}{R_L + R_C}\cdot\frac{R_S}{r_L + R_S}, \quad \begin{cases} Q_C = R_C\, C\, \omega_0 \\ Q_L = \dfrac{\omega_0 L}{r_L} \end{cases}$$

$$\downarrow$$

$$R_C = \frac{Q_C}{\omega_0 C}, \quad r_L = \frac{\omega_0 L}{Q_L}$$

T

$$I.L = \frac{1}{1 + R_L/R_c} \cdot \frac{1}{1 + r_L/R_S}$$

$$IL = \frac{1}{1 + \frac{R_L \omega_0 C}{Q_c}} \cdot \frac{1}{1 + \frac{\omega_0 L}{R_S Q_L}}$$

$$\text{(I), (II)} \quad IL = \frac{1}{1 + \frac{\cancel{R_L}}{Q_c} \cdot \frac{\sqrt{N^2-1}}{\cancel{R_L}}} \cdot \frac{1}{1 + \frac{1}{\cancel{R_S} Q_L} \cancel{R_S}\sqrt{N^2-1}} = \boxed{\frac{1}{1 + \frac{\sqrt{N^2-1}}{Q_c}} \cdot \frac{1}{1 + \frac{\sqrt{N^2-1}}{Q_L}}}$$

(III)

In this example: $Q_c = 30$, $Q_L = 15$, $R_L/R_S = N^2 = 5$ $\Rightarrow$ Using (III): $\boxed{IL = 0.8272 \equiv +0.82\,dB}$

From ADS Simulation: $\boxed{I.L. = +0.835\,dB}$

∴ Hand Analysis is in very good agreement with simulation

Q4

$\frac{b_2}{a_1} = ?$

$$b_2 = S_{22} \cancel{a_2}^{\,0} + S_{23}\, a_3 \quad \text{(I)}$$

$$a_3 = b_4 e^{-j\pi/4} \quad \text{(II)}$$

$$b_4 = S_{41} a_1 + S_{44} a_4 \quad \text{(III)}$$

$$a_4 = b_3 e^{-j\pi/4} \quad \text{(IV)}$$

$$b_3 = S_{32} \cancel{a_2}^{\,0} + S_{33} a_3 \quad \text{(V)}$$

$$\text{(II)} \,\&\, \text{(III)} \Rightarrow a_3 = e^{-j\pi/4}\left(S_{41} a_1 + S_{44} \underset{}{\overset{\text{IV}}{a_4}}\right)$$

$$= e^{-j\pi/4}\left(S_{41} a_1 + S_{44} \underset{\text{V}}{b_3} e^{-j\pi/4}\right)$$

$$\Rightarrow a_3 = e^{-j\pi/4}\left(S_{41} a_1 + S_{44} S_{33} e^{-j\pi/4} a_3\right)$$

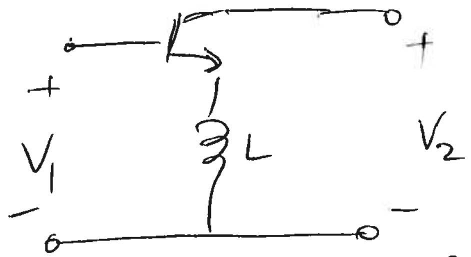
$$\Rightarrow \boxed{a_3 = \frac{e^{-j\pi/4} S_{41}}{1 + j S_{44} S_{33}}\, a_1} \quad \text{(VI)}$$

$$\text{(I, VI)} \Rightarrow \frac{b_2}{a_1} = \frac{e^{-j\pi/4} S_{41} S_{23}}{1 + j S_{44} S_{33}}$$

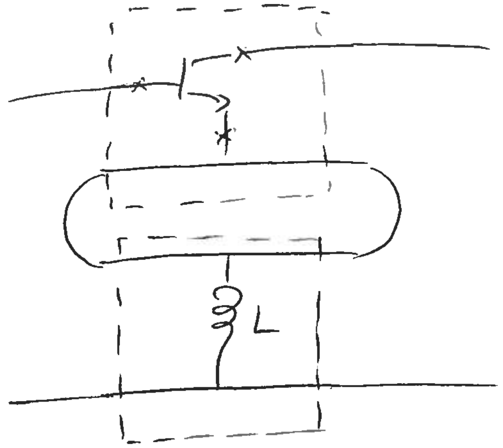
Substitution $\rightarrow$ $\boxed{\frac{b_2}{a_1} = 0.4 \angle -135^\circ}$ $\rightarrow$ $IL = 7.96\ dB$

$\rightarrow$ delay $= 135^\circ$

Q5



$\equiv$



$L = 0.1\,nH$, $f = 10\,GHz$

Series Connection

$$Z_T = Z_{\cancel{tr}} + Z_\ell$$

$$Z_\ell = \begin{bmatrix} j\omega L & j\omega L \\ j\omega L & j\omega L \end{bmatrix} = \begin{bmatrix} j6.28 & j6.28 \\ j6.28 & j6.28 \end{bmatrix}$$

$$[S]_{tr} \longrightarrow [Z]_{tr} = \begin{bmatrix} 26.56\angle -68.42^\circ & 0 \\ 139.958\angle -143.128^\circ & 98.62\angle -69.68^\circ \end{bmatrix}$$

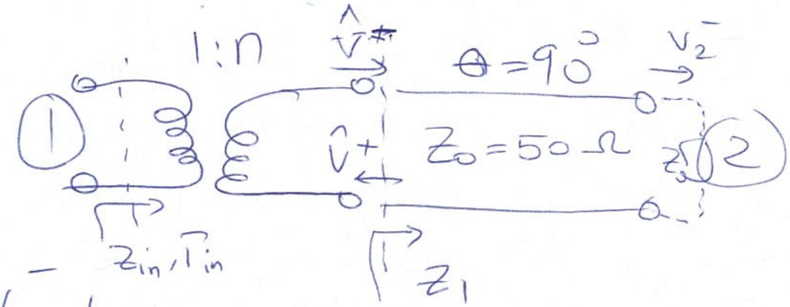
$$[Z_T] = [Z]_{tr} + [Z]_\ell$$

$$\therefore [Z_T] = \begin{bmatrix} 20.84\angle -62.06^\circ & 6.28\angle 90^\circ \\ 136.28\angle -145.24^\circ & 92.7\angle -68^\circ \end{bmatrix}$$

$(Z_0 = 50\,\Omega)$

$$[S]_T = \begin{bmatrix} 0.906\angle -141.9 & 0.094\angle 153.7 \\ 2.037\angle -81.5 & 0.768\angle -62.2 \end{bmatrix}$$

Q6



(1) $S_{11} = \left.\dfrac{V_1^-}{V_1^+}\right|_{V_2^+=0} = \left.\Gamma_{in}\right|_{\text{Port 2 is matched}} = ?$

$Z_1 = Z_0 \rightarrow Z_{in} = Z_0/n^2 \Rightarrow \Gamma_{in} = \dfrac{Z_0/n^2 - Z_0}{Z_0/n^2 + Z_0}$

$\therefore\ S_{11} = \dfrac{1-n^2}{1+n^2}$

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(2) $S_{21} = \left.\dfrac{V_2^-}{V_1^+}\right|_{V_2^+=0} = ?$

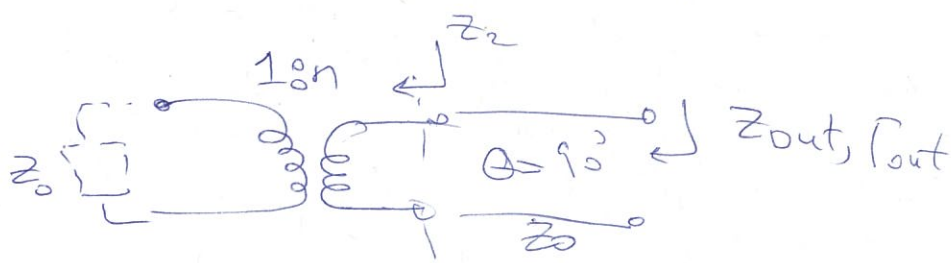
$V_2^- = \hat{V}^- e^{-j\pi/2}$ (I)

$\begin{cases} \hat{V} = V_1 n \Rightarrow \hat{V}^+ + \hat{V}^- = (V_1^+ + V_1^-) n \\ \hat{V}^+ = V_2^+ e^{-j\pi/2} = 0 \end{cases}$

$\Rightarrow \hat{V}^- = V_1^+ (1 + S_{11}) n$ (II)

(I, II) $\Rightarrow S_{21} = -jn^2(1+S_{11}) = -j\dfrac{2n}{1+n^2} = S_{12}$

Q6) ③ $S_{22} = \left.\dfrac{V_2^-}{V_2^+}\right|_{V_1^+=0} = \Gamma_{out}\Big|_{\text{Port 1 matched}} = ?$



$$Z_2 = n^2 Z_0\,,\quad Z_{out} = Z_0^2/Z_2 = Z_0/n^2 = Z_{in}$$

$$\Rightarrow S_{22} = \frac{1-n^2}{1+n^2}$$