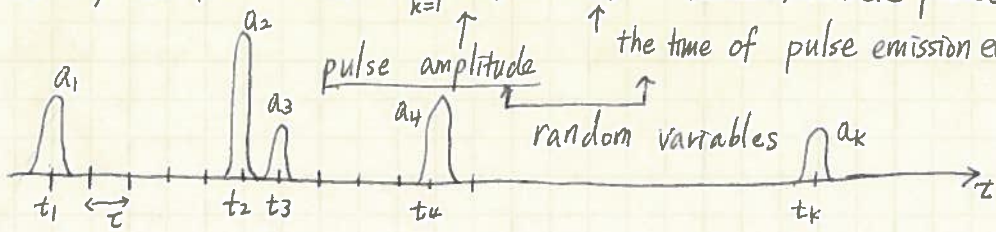
Tue

Carson's thm

A noisy waveform $x(t) = \sum_{k=1}^{K} a_k f(t - t_k)$ random, discrete pulses

a fixed fn

a_k: pulse amplitude; t_k: the time of pulse emission event — random variables



$$x(t) = \sum_{k=1}^{K} a_k f(t - t_k)$$

$$X(i\omega) = \int_{-\infty}^{\infty} x(t) e^{-i\omega t} dt ,$$

F.T

$$X(i\omega) = F(i\omega) \sum_{k=1}^{K} a_k e^{-i\omega t_k}$$

Unilateral power spectral density

$$S_x(\omega) = \lim_{T\to\infty} \frac{2\langle |X(i\omega)| \rangle^2}{T} = \lim_{T\to\infty} \frac{2|F(i\omega)|^2}{T} \sum_{k,m=1}^{K} \langle a_k a_m e^{-i\omega(t_k - t_m)} \rangle$$

$$= \lim_{T\to\infty} \frac{2|F(i\omega)|^2}{T} \Big[\underbrace{\sum_{k=1}^{K} \langle a_k^2 \rangle}_{k=m} + \sum_{k\neq m} \langle a_k a_m e^{-i\omega(t_k - t_m)} \rangle$$

suppose $\lim_{T\to\infty} \frac{K}{T} = \nu$ (average rate of pulse emission)

$$\langle a^2 \rangle = \lim_{T\to\infty} \frac{1}{K} \sum_{k=1}^{K} \langle a_k^2 \rangle \Leftarrow \text{the mean-square of pulse amplitude}$$

∴ First term of the RHS

$$\lim_{T\to\infty} \frac{2|F(i\omega)|^2}{T} \sum_{k=1}^{K} \langle a_k^2 \rangle = \lim_{T\to\infty} \frac{2|F(i\omega)|^2}{K} \left(\frac{K}{T}\right) \sum_{k=1}^{K} \langle a_k \rangle^2 = \boxed{2|F(i\omega)|^2 \nu \langle a^2 \rangle}$$

∴ 2nd-term of the RHS

→ suppose the pulse emission occurs ~~the~~ completely independently,

$$\lim_{T\to\infty} \frac{2|F(i\omega)|^2}{T} \sum_{k\neq m} \langle a_k a_m e^{-i\omega(t_k - t_m)} \rangle = \lim_{T\to\infty} \frac{2|F(i\omega)|^2}{T} \sum_{k\neq m} \langle a_k \rangle \langle a_m \rangle \langle e^{-i\omega t_k} \rangle \langle e^{i\omega t_m} \rangle$$

$$= \lim_{T\to\infty} \frac{2|F(i\omega)|^2}{T} \sum_{k\neq m} \langle a \rangle^2 \frac{\sin^2\left(\frac{\omega T}{2}\right)}{\left(\frac{\omega T}{2}\right)^2}$$

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2nd-term of the RHS

Suppose the pulse emission occurs completely independently.

$$\lim_{T\to\infty} \frac{2|F(i\omega)|^2}{T} \sum_{k\neq m} \langle a_k a_m e^{-i\omega(t_k - t_m)} \rangle = \lim_{T\to\infty} \frac{2|F(i\omega)|^2}{T} \sum_{k\neq m} \langle a_k\rangle\langle a_m\rangle \langle e^{-i\omega t_k}\rangle\langle e^{i\omega t_m}\rangle$$

$$\langle a_k\rangle = \langle a_m\rangle = \langle a\rangle$$

$$\langle e^{-i\omega t_k}\rangle = \frac{\int_{-T/2}^{T/2} e^{-i\omega t_k} dt_k}{T} = \frac{e^{-i\omega T/2} - e^{i\omega T/2}}{T(-i\omega)}$$

$$\langle e^{i\omega t_m}\rangle = \frac{\int_{-T/2}^{T/2} e^{i\omega t_m} dt_m}{T} = \frac{e^{i\omega T/2} - e^{-i\omega T/2}}{T(i\omega)}$$

$$\langle e^{-i\omega t_k}\rangle\langle e^{i\omega t_m}\rangle = \frac{1 - e^{-i\omega T} - e^{i\omega T} + 1}{\omega^2 T^2} = \frac{2 - (e^{i\omega T} + e^{-i\omega T})}{\omega^2 T^2}$$

$$= \frac{2 - 2\cos\omega T}{(\omega T)^2} = \frac{2\cdot 2\sin^2\left(\frac{\omega T}{2}\right)}{(\omega T)^2} = \frac{\sin^2\left(\frac{\omega T}{2}\right)}{\left(\frac{\omega T}{2}\right)^2}$$

$$\therefore \lim_{T\to\infty} \frac{2|F(i\omega)|^2}{T} \sum_{k\neq m} \langle a\rangle^2 \frac{\sin^2\left(\frac{\omega T}{2}\right)}{\left(\frac{\omega T}{2}\right)^2}$$

$$\langle a\rangle = \lim_{K\to\infty} \frac{1}{K}\sum_{k=1}^{K} a_k$$

$$= 4\pi \langle x(t)\rangle^2 \delta(\omega) \leftarrow \langle x(t)\rangle = \nu\langle a\rangle \int_{-\infty}^{\infty} f(t)\,dt$$

$$\leftarrow \lim_{T\to\infty} \frac{\sin^2(\omega T/2)}{\left(\frac{\omega^2 T}{4}\right)} = \boxed{2\pi\delta(\omega)}$$

$$\lim_{T\to\infty} \frac{2\left|\int_{-\infty}^{\infty} f(t) e^{-i\omega t} dt\right|^2}{T^2} \frac{K^2}{K^2} \sum_{k\neq m} \langle a\rangle^2 \, 2\pi\delta(\omega)$$

$$= \boxed{4\pi \left(\nu \langle a\rangle \int_{-\infty}^{\infty} f(t)\,dt\right)^2 \delta(\omega)}$$

Carson's theorem $\boxed{S_x(\omega) = 2\nu\langle a^2\rangle |F(i\omega)|^2 + 4\pi\left(\nu\langle a\rangle\int_{-\infty}^{\infty} f(t)\,dt\right)^2\delta(\omega)}$

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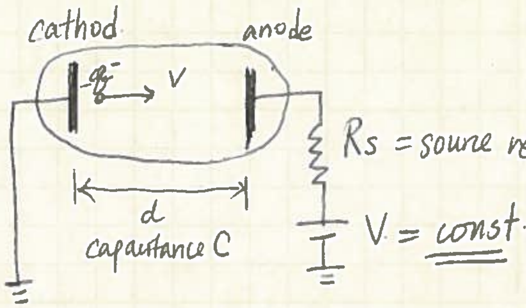
Shot Noise in a Vacuum Diode

← not intrinsic noise.

← Due to fluctuations in the intensity of the stream of electrons flowing from the cathode to the anode



☆ constant voltage operation
no voltage fluctuations
but current fluctuations.

$$i(t) = \frac{Q}{t}$$

circuit current $i(t) = \frac{q v(t)}{d} = \boxed{\frac{q}{\left(\frac{d}{v(t)}\right)}}$.

2-time scales

① the electron transit time τ_t

② the circuit relaxation time $\boxed{\tau_c = R_s C}$

Case 1 $\tau_t \ll \tau_c$

(voltage drop due to the electron transit occurs "instantly"
but the relaxation through the external circuit = very slow)

Right after the transit,

$$V - \frac{q}{C} = \text{the voltage across the vacuum diode}$$
$$= V_A(t) \quad @\ t=0 \quad \text{anode voltage}$$

Using the Kirchoff's law,

$$\frac{d}{dt} V_A(t) = \frac{V}{R_s C} - \frac{V_A(t)}{R_s C}$$

$$\boxed{V_A(t) = V - \frac{q}{C} e^{-t/R_s C}}$$

$$i(t) = \frac{V - V_A}{R_s} = \frac{q}{R_s C} e^{-t/R_s C}$$

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Now compute the surface charges of the cathode and the anode

3 cases

① the electron drift velocity = constant over τ_t, $\tau_c \ll \tau_t$ $R_s = 0$?

② $(\text{drift velocity})_i = 0$, velocity is accelerated by $\vec{E}$ = constant, $\tau_c \ll \tau_t$.

③ $\tau_c \gg \tau_t$, the transit = an impulsive event.

(Initially ~~the~~ V = the voltage across the vacuum diode.

∴ cathode surface charge $= -CV$
anode " $= CV$.)

When an electron $(-q)$ is emitted from the cathode, the net charge $+q$ is induced on the cathod.

~~① case 1~~ : cathod surface charge

$$Q_c(t) = \boxed{-CV} + q - \int_0^t dt' i(t')$$

① case 1: $i(t) = \frac{q v(t)}{d} = \frac{qv}{d}$. v = constant.

$\tau_t = \frac{d}{v}$.

$0 < t < d/v$. $Q_c(t) = \begin{cases} -CV + q - \int_{t_1}^{t_2} \frac{qv}{d} t\, dt = -CV + q - \frac{qv}{d} t & 0 < t < \frac{d}{v} \\ -CV & \text{otherwise} \end{cases}$

$Q_A(t) = \begin{cases} \boxed{CV + q\frac{v}{d} t} & \text{if } 0 < \tau_t < \frac{d}{v} \\ CV & \text{otherwise} \end{cases}$ ← not necessary

Using the external current

② Case 2 acceleration by the $\vec{E}$-field

$E = \frac{V}{d}$

$v(t) = \frac{qE}{m} t$

transit time $\int_0^d dr = \int_0^{\tau_t} dt' v(t') \Rightarrow \tau_1 = \sqrt{\frac{2md}{qE}}$, $I(t) = \frac{q}{d} v(t) = \frac{q^2 V}{md^2} t$

$d = \frac{qE}{m}\frac{t^2}{2}$ ∴ $t = \frac{2md}{qE}$

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$$\therefore Q_c(t) = -CV + q - \int_0^t \frac{q^2 V}{md^2} t' dt'$$

$$= -CV + q - \frac{q^2 V}{md^2}\frac{t^2}{2} = -CV + q - q \cdot \frac{1}{2d}\left(\frac{qV}{md} t^2\right)$$

$$= \begin{cases} -CV + q\left(1 - \frac{1}{2d}\frac{qV}{md}t^2\right) = -CV + q\left(1 - \frac{1}{2d} v(t) \cdot t\right) & 0 < t < \tau_t \\ -CV & \text{otherwise} \end{cases}$$

$$\because v(t) = \frac{qE}{m} t = \frac{qV}{md} t.$$

$$Q_A(t) = CV + \int_0^t \frac{q^2 V}{md^2} t' dt'$$

$$= CV + \frac{q^2 V}{md^2}\frac{t^2}{2} = \begin{cases} CV + \frac{q^2 qV}{md} t \cdot \frac{1}{2d} = \boxed{CV + \frac{q}{2d} v(t)\, t} \\ CV & \text{otherwise} \end{cases}$$

③ Case 3 : Impulsive electron transit

$$Q_A(t) = CV_A(t) = \begin{cases} CV - qe^{-t/RC} & (t>0) \\ CV & (t<0) \end{cases}$$

$$Q_C(t) = -CV_A(t) = \begin{cases} -CV + qe^{-t/RC} & t>0 \\ -CV & t<0,\ \alpha(0) \end{cases}$$

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Current Noise

Suppose each electron emission event and its transport process are mutually independent.

Let's calculate the external current noise spectra for 3-cases using the Carson's thm.

① Case 1: $\tau_c \ll \tau_t$ & $v = \text{const}$.

Carson's thm.

$$\begin{cases} i(t) = \sum_{k=1}^{K} a_k f(t-t_k) \\ S(\omega) = 2\nu\langle a^2\rangle |F(i\omega)|^2 + 4\pi\left[\nu\langle a\rangle\int_{-\infty}^{\infty} dt\, f(t)\right]^2 \delta(\omega) \end{cases}$$

ν = the average rate.

$$f(t) = \begin{cases} q\frac{v}{d} & 0 < t < \frac{d}{v} \\ 0 & \text{otherwise} \end{cases}$$

$$\therefore F(i\omega) = \int_0^{\frac{d}{v}} q\frac{v}{d} e^{-i\omega t}\, dt = q\frac{v}{d}\left[\frac{1}{-i\omega}\right]\left(e^{-i\omega\frac{d}{v}} - 1\right)$$

$\cos x + i\sin x - \cos x + i\sin x$

$$\sin x = \frac{e^{ix} - e^{-ix}}{2i} = \frac{e^{ix}\left(1 - e^{-2ix}\right)}{2i}$$

$$= q\frac{v}{d}\frac{1 - e^{-i\omega d/v}}{i\omega} = q\frac{e^{-i\frac{\omega d}{2v}}}{\left(\frac{\omega d}{2v}\right)}\sin\left(\frac{\omega d}{2v}\right)$$

$$\int_{-\infty}^{\infty} dt\, f(t) = q\frac{v}{d}\frac{d}{v} = q$$

$$\therefore S(\omega) = 2\nu \cdot q^2 \frac{\sin^2\left(\frac{\omega d}{2v}\right)}{\left(\frac{\omega d}{2v}\right)^2} + 4\pi\nu^2 q^2\delta(\omega)$$

$\underline{\langle a^2\rangle = 1}$

Note that ν = the average electron emission rate.

$$\therefore \langle I\rangle = q\nu \qquad \langle I\rangle = \text{average current}$$

$$\therefore S(\omega) = 2q\langle I\rangle \cdot \operatorname{sinc}^2\left(\frac{\omega d}{2v}\right) + 4\pi I^2\delta(\omega)$$

In the low frequency limit, $\boxed{0 < \omega \ll v/d}$, $\quad \operatorname{sinc} x \longrightarrow 1$

$$\therefore S(\omega) = 2q\langle I\rangle \qquad \underline{\text{full shot noise}}$$

③ Case 2 : $\tau_c \ll \tau_t$, & accelerated velocity

each pulse is given by

$$a = \frac{q^2 V}{md^2}, \quad f(t) = \begin{cases} t & 0 < t < \tau_t = \frac{d}{v} \\ 0 & \text{otherwise} \end{cases}$$

$$\langle a^2 \rangle = a^2$$

$$F(i\omega) = \int_{-\infty}^{\infty} f(t) e^{-i\omega t} dt = \int_0^{\tau_t} t e^{-i\omega t} dt$$

$$= i\tau_t \frac{e^{-i\omega\tau_t}}{\omega} - \frac{1 - e^{-i\omega\tau_t}}{\omega^2}$$

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$$|F(i\omega)|^2 = \frac{2 + \omega^2\tau_t^2 - 2\omega\tau_t \sin(\omega\tau_t) - 2\cos(\omega\tau_t)}{\omega^4}$$

$$\int_{-\infty}^{\infty} f(t)\,dt = \int_0^{\frac{d}{v}} t\,dt = \frac{1}{2}\left(\frac{d}{v}\right)^2$$

Using the Carson's thm.

$$\boxed{S_i(\omega) = 2\nu \left(\frac{q^2 V}{md^2}\right)^2 \cdot \frac{2 + \omega^2\tau_t^2 - 2\omega\tau_t \sin(\omega\tau_t) - 2\cos(\omega\tau_t)}{\omega^4} + 4\pi\nu^2 q^2 \delta(\omega)}$$

$$\langle I \rangle = \frac{1}{2} \frac{q^2 V}{md^2} \tau_t^2,$$

in the low-frequency limit, $\left(\omega \ll \frac{1}{\tau_t}\right)$

$$\sin(\omega\tau_t) = \omega\tau_t - \frac{1}{3!}(\omega\tau_t)^3 + O(\omega^5)$$

$$\cos(\omega\tau_t) = 1 + \frac{1}{2!}(\omega\tau_t)^2 + \frac{1}{4!}(\omega\tau_t)^4 + O(\omega^6)$$

$\therefore S_i(\omega) = 2q\langle I \rangle$ full shot noise

③ $\tau_t \ll \tau_c$ impulsive electron transit.

each current pulse is given by

$$f(t) = \begin{cases} \frac{q}{CR_s} e^{-t/R_sC} & t>0 \\ 0 & t<0 \end{cases}$$

$\downarrow$ F.T.

$$F(i\omega) = \frac{q}{1+i\omega R_s C}$$

$$S_i(\omega) = 2q\langle I\rangle \frac{1}{1+\omega^2 R_s^2 C^2} + 4\pi\langle I\rangle^2\delta(\omega)$$

in the low frequency limit, $\left(\omega \ll \frac{1}{R_s C}\right)$

$$S_i(\omega) = 2q\langle I\rangle \quad \text{full shot noise}$$

The origin of shot noise in a vacuum diode

= the statistical independence of electron emission events at the cathode.

<u>If</u> ∃ statistical dependence btw the electron emission events,

the dependence <u>manifests</u> itself as a negative feedback process in which subsequent electron emissions are modulated by <u>earlier events</u>.

a) <u>a space-charge effect</u> in <u>$\tau_t \gg \tau_c$</u> limit

the existence of many electrons in the vacuum diode creates a potential modulation such that the rate of electron emissions is substantially smoothed.

b) a <u>memory-effect</u> in the external circuit in $\tau_c \gg \tau_t$ limit.

the slow recovery of the voltage across the vacuum diode suppresses the rate of the subsequent electron emissions

a) b) ⇒ sub-shot noise (constant current operation)

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Partition Noise

Suppose there are scatterers along the path of electrons or charge carriers. Then electrons are scattered by them, to cause a new source of noise

This is called "partition noise".

This effect happens in a mesoscopic conductor.
↳ due to the wave nature of electrons

① Ballistic

$\boxed{\text{"Coherent scattering theory"}}$ developed by Landauer–Büttiker
↳ define single electron states based on the scattering matrix

for each electron state,
partitioning by the scatterer introduces intrinsic quantum fluctuations of the appropriately defined number and phase difference operators for the electron

Simply, T = the probability that the electron is transmitted at the scatterer.
Then the process is binomial distribution, yielding the (1−T) fluctuations

$$S_I(\omega \to 0) = 2q\langle I\rangle(1-T)$$

$$\downarrow T \ll 1$$

$$2q\langle I\rangle \quad \text{full shot noise}$$

this scattering is elastic by conserving energy & momentum
∴ electron phase is preserved.
i.e. phase-coherent process

② Diffusive transport (distributed elastic scattering case)

$$S_I(\omega \to 0) = 2q\langle I\rangle \frac{1}{3}$$

"Pauli-exclusion" principle
Fermionic nature

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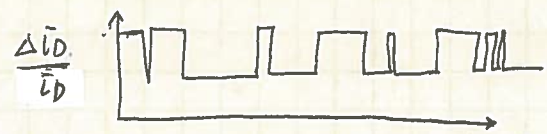
1/f - Noise & Random Telegraph Signals

- all electronic & optical devices show the excess noise obeying the inverse frequency power law
- a physical mechanism for 1/f noise has not been completely identified.
  - One physical mechanism to lead the 1/f power law, but it's not necessarily a unique physical mechanism.

Random Telegraph Signal (RTS)



in very small electronic devices,

the alternate capture and emission of carriers at an individual defect site generates discrete switching in the device resistance

→ a possible microscopic origin of low-frequency noise

Focus of this section

to study the relationship between the RTS associated with these defects in small devices and the 1/f noise found in large devices.

e.g. in MOSFETs, Metal-insulator-metal diodes,

1/f noise may be caused by the summation of many RTS due to the defects in the insulator

Characteristics of 1/f Noise

① Scale Invariance

A power spectral density function of a 1/f noise

$$S_x(\omega) = \frac{C}{\omega} \quad \text{where } C = \text{const.}$$

The integrated power in the spectrum between ω_1 and ω_2 is

$$P_x(\omega_1, \omega_2) = \frac{1}{2\pi}\int_{\omega_1}^{\omega_2} S_x(\omega)\, d\omega$$

$$= \frac{C}{2\pi} \ln\left(\frac{\omega_2}{\omega_1}\right)$$

$\therefore$ a fixed freq. ratio $\frac{\omega_2}{\omega_1}$ $\Rightarrow$ the integrated noise power is constant.

$$P_x\left(\frac{0.1\,\text{Hz}}{1\,\text{Hz}}\right) = P_x\left(\frac{1\,\text{Hz}}{10\,\text{Hz}}\right) = P_x\left(\frac{10\,\text{Hz}}{100\,\text{Hz}}\right) \cdots\cdots$$

"Scale Invariance"

② Stationarity

$$S_x(\omega) = \begin{cases} C/\omega & \text{for } \omega_1 \le \omega \le \omega_2 \\ 0 & \text{otherwise} \end{cases} \quad \leftarrow \text{band-pass-filtered power}$$

Using W.-K. thm, the autocorrelation function

$$\phi_x(\tau) = \frac{C}{2\pi}\int_{\omega_1}^{\omega_2} \frac{\cos \omega\tau}{\omega}\, d\omega = \frac{C}{2\pi}\left[C_i(\omega_2\tau) - C_i(\omega_1\tau)\right]$$

where $C_i(z) = \int_{\infty}^{z} \frac{\cos y}{y}\, dy$ cosine integral

$$= \underbrace{\gamma}_{\text{Euler's constant}} + \ln(z) + \sum_{k=1}^{\infty} \frac{(-1)^k z^{2k}}{(2k)!\, 2k}$$

in the limit of $z \to 0$,

$$C_i(z) \simeq \ln z$$

$$\therefore \phi_x(\tau = 0) = \frac{C}{2\pi}\ln\left(\frac{\omega_2}{\omega_1}\right) \quad \leftarrow \text{mean-square of } x(t)$$

$\therefore$ it's the statistically stationary(?) but no experimental evidence for the existence of the low freq. limit ω_1!