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1/f noise.

✓ pink noise, flicker noise, → $\alpha = 1$

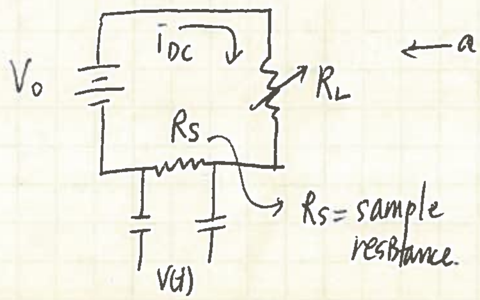
✓ form. $S(f) \propto \dfrac{1}{f^{\alpha}}$ $\quad 0 < \alpha < 2$, typically $\alpha \approx 1$.

$\underline{0 < \alpha \leq 3}$

✓ ubiquitous in nature & man-made processes

solids. condensed matter, electronic devices e.g. resistors, op-amps, etc.

· Description of the phenomenon.



← a circuit to measure voltage noise.

$V(t)$ = the instantaneous voltage drop across R_{sample}

observation ① i_{DC} = const, $V(t)$ fluctuates about $\langle V \rangle = V_{DC}$.

$$S_V(\omega) = 4\int_0^{\infty} \underbrace{\phi_V(\tau)}_{\text{autocorrelation}} \cos(\omega\tau)\, d\tau.$$

When $i_{DC}=0$, $V_{DC}=0$, $S_V(\omega)$ · Johnson-Nyquist noise

$= 4k_B\Theta R(\omega)$ $\quad \underline{(k_B\Theta \gg \hbar\omega)}$

for $f < 10^{10}$ Hz, $R(f) = R(0)$ for most conductors

$\therefore S_V(\omega) = 4k_B\Theta R$ (freq. indep. at low freq.)

② $i_{DC} \neq 0$, $V(t)$ fluctuations are observed to increase over the eq. value, $4k_B\Theta R(\omega)$

~~there~~ $\exists$ two frequently observed sources of current-induced noise

1) shot noise $\propto$ $\underline{i_{DC}}$, arising from the finite size of the electron charge

↳ as a fraction of the dc power, shot noise is larger at low currents where the discreteness of the charge is more important

white noise at low frequencies

2) flicker or 1/f noise : at sufficiently low frequencies

→ extra low-freq. noise

Note: the shape of the power spectrum uniquely characterizes the process only if gaussian & stationary process

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1/f noise phenomenological eq. by Hooge (1969)

$$\frac{S_V(f)}{V^2} = \frac{S_I(f)}{I^2} = \frac{S_R(f)}{R^2} = \frac{S_G(f)}{G^2} = \frac{\gamma}{N_c f^{\alpha}}.$$

N_c = # of the charge ~~charge~~ carriers in the sample

γ = constant, dimensionless only if $\alpha = 1$ & $\beta = 0$

Hooge unified the case of metals & semiconductors by postulating the inverse dependence on N_c & $\gamma \cong 2\times10^{-3}$ } postulate

→ the spectral density is indep. of temperature and material properties

↳ a powerlaw at all freq.

$\boxed{\text{reality}}$ not all exp. data give the relation $\frac{\gamma}{N_c f^\alpha}$ w/ $\gamma = 2\times10^{-3}$

$\because$ different origins & properties ⇒ different source of excess noise

At least w/n the bulk metals //

① $S_V \propto \langle V^2 \rangle$ ⇒ current does not drive fluctuations but merely makes resistance fluctuations

1/f noise due to turbulent convection in ionic solution

② $S_V \propto \frac{1}{N}$ ⇒ bulk property

③ $S_V \propto \frac{1}{f^\alpha}$ w/ $0.9 \le \alpha \le 1.4$

Several theoretical Approaches to explain the 1/f noise

Ⓐ Activated random process

a random process w/ a characteristic time τ.

$$S(\omega) \propto \frac{\tau}{1+(\omega\tau)^2}$$ Debye-Lorentzian spectrum

↑ generated by postulating an appropriate distribution $D(\tau)$ of the characteristic times w/n the sample

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① If the sample was inhomogeneous & $D(\tau) \propto \frac{1}{\tau}$ for $\tau_1 \le \tau \le \tau_2$.

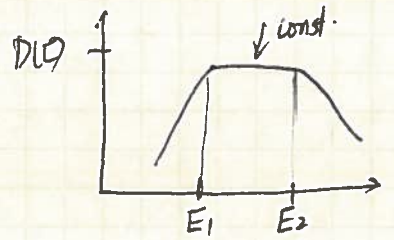
$$S(\omega) \propto \int \frac{\tau}{(\omega\tau)^2+1} D(\tau)\, d\tau = \int \frac{1}{1+(\omega\tau)^2} d\tau$$

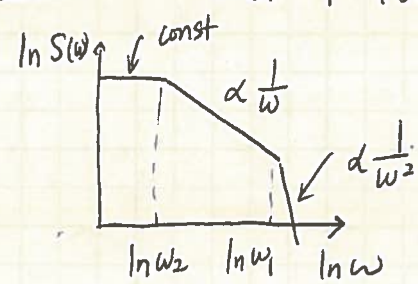
$$\boxed{S(\omega) \propto \frac{1}{\omega} \quad \text{for} \quad \frac{1}{\tau_2} \ll \omega \ll \frac{1}{\tau_1}}$$

② If τ is thermally activated.

$$\tau = \tau_0 e^{E/k_B\theta}$$

$\therefore D(E)$. the required energy distribution = const. for $k_B\theta \ln\left(\frac{\tau_1}{\tau_0}\right) \le E \le k_B\theta \ln\left(\frac{\tau_2}{\tau_0}\right)$





B. A random telegraph signal due to the trapping centers in system to capture and release electrons or holes in a random fashion.

Consider generation-recombination noise.

a physical quantity X, its fluctuation ΔX.

Suppose ΔX decays w/ time scale τ.

then the differential equation of the decay is given by

$$-\frac{d\,\Delta X(t)}{dt} = \frac{\Delta X(t)}{\tau}$$

↓ integration

$$\boxed{\Delta X(t) = \Delta X(t_0)\, e^{-\frac{1}{\tau}(t-t_0)}}$$

then, the correlation fn of X is

$$\phi_X(t) = \langle \Delta X(t_0)\, \Delta X(t_0+t) \rangle = \langle \Delta X(t_0) \Delta X(t_0)\, e^{-t/\tau} \rangle$$

$$= \langle (\Delta X)^2 \rangle\, e^{-t/\tau}$$

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W-K thm.

$$\to S_x(f) = 4\int_0^\infty \phi_x(t)\cos(2\pi f t)\,dt$$

$$= 4\int_0^\infty \langle(\Delta X)^2\rangle e^{-t/\tau}\cos(2\pi f t)\,dt$$

$$= 4\langle(\Delta X)^2\rangle \frac{\tau}{1+(2\pi f\tau)^2}$$

$\therefore$ the independent-electron process gives "Lorentzian spectrum"

$$\begin{cases} \text{if } f\tau \ll 1, & S_x(f) \to \text{white const.} \\ \text{if } f\tau \gg 1, & S_x(f) \propto \frac{1}{f^2} \end{cases}$$

If there are a large # of Lorentzian spectra,

the overall noise is from the summation of independent process.

Mathematically, we need a weighting factor $g(\tau) \propto \frac{1}{\tau}$ ($\because$ difficulties at $f=0$ or $f=\infty$)

for the Lorentzian spectra w/ the relaxation time τ,

$\tau_1 < \tau < \tau_2$, $g(\tau)\,d\tau = \frac{1}{\ln\left(\frac{\tau_2}{\tau_1}\right)}\frac{1}{\tau}\,d\tau$ ← statistical weights.

[← When the spectrum is about $1/f$ in a limited freq. range,
the correlation fn goes $\approx \ln t$ in a limited time interval

this means $-\frac{d\Delta X}{dt} = A e^{B\Delta X}$ w/ $B\Delta X \gg 1$]

$$S_x(f) = \int_{\tau_1}^{\tau_2} g(\tau)\frac{\langle(\Delta X)^2\rangle 4\tau}{1+(2\pi f\tau)^2}\,d\tau = \frac{\langle(\Delta X)^2\rangle}{\ln(\tau_2/\tau_1)}\cdot\frac{2}{\pi}\left(\tan^{-1}2\pi f\tau_2 - \tan^{-1}2\pi f\tau_1\right)\frac{1}{f}$$

$$\begin{cases} f < \frac{1}{2\pi\tau_2}, & S_x(f) = \frac{4\tau_2\langle(\Delta X)^2\rangle}{\ln \tau_2/\tau_1} \quad \text{white} \\ \frac{1}{2\pi\tau_2} < f < \frac{1}{2\pi\tau_1}, & S_x(f) = \frac{\langle(\Delta X)^2\rangle}{\ln(\tau_2/\tau_1)}\cdot\frac{1}{f} \\ f > \frac{1}{2\pi\tau_1}, & S_x(f) = \frac{\langle(\Delta X)^2\rangle}{\ln(\tau_2/\tau_1)\pi^2\tau_1}\cdot\frac{1}{f^2} \end{cases}$$

if $\tau_2 \gg 1$, the total Integral from $f=0$ & $f=\infty$, only $S_x(f) \propto \frac{1}{f}$ dominates

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<u>Idea</u>

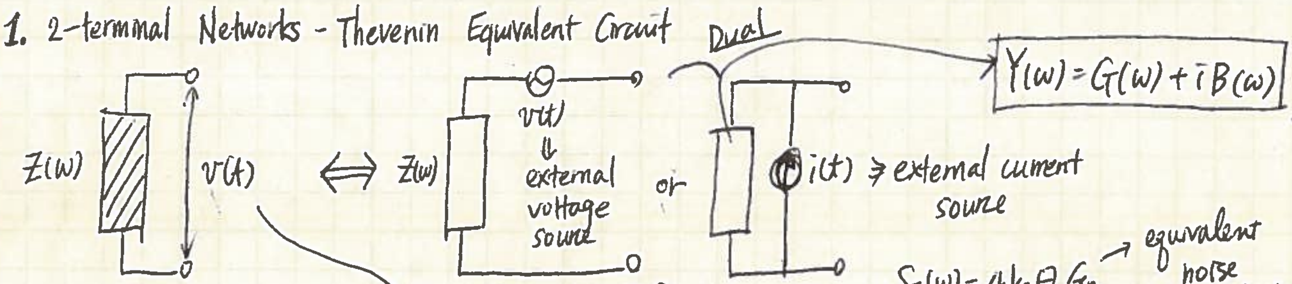
A noisy electrical network = a noise-free network + external noise generator

Model of external noise generator
↳ equivalent noise resistance or equivalent noise temperature

~~for a lumped circuit~~,

A <u>Noise figure</u> = a figure of merit to describe the inherent noiseness of the circuit
for a <u>2-port (4-terminal) network</u>.
→ this technique is useful. for a cascaded amplifier system

1. 2-terminal Networks - Thevenin Equivalent Circuit

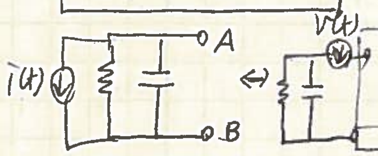


noisy 2-terminal network
$\boxed{Z(\omega)=R(\omega)+iX(\omega)}$

$S_V(\omega)=4k_B\Theta R_n$
$v(t)$ = open circuit voltage fluctuation

$S_I(\omega)=4k_B\Theta G_n$ → equivalent noise conductance

Thevenin equivalent circuits



$$S_V(\omega)=\frac{R^2}{1+\omega^2(CR)^2}S_I(\omega) = \frac{4k_B\Theta R}{1+(\omega CR)^2}$$

$\boxed{\text{Assume}}$ $S_I(\omega)=\text{const}$ & flat in a freq. range of interest.

$\boxed{\text{W-K thm}}$

$$\langle v^2\rangle=\frac{1}{2\pi}\int_0^\infty S_V(\omega)\,d\omega = \frac{4k_B\Theta}{2\pi}\int_0^\infty\frac{R}{1+(\omega CR)^2}\,d\omega=\frac{k_B\Theta}{C}$$

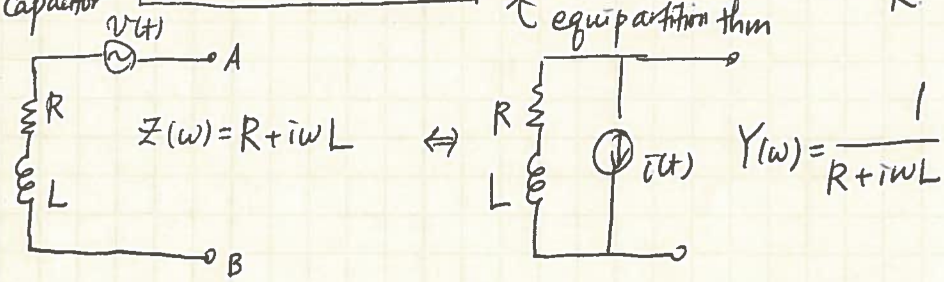
For a linear & passive network, no energy flow ∴ circuit = thermal eq.

fluctuation energy stored in capacitor

$$\boxed{\frac{1}{2}C\langle v^2\rangle} = \frac{1}{2}k_B\Theta \qquad \therefore S_I(\omega)=\frac{4k_B\Theta}{R}$$

↑ equipartition thm

Suppose



$$S_V(\omega)=4k_B\Theta\,\mathrm{Re}(Z(\omega)) \qquad S_I(\omega)=4k_B\Theta\,\mathrm{Re}(Y(\omega))$$

$$=4k_B\Theta R \qquad \Leftrightarrow \qquad =4k_B\Theta\frac{R}{R^2+(\omega L)^2}$$

F. T. $V(\omega)=Z(\omega)I(\omega)$ $\qquad = S_V(R)\dfrac{1}{R^2+\omega^2L^2}$

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W-K. Thm

$$\hookrightarrow \langle i(t)^2 \rangle = \frac{1}{2\pi}\int_0^\infty S_i(\omega)\, d\omega$$

$$= \frac{4k_B\theta}{2\pi}\int_0^\infty \frac{1}{1+\left(\frac{\omega L}{R}\right)^2} d\omega$$

$$= \frac{k_B\theta}{L}.$$

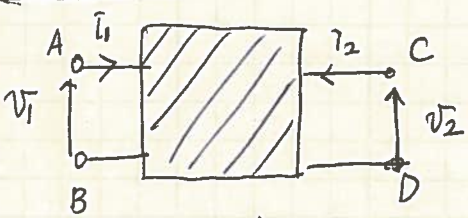
$\therefore$ fluctuation energy stored in the inductor

$$\boxed{\frac{1}{2}L\langle i(t)^2\rangle = \frac{1}{2}k_B\theta}$$ equipartition thm

Note

The resistance R does not affect the total fluctuation energy $\frac{1}{2}k_B\theta$ per DOF, but R determines the magnitude and the bandwidth of the spectral density

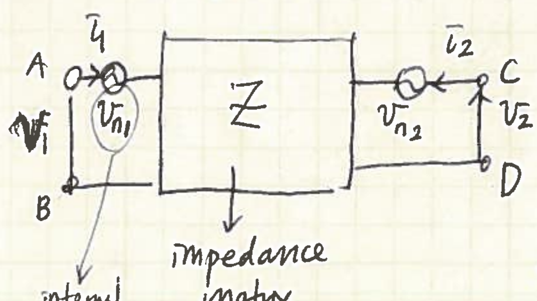
Linear 2-ports (= 4-terminal Network)



linear 2-port with internal noise generators

$\Downarrow$ Thevenin's thm.

①

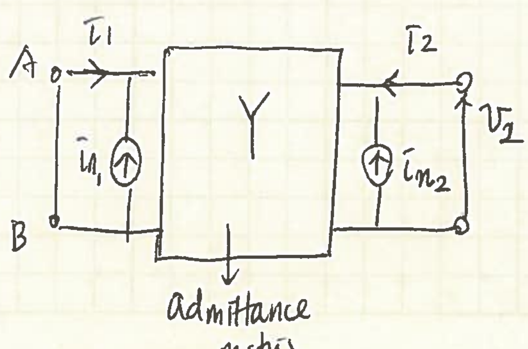


$$\begin{bmatrix} V_1 + V_{n_1} \\ V_2 + V_{n_2} \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Fourier Transforms of $v(t), v_n(t)$

F.T. of $i(t)$

②



$$\begin{bmatrix} I_1 + I_{n_1} \\ I_2 + I_{n_2} \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

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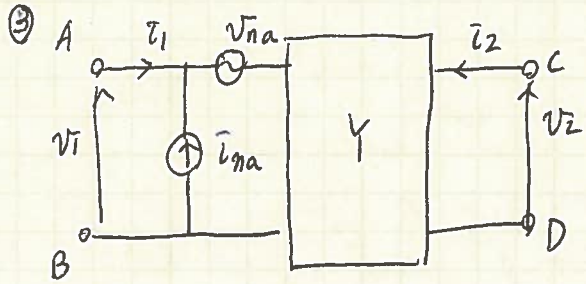
③ 

$$\begin{bmatrix} I_1 + I_{na} \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 + V_{na} \\ V_2 \end{bmatrix}$$

in this case
$$\boxed{\begin{aligned} V_{na} &= -\frac{I_{n2}}{Y_{21}} \\ I_{na} &= I_{n1} - \frac{Y_{11}}{Y_{21}} I_{n2} \end{aligned}}$$

This last equivalent circuit is valid only for calculating the noise in the output circuit. It does not give the correct description for the input port. ($i_{n1} \neq i_{na}$)

Suppose a weak signal is amplified. Then the noise associated with the signal is also amplified by a same factor.

If the amplifier is free from internal noise, the signal-to-noise (S/N) ratio is preserved.

But, since an amplifier has alway the internal noise that is added to the output signal, S/N ratio is ↓ by an amplification process.

The reason to use an amplifier is that the background noise of decision circuit are always larger than the input noise.

Thus w/ trading off S/N ratio, we are using an amplifier

A noise figure = the noisiness of a linear amplifier.

$$F = \frac{(S/N)_{in}}{(S/N)_{out}} = \frac{\text{input SNR}}{\text{output SNR}}$$

a linear 2-port case

How? $\overset{\downarrow}{=}$ $\dfrac{\text{total output noise power per unit bandwidth}}{\text{output noise power per unit bandwidth due to input noise}}$

at a specific freq. and temperature.

$$F = \frac{(S/N)_{in}}{(S/N)_{out}} \rightarrow 1 \ (0\,dB) \text{ for a noise-free amplifier}$$

$$(S/N)_{in} = \frac{\langle i_s \rangle^2 \leftarrow \text{input signal}}{\langle i_{ns}^2 \rangle \leftarrow \text{input noise}}$$

$$(S/N)_{out} = \frac{G\langle i_s \rangle^2 \leftarrow \text{output signal}}{G\langle i_{ns}^2 \rangle + \underbrace{\langle i_{no}^2 \rangle}_{\text{amplifier internal noise}} \leftarrow \text{output noise}} \qquad G = \text{gain}$$

$$\therefore F = \frac{\dfrac{\cancel{\langle i_s \rangle^2}}{\langle i_{ns}^2 \rangle}}{\dfrac{G\cancel{\langle i_s \rangle^2}}{G\langle i_{ns}^2 \rangle + \langle i_{no}^2 \rangle}} = \frac{G\langle i_{ns}^2 \rangle + \langle i_{no}^2 \rangle}{G\langle i_{ns}^2 \rangle} = 1 + \frac{\langle i_{no}^2 \rangle}{G\langle i_{ns}^2 \rangle} \equiv 1 + M$$

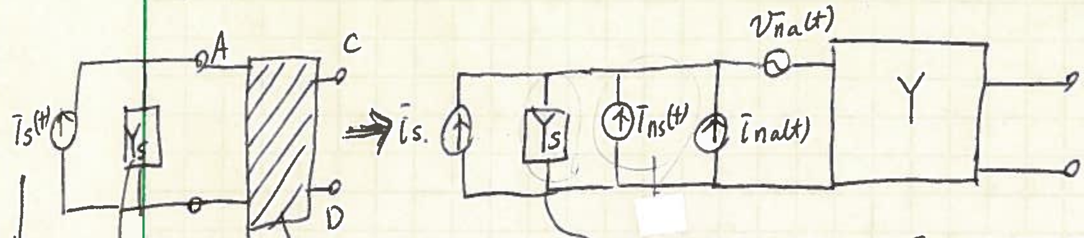
M: noise measure

for a noise-free amplifier

$$= \frac{\text{total output noise power}}{\text{output noise power due to input noise}} \quad @ \ \underline{\text{2-port}} \text{ case.}$$

if $F = 1$ ($= 0$dB) for a noise-free amplifier

A circuit for calculating F of a 2-port network



noise-free signal (ideal case)

input termination + input noise (thermal noise)

noisy linear 2-port

generated in the input admittance

$I_{ns}(t), I_{na}(t), V_{na}(t)$ are independent!

$\therefore \langle I_{ns}\rangle \langle I_{na}\rangle = 0$

$$F = \frac{\langle |I_{ns} + I_{na} + Y_s V_{na}|^2\rangle}{\langle I_{ns}^2\rangle} \longleftarrow \text{Thevenin's thm}$$

$$= 1 + \frac{S_{ia}(\omega)}{S_{is}(\omega)} + |Y_s|^2 \frac{S_{va}(\omega)}{S_{is}(\omega)} + 2\,\mathrm{Re}(\Gamma_{iv} Y_s^*)\frac{\sqrt{S_{ia}(\omega) S_{va}(\omega)}}{S_{is}(\omega)}$$

$\left.\begin{matrix} S_{ia}(\omega) \\ S_{va}(\omega) \\ S_{is}(\omega)\end{matrix}\right\}$ = power spectral density of $\left\{\begin{matrix} i_{na}(t) \\ V_{na}(t) \\ i_{ns}(t)\end{matrix}\right.$

Γ_{iv} = normalized cross-correlation spectral density (coherence ftn) btw $I_{na}(t)$ and $V_{na}(t)$

$$\therefore \Gamma_{iv}^*(\omega) = \frac{\langle I_{na}^* V_{na}\rangle}{\sqrt{\langle I_{na}^2\rangle\langle V_{na}^2\rangle}} = \frac{S_{iva}(\omega)}{\sqrt{S_{ia}(\omega) S_{va}(\omega)}}$$

Since $S_{ia}(\omega) = 4k_B\Theta G_{ni}$

$S_{va}(\omega) = \dfrac{4k_B\Theta}{G_{nv}}$

$\begin{pmatrix} G_{ni} \\ G_{nv}\end{pmatrix}$ = equivalent noise conductances, not necessarily actual conductances

$G_s = \mathrm{Re}(Y_s)$ = actual source conductance

Note that $I_{na}(t)$ is split into 2-ports

① one port which is uncorrelated with $V_{na}(t)$

② the other port which is fully correlated w/ $V_{na}(t)$

$\Rightarrow \boxed{I_{na} = I_{nb} + Y_c V_{na}}$

correlation admittance of $i_{na}(t)$ & $V_{na}(t)$

Since $\langle I_{nb} V_{na}^* \rangle = 0$ (independent.)

$I_{na} = I_{nb} + Y_c V_{na} \rightarrow 0$

$\langle I_{na} V_{na}^* \rangle = \langle \cancel{I_{nb} V_{na}^*} \rangle + \langle Y_c V_{na} V_{na}^* \rangle$ ✓

$$\Gamma_{iv} \equiv \frac{\langle I_{na} V_{na}^* \rangle}{\sqrt{\langle I_{na}^2 \rangle \langle V_{na}^2 \rangle}} = Y_c \sqrt{\frac{|V_{na}|^2}{|I_{na}|^2}} = \frac{Y_c}{\sqrt{G_{ni} G_{nv}}}$$

HW

$$\therefore F = 1 + \frac{G_{ni}}{G_s} + \frac{(G_s + G_c)^2 + (B_s + B_c)^2 - (G_s^2 + B_c^2)}{G_{nv} G_s}$$

where G_c = the real part of Y_c.

B_c = the imaginary part of Y_c

To find the optimal source admittance to minimize F,

$$\frac{\partial F}{\partial B_s} = 0 \quad \& \quad \frac{\partial F}{\partial G_s} = 0$$

$$\rightarrow F = F_0 + \frac{(G_s - G_{s0})^2 + (B_s - B_{s0})^2}{G_{nv} G_s}$$

where $F_0 = 1 + \frac{2}{G_{nv}} (G_{s0} + G_c)$ min. noise figure achieved

when $\underbrace{G_s, B_s}_{\text{source admittance}}$ satisfy the following matching condition

$$\left.\begin{aligned} G_s &= G_{s0} = \sqrt{(G_{nv} G_{ni} - B_c^2)} \\ B_s &= B_{s0} = -B_c \end{aligned}\right\} \text{noise tuning or noise matching.}$$

F increases quadratically when G_s & B_s are deviated from the optimum value

4 parameters $F_0, G_{s0}, B_{s0}, G_{nv}$ $\Rightarrow$ completely characterize the noise of the 2-port network.