2 QIC 890 Problem Set 2. Due March 1, 2016.

The assignment mark may be determined from answers to a subset of these questions.

1) Calculate the visibility of a Michelson interference pattern as a function of τ for a doppler-broadened source treating the light classically.

2) Calculate the $g^{(2)}(\tau = 0)$ for single-mode squeezed vacuum. Calculate the limits for both very weak and strong squeezing and explain these results. Does this state show a violation of the classical bounds on $g^{(2)}$?

3) a) Show that a Poissonian photon distribution remains Poissonian after linear loss.
b) If a coherent state in one mode is split at a beamsplitter, show that the output can be written as a product of coherent states.

4) Derive the momentum form of the Wigner function,

$$W(q,p) = \frac{1}{2\pi} \int e^{iqy} \langle p + y/2|\rho|p - y/2\rangle dy. \tag{2.1}$$

5) Calculate the Wigner function, $W(\alpha)$ for Schroedinger Cat state, $|\psi\rangle = \frac{1}{\sqrt{2}}(|\beta\rangle + |-\beta\rangle)$, where $|\pm\beta\rangle$ are coherent states and β is real. Compare this function to that of a probabilistic mixture of $|\beta\rangle$ and $|-\beta\rangle$ and comment on the differences. Plot your results.

6) Repeat question 5 using the Q distribution. Plot your results. Show that most (not all) of the distinguishing features obervable in the Wigner functions are effectively washed out by convolution.

7) Calculate the Wigner function for the quantum state $\frac{1}{\sqrt{2}}(|0\rangle + |3\rangle)$ and make a comment on the symmetry. What do you think the Wigner function for the state $\frac{1}{\sqrt{2}}(|0\rangle + |n\rangle)$ would look like? (This state has been referred to as a 'star state'; similar calculations have been done with states that result from application of the cubic version of a squeezing operator, $\exp\left[g^* a^{\dagger 3} - g a^3)\right]$).

8) Show that a state with a positive P function cannot violate the classical bounds on $g^2(0)$, i.e., cannot exhibit sub-poissonian statistics.

9) The Q-function can be generalized to two-mode states using,

$$Q(\alpha, \beta) = \frac{1}{\pi^2} \langle\alpha|_1 \langle\beta|_2 \rho |\alpha\rangle_1 |\beta\rangle_2 \tag{2.2}$$

(see Barnett and Knight, Journal of Modern Optics 34, 841 (1987).) Find the two-mode Q function for the two-mode squeezed vacuum state (with real ξ). Can you see the EPR correlations in the Q-function?

Unmarked questions

Find the unitary transform for the beamsplitter from a different Hamiltonian than we considered in class that leads to entirely real transform coefficients when applied to the input modes?

Sometimes experimentalists like to refer to fringe contrast as 10:1 (10 times more intensity in the peak than trough). Convert the ratio X:1 to visibility. What visibility does 10:1, 100:1, 1000:1 correspond to?

Consider the action of the displacement and squeezing operator on the quadrature operators, q and p. Show that these operators have analogous actions on the Wigner function.

Show that moments of the Q function can be used to calculate expectation value of antinormally ordered field operators.

Calculate $\langle n \rangle$ from the P function for the number state.

In the two-mode squeezed state, change the phase of ξ to make strong EPR correlations in $q_1 - q_2$ and $p_1 + p_2$ instead of $q_1 + q_2$ and $p_1 - p_2$ in the two-mode squeezed state.

Show that two-mode product states have factorizable wigner functions, $W(\alpha, \beta) = W_1(\alpha)W_2(\beta)$.

Calculate the normalized $g^{(3)}(0)$ for single mode thermal light. Compare this to the normalized classical 3rd-order correlation for chaotic light. Generalize these results to nth order.