# ECE 770-T14/ QIC 885: Quantum Electronics & Photonics

# Problem Set 2, University of Waterloo, Winter 2013

Instructor: A. Hamed Majedi

**Problem 1-** A qubit (quantum bit) can be represented by a two-level energy system with two independent states:

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

and

$$|1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

The most general state is a normalized linear combination of these two states as: $|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle$.

a) Find the relationship between $\alpha$ and $\beta$.

b) Suppose the Hamiltonian of the system is expressed by a Hermitian matrix with the following form:

$$H = \begin{pmatrix} a & b \\ b & a \end{pmatrix}$$

where $a$ and $b$ are real numbers. Solve the time-independent Schrodinger equation to find the eigenstates and their associated eigenvalues of the Hamiltonian.

c) Construct the time-dependent solution to Schrodinger equation.

d) If the system starts out at $t = 0$ in state $|0\rangle$, what is its state after time $t$.

**Problem 2-** Consider a Multiple Quantum Well (MQW) composed of 8 wells with the width of 6.25 nm that are separated by a potential barrier with a thickness of 3.75 nm and the barrier potential energy of 0.9 eV. If the effective mass of the electron in the MQW is $0.07m_0$

a) Find the first seventeen energy eigenvalues of an electron in the MQW by considering periodic boundary conditions.

b) How many energy band gaps are within the first seventeen energy eigenvalues and what are their values?

c) Plot the highest energy eigenfunctions of the first band and the lowest energy eigenfunction of the second band in one figure for comparison. Explain how you get the plots. Include your computer program.

**Problem 3-** A simple harmonic oscillator (SHO) is initially (at time $t = 0$) in a state with a wavefunction, $|\Psi\rangle = A\sum_{n=0}^{\infty} c^n|\psi_n\rangle$ where $|\psi_n\rangle$ are the SHO energy eigenfunctions, $c$ is a complex number and $|c| < 1$.

a) Determine the normalization constant $A$.

b) Find the wavefunction of the system at a later time $t > 0$, i.e. $|\Psi(t)\rangle$.

c) Compute the probability of finding the system again in the initial state at a later time, $t > 0$.

d) Calculate the expectation value of the total energy of the system.

**Problem 4-** Given the Hamiltonian operator $\hat{H} = \dfrac{\hat{p}^2}{2m} + V(x)$

where the operators $\hat{x}$ and $\hat{p}$ satisfy the commutation relationship $[\hat{x}, \hat{p}] = \hat{x}\hat{p} - \hat{p}\hat{x} = i\hbar$, find the following commutators

a) $[\hat{x}^2, \hat{p}]$.

b) $[\hat{x}^n, \hat{p}]$. Hint: $n$ is natural number. Use Induction for proof.

c) $[g(\hat{x}), \hat{p}]$.

d) $[\hat{x}, \hat{H}]$

e) $[\hat{p}, \hat{H}]$.

**Problem 5-** Consider an atomic structure that is made up of three identical atoms at the corners of an equilateral triangle, such as Carbon and the atomic structure looks like graphene unit cell. Each atom contributes one electron with self-energy $E_0$. Suppose the matrix element of the Hamiltonian for the electron on two adjacent sites $i$, $j$ is $\langle i|\hat{H}|j\rangle$=-a for $i \neq j$. Note that $i$ and $j$ can be 1, 2, and 3 and you can define three state vectors describing each atom at its own site.

a) Calculate the energy eigenvalues of this system that represent energy splittings.

b) Suppose an electric field is applied to this system such that potential energy of the electron on top of the triangle is lowered by $V_o << a$. Now calculate the energy levels.

c) Suppose electron is in the ground level of the system. Suddenly the field is rotated 120 degrees and points toward one of the atom on the base of the triangle. calculate the probability for the electron to remain in the ground state.

**Due: Wednesday Feb. 13, 2013. (before starting the class)**