Lecture 17: Decoherence with low frequency noise and driving, characterization of decoherence

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Decoherence in superconducting qubits arises from interaction with a complex environment. The physics associated with decoherence in superconducting devices is very rich. Intense effort in recent years led to more understanding on the way in which different decoherence channels couple to qubits. This understanding led to better qubit designs, in which coherence is optimized by avoiding coupling to noise channels while still allowing for sufficient coupling to implement control. One recent relevant example of such an optimization procedure is the transmon [1]. Only 10 years after the first observation of coherent oscillations in a superconducting device, the coherence times improved by more that three orders of magnitude.

Despite these advances, many sources of decoherence are still only partially understood. More research is needed to understand both the origin of energy relaxation and dephasing. This is a very active area of current research.

In this lecture we will consider the following topics: low frequency noise (for which the treatment based on master equations is not sufficient), decoherence during driving, and procedures to experimentally characterize decoherence.

I. TREATMENT OF LOW-FREQUENCY NOISE

Low-frequency noise is present in all superconducting qubits. Low frequency noise is most commonly noise for which the power spectral density varies inversely proportionally with frequency. So for a noise process $\xi(t)$ the spectral density varies as a funtion of ω as

$$S_\xi(\omega) = A/\omega \tag{1}$$

with A a constant, and more generally as

$$S_\xi(\omega) = A/\omega^\alpha \tag{2}$$

with α a positive number, very often close to one.

Treatment of low frequency noise requires techniques which go beyond what can be done using master equations. The reason for this will become clear in the treatment below. We assume the noise to be "classical". That is we take the qubit Hamiltonian to be

$$H_{qb} = -\hbar\frac{\omega_{01} + \xi(t)}{2}\sigma_z. \tag{3}$$

The possibility exists that the noise ξ_t couples as well through a transverse operator (σ_x or σ_y). However, since the variations are slow (Hz to kHz) compared to qubit frequencies, no relaxation will take place. However, pure dephasing occurs; this is explored theoretically and experimentally in detail in *e.g.* [2].

The effect of the "coherent" part of the Hamiltonian 3 is that the phase of a superposition of states $|0\rangle$ and $|1\rangle$ acquires a phase $\Delta\phi_{01}(\tau) = -\omega_{01}\tau$. This is a deterministic contribution which can be simply left aside. The effect of the noise part is to lead to a phase

$$\Delta\phi_{01}(\tau) = -\int_0^\tau dt\xi(t). \tag{4}$$

Since $\xi(t)$ is a random process, $\Delta\phi_{01}(\tau)$ is a random variable. It is relevant to calculate the expectation value of $e^{i\Delta\phi_{01}(\tau)}$, since it is this term which described the decay of the coherence of the density matrix. In experiments, these different realizations correspond to different runs of a sequence in which the qubit is prepared in a superposition, for example $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$, and the phase is measured after free precession over time τ. These different realizations are independent if the repetition time T_{rep} is much longer than the correlations time of the noise. This correlation time is the time at which $\langle\xi(t)\xi(0)\rangle$ tends to zero. This is a first problem with 1/f noise. Without a lower frequency cutoff, the correlation time would be infinite. Assuming this problem is solved we consider the calculation of $\langle e^{i\Delta\phi_{01}(\tau)}\rangle$.

We use an important result of the theory of random processes [3]. For a noise $\xi(t)$ which is Gaussian [1] and zero average

$$\langle e^{-i\int_0^\tau dt\xi(t)}\rangle = e^{-\frac{1}{2}\int_0^\tau dt_1\int_0^\tau dt_2\langle\xi(t_1)\xi(t_2)\rangle}. \tag{5}$$

It is useful to express this as a function of the Fourier transform of the correlation function

$$S_\xi(\omega) = \int_{-\infty}^{\infty} d\tau e^{i\omega\tau}\langle\xi(\tau)\xi(0)\rangle \tag{6}$$

through its inverse

$$\langle\xi(\tau)\xi(0)\rangle = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\omega e^{-i\omega\tau}S(\omega). \tag{7}$$

We have

$$\langle e^{i\Delta\phi_{01}(\tau)}\rangle = e^{-\frac{\tau^2}{2}\frac{1}{2\pi}\int d\omega S(\omega)\mathrm{sinc}^2\left(\frac{\omega\tau}{2}\right)}. \tag{8}$$

[1] The notion of a Gaussian process is an extension of the notion of Gaussian variable. For a sample of the process, $\{\xi(t_1)\xi(t_2)..\xi(t_n)\}$ the probability to have $\xi(t_1) = \xi_1$, $\xi(t_2) = \xi_2$, .., $\xi(t_n) = \xi_n$ is e to power a quadratic positive form of the variables ξ_1, ξ_2, .. , ξ_n.

We first apply 8 to the situation of white noise, that is $S(\omega) = S(0)$ is a constant. The function $\mathrm{sinc}^2 x$ in the integrand of 8 has a width of $\approx \pi$ and an integral

$$\int_{-\infty}^{\infty} dx \mathrm{sinc}^2 x = \pi. \tag{9}$$

Since the spectral density is a constant, it can be pulled outside the integral to give

$$\langle e^{i\Delta\phi_{01}(\tau)} \rangle = e^{-\frac{S(0)}{2}\tau}. \tag{10}$$

The coherence of the density matrix decays with a rate $S(0)/2$. This is the same as what is obtained from the master equation, where we found as well that the pure dephasing rate depends on noise at zero frequency. A deviation from the exponential law occurs for example for a spectral density which is constant up to frequency ω_c and decreases to zero after this value. In this case, the decay is slower than exponential at short times.

We next deal with $1/f$ noise, following Ithier *et al.* We include both a low-frequency cutoff ω_{ir} and the high frequency cutoff ω_c. The low frequency cutoff is necessary to guarantee convergence. Even in the absence of an actual cutoff, an effective low frequency cutoff arises from the finite total duration of the experiment (including all the repetitions required to extract statistics) t_{exp}, as $\omega_{ir} \approx 1/t_{exp}$. The result for the decay function, in the limit $t \ll 1/\omega_{ir}$ is

$$\langle e^{i\Delta\phi_{01}(\tau)} \rangle = \exp\left[-t^2 A \left(\ln \frac{1}{\omega_{ir} t} + O(1)\right)\right]. \tag{11}$$

The decay is found to be Gaussian, except for a small logarithmic correction.

We finally note one point about how to make use of the treatment here of low frequency noise, seen as a classical signal, in combination with the treatment based on master equations. We can use a transformation to a rotating frame as used in the derivation of the master equations, which is with the evolution operatorexp $\left[\frac{i}{\hbar}\left(H_s + H_{bath}\right) t\right]$ with H_s the deterministic component of the system Hamiltonian and H_{bath} the bath Hamiltonian. In this frame the density matrix ρ of the system varies according to the following equation:

$$\dot{\rho}(t) = -\frac{i}{\hbar}\left[H_{qb,random}, \rho\right] + \text{dissipative terms}, \tag{12}$$

where

$$H_{qb,random} = -\hbar \frac{\xi(t)}{2} \sigma_z \tag{13}$$

is the random component of the qubit dynamics and the reminder of the RHS is given by the usual terms in the master equation possibly including "behaved" low-frequency noise. The equation satisfied by the diagonal elements of the density matrix is unchanged by the addition of the random component to the Hamiltonian. For the off-diagonal component one has

$$\dot{\rho}_{01} = i\xi(t)\rho_{01} - \Gamma_2\rho_{01}. \tag{14}$$

With a transformation

$$\rho_{01}(t) = e^{i\int_0^t \xi(t')dt'}\widetilde{\rho}_{01}(t). \tag{15}$$

Inserting this in 14 we obtain

$$\dot{\widetilde{\rho}}_{01} = -\Gamma_2\widetilde{\rho_{01}} \tag{16}$$

which leads to

$$\rho_{01}(t) = \rho_{01}(0)e^{-\Gamma_2 t}e^{-i\int_0^t \xi(t')dt'}. \tag{17}$$

This expression for the off-diagonal element of the density matrix already contains the quantum average and the corresponding Γ_2 decay due to the interaction with the bath. One can take in addition an average over different realizations of the ξ noise process, which is independent of the quantum average (due to interaction with the bath). The decay of the coherence due to the noise process is simply an additional factor; this is the assumption taken in the interpretation of the data in the experiments of Ithier *et al* [2].

II. DECOHERENCE FOR A DRIVEN SYSTEM

A system driven by a resonant field decoheres in a way which is different of decoherence without driving. In the following we analyse a simple instance of this effect, namely a qubit coupled to its environment only through dephasing processes and driven with a field on resonance.

We assume the following Hamiltonian:

$$H = -\frac{\hbar\omega_{01}}{2}\sigma_z + A_x\cos\omega_{01}t\sigma_x + \sigma_z f_x + H_{bath} \tag{18}$$

where the Pauli matrices are operators corresponding to the qubit written in the eigenenergy basis, A_x is the amplitude of the driving term, f is a bath operator and H_{bath} is the bath

Hamiltonian. We apply a frame transformation which undoes the qubit evolution under its own Hamiltonian. In this frame the Hamiltonian is

$$\widetilde{H} = \frac{\hbar A_x}{2}\sigma_x + \sigma_z f_x + H_{bath} \tag{19}$$

where we also neglected fast rotating terms (the rotating wave approximation). In the rotating frame the qubit Hamiltonian is along the x axis whereas the noise couples to the qubit through the σ_z operator. Since this is transverse, the effect of the coupling to the environment will be relaxation in the new frame, which is proportional to the noise spectral density at frequency $A_x/2$. In the lab frame, energy relaxation is decoherence. We thus obtain the result that the qubit decoheres during driving with a rate which depends on the noise spectral density at the Rabi frequency. The rotation of the qubit does in a sense a conversion of the noise at the rotation frequency to noise at zero frequency which is what produces dephasing in the lab frame. This is similar to decoherence with dynamical decoupling, which we will analyze below.

Both decoherence and relaxation rates are affected by driving. These effects have been observed in experiment by Ithier *et al* [2] and others, and treated in extensive theoretical details by Smirnov [4]

III. EXPERIMENTAL CHARACTERIZATION OF DECOHERENCE

A. Use of MW pulses for qubit control

A qubit Hamiltonian is in general given by

$$H_{qb} = -\frac{\hbar\omega_{01}}{2}\sigma_z + H_{ctr} \tag{20}$$

with H_{ctr} the control Hamiltonian which contains the following terms:

1. driving terms of the form $2\Omega_R(t)\cos(\omega t + \phi(t))\sigma_x$. $\Omega_R(t)$ (which is the Rabi frequency for driving on resonance) and $\phi(t)$ are slow (compared to $1/\omega_{01}$) functions of time. The frequency is taken either equal or close to the rotation frequency. In a frame rotating at frequency ω, after the RWA, one is left with a static term of the form $\Omega_R(t)\cos\phi(t)\sigma_x - \Omega_R(t)\sin\phi(t)\sigma_y(t)$. This allows rotation over any axis in the equatorial plane of the Bloch sphere. Note that adding a driving term proportional to σ_y does not allow for

additional freedom as long as it is possible to change the phase ϕ. If this fast driving term couples as well to the σ_z component the effect is negligible as long as the Rabi frequency is small compared to the energy level splitting.

2. a slowly varying term of the form $-\frac{\hbar\overline{\omega}_{01}(t)}{2}\sigma_z$. In the frame rotating with respect to omega this term remains. The variation of $\overline{\omega}_{01}(t)$ allows for the implementation of rotations around the z axis.

A rotation of angle θ around the $\boldsymbol{n}$ axis is defined in the following way

$$\mathcal{R}_{\boldsymbol{n}}(\theta) = e^{-i\theta\frac{\sigma_{\boldsymbol{n}}}{2}} \tag{21}$$

with

$$\sigma_{\boldsymbol{n}} = \boldsymbol{\sigma}\boldsymbol{n}. \tag{22}$$

For a Hamiltonian

$$H_{qb} = -\frac{\omega}{2}\sigma_{\boldsymbol{n}} \tag{23}$$

a rotation takes place around the $\boldsymbol{n}$ axis with angle $\theta = -\omega t$ during a time interval t. Note that a positive relation is in the trigonometric direction when looking in the negative direction of the axis. Examples of standard rotations around different axes are shown in Fig. 1.

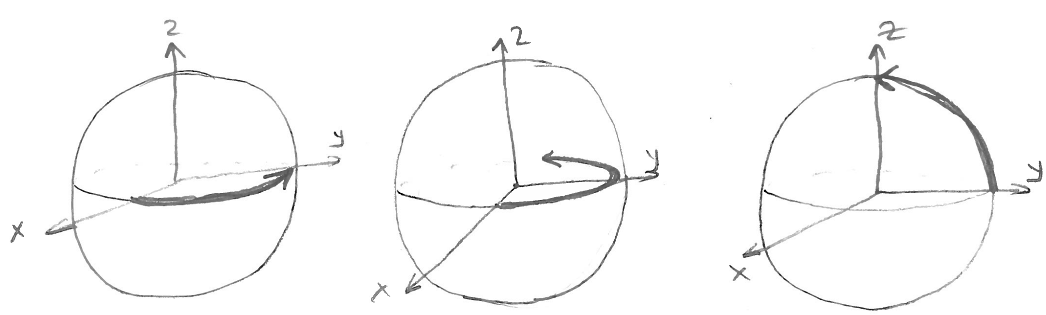

FIG. 1. Representation of rotations: (a) $\pi/2$ around z axis, (b) π rotations around the z axis, and (c) $\pi/2$ rotation around x axis.

A theorem in quantum control shows that any unitary evolution (which is a rotation of some angle around some axis) can be decomposed into rotations about two perpendicular axes, with no requirement on the two axes other than they are perpendicular to each other [5].

So the possibility to do rotations around x, y, and z axis which is possible by rotations of controllable phase (for the first two) and by changing the energy (for the third) provide in fact more than is necessary for full control. Any two of these knobs are sufficient.

B. Ensemble measurements

In general, characterization of decoherence requires some form of ensemble average. This is a temporal ensemble for superconducting qubits. The general unit sequence used in the experiments is illustrated in Fig. 2.

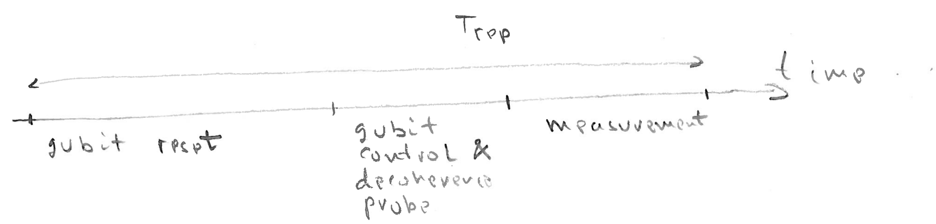

FIG. 2. Unit sequence used for characterization of decoherence in superconducting qubits.

In general many repetitions are needed to reduce statistical fluctuations (scaling as the inverse of the square root of runs) of single qubit measurement. Detector readout errors add to this error. Depending on the efficiency of the single measurement and required accuracy, the number of repetitions is typically 10^{3-6}. Qubit reset is most commonly done through energy relaxation.

C. T_1 measurements

The pulse sequence is shown in Fig. 3. The sequence is repeated a number of times for a set of delay times.

D. Free induction decay

This type of measurement is called either a free induction decay (NMR language) or Ramsey fringe (in atomic physics). A first $\left(\frac{\pi}{2}\right)_x$ brings the polarization in the equatorial

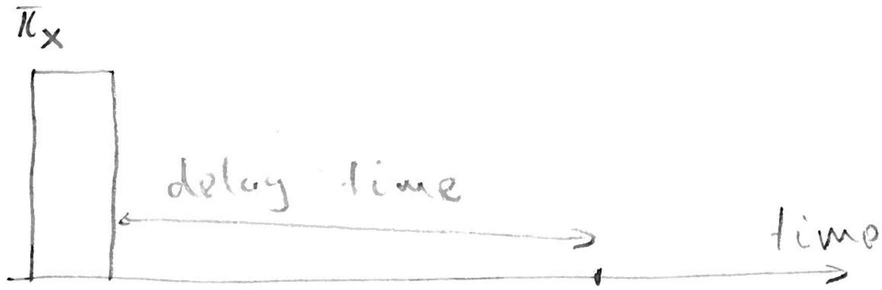

FIG. 3. Control and probe sequence for T_1 measurements.

plane. During the delay time the polarization moves in the equatorial plane as a result of dephasing (in the absence of dephasing the polarization would be a fixed quantity). A second pulse, $\left(-\frac{\pi}{2}\right)_x$, maps the random phase to a population (that is a σ_z "signal") which can be measured. In most experiments, the experiments are done at some fixed value of the detuning of the drive with respect to the energy level splitting. This has the effect of removing random experimental detuning which can lead to experimental artefacts. With detuning, the observed signal is a decaying sinusoidal average population versus time; the envelope represents the relevant decay signal.

IV. SPIN ECHO

This is a sequence formed by:

- a $\left(\frac{\pi}{2}\right)_x$ pulse which brings polarization around the $-y$ axis.
- free precession over a time $t_{delay}/2$
- a $(\pi)_x$ pulse. This reverses all the phases.
- free precession over a time $t_{delay}/2$
- a $\left(\frac{\pi}{2}\right)_x$ pulse which maps the acquired phase back to a population

For a single experiment the total phase of the precession is $\phi_2 - \phi_1$ with ϕ_1 (ϕ_2) the phase accumulated during the first (second) free precession interval. If low-frequency fluctuations are slow, then $\phi_1 = \phi_2$. As a result decoherence is fully recovered. In general, for any noise process a spin echo sequence reduces the difference between T_2 and $2T_1$. In many cases the limit $T_2 = 2T_1$ is found with such an experiment.

The usefulness of the spin-echo sequence is two-fold. First of all for studying decoherence, the difference between dephasing with and without spin echo gives useful information on the environmental noise. Secondly, the spin echo sequence can be used to improve the coherence of a qubit which is "idle" in some step of a quantum protocol.

A. CPMG

The CPMG (Carr-Purcell-Meibomm-Gill) sequence is a generalization of the spin-echo sequence. It is a sequence of pulses of the form $\left(\frac{\pi}{2}\right)_x$–$(\pi)_x^N$-$\left(\frac{\pi}{2}\right)_x$, with $(\pi)_x^N$ signifying a succession of N $(\pi)_x$ pulses. The time delay between any two consecutive pulses is the same. Increases the number of pulses within a total time interval has the effect that faster and faster environmental fluctuations can be removed. This has been used also as a probe of environmental noise up to relatively large frequency in [6].

V. FURTHER READING

For detailed explanations on control and studies of decoherence in nuclear magnetic resonance, see [7]

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