Values used everywhere

In[60]:=
```
S11 = .1 Exp[-I 30 °];
S12 = .4 Exp[-I 75 °];
S21 = .95 Exp[-I 45 °];
S22 = .15 Exp[-I 10 °];
Vs = 20 V;
Zs = 100 Ω;
Zc = 50 Ω;
Z1 = 50 Ω;
```

Attempt at a reference-impedance-free solution:

In[63]:=
```
Gammas =
  FullSimplify[Solve[{Γins == S11 - S12 S21 Γouts / (1 + S22 Γouts), Γouts == S22 - S12 S21 Γins / (1 + S11 Γins)}, {Γins, Γouts}]];
{Γin1, Γout1} = ({Γins, Γouts} /. Gammas)[[1]]
{Γin2, Γout2} = ({Γins, Γouts} /. Gammas)[[2]]
Zin = Zs (1 + Γin2) / (1 - Γin2)
Γin = Γin2
Γ1 = -Γout2
```

Solve::ratnz : Solve was unable to solve the system with inexact coefficients.
The answer was obtained by solving a corresponding exact system and numericizing the result. >>

Out[64]= {-8.78584 + 0.458246 i, -5.95836 + 0.433468 i}

Out[65]= {0.113511 + 0.00592043 i, 0.166948 + 0.0121454 i}

Out[66]= (125.599 + 1.50666 i) Ω

Out[67]= 0.113511 + 0.00592043 i

Out[68]= -0.166948 - 0.0121454 i

In[69]:=
```
a1 = Vs / (2 √Zc)
```

Out[69]= $\sqrt{2}\ \mathrm{V}/\sqrt{\Omega}$

In[70]:= $b_1 = \frac{V_s}{2\sqrt{Z_c}}\,\Gamma_{in}$

Out[70]= $(0.160528 + 0.00837275\,i)\ V/\sqrt{\Omega}$

In[71]:= $a_2 = \frac{V_s}{2\sqrt{Z_c}}\,\frac{S_{21}\,\Gamma_L}{1 - S_{22}\,\Gamma_L}$

Out[71]= $\frac{\left((0.95 - 0.95\,i)\ V/\sqrt{\Omega}\right)\Gamma_L}{1 - (0.147721 - 0.0260472\,i)\,\Gamma_L}$

In[72]:= $b_2 = \frac{V_s}{2\sqrt{Z_c}}\,\frac{S_{21}}{1 - S_{22}\,\Gamma_L}$

Out[72]= $\frac{(0.95 - 0.95\,i)\ V/\sqrt{\Omega}}{1 - (0.147721 - 0.0260472\,i)\,\Gamma_L}$

In[73]:= $P_{load} = \text{UnitConvert}\left[\frac{1}{2}\,\text{Re}\left[\sqrt{Z_c}\,(a_2 + b_2)\,\text{Conjugate}\left[\frac{\sqrt{Z_c}\,(a_2 + b_2)}{Z_L}\right]\right],\ \text{"milliWatts"}\right]$

Thread::tdlen : Objects of unequal length in {{0.852279 − 0.0260472 *i*, 1 + Conjugate[Γ$_L$]}, Quantity[0.95 + 0.95 *i*, {Ohms, Volts}]} + {{0.852279 − 0.0260472 *i*, 1 + Conjugate[Γ$_L$]}, Conjugate[Γ$_L$], Quantity[0.95 + 0.95 *i*, {Ohms, Volts}]} cannot be combined. >>

Thread::tdlen : Objects of unequal length in {Quantity[0.95 − 0.95 *i*, {Ohms, Volts}], {0.852279 + 0.0260472 *i*, 1 + Γ$_L$}} + {Quantity[0.95 − 0.95 *i*, {Ohms, Volts}], {0.852279 + 0.0260472 *i*, 1 + Γ$_L$}, Γ$_L$} cannot be combined. >>

Thread::tdlen : Objects of unequal length in {{0.852279, 1 + Re[Γ$_L$]}, Re[Quantity[0.95 + 0.95 *i*, {Ohms, Volts}]]} + {{0.852279, 1 + Re[Γ$_L$]}, Re[Γ$_L$], Re[Quantity[0.95 + 0.95 *i*, {Ohms, Volts}]]} cannot be combined. >>

General::stop : Further output of Thread::tdlen will be suppressed during this calculation. >>

Out[73]= $\text{UnitConvert}\left[\frac{1}{2}\,\text{Re}\left[\frac{1}{\text{Conjugate}[Z_L]}\left(\frac{(0.95 + 0.95\,i)\ V/\sqrt{\Omega}}{1 - (0.147721 + 0.0260472\,i)\,\text{Conjugate}[\Gamma_L]} + \frac{\text{Conjugate}[\Gamma_L]\left((0.95 + 0.95\,i)\ V/\sqrt{\Omega}\right)}{1 - (0.147721 + 0.0260472\,i)\,\text{Conjugate}[\Gamma_L]}\right)(50\,\Omega)\left(\frac{(0.95 - 0.95\,i)\ V/\sqrt{\Omega}}{1 - (0.147721 - 0.0260472\,i)\,\Gamma_L} + \frac{\left((0.95 - 0.95\,i)\ V/\sqrt{\Omega}\right)\Gamma_L}{1 - (0.147721 - 0.0260472\,i)\,\Gamma_L}\right)\right],\ \text{milliWatts}\right]$

In[74]:= $P_{ref} = \text{UnitConvert}\left[\frac{1}{2}\frac{\text{Abs}[V_s]^2}{\text{Abs}[Z_s + Z_{in}]^2}\text{Re}[Z_s], \text{"milliWatts"}\right]$

Out[74]= 392.949 mW

In[75]:= $P_{used} = \text{UnitConvert}\left[\frac{1}{2}\text{Re}\left[V_s\,\text{Conjugate}\left[\frac{(a_1 + b_1)}{Z_{in}}\text{Sqrt}[Z_c]\right]\right], \text{"milliWatts"}\right]$

Out[75]= 886.489 mW

In[76]:= $P_{network} = \text{UnitConvert}\left[\frac{1}{2}\text{Re}\left[\left(V_s\frac{Z_{in}}{Z_{in} + Z_s}\right)\text{Conjugate}\left[\frac{V_s}{Z_{in} + Z_s}\right]\right], \text{"milliWatts"}\right]$

Out[76]= 493.54 mW

In[77]:= $P_{network} + P_{ref}$

Out[77]= 886.489 mW

In[78]:=

Reference Impedance Solution

Out[78]= Reference Impedance Solution

In[79]:= $Z_{c_1} = 100\ \Omega\ ;\ Z_{c_2} = 50\ \Omega\ ;\ \Gamma_1 = \frac{Z_1 - Z_{c_2}}{Z_1 + Z_{c_2}}\ ;\ \Gamma_{in} = S_{11} + \frac{S_{12}\,S_{21}\,\Gamma_1}{1 - S_{22}\,\Gamma_1}\ ;\ \Gamma_s = \frac{Z_s - Z_{c_1}}{Z_s + Z_{c_1}}\ ;$

$Z_{in} = Z_{c_1}\frac{(1 + \Gamma_{in})}{1 - \Gamma_{in}}\ ;$

$a_1 = \text{UnitConvert}\left[N\left[\frac{V_s}{\sqrt{Z_{c_1}}}\frac{Z_{in}}{Z_s + Z_{in}}\frac{1}{1 + \Gamma_{in}}\right], \text{"SI"}\right]$

Out[81]= $\left(1. + 1.38778 \times 10^{-17}\,i\right)\ V/\sqrt{\Omega}$

In[82]:= $b_1 = \Gamma_{in}\,a_1$

Out[82]= $\left(0.0866025 - 0.05\,i\right)\ V/\sqrt{\Omega}$

In[83]:= $b_2 = a_1\frac{S_{21}}{1 - \Gamma_1\,S_{22}}$

Out[83]= $\left(0.671751 - 0.671751\,i\right)\ V/\sqrt{\Omega}$

In[84]:= $a_2 = b_2\,\Gamma_1$

Out[84]= $\left(0. + 0.\,i\right)\ V/\sqrt{\Omega}$

In[85]:= $P_{load} = \text{UnitConvert}\left[\frac{1}{2}\,\text{Re}\left[\sqrt{Z_{c_2}}\,(a_2 + b_2)\,\text{Conjugate}\left[\sqrt{Z_{c_2}}\,\frac{a_2 + b_2}{Z_1}\right]\right], \text{"milliWatts"}\right]$

Out[85]= 451.25 mW

In[86]:= $P_{source} = \text{UnitConvert}\left[\frac{1}{2}\,\text{Re}\left[\left(V_s\,\frac{Z_s}{Z_{in} + Z_s}\right)\,\text{Conjugate}\left[\frac{V_s}{Z_{in} + Z_s}\right]\right], \text{"milliWatts"}\right]$

Out[86]= 418.397 mW

In[87]:= $P_{network} = \text{UnitConvert}\left[\frac{1}{2}\,\text{Re}\left[(a_1 + b_1)\,\text{Conjugate}[a_1 - b_1]\right], \text{"milliWatts"}\right]$

Out[87]= 495. mW

In[89]:= $P_{del} = \text{UnitConvert}\left[\frac{1}{2}\,\text{Re}\left[\frac{V_s^2}{\text{Conjugate}[Z_{in} + Z_s]}\right], \text{"milliWatts"}\right]$

Out[89]= 913.397 mW

In[90]:= $P_{network} + P_{source}$

Out[90]= 913.397 mW