Final Exam

ECE 770-T14/QIC 885: Quantum Electronics & Photonics

University of Waterloo

Instructor: A. Hamed Majedi, April 13 2013 , Duration: 2.5 hours

Problem 1 (50 points):

Consider a spinless object of mass m and charge q that is constrained to move in a circle of radius a in the $x-y$ plane.

a) Determine the quantized energy levels for the following cases:

a1) The motion of the object is nonrelativistic with a constant linear momentum.

a2) The motion of the object is strongly relativistic, namely $E \approx pc$, c is the speed of light.

a3) The nonrelativistic motion of the object is influenced by a constant magnetic field, B_o, perpendicular to the plane of the circle, e.g. in $+z$ direction.

Hint: Use the circular gauge for the vector magnetic potential.

b) Now consider the motion of the object that is influenced by a strong electric field, $\mathbf{E} = E_o\hat{x}$, in $+x$ direction so that $qE_o >> \frac{\hbar^2}{ma^2}$.

b1) Write down the Hamiltonian of the system in a cylindrical coordinate system. Note that the electric potential energy can be written as $V = -\int q\mathbf{E}\cdot d\mathbf{r}$.

Note 1: $(x = \rho\cos\phi, y = \rho\sin\phi)$ and (ρ, ϕ) are cartesian and cylindrical coordinates, respectively.

Note 2: $\nabla^2 = \frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho\frac{\partial}{\partial\rho}) + \frac{1}{\rho^2}\frac{\partial^2}{\partial\phi^2} + \frac{\partial^2}{\partial z^2}$.

b2) Given the electric field is strong, the probability that the object is moving around $\phi = 0$ is large and $\cos\phi \approx 1 - \frac{\phi^2}{2}$, rewrite the Hamiltonian in part b1).

b3) Write down the time-independent Schrodinger equation for the wavefunction of the system, namely $\Psi(\phi)$.

b4) Determine the eigenstates of the system.

b5) Determine the quantized energy levels.

c) If there is an experiment that determines few quantized energy levels, such as spectroscopy, how can you identify that the system under the experiment is falling into each categories, i.e. a1), a2), a3) or b5) ?

Problem 2 (20 points):

Consider a system of free electron gas. A fraction p is known to have their z-component of spin in the up direction, i.e. $|\uparrow\rangle$ state, and remainder are randomly in both up and down direction with equal probability.

a) Write down the density matrix describing the system.

b) Find the expectation value of spin in x, y and z directions.

Problem 3 (30 points):

Consider an electromagnetic field intensity operator $\hat{I}(z) = I_o \exp(gz\hat{N})$ representing an intensity of a traveling wave along $z > 0$ direction with constant I_o and g, where $\hat{N}$ is the number operator. Calculate the uncertainty in the measurement of the field intensity operator in a coherent state basis.