Multilayer Josephson Junctions as a Multiple Quantum Well Structure

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***Abstract*—The Macroscopic Quantum Model in conjunction with the Transfer Matrix Method and Bloch wave analysis have been used to compute the energy dispersion equation for Cooper pairs in finite Superconducting Quantum Well (SQW) structures. The Energy dispersion equation has been numerically solved by the Cauchy Integration method to obtain the energy levels of quantum states in SQW. The possibility of energy levels and subbands formation for Cooper pairs within the bulk energy gap has been shown. Energy gap squeezing has been derived when the Bragg condition for Bloch wave number is satisfied.**

***Index Terms*—Multilayer Josephson Junctions, quantum well structures, superconducting devices, THz devices.**

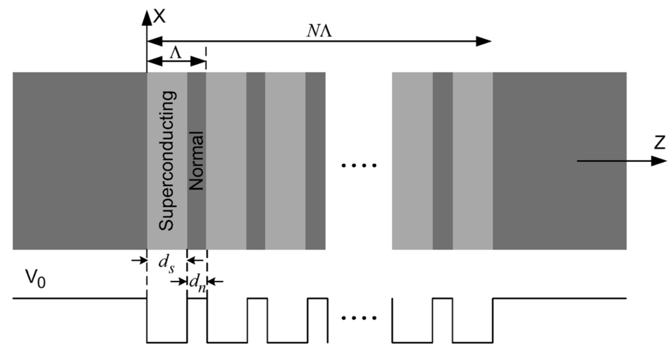

Fig. 1. N-layered superconducting quantum well structure.

I. Introduction

QUANTUM well structures are ultrafine layered media capable of confining electrons and quasi-particles with quantized energy levels. Electronic transitions between sets of discrete energy levels and bands have long been employed to generate and detect electromagnetic radiation for various frequencies. Semiconductor Quantum well Infra Red (IR) detectors and THz quantum cascade lasers are among the paramount examples of optoelectronic devices that benefit from such structures. Incorporating superconducting layers in such structures was discussed theoretically and experimentally by a number of people and groups from the perspective of electronic bandgap structures [1] as well as stacked Josephson Junctions to generate high-frequency Josephson radiation [2]. The recent advancement of nanofabrication technology and characterization techniques has revitalized the applications of such Superconducting Quantum Well (SQW) structures in superconducting THz optoelectronics [3]. Conventionally, the Gor'kov and Bogoliubov-de Gennes equations have been employed to find the quasi-particle energy states in the SQW structure and superconducting superlattices [4], [5]. In this paper, we present an intuitive model based on the Macroscopic Quantum Model [6] of superconductivity to come up with an energy dispersion equation for a finite SQW structure. The energy dispersion equation is then solved by the Cauchy Integration Method to obtain the allowed energy levels of the quantum states for Cooper pairs. SQWs can be engineered to form energy levels for Cooper pairs within the bulk energy gap. The depopulation of carriers results in electronic transitions between various allowed energy levels useful for applications in designing mm-wave and THz devices.

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II. Analysis and Discussion

The structure under consideration has alternating superconducting and normal layers with thicknesses d_s, and d_n, respectively. This structure has a periodicity length, $\Lambda = d_s + d_n$ in the z direction and includes N layers attached from both ends to the semi-infinite normal layers, as shown in Fig. 1. This stacked configuration might represent a series of Superconductor-Normal-Superconductor (SNS) or Superconductor-Insulator-Superconductor (SIS) Josephson junctions that can be built in-plane or out-of-plane in a waveguide system. Similar to [3], a normal-superconductor-normal structure is here called a superconducting quantum well (SQW) structure and in the case of a large number of layers, a superconducting superlattice structure. The thickness of the superconducting layer is assumed to be greater than its coherence length, to safely define a macroscopic quantum wave function, $\Psi(z)$ for the Cooper pair condensate. In structures with a very thin superconducting and normal layers, the proximity effects such as Andreev reflection should be further included in the modeling [7]. Using the Macroscopic Quantum Model (MQM), the Cooper pair number density, $n_s(z)$, is related to its wave function in each layer as:

$$\Psi(z) = \sqrt{n_s(z)}e^{j\theta(z)} \quad (n-1)\Lambda < z < (n-1)\Lambda + d_s \tag{1}$$

where $n = 1, 2, \ldots, N$. The normal layer is characterized by a barrier voltage $V_o = \alpha\Delta_o$, in which Δ_o is the energy gap in the superconductor and $\alpha \geq 1$ measures the relative strength of the barrier with respect to the energy gap, as shown in Fig. 1. Based on the Kronig-Penny model, the barrier can be considered as:

$$V(z) = \begin{cases} 0 & (n-1)\Lambda < z < (n-1)\Lambda + d_s, \quad n = 1, 2, \ldots, N \\ V_o & \text{otherwise} \end{cases} \tag{2}$$

The macroscopic wave function satisfies the time-independent Schrödinger equation,

$$-\frac{\hbar^2}{2m^*}\nabla^2\Psi(z)=[E-V(z)]\,\Psi(z) \tag{3}$$

where m^* is the mass of Cooper pair and E is the allowed energy of quantum states for Cooper pairs. The solution of (3) in the SQW structure can be written as:

$$\Psi(z)=\begin{cases} C_0e^{-qz}+D_0e^{qz} & z<0\\ A_1e^{-jkz}+B_1e^{jkz} & 0<z<d_s\\ C_1e^{-q(z-d_s)}+D_1e^{q(z-d_s)} & d_s<z<\Lambda\\ A_2e^{-jk(z-\Lambda)}+B_2e^{jk(z-\Lambda)} & \Lambda<z<\Lambda+d_s\\ \vdots & \\ \vdots & \\ A_Ne^{-jk[z-(N-1)\Lambda]}+B_Ne^{jk[z-(N-1)\Lambda]} & \\ \qquad (N-1)\Lambda<z<(N-1)\Lambda+d_s & \\ C_Ne^{-q[z-(N-1)\Lambda-d_s]}+D_Ne^{q[z-(N-1)\Lambda-d_s]} & \\ \qquad z>(N-1)\Lambda+d_s & \end{cases} \tag{4}$$

where $k=(1/\hbar)\sqrt{2m^*E}$ and $q=(1/\hbar)\sqrt{2m^*(V_o-E)}$. To be acceptable solutions, Ψ/n^* and $D(d\Psi/dz)$ where n^* and D represent the density of states and the Diffusion coefficient in the superconducting and normal layers, respectively, must be continuous at the interfaces [8]. However, here we consider the approximation of the continuity of the macroscopic wave function and its derivative with respect to z for simplicity. Moreover, the macroscopic wave function has to vanish at infinity from both ends, leading to $C_0=D_N=0$. The goal here is to find the dispersion equation to solve and to find the allowed Cooper pair energy levels. The allowed energy levels of the quantum states physically represent the coherent macroscopic tunneling of the Cooper pairs through the SQW structure, and the spatial distribution and confinement of Cooper pairs across the structure. This phenomenon is quite similar to the reflection/transmission of electromagnetic waves in multilayer structures.

In order to find out the dispersion equation, the Transfer Matrix Method in conjunction with Bloch wave analysis has been used, similar to the method presented in [9]. The translation matrix, $ABCD$, can be found as:

$$\begin{pmatrix}C_{n-1}\\D_{n-1}\end{pmatrix}=\begin{pmatrix}A&B\\C&D\end{pmatrix}\begin{pmatrix}C_n\\D_n\end{pmatrix} \tag{5}$$

where

$$A=e^{qd_n}\left[\cos kd_s-\frac{1}{2}\left(\frac{k}{q}-\frac{q}{k}\right)\sin kd_s\right] \tag{6}$$

$$B=\frac{j}{2}e^{qd_n}\left[\left(\frac{k}{q}+\frac{q}{k}\right)\sin kd_s\right] \tag{7}$$

$$C=-\frac{j}{2}e^{-qd_n}\left[\left(\frac{k}{q}+\frac{q}{k}\right)\sin kd_s\right] \tag{8}$$

$$D=e^{-qd_n}\left[\cos kd_s+\frac{1}{2}\left(\frac{k}{q}-\frac{q}{k}\right)\sin kd_s\right] \tag{9}$$

According to the Floquet theorem, solutions of the wave function for a periodic medium have to satisfy the following eigenvalue problem:

$$\begin{pmatrix}A&B\\C&D\end{pmatrix}\begin{pmatrix}C_n\\D_n\end{pmatrix}=e^{jK\Lambda}\begin{pmatrix}C_n\\D_n\end{pmatrix} \tag{10}$$

where K is the Bloch wave number. The phase factor $e^{jk\Lambda}$ is the eigenvalue of the unimodular translation matrix, $ABCD$ and is given by:

$$\cos K\Lambda=\frac{1}{2}(A+D) \tag{11}$$

Real and imaginary values of K represent respectively propagating and evanescent waves in the structure. These determine respectively the allowed and forbidden energy levels for Cooper pairs. The band edges can be calculated where $(1/2)|(A+D)|=1$. The translation matrix for the N layer SQW can be found as:

$$\begin{pmatrix}C_0\\D_0\end{pmatrix}=\begin{pmatrix}A&B\\C&D\end{pmatrix}^N\begin{pmatrix}C_N\\D_N\end{pmatrix}=\begin{pmatrix}AU_{N-1}-U_{N-2} & BU_{N-1}\\ CU_{N-1} & DU_{N-1}-U_{N-2}\end{pmatrix}\begin{pmatrix}C_N\\D_N\end{pmatrix} \tag{12}$$

where $U_n=(\sin(n+1)K\Lambda)/\sin K\Lambda$. Setting the boundary conditions $C_0=D_N=0$, the dispersion equation for the Bloch wave number or the allowed energy state can be obtained as [9]:

$$A\frac{\sin NK\Lambda}{\sin K\Lambda}=\frac{\sin(N-1)K\Lambda}{\sin K\Lambda} \tag{13}$$

Solution of (13) yields the allowed energy E of the quantum states for the Cooper pairs. Equation (13) is formally the same as the dispersion equation for Transverse Electric (TE) Bloch electromagnetic waves in identical multilayer structure in which k and q represent the wave propagation constants in each layer.

The Cauchy Integration Method (CIM) has been used to solve the dispersion (13) [10]. CIM is fast, robust and an efficient numerical method without an initial guess requirement. Solutions of various structures with different numbers of layers, and various thicknesses for both superconducting and normal layers for different materials reveal the possibility of creating allowed energy levels and subbands for Cooper pair condensate within the bulk energy gap. Let's discuss some of the consequences of the analytical and numerical investigations over the dispersion (13). If V_o is large enough, there are exactly N quantum states in each region where $K\Lambda$ varies from $m\pi$ to $(m+1)\pi$. In this case, the SQW structure can be envisioned as N interacting potential wells. Each SQW can support a number of allowed energy states within or outside of the energy gap. By bringing these wells together, the presence of each SQW affects the state of each individual well. This leads to the splitting of each of the N-fold individual wells. In the case of a superlattice structure with a large number of layers, N nondegenerate energy levels fill the region where $K\Lambda$ varies between $m\pi$ and $(m+1)\pi$. For instance, the energy band diagrams of a trilayer, $N=1$ and 11-layer, $N=5$ SQW structure with thickness $d_S=20$ nm,

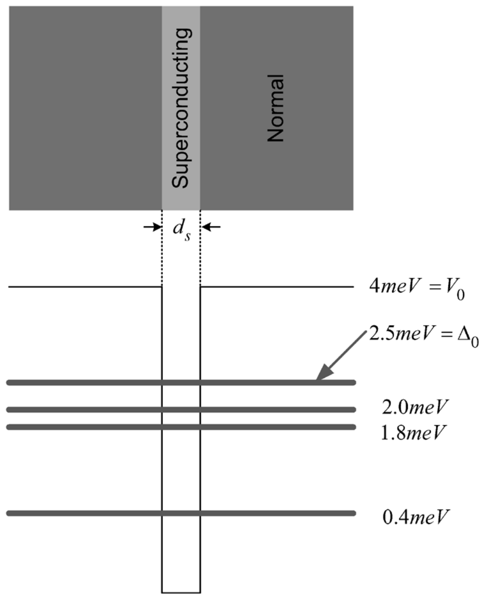

Fig. 2. Energy band diagram for 3-layer, $N = 1$, NSN SQW structure.

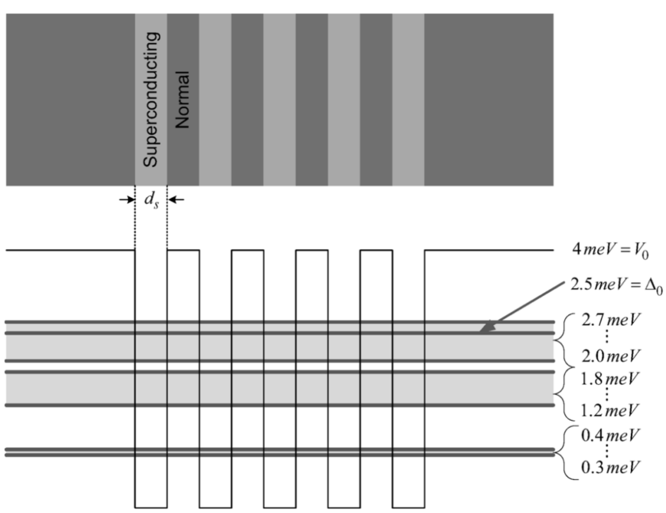

Fig. 3. Energy band diagram for 11-layer, $N = 5$ SNS SQW structure.

$d_N = 15$ nm, $\Delta_o = 2.5$ meV, $V_o = 4$ meV and coherence length 5 nm have been sketched in Figs. 2 and 3.

In a trilayer NSN junction, there are three energy levels available within the bulk energy gap of the superconductor. By increasing the number of layers, the three energy levels below the bulk energy gap become subbands. By a further increase in the number of layers more energy levels are introduced in each band. This indicates the possibility of bandgap engineering of the SQW structure for the design of mm-wave and THz sources and detectors, similar to the techniques used for semiconductor-based quantum well structures. For example, Cooper pairs can be depopulated from each energy state by exposure to electromagnetic radiation of the correct frequency. Note that in the 11-layered SQW structure, the third energy band cuts the energy gap, therefore the Cooper pairs undergo a breaking process and any energy levels above energy gap do not introduce any allowed quantum state for Cooper pairs. Considering thermal

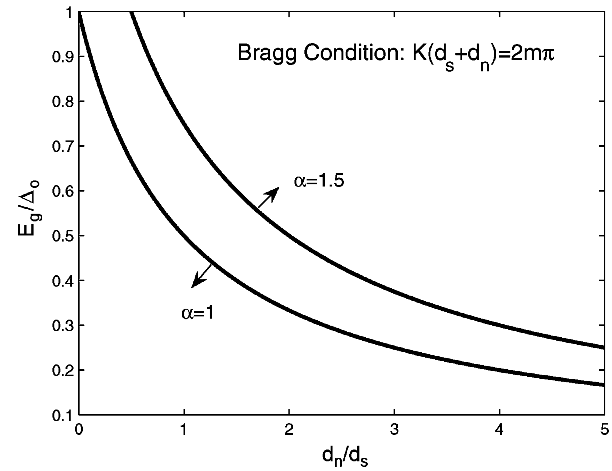

Fig. 4. Relative energy gap with respect to the relative thickness of normal and superconducting layers.

fluctuations, it is desired to have the separation between the energy bands larger than $k_B T$. The band edges can be calculated by imposing the Bragg condition, $K = 2m\pi/\Lambda$, on the Bloch wave number given that $kd_s < 1$, $qd_n < 1$ and V_o is slightly larger than Δ_o. The energy gap of the SQW structure, defined as the difference between the barrier height and the lowest allowed energy of the Cooper pair condensate in its last energy subband, can be written as (Appendix A):

$$E_g \approx \frac{\alpha \Delta_o}{1 + \frac{d_n}{d_s}} \tag{14}$$

Fig. 4 shows the variation of the relative energy gap with respect to the thickness of the normal layer relative to that of the superconducting layer, in full agreement with the analysis presented in [11].

III. Conclusion

The stacked SNS Josephson Junctions has been analysed in the context of the Macroscopic Quantum Model. The Transfer Matrix method with a Bloch wave analysis has been employed to find the energy dispersion equation. Solution of the dispersion equation with the Cauchy Integration Method shows the possibility of energy level and subbands formation in SQW structures and Superconducting superlattices. Although the method presented here does not include the detailed physics of the multilayer SNS junctions such as formation of Andreev bound states and quasi-particle quantum states, it has the benefit of straightforward analysis; it is fast and reliable numerical technique for finite structures with any number of layers. Bandgap engineering of the superconducting condensate within its bulk energy gap can be used as a basis for designing mm-wave and THz devices, similar to the application of quantum well structures to design IR detectors and quantum cascade lasers. The possibility of in-plane and out-of-plane integration of SQWs with the waveguiding structure can be potentially useful for mm-wave and THz integrated circuits.

APPENDIX A

At the lower edge of each energy band, $\cos K\Lambda = 1$, or, $K\Lambda = 2m\pi$, where m is a positive/negative integer. This is the so-called Bragg-condition. Equation (11) in conjunction with (6) and (9) can be written as:

$$\cos K\Lambda = \cosh qd_n \cos kd_s - \frac{1}{2}\left(\frac{k}{q} - \frac{q}{k}\right)\sinh qd_n \sin kd_s \tag{A-1}$$

Using the small argument approximation of the trigonometric functions in (A-1) under the condition of $qd_n \ll 1$ and $kd_s \ll 1$ when $K\Lambda = 2m\pi$, (A-1) becomes:

$$\left(k_B^2 - q_B^2\right) d_s d_n \approx q_B^2 d_n^2 - k_B^2 d_s^2 \tag{A-2}$$

Plugging $k_B = (1/\hbar)\sqrt{2m^* E_B}$ and $q_B = (1/\hbar)\sqrt{2m^*(V_o - E_B)}$, in which E_B represents the Bragg energy level, into (A-2) yields:

$$E_B \approx \frac{V_o}{1 + \frac{d_s}{d_n}} \tag{A-3}$$

If E_B is the energy associated with the lower edge of the last energy band below the bulk energy gap of the superconductor, the energy gap in the SQW structure might be defined as $E_g = V_o - E_B$, which results in (14). Note that E_g has to be smaller than the bulk energy gap, therefore $\alpha \leq 1 + (d_n/d_s)$.

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