L24

Measurement of microwave photons

Introduction

This lecture will address two topics:

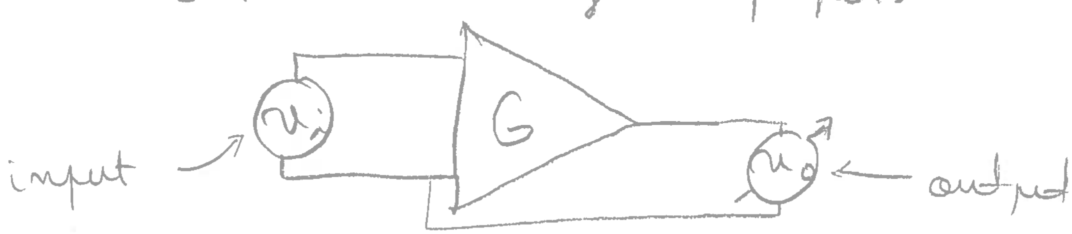
- amplification of bosonic fields (e.g. photons) using <u>linear amplifiers</u>
- measurement of bosonic modes, in particular photon counters.

Linear amplifiers

Introduction

The concept of linear amplifier is connected with "classical" amplifiers for electrical signals, such as vacuum-tube and transistor based amplifiers.

Take a voltage amplifier:



(1) $$v_o(t) = G\, v_i(t) + \underbrace{v_n(t)}_{\text{noise}} = G\left[v_i(t) + \underbrace{\tilde{v}_n(t)}_{\text{noise referred to the input}}\right].$$

Electrical signals are photons, so they have bosonic character. The treatment of a transmission line showed that a classical variable such as the voltage becomes, in the quantum world, a operator.

$$(2) \qquad V(t) \rightarrow \hat{V}.$$

For a transmission line, the voltage operator is a linear combination of creation and annihilation operators:

$$(3) \qquad \hat{V}(z) = C \sum_n \left(f_n(z)\hat{a}_n + f_n^*(z)\hat{a}_n^\dagger \right)$$

constant

mode function

Disregarding spatial coordinates, this can be written as

$$(4) \qquad V = \sum_n \left(f_n a_n + f_n^* a_n^\dagger \right).$$

Following relation (1) for a conventional voltage amplifier, we find that for <u>any</u> m in the output

$$(5) \qquad a_n|_{out} = \mathcal{L}_n\left(\{ a_m|_{in} \} \right),$$

with $\mathcal{L}$ a linear function of the input operators. This relation establishes the notion of linear amplifier in the quantum domain.

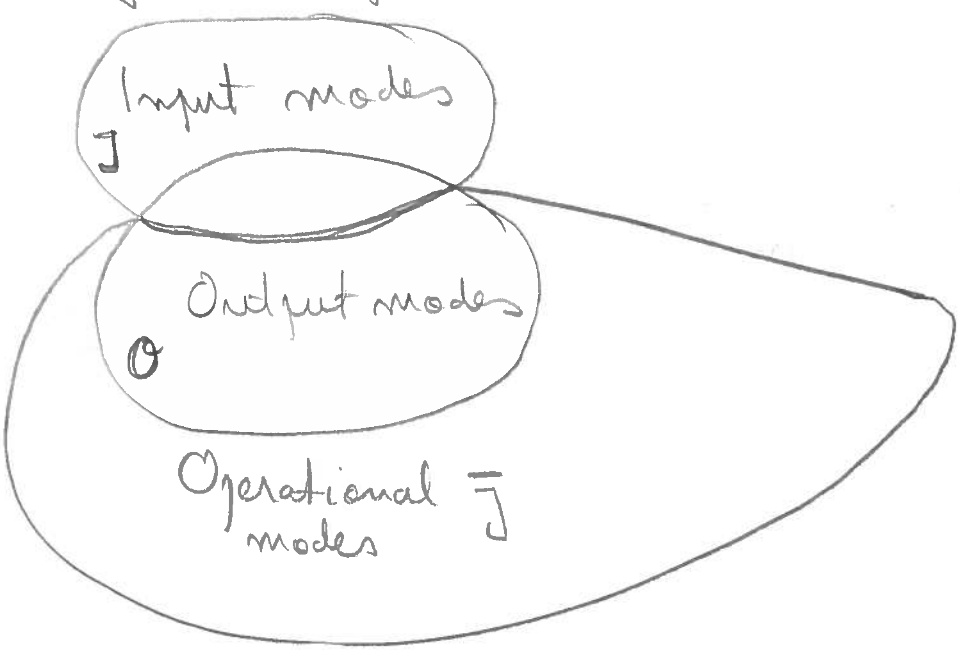
Caves' theory of linear amplifiers

Seminal paper: Caves, PR D 26, 1817 (1982).

This paper contains a detailed theoretical treatment of linear amplifiers. Some of the ideas go back to previous work on the MASER & parametric amplifiers, as reviewed in the introduction of Caves' paper.

Amplifier model:

- Physical system



Modes can be fixed (cavity) or propagating (transmission line).

- "Before" and "after" states

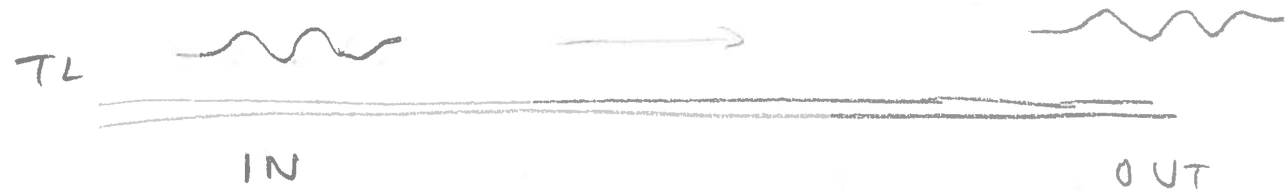
Also known as (in) a_α $(\alpha \in \mathcal{I})$; (out) b_α $(\alpha \in \mathcal{O})$.

Separation relies on the fact that for propagating modes there is a finite range (<=> time) of interaction.

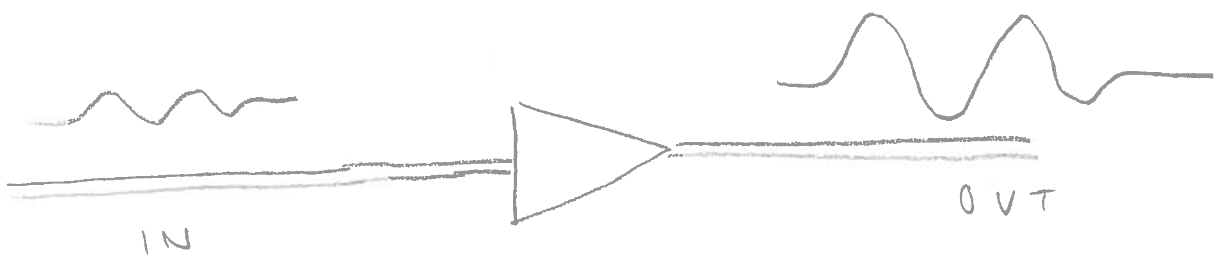
- An arbitrary phase in the evolution of the modes can be removed

(6) $\tilde{a}_\alpha = e^{-i\varphi_\alpha} a_\alpha$

(7) $\tilde{b}_\alpha = e^{-i\theta_\alpha} b_\alpha$



$b(t) = a(0)\,e^{-i\omega t}$ => a phase transformation makes these fields equivalent



In this case a phase transformation on the OUT state does not result in the IN state. ~~Amplification occured~~.

- The amplifier operational modes state and the input state are independent

Initially $\rho = \rho_I\,\rho_{op}$ (separable state)
(8)

- Linearity conditions

(9) $b_\alpha = \sum_{\beta\in J} \left(M_{\alpha\beta} a_\beta + L_{\alpha\beta} a_\beta^\dagger\right) + F_\alpha \quad (\alpha \in O)$

- The evolution of the system is unitary

Analysis of a single mode amplifier

(10) $b_0 = M a_I + L a_I^+ + \mathcal{F}$

(11) $[b_0, b_0^+] = 1$ (unitary evolution preserves the commutation relations)

(12) $1 = |M|^2 - |L|^2 + [\mathcal{F}, \mathcal{F}^+]$

For a bosonic mode (annihilation operator a), one can define the two quadratures

(13) $a = X_1 + i X_2$ (X_1 & X_2 are Hermitian)

Heisenberg uncertainty relation for X_1 & X_2

(14) $\Delta X_1 \Delta X_2 \geq \frac{1}{4}$ (In general, for A & B Hermitian, $\Delta A \times \Delta B \geq \frac{1}{2} |\langle [A, B] \rangle|$)

For a non-Hermitian operator, R, the mean square fluctuations are defined as

(15) $|\Delta R|^2 \equiv \frac{1}{2} \langle R R^+ + R^+ R \rangle - \langle R \rangle \langle R^+ \rangle.$

It can be shown that

(16) $|\Delta R|^2 = (\Delta R_1)^2 + (\Delta R_2)^2$ (with $R = R_1 + i R_2$, with R_1 & R_2 Hermitian)

The fluctuations for the annihilation of a:

(17) $|\Delta a|^2 = (\Delta X_1)^2 + (\Delta X_2)^2 \geq 2 \Delta X_1 \Delta X_2 \geq \frac{1}{2}$

It is useful to define as well the moment matrix, a measure of fluctuations for a given state:

(18) $\sigma_{pq} = \frac{1}{2} \langle X_p X_q + X_q X_p \rangle - \langle X_p \rangle \langle X_q \rangle, \; p, q = \overline{1, 2}$

Phase insensitive noise states are states for which σ_{pq} does not depend on a phase transformation ($a \to a e^{-i\varphi}$). In this case

(19) $$\sigma_{pq} = \frac{1}{2} |\Delta a|^2 \delta_{pq}.$$

o Case A: Phase-insensitive amplifiers

a) The relation (10) is invariant with respect to a phase change
i) $\varphi_I = \theta_0$
ii) $\varphi_I = -\theta_0$

b) Phase insensitive noise @ input $\Rightarrow$ phase insensitive noise at the output

Two possible situations:
i) $L=0$ (phase preserving amplifier)
ii) $M=0$ (phase conjugating amplifier)

The added amplifier noise is (use (12))

(20) $$|\Delta \mathcal{F}|^2_{of} \geq \frac{1}{2} |\langle[\mathcal{F},\mathcal{F}^+]\rangle \frac{1}{2}| \, 1 - |M|^2 + |L|^2|$$

which gives, when referred to the input

(21) $$A \equiv \frac{|\Delta \mathcal{F}|_{of}^2}{G} = \frac{1}{2G} |1 - |M|^2 + |L|^2| = \begin{cases} \frac{1}{2}|1 - \frac{1}{G}|, & \text{phase preserving} \\ \frac{1}{2}|1 + \frac{1}{G}|, & \text{phase conjugating} \end{cases}$$

- Case B: phase-sensitive amplifiers

These are non-phase-insensitive amplifiers. With a phase transformation, one can obtain

$$(22)\quad Y_1 = (M+L)X_2 + \mathcal{F}_1 \quad , \quad \mathcal{F}_1 = \tfrac{1}{2}(\mathcal{F} + \mathcal{F}^+)$$

$$(23)\quad Y_2 = (M-L)X_2 + \mathcal{F}_2 \quad , \quad \mathcal{F}_2 = -\tfrac{1}{2}i(\mathcal{F} - \mathcal{F}^+)$$

with M & L real and positive.

The added noise number, per quadrature, is

$$(24)\quad A_p = \frac{(\Delta \mathcal{F}_p)^2_{op}}{G_p} \quad , \quad \begin{array}{l} G_1 = (M+L)^2 \\ G_2 = (M-L)^2 \end{array}$$

We have

$$A_1 A_2 = \frac{(\Delta \mathcal{F}_1)^2_{op}}{G_1}\frac{(\Delta \mathcal{F}_2)^2_{op}}{G_2} \geqslant \frac{1}{G_1 G_2}\frac{1}{4}\left|\langle[\mathcal{F}_1, \mathcal{F}_2]\rangle\right|^2 = \frac{1}{G_1 G_2}\frac{1}{4}\frac{1}{4}\left|\langle[\mathcal{F}, \mathcal{F}^+]\rangle\right|^2$$

$$= \frac{1}{16 G_1 G_2}\left|1 - |M|^2 + |L|^2\right|^2 = \frac{1}{16}\left(\frac{1}{\sqrt{G_1 G_2}} - \frac{|M|^2 - |L|^2}{\sqrt{G_1 G_2}}\right)$$

The final result

$$(25)\quad \sqrt{A_1 A_2} \geqslant \frac{1}{4}\left(1 \mp \frac{1}{\sqrt{G_1 G_2}}\right) \begin{cases} \text{"}-\text{"}, & |M| > |L| \\ \text{"}+\text{"}, & |M| < |L| \end{cases}$$

This result is interesting because for $G_1 G_2 = 1$, corresponding to e.g. $G_1 \gg 1$ & $G_2 \ll 1$, one can have a negligible amount of noise in one of the two quadratures.

An example of such a "noiseless" device is the degenerate parametric amplifier (DPA). The DPA is an oscillator for which the frequency is modulated at twice its resonant frequency. The in-out relation for a DPA is

(26) $$b = a \cosh(r) + a^{+} \sinh(r).$$

With a phase transformation

(27) $$Y_2 = e^{r} X_2$$

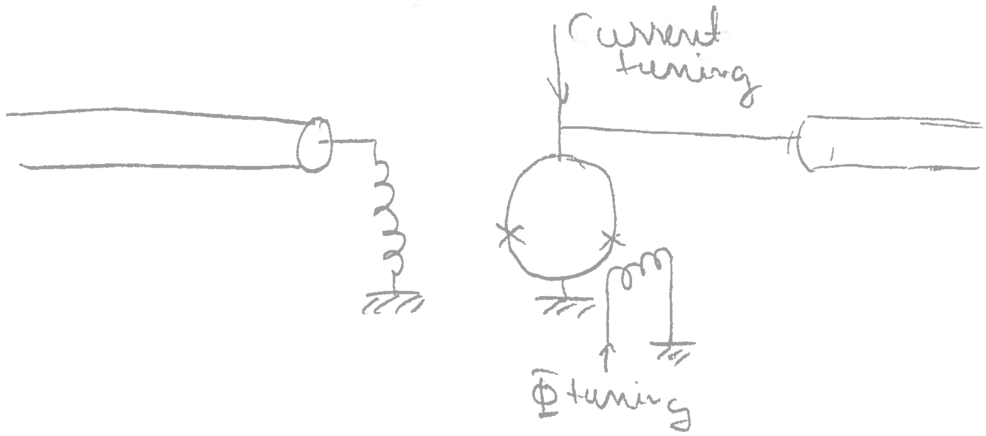
(28) $$Y_2 = e^{-r} X_1$$

There is <u>no noise</u> term!

Superconductor-based (nearly) quantum limited amplifiers

The DC-SQUID based amplifier for ~GHz range: a phase-insensitive amplifier

Muck, Kycia, and Clarke, APL 78, 967 (2001)



Operation frequency : ~0.5 GHz
Operation temperature : 20 mK
Gain : 30 dB

The noise performance is defined in terms of noise temperature. The overall system noise is

$$\frac{P_n}{B} = \underbrace{\frac{h\nu}{2}\coth\left(\frac{h\nu}{2k_BT}\right)}_{\text{thermal noise of the source}} + \underbrace{k_BT_n}_{\text{added noise}} + \underbrace{k_B\frac{T_p}{G}}_{\text{noise of subsequent stages (negligible)}} \quad (29)$$

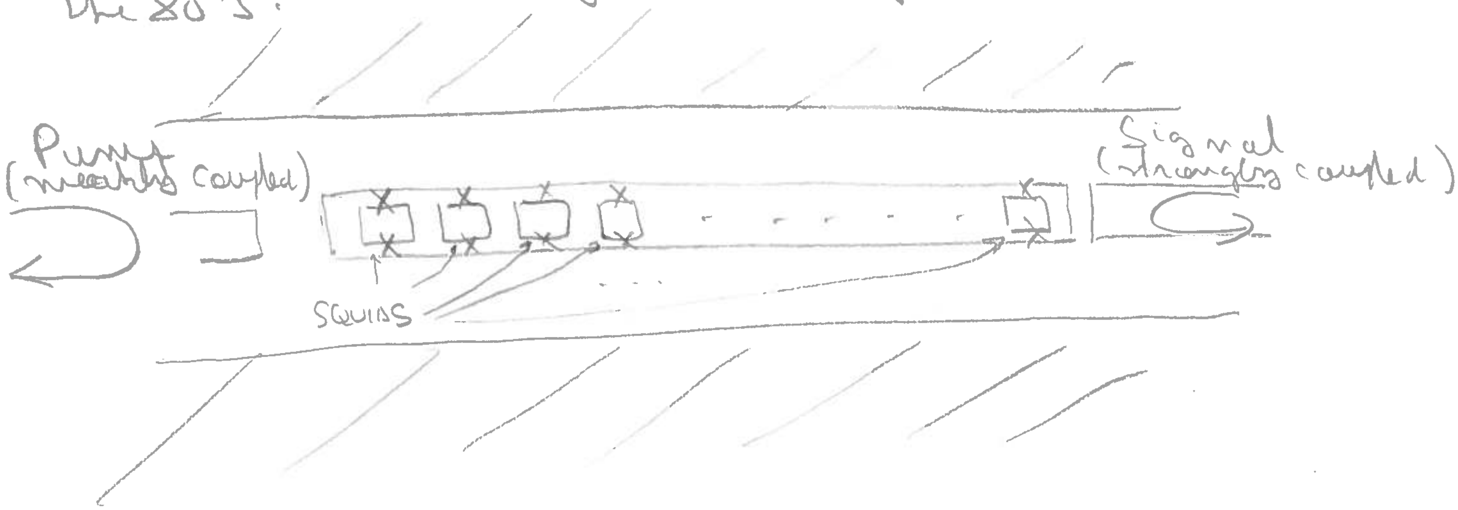
The noise temperature is only a factor 2 larger than the quantum limit

$$T_Q = \frac{h\nu}{k_B} \simeq 25\,\mathrm{mK} \quad (30)$$

The measured $T_n = 50\,\mathrm{mK}$

The cavity-based Josephson parametric amplifier: a phase-sensitive amplifier

Castellanos-Beltran et al, Nature Physics 4, 928 (2008)
See also pioneering work by Yurke & collaborators in the 80's.



The role of the pump: a current in the signal line of the CPW resonator, at frequency ν_p makes the Josephson inductance L_J of each SQUID vary at frequency $2\nu_p$. This is because

(31) $$I = I_c \sin\gamma$$

(32) $$L_J = \frac{\varphi_0}{I_c \cos\gamma}$$

When $\nu_p = \nu_{res}$ (the resonance frequency) a parametric amplifier is obtained.

Achieved performance:

- gain up to 32 dB
- 10 dB squeezing of quantum fluctuations.

Photon detection/counting (in the MW range)

For a single mode of the electromagnetic field, the quantum state can be written as

(33) $$|\psi\rangle = \sum_n c_n |n\rangle .$$

- Photon detector: discrimination of state $|0\rangle$ of a state $|n\rangle$ $(n \neq 0)$.
- Photon counter: discrimination of any two states $|m\rangle$ & $|n\rangle$ with $m \neq n$.

The notion of photon detection can be extended to the case of a multi-mode field

(34) $$|\psi\rangle = \sum_{\omega \in (\omega_0 - B,\, \omega_0 + B)} \sum_n c_{n\omega} |n\rangle_\omega .$$

In this case photon detection/counting outputs can be related to the total photon number operator

(35) $$N_{total} = \sum_n N_n .$$

The fact that a detector is in general sensitive to the number of photons and less to photon energy is related to the typical detector structure

Two photons of frequencies ω_A, ω_B larger than the gap Δ produce a transition to different states in the continuum. This transition is followed by an avalanche process which is forgetful of the initial state in the continuum.

No single photon detectors have been demonstrated so far for the low GHz range.
A theoretical proposal appeared in 2009:
Romero et al, PRL 102, 173602 (2009).

More recently, experiments were done using detection through resonant activation of a Josephson junction (Chen et al, PRL 107, 217401, (2011)

Homodyne meas: see Walls & Milburn p45-46