From Classical To Quantum Channel Capacities

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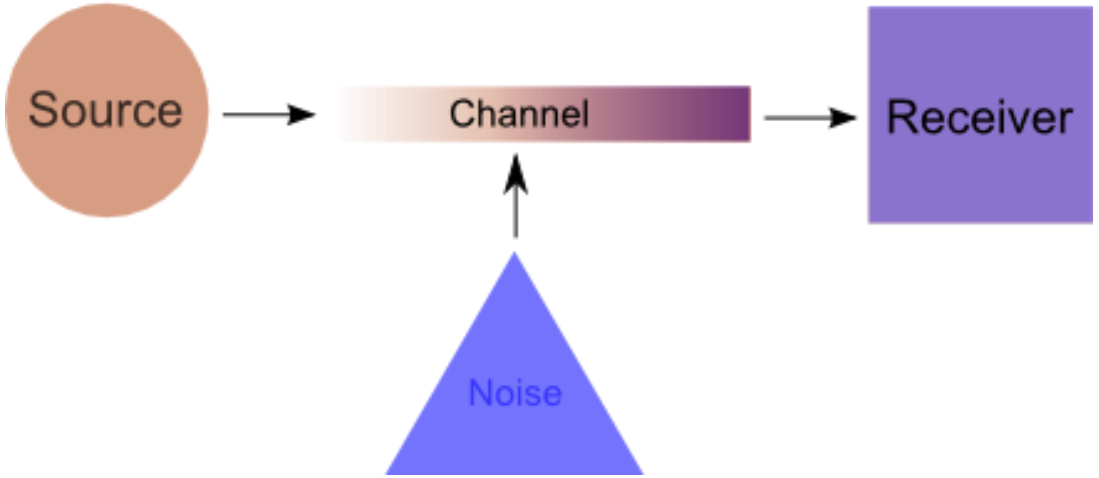
Outline

1. Classical Information Theory (Framework)
    1. Shannon Entropy
    2. Classical Channel Capacity
2. Quantum Information Theory
    1. Quantum Channel Capacity
        1. Classical Zero-Error Capacity
        2. Entanglement-Assisted Quantum Capacity
    2. Specific Channels
        1. Erasure Channel
        2. Dephasing Channel
        3. Amplitude Damping Channel
        4. Ideal Channel

Communication Protocol



Definitions

Channel Capacity

$$C = \lim_{T \to \infty} \frac{Log(N(T))}{T} \tag{1}$$

Shannon Entropy

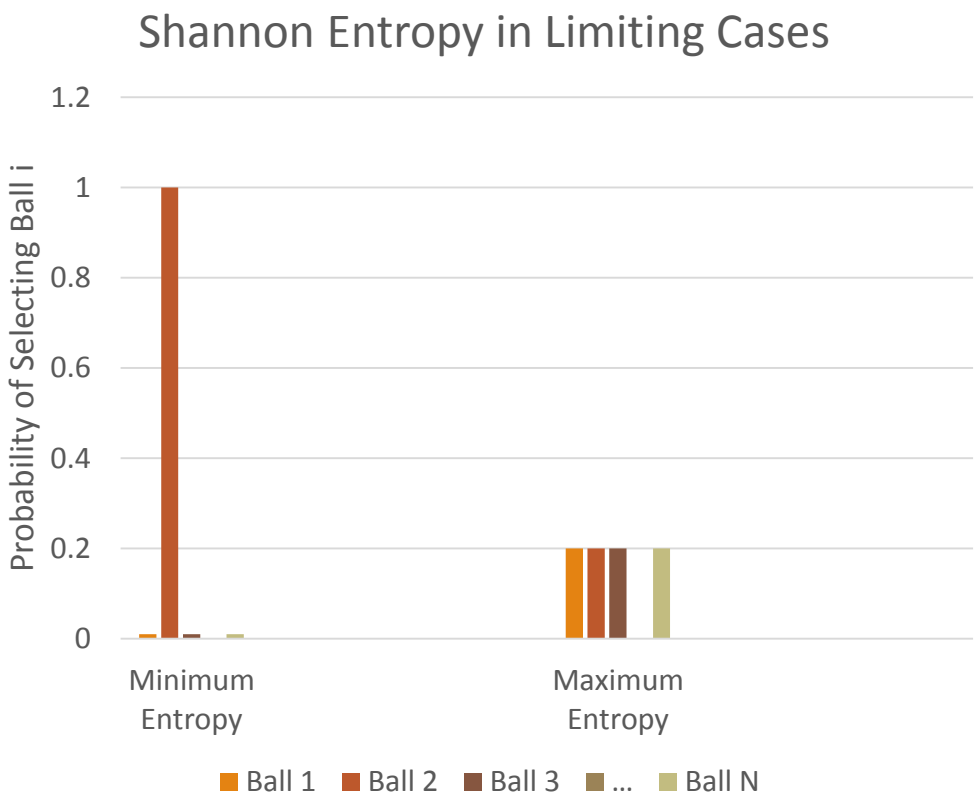
$$H = -\sum_{i=1}^{n} p_i log(p_i) \tag{2}$$

Shannon Entropy (cont'd)

1. The entropy only vanishes when only one ball is available for selection

2. The entropy is largest when all of the balls are selected with equal probability

$$H = -\sum_{i=1}^{n} p_i log(p_i)$$



Shannon Entropy (cont'd)

3. Considering two events X and Y the entropy of the joint system formed by $X \cap Y$ is

$$H(X \cap Y) = -\sum_i \sum_j p(i \cap j) \log\{p(i \cap j)\}$$

$$H(X/Y) = -\sum_{i/j} \overline{p}(i/j) \log \overline{p}(i/j) \quad , \quad \overline{p}(i/j) = \sum_{j/i} p(i \cap j)$$

$$H_X(Y) = -\sum_i \sum_j p(i \cap j) \log p_i(j)$$

Combining the above:

$$H(X \cap Y) = H(X) + H_X(Y) = H(Y) + H_Y(X) \qquad (1)$$

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Classical Channel Capacity

- $C = Max(H(X) - H_Y(X))$ $\qquad H_Y(X) = -\sum_i \sum_j p(i \cap j) \log p_j(i)$
- Small, non-vanishing error probability
- Can not preserve superposition (modeled as a complete dephasing channel)

$$\frac{1}{2}(|0\rangle\langle 0| + |0\rangle\langle 1| + |1\rangle\langle 0| + |1\rangle\langle 1|) \xrightarrow{classical\ channel} \frac{1}{2}(|0\rangle\langle 0| + |1\rangle\langle 1|)$$

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Classical Zero-Error Capacity

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Entanglement-Assisted Quantum Capacity

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Erasure Channel

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Dephasing Channel

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Amplitude Damping Channel

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Ideal Channel

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      4. ~~Ideal Channel~~

Thank you all for coming

References