ECE770 T14/QIC 885: Quantum Electronics & Photonics

Lecture 9

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Abstract and Objectives of Lecture 9

Quantum treatment of electromagnetic (EM) radiation demands a good review of classical electrodynamics starting by a brief review of the Maxwell's equations in free-space. By stating the conservation laws of EM radiation, namely EM energy, EM momentum and EM angular momentum, we put EM radiation in a context of classical mechanics. We then discuss the EM potentials and Coulomb and Lorentz gauge transformations. This will equip us to solve the wave equation for magnetic vector potential in which it suffices to describe EM radiation in a Coulomb gauge. The properties of the solution to wave equations can provide the opportunity to define a a Hilbert space of orthonormal EM modes, similar to Hilbert space defined for ket vectors in quantum mechanics. Using the Fourier transformation of fields, we can describe the EM radiation and propagation in reciprocal space, $(\mathbf{k}, \omega)$ instead of space-time, $(\mathbf{r}, t)$. EM field modes in cavity resonator and free-space are discussed where the standing-waves similar to bound states and free-EM propagation similar to free motion of massive objects will be our main focus. Calculating the EM energy based on these modes allows us to treat EM radiation and free-space propagation similar to simple harmonic oscillator, where the quantization road has been already paved perfectly. We will introduce the number representation of quantized EM field and discuss the concept of a photon. By introducing the EM field operators and their properties in photon number state we will end lecture 9.

Topics of Lecture 9

7-1) EM Field in Free-Space

7-2) EM Conservation Laws

7-3) EM Potentials & Gauge Transformations

7-4) EM wave Equation in Free-Space

7-5) Hilbert Space of Orthonormal EM Modes

7-6) EM Field in Reciprocal Space

7-7) EM Field Modes in Cavity

7-8) EM Field Modes in Free-Space

7-9) EM Energy & Simple Harmonic Oscillator

7-10) Quantization of EM Field Modes

7-11) Photon Number (Fock) State

7-12) Properties of EM Field Operators in Photon Number State

References & Suggested Readings

1- D.J. Griffith, *Introduction to Electrodynamics*, 3^{rd} ed., PH, 1999, Chapters 7, 8, 10.

2- J. D. Jackson, *Classical Electrodynamics*, 3^{rd}, JW, 1999, Chapter 6

3- J. Schwinger, L.L. D. Raad, Jr., K. A. Milton, W. Tsi, *Classical Electrodynamics*, ABP, 1998, Chapters 1, 3, 4 .

4- J. C. Garison, R.Y. Chiao, *Quantum Optics*, Oxford, 2008, Chapter 2, Appendix B.

5- H. Bachor and T.C. Ralph, *A Guide To Experimental Quantum Optics*, 2^{nd} ed., 2004, Sections 2.1, 4.1 and 4.2.

6- M. Fox, *Quantum Optics, An Introduction*, Oxford, 2006, Chapter 3.

7- U. Leonhardt, *Measuring the Quantum State of Light*, Cambridge, 1997, Chapter 2

8- W.P. Schliech, *Quantum Optics in Phase Space*, 2001, Chapter 10.

9- P.W. Milonni, *The Quantum Vacuum, An Introduction to Quantum Electrodynamics*, AP, 1994, Chapter 2.