Tue

Quantum noise spectral density

$$S_{xx}[\omega] = \int_{-\infty}^{\infty} dt\, e^{i\omega t} \langle \hat{x}(t)\hat{x}(0)\rangle$$

↳ q. statistical average

<u>Example</u> a simple H.O. w/ m & ω_0 in eq. at Θ w/ a large heat bath

position autocorrelation fn is

$$\phi_{xx}(t) = \langle \hat{x}(t)\hat{x}(0)\rangle$$

Hamilton's equation of motion

$\because$ $x(t) = x(0)\cos(\omega_0 t) + p(0)\frac{1}{m\omega_0}\sin(\omega_0 t)$

$$= \langle \hat{x}(0)\hat{x}(0)\rangle \cos(\omega_0 t) + \langle \hat{p}(0)\hat{x}(0)\rangle \frac{1}{m\omega_0}\sin(\omega_0 t)$$

classically $\langle \hat{p}(0)\hat{x}(0)\rangle = 0$ $\because$ uncorrelated

$$\langle \hat{x}(0)\hat{p}(0)\rangle - \langle \hat{p}(0)\hat{x}(0)\rangle = i\hbar$$

$$\left.\begin{aligned} \langle \hat{p}(0)\hat{x}(0)\rangle &= -\frac{i\hbar}{2} \\ \langle \hat{x}(0)\hat{p}(0)\rangle &= +\frac{i\hbar}{2} \end{aligned}\right\} \text{ in thermal eq.}$$

even if $\hat{x}$ is an hermitian operator, its autocorrelation fn is <u>complex</u>.

due to non-commuting nature btw $\hat{x}(0)$ and $\hat{x}(t)$

$[\hat{x}(0), \hat{x}(t)] \neq 0$

$$\therefore \phi_{xx}(t) = \left(\frac{\hbar}{2m\omega_0}\right)^2 \left\{ \underline{n_B(\hbar\omega_0)} e^{i\omega_0 t} + [n_B(\hbar\omega_0)+1] e^{-i\omega_0 t} \right\}$$

rms zero-point uncertainty of x in the $|0\rangle$

BE ~~occ~~ occupation factor.

Since $\phi_{xx}(t)$ is complex, ✓

the q. power spectral density is not <u>symmetric</u> in ω!

cf) classical case: <u>$\phi_{xx}(t) = \text{real}$</u> ; PSD is always symmetric in ω.

$$S_{xx}[\omega] = 2\pi\left(\frac{\hbar}{2m\omega_0}\right)^2 \left\{ \underline{n_B(\hbar\omega_0)\,\delta(\omega+\omega_0)} + \underline{[n_B(\hbar\omega_0)+1]\,\delta(\omega-\omega_0)} \right\}$$

if $k_B\Theta \gg \hbar\omega_0$, $\quad n_B(\hbar\omega) \sim n_B(\hbar\omega_0)+1 \sim \frac{k_B\Theta}{\hbar\omega_0}$

high T. limit

$\therefore$ $S_{xx}(\omega)$ becomes symmetric $\Leftarrow$ similar to the classical case

$$S_{xx}[\omega] = \pi \frac{k_B\Theta}{m\omega_0^2}\left[\delta(\omega-\omega_0) + \delta(\omega+\omega_0)\right] \checkmark$$

Tue

In 1949, W. Shockley's paper Bell Syst. Tech. J. 28.435 (1949)

⇒ "Basic transport processes of a pn junction diode and transistor"

R.M. Ryder & R.J. Kircher Bell Syst. Tech. J. 28, 367 (1949)

⇒ "noise of a point contact transistor"

observed noise figure = 50-70 dB above the intrinsic limit

≈ 60 years later, reduce 50-70 dB down ~~to~~ close to the theoretical limit

(mainly due to 1/f noise & surface recombination. which can be reduced!)

Intrinsic noise sources of a pn junction device

① thermal noise in the bulk region

② the shot noise in the pn junction

* this noise study of pn junction diodes sets a fundamental limit on the noise performance of various semiconductor pn junction devices

e.g. a semiconductor laser
a photodiode
an avalanche photodiode
a bipolar transistor

How to operate a pn-junction diode

① constant voltage bias : R_d (junction differential resistance) $\equiv \left(\frac{dI}{dV}\right)^{-1} \gg R_s$

⇒ no fluctuation in the junction voltage due to CR_s is fast

but ∃ a fluctuation in the junction current I.

② constant current bias : $R_d \ll R_s$

⇒ no fluctuation in the junction current

but ∃ a fluctuation in the junction voltage V due to slow CR_s.

Junction size

① macroscopic junction $k_B\Theta \gg \frac{q^2}{2C}$

② mesoscopic junction $k_B\Theta \ll q^2/2C$

Tue

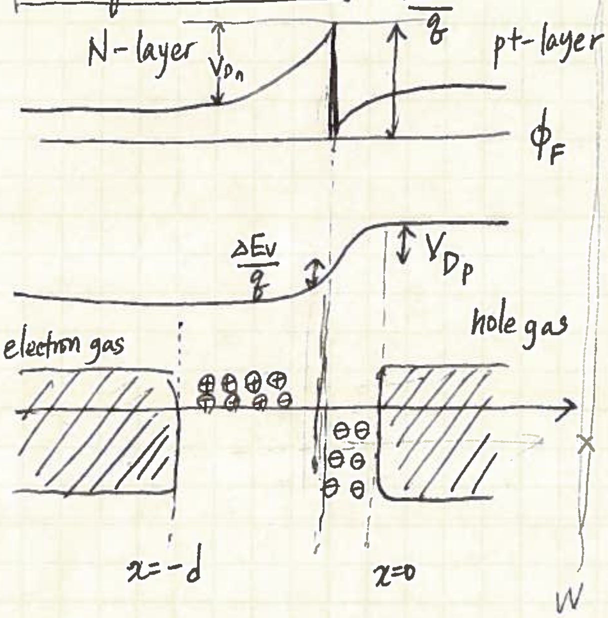
pn Junction Diodes Under Const. Voltage Operation

① current - Voltage and capacitance - voltage characteristics

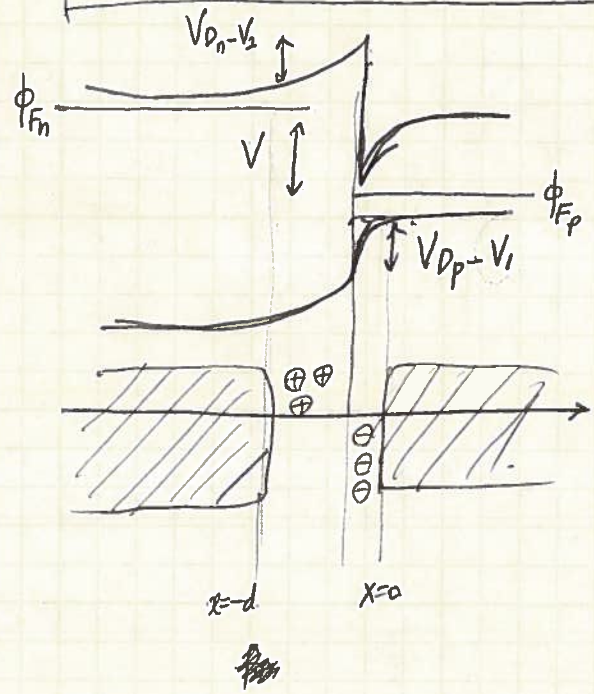
a p^+–N heterojunction w/ a heavily p-doped narrow bandgap material and lightly n-doped wide bandgap material.

Band diagram

At equilibrium ($V=0$)



Under a forward bias condition ($V>0$)



- V_D = the built-in potential.
- for $V\neq 0$, $V_{D_p} = \frac{V_D}{K}$, $V_{D_n} = V_D\left(1-\frac{1}{K}\right)$

where $K = 1 + \frac{\varepsilon_1 (N_{A1}^- - N_{D1}^+)}{\varepsilon_2 (N_{D_2}^+ - N_{A2}^-)}$ ← p+-layer ← N-layer

↑ donar concentration ↑ acceptor concentration

Since $\varepsilon_1 > \varepsilon_2$, $N_{A1}^- - N_{D1}^+ \gg N_{D2}^+ - N_{A2}^-$ (p^+N diode)

$\therefore \boxed{K \gg 1}$

$\therefore \boxed{V_{D_p} \approx 0, \ V_{D_n} \approx V_D}$

Tue

· for $V>0$, $V_{Dn} - V_2 \approx V_D - V$ $\because V_2 = V\left(1-\frac{1}{K}\right) \approx V$ for $\underline{\underline{K \gg 1}}$.

n_p = the electron density at the edge of the depletion layer ($x=0$) in the p^+-layer.

$$n_p = \underset{\substack{\uparrow \\ \text{transmission coeff.} \\ \text{of an electron at the} \\ \text{heterojunction interface}}}{X}\ n_{N0} \exp\left(-\frac{V_D - V}{V_T}\right) \equiv n_{p0} \exp\left(+\frac{V}{V_T}\right) \quad \text{where } n_{p0} = X n_{N0} \exp\left(-\frac{V_D}{V_T}\right)$$

n_{N0} ← the electron density at the edge of the depletion layer ($x=-d$) in the N-layer

$\boxed{V_T = \frac{k_B \theta}{q}}$ = the thermal voltage.

<u>Now</u> n_p, the excess electron density at $x=0$, diffuses towards $x=W$, where a p-side metal contact is located.

∴ The distribution of the excess electron density obeys

$$\frac{\partial n(x,t)}{\partial t} = -\frac{n(x,t) - n_{p0}}{\tau_n} - \frac{1}{q}\frac{\partial}{\partial x} i_n(x,t)$$

, where τ_n = the electron lifetime.

Since there is <u>no</u> electric field in the neutral p^+-layer,

the current $i_n(x,t)$ is carried only by a diffusion component

$$i_n(x,t) = +q \underset{\substack{\uparrow \\ \text{the electron diffusion constant}}}{D_n} \frac{\partial}{\partial x} n(x,t)$$

+ boundary conditions $n_p = \begin{cases} n_{p0} \exp\left(\frac{V}{V_T}\right) & \text{at } x=0. \\ n_{p0} & \text{at } x \gg L_n \end{cases}$

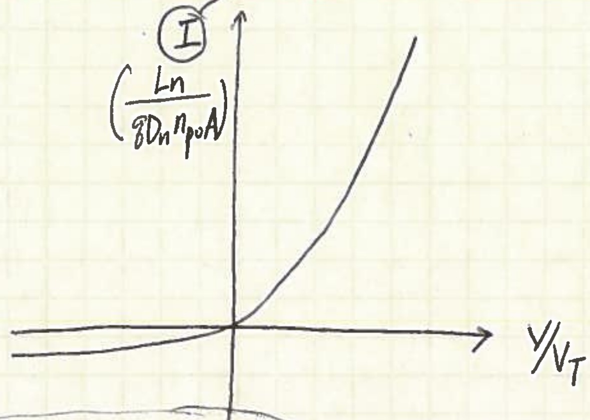
∴ the steady-state solution for $n(x)$ is

$$n_p(x) = n_{p0} + (n_p - n_{p0})\, e^{-x/L_n}$$

, where $L_n = \sqrt{D_n \tau_n}$, the electron diffusion length.

The junction current density at $x=0$ $\quad i \equiv i_n(x=0) = \frac{q D_n}{L_n}(n_p - n_{p0}) = \frac{q D_n n_{p0}}{L_n}\left(e^{\frac{V}{V_T}} - 1\right)$ ✓

Tue



$I = Ai$
↑
cross-sectional area.

$$R_d \equiv \left(\frac{dI}{dV}\right)^{-1} \approx \frac{V_T}{I}$$

C_{diff} = the diffusion capacitance $\equiv \frac{d}{dV} Q_{\text{(minority carrier)}} = A\frac{d}{dV}\left[q\int_0^\infty [n_p(x) - n_{po}]\,dx\right]$

$$= \frac{AqL_n n_{po}}{V_T} e^{\frac{V}{V_T}} \approx \frac{I}{V_T}\tau_n$$

the CR time constant characterized by R_d & C_{diff} $= \tau_n$, the electron lifetime.

The depletion-layer capacitance C_{dep} of the diode is defined as the voltage derivative of the total space charge in the depletion region.

$$C_{dep} \equiv \frac{d}{dV} Q_{\text{(space charge)}}$$

$$= \varepsilon_2 \cdot \frac{A}{W_{dep}}$$

$$= \sqrt{\frac{q\varepsilon_2 N_{D_2}}{2(V_D - V)}}\,A \quad \text{where } W_{dep} = \sqrt{\frac{2\varepsilon_2}{qN_{D_2}}(V_D - V)}$$

= depletion layer width in the N-layer

here, the capacitance contributed by the depletion layer in the p+ layer is neglected.

$R_d \cdot C_{dep} = \tau_{te}$, the thermionic emission time.

a key parameter for determing the noise characteristics of a pn junction diode under weak forward bias.

$R_d C_{diff} = \tau_n$, the minority-carrier life time

a key parameter for determing the noise characteristics of a pn junction diode under strong forward bias

Tue

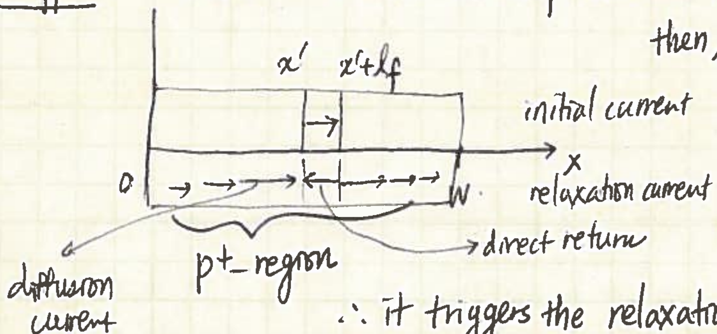
- Thermal Diffusion Noise

For a constant voltage bias,

boundary conditions
at $x=0$, $n_p = n_{po} e^{V/V_T} = \text{const}$
at $x=W$, $n_p = n_{po} = \text{const}$.

but $0 < x < W$, the electron density fluctuates due to microscopic random electron motion induced by thermal agitation and by generation and recombination processes.

Suppose an electron transits over l_f between collisions with the lattice.



then, $\exists$ an instantaneous current $q\delta(t)$ flows x' & $x'+l_f$.

$\downarrow$

disturbs the steady-state electron distribution.

$\therefore$ it triggers the relaxation current to remove this deviation which restores the steady-state electron distribution after a reasonably short time.

$n'(x,t) = n(x,t) - n_{po}(x)$ = deviation → satisfies the diffusion eq. & boundary conditions

$n_{po}(x)$: steady-state

$n'(x,t) = 0$ at $x=0$ & $x=W$ ($\because$ for constant voltage).

Fourier transform of the diffusion eq.

$$\frac{\partial^2}{\partial x^2} N'(i\omega) = \frac{1}{L^2} N'(i\omega) \quad \text{where } L^2 = \frac{L_n^2}{1+i\omega\tau_n}.$$

$N'(i\omega)$: the F.T. of $n'(x,t)$

Tue

① F.T. of the relaxation currents $i_1'(t)$ at $x=x'$
$i_2'(t)$ at $x=x'+l_f$.

where $N_1'(i\omega)$ = F.T. of electron density deviation at $x=x'$
$N_2'(i\omega)$ = F.T. of electron density deviation at $x=x'+l_f$

$$I_1'(i\omega) = qD_n \left.\frac{\partial N'(i\omega)}{\partial x}\right|_{x=x'} = \frac{qD_n}{L}\coth\left(\frac{x'}{L}\right) N_1'(i\omega)$$

$$I_2'(i\omega) = qD_n \left.\frac{\partial N'(i\omega)}{\partial x}\right|_{x=x'+l_f} = -\frac{qD_n}{L}\coth\left(\frac{W-x'}{L}\right) N_2'(i\omega)$$

② F.T. of the direct return currents $i_{r_1}'(t)$
$i_{r_2}'(t)$

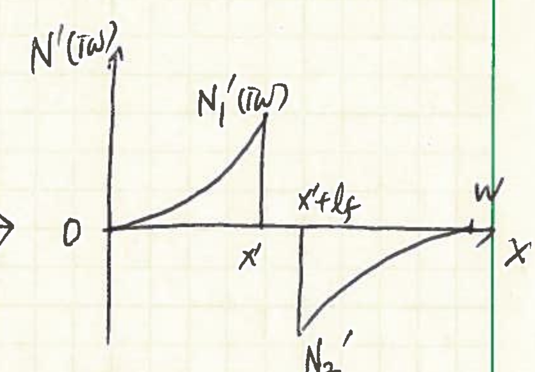
$$I_{r_1}'(i\omega) = I_{r_2}'(i\omega) = -\frac{qD_n}{l_f}\left[N_1'(i\omega) - N_2'(i\omega)\right]$$

Note that $\exists$ no charge accummulation in the entire p^+-layer

$\Rightarrow$ current should be continuous.

$$I_1'(i\omega) + I_{r_1}'(i\omega) + q = 0$$

$$I_2'(i\omega) + I_{r_2}'(i\omega) + q = 0.$$



$$\therefore N_1'(i\omega) = \frac{l_f}{D_n}\cdot\frac{\frac{qD_n}{L}\coth\left(\frac{x'}{L}\right)}{\frac{qD_n}{L}\left(\coth\left(\frac{x'}{L}\right)+\coth\left(\frac{W-x'}{L}\right)\right)} \equiv \frac{l_f}{D_n}\cdot\frac{k_1}{k_1+k_2}$$

$$N_2'(i\omega) = -\frac{l_f}{D_n}\cdot\frac{\frac{qD_n}{L}\coth\left(\frac{W-x'}{L}\right)}{\frac{qD_n}{L}\left(\coth\left(\frac{x'}{L}\right)+\coth\left(\frac{W-x'}{L}\right)\right)} \equiv -\frac{l_f}{D_n}\cdot\frac{k_2}{k_1+k_2}.$$

$$k_1 = \coth\left(\frac{x'}{L}\right)\left(\frac{qD_n}{L}\right), \quad k_2 = \frac{qD_n}{L}\coth\left(\frac{W-x'}{L}\right)$$

Tue

The circuit current flows in the external circuit by 2 relaxation currents at $x=0$ & $x=W$

$$I_T'(i\omega) = \underbrace{I_0'(i\omega)}_{\text{relaxation current at } x=0} - \underbrace{I_W'(i\omega)}_{x=W}$$

where
$$I_0'(i\omega) = qD_n \frac{\partial}{\partial x} N'(i\omega)\Big|_{x=0} = \frac{l_f}{D_n} \cdot \frac{\frac{qD_n}{L_n}\sqrt{\frac{qD_n}{L_n}}\coth\left(\frac{W-x'}{L}\right)}{\frac{qD_n}{L}\left[\coth\left(\frac{x'}{L}\right)+\coth\left(\frac{W-x'}{L}\right)\right]} = \frac{l_f}{D_n}\cdot\frac{k_0 k_2}{k_1+k_2}$$

cosech$\left(\frac{x'}{L}\right)$

$$I_W'(i\omega) = qD_n \frac{\partial}{\partial x} N'(i\omega)\Big|_{x=W} = \frac{l_f}{D_n} \cdot \frac{\frac{qD_n}{L_n}\,\mathrm{cosech}\left(\frac{W-x'}{L}\right)}{\frac{qD_n}{L}\left[\coth\left(\frac{x'}{L}\right)+\coth\left(\frac{W-x'}{L}\right)\right]} = \frac{l_f}{D_n}\cdot\frac{k_W k_1}{k_1+k_2}$$

the F.T. of the circuit current pulse due to a single electron event in the p+ layer

∴ The average # of thermal diffusive transit events per second in a small volume $A\Delta x$ (A = cross-section, Δx = the small distance along x)

$$\gamma_T = \frac{n(x)A\Delta x}{\overline{\tau_f}}$$

$\overline{\tau_f}$ = mean free time of the electron in the p+ region.

Since each thermal diffusive event occurs independently

Using the Carson's theorem

$$\Delta S_{I_T'}(\omega) = 2\gamma_T \overline{|I_T'(i\omega)|^2}$$

$$= \frac{2n(x)A\Delta x}{\overline{\tau_f}} \cdot \frac{\overline{l_f^2}}{D_n^2} \left|\frac{k_0k_2 - k_Wk_1}{k_1+k_2}\right|^2$$

$$= \frac{4A}{D_n} n(x) \left|\frac{k_0k_2-k_Wk_1}{k_1+k_2}\right|^2 \Delta x$$

$\because \overline{l_f^2} = 2D_n\overline{\tau_f}$

Tue

∴ The total current fluctuation power spectral density by the thermal Diffusion noise

$$S_{I_T'}(\omega) = \frac{4A}{D_n}\int_0^W n(x)\cdot\left|\frac{k_0 k_2 - k_W k_1}{k_1+k_2}\right|^2 dx.$$

$$\approx \frac{4q^2 A D_n}{L_n}\left(\frac{n_p - n_{po}}{3} + \frac{n_{po}}{2}\right) \leftarrow \text{thermal diffusion noise}$$

$$\boxed{\begin{array}{l} W \gg L_n, \\ \omega\tau_n \ll 1 \end{array}} \begin{array}{l} \leftarrow \text{long diode} \\ \leftarrow \text{low-freq. fluctuation component} \end{array}$$

$k_1 = \frac{qD_n}{L}\coth\left(\frac{x'}{L}\right)$

$k_2 = \frac{qD_n}{L}\coth\left(\frac{W-x'}{L}\right)$

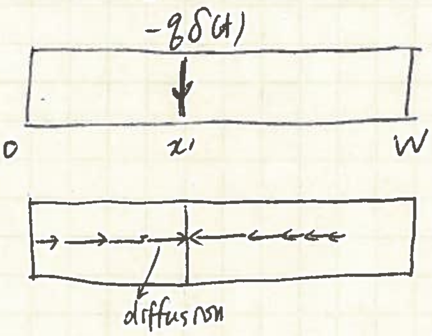
$k_0 = \frac{qD_n}{L_n}\operatorname{cosech}\left(\frac{x'}{L}\right)$

$k_W = \frac{qD_n}{L_n}\operatorname{cosech}\left(\frac{W-x'}{L}\right)$

② Generation-Recombination Noise

← "instantenous appearance or disappearance of an electron"

Suppose an electron is generated at $x = x'$, an instantaneous current $-q\delta(t)$ occurs suddenly

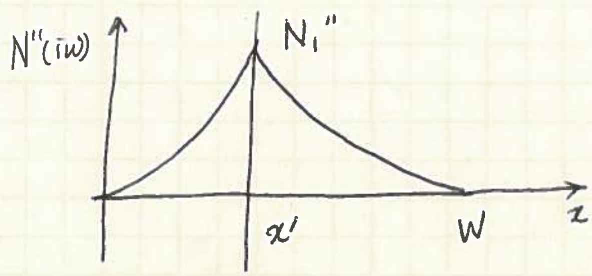


Solving Fourier transformed diffusion equations with the b.c. $N''(i\omega) = 0$ at $x=0$, $x=W$

$N''(i\omega) = N_1''$ at $x = x'$

$$\frac{\partial^2}{\partial x^2}N''(i\omega) = \frac{1}{L^2}N''(i\omega)$$

$$N''(i\omega) = \begin{cases} \dfrac{N_1''}{e^{x'/L} - e^{-x'/L}}\left(e^{x/L} - e^{-x/L}\right), & 0 \le x \le x' \\[2ex] \dfrac{N_1''}{e^{(W-x')/L} - e^{-(W-x)/L}}\left(e^{(W-x)/L} - e^{-(W-x)/L}\right), & x' \le x \le W \end{cases}$$



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$\therefore$ The counter propagating relaxation currents $I_1''(i\omega)$ & $I_2''(i\omega)$ at $x=x'$ are

$$I_1''(i\omega) = qD_n \left.\frac{\partial N''(i\omega)}{dx}\right|_{x=x'-0} = k_1 N_1''(i\omega), \quad k_1 = \frac{qD_n}{L}\coth\left(\frac{x'}{L}\right)$$

$$I_2''(i\omega) = qD_n \left.\frac{\partial N''(i\omega)}{dx}\right|_{x=x'+0} = -k_2 N_1''(i\omega), \quad k_2 = \frac{qD_n}{L}\coth\left(\frac{W-x'}{L}\right)$$

$\therefore$ The current condition at $\underline{x=x'}$ imposes.

$$I_1''(i\omega) - I_2''(i\omega) - q = 0$$

$$\therefore \quad N_1''(\omega) = \frac{q}{k_1+k_2}$$

$$\therefore \quad I_0''(i\omega) = qD_n \left.\frac{\partial N''(i\omega)}{\partial x}\right|_{x=0} = q\frac{k_0}{k_1+k_2}$$

$$I_W''(i\omega) = qD_n \left.\frac{\partial N''(i\omega)}{\partial x}\right|_{x=W} = -q\frac{k_W}{k_1+k_2}.$$

The external circuit current is again given by.

$$I_T''(i\omega) = I_0''(i\omega) - I_W''(i\omega)$$

$$= q\frac{k_0+k_W}{k_1+k_2}$$

The average # of recombination event in a small volume $A\Delta x$ is

$$\gamma_R = \frac{n(x)A\Delta x}{\tau_n}$$

The average # of generation event is

$$\gamma_G = \frac{n_{p0}A\Delta x}{\tau_n}$$

At $V=0$, $n(x)=n_{p0}$ $\quad\therefore \gamma_R = \gamma_G$ in the thermal eq. $\Leftarrow$ "detailed balance"

Tue

The current fluctuation PSD due to the G-R events in this small volume is

$$\Delta S_{I_T''}(\omega) = 2(\gamma_G + \gamma_R)\overline{|I_T''(i\omega)|^2}$$

$$= 2 \cdot \frac{[n(x') + n_{po}]A\Delta x}{\tau_n} \, q^2 \left|\frac{k_0 + k_W}{k_1 + k_2}\right|^2$$

∴ The total current fluctuation PSD is to calculate by integration from $x=0$ to $x=W$

$$S_{I_T''}(\omega) = \frac{2Aq^2}{\tau_n}\int_0^W (n(x') + n_{po})\left|\frac{k_0 + k_W}{k_1 + k_2}\right|^2 dx'$$

$$\approx \frac{2Aq^2 D_n}{L_n}\left[\frac{n_p - n_{po}}{3} + n_{po}\right]$$ if $W \gg L_n$, long-diode limit; $\omega\tau_n \ll 1$, low-freq. limit

---

Finally, the total current fluctuation PSD is the sum of thermal diffusion noise & the G-R noise.

$$S_{I_T}(\omega) = \underbrace{\frac{4Aq^2 D_n}{L_n}\left(\frac{n_p - n_{po}}{3} + \frac{n_{po}}{2}\right)}_{\text{thermal}} + \underbrace{\frac{2Aq^2 D_n}{L_n}\left(\frac{n_p - n_{po}}{3} + n_{po}\right)}_{\text{G-R Noise}}$$

Let's check in 3-bias regions

① $V=0$.

$n_p = n_{po}$ ∴ $S_{I_T}(\omega) = \frac{4Aq^2 D_n n_{po}}{L_n} = \frac{4k_B\theta}{R_d(V=0)}$ ← Johnson-Nyquist thermal noise

where $R_d(V=0) = \frac{L_n k_B \theta}{Aq^2 D_n n_{po}}$, differential resistance at $V=0$

* remember that half comes from standard thermal diffusion noise

and half " G-R noise.

by lattice vibration (thermal phonon reservoirs)

or E&M field (thermal photon reservoirs)

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② $V>0$ forward bias.

$$S_{I_T}(\omega) = \frac{2Aq^2D_n}{L_n}(n_p + n_{po}) = 2q(I + 2I_s)$$ ✓

$\frac{2}{8}+\frac{1}{6}=\frac{1}{2}$

$\frac{8}{8}+\frac{4}{6}=\frac{12}{6}=2\frac{1}{2}$

$-\frac{2}{8}+\frac{1}{2}-\frac{1}{6}+\frac{1}{2}$
$=1-\frac{3}{6}=\boxed{\frac{1}{2}}$

where I = forward bias $= \frac{AqD_n}{L_n}(n_p - n_{po})$

I_s = the reverse saturation current $= \frac{AqD_n}{L_n} n_{po}$

If a reasonably high forward bias voltage, $I \gg I_s$

$S_{I_T}(\omega) \approx \underline{2qI}$ full shot noise

- 2/3 from thermal diffusion noise
- 1/3 from G-R noise
  - ↑ by radiative recombination noise (spon. emission) for a direct bandgap
  - thus it's q. noise

$\frac{1}{6}$ $\quad \frac{2}{6}=\frac{1}{3}$

$-\frac{2}{8}+\frac{3}{8}-\frac{1}{6}+\frac{3}{8}$

③ $V<0$, reverse bias

$n_p \ll n_{po}$ $\quad \therefore S_{I_T}(\omega) = \frac{2Aq^2D_n}{L_n} n_{po} = 2qI_s$

← "dark current shot noise"

by the dominant noise source of a reverse-biased photodiode and avalanche photodiode

- 2/3 from G-R noise ← due to the thermal photon absorption
- 1/3 from thermal noise.

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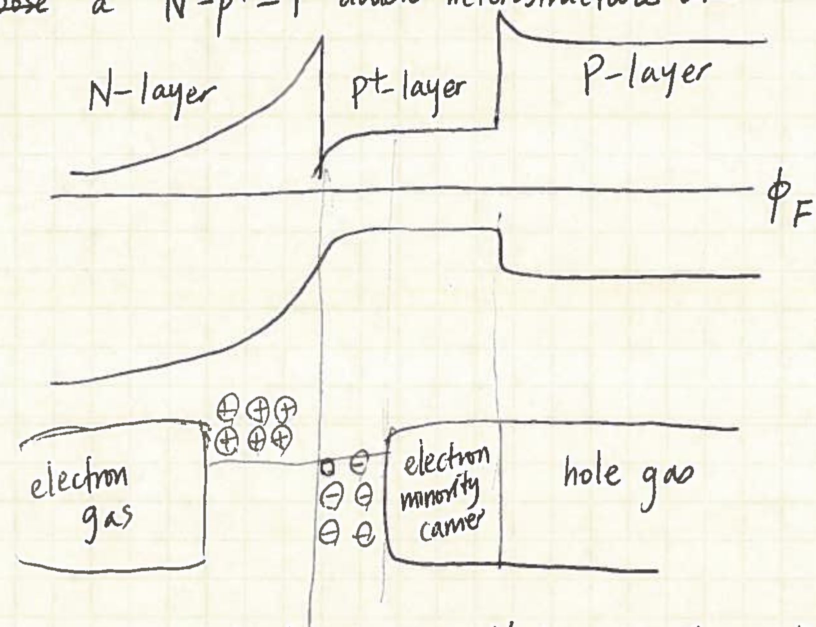
$\boxed{\text{Short Diode}}$ cf) Long diode limit so far
$W \gg L_n$ = diffusion length

For a double heterostructure semiconductor laser diode
a heterojunction bipolar transistor,

p^+-layer is much thinner than L_n ($W \ll L_n$)

Consider ~~Suppose~~ a $N-p^+-P$ double heterostructure diode

V=0



Note an injected electron from N-layer to the p^+-layer cannot diffuse freely toward the p-side metal
∵ the conduction band discontinuity at p^+-P heterojunction

∴ A junction current is not carried by a thermal diffusion process
but crosses an "imaginary plane" between the conduction and the valence bands by a "recombination process"

the electron density is uniform in the p^+-layer ∵ $W \ll L_n$

for the small bias $n_p = n_{po} \exp\left(\frac{V}{V_T}\right)$

$$\therefore I = q\frac{N_e}{\tau_n} = q\frac{AWn}{\tau_n}$$

$$\therefore R_d = \left(\frac{dI}{dV}\right)^{-1} = \frac{V_T \tau_n}{qN_e}, \quad C_{diff} = \frac{dQ}{dV} = \frac{qN_e}{V_T} \Rightarrow R_d C_{diff} = \tau_n$$

(Q → minority carrier)

Tue

$$C_{depletion} \overset{\text{same}}{=} \frac{d}{dV} Q_{space\ charge} = \varepsilon_2 \frac{A}{W_{dep}} = \sqrt{\frac{q\varepsilon_2 N_{D_2}}{2(V_D - V)}}\, A$$

When $W \ll L_n$,

$$\frac{k_0 k_2}{k_1 + k_2} \approx \frac{k_W k_1}{k_1 + k_2} \approx \frac{q D_n}{W}$$

thermal diffusion noise

$$\therefore I_0'(i\omega) \simeq I_W'(i\omega) = \frac{q l_f}{W}$$

At each boundary,

$$S_{I_0'}(\omega) = S_{I_W'}(\omega) = \int_0^W \frac{2n_p A}{\overline{\tau_f}} \overline{\left(\frac{q l_f}{W}\right)^2}\, ds$$

$$= 4qI\left(\frac{L_n}{W}\right)^2 \quad \because D_n = \frac{\overline{l_f^2}}{2\overline{\tau_f}} = \frac{L_n^2}{\tau_n}$$

Since $L_n/W \gg 1$, the current noise is much larger than the full shot noise

__But__ $I_0'(i\omega)$ & $I_W'(i\omega)$ are identical, thus it cancel out ~~the~~

$$\therefore I_T'(i\omega) = I_0'(i\omega) - I_W'(i\omega) = 0$$

$\Rightarrow$ the total external circuit current ~~fluctuation~~ fluctuation is __zero__.

When $W \ll L_n$, $\frac{k_0}{k_1 + k_2} \approx 1 - \frac{x'}{W}$, $\frac{k_W}{k_1 + k_2} \approx \frac{x'}{W}$

$$\therefore I_0''(i\omega) = q\left(1 - \frac{x'}{W}\right)$$

$$I_W''(i\omega) = -q\frac{x'}{W}$$

$$\therefore I_T''(i\omega) = I_0''(i\omega) - I_W''(i\omega) = -q$$

each event of electron G-R results in "indep. current pulses" with a time-integrated area $= q$ in the external circuit $\therefore S_{I_T''}(\omega) = \frac{2q^2(N_e + N_{e0})}{\tau_n}$

Tue

① $V=0$ zero-bias $\underline{N_e = N_{e0}}$.

$$S_{I_T}''(\omega) = \frac{4q^2 N_{e0}}{\tau_n} = \frac{4k_B\Theta}{R_d(V=0)}$$

a pn-junction is in eq. w/ thermal photon reservoir

(1/2 thermal noise from thermal photon absorption
(1/2 radiative recombination (spontaneous emission)

② $V>0$, forward bias

$$S_{I_T}''(\omega) = 2q^2\frac{N_e}{\tau_n} = 2qI$$ full shot noise from radiative recombination / spontaneous emission

← QM origin

③ $V<0$ reverse bias

$$S_{I_T}''(\omega) = 2q^2\frac{N_{e0}}{\tau_n} = 2qI_s$$ full shot noise due to generation (thermal photon absorption) process

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Summary of constant voltage bias

The origin of current noise

① thermal diffusive transit btw collisions with the lattice

② generation-recombination of a minority carrier (electron in p-layer, hole in n-layer)

→ introduce "the relaxation current" to restore the steady-state distribution of minority carriers

↓ causes deviation of minority carrier distribution in the depletion layer edge

<u>example</u>. n-p^+ layer, if n_p is temorarily decreased

→ ∃ forward thermionic emission from N-layer to p^+

∴ it causes n_N reduction in the N-layer depletion layer edge

but this is recovered immediately by a majority carrier flow in the N-layer by the external circuit

if ∃ n_p excess in the p^+ layer,

n_N increases, then external current flows oppositely

since $R_s \ll 0$ ⇒ this happens immediately w/o memory ∴ treat as "independent" → full shot noise

Tue

pn Junction Diodes Under Constant Current bias

① Source Resistence effect

$R_s \gg R_d$.

the modulation in n_N induced by excess forward or backward thermionic emission of an e^-
cannot be instantaneously eliminated by the external circuit

∴ ∃ junction voltage fluctuation by thermal diffusive transit
& G-R events.

if the recombination events for e^-s on the p^+-layer exceed the average value,
junction voltage ↓ due to excess forward the thermionic emission of electron

since $R_s \gg R_d$,
this junction voltage decrease ~~does~~ is not immediately recovered by the external circuit
∴ the forward thermionic emission rate ↓
∴ lower recombination

"sort-of" self-feedback stabilization mechanism

While the external current is smoothed due to overlapping pulses
with a long relaxation time CR_s.

the Kirchhoff circuit eq. for this noise eq circuit is

$$\underbrace{\frac{dv_n}{dt}}_{\text{junction voltage fluctuation}} = \underbrace{-\frac{v_n}{CR_s}}_{\text{relaxation rate due to external circuit}} + \underbrace{i_s}_{\text{J-N noise by } R_s} \underbrace{-\frac{v_n}{CR_d}}_{\text{relaxation rate due to thermionic emission and/or recombination}} + \underbrace{i}_{\text{noise due to thermionic emission and/or recombination}}$$

$C = C_{diffusion} + C_{depletion}$
= total junction cap

F.T. ↓

② current noise PSD

Tue

F.T.

$\hookrightarrow \left(i\omega C + \frac{1}{R_s} + \frac{1}{R_d} \right) V_n(\omega) = I(\omega) + I_s(\omega)$

J-N noise

The external current noise $I_n = I_s - \frac{V_n}{R_s}$. from Kirchhoff noise

$$\therefore I_n(\omega) = \frac{-I(\omega) + \left(i\omega C R_s + \frac{R_s}{d}\right) I_s(\omega)}{i\omega C R_s + \frac{R_s}{R_d} + 1}$$

$$\therefore S_{in}(\omega) \propto |I_n(\omega)|^2$$

$$= \frac{S_i(\omega) + \left[\left(\frac{R_s}{R_d}\right)^2 + (\omega C R_s)^2\right] S_{I_s}(\omega)}{\left(1 + \frac{R_s}{R_d}\right)^2 + (\omega R_s C)^2}$$

$S_i(\omega) = 2qI, \quad S_{I_s}(\omega) = \frac{4k_B\theta}{R_s}$

$\leftarrow$ we assume $I \sim I_s e^{qV/k_B\theta} \Rightarrow 2qI = \frac{2k_B\theta}{R_d}$.

i) constant voltage source $R_s \ll R_d$, $2qI = \frac{2k_B\theta}{R_d} \ll 4k_B\theta/R_s$

$$S_{in}(\omega) = \frac{2qI + (\omega C R_s)^2 \frac{4k_B\theta}{R_s}}{1 + (\omega C R_s)^2} \left[\begin{array}{ll} 2qI & \text{if } \omega C R_s \ll 1 \quad \text{low-f. limit} \\ \frac{4k_B\theta}{R_s} & \text{if } \omega C R_s \gg 1 \quad \text{high-f limit} \end{array} \right.$$

$\downarrow$

C is shorted

$\therefore$ internal noise cannot be extracted

ii) constant current source $R_s \gg R_d$, $2qI = \frac{2k_B\theta}{R_d} \gg \frac{4k_B\theta}{R_s}$

$S_{in}(\omega) = \frac{4k_B\theta}{R_d}$ always <u>thermal noise limit</u>.

Tue

② Voltage Noise PSD

$$V_n(\omega) = \frac{I(\omega) + I_s(\omega)}{i\omega C + \frac{1}{R_s} + \frac{1}{R_d}}$$

$$\therefore S_{V_n}(\omega) = \frac{2qI R_d^2 + \frac{4k_B\theta}{R_s} R_d^2}{(\omega C R_d)^2 + \left(1+\frac{R_d}{R_s}\right)^2}$$

→ ① constant voltage bias ($R_s \ll R_d$)

$$S_{V_n}(\omega) = \frac{2qIR_s^2 + 4k_B\theta R_s}{1+(\omega R_s C)^2} \xrightarrow{2qIR_s^2 \ll 4k_B\theta R_s} \frac{4k_B\theta R_s}{1+(\omega R_s C)^2} \longrightarrow 0$$

↑ thermal noise w/ R_s

↑ if $R_s = 0$

→ ② constant current bias ($R_s \gg R_d$)

$$S_{V_n}(\omega) = \frac{2qIR_s^2}{1+(\omega R_d C)^2}$$

← Both electron emission & external current is regulated.