Introduction to Noise Processes
ECE730/QIC890-T33
Instructor: Professor Na Young Kim

Problem Set 1

Due: May 9, 2016, 5:00 pm : QNC 4118

1. **Statistically Stationary vs. Statistically Non-Stationary Processes**

   In class, we discussed stationarity. If $\langle x(t_1)\rangle$ and $\langle x(t_1)^2\rangle$ are independent of the time t_1 and if $\langle x(t_1)x(t_2)\rangle$ is independent of absolute times t_1 and t_2 but dependent only on the time difference $\tau = t_2 - t_1$, such a noise process is called as "statistically stationary process". The "statistically-nonstationary" process does not hold the aforementioned conditions. Hence, the concept of ensemble averaging is valid, whereas the concept of time averaging fails.

   (1) Please show the example of statistically stationary and statistically non-stationary processes.

   (2) Can you make a statistically non-stationary process to a statistically stationary process? If yes, how? If no, why?

   (3) Here is a member function $x(t) = \sin(\omega t + \theta)$ from a stochastic process specified by a transformation of variables. θ is a random variable with uniform distribution over the internal $0 < \theta < 2\pi$, with the probability $P(\theta) = \frac{1}{2\pi}$. Is $x(t)$ ergodic in the mean?
   Is $x(t)$ ergodic in the auto-correlation? Please justify your answers.

2. **Ergodicity**

   We learned ergodicity in the mean and in the autocorrelation in class.

   (1) Please write down an example of the processes which are ergodic in the mean and/or the autocorrelation.

   (2) Suppose the autocorrelation of a noise process is ergodic. Is the process ergodic in the mean? Please justify your answers.

3. **Parseval Theorem**

   (1) If $x_1(t)$ and $x_2(t)$ have Fourier transform $X_1(i\omega)$ and $X_2(i\omega)$, prove the equality of the equation:

   $$\int_{-\infty}^{\infty} x_1(t)x_2^*(t)dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} X_1(i\omega)X_2^*(i\omega)d\omega.$$

   (2) Suppose $x_1(t) = x_T(t+\tau)$ and $x_2(t) = x_T(t)$. The notation $x_T(t)$ implies $x(t)$ if $|t| \le T/2$ and 0 if $|t| > T/2$, where T is a finite-valued measurement time interval. Show that

   $$\int_{-\infty}^{\infty} x_T(t+\tau)x_T(t)dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |X_T(i\omega)|^2 \exp(i\omega\tau)d\omega.$$