May 25th Assignment

Here is something to work on for the next two weeks. You can do it alone or together with other people. You can do it by hand or on the computer. We've covered a lot of material in the last couple of weeks and not everyone has been here. So it's likely that you won't understand part of what's here. Please email me with questions. I expect there to be quite a few of them, and I will be reading email. If you feel comfortable doing so, then copy the whole group. If you copy the group on your question, then I will also copy them on the answer. That way each question can just be answered once, and people can also discuss it. But, if you don't feel comfortable doing that, still send me questions.

This assignment is mainly about the so called Bell States:

$$(1)\ \ |\phi^+\rangle = \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle = \frac{1}{\sqrt{2}}|+z,+z\rangle + \frac{1}{\sqrt{2}}|-z,-z\rangle$$

$$(2)\ \ |\phi^-\rangle = \frac{1}{\sqrt{2}}|00\rangle - \frac{1}{\sqrt{2}}|11\rangle = \frac{1}{\sqrt{2}}|+z,+z\rangle - \frac{1}{\sqrt{2}}|-z,-z\rangle$$

$$(3)\ \ |\psi^+\rangle = \frac{1}{\sqrt{2}}|01\rangle + \frac{1}{\sqrt{2}}|10\rangle = \frac{1}{\sqrt{2}}|+z,-z\rangle + \frac{1}{\sqrt{2}}|-z,+z\rangle$$

$$(4)\ \ |\psi^-\rangle = \frac{1}{\sqrt{2}}|01\rangle - \frac{1}{\sqrt{2}}|10\rangle = \frac{1}{\sqrt{2}}|+z,-z\rangle - \frac{1}{\sqrt{2}}|-z,+z\rangle$$

A few things to note:

1. I've written out each state first in its "information theory" form (using ones and zeros) and then in its "electron spin" form (using plus and minus spin on the z axis). In both cases the math is exactly the same, since $|+z\rangle = |0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $|-z\rangle = |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$.

2. The pluses and minuses in the names of the Bell states have nothing to do with spin or eigenvalues. The plus in $|\phi^+\rangle$ is a reminder of the "+" in the middle of the state. The same for the minus in $|\phi^-\rangle$, and so on.

3. I gave each state both a name and a number. So, for example, "Bell 3" and ψ^+ refer to the same state. The main reason for using ϕ and ψ is that it appears to be pretty common practice. Take a look at the Wikipedia entry for "Bell States" and make sure that you can see the equivalence between their notation and what's above.

For everything that follows, we are going to assume that each of the Bell states represents an entangled pair of either (quantum) bits or electrons. In either case, the bit or electron on the left belongs to Alice and the one on the right belongs to Bob. So, for example, the state $|01\rangle$ would mean that Alice has a zero and Bob has a one.

Problem 1:

For each of the Bell states, if Alice does a measurement resulting in a zero, then what is the probability of Bob also getting a zero? If Bob measures a -z, then what is Alice's probability of gettng a +z?

Problem 2:

Express each of the four Bell states as a column vector. You may be able to recognize the "pattern" of the kets, but if all else fails you can take the tensor products of all the individual ones and zeros and add everything up. For example, Bell 1 should come out to:

$$\text{Either:} \quad \begin{pmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ 0 \\ \frac{1}{\sqrt{2}} \end{pmatrix}, \quad \text{or:} \quad \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}.$$

Problem 3:

Take the four column vectors, from the last problem, and change them to the *x basis*. In order to do this, you will have to project each of the vectors on to all four of the x basis vectors. If you don't know what the four "two bit" x basis vectors are, you can find them by taking the appropriate tensor products of the two "1 bit" x basis vectors: $|+x\rangle$ and $|-x\rangle$.

Problem 4:

Write out all four Bell states, in the x basis, in ket form. Here is the first one:

$$|\phi^+\rangle \;=\; \frac{1}{\sqrt{2}}|+z,+z\rangle + \frac{1}{\sqrt{2}}|-z,-z\rangle \;=\; \frac{1}{\sqrt{2}}|+x,+x\rangle + \frac{1}{\sqrt{2}}|-x,-x\rangle;$$

So for Bell 1 the pattern doesn't change at all when you switch to the x basis.

Problem 5:

For all four states, discuss the correlations between the measurements, and how the correlations differ depending on whether you do a z or an x measurement. For example, we just saw that when we changed ϕ^+ from the z to the x basis the correlations didn't change at all. Alice and Bob's measurement were positively correlated in both z and x. What I mean by positively correlated is that Alice and Bob are guaranteed to get the same result, if they do the same type of measurement. In the (z basis) ψ states, however, the measurement are negatively correlated. Alice and Bob are guaranteed to get different results for the same type of measurement. So your job here is to figure out what type of correlations there are for all the states in both the x and z bases, and find out when a change of basis results in a change in correlations.

Note:

It "should" be possible to do all the above problems by hand. But doing the 45° calculations below will require using the computer, in one way or another. (At least to get the eigenvectors for the 45° measurement matrix.)

Problem 6:

Find the 1-bit basis: $|+45\rangle, |-45\rangle$ associated with the spin vector that is exactly half way between the x and z axes. If you can't figure out how to do this, see the "Electron Spin" notebook.

Problem 7:

Construct the 2-bit 45° basis by taking the appropriate tensor products of the 1-bit vectors. Remember you can use the computer, and you can use decimal approximations.

Problem 8:

Construct all four of the Bell states in the 45° basis, using same procedure that worked for the x basis in problem 3.

Problem 9:

Write out all the 45° states as kets like we did for the x states in problem 4.

Problem 10:

Analyze the correlations for the 45° basis, like we did for the x basis in problem 5.

Good luck on this. I think it would be great if people discussed and worked these problems together. But whatever works for you. I know that we've covered a lot of material very quickly. So if you're lost, just go back and work on one of the previous problems sets, or review the notes. We can always do more review of the basics, if that seems like a good idea. And email me as much as you like for help or ideas. I will not actually be all that busy most of the time we're away.