Week 7 - Review

(Version: March 1, 2014)

1 Part One

Everyone should definitely be able to do the problems in part one.
Note: The numbers in bold parentheses, **( )** are the problem numbers.

1.1 Vectors

Let $a = \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \quad b = \begin{pmatrix} 3 \\ 4 \end{pmatrix}, \quad c = \begin{pmatrix} -2 \\ 5 \end{pmatrix}$

Multiply the vectors by the indicated scalar, giving the answer as a column:

(1) $3|a\rangle$ **(2)** $6|b\rangle$ **(3)** $k|c\rangle$

Do the additions/subtractions (end up with a single row or column):

(4) $|a\rangle + |b\rangle$ **(5)** $\langle b| + \langle c|$ **(6)** $|a\rangle - |c\rangle$

Do the linear combinations (ending with a single row or column):

(7) $2|a\rangle + 3|b\rangle$ **(8)** $3|b\rangle - 2|c\rangle + |a\rangle$ **(9)** $-2\langle b| + 2\langle c|$

Work out these inner products:

(10) $\langle a|b\rangle$ **(11)** $\langle b|c\rangle$ **(12)** $\langle c|a\rangle$

1.2 Multiplying a matrix times a vector

Note: When you multiply a matrix times a column vector, the result is a column:

(13) $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \begin{pmatrix} 1 \\ -1 \end{pmatrix}$ **(14)** $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ **(15)** $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix}$

1.3 Complex Numbers

Let $z_1 = 2 + 3i, \quad z_2 = 1 - i, \quad z_3 = 5i$

Do the sums: **(16)** $z_1 + z_2$ **(17)** $z_1 - z_3$ **(18)** $z_1 + z_2 - z_3$

Do the products: **(19)** $z_1 z_2$ **(20)** $z_2 z_3$ Conjugate: **(21)** z_1^* **(22)** z_2^*

Find the absolute values: **(23)** $|z_1|$ **(24)** $|z_2|$ **(25)** $|z_3|$

2 Part Two

It would be good if you could do these now, but we'll be getting more practice.

2.1 Matrices

Find the indicated transpose:

(26) $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}^T$ **(27)** $\begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 3 \\ 3 & 4 & 5 \end{pmatrix}^T$ **(28)** $\begin{pmatrix} w & x \\ y & z \end{pmatrix}^T$

Multiplication:

(29) $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} 4 & 3 \\ 2 & 1 \end{pmatrix}$ **(30)** $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ **(31)** $\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} w & x \\ y & z \end{pmatrix}$

(32) $\begin{pmatrix} 1 & 2 \end{pmatrix}\begin{pmatrix} 4 \\ 3 \end{pmatrix}$ **(33)** $\begin{pmatrix} 1 \\ 2 \end{pmatrix}\begin{pmatrix} 4 & 3 \end{pmatrix}$

(34) Which of the following is the inverse of $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

(a) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ (b) $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ (c) $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ (d) None of them

(35) Which of the following is the inverse of $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$

(a) $\begin{pmatrix} 1 & 1/2 \\ 1/3 & 1/4 \end{pmatrix}$ (b) $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ (c) $\begin{pmatrix} -2 & 1 \\ 3/2 & -1/2 \end{pmatrix}$ (d) None of them

(36) Which of the following is the inverse of $\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$

(a) $\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ (b) $\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$ (c) $\begin{pmatrix} -1 & 0 \\ 0 & 0 \end{pmatrix}$ (d) None of them

2.2 Change of basis

Given the two bases: $b_1 = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}, \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$ and $b_2 = \frac{1}{\sqrt{5}}\begin{pmatrix} 2 \\ 1 \end{pmatrix}, \frac{1}{\sqrt{5}}\begin{pmatrix} -1 \\ 2 \end{pmatrix}$

(37) Find the vector $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$ in the b_1 basis

(38) Find the vector $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$ in the b_2 basis

(39) Find the vector $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ in the b_2 basis

(40) Find the vector $\begin{pmatrix} 5 \\ 5 \end{pmatrix}$ in the b_1 basis

3 Part Three

You don't *need* to do this part at all. But I'm curious how many of these anybody *can* do.

3.1 What is the dimension of the following objects:

(41) $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ **(42)** $\begin{pmatrix} a & b \end{pmatrix}$ **(43)** $\begin{pmatrix} 1 & 2 & 3 \\ a & b & c \\ x & x & z \end{pmatrix}$ **(44)** $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$

3.2 Commutativity

An operation is *commutative* if the order of the operation doesn't matter. Examples: $2+3 = 3+2 = 5$, so addition of real scalars is commutative. "Pillage then burn" - not commutative.

(45) Is addition of vectors commutative?

(46) Is addition of matrices commutative?

(47) Is multiplication of matrices commutative?

(48) Is addition of complex numbers commutative?

(49) Is multiplication of complex numbers commutative?

(50) If two vectors both contain real numbers, then is their inner product commutative?

(51) Can you *prove* that any of the above statements is either true or false? You would prove it true by solving the "general case" (all variables, no actual numbers - so that the result would work for any numbers). You would prove it false simply by finding a counter example.

3.3 Orthonormality

For each problem below, state whether the list of vectors is orthonormal or not.

(52) $\begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ **(53)** $\begin{pmatrix} 2 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 2 \end{pmatrix}$ **(54)** $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}, \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$

(55) $\begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ -1 \end{pmatrix}$ **(56)** $\frac{1}{\sqrt{5}}\begin{pmatrix} 2 \\ 1 \end{pmatrix}, \frac{1}{\sqrt{5}}\begin{pmatrix} -1 \\ 2 \end{pmatrix}$ **(57)** $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}, \frac{1}{\sqrt{5}}\begin{pmatrix} -1 \\ 2 \end{pmatrix}$