## Length Contraction

1. Now put two observers on either end of the ship and one on the ground.
2. The observers on the ship each note the time that they pass the observer on the ground and they calculate $t_s$ as the difference between the two.
3. The observer on the ground notes the time between the passage of the two ends of the ship and calls that $t_g$.
4. The "events" here are the times that each end of the ship passed the observer on the ground. So that observer sees the proper time: $t_g = t_0$.
5. Hence $t_s = \dfrac{t_g}{\sqrt{1 - \frac{v^2}{c^2}}}$.
6. The observers on the ship calculate $L_s = v \times t_s$ an the one on the ground calculates $L_g = v \times t_g$.
7. You might think that $L_g$ would be the "proper" length because it was calculated with the proper time, but no. The *proper length* is defined as the length of the object measured in *that object's own frame of reference* (which makes sense when you think about it, since this is the only way of defining it uniquely).
8. We want to find the multiplier which will take us from the proper lentgh $L_0 = L_s$ to the other length $L_g$.
9. So we want: $v\dfrac{t_g}{\sqrt{1 - \frac{v^2}{c^2}}} \times (?) = vt_g$
10. Cancel $vt_g$ from both sides and it's obvious that we have to multiply by $\sqrt{1 - \dfrac{v^2}{c^2}}$.
11. So: $L = L_0\sqrt{1 - \dfrac{v^2}{c^2}}$.

## Summary

1. The *proper time between two events*, $t_0$, is the time measured by an observer who is present at both events.
2. The *proper length of an object*, $L_0$, is the length as measured in the object's own frame of reference.
3. $t = \dfrac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$.
4. $L = L_0\sqrt{1 - \dfrac{v^2}{c^2}}$.