Topic 21: Light Waves and Optics

Advanced Placement Physics

Dr. Timothy Leung

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Olympiads School
Toronto, Ontario, Canada

Huygens

Huygens' Principle

In the 1600's there were two competing theories of light...

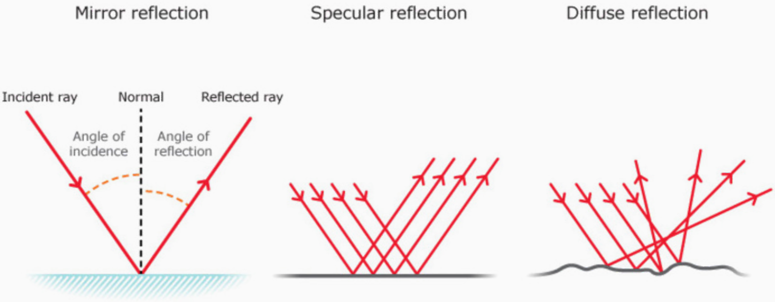
- Some, including Issac Newton, believed that light is a particle
- Others, including Christiaan Huygen (Dutch) and Augustin-Jean Fresnel (French), believed that light is a wave

Huygen's Princple: all waves are in fact an infinite series of circular wavelets

Reflection

Reflection of Light

In the **law of reflection**, the incident ray, the reflected ray, and the normal to the surface of the mirror all lie in the same plane, and the angle of reflection is equal to the angle of incidence



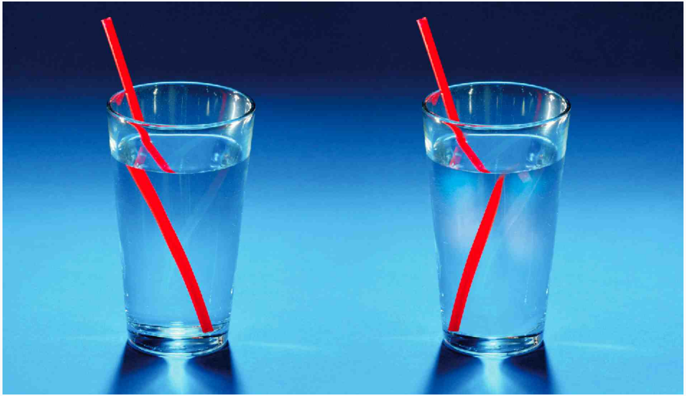
Specular Reflection Example

This photo of Lake Matheson shows specular reflection in the water of the lake with reflected images of Aoraki/Mt Cook (left) and Mt Tasman (right). The very still lake water provides a perfectly smooth surface for this to occur.

Refraction

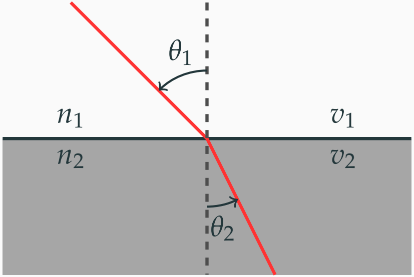
Refraction of Light Through a Medium

- When a wave enters another medium, the wave speed changes
- When entering at an angle, the change of speed causes the wave to change direction (e.g. from air to water, air to glass, glass to air etc)
- The amount of bending depends on the **indices of refraction of the two media**
- Responsible for **image formation** by lenses and the eye



Refraction of Light Through a Medium

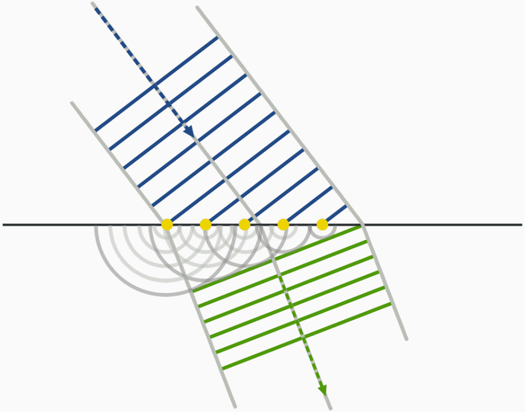
Light entering from one medium to another at an angle results in refraction In the diagram, light could be going in either direction, from top to bottom (n_1 to n_2) or from bottom to top (n_2 to n_1)



Snell's law relates the refractive indices n of the two media to the directions of propagation in terms of the angles θ to the normal

$$\boxed{n_1 \sin\theta_1 = n_2 \sin\theta_2}$$

Refraction and Huygens Principle



We can explain the refraction phenomenon using Huygens' Principle

Index of Refraction

Index of refraction (n) (or **refractive index**, or just **index**) is the ratio of the speed of light in a vacuum (c_0) to the speed of light in the medium (c):

$$\boxed{n = \frac{c_0}{c} = \frac{\lambda_{\text{vacuum}}}{\lambda}}$$

When light enters a second medium, the *frequency* remains unchanged but since the speed changes, the *wavelength* also changes:

$$\boxed{\frac{n_1}{n_2} = \frac{\lambda_2}{\lambda_1}}$$

You can work this out using the univesal wave equation: $v = f\lambda$

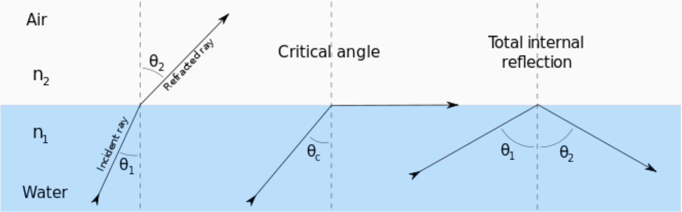
Index of Refraction of Common Materials

Material	n	Material	n
Vacuum	1	Ethanol	1.362
Air	1.000277	Glycerine	1.473
Water at 20 °C	1.33	Ice	1.31
Carbon disulfide	1.63	Polystyrene	1.59
Methylene iodide	1.74	Crown glass	1.50-1.62
Diamond	2.417	Flint glass	1.57-1.75

The values given are *approximate* and do not account for the small variation of index with light wavelength. That's called **dispersion**.

Total Internal Reflection

Total internal reflection ("TIR") occurs when light enters from a medium with high index to another medium with low index (i.e. $n_1 > n_2$). Snell's law still holds, but something weird can happen:

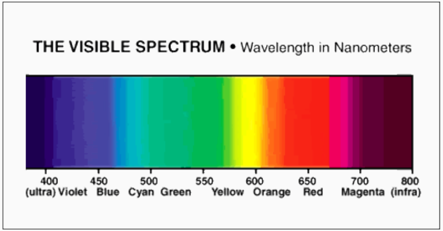


Critical angle θ_c for water-air interface is $48.6°$. TIR occurs when the incident angle is greater $\theta_1 > \theta_c$.

Dispersion

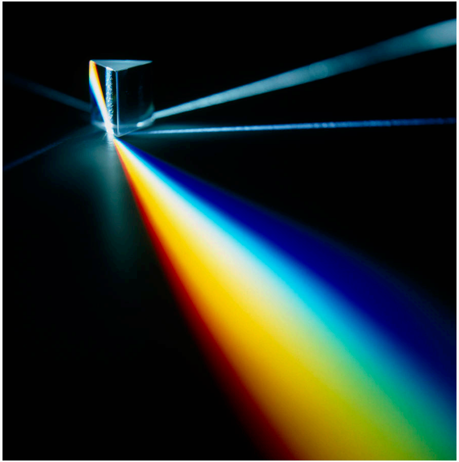
Color of Light and Wavelength

Human eyes perceive different frequencies of light as different colors. The visible spectrum of light:



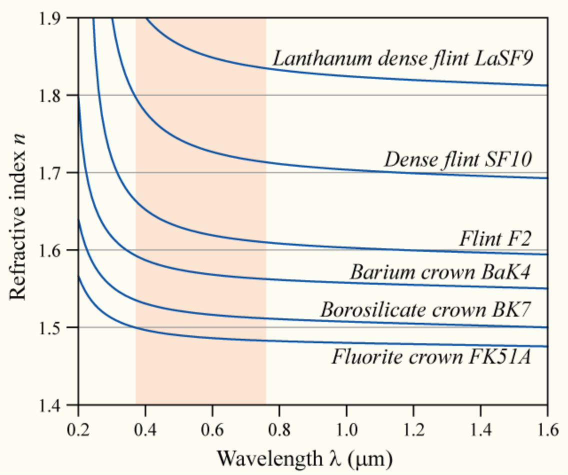
- The *color* of the light depends on its frequency (& wavelength when it's in a vacuum)
- *White light* is light that contains waves in all frequencies.

Dispersion of Light Through Refraction



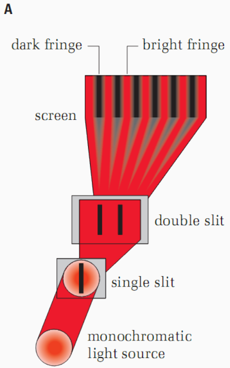
- When white light passes through a prism it is separated into different colors (spectrum) through refraction.
- This is because the index of refraction n is slightly different for different wavelengths
- Otherwise, we will never see a rainbow

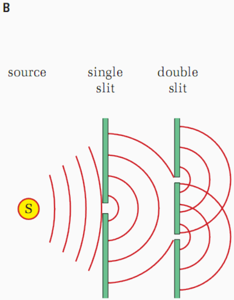
Wavelength Dependency of Index of Refraction



Interference

Thomas Young's Double-Slit Experiment

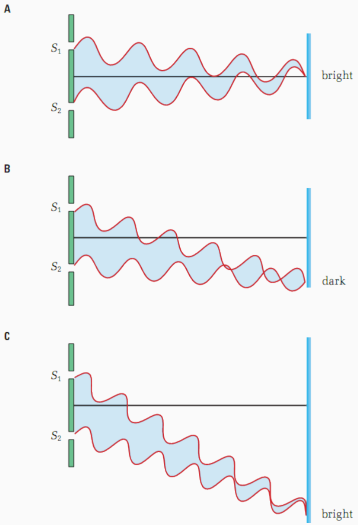




- **Monochromatic light** light with a single color (frequency); the light source can be a laser, LED , or gas lamp (most likely what Young used)
- **Slit:** an opening; also called an **aperture**
- The **screen** far away from the slits is also called the **projection**

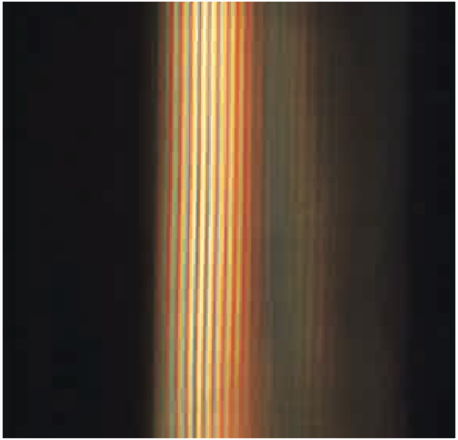
Double-slit experiment showed that light causes interference, just like any other wave

Thomas Young's Double-Slit Experiment



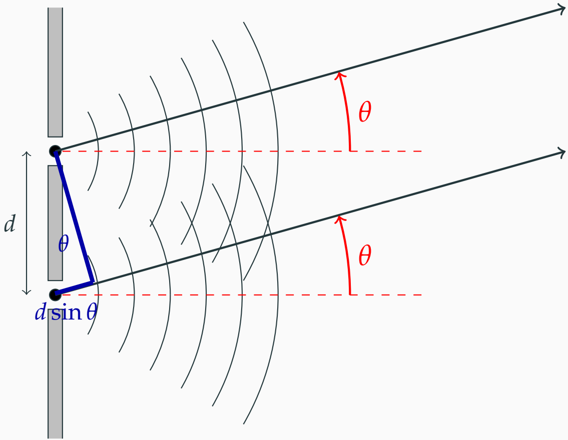
- At **A**, the path from slits S_1 and S_2 are the same, therefore we have **constructive interference** and the projection is bright
- At **B**, the path from S_1 and S_2 are diffed by half a wavelength, and therefore there is **destructive interference** and the projection is dark
- At **C**, the path from S_1 and S_2 are diffed by one wavelength, and therefore there is **constructive interference** again, and again, the projection is bright

Interference Pattern: Bright and Dark Fringes



The “bright fringes” are from constructive interference; the “dark fringes” are from destructive interference.

Let's Work This Out!



- We have two slits at distance d apart, emitting *coherent* light
- Huygens' Principle: light passing through the slits become point sources
- Assume that the projection (screen) is far enough from the slits that we can treat the two beams of light from the slits as being parallel
- Using basic geometry, we can see that the path difference from the two slit to the projection is $d \sin \theta$

Contructive Interference with Double-Slits

A bright fringe (constructive interference) happens if the path difference ($d \sin \theta$) is an integer (n) multiple of wavelength (λ):

$$\boxed{n\lambda = d \sin \theta_n}$$

where $n = 0, 1, 2, 3 \ldots$

Quantity	Symbol	SI Unit
Integer number of full wavelengths	n	(none)
Wavelength of light	λ	m
Distance between slits	d	m
Deflection angle	θ	(unit less)

The last fringe $n_{\max}$ can be found by setting $\sin \theta = 1$. Therefore the total nubmer of bright fringes N:

$$N = 2n_{\max} + 1$$

Destructive Interference with Double-Slits

Conversely, dark fringes (destructive interference) occurs when the path difference ($d \sin \theta$) is an half-number ($n + \frac{1}{2}$) multiple of wavelength (λ):

$$\boxed{\left(n + \frac{1}{2}\right) \lambda = d \sin \theta_n}$$

where $n = 0, 1, 2, 3 \ldots$. Again, the last dark fringe $n_{\max}$ can be found by setting $\sin \theta = 1$, and the total nubmer of dark fringes N:

$$N = 2n_{\max}$$

Approximation of The Wavelength of Light

We can actually estimate the wavelength of light based on the distances between bright fringes, by applying the **small-angle approximation**:

$$\theta \approx \tan\theta \approx \sin\theta$$

We can already relate the distance from slits to the screen (x), and the distance of the n-th bright fringe from the centre (y_n) to θ_n. Applying the approximation, we have:

$$\tan\theta_n = \frac{y_n}{x} \approx \sin\theta_n$$

Substitute this approximation into the constructive interference equation:

$$n\lambda \approx \frac{y_n d}{x} \quad \longrightarrow \quad \boxed{\lambda \approx \frac{\Delta y d}{x}}$$

Approximation of The Wavelength of Light

This equation applies equally to dark fringes (nodal lines) as well as bright fringes.

$$\boxed{\lambda \approx \frac{\Delta y d}{x}}$$

Quantity	Symbol	SI Unit
Wavelength	λ	m
Distance between fringes	Δy	m
Distance between slits	d	m
Distance from source to screen	x	m

Since the approximation is based on small angles, we must apply this equation to where Δy close to the centre, where light from both slits are deflected by a small angle.

Important Notes

- We have applied the double-slit problem specifically to light, but it can be applied to any mechanical wave (e.g. sound waves, ocean waves) as well
- The sources don't actually need to be slits; any point source will do
- The projection/screen doesn't need to be a real screen either; it just has to be a line where wave intensity can be measured

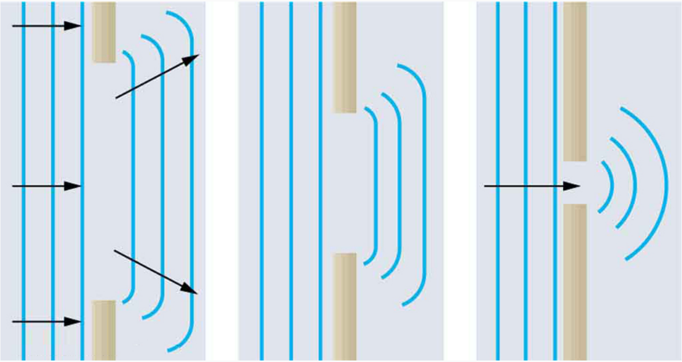
Diffraction

Diffraction of Waves

When a wave goes through an small opening, it **diffracts**. This happens with sound waves, ocean waves...and light.

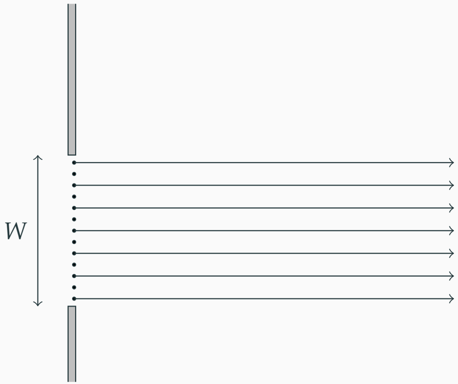
The photo is from the Port of Alexandria in Egypt. The shape of the entire harbor is created because of diffraction of ocean wave.

Diffraction of Waves



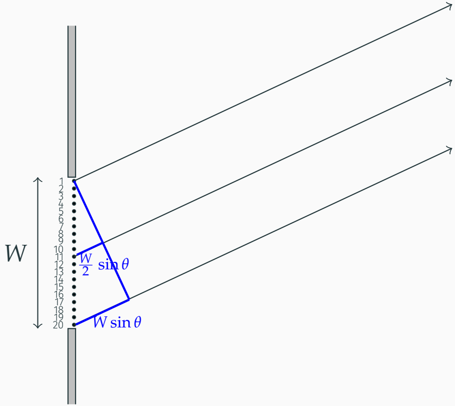
The smaller the opening (compared to the wavelength of the incoming wave) the greater the diffraction effects.

Equations for Diffraction



- Similar to the double-slit problem, we apply Huygens' Principle again
- This time we treat the slit as wide enough that there is a series (an infinite series, actually) of point waves at the slit
- We can easily see that the light from the wavelet that travel perpendicular to the slit (aperture) will not interfere with one another
- i.e. a bright fringe at the middle. **This is called the central maximum.**

At Some Other Angle θ



- Like what we did with double-slit, we can find the path difference between the wavelet on the top (1) and bottom (20): $W \sin\theta$
- At some θ, the path difference between 1 and 20 will be an integer multiple of the wavelength ($m\lambda$)
- In this case, the path difference between 1 and 11 is a half-number multiple of the wavelength (i.e. destructive interference) and they cancel each other
- Similarly, 2 cancels 12, 3 cancels 13...

RESULT: COMPLETE DESTRUCTIVE INTERFERENCE

Dark Fringes: Destructive Interference

Dark fringes exists on the screen at regular, whole-numbered intervals ($m = 1, 2, 3 \ldots$):

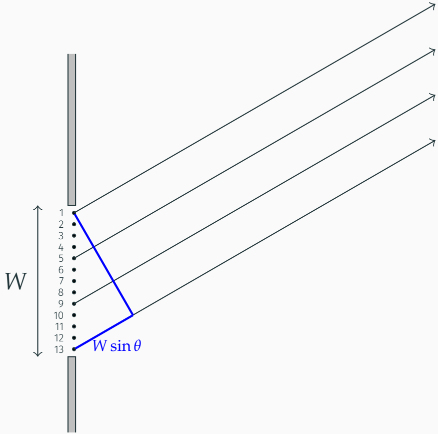
$$\boxed{m\lambda = W \sin\theta_m}$$

Applying the small-angle approximation equation, we end up with:

$$\boxed{y_m = \frac{m\lambda L}{W}}$$

This equation looks very similar to the double-slit equation for *bright* fringes, so be *very* careful when you use them!

At Some Other Angle θ



- Again, we follow what we did with the the previous case, and we find that at some angle θ, the path difference between the top and bottom is $W \sin \theta = \frac{3}{2}\lambda$
- Beam from (1) and (5) differ by $\frac{\lambda}{2}$, so they have destructive interference; similarly 2 and 6, 3 and 7, 4 and 8, 9 and 13 will all interfere destructively
- But some of the beams will not, so we have a bright fringe at the projection
- This bright fringe is not as bright as the central one because of the destructive interference

Bright Fringes: Constructive Interference

Bright fringes exist on the screen at regular, half-numbered intervals ($m = 1, 2, 3 \ldots$):

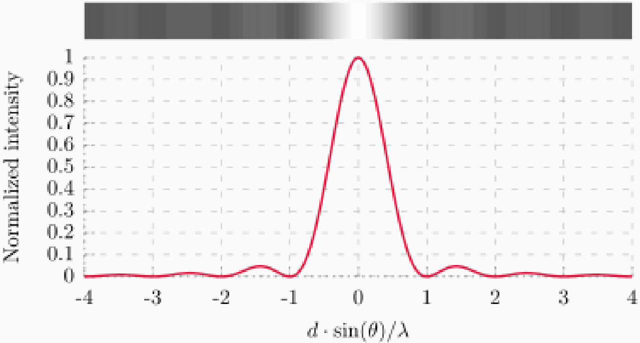
$$\boxed{\left(m + \frac{1}{2}\right) \lambda = W \sin \theta_m}$$

Again, similar to the dark fringes, we apply our small-angle approximation equation:

$$\boxed{y_m = \left(m + \frac{1}{2}\right) \frac{\lambda L}{W}}$$

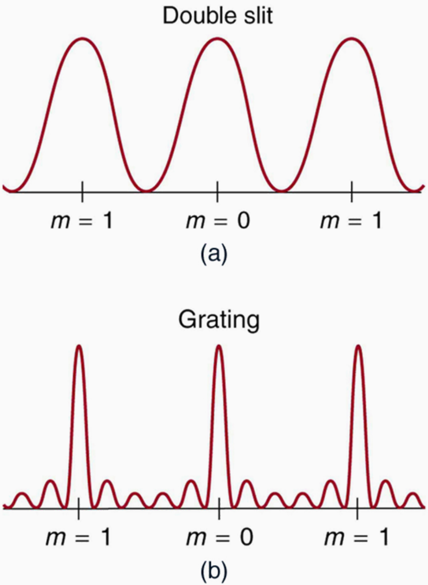
Single-Slit Diffraction, A Summary

- Similar to the double-slit interference, single-slit diffraction projects a series of alternating bright fringes ("maxima") and dark fringes ("minima") in the far field
- The bright fringe in the middle ("central maximum") is twice as wide and very bright
- Subsequent bright fringes on either side ("higher-order maxima") are much dimmer because of the partial destructive interference



Grating

Diffraction Grating: What if there are more than 2 slits?



- We can apply the same analysis from double-slit to a diffraction grating
- Use equation for double-slit interference to locate bright fringes

$$n\lambda = d \sin \theta_n$$

- Interference pattern is sharper
- Bright fringes are narrower

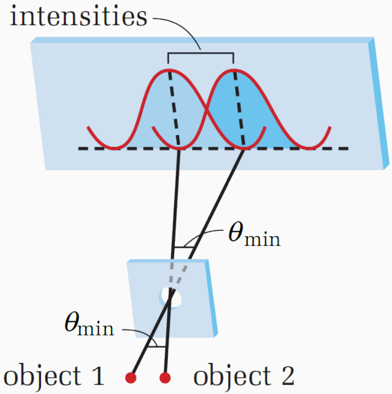
Applications

Optical Resolution

The ability of an optical instrument (e.g. the human eye, microscope, camera) to distinguish two distinct objects

When light from any object passes through an “optical instrument”, it **diffracts**, therefore “blurring” the object

Optical Resolution



Rayleigh limit: Two objects are resolved if the angle $\theta > \theta_{\min}$, where $\theta_{\min}$ is when the first minimum (dark fringe) from object 1 overlaps with the central maximum (bright fringe in the middle) from object 2.

Resolving Power

To resolve two objects, the minimum angle between rays from the two objects passing through an aperture is given by: D of the aperture.

Rectangular aperture:

$$\boxed{\theta_{\text{min}} = \frac{\lambda}{W}}$$

Circular aperture:

$$\boxed{\theta_{\text{min}} = \frac{1.22\lambda}{D}}$$

where W is the width of the aperture, and D is the diameter of the aperture. The angle θ_{min} is measured in **radians**.