Class 22a: Special Relativity

Advanced Placement Physics

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Introduction

The slides on special relativity is a condensed version of the slides used for Physics 12. For some of you, this will be a review.

Newtonian (Classical) Relativity

- In Newtonian physics, space and time are absolute:
  - 1 m is 1 m no matter where you are in the universe
  - 1 s is 1 s no matter where you are in the universe
  - 1 kg is 1 kg no matter where you are in the universe
- Space and time are absolute, therefore velocities are relative: *all measured velocities depend on the observer's relative motion*
- Our kinematic and dynamic equations applies to all *inertial* frames of reference

Maxwell's Equations in a Vacuum

Everything Comes Back to This

$$\nabla \cdot \mathbf{E} = 0$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \mu_o \varepsilon_o \frac{\partial \mathbf{E}}{\partial t}$$

Disturbances in $\mathbf{E}$ and $\mathbf{B}$ travel as an "electromagnetic wave", with a speed:

$$c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} = 299\,792\,458\ \mathrm{m/s}$$

Peculiar features of Maxwell’s equation

- Makes no mention of the *medium* in which EM waves travels
- When applying *Galilean transformation* (classical equation for calculating *relative velocity*) to Maxwell’s equations, they fail
- Gauss’s law for magnetism break down: magnetic field lines appear to have beginnings/ends
- In *some* inertial frames of reference, Maxwell’s equations are valid and elegant, but in another inertial frame of reference, they are ugly
- Physicists at the time theorized that—perhaps—there is/are actually *preferred* inertial frame(s) of references
- This violate the long-standing *principle of relativity*, which says that *the laws of physics are equal in all inertial frames of reference*

Making The Equations Work Again

Maxwell's equations didn't "fail"; it was our understanding of space and time that needed to change

- Albert Einstein believed in the principle of relativity, and rejected the concept of a preferred frame of reference
- The speed of an electromagnetic wave (speed of light) must be independent of the frame of reference
- The failure of the Michelson-Morley experiment which sought to experimentally demonstrate the flow of "ether" merely
- In order to make the equations to work again, Einstein revisited two most fundamental concepts in physics: space and time

Einstein's Postulates on Relativity

The Principle of Relativity

All laws of physics must apply equally in all inertial frames of reference.

The Principle of Invariant Light Speed

As measured in any inertial frame of reference, light is always propagated in empty space with a definite velocity c that is independent of the state of motion of the emitting body.

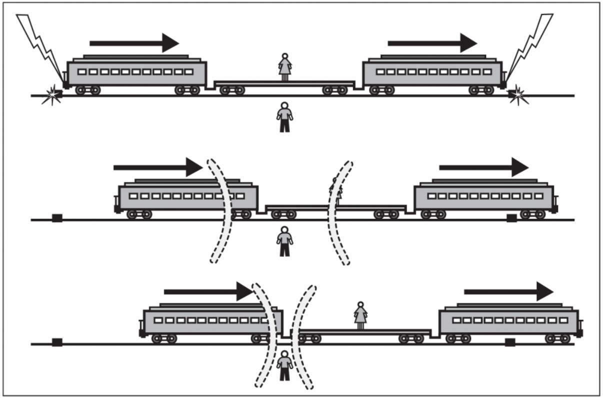
Published in 1905 in the article *On the Electrodynamics of Moving Bodies* when Einstein was 26 years old working as a patent clerk in Switzerland

What's so Special About Special Relativity?

- **Classical (Newtonian) relativity:** space and time are absolute—speed of light must be relative to the observer
- **Einstein relativity:** speed of light is absolute—space and time must be relative to the observer
- We must modify our traditional concepts:
    - Measurement of space (our coordinate system)
    - Measurement of time (our clock)
    - Concept of simultaneity (whether or not two events happens at the same time)

Simultaneity: Thought Experiment

Lightning bolt strikes the ends of a moving train



- The man on the ground sees the lightning bolt striking at the same time
- The woman on the moving train sees the lightning bolt on the right first

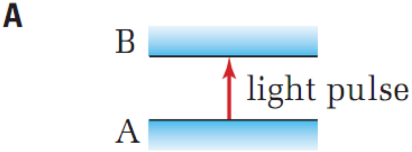
Simultaneity

- Both observers cannot agree on the result, but
  - Neither person is wrong
  - Neither person is misinformed
- Both are valid inertial frames of reference

This means that:

- Simultaneity depends on your motion
- **Events that are simultaneous in one inertial frame of reference are generally not simultaneous in another.**

Time Dilation: A Thought Experiment

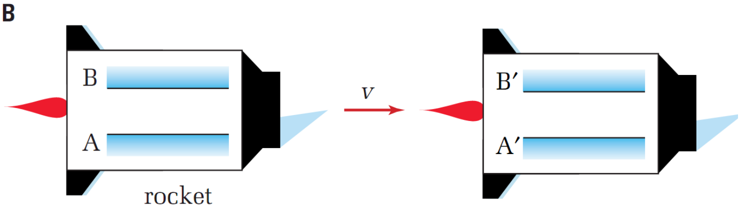


I'm on a spaceship travelling in deep space, and I shine a light from A to B. The distance between A and B is really just:

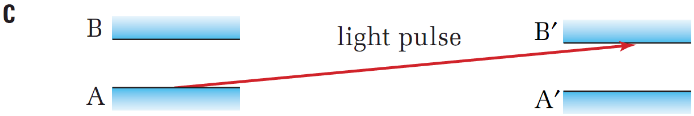
$$|AB| = c\Delta t_0$$

I know the speed of light c, and I know how long it took for the light pulse to reach B. (The reason I used Δt_0 will be obvious later.)

Time Dilation: A Thought Experiment

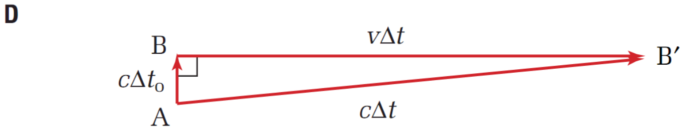


You are in space station watching my spaceship go past you at speed v. You would see that same beam of light travel from A to B' instead.



Abandoning Concept of Absolute Time: Time Dilation

A "thought experiment"



$$c^2\Delta t^2 = v^2\Delta t^2 + c^2\Delta t_0^2$$

$$\left(c^2 - v^2\right)\Delta t^2 = c^2\Delta t_0^2$$

$$\left(1 - \frac{v^2}{c^2}\right)\Delta t^2 = \Delta t_0^2$$

$$\Delta t = \frac{\Delta t_0}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$$

Time Dilation

$$\boxed{\Delta t = \frac{\Delta t_0}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}}$$

- Δt_0 is called the **proper time**. It is the time measured by a person at rest relative to the object or event.
- Δt is called the expanded time or **dilated time**. It is the time measured by a moving observer in another inertial frame of reference. Since $\sqrt{1 - \left(\frac{v}{c}\right)^2}$ is always smaller than 1, Δt is always greater than Δt_0.

Example Problem (A Simple One)

Example 1a: A rocket speeds past an asteroid at $0.800c$. If an observer in the rocket sees $10.0\,\mathrm{s}$ pass on her watch, how long would that time interval be as seen by an observer on the asteroid?

Example Problem (A Simple One)

Example 1a: A rocket speeds past an asteroid at $0.800c$. If an observer in the rocket sees $10.0\,\mathrm{s}$ pass on her watch, how long would that time interval be as seen by an observer on the asteroid?

Example 1b: A rocket speeds past an asteroid at $0.800c$. If an observer in the *asteroid* sees $10.0\,\mathrm{s}$ pass on his watch, how long would that time interval be as seen by an observer on the *rocket*?

Example Problem (A Simple One)

Example 1a: A rocket speeds past an asteroid at $0.800c$. If an observer in the rocket sees 10.0 s pass on her watch, how long would that time interval be as seen by an observer on the asteroid?

Example 1b: A rocket speeds past an asteroid at $0.800c$. If an observer in the *asteroid* sees 10.0 s pass on his watch, how long would that time interval be as seen by an observer on the *rocket*?

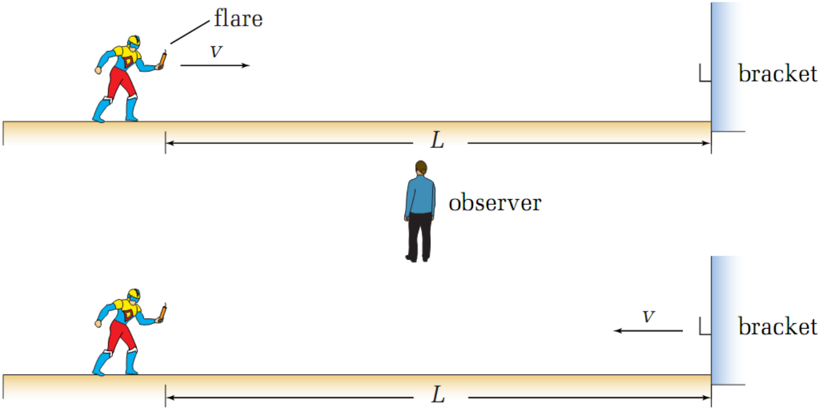
How can that be?!

Length Contraction

Captain Quick is a comic book hero who can run at nearly the speed of light. In his hand, he is carrying a flare with a lit fuse set to explode in 1.5 µs. The flare must be placed into its bracket before this happens. The distance (L) between the flare and the bracket is 402 m.

Length Contraction

Another Example



Length Contraction

If Captain Quick runs at 2.00×10^8 m/s, according to classical mechanics, he will not make it in time:

$$\Delta t = \frac{L}{v} = \frac{402\,\mathrm{m}}{2.00 \times 10^8\,\mathrm{m/s}} = 2.01 \times 10^{-6}\,\mathrm{s} = 2.01\,\mu\mathrm{s}$$

But according to relativistic mechanics, he makes it just in time...

Length Contraction

To a stationary observer, the time on the flare is slowed:

$$\Delta t = \frac{\Delta t_0}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} = \frac{1.5 \times 10^{-6}\,\mathrm{s}}{\sqrt{1 - \left(\frac{2}{3}\right)^2}} = \frac{1.5 \times 10^{-6}\,\mathrm{s}}{0.7454} = 2.01 \times 10^{-6}\,\mathrm{s}$$

The stationary observer sees a passage of time of $\Delta t = 2.01\,\mu\mathrm{s}$, but Captain Quick, who is in the same reference frame as the flare, experiences a passage of time of $\Delta t_0 = 1.50\,\mu\mathrm{s}$, precisely the time for the flare to explode.

Length Contraction

- So, if Captain Quick sees only $\Delta t_0 = 1.50\,\mu\text{s}$, then how far did he travel?
- He isn't travelling any faster, so he only other possibility is that **the distance actually got shorter** (in his frame of reference).
- How much did the distance contract?

$$\boxed{L = L_0\sqrt{1 - \left(\frac{v}{c}\right)^2}}$$

For this example:

$$L = L_0\sqrt{1 - \left(\frac{v}{c}\right)^2} = (402\,\text{m}) \cdot \sqrt{1 - \left(\frac{2}{3}\right)^2} = 300\,\text{m}$$

Lorentz Factor

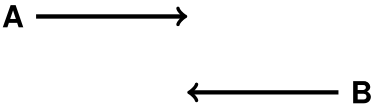
The **Lorentz factor** γ is a short-hand for writing length contraction, time dilation and relativistic mass:

$$\boxed{\gamma = \frac{1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}}$$

Then time dilation and length contraction can be written simply as:

$$\boxed{\Delta t = \gamma \Delta t_o} \qquad \boxed{L = \frac{L_o}{\gamma}}$$

Summary



If Person A and Person B are moving at constant speed with respect to one another (doesn't matter if they're moving towards, or away from each other)

- They cannot agree whether any events happens at the same time or not
- Each sees the other's clock running slow
- Each sees the other "contracted" in length along the direction of motion

Relativistic Momentum

In Unit 2, you were taught that momentum is mass times velocity. And back in Physics 11, you were taught that velocity is displacement over time:

$$\mathbf{p} = m\mathbf{v} = m\frac{d\mathbf{x}}{dt}$$

Now that you know both $\mathbf{x}$ and t depends on the motion, we can find the "relativistic version" of momentum for when $\mathbf{v}$ is high compared to c):

$$\mathbf{p} = m_0\frac{d\mathbf{x}}{dt_0} = \frac{m_0 d\mathbf{x}}{\sqrt{1-\left(\frac{v}{c}\right)^2}dt} = \frac{m_0\mathbf{v}}{\sqrt{1-\left(\frac{v}{c}\right)^2}}$$

Relativistic Mass

Based on the momentum equation, we can see that mass is also relativistic:

$$\boxed{m = \frac{m_0}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}}$$

Mass increases with velocity. At $v = c$, $m = \infty$! But is it *real*?

Force and Work

Knowing the relationship between force and momentum, and the definition of work by a force:

$$\mathbf{F} = \frac{d\mathbf{p}}{dt} \qquad W = \int \mathbf{F} \cdot d\mathbf{x}$$

Substitute the expression for force into the definition of work, then substitute expression for relativistic momentum, and after some calculus, we get an expression for kinetic energy:

$$W = \Delta K = \frac{m_0 c^2}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} - m_o c^2$$

Relativistic Energy

$$\boxed{K = mc^2 - m_0c^2}$$

where

- m_0 = rest mass = mass as measured in a stationary frame of reference
- m = relativistic mass = $m_0 / \sqrt{1 - \left(\frac{v}{c}\right)^2}$
- K = kinetic energy
- c = speed of light

Relativistic Energy

$$\boxed{K = mc^2 - m_0c^2}$$

- An object of mass m has energy $E_0 = m_0c^2$ even when it is not moving (this is called *rest energy*
- If it *is* moving, then it has a *total* energy of $E_T = mc^2$
- Kinetic energy K is the difference between total energy and rest energy
- Whenever there is a change of energy, there is also a change of mass
  - "Conservation of mass" and "conservation of energy" must be combined into a single concept of **conservation of mass-energy**

Kinetic Energy: Classical vs. Relativistic

Relativistic:

$$K = \frac{m_0 c^2}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} - m_o c^2$$

Newtonian:

$$K = \frac{1}{2} m v^2$$

But are they really different? If we do a series expansion of the square-root term, we get:

$$K = m_0 c^2 \left(1 + \frac{1}{2}\frac{v^2}{c^2} + \cdots\right) - m_0 c^2 \approx \frac{1}{2} m v^2 + \cdots$$

When v is small compared to c

Comparing Classical and Relativistic Energy

Classical mechanics:

$$K = \frac{1}{2}mv^2$$

Relativistic mechanics:

$$K = mc^2 - m_o c^2$$