Topic 18: Heat Capacity

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1 Heat Capacity

The **heat capacity** C of an object is defined to be the amount of heat needed to raise its temperature by 1 K, i.e.:

$$C = \frac{Q}{\Delta T} \tag{1}$$

Of course, this depends on *what substance* the objects is made of, and *how much* of the sustance (i.e. its mass) there is in the object. Therefore, a better quantity to use is the **specific heat capacity** c,[1] which is defined as:

$$c = \frac{C}{m} = \frac{Q}{m\Delta T} \tag{2}$$

Note that when heat is added to a system, by the first law of thermodynamics, the internal energy can change (ΔU), and mechanical work W may also be done as well.[2] so Equation 1 is best expressed as:

$$C = \frac{\Delta U - W}{\Delta T} \tag{3}$$

Since mechanical work can be *anything* while heat is added, C is not necessarily a well defined property. In this case, we examine two situations:

- Heat capacity at constant volume (C_V)
- Heat capacity at constant pressure (C_P)

2 Heat Capacity at Constant Volume

At constant volume, no work is being done, and therefore all the heat added to the thermodynamic system goes to the internal energy of the system. Therefore, we define the **heat capacity at constant volume** as:

$$C_V = \left(\frac{\Delta U}{\Delta T}\right)_V = \left(\frac{\partial U}{\partial T}\right)_V \tag{4}$$

The subscript in the symbol C_V indicates that the partial derivative is taken at constant volume V.

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[1]Note that the symbol for heat capacity is in *uppercase*, while the symbol for specific heat capacity is in lowecase

[2]In this handout, we assume that mechanical work is *positive* when it is done from the surrounding *to* the system, and *negative* when it is done *by* the system to the surroundings

3 Heat Capacity at Constant Pressure

In everyday life, objects usually *expand* as they are heated, pushing against the atmosphere at constant pressure as it expands. The work done by the system to the surrounding is therfore negative:

$$W = -P\Delta V \tag{5}$$

And subsequently, the Equation 3 can now be expressed for the **heat capacity at constant pressure**:

$$C_P = \frac{\Delta U - W}{\Delta T} = \frac{\Delta U - (-P\Delta T)}{\Delta T} = \boxed{\left(\frac{\partial U}{\partial T}\right)_P + P\left(\frac{\partial V}{\partial T}\right)_P} \tag{6}$$