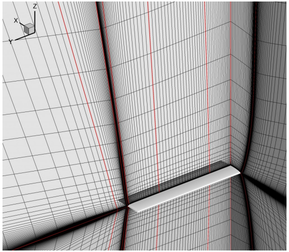
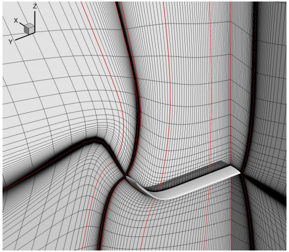
WELCOME TO AP PHYSICS

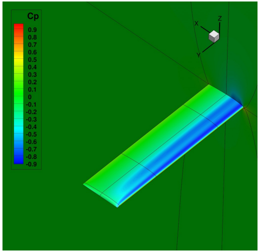
Hi, My Name is Tim

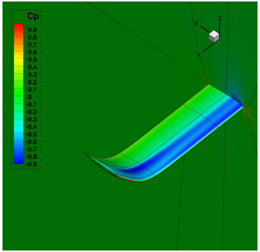
- B.A.Sc. in Engineering Physics (UBC)
  - Won the Roy Nodwell Prize for my design of a solar car
- M.A.Sc. and Ph.D. in Aerospace Engineering (UTIAS)
  - “Computational Fluid Dynamics” (CFD)
  - “Aerodynamic shape optimization”
  - Aircraft design
- Also spent a year in Vancouver as a professional violinist. . .

Tim’s Past Research Work









Classroom Rules

If you have been in my Grade 11 and 12 classes before, the rules are the same:

- Treat each other with respect, and I'll treat you like an adult.
- If you need to go to the bathroom, you don't need my permission
- Raise your hands if you have a question. Don't wait too long
- E-mail me at `tim@timleungjr.ca` for any questions related to physics and math and engineering
- Do ***not*** try to find me on social media

1. Kinematics, With Calculus

Advanced Placement Physics

Dr. Timothy Leung

Olympiads School

Fall 2017

Pre-requisites

- **Physics 11 and 12** As AP Physics is primarily taught at the first-year university level, you wil need to be comfortable with the topics covered in high-school physics courses.
- **Calculus** The AP Physics C exams are calculus based, and you will be required to perform basic differentiation and integration. You don't need to be a caclulus expert, but some basic knowledge is required. The differentiation and integration in the course are generally not difficult, but there are occassional challenges.
- **Vectors** You need to be comfortable with vector operations, including addition and subtraction, multiplication and division by constants, as well as dot products and cross products.

The AP Physics Exams

- Offered in May of each year.
- There are 4 AP Physics courses:
  - Physics 1
  - Physics 2
  - Physics C–Mechanics
  - Physics C–Electricity and Magnetism

Files for You to Download

- `00-courseOutline.pdf`—The course outline
- `01-Calculus-2x2.pdf`—The slides that I am using right now
- `02-Homework.pdf`—The homework assignment for Topics 1 (Kinematics) and 2 (Dynamics). The homework assignment is due at *the class after we have finished Topic 2*.

Please download/print the PDF file for the class slides before each class. There is no point copying notes that are already printed out for you. Instead, take notes on things I say that aren't necessarily on the slides.

Writing Vectors

In Physics 11 and 12, vectors are generally written by separating the magnitude from the direction, e.g. a velocity vector can be written as:

$$\mathbf{v} = 4.5\,\mathrm{m/s}\ [\mathrm{N}\ 55^\circ\ \mathrm{E}]$$

This format is intuitive for describing *one* vector in 2D, but extending into 3D space, and performing vector operations are difficult. Instead, vectors in this class are generally written in components using the “ijk notation”, like this:

Dot Products

Cross Products

Calculus is Invented for Physics

Thanks, Issac Newton

- We cannot learn physics properly without calculus (you have got away with it long enough in Grade 11 and 12 Physics classes. . .)
- Differential calculus was “invented” so that we can understand motion, especially on non-constant velocity and acceleration.
- You may have already noticed that a lot of the word problems in calculus are really physics problems

Differentiation and Integration

- **Differential Calculus**
  - Finding how quickly something is changing (“rate of change” of a quantity)
  - Math: slopes of functions
  - Physics: how quickly a physical quantity is changing in time and/or space
  - Examples: velocity (how quickly position changes), acceleration (how quickly velocity changes), power (how quickly work is done)
- **Integral Calculus**
  - The opposite of differentiation
  - We use it to compute the area under a curve, or
  - Summation of many small terms
  - Examples: area under the $\mathbf{v}$-t graph (displacement), area under the $\mathbf{F}$-t graph (impulse), area under the F-d graph (work)

Derivative

For any arbitrary function $f(x)$, the derivative with respect to (“w.r.t.”) x is:

$$\boxed{f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}}$$

The “limit as h approaches 0” is the mathematical way of making h a very small number

Know the Tricks for Differentiaion

The derivative of a constant ("C") with respect to any variable is zero. (This is obvious, since the slope of the function $f(x) = C$ is zero.)

$$\frac{dC}{dt} = 0$$

A constant multiple of any function f can be factored outside the derivative:

$$\frac{d}{dx}(af) = a\frac{df}{dx}$$

The derivative of a sum is the sum of a derivative:

$$\frac{d}{dt}(f(t) + g(t)) = \frac{df}{dt} + \frac{dg}{dt}$$

Time-Saving Rules for Differentiation

Power Rule:

$$\frac{d}{dt}\left(t^n\right) = nt^{n-1} \quad \text{for} \quad n \neq 0$$

Sines and cosines:

$$\frac{d}{dt}\sin t = \cos t$$
$$\frac{d}{dt}\cos t = -\sin t$$

Partial Derivatives

Common Integrals in Physics

Integrating a function can be a very daunting task (even though it's often necessary), but the integral you'll see in AP Physics are relatively straightforward. These examples should help in most cases:

- Power Rule in reverse:

$$\int x^n dx = \frac{1}{n+1} x^{n+1} + C$$

- Natural logarithm:

$$\int \frac{1}{x} dx = \ln |x| + C$$

We can "ignore" (i.e. cancel) the constant of integration C for definite integrals.

Common Integrals in Physics

- Sines and cosines:

$$\int \cos x dx = \sin x + C$$
$$\int \sin x dx = -\cos x + C$$

We can “ignore” (i.e. cancel) the constant of integration C for definite integrals.

Definite vs. Indefinite Integral

This Should Be a Review

- Integral can be either **indefinite** or **definite**
- An "indefinite" integral is another function, e.g. position $\mathbf{s}(t)$ as a function of time is found by integrating velocity $\mathbf{v}(t)$:

$$\mathbf{s}(t) = \int \mathbf{v}(t)dt = \cdots + \mathbf{C}$$

- A "constant of integration" $\mathbf{C}$ is added to the integral $\mathbf{s}(t)$. It is obtained through applying "initial condition" to the problem.

Definite Integrals

A **definite** intgral has lower and upper bounds. e.g., given $\mathbf{v}(t)$, the displacement between t_1 and t_2 can be found:

$$\Delta\mathbf{s} = \int_{t_1}^{t_2} \mathbf{v}(t)dt$$

Once we have computed the integral, we have to evaluate between the limits:

$$\Delta\mathbf{s} = \mathbf{s}(t)\Big|_{t_1}^{t_2} = \mathbf{s}(t_2) - \mathbf{s}(t_1)$$

We do not have to bother with the constant of integration $\mathbf{C}$, since it cancels when we evaluate the upper and lower bound.

Instantaneous Velocity

Time Derivative of Position

The *instantaneous* velocity of an object is the time derivative of its position:

$$\boxed{\mathbf{v}(t) = \frac{d\mathbf{s}(t)}{dt}}$$

Since position $\mathbf{s}$ has x, y and z components in the $\hat{\imath}$, $\hat{\jmath}$ directions, and $\hat{k}$ directions, the velocity is therefore:

$$\mathbf{v}(t) = \frac{d\mathbf{s}}{dt} = \frac{ds_x}{dt}\hat{\boldsymbol{\imath}} + \frac{ds_y}{dt}\hat{\boldsymbol{\jmath}} + \frac{ds_z}{dt}\hat{\boldsymbol{k}}$$

s_x, s_y and s_z can be functions of time and of x, y and z coordinates as well.

Integrating Velocity to Get Position/Displacement

If velocity is the time derivative of position, then position is the time integral of velocity (fundamental theorem of calculus):

$$\mathbf{s}(t) = \int \mathbf{v}(t)dt + S_0$$

As both $\mathbf{s}$ and $\mathbf{v}$ are vectors, we need to integrate in each direction:

$$\mathbf{s}(t) = \left(\int v_x \hat{\boldsymbol{\imath}} + \int v_y \hat{\boldsymbol{\jmath}} + \int v_z \hat{\mathbf{k}} \right) dt$$

Instantaneous Acceleration

In the same way that velocity is the time derivative of position, **acceleration is the time derivative of velocity**, i.e.:

$$\boxed{\mathbf{a}(t) = \frac{d\mathbf{v}(t)}{dt} = \frac{d^2\mathbf{s}(t)}{dt^2}}$$

Acceleration is the second derivative of position, i.e.

1. Take derivative of $\mathbf{s}(t)$ to get $\mathbf{v}(t) = \mathbf{s}'(t)$
2. Take derivative again of $\mathbf{v}(t)$ to get $\mathbf{a}(t) = \mathbf{v}'(t)$

Integrating Acceleration to Get Velocity

Similar to the relationship between velocity and position, we also know that velocity $\mathbf{v}(t)$ is the time integral of acceleration $\mathbf{a}(t)$:

$$\mathbf{v}(t) = \int \mathbf{a}(t)dt$$

Again, since both $\mathbf{v}$ and $\mathbf{a}$ are vectors, we need to integrate in each direction:

$$\mathbf{v}(t) = \left(\int a_x\hat{\boldsymbol{\imath}} + \int a_y\hat{\boldsymbol{\jmath}} + \int a_z\hat{\mathbf{k}} \right) dt$$

Kinematic Equations For Constant Acceleration

Even though *some* of the problems require calculus, these kinematic equations are still a very powerful tool, as there will be constant acceleration in many cases.

$$\Delta \mathbf{s} = \mathbf{v}_1 \Delta t + \frac{1}{2}\mathbf{a}\Delta t^2$$

$$\Delta \mathbf{s} = \mathbf{v}_2 \Delta t - \frac{1}{2}\mathbf{a}\Delta t^2$$

$$\Delta \mathbf{s} = \frac{\mathbf{v}_1 + \mathbf{v}_2}{2}\Delta t$$

$$\mathbf{v}_2 = \mathbf{v}_1 + \mathbf{a}\Delta t$$

$$v_2^2 = v_1^2 + 2a\Delta d$$

Solving Kinematic Problems

The variables of interests are:

$$\Delta \mathbf{s} \quad \mathbf{v}_1 \quad \mathbf{v}_2 \quad \Delta t \quad \mathbf{a}$$

- For single-object problems, you are usually given 3 of the 5 variables, and you are asked to find a 4th one
- For two-object problems, the motion of the two objects are connected by time interval Δt and displacement $\Delta \mathbf{s}$