# Classes 19: Fluid Mechanics

## AP Physics

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March 2018

# Files for You to Download

Download from the school website:

1. `19-fluidMechanics.pdf`—This presentation. If you want to print the slides on paper, I recommend printing 4 slides per page.
2. `20-Homework.pdf`—Homework assignment for Classes 19 and 20, which cover Fluid Mechanics and Thermodynamics

Please download/print the PDF file before each class. When you are taking notes, pay particular attention to things I say that aren't necessarily on the slides.

# Disclaimer

## Use of Calculus

Fluid mechanics is part of the AP Physics 2 Exam, which does not require calculus. However, in the interest in completeness, *some* calculus will still be used when deriving equations.

# What is a Fluid

- **The simplistic explanstion:** anything that flows
- **The scientific explanation:** Any substancs that deform *continuously* under oblique stress

# Properties of Fluids

## Density

# Continuity

A fluid is considered to be continuous in space.

# Properties of Fluids

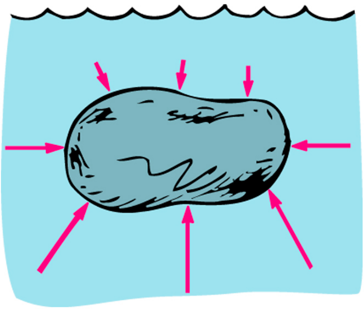
## Viscosity

# Hydrostatics

# Buoyancy

## Everything Floats a Little

When an object is submerged inside a fluid (e.g. water, air, etc), the fluid exerts a pressure at the surface of the object. We can integrate the pressure over the entire surface area and find the total force the fluid exerts on the object.



# Derivation of Buoyance Force

We can integrate the pressure over the entire surface to find the total force, or take some knowledge of vector calculus (divergence theorem):

$$\mathbf{B} = -\oint_S p\mathbf{n}\,dS = -\iiint \nabla p\,dV$$

Since pressure is given by $p = \rho g z$—a function in $z$ only—the gradient easy to compute: $\nabla p = \rho g \hat{\mathbf{k}}$, giving us

$$\mathbf{B} = \rho_{\text{fluid}} g \hat{\mathbf{k}} \iiint dV = \rho_{\text{fluid}} g V \hat{\mathbf{k}}$$

# Derivation of Buoyance Force

Although the derivation required a lot of calculus, the expression of buoyance force is *very* straightforward:

$$\boxed{\mathbf{B} = \rho_{\text{fluid}} g V \hat{\mathbf{k}} = m_{\text{fluid}} g \hat{\mathbf{k}}}$$

where $\rho_{\text{fluid}}$ is the density of the displaced fluid, and $V$ is the volume displaced. This equation is known as **Archimedes' principle**.

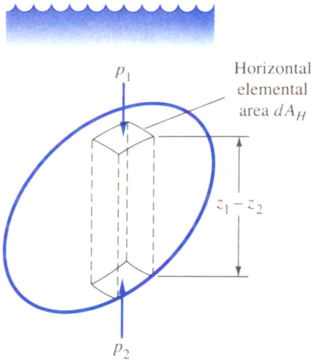
**Buoyance force has a magnitude that equals to the weight of the fluid displaced by the submerged object, pointing upward.**

# An Easier Explanation of Buoyancy

## Not Much Calculus

There is a simpler way to find the buoyance force, by taking an infinitesimal "tube" of the object, and finding the pressure difference between the top and bottom of the tube:

$$\mathbf{B} = \int (p_2 - p_1) dA$$
$$= \rho g \int (z_2 - z_1) dA$$
$$= \rho g V$$




which is the same expression that we got with calculus.

# Buoyancy

Note that buoyancy does not depend on:

- the mass of the immersed object, or
- the density of the immersed object

Objects immersed in a fluid have an “apparent weight” that is reduced by the buoyance force:

$$\mathbf{W} = \mathbf{W} - \mathbf{B}$$

$$\mathbf{W} = (\rho_{\text{obj}} - \rho_{\text{fluid}})\mathbf{g}V$$

$\mathbf{W}'$ is proportional to the relative density $(\rho' = \rho_{\text{obj}} - \rho_{\text{fluid}})$

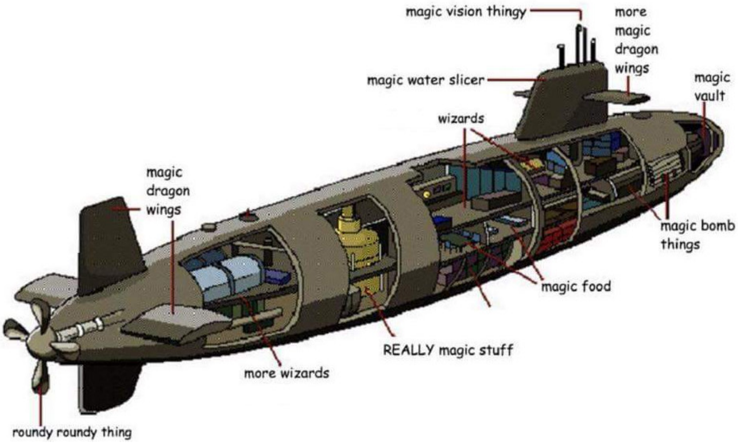
# Buoyancy

For a submerged object:

| Densities | $B > W_{\text{obj}}$ | $B = W_{\text{obj}}$ | $B < W_{\text{obj}}$ |
|---|---|---|---|
| $\rho_{\text{obj}} < \rho_{\text{fluid}}$ | object rises | float on surface | |
| $\rho_{\text{obj}} = \rho_{\text{fluid}}$ | | neutral buoyancy | |
| $\rho_{\text{obj}} > \rho_{\text{fluid}}$ | | | object sinks |

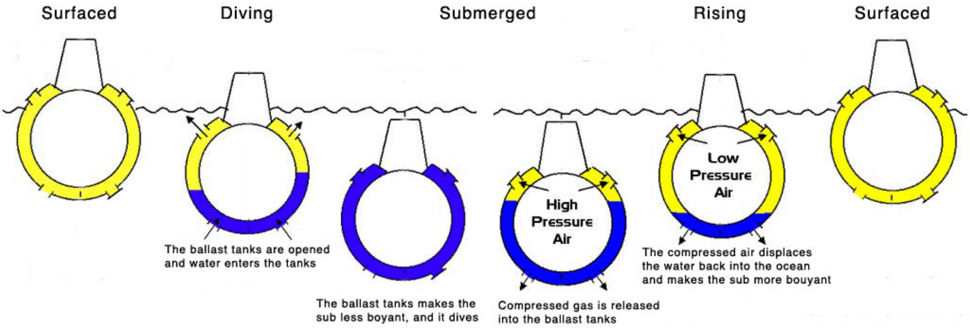
# How Submarines Work

## Like this?



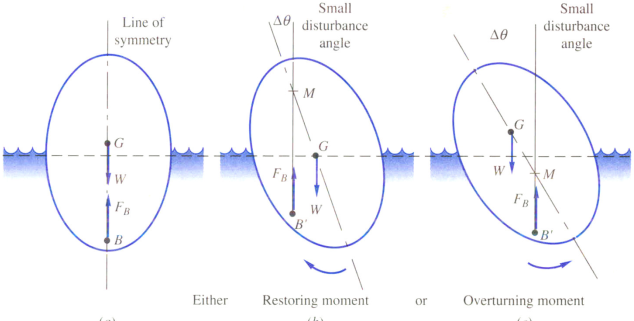
# How Submarines Work

Like most ships, a submarine does not naturally sink because of the buoyance force. When a submarine submerges, water needed to be pumped inside “ballast tanks” to make the ship heavier.



# Stable? Or unstable?

- Buoyance force $\mathbf{B}$ acts at the *center of buoyancy* (CB) of the submerged object
  - The CB is the CG *if the object has constant density*
  - The actual CG of the object may be at a different position
  - Sometimes the object is not fully submerged
- Therefore $\mathbf{F}_g$ and $\mathbf{B}$ may act at different points, creating a torque/moment on the object



# Fluid Flow

Flow of fluid out of a surface requires us to look at the flux function again: Volume flux is defined as:

$$\boxed{\Phi_{\mathrm{V}} = \int \mathbf{V} \cdot d\mathbf{A}}$$

where $\mathbf{V}$ is the velocity (vector field) at the surface, and $d\mathbf{A}$ is the infinitesimal area pointing **outwards.** We can also expressed volume flux using the outward normal unit vector $\hat{\mathbf{n}}$:

$$\boxed{\Phi_{\mathrm{V}} = \int \mathbf{V} \cdot \hat{\mathbf{n}} dA}$$

# Bernoulli Equation

$$\boxed{p_1 + \frac{1}{2}\rho v_1^2 + \rho g z_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g z_2}$$

The term $\frac{1}{2}\rho v^2$ is called “dynamic pressure”

# Bernoulli Equation

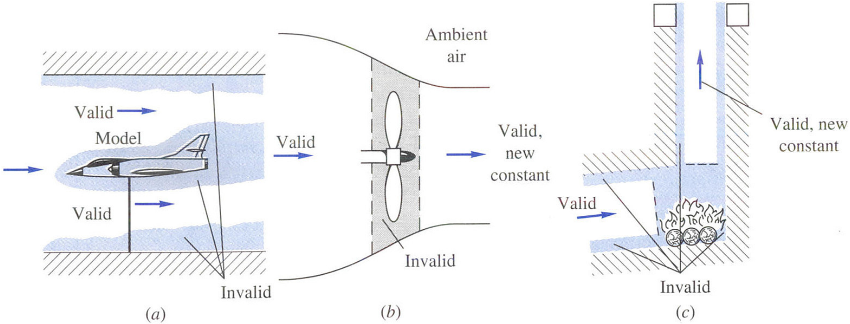
$$\boxed{p_1 + \frac{1}{2}\rho v_1^2 + \rho g z_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g z_2}$$

Bernoulli's equation is valid when

- the flow is **steady** (independent of time)
- the flow is **incompressible**–compressibility (i.e. changes in density of the fluid) effects are negligible for Mach number $M < 0.30$
- the flow **along a single streamline**
- there is **no shaft work** done along the streamline between 1 and 2
- there is **no heat transfer** along the streamline between 1 and 2

# Bernoulli Equation

Regions where Bernoulli equation is valid:



# How Does A Wing Work?

When air flows past a wing, a force is generated