Topic 23: Special Relativity

Advanced Placement Physics

Dr. Timothy Leung

Olympiads School

Summer 2018

Introduction

These slides for this unit are an expanded version of Grade 12 Physics slides (with some additional calculus). Two versions of the slides are downloadable from the school website:

- The long version
  - More background information (more than needed for the AP 2 Exam) and derivations and integrations
  - May answer some of your questions about the specifics of the theory
  - `23a-relativity_long.pdf`
- The short version
  - More "to the point"
  - The version that is used during class
  - `23a-relativity_short.pdf`

There is also a handout on how to solve and interpret the time dilation problem

Frame of Reference

A Quick Review

- A **frame of reference** is a hypothetical, perfect, mobile “laboratory” an observer uses to make measurements (mass, lengths, time, etc). At a minimum, it includes:
    - A ruler to measure lengths
    - A clock to measure the passage of time
    - A scale to compare forces
    - A balance to measure masses
- What matters is the *motion* (at rest, uniform motion, acceleration etc) of your laboratory, and how it affects the measurement that you make
- An **inertial frame of reference** is one that is moving in uniform motion

The Principle of Relativity

The laws of motion must apply equally in all inertial frames of reference.

Newtonian (Classical) Relativity

In Newtonian physics, space and time are *absolute*:

- $1\,\mathrm{m}$ is $1\,\mathrm{m}$ no matter where you are in the universe
- $1\,\mathrm{s}$ is $1\,\mathrm{s}$ no matter where you are in the universe
- Measurements of space and time do not depend on motion

If space and time are absolute, then *all* velocities are relative

- Measured velocities depend on the motion of the observer

Maxwell's Equations in a Vacuum

Everything Comes Back to This

$$\nabla \cdot \mathbf{E} = 0$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

Disturbances in $\mathbf{E}$ and $\mathbf{B}$ travel as an "electromagnetic wave", with a speed:

$$c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} = 299\,792\,458\,\mathrm{m/s}$$

Maxwell's Equations

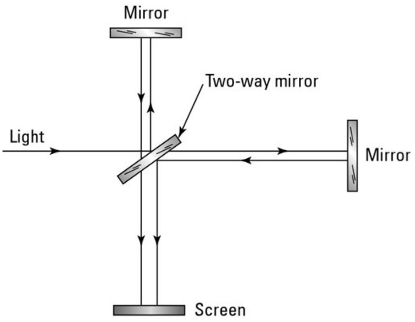
- Does not mention the *medium* in which EM waves travels
- When applying *Galilean transformation* (classical equation for adding velocities) to Maxwell's equations, asymmetry is introduced
    - Gauss's law for magnetism break down: magnetic field lines appear to have beginnings/ends
    - In *some* inertial frames of reference, Maxwell's equations are simple and elegant, but in another inertial frame of reference, they are ugly and complex
- Physicists at the time theorized that—perhaps—there is/are actually *preferred* inertial frame(s) of references
    - This violates the *principle of relativity*

The Illusive Aether

- Maxwell's hypothesis: the speed of light c_0 is relative to a hypothetical "luminiferous aether"
- In order for this "aether" (or "ether") to exist, it must have some fantastic (as in, a fantasy, too good to be true!) properties:
  - *All* space is filled with aether
  - Massless
  - Zero viscosity
  - Non-dispersive
  - Incompressible
  - Continuous at a very small (sub-atomic) scale

The Michelson-Morley Experiment

The Michelson-Morley experiement attempted to measure the flow of ether by observing how the speed of light would change.



- A beam of light is split into two using a two-way (half-silvered) mirror
- The two beams are reflected off mirrors and finally arriving at the screen where interference patterns are observed
- The two paths are the same length, so if the *speed* of the light changes, we should see an interference pattern
- **Except none were ever found!**

Making Maxwell's Equations Work

Einstein in 1905

- Albert Einstein was 26, working as a patent clerk in Switzerland
  - believed in the principle of relativity, and therefore
  - rejected the concept of a preferred frame of reference
- The failure of the Michelson-Morley experiment to find the flow of ether proves that it does not exist
- In order to make Maxwell's equations to work again, Einstein revisited two most fundamental concepts in physics: *space* and *time*

Published in the journal *Annalen der Physik* on September 26, 1905 in the article *On the Electrodynamics of Moving Bodies*

Postulates of Special Relativity

The Principle of Relativity

All laws of physics must apply equally in all inertial frames of reference.

- Reaffirms the principle in which all physics is based on
- Extend the principle to include electrodynamics

The Principle of Invariant Light Speed

As measured in any inertial frame of reference, light is always propagated in empty space with a definite velocity c that is independent of the state of motion of the emitting body.

- Reaffirms the results from Michelson-Morley experiment
- Discounts the existances of the hypothetical ether

What's so Special About Special Relativity?

Classical (Newtonian) relativity:

- Space and time are absolute (invariant), therefore
- The speed of light must be relative to the observer

Einstein's special relativity:

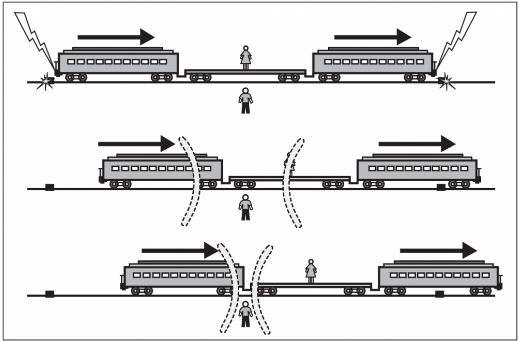
- The speed of light is absolute (invariant), therefore
- Space and time must be relative to the observer

We must modify our traditional concepts:

- Measurement of space (our ruler in the x-, y- and z-directions)
- Measurement of time (our clock)
- Concept of simultaneity (whether two events happens at the same time)

Simultaneity: A Thought Experiment

Lightning bolt strikes the ends of a moving train



- The man on the ground sees the lightning bolt striking at the same time
- The woman on the moving train sees the lightning bolt on the right first

Simultaneity: A Thought Experiment

- The two observers disagree on the result, but
  - Neither person is wrong
  - Neither person is misinformed
- Both observers are valid *inertial* frames of reference
- This means that simultaneity depends on your motion

Events that are simultaneous in one inertial frame of reference are not simultaneous in another.

Time Dilation

A Moving Clock Runs Slow

Time on a moving object, as perceived by a stationary observer, appears to slow down:

$$\boxed{t' = \frac{t}{\sqrt{1-\left(\frac{v}{c}\right)^2}} = \gamma t}$$

- t is called the **proper time**. It is the time measured by a person at rest relative to the object or event.
- t' is called the **ordinary time**, **expanded time**, or **dilated time**. It is the time measured by a moving observer in another inertial frame of reference.
- Since $\sqrt{1-\left(\frac{v}{c}\right)^2}$ is always smaller than 1, t' is always greater than t.

Example Problem

This example problem can show clearly whether you have a handle on time dilation problem:

Example 1a: A rocket speeds past an asteroid at $0.600c$. If an observer in the rocket sees $10.0\,\mathrm{s}$ pass on her watch, how long would that time interval be as seen by an observer on the asteroid?

Example Problem

This example problem can show clearly whether you have a handle on time dilation problem:

Example 1a: A rocket speeds past an asteroid at $0.600c$. If an observer in the rocket sees $10.0\,\mathrm{s}$ pass on her watch, how long would that time interval be as seen by an observer on the asteroid?

Example 1b: A rocket speeds past an asteroid at $0.600c$. If an observer in the *asteroid* sees $10.0\,\mathrm{s}$ pass on his watch, how long would that time interval be as seen by an observer on the *rocket*?

Example Problem

This example problem can show clearly whether you have a handle on time dilation problem:

Example 1a: A rocket speeds past an asteroid at $0.600c$. If an observer in the rocket sees $10.0\,\mathrm{s}$ pass on her watch, how long would that time interval be as seen by an observer on the asteroid?

Example 1b: A rocket speeds past an asteroid at $0.600c$. If an observer in the *asteroid* sees $10.0\,\mathrm{s}$ pass on his watch, how long would that time interval be as seen by an observer on the *rocket*?

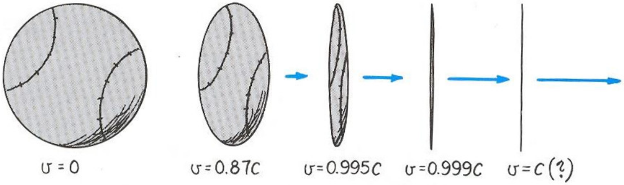
How can that be?!

Length Contraction

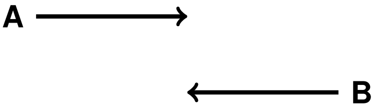
The length of an object, as measured by a moving observer, is contracted in length:

$$\boxed{L' = L\sqrt{1 - \left(\frac{v}{c}\right)^2} = \frac{L}{\gamma}}$$

This length contraction only occurs along the direction of motion



Let’s Summarize



If Person A and Person B are moving at constant velocity relative to one another (doesn’t matter if they’re moving towards, or away from each other)

- They cannot agree whether any events happens at the same time or not
- Each sees the other’s clock running slow
- Each sees the other “contracted” in length along the direction of motion

Example Problem

Example 2: A spacecraft passes Earth at a speed of $2.00 \times 10^8\,\mathrm{m/s}$. If observers on Earth measure the length of the spacecraft to be $554\,\mathrm{m}$, how long would it be according to its passengers?

Lorentz Transformation

The equations for time dilation and length contraction only tell part of the story. In order to account for the loss of simultaneity from one reference frame to another, we need to use the **Lorentz transformation:**

$$x' = \gamma(x - vt)$$

$$y' = y$$

$$z' = z$$

$$t' = \gamma\left(t - \frac{vx}{c^2}\right)$$

The Lorentz transformation "solves" many paradoxes (e.g. the twin paradox) from the time-dilation and length-contraction equations, but aren't really there.

Lorentz Transformation

For slow speeds $v \ll c$, Lorentz transformation reduces to the Galilean transformation from classical mechanics.

$$x' = x - vt$$
$$y' = y$$
$$z' = z$$
$$t' = t'$$

Relative Velocity

- Unlike in classical mechanics, velocities (speeds) do not simply add
- If the observer in frame S measures an object moving along the x-axis at velocity u, then the observer in the referenfe frame S that is moving at velocity v in the x-direction with respect to S, will measure the object moving with velocity u:

$$u' = \frac{dx'}{dt'} = \frac{\gamma(dx - vdt)}{\gamma(dt - vdx/c^2)} = \frac{(dx/dt) - v}{1 - (v/c^2)(dx/dt)} = \frac{}{1}$$

- The other frame S will measure:

$$u = \frac{dx}{dt} = \frac{\gamma(dx' + vdt')}{\gamma(dt' + vdx'/c^2)} = \frac{(dx'/dt') + v}{1 + (v/c^2)(dx'/dt')} =$$

Relativistic Momentum

The definition of momentum has not changed, but in relativistic momentum, we must now include the effects of time dilation and/or length contraction:

$$\boxed{\mathbf{p} = \frac{m\mathbf{v}}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} = \gamma m \mathbf{v}}$$

Relativistic Mass

Relativistic momentum equation show that there is a relativistic aspect to mass. The apparent mass m' as measured by a moving observer is related to its rest mass (intrinsic mass, invariant mass) m by the Lorentz factor:

$$\boxed{m' = \frac{m}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} = \gamma m}$$

The intrinsic mass has not increased, but a moving observer will see the object behave as if it is more massive. As $v \to c$, $m \to \infty$.

Work and Energy

Recall the definition of work. **This definition has not changed.**

$$W = \int \mathbf{F} \cdot d\mathbf{x} = \int \frac{d\mathbf{p}}{dt} \cdot \mathbf{dx}$$

Since we now have a relativistic expression for momentum, we substitute that new expression into the expression for force, and then integrate.

Doing the integral gives a surprisingly simple expression for kinetic energy K:

$$W = \gamma mc^2 - mc^2$$

Work and Energy

We know from the work-kinetic energy theorem that the work W done is equal to the change in kinetic energy K, therefore

$$\boxed{K = m'c^2 - mc^2}$$

Variable	Symbol	SI Unit
Kinetic energy of an object	K	J
Relativistic mass (measured in moving frame)	m'	kg
Rest mass (measured in stationary frame)	m	kg
Speed of light	c_0	m/s

The full integration is shown in the long version of the slides. The integral is not very complicated, but it does require some experience with intgral calculus.

Relativistic Energy

What This All Means

$$\boxed{K = m'c^2 - mc^2}$$

- **rest energy**:

$$E_0 = mc^2$$

- **total energy**:

$$E_T = m'c^2 = \gamma mc^2$$

- Kinetic energy is the difference between total energy and rest energy:

$$K = E_T - E_0$$

Relativistic Energy

What This All Means

$$\boxed{E = mc^2}$$

Mass-energy equivalence:

- Whenever there is a change of energy, there is also a change of mass
- “Conservation of mass” and “conservation of energy” must be combined into a single concept of **conservation of mass-energy**
- Mass-energy equivalence doesn’t merely mean that mass can be converted into energy, and vice versa (although this is true), but rather, one can be converted into the other **because they are fundamentally the same thing**

Example Problem

Example 3: An electron has a rest mass of 9.11×10^{-31} kg. In a detector, it behaves as if it has a mass of 12.55×10^{-31} kg. How fast is that electron moving relative to the detector?

Energy-Momentum Relation

The **energy-momentum relation** relates an object's rest (intrinsic) mass m, total energy E, and momentum p:

$$\boxed{E^2 = p^2c^2 + m^2c^4}$$

Quantity	Symbol	SI Unit
Total energy	E	J (joules)
Momentum	p	kgm/s (kilogram meters per second)
Rest mass	m	kg (kilogram)
Speed of light	c	m/s (meter per second)

The derivation of this expression is shown in the long version of the slides.

Kinetic Energy–Classical vs. Relativistic

Relativistic:

$$K = \frac{mc^2}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} - mc^2$$

Newtonian:

$$K = \frac{1}{2}mv^2$$

If space and time are indeed relative quantities, then the relativistic equation for K must apply to all velocities

Kinetic Energy—Classical vs. Relativistic

Applying the **binomial series expansion** to γ gives a series representation of kinetic energy K:

$$K = mc^2\left(1 + \frac{1}{2}\frac{v^2}{c^2} + \frac{3}{8}\frac{v^4}{c^4} + \cdots\right) - mc^2$$
$$\approx \frac{1}{2}mv^2 + \frac{3mv^4}{8c^2} + \cdots$$

For $v \ll c$, we can ignore the high-order terms. The leading term reduces to the Newtonian expression.

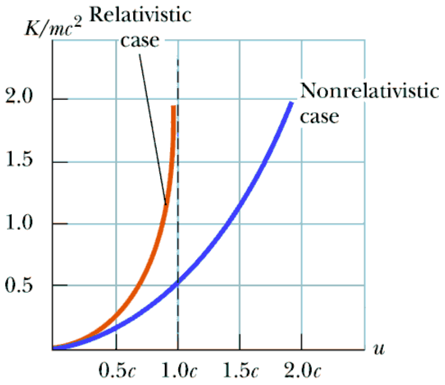
Comparing Classical and Relativistic Energy

In classical mechanics:

$$K = \frac{1}{2}mv^2$$

In relativistic mechanics:

$$K = \gamma mc^2 - mc^2$$



© 2006 Brooks/Cole - Thomson

The classical expression is accurate for speeds up to $v \approx 0.3c$.

Example Problem

Example 4: A rocket car with a mass of 2.00×10^3 kg is accelerated from rest to 1.00×10^8 m/s. Calculate its kinetic energy:

1. Using the classical equation
2. Using the relativistic equation