AP PHYSICS 1 & C: WORK AND ENERGY

Directions: Each of the questions or incomplete statements below is followed by five suggested answers or completions. Select the one that is best in each case and place the letter of your choice in the corresponding box on the student answer sheet.

Note: To simplify calculations, you may use $g = 10\,\mathrm{m/s^2}$ in all problems.

1. A 1 kg ball is thrown vertically downward from a 50-meter-high tower with an initial speed of 4 m/s. Just before striking the ground, the speed of the ball is 20 m/s. The energy lost to air friction is most nearly

   $U_g + K + W_{nc} = K'$

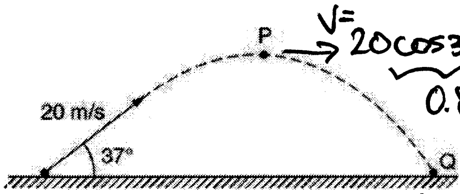
   $mgh + \frac{1}{2}mv^2 + W_{nc} = \frac{1}{2}mv'^2$

   $(10)(50) + \frac{1}{2}(4)^2 + W_{nc} = \frac{1}{2}(20)^2$

   $W_{nc} = -308\,J$

   (A) 101 J
   (B) 210 J
   (C) 308 J
   (D) 406 J
   (E) 508 J

Questions 2–3. A 2 kg projectile is launched with a speed of 20 m/s from horizontal ground at an angle of 37° to the horizontal as shown. Point P is at the top of the path, and point Q is at the end of the path, just before the projectile again reaches the ground.



$V = 20\cos 37° = 16$

0.8

$V = 20\,m/s$

2. The kinetic energy of the projectile at point P is

   $\frac{1}{2}mv^2$

   (A) 108 J
   (B) 225 J
   (C) 256 J
   (D) 400 J
   (E) 525 J

3. The kinetic energy of the projectile at point Q is

   (A) 108 J
   (B) 225 J
   (C) 256 J
   (D) 400 J
   (E) 525 J

4. If a projectile thrown directly upward reaches a maximum height h and spends a total time in the air of T, the average power of the gravitational force during the trajectory is

   (A) $P = 2mgh/T$
   (B) $P = -2mgh/T$
   (C) 0
   (D) $P = mgh/T$
   (E) $P = -mgh/T$

   no work done.

5. Given that the constant net force on an object and the object's displacement, which of the following quantities can be calculated?

   (A) the net change in the object's velocity
   (B) the net change in the object's mechanical energy
   (C) the average acceleration
   (D) the net change in the object's kinetic energy
   (E) the net change in the object's potential energy

   not if you don't know mass

   Work energy theorem

   only if it's conservative

6. The force acting on an object varies with the equation $F(x) = -3x^2 - 2x - 4$, where force is in newtons and displacement is in meters. The potential energy at $x = 2$ m is

   (A) zero
   (B) 20 J
   (C) 40 J
   (D) −20 J
   (E) −40 J

7. If the only force acting on an object is given by the equation $F(x) = 2 - 4x$ (where the force is measured in newtons and position in meters), what is the change in the object's kinetic energy as it moves from $x = 2$ to $x = 1$?

   $\int_2^1 F(x)dx = \int_2^1 (2-4x)dx$

   (A) +4 J
   (B) −4 J
   (C) +2 J
   (D) −2 J
   (E) +8 J

8. The potential energy of an object varies with the equation $U(x) = 2x^2 + x - 6$, where force is in newtons and displacement is in meters. A force F vs. displacement x graph would yield which of the following?

   (A) A straight, horizontal line
   (B) A parabola
   (C) An exponential decay curve
   (D) A straight line with a positive slope
   (E) A straight line with a negative slope

$F = -\frac{dU}{dx}$
$= -4x - 1$

9. A particle of mass m moves according to the displacement equation $x = 2t^{5/2}$. The kinetic energy of the particle as a function of time is

   (A) $10mt^{5/2}$
   (B) $10mt^{3/2}$
   (C) $\frac{25}{2}mt^3$
   (D) $5mt^2$
   (E) $2mt^{3/2}$

$\frac{25}{2}mt^3$

$V = \frac{dx}{dt} = \left(\frac{5}{2}\right)2t^{3/2} = 5t^{3/2}$

$K = \frac{1}{2}mv^2 = \frac{1}{2}m\left(5t^{3/2}\right)^2$

10. An electron travels in a circle around a hydrogen nucleus at a very high speed. The work done by the electrostatic force acting on the electron after one complete revolution is

   (A) zero
   (B) positive
   (C) negative
   (D) equal to the kinetic energy of the electron
   (E) equal to the potential energy of the electron

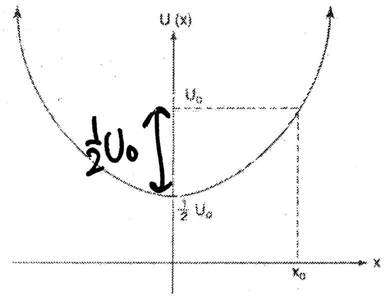
force $\perp$ velocity.

11. An object is moved from rest at point P to rest at point Q in a gravitational field. The net work against the gravitational field depends on the

   (A) mass of the object and the positions of P and Q
   (B) mass of the object only
   (C) positions of P and Q only
   (D) length moved between points P and Q
   (E) coefficient of friction

gravitational force is conservative, i.e: path independent

Questions 12–13. Consider the potential energy function shown below.



12. Assuming that no non-conservative forces are present, if a particle of mass m is released from position x_0, what is the maximum speed it will achieve?

   (A) $\sqrt{4U_0/m}$
   (B) $\sqrt{2U_0/m}$
   (C) $\sqrt{U_0/m}$
   (D) $\sqrt{U_0/2m}$
   (E) The particle will achieve no maximum speed but instead will continue to accelerate indefinitely.

$\frac{1}{2}U_0 = \frac{1}{2}mv^2$

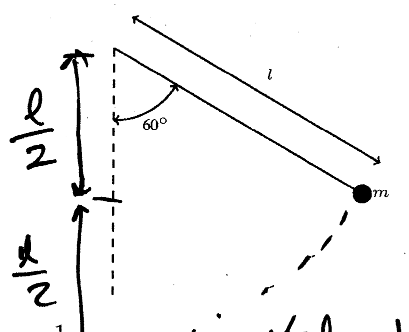
$v = \sqrt{\frac{U_0}{m}}$

13. Which of the following is the most accurate description of the system introduced in the previous question?

   (A) stable equilibrium
   (B) unstable equilibrium
   (C) neutral equilibrium
   (D) a bound system
   (E) There is a linear restoring force

equilibrium only if K=0 at the bottom, which is not the case for this problem.

14. A pendulum bob of mass m is released from rest as shown in the figure below. What is the tension in the string as the pendulum swings through the lowest point of its motion?



$\cancel{m}g\frac{l}{2}=\frac{1}{2}\cancel{m}v^2$

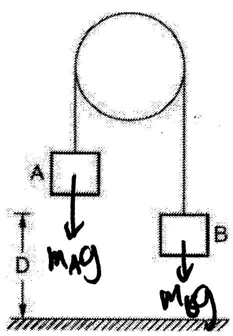
$v^2=gl$

(A) $T=\frac{1}{2}mg$
(B) $T=mg$
(C) $T=\frac{3}{2}mg$
(D) $T=2mg$
(E) None of the above

$T-mg=F_c$ $\quad F_c=\frac{mv^2}{l}=\frac{mg\cancel{l}}{\cancel{l}}=mg$

$T=2mg$

15. Two blocks of mass m_A and m_B are connected by a string that passes over a light pulley. The mass of A is larger than the mass of B. The speed of mass A just before reaching the floor is:

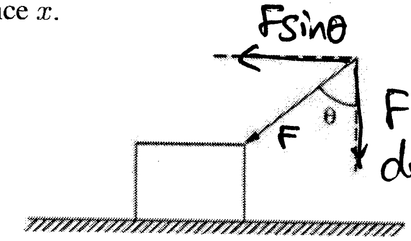


$\Sigma F=m_Ag-m_Bg=ma$

$a=\frac{m_A-m_B}{m_A+m_B}g$

(A) $\sqrt{\frac{m_A-m_B}{m_A+m_B}gD}$
(B) $\sqrt{\frac{m_A+m_B}{m_A-m_B}gD}$
(C) $\sqrt{\frac{m_A}{m_A+m_B}gD}$
(D) $\sqrt{\frac{m_B}{m_A+m_B}gD}$
(E) $\sqrt{\frac{m_A}{m_B}gD}$

Questions 16–17. A force is applied to a block of mass m at a downward angle of θ to the vertical as shown. The block moves with a constant speed across a rough floor for a distance x.

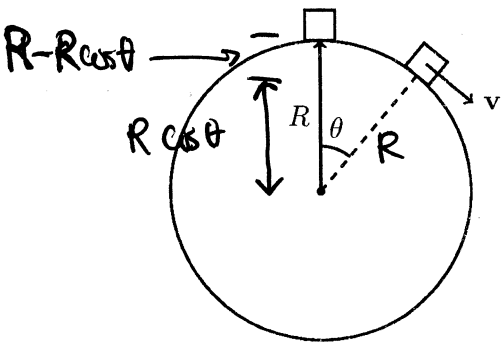


16. The work done by the applied force on the block is
(A) $Fx\sin\theta$
(B) $Fx\cos\theta$
(C) $Fmx\sin\theta$
(D) $Fmx\cos\theta$
(E) zero

17. The coefficient of friction between the block and the floor is
(A) $\frac{F}{mg}$
(B) $\frac{F\cos\theta}{mg}$
(C) $\frac{F\cos\theta}{F\sin\theta+mg}$
(D) $\frac{F\sin\theta}{F\cos\theta+mg}$
(E) $\frac{F\cos\theta}{F\sin\theta}$

$\mu=\frac{F_f}{F_N}=\frac{F\cos\theta}{F\sin\theta+mg}$

Questions 18–19. A small block rests on the top of a smooth sphere of radius R when it is given a light tap so that it just begins sliding on the sphere. When the block reaches the angle θ, it loses contact with the surface of the sphere.



18. The kinetic energy of the block as it leaves the surface of the sphere is
    (A) mgR
    (B) $mgR\cos\theta$
    (C) $mgR\sin\theta$
    (D) $mg(R - R\cos\theta)$
    (E) $mg(R - R\sin\theta)$

19. The speed of the block as it leaves the surface of the sphere is
    (A) $\sqrt{2gm}$
    (B) $\sqrt{2gRm}$
    (C) $2gR\cos\theta$
    (D) $2g(R - R\cos\theta)$
    (E) $2g(R - R\sin\theta)$

?

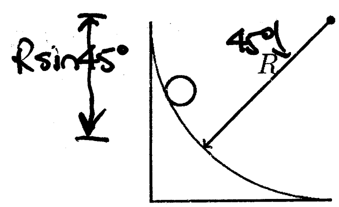
$mg(R - R\cos\theta) = \frac{1}{2}mv^2$

20. A machine can lift large weights according to the power equation $P(t) = 4t^3 + 3t^2 - 2$, where power is in watts and time is in seconds. The energy expended by the machine from $t = 0$ to $t = 10\,\text{s}$ is
    (A) 1260 J
    (B) 3630 J
    (C) 9240 J
    (D) 10,980 J
    (E) 18,150 J

$E = \int_0^{10} P\,dt = t^4 + t^3 - 2t \Big|_0^{10}$

$= 10^4 + 10^3 - 2(10)$

21. A small ball starts from rest and rolls down a quarter-circle ramp of radius R. The speed of the ball at the point halfway down the ramp is most nearly



    (A) gR
    (B) $2gR$
    (C) $\sqrt{gR\sin 45°}$
    (D) $\sqrt{2gR\sin 45°}$
    (E) The speed cannot be determined without knowing the mass of the ball.

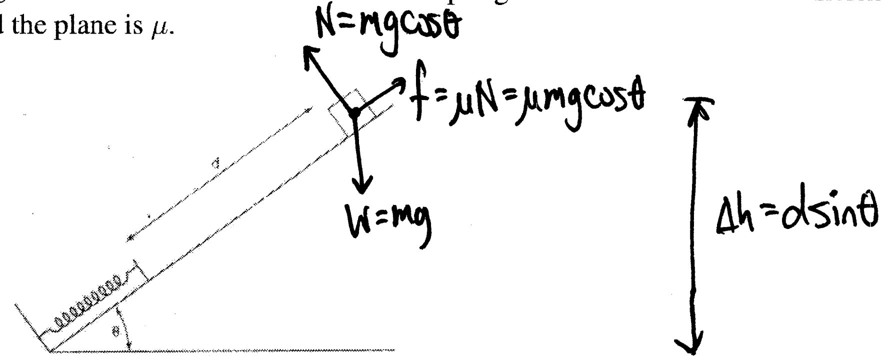
$mgR\sin 45° = \frac{1}{2}mv^2$

$v = \sqrt{2gR\sin 45°}$

AP PHYSICS 1 & C: MOMENTUM AND ENERGY
SECTION II
5 Questions

Directions: Answer all questions. The suggested time is about 10 minutes for answering cach of the questions. The parts within a question may not have equal weight. All final numerical answers should include appropriate units. Credit depends on the quality of your solutions and explanations, so you should show your work. Credit also depends on demonstrating that you know which physical principles would be appropriate to apply in a particular situation. Therefore, you should clearly indicate which part of a question your work is for.

1. A mass m is placed on an incline of angle θ at a distance d from the end of a spring as shown below. The coefficient of kinetic friction between the mass and the plane is μ.



(a) The mass is released from rest at the position shown. Using Newton's laws, calculate the block's speed when it reaches the spring.

$F_{net} = mg\sin\theta - \mu mg\cos\theta = ma \rightarrow a = g(\sin\theta - \mu\cos\theta)$

$v^2 = \cancel{v_0^2} + 2ad$

$v = \sqrt{2ad} = \boxed{\sqrt{2g(\sin\theta - \mu\cos\theta)d}}$

they should be the same of course!

(b) Using energy conservation, alculate the block's speed when it reaches the spring.

$\cancel{K} + U + W_{nc} = K' + \cancel{U'}$

$\cancel{m}gd\sin\theta - \mu \cancel{m}g\cos\theta\, d = \frac{1}{2}\cancel{m}v^2 \longrightarrow \boxed{v = \sqrt{2gd(\sin\theta - \mu\cos\theta)}}$

work done by friction is negative

(c) The spring has spring constant k. At what value x of the compression of the spring does the object reach its maximum speed?

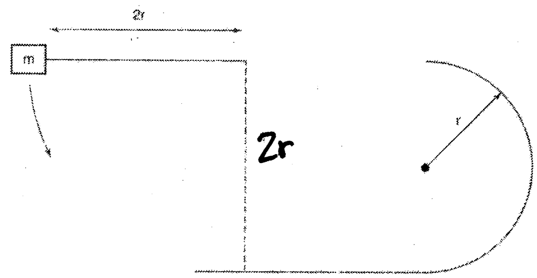
at max speed acceleration = 0.
F_{net} along direction of motion = 0.

$mg\sin\theta = \mu mg\cos\theta + kx$

$\boxed{x = \frac{mg}{k}(\sin\theta - \mu\cos\theta)}$

2. A mass m attached to a string of length $2r$ swings, starting at rest when the string is horizontal, until the string is vertical. At the instant the string is vertical, the mass makes contact with the horizontal surface, the string is cut, and the mass continues along a frictionless track as shown below.

total energy $= 2mgr$



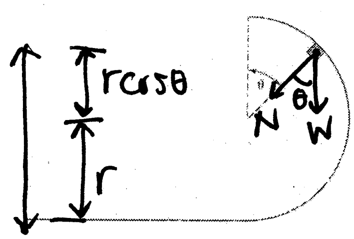
(a) What is the speed of the mass attached to the string the instant the string is cut?

$$\cancel{m}g(2r) = \frac{1}{2}\cancel{m}v^2$$

$$v^2 = 4gr \qquad v = \boxed{2\sqrt{gr}}$$

(b) Sketch the forces acting on the mass when it is in the position shown below.

$h = r(1+\cos\theta)$



there are only two forces because there is no friction.

When the mass is in the position shown above,

(c) Find the object's speed as a function of θ

$$\frac{1}{2}\cancel{m}v^2 = 2\cancel{m}gr - \cancel{m}gr(1+\cos\theta) = \overbrace{2gr - gr}^{gr} - gr\cos\theta = rg(1-\cos\theta)$$

$$\boxed{v = \sqrt{2gr(1-\cos\theta)}}$$

(d) Find the object's centripetal acceleration as a function of θ

$$a_c = \frac{v^2}{r} = \boxed{2g(1-\cos\theta)} \qquad F_c = ma_c = 2mg(1-\cos\theta)$$

(e) Determine at what angle θ the mass will fall of the track

just before it falls off $N = 0$.

$$F_c = ma_c = \cancel{mg}\cos\theta = 2\cancel{mg}(1-\cos\theta)$$

provided by the ~~ka~~ centripetal component of weight.

$$\cos\theta = 2(1-\cos\theta) = 2 - 2\cos\theta$$

$$3\cos\theta = 2$$

$$\cos\theta = \frac{2}{3} \qquad \theta = 48.2^\circ$$

GO ON TO THE NEXT PAGE.

3. The theoretical formula for the potential energy associated with the nuclear force between two protons, two neutrons, or a proton and a neutron is the *Yukawa potential*:

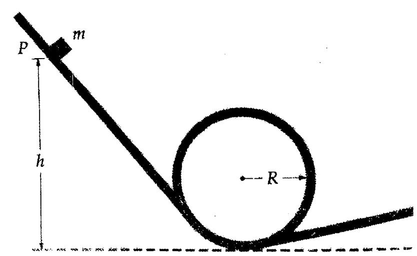
$$U = -U_0 \underbrace{\left(\frac{a}{x}\right)}_{f(x)} \underbrace{e^{-x/a}}_{g(x)}$$

where U_0 and a are constants.

(a) Sketch U versus x using $U_0 = 4\,\text{pJ}$ and $a = 2.5\,\text{fm}$.
(b) Find the force F_x.
(c) Compare the magnitude of the force at the separation $x = 2a$ to that at $x = a$.
(d) Compare the magnitude of the force at the separation $x = 5a$ to that at $x = a$.

(b) $F_x = -\frac{dU}{dx} = \frac{d}{dx}\left[U_0\left(\frac{a}{x}\right)e^{-x/a}\right]$

4. A small block of mass m slides without friction along the loop-the-loop track shown in the figure. The block starts from point P a distance h above the bottom of the loop.



(a) What is the kinetic energy of the block when it reaches the top of the loop?
(b) What is the acceleration at the top of the loop, assuming that it stays on the track?
(c) What is the least value of h for which the block will reach the top of the loop without leaving the track?

(a) $K = \Delta U_g = mgh - mg(2R) = mg(h-2R)$ $\quad h > 2R$

(b) acceleration is centripetal acceleration $\frac{v^2}{R}$

$\cancel{m}g(h-2R) = \frac{1}{2}\cancel{m}v^2$

$v^2 = 2g(h-2R)$

$a_c = \frac{v^2}{R} = \boxed{\frac{2g}{R}(h-2R)}$

(c) centripetal force from weight alone

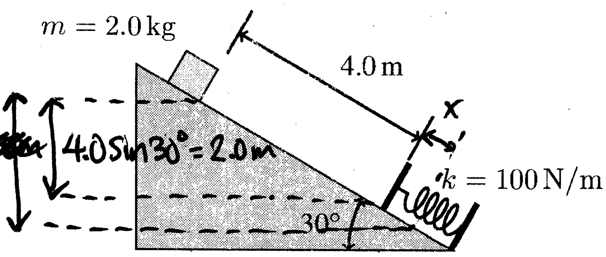
$\cancel{mg} = ma_c = \frac{2\cancel{mg}}{R}(h-2R)$

$1 = \frac{2}{R}(h-2R) = \frac{2h}{R} - 4$

$5 = \frac{2h}{R} \rightarrow \boxed{h = \frac{5R}{2}}$

5. A 2.0-kg block is released 4.0 m from a massless spring with a spring constant of 100 N/m that is fixed along a frictionless plane inclined at 30°, as shown in the figure below. Plese give answer to 3 significant figures.

m k



$(4.0+x)\sin 30°$
$= 2.0 + \frac{x}{2}$

μ

(a) Find the maximum compression of the spring.
(b) If the plane is not frictionless, and the coefficient of kinetic friction between it and the block is 0.20, find the maximum compression.
(c) For the rough incline ($\mu = 0.20$), how far up the incline will the block travel after leaving the spring?

(a)

$U_g + \cancel{U_e} + \cancel{K} = \cancel{U_g'} + U_e' + \cancel{K'}$

=0 at maximum Compressin

$mg\left(2.0+\frac{x}{2}\right) = \frac{1}{2}kx^2$

$2mg + \frac{mg}{2}x = \frac{1}{2}kx^2$

$\frac{1}{2}kx^2 - \frac{mg}{2}x - 2mg = 0$

$50x^2 - 10x - 40 = 0$

$5x^2 - x - 4 = 0$

$\boxed{x = 1.0\,m}$

$\cancel{-0.8}$

(b)

$N = mg\cos 30°$

f_k

mg

$f_k = \mu N$
$= \mu mg\cos 30°$
$= 3.464\,N$

$U_g + \cancel{U_e} + \cancel{K} + W_{nc} = \cancel{U_g'} + U_e' + \cancel{K'}$

$mg\left(2.0+\frac{x}{2}\right) - f_k(4.0+x) = \frac{1}{2}kx^2$

$2mg + \frac{mg}{2}x - 4f_k - xf_k = \frac{1}{2}kx^2$

$\left[\frac{1}{2}k\right]x^2 - \left(\frac{mg}{2} - f_k\right)x - \left[2mg - 4f_k\right] = 0$

50 10 3.464 40 13.856

$x^2 - \underset{6.536}{[\,]}x - \underset{26.144}{[\,]} = 0$

$\boxed{x = 0.791\,m}$

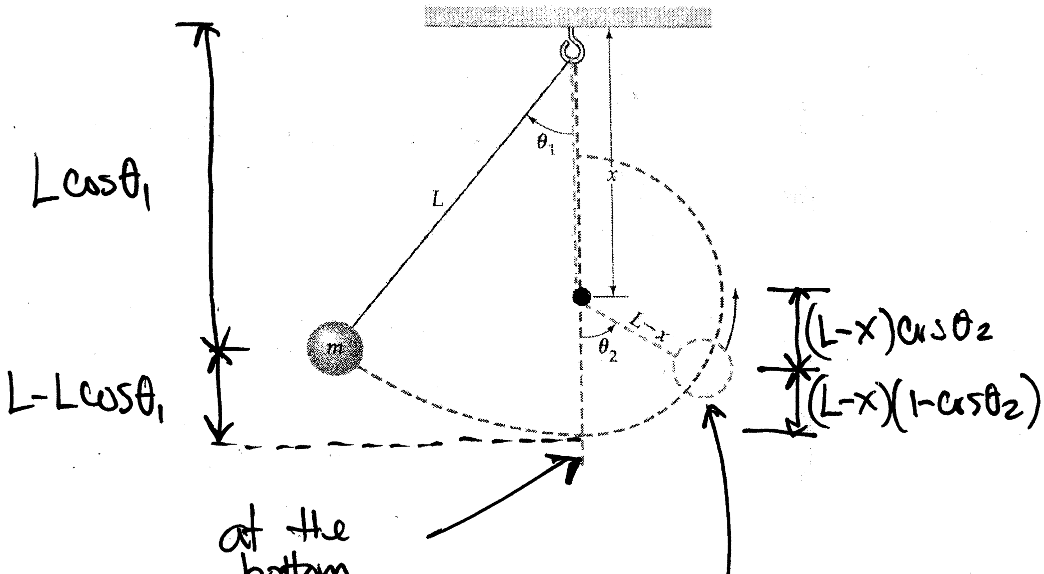
-0.661

we expect a decrease in Compression

(c) see after page 9

GO ON TO THE NEXT PAGE.

6. A pendulum of length L has a bob of mass m. It is released from some angle θ_1. The string hits a peg at a distance x directly below the pivot, as shown in the figure below, effectively shortening the length of the pendulum. Find the maximum angle θ_2 between the string and the vertical when the bob is to the right of the peg.



$$K = \Delta U_g = mgL(1-\cos\theta_1)$$

at this point all the kinetic energy converted back to U_g

$$U_g = K$$

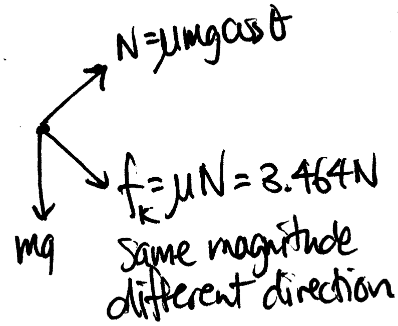
$$\cancel{m}\cancel{g}(L-x)(1-\cos\theta_2) = \cancel{m}\cancel{g}L(1-\cos\theta_1)$$

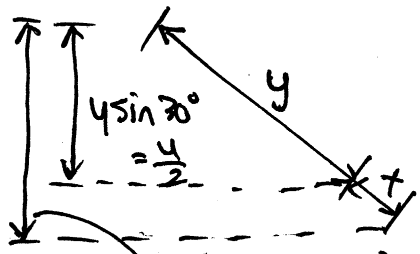
$$1-\cos\theta_2 = \frac{L}{L-x}(1-\cos\theta_1)$$

$$\cos\theta_2 = 1 - \frac{L}{L-x}(1-\cos\theta_1)$$

$$\boxed{\theta_2 = \cos^{-1}\left[1-\frac{L}{L-x}(1-\cos\theta_1)\right]}$$

5(c)





$(y+x)\sin 30° = \frac{y}{2} + \frac{x}{2} = \frac{x+y}{2}$

$X = 0.7914$ from part (b)

$\cancel{U_g} + U_e + \cancel{K} + W_{nc} = U_g' + \cancel{U_e'} + \cancel{K'}$

$\cancel{\frac{1}{2}kx^2 = mg\left(\frac{y}{2}+\frac{x}{2}\right) = \frac{mg}{2}(y+x)}$

$\cancel{\frac{kx^2}{mg} = y + x}$

$\cancel{y = \frac{kx^2}{mg} - x}$

$\cancel{= \frac{(100)(0.235)^2}{(2)(10)} - 0.235}$

$\frac{1}{2}kx^2 - f_k(x+y) = \frac{mg}{2}(x+y)$

$\frac{1}{2}kx^2 = \left(\frac{mg}{2} + f\right)(x+y)$

$31.316 = 13.464(0.7914 + y)$

$\boxed{y = 1.53\,m}$