

# Hall Effect

## Advanced Placement Physics

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# Current Through the Conductor

The (instantaneous) electric current through conductor is the rate at which charge carriers pass through a point in the conductor:

$$\boxed{I = \frac{dQ}{dt}} = \frac{dQ}{dt} = \left( \frac{Q}{V} \right) \frac{dV}{dt} = [ne] [Av_d]$$

where

- $Q/V$  is the amount of charges *per volume*, which is just the charge carrier density  $n$  times the elementary charge  $e$
- $dV/dt$  is the rate the volume of charges moves through the conductor, give by the cross-section area of the conductor  $A$  times the **drift velocity**  $v_d$  of the charge carrier

For simplicity, we *assume* that charge carriers are positive. While the opposite is true, the behavior will be almost identical.

# Current Through the Conductor

Combining the terms:

$$I = \frac{dQ}{dt} = neAv_d$$

Quantity	Symbol	SI Unit
Current	$I$	A
Charge carrier density (carriers per volume)	$n$	/m <sup>3</sup>
Elementary charge	$e$	C
Cross-section area of the conductor	$A$	m <sup>2</sup>
Drift velocity of the charge carriers	$v_d$	m/s

The calculation for the charge carrier density  $n$  requires some additional thoughts.

# Charge Carrier Density

The charge carrier density in a *metal* conductor involves some physical information about the metal:

1. Divide the metals density  $\rho$  by the metal's molar mass  $M$  to find the *number of moles of atoms per unit volume*
2. Multiply by Avagadro's number  $N_A$  to find *number of atoms per unit volume*
3. Multiply by *the number of free electrons per atom  $k$*  for that particular metal

# Charge Carrier Density

Collecting all the terms from the last slide, we have:

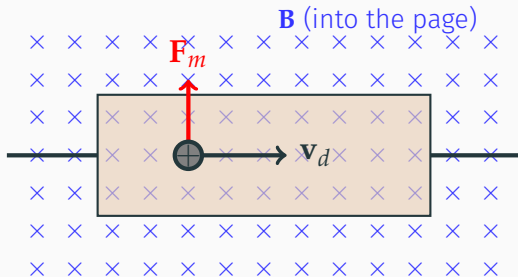
$$n = \frac{\rho k N_A}{M}$$

Quantity	Symbol	SI Unit
Charge carrier density	$n$	$/\text{m}^3$
Density of material	$\rho$	$\text{kg}/\text{m}^3$
Number of free electrons per atom	$k$	
Avogadro's number	$N_A$	$/\text{mol}$
Molar mass	$M$	$\text{kg}/\text{mol}$

For copper,  $M = 63.54 \times 10^{-3} \text{ kg/mol}$ ,  $\rho = 9.0 \times 10^3 \text{ kg/m}^3$ ,  $k = 1$  and therefore  $n = 8.5 \times 10^{28} / \text{m}^3$ .

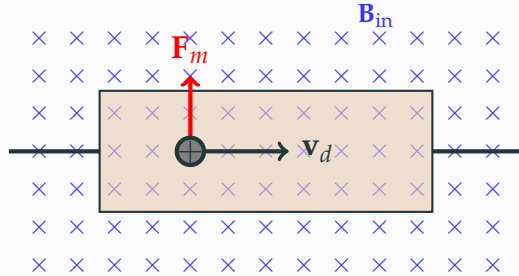
# Hall Effect

When a current  $I$  flows through a conductor in a magnetic field  $\mathbf{B}$ , the magnetic field exerts a transverse (i.e. perpendicular to motion) magnetic force  $\mathbf{F}_m$  on the moving charges which pushes them toward one side of the conductor. This is called **Hall effect**.



This is most evident in a *thin, flat* conductor as illustrated.

# Magnetic Force

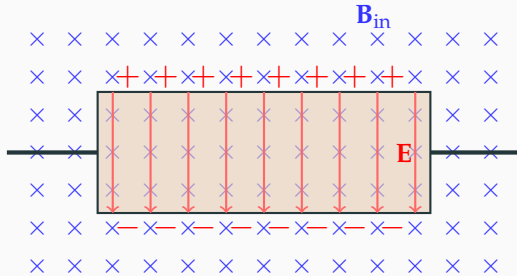


As the charges enter the magnetic field,  $\mathbf{F}_m$  is directed toward the top:

$$\mathbf{F}_m = e\mathbf{v}_d \times \mathbf{B} = \frac{e\mathbf{I} \times \mathbf{B}}{neA}$$

leading to a surplus of positive charges on the top edge of the conductor, and negative charges on the bottom.

# Hall Voltage

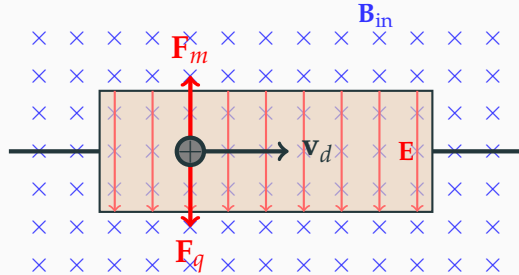


The charge imbalance on the conductor creates an electric field  $\mathbf{E}$ , pointing toward the bottom, and therefore a voltage across two sides of the conductor (width  $W$ ), called the **Hall voltage**:

$$V_H = EW$$



# Balancing Electrostatic & Magnetic Forces



Subsequently, charge carriers entering the magnetic field will experience both a magnetic force and an electrostatic force. At equilibrium, the two forces are balanced:

$$\mathbf{F}_m + \mathbf{F}_q = \mathbf{0}$$

## Calculating Hall Voltage

The electrostatic force on the charge carrier can be expressed in terms of the Hall voltage  $V_H$  across the two sides of the plate:

$$F_q = eE = \frac{eV_H}{W}$$

Equating the magnitudes of electrostatic and magnetic forces, we can solve for the Hall voltage:

$$F_m = F_q \quad \rightarrow \quad \frac{IB}{nA} = \frac{eV_H}{W}$$

# Hall Voltage

Cancelling terms and noting that the thickness of the conductor is

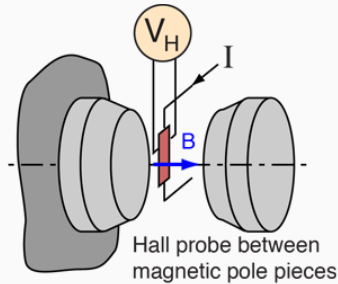
$$d = \frac{A}{W}$$

we find the expression for the Hall voltage  $V_H$ :

$$V_H = \frac{IB}{ned}$$

# Hall Probe

Large magnetic fields ( $\sim 1\text{ T}$ ) is often measured using a **Hall probe**. A thin film Hall probe is placed in the magnetic field and the transverse voltage (usually measured in on the order of  $10^{-6}\text{ V}$ ) is measured.



The polarity of the Hall voltage for a copper probe shows that electrons (negative charge) are the charge carriers.