Class 13: Magnetism

AP Physics

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Files for You to Download

Download from the school website:

1. `12-Magnetism_print.pdf`—The "print version" of this presentation. If you want to print the slides on paper, I recommend printing 4 slides per page.
2. `13-Homework.pdf`—Homework assignment for Class 12 and 13. Please note the new formatting style

Please download/print the PDF file before each class. When you are taking notes, pay particular attention to things I say that aren't necessarily on the slides.

Review of Magnetic Field

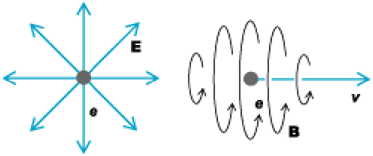
Remember Physics 12?

- A magnetism is generated by moving charged particles, e.g. a single charge, or an electric current
- It can also be generated by permanent magnets, or Earth

Review of Magnetic Field

- Magnetism affects other *moving* charged particles
- The vector field is called the **magnetic field**
- Magnetic field has unit **tesla**
- Magnetic field lines have ends—they always run in a loop

Magnetic Field Generated by a Moving Point Charge



A point charge generates an electric field $\mathbf{E}$. When it's moving, it also generates a magnetic field $\mathbf{B}$, given by the equation:

$$\boxed{\mathbf{B} = \frac{\mu_o}{4\pi}\frac{q\mathbf{v} \times \hat{\mathbf{r}}}{r^2}}$$

The direction of $\mathbf{B}$ can be obtained by applying the "right hand rule" if you are not confident with cross products.

Reminder on the cross product

Whenever the "right hand rule" is mentioned, it usually means that the equation has a cross product in it. Just a reminder on a few properties of the cross product:

- If $\mathbf{C} = \mathbf{A} \times \mathbf{B}$, then $\mathbf{C}$ is perpendicular to both $\mathbf{A}$ and $\mathbf{B}$.
- The length of the cross product of two vectors is:

$$\left|\mathbf{A} \times \mathbf{B}\right| = \left|\mathbf{A}\right|\left|\mathbf{B}\right| \sin\theta$$

  where θ is the angle between $\mathbf{A}$ and $\mathbf{B}$

- Cross products are anti-commutable:

$$\mathbf{A} \times \mathbf{B} = -\mathbf{B} \times \mathbf{A}$$

Magnetic Field Generated by a Moving Point Charge

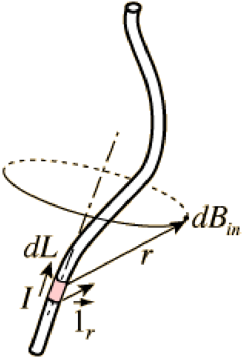
$$\boxed{\mathbf{B} = \frac{\mu_o}{4\pi} \frac{q\mathbf{v} \times \hat{\mathbf{r}}}{r^2}}$$

Quantity	Symbol	SI Unit
Magnetic field	$\mathbf{B}$	T (teslas)
Charge	q	C (coulombs)
Velocity of the charge	$\mathbf{v}$	m/s (metres per second)
Distance from the moving charge	r	m (metres)
Radial unit vector from the charge	$\hat{\mathbf{r}}$	no units
Permeability of free space	μ_0	T m/A (tesla metres per ampere)

Permeability of free space is a constant with a value of $\mu_0 = 4\pi \times 10^{-7}$ T m/A

Magnetic Generated By a Current

Biot-Savart Law

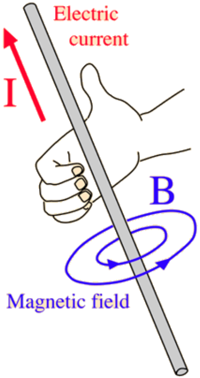


An electric current is really many charges particles moving along a wire; each charge creating its own magnetic field. The total magnetic field in the wire is the integral of the contribution ($d\mathbf{B}$) of the current (I) from each infinitesimal sections ($d\mathbf{L}$) of the wire, given by the **Biot-Savart Law**:

$$\boxed{d\mathbf{B} = \frac{\mu_o}{4\pi}\frac{I d\mathbf{L} \times \hat{\mathbf{r}}}{r^2}}$$

The magnetic field in the diagram goes *into* the page

Magnetic Field Generated By an Infinitely Long Wire



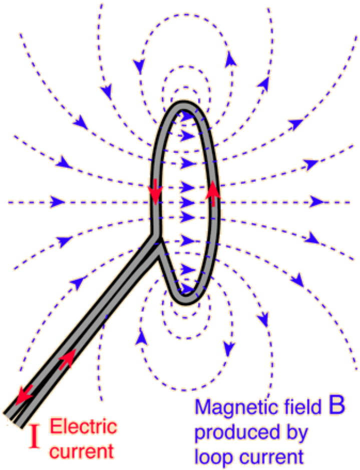
Integrating Biot-Savart law for a point at radial distance r from an *infinitely long wire* gives a simple expression:

$$\boxed{\mathbf{B} = \frac{\mu_0(\mathbf{I} \times \hat{\mathbf{r}})}{2\pi r}} \quad \text{or} \quad \boxed{B = \frac{\mu_0 I}{2\pi r}}$$

The magnitude and direction current "vector" $\mathbf{I}$ is straight forward

Quantity	Symbol	SI Unit
Magnetic field	$\mathbf{B}$	T (teslas)
Current	$\mathbf{I}$	A (amperes)
Radial direction from the wire	$\hat{\mathbf{r}}$	(no units)
Radial distance from the wire	r	m (metres)

Current-Carrying Wire Loop

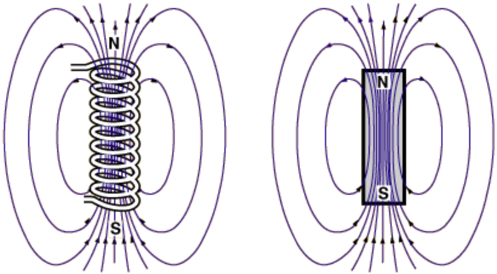


When we shape the current-carrying wire into a loop, we can (again) use the Biot-Savart law to find the magnetic field away from it.

One loop isn't very interesting (except when you're integrating Biot-Savart law) but what if we have many loops

Wounding Wires Into a Coil

- A **solenoid** is when you wound a wire into a coil
- You create a magnet very similar to a bar magnet, with an effective north pole and a south pole
- Magnetic field inside the solenoid is uniform
- Magnetic field strength can be increased by the addition of an iron core

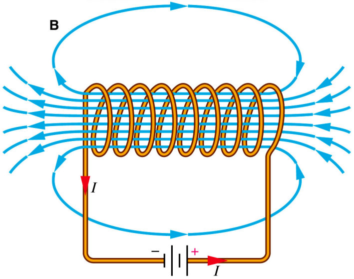


A Practical Solenoid

A practical solenoid usually has hundreds or thousands of turns:

This "air core" coil is used for high school and university experiments. It has approximately 600 turns of copper wire wound around a plastic core.

Magnetic Field Inside a Solenoid



The magnetic field **inside** the solenoid given by:

$$\boxed{B = \mu n I}$$

Direction of $\mathbf{B}$ determined by **right hand rule**

Quantity	Symbol	SI Unit
Magnetic field intensity	B	T (teslas)
Number of coils	n	integer, no units
Current	I	A (amperes)
Effective permeability	μ	T m / A

So What Does the Magnetic Field Do?

In Classical Physics

Gravitational Field $\mathbf{g}$

- Generated by objects with mass
- Affects objects with mass

Electric Field $\mathbf{E}$

- Generated by charged particles
- Affects charged particles

Magnetic Field $\mathbf{B}$

- Generated by *moving* charged particles
- Affects moving charged particles

Lorentz Force Law

Since a moving charge or current create both electric and magnetic fields, another moving charge is therefore affected by both $\mathbf{E}$ and $\mathbf{B}$. The total effect is given by the **Lorentz Force Law**:

$$\boxed{\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})}$$

$\mathbf{F}_q = q\mathbf{E}$ is the electrostatic force, and $\mathbf{F}_M = q\mathbf{v} \times \mathbf{B}$ is the magnetic force.

Quantity	Symbol	SI Unit
Total force on the moving charge	$\mathbf{F}$	N (newtons)
Charge	q	C (coulombs)
Velocity of the charge	$\mathbf{v}$	m/s (metres per second)
Magnetic field	$\mathbf{B}$	T (teslas)
Electric field	$\mathbf{E}$	N/C (newtons per coulomb)

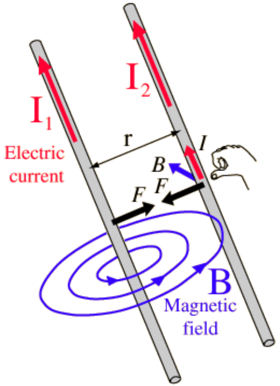
Force on a Current-Carrying Conductor in a Magnetic Field

Likewise, $\mathbf{B}$ exerts a force on another current-carrying conductor.

$$\boxed{F_M = \mathbf{I}l \times \mathbf{B}}$$

Quantity	Symbol	SI Unit
Magnetic force on the conductor	$\mathbf{F}_M$	N (newtons)
Electric current in the conductor	$\mathbf{I}$	A (amperes)
Length of the conductor	l	m (metres)
Magnetic field strength	$\mathbf{B}$	T (teslas)

Magnetic Force on Two Current-Carrying Wires



Two parallel current carrying wires are at a distance r apart. Magnetic field at wire 2 from current I_1 has strength:

$$B_1 = \frac{\mu_0 I_1}{2\pi r}$$

which is constant everywhere along wire 2. The force of B_1 on I_2 is:

$$F = I_2 L B_1 = \frac{\mu_0 I_1 I_2 L}{2\pi r} \quad \rightarrow \quad \boxed{\frac{F}{L} = \frac{\mu_0 I_1 I_2}{2\pi r}}$$

Similarly, I_1 exerts the same force on I_2, pulling the wires toward each other.

Circular Motion Caused by a Magnetic Field

When a charged particle enters a magnetic field at right angle...

- Magnetic force $\mathbf{F}_M$ perpendicular to both velocity $\mathbf{v}$ and magnetic field $\mathbf{B}$.
- Results in circular motion

Centripetal force $\mathbf{F}_c$ is provided by the magnetic force $\mathbf{F}_M$. Equating the two expressions:

$$\frac{mv^2}{r} = qvB$$

We can solve for r get the radius for a charge with a known mass, or solve for mass m of a charged particle based on its radius:

$$r = \frac{mv}{qB} \qquad m = \frac{qrB}{v}$$

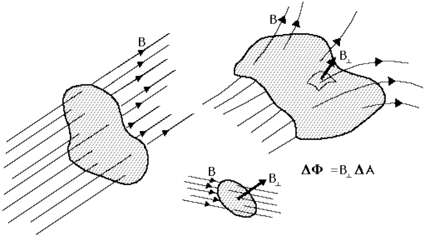
Magnetic Flux

Question: If a current-carrying wire can generate a magnetic field, can a magnetic field affect the current in a wire?

Answer: Yes, sort of. . .

To understand how to *induce* a curent by a magnetic field, we need to look at fluxes again.

Magnetic Flux



Not surprisingly, the magnetic flux is defined similar to electric flux:

$$\boxed{\Phi_{\text{magnetic}} = \int \mathbf{B} \cdot d\mathbf{A}}$$

where $\mathbf{B}$ is the magnetic field, and $d\mathbf{A}$ is the infinitesimal area with its direction point outward.

Magnetic Flux Over a Closed Surface

The magnetic flux over a closed surface is always zero:

$$\boxed{\oint \mathbf{B} \cdot d\mathbf{A} = 0}$$

Since magnetic field exists in a loop only, what every flux that leaves the surface has to eventually come back.

Changing Magnetic Flux

Changes to magnetic flux can be due to a number of reasons:

1. **Changing magnetic field**. . . if the magnetic field is created by a time-dependent source (e.g. alternating current)
2. **Changing orientation of magnetic field** either because the surface area is moving relative to the magnetic field.
3. **Changing area** the surface area from which the flux is calculated is changing.

Faraday's Law

Faraday's law states that the rate of change of magnetic flux produces an electromotive force:

$$\boxed{\mathcal{E} = -\frac{d\Phi}{dt}}$$