# Light Wave (Supplemental Slides)

## Advanced Placement Physics

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# What These Slides Are About

These slides are condensed from the Grade 12 Physics slides for the unit on *The Wave Nature of Light*. Some of the information are also presented in other classes.

# Huygens' Principle

In the 1600's there were two competing theories of light. . .

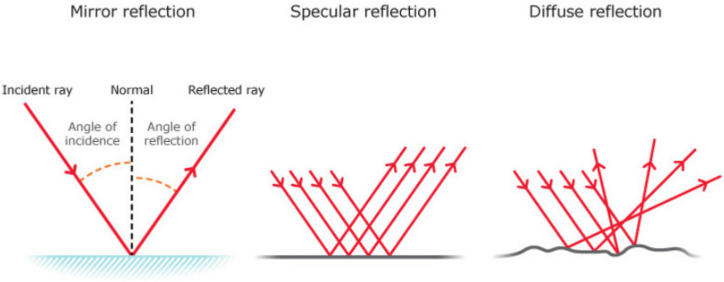
- Some, including Issac Newton, believed that light is a particle
- Others, including Christiaan Huygen (Dutch) and Augustin-Jean Fresnel (French), believed that light is a wave

Huygen's Princple: all waves are in fact an infinite series of circular wavelets

# Reflection of Light

**Law of Reflection**

The incident ray, the reflected ray, and the normal to the surface of the mirror all lie in the same plane, and the angle of reflection is equal to the angle of incidence.



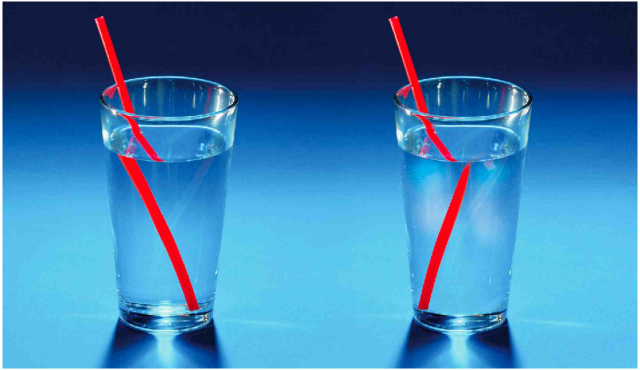
# Specular Reflection

## Example: Lake Reflection

This photo of Lake Matheson shows specular reflection in the water of the lake with reflected images of Aoraki/Mt Cook (left) and Mt Tasman (right). The very still lake water provides a perfectly smooth surface for this to occur.

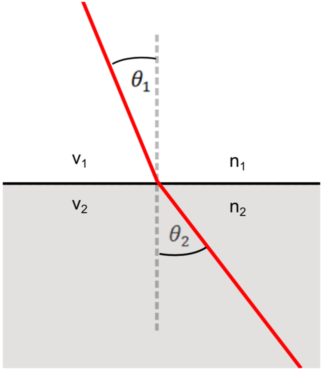
# Refraction of Light Through a Medium

- When a wave enters another medium, the wave speed changes
- When entering at an angle, the change of speed causes the wave to change direction (e.g. from air to water, air to glass, glass to air etc)
- The amount of bending depends on the **indices of refraction of the two media**
- Responsible for **image formation** by lenses and the eye



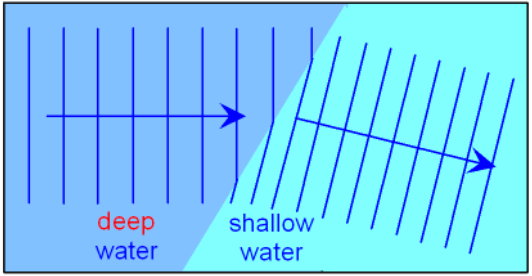
# Refraction of Light Through a Medium

You have probably all seen this diagram of light entering from one medium to another.



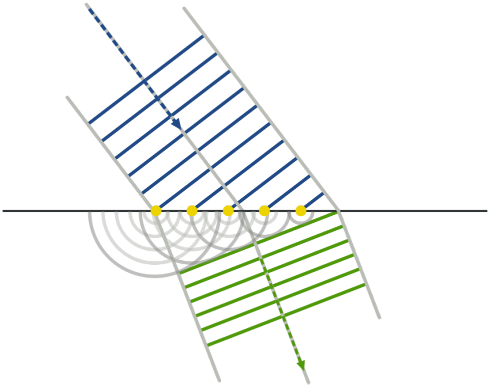

Light could be going in either direction, from top to bottom ($n_1$ to $n_2$) or from bottom to top ($n_2$ to $n_1$)

# Refraction Happens in Ocean Waves Too!




Refraction happens not only with light, we see the same behaviour in ocean waves, when the wave travel from deeper water (faster waves) to shallow depths.

# Refraction and Huygens Principle



We can explain the refraction phenomenon using Huygens' Principle

# Snell's Law

**Snell's law** relates the indices of refraction $n$ of the two media to the directions of propagation in terms of the angles to the normal.

$$\boxed{n_1 \sin\theta_1 = n_2 \sin\theta_2}$$

| Variable | Symbol | SI Unit |
|---|---|---|
| Indices of refraction of the media | $n_1$, $n_2$ | (no units) |
| Incident angle of light | $\theta_1$ | (no units) |
| Refraction angle of light | $\theta_2$ | (no units) |

# Index of Refraction

**Index of refraction** ($n$) is defined as the speed of light in vacuum ($c$) divided by the speed of light in the medium ($v$).

$$\boxed{n = \frac{c}{v} = \frac{\lambda_{\text{vacuum}}}{\lambda}}$$

When light enters a second medium, the *frequency* remains unchanged (i.e. the colour doesn't change!) but since the speed changes, the *wavelength* also changes:

$$\boxed{\frac{n_1}{n_2} = \frac{\lambda_2}{\lambda_1}}$$

# Index of Refraction of Common Materials

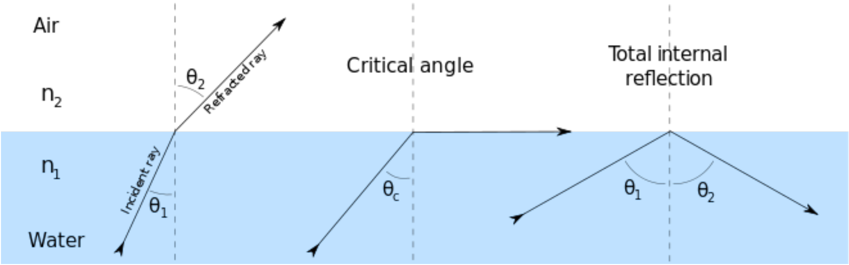
| Material | n | Material | n |
|---|---|---|---|
| Vacuum | 1 | Ethanol | 1.362 |
| Air | 1.000277 | Glycerine | 1.473 |
| Water at $20\,^{\circ}\mathrm{C}$ | 1.33 | Ice | 1.31 |
| Carbon disulfide | 1.63 | Polystyrene | 1.59 |
| Methylene iodide | 1.74 | Crown glass | 1.50-1.62 |
| Diamond | 2.417 | Flint glass | 1.57-1.75 |

The values given are *approximate* and do not account for the small variation of index with light wavelength which is called **dispersion**. We'll get to that later!

# Total Internal Reflection

## From High Index to Low Index

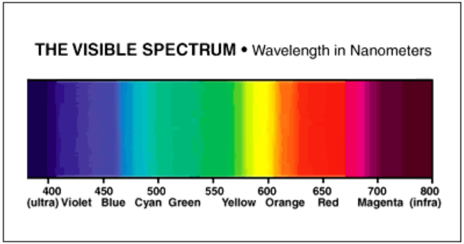
Snell's law still holds, but something weird can happen:




Critical angle $\theta_c$ for water-air interface is $48.6^\circ$. If incident angle is greater $\theta_1 > \theta_c$, we have **total internal reflection**. TIR can only happen going from a higher index to a lower index, $n_1 > n_2$.

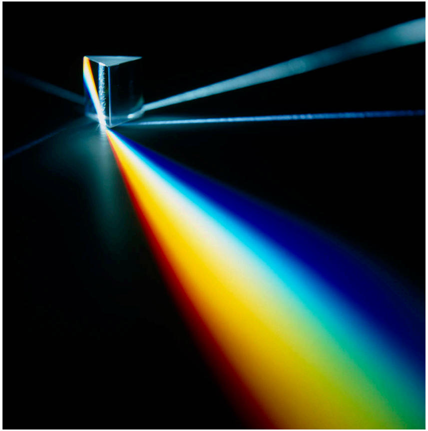
# Colour of Light and Wavelength

Human eyes perceive different frequencies of light as different colours. The visible spectrum of light:



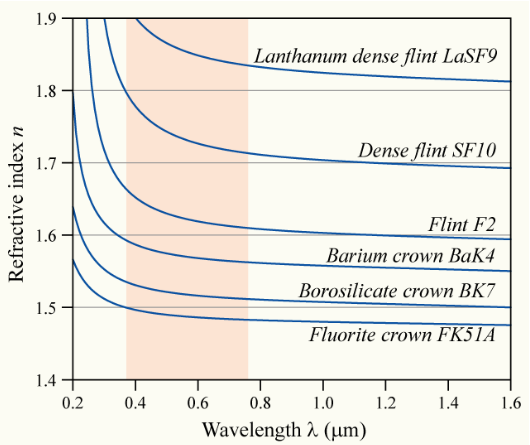

- The “colour” of the light depends on its frequency (& wavelength when it’s in a vacuum)
- *White light* is light that contains waves in all frequencies.

# Dispersion of Light Through Refraction



- When white light passes through a prism it is separated into different colours (spectrum) through refraction.
- This is because the index of refraction $n$ is slightly different for different wavelengths
- Otherwise, we will never see a rainbow

# Wavelength Dependency of Index of Refraction



# Chromatic Aberration

## This is What Disperson Can Do!

When looking at an image through a low-quality binocular, magnifying glass, or telescope, we often see the edges of images blurred a bit. Sometimes we see a rainbow-coloured edge:

Chromatic Aberration can occur even with high-quality camera lenses, particularly with wide-angle lenses where light from high angles have to bend towards the camera sensor/film, requiring high lens curvature.

# Chromatic Aberration

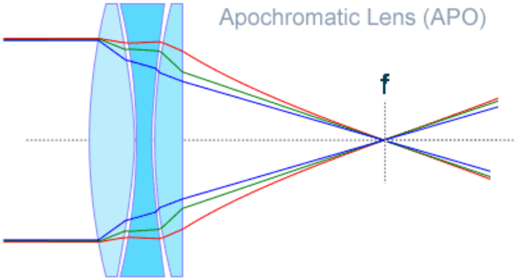
## This is What Disperson Can Do!

The reason that chromatic aberration happens is the same reason that prisms work: **dispersion of light**

The focal lengths for different frequencies (colour) of light are different, thus blurring the image. So how do we fix it?

# Chromatic Aberration: Camera Lens Design

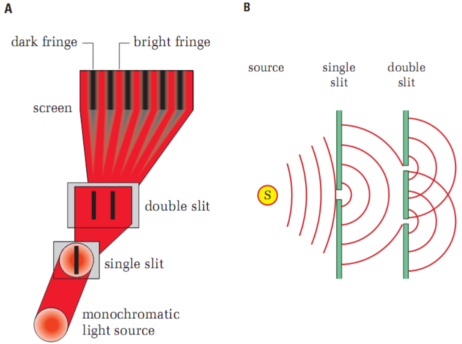
By lining different lenses of different materials and geometries, we can correct for the chromatic aberration.




- Lens design is a closely guarded secret by camera companies
- Shape of the lens, material and coating are all factors
- A “lens” on a DSLR camera can have up to 30 lens “elements”

# Thomas Young's Double-Slit Experiment

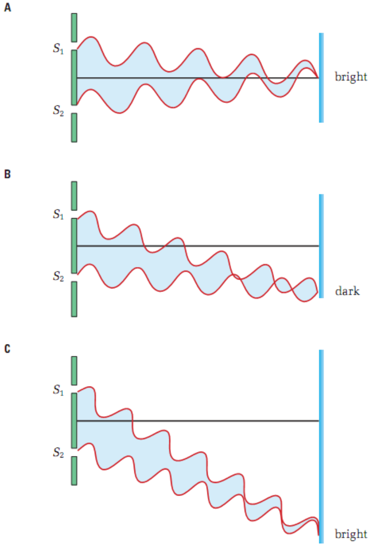
## First definitive evidence that light is a wave




- **Monochromatic light** light with a single colour (frequency); the light source can be a laser, LED , or gas lamp (most likely what Young used)
- **Slit:** an opening; also called an **aperture**
- The **screen** far away from the slits is also called the **projection**

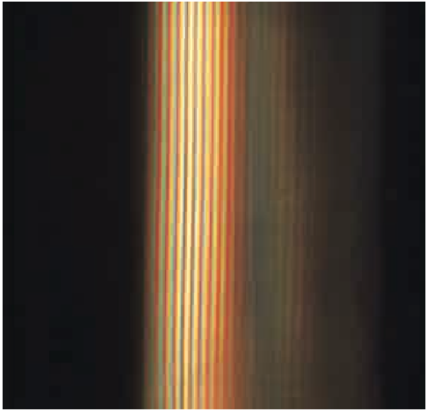
Double-slit experiment showed that light causes interference, just like any other wave

# Thomas Young's Double-Slit Experiment



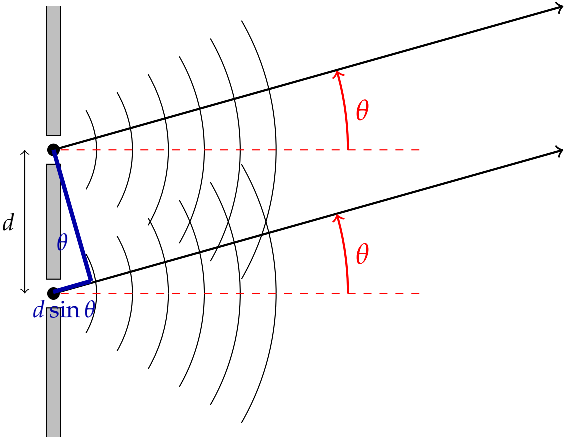

- At **A**, the path from slits $S_1$ and $S_2$ are the same, therefore **constructive interference** (bright fringe)
- At **B**, the path from $S_1$ and $S_2$ differ by half a wavelength, and therefore **destructive interference** (dark fringe)
- At **C**, the path from $S_1$ and $S_2$ differ by one wavelength, and therefore **constructive interference** (bright fringe) again

# Interference Pattern: Bright and Dark Fringes



The “bright fringes” are from constructive interference; the “dark fringes” are from destructive interference.

# Double-Slit Interference




- We have two slits at distance $d$ apart, emitting *coherent* light
- Huygens' Principle: light passing through the slits become point sources
- Assume that the projection (screen) is far enough from the slits that we can treat the two beams of light from the slits as being parallel
- Using basic geometry, we can see that the path difference from the two slit to the projection is $d \sin \theta$

# Double-Slit Interference

## Constructive Interference

A bright fringe (constructive interference) occurs when the path difference ($d\sin\theta$) is an integer ($n$) multiple of wavelength ($\lambda$), i.e.

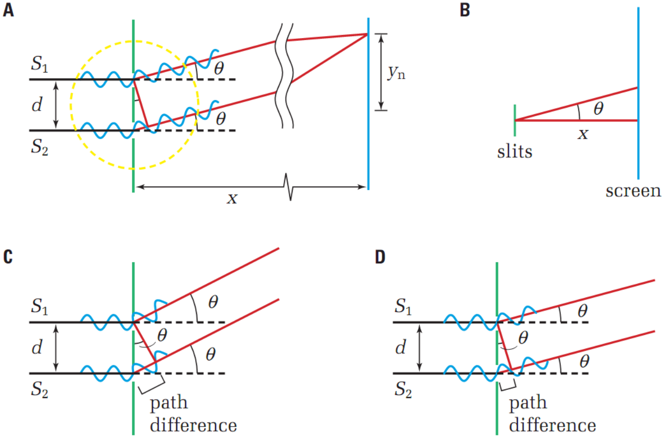
$$\boxed{\pm n\lambda = d\sin\theta_n}$$

Conversely, a dark fringe (destructive interference) occurs when the path difference ($d\sin\theta$) is a half-number ($n+\frac{1}{2}$) multiple of wavelength ($\lambda$), i.e.

$$\boxed{\pm\left(n+\frac{1}{2}\right)\lambda = d\sin\theta_n}$$

where $n = 0, 1, 2, 3\ldots$

# Double-Slit Interference



# Approximation of The Wavelength of Light

We can estimate the wavelength of light based on the distances between bright fringes, by applying the **small-angle approximation** (make sure $\theta$ is in *radians*):

$$\theta \approx \tan\theta \approx \sin\theta$$

The distance from slits to the screen ($x$), and the distance of the $n$-th bright fringe from the centre ($y_n$) to $\theta_n$ can be approximated by:

$$\tan\theta_n = \frac{y_n}{x} \approx \sin\theta_n$$

We can substitute our approximation into the constructive interference equation:

$$n\lambda \approx \frac{y_n d}{x} \longrightarrow \boxed{\lambda \approx \frac{\Delta y d}{x}}$$

# Approximation of The Wavelength of Light

This equation applies equally to dark fringes (nodal lines) as well as bright fringes.

$$\boxed{\lambda \approx \frac{\Delta y d}{x}}$$

| Quantity | Symbol | SI Unit |
|---|---|---|
| Wavelength | $\lambda$ | m (metre) |
| Distance between fringes | $\Delta y$ | m (metre) |
| Distance between slits | $d$ | m (metre) |
| Distance from slits to screen | $x$ | m (metre) |

Since the approximation is based on small angles, we generally apply this to $\Delta y$ close to the centre, where light from both slits are deflected by a small angle.

# Important Notes

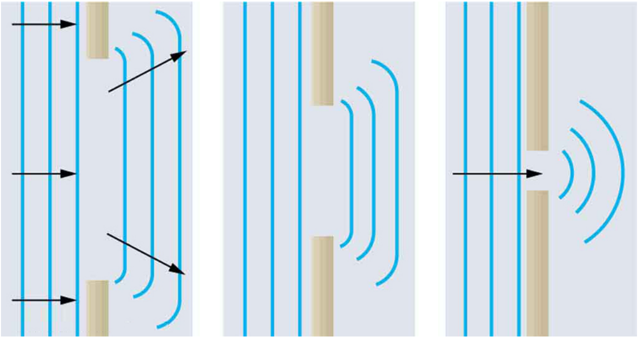
- We have applied the double-slit problem specifically to light, but it can be applied to any wave (e.g. ocean waves) as well
- The "slits" don't actually need to be slits; any point source will do
- The projection/screen doesn't need to be a real screen either; it just has to be a line where wave intensity can be measured

# Diffraction of Waves

When a wave goes through an small opening, it **diffracts**. This happens with sound waves, ocean waves. . . and light.


(The photo is from the Port of Alexandria in Egypt. The shape of the entire harbour is created because of diffraction of ocean wave.)

# Diffraction of Waves



The smaller the opening (compared to the wavelength of the incoming wave) the greater the diffraction effects.

# There are Two Types of Diffraction

- **Fresnel diffraction**
  - “Near-field” diffraction
  - The distance between aperture and the projection is small
  - The short distance to the projection causes the diffraction pattern observed to differ in size and shape
- **Fraunhofer diffraction**
  - “Far-field diffraction”
  - The distance between the aperture and the projection is large
  - Will only focus on this form of diffraction in Physics 12 because the pattern is easier to understand

# Fresnel Number

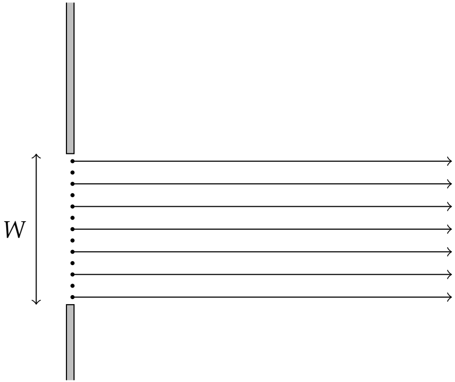
The Fresnel number tell us when to use Fresnel diffraction (difficult) and when to use Fraunhofer (easier):

$$F = \frac{W^2}{\lambda L}$$

| Quantity | Symbol | SI Unit |
|---|---|---|
| Fresnel Number | $F$ | (no units) |
| Characteristic length of the aperture | $W$ | m (metres) |
| Wavelength of light | $\lambda$ | m (metres) |
| Distance from aperture to projection | $L$ | m (metres) |

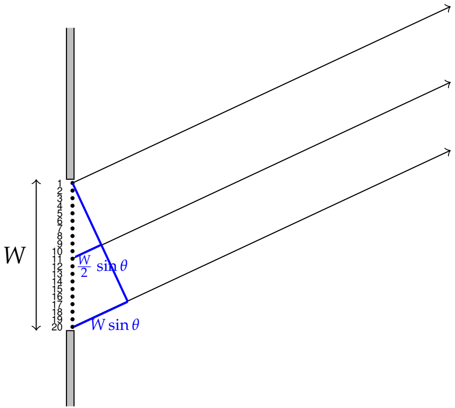
- Fresnel diffraction if $F \gg 1$; Fraunhofer diffraction if $F \ll 1$
- In Physics 12, we will only deal with Fraunhofer diffraction

# Let's Work This Out Again!




- Similar to the double-slit problem, we apply Huygens' Principle again
- Treat the slit as wide enough that there is an infinite series of point waves at the slit
- The light from the wavelet that travel perpendicular to the aperture will not interfere with one another
- i.e. a bright fringe at the middle called the **central maximum**.

# At Some Angle $\theta$




- Repeating the analyis as double-slit, we can find the path difference between the wavelet on the top (1) and bottom (20): $W \sin\theta$
- At some $\theta$, the path difference between 1 and 20 will be one wavelength ($\lambda$)
- In this case, the path difference between 1 and 11 is half of the wavelength (i.e. destructive interference) and they cancel each other
- Similarly, 2 cancels 12, 3 cancels 13. . . , resulting in **complete desctructive interference**

# Dark Fringes: Destructive Interference

Dark fringes exists on the screen at regular, whole-numbered intervals ($m = 1, 2, 3\ldots$):

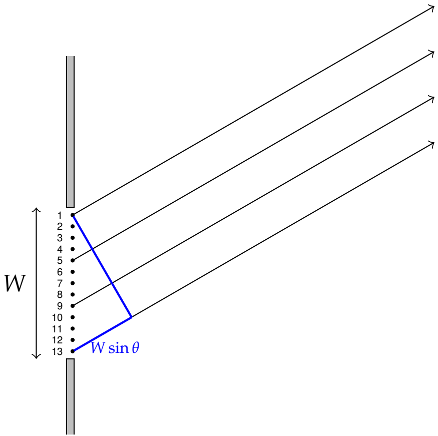
$$\boxed{\pm m\lambda = W \sin\theta_m}$$

Applying small-angle approximation equation, we end up with:

$$\boxed{y_m = \frac{m\lambda L}{W}}$$

**Pro-tip:** This equation looks very similar to the double-slit equation for *bright* fringes, so be *very* careful when you use them!

# At Some Other Angle $\theta$




- Again, we follow what we did with the the previous case, and we find that at some angle $\theta$, the path difference between the top and bottom is $W \sin \theta = \frac{3}{2}\lambda$
- Beam from (1) and (5) differ by $\frac{\lambda}{2}$, so they have destructive interference; similarly 2 and 6, 3 and 7, 4 and 8, 9 and 13 will all interfere destructively
- But some of the beams will not, so we have a bright fringe at the projection
- This bright fringe is not as bright as the central one because of the destructive interference

# Bright Fringes: Constructive Interference

Bright fringes exist on the screen at regular, half-numbered intervals ($m = 1, 2, 3\ldots$):

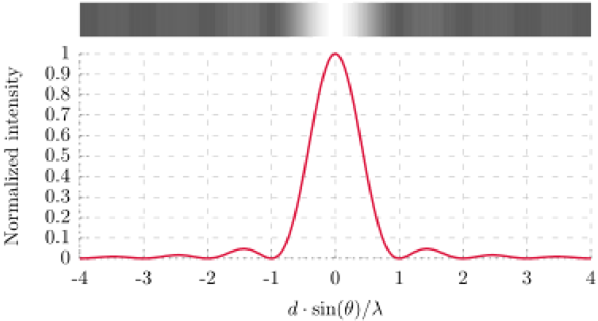
$$\boxed{\pm\left(m + \frac{1}{2}\right)\lambda = W\sin\theta_m}$$

Again, similar to the dark fringes, we apply our small-angle approximation equation:

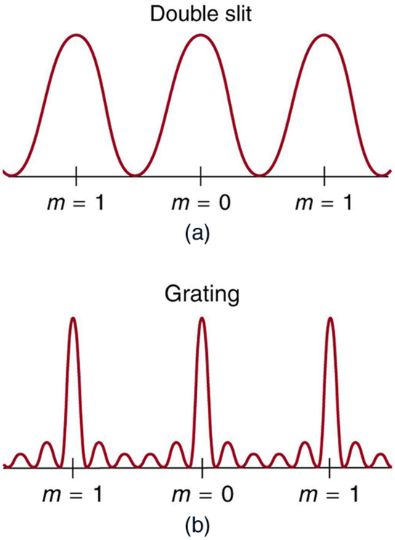
$$\boxed{y_m = \pm\left(m + \frac{1}{2}\right)\frac{\lambda L}{W}}$$

# Single-Slit Diffraction, A Summary

- Similar to the double-slit interference, single-slit diffraction projects a series of alternating bright fringes (“maxima”) and dark fringes (“minima”) in the far field
- The bright fringe in the middle (“central maximum”) is twice as wide and very bright
- Subsequent bright fringes on either side (“higher-order maxima”) are much dimmer because of the partial destructive interference



# Diffraction Grating: What if there are more than 2 slits?




- We can apply the same analysis from double-slit to a diffraction grating
- Use equation for double-slit interference to locate bright fringes

$$n\lambda = d \sin \theta_n$$

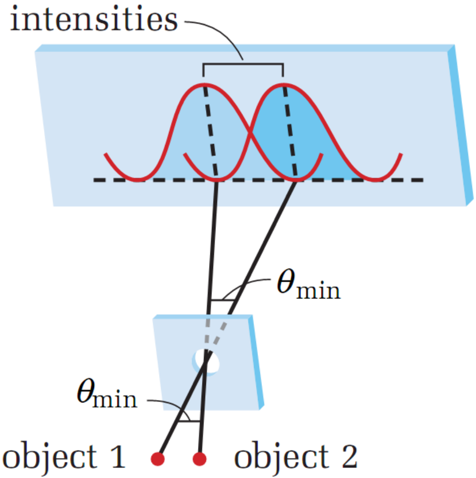
- Interference pattern is sharper
- Bright fringes are narrower

# Resolving Power

The ability of an optical instrument (e.g. the human eye, microscope, camera) to distinguish two distinct objects.

**WHY?** When light from any object passes through an “optical instrument”, it **diffracts**, therefore “blurring” the object.

# Resolving Power




**Rayleigh limit**: Two objects are resolved if the angle $\theta > \theta_{\mathrm{min}}$, where $\theta_{\mathrm{min}}$ is when the first minimum (dark fringe) from object 1 overlaps with the central maximum (bright fringe in the middle) from object 2.

# Resolving Power

In order to resolve two objects, the minimum angle between rays from the two objects passing through a rectangular aperture is the quotient of the wavelength and the width $W$ of the aperture. For a circular aperture, the minimum angle is the quotient of $1.22$ times the wavelength and the diameter $D$ of the aperture.

Rectangular aperture:

$$\boxed{\theta_{\text{min}} = \frac{\lambda}{W}}$$

Circular aperture:

$$\boxed{\theta_{\text{min}} = \frac{1.22\lambda}{D}}$$

**Note:** The angle $\theta_{\text{min}}$ is measured in **radians** not degree.

# Dispersion of Light Through Diffraction

The examples for single- and double-slit patterns that have all been based on a single wavelength of light, but we know that the equations depends on wavelength. So what happens to our diffraction pattern when the light source is a white light?