One-Dimensional Motion Graphs for Constant Acceleration

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THIS IS THE FIRST DRAFT OF A LONGER HANDOUT. AT THE MOMENT, THERE ARE STILL SOME INFORMATION MISSING.

In analyzing one-dimensional motion, particularly in experiments, we often use **motion graphs** to graphically express how motion quantities (position, velocity, acceleration) evolves in time. The basic motion graphs most familiar to physics students are:

- position vs. time (or displacement vs. time)
- velocity vs. time
- acceleration vs. time

Since velocity is the time derivative of position ($v = \dot{x}$), and acceleration is the time derivative of velocity ($a = \dot{v} = \ddot{x}$), the relationship between the graphs are straightforward: the velocity graph is the slope of the position graph, and the acceleration graph is the slope of the velocity graph. We can also use the area under the acceleration graph to find the change in velocity ($\int a(t)dt = \Delta v$) and the area under the velocity graph to find displacement ($\int v(t)dt = \Delta x$).

1 Graphs for Uniform Motion & Uniform Acceleration

For uniform motion (constant velocity), the basic motion graphs are shown in Figure 1. All the graphs are linear. Computing the slopes (derivatives) and areas (integrals) are straightforward exercises.

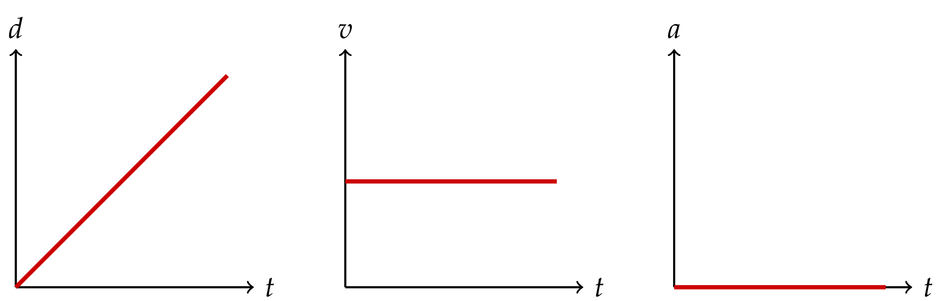

Figure 1: Position, velocity and acceleration are all linear functions of time for uniform motion.

For uniform acceleration, the position graph is a parabola, while velocity and acceleration graphs are linear, as shown in Figure 2. Computing the area under the velocity and acceleration graphs are still straightforward, as is finding the slope of the velocity graph, however, finding the acceleration using only the position graph is much more difficult.

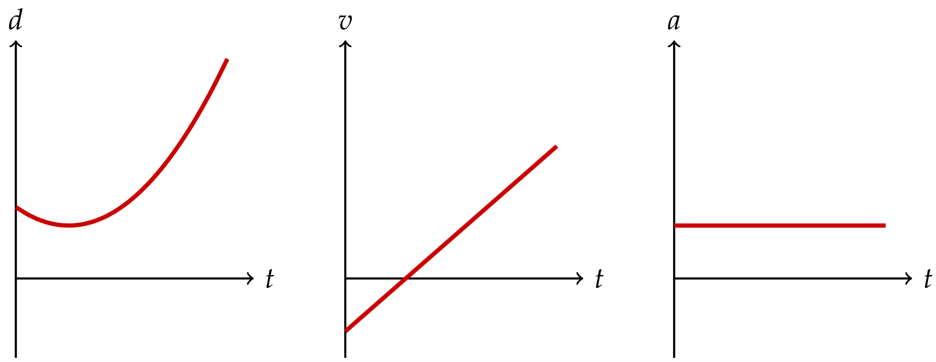

Figure 2: Displacement, velocity and acceleration as functions of time for uniformly accelerated motion.

2 Position vs. Time Squared

In many cases, the position graph in Figure 2 would be obtained by plotting experimental values of x and t. For example, and then making curve fit of the data, by making an educated guess that acceleration is uniform[1], and therefore the graph is a parabola. However, one of the weaknesses of this graph is the difficulty in determining the acceleration of the object by examining the graph itself. It is possible to take the second time derivative of the fitted curve to calculate the acceleration, which must be known algebraically in order to be plotted. Inevitably, some accuracy will be lost.

One way to get around the problem is that, *if initial velocity is zero*, i.e. $v_0 = 0$, then instead of plotting position x directly against time t, we re-interpret the the kinematic equation as a linear function in the form of $y = mx + b$:

$$\underset{\mathrm{y}}{x} = \underset{\mathrm{m}}{\left(\tfrac{1}{2}a\right)}\underset{\mathrm{x}}{t^2} + \cancel{v_0} t + \underset{\mathrm{b}}{x_0} \tag{1}$$

and plot x against the *square* of time, t^2. If acceleration is indeed uniform, then the graph would be linear. That constant acceleration is twice slope, i.e.:

$$m = \frac{1}{2}a \quad \rightarrow \quad a = 2m$$

The x-intercept of both graphs are still the initial position x_0. One example is shown in Figure 3. The x vs. t and the x vs. t^2 graphs both describe the same motion.

[1] This would have to be done through a thorough analysis of all the forces acting on an object.

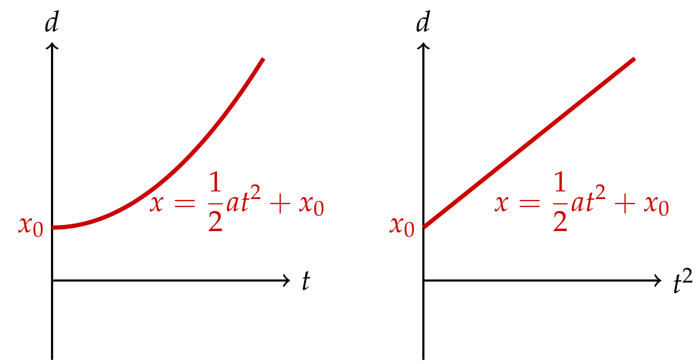

Figure 3: Position plotted against time t and time squared t^2 for uniformly accelerated motion.

3 Velocity Squared vs. Displacement

Likewise, some experimental equation is able to capture velocity information as an object moves in *space*. In other words, velocity and position information is available experimentally but not time, then instead of plotting velocity vs. time, position vs. time, we can plot velocity as a function of *position*. Again, even if acceleration is uniform, finding a from this graph is not straightforward. However, another option is to, like in the previous section, re-interpret the kinematic equation as a linear function:

$$\underset{y}{\underset{\uparrow}{v^2}} = \underset{\mathrm{b}}{\underset{\uparrow}{v_0^2}} + \underset{\mathrm{m}}{\underset{\uparrow}{(2a)}}\underset{\mathrm{x}}{\underset{\uparrow}{(x - x_0)}} \tag{2}$$

and plot the *square* of velocity (i.e. v^2) as a function of displacement $(x - x_0)$. If acceleration is uniform (constant a), the graph would be linear. The acceleration is half the slope m of the graph: $a = \frac{m}{2}$ and the y-intercept is the square of the initial velocity (i.e. v_0^2).