Dynamics

Now that we can mathematically describe the motion of any object, we have to be able describe *what* causes motion.

- Newton's three laws of motion

Newton's First Law

An object at rest or in uniform motion will remain at rest or in uniform motion unless acted on by a net external force.

- Uniform motion: constant velocity
- e.g. spacecraft in "deep space"
- e.g. hockey puck sliding on very smooth ice

Newton's Second Law

The sum of the forces acting on an object is proportional to its mass and its acceleration.

$$\boxed{\mathbf{F}_{\text{net}} = \Sigma\mathbf{F} = m\mathbf{a}}$$

Quantity	**Symbol**	**SI Unit**
Net force (sum of all forces)	$\mathbf{F}_{\text{net}}$	N
Mass	m	kg
Acceleration	$\mathbf{a}$	m/s^2

(Actually, we will find out later that this isn't exactly what Newton said. But it requires us to learn another concept first, which we will do in the next unit!)

Newton's Third Law

For every action there is an equal and opposite reaction.

For every action force on an object (B) due to another object (A), there is a reaction force which is equal in magnitude but opposite in direction, on object (A), due to object (B):

$$\boxed{\mathbf{F}_{\mathrm{A\ on\ B}} = -\mathbf{F}_{\mathrm{B\ on\ A}}}$$

The reaction forces act on different objects!

Example Problem

A Blast From the Past

Example 7: In old-style television picture tubes and computer monitors (cathode ray tubes), light is produced when fast-moving electrons collide with phosphor molecules on the surface of the screen. The electrons (mass $m = 9.1 \times 10^{-31}$ kg) are accelerated from rest in the electron "gun" at the back of the vacuum tube. Find the velocity of an electron when it exits the gun after experiencing an electric force of 5.8×10^{-15} N over a distance of 3.5 mm.

Forces

- **Force** is the interaction between the objects.
  - When there is interaction, then forces are created
  - A “push” or a “pull”
- Newton considered all forces acting at a single point of an object called the centre of gravity (“CG”)
  - The centre of gravity is also called the centre of mass (“CM”)
  - If the density of an object is constant, then the CG is also the geometric centre (centroid) of the object

Forces

- There are two types of forces:
  - **Contact forces** act between two objects that are in contact with one another
  - **Non-contact forces** act between two objects without them touching each other. They are also called "action-at-a-distance" force
- If the net force on an object is zero ($\Sigma\mathbf{F} = \mathbf{0}$) then the object is in a *state of equilibrium*
  - Dynamic equilibrium: the object is moving relative to us
  - Static equilibrium: the object is not moving relative to us

Common Forces

Common everyday forces that we encounter in Physics 12 include:

- Gravitational force (weight) $\mathbf{F}_g$
- Normal force $\mathbf{F}_N$
- Friction (static $\mathbf{F}_s$ and kinetic $\mathbf{F}_k$)
- Tension $\mathbf{F}_T$
- Applied force $\mathbf{F}_a$
- Air resistance (drag) $\mathbf{D}$
- Electrostatic force $\mathbf{F}_q$ (discussed in Unit 4)
- Magnetic force $\mathbf{F}_M$ (discussed in Unit 4)

Gravity

- The force of attraction between all objects with mass
- Near the surface of Earth:

$$\boxed{\mathbf{F}_g = m\mathbf{g}} \quad \text{where} \quad \mathbf{g} = 9.81\,\mathrm{m/s^2}\ [\text{down}]$$

- $\mathbf{F}_g$ always points **down**
- Newton's law of universal gravity:

$$\boxed{F_g = \frac{Gm_1m_2}{r^2}} \quad \text{where} \quad G = 6.67 \times 10^{-11}\,\mathrm{m^3/kg\,s^2}$$

Friction

- A force that opposes the sliding of two surface across one another
- Always act in a the direction opposite to motion or attempted motion
- Two types: *static friction* and *kinetic friction*

Static Friction

- The friction between the two surfaces when there is no relative motion between them
- Static friction increases with increasing applied force F_a. It is at maximum when the object is just about to move.

$$\boxed{\max F_s = \mu_s F_N}$$

- μ_s is the static friction coefficient (does not have a unit)
- F_N is the normal force
- I didn't use vector notation. This equation deals with the magnitude only

(**Pro tip:** The symbol for the coefficient of friction μ is the Greek letter *mu*)

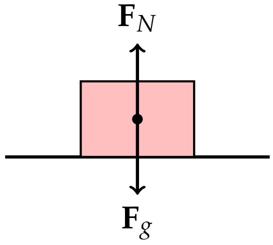
Kinetic Friction

$$\boxed{F_k = \mu_k F_N}$$

- The friction between two surfaces moving relative to each other
- F_k is constant along the path of movement as long as the normal force stays constant
- μ_k is the kinetic friction coefficient (does not have a unit). It is always lower than the static coefficient, otherwise nothing will ever move

$$\mu_k \leq \mu_s$$

Normal Force

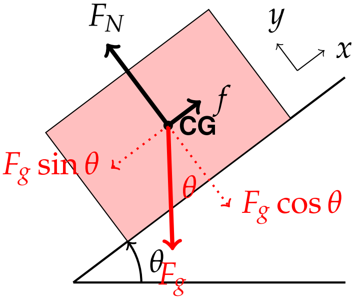


$$\mathbf{F}_g = m\mathbf{g} = -\mathbf{F}_N$$

- A force a surface exerts on another object that it is in contact with
- Always **perpendicular** to the contact surface
- **Special case:** When an object is on a horizontal surface with no additional applied force, the magnitude of the normal force is equal to the magnitude of the weight of the object, i.e. $F_N = F_g$

Normal Forces on a Slope

For this case, we label the x-axis to be along the slope, and y-axis to be perpendicular to the slope.



- If on a slope: $F_N = F_g \cos\theta$
    - F_N decreases as ramp angle θ increases
    - Obviously, at $\theta = 90^\circ$, $F_N = 0$!
- F_g has a component along the ramp $F_g \sin\theta$ that wants to slide the block down.
- Friction force f opposes the motion
    - Be careful: if the block is moving *up* the ramp with an applied force, then f will point *down* the ramp

Example Problem

Example 8: To move a $45\,\mathrm{kg}$ wooden crate across a wooden floor ($\mu = 0.20$), you tie a rope onto the crate and pull on the rope. While you are pulling the rope with a force of $115\,\mathrm{N}$, it makes an angle of 15° with the horizontal. How much time elapses between the time at which the crate just starts to move and the time at which you are pulling it with a velocity of $1.4\,\mathrm{m/s}$?

Example Problem

Example 9: You are holding an $85\,\mathrm{kg}$ trunk at the top of a ramp that slopes from a moving van to the ground, making an angle of 35° with the ground. You lose your grip and the trunk begins to slide.

- If the coefficient of friction between the trunk and the ramp is 0.42, what is the acceleration of the trunk?
- If the trunk slides $1.3\,\mathrm{m}$ before reaching the bottom of the ramp, for what time interval did it slide?

Example: Vertical Motion

Example 10: A $55\,\mathrm{kg}$ person is standing on a scale in an elevator. If the scale is calibrated in *newtons*, what is the reading on the scale when the elevator is not moving? If the elevator begins to accelerate upward at $0.75\,\mathrm{m/s^2}$, what will be the reading on the scale?

Tension in a Cable

Tension: The magnitude of the force exerted on and by a cable, rope, or string. How do engineers determine the amount of tension needed for a specific object (bridges, floors or light fixtures)?

- You can't push on a rope
- Assume the cable/rope/string to be mass less
- Force can change direction when used with pulleys

Example Problem

Example 11: An elevator filled with people has a total mass of $2245\,\mathrm{kg}$. As the elevator begins to rise, the acceleration is $0.55\,\mathrm{m/s^2}$. What is the tension in the cable that is lifting the elevator?

Applying Newton's Third Law on Connected Bodies

- Usually the objects are connected by a cable or a solid linkage with negligible mass
- All object have the same acceleration
- Require multiple free-body diagrams

Solving Connected-Bodies Problems

To solve a connected-bodies problem, you can follow these procedures:

1. Draw a FBD on each of the objects
2. Sum all the forces on all the objects along the direction of motion
   - Direction of motion are usually very obvious
   - All the tension forces should cancel, because they are “internal” forces and not “external forces”
3. Compute the acceleration of the entire system using Newton’s second law
   - Remember that every object has the same acceleration!
4. Go back to the FBD of each of the objects and compute the unknown forces (usually tension)

Connected Bodies: Example

Example 12: A tractor-trailer pulling two trailers starts from rest and accelerates to a speed of $16.2\,\mathrm{km/h}$ in $15\,\mathrm{s}$ on a straight, level section of highway. The mass of the truck itself (T) is $5450\,\mathrm{kg}$, the mass of the first trailer (A) is $31\,500\,\mathrm{kg}$, and the mass of the second trailer (B) is $19\,600\,\mathrm{kg}$.

- What magnitude of force must the truck generate in order to accelerate the entire vehicle?
- What magnitude of force must each of the trailer hitches withstand while the vehicle is accelerating?

For this problem we will assume that frictional forces are negligible in comparison with the forces needed to accelerate the large masses.

Example Problem: Atwood Machine

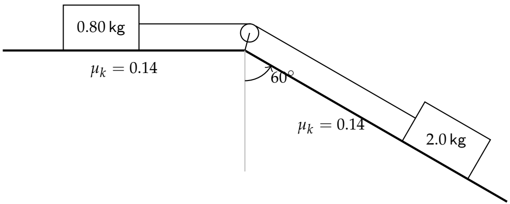
An **Atwood machine** is made of two objects connected by a rope that runs over a pulley. The pulley allows the direction of force and direction of motion to change between two objects.

Example 13: The object on the left (m_1) has a mass of $8.5\,\mathrm{kg}$ and the object on the right (m_2) has a mass of $17\,\mathrm{kg}$.

- What is the acceleration of the masses?
- What is the tension in the rope?

A More Typical Problem

Example 14: More typically, an Atwood machine problem is one where two objects are sliding on a surface. These surfaces may have (or may not) have friction. In this example, two blocks are connected by a mass-less string over a friction-less pulley as shown in the diagram.



(a) Determine the acceleration of the blocks.

(b) Calculate the tension in the string.

(c) If the string broke, for what minimum value of the coefficient of static friction would the $2.0\,\mathrm{kg}$ block not begin to slide?