Topic 10: Electric Field

Advanced Placement Physics

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Olympiads School

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Files for You to Download

Download from the school website:

1. `PhysAP-10-ElectricField.pdf`

Please download/print the PDF file before each class. There is no point copying notes that are already printed out for you. Instead, take notes on things I say that aren't necessarily on the slides. If you want to print on paper, I recommend printing 4 pages per side.

The Charges Are

Let's Review Some Basics

We already know a bit about charge particles:

- A **proton** carries a **positive** charge
- An **electron** carries a **negative** charge
- A *net charge* of an object means an excess of protons or electrons
- Similar charges are repel, opposite charges attract

We start with electrostatics:

- Charges that are not moving relative to one another

Coulomb's Law for Electrostatic Force

The **electrostatic force** (or **coulomb force**) between two point charges is given by:

$$\boxed{\mathbf{F}_q = -\frac{kq_1q_2}{r^2}\hat{\mathbf{r}}_{12}}$$

Quantity	Symbol	SI Unit
Electrostatic force	$\mathbf{F}_q$	N (newtons)
Coulomb's constant (electrostatic constant)	k	$\mathrm{N\,m^2/C^2}$
Point charges 1 and 2 (occupies no space)	q_1, q_2	C (coulombs)
Distance between point charges	r	m (meters)
Unit vector of direction between point charges	$\hat{\mathbf{r}}$	

$k = \dfrac{1}{4\pi\epsilon_0} = 8.99 \times 10^9\ \mathrm{N\,m^2/C^2}$ where $\epsilon_0 = 8.85 \times 10^{-12}\ \mathrm{C^2/N\,m^2}$ is called the "permittivity of free space"

Think Electric Field

We can get **electric field** by repeating the same procedure as with gravitational field, by grouping the variables in Coulomb's equation:

$$F_q = \underbrace{\left[\frac{kq_1}{r^2}\right]}_{E} q_2$$

We can say that charge q_1 creates an "electric field" (E) with an intensity

$$E = \frac{kq_1}{r^2}$$

The electric field $\mathbf{E}$ created by q_1 is a function ("vector field") that shows how it influences other charged particles around it

Electric Field Intensity Near a Point Charge

In vector form, the electric field a distance r away from a point charge q_s is:

$$\boxed{\mathbf{E} = \frac{kq_s}{r^2}\hat{\mathbf{r}}}$$

Quantity	Symbol	SI Unit
Electric field intensity	E	N / C
Coulomb's constant	k	N m^2 / C^2
Source charge	q_s	C
Distance from source charge	r	m
Outward unit vector from point source	$\hat{\mathbf{r}}$	

The direction of the field is radially outward from a positive point charge and radially inward toward a negative charge.

Think Electric Field

$\mathbf{E}$ *doesn't do anything* until another charge interacts with it. And when there is a charge q, the electrostatic force $\mathbf{F}_q$ that it experiences in the presence of $\mathbf{E}$ is:

$$\boxed{\mathbf{F}_q = \mathbf{E}q}$$

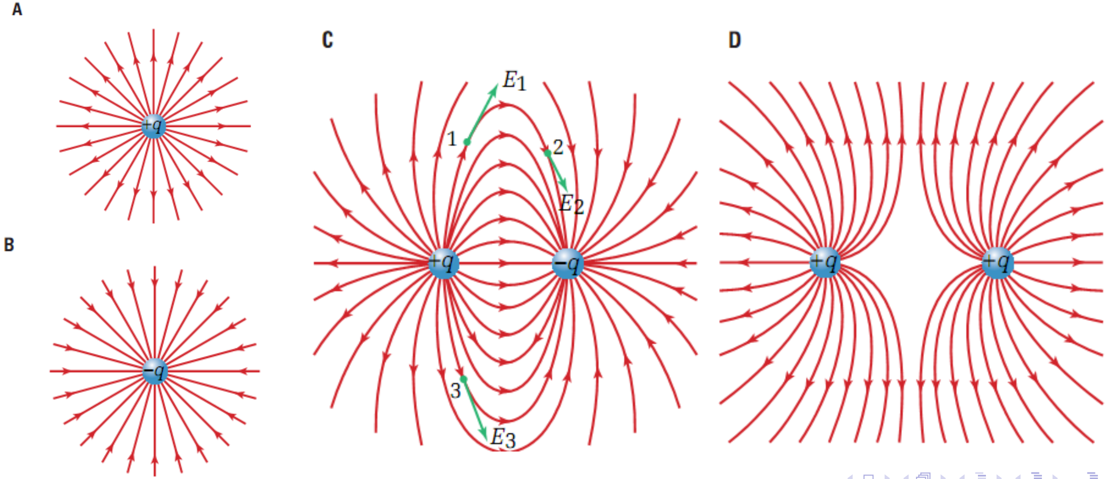
$\mathbf{F}_q$ and $\mathbf{E}$ are vectors, and following the principle of superposition, i.e.

$$\mathbf{F}_q = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \mathbf{F}_4 \ldots$$
$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 + \mathbf{E}_3 + \mathbf{E}_4 \ldots$$

This understanding is especially important when we want to find $\mathbf{F}_q$ and $\mathbf{E}$ some distance from a continuous distribution of charges

Electric Field Lines

If you place a positive charge in an electric field, the electrostatic force on the charge will be in the direction of the electric field.

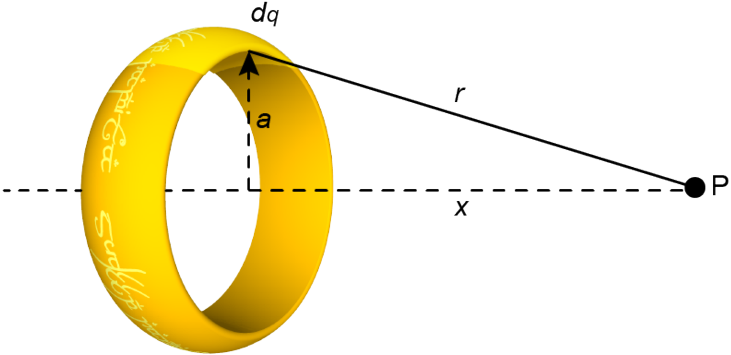


Not a Point Charge

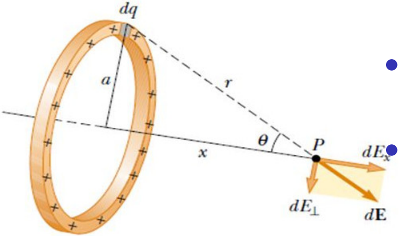
What if the charge configuration does not consist of point charges?

Lord of the Ring Charge

You've been given The One Ring To Rule Them All, and you found out that it is charged! What is its electric field at point P along its axis?



Electric Field Along Axis of a Ring Charge



- Separate the electric field $d\mathbf{E}$ from charge dq into axial (dE_x) and radial ($dE_\perp$) components
- Based on symmetry, $dE_\perp$ does not contribute to anything; but finding dE_x is straightforward:

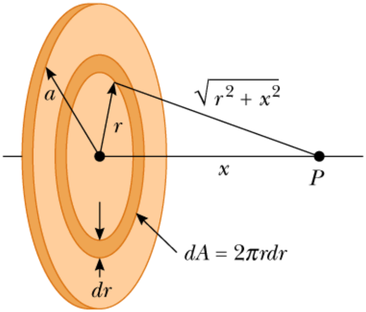
$$dE_x = \frac{kdq}{r^2}\cos\theta = \frac{kdq}{r^2}\frac{x}{r} = \frac{kxdq}{(x^2+a^2)^{3/2}}$$

Integrating this over all charges dq, we have:

$$E_x = \frac{kx}{(x^2+a^2)^{3/2}}\int dq = \boxed{\frac{kQx}{(x^2+a^2)^{3/2}}}$$

Electric Field Along Axis of a Uniformly Charged Disk

Let's extend what we know to a disk of radius a and charge density σ



- We start with the solution from the ring problem, and replace Q with $dq = 2\pi\sigma a da$:

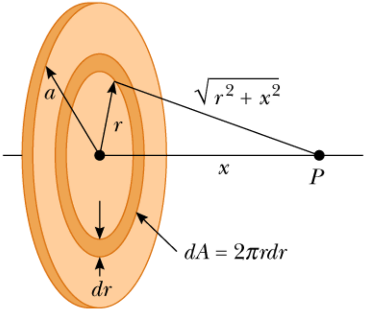
$$dE_x = \frac{2\pi k x \sigma a da}{(x^2 + a^2)^{3/2}}$$

- Integrating over the entire disk:

$$E_x = \pi k x \sigma \int \frac{2a da}{(x^2 + a^2)^{3/2}}$$

This is not an easy integral!

Eclectic Field Along Axis of a Uniformly Charged Disk



- Luckily for us, the integral is in the form of $\int u^n du$, with $u = x^2 + a^2$ and $n = \frac{-3}{2}$.
- You can find the integral in any math textbook:

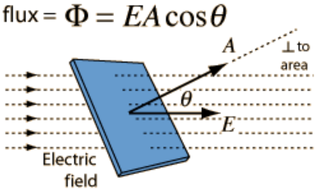
$$\boxed{E_x = 2\pi k\sigma \left(1 - \frac{x}{\sqrt{x^2 + R^2}}\right)}$$

Flux

Flux is an important concept in many disciplines in physics. The flux of a vector quantity $\mathbf{X}$ is the amount of that quantity flowing through a surface. In integral form:

$$\Phi = \int \mathbf{X} \cdot d\mathbf{A} \quad \text{or} \quad \Phi = \int (\mathbf{X} \cdot \hat{\mathbf{n}}) dA$$

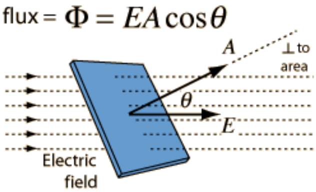
The direction of the infinitesimal area $d\mathbf{A}$ is **outward normal** to the surface.



Flux

Φ can be something physical, like water, or bananas, or something abstract, like electric field (which is what we are interested in). We can compute a flux as long as there is a vector field i.e. $\mathbf{X} = \mathbf{X}(x, y, z)$. In the case of **electric flux**, the quantity $\mathbf{X}$ is just the electric field, i.e.:

$$\Phi_q = \int \mathbf{E} \cdot d\mathbf{A}$$



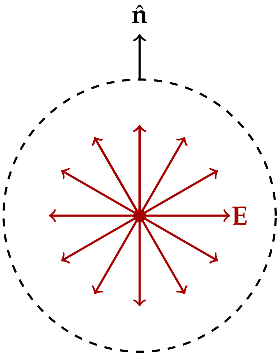
Electric Flux and Gauss's Law

Gauss's law tells us that if we have a closed surface (think of the surface of a balloon), the total electric flux is very well defined:

$$\boxed{\Phi_{\text{total}} = \oint \mathbf{E} \cdot d\mathbf{A} = \frac{Q_{\text{encl}}}{\epsilon_0}}$$

where Q_{encl} is the charge enclosed by the surface, and $\epsilon_0 = 8.85 \times 10^{-12}\ \mathrm{C^2/N\,m^2}$ is the permittivity of free space. That closed surface is called a **Gaussian surface**.

Electric Field from a Positive Point Charge



By symmetry, electric field lines are radially outward from the charge, so the integral reduces to:

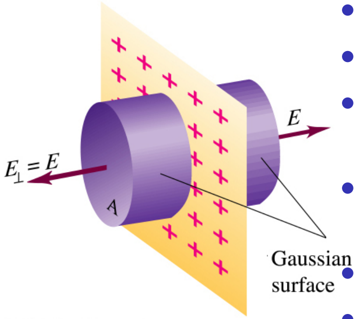
$$\Phi = \oint \mathbf{E} \cdot d\mathbf{A} = EA = \frac{q}{\epsilon_0}$$

Since area of a sphere is $A = 4\pi r^2$, we recover Coulomb's law:

$$E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2} = \frac{kq}{r^2}$$

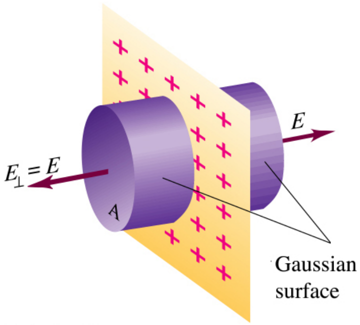
In fact, it was through studying point charges that Gauss's law was discovered, so it should not be a surprise that they agree.

Electric Field Near an Infinite Plane of Charge



- Charge density (charge per unit area) σ
- By symmetry, $\mathbf{E}$ must be perpendicular to the plane
- Our Gaussian surface is a cylinder shown in the left with an area A; the height of the cylinder is unimportant
- We can see that nothing "flows out" of the side of the cylinder, only at the ends.
- The total flux is $\Phi = E(2A)$
- The enclosed charge is $Q_{\text{encl}} = \sigma A$.

Electric Field Near an Infinite Plane of Charge



Gauss's law simplifies to:

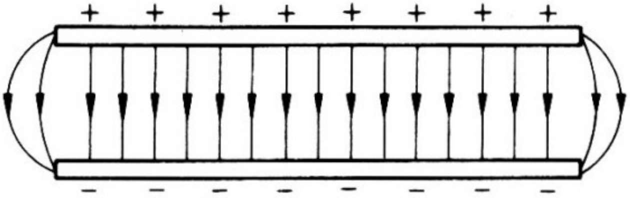
$$\oint \mathbf{E} \cdot d\mathbf{A} = \frac{Q_{\mathrm{encl}}}{\epsilon_0} \quad \rightarrow \quad E(2A) = \frac{\sigma A}{\epsilon_0}$$

Solving for E, we get:

$$\boxed{E = \frac{\sigma}{2\epsilon_0}}$$

- E is a constant
- Independent of distance from the plane
- Both sides of the plane are the same

Electric Field between Two Infinite Parallel Plates



- Two plates, each producing an electric field pointing in the same direction
- The total electric field is twice the value that we found on the last slide

$$\boxed{E = \frac{\sigma}{\epsilon_0}}$$