Topic 22: Wave-Particle Duality

Advanced Placement Physics
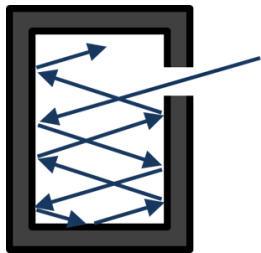
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Olympiads School

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Anyone who is not shocked by quantum theory has not understood it.

- Niels Bohr

Blackbody Radiation

- An idealized physical object that absorbs all incident electromagnetic radiation, regardless of frequency or angle of incidence

- A blackbody is in thermal equilibrium, i.e. all absorbed radiation (energy) is immediately radiated back

- A blackbody at room temperature appears black, as most of the energy it radiates is infrared and cannot be perceived by the human eye

- Thermal radiation spontaneously emitted by many ordinary objects can be approximated as blackbody radiation

- The concept was coined by Gustav Kirchkoff in 1860.

Raleigh-Jeans Law and the Ultraviolet Catastrophe

- Based on classical thermodynamics

$$P(\lambda, T) = 8\pi kT\lambda^{-4}$$

T=temperature, λ=wavelength, k=Boltzmann's constant

- Agrees with experimental results for long wavelengths

- But disagrees violently for short wavelengths:
  - Shorter wavelengths (e.g. ultraviolet waves) seem to have infinite intensity
  - Known as **"Ultraviolet catastrophe"**
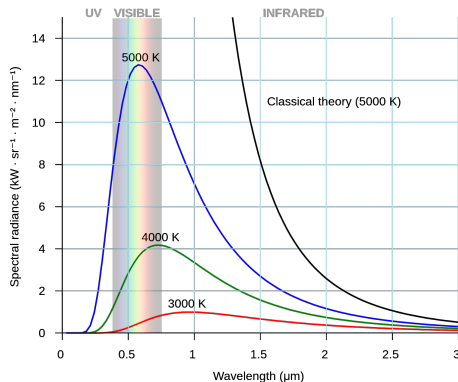
"Quantization" of Energy

Max Planck

- Made a strange modification in the classical calculations
- Derived a function of $P(\lambda, T)$ that agreed with experimental data for all wavelengths
- First found an empirical function to fit the data
- Then searched for a way to modify the usual calculations
- Energy emitted by blackbody not continuous but discrete
- When energy is emitted from the harmonic oscillator, it drops to the next lower energy level
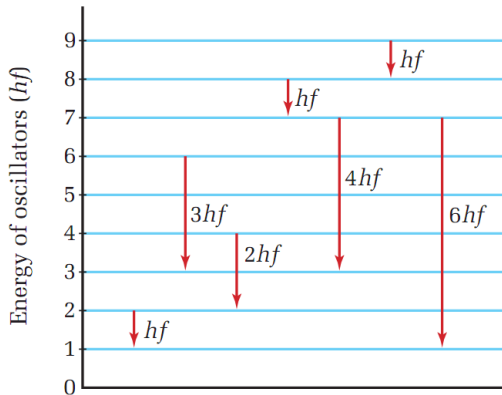
Quantization of Energy

$$E = hf$$

where

- $h = 6.626 \times 10^{-34}$ J s is "Planck's constant", and
- f is the frequency

When a blackbody emits radiation, it has to drop down one or more levels and emit a unit of energy equal to the difference between the allowed energy levels of the oscillator.

Planck's Law

As for his formula, it's called **Planck's law**:

$$P(\lambda, T) = \frac{2hc^2}{\lambda^5} \frac{1}{e^{\frac{hc}{\lambda k_B T}} - 1}$$

Maxwell's Equations in a Vacuum

$$\nabla \cdot \mathbf{E} = 0$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$
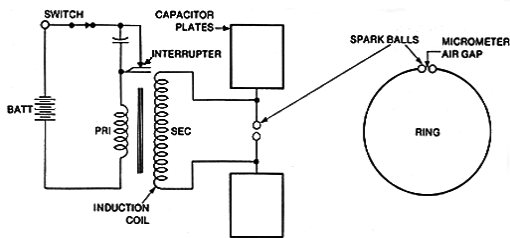
$$\nabla \times \mathbf{B} = \mu_o \varepsilon_o \frac{\partial \mathbf{E}}{\partial t}$$

Disturbances in $\mathbf{E}$ and $\mathbf{B}$ travel as an "electromagnetic wave", with a speed:

$$c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} = 299\,792\,458\,\text{m/s}$$

The Spark Gap Experiment

To prove that light is a electromagnetic in nature, German physicist Heinrich Hertz devised a "spark gap experiment" to generate frequencies in the range of 10^{14} Hz

- Also showed that light has the same wavelengths as predicted by Maxwell's equations
- Discovered **photoelectric effect** that was caused by ultraviolet radiation
- Physicist who repeated his experiments did not have an explanation

Photoelectric Effect

When electromagnetic waves (e.g. light) hits certain metals, electrons are knocked off the surface
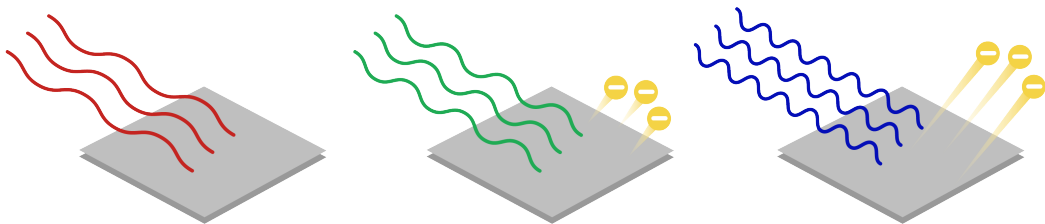
- Increasing intensity of light knocked off more electrons, but doesn't change their kinetic energy, but
- Changing the frequency of the light did change K though, although
- Below a certain frequency, *no* electrons were emitted

The Photon: Packets of Energy

Einstein argued that light is not a continuous wave, but a collection of discrete energy packets (photons), each with energy $E = hf$

$$K_{\max} = \begin{cases} hf - \varphi & \text{if } hf > \varphi \\ 0 & \text{otherwise} \end{cases}$$

Quantity	Symbol	SI Unit
Maximum kinetic energy of	$K_{\max}$	J
Planck's constant	h	J s
Frequency of the EM wave	f	Hz
Work function of the metal	φ	J

Einstein may have been alerted to the fact that the blackbody radiation curve resembles the distribution of energies in a gas.

Work Function
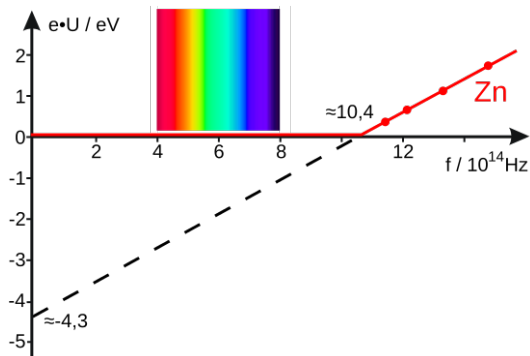
Work function the minimum energy required to remove an electron from a solid to a point immediately outside the solid surface. The minimum frequency at which electrons will be ejected is called the **threshold frequency**.

Slope is h no matter what metal it is.

Work Functions of Different Materials

The work function φ depends on the metal. They are generally presented in electron volts.

| Metal	Work function (eV)
aluminum	4.28
calcium	2.87
cesium	2.14
copper	4.65
iron	4.50
lead	4.25
lithium	2.90
nickel	5.15
platinum	5.65
potassium	2.30
tin	4.42
tungsten	4.55
zinc	4.33

Compton Scattering

American physicist Arthur Holly Compton studied x-ray scattering by free electrons

- Classical theory cannot account for the scattering behaviour
- Frequency shift only depends on scattering angle
- Prediction possible if treating the x-ray as photons with momentum, just like a particle. Compton used Einstein's invariant and applied to a massless particle:

$$p = \frac{E}{c} = \frac{hf}{c} = \frac{h}{\lambda}$$

Compton Scattering
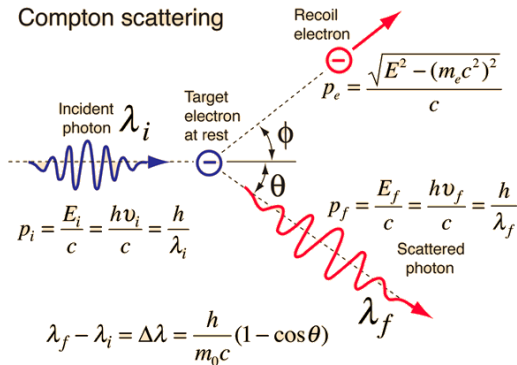
If we treat the x-ray as a photon with momentum $p = h/\lambda$ then we can use Newton's laws of motion to predict both the recoil electron and scattered x-ray!

Momentum of a Photon

The momentum of a photon is proportional to Planck's constant and inversely proportional to its wavelength.

$$p = \frac{h}{\lambda}$$

Quantity	Symbol	SI Unit
Momentum	p	kg m / s
Planck's constant	h	J s
Wavelength	λ	m

This is a very odd expression, which treats photon both as a particle (with momentum) and a wave (with a wavelength λ).

Matter Waves

If electromagnetic waves are really particles of energy, then are particles (e.g. electrons) a wave of some sort?

- The De Broglie hypothesis in 1924: a particle can also have a wavelength
- Confirmed accidentally by the Davisson-Germer Experiment in 1927 (beam of electron scattering on nickel crystal surface)

If a particle is also a wave, what *kind* of a wave is it then?

Electron Interference

If I perform a double-slit experiment with a beam of electrons, will I get an interference pattern?
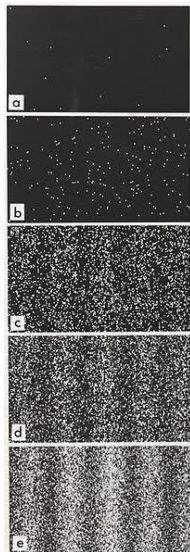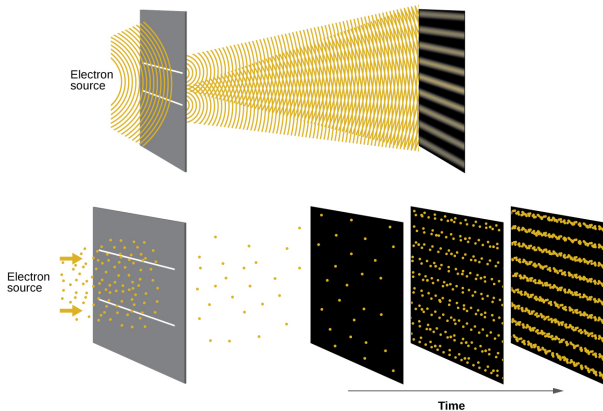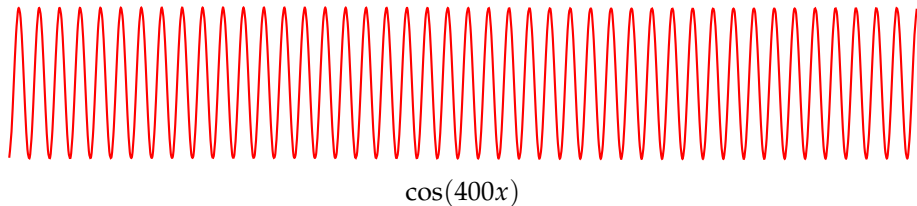
De Broglie Wavelength

If matter is also a wave, then what would be its wavelength? Solving the momentum equation for λ:

$$p = \frac{h}{\lambda} \quad \rightarrow \quad \lambda = \frac{h}{p} \quad \rightarrow \quad \boxed{\lambda = \frac{h}{mv}}$$

Quantity	Symbol	SI Unit
Wavelength of a particle	λ	m
Planck's constant	h	J s
Mass	m	kg
Velocity	v	m/s

Uncertainty Principle

If a particle is a wave, how do you tell where it is?

$$\cos(400x)$$

- This wave has a single value of wavelength λ (therefore momentum p)
- This wave has no distinguishing features that can tell you its location x
- When we have precise knowledge of the particle wave's momentum (i.e. its wavelength), then we have no idea where it is.

Uncertainty Principle

If a particle is defined as waves with small variations of wavelengths, when we add up the different waves together, we begin to see a **packet**
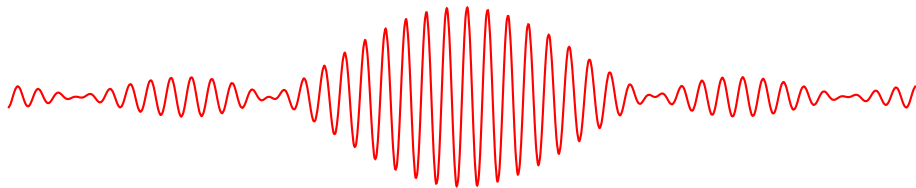
$$\cos(380x) + \cos(382.5x) + \cdots + \cos(400x) + \cos(402.5x) + \cdots + \cos(420x)$$

In order to know more about the location of the particle, we *must* lose information about its momentum.

Heisenberg Uncertainty Principle

Because of the wave properties of particles, you can never be completely certain of the relationship between an object's momentum p and position x:

$$\sigma_p \sigma_x \geq \frac{h}{4\pi}$$

Quantity	Symbol	SI Unit
Uncertainty in momentum	σ_p	kg m / s
Uncertainty in position	σ_x	m
Planck's constant	h	J s

The more you know about an object's position, the less you know about its momentum, and vice versa.

Bohr Atomic Model

The "orbital" model of electrons does not work, because as the electron orbits (accelerates around) the nucleus, it radiates electromagnetic radiation, and therefore lose energy. The orbit will eventually collapse.

Bohr postulated that electron can move in certain "non-radiating" orbits, corresponding to energy levels:

$$E_n = -\frac{k^2 e^4 m}{2\hbar^2} \frac{Z^2}{n^2}$$

From the wave-particle duality perspective, the "orbits" correspond more to a standing wave around the nucleus (a standing wave does not lose energy)

Bohr Atomic Model

$$E_n = -\frac{k^2 e^4 m}{2\hbar^2} \frac{Z^2}{n^2}$$

Quantity	Symbol	SI Unit
Energy at level n	E_n	J
Coulomb's constant	k	$N\,m^2/C^2$
Elementary charge	e	C
Atomic mass	m	kg
Reduced Planck's constant	$\hbar$	J s (joule seconds)
Atomic number	Z	integer; no units
Energy level	n	integer; no units

Bohr Atomic Model

Successful in describing the behaviour of the hydrogen atom—but fails for heavier atoms—although it still relies on

- Coulomb forces between electrons and protons (classical)
- Centripetal forces (classical)
- Quantization of energy (new physics!)

De Broglie's hypothesis gives us a glimpse of what Bohr is missing

- The "orbits" correspond to a standing wave around the nucleus
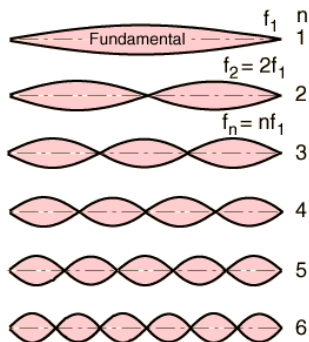- A standing wave does not lose energy

Hydrogen Emission

- Lyman series:
  - the EM emissions when the electrons drop from a higher energy state (E_n) to the ground state $n = 1$ (i.e. E_1)
  - The frequency is given by:

  $$f = \frac{E_1 - E_n}{h}$$

  - We can apply universal wave equation to get the wavelengths

- Balmer series–dropping to E_2
- Paschen series–dropping to E_3
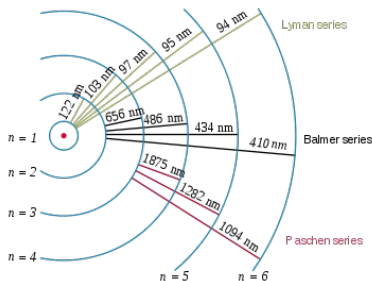
Standing Wave on a String

- We have studied standing waves in Grade 11
- If electron is to be in a "stable orbit" around a nucleus, it has to be in a standing wave pattern
- Otherwise, it will interfere with itself

Circular Standing Wave

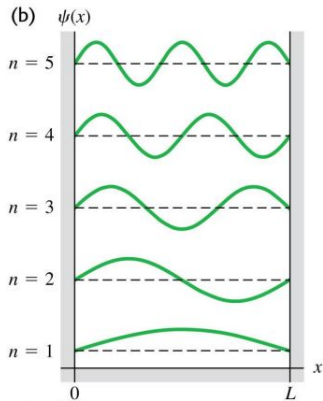Electron resonance states $n = 3, 4, 5, 6$

Particle in a Box

A particle in a 1D box has to behave like a standing wave. The resonance modes (frequencies where a stable standing wave exists) correspond to the wavelengths:

$$\lambda = \frac{2L}{n} \quad n = 1, 2, 3, 4, \ldots$$

and the momentum of the particle is:

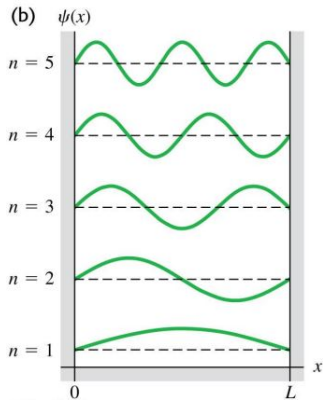$$p = \frac{h}{\lambda} = \frac{nh}{2L}$$

Particle in a Box

(b) $\psi(x)$

$n = 5$

$n = 4$

$n = 3$

$n = 2$

$n = 1$

$0 \qquad L$

Kinetic energy of the particle can be expressed in terms of momentum:

$$E_n = \frac{1}{2}mv^2 = \frac{p^2}{2m} = \frac{n^2h^2}{8mL^2}$$

If a particle is a standing wave, then kinetic energy of the particle can never be zero, as long as it is confined inside the box, therefore

- It cannot have zero velocity
- The lowest energy level ($n = 1$) is called the **zero-point energy**

Example

Example 3: A 0.150 kg billiard ball is confined to the pool table 1.42 m wide. How long (in seconds) will it take to travel from one side of the table to the other? (Use fundamental mode.)