WELCOME TO AP PHYSICS

Pre-requisites

- **Physics 11 and 12** You will need to be comfortable with the topics covered in high-school physics courses.
- **Calculus** The AP Physics C exams are calculus based, and you will be required to perform basic differentiation and integration. You don't need to be an expert, but basic knowledge is required. Differentiation and integration in the course are generally not difficult, but there are occasional challenges.
- **Vectors** You need to be comfortable with vector operations, including addition and subtraction, multiplication and division by constants, as well as dot products and cross products.

The AP Physics Exams

There are 4 AP Physics exams:

- Physics 1
- Physics 2
- Physics C–Mechanics
- Physics C–Electricity and Magnetism

Offered in first or second week of May of each year

Classroom Rules

Same as in Physics 11 and 12

- Treat each other with respect
- Raise your hands if you have a question. Don't wait too long
- E-mail me at `tleung@olympiadsmail.ca` for any questions related to physics and math and engineering
- Do ***not*** try to find me on social media

Topic 1: Introduction & Kinematics

Advanced Placement Physics

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Olympiads School

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Files for You to Download

- `PhysAP-courseOutline.pdf`—The course outline that I am handing to you now.
- `PhysAP-equationSheet.pdf`—An equation sheet that you will be using during the exams.
- `PhysAP-01-kinematics.pdf`—The slides that I am using right now.
- `PhysAP-02-dynamics.pdf`—The slides that I will be using next class.
- `PhysAP-03-Homework.pdf`—Homework problems for Topics 1 & 2.

Please download/print the PDF file for the class slides before each class. There is no point copying notes that are already printed out for you. Instead, take notes on things I say that aren't necessarily on the slides. If you wish to print the slides, we recommend printing 4 slides per page.

Notes on Writing Vectors

In this course, vectors are *printed* using a bold face font, while the “arrow on top” notation is used when *writing*.

In print (books, journal papers)

$$\mathbf{v} \quad \mathbf{F}_g \quad \mathbf{p} \quad \mathbf{I}$$

Handwritten (used by some books)

$$\vec{v} \quad \vec{F}_g \quad \vec{p} \quad \vec{I}$$

The magnitude of these vectors are expressed in one of two ways:

With absolute-value sign

$$|\mathbf{v}| \quad |\mathbf{F}_g| \quad |\mathbf{p}| \quad |\mathbf{I}|$$

Or as a scalar

$$v \quad F_g \quad p \quad I$$

Writing Vectors

In Physics 11 and 12, vectors are often written by separating the magnitude from the direction, e.g. a velocity vector can be written as:

$$\mathbf{v} = 4.5\,\mathrm{m/s}\ [\mathrm{N}\ 55^\circ\ \mathrm{E}]$$

- Intuitive for describing *one* vector in 2D
- Complicated to describe direction when extended into 3D
- Difficult to perform vector arithmetic

IJK Vector Notation

Vectors in 2D/3D Cartesian space are generally written in their x, y & z components using the "IJK notation":

$$\mathbf{A} = A_x\hat{\boldsymbol{\imath}} + A_y\hat{\boldsymbol{\jmath}} + A_z\hat{\boldsymbol{k}}$$

- $\hat{\imath}$, $\hat{\jmath}$ and $\hat{k}$ are **basis vectors** indicating the directions of the x, y and z axes. Basis vectors are **unit vectors** (i.e. length 1).
- The IJK notation does not give the magnitude of the vector, which needs to be calculated:

$$A = |\mathbf{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

Vector Addition and Subtraction

Adding and subtracting vectors is straightforward:

$$\mathbf{A} \pm \mathbf{B} = (A_x \pm B_x)\hat{\boldsymbol{\imath}} + (A_y \pm B_y)\hat{\boldsymbol{\jmath}} + (A_z \pm B_z)\hat{\boldsymbol{k}}$$

Dot Product

The **dot product** is the scalar multiplication of two vectors. It is determined by the magnitude of the two vectors and the cosine of the angle between them:

$$C = \mathbf{A} \cdot \mathbf{B} = \mathbf{B} \cdot \mathbf{A} = AB\cos\theta$$

- You have been using this in the calculation of mechanical work
- C is the *projection* of the vector $\mathbf{A}$ onto $\mathbf{B}$, or the component of $\mathbf{A}$ along $\mathbf{B}$
- $\hat{\imath} \cdot \hat{\imath} = 1$, $\hat{\jmath} \cdot \hat{\jmath} = 1$, and $\hat{k} \cdot \hat{k} = 1$
- For vectors written in IJK notation, and you don't immediately know the magnitude or the angle between them, then:

$$\mathbf{A} \cdot \mathbf{B} = A_x B_x + A_y B_y + A_z B_z$$

Cross Products

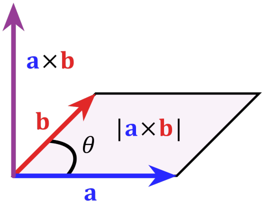
The **cross product** is the vector multiplication of two vectors:

$$\mathbf{C} = \mathbf{A} \times \mathbf{B}$$

- The magnitude of the cross product is determined by the magnitude of $\mathbf{A}$ and $\mathbf{B}$ and the angle between them:

$$C = AB \sin\theta$$

- $\mathbf{C}$ is perpendicular to both $\mathbf{A}$ and $\mathbf{B}$; its direction given by the right hand rule
- Used in extensively in rotational motion and in electromagnetism



Cross Products

The cross product of any two vectors in 3D space is the determinant of this 3×3 matrix:

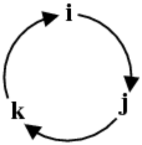
$$\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

The order of the cross product is important. (This is why you have to get the right hand rule correctly.)

$$\mathbf{A} \times \mathbf{B} = -\mathbf{B} \times \mathbf{A}$$

Cross Product

Most cross products applications in AP Physics are simpler, so we only have to remember this circle:



- The direction of the arrow gives the index of the cross product (e.g. $\hat{\imath} \times \hat{\jmath} = \hat{k}$)
- Going against the direction of the arrow gives the negative of the next index (e.g. $\hat{k} \times \hat{\jmath} = -\hat{\imath}$)

Calculus is Invented for Physics

Thanks, Issac Newton

- We cannot learn physics properly without calculus
- Calculus was “invented” so that we can understand motion, especially non-constant velocities and accelerations
- You may have already noticed that a lot of the word problems in calculus are really physics problems

Differentiation and Integration

- **Differential Calculus**
  - How quickly something is changing (“rate of change” of a quantity)
  - Math: slopes of functions
  - Physics: how quickly a physical quantity is changing in time and/or space
  - Examples: velocity (how quickly position changes with time), acceleration (how quickly velocity changes with time), power (how quickly work is done), electric fields (how electric potential changes in space)
- **Integral Calculus**
  - The opposite of differentiation
  - We use it to compute the area under a curve, or
  - Summation of many small terms
  - Examples: area under the $\mathbf{v}$-t graph (displacement), area under the $\mathbf{F}$-t graph (impulse), area under the F-d graph (work)

Derivative

For any arbitrary function $f(x)$, the derivative with respect to ("w.r.t.") x is:

$$\boxed{f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}}$$

The "limit as h approaches 0" is the mathematical way of making h a very small number

Know the Tricks for Differentiation

The derivative of a constant ("C") w.r.t. any variable is zero. (Obviously, the slope of any function $f(x) = C$ is zero.)

$$\frac{dC}{dx} = 0$$

A constant multiple of any function f can be factored outside the derivative:

$$\frac{d}{dx}(af) = a\frac{df}{dx}$$

The derivative of a sum is the sum of a derivative:

$$\frac{d}{dt}(f(t) + g(t)) = \frac{df}{dt} + \frac{dg}{dt}$$

Time-Saving Rules for Differentiation

Power Rule:

$$\frac{d}{dt}\left(t^n\right) = nt^{n-1}$$

Product Rule:

$$\frac{d}{dx}\left(f(x)g(x)\right) = f'(x)g(x) + f(x)g'(x)$$

Chain Rule:

$$\frac{d}{dx}f\left(g(x)\right) = f'(g(x))g'(x)$$

Time-Saving Rules for Differentiation

Quotient Rule:

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)g(x) - g'(x)f(x)}{(g(x))^2}$$

Elementary Derivatives

Note how sines and cosines are related:

$$\frac{d}{dt}\sin t = \cos t \qquad \frac{d}{dt}\cos t = -\sin t$$

And exponential:

$$\frac{d}{dt}e^t = e^t$$

Partial Derivatives

For functions with many variables, for example, gravitational potential energy U_g has three variables: masses m_1 and m_2 and the distance r between them:

$$U_g(m_1, m_2, r) = -\frac{Gm_1m_2}{r}$$

Differentiating w.r.t. one variable while holding others constant gives its **partial derivative**. (We use the ∂ symbol). e.g. the partial derivative of U_g w.r.t. r is

$$\frac{\partial U_g}{\partial r} = \frac{Gm_1m_2}{r^2}$$

(By the way, this is how we relate U_g to F_g.)

Integration

If $F(x)$ is the anti-derivative of $f(x)$, they are related this way:

$$\frac{d}{dx}F(x) = f(x) \quad \longrightarrow \quad F(x) = \int f(x)dx$$

The mathematical proof is the **fundamental theorem of calculus.**

Common Integrals in Physics

Integration, while often necessary, can be very daunting, but integrals in AP Physics are generally straightforward. These rules should help in most cases:

- Power rule in reverse:

$$\int x^n dx = \frac{1}{n+1}x^{n+1} + C$$

- Natural logarithm:

$$\int \frac{1}{x} dx = \ln|x| + C$$

Common Integrals in Physics

- Sines and cosines:

$$\int \cos x dx = \sin x + C$$
$$\int \sin x dx = -\cos x + C$$

Definite vs. Indefinite Integral

This Should Be a Review

- Integral can be either **indefinite** or **definite**
- An "indefinite" integral is another function, e.g. position $\mathbf{x}(t)$ as a function of time is found by integrating velocity $\mathbf{v}(t)$:

$$\mathbf{x}(t) = \int \mathbf{v}(t)dt = \cdots + \mathbf{C}$$

- A *constant of integration* $\mathbf{C}$ is added to the integral $\mathbf{x}(t)$. It is obtained through applying "initial condition" to the problem.

Definite Integrals

A **definite** integral has lower and upper bounds. e.g., given $\mathbf{v}(t)$, the displacement between t_1 and t_2 can be found:

$$\Delta\mathbf{x} = \int_{t_0}^{t_1} \mathbf{v}(t)dt$$

Once we have computed the integral, we evaluate the limits:

$$\Delta\mathbf{x} = \mathbf{x}(t)\Big|_{t_0}^{t_1} = \mathbf{x}(t_1) - \mathbf{x}(t_0) = \mathbf{x}_1 - \mathbf{x}_0$$

The constant of integration $\mathbf{C}$ cancels when we evaluate the upper and lower bounds.

Kinematics

- Describing the motion of points, bodies (objects), and systems of bodies (groups of objects)
- Relationship between
  - Position
  - Displacement
  - Distance
  - Velocity
  - Speed
  - Acceleration
- Kinematics does not deal with what causes motion

Position

Position is a vector describing the location of an object in a coordinate system (usually *Cartesian*; can also be *cylindrical* or *polar*). The origin of the coordinate system is the “reference point”.

$$\mathbf{x}(t) = x(t)\hat{\boldsymbol{\imath}} + y(t)\hat{\boldsymbol{\jmath}} + z(t)\hat{\boldsymbol{k}}$$

- The SI unit for position is a **meter**, m
- The components x, y and z are the coordinates along those axes
- The vector is a function of time t

Displacement

Displacement is the change in position from 1 to 2:

$$\Delta \mathbf{x} = \mathbf{x}_2 - \mathbf{x}_1 = (x_2 - x_1)\hat{\boldsymbol{\imath}} + (y_2 - y_1)\hat{\boldsymbol{\jmath}} + (z_2 - z_1)\hat{\boldsymbol{k}}$$

- IJK notation makes vector addition and subtraction less prone to errors
- Since reference point $\mathbf{x}_{\text{ref}} = \mathbf{0}$, the position $\mathbf{x}$ is also its displacement from the reference point

Instantaneous Velocity

Time Derivative of Position

If position $\mathbf{x}$ is a continuously differentiable function in time t, then velocity $\mathbf{v}$ can be found at any time t. The **instantaneous velocity** of an object is the rate of change of its position vector:

$$\boxed{\mathbf{v}(t) = \frac{d\mathbf{x}(t)}{dt}}$$

Since $\mathbf{x}$ has x, y and z components in the $\hat{\imath}$, $\hat{\jmath}$ and $\hat{k}$ directions, we can take the derivative with respect to time in every component:

$$\mathbf{v}(t) = \frac{d\mathbf{x}}{dt} = \frac{dx}{dt}\hat{\boldsymbol{\imath}} + \frac{dy}{dt}\hat{\boldsymbol{\jmath}} + \frac{dz}{dt}\hat{\boldsymbol{k}} = v_x\hat{\boldsymbol{\imath}} + v_y\hat{\boldsymbol{\jmath}} + v_z\hat{\boldsymbol{k}}$$

Integrating Velocity to Get Position/Displacement

If instantaneous velocity $\mathbf{v}$ is the rate of change of position $\mathbf{x}$ with respect to time t, then $\mathbf{x}$ is the time integral of $\mathbf{v}$:

$$\mathbf{x}(t) = \int \mathbf{v}(t)dt + \mathbf{x}_0$$

The constant of integration $\mathbf{x}_0 = \mathbf{x}(0)$ is the *initial position* at $t = 0$. As both $\mathbf{x}$ and $\mathbf{v}$ are vectors, we integrate each component to get $\mathbf{x}$:

$$\mathbf{x}(t) = \left(\int v_x\hat{\boldsymbol{\imath}} + \int v_y\hat{\boldsymbol{\jmath}} + \int v_z\hat{\boldsymbol{k}}\right) dt + \mathbf{x}_0$$

Average Velocity

The **average velocity** of an object is the change in position $\Delta \mathbf{x}$ over a finite time interval Δt:

$$\boxed{\bar{\mathbf{v}} = \frac{\Delta \mathbf{x}}{\Delta t}}$$

Like instantaneous velocity, we can find the x, y and z components of average velocity by separating components in each direction:

$$\bar{\mathbf{v}} = \frac{\Delta x}{\Delta t}\hat{\boldsymbol{\imath}} + \frac{\Delta y}{\Delta t}\hat{\boldsymbol{\jmath}} + \frac{\Delta z}{\Delta t}\hat{\boldsymbol{k}}$$

Path

Sometimes instead of explicitly describing the position $x = x(t)$ and $y = y(t)$, the path of an object can be given in terms of x coordinate $y = y(x)$, while giving the x (or y) coordinate as a function of time.

- In this case, substitute the expression for $x(t)$ into $y = y(x)$ to get an expression of $y = y(t)$
- Take derivative using chain rule to get $v_y = v_y(t)$

Instantaneous Acceleration

In the same way that velocity rate of change in position, **acceleration** is the rate of change in velocity:

$$\boxed{\mathbf{a}(t) = \frac{d\mathbf{v}(t)}{dt} = \frac{d^2\mathbf{x}(t)}{dt^2}}$$

Acceleration is the second derivative of position, i.e.

1. Take derivative of $\mathbf{x}(t)$ to get $\mathbf{v}(t) = \mathbf{x}'(t)$
2. Take derivative again of $\mathbf{v}(t)$ to get $\mathbf{a}(t) = \mathbf{v}'(t)$

Special Notation When Differentiating With Time

Physicists and engineers use a special notation when the derivative is taken with respect to time, by writing a dot above the variable:

- Velocity:

$$\boxed{\mathbf{v}(t) = \dot{\mathbf{x}}(t)}$$

- Acceleration:

$$\boxed{\mathbf{a}(t) = \dot{\mathbf{v}}(t) = \ddot{\mathbf{x}}(t)}$$

We will use this notation when it is convenient

Integrating Acceleration to Get Velocity

Velocity $\mathbf{v}(t)$ is the time integral of acceleration $\mathbf{a}(t)$:

$$\mathbf{v}(t) = \int \mathbf{a}(t)dt + \mathbf{v}_0$$

Again, since both $\mathbf{v}$ and $\mathbf{a}$ are vectors, we need to integrate in each direction:

$$\mathbf{v}(t) = \left(\int a_x \hat{\boldsymbol{\imath}} + \int a_y \hat{\boldsymbol{\jmath}} + \int a_z \hat{\mathbf{k}} \right) dt + \mathbf{v}_0$$

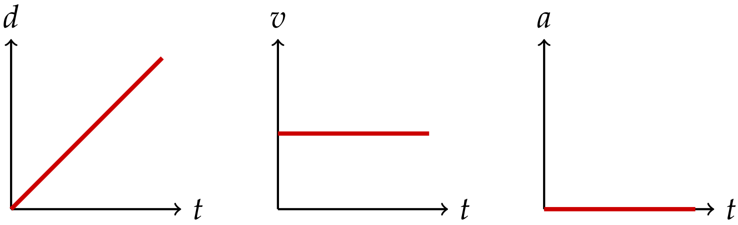
Motion Graphs

For 1D motion, we can describe motion graphically using motion graphs, by plotting

- Position vs. time ($x - t$) graph
- Velocity vs. time ($v - t$) graph
- Acceleration vs. time ($a - t$) graph

Motion Graphs

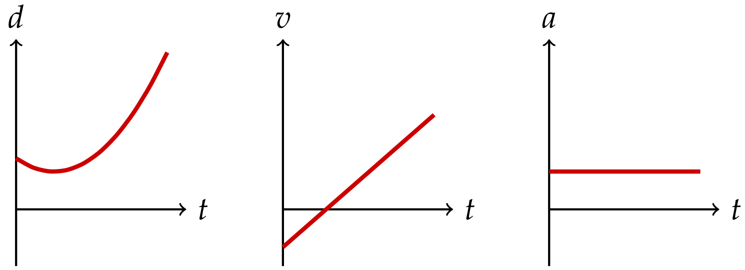
Uniform Motion (Constant Velocity)



- Constant velocity has a straight line in the $d - t$ graph
- The slope of the $d - t$ graph is the velocity v
- The slope of the $v - t$ graph is the acceleration a, which is zero in this case

Motion Graphs

Uniform (Constant) Acceleration

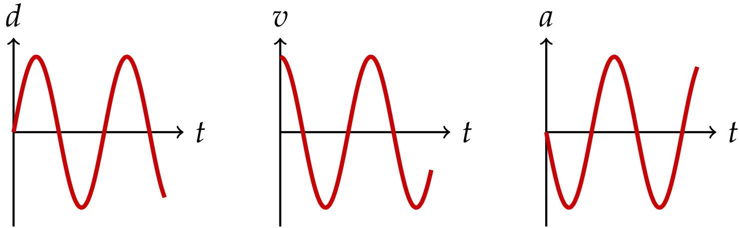


- The $d - t$ graph for motion with constant acceleration is part of a *parabola*
  - If the parabola is *convex*, then acceleration is positive
  - If the parabola is *concave*, then acceleration is negative
- The $v - t$ graph is a straight line; its slope (a constant) is the acceleration

Motion Graphs

Simple Harmonic Motion

For oscillatory motion, or **simple harmonic motion** (we will study this more in-depth later), neither position, velocity nor acceleration are constant:

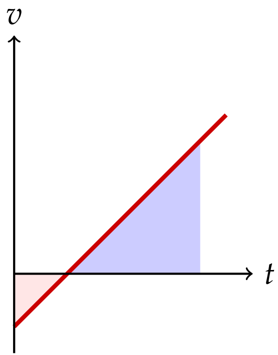


Bottom line: regardless of the type motion,

- The $v - t$ graph is the slope of the $d - t$ graph
- The $a - t$ graph is the slope of the $v - t$ graph

Area Under $v - t$ Graph

The area under the $v - t$ graph is the displacement $x - x_0$. (This should be obvious, since x is the time integral of v.)



- If the area is *below* the x (time) axis, then the displacement is negative;
- If the area is *above* the time axis, then displacement is positive

For Those Who Are Curious

We are not using these in AP Physics

The time derivative of acceleration is called **jerk**, with a unit of $\mathrm{m/s^3}$:

$$\mathbf{j} = \frac{d\mathbf{a}}{dt} = \frac{d^2\mathbf{v}}{dt^2} = \frac{d^3\mathbf{x}}{dt^3}$$

The time derivative of jerk is **jounce**, or **snap**, with a unit of $\mathrm{m/s^4}$:

$$\mathbf{s} = \frac{d\mathbf{j}}{dt} = \frac{d^2\mathbf{a}}{dt^2} = \frac{d^3\mathbf{v}}{dt^3} = \frac{d^4\mathbf{x}}{dt^4}$$

The next two derivatives of snap is facetiously called **crackle** and **pop**, but these higher derivatives of position vector are rarely seen.

Kinematic Equations For Constant Acceleration

Although kinematic problems in AP Physics often require calculus, these basic kinematic equations are still a very powerful tool.

$$\mathbf{x} = \mathbf{x}_0 + \mathbf{v}_0 t + \frac{1}{2}\mathbf{a}t^2$$

$$\mathbf{v} = \mathbf{v}_0 + \mathbf{a}t$$

$$v^2 = v_0^2 + 2a(x - x_0)$$

- The variables of interests are:

$$\mathbf{x}_0 \quad \mathbf{x} \quad \mathbf{v}_0 \quad \mathbf{v} \quad t \quad \mathbf{a}$$

- Only applicable for constant acceleration

Projectile Motion

- For 2D problems, resolve the problem into its horizontal (x) and vertical (y) directions, and apply kinematic equations independently
- For projectile motion, there is no acceleration in the x direction, i.e. $a_x = 0$, therefore the kinematic equations reduce to just

$$x = v_x t \hat{\boldsymbol{\imath}}$$

- The only acceleration is in the $\hat{\jmath}$ direction. In the standard Cartesian coordinate system, this usually means that $\hat{\jmath}$ direction is *up*:

$$a_y = -g\hat{\boldsymbol{\jmath}}$$

- The variable that connects the two directions is time t

Symmetric Trajectory

Trajectory is symmetric if the object lands at the same height as when it started.

- Time of flight

$$t_{\max} = \frac{2v_i \sin\theta}{g}$$

- Range

$$R = \frac{v_i^2 \sin(2\theta)}{g}$$

- Maximum height

$$h_{\max} = \frac{v_i^2 \sin^2\theta}{2g}$$

The angle θ is measured **above the the horizontal**

Maximum Range

$$R = \frac{v_i^2 \sin(2\theta)}{g}$$

- For a given initial speed v_i, maximum range occurs at $\theta = 45^\circ$
- For a given initial speed v_i and range R, I can find a launch angle θ that gives the required range:

$$\theta_1 = \frac{1}{2} \sin^{-1}\left(\frac{Rg}{v_i^2}\right)$$

- But there is another angle that *gives the same range*!

$$\theta_2 = 90^\circ - \theta_1$$

Relative Motion

Notation

When expressing relative motion, the first subscript (A) represents the moving object, and the second subscript (B) represents the frame of reference:

$$\mathbf{v}_{AB}$$

If an airplane ("P") is traveling at $251\,\mathrm{km/h}$ [N] relative to Earth ("E"), its velocity is expressed as:

$$\mathbf{v}_{PE} = 251\,\mathrm{km/h}\ [\mathrm{N}]$$

Relative Motion

If the airplane flies in windy air ("A") we must consider the velocity of the airplane relative to air $\mathbf{v}_{PA}$ and the velocity of the air relative to Earth $\mathbf{v}_{AE}$. The velocity of the airplane relative to Earth is therefore

$$\mathbf{v}_{PE} = \mathbf{v}_{PA} + \mathbf{v}_{AE}$$

If an airplane is flying at a constant velocity of $253\,\mathrm{km/h}$ [S] relative to the air and the air velocity is $24\,\mathrm{km/h}$ [N], what is the velocity of the airplane relative to Earth?

Relative Motion

In classical mechanics, the equation for relative motion follows the **Galilean velocity addition rule**[1]:

$$\boxed{\mathbf{v}_{AC} = \mathbf{v}_{AB} + \mathbf{v}_{BC}}$$

The velocity of A relative to reference frame C is the velocity of A relative to reference frame B, plus the velocity of B relative to C.

If we add another frame of reference ("D"), the equation becomes:

$$\mathbf{v}_{AD} = \mathbf{v}_{AB} + \mathbf{v}_{BC} + \mathbf{v}_{CD}$$

[1] This equation was thought to be so obvious that no one bother to give it a name until Einstein proved that it was incorrect for speeds close to the speed of light

Typical Problems

For both AP Physics 1 and AP Physics C exams, questions involving kinematics usually appear in the multiple-choice section. The problems themselves are not very different compared to the Grade 12 Physics problems, but:

- You have to solve problems faster because of time constraint
- You can use $g = 10\,\mathrm{m/s^2}$ to make your lives simpler
- A lot of problems are *symbolic*, which means that they deal with the equations, not actual numbers
- Will be coupled with other types (e.g. dynamics and rotational) in the free-response section
- You *will* be given an equation sheet