Topic 9: Electrostatics

Advanced Placement Physics

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The Charges Are

Let's Review Some Basics

We already know a bit about charge particles:

- A **proton** carries a **positive** charge
- An **electron** carries a **negative** charge
- A *net charge* of an object means an excess of protons or electrons
- Similar charges are repel, opposite charges attract

We start with electrostatics:

- Charges that are not moving relative to one another

Coulomb's Law for Electrostatic Force

The **electrostatic force** between two point charges is given by:

$$\boxed{F_q = -\frac{kq_1q_2}{r^2}\hat{\mathbf{r}}_{12}}$$

Quantity	Symbol	SI Unit
Electrostatic force	F_q	N (newtons)
Coulomb's constant (electrostatic constant)	k	$\mathrm{N\,m^2/C^2}$
Point charges 1 and 2 (occupies no space)	q_1, q_2	C (coulombs)
Distance between point charges	r	m (meters)
Unit vector of direction between point charges	$\hat{\mathbf{r}}$	

$k = \dfrac{1}{4\pi\epsilon_0} = 8.99 \times 10^9\ \mathrm{N\,m^2/C^2}$ where $\epsilon_0 = 8.85 \times 10^{-12}\ \mathrm{C^2/N\,m^2}$ is called the "permittivity of free space"

Think Electric Field

We can get **electric field** by repeating the same procedure as with gravitational field. Again, let's group the variables in Coulomb's equation:

$$F_q = \underbrace{\left[\frac{kq_1}{r^2}\right]}_{=E} q_2$$

We can say that charge q_1 creates an "electric field" (E) with an intensity

$$E = \frac{kq_1}{r^2}$$

This electric field $\mathbf{E}$ created by q_1 is a function ("vector field") that shows how it influences other charged particles around it

Electric Field Intensity Near a Point Charge

The electric field a distance r away from a point charge is given by:

$$\boxed{\mathbf{E} = \frac{kq_s}{r^2}\hat{\mathbf{r}}}$$

Quantity	**Symbol**	**SI Unit**
Electric field intensity	E	N/C (newtons per coulomb)
Coulomb's constant	k	$\mathrm{N\,m^2/C^2}$
Source charge	q_s	C (coulombs)
Distance from source charge	r	m (meters)

The direction of the field is radially outward from a positive point charge and radially inward toward a negative charge.

Think Electric Field

$\mathbf{E}$ *doesn't do anything* until another charge interacts with it. And when there is a charge q, the electrostatic force $\mathbf{F}_q$ that it experiences in the presence of $\mathbf{E}$ is:

$$\boxed{\mathbf{F}_q = \mathbf{E}q}$$

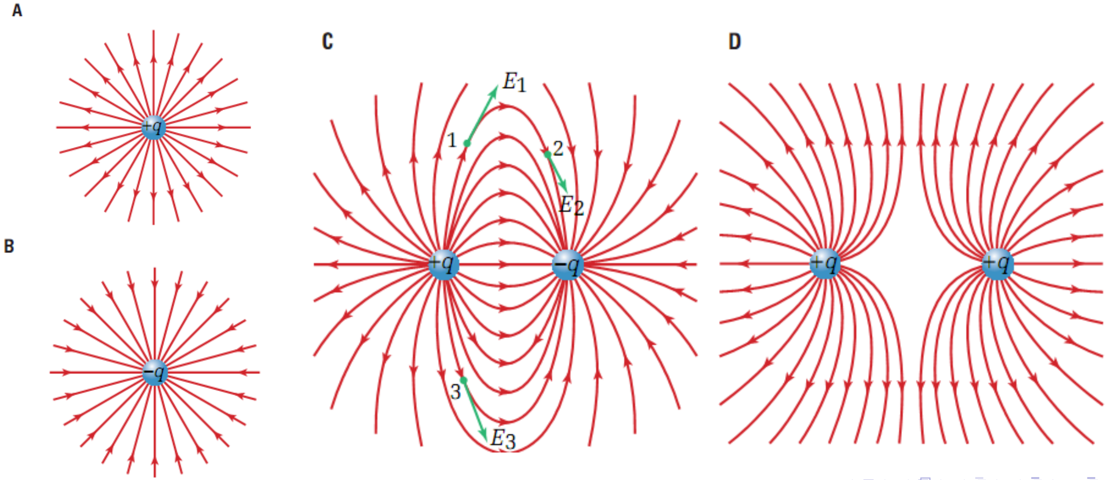
$\mathbf{F}_q$ and $\mathbf{E}$ are vectors, and following the principle of superposition, i.e.

$$\mathbf{F}_q = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \mathbf{F}_4 \dots$$
$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 + \mathbf{E}_3 + \mathbf{E}_4 \dots$$

This understanding is especially important when we want to find $\mathbf{F}_q$ and $\mathbf{E}$ some distance from a continuous distribution of charges

Electric Field Lines

If you place a positive charge in an electric field, the force on the charge will be in the direction of the electric field.



Electrical Potential Energy

(Follow the Same Work on Gravitational Potential Energy)

If we move a charged particle against the electrostatic force, work must be done (either positive or negative, depending on which way the particle moves):

$$W = \int \mathbf{F}_q \cdot d\mathbf{r} = -kq_1q_2 \int_{r_1}^{r_2} \frac{dr}{r^2} = -\frac{kq_1q_2}{r}\bigg|_{r_1}^{r_2} = -\Delta U_q$$

Electrical potential energy is defined as:

$$\boxed{U_q = \frac{kq_1q_2}{r}}$$

U_q can be (+) or (-), because charged particles can be either (+) or (-)

How it Differs from Gravitational Potential

Two positive charges:	Two negative charges:	One positive and one negative charge:
$U_q > 0$	$U_q > 0$	$U_q < 0$

- $U_q > 0$ means positive work is done to bring two charges together from $r = \infty$ to r (both charges of the same sign)
- $U_q < 0$ means negative work (the charges are opposite signs)
- For gravitational potential U_g is always < 0

Electric Potential

Start with an Analogy

When I move an object of mass m against a gravitational force from one point to another, the work that I do is directly proportional to m, i.e. there is a “constant” in that scales with *any* mass, as long as they move between those same two points:

$$W = \Delta U_g = Km$$

In the trivial case (small changes in height, no change in g), this constant is just

$$\frac{\Delta U_g}{m} = g\Delta h$$

(We have actually looked at this briefly in our discussion on universal gravitation.)

Electric Potential

This is also true for moving a charged particle q against an electric electric field created by q_s, and the “constant” is called the **electric potential**. For a point charge, it is defined as:

$$\boxed{V = \frac{U_q}{q} = \frac{kq_s}{r}}$$

The unit for electric potential is a *volt* which is *one joule per coulomb*:

$$1\,\mathrm{V} = 1\,\mathrm{J/C}$$

We can easily that there is also a relationship between electrical potential V and electric field $\mathbf{E}$:

$$\boxed{\Delta V = \int \mathbf{E} \cdot d\mathbf{r}}$$

Potential Difference (Voltage)

The change in electric potential is called the **electric potential difference** or **voltage**:

$$\boxed{\Delta V = \frac{\Delta U_q}{q}} \quad \text{and} \quad \boxed{dV = \frac{dU_q}{q}}$$

Here, we can relate ΔV to an equation that we knew from Physics 11, which related to the energy dissipated in a resistor in a circuit ΔU to the voltage drop ΔV:

$$\boxed{\Delta U = q\Delta V}$$

Electric potential difference also has the unit *volts* (V)

Getting Those Names Right

Remember that these three quantities are all scalars, as opposed to electrostatic force $\mathbf{F}_q$ and electric field $\mathbf{E}$ which are vectors

- Electric potential energy:

$$U = \frac{kq_1q_2}{r}$$

- Electric potential:

$$V = \frac{kq}{r}$$

- Electric potential difference (voltage):

$$\Delta V = \frac{\Delta U_q}{q}$$

Relating U_q, $\mathbf{F}_q$ and $\mathbf{E}$

Our Integrals In Reverse

Using vector calculus, we can relate electrostatic force ($\mathbf{F}_q$) to electric potential energy (U_q), and electric field ($\mathbf{E}$) to the electric potential (V):

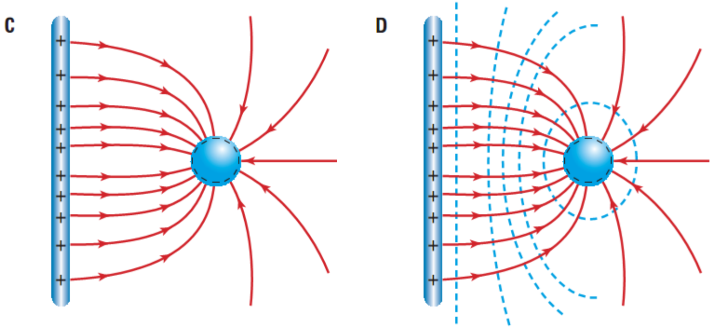
$$\mathbf{F}_q = -\nabla U_q = -\frac{\partial U_q}{\partial r}\hat{\mathbf{r}} \qquad \mathbf{E} = -\nabla V = -\frac{\partial V}{\partial r}\hat{\mathbf{r}}$$

- Electrostatic force $\mathbf{F}_q$ always points from high potential to low potential energy
- Electric field can also be expressed as the change of electric potential per unit distance, which has the unit

$$1\,\mathrm{N/C} = 1\,\mathrm{V/m}$$

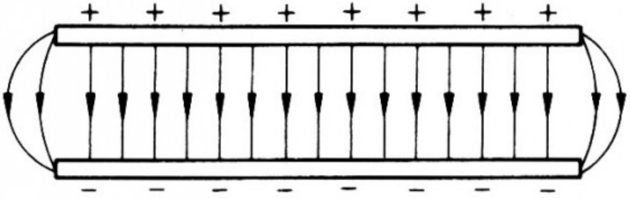
- Electric field is also called “potential gradient”

Equipotential Lines



The dotted blue lines are called **equipotential lines.** They are always *perpendicular* to the electric field lines. Charges moving in the direction of the equipotential lines have constant electric potential

Electric Field between Two Parallel Plates



- $\mathbf{E}$ is uniform at all points between the parallel plates, independent of position
- E is proportional to the charge density (charge per unit area) on the plates:

$$E \propto \sigma \quad \text{where} \quad \sigma = \frac{q}{A}$$

- E outside the plates is very low (close to zero), except for the fringe effects at the edges of the plates.

Electric Field and Electric Potential Difference

The relationship between electric field ($\mathbf{E}$) and electric potential difference (V):

$$\mathbf{E} = -\frac{\partial V}{\partial r}$$

In a uniform electric field (e.g. parallel plate) it simplifies to a very simple equation:

$$\boxed{E = \frac{\Delta V}{d}}$$

Quantity	Symbol	SI Unit
Electric field intensity	E	N/C (newtons per coulomb)
Electric potential difference between plates	ΔV	V (volts)
Distance between plates	d	m (meters)