

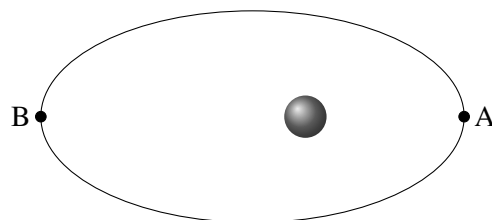
**AP PHYSICS 1 & C: UNIVERSAL GRAVITATION**

**Directions:** Each of the questions or incomplete statements below is followed by five suggested answers or completions. Select the one that is best in each case and place the letter of your choice in the corresponding box on the student answer sheet.

**Note:** To simplify calculations, you may use  $g = 10 \text{ m/s}^2$  in all problems.

1. Two satellites of equal mass orbit a planet. Satellite B orbits at twice the orbital radius of Satellite A. Which of the following statements is true?
  - (A) The gravitational force on Satellite A is four times less than that on Satellite B.
  - (B) The gravitational force on Satellite A is two times less than that on Satellite B.
  - (C) The gravitational force on the satellites is equal.
  - (D) The gravitational force on Satellite A is two times greater than that on Satellite B.
  - (E) The gravitational force on Satellite A is four times greater than that on Satellite B.
2. A 70-kg astronaut floats at a distance of 10 m from a 50000-kg spacecraft. What is the force of attraction between the astronaut and spacecraft?
  - (A)  $2.4 \times 10^{-6} \text{ N}$
  - (B)  $2.4 \times 10^{-5} \text{ N}$
  - (C) Zero; there is no gravity in space.
  - (D)  $2.4 \times 10^5 \text{ N}$
  - (E)  $2.4 \times 10^6 \text{ N}$
3. The centripetal acceleration on 1000 kg car in a turn is  $1 \times 10^5 \text{ m/s}^2$ . The radius of the turn is 10 m. What is the car's speed?
  - (A)  $1 \times 10^1 \text{ m/s}$
  - (B)  $1 \times 10^2 \text{ m/s}$
  - (C)  $1 \times 10^3 \text{ m/s}$
  - (D)  $1 \times 10^4 \text{ m/s}$
  - (E)  $1 \times 10^5 \text{ m/s}$
4. A proposed "space elevator" can lift a 1000 kg payload to an orbit of 150 km above the Earth's surface. The radius of the Earth is  $6.4 \times 10^6 \text{ m}$ , and the Earth's mass is  $6.0 \times 10^{24} \text{ kg}$ . What is the gravitational potential energy of the payload when it reaches orbit?
  - (A)  $1.0 \times 10^3 \text{ J}$
  - (B)  $2.7 \times 10^6 \text{ J}$
  - (C)  $6.1 \times 10^{10} \text{ J}$
  - (D)  $2.7 \times 10^{12} \text{ J}$
  - (E)  $1.0 \times 10^{15} \text{ J}$
5. The Earth is at an average distance of 1 AU from the Sun and has an orbital period of 1 year. Jupiter orbits the Sun at approximately 5 AU. About how long is the orbital period of Jupiter?
  - (A) 1 year
  - (B) 2 years
  - (C) 5 years
  - (D) 11 years
  - (E) 125 years
6. A satellite orbits the Earth at a distance of 200 km. If the mass of the Earth is  $6.0 \times 10^{24} \text{ kg}$  and the Earth's radius is  $6.4 \times 10^6 \text{ m}$ , what is the satellite's speed?
  - (A)  $1.0 \times 10^3 \text{ m/s}$
  - (B)  $3.5 \times 10^3 \text{ m/s}$
  - (C)  $7.8 \times 10^3 \text{ m/s}$
  - (D)  $5 \times 10^6 \text{ m/s}$
  - (E)  $6.1 \times 10^7 \text{ m/s}$

7. Mars orbits the Sun at a distance of  $2.3 \times 10^{11}$  m. The mass of the Sun is  $2.0 \times 10^{30}$  kg, and the mass of Mars is  $6.4 \times 10^{23}$  kg. Approximately what is the gravitational force that the Sun exerts on Mars?
- (A)  $1.6 \times 10^{20}$  N  
 (B)  $1.6 \times 10^{21}$  N  
 (C)  $3.7 \times 10^{21}$  N  
 (D)  $3.7 \times 10^{32}$  N  
 (E)  $3.7 \times 10^{42}$  N
8. When climbing from sea level to the top of Mount Everest, a hiker changes elevation by 8848 m. By what percentage will the gravitational field of the Earth change during the climb? (The Earth's mass is  $6.0 \times 10^{24}$  kg, and its radius is  $6.4 \times 10^6$  m.)
- (A) It will increase by approximately 0.3 %.  
 (B) It will decrease by approximately 0.3 %.  
 (C) It will increase by approximately 12 %.  
 (D) It will decrease by approximately 12 %.  
 (E) The gravitational field strength will not change.
9. Four planets, A through D, orbit the same star. The relative masses and distances from the star for each planet are shown in the table. For example, Planet A has twice the mass of Planet B, and Planet D has three times the orbital radius of Planet A. Which planet has the highest gravitational attraction to the star?
- | Planet | Relative mass | Relative distance |
|--------|---------------|-------------------|
| A      | $2m$          | $r$               |
| B      | $m$           | $0.1r$            |
| C      | $0.5m$        | $2r$              |
| D      | $4m$          | $3r$              |
- (A) Planet A  
 (B) Planet B  
 (C) Planet C  
 (D) Planet D  
 (E) All have the same gravitational attraction to the star.
10. A satellite orbits the Earth at a distance that is four times the radius of the Earth. If the acceleration due to gravity near the surface of the Earth is  $g$ , the acceleration of the satellite is most nearly
- (A) zero  
 (B)  $g/2$   
 (C)  $g/4$   
 (D)  $g/8$   
 (E)  $g/16$
11. The mass of a planet is  $1/4$  that of Earth and its radius is half of Earth's radius. The acceleration due to gravity on this planet is most nearly
- (A)  $2 \text{ m/s}^2$   
 (B)  $4 \text{ m/s}^2$   
 (C)  $5 \text{ m/s}^2$   
 (D)  $10 \text{ m/s}^2$   
 (E)  $20 \text{ m/s}^2$
12. A satellite orbits the Earth in an elliptical orbit, with point A being close to the Earth and point B farther away. As the satellite moves from point A to point B, which of the following is true of the angular momentum and kinetic energy of the satellite?

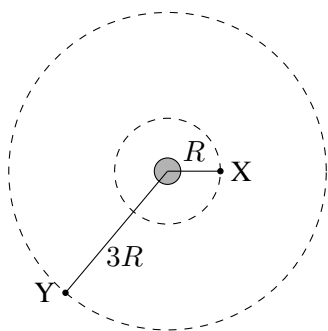


- |     | <u>Angular momentum</u> | <u>Kinetic energy</u> |
|-----|-------------------------|-----------------------|
| (a) | Increases               | Remains constant      |
| (b) | Remains constant        | Increases             |
| (c) | Decreases               | Remains constant      |
| (d) | Remains constant        | Decreases             |
| (e) | Remains constant        | Remains constant      |

13. Two planets of mass  $M$  and  $9M$  are in the same solar system. The radius of the planet of mass  $M$  is  $R$ . In order for the acceleration due to gravity to be the same for each planet, the radius of the planet of mass  $9M$  would have to be

(A)  $R/2$   
 (B)  $R$   
 (C)  $2R$   
 (D)  $3R$   
 (E)  $9R$

14. Two planets, X and Y, orbit a star. Planet X orbits at a radius  $R$ , and Planet Y orbits at a radius  $3R$ . Which of the following best represents the relationship between the acceleration  $a_X$  of Planet X and the acceleration  $a_Y$  of Planet Y?



(A)  $a_X = 9a_Y$   
 (B)  $9a_X = a_Y$   
 (C)  $a_X = 3a_Y$   
 (D)  $3a_X = a_Y$   
 (E)  $a_X = a_Y$

15. A satellite is in a stable circular orbit around the Earth at a radius  $R$  and speed  $v$ . At what radius would the satellite travel in a stable orbit with a speed  $2v$ ?

(A)  $14 R$   
 (B)  $12 R$   
 (C)  $R$   
 (D)  $2R$   
 (E)  $4R$

16. The Earth and the moon apply a gravitational force to each other. Which of the following statements is true?

(A) The Earth applies a greater force on the moon than the moon exerts on the Earth.  
 (B) The Earth applies a smaller force on the moon than the moon exerts on the Earth.  
 (C) The Earth applies a force on the moon, but the moon does not exert a force on the Earth.  
 (D) The Earth does not apply a force on the moon, but the moon exerts a force on the Earth.  
 (E) The force the Earth applies to the moon is equal and opposite to the force the moon applies to the Earth.

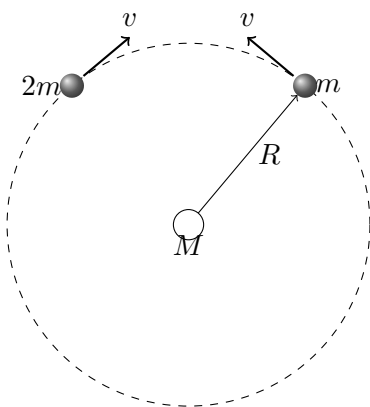
17. Two masses exert a gravitational force  $F$  on each other. If one of the masses is doubled, and the distance between the masses is tripled, the new force between them is

(A)  $6F$   
 (B)  $2F/3$   
 (C)  $2F/9$   
 (D)  $3F/2$   
 (E)  $4F/9$

18. A planet orbits at a radius  $R$  around a star of mass  $M$ . The period of orbit of the planet is

(A)  $\sqrt{\frac{4\pi^2 R^2}{GM}}$   
 (B)  $\frac{4\pi^2 R^3}{GM}$   
 (C)  $\sqrt{\frac{4\pi^2 R^3}{GM}}$   
 (D)  $\sqrt{\frac{4\pi^2 R}{GM}}$   
 (E)  $\frac{GM}{4\pi^2 R}$

19. A moon orbits a large planet in an elliptical orbit, with its closest approach at a distance  $a$ , and its farthest distance  $b$ . The speed of the moon at point  $b$  is  $v$ . The speed at point  $a$  is
- (A)  $\frac{av}{b}$   
 (B)  $\frac{bv}{a}$   
 (C)  $\frac{(a+b)v}{b}$   
 (D)  $\frac{(b-a)v}{b}$   
 (E)  $\frac{2bv}{a}$
20. A satellite orbits the Earth in an elliptical orbit. Which of the following statements is true?
- (A) The angular velocity of the satellite increases as it travels farther from the Earth.  
 (B) The acceleration of the satellite increases as it travels closer to the Earth.  
 (C) The angular momentum of the satellite increases as it travels closer to the Earth.  
 (D) The potential energy of the satellite is equal to its kinetic energy at all points in the orbit.  
 (E) The speed of the satellite must remain constant for it to remain in orbit around the Earth.
21. Two moons of mass  $m$  and  $2m$  orbit a planet of mass  $M$  at the same radius  $R$  and speed  $v$  toward each other, as shown. The moons collide and stick together without destroying either moon. The total momentum of the moons after the collision is
22. The velocity of the two masses after the collision above is
- (A)  $v$  counterclockwise  
 (B)  $v/2$  counterclockwise  
 (C)  $v/2$  clockwise  
 (D)  $v/3$  counterclockwise  
 (E)  $v/3$  clockwise
23. Consider a two-star system shown above, which consists of two stars of mass  $m$  rotating in a circle of radius  $r$  about their center of mass. What is the total energy of the two-star system?
- (A)  $-Gm^2/2r$   
 (B)  $Gm^2/2r$   
 (C)  $Gm^2/4r$   
 (D)  $3Gm^2/4r$   
 (E)  $-Gm^2/4r$
24. If a planet has twice the radius of Earth and half of Earth's density, what is the acceleration due to gravity on the surface of the planet (in terms of the gravitational acceleration  $g$  on the surface of Earth)?
- (A)  $4g$   
 (B)  $2g$   
 (C)  $g$   
 (D)  $g/2$   
 (E)  $g/4$



- (A)  $mv$   
 (B)  $2mv$   
 (C)  $3mv$   
 (D)  $6mv$   
 (E) zero

**AP PHYSICS 1 & C: UNIVERSAL GRAVITATION**  
**SECTION II**  
**5 Questions**

**Directions:** Answer all questions. The suggested time is about 10 minutes for answering each of the questions. The parts within a question may not have equal weight. All final numerical answers should include appropriate units. Credit depends on the quality of your solutions and explanations, so you should show your work. Credit also depends on demonstrating that you know which physical principles would be appropriate to apply in a particular situation. Therefore, you should clearly indicate which part of a question your work is for.

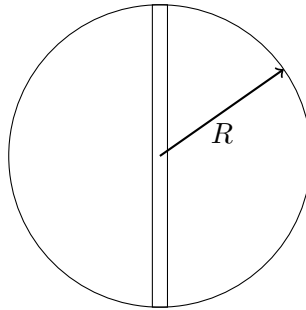
1. Two meteoroids are 250,000 km from Earth's center and moving at 2.1 km/s. One is headed straight for Earth, while the other is on a path that will come within 8500 km of Earth's center. Find the speed of
  - (a) the first meteoroid when it strikes Earth, and
  - (b) the second meteoroid at its closest approach.
  - (c) Will the second meteoroid ever return to Earth's vicinity?

2. Two stars of equal mass  $M$  are orbiting each other in a circular path. Show that the orbital period is given by:

$$T^2 = \frac{2\pi^2 d^3}{GM}$$

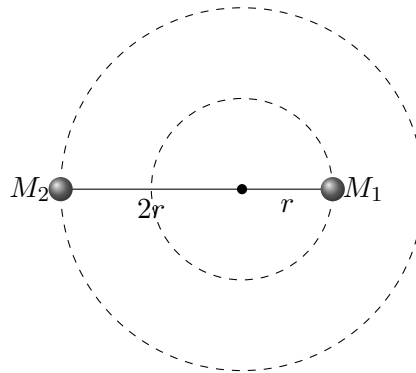
where  $d$  is the distance between the stars.

3. A planet of mass  $M$ , radius  $R$ , and uniform density has a small tunnel drilled through the center of the planet, as shown below. When the mass is inside the tunnel, it experiences a force of  $F = (GmM/R^3)r$ , whereas when the mass is outside of the planet, it experiences a gravitational force of  $F = GmM/r^2$ .



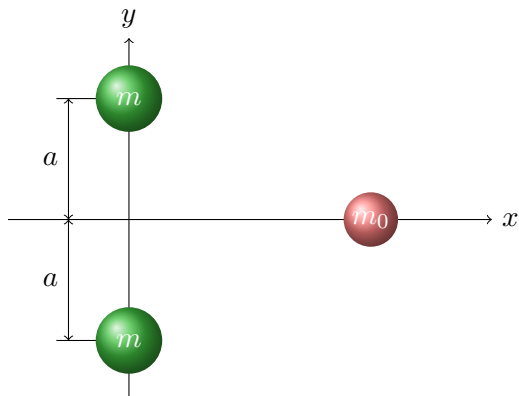
- (a) Setting the potential energy of the mass to be zero at the planet's center, calculate the mass's potential energy as a function of distance from the center of the planet  $U(r)$ , for values  $r < R$ . Sketch this potential function.
- (b) If the mass is dropped from  $R$  from the center of the planet, how long will it take until it returns to its original position?
- (c) If the mass is dropped from  $R/2$  from the center of the planet, will it require more, or less, or the same amount of time to return to its original position compared to if it was dropped from  $R$ ?
- (d) If the mass is dropped from  $2R$  from the center of the planet, will it require more, or less, or the same amount of time to return to its original position compared to if it was dropped from  $R$ ?

4. Two stars of unequal mass orbit each other about their common center of mass as shown. The star of mass  $M_1$  orbits in a circle of radius  $r$ , and the star of mass  $M_2$  orbits in a circle of radius  $2r$ .



- (a) Determine the ratio of masses  $M_1/M_2$ .
- (b) Determine the ratio of the acceleration  $a_1$  of  $M_1$  to the acceleration  $a_2$  of  $M_2$ .
- (c) Determine the ratio of the period  $T_1$  of  $M_1$  to the period  $T_2$  of  $M_2$ .
5. As a member of the 2240 Olympic Committee, you are considering a new sport: asteroid jumping. On Earth, world-class high jumpers routinely clear 2 m. Your job is to make sure athletes jumping from asteroids will return to the asteroid. Make the simplifying assumption that asteroids are spherical, with an average density of  $2500 \text{ kg/m}^3$ . For safety, make sure that even a jumper capable of 3 m on Earth will return to the surface. What do you report for the minimum asteroid diameter?

6. Two point particles of mass  $m$  are on the  $y$  axis at  $y = a$  and  $y = -a$ , as shown in the figure below.



- (a) Derive the expression for the gravitational force exerted by these two particles on a third particle of mass  $m_0$  located on the  $x$  axis at a distance  $x$  away from the origin.
- (b) What is the gravitational field  $\mathbf{g}$  on the  $x$ -axis due to the two particles?
- (c) Show that  $g_x$  (the  $x$  component of  $\mathbf{g}$ ) due to the two particles on the  $y$  axis is approximately  $-\frac{2Gm}{x^2}$  when  $x$  is much greater than  $a$ .
- (d) Show that the maximum value of  $|g_x|$  occurs at the point  $x = \frac{\pm a}{\sqrt{2}}$ .



7. Five equal masses  $M$  are equally spaced on the arc of a semicircle of radius  $R$  as shown in the figure below. A mass  $m$  is located at the center of curvature of the arc. If  $M$  is 3 kg,  $m$  is 21 kg, and  $R$  is 10 cm, what is the force on  $m$  due to the five masses?

