Student #: ___________ Student Name: ______________________

AP Physics Class 8: Universal Gravitation

The questions in this homework assignment cover AP 1 and C exams.

C 1. A car moves in a horizontal circle with a radius of 10 m. The tangential velocity of the car is 30 m/s. What is the car's acceleration?

(a) $3\,m/s^2$ toward the center
(b) $3\,m/s^2$ away from the center
(c) $90\,m/s^2$ toward the center
(d) $90\,m/s^2$ away from the center
(e) $270\,m/s^2$ toward the center

$a = \frac{v^2}{r} = \frac{30^2}{10} = 90$

___ 2. A satellite orbits the Earth at a distance of 100 km. The mass of the satellite is 100 kg, while the mass of the Earth is approximately 6.0×10^{24} kg. The radius of the Earth is approximately 6.4×10^6 m. What is the approximate force of gravity acting on the satellite?

(a) 4×10^4 N
(b) 6.2×10^6 N
(c) 4×10^8 N
(d) 6.2×10^9 N
(e) 4×10^{14} N

None of these are correct.

~~6.2×10^2 N~~

947N.

e 3. Two satellites of equal mass orbit a planet. Satellite B orbits at twice the orbital radius of Satellite A. Which of the following statements is true?

(a) The gravitational force on Satellite A is four times less than that on Satellite B.
(b) The gravitational force on Satellite A is two times less than that on Satellite B.
(c) The gravitational force on the satellites is equal.
(d) The gravitational force on Satellite A is two times greater than that on Satellite B.
(e) The gravitational force on Satellite A is four times greater than that on Satellite B.

a 4. A 70 kg astronaut floats at a distance of 10 m from a 50 000 kg spacecraft. What is the force of attraction between the astronaut and spacecraft?

(a) 2.4×10^{-6} N
(b) 2.4×10^{-5} N
(c) Zero; there is no gravity in space.
(d) 2.4×10^5 N
(e) 2.4×10^6 N

doesn't matter! a

C 5. The centripetal acceleration on 1,000 kg car in a turn is $1 \times 10^5\,m/s^2$. The radius of the turn is 10 m. What is the car's speed?

(a) 1×10^1 m/s
(b) 1×10^2 m/s
(c) 1×10^3 m/s
(d) 1×10^4 m/s
(e) 1×10^5 m/s

$a = \frac{v^2}{r}$ $v = \sqrt{ar} = \sqrt{1 \times 10^6} = 1 \times 10^3$

_____ 6. A proposed "space elevator" can lift a 1,000 kg payload to an orbit of 150 km above the Earth's surface. The radius of the Earth is 6.4×10^6 m, and the Earth's mass is 6×10^{24} kg. What is the gravitational potential energy of the payload when it reaches orbit?

(a) 1.0×10^3 J
(b) 2.7×10^6 J
(c) 6.1×10^{10} J   C?
(d) 2.7×10^{12} J
(e) 1.0×10^{15} J

a_1

d 7. The Earth is at an average distance of 1 AU from the Sun and has an orbital period of 1 year. Jupiter orbits the Sun at approximately 5 AU. About how long is the orbital period of Jupiter?

T_1 a_2

(a) 1 year
(b) 2 years
(c) 5 years
(d) 11 years
(e) 125 years

$$\frac{a_1^3}{T_1^2} = \frac{a_2^3}{T_2^2}$$

2.0×10^5 m   M

c 8. A satellite orbits the Earth at a distance of 200 km. If the mass of the Earth is 6.0×10^{24} kg and the Earth's radius is 6.4×10^6 m, what is the satellite's speed?

$\rightarrow r = 6.6 \times 10^6$ m

(a) 1×10^3 m/s
(b) 3.5×10^3 m/s
(c) 7.8×10^3 m/s
(d) 5×10^6 m/s
(e) 6.1×10^7 m/s

r M_1

b 9. Mars orbits the Sun at a distance of 2.3×10^{11} m. The mass of the Sun is 2×10^{30} kg, and the mass of Mars is 6.4×10^{23} kg. Approximately what is the gravitational force that the Sun exerts on Mars?

M_2

(a) 1.6×10^{20} N
(b) 1.6×10^{21} N
(c) 3.7×10^{21} N
(d) 3.7×10^{32} N
(e) 3.7×10^{42} N

$$F = \frac{G m_1 m_2}{r^2}$$

b 10. When climbing from sea level to the top of Mount Everest, a hiker changes elevation by 8848 m. By what percentage will the gravitational field of the Earth change during the climb? (The Earth's mass is 6.0×10^{24} kg, and its radius is 6.4×10^6 m.)

(a) It will increase by approximately 0.3%.
(b) It will decrease by approximately 0.3%.
(c) It will increase by approximately 12%.
(d) It will decrease by approximately 12%.   that's a lot!
(e) The gravitational field strength will not change.

b 11. Four planets, A through D, orbit the same star. The relative masses and distances from the star for each planet are shown in the table. For example, Planet A has twice the mass of Planet B, and Planet D has three times the orbital radius of Planet A. Which planet has the highest gravitational attraction to the star?

Planet	Relative mass	Relative distance
A	$2m$	r
B	m	$0.1r$
C	$0.5m$	$2r$
D	$4m$	$3r$

$\frac{g}{2}$
100
$1/8$
$4/9$

(a) Planet A
(b) Planet B
(c) Planet C
(d) Planet D
(e) All have the same gravitational attraction to the star.

e 12. A satellite orbits the Earth at a distance that is four times the radius of the Earth. If the acceleration due to gravity near the surface of the Earth is g, the acceleration of the satellite is most nearly

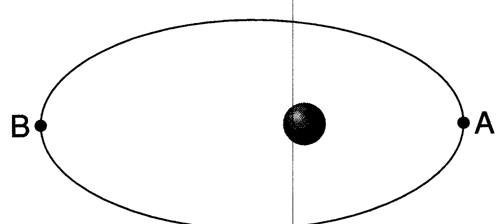
(a) zero
(b) $g/2$
(c) $g/4$
(d) $g/8$
(e) $g/16$

$$g = \frac{GM}{R^2} \rightarrow \frac{GM}{(4R)^2} = \frac{GM}{16R^2} \rightarrow \frac{1}{16} g$$

d 13. The mass of a planet is 1/4 that of Earth and its radius is half of Earth's radius. The acceleration due to gravity on this planet is most nearly

(a) $2\,\mathrm{m/s^2}$
(b) $4\,\mathrm{m/s^2}$
(c) $5\,\mathrm{m/s^2}$
(d) $10\,\mathrm{m/s^2}$
(e) $20\,\mathrm{m/s^2}$

$$g = \frac{GM}{R^2} \rightarrow \frac{G(\frac{1}{4}M)}{(\frac{1}{2}R)^2} = \frac{GM}{R^2} = g\ !$$



d 14. A satellite orbits the Earth in an elliptical orbit, with point A being close to the Earth and point B farther away. As the satellite moves from point A to point B, which of the following is true of the angular momentum and kinetic energy of the satellite?

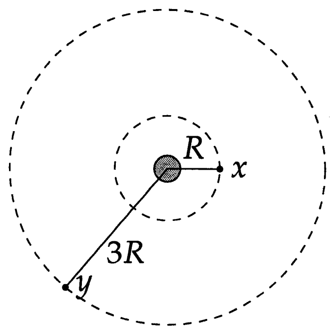
	Angular momentum	Kinetic energy
(a)	Increases	Remains constant
(b)	Remains constant	Increases
(c)	Decreases	Remains constant
(d)	Remains constant	Decreases
(e)	Remains constant	Remains constant

d 15. Two planets of mass M and $9M$ are in the same solar system. The radius of the planet of mass M is R. In order for the acceleration due to gravity to be the same for each planet, the radius of the planet of mass $9M$ would have to be

(a) $1/2\,R$
(b) R
(c) $2R$
(d) $3R$
(e) $9R$

$\frac{G(9M)}{(3R)^2} = 3R$

↑ new R.



a 16. Two planets, X and Y, orbit a star. Planet X orbits at a radius R, and Planet Y orbits at a radius $3R$. Which of the following best represents the relationship between the acceleration a_X of Planet X and the acceleration a_Y of Planet Y?

(a) $a_X = 9a_Y$
(b) $9a_X = a_Y$
(c) $a_X = 3a_Y$
(d) $3a_X = a_Y$
(e) $a_X = a_Y$

$g_x = \frac{GM}{R^2}$

$g_x = 9g_y$

$g_y = \frac{GM}{(3R)^2} = \frac{GM}{9R^2} = \frac{1}{9}g_x$

a 17. A satellite is in a stable circular orbit around the Earth at a radius R and speed v. At what radius would the satellite travel in a stable orbit with a speed $2v$?

(a) $1/4\,R$
(b) $1/2\,R$
(c) R
~~(d) $2R$~~
(e) $4R$

e 18. The Earth and the moon apply a gravitational force to each other. Which of the following statements is true?

(a) The Earth applies a greater force on the moon than the moon exerts on the Earth.
(b) The Earth applies a smaller force on the moon than the moon exerts on the Earth.
(c) The Earth applies a force on the moon, but the moon does not exert a force on the Earth.
(d) The Earth does not apply a force on the moon, but the moon exerts a force on the Earth.
(e) The force the Earth applies to the moon is equal and opposite to the force the moon applies to the Earth.

C 19. Two masses exert a gravitational force F on each other. If one of the masses is doubled, and the distance between the masses is tripled, the new force between them is

(a) $6F$
(b) $2/3\ F$
(c) $2/9\ F$
(d) $3/2\ F$
(e) $4/9\ F$

$$F_g = \frac{GM_1M_2}{R^2} \rightarrow \frac{G(2M_1)M_2}{(3R)^2} \rightarrow \frac{2}{9}F_g$$

C 20. A planet orbits at a radius R around a star of mass M. The period of orbit of the planet is

(a) $\sqrt{\frac{4\pi^2R^2}{GM}}$
(b) $\frac{4\pi^2R^3}{GM}$
(c) $\sqrt{\frac{4\pi^2R^3}{GM}}$
(d) $\sqrt{\frac{4\pi^2R}{GM}}$
(e) $\frac{GM}{4\pi^2R}$

$$v = \sqrt{\frac{GM}{R^2}} = \frac{2\pi R}{T} \rightarrow T = \sqrt{\frac{4\pi^2R^3}{GM}}$$

b 21. A moon orbits a large planet in an elliptical orbit, with its closest approach at a distance a, and its farthest distance b. The speed of the moon at point b is v. The speed at point a is

(a) $\frac{av}{b}$
(b) $\frac{bv}{a}$
(c) $\frac{(a+b)v}{b}$
(d) $\frac{(b-a)v}{b}$
(e) $\frac{2bv}{a}$

angular momentum conserves.

$\cancel{m}v_b b = \cancel{m}v_a a \qquad V_a = V\frac{b}{a} \rightarrow$

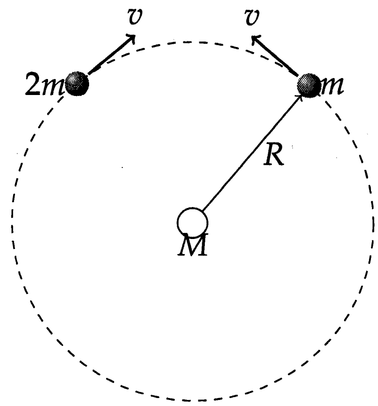
b 22. A satellite orbits the Earth in an elliptical orbit. Which of the following statements is true?

(a) The angular velocity of the satellite increases as it travels farther from the Earth.
(b) The acceleration of the satellite increases as it travels closer to the Earth.
(c) The angular momentum of the satellite increases as it travels closer to the Earth.
(d) The potential energy of the satellite is equal to its kinetic energy at all points in the orbit.
(e) The speed of the satellite must remain constant for it to remain in orbit around the Earth.

no, stays the same!

no!

no!



a 23. Two moons of mass m and $2m$ orbit a planet of mass M at the same radius R and speed v toward each other, as shown. The moons collide and stick together without destroying either moon. The total momentum of the moons after the collision is

(a) mv
(b) $2mv$
(c) $3mv$
(d) $6mv$
(e) zero

$mv \circlearrowleft$

e 24. The velocity of the two masses after the collision above is

(a) v counterclockwise
(b) $v/2$ counterclockwise
(c) $v/2$ clockwise
(d) $v/3$ counterclockwise
(e) $v/3$ clockwise

$v' = \frac{mv}{3m} = \frac{v}{3}$

____ 25. Consider a two-star system shown above, which consists of two stars of mass m rotating in a circle of radius r about their center of mass. What is the total energy of the two-star system?

(a) $-Gm^2/2r$
(b) $Gm^2/2r$
(c) $Gm^2/4r$
(d) $3Gm^2/4r$
(e) $-Gm^2/4r$

c 26. If a planet has twice the radius of Earth and half of Earth's density, what is the acceleration due to gravity on the surface of the planet (in terms of the gravitational acceleration g on the surface of Earth)?

(a) $4g$
(b) $2g$
(c) g
(d) $g/2$
(e) $g/4$

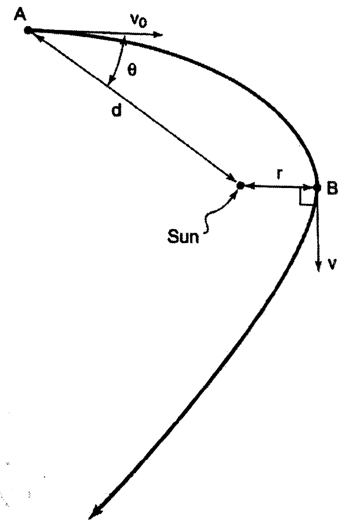
$m_1 = \frac{4}{3}\rho\pi r^3$

$m_2 = \frac{4}{3}\left(\frac{\rho}{2}\right)\pi(2r)^3 = 4m_1$

$g = \frac{GM}{R^2} \rightarrow \frac{G(4M)}{(2R)^2}$

Free-Response Questions:

1. A spacecraft moving with an initial velocity $\mathbf{v}_0$, shown below, "slingshots" around the sun in order to reverse its direction. The sun's mass is m_{sun} and you can make the assumption that the sun remains stationary.



(a) What is the minimum initial speed required by the spacecraft to escape the sun's gravitational field and move in a trajectory toward infinity?

(b) What is the minimum initial speed v_0 that the spacecraft must have in order to avoid falling into the sun? (Treat the sun and the spacecraft as points.)

(c) Repeat the previous questions, but now the sun has a radius R.

(d) Write down the equations required to calculate the initial angle θ in terms of v_0, d, m_{sun}, G, and r.

↓ equation from c, plus

$$\boxed{\frac{1}{2} m_{shuttle} v_0^2 - \frac{G m_{shu.} m_s}{d} = \frac{1}{2} m_{shuttle} v^2 - \frac{G m_{shu.} m_{sun}}{r}}$$

(a) it's just escape velocity.

$$v_0 = \sqrt{\frac{G m_{sun}}{d}}$$

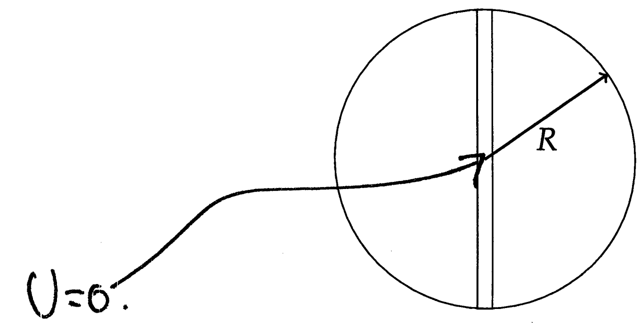
(b) conservation of angular momentum

$$v_0 = 0$$

(c) $p = \cancel{m} v_0 \sin\theta = \cancel{m} v R$

$$v = \frac{v_0 d}{vR} \sin\theta$$

2. A planet of mass M, radius R, and uniform density has a small tunnel drilled through the center of the planet, as shown below. When the mass is inside the tunnel, it experiences a force of $F = (GmM/R^3)r$, whereas when the mass is outside of the planet, it experiences a gravitational force of $F = GmM/r^2$.



(a) Setting the potential energy of the mass to be zero at the planet's center, calculate the mass's potential energy as a function of distance from the center of the planet $U(r)$, for values $r < R$. Sketch this potential function.

(b) If the mass is dropped from R from the center of the planet, how long will it take until it returns to its original position?

(c) If the mass is dropped from $R/2$ from the center of the planet, will it require more, or less, or the same amount of time to return to its original position compared to if it was dropped from R?

(d) If the mass is dropped from $2R$ from the center of the planet, will it require more, or less, or the same amount of time to return to its original position compared to if it was dropped from R?

(a) $U(r) = -\int_0^r -F_g\,dr = \int_0^r \underbrace{\frac{GMm}{R^3}}_{\text{Constant}} r\,dr = \frac{GMm}{2R^3} r^2$

quadratic equation.

(b) $T = 2\pi\sqrt{\frac{m}{(GmM/R^3)}} = 2\pi\sqrt{\frac{R^3}{GM}}$

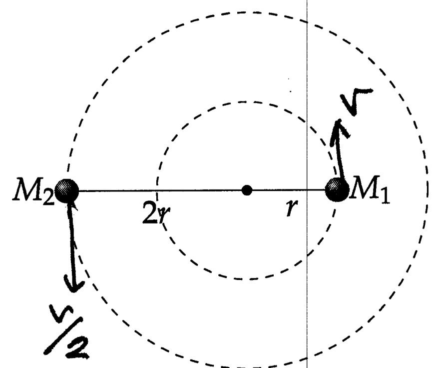
$F_g = kx$ at $r=R$

$k = \frac{F_g}{R}$

(d) more amount of time.

(c) same period !!

3. Two stars of unequal mass orbit each other about their common center of mass as shown. The star of mass M_1 orbits in a circle of radius r, and the star of mass M_2 orbits in a circle of radius $2r$.



(a) Determine the ratio of masses M_1/M_2. on the surface.
(b) Determine the ratio of the acceleration a_1 of M_1 to the acceleration a_2 of M_2.
(c) Determine the ratio of the period T_1 of M_1 to the period T_2 of M_2.

(c)

(a) the period must be the same in order for the orbit to be stable!

$T_1 = T_2$ 8

$$\frac{\cancel{4\pi^2} r^3}{GM_1} = \frac{\cancel{4\pi^2}(2r)^3}{GM_2}$$

$$\frac{M_1}{M_2} = 8$$

(b) $g_1 = \frac{GM}{r^2}$ $g_2 = \frac{GM}{(2r)^2}$

$\frac{g_1}{g_2} = 4.$

not sure if this is right.....

$a \cancel{g_1} = \frac{v^2}{r}$ $\frac{\cancel{g_2}}{a_2} = \frac{\left(\frac{v}{2}\right)^2}{(2r)} = \frac{1}{8}\cancel{g_1}$

$\cancel{\frac{g_1}{g_2} = 8}$

$\frac{a_1}{a_2} = 8.$