Classes 19: Fluid Mechanics

AP Physics

Dr. Timothy Leung

Olympiads School

March 2018

Files for You to Download

Download from the school website:

1. `19-fluidMechanics.pdf`—This presentation. If you want to print the slides on paper, I recommend printing 4 slides per page.
2. `20-Homework.pdf`—Homework assignment for Classes 19 and 20, which cover Fluid Mechanics and Thermodynamics

Please download/print the PDF file before each class. When you are taking notes, pay particular attention to things I say that aren't necessarily on the slides.

Disclaimer

Use of Calculus

Fluid mechanics is part of the AP Physics 2 Exam, which does not require calculus. However, in the interest in completeness, *some* calculus will still be used when deriving equations.

What is a Fluid

- **The simplistic explanstion:** anything that flows
- **The scientific explanation:** Any substancs that deform *continuously* under oblique stress

Properties of Fluids

Density

Continuity

A fluid is considered to be continuous in space.

Properties of Fluids

Viscosity

Hydrostatics

Buoyancy

Everything Floats a Little

When an object is submerged inside a fluid (e.g. water, air, etc), the fluid exerts a pressure at the surface of the object. We can integrate the pressure over the entire surface area and find the total force the fluid exerts on the object.

Derivation of Buoyance Force

We can integrate the pressure over the entire surface to find the total force, or take some knowledge of vector calculus (divergence theorem):

$$\mathbf{B} = -\oint_S p\mathbf{n}dS = -\iiint \nabla p dV$$

Since pressure is given by $p = \rho g z$—a function in z only—the gradient easy to compute: $\nabla p = \rho g \hat{\mathbf{k}}$, giving us

$$\mathbf{B} = \rho_{\text{fluid}} g \hat{\mathbf{k}} \iiint dV = \rho_{\text{fluid}} g V \hat{\mathbf{k}}$$

Derivation of Buoyance Force

Although the derivation required a lot of calculus, the expression of buoyance force is *very* straightforward:

$$\boxed{\mathbf{B} = \rho_{\mathrm{fluid}} g V \hat{\mathbf{k}} = m_{\mathrm{fluid}} g \hat{\mathbf{k}}}$$

where ρ_{fluid} is the density of the displaced fluid, and V is the volume displaced. This equation is known as **Archimedes' principle**.

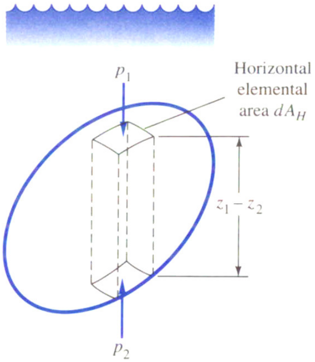
Buoyance force has a magnitude that equals to the weight of the fluid displaced by the submerged object, pointing upward.

An Easier Explanation of Buoyancy

Not Much Calculus

There is a simpler way to find the buoyance force, by taking an infinitesimal "tube" of the object, and finding the pressure difference between the top and bottom of the tube:

$$\mathbf{B} = \int (p_2 - p_1) dA$$
$$= \rho g \int (z_2 - z_1) dA$$
$$= \rho g V$$



which is the same expression that we got with calculus.

Buoyancy

Note that buoyancy does not depend on:

- the mass of the immersed object, or
- the density of the immersed object

Objects immersed in a fluid have an "apparent weight" that is reduced by the buoyance force:

$$\mathbf{W} = \mathbf{W} - \mathbf{B}$$

$$\mathbf{W} = (\rho_{\text{obj}} - \rho_{\text{fluid}})\mathbf{g}V$$

$\mathbf{W}'$ is proportional to the relative density ($\rho' = \rho_{\text{obj}} - \rho_{\text{fluid}}$)

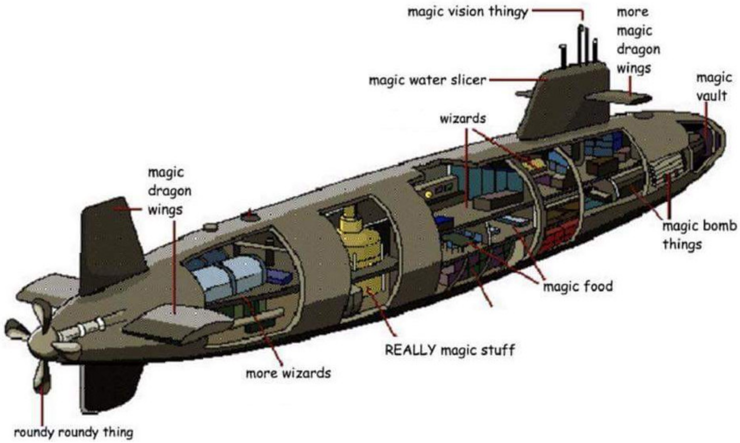
Buoyancy

For a submerged object:

Densities	$B > W_{\text{obj}}$	$B = W_{\text{obj}}$	$B < W_{\text{obj}}$
$\rho_{\text{obj}} < \rho_{\text{fluid}}$	object rises	float on surface	
$\rho_{\text{obj}} = \rho_{\text{fluid}}$		neutral buoyancy	
$\rho_{\text{obj}} > \rho_{\text{fluid}}$			object sinks

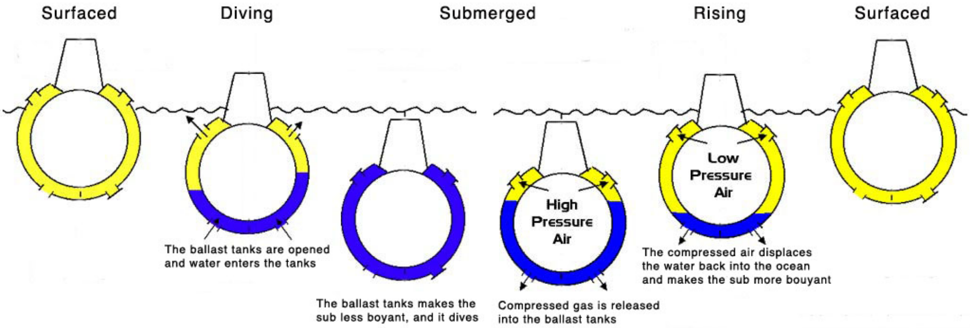
How Submarines Work

Like this?



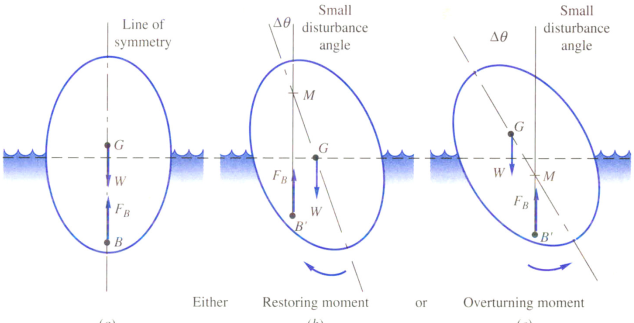
How Submarines Work

Like most ships, a submarine does not naturally sink because of the buoyance force. When a submarine submerges, water needed to be pumped inside "ballast tanks" to make the ship heavier.



Stable? Or unstable?

- Buoyance force $\mathbf{B}$ acts at the *center of buoyancy* (CB) of the submerged object
  - The CB is the CG *if the object has constant density*
  - The actual CG of the object may be at a different position
  - Sometimes the object is not fully submerged
- Therefore $\mathbf{F}_g$ and $\mathbf{B}$ may act at different points, creating a torque/moment on the object



Fluid Flow

Flow of fluid out of a surface requires us to look at the flux function again: Volume flux is defined as:

$$\boxed{\Phi_{\mathrm{V}} = \int \mathbf{V} \cdot d\mathbf{A}}$$

where $\mathbf{V}$ is the velocity (vector field) at the surface, and $d\mathbf{A}$ is the infinitesimal area pointing **outwards.** We can also expressed volume flux using the outward normal unit vector $\hat{\mathbf{n}}$:

$$\boxed{\Phi_{\mathrm{V}} = \int \mathbf{V} \cdot \hat{\mathbf{n}} dA}$$

Bernoulli Equation

$$\boxed{p_1 + \frac{1}{2}\rho v_1^2 + \rho g z_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g z_2}$$

The term $\frac{1}{2}\rho v^2$ is called "dynamic pressure"

Bernoulli Equation

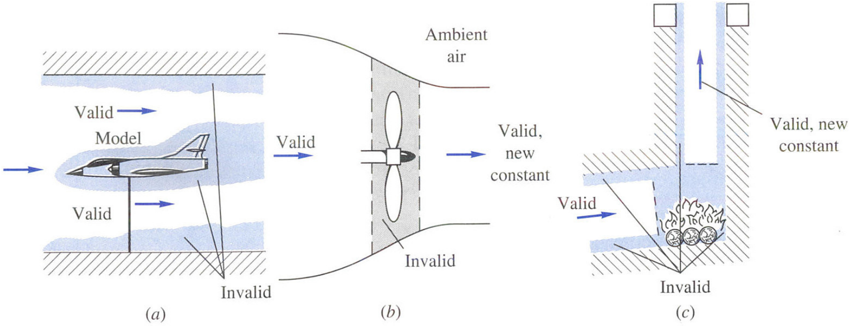
$$\boxed{p_1 + \frac{1}{2}\rho v_1^2 + \rho g z_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g z_2}$$

Bernoulli's equation is valid when

- the flow is **steady** (independent of time)
- the flow is **incompressible**–compressibility (i.e. changes in density of the fluid) effects are negligible for Mach number $M < 0.30$
- the flow **along a single streamline**
- there is **no shaft work** done along the streamline between 1 and 2
- there is **no heat transfer** along the streamline between 1 and 2

Bernoulli Equation

Regions where Bernoulli equation is valid:



How Does A Wing Work?

When air flows past a wing, a force is generated