Class 21. Light and Optics

Advanced Placement Physics

Olympiads School

April 2018

In This Class

We will be discussing some important properties of light in this class.

- Reflection
- Refraction
- Dispersion
- Diffraction
- Interference
- Optical resolution
- Electromagnetic waves
- Polarization of light

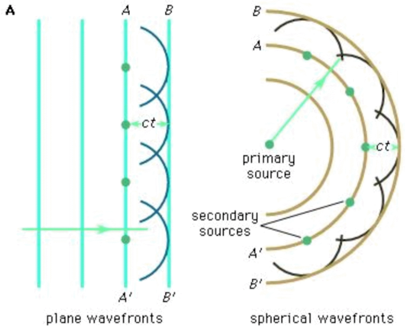
Most of what we discuss should be reviews from Grade 10 Science and Physics 12.

Huygen’s Principle

In the 1600’s there were two competing theories of light. . .

- Some, including Issac Newton, believed that light is a particle
- Others, including Christiaan Huygen (Dutch) and Augustin-Jean Fresnel (French), believed that light is a wave

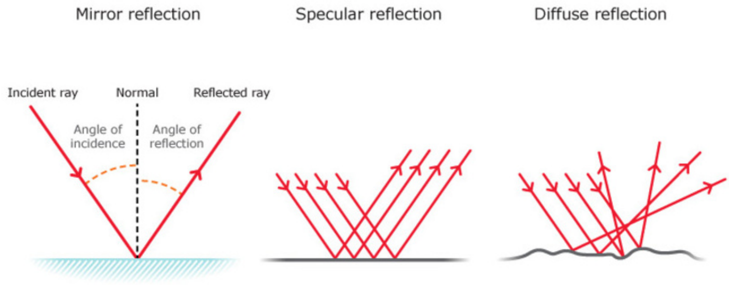
Huygen’s Princple: all waves are in fact an infinite series of circular wavelets



Reflection of Light

Law of Reflection

The incident ray, the reflected ray, and the normal to the surface of the mirror all lie in the same plane, and the angle of reflection is equal to the angle of incidence.



Specular Reflection

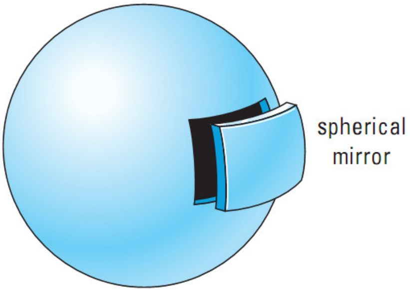
Example: Lake Reflection

This photo of Lake Matheson shows specular reflection in the water of the lake with reflected images of Aoraki/Mt Cook (left) and Mt Tasman (right). The very still lake water provides a perfectly smooth surface for this to occur.

Some Terminology

- **Object:** the tangible item that you see in the absence of any optical devices
- **Real image:** Light rays are actually converging at a point then continuing on beyond that point and diverging.
- **Virtual image:** No actual light rays converging on the image position

Spherical Mirror

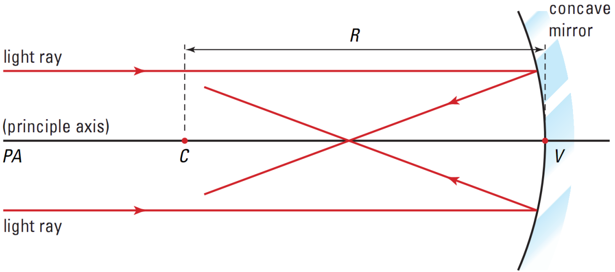


A sphere with smooth surface on inside and out.

- **Concave Mirror:** Inside of a Spherical Mirror
- **Convex Mirror:** Outside of a Spherical Mirror

More Keywords

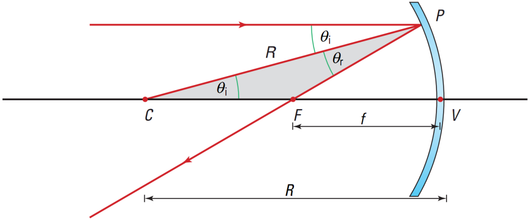
- **Center of Curvature** C is the center of an imaginary sphere with the same curvature as the mirror.
- **Radius of Curvature** R is any straight line drawn from the center of curvature to the curved surface.
- **Vertex** V is the geometric center of the actual curved mirror surface
- **Principal Axis** PA is a straight line that passes through V and C.



More Keywords

- **Focal Point** F is a point halfway between the center of curvature and the vertex of the mirror.
- **Focal Length** f is the distance from the focal point to the vertex. It it one half the radius of curvature, R:

$$\boxed{f = \frac{1}{2}R}$$



Spherical Aberration

The light rays that are not close to the principal axis (PA) does not reflect to the focal point (Figure A) To focus properly, a *parabolic* mirror (Figure B) is needed!

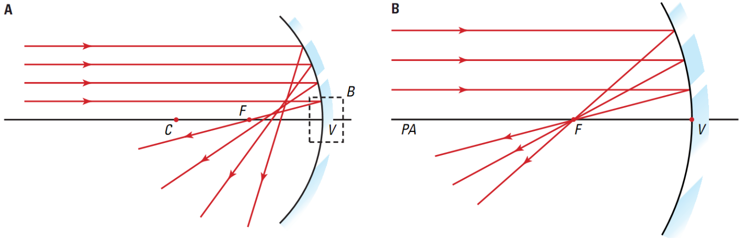


Image Outside Center

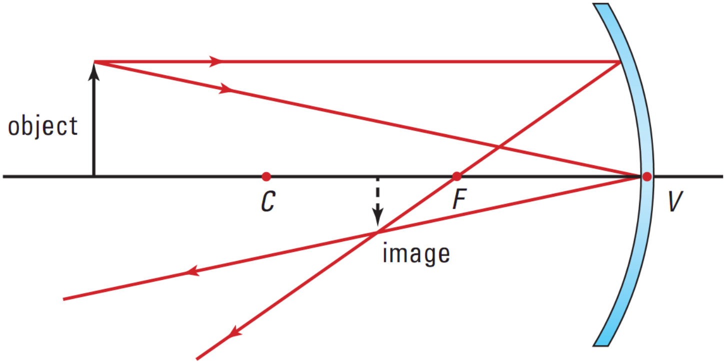


Image Between Center and Focal Point

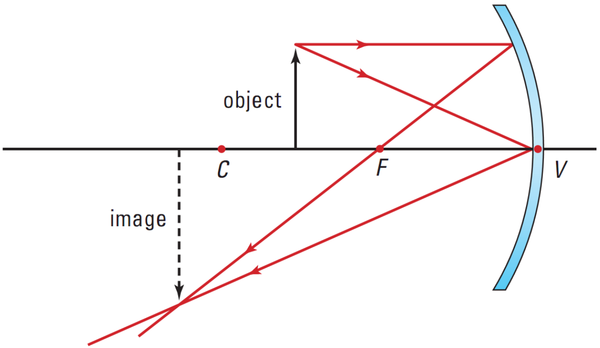
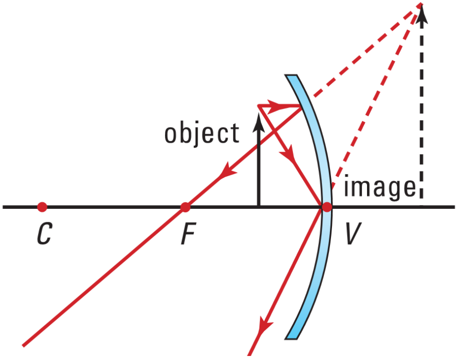
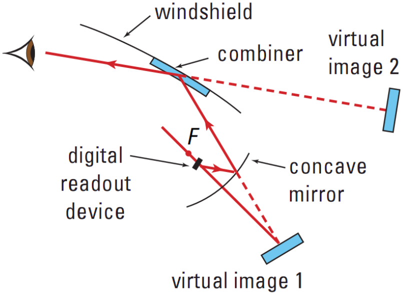


Image Between Focal Point and Vertex



Practical Usage: HUD

- Head's-Up Display (HUD)
- Used majority in military aircraft
- Becoming popular in many cars and trucks



Thin-Lens Equation

Magnification:

$$\boxed{M = \frac{h_i}{h_o} = -\frac{d_i}{d_o}}$$

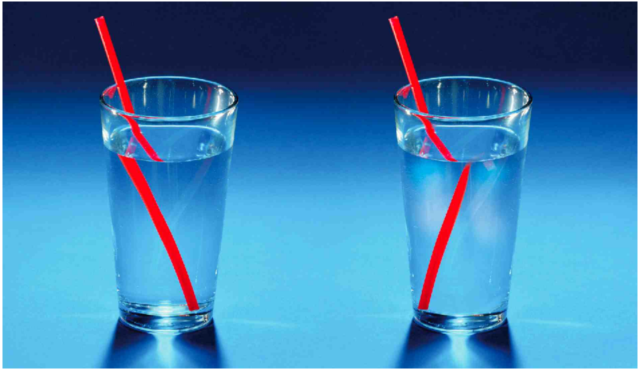
Mirror/lens equation:

$$\boxed{\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}}$$

Variable	Symbol	SI Unit
Object and image height	h_o, h_i	m (meters)
Object and image distance to lens	d_o, d_i	m (meters)
Focal length	f	m (meters)
Magnification factor	M	no units

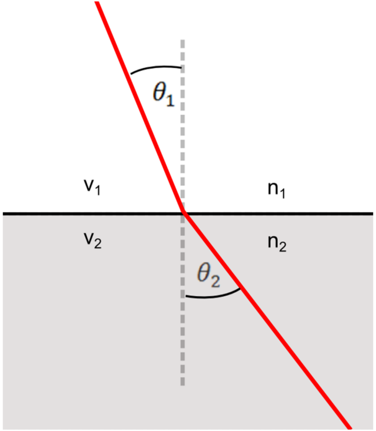
Refraction of Light Through a Medium

- When a wave enters another medium, the wave speed changes
- When entering at an angle, the change of speed causes the wave to change direction (e.g. from air to water, air to glass, glass to air etc)
- The amount of bending depends on the **indices of refraction of the two media**
- Responsible for **image formation** by lenses and the eye



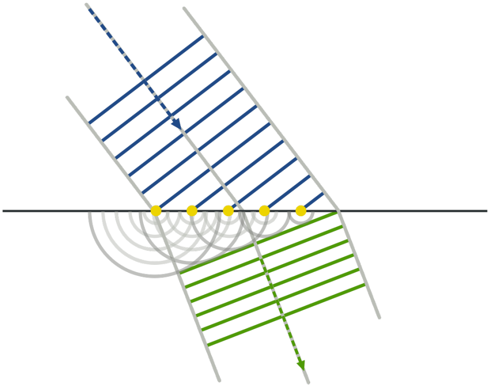
Refraction of Light Through a Medium

You have probably all seen this diagram of light entering from one medium to another.



Light could be going in either direction, from top to bottom (1 to 2) or or from bottom to top (2 to 1)

Refraction and Huygen's Principle



We can explain the refraction phenomenon using Huygen's Principle

Snell's Law

Snell's Law relates the indices of refraction n of the two media to the directions of propagation in terms of the angles to the normal.

$$\boxed{n_1 \sin\theta_1 = n_2 \sin\theta_2}$$

Variable	Symbol	SI Unit
Indices of refraction of the media	n_1, n_2	(no units)
Incident angle of light	θ_1	(no units)
Refraction angle of light	θ_2	(no units)

Index of Refraction

Index of refraction (n) is defined as the speed of light in vacuum (c) divided by the speed of light in the medium (v).

$$\boxed{n = \frac{c}{v}}$$

When light enters a second medium, the *frequency* remains unchanged (i.e. the colour doesn't change!) but since the speed changes, the *wavelength* also changes:

$$\boxed{\frac{n_1}{n_2} = \frac{\lambda_2}{\lambda_1}}$$

You can work this out using the universal wave equation: $v = f\lambda$

Index of Refraction of Common Materials

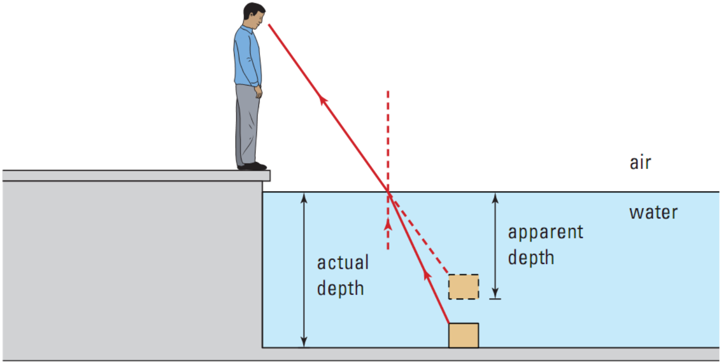
Material	n	Material	n
Vacuum	1	Ethanol	1.362
Air	1.000277	Glycerine	1.473
Water at 20 °C	1.33	Ice	1.31
Carbon disulfide	1.63	Polystyrene	1.59
Methylene iodide	1.74	Crown glass	1.50-1.62
Diamond	2.417	Flint glass	1.57-1.75

The values given are *approximate* and do not account for the small variation of index with light wavelength. That's called **dispersion**.

Effects of Refraction

Apparent Depth

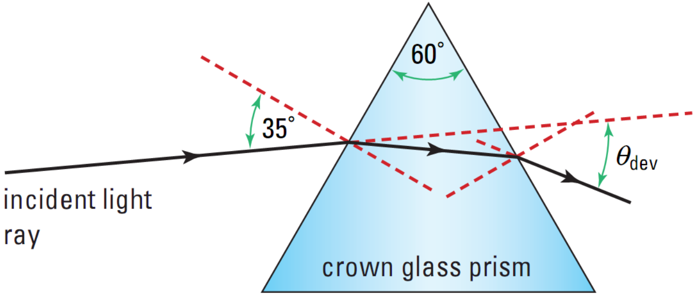
Objects in water appear to be at a shallower depth:



Effects of Refraction

Deviation

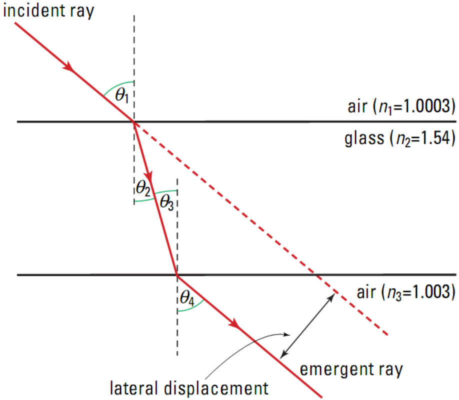
Light through chandeliers

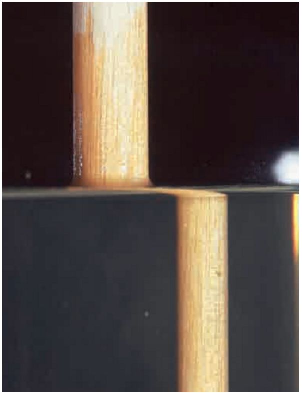


Effects of Refraction

Lateral Displacement

Light travels through a medium which has the two sides parallel to each other.

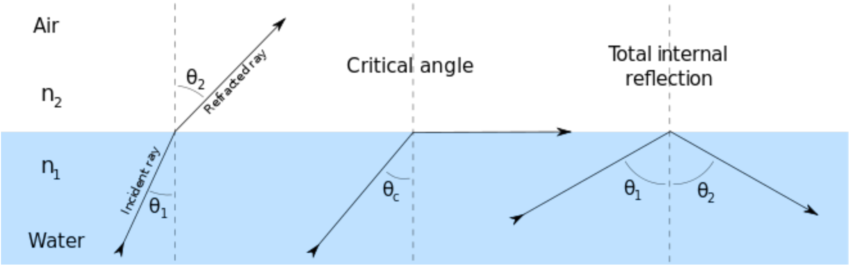




Total Internal Reflection

From High Index to Low Index

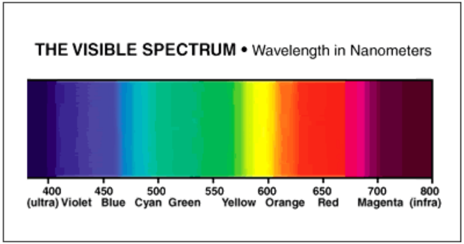
Snell's Law still holds, but something weird can happen:



Critical angle θ_c for water-air interface is 48.6°. If incident angle is greater $\theta_1 > \theta_c$, we have **total internal reflection**. TIR can only happen going from a higher index to a lower index, $n_1 > n_2$.

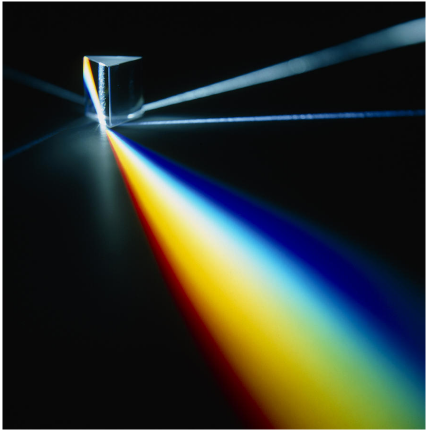
Colour of Light and Wavelength

Human eyes perceive different frequencies of light as different colours. The visible spectrum of light:



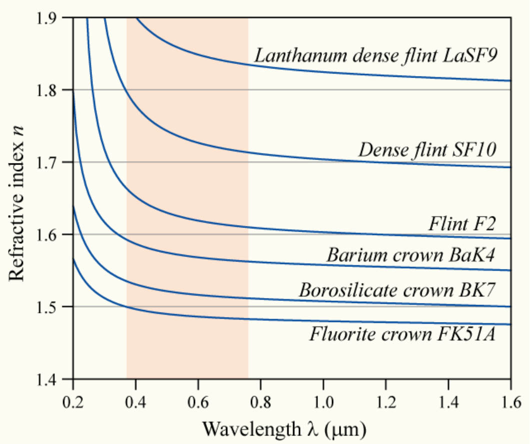
- The *color* of the light depends on its frequency (& wavelength when it's in a vacuum)
- *White light* is light that contains waves in all frequencies.

Dispersion of Light Through Refraction

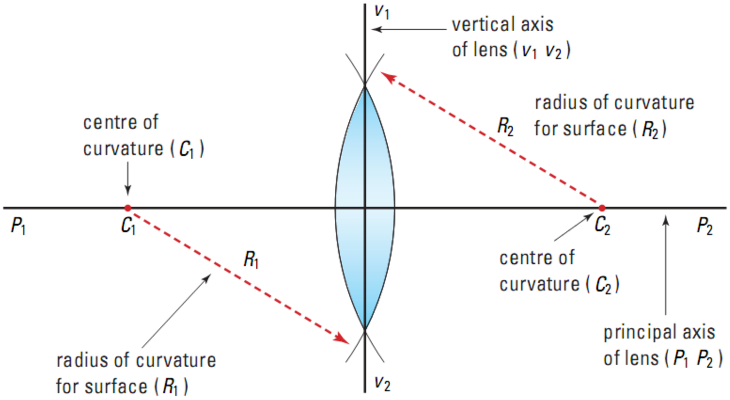


- When white light passes through a prism it is separated into different colours (spectrum) through refraction.
- This is because the index of refraction n is slightly different for different wavelengths
- Otherwise, we will never see a rainbow

Wavelength Dependency of Index of Refraction



Lenses and Ray Diagrams



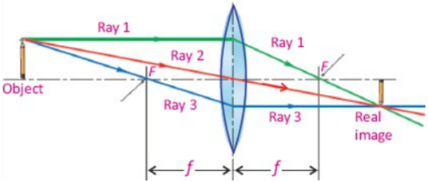
Lensmaker's Equation

- Used to determine the focal length (the distance from the lens at which the image is focused on) of the lens
- The reciprocal of the focal length is the product of one less than the index of refraction and the sum of the reciprocals of the radii of curvature.

$$\boxed{\frac{1}{f} = (n-1)\left[\frac{1}{R_1} + \frac{1}{R_2}\right]}$$

Variable	Symbol	SI Unit
Focal length	f	m (meters)
Index of refraction	n	(no units)
First radius of curvature	R_1	m (meters)
Second radius of curvature	R_2	m (meters)

Convex Lenses



Thin-Lens Equation

Same Equations as Mirrors

Magnification:

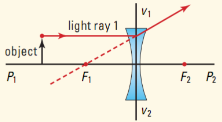
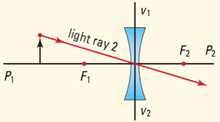
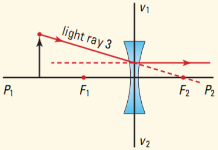
$$\boxed{M = \frac{h_i}{h_o} = -\frac{d_i}{d_o}}$$

Mirror/lens equation:

$$\boxed{\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}}$$

Variable	Symbol	SI Unit
Object and image height	h_o, h_i	m (meters)
Object and image distance to lens	d_o, d_i	m (meters)
Focal length	f	m (meters)

Concave Lenses

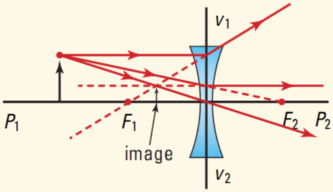
Light ray #1 Draw a ray from the top of the object to the lens parallel to the principal axis. Draw the refracted ray as though it were coming from the focal point on the same side of the lens as the object.	By the definition of the focal point, any ray entering the lens parallel to the principal axis will diverge as though it were coming from the focal point on the same side of the lens as the object.	
Light ray #2 Draw a ray from the top of the object through the centre of the lens and extend it straight through to the far side of the lens.	At the very centre of the lens, the sides are parallel. When light passes through a refracting medium with parallel sides, the emergent ray is parallel to the incident ray but displaced to the side. In the case of thin lenses, the displacement is so small that it can be ignored.	
Light ray #3 Draw a ray from the top of the object directly toward the focal point on the opposite side of the lens. Draw the refracted ray on the far side of the lens parallel to the principal axis.	As in the case of ray #1, any ray entering the lens parallel to the principal axis will diverge as though it were coming from the focal point on the object side of the lens. Then apply the principle of reversibility of light. The result is that any ray entering a lens toward the opposite focal point will leave parallel to the principal axis.	

Concave Lenses

Identify the Image

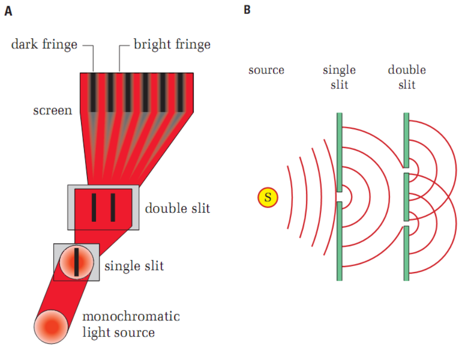
Refracted rays always diverge. Therefore, extend the rays backward with dashed lines until they intersect.

The point at which the extended rays meet is the position of the top of the image. The image is always virtual.



Thomas Young's Double-Slit Experiment

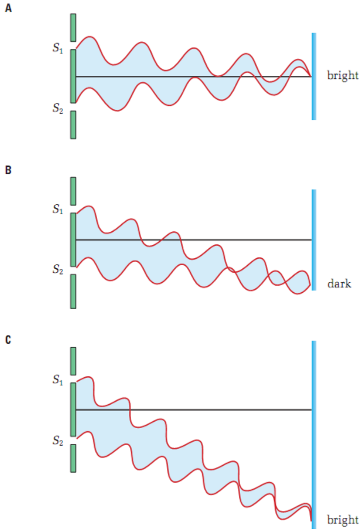
First definitive evidence that light is a wave



- **Monochromatic light** light with a single colour (frequency); the light source can be a laser, LED , or gas lamp (most likely what Young used)
- **Slit:** an opening; also called an **aperture**
- The **screen** far away from the slits is also called the **projection**

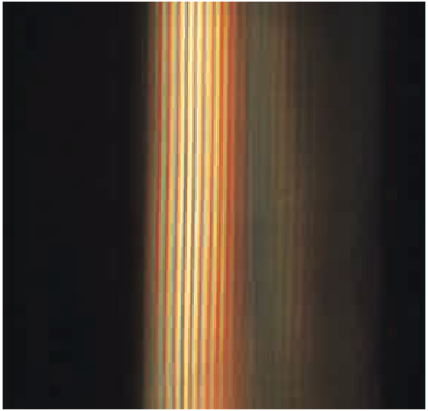
Double-slit experiment showed that light causes interference, just like any other wave

Thomas Young's Double-Slit Experiment



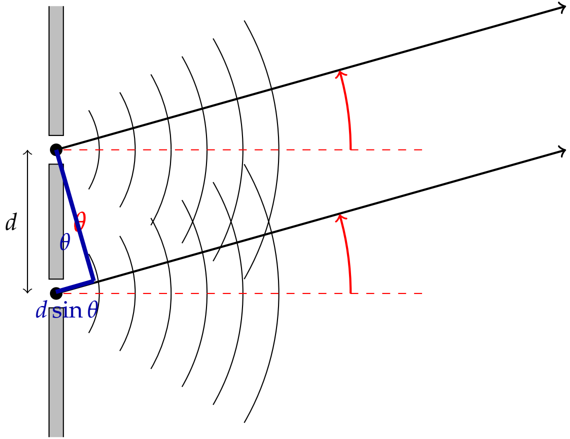
- At **A**, the path from slits S_1 and S_2 are the same, therefore we have **constructive interference** and the projection is bright
- At **B**, the path from S_1 and S_2 are diffed by half a wavelength, and therefore there is **destructive interference** and the projection is dark
- At **C**, the path from S_1 and S_2 are diffed by one wavelength, and therefore there is **constructive interference** again, and again, the projection is bright

Interference Pattern: Bright and Dark Fringes



The “bright fringes” are from constructive interference; the “dark fringes” are from destructive interference.

Let's Work This Out!



- We have two slits at distance d apart, emitting *coherent* light
- Huygens' Principle: light passing through the slits become point sources
- Assume that the projection (screen) is far enough from the slits that we can treat the two beams of light from the slits as being parallel
- Using basic geometry, we can see that the path difference from the two slit to the projection is $d \sin\theta$

Double-Slit Interference

Constructive Interference

A bright fringe (constructive interference) will happen if the path length difference ($d \sin \theta$) is an integer (n) multiple of wavelength (λ), i.e.

$$\boxed{\pm n\lambda = d \sin \theta_n}$$

where $n = 0, 1, 2, 3 \ldots$

Quantity	Symbol	SI Unit
Integer number of full wavelengths	n	(none)
Wavelength of light	λ	m (meters)
Distance between slits	d	m (meters)
Angle between slit separation and line perpendicular to light rays	θ	(unit less)

Double-Slit Interference

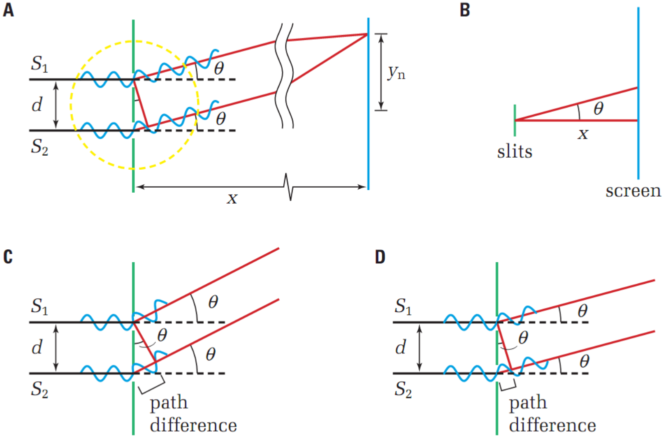
Destructive Interference

Conversely, a dark fringe (destructive interference) will happen if the path length difference ($d\sin\theta$) is an half-number ($n+\frac{1}{2}$) multiple of wavelength (λ), i.e.

$$\boxed{\pm\left(n+\frac{1}{2}\right)\lambda = d\sin\theta_n}$$

where $n = 0, 1, 2, 3\ldots$

Double-Slit Interference



Approximation of The Wavelength of Light

We can actually estimate the wavelength of light based on the distances between bright fringes, by applying the **small-angle approximation** for angles measured in *radians*:

$$\theta \approx \tan\theta \approx \sin\theta$$

We can already relate the distance from slits to the screen (x), and the distance of the n-th bright fringe from the centre (y_n) to θ_n. Applying the approximation, we have:

$$\tan\theta_n = \frac{y_n}{x} \approx \sin\theta_n$$

Substitute this approximation into the constructive interference equation:

$$n\lambda \approx \frac{y_n d}{x} \quad \longrightarrow \quad \boxed{\lambda \approx \frac{\Delta y d}{x}}$$

Approximation of The Wavelength of Light

This equation applies equally to dark fringes (nodal lines) as well as bright fringes.

$$\boxed{\lambda \approx \frac{\Delta y d}{x}}$$

Quantity	Symbol	SI Unit
Wavelength	λ	m (meter)
Distance between fringes	Δy	m (meter)
Distance between slits	d	m (meter)
Distance from source to screen	x	m (meter)

Since the approximation is based on small angles, we generally apply this to Δy close to the centre, where light from both slits are deflected by a small angle.

Important Notes

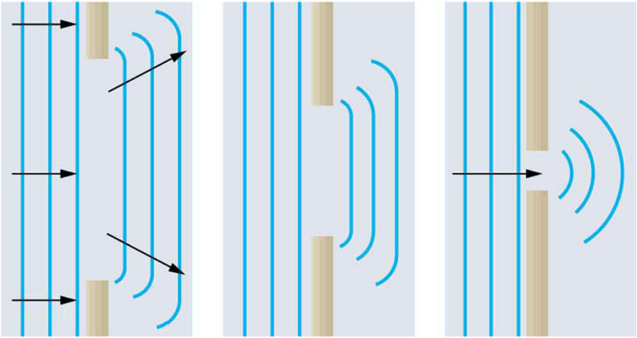
- We have applied the double-slit problem specifically to light, but it can be applied to any wave (e.g. ocean waves) as well
- The sources don't actually need to be slits; any point source will do
- The projection/screen doesn't need to be a real screen either; it just has to be a line where wave intensity can be measured

Diffraction of Waves

When a wave goes through an small opening, it **diffracts**. This happens with sound waves, ocean waves. . . and light.

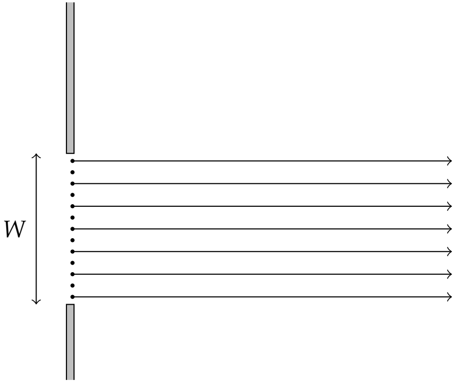
(The photo is from the Port of Alexandria in Egypt. The shape of the entire harbour is created because of diffraction of ocean wave.)

Diffraction of Waves



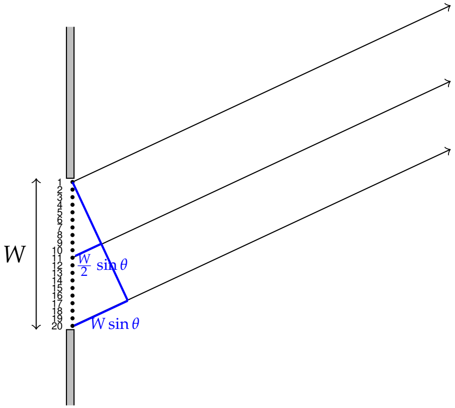
The smaller the opening (compared to the wavelength of the incoming wave) the greater the diffraction effects.

Equations for Diffraction



- Similar to the double-slit problem, we apply Huygens' Principle again
- This time we treat the slit as wide enough that there is a series (an infinite series, actually) of point waves at the slit
- We can easily see that the light from the wavelet that travel perpendicular to the slit (aperture) will not interfere with one another
- i.e. a bright fringe at the middle. **This is called the central maximum.**

At Some Other Angle θ



- Like what we did with double-slit, we can find the path difference between the wavelet on the top (1) and bottom (20): $W\sin\theta$
- At some θ, the path difference between 1 and 20 will be an integer multiple of the wavelength $(m\lambda)$
- In this case, the path difference between 1 and 11 is a half-number multiple of the wavelength (i.e. destructive interference) and they cancel each other
- Similarly, 2 cancels 12, 3 cancels 13. . .

RESULT: COMPLETE DESTRUCTIVE INTERFERENCE

Dark Fringes: Destructive Interference

Dark fringes exists on the screen at regular, whole-numbered intervals ($m = 1, 2, 3 \ldots$):

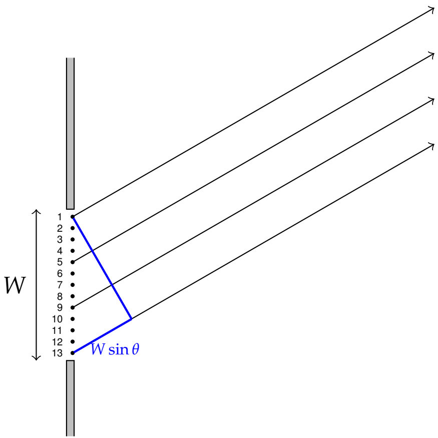
$$\boxed{\pm m\lambda = W \sin\theta_m}$$

Applying the small-angle approximation equation, we end up with:

$$\boxed{y_m = \frac{m\lambda L}{W}}$$

This equation looks very similar to the double-slit equation for *bright* fringes, so be *very* careful when you use them!

At Some Other Angle θ



- Again, we follow what we did with the the previous case, and we find that at some angle θ, the path difference between the top and bottom is $W\sin\theta = \frac{3}{2}\lambda$
- Beam from (1) and (5) differ by $\frac{\lambda}{2}$, so they have destructive interference; similarly 2 and 6, 3 and 7, 4 and 8, 9 and 13 will all interfere destructively
- But some of the beams will not, so we have a bright fringe at the projection
- This bright fringe is not as bright as the central one because of the destructive interference

Bright Fringes: Constructive Interference

Bright fringes exist on the screen at regular, half-numbered intervals ($m = 1, 2, 3 \ldots$):

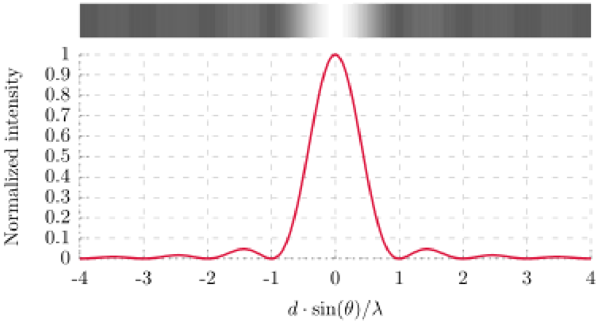
$$\boxed{\pm\left(m + \frac{1}{2}\right)\lambda = W \sin\theta_m}$$

Again, similar to the dark fringes, we apply our small-angle approximation equation:

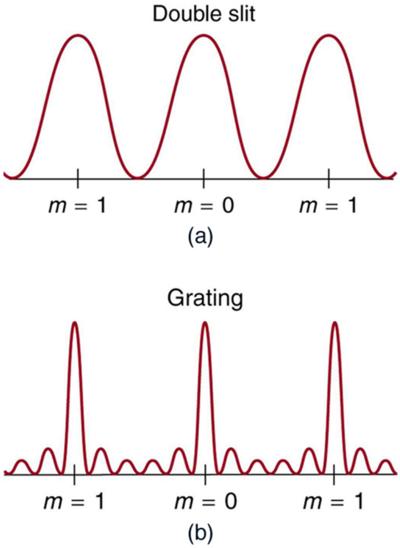
$$\boxed{y_m = \pm\left(m + \frac{1}{2}\right)\frac{\lambda L}{W}}$$

Single-Slit Diffraction, A Summary

- Similar to the double-slit interference, single-slit diffraction projects a series of alternating bright fringes ("maxima") and dark fringes ("minima") in the far field
- The bright fringe in the middle ("central maximum") is twice as wide and very bright
- Subsequent bright fringes on either side ("higher-order maxima") are much dimmer because of the partial destructive interference



Diffraction Grating: What if there are more than 2 slits?



- We can apply the same analysis from double-slit to a diffraction grating
- Use equation for double-slit interference to locate bright fringes

$$n\lambda = d\sin\theta_n$$

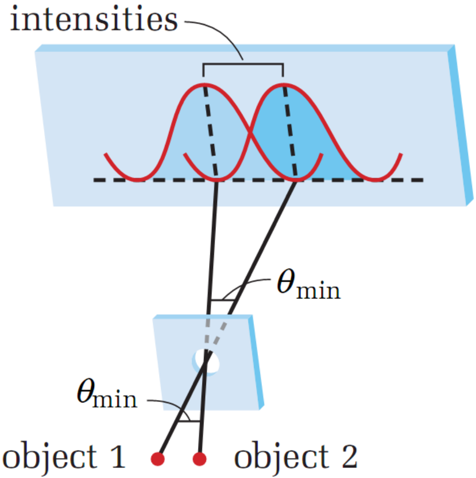
- Interference pattern is sharper
- Bright fringes are narrower

Resolving Power

The ability of an optical instrument (e.g. the human eye, microscope, camera) to distinguish two distinct objects.

WHY? When light from any object passes through an "optical instrument", it **diffracts**, therefore "blurring" the object.

Resolving Power



Rayleigh limit: Two objects are resolved if the angle $\theta > \theta_{\min}$, where $\theta_{\min}$ is when the first minimum (dark fringe) from object 1 overlaps with the central maximum (bright fringe in the middle) from object 2.

Resolving Power

In order to resolve two objects, the minimum angle between rays from the two objects passing through a rectangular aperture is the quotient of the wavelength and the width W of the aperture. For a circular aperture, the minimum angle is the quotient of 1.22 times the wavelength and the diameter D of the aperture.

Rectangular aperture:

$$\boxed{\theta_{\min} = \frac{\lambda}{W}}$$

Circular aperture:

$$\boxed{\theta_{\min} = \frac{1.22\lambda}{D}}$$

The angle $\theta_{\min}$ is measured in **radians** not degree.

Maxwell's Equations

We have already studied them

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = -\mu_0 \mathbf{J} + \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

Maxwell's Equations

Major Findings

- Electric fields starts/ends at a charge
- Magnetic fields runs in a loop, and has no beginning or ends
- A changing electric field creates a magnetic field
- A changing magnetic field creates an electric field
- Disturbances in the electric and magnetic fields propagate as a wave with speed

$$\boxed{c = \frac{1}{\sqrt{\varepsilon_o \mu_o}} = 2.998 \times 10^8 \ \mathrm{m/s}}$$

...the speed of light!

Speed of Electromagnetic Radiation

Electric Permittivity ε_0

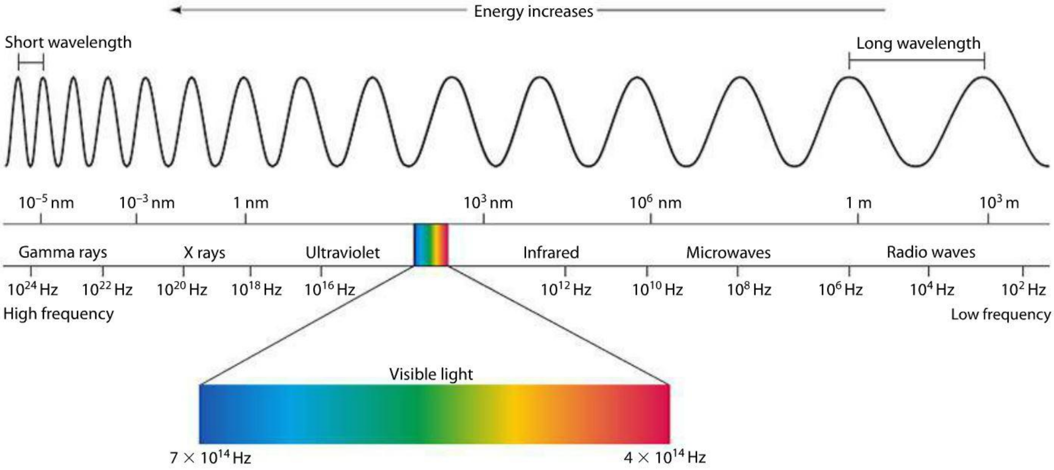
The ability of a medium to resist the formation of an electric field within it. The constant is directly related to the Coulomb constant in Coulomb's law.

Magnetic Permeability μ_o

A measure of the ability of the medium to become magnetized.

- Scientist have previously measured the speed of light to good accuracy
- Maxwell's Equations show that light is (probably) an electromagnetic ("EM") wave
- $\mathbf{E}$ and $\mathbf{B}$ fields of an EM wave are always perpendicular to one another, according to Maxwell's Equations

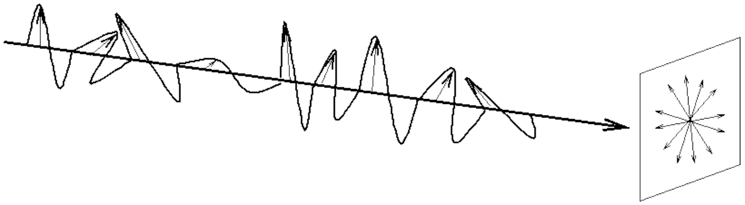
The Electromagnetic Spectrum



On Polarization of Light

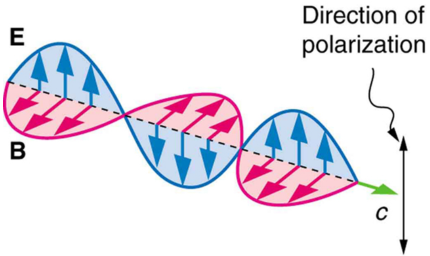
A charged particle can vibrate in any direction, so the oscillating $\mathbf{E}$ and $\mathbf{B}$ can look quite chaotic. We can only guarantee that no matter what happen, $\mathbf{E}$ and $\mathbf{B}$ are:

- Always perpendicular to each other
- Always perpendicular to the direction of wave travel
- This kind of light (or general EM wave) is "unpolarized"
- Most EM waves you experience in life are this kind



On Polarization of Light

But if we can confine $\mathbf{E}$ and $\mathbf{B}$ to one plane, then we have a “polarized” light:

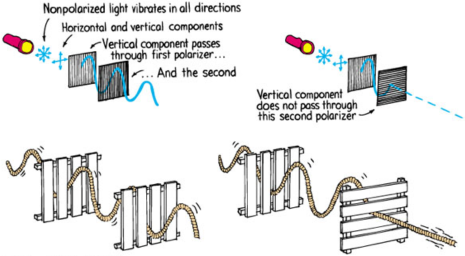


There are a few ways to do this. . .

On Polarization of Light

Using Polarizer

- A polarizer is really just a grill that only lets in vibration in one direction through:

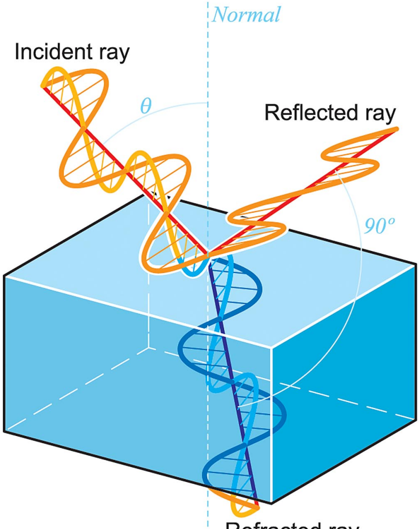


Hewitt, *Conceptual Physics*, Ninth Edition.

- The incoming wave can be vibrating in any direction, but outgoing wave only vibrates in one direction.
- Sunglasses with polarizing lens
- Polarizer filters on cameras

On Polarization of Light

Polarization by Reflection



At **Brewster's angle**, the light reflected off a medium (e.g. glass, water) is also polarized

$$\theta_B = \tan^{-1}\left(\frac{n_2}{n_1}\right)$$

- Incident light is non-polarized
- Reflected light is polarized
- Refracted light is partially polarized
- For water ($n = 1.33$), $\theta_B = 53^\circ$
- For glass ($n = 1.5$), $\theta_B = 56^\circ$