Student #: ____________ Student Name: __________________________

AP Physics Class 5: Circular and Rotational Motion

d 1 Linear acceleration is to force as angular acceleration is to

(a) kinetic energy
(b) angular velocity
(c) rotational inertia
(d) torque
(e) angular momentum

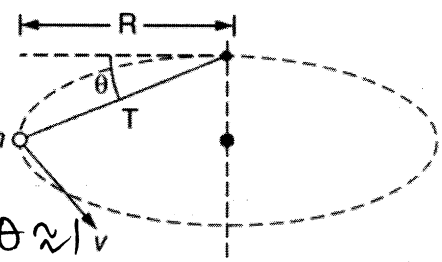
b 2 A girl stands on a rotating merry-go-round without holding on to a rail. The force that keeps her moving in a circle is the

(a) frictional force on the girl directed away from the center of the merry-go-round
(b) frictional force on the girl directed toward the center of the merry-go-round
(c) normal force on the girl directed away from the center of the merry-go-round
(d) normal force on the girl directed toward the center of the merry-go-round
(e) weight of the girl

b 3 A $0.5\,\mathrm{kg}$ ball on the end of a $0.5\,\mathrm{m}$ long string is swung in a horizontal circle. What would the speed of the ball have to be for the tension in the string to be $9.0\,\mathrm{N}$?

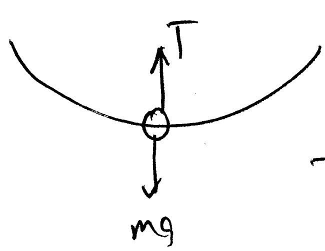
(a) 1.0 m/s
(b) 3.0 m/s
(c) 6.0 m/s
(d) 9.0 m/s
(e) 12.0 m/s

assume that θ is relatively small: ie $\cos\theta \approx 1$



b 4 A ball of mass m is swung in a vertical circle of radius R. The speed of the ball at the bottom of the circle is v. The tension in the string at the bottom of the circle is

(a) mg
(b) $mg + \dfrac{mv^2}{R}$
(c) $mg - \dfrac{mv^2}{R}$
(d) $\dfrac{mv^2}{R}$
(e) zero



$F_c = T - mg$

$T = F_c + mg = \frac{mv^2}{r} + mg$

d 5 A car of mass m drives on a flat circular track of radius R. To maintain a constant speed v on the track, the coefficient of friction μ between the tires and the road must be

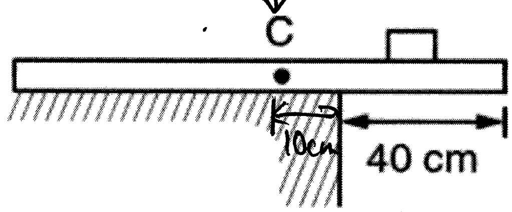
(a) mg
(b) $mg + \dfrac{mv^2}{R}$
(c) $mg - \dfrac{mv^2}{R}$
(d) $\dfrac{v^2}{gR}$
(e) $\sqrt{\dfrac{v^2}{gR}}$

$F_f = F_c$

$\mu \cancel{m} g = \dfrac{\cancel{m} v^2}{R} \longrightarrow \mu = \dfrac{v^2}{gR}$

d 6 A meter stick of mass 0.1 kg rests on a table as shown. A length of 40 cm extends over the edge of the table. How far from the edge of the table could a 0.05 kg mass be placed on the meter stick so that the stick just begins to tip? $(0.1)(10) = (0.05)(r) \rightarrow r = 20$

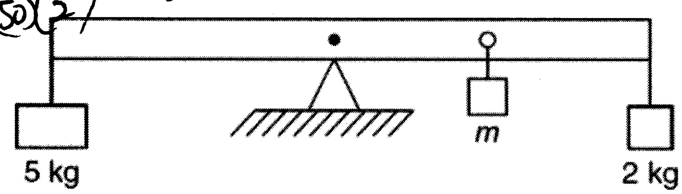
(a) 5 cm
(b) 10 cm
(c) 15 cm
(d) 20 cm
(e) 30 cm



b 7 A meter stick is balanced on a fulcrum at its center, as shown. A mass of 5 kg is hung on the left end of the stick, and a mass of 2 kg is hung on the right end. In order to balance the system, a mass m is hung at the 25-cm mark on the right side. What is the value of the mass m?

30

(a) 12 kg $(50)(5) = (25)(m) + (50)(2)$
(b) 6 kg
(c) 4 kg $m = 6$
(d) 3 kg
(e) 2 kg



e 8 A ball on the end of a string is swung in a circle of radius 2 m according to the equation $\theta = 4t^2 + 3t$, where θ is in radians and t is in seconds. The angular acceleration of the ball is

(a) $6\ \mathrm{rad/s^2}$
(b) $4t^2 + 3t\ \mathrm{rad/s^2}$
(c) $8t + 3\ \mathrm{rad/s^2}$
(d) $\frac{3}{4}t^3 + 3t^2\ \mathrm{rad/s^2}$
(e) $8\ \mathrm{rad/s^2}$

$\omega = \dfrac{d\theta}{dt} = 8t + 3$

$\alpha = \dfrac{d\omega}{dt} = 8$

b 9 The linear speed v of the ball (in the previous question) at $t = 3\,\text{s}$ is

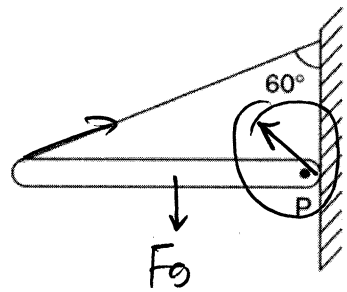
(a) $27\,\text{m/s}$
(b) $54\,\text{m/s}$
(c) $108\,\text{m/s}$
(d) $135\,\text{m/s}$
(e) $210\,\text{m/s}$

$v = r\omega = (2)(8t+3) = 16t+6$

at $t=3$, $v = 16(3)+6 = 54$

d 10 A metal bar of constant density and weight W is attached to a pivot on the wall at point P and supported by a rope that makes an angle of $60°$ with the vertical wall. The reaction force exerted by the pivot on the bar at point P is best represented by which arrow?

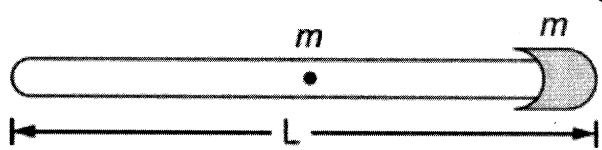
(a) ↗
(b) ↑
(c) ↓
(d) ↖
(e) ↘



a 11 A uniform rod of length L and mass m has a rotational inertia of $\frac{1}{12}mL^2$ about its center. A particle, also of mass m, is attached to one end of the stick. The combined rotational inertia of the stick and particle about the center of the rod is

(a) $\frac{mL^2}{3}$
(b) $\frac{12mL^2}{13}$
(c) $\frac{13mL^2}{12}$
(d) $\frac{mL^2}{156}$
(e) $\frac{13mL^2}{156}$

$I = \frac{1}{12}mL^2 + m\left(\frac{L}{2}\right)^2 = \frac{1}{12}mL^2 + \frac{1}{4}mL^2 = \frac{1}{3}mL^2$

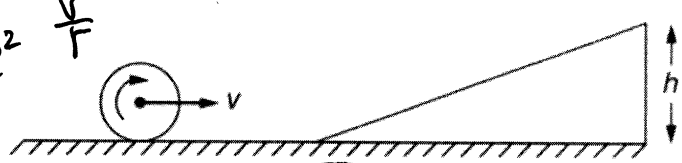


A 12 A hoop of radius R and mass m has a rotational inertia of mR^2. The hoop rolls without slipping along a horizontal floor with a constant speed v and then rolls up a long incline. The hoop can roll up the incline to a maximum vertical height of

(a) $\frac{v^2}{g}$
(b) $\frac{2v^2}{g}$
(c) $\frac{v^2}{2g}$
(d) $\frac{4v^2}{g}$
(e) $\frac{v^2}{4g}$

$\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = mgh$

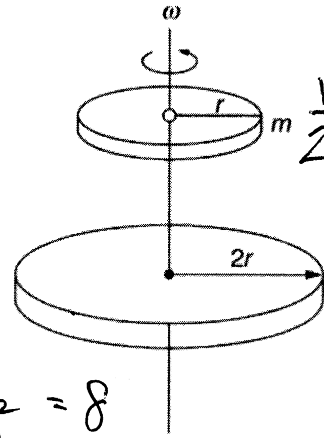
mR^2 $\quad \frac{v^2}{r^2}$



$\frac{1}{2}mv^2 + \frac{1}{2}mv^2 = mgh$

$h = \frac{v^2}{g}$

e 13 Two disks are fixed to a vertical axle that is rotating with a constant angular speed ω. The smaller disk has a mass m and a radius r, and the larger disk has a mass $2m$ and radius $2r$. The general equation for the rotational inertia of a disk of mass M and radius R is $\frac{1}{2}MR^2$. The ratio of the angular momentum of the larger disk to the smaller disk is



$\frac{1}{2}mr^2$

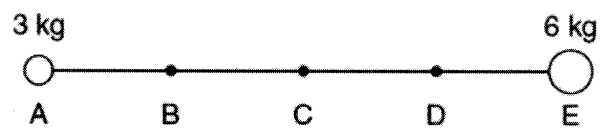
$\frac{1}{2}(2m)(2r)^2 = 4mr^2$

(a) 1/4
(b) 4/1
(c) 1/2
(d) 2/1
(e) 8/1

$$\frac{L_1}{L_2} = \frac{I\omega}{I\omega} = \frac{I_1}{I_2} = \frac{\frac{1}{2}mr^2}{\frac{1}{2}(2m)(2r)^2} = 8$$

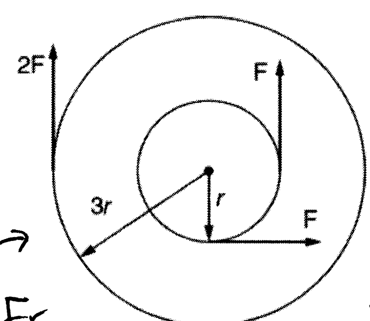
a 14 A light rod has a mass attached at each end. At one end is a 6 kg mass, and at the other end is a 3 kg mass. An axis can be placed at any of the points shown. Through which point should an axis be placed so that the rotational inertia is the greatest about that axis?

(a) A
(b) B
(c) C
(d) D
(e) E



d 15 Two wheels are attached to each other and fixed so that they can only turn together. The smaller wheel has a radius of r and the larger wheel has a radius of $3r$. The two wheels can rotate together on a frictionless axle. Three forces act tangentially on the edge of the wheels as shown. The magnitude of the net torque acting on the system of wheels is

(a) Fr
(b) $2Fr$
(c) $3Fr$
(d) $4Fr$
(e) $6Fr$



6Fr

2Fr

e 16 Astronauts are conducting an experiment in a negligible gravity environment. Two spheres of mass m are attached to either end of a light rod. As the rod and spheres float motionless in space, an astronaut launches a piece of sticky clay, also of mass m, toward one of the spheres so that the clay strikes and sticks to the sphere perpendicular to the rod. Which of the following statements is true of the motion of the rod, clay, and spheres after the collision?

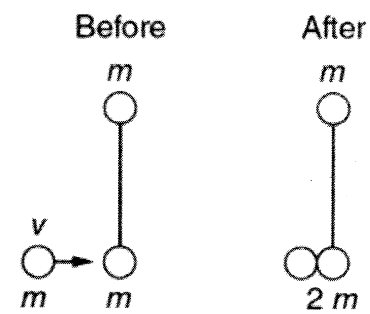
(a) Linear momentum is not conserved, but angular momentum is conserved.

(b) Angular momentum is not conserved, but linear momentum is conserved.

(c) Kinetic energy is conserved, but angular momentum is not conserved.

(d) Kinetic energy is conserved, but linear momentum is not conserved.

(e) Both linear momentum and angular momentum are conserved, but kinetic energy is not conserved.



b 17 One end of a stick of length L, rotational inertia I, and mass m is pivoted on an axle with negligible friction at point P. The other end is tied to a string and held in a horizontal position. When the string is cut, the stick rotates counterclockwise. The angular speed ω of the stick when it reaches the bottom of its swing is

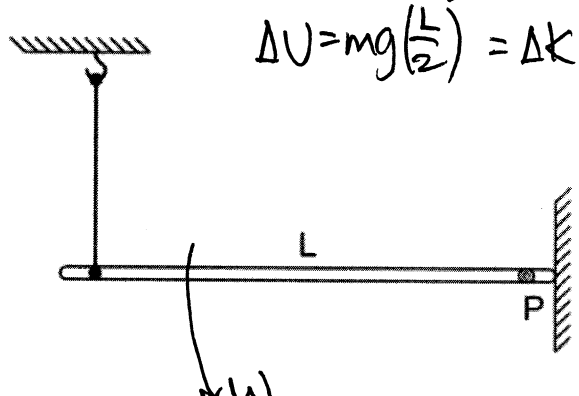
(a) $\frac{mgL}{I}$

(b) $\sqrt{\frac{mgL}{I}}$

(c) $\sqrt{\frac{2mgL}{I}}$

(d) $\sqrt{\frac{mgL}{2I}}$

(e) $\sqrt{\frac{4mgL}{I}}$



centre of mass

$\Delta U = mg\left(\frac{L}{2}\right) = \Delta K = \frac{1}{2} I\omega^2$

$mgL = I\omega^2$

$\omega = \sqrt{\frac{mgL}{I}}$

c 18 A belt is wrapped around two wheels as shown. The smaller wheel has a radius r, and the larger wheel has a radius $2r$. When the wheels turn, the belt does not slip on the wheels, and gives the smaller wheel an angular speed ω. The angular speed of the larger wheel is

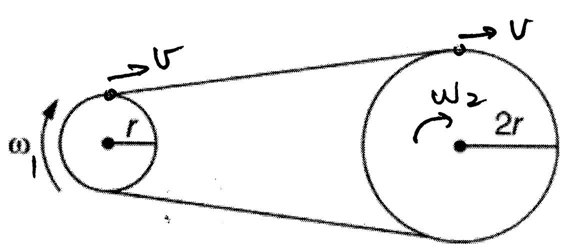
(a) ω

(b) 2ω

(c) $\frac{1}{2}\omega$

(d) $\frac{1}{4}\omega$

(e) 4ω



$v = \omega_1 r$

$v = 2r\omega_2$

$\omega_2 = \frac{1}{2}\omega_1$

c 19 A disk is mounted on a fixed axle. The rotational inertia of the disk is I. The angular velocity of the disk is decreased from ω_0 to ω_f during a time Δt due to friction in the axle. The magnitude of the average net torque acting on the wheel is

(a) $\dfrac{\omega_f - \omega_0}{\Delta t}$

(b) $\dfrac{(\omega_f - \omega_o)^2}{\Delta t}$

(c) $\dfrac{I(\omega_f - \omega_o)}{\Delta t}$

(d) $\dfrac{I(\omega_f - \omega_o)^2}{\Delta t}$

(e) $\dfrac{I(\omega_f - \omega_o)}{\Delta t^2}$

$$\tau = I\alpha = I\frac{\Delta\omega}{\Delta t} = I\frac{(\omega_f - \omega_0)}{\Delta t}.$$

d 20 The average power developed by the friction in the axle of the disk from the previous question to bring it to a complete stop is

(a) $\dfrac{\omega_o}{\Delta t}$

(b) $\dfrac{(\omega_o)^2}{\Delta t}$

(c) $\dfrac{I(\omega_f - \omega_o)}{\Delta t}$

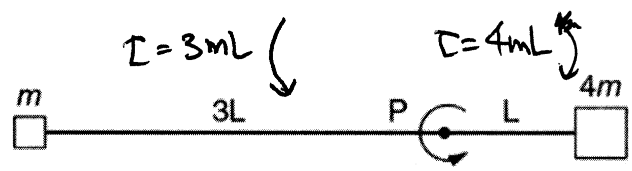
(d) $\dfrac{I\omega_o^2}{\Delta t}$

(e) $\dfrac{I(\omega_f - \omega_o)}{\Delta t^2}$

$$P = \tau\omega = \frac{I\omega}{\Delta t}\omega = \frac{I\omega^2}{\Delta t}$$

e 21 A light rod of negligible mass is pivoted at point P a distance L from one end as shown. A mass m is attached to the left end of the rod at a distance of $3L$ from the pivot, and another mass $4m$ is attached to the other end a distance L from the pivot. The system begins from rest in the horizontal position. The net torque acting on the system due to gravitational forces is

(a) $4mgL$ clockwise
(b) $3mgL$ clockwise
(c) $3mgL$ counterclockwise
(d) mgL counterclockwise
(e) mgL clockwise



d 22 The angular acceleration of the system when it is released from rest is

(a) zero

(b) $\dfrac{g}{5L}$

(c) $\dfrac{g}{4L}$

(d) $\dfrac{g}{13L}$

(e) $\dfrac{g}{L}$

$$\tau = I\alpha$$

$$\alpha = \frac{\tau}{I} = \frac{\cancel{m}gL}{4\cancel{m}L^2 + 9\cancel{m}L^2} = \frac{g}{13L}$$

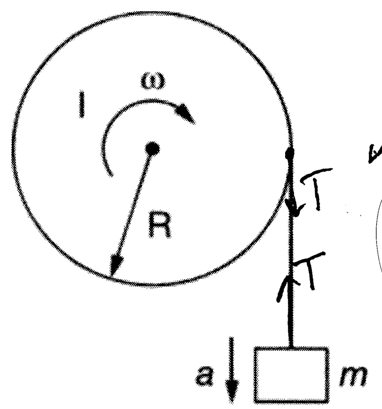
$$I = mr^2$$

Free-Response Questions:

1. A mass m is hung on a string that is wrapped around a disk of radius R and rotational inertia I. The mass is released from rest and accelerates downward with an acceleration a.

   (a) Determine the tension in the string as the mass accelerates downward in terms of the given quantities.

   (b) In terms of the tension T and the other given quantities, determine the rate of change of the angular speed of the disk.



the string has the same acceleration as the mass

$\alpha = \frac{a}{R}$

$$\tau = TR = I\alpha = I\left(\frac{a}{R}\right)$$

$$\boxed{(a)\ T = \frac{Ia}{R^2}}$$

$$\boxed{\alpha = \frac{TR}{I}}$$

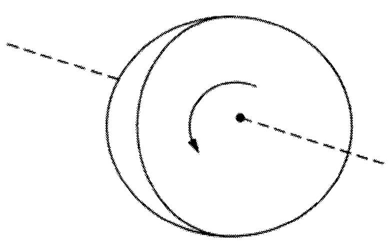
I

2. A disk having a rotational inertia of 2 kg m^2 rotates about a fixed axis through its center. The disk begins from rest at $t = 0$, and at time $t = 2$ s, its angular velocity is 2 rad/s.

   (a) Determine the angular momentum of the disk at $t = 2$ s.

   (b) What is the angular acceleration of the disk between $t = 0$ and $t = 2$ s?

   (c) What is the kinetic energy of the disk at $t = 2$ s?



(b) $\alpha = \frac{\Delta\omega}{\Delta t} = \frac{2}{2} = 1\ \frac{\text{rad}}{s^2}$

(a) $L = I\omega$

$= (2)(2) = \boxed{4\ \frac{\text{kgm}^2}{s}}$

(c) $K = \frac{1}{2}I\omega^2 = \frac{1}{2}(2)(2)^2 = 4\text{J}$