

# 23. Mechanical Waves

Advanced Placement Physics

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Olympiads School

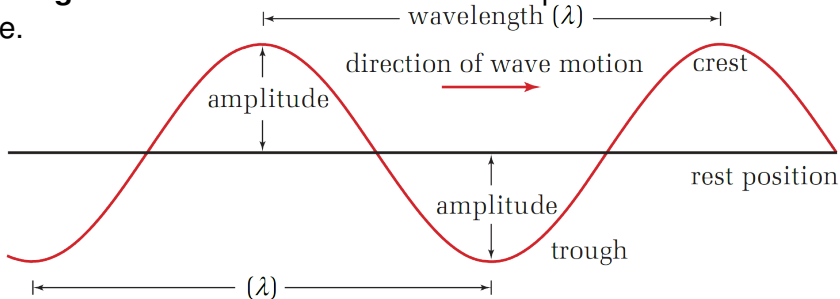
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# What is a wave?

- When a disturbance (vibration) causes vibrations in its vicinity, a wave is created
- A wave transfers energy through a medium (there is one exception)
  - The medium vibrates and have a net displacement of zero.
  - Each particle vibrates instead of moving horizontally, and the vibration get transferred to the next particle.

# Features of a Wave

- **Crest:** Highest point
- **Trough:** Lowest point
- **Wavelength:** Shortest distance between two points in the medium that are in phase.



(The easiest way to measure wavelength is from crest to crest, or from trough to trough.)

# Frequency and Speed of A Wave

## Frequency of A Wave ( $f$ )

- The number of complete wavelengths that pass a point in a given amount of time
- Unit: hertz (Hz)
- Same as the frequency of the disturbance that generated the wave
- **Does not depend on the medium**, only the source that produces the wave.

## Speed of A Wave ( $v$ )

- The speed at which the wave fronts are moving
- **Depends only on the medium**

## Equation

A harmonic wave can be described as a sinusoidal function:

$$y(x, t) = A \sin(kx - \omega t)$$

Quantity	Symbol	SI Unit
Displacement of the medium	$y$	m (meters)
Wave number	$k$	/m (per meter)
Distance from the source	$x$	m (meters)
Time	$t$	s (seconds)
Angular frequency	$\omega$	/s (per second)

## Equation

$$y(x, t) = A \sin(kx - \omega t)$$

If the wave is generated by a mass on a spring, then  $k$  is the spring constant of the spring. It is related to the wavelength by:

$$k = \frac{2\pi}{\lambda}$$

The angular frequency (angular velocity) is related to the frequency  $f$  and period  $T$  of the wave by:

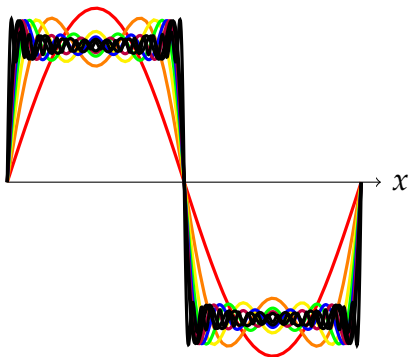
$$\omega = \frac{2\pi}{T} = 2\pi f$$

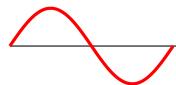
## Why Sine and Cosines

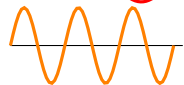
French mathematician Joseph Fourier discovered that *all* periodic functions are infinite series of sin and/or cos functions:

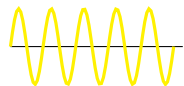
$$\begin{aligned} f(x) &= a_1 \sin(x) + a_2 \sin(2x) + a_3 \sin(3x) + \dots + \\ &\quad b_1 \cos(x) + b_2 \cos(2x) + b_3 \cos(3x) + \dots \\ &= \sum_{n=1}^{\infty} a_n \sin(nx) + \sum_{n=1}^{\infty} b_n \cos(nx) \end{aligned}$$

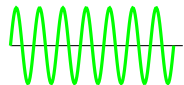
# Making a Square Wave with Sine Waves

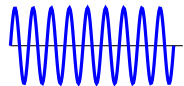



$$f_1 = \sin(x)$$


$$f_1 = \frac{1}{3} \sin(3x)$$

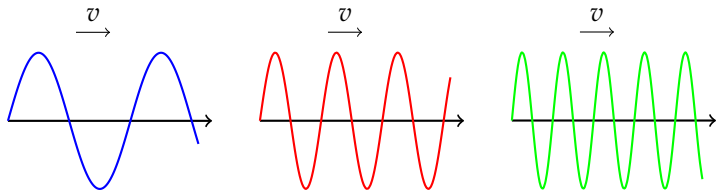

$$f_1 = \frac{1}{5} \sin(5x)$$


$$f_1 = \frac{1}{7} \sin(7x)$$


$$f_1 = \frac{1}{9} \sin(9x)$$



# Fourier Series and Harmonic Frequencies



- The first wave—with the longest wavelength and lowest frequency—is called the **fundamental frequency**, or **first harmonic**
- The second term has half the wavelength and twice the frequency. It's called the **second harmonic**, the **first overtone**
- Also, third, fourth, fifth. . . harmonics

# Harmonic Frequencies

- When a musical instrument produces a sound, the frequency that is “heard” is the fundamental frequency
- Every whole-number multiples of the fundamental frequency  $f_1$  is its harmonic frequency, i.e. the  $n$ -th harmonic is:

$$\boxed{f_{\text{harm},n} = n f_1} \quad \text{where} \quad n \geq 1$$

# Universal Wave Equation

When combining the wave number and angular frequency, we can find that the speed of a wave is the product of the wavelength and the frequency:

$$v = f\lambda$$

Quantity	Symbol	SI Unit
Speed	$v$	m/s (meters per second)
Frequency	$f$	Hz (hertz)
Wavelength	$\lambda$	m (meters)

The universal wave equation applies to *all* waves. For sound waves,  $v = v_{\text{sound}}$ ; for electromagnetic waves  $v = c$ .

# Two Types of Wave

There are two types of waves

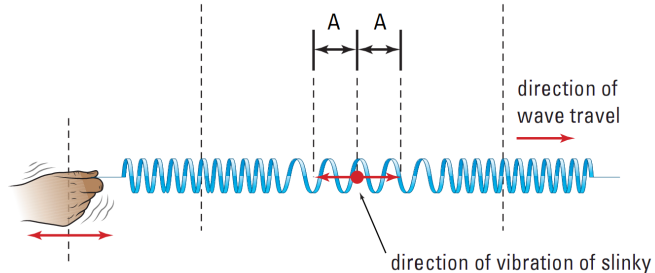
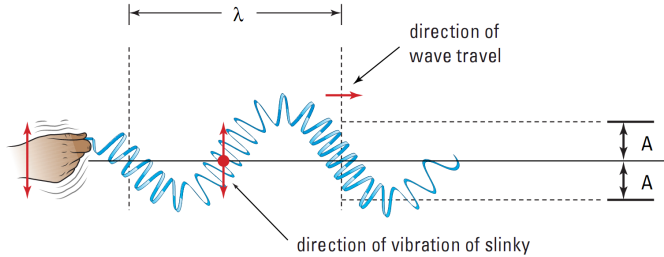
- **Transverse waves:**

- Particles of a medium vibrate at right angles to the direction of the motion.
- e.g.: ocean waves, electromagnetic waves

- **Longitudinal waves:**

- Particles of a medium vibrate parallel to the direction of the motion of the wave
- e.g.: sound waves

# Transverse Wave vs. Longitudinal Wave



# Wave Simulation

A helpful simulation can be found on the PhET website at University of Colorado.

**Click for external link:**  
wave on a string simulation

## Wave on a String

The speed of a travelling wave on a stretched string is given by:

$$v = \sqrt{\frac{F_T}{\mu}} \quad \text{where} \quad \mu = \frac{m}{L}$$

Quantity	Symbol	SI Unit
Wave speed	$v$	m/s (meters per second)
Tension	$F_T$	N (newtons)
Linear mass density	$\mu$	kg/m (kilograms per meter)
Mass of the string	$m$	kg (kilograms)
Length of the string	$L$	m (meters)

## Power Transmitted by a Harmonic Wave

Then the power transmitted by a harmonic wave is through a travelling wave on a string is determined by the linear mass density  $\mu$ , the angular frequency  $\omega$ , amplitude  $A$  and wave speed  $v$ :

$$P = \frac{1}{2} \mu \omega^2 A^2 v$$



## The Decibel

The decibel is defined as by the intensity of sound  $I$  compared to the *threshold of hearing* intensity  $I_0$ :

$$\beta = 10 \log_{10} \left[ \frac{I}{I_0} \right] \quad \text{where} \quad I_0 = 10^{-12} \text{ W/m}^2 \quad \text{and} \quad I = \frac{P_{\text{ave}}}{4\pi r^2}$$

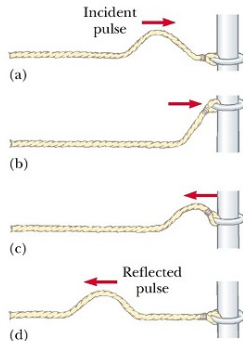
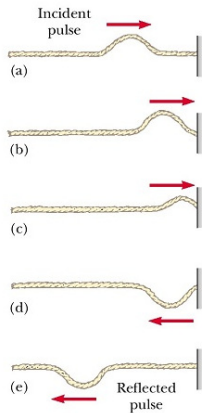
Quantity	Symbol	SI Unit
Intensity of sound	$\beta$	dB (decibels)
Intensity of sound	$I$	W/m <sup>2</sup> (watts per square meters)
Threshold intensity	$I_0$	W/m <sup>2</sup> (Watts per square meters)
Average power of the source	$P_{\text{ave}}$	W (watts)
Distance from the source	$r$	m (meters)

The *threshold of pain* for human ears is defined at 120 dB.

# Reflection of Wave at a Boundary

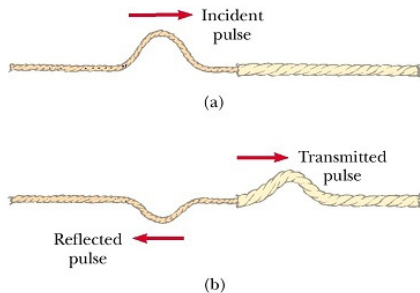
When a wave on a string reflects at a boundary, how the reflected wave looks depends on the type of boundary

- At a *fixed end* (left), the reflected wave is *inverted*, i.e. a crest becomes a trough
- At a *free end* (right), the reflected wave is upright



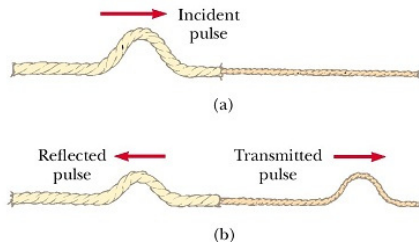
# Transmission of Waves: Fast to Slow Medium

- Reflected wave:
  - Inverted, like a fixed end
  - Same frequency and wavelength as the incoming wave
  - The amplitude is decreased
- Transmitted wave:
  - Upright
  - Same frequency as incoming wave, but has a shorter wavelength because the wave slowed down



# Transmission of Waves: Slow to Fast Medium

- Reflected wave:
    - Upright, like a free end
    - Same frequency and wavelength as the incoming wave
    - The amplitude is decreased
  - Transmitted wave:
    - Upright
    - Same frequency as incoming wave, but has a longer wavelength because the wave sped up
- Note that the transmitted wave is *always* upright.



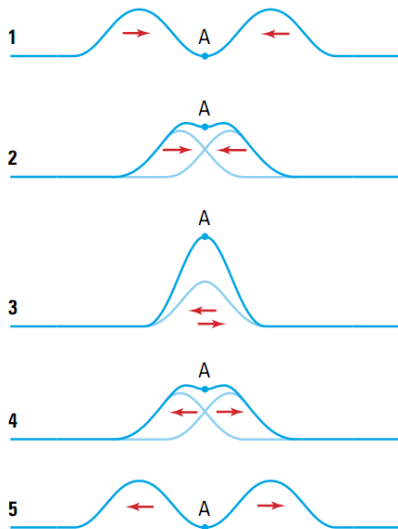
# Superposition of Waves

- **Principle of Superposition:** When multiple waves pass through the same point, the resultant wave is the *sum* of the waves
  - A fancy way of saying that waves add together
- The consequence of the principle of superposition is *interference of waves*. There are two kinds of interference:
  - **Constructive interference:** Two wave fronts (crests) passing through creates a wave front with greater amplitude
  - **Destructive interference:** A crest and trough will cancel each other

# Superposition of Waves

**Constructive interference:** In-phase wave fronts sum together

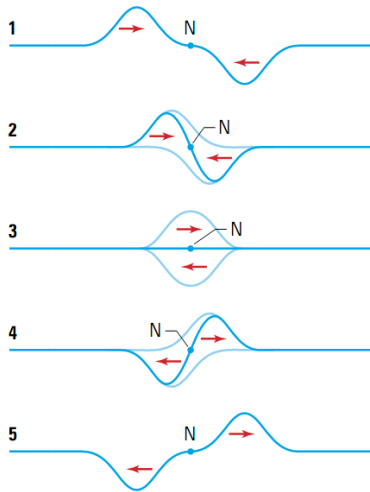
- In this example, two identical pulses move towards each other
- Their crests pass through A at the same time
- The amplitude at A when the waves pass through is higher



# Superposition of Waves

**Destructive interference:** Out-of-phase wave fronts shows the difference of the wave fronts

- Two pulses move towards each other, one a crest, the other a trough
- They both pass through A at the same time
- Two waves cancel each other at A



# Standing Waves

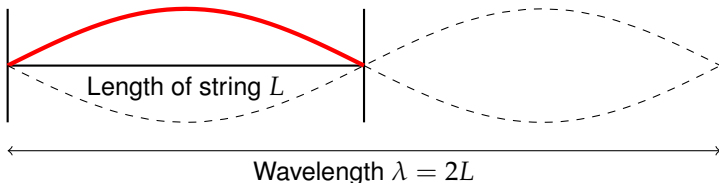
If two waves of the same frequency meet up under the right conditions, they may appear to be “standing still”. This is called a standing wave

- Node: A point that never moves
- Anti-node: A point which moves/vibrates maximally



## Standing Waves On a String Length $L$

- A “vibrating” string is actually a standing wave on a string
- Both ends of the string are nodes
- **Resonance frequency** is a frequency that allows a standing wave to be created on the string. The first resonance (fundamental) frequency occurs when  $\lambda = 2L$ :

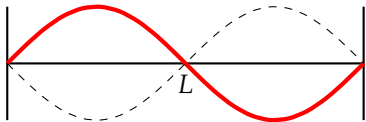


- Fundamental frequency is based on the speed of the travelling wave along the string  $v_{\text{str}}$ :

$$f_1 = \frac{v_{\text{str}}}{\lambda} = \frac{v_{\text{str}}}{2L}$$

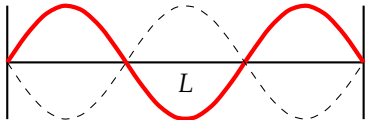
## Standing Waves On a String Length $L$

A second resonance frequency happens when  $L = \lambda$ :



$$f_{\text{res},2} = \frac{v_{\text{str}}}{\lambda} = \frac{v_{\text{str}}}{L} = 2f_1$$

And again, a third resonance frequency occurs at  $L = \frac{3}{2}\lambda$ :



$$f_{\text{res},3} = \frac{3v_{\text{str}}}{2L} = 3f_1$$

## Standing Waves On a String Length $L$

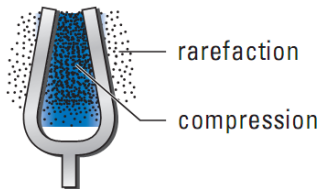
In fact, the  $n$ -th resonance frequency of a wave on string is just:

$$\boxed{f_{\text{res},n} = n f_1} \quad (\text{standing wave on string})$$

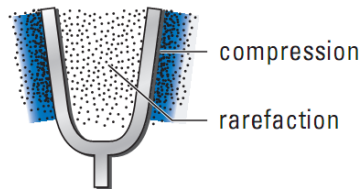
- $f_1$  is the fundamental frequency, and  $n$  is a whole-number multiple
- This equation is *identical* to the equation for harmonic frequencies, meaning that every harmonic is a resonance frequency
- It has a “full set of harmonics”

# Transfer of Sound Wave

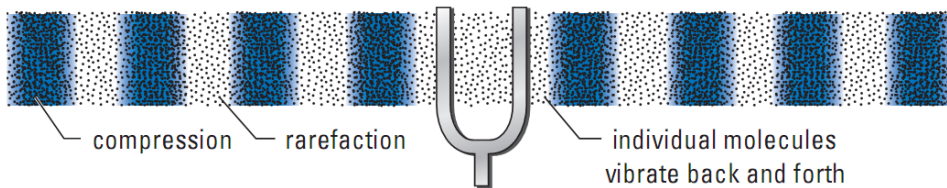
Example: tuning fork



prongs coming together



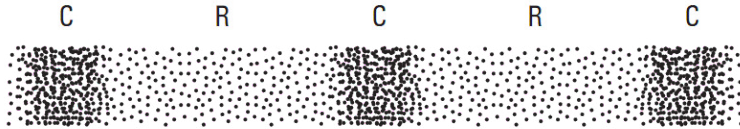
prongs spreading apart



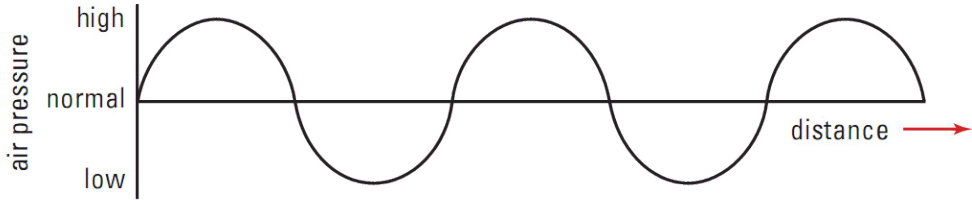
# Transfer of Sound Wave

## Schematic Diagram vs. Wave Graph

We can also express the amplitude of the sound wave by plotting the change in *air pressure*:



schematic representation of the density of air molecules



## Speed of Sound in a Gas

The equation for the speed of sound in a gas (e.g. air) is given by:

$$v_s = \sqrt{\frac{\gamma RT}{M}}$$

Quantity	Symbol	SI Unit
Speed of sound	$v_s$	m/s (meters per second)
Temperature	$T$	K (kelvin)
Universal gas constant	$R$	J/mol K (joule per mol per kelvin)
Molar mass	$M$	kg/mol (kilograms per mol)
Adiabatic constant	$\gamma$	(no units)

For air  $\gamma = 1.4$ , and  $M = 29 \times 10^{-3}$  kg/mol.

# Mach Number

When working with sound, it is useful to express speed in terms of its ratio to the speed of sound. This is called the **Mach number**:

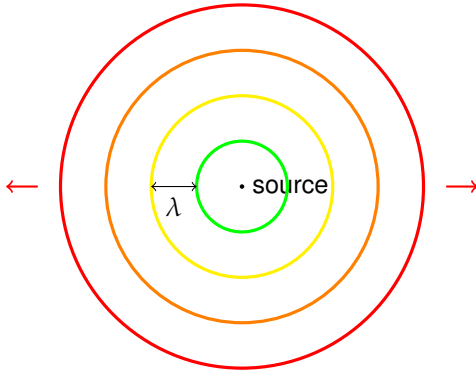
$$M = \frac{v}{v_s}$$

Quantity	Symbol	SI Unit
Mach Number	$M$	no units
Speed of the object	$v$	m/s (meters per second)
Local speed of sound	$v_s$	m/s (meters per second)

- When an object is travelling at  $M < 1$ , it is travelling at a *subsonic* speed
- When an object is travelling at  $M > 1$ , it is travelling at a *supersonic* speed

## Sound from a Moving Source

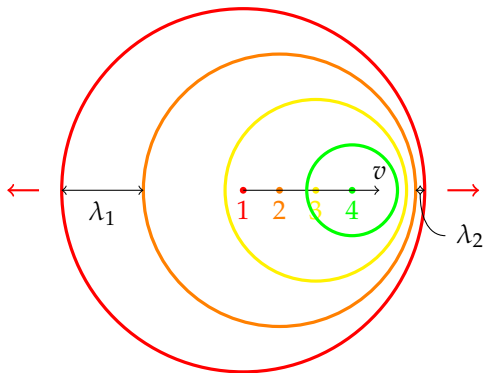
When a sound is emitted from a point source, the sound wave moves radially outward from the point of origin. In this diagram, the source is stationary:





## Sound from a Moving Source

But when sound is emitted from a *moving* source, the diagram looks different. In this case, the sound source is moving to the right, from 1 to 4:



- When the sound source is moving *towards you*, the wavelength  $\lambda_2$  decreases, and the frequency increases.
- When the sound source is moving *away from you*, the wavelength  $\lambda_1$  increases, and the frequency decreases.

This is called the **Doppler Effect**.

## Doppler Effect

When a wave source is moving at a speed  $v_{\text{src}}$  and the observer is moving at  $v_{\text{ob}}$ , the frequency perceived by the observer is shifted to  $f'$ :

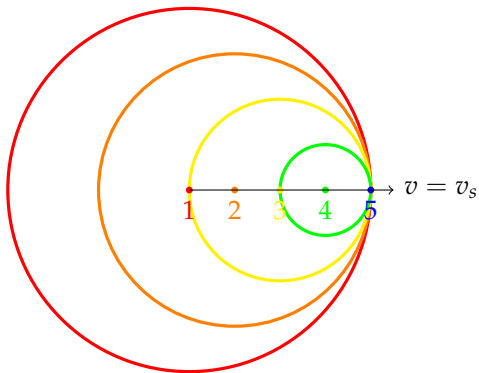
$$f' = \frac{v_s + v_{\text{ob}}}{v_s - v_{\text{src}}} f$$

Quantity	Symbol	SI Unit
Apparent frequency	$f'$	hertz (hertz)
Actual frequency	$f$	hertz (hertz)
Speed of sound	$v_s$	m/s (meters per second)
Speed of source	$v_{\text{src}}$	m/s (meters per second)
Speed of observer	$v_{\text{ob}}$	m/s (meters per second)

The Doppler effect equation works for all types of waves, including sound waves *and* electromagnetic waves.

# Sound from a Source Moving At Sonic Speed

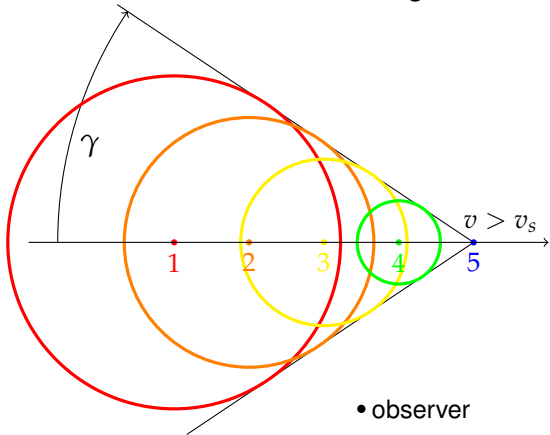
Doppler effect is more interesting is when sound source is moving at  $M = 1$ , the speed of sound:



- The wave fronts (crests) from all the waves are bunched up just in front of the source
- Since sound wave is a pressure wave, right in front of the sound source, there is a large change in pressure (called a shock wave)
- When the shock passes an observer, an loud bang can be heard (aka **sonic boom**)

# Sound from a Supersonic Source

When sound source is moving at  $M > 1$ :



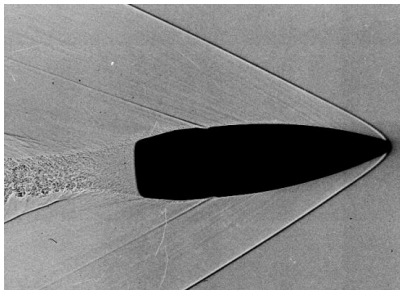
An *oblique shock* is formed at an angle (called the **Mach angle**) given by:

$$\gamma = \sin^{-1} \left( \frac{1}{M} \right)$$

A stationary observer does not hear the sound source coming until it has gone past!

## Bullet in Supersonic Flight

Generating a shock doesn't require an actual sound source. Any object moving through air creates a pressure disturbance. This is a 7.62 mm NATO bullet in supersonic flight.



This bullet was not fired from a gun. Instead, it was placed in a shock tube that generates a short burst of supersonic flow, and a high-speed camera is then used to take the photo.

## Duck in Water

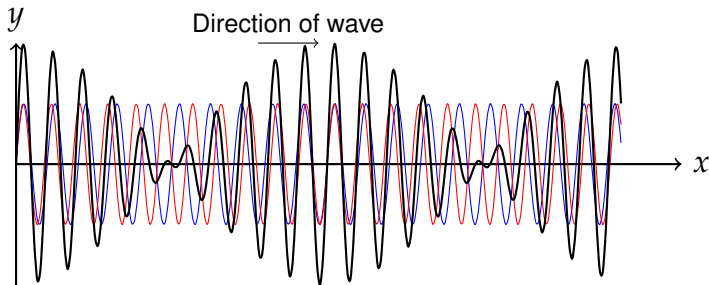
No sonic booms here (just a duck swimming), but a similar shock behaviour is observed. The duck swims faster than the speed of the water wave, and it also creates a cone shape.



## Beat Frequency

When waves of two different frequencies are added together, there is both constructive and destructive interference

- Plotting two functions representing two waves with equal magnitude and wave speed  $v_s$ :  $y = \sin(x)$  and  $y = \sin(1.1x)$



- The thick black line is the sum:  $y = \sin(x) + \sin(1.1x)$

# Beat Frequency

The *beat frequency* is the absolute value of the difference of the frequencies of the two component waves:

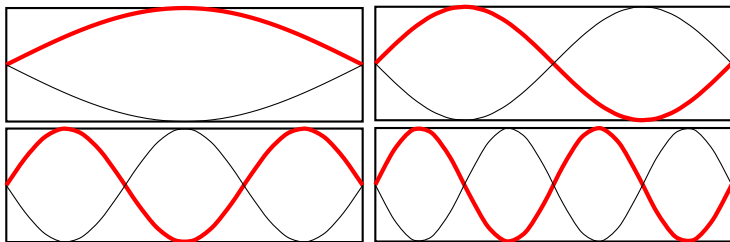
$$f_{\text{beat}} = |f_1 - f_2|$$

Quantity	Symbol	SI Unit
Beat frequency	$f_{\text{beat}}$	Hz (hertz)
Frequency of 1st component wave	$f_1$	Hz (hertz)
Frequency of 2nd component wave	$f_2$	Hz (hertz)



## Standing Waves in a Closed Pipe

A standing-wave patterns can be found on pipes that have both ends closed:



## Standing Waves in Closed Pipes

Like strings, pipes that are *closed at both ends* also have a full set of harmonics. The  $n$ -th resonance frequency is given by:

$$\boxed{f_{\text{res},n} = n f_1} \quad (\text{closed pipe})$$

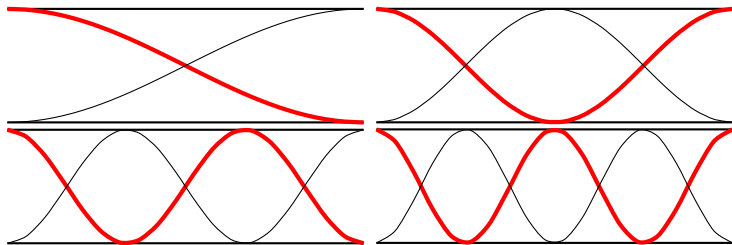
where  $n$  is a whole-number multiple of the fundamental frequency  $f_1$ :

$$\boxed{f_1 = \frac{v_s}{2L}}$$

The difference between a closed pipe and a string is that the wave speed is now the speed of sound  $v_s$  inside the pipe.

# Standing Waves in Open Pipes

- Example: Some organ pipes, flute
- Both ends of the pipes are anti-nodes



First resonance at  $\lambda = 2L$

$$f_1 = \frac{v_s}{\lambda} = \frac{v_s}{2L}$$

Second resonance at  $\lambda = L$

$$f_2 = \frac{v_s}{\lambda} = \frac{v_s}{L} = 2f_1$$

## Standing Waves in Open Pipes

Open pipes also have a “full set of harmonics”. The  $n$ -th resonance frequency is given by:

$$f_{\text{res},n} = n f_1 \quad (\text{open pipe})$$

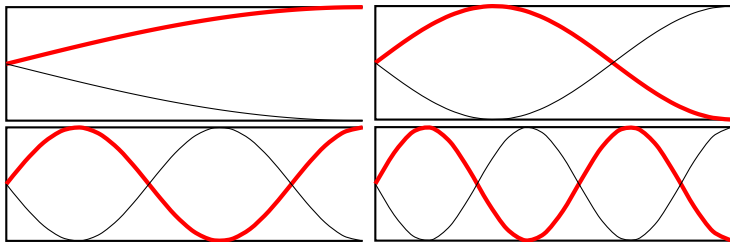
where  $n$  is a whole-number multiple of fundamental frequency  $f_1$ :

$$f_1 = \frac{v_s}{2L}$$

# Standing Waves in Semi-Open Pipes

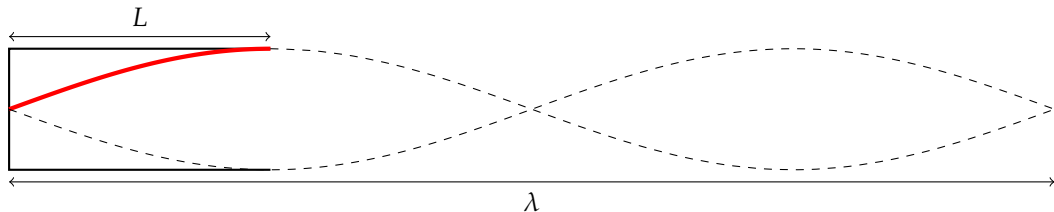
This is when things get a bit more interesting. . .

- Examples: Most organ pipes, clarinet, oboes, brass instruments
- Closed end: node (like in the closed pipes)
- Open end: anti-node (like in the open pipes)



## Standing Waves in Semi-Open Pipes

Again starting with the fundamental frequency (lowest frequency where a standing wave can form inside the pipe). This occurs at  $\lambda = 4L$ :

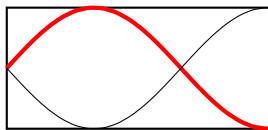


Fundamental frequency  $f_1$  differs from the open-pipe and closed-pipe configurations by a factor of 2:

$$f_1 = \frac{v_s}{\lambda} = \frac{v_s}{4L}$$

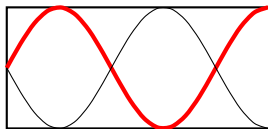
## Standing Waves in Semi-Open Pipes

Likewise, second resonance can be found at  $\lambda = \frac{4}{3}L$ :



$$f_{\text{res},2} = \frac{v_s}{\lambda} = \frac{3v_s}{4L} = 3f_1$$

And then, third resonance at  $\lambda = \frac{4}{5}L$ :



$$f_{\text{res},3} = \frac{v_s}{\lambda} = \frac{5v_s}{4L} = 5f_1$$

We can repeat that for 4th, 5th... resonances.

## Standing Waves in Semi-Open Pipes

Only **odd-number multiples** of the fundamental frequency are resonance frequencies in a semi-open pipe. We say that semi-open pipes have an *odd* set of harmonics.

$$f_{\text{res},n} = (2n - 1)f_1 \quad (\text{semi-open pipes})$$

Because fundamental frequency  $f_1$  is lower than open-pipe and closed-pipe configurations by a factor of 2 for the same length  $L$ , it has advantages when designing an organ pipe.

$$f_1 = \frac{v_s}{\lambda} = \frac{v_s}{4L}$$

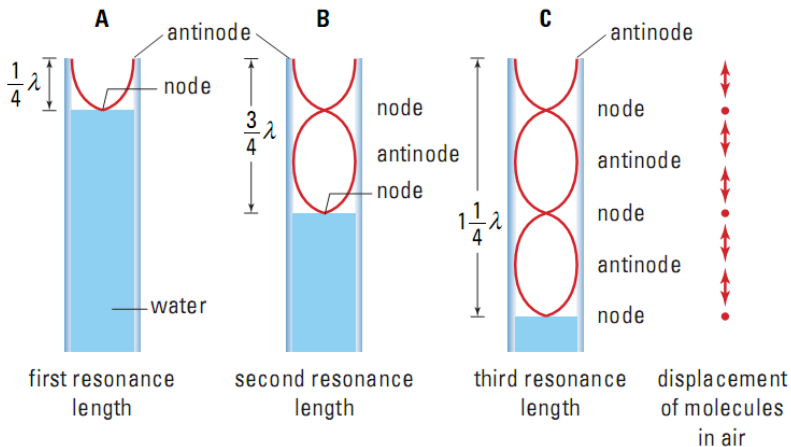


## Resonance *Length* in a Semi-Open Pipe

- Now that we have looked at resonance *frequencies*, we'll look at resonance *lengths*
- We produce a single frequency in the pipe, and vary the length of the pipe until we have resonance

# Resonance Length in a Semi-Open Pipe

Let's submerge a part of this pipe in water...



## Resonance Length in a Semi-Open Pipe

The resonance lengths are **odd whole-number multiples** of the first resonance length  $L_1$ :

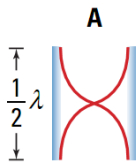
$$L_{\text{res},n} = (2n - 1)L_1$$

where

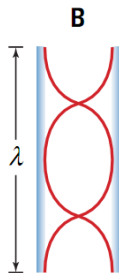
$$L_1 = \frac{\lambda}{4}$$

## Resonance in an Open Pipe

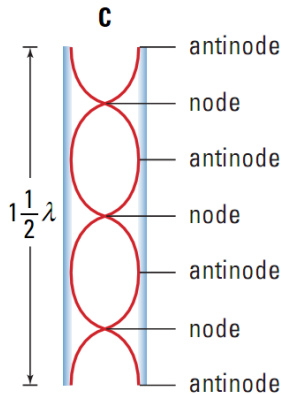
We can also repeat this with pipes that are open on both ends.



first resonance  
length



second resonance  
length



third resonance  
length

## Resonance in an Open Pipe

Resonance lengths of an open pipe are **whole-number multiples** of the first resonance length  $L_1$ :

$$L_{\text{res},n} = nL_1 \quad (\text{open pipe})$$

where first resonance length is given by:

$$L_1 = \frac{\lambda}{2}$$

Be careful! This equation looks a lot like the resonance frequency equation!