Topic 3: Work and Energy

Advanced Placement Physics

Dr. Timothy Leung

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Olympiads School

Files for You to Download

- Slides for this week and next
  - `PhysAP-03-workEnergy.pdf`–This week's slides on work and energy
  - `PhysAP-03-momentumImpulse.pdf`–Next week's slides on momentum, impulse and general collisions.
- `PhysAP-04-Homework.pdf`–Homework problems for Topics 3 and 4.

Please download/print the PDF file for the class slides before each class. There is no point copying notes that are already printed out for you. Instead, take notes on things I say that aren't necessarily on the slides. If you want to print the slides, we recommend that you print 4 slides per page to save paper.

Work and Energy

We start with some definition at are (unfortunately) not very useful:

- **Energy** is the ability to do work.
- **Work** is the mechanism in which energy is transformed.

Luckily, we can also use equations to define these concepts.

Work

Mechanical work is performed when a force $\mathbf{F}$ is used to displace an object by an infinitesimal amount $d\mathbf{r}$. If a varying force is applied to move an object from r_1 to r_2 along a path, then the total work done by the force is defined by the integral:

$$\boxed{W = \int_{r_1}^{r_2} \mathbf{F}(r) \cdot d\mathbf{r}}$$

- No work done if the force is perpendicular to displacement (i.e. $\mathbf{F} \cdot d\mathbf{r} = 0$)
- No work done if no displacement ($d\mathbf{r} = \mathbf{0}$)
- Work can be positive or negative depending on the dot product
- When there are multiple forces acting on an object, we can compute the work done by each *each* force

Work by Constant Force

If the force is constant, and the object moves along straight path, the integral simplifies to:

$$\boxed{W = \mathbf{F} \cdot \Delta \mathbf{r}}$$

Or in a form that is more familiar in Grade 11 and 12 Physics courses:

$$\boxed{W = F \Delta r \cos \theta}$$

where θ is the angle between the force vector and displacement vector.

Definition of Work

- **Work done by a force**
  - We can quantify work by calculating the work done by a specific force
  - Example: A boy pushes a cart forward. The “work done by the boy” is the work done by the applied force.
- **Work done on an object**
  - There may be more than one force acting on an object
  - The *sum* of all the work done on the object by each force
  - The work done by the net force
  - Also called the **net work** W_{net}

Kinetic Energy

When a net force on an object accelerates it, the resulting amount of work done on the object (net work W_{net}) is given by:

$$W_{\text{net}} = \int \mathbf{F}_{\text{net}} \cdot d\mathbf{x} = \int m\mathbf{a} \cdot d\mathbf{x} = m \int \frac{d\mathbf{v}}{dt} \cdot d\mathbf{x}$$

Since both $\mathbf{v}$ and $\mathbf{x}$ are continuous functions in time, we can switch the order of differentitation, i.e.:

$$= m \int \frac{d\mathbf{x}}{dt} \cdot d\mathbf{v} = m \int \mathbf{v} \cdot d\mathbf{v} = \int_{v_1}^{v_2} mv dv$$

and since $\mathbf{v}$ and $d\mathbf{v}$ must be in the same direction, the dot product is trivial: $\mathbf{v} \cdot d\mathbf{v} = vdv$

Kinetic Energy

Continuing from the last slide, this integral, when integrated from v_1 (initial velocity) to v_2 (final velocity):

$$m \int_{v_1}^{v_2} v dv = \frac{1}{2} m v^2 \Big|_{v_1}^{v_2} = \Delta K$$

where K is defined as the **translational kinetic energy**:

$$\boxed{K = \frac{1}{2} m v^2}$$

Work and Kinetic Energy

In fact, the *definition* of kinetic energy came from this integration, in that we want to say that work equals to the change in *something*, and we called that kinetic energy. This is the **work-energy theorem**:

$$\boxed{W_{\text{net}} = \Delta K}$$

- ΔK can be positive or negative depending on the dot product
- There may be multiple forces acting on an object; each of the forces can add or take away kinetic energy from an object
- Therefore we use the “net” amount of work done in the above equation

Example

Example 1: A force $\mathbf{F} = 4.0x\hat{\boldsymbol{\imath}}$ (in newtons) acts on an object of mass $2.0\,\mathrm{kg}$ as it moves from $x = 0$ to $x = 5.0\,\mathrm{m}$. Given that the object is at rest at $x = 0$,

(a) Calculate the net work

(b) What is the final speed of the object?

Gravitational Force & Potential Energy

The gravitational force (weight) of an object is defined as:

$$\boxed{\mathbf{F}_g = m\mathbf{g}}$$

For objects near the surface of Earth, we assume that $\mathbf{g} = -g\hat{\jmath} = -10\hat{\jmath}$ (in m/s^2) is a constant. The work done to move an object from height h_1 to h_2 is therefore:

$$W = \int \mathbf{F}_g \cdot d\mathbf{h} = \int_{h_1}^{h_2} -mg\hat{\jmath} \cdot dh\hat{\jmath} = -mgh\Big|_{h_1}^{h_2} = -\Delta U_g$$

where U_g is the **gravitational potential energy**:

$$\boxed{U_g = mgh}$$

Spring Force & Elastic Potential Energy

The spring force $\mathbf{F}_e$ is the force a compressed or stretched spring exerts onto objects connected to it. It obeys Hooke's law:

$$\boxed{\mathbf{F}_e = -k\mathbf{x}}$$

When applied to the work equation, we can find the work done to displace a spring:

$$W = \int \mathbf{F}_e \cdot d\mathbf{x} = -k \int x dx = -\frac{1}{2}kx^2\Big|_{x_1}^{x_2} = -\Delta U_e$$

where $U_e = \frac{1}{2}kx^2$ is the **elastic potential energy**:

$$\boxed{U_e = \frac{1}{2}kx^2}$$

Conservative Forces

Gravitational force, spring force, electrostatic force (later in the course) are called **conservative forces**

- The work done by these forces relate to a change of another quantity called *potential energy*
- Since the potential energy is evaluated at the end points, the work done by a conservative force is *path independent*

Conservative Forces

Since the expressions for potential energies are obtained by integrating the work done by the conservative forces, these forces are therefore the negative gradient of the potential energies:

$$\boxed{\mathbf{F} = -\nabla U = -\frac{\partial U}{\partial x}\hat{\boldsymbol{\imath}} - \frac{\partial U}{\partial y}\hat{\boldsymbol{\jmath}} - \frac{\partial U}{\partial z}\hat{\boldsymbol{k}}}$$

The direction of a conservative force *always* decreases the potential energy

Work and Potential Energy

Like kinetic energy, the expressions for potential energies come from integrating the work equation, in that work equals to the change in *something*, and we called that potential energy.

$$\boxed{W = -\Delta U}$$

- ΔU can be positive or negative depending on the direction of the (conservative) force
- Positive work *decreases* the related potential energy
- Negative work increases the related potential energy

Conservation of Mechanical Energy

Positive work done by conservative forces on an object does two things:

1. Decrease its potential energy, while
2. Increase its kinetic energy by the same amount

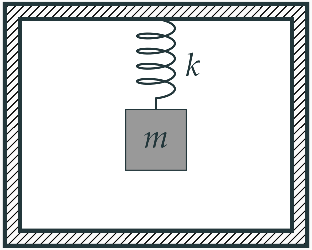
Mathematically, this shows that mechanical energy must always be conserved when there are only conservative forces:

$$W = -\Delta U = \Delta K \quad \longrightarrow \quad \boxed{\Delta K + \Delta U = 0}$$

That's why those forces are called conservative forces!

Isolated Systems and the Conservation of Energy

An **isolated system** is a system of objects that does not interact with the surrounding Think of an isolated system as a bunch of objects inside an insulated box.

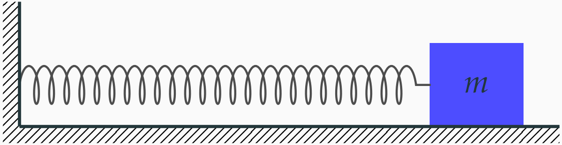


Isolated Systems and Conservation of Energy

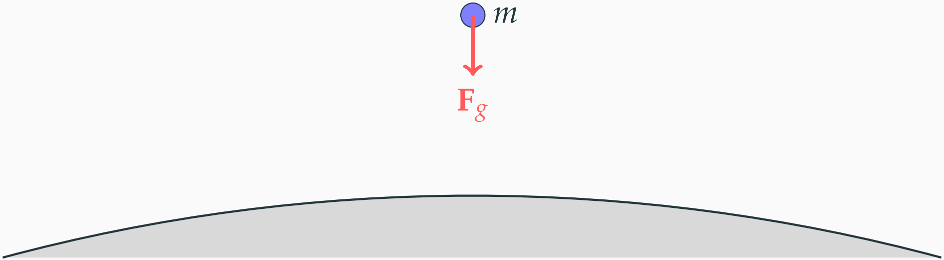
- Since the system is isolated from the surrounding environment, the environment can't do any work on it, by definition!
- Likewise, the energy inside the system cannot escape either
- Therefore energy of the system is conserved
- There are *internal* force inside the system that is doing work, but the work only converts kinetic energy into potential energies, and vice versa.

Example: Mass sliding on a spring

- Assuming that there is no friction in any part of the system
- The isolated system consists of the mass and the spring
- Energies:
  - Kinetic energy of the mass
  - Elastic potential energy stored in the spring

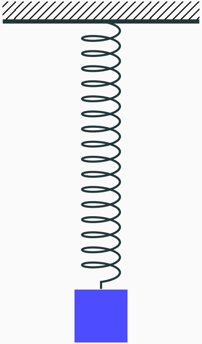


Example: Gravity



- The isolated system consists of the mass and Earth
- Assuming no friction
- Energies:
    - Kinetic energy of the mass
    - Gravitational potential energy of the mass

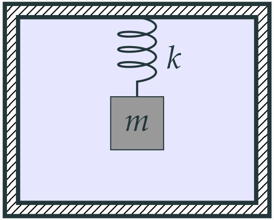
Example: Vertical spring-mass system



- The system consists of a mass, a spring and Earth
- Energies:
    - Kinetic energy of the mass
    - Gravitational potential energy of the mass
    - Elastic potential energy stored in the spring
- The total energy of the system is conserved if there is no friction

What if there is friction?

Energy is always conserved as long as your system is defined properly



- The system consists of a mass, a spring, Earth and all the air particles inside the box
- As the mass vibrates, friction with air slows it down
- While the mass loses energy, the temperature of the air rises due to friction
- Energies:
    - Kinetic and gravitational potential energies of the mass
    - Elastic potential energy stored in the spring
    - Kinetic energy of the vibration of the air molecules
- Total energy is conserved even as the mass stops moving

Conservation of Energy

If there are only conservative forces, mechanical energy (i.e. $K + U$) is always conserved:

$$\boxed{K + U = K' + U'}$$

When non-conservative forces are also doing work, instead of *trying* to isolate the system, we can calculate the work done by them W_{nc} and add it to the total energy of the system

$$\boxed{K + U + W_{\mathrm{nc}} = K' + U'}$$

Work By Non-Conservative Force

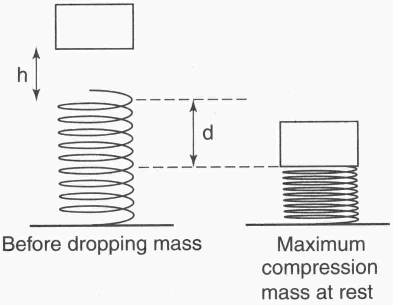
Examples of non-conservative forces include:

- Work done by these forces are *usually* negative because they oppose the direction of motion
  - Drag (fluid resistance)
  - Kinetic friction
- The work done by these forces may be positive or negative, depending on the problem
  - Applied force
  - Tension force
  - Normal force

Note that the work-kinetic energy theorem still applies when non-conservative forces are present

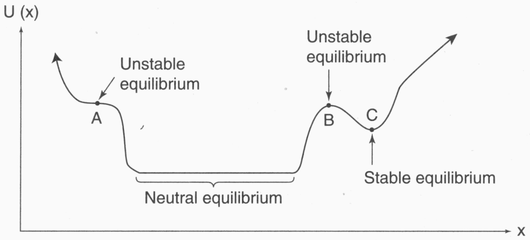
Example

Example 2: A mass m is dropped from a height of h above the equilibrium position of a spring. Set up the equation that determines the spring's compression d when the object is instantaneously at rest.



Energy Diagrams

- Plots of potential energy (U) vs. position for a conservative force



- If more than one conservative force, they can be combined into one graph
- Where slope is zero means no force acting on it: it is in a state of **equilibrium**
- An object placed at an equilibrium point with $K = 0$ will remain there