Class 8: Gravitation

AP Physics

Timothy Leung, Ph.D.

Olympiads School

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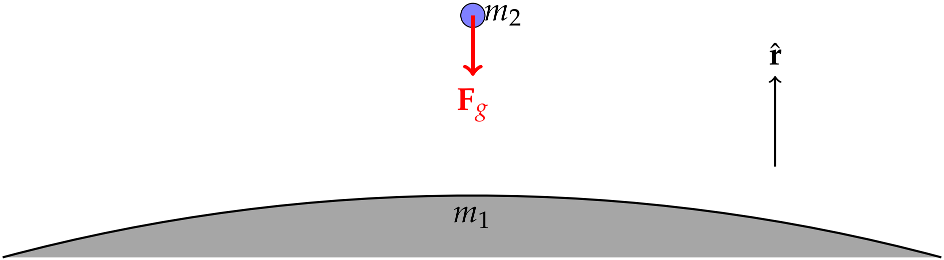
Today's Plan

Gravitational Force



$$\mathbf{F}_g = -\frac{Gm_1m_2}{r^2}\hat{\mathbf{r}}$$

Gravitational Potential Energy

- The gravitational potential energy is defined as:

$$\boxed{U_g = -\frac{Gm_1m_2}{r}}$$

- It has a very similar form to the the equation for $\mathbf{F}_g$
- $U_g = 0$ at $r = 0$ and *decrease* as r decreases

Relating Gravitational Potential Energy to Force

- If you know *vector* calculus, you can easily see that gravitational force ($\mathbf{F}_g$) is the negative gradient of the gravitational potential energy (U_g):

$$\mathbf{F}_g = -\nabla U_g = -\frac{\partial U_g}{\partial r}\hat{\mathbf{r}}$$

- Even without using vector calculus, you should still see that, like all conservative force, $\Delta F_g = -\Delta U_g$, as we have seen in Class 3
- The direction of $\mathbf{F}_g$ always points from high to low potential
  - A falling object is always decreasing in U_g
  - “Steepest descent”: the direction of $\mathbf{F}$ is the shortest path to decrease U_g
  - Objects traveling perpendicular to $\mathbf{F}$ has constant U_g

Gravitational Field

A Review

- The concept of gravitational field was studied in Grade 12 Physics, so this *should* be a review

Think Gravitational Field: What is g?

- We generally describe the force of gravity as

$$\mathbf{F}_g = m\mathbf{g}$$

- To find the magnitude of g, we group the variables in Newton's universal gravitation equation:

$$F_g = \underbrace{\left[\frac{Gm_1}{r^2}\right]}_{=g} m_2 = m_2 g$$

- On the surface of Earth, we use use $m_1 = m_{\text{Earth}}$ and $r = r_{\text{Earth}}$ to compute $g = 9.81\,\text{m/s}^2$, or $g = 9.81\,\text{N/kg}$ (both units are equivalent)

Gravitational Field

- The intensity of the **gravitational field $\mathbf{g}$** generated by a source mass m_s is defined by:

$$\boxed{g(m_s, r) = \frac{G m_s}{r^2}}$$

- Mapping of how m_s influences the gravitational forces on other masses

Quantity	**Symbol**	**SI Unit**
Gravitational field intensity	g	$\mathrm{N/kg}$
Universal gravitational constant	G	$\mathrm{N\,m^2/kg^2}$
Mass of source (a point mass)	m_s	kg
Distance from centre of source	r	m

Relating Gravitational Field & Gravitational Force

- $\mathbf{g}$ itself doesn't do anything until there is another mass m. At which point, m experiences a gravitational force related to $\mathbf{g}$ by:

$$\boxed{\mathbf{g} = \frac{\mathbf{F}_g}{m}}$$

- $\mathbf{F}_g$ and $\mathbf{g}$ are *vectors* in the same direction: toward the centre of the source mass that created the field
- All vector operations apply

Quantity	Symbol	SI Unit
Gravitational field	$\mathbf{g}$	N / kg
Gravitational force on a mass	$\mathbf{F}_g$	N
Mass inside the gravitational field	m	kg

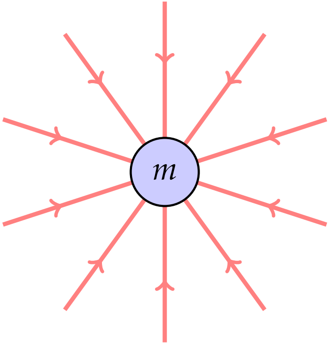
Relating U_g, $\mathbf{F}_g$ and $\mathbf{g}$

- Knowing that $\mathbf{F}_g$ and $\mathbf{g}$ only differ by a constant, we can also relate gravitational field to U_g by the gradient operator:

$$\mathbf{g} = \frac{\mathbf{F}_g}{m} = -\nabla\left(\frac{U_g}{m}\right) = -\frac{\partial}{\partial r}\left(\frac{U_g}{m}\right)\hat{\mathbf{r}}$$

- We already know that the direction of $\mathbf{g}$ is the same as $\mathbf{F}_g$, i.e.
  - The direction of $\mathbf{g}$ is the shortest path to decrease U_g
  - Objects traveling perpendicular to $\mathbf{g}$ has constant U_g

Gravitational Field Lines



- The direction of $\mathbf{g}$ is towards the centre of the object that created it
- Field lines do not tell the intensity (i.e. magnitude) of $\mathbf{g}$, only the direction

Orbital Velocity

Orbital Energies

- Kinetic Energy
- Gravitational Potential Energy
- Total Energy

Escape Velocity

Kepler's Law of Planetary Motion