Classes 14 & 15: Magnetism, Part 2

AP Physics

Dr. Timothy Leung

Olympiads School

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Files for You to Download

Download from the school website:

1. `14-Magnetism2.pdf`—The "print version" of this presentation. If you want to print the slides on paper, I recommend printing 4 slides per page.

Please download/print the PDF file before each class. When you are taking notes, pay particular attention to things I say that aren't necessarily on the slides.

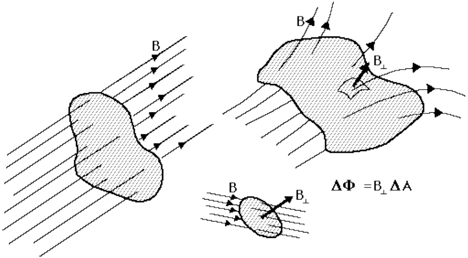
Magnetic Flux

Question: If a current-carrying wire can generate a magnetic field, can a magnetic field affect the current in a wire?

Answer: Yes, sort of. . .

To understand how to *induce* a curent by a magnetic field, we need to look at fluxes again.

Magnetic Flux



Magnetic flux is defined as:

$$\boxed{\Phi_{\text{magnetic}} = \int \mathbf{B} \cdot d\mathbf{A}}$$

where $\mathbf{B}$ is the magnetic field, and $d\mathbf{A}$ is the infinitesimal area pointing **outwards**. If you are uncomfortable with using vector surfaces, note that magnetic flux can also be expressed as:

$$\boxed{\Phi_{\text{magnetic}} = \int \mathbf{B} \cdot \hat{\mathbf{n}} dA}$$

where $\hat{\mathbf{n}}$ is the outward normal direction

Magnetic Flux Over a Closed Surface

The unit for magnetic flux is a "weber" (Wb), in honor of German physicist Wilhelm Weber, who invented the electromagnetic telegraph with Carl Gauss. It is defined as:

$$1\,\mathrm{Wb} = 1\,\mathrm{T\,m^2}$$

The magnetic flux over a closed surface is always zero:

$$\boxed{\oint \mathbf{B} \cdot d\mathbf{A} = 0}$$

Since magnetic field lines only exist as a loop, that means there should be equal amount of "flux" flowing out of a closed surface as entering the surface.

Changing Flux

We can

Unlike in a basic circuit, where the *emf* is concentrated at the terminals of the battery, the induced *emf* is spread across the entire wire.

Faraday's Law

Faraday's law states that the rate of change of magnetic flux produces an electromotive force:

$$\boxed{\mathcal{E} = \oint \mathbf{E} \cdot d\mathbf{l} = -\frac{d\Phi}{dt}}$$

How Can Magnetic Flux Change

Magnetic flux can change due to a number of reasons:

1. **Changing magnetic field strength** e.g. if $\mathbf{B}$ is created by a time dependent current source like an alternating current
2. **Changing orientation of magnetic field** because the surface area is moving (translation and/or rotation) relative to $\mathbf{B}$
3. **Changing area** the surface area from which the flux is calculated is changing

Self Inductance

A solenoid carrying a current generates a magnetic field; its strength given by Biot-Savart Law.

MISSING DIAGRAM HERE!

Since $\mathbf{B} \propto I$, the magnetic flux through the coil is therefore also proportional to I, i.e.:

$$\boxed{\Phi_{\text{magnetic}} = LI}$$

where L is the called the **self inductance** of the coil.

Self Inductance

For a solenoid, we can see that the self inductance is given by:

$$\boxed{L = \frac{\Phi_{\text{magnetic}}}{I} = \mu_0 n^2 A l}$$

where μ_o is the magnetic permeability of free space, n is the number of coil turns per unit length, and A and l are the cross-section and length of the solenoid. (i.e. Al is the enclosed volume.)

Self Inductance and Induced EMF

If the current changes, the magnetic flux changes as well, therefore inducing an electromotive force in the circuit! According Faraday's law:

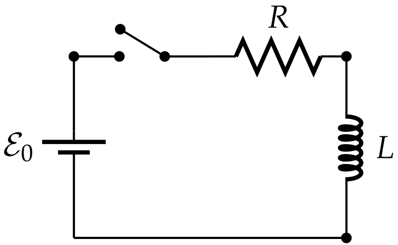
$$\boxed{\mathcal{E} = -\frac{d\Phi}{dt} = -L\frac{dI}{dt}}$$

The self-induced emf is proportional to the rate of change of current.

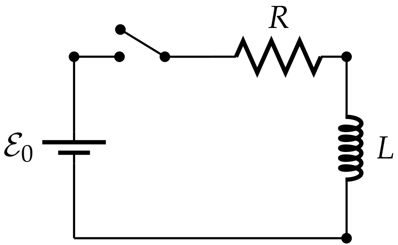
Mutual Inductance

Circuits with Inductors

- Coils and solenoids in circuits are known as "inductors" and have large self inductance L
- Self inductance prevents currents rising and falling instantaneously
- A basic circuit containing a resistor and an inductor is called an ***LR* circuit**:



Analyzing LR Circuits



Applying Kirchkoff's voltage law:

$$\mathcal{E}_0 - IR - L\frac{dI}{dt} = 0$$

We follow the same procedure as charging a capacitor to find the time dependent current:

$$I = \frac{\mathcal{E}_0}{R}\left(1 - e^{-Rt/L}\right)$$

The time constant for an LR circuit is

$$\tau = \frac{L}{R}$$

Magnetic Energy

Just as a capacitor stores energy, the energy stored by an inductor carrying a current I in given by:

$$\boxed{U_m = \frac{1}{2}LI^2}$$

Maxwell's Equations in Integral Form

James Clerk Maxwell recognized the relationship between electricity and magnetism, and combined the few laws into a unifying set of equations, now known as **Maxwell's equations** for electrodynamics:
By manipulating the equations, we can show the existence of a "electromagnetic wave" that travels at the speed of light.

Maxwell's Equations in Differential Form