Classes 19: Fluid Mechanics

AP Physics

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Olympiads School

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Files for You to Download

Download from the school website:

1. `19-fluidMechanics.pdf`—This presentation. If you want to print the slides on paper, I recommend printing 4 slides per page.
2. `20-Homework.pdf`—Homework assignment for Classes 19 and 20, which cover Fluid Mechanics and Thermodynamics

Please download/print the PDF file before each class. When you are taking notes, pay particular attention to things I say that aren't necessarily on the slides.

Disclaimer

Use of Calculus

Fluid mechanics is part of the AP Physics 2 Exam, which does not require calculus. However, in the interest in completeness, *some* calculus will still be used when deriving equations.

What is a Fluid

- **The simple explanstion:** anything that flows, which covers most *gases* and *liquids*
- **The scientific explanation:** Any substancs that deform *continuously* under oblique stress

Properties of Fluids

Density

Continuity

A fluid is considered to be continuous in space.

Properties of Fluids

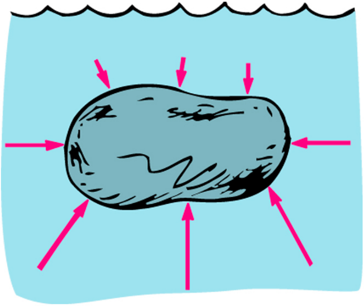
Viscosity

Hydrostatics

Buoyancy

Everything Floats a Little

When an object is submerged inside a fluid (e.g. water, air, etc), the fluid exerts a pressure at the surface of the object. We can integrate the pressure over the entire surface area S to find the total force $\mathbf{B}$ the fluid exerts on the object.



Derivation of Buoyance Force

Integrate the pressure$p$ over the entire surface S to find the total force $\mathbf{B}$, or take some knowledge of vector calculus (divergence theorem):

$$\mathbf{B} = -\oint_S p\mathbf{n}dS = -\iiint \nabla p dV$$

Since pressure $p = \rho g z$ is a function in z only, the gradient easy to compute: $\nabla p = \partial p/\partial z = \rho g \hat{\mathbf{k}}$, giving us

$$\mathbf{B} = \rho_{\text{fluid}} g \hat{\mathbf{k}} \iiint dV = \rho_{\text{fluid}} g V \hat{\mathbf{k}}$$

Derivation of Buoyance Force

Although the derivation required a lot of calculus, the expression of buoyance force is straightforward:

$$\boxed{\mathbf{B} = \rho_{\text{fluid}} g V \hat{\mathbf{k}} = m_{\text{fluid}} g \hat{\mathbf{k}}}$$

where ρ_{fluid} is the density of the displaced fluid, and V is the volume displaced. This equation is known as **Archimedes' principle**.

Buoyance force has a magnitude that equals to the weight of the fluid displaced by the submerged object, pointing upward.

An Easier Explanation of Buoyancy

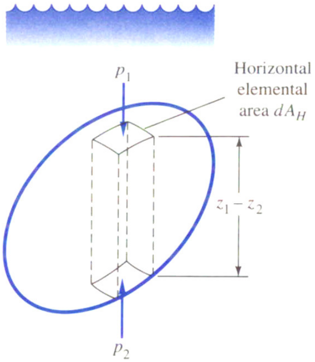
Not Much Calculus

There is a simpler way to find the buoyance force, by taking an infinitesimal “tube” of the object, and finding the pressure difference between the top and bottom of the tube:

$$\mathbf{B} = \int (p_2 - p_1)\,dA$$

$$= \rho_{\text{fluid}}\, g \int (z_2 - z_1)\,dA$$

$$= \rho_{\text{fluid}}\, g V$$



which is the same expression that we got with calculus.

Buoyancy

Note that buoyancy does not depend on:

- the mass of the immersed object, or
- the density of the immersed object

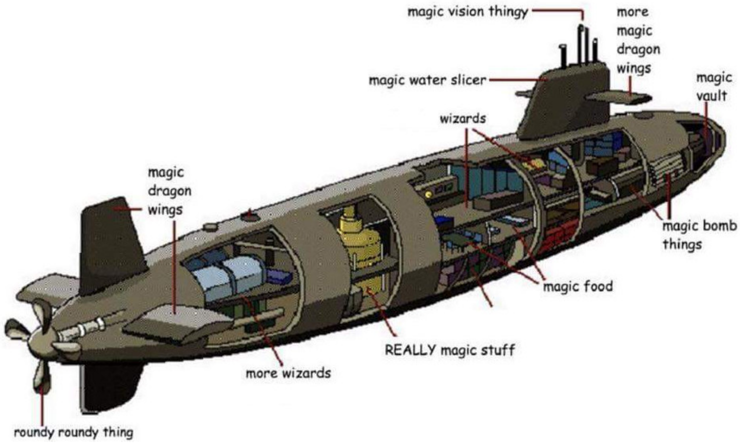
Objects immersed in a fluid have an “apparent weight” $\mathbf{W}'$ that is reduced by the buoyance force:

$$\mathbf{W}' = \mathbf{W} - \mathbf{B} = \rho' \mathbf{g} V$$

where $\rho' = \rho_{\text{obj}} - \rho_{\text{fluid}}$ is the relative density

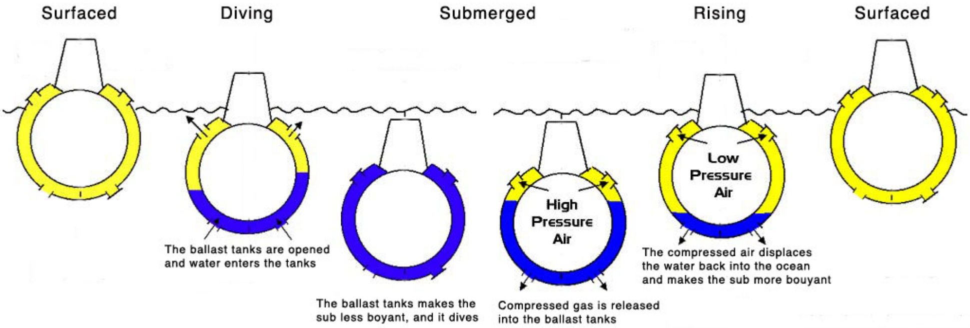
How Submarines Work

Like this?



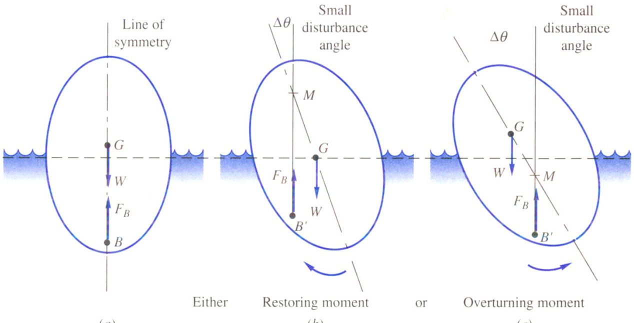
How Submarines Work

Like most ships, a submarine does not naturally sink because of the buoyance force. When a submarine submerges, water needed to be pumped inside "ballast tanks" to make the ship heavier.



Stable? Or unstable?

- Buoyance force $\mathbf{B}$ acts at the *center of buoyancy* (CB) of the submerged object
  - The CB is the CG *if the object has constant density* and is fully submerged
  - The actual CG of the object may be at a different position
  - Sometimes the object is not fully submerged
- $\mathbf{F}_g$ and $\mathbf{B}$ may act at different points, creating a torque/moment on the object



Fluid Flow: Continuity

In a fixed volume (known as a "control volume", or CV) we can quantify how fluid mass changes in the CV:

Rate of decrease in mass in the CV = mass flux out of the CV

On the left hand side, the fluid mass in the CV is the integral of density over the volume:

$$\int_{CV} \rho dV$$

The rate of decrease is therefore the negative of the time derivative:

$$-\frac{\partial}{\partial t}\int_{CV} \rho dV$$

Fluid Flow: Continuity

The mass flux out of the surfaces of the control volume the volume flux multiplied by the fluid density at the surface:

$$\int_{CS} \rho \mathbf{v} \cdot d\mathbf{A}$$

Combining the LHS and RHS terms, we have the *integral* form of the continuity equation:

$$\boxed{\int_{CV} \frac{\partial \rho}{\partial t} dV + \int_{CS} \rho \mathbf{v} \cdot d\mathbf{A} = 0}$$

Fluid Flow: Continuity

With some clever use of vector calculus, we get the *differential form* of the continuity equation:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0$$

…which is still too difficult. So in AP Physics we usually only look at simple cases where

- Steady flow (time independent)
- Constant density
- Flow perpendicular to control surfaces

Inlet Outlet Flow

Diagram

In this example, the mass flowing at the inlet is the same as the flow out of it:

$$\rho_1 v_1 A_1 = \rho_2 v_2 A_2$$

And if density is constant, the ρ terms will cancel.

Example: Multiple Inlet & Outlets

examples

Governing Equations for Fluid Dynamics

To properly describe fluid flows, there are three conservation equations:

- continuity
- momentum, and
- energy

Fluid Flow: Momentum Equation

In the momentum equation, the rate of decrease of total momentum inside the control volume CV is the net momentum flux of the fluid out of the control volume plus the normal and shear forces acting on the fluid:

$$\frac{\partial(\rho\mathbf{v})}{\partial t} + \rho(\mathbf{v}\cdot\nabla)\mathbf{v} = -\nabla p + \mathbf{f} + \mu\nabla^2\mathbf{v}$$

(That's pretty hard, so thankfully you won't need this equation for AP Physics.)

Fluid Flow: Energy Equation

The energy equation follows a similar thought process as the previous two equations, but the terms are even more complicated:

Navier-Stokes Equations

The three conservation equations combined together is called the **Navier-Stokes equations.** In differential form, they are usually written as: Even for a 2nd-year engineering student with lots of experience with calculus, solving the N-S equation is still a daunting task.

Let's Make Some Assumptions

If we can make these assumptions:

- the flow is steady
  - all derivatives w.r.t. time are zero
- the flow is inviscid
  - "inviscid" means zero viscosity
  - no friction
  - no shear stresses
  - Only forces are pressure at the surface
- there is **no shaft work** done along the streamline
- there is **no heat transfer** along the streamline

Then the N-S equations reduces to the **Bernoulli equation**

$$\boxed{p_1 + \frac{1}{2}\rho v_1^2 + \rho g z_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g z_2}$$

Bernoulli Equation

$$\boxed{p_1 + \frac{1}{2}\rho v_1^2 + \rho g z_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g z_2}$$

The term $\frac{1}{2}\rho v^2$ is called “dynamic pressure”