Topic 2: Dynamics

Advanced Placement Physics

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Olympiads School

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Dynamics

While we use **kinematics** to describe the motion of any object mathematically, we use **dynamics** to describe *what* causes motion.

- Newton's three laws of motion

Newton's First Law

An object at rest or in uniform motion will remain at rest or in uniform motion unless acted on by a net external force.

- Uniform motion: constant velocity
- An object at rest is also in uniform motion: it has a velocity of zero
- Common examples:
  - spacecraft in "deep space"
  - hockey puck sliding on very smooth ice

Newton's Second Law

The sum of the forces acting on an object is proportional to its mass and its acceleration.

$$\boxed{\mathbf{F}_{\text{net}} = \Sigma\mathbf{F} = m\mathbf{a}}$$

Quantity	**Symbol**	**SI Unit**
Net force (sum of all forces)	$\mathbf{F}_{\text{net}}$	N (newtons)
Mass	m	kg (kilograms)
Acceleration	$\mathbf{a}$	m/s^2 (meters per second squared)

Actually, this isn't exactly what Newton said. He wrote that force is the rate of change of the "quantity of motion" (momentum). We will be looking into that in the next topic.

Newton's Third Law

For every action there is an equal and opposite reaction.

For every action force on an object (B) due to another object (A), there is a reaction force which is equal in magnitude but opposite in direction, on object (A), due to object (B):

$$\boxed{\mathbf{F}_{\text{A on B}} = -\mathbf{F}_{\text{B on A}}}$$

The reaction forces act on different objects!

Example Problem

A Blast From the Past

This problem can get you started to remember how to do these problems, but it is a bit too easy for AP Physics.

Example 1: In old-style television picture tubes and computer monitors (cathode ray tubes), light is produced when fast-moving electrons collide with phosphor molecules on the surface of the screen. The electrons (mass $m = 9.1 \times 10^{-31}$ kg) are accelerated from rest in the electron "gun" at the back of the vacuum tube. Find the velocity of an electron when it exits the gun after experiencing an electric force of 5.8×10^{-15} N over a distance of 3.5 mm.

Forces

A **force** is the interaction between the objects.

- When there is interaction, then forces are created
- A "push" or a "pull"

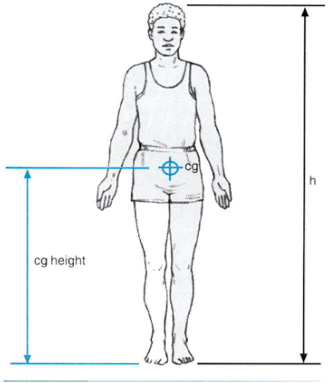
There are two broad categories of forces:

- **Contact forces** act between two objects that are in contact with one another
- **Non-contact forces** act between two objects without them touching each other. They are also called "action-at-a-distance" force

Center of Mass

Newton considered all forces acting at a single point of an object called the center of mass ("CM")

- The center of mass is also called the center of gravity ("CG")
- If the density of an object is constant, then the CM/CG is also the geometric center (centroid) of the object
- Sooner we will look into how to compute the centers of mass of different objects



Static & Dynamic Equilibrium

If the net force on an object is zero ($\Sigma\mathbf{F} = \mathbf{0}$) then the object is in a *state of equilibrium*

- Dynamic equilibrium: the object is moving relative to us
- Static equilibrium: the object is not moving relative to us

Common Forces

Common everyday forces that we encounter in Physics 12 include:

- Gravitational force (weight) $\mathbf{F}_g$
- Normal force $\mathbf{F}_N$
- Friction (static $\mathbf{F}_s$ and kinetic $\mathbf{F}_k$)
- Tension $\mathbf{F}_T$
- Applied force $\mathbf{F}_a$
- Spring force $\mathbf{F}_e$
- Drag $\mathbf{F}_d$ (fluid resistance, then again in fluid mechanics)
- Buoyant Force $\mathbf{F}_B$ (discussed in fluid mechanics)
- Electrostatic force $\mathbf{F}_q$ (discussed in E & M exam)
- Magnetic force $\mathbf{F}_M$ (discussed E & M exam)

Gravity

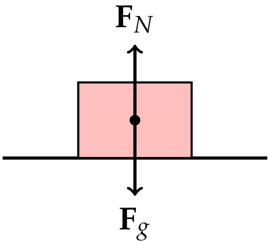
Gravity is the force of attraction between all objects with mass

$$\boxed{\mathbf{F}_g = m\mathbf{g}}$$

- Near surface of Earth, use $g = 9.81\,\mathrm{m/s^2}$ or $g = 10\,\mathrm{m/s^2}$
- You may be asked to find the value of g on some "unknown planet".
- $\mathbf{F}_g$ always points **down** (the direction of $\mathbf{F}_g$ is how "down" is defined)
- Newton's law of universal gravity:

$$\boxed{F_g = \frac{Gm_1m_2}{r^2}} \quad \text{where} \quad G = 6.67 \times 10^{-11}\,\mathrm{N\,m^2/kg^2}$$

Normal Force

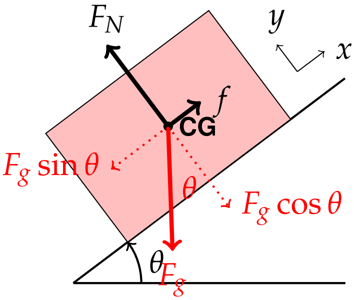


$$\mathbf{F}_g = m\mathbf{g} = -\mathbf{F}_N$$

- A force a surface exerts on another object that it is in contact with
- Always **perpendicular** to the contact surface
- **Special case:** When an object is on a horizontal surface with no additional applied force, the magnitude of the normal force is equal to the magnitude of the weight of the object, i.e. $F_N = F_g$

Normal Force on a Slope

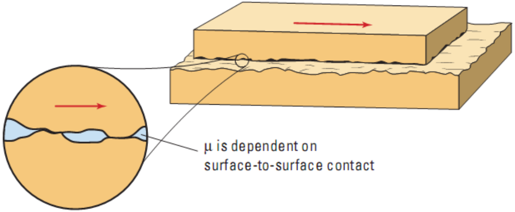
For this case, we label the x-axis to be along the slope, and y-axis to be perpendicular to the slope.



- If on a slope: $F_N = F_g \cos\theta$
    - F_N decreases as ramp angle θ increases
    - Obviously, at $\theta = 90°$, $F_N = 0$!
- F_g has a component along the ramp $F_g \sin\theta$ that wants to slide the block down.
- Friction force f opposes the motion
    - Be careful: if the block is moving *up* the ramp with an applied force, then f will point *down* the ramp

Friction

- A force that opposes the sliding of two surface against one another
- Always act in a direction that opposes motion or attempted motion
- Depends on:
  - The amount of force the two surfaces are pressed against each other, reflected in the normal force F_N
  - Type of surfaces reflected by "coefficient of friction" (μ_s for static, μ_k for kinetic), which itself depends on
  - The material(s) the surfaces are made of, and
  - Whether a lubricant is used



Static Friction

Static friction between the two surfaces is when there is no relative motion between them

- Increases with increasing applied force
- Maximum when the object is just about to move

$$\boxed{F_s \leq \mu_s F_N}$$

Quantity	Symbol	SI Unit
Static friction	$\mathbf{F}_s$	N (newtons)
Static friction coefficient	μ_s	no units
Normal force	$\mathbf{F}_N$	N (newtons)

Kinetic Friction

Kinetic friction between two surfaces is when they are moving relative to each other

- F_k is constant along the path of movement as long as the $\mathbf{F}_N$ stays constant

$$\boxed{F_k = \mu_k F_N}$$

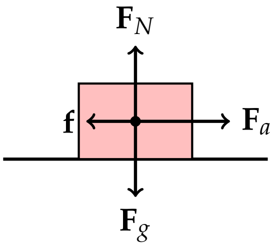
Quantity	Symbol	SI Unit
Kinetic friction	$\mathbf{F}_k$	N (newtons)
Kinetic friction coefficient	μ_k	no units
Normal force	$\mathbf{F}_N$	N (newtons)

Static and Kinetic Coefficients of Friction

Kinetic friction coefficient is always lower than the static coefficient, otherwise nothing will ever move:

$$\mu_k \leq \mu_s$$

Consider a simple case of a box being pulled along a level floor. The free-body diagram is simple (left). How do the magnitudes of the applied force F_a and friction f compare?



Drag

- The force opposing to the motion of an object moving in a fluid
- Unlike kinetic friction, drag depend on velocity[1]
- Sources of drag:
  - **Form drag** is due to the shape of the object
  - **Skin friction** is due to the friction of the fluid against the surface of the object moving through it
  - **Interference drag** is due to when airflow around one part of an object occupying the same space as the airflow around another part (e.g. fuselage and wing of an airplane)

[1] Some high-school textbooks incorrectly state that “air resistance is a force of friction”. While drag does includes a friction component, the primary contribution to drag is due to the shape.

Drag

The direction of the drag force $\mathbf{F}_d$ moving in a fluid is opposite to the velocity vector, and its magnitude is:

$$F_d = \frac{1}{2}\rho V_\infty^2 A_{\text{ref}} C_d$$

Quantity	Symbol	SI Unit
Air resistance (Drag)	F_d	N (newtons)
Density of the fluid	ρ	$\mathrm{kg/m^3}$ (kilograms per meters cubed)
Far-field fluid velocity	V_∞	$\mathrm{m/s}$ (meters per second)
Reference area	A_{ref}	m^2 (meters squared)
Drag coefficient	C_d	(no unit)

Drag coefficient depends on the shape and surface smoothness of the object. For blunt objects A_{ref} is the frontal area; for streamlined objects A_{ref} is the planform (top-view) area

Drag

In AP Physics you are *not* asked to know the drag equation. However, you should know that drag (air resistance) depends on the motion of the object and is not a constant.

Terminal Velocity

When we take drag force into account, we understand that the drag force increases as an object speeds up, and therefore a free-falling object does *not* accelerate infinitely. Instead it reaches a **terminal velocity**.

There is no air resistance just as the object *begins* to fall. Acceleration is due to gravity alone.

Drag increases as v increases. Magnitude of acceleration decreases, but the object continues to gather speed

Terminal velocity is reached when the drag force equals the object's weight. Not net force; no acceleration.

Tension in a Cable

Tension is the force exerted on and by a cable, rope, or string.

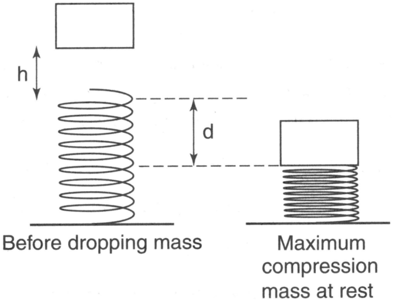
- You can't push on a rope
- Assume the cable/rope/string to be mass less
- Force can change direction when used with pulleys

Spring Force

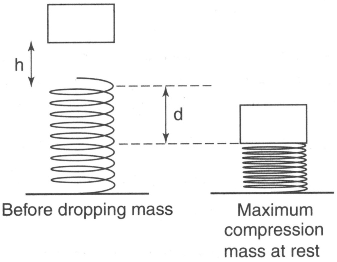
The spring force F_e is the force a compressed or stretched spring exerts onto objects connected to it. It obeys Hooke's Law:

$$\mathbf{F}_e = -k\mathbf{x}$$

where $\mathbf{x}$ is the relative displacement of the ends of the spring.

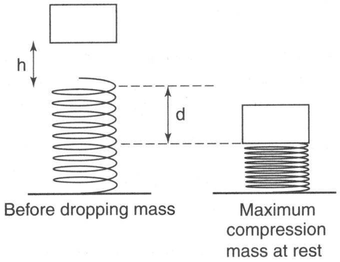


Spring Force



- As the object falls onto the spring, the spring begins to compress
- As the spring compresses, the spring force (pointing up!) increases linearly (Hooke's law)
- At some point, the spring force balances the weight of the block
  - At this point, the *acceleration* is zero
  - But the velocity continues to be downward
- The spring continues to compress until velocity is zero

Spring Force

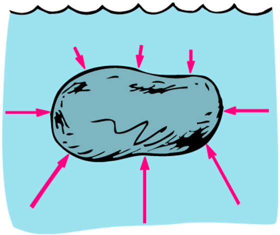


- Solving this problem using dynamics is difficult, because:
  - Spring force scales linearly with *displacement*, but
  - Net force scales linearly with *acceleration* (2nd time derivative of displacement)
- Note that the block continues to *increase* velocity even after it starts to compress the spring.
- Acceleration is zero only after the spring has compressed some amount

Buoyancy

Everything Floats a Little

When an object is submerged inside a fluid (e.g. water, air, etc), the fluid exerts a pressure at the surface of the object. We can integrate the pressure over the entire surface area S to find the total force $\mathbf{B}$ the fluid exerts on the object.



Buoyancy

Everything Floats a Little

The buoyant force is given by:

$$\mathbf{B} = \rho_{\mathrm{fluid}} g \hat{\boldsymbol{k}} \iiint dV = \rho_{\mathrm{fluid}} g V \hat{\boldsymbol{k}}$$

Quantity	Symbol	SI Unit
Buoyant force	$\mathbf{B}$	N (newtons)
Density of the fluid	ρ_{fluid}	$\mathrm{kg/m^3}$ (kilograms per meters cubed)
Volume of the object	V	$\mathrm{m/s}$ (meters per second)

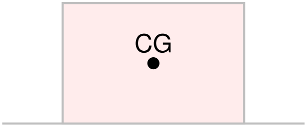
We will discuss this later in the course, when we deal with fluid mechanics.

Free Body Diagrams

- Acceleration (if there is going to be any at all) depends on net force $\mathbf{F}_{\text{net}}$)
- Without a vector sum of all the forces, we cannot determine the magnitude, direction of the acceleration, or how acceleration will evolve in time
- We use **free body diagrams** (FBD) to represent all the forces.
  - Very important in solving any dynamics problems
  - Don't try to save this step, even if the problem does not ask for it
  - Always draw FBD for solving classical mechanics problem

Free Body Diagrams

Step 1: Draw a “big dot” to represent the CG of the object (This makes sense, because we assume that all masses are point masses anyway)



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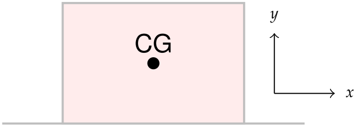


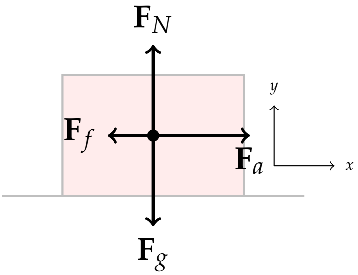
Step 2: Define a coordinate system (x and y axes)

- We can define the axes in any arbitrary direction, *but we want to simplify our problem, not to make it more complicated*, so instead:
- Define them such that the sum of along one axis (usually y) is always zero

Free Body Diagrams

Step 3: Identify all the forces are acting on the object

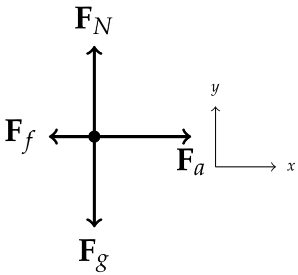
- Make note of the direction of the forces.
- Gravity is a must!



Free Body Diagrams

Step 4: The free-body diagram itself does not require drawing the object itself

- Make sure the arrows representing the forces originate at the CG
- It is generally a good practice to approximately scale the lengths of the arrows to the magnitude of the forces



Solving Force Problem

- If you have chosen your coordinate system properly, you should have
  - Normal force along the $\hat{\jmath}$ direction
  - Friction force along the $\hat{\imath}$ axis
- Break down the forces into the x and y components
- Sum the forces in the direction that doesn't have a net force (usually y axis)
- Sum the forces in the other axis, and find out what the acceleration is
- Solve the motion of the object

Applying Newton's Third Law on Connected Bodies

- The objects are connected by a cable or a solid linkage with negligible mass
- All objects (usually) have the same acceleration
- Require multiple free-body diagrams

Solving Connected-Bodies Problems

To solve a connected-bodies problem, you can follow these procedures:

1. Draw a FBD on each of the objects
2. Sum all the forces on all the objects along the direction of motion
    - Direction of motion is usually very obvious
    - All internal forces should cancel and do not figure into the acceleration of the system
3. Compute the acceleration of the entire system using Newton's second law
    - Remember that (usually) every object has the same acceleration!
4. Go back to the FBD of each of the objects and compute the unknown forces (usually tension)

Connected Bodies: Example

Example: A tractor-trailer pulling two trailers starts from rest and accelerates with an acceleration a on a straight, level road. The mass of the truck itself (T) is $5450\,\mathrm{kg}$, the mass of the first trailer (A) is $31\,500\,\mathrm{kg}$, and the mass of the second trailer (B) is $19\,600\,\mathrm{kg}$.

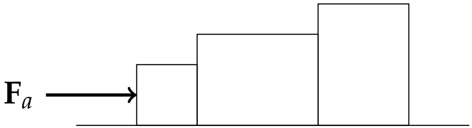
- What magnitude of force must the truck generate in order to accelerate the entire vehicle?
- What magnitude of force must each of the trailer hitches withstand while the vehicle is accelerating?

For this problem we will assume that frictional forces are negligible in comparison with the forces needed to accelerate the large masses.

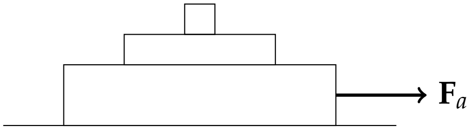
Connected Bodies

Other type of connected bodies problem may be like this

Multiple objects pressed against one another:

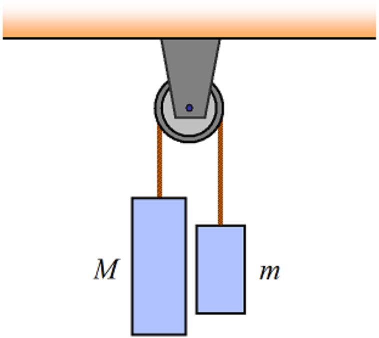


Multiple objects stacked on top of one another:



Example Problem: Atwood Machine

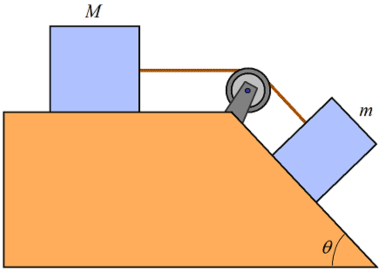
An **Atwood machine** is made of two objects connected by a rope that runs over a pulley. The pulley allows the direction of force and direction of motion to change between two objects.



Example: The object on the left has a mass of M and the object on the right has a mass of m.

- What is the acceleration of the masses?
- What is the tension in the rope?

A Slightly More Difficult Problem

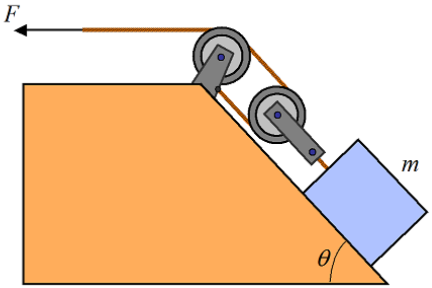


Two blocks of mass m and M are connected via pulley with a configuration as shown on the left. The coefficient of static friction is μ_s, between blocks and surface. What is the maximum mass m so that no sliding occurs?

Multiple Pulleys

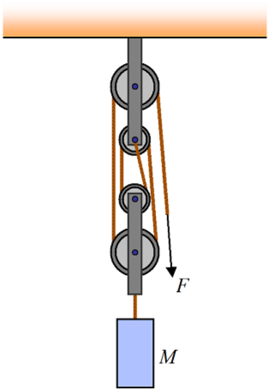
When there are multiple pulleys involved, we have to remember that tension force is distributed evenly along the cable.

Example: A block of mass m is pulled, via two pulleys as shown, at constant velocity along a surface inclined at angle θ. The coefficient of kinetic friction is μ_k, between block and surface. Determine the pulling force F. Ignore the mass of the pulleys.



One More!

Example: A block of mass M is lifted at constant velocity, via an arrangement of pulleys as shown. Determine the pulling force F. Ignore the mass of the pulleys.



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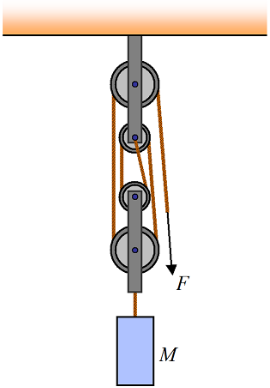

Example: The pulling force is replaced by a $10M$ mass, and was let go. What are the accelerations of the M and the $10M$ mass?