WELCOME TO AP PHYSICS

Pre-requisites

- **Physics 11 and 12** As AP Physics is primarily taught at the first-year university level, you will need to be comfortable with the topics covered in high-school physics courses.
- **Calculus** The AP Physics C exams are calculus based, and you will be required to perform basic differentiation and integration. You don't need to be an expert, but basic knowledge is required. Differentiation and integration in the course are generally not difficult, but there are occasional challenges.
- **Vectors** You need to be comfortable with vector operations, including addition and subtraction, multiplication and division by constants, as well as dot products and cross products.

The AP Physics Exams

There are 4 AP Physics exams:

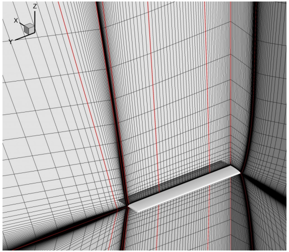
- Physics 1
- Physics 2
- Physics C–Mechanics
- Physics C–Electricity and Magnetism

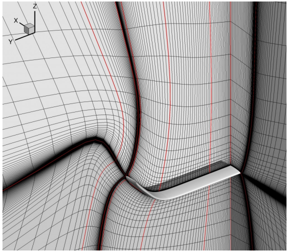
Offered in first or second week of May of each year

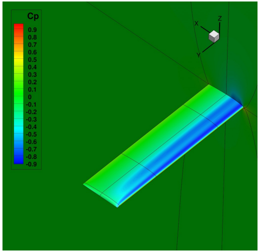
Hi, My Name is Tim

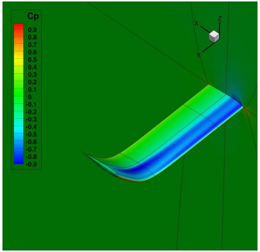
- B.A.Sc. in Engineering Physics (UBC)
  - Won the Roy Nodwell Prize for my design of a solar car
- M.A.Sc. and Ph.D. in Aerospace Engineering (UTIAS)
  - "Computational Fluid Dynamics" (CFD)
  - "Aerodynamic shape optimization"
  - Aircraft design
- Also spent a year in Vancouver as a professional violinist. . .

Tim's Past Research Work









Classroom Rules

The rules are the same as in my Grade 11 and 12 Physics classes.

- Treat each other with respect, and I'll treat you like an adult.
- If you need to go to the bathroom, you don't need my permission
- Raise your hands if you have a question. Don't wait too long
- E-mail me at `tleung@olympiadsmail.ca` for any questions related to physics and math and engineering
- Do ***not*** try to find me on social media

1. Kinematics, With Calculus

Advanced Placement Physics

Dr. Timothy Leung

Olympiads School

Summer 2018

Notes on Writing Vectors

Like in my Physics 12 class (and most university textbooks), we *print* vectors using a bold face font, while using use the “arrow on top” format when *writing*.

In print (books, journal papers)

$$\mathbf{v} \quad \mathbf{F}_g \quad \mathbf{p} \quad \mathbf{I}$$

Handwritten (used by some books)

$$\vec{v} \quad \vec{F}_g \quad \vec{p} \quad \vec{I}$$

When we write the magnitude of these vectors, we have two options:

With absolute-value sign

$$|\mathbf{v}| \quad |\mathbf{F}_g| \quad |\mathbf{p}| \quad |\mathbf{I}|$$

Or as a scalar value

$$v \quad F_g \quad p \quad I$$

This actually makes sense because the magnitude of a vector is a scalar

Writing Vectors

In Physics 11 and 12, vectors are written by separating the magnitude from the direction, e.g. a velocity vector can be written as:

$$\mathbf{v} = 4.5\,\mathrm{m/s}\ [\mathrm{N}\ 55^\circ\ \mathrm{E}]$$

- Intuitive for describing *one* vector in 2D
- Complicated to describe direction when extended into 3D
- Difficult to perform vector arithmetic

IJK Vector Notation

For vectors in 2D/3D Cartesian space, we generally write them in their x, y and z components using the “IJK notation”:

$$\mathbf{A} = A_x\hat{\boldsymbol{\imath}} + A_y\hat{\boldsymbol{\jmath}} + A_z\hat{\boldsymbol{k}}$$

- $\hat{\imath}$, $\hat{\jmath}$ and $\hat{k}$ are *basis vectors* indicating the directions of the x, y and z axes. Basis vectors are “unit vectors” (i.e. length 1).
- The IJK notation does not give the magnitude of the vector, which needs to be calculated:

$$A = |\mathbf{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

Vector Addition and Subtraction

Adding and subtracting vectors is straightforward:

$$\mathbf{A} \pm \mathbf{B} = (A_x \pm B_x)\hat{\boldsymbol{\imath}} + (A_y \pm B_y)\hat{\boldsymbol{\jmath}} + (A_z \pm B_z)\hat{\boldsymbol{k}}$$

Dot Product

The **dot product** is the scalar multiplication of two vectors. It is determined by the magnitude of the two vectors and the cosine of the angle between them:

$$C = \mathbf{A} \cdot \mathbf{B} = \mathbf{B} \cdot \mathbf{A} = AB\cos\theta$$

- You have been using this in the calculation of mechanical work
- Think of C as the *projection* of the vector $\mathbf{A}$ onto the direction of $\mathbf{B}$, or the component of $\mathbf{A}$ along $\mathbf{B}$
- $\hat{\imath} \cdot \hat{\imath} = 1$, $\hat{\jmath} \cdot \hat{\jmath} = 1$, and $\hat{k} \cdot \hat{k} = 1$
- If the vectors are written in IJK notation, and you don't immediately know the magnitude or the angle between them, then:

$$\mathbf{A} \cdot \mathbf{B} = A_x B_x + A_y B_y + A_z B_z$$

Cross Products

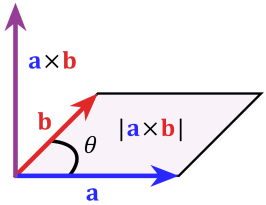
The **cross product** is the vector multiplication of two vectors:

$$\mathbf{C} = \mathbf{A} \times \mathbf{B}$$

- The magnitude of the cross product is determined by the magnitude of $\mathbf{A}$ and $\mathbf{B}$ and the angle between them:

$$C = AB \sin\theta$$

- $\mathbf{C}$ is perpendicular to both $\mathbf{A}$ and $\mathbf{B}$; its direction given by the right hand rule
- In AP Physics, cross products are used in extensively in rotational motion and in electromagnetism



Cross Products

The general notation for the cross product in 3D space is computed by doing the determinant of this 3×3 matrix:

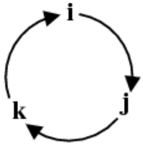
$$\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

and the order of the cross product is important. (This is why you have to get the right hand rule correctly.)

$$\mathbf{A} \times \mathbf{B} = -\mathbf{B} \times \mathbf{A}$$

Cross Product

Luckily, most cross products applications in AP Physics are simpler, so we only have to remember this circle:



- The direction of the arrow gives the index of the cross product (e.g. $\hat{\imath} \times \hat{\jmath} = \hat{k}$)
- Going against the direction of the arrow gives the negative of the next index (e.g. $\hat{k} \times \hat{\jmath} = \hat{\imath}$)

Calculus is Invented for Physics

Thanks, Issac Newton

- We cannot learn physics properly without calculus (you got away with it long enough in Grade 11 and 12 Physics classes. . .)
- Calculus was "invented" so that we can understand motion, especially non-constant velocities and accelerations
- You may have already noticed that a lot of the word problems in calculus are really physics problems

Differentiation and Integration

- **Differential Calculus**
  - Finding how quickly something is changing (“rate of change” of a quantity)
  - Math: slopes of functions
  - Physics: how quickly a physical quantity is changing in time and/or space
  - Examples: velocity (how quickly position changes with time), acceleration (how quickly velocity changes with time), power (how quickly work is done), electric fields (how electric potential changes in space)
- **Integral Calculus**
  - The opposite of differentiation
  - We use it to compute the area under a curve, or
  - Summation of many small terms
  - Examples: area under the $\mathbf{v}$-t graph (displacement), area under the $\mathbf{F}$-t graph (impulse), area under the F-d graph (work)

Derivative

For any arbitrary function $f(x)$, the derivative with respect to ("w.r.t.") x is:

$$\boxed{f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}}$$

The "limit as h approaches 0" is the mathematical way of making h a very small number

Know the Tricks for Differentiation

The derivative of a constant ("C") with respect to any variable is zero. (Obviously, the slope of any function $f(x) = C$ is zero.)

$$\frac{dC}{dx} = 0$$

A constant multiple of any function f can be factored outside the derivative:

$$\frac{d}{dx}(af) = a\frac{df}{dx}$$

The derivative of a sum is the sum of a derivative:

$$\frac{d}{dt}(f(t) + g(t)) = \frac{df}{dt} + \frac{dg}{dt}$$

Time-Saving Rules for Differentiation

Power Rule:

$$\frac{d}{dt}\left(t^n\right) = nt^{n-1}$$

Product Rule:

$$\frac{d}{dx}\left(f(x)g(x)\right) = f'(x)g(x) + f(x)g'(x)$$

Chain Rule:

$$\frac{d}{dx}f\left(g(x)\right) = f'(g(x))g'(x)$$

Time-Saving Rules for Differentiation

Quotient Rule:

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)g(x) - g'(x)f(x)}{(g(x))^2}$$

Elementary Derivatives

Note how sines and cosines are related:

$$\frac{d}{dt}\sin t = \cos t \qquad \frac{d}{dt}\cos t = -\sin t$$

And exponential:

$$\frac{d}{dt}e^t = e^t$$

Partial Derivatives

Equations in physics are often functions with many variables. For example, gravitational potential energy U_g has three variables: the masses m_1 and m_2 and their distance r:

$$U_g(m_1, m_2, r) = -\frac{Gm_1m_2}{r}$$

Differentiating with respect to one variable while holding other variables constant gives its **partial derivative**. We use the ∂ symbol. In this example, the partial derivative of U_g w.r.t. r is

$$\frac{\partial U_g}{\partial r} = \frac{Gm_1m_2}{r^2}$$

(By the way, this is how we relate U_g to F_g.)

Integration

If $F(x)$ is the anti-derivative of $f(x)$, they are related this way:

$$\frac{d}{dx}F(x) = f(x) \quad \longrightarrow \quad F(x) = \int f(x)dx$$

The mathematical proof is actually the **fundamental theorem of calculus.**

Common Integrals in Physics

Integration can be a very daunting task (even though it's often necessary), but the integrals you'll encounter in AP Physics are relatively straightforward. These examples should help in most cases:

- Power Rule in reverse:

$$\int x^n dx = \frac{1}{n+1} x^{n+1} + C$$

- Natural logarithm:

$$\int \frac{1}{x} dx = \ln|x| + C$$

Common Integrals in Physics

- Sines and cosines:

$$\int \cos x dx = \sin x + C$$
$$\int \sin x dx = -\cos x + C$$

We can "ignore" (i.e. cancel) the constant of integration C for definite integrals.

Definite vs. Indefinite Integral

This Should Be a Review

- Integral can be either **indefinite** or **definite**
- An "indefinite" integral is another function, e.g. position $\mathbf{x}(t)$ as a function of time is found by integrating velocity $\mathbf{v}(t)$:

$$\mathbf{x}(t) = \int \mathbf{v}(t)dt = \cdots + \mathbf{C}$$

- A *constant of integration* $\mathbf{C}$ is added to the integral $\mathbf{x}(t)$. It is obtained through applying "initial condition" to the problem.

Definite Integrals

A **definite** integral has lower and upper bounds. e.g., given $\mathbf{v}(t)$, the displacement between t_1 and t_2 can be found:

$$\Delta \mathbf{x} = \int_{t_1}^{t_2} \mathbf{v}(t) dt$$

Once we have computed the integral, we evaluate the limits:

$$\Delta \mathbf{x} = \mathbf{x}(t) \Big|_{t_1}^{t_2} = \mathbf{x}(t_2) - \mathbf{x}(t_1)$$

The constant of integration $\mathbf{C}$ cancels when we evaluate the upper and lower bounds.

Kinematics

- Describing the motion of points, bodies (objects), and systems of bodies (groups of objects)
- Relationship between
  - Position & displacement
  - Velocity
  - Acceleration
- Kinematics does not deal with what causes the motion

Position

Position is a vector describing the location of an object in a coordinate system (usually *Cartesian*; can also be *cylindrical* or *polar*). The origin of the coordinate system is the “reference point”.

$$\mathbf{x} = x\hat{\boldsymbol{\imath}} + y\hat{\boldsymbol{\jmath}} + z\hat{\boldsymbol{k}}$$

- The SI unit for position is a **meter**, m
- The components x, y and z are the coordinates along those axes
- The vector is a function of time t

Displacement

Displacement is the change in position from 1 to 2 (make sure you use the same coordinate system!):

$$\Delta \mathbf{x} = \mathbf{x}_2 - \mathbf{x}_1 = (x_2 - x_1)\hat{\boldsymbol{\imath}} + (y_2 - y_1)\hat{\boldsymbol{\jmath}} + (z_2 - z_1)\hat{\boldsymbol{k}}$$

- IJK notation makes vector addition and subtraction less prone to errors
- Since reference point $\mathbf{x}_{\mathrm{ref}} = \mathbf{0}$, the position $\mathbf{x}$ is also its displacement from the reference point

Instantaneous Velocity

Time Derivative of Position

If the position of an object $\mathbf{x}$ can be described for any time t, then velocity $\mathbf{v}$ can be found at any time t. The **instantaneous velocity** of an object is the rate of change of its position:

$$\boxed{\mathbf{v}(t) = \frac{d\mathbf{x}(t)}{dt}}$$

Since $\mathbf{x}$ has x, y and z components in the $\hat{\imath}$, $\hat{\jmath}$ and $\hat{k}$ directions, we can take the derivative with respect to time in every component:

$$\mathbf{v}(t) = \frac{d\mathbf{x}}{dt} = \frac{dx}{dt}\hat{\boldsymbol{\imath}} + \frac{dy}{dt}\hat{\boldsymbol{\jmath}} + \frac{dz}{dt}\hat{\mathbf{k}} = v_x\hat{\boldsymbol{\imath}} + v_y\hat{\boldsymbol{\jmath}} + v_z\hat{\mathbf{k}}$$

Integrating Velocity to Get Position/Displacement

If instantaneous velocity $\mathbf{v}$ is the rate of change of position $\mathbf{x}$ with respect to time t, then $\mathbf{x}$ is the time integral of $\mathbf{v}$ (by the fundamental theorem of calculus):

$$\mathbf{x}(t) = \int \mathbf{v}(t)dt + \mathbf{x}_0$$

where $\mathbf{x}_0 = \mathbf{x}(0)$ is the *initial position* at $t = 0$. As both $\mathbf{x}$ and $\mathbf{v}$ are vectors, we integrate each component to get $\mathbf{x}$:

$$\mathbf{x}(t) = \left(\int v_x \hat{\boldsymbol{\imath}} + \int v_y \hat{\boldsymbol{\jmath}} + \int v_z \hat{\boldsymbol{k}} \right) dt + \mathbf{x}_0$$

Average Velocity

The **average velocity** of an object is the the change in position $\Delta\mathbf{v}$ over a finite time interval:

$$\boxed{\mathbf{v}_{\text{ave}} = \frac{\Delta\mathbf{x}}{\Delta t}}$$

Just like for instantaneous velocity, we can find the x, y and z components of average velocity by separating components in each direction:

$$\mathbf{v}_{\text{ave}} = \frac{\Delta x}{\Delta t}\hat{\boldsymbol{\imath}} + \frac{\Delta y}{\Delta t}\hat{\boldsymbol{\jmath}} + \frac{\Delta z}{\Delta t}\hat{\boldsymbol{k}}$$

Path

Sometimes instead of explicitly describing the position $x = x(t)$ and $y = y(t)$, the path of an object can be given in terms of x coordinate $y = y(x)$, while giving the x (or y) coordinate as a function of time.

- In this case, substitute the expression for $x(t)$ into $y = y(x)$ to get an expression of $y = y(t)$
- Take derivative using chain rule to get $v_y = v_y(t)$

Instantaneous Acceleration

In the same way that velocity is the time derivative of position, **acceleration is the rate of change in velocity**, i.e.:

$$\boxed{\mathbf{a}(t) = \frac{d\mathbf{v}(t)}{dt} = \frac{d^2\mathbf{x}(t)}{dt^2}}$$

Acceleration is the second derivative of position, i.e.

1. Take derivative of $\mathbf{x}(t)$ to get $\mathbf{v}(t) = \mathbf{x}'(t)$
2. Take derivative again of $\mathbf{v}(t)$ to get $\mathbf{a}(t) = \mathbf{v}'(t)$

Integrating Acceleration to Get Velocity

Similar to the relationship between velocity and position, we also know that velocity $\mathbf{v}(t)$ is the time integral of acceleration $\mathbf{a}(t)$:

$$\mathbf{v}(t) = \int \mathbf{a}(t)dt + \mathbf{v}_0$$

Again, since both $\mathbf{v}$ and $\mathbf{a}$ are vectors, we need to integrate in each direction:

$$\mathbf{v}(t) = \left(\int a_x \hat{\mathbf{\imath}} + \int a_y \hat{\mathbf{\jmath}} + \int a_z \hat{\mathbf{k}} \right) dt + \mathbf{v}_0$$

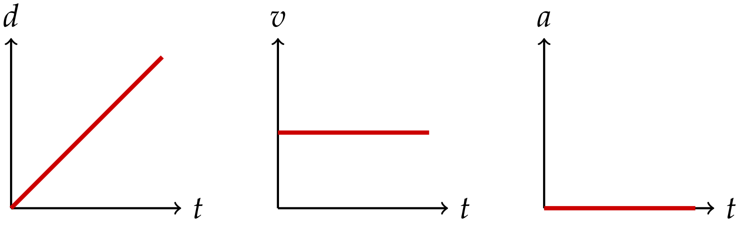
Motion Graphs

For one-dimensional motion, we can describe motion graphically using motion graphs, by plotting

- Position vs. time ($x - t$) graph
- Velocity vs. time ($v - t$) graph
- Acceleration vs. time ($a - t$) graph

Converting Motion Graphs

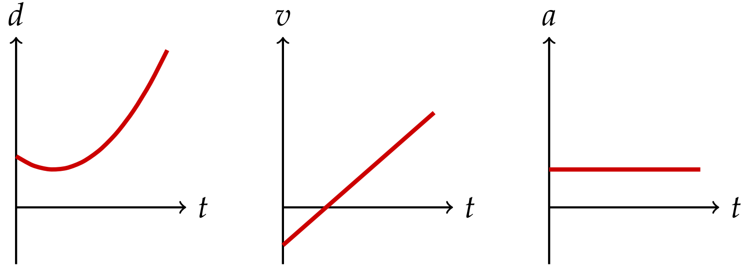
For constant velocity (uniform motion), the motion graphs are like this:



- Constant velocity has a straight line in the $d - t$ graph
- The slope of the $d - t$ graph is the velocity v
- There is no acceleration so $a = 0$ for all t

Converting Motion Graphs

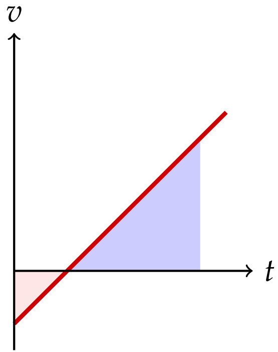
For constant acceleration (uniform acceleration), the motion graphs are like this:



- The $d-t$ graph for motion with constant acceleration is part of a *parabola*
  - If the parabola is *convex*, then acceleration is positive
  - If the parabola is *concave*, then acceleration is negative
- The $v-t$ graph is a straight line; the slope of this graph (a constant) is the acceleration

Area Under $v - t$ Graph

Let's not forget the area under the $v - t$ graph is the displacement. (This should be obvious, since x is the time integral of v.)



- If the area is *below* the x (time) axis, then the displacement is negative;
- If the area is *above* the time axis, then displacement is positive

Kinematic Equations For Constant Acceleration

$$\Delta \mathbf{x} = \mathbf{v}_1 \Delta t + \frac{1}{2}\mathbf{a}\Delta t^2$$

$$\Delta \mathbf{x} = \mathbf{v}_2 \Delta t - \frac{1}{2}\mathbf{a}\Delta t^2$$

$$\Delta \mathbf{x} = \frac{\mathbf{v}_1 + \mathbf{v}_2}{2}\Delta t$$

$$\mathbf{v}_2 = \mathbf{v}_1 + \mathbf{a}\Delta t$$

$$v_2^2 = v_1^2 + 2a\Delta x$$

Even though *some* of the kinematic problems that you encounter in AP Physics require calculus, these basic kinematic equations are still a very powerful tool.

- The variables of interests are:

$$\Delta \mathbf{x} \quad \mathbf{v}_1 \quad \mathbf{v}_2 \quad \Delta t \quad \mathbf{a}$$

- Only applicable for constant acceleration

Solving Typical Kinematics Problems

One object: the problem provides 3 of the 5 variables, and you are asked to find a 4th one.

- Define the positive direction (usually very obvious)
- Apply the correct kinematic equation and solve the problem!

Two objects: two objects are in motion. Usually one of them is moving at constant velocity while the other is accelerating.

- Time interval Δt and displacement $\Delta \mathbf{x}$ of the two objects are related
- Examples:
  - Police car chasing a speeder
  - Two football players running towards each other
  - A person trying to catch the bus

Projectile Motion

- For 2D problems, resolve the problem into its horizontal (x) and vertical (y) directions, and apply kinematic equations independently
- For projectile motion, there is no acceleration in the x direction, i.e. $a_x = 0$, therefore the kinematic equations reduce to just

$$\Delta x = v_x \Delta t$$

- The only acceleration is in the y direction:

$$a_y = 9.81\,\mathrm{m/s^2}\ [\mathrm{down}]$$

We *usually* define the (+) direction to be [up], so $g = -9.81\,\mathrm{m/s^2}$, but it can change depending on the problem

- The variable that connects the two directions is the time interval Δt

Symmetric Trajectory

Trajectory is symmetric if the object lands at the same height as when it started.

- Time of flight

$$t_{\max} = \frac{2v_i \sin\theta}{g}$$

- Range

$$R = \frac{v_i^2 \sin(2\theta)}{g}$$

- Maximum height

$$h_{\max} = \frac{v_i^2 \sin^2\theta}{2g}$$

The angle θ is measured **above the the horizontal**

Maximum Range

$$R = \frac{v_i^2 \sin(2\theta)}{g}$$

- For a given initial speed v_i, maximum range occurs at $\theta = 45^\circ$
- For a given initial speed v_i and range R, I can find a launch angle θ that gives the required range:

$$\theta_1 = \frac{1}{2}\sin^{-1}\left(\frac{Rg}{v_i^2}\right)$$

- But there is another angle that *gives the same range*!

$$\theta_2 = 90^\circ - \theta_1$$

Typical Problems

For both AP Physics 1 and AP Physics C exams, questions involving kinematics usually appear in the multiple-choice section. The problems themselves are not very different compared to the Grade 12 Physics problems, but:

- You have to solve problems faster because of time constraint
- You can use $g = 10\,\mathrm{m/s^2}$ to make your lives simpler
- A lot of problems are *symbolic*, which means that they deal with the equations, not actual numbers
- Will be coupled with other types (e.g. dynamics and rotational) in the free-response section
- You *will* be given an equation sheet