Classes 19: Fluid Mechanics

AP Physics

Dr. Timothy Leung

Olympiads School

March 2018

Files for You to Download

Download from the school website:

1. `19-fluidMechanics.pdf`—This presentation. If you want to print the slides on paper, I recommend printing 4 slides per page.
2. `20-Homework.pdf`—Homework assignment for Classes 19 and 20, which cover Fluid Mechanics and Thermodynamics

Please download/print the PDF file before each class. When you are taking notes, pay particular attention to things I say that aren't necessarily on the slides.

Disclaimer

Use of Calculus

Fluid mechanics is part of the AP Physics 2 Exam, which does not require calculus. However, in the interest in completeness, *some* calculus will still be used when deriving equations.

What is a Fluid

- **The simple explanation:** anything that flows, which covers most *gases* and *liquids* and *plasmas*
- **The scientific explanation:** Any substancs that deform *continuously* under oblique stress

In fluid mechanics, we assume that a fluid is continuous: it will fill all available space without gaps

Some Properties of Fluids

Density ρ of a fluid is defined as the mass m per unit volume V:

$$\rho = \frac{m_{\text{fluid}}}{V_{\text{fluid}}}$$

Viscosity μ measures how "thick" a fluid is; e.g. honey is more viscous than water. It relates the rate of deformation $(\partial u / \partial y)$ of the fluid to the shear stress τ that it experiences:

$$\tau = \mu \frac{\partial u}{\partial y}$$

Shear stress defined as $\tau = F/A$ which has the same unit (Pa) as pressure. In AP Physics, we will, for now, ignore viscous effects, as important as they are

Hydrostatics

- The pressure of fluid on an object depends on the density and depth of the object:

$$\boxed{p = p_0 + \rho_{\text{fluid}} g z}$$

  where g is the acceleration due to gravity, z is the depth below the surface, and $p_0 = 1.01 \times 10^5$ Pa is the atmospheric pressure at the surface.
- Pressure is the same in all directions
- Pressure is defined as force per unit area, and the unit is pascal:

$$\boxed{p = \frac{F}{A}} \qquad 1\,\text{Pa} = 1\,\text{N}/\text{m}^2$$

Pascal's Principle

If force is applied somewhere on a container holding fluid, the pressure increases *everywhere* in the fluid, not just where the force is applied.

i.e. the pressure of the force will be transmitted into the fluid.

A Simple Example

Example 1: An aquarium is filled with water. The lateral wall of the aquarium is 40 cm long and 30 cm high. Using 10 m/s^2 for the acceleration due to gravity, and 1 g/cm^3 for density of water, the force on the lateral wall of the aquarium is:

(a) 36 N

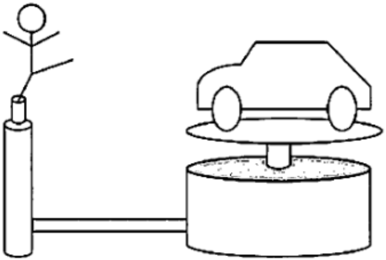
(b) 90 N

(c) 180 N

(d) 1500 N

Example

Example 2: Consider the hydraulic jack in the diagram. A person stands on a piston that pushes down on a thin cylinder full of water. The cylinder is connected via pipes to a wide platform on top of which rests a 1-ton (1000 kg) car. The area of the platform under the car is $25\,\mathrm{m}^2$; the person stands on a $0.3\,\mathrm{m}^2$ piston. What is the lightest weight of a person who could successfully lift the car?

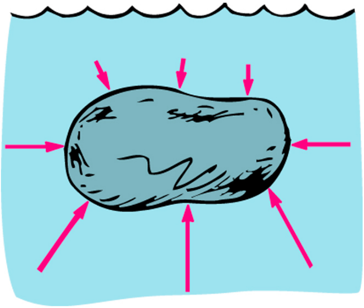


Believe it or not, there *is* someone who draws worse diagrams than Tim!

Buoyancy

Everything Floats a Little

When an object is submerged inside a fluid (e.g. water, air, etc), the fluid exerts a pressure at the surface of the object. We can integrate the pressure over the entire surface area S to find the total force $\mathbf{B}$ the fluid exerts on the object.



Derivation of Buoyance Force

Integrate the pressure p over the entire surface S to find the total force $\mathbf{B}$, or take some knowledge of vector calculus (divergence theorem) so that you don't have to:

$$\mathbf{B} = -\oint_S p\hat{\mathbf{n}}dS = -\iiint \nabla p dV$$

Since pressure $p = \rho g z$ is a function in z only, the gradient easy to compute: $\nabla p = dp/dz = \rho g \hat{\boldsymbol{k}}$, giving us

$$\mathbf{B} = \rho_{\text{fluid}} g \hat{\boldsymbol{k}} \iiint dV = \rho_{\text{fluid}} g V \hat{\boldsymbol{k}}$$

Derivation of Buoyance Force

Although the derivation required a lot of calculus, the expression of buoyance force is straightforward (and *this* is what you need to remember):

$$\boxed{\mathbf{B} = \rho_{\text{fluid}} g V \hat{\boldsymbol{k}} = m_{\text{fluid}} g \hat{\boldsymbol{k}}}$$

where ρ_{fluid} is the density of the displaced fluid, and V is the volume displaced. This equation is known as **Archimedes' principle.**

Buoyance force has a magnitude that equals to the weight of the fluid displaced by the submerged object, pointing upward.

An Easier Explanation of Buoyancy

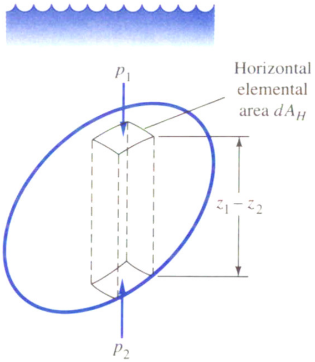
Not Much Calculus

There is a simpler way to find the buoyance force, by taking an infinitesimal "tube" of the object, and finding the pressure difference between the top and bottom of the tube:

$$\mathbf{B} = \int (p_2 - p_1) dA$$

$$= \rho_{\text{fluid}} g \int (z_2 - z_1) dA$$

$$= \rho_{\text{fluid}} g V$$



which is the same expression that we got with calculus.

Buoyancy

Note that buoyancy does not depend on:

- the mass of the immersed object, or
- the density of the immersed object

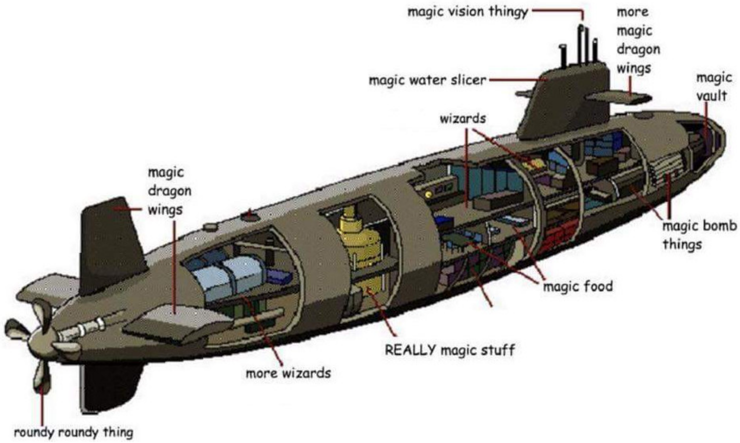
Objects immersed in a fluid have an “apparent weight” $\mathbf{W}'$ that is reduced by the buoyance force:

$$\mathbf{W}' = \mathbf{W} - \mathbf{B} = \rho' \mathbf{g} V$$

where $\rho' = \rho_{\text{obj}} - \rho_{\text{fluid}}$ is the relative density

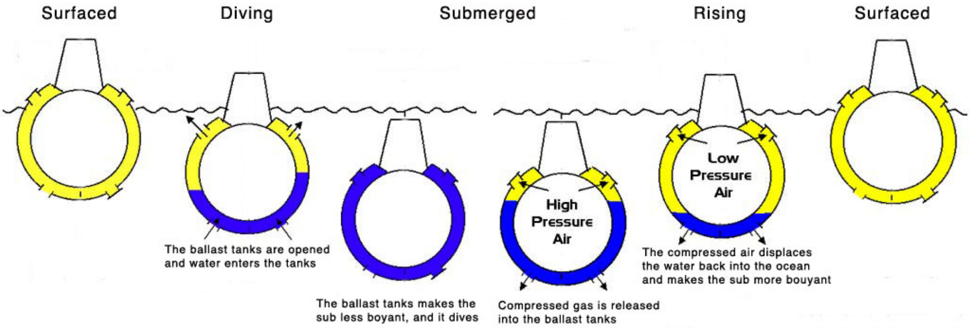
How Submarines Work

Like this?



How Submarines Work

Like all ships, a submarine does not naturally sink due to buoyancy. When a submarine submerges, water needed to be pumped into the “ballast tanks” in the hull to make the ship heavier.

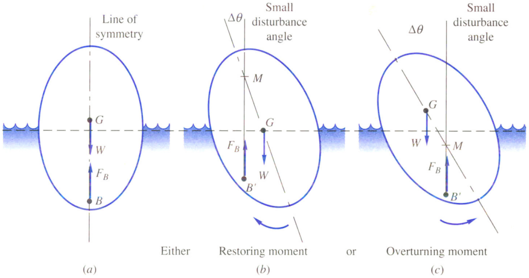


Stable? Or unstable?

Buoyance force $\mathbf{B}$ acts at the *center of buoyancy* (CB) of a submerged object

- The CB is the CG *if the object has constant density* and is fully submerged
- The actual CG of the object may be at a different position
- Sometimes the object is not fully submerged

$\mathbf{F}_g$ and $\mathbf{B}$ may act at different points, creating a torque/moment on the object



Example

Example 3: An apple is held completely submerged just below the surface of a container of water. The apple is then moved to a deeper point in the water. Compared with the force needed to hold the water just below the surface, what is the force needed to hold it at a deeper point?

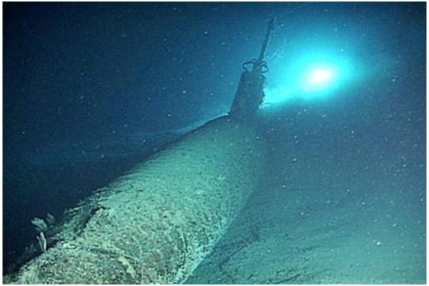
(a) Larger

(b) The same

(c) Smaller

(d) Impossible to determine

Example



Example 4: A salvage ship tries to raise a sunken miniature submarine from the bottom of Lake Superior. The submarine and its contents have a mass of 72 000 kg and a volume of 18.9 m^3. What upward force must be applied to raise the submarine? The density of water is 1000 kg/m^3.

(a) 1.8×10^5 N

(b) 2.0×10^5 N

(c) 4.8×10^5 N

(d) 5.2×10^5 N

Fluid Flow

As important as it is to understand hydrostatics,
it's way more interesting when the fluid is moving!

Fluid Flow: Continuity

In a fixed volume (known as a "control volume", or CV) we can quantify how fluid mass changes in the CV:

rate of decrease in mass in the CV = mass flux out of the CV

The fluid mass in the CV is the integral of density over the volume:

$$\int_{CV} \rho dV$$

The rate of decrease is therefore the negative of the time derivative:

$$-\frac{\partial}{\partial t}\int_{CV} \rho dV$$

Fluid Flow: Continuity

The mass flux out of the surfaces of the control volume the volume flux multiplied by the fluid density at the surface:

$$\int_{CS} \rho \mathbf{v} \cdot d\mathbf{A}$$

Combining the LHS and RHS terms, we have the *integral* form of the continuity equation:

$$\boxed{\int_{CV} \frac{\partial \rho}{\partial t} dV + \int_{CS} \rho \mathbf{v} \cdot d\mathbf{A} = 0}$$

Fluid Flow: Continuity

With some clever use of vector calculus, we get the *differential form* of the continuity equation:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0$$

. . . which is still too difficult. So in AP Physics we usually only look at simple cases where

- Steady flow (time independent)
- Constant density
- Flow perpendicular to control surfaces

Inlet Outlet Flow

Diagram

In this example, the mass flowing at the inlet is the same as the flow out of it:

$$\boxed{\rho_1 v_1 A_1 = \rho_2 v_2 A_2}$$

And if fluid density is constant (incompressible flow), the ρ terms on both sides of the equation will cancel:

$$\boxed{v_1 A_1 = v_2 A_2}$$

Example: Multiple Inlet & Outlets

Example 5:

Governing Equations for Fluid Dynamics

To properly describe fluid flows, there are three conservation equations:

- continuity
- momentum, and
- energy

Fluid Flow: Momentum & Energy Equations

In the momentum equation, the rate of decrease of total fluid momentum inside the control volume CV is the net momentum flux of the fluid out of the CV all the forces (pressure, body, shear) acting on the fluid:

$$\frac{\partial(\rho\mathbf{v})}{\partial t} + \nabla(\rho\mathbf{v} \otimes \mathbf{v}) = -\nabla p + \mathbf{f} + \mu\nabla^2\mathbf{v}$$

(This is even more complicated than the continuity equation, so thankfully you won't need this equation for AP Physics!)

The energy equation follows a similar thought process as the previous two equations, but the terms are even more complicated.

Navier-Stokes Equations

Together, the three conservation equations are called the **Navier-Stokes equations.** In differential form, they are usually written as:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0$$

$$\frac{\partial (\rho \mathbf{v})}{\partial t} + \nabla (\rho \mathbf{v} \otimes \mathbf{v}) = -\nabla p + \mathbf{f} + \mu \nabla^2 \mathbf{v}$$

$$\frac{\partial e}{\partial t} + \nabla \cdot (e \mathbf{v}) = -\nabla \cdot p + \frac{1}{Re\ Pr} \nabla q + \frac{1}{Re} \nabla \cdot (\tau \cdot \mathbf{v})$$

Even for a 2nd-year engineering student experienced with calculus, solving the N-S equations is still a daunting task, so let's make some assumptions!

Let's Make Some Assumptions

For an "ideal fluid flow"

The flow is **steady**

- Flow is "time independent", i.e. does not change with time
- All derivatives w.r.t. time are zero

The flow is **inviscid**

- The fluid has no viscosity
- No friction between the fluid and the surrounding, and therefore
- No shear stresses on the fluid
- Only forces are pressure at the surface, and body forces from gravity

The flow is **incompressible**

- Density is constant throughout
- The compressibility of fluid usually depends on its flow velocity, at Mach number of $M \approx 0.3$, fluid becomes incompressible

Let's Make Some Assumptions

For an "ideal fluid flow"

We will also assume that

- there is **no shaft work** done along the streamline
- there is **no heat transfer** along the streamline

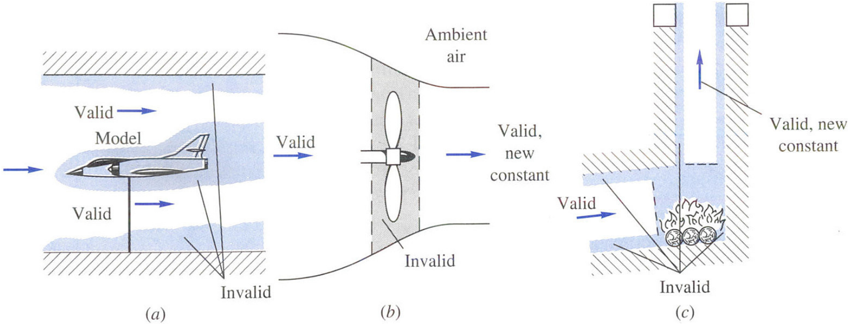
Then the N-S equations reduces to the **Bernoulli equation**

$$\boxed{p_1 + \frac{1}{2}\rho v_1^2 + \rho g z_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g z_2}$$

The term $\frac{1}{2}\rho v^2$ is called "dynamic pressure", and $\rho g z$ is the "hydrostatic pressure"

Bernoulli Equation

Regions where Bernoulli equation is valid:



Example

Example 6:

How Does A Wing Work?

When air flows past a wing, a force is generated