Sound Waves and Music

Advanced Placement Physics

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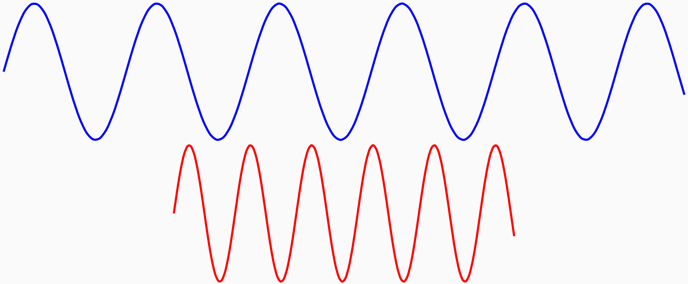
Olympiads School
Toronto, Ontario, Canada

Properties

Amplitude: Loudness



General waves	Sound Waves
Amplitude	Loudness
	Perceived **intensity**
	“volume”

- Low amplitude: soft/quiet
- High amplitude: loud

Intensity of Sound

The energy of a sound wave is proportional to the square of the amplitude (pressure difference):

$$P \propto A^2$$

The intensity is the power (P) divided by the area (A) that the sound wave passes through:

$$\boxed{I = \frac{P}{A} \propto \frac{1}{r^2}}$$

When a sound is emitted from a stationary source, the area that the wavefront passes through is $A = 4\pi r^2$, where r is the distance from the source.

Threshold of Hearing

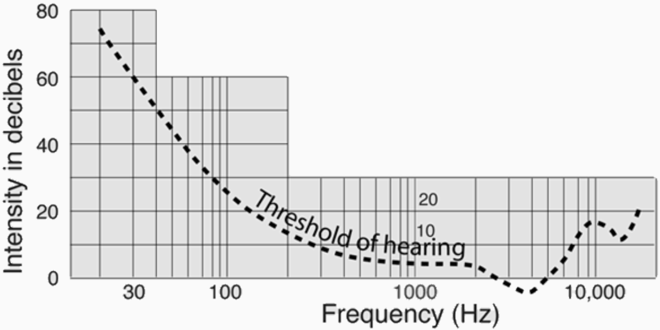
The smallest detectable sound intensity is called the **threshold of hearing** I_0, defined as:

$$I_0 = 1 \times 10^{-12}\,\mathrm{W/m^2}$$

While the sound intensity at the **threshold of pain** can damage a human ear:

$$I_p = 1\,\mathrm{W/m^2}$$

Threshold of Hearing



- The threshold of hearing is actually experimentally determined to be about $4 \times 10^{-12}\,\mathrm{W/m^2}$ at $1000\,\mathrm{Hz}$.
- Maximum sensitivity to sound is 3500 to $4000\,\mathrm{Hz}$, corresponding to the resonance of the auditory canal.

The Decibel

The **decibel** is defined as by the intensity of sound I compared to the pre-defined threshold of hearing:

$$\beta = 10 \log_{10} \left[\frac{I}{I_0} \right]$$

- By definition, the threshold of hearing is $0\,\mathrm{dB}$ while the threshold of pain is $120\,\mathrm{dB}$.
- Humans perceive a doubling of loudness when intensity is increases by a factor of 10 (an increase of $10\,\mathrm{dB}$)

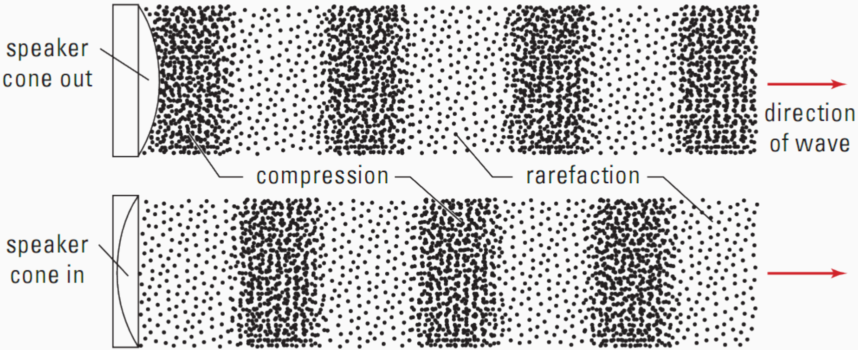
Frequency and Pitch

- Unit for frequency is **hertz**: Hz
  - Sound frequencies are usually referred to as its **pitch**
- Audible range for human ears is approximately 20 Hz to 20,000 Hz
- **Infrasound**: below the audible range
- **Ultrasound**: above the audible range, e.g.
  - Dog whistles
  - Medical ultrasound devices

Medical ultrasound devices usually use frequencies between 1 MHz to 20 MHz.

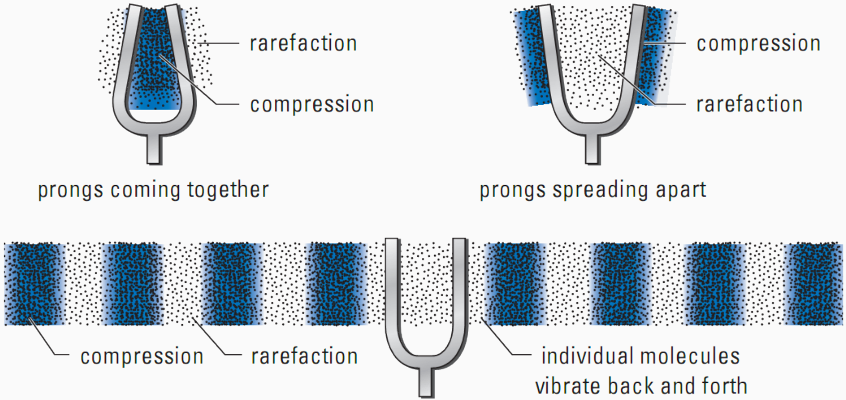
Transfer of Sound Wave in Air

- Sound wave is a **longitudinal wave**
- Compression and rarefaction (expansion) of the air molecules
- Example below: speaker of a stereo system



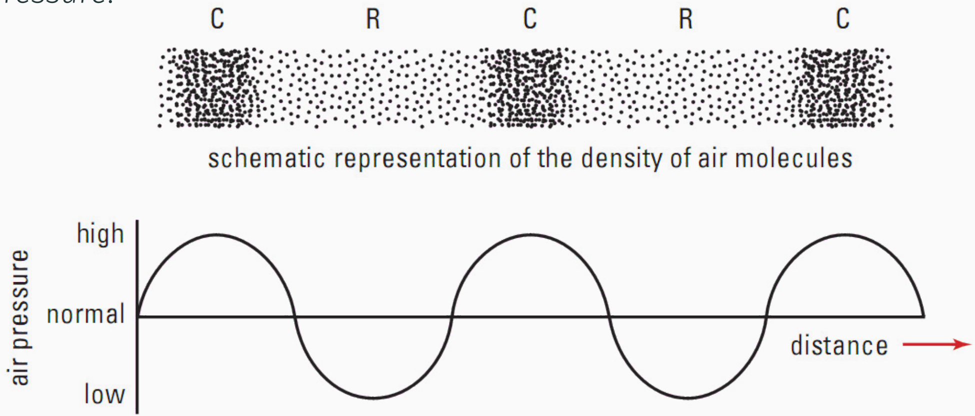
Transfer of Sound Wave

Example: tuning fork



Transfer of Sound Wave

We can also express the amplitude of the sound wave by plotting the change in *air pressure*:



Transmission

Speed of Sound in a Gas

The speed of sound in a gas (e.g. air) depends on temperature and its composition:

$$\boxed{v_s = \sqrt{\frac{\gamma RT}{M}}}$$

Quantity	Symbol	SI Unit
Speed of sound	v_s	m/s
Termodynamic temperature	T	K
Universal gas constant	R	J/mol · K
Molar mass	M	kg/mol
Adiabatic constant (C_v/C_p)	γ	(no units)

For diatomic gases $\gamma = 1.4$, and $M = 29 \times 10^{-3}$ kg/mol

Speed of Sound in Air

The speed of sound in air near room temperature ($\approx 300\,\mathrm{K}$) can be approximated as function that varies linearly with temperature in celsius:

$$\boxed{v_s = 331 + 0.59 T_C}$$

Quantity	Symbol	SI Unit
Speed of sound	v_s	m/s
Temperature of air in celsius	T_C	N/A

Speed of Sound in Liquids and Solids

Speed of sound in a liquid depends on the “bulk modulus” K of the liquid, and density ρ:

$$v = \sqrt{\frac{K}{\rho}}$$

Speed of sound in a solid depends on the “Young’s modulus” E of the solid and density ρ:

$$v = \sqrt{\frac{E}{\rho}}$$

Speed of Sound in Different Media

Material	Speed (m/s)
Gases (0 °C, 101 kPa)	
Carbon dioxide	259
Oxygen	316
Air	331
Helium	965
Liquids (20 °C)	
Ethanol	1162
Fresh water	1482
Seawater (depends on depth and salinity)	1440 – 1500
Solids	
Copper	5010
Glass (heat-resistant)	5640
Steel	5960

Mach Number

Speeds close to the speed of sound is often expressed in terms of its ratio to the speed of sound. This is called the **Mach number** (M):

$$\boxed{M = \frac{v}{v_s}}$$

Quantity	Symbol	SI Unit
Mach number	M	no units
Speed of the object	v	m/s
Local speed of sound	v_s	m/s

- When an object is travelling at $M < 1$, it is travelling at a *subsonic* speed
- When an object is travelling at $M > 1$, it is travelling at a *supersonic* speed

Sound from a Stationary Source

When a sound is emitted from a stationary point source, the sound wave moves radially outward from the origin:

• source

- Sound intensity (amplitude) drops farther away from the source
- All points hear the same wavelength (and frequency) of sound

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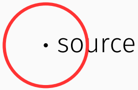

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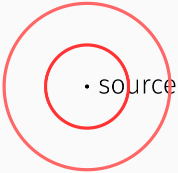

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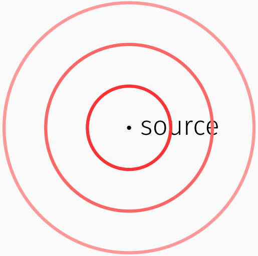

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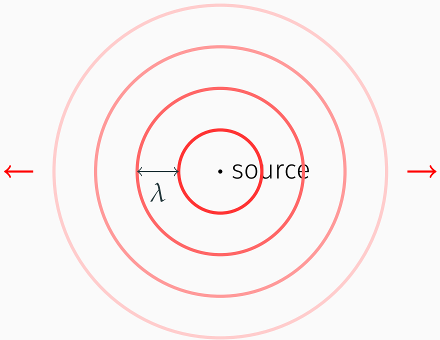

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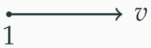
Sound from a Moving Source

When sound is emitted from a *moving* source, the diagram looks different. In this case, the sound source is moving to the right, from 1 to 4:



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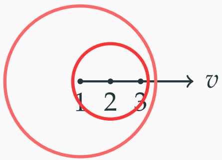

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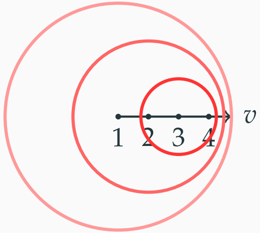

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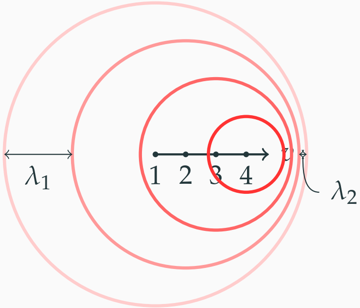


- When the source is moving *towards you*, the wavelength λ_2 decreases, and the apparent frequency increases.
- When the source is moving *away from you*, the wavelength λ_1 increases, and the aparent frequency decreases.

This is called the **Doppler effect**.

Doppler Effect

We all experience Doppler effect every time an ambulance speeds by us with its sirens on.

When it is moving towards us, the pitch of the siren is high, but the moment it passes us, the pitch decreases.

Doppler Effect

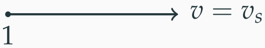
When a wave source is moving at a speed v_{src} and an observing is moving at observer v_{ob}, the perceived frequency is shifted:

$$\boxed{f' = \frac{v_s \pm v_{\text{ob}}}{v_s \mp v_{\text{src}}} f}$$

Quantity	Symbol	SI Unit
Apparent frequency	f'	Hz
Actual frequency	f	Hz
Speed of sound	v_s	m/s
Speed of source	v_{src}	m/s
Speed of observer	v_{ob}	m/s

Sound Source at Sonic Speed

Doppler effect is even more interesting is when sound source is moving at the speed of sound ($M = 1$):



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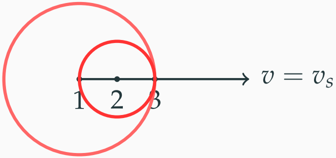

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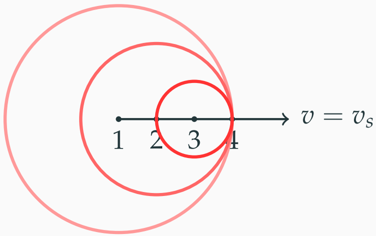

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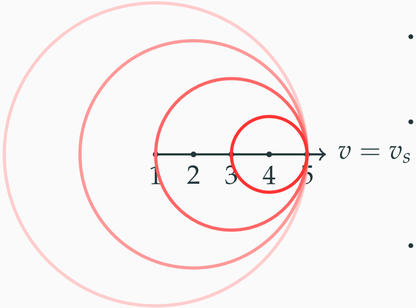

- The wavefronts (crests) from all the waves are bunched up just in front of the source
- Since sound wave is a pressure wave, right in front of the sound source, there is a large change in pressure (called a shock wave)
- When the shock passes an observer, an loud bang can be heard (aka **sonic boom**)

Sound from a Supersonic Source

When sound source is moving at $M > 1$, it out runs the sound that it makes:

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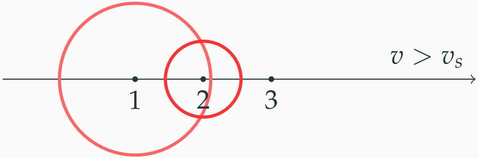

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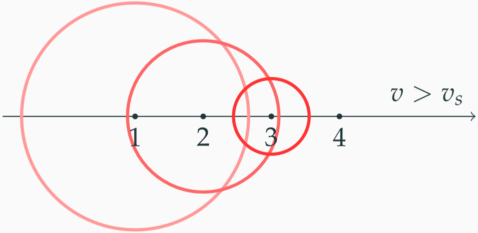

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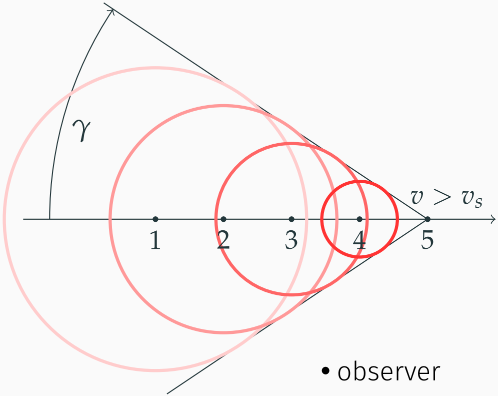


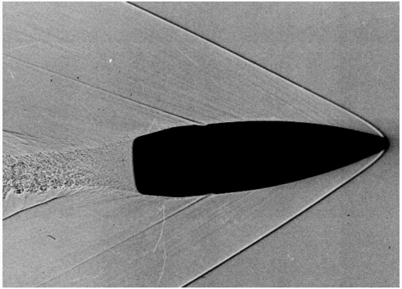
An *oblique shock* is formed at an angle (called the **Mach angle**) given by:

$$\gamma = \sin^{-1}\left(\frac{1}{M}\right)$$

An observer does not hear the sound source until it has gone past!

Bullet in Supersonic Flight

Generating a shock doesn't require an actual sound source. Any object moving through air creates a pressure disturbance. This is a $7.62\,\mathrm{mm}$ NATO bullet in supersonic flight.



The flow around this bullet is taken inside a *shock tube* that generates a short burst of supersonic flow. A high-speed camera is used to take the photo.

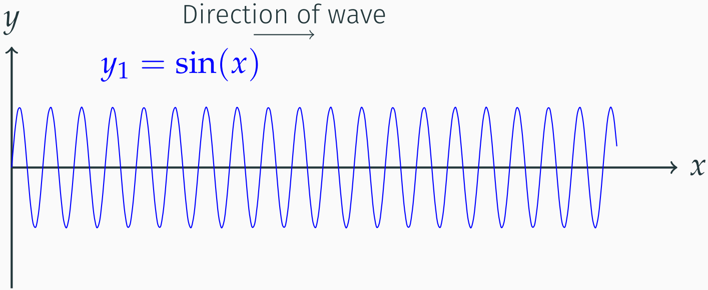
Duck in Water

A similar shock behaviour is observed when the duck swims in water, because the duck swims faster than the speed of the water wave, it also creates a cone shape.

Wave Interference

Visualizing Beat Frequency

We plot two waves moving in the same medium, but with slightly different frequencies (Think of them as the two musical instruments playing two slightly different pitch)



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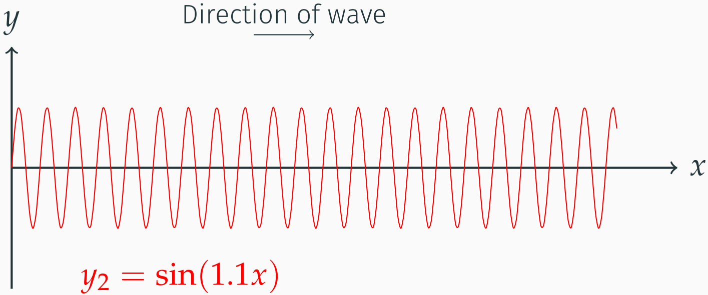

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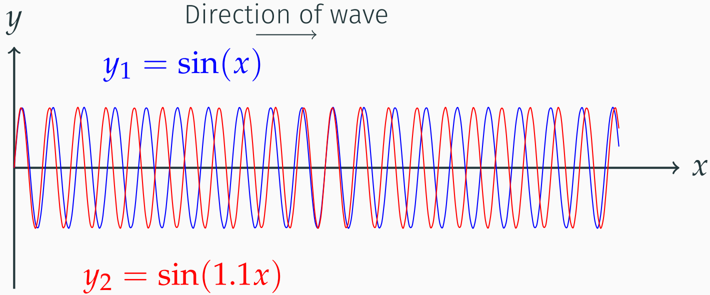

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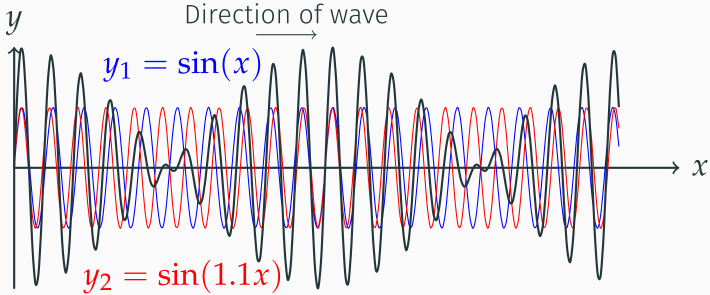


The thick black line shows the sum of the waves: $y = \sin(x) + \sin(1.1x)$

Beat Frequency

The **beat frequency** is the absolute value of the difference of the frequencies of the two component waves:

$$\boxed{f_{\text{beat}} = |f_2 - f_1|}$$

Quantity	Symbol	SI Unit
Beat frequency	f_{beat}	Hz
Frequency of 1st component wave	f_1	Hz
Frequency of 2nd component wave	f_2	Hz

Music

Music vs. Noise

Difference between *noise* and *music* is often difficult to distinguish. Generally, the concept of music is based on:

- Organized combinations of different frequencies
- Harmonics of dominated frequencies, which are
- Whole-number multiples of the lowest (fundamental) frequency

Musical Instruments

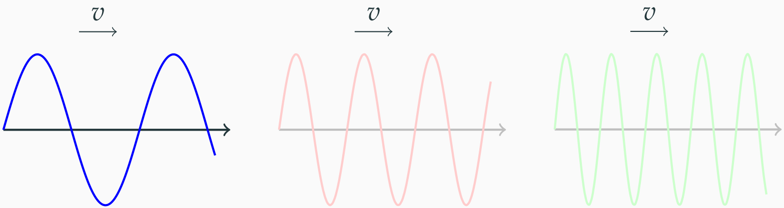
When a musical instrument produces a sound at a certain frequency, it also produces higher frequency sounds

- The higher frequency sounds are *whole-number multiples* of the lowest frequency
- e.g. a violin playing at $440\,\mathrm{Hz}$ produces sound waves at

$$440\,\mathrm{Hz}, 880\,\mathrm{Hz}, 1320\,\mathrm{Hz}, 1760\,\mathrm{Hz}, 2200\,\mathrm{Hz}, 2640\,\mathrm{Hz}\ldots$$

- Generally, the higher the frequency, the smaller the amplitude
- The overall quality of the sound is the sum of all the waves. This is why a violin and a trumpet can both play the same note, but sounding different

Harmonic Frequencies

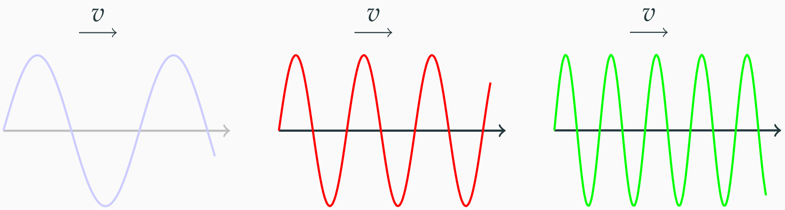


The sound wave with the longest wavelength and lowest frequency is:

- **fundamental frequency**, which is also called
- *first harmonic*
- *first partial*

Generally, when a musical instrument produces a sound, the fundamental frequency is the one that is “heard”

Harmonic Frequencies



A second wave has half the wavelength and twice the frequency. It's called:

- *second harmonic*
- *second partial*
- *first overtone*

From there we have the third, fourth, fifth...harmonics

Harmonic Frequencies

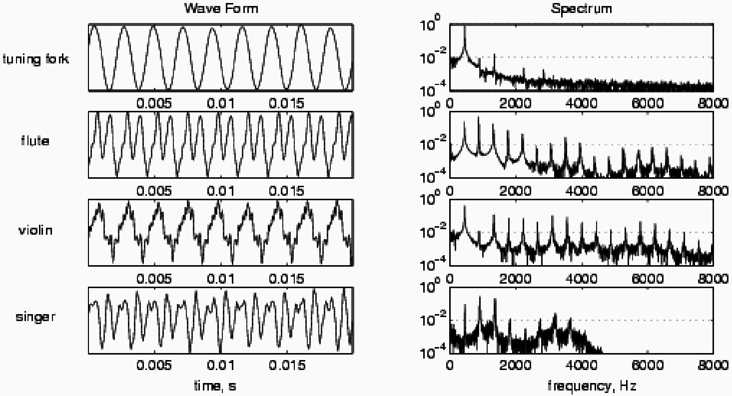
Whole-number multiples of the fundamental frequency f_1 are its **harmonic frequencies**, i.e. the n-th harmonic is:

$$\boxed{f_{h,n} = nf_1} \qquad n \geq 1$$

Pro-tip: Different physics and musical traditions use slightly different terminologies when talking about "harmonics". The words *overtones* and *partials* are often use, not always correctly. If you hear these terms, clarify with the person what they mean.

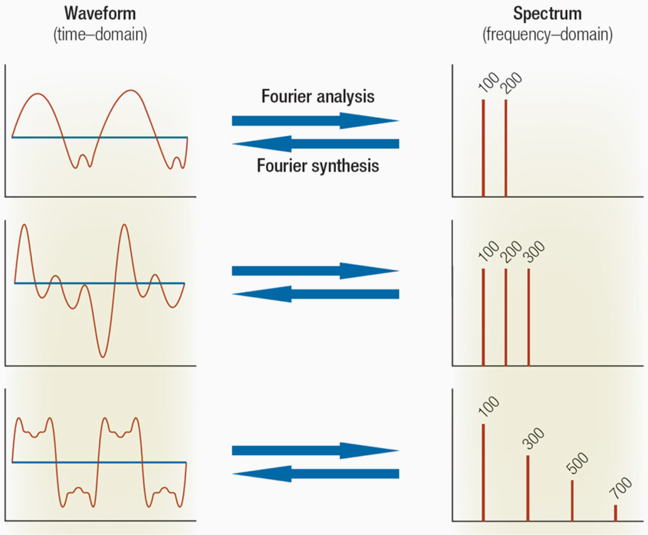
Different Musical Instruments

Waveforms from different musical instruments at $440\,\mathrm{Hz}$:



The graph on the right correspond to the amplitudes at different frequencies. Note the peaks at regular intervals. Those are the harmonic frequencies.

Fourier Analysis and Synthesis



Your Piano is Always Out of Tune!

- The concept of harmonic frequencies is the reason why some musical instruments will never play well in harmony (e.g. pianos, modern organs).
- **Example:** Find all the harmonics of a fundamental frequency of $110\,\mathrm{Hz}$ and then compare them to the values on the table.

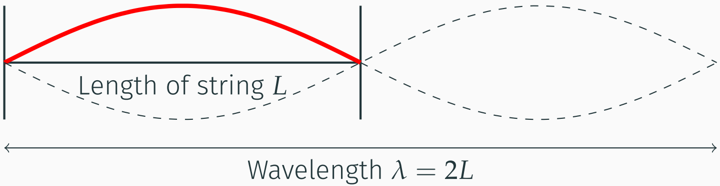
Note	Hz	Note	Hz	Note	Hz	Note	Hz	Note	Hz	Note	Hz	Note	Hz
C1	32.7	C2	65.4	C3	130.8	C4	261.6	C5	523.3	C6	1046.5	C7	2093.0
C#1	34.6	C#2	69.3	C#3	138.6	C#4	277.2	C#5	554.4	C#6	1108.7	C#7	2217.5
D1	36.7	D2	73.4	D3	146.8	D4	293.7	D5	587.3	D6	1174.7	D7	2349.3
D#1	38.9	D#2	77.8	D#3	155.6	D#4	311.1	D#5	622.3	D#6	1244.5	D#7	2489.0
E1	41.2	E2	82.4	E3	164.8	E4	329.6	E5	659.3	E6	1318.5	E7	2637.0
F1	43.7	F2	87.3	F3	174.6	F4	349.2	F5	698.5	F6	1396.9	F7	2793.8
F#1	46.2	F#2	92.5	F#3	185.0	F#4	370.0	F#5	740.0	F#6	1480.0	F#7	2960.0
G1	49.0	G2	98.0	G3	196.0	G4	392.0	G5	784.0	G6	1568.0	G7	3136.0
G#1	51.9	G#2	103.8	G#3	207.7	G#4	415.3	G#5	830.6	G#6	1661.2	G#7	3322.4
A1	55.0	A2	110.0	A3	220.0	A4	440.0	A5	880.0	A6	1760.0	A7	3520.0
A#1	58.3	A#2	116.5	A#3	233.1	A#4	466.2	A#5	932.3	A#6	1864.7	A#7	3729.3
B1	61.7	B2	123.5	B3	246.9	B4	493.9	B5	987.8	B6	1975.5	B7	3951.1

Standing Waves On a String

- A “vibrating” string is actually a standing wave on a string
- Both ends of the string are nodes
- As the string vibrates, the air around it vibrates at the same frequency
- The vibration travels as a sound wave towards your ears
- Examples:
    - Plucking a guitar or violin string
    - Hitting a key on a piano/harpsichord

Standing Waves On a String of Length L

Resonance frequencies are frequencies where a standing wave can be created. The first resonance (fundamental) frequency at occurs when $\lambda = 2L$:

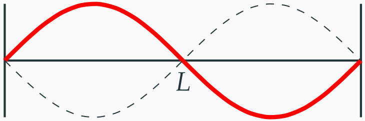


The fundamental frequency is based on the speed of the travelling wave along the string v_{str}:

$$\boxed{f_{r,1} = \frac{v_{\text{str}}}{\lambda} = \frac{v_{\text{str}}}{2L}}$$

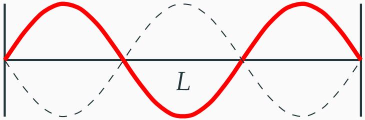
Standing Waves On a String of Length L

A second resonance frequency occurs when $L = \lambda$:



$$f_{r,2} = \frac{v_{\text{str}}}{\lambda} = \frac{v_{\text{str}}}{L} = 2f_{r,1}$$

And a third resonance frequency occurs at $L = \frac{3}{2}\lambda$:



$$f_{r,3} = \frac{3v_{\text{str}}}{2L} = 3f_{r,1}$$

Standing Waves On a String of Length L

In fact, the n-th resonance frequency of a wave on string is:

$$\boxed{f_{r,n} = n f_{r,1}} \quad \text{(standing wave on string)}$$

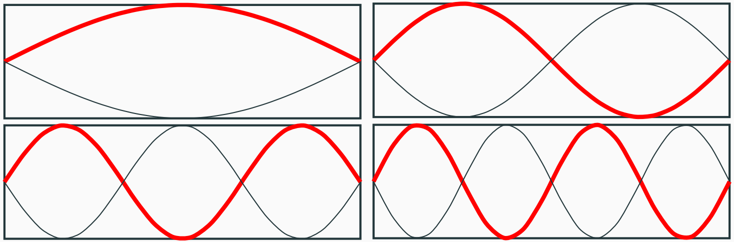
- where the fundamental frequency is

$$\boxed{f_{r,1} = \frac{v_{\mathrm{str}}}{2L}}$$

- n is a whole-number multiple
- This equation is *identical* to the equation for harmonic frequencies, meaning that on a string, every harmonic is a resonance frequency
- A vibrating string is said to have a "full set of harmonics"

Standing Waves in Closed Pipes

The same standing-wave patterns (nodes on both ends of the pipe) can be found on pipes that have both ends closed:



No musical instruments are built this way, but you can model standing waves inside a concert hall this way.

Standing Waves in Closed Pipes

Like strings, pipes that are *closed at both ends* also have a full set of harmonics. The n-th resonance frequency is given by:

$$\boxed{f_{r,n} = n f_{r,1}} \qquad \text{(closed pipe)}$$

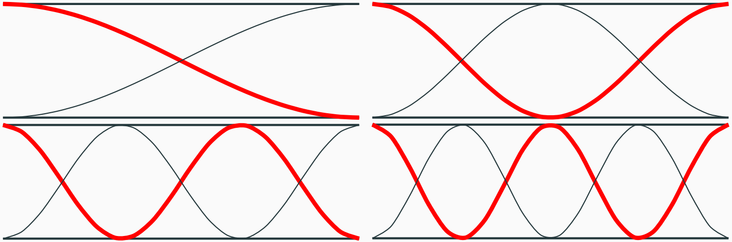
where n is a whole-number multiple of the fundamental frequency $f_{r\text{res},1}$:

$$\boxed{f_{r,1} = \frac{v_s}{2L}}$$

The difference between a closed pipe and a string is that the wave speed is now the speed of sound v_s inside the pipe.

Standing Waves in Open Pipes

- Example: Some organ pipes, flute
- Both ends of the pipes are anti-nodes



First resonance at $\lambda = 2L$

$$f_{r,1} = \frac{v_s}{\lambda} = \frac{v_s}{2L}$$

Second resonance at $\lambda = L$

$$f_{r,2} = \frac{v_s}{\lambda} = \frac{v_s}{L} = 2f_{r,1}$$

Standing Waves in Open Pipes

Like strings and closed pipes, open pipes also have a “full set of harmonics”. The n-th resonance frequency is given by:

$$\boxed{f_{r,n} = n f_{r,1}} \qquad \text{(open pipe)}$$

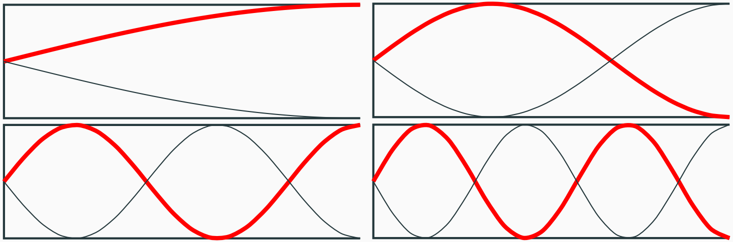
where n is a whole-number multiple of fundamental frequency $f_{r,1}$:

$$\boxed{f_{r,1} = \frac{v_s}{2L}}$$

This is not the case for every configuration. Some common configurations do not have a full set of harmonics.

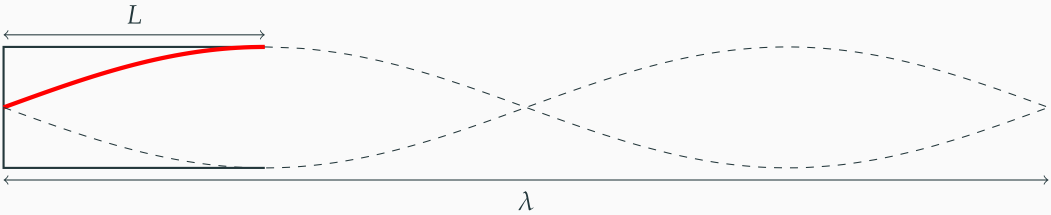
Standing Waves in Semi-Open Pipes

- Examples: Most organ pipes, clarinet, oboes, brass instruments
- Closed end: node (like in the closed pipes)
- Open end: anti-node (like in the open pipes)



Standing Waves in Semi-Open Pipes

Again starting with the fundamental frequency (lowest frequency where a standing wave can form inside the pipe). This occurs at $\lambda = 4L$:

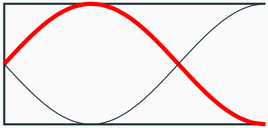


Fundamental frequency $f_{r,1}$ differs from the open-pipe and closed-pipe configurations by a factor of 2:

$$\boxed{f_{r,1} = \frac{v_s}{\lambda} = \frac{v_s}{4L}}$$

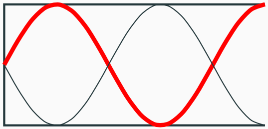
Standing Waves in Semi-Open Pipes

Likewise, second resonance occurs at $\lambda = \frac{4}{3}L$:



$$f_{r,2} = \frac{v_s}{\lambda} = \frac{3v_s}{4L} = 3f_{r,1}$$

And a third resonance occurs at $\lambda = \frac{4}{5}L$:



$$f_{r,3} = \frac{v_s}{\lambda} = \frac{5v_s}{4L} = 5f_{r,1}$$

We can repeat that for 4th, 5th...resonances.

Standing Waves in Semi-Open Pipes

Only *odd-number multiples* of the fundamental frequency are resonance frequencies in a semi-open pipe. We say that semi-open pipes have an *odd* set of harmonics.

$$\boxed{f_{r,n} = (2n-1) f_{r,1}} \qquad \text{(semi-open pipes)}$$

Because fundamental frequency $f_{r,1}$ is lower than open-pipe and closed-pipe configurations by a factor of 2 for the same length L, it has advantages when designing organ pipes that produces low frequencies.

$$\boxed{f_{r,1} = \frac{v_s}{\lambda} = \frac{v_s}{4L}}$$

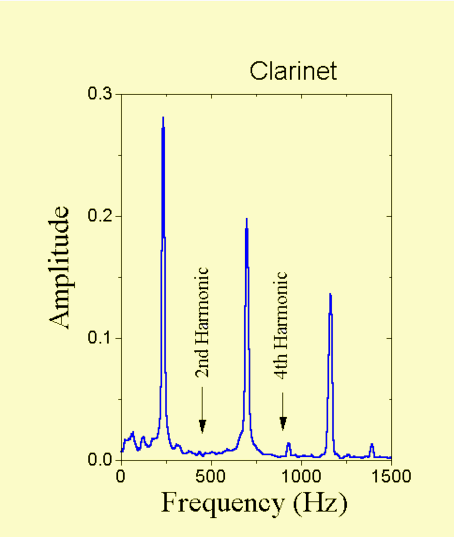
Standing Waves in Semi-Open Pipes

Remember: harmonic frequencies are multiples of the fundamental frequency. This means that:

- 2nd resonance frequency = 3rd harmonic frequency
- 3rd resonance frequency = 5th harmonic frequency
- 4rd resonance frequency = 7th harmonic frequency

I mentioned that the **clarinet** has a wave pattern of this type...

Standing Waves in Semi-Open Pipes



Yes it is! There are no peaks at the even number harmonics!