Topic 20: Light Waves and Optics

Advanced Placement Physics

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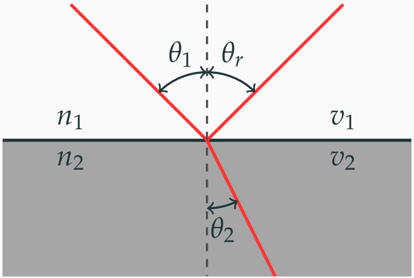
Reflection and Refraction

Reflection and Refraction

We begin with a model of a beam of light traveling toward the interface between two "indexed material":



The **index of refraction** (or **refractive index**, **index**) of the two media is defined as the ratio of speeds of light in a vaccuum and in the medium:

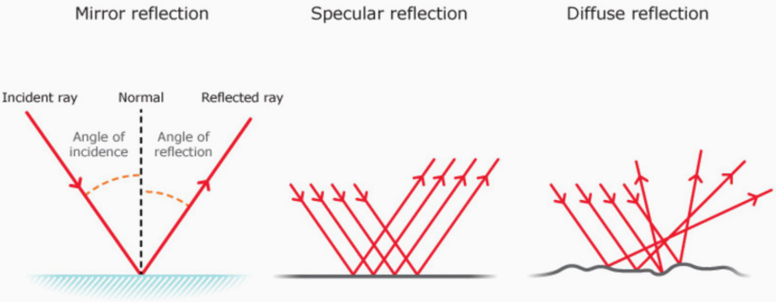
$$n = \frac{c}{v}$$

The index of a material is always greater than 1, since $v < c$.

Reflection of Light

In the **law of reflection**, the incident ray, the reflected ray, and the normal to the surface of the mirror all lie in the same plane, and the angle of reflection θ_r is equal to the angle of incidence θ_1:

$$\boxed{\theta_r = \theta_1}$$



Specular Reflection Example

This photo of Lake Matheson shows specular reflection in the water of the lake with reflected images of Aoraki/Mt Cook (left) and Mt Tasman (right). The very still lake water provides a perfectly smooth surface for this to occur.

Intensity of Light Reflecting from

In the special case where the incident and reflected angles normal, i.e. $\theta_1 = \theta_r = 0$, the reflected intensity of light I is related to the incident intensity I_0 by the indices of the two material (n_1 and n_2):

$$I = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2 I_0$$

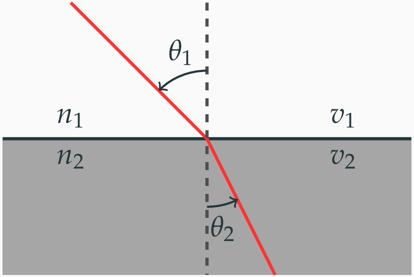
Remember that the intensity of a wave is the power P over the area A that the wave passes through:

$$I = \frac{P}{A}$$

The reflected intensity is lower than the incident, indicating that some of the energy from the incident wave is transmitted into the second medium.

Refraction of Light Through a Medium

Refraction occurs when light is transmitted from one medium to another at an oblique angle. The wave changes in direction due to the difference in the speed of light in the two media.



Law of Refraction

Snell's law (or **law of refraction**) relates the refractive indices n of the two media to the directions of propagation in terms of the angles θ to the normal

$$\boxed{n_1 \sin\theta_1 = n_2 \sin\theta_2}$$

This equation holds for the refraction of any kind of wave incident on a boundary surface separating two media (e.g. surface ocean wave at two depths)

Index of Refraction

When light enters a new medium, the *frequency* remains the same: the atoms in the new medium would absorb and then radiate the light at the same frequency. However, the *speed* of the radiated wave is different, therefore a different *wavelength* is observed:

$$\boxed{\frac{n_1}{n_2} = \frac{\lambda_2}{\lambda_1}}$$

You can work this out using the univesal wave equation: $v = f\lambda$

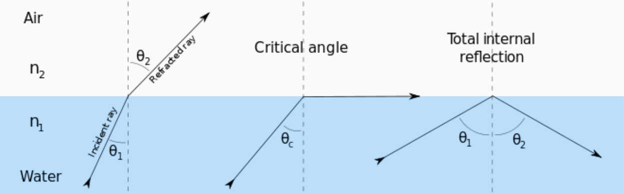
Index of Refraction of Common Materials

Material	n	Material	n
Vacuum	1	Ethanol	1.362
Air	1.000277	Glycerine	1.473
Water at 20 °C	1.33	Ice	1.31
Carbon disulfide	1.63	Polystyrene	1.59
Methylene iodide	1.74	Crown glass	1.50-1.62
Diamond	2.417	Flint glass	1.57-1.75

The values given are *approximate* and do not account for the small variation of index with light wavelength. That's called **dispersion**.

Total Internal Reflection

Total internal reflection ("TIR") can occur when light enters from a medium with high index to another with low index (i.e. $n_1 > n_2$). Snell's law still holds, but something weird can happen:



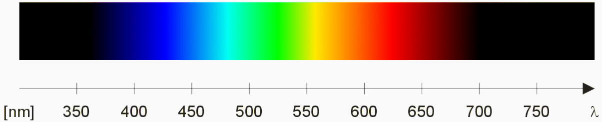
The **critical angle** can be found for when the refracted angle is 90°:

$$\boxed{\theta_c = \frac{n_2}{n_1}}$$

For water-air interface, θ_c=48.6°

Color of Light and Wavelength

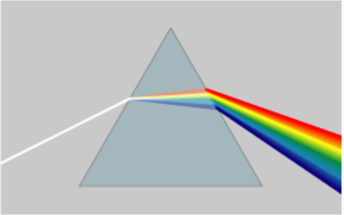
Human eyes perceive different frequencies of light as different colors. The visible spectrum of light range from about 380 nm (violet) to about 700 nm (red).



A good question to ask: where is *purple*?

Dispersion of Light

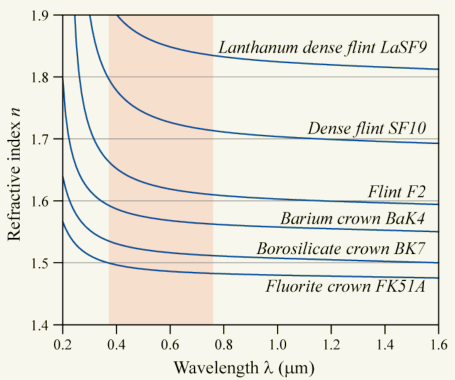
The dependence of the index of refraction n on wavelength λ is called **disperson**. The dispersion of light through a prism is why we can see the rainbow colors from a beam of white light:



In glass, the index of refraction for shorter wavelengths is always higher than longer wavelengths.

Wavelength Dependency of Index of Refraction

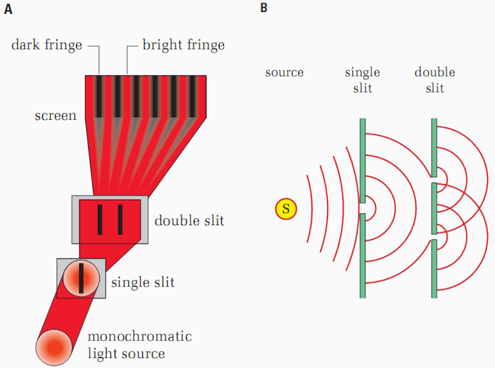
We can see that for different kinds of glass, the index of refraction can vary significantly through the visible spectrum.



Lenses and Mirrors

Interference of Light

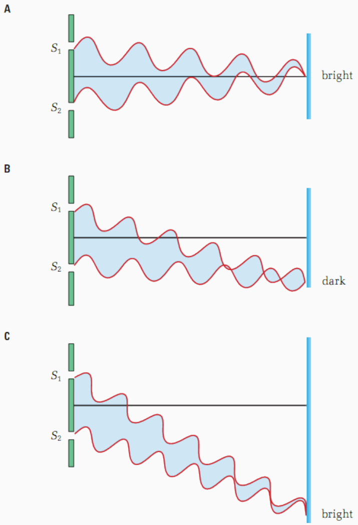
Thomas Young’s Double-Slit Experiment



- **Monochromatic light** light with a single color (frequency); the light source can be a laser, LED , or gas lamp (most likely what Young used)
- **Slit:** an opening; also called an **aperture**
- The **screen** far away from the slits is also called the **projection**

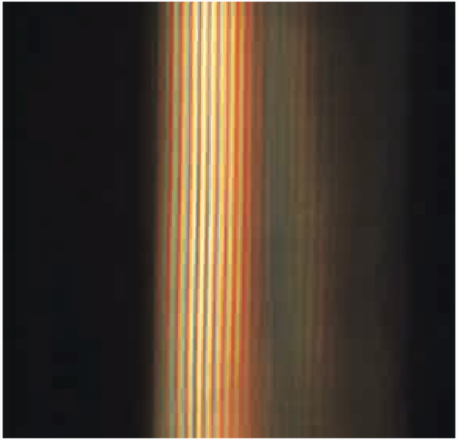
Double-slit experiment showed that light causes interference, just like any other wave

Thomas Young's Double-Slit Experiment



- At **A**, the path from slits S_1 and S_2 are the same, therefore we have **constructive interference** and the projection is bright
- At **B**, the path from S_1 and S_2 are diffed by half a wavelength, and therefore there is **destructive interference** and the projection is dark
- At **C**, the path from S_1 and S_2 are diffed by one wavelength, and therefore there is **constructive interference** again, and again, the projection is bright

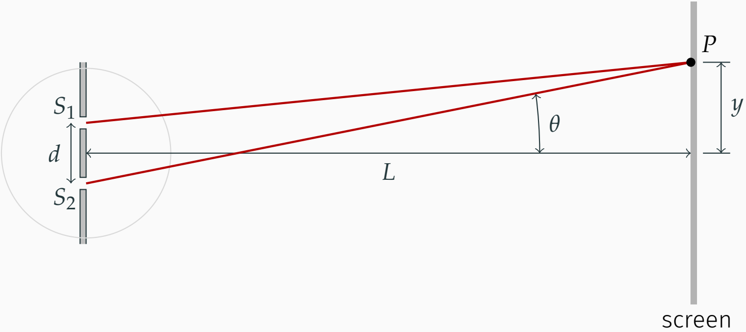
Interference Pattern: Bright and Dark Fringes



The “bright fringes” are from constructive interference; the “dark fringes” are from destructive interference.

Geometry of the Two-Slit Interference

Two coherent (in phase) sources pass through two narrow slits (that can be treated as point sources). The light from the slits emerge at the screen P at a distance L away.

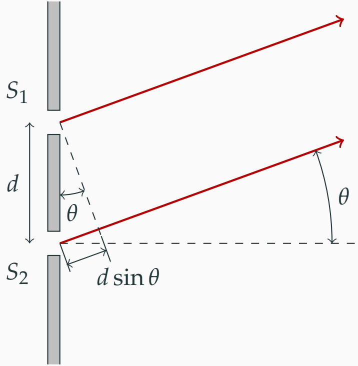


The two beams travel a slightly different distance:

- **Constuctive inteference:** path difference is an integer multiple of λ
- **Destructive interference:** path difference is a half-integer multiple

Geometry of the Two-Slit Interference

Using basic geometry, we can see that the path difference from the two slit to the projection is $d\sin\theta$.



- **Constructive maxima** occur when:

$$\boxed{n\lambda = d\sin\theta_n}$$

- Conversely, **destructive minima** occur when:

$$\boxed{\left(n + \frac{1}{2}\right)\lambda = d\sin\theta_n}$$

- $n = 0, 1, 2, 3 \ldots n_{\mathrm{max}}$ is called the **order number**
- n_{max} can be found by setting $\sin\theta = 1$
  - The nubmer of maxima is $2n_{\mathrm{max}} + 1$
  - The number of minima is $2n$

Approximation of The Wavelength of Light

For small angles, we can apply the **small-angle approximation** where

$$\theta \approx \tan\theta \approx \sin\theta$$

to find that the n-th bright fringe is located at: (replace n with $n + \frac{1}{2}$ for dark fringes)

$$\boxed{y_n = n\frac{\lambda L}{d}}$$

And the distance between frindges is given by:

$$\boxed{\Delta y = \frac{\lambda L}{d}}$$

This equation is used to estimate the wavelength of light based on the distances between bright fringes (or dark fringes).

Important Notes

- We usually apply the double-slit problem to light, but the problem can be applied to any wave (e.g. EM waves, sound waves, ocean waves) as well
- The sources don't actually need to be slits; any point source will do
- The projection/screen doesn't need to be a real screen either; it just has to be a line where wave intensity can be measured

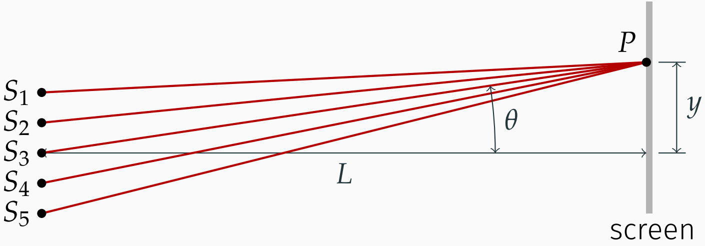
Remember:

- integer multiple = constructive maxima
- half-integer multiple = destructive minima

Interference of Multiple Equally Spaced Point sources

When there are multiple equally-spaced point sources (e.g. diffraction grating) we can just use the equation for two-slits:

$$n\lambda = d\sin\theta$$



The interference pattern gets sharper with increasing number of sources, and the bright fringes are narrower.

Diffraction

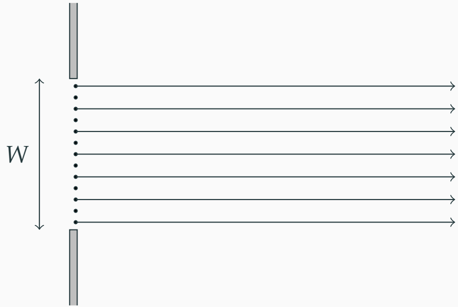
Diffraction of Waves

When a wave goes through an small opening, it **diffracts**. This happens with sound waves, ocean waves...and light.

The photo is from the Port of Alexandria in Egypt. The shape of the entire harbor is created because of diffraction of ocean wave.

Geometry for Single-Slit Diffraction

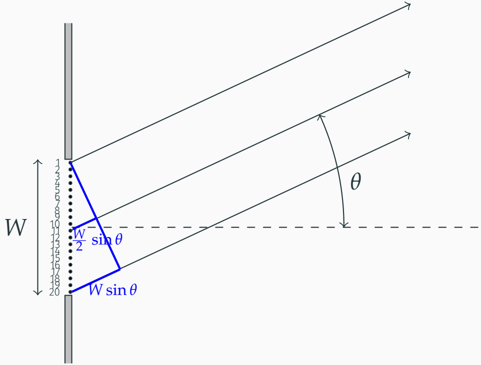
To examine the geometry for the single-slit diffraction problem, we treat the wave passing through the slit of width W as an infinite series of point sources at the slit.



- Light from the wavelets traveling perpendicular to the aperture do not interfere with one another
- Therfore, there is a bright fringe at the middle, called the **central diffraction maximum**.

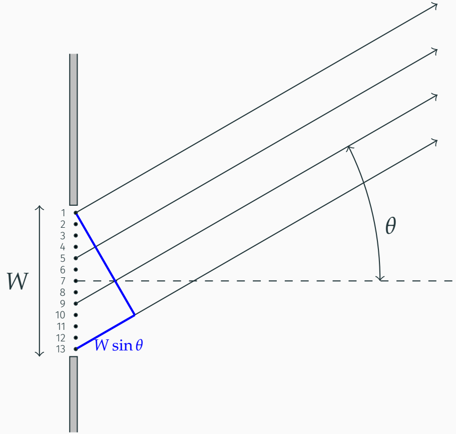
Geometry for Single-Slit Diffraction

Using the same analysis from the double-slit problem, we find that the path difference the wavelets at the top and bottom edges is $W \sin\theta$



- At some angle θ, the path difference between 1 and 20 will be an integer multiple of the wavelength $(m\lambda)$
- In this case, the path difference between 1 and 11 is a half-number multiple of the wavelength (i.e. destructive interference) and they cancel each other
- Similarly, 2 cancels 12, 3 cancels 13...
- **RESULT: Complete destructive interference**

Geometry for Single-Slit Diffraction



- At some other angle θ, the path difference between the top and bottom is $W\sin\theta = \frac{3}{2}\lambda$
- 1 and 5 differ by $\dfrac{\lambda}{2}$, so they cancel (as do 2 and 6, 3 and 7, 4 and 8, 9 and 13)
- But some of the beams will not, so we have a "bright fringe" at the projection
- This bright fringe is not as bright as the central one because of the destructive interference

Dark and Bright Fringes

Dark fringes exists on the screen at regular, whole-numbered intervals ($m = 1, 2, 3 \ldots$):

$$\boxed{m\lambda = W \sin\theta_m}$$

while bright fringes exist on the screen at regular, half-numbered intervals:

$$\boxed{\left(m + \frac{1}{2}\right)\lambda = W \sin\theta_m}$$

The equations look very similar to the double-slit equations for, but with dark and bright fringes in reverse, so be *very* careful when you use them!

Bright and Dark Fringes

The location of the bright fringes on the screen is determined by applying the small-angle approximation equation:

$$\boxed{y_m = \left(m + \frac{1}{2}\right)\frac{\lambda L}{W}}$$

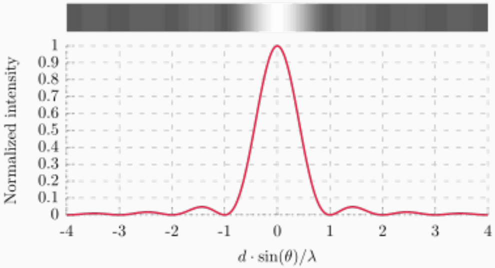
While for the dark fringes,

$$\boxed{y_m = \frac{m\lambda L}{W}}$$

Again, the equations look to be the reverse of the two-slit problem.

Single-Slit Diffraction, A Summary

- Similar to the double-slit interference, single-slit diffraction projects a series of alternating bright fringes (“maxima”) and dark fringes (“minima”) in the far field
- The bright fringe in the middle (“central maximum”) is twice as wide and very bright
- Subsequent bright fringes on either side (“higher-order maxima”) are much dimmer because of the partial destructive interference



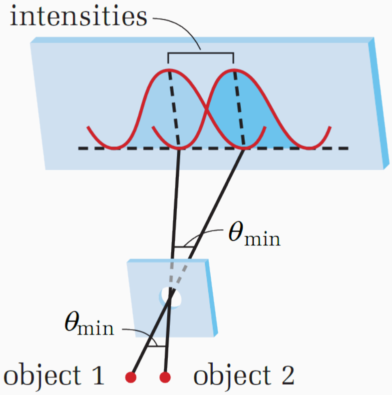
Optical Resolution

Optical Resolution

The ability of an optical instrument (e.g. the human eye, microscope, camera) to distinguish two distinct objects

When light from any object passes through an “optical instrument”, it *diffracts*, therefore “blurring” the object

Optical Resolution



Rayleigh limit: Two objects are resolved if the angle $\theta > \theta_{\text{min}}$, where θ_{min} is when the first minimum (dark fringe) from object 1 overlaps with the central maximum (bright fringe in the middle) from object 2.

Resolving Power

To resolve two objects, the minimum angle between rays from the two objects passing through an aperture is given by: D of the aperture.

Rectangular aperture:

$$\boxed{\theta_{\text{min}} = \frac{\lambda}{W}}$$

Circular aperture:

$$\boxed{\theta_{\text{min}} = \frac{1.22\lambda}{D}}$$

where W is the width of the aperture, and D is the diameter of the aperture. The angle θ_{min} is measured in **radians**.