Class 22b. Wave-Particle Duality

Advanced Placement Physics

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You Have Seen This Before

A significant portion of these slides are condensed from Physics 12. For some of you this is a review.

Maxwell's Equations in a Vacuum

Everything Comes Back to This

$$\nabla \cdot \mathbf{E} = 0$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \mu_o \varepsilon_o \frac{\partial \mathbf{E}}{\partial t}$$

Disturbances in $\mathbf{E}$ and $\mathbf{B}$ travel as an "electromagnetic wave", with a speed:

$$c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} = 299\,792\,458\ \mathrm{m/s}$$

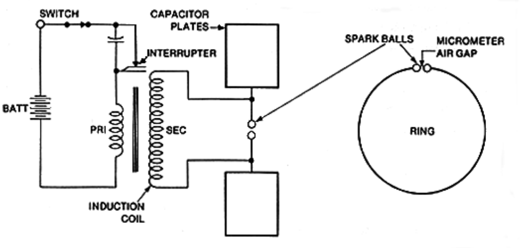
Maxwell's Equations

Physicists already have an estimate of the speed of light (within about 10 %, so is light an electromagnetic wave then?

- In order to prove that light is an electromagnetic wave, we must generate an alternating current with a frequency of 10^{14} Hz
- Technology of that time can only generate frequencies around 10^{8} Hz (already much higher than the 60 Hz that our electrical outlet uses, but still 10^{6} times too low)

The Spark Gap Experiment

German physicist Heinrich Hertz devised a “spark gap experiment” to generate high frequencies



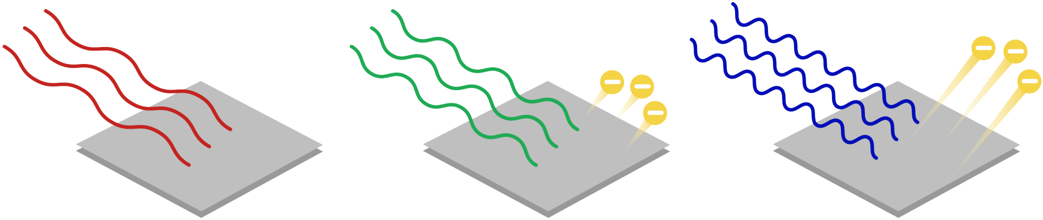
- Produced EM waves with frequency 10^{14} Hz
- Also showed that light has the same wavelengths as predicted by Maxwell’s equations

Photoelectric Effect

- Terse remark in Hertz's results:
  It is essential that the pole surfaces of the spark gap should be frequently repolished to ensure reliable operation of the spark.
- This is now known as the **photoelectric effect** caused by ultraviolet radiation
- Physicist who repeated his experiments did not have an explanation

Photoelectric Effect

When electromagnetic waves (e.g. light) hits certain metals, electrons are knocked off the surface



- Increasing intensity of light knocked off more electrons, but doesn't change their kinetic energy, but
- Changing the frequency of the light did change K though, although
- Below a certain frequency, *no* electrons were emitted

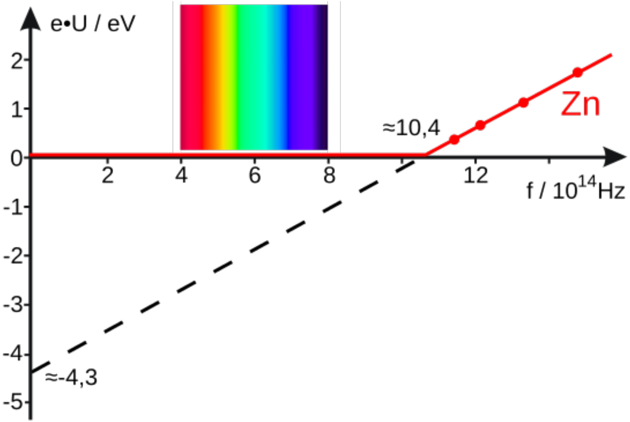
The Photon: Packets of Energy

- Light is not a continuous wave, but
- A collection of discrete energy packets (photons)
- Each photon has energy $E = hf$

$$\boxed{K_{\text{max}} = \begin{cases} hf - \varphi & \text{if } hf > \varphi \\ 0 & \text{otherwise} \end{cases}}$$

Quantity	**Symbol**	**SI Unit**
Maximum kinetic energy of “photoelectrons”	K	J
Planck’s constant	h	J s
Frequency of the EM wave	f	Hz
Work function of the metal	φ	J

Work Function φ



Slope is h no matter what metal it is.

Work Functions of Different Materials

The work function φ depends on the metal.

Metal	Work function (eV)
aluminum	4.28
calcium	2.87
cesium	2.14
copper	4.65
iron	4.50
lead	4.25
lithium	2.90
nickel	5.15
platinum	5.65
potassium	2.30
tin	4.42
tungsten	4.55
zinc	4.33

Work Function for Common Metals:

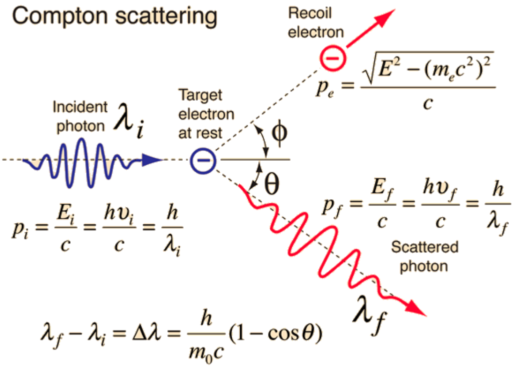
- The minimum energy required to remove an electron from a solid to a point immediately outside the solid surface.

Compton Scattering

- American physicist Arthur Compton studied x-ray scattering by free electrons
- Classical theory cannot account for the scattering behaviour
- Frequency shift only depends on scattering angle
- Prediction possible if treating the x-ray as photons with momentum–just like a particle

$$\boxed{p = \frac{E}{c} = \frac{hf}{c} = \frac{h}{\lambda}}$$

Compton Scattering



If we treat the x-ray as a photon with momentum $p = h/\lambda$ then we can use Newton's laws of motion to predict both the recoil electron and scattered x-ray!

Momentum of a Photon

The momentum of a photon is prpoortional to Planck's constant and inversely proportional to its wavelength.

$$\boxed{p = \frac{h}{\lambda}}$$

Quantity	Symbol	SI Unit
Momentum	p	kg m/s
Planck's constant	h	J s
Wavelength	λ	m (meters)

This is an odd expression, which treats photon both as a particle (with momentum) and a wave (with a wavelength λ).

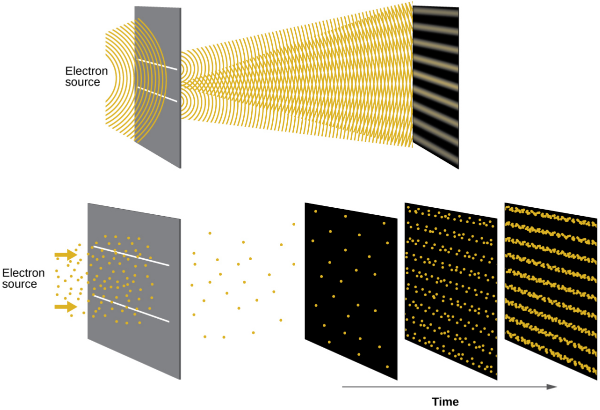
Matter Waves

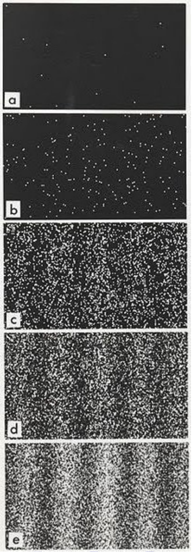
If electromagnetic waves are really particles of energy, then are particles (e.g. electrons) a wave of some sort?

- The De Broglie hypothesis in 1924: a particle can also have a wavelength
- Confirmed by the Davisson-Germer Experiment in 1927 (beam of electron scattering on nickel crystal surface)

Electron Interference

If I perform a double-slit experiment with a beam of electrons, will I get an interference pattern?





De Broglie Wavelength

If matter is also a wave, then what would be its wavelength? Let's solve momentum equation for λ:

$$p = \frac{h}{\lambda} \quad \rightarrow \quad \lambda = \frac{h}{p} \quad \rightarrow \quad \boxed{\lambda = \frac{h}{mv}}$$

Quantity	**Symbol**	**SI Unit**
Wavelength of a particle	λ	m (meters)
Planck's constant	h	J s (joule seconds)
Mass	m	kg (kilograms)
Velocity	v	m/s (meters per second)

Heisenberg Uncertainty Principle

Because of the wave properties of particles, you can never be completely certain of the relationship between an object's momentum p and position x:

$$\boxed{\Delta p \Delta x \leq \frac{1}{2}\hbar}$$

where $\hbar$ is the **reduced Plank constant**, or **Dirac constant**:

$$\hbar = \frac{h}{2\pi} = 1.054 \times 10^{-34}\ \mathrm{J\,s}$$

Bohr Atomic Model

- The "orbital" model of electrons does not work, because
- As the electron orbits a nucleus, it radiates EM radiation, and lose energy
- The orbit will eventually collapse
- Bohr postulated that electron can move in certain "non-radiating" orbits, corresponding to energy levels:

$$\boxed{E_n = -\frac{k^2 e^4 m}{2\hbar^2}\frac{Z^2}{n^2}}$$

- From the wave-particle duality perspective, the "orbits" correspond more to a standing wave around the nucleus (remember that a standing wave does not lose energy)

Hydrogen Emission

- Lyman series:
  - the EM emissions when the electrons drop from a higher energy state (E_n) to the ground state $n = 1$ (i.e. E_1)
  - The frequency is given by:

$$f = \frac{E_1 - E_n}{h}$$

  - We can apply universal wave equation to get the wavelengths
- Balmer series–dropping to E_2
- Paschen series–dropping to E_3