Classes 14: Magnetism, Part 2

AP Physics

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Olympiads School

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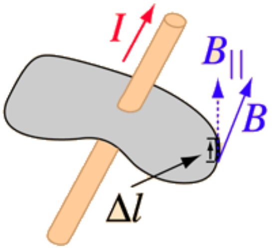
Files for You to Download

Download from the school website:

1. `14-Magnetism2.pdf`—The "print version" of this presentation. If you want to print the slides on paper, I recommend printing 4 slides per page.

Please download/print the PDF file before each class. When you are taking notes, pay particular attention to things I say that aren't necessarily on the slides.

Ampère's Law



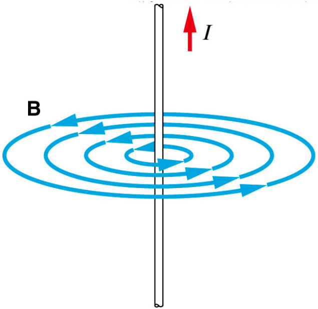
Like Gauss's law to calculate electric fields for symmetric configurations, **Ampère's law** can be used to calculate the magnetic field for symmetric configurations:

$$\boxed{\oint_C \mathbf{B} \cdot d\boldsymbol{\ell} = \mu_o I_C}$$

where

- C is a closed curve around a current ("Amperian lop")
- $d\ell$ is an infinitesimal length along the closed curve
- I_c is the net current that penetrates the area bounded by C

Application of Ampère's Law: Infinitely Long Wire



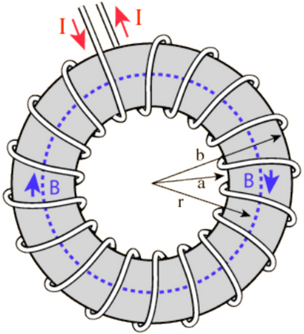
An *infinitely* long wire must generate a magnetic field that only depend on radial distance. We place our loop as a circle of radius r around the wire. Ampère's law reduces to:

$$\oint_C \mathbf{B} \cdot d\boldsymbol{\ell} = \mu_o I_C \;\rightarrow\; B(2\pi r) = \mu_o I$$

From this, we get our expression of the magnetic field from an infinitely long wire:

$$B = \frac{\mu_o I}{2\pi r}$$

Toroid



A toroid consists of a current-carrying wire wound around a donut-shaped core

Another application of Ampère's Law is the **toroid**. This time, we put our loop at $a < r < b$ inside the toroid. Once again, because of symmetry, Ampère's law reduces to:

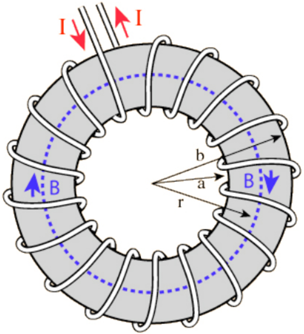
$$\oint_C \mathbf{B} \cdot d\boldsymbol{\ell} = \mu_o I_C$$

$$B(2\pi r) = \mu_o N I$$

$$B = \frac{\mu_o N I}{2\pi r}$$

where N is the number of times the wire is wound around the core

Toroid



More interestingly, when the loop is placed at $r < a$:

$$B = 0 \quad \text{for} \quad r < a$$

When the loop is placed at $r > b$, the amount of current penetrating the loop is the same in both direction, i.e. $I_c = 0$, and

$$B = 0 \quad \text{for} \quad r > b$$

In fact, the *only* place that a magnetic field exists is inside the core, between a and b, where

$$B = \frac{\mu_o N I}{2\pi r} \quad \text{for} \quad a \le r \le b$$

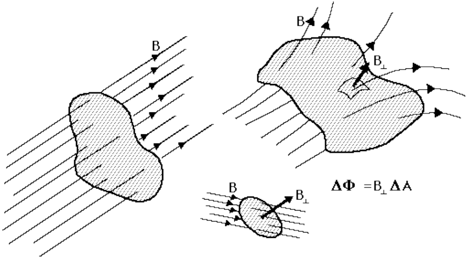
Magnetic Flux

Question: If a current-carrying wire can generate a magnetic field, can a magnetic field affect the current in a wire?

Answer: Yes, sort of...

To understand how to *induce* a current by a magnetic field, we need to look at fluxes again.

Magnetic Flux



Magnetic flux is defined as:

$$\boxed{\Phi_{\text{magnetic}} = \int \mathbf{B} \cdot d\mathbf{A}}$$

where $\mathbf{B}$ is the magnetic field, and $d\mathbf{A}$ is the infinitesimal area pointing **outwards**. If you are uncomfortable with using vector surfaces, note that magnetic flux can also be expressed as:

$$\boxed{\Phi_{\text{magnetic}} = \int \mathbf{B} \cdot \hat{\mathbf{n}} dA}$$

where $\hat{\mathbf{n}}$ is the outward normal direction

Magnetic Flux Over a Closed Surface

The unit for magnetic flux is a "weber" (Wb), in honor of German physicist Wilhelm Weber, who invented the electromagnetic telegraph with Carl Gauss. The unit is defined as:

$$1\,\mathrm{Wb} = 1\,\mathrm{T\,m^2}$$

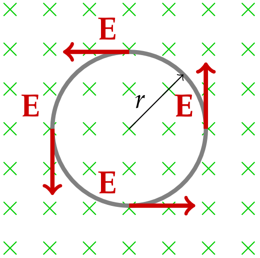
The magnetic flux over a closed surface is always zero:

$$\boxed{\oint \mathbf{B} \cdot d\mathbf{A} = 0}$$

Since magnetic field lines only exist as a loop, that means there should be equal amount of "flux" flowing out of a closed surface as entering the surface.

When Magnetic Flux is Changing

- When the magnetic flux Φ_{magnetic} is changing, an electromotive force (*emf*, $\mathcal{E}$) is created in the wire.
- Unlike in a circuit, where the *emf* is concentrated at the terminals of the battery, the induced *emf* is spread across the entire wire.



- Since *emf* is work per unit charge, that means that there is an electric field inside the wire to move the charges.
- In this example:
  - Magnetic field $\mathbf{B}$ into the page
  - The direction of the electric field $\mathbf{E}$ corresponds to an *increase* in magnetic flux

Faraday's Law

Faraday's law states that the rate of change of magnetic flux produces an electromotive force:

$$\boxed{\mathcal{E} = \oint \mathbf{E} \cdot d\mathbf{l} = -\frac{d\Phi}{dt}}$$

The negative sign highlighted in red is the result of Lenz's law, which is related to the conservation energy

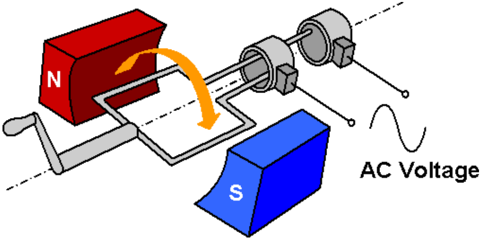
How Can Magnetic Flux Change

Magnetic flux can change due to a number of reasons:

1. **Changing magnetic field strength** e.g. if $\mathbf{B}$ is created by a time dependent current source like an alternating current
2. **Changing orientation of magnetic field** because the surface area is moving (translation and/or rotation) relative to $\mathbf{B}$
3. **Changing area** the surface area from which the flux is calculated is changing

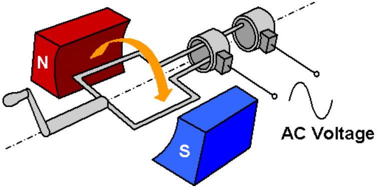
AC Generators

A simple AC (alternating current) generator makes use of the fact that a coil rotating against a fixed magnetic field has a changing flux.



Let's say the permanent magnets produce a uniform magnetic field B, and the coil between them has N turns, and an area A. Now let's say that the coil is rotating with an angular frequency ω.

AC Generators



When the coil is turning, the angle between the coil and the magnetic field is:

$$\theta = \omega t + \delta$$

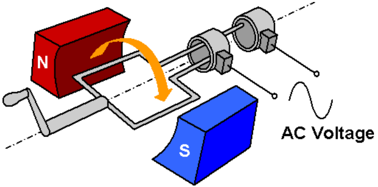
where δ is the initial angle, the magnetic flux through the coil is given by

$$\Phi = NBA\cos\theta = NBA\cos(\omega t + \delta)$$

when motion starts. The *emf* produced is therefore:

$$\mathcal{E} = -\frac{d\Phi}{dt} = -NBA\omega\sin(\omega t + \delta)$$

AC Generators



We commonly write it this way instead:

$$\mathcal{E} = \mathcal{E}_{\max} \sin(\omega t + \delta)$$

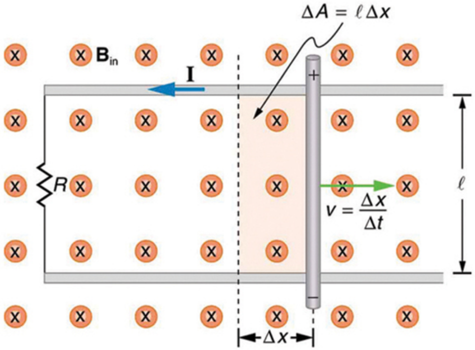
where

$$\mathcal{E}_{\max} = NBA\omega$$

What happened to the negative sign? We got rid of it by choosing a different value for δ.

Motional EMF

What happens when I slide the rod to the right?



When sliding the rod to the right with speed v, the magnetic flux through the loop (and its rate of change) is:

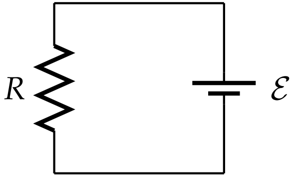
$$\Phi = BA = B\ell x$$

$$\frac{d\Phi}{dt} = B\ell\frac{dx}{dt} = \boxed{B\ell v = \mathcal{E}}$$

We can use the Lorentz force law on the charges on the rod to find the direction of the current I.

Motional EMF

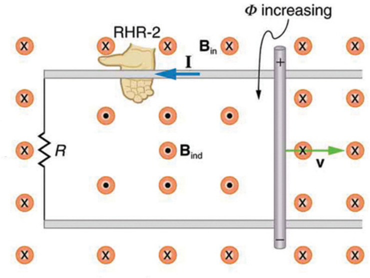
What happens when I slide the rod to the right?



- An equivalent circuit is shown on the left
- The amount of current can be found using Ohm's law
- Note that the "motional emf" produced is spread over the entire circuit

Lenz's Law

Something very interesting happens when the current starts running on the wire.



It produces an "induced magnetic field" out of the page, in the opposite direction as the field that generated the current in the first place!

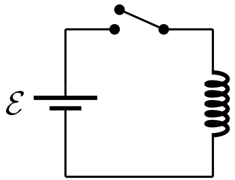
Lenz's Law

LENZ'S LAW
The induced *emf* and induced current are in such are direction as to oppose the change that produces them

So basically, the conservation of energy

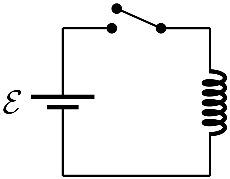
Back *emf*

Consider a very simple circuit consisting of a voltage source and a coil



- When the switch is closed an the current begin to flow, the coil begin to generate a magnetic flux inside
- As the current changes (initially increasing with time), it self-induces a "back emf" that opposes the change in current
- A current can't jump from zero to some value (or from some value to zero) instantaneously

Back *emf*



- If you try to break the circuit, you change the magnetic flux very rapidly
- Change of Φ creates a huge induced "back emf" that is proportional to $d\Phi/dt$
- The back emf creates a large voltage drop across the switch
- Large voltage across two metal contact produces a very strong electric field–strong enough to tear electrons away from air molecules ("dielectric breakdown")
- Air conducts electricity in the form of a "spark"

Self Inductance

A solenoid carrying a current generates a magnetic field; its strength given by Biot-Savart law (or Ampère's law):

$$B = \frac{\mu_0 N I}{L}$$

Since $\mathbf{B} \propto I$, the magnetic flux through the solenoid (really $\Phi = NBA$ where A is the cross-sectional area of the solenoid and N is the number of coils) is therefore also proportional to I, i.e.:

$$\boxed{\Phi_{\text{magnetic}} = LI}$$

where L is the called the **self inductance** of the coil.

Self Inductance

For a solenoid, we can see that the self inductance is given by:

$$\boxed{L = \frac{\Phi_{\text{magnetic}}}{I} = \mu_0 n^2 A l}$$

where μ_o is the magnetic permeability of free space, n is the number of coil turns per unit length, and A and l are the cross-section and length of the solenoid. (i.e. Al is the enclosed volume.)

Self Inductance and Induced EMF

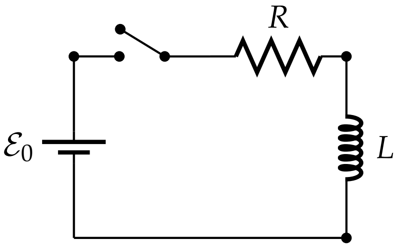
If the current changes, the magnetic flux changes as well, therefore inducing an electromotive force in the circuit! According Faraday's law:

$$\boxed{\mathcal{E} = -\frac{d\Phi}{dt} = -L\frac{dI}{dt}}$$

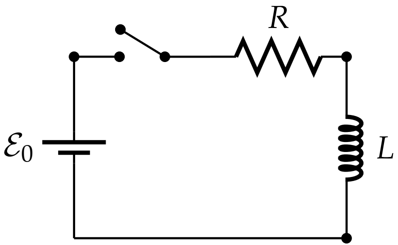
The self-induced emf is proportional to the rate of change of current.

Circuits with Inductors

- Coils and solenoids in circuits are known as “inductors” and have large self inductance L
- Self inductance prevents currents rising and falling instantaneously
- A basic circuit containing a resistor and an inductor is called an ***LR* circuit**:



Analyzing LR Circuits



Applying Kirchkoff's voltage law:

$$\mathcal{E}_0 - IR - L\frac{dI}{dt} = 0$$

We follow the same procedure as charging a capacitor to find the time dependent current:

$$I = \frac{\mathcal{E}_0}{R}\left(1 - e^{-Rt/L}\right)$$

The time constant for an LR circuit is

$$\tau = \frac{L}{R}$$

Magnetic Energy

Just as a capacitor stores energy, the energy stored by an inductor carrying a current I in given by:

$$\boxed{U_m = \frac{1}{2} L I^2}$$

We can also define a **magnetic energy density**:

$$\boxed{\eta_m = \frac{B^2}{2\mu_0}}$$