AP PHYSICS 1 & C: CIRCULAR MOTION AND SIMPLE HARMONIC MOTION (Topics 5 & 6)

Directions: Each of the questions or incomplete statements below is followed by five suggested answers or completions. Select the one that is best in each case and place the letter of your choice in the corresponding box on the student answer sheet.

Note: To simplify calculations, you may use $g = 10\,\mathrm{m/s^2}$ in all problems.

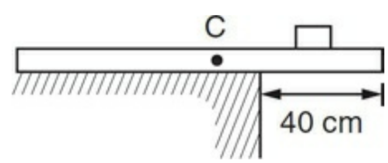
1. Linear acceleration is to force as angular acceleration is to
   (A) kinetic energy
   (B) angular velocity
   (C) rotational inertia
   (D) torque
   (E) angular momentum

2. A girl stands on a rotating merry-go-round without holding on to a rail. The force that keeps her moving in a circle is the
   (A) frictional force on the girl directed away from the center of the merry-go-round
   (B) frictional force on the girl directed toward the center of the merry-go-round
   (C) normal force on the girl directed away from the center of the merry-go-round
   (D) normal force on the girl directed toward the center of the merry-go-round
   (E) weight of the girl

3. A ball of mass m is swung in a vertical circle of radius R. The speed of the ball at the bottom of the circle is v. The tension in the string at the bottom of the circle is
   (A) mg
   (B) $mg + \dfrac{mv^2}{R}$
   (C) $mg - \dfrac{mv^2}{R}$
   (D) $\dfrac{mv^2}{R}$
   (E) zero

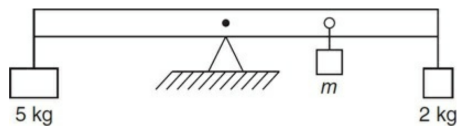
4. A car of mass m drives on a flat circular track of radius R. To maintain a constant speed v on the track, the coefficient of friction μ between the tires and the road must be
   (A) mg
   (B) $mg + \dfrac{mv^2}{R}$
   (C) $mg - \dfrac{mv^2}{R}$
   (D) $\dfrac{v^2}{gR}$
   (E) $\sqrt{\dfrac{v^2}{gR}}$

5. A meter stick of mass 0.1 kg rests on a table as shown. A length of 40 cm extends over the edge of the table. How far from the edge of the table could a 0.05 kg mass be placed on the meter stick so that the stick just begins to tip?



   (A) 5 cm
   (B) 10 cm
   (C) 15 cm
   (D) 20 cm
   (E) 30 cm

6. A meter stick is balanced on a fulcrum at its center, as shown. A mass of 5 kg is hung on the left end of the stick, and a mass of 2 kg is hung on the right end. In order to balance the system, a mass m is hung at the 25-cm mark on the right side. What is the value of the mass m?



(A) 12 kg
(B) 6 kg
(C) 4 kg
(D) 3 kg
(E) 2 kg

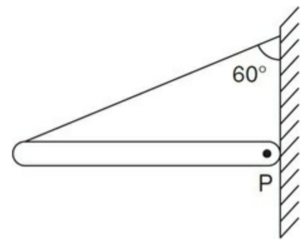
7. A ball on the end of a string is swung in a circle of radius 2 m according to the equation $\theta = 4t^2 + 3t$, where θ is in radians and t is in seconds. The angular acceleration of the ball is

(A) 6 rad/s^2
(B) $4t^2 + 3t$ rad/s^2
(C) $8t + 3$ rad/s^2
(D) $\frac{3}{4}t^3 + 3t^2$ rad/s^2
(E) 8 rad/s^2

8. The linear speed v of the ball (in the previous question) at $t = 3$ s is

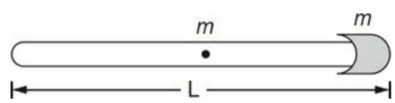
(A) 27 m/s
(B) 54 m/s
(C) 108 m/s
(D) 135 m/s
(E) 210 m/s

9. A metal bar of constant density and weight W is attached to a pivot on the wall at point P and supported by a rope that makes an angle of 60° with the vertical wall. The reaction force exerted by the pivot on the bar at point P is best represented by which arrow?



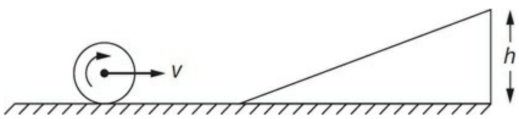
(A) $\nearrow$
(B) $\uparrow$
(C) $\downarrow$
(D) $\nwarrow$
(E) $\searrow$

10. A uniform rod of length L and mass m has a rotational inertia of $\frac{1}{12}mL^2$ about its center. A particle, also of mass m, is attached to one end of the stick. The combined rotational inertia of the stick and particle about the center of the rod is



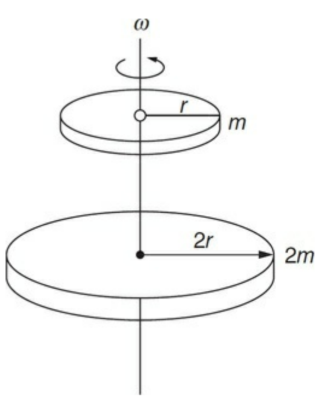
(A) $\frac{mL^2}{3}$
(B) $\frac{12mL^2}{13}$
(C) $\frac{13mL^2}{12}$
(D) $\frac{mL^2}{156}$
(E) $\frac{13mL^2}{156}$

11. A hoop of radius R and mass m has a rotational inertia of mR^2. The hoop rolls without slipping along a horizontal floor with a constant speed v and then rolls up a long incline. The hoop can roll up the incline to a maximum vertical height of



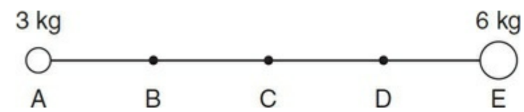
(A) $\frac{v^2}{g}$
(B) $\frac{2v^2}{g}$
(C) $\frac{v^2}{2g}$
(D) $\frac{4v^2}{g}$
(E) $\frac{v^2}{4g}$

12. Two disks are fixed to a vertical axle that is rotating with a constant angular speed ω. The smaller disk has a mass m and a radius r, and the larger disk has a mass $2m$ and radius $2r$. The general equation for the rotational inertia of a disk of mass M and radius R is $\frac{1}{2}MR^2$. The ratio of the angular momentum of the larger disk to the smaller disk is



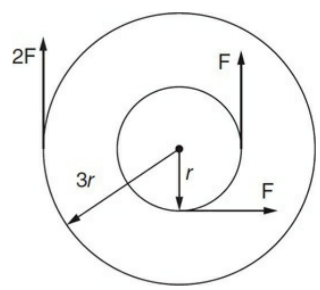
(A) 1 : 4
(B) 4 : 1
(C) 1 : 2
(D) 2 : 1
(E) 8 : 1

13. A light rod has a mass attached at each end. At one end is a 6 kg mass, and at the other end is a 3 kg mass. An axis can be placed at any of the points shown. Through which point should an axis be placed so that the rotational inertia is the greatest about that axis?



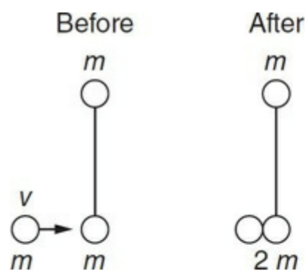
(A) A
(B) B
(C) C
(D) D
(E) E

14. Two wheels are attached to each other and fixed so that they can only turn together. The smaller wheel has a radius of r and the larger wheel has a radius of $3r$. The two wheels can rotate together on a frictionless axle. Three forces act tangentially on the edge of the wheels as shown. The magnitude of the net torque acting on the system of wheels is



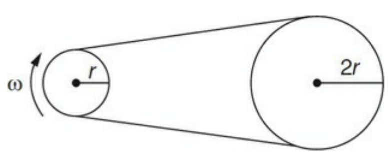
(A) Fr
(B) $2Fr$
(C) $3Fr$
(D) $4Fr$
(E) $6Fr$

15. Astronauts are conducting an experiment in a negligible gravity environment. Two spheres of mass m are attached to either end of a light rod. As the rod and spheres float motionless in space, an astronaut launches a piece of sticky clay, also of mass m, toward one of the spheres so that the clay strikes and sticks to the sphere perpendicular to the rod. Which of the following statements is true of the motion of the rod, clay, and spheres after the collision?



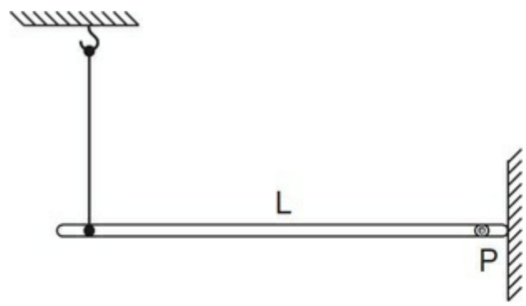
(A) Linear momentum is not conserved, but angular momentum is conserved.
(B) Angular momentum is not conserved, but linear momentum is conserved.
(C) Kinetic energy is conserved, but angular momentum is not conserved.
(D) Kinetic energy is conserved, but linear momentum is not conserved.
(E) Both linear momentum and angular momentum are conserved, but kinetic energy is not conserved.

16. A belt is wrapped around two wheels as shown. The smaller wheel has a radius r, and the larger wheel has a radius $2r$. When the wheels turn, the belt does not slip on the wheels, and gives the smaller wheel an angular speed ω. The angular speed of the larger wheel is



(A) ω
(B) 2ω
(C) $\frac{1}{2}\omega$
(D) $\frac{1}{4}\omega$
(E) 4ω

17. One end of a stick of length L, rotational inertia I, and mass m is pivoted on an axle with negligible friction at point P. The other end is tied to a string and held in a horizontal position. When the string is cut, the stick rotates counterclockwise. The angular speed ω of the stick when it reaches the bottom of its swing is



(A) $\frac{mgL}{I}$
(B) $\sqrt{\frac{mgL}{I}}$
(C) $\sqrt{\frac{2mgL}{I}}$
(D) $\sqrt{\frac{mgL}{2I}}$
(E) $\sqrt{\frac{4mgL}{I}}$

18. A disk is mounted on a fixed axle. The rotational inertia of the disk is I. The angular velocity of the disk is decreased from ω_0 to ω_f during a time Δt due to friction in the axle. The magnitude of the average net torque acting on the wheel is

(A) $\frac{\omega_f - \omega_0}{\Delta t}$
(B) $\frac{(\omega_f - \omega_o)^2}{\Delta t}$
(C) $\frac{I(\omega_f - \omega_o)}{\Delta t}$
(D) $\frac{I(\omega_f - \omega_o)^2}{\Delta t}$
(E) $\frac{I(\omega_f - \omega_o)}{\Delta t^2}$

19. The average power developed by the friction in the axle of the disk from the previous question to bring it to a complete stop is
   (A) $\dfrac{\omega_o}{\Delta t}$
   (B) $\dfrac{(\omega_o)^2}{\Delta t}$
   (C) $\dfrac{I(\omega_f - \omega_o)}{2\Delta t}$
   (D) $\dfrac{I\omega_o^2}{2\Delta t}$
   (E) $\dfrac{I(\omega_f - \omega_o)}{\Delta t^2}$

20. A light rod of negligible mass is pivoted at point P a distance L from one end as shown. A mass m is attached to the left end of the rod at a distance of $3L$ from the pivot, and another mass $4m$ is attached to the other end a distance L from the pivot. The system begins from rest in the horizontal position. The net torque acting on the system due to gravitational forces is

   (A) $4mgL$ clockwise
   (B) $3mgL$ clockwise
   (C) $3mgL$ counterclockwise
   (D) mgL counterclockwise
   (E) mgL clockwise

21. The angular acceleration of the system when it is released from rest is
   (A) zero
   (B) $\dfrac{g}{5L}$
   (C) $\dfrac{g}{4L}$
   (D) $\dfrac{g}{13L}$
   (E) $\dfrac{g}{L}$

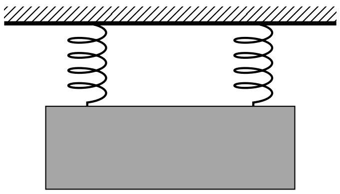
22. A simple pendulum has a mass m, length L, and period T. If the pendulum mass is replaced by a mass of $2m$, the period will be
   (A) doubled
   (B) halved
   (C) quartered
   (D) quadrupled
   (E) unchanged

23. A mass oscillates on the end of a spring that obeys Hooke's law. Which of the following statements is true?
   (A) The amplitude of oscillation is equal to the potential energy of the spring.
   (B) The kinetic energy of the oscillating mass is constant.
   (C) Maximum potential energy occurs when the mass reaches the equilibrium position.
   (D) The potential energy of the spring at the amplitude is equal to the kinetic energy at the equilibrium position.
   (E) The kinetic energy of the spring at the amplitude is equal to the potential energy at the equilibrium position.

24. A superball is dropped from a height of 5.0 m above a floor. The ball bounces off the floor in a perfectly elastic collision so that it rises to the same height with each bounce. The motion of the ball can be described as
   (A) harmonic motion with a period of 2 s
   (B) harmonic motion with a period of 1 s
   (C) harmonic motion with a period of $1/2$ s
   (D) motion with a constant velocity
   (E) motion with a constant momentum

25. An object oscillates in simple harmonic motion along the x-axis according to the equation $x = 6\cos(4t)$. The period of oscillation of the object is
   (A) $1/4$ s
   (B) 4 s
   (C) $\pi/4$ s
   (D) $\pi/2$ s
   (E) 4π s

26. A mass m oscillates on the end of a string of length L. The frequency of the pendulum is f. How would you increase the frequency of the pendulum to $2f$?



(A) Increase the length of the pendulum to $4L$
(B) Decrease the length of the pendulum to $L/4$
(C) Increase the length of the pendulum to $2L$
(D) Decrease the length of the pendulum to $L/2$
(E) Decrease the mass of the pendulum to $m/2$

27. A mass hangs from two parallel springs, each with the same spring constant k. Compared to the period T of the same mass oscillating on one of the springs, the period of oscillation of the mass with both springs connected to it is

(A) $1/4T$
(B) $1/2T$
(C) T (unchanged)
(D) $2T$
(E) $4T$

28. Which of the following is generally true for an object in simple harmonic motion on a spring of constant k?

(A) The greater the spring constant k, the greater the amplitude of the motion.
(B) The greater the spring constant k, the greater the period of the motion.
(C) The greater the spring constant k, the greater the frequency of the motion.
(D) The lower the spring constant k, the greater the frequency of the motion.
(E) The lower the spring constant k, the greater the kinetic energy of the motion.

Questions 30-32: A harmonic oscillator follows the equation $\frac{d^2x}{dt^2} = -4x$. The spring constant k is 4 N/m.

29. The angular frequency ω of the harmonic motion is

(A) zero
(B) 2 rad/s
(C) 4 rad/s
(D) 8 rad/s
(E) 16 rad/s

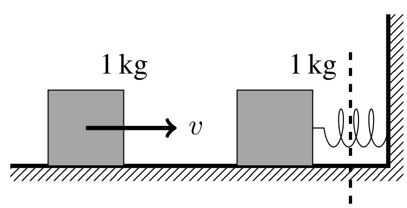
30. The mass m oscillating on the spring is

(A) 1 kg
(B) 2 kg
(C) 4 kg
(D) 8 kg
(E) 16 kg

31. The period T of oscillation is

(A) zero
(B) $\pi/4$s
(C) $\pi/2$s
(D) π s
(E) 2π s

32. A pendulum of length L has a period of 2 s on Earth. A planetary explorer takes the same pendulum of length L to another planet where its period is 1 s. The gravitational acceleration on the surface of this planet is most nearly

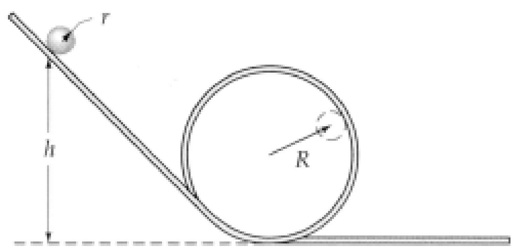


(A) $8g$
(B) $4g$
(C) $2g$
(D) $g/2$
(E) $g/4$

AP PHYSICS 1 & C: CIRCULAR MOTION AND SIMPLE HARMONIC MOTION
SECTION II
6 Questions

Directions: Answer all questions. The suggested time is about 10 minutes for answering each of the questions. The parts within a question may not have equal weight. All final numerical answers should include appropriate units. Credit depends on the quality of your solutions and explanations, so you should show your work. Credit also depends on demonstrating that you know which physical principles would be appropriate to apply in a particular situation. Therefore, you should clearly indicate which part of a question your work is for.

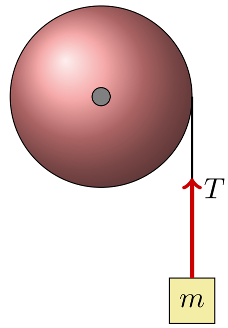
1. A uniform ball of radius r rolls without slipping along the loop-the-loop track in the figure below. The ball starts at rest at a height of h above the bottom of the loop.



(a) If it is not to leave the track at the top of the loop, what is the least value h can have (in terms of radius R of the loop)?

(b) What would h have to be if, instead of rolling, the ball slides without friction?

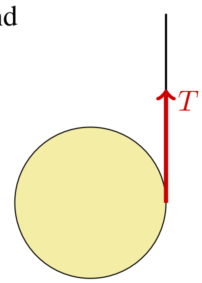
2. A uniform sphere of mass M and radius R is free to rotate, without friction, about a horizontal axis through its center. A string is wrapped around the sphere and is attached to a body of mass m as shown in the figure below. Find



(a) the acceleration of the body, and

(b) the tension in the string.

3. A uniform cylinder of mass M and radius R has a string wrapped around it. The string is held fixed, and the cylinder falls vertically as shown in the figure below. Find



(a) the acceleration of the body, and

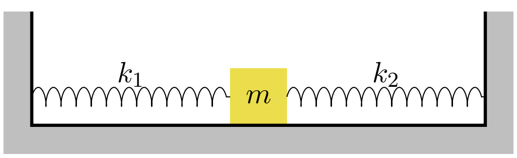
(b) the tension in the string.

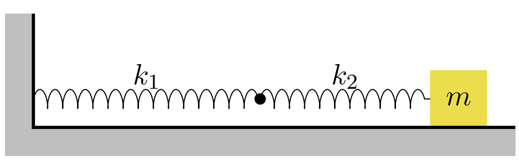
4. In heavy seas, the bow of a battle ship undergoes a simple harmonic vertical pitching motion with a period of 8.0 s and an amplitude of 2.0 m.
   (a) What is the maximum vertical velocity of the battle ship's bow?

   (b) What is its maximum acceleration?

   (c) An 80 kg sailor is standing on the scale in the bunk room in the bow. What are the maximum and minimum reading on the scale in newtons?

5. Show that for the situations in the figures below, the object of mass m oscillates with a frequency of $f = \frac{1}{2\pi}\sqrt{\frac{k_{\text{eff}}}{m}}$ where k_{eff} is given by (a) $k_{\text{eff}} = k_1 + k_2$ and (b) $\frac{1}{k_{\text{eff}}} = \frac{1}{k_1} + \frac{1}{k_2}$. Hint: find the net force on the mass and write $F = -k_{\text{eff}}x$. Note that in (b), the springs stretch by different amounts, the sum of which is x.

(a) 

(b) 

6. A simple pendulum of length L is released from rest from an angle of θ_0.
   (a) Assuming the motion of the pendulum to be simple harmonic motion, find its speed as it passes through $\theta = 0$.

   (b) Using the conservation of energy, find this speed exactly.

   (c) Show that your results for (a) and (b) are the same when θ_0 is small.

   (d) Find the difference in your results for $\theta_0 = 0.20\,\mathrm{rad}$ and $L = 1\,\mathrm{m}$.