Hall Effect

Special CAP/AP Lecture

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Current Through the Conductor

The current in the conductor is:

$$\boxed{I = \frac{dQ}{dt} = neAv_d}$$

Quantity	Symbol	SI Unit
Current	I	A
Charge carrier density (carriers per volume)	n	$/\mathrm{m}^3$
Elementary charge	e	C
Cross-section area of the conductor	A	m^2
Drift velocity of the charge carriers	v_d	m/s

For simplicity, we *assume* that the charge carriers are positive. While the opposite is true, the behaviour will be identical.

Charge Carrier Density

Density of free electrons in a metal involves basic physical data about the metal:

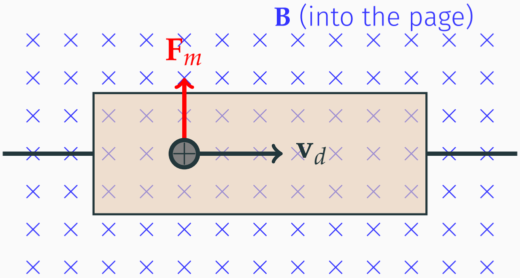
$$\boxed{n = \frac{\rho k N_A}{M}}$$

Quantity	Symbol	SI Unit
Charge carrier density	n	$/\mathrm{m}^3$
Density of material	ρ	kg/m^3
Number of free electrons per atom	k	
Avogadro's number	N_A	$/\mathrm{mol}$
Molar mass	M	kg/mol

For copper, $M = 63.54 \times 10^{-3}\,\mathrm{kg}/\mathrm{mol}$, $\rho = 9.0 \times 10^3\,\mathrm{kg}/\mathrm{m}^3$, $k = 1$ and therefore $n = 8.5 \times 10^{28}\ /\mathrm{m}^3$.

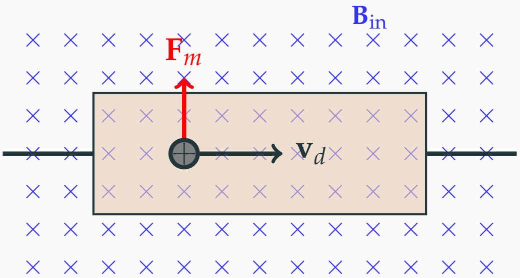
Hall Effect

When a current I flows through a conductor in a magnetic field $\mathbf{B}$, the magnetic field exerts a transverse force on the moving charges which pushes them toward one side of the conductor. This is called **Hall effect**.



This is most evident in a thin flat conductor as illustrated.

Magnetic Force

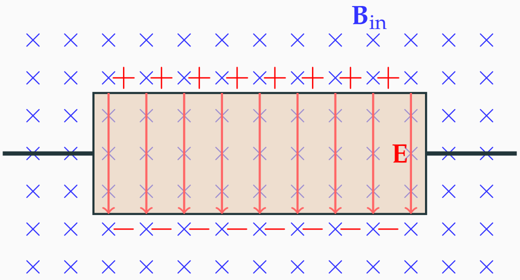


As the charges enter the magnetic field, they experience a magnetic force directed toward the top:

$$\mathbf{F}_m = e\mathbf{v}_d \times \mathbf{B} = \frac{e\mathbf{I} \times \mathbf{B}}{neA}$$

leading to a surplus of positive charges on the top, and negative charges on the bottom.

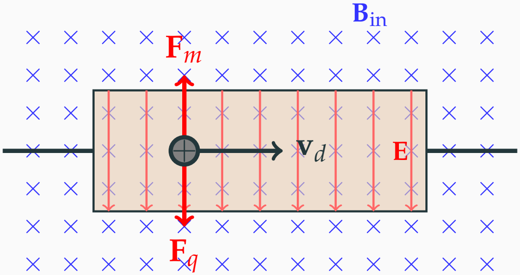
Hall Voltage



The charge imbalance caused by the magnetic force on the charge carriers creates an electric field $\mathbf{E}$, pointing toward the bottom of the page, and therefore a voltage across two sides of the conductor, called the **Hall voltage**:

$$V_H = |\mathbf{E}|W$$

Balancing Electrostatic & Magnetic Forces



Subsequently, charge carriers entering the magnetic field will experience both a magnetic force and an electrostatic force. At equilibrium, the two forces are balanced, i.e.:

$$\mathbf{F}_m = -\mathbf{F}_q$$

Calculating Hall Voltage

The electrostatic force on the charge carrier can be expressed in terms of the Hall voltage V_H across the two sides of the plate:

$$F_q = eE = \frac{eV_H}{W}$$

Equating this expression to the magnetic force, we can solve for the Hall voltage:

$$F_m = F_q \quad \rightarrow \quad \frac{IB}{nA} = \frac{eV_H}{W}$$

Hall Voltage

Cancelling terms and noting that the thickness of the conductor is

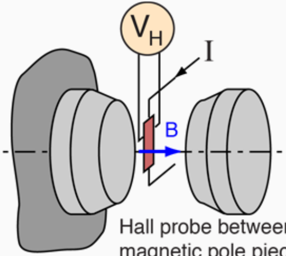
$$d = \frac{A}{W}$$

we find the expression for the Hall voltage V_H:

$$\boxed{V_H = \frac{IB}{ned}}$$

Hall Probe

Large magnetic fields ($\sim 1\,\text{T}$) is often measured using a **Hall probe**. A thin film Hall probe is placed in the magnetic field and the transverse voltage (usually measured in on the order of $10^{-6}\,\text{V}$) is measured.



Hall probe between magnetic pole pieces

The polarity of the Hall voltage for a copper probe shows that electrons (negative charge) are the charge carriers.