Files for You to Download

- `00-outline.pdf`—The course outline
- `01-Calculus-print.pdf`—The slides that I used last week
- `01-integration.pdf`—The slides that I am using right now
- `01-Homework.pdf`—Last/this week's homework assignment

Please download/print the PDF file for the class slides before each class.

On Differential Calculus

A quick review

- Finding out how quickly a physical quantity is changing ("rate of change" of that quantity)
- Math: slopes of functions
- Terminology:
    - A ***derivative***: The slope of a function (noun)
    - To ***differentiate***: Finding the derivative with respect to a variable (verb)
- Last class: went through the rules and some examples of derivatives

Examples of Derivatives in Physics

- Instantaneous velocity $\mathbf{v}(t)$ is the derivative of position $\mathbf{s}(t)$

$$\mathbf{v}(t) = \frac{d\mathbf{s}}{dt}$$

- Instantaneous acceleration $\mathbf{a}(t)$ is the derivative of velocity $\mathbf{v}(t)$. It's also the "second derivative" (derivative of a derivative) of position $\mathbf{s}$ with respect to time

$$\mathbf{a}(t) = \frac{d\mathbf{v}}{dt} = \frac{d^2\mathbf{s}}{dt^2}$$

- Instantaneous force $\mathbf{F}(t)$ is the derivative of momentum $\mathbf{p}$ (Newton's second law of motion)

$$\mathbf{F}(t) = \frac{d\mathbf{p}}{dt}$$

They're All Vectors

Resolve them into components

- Notice that position $\mathbf{s}$, velocity $\mathbf{v}$, acceleration $\mathbf{a}$, momentum $\mathbf{p}$, force $\mathbf{F}$ are all vector quantities with x, y and z components
- In this case, we take the derivative separately in each direction.

$$\mathbf{v}(t) = \frac{d\mathbf{s}}{dt} = \frac{d}{dt}\left(s_x(t)\mathbf{i} + s_y(t)\mathbf{j} + s_z(t)\mathbf{k}\right)$$

where s_x, s_y and s_z are the x-, y- and z-components of $\mathbf{s}$

- In AP or 1st-year physics, s_x, s_y and s_z are functions of time only, but in practical problems in physics and engineering, they are often functions of x, y and z coordinates as well. (This is *multi-variable calculus*. It's a lot of fun!)

What the Notation Tell Us

- When we say we that **velocity is the time rate of change of position**

$$\mathbf{v} = \frac{d\mathbf{s}}{dt}$$

- We are really asking **what is the (small) change in position ds for an infinitesimal (infinitely small) change in time dt?**

NOW ON TO INTEGRATION

Integration: Area Under the Curve

Let's start with an example

- A car is moving with speed $v(t) = 5t$. What is its displacement at $t = 5$?
- We know that if on a v-t graph, and the area under that curve is the displacement. So how do we find the area?
- If we divide 5 into many small time intervals:

$$\Delta t_1,\ \Delta t_2,\ \Delta t_3,\ \Delta t_4, \ldots,\ \Delta t_n$$

  We can find the displacement in teach of these Δt_i, and
- In this example, the total displacement would be

$$d(5) = \lim_{n\to\infty} \sum_{i=1}^{n} v(t_i)\Delta t_i = \int_{t_1}^{t_2} v(t)dt = \int_{t=0}^{5} 5t\ dt = \frac{5}{2}t^2\Big|_0^5 = \frac{125}{2}$$

Remember Our Kinematic Equations?

- In Physics 11 and 12, you were introduced to a set of 5 kinematic equations, which applies to constant acceleration.
- Now that we know something about integration, we can understand these equations a little bit better
- We start with a constant acceleration a. The velocity is the integral:

$$v(t) = \int a dt = at + C$$

- We know that at $t = 0$, $v = v_0$ ("initial value"). Substituting those allow to find $C = v_0$, and therefore

$$\boxed{v(t) = v_0 + at}$$

Remember Our Kinematic Equations?

- Now we integrate $v(t)$ again to get position $s(t)$:

$$s(t) = \int v(t)dt = \int (v_0 + at)dt = v_o t + \frac{1}{2}at^2 + C$$

- Again, we take advantage of know our initial position, so $C = s_o$, and we have:

$$\boxed{s(t) = s_0 + v_o t + \frac{1}{2}at^2}$$

- You may be more familiar with this expression, where we use *displacement* $\Delta s(t) = s(t) - s_0$ instead of position s:

$$\Delta s(t) = v_o t + \frac{1}{2}at^2$$

Remember Our Kinematic Equations?

- In practical situations, acceleration is *not* constant, and we generally have to differentiate or integrate to find your answers.