Hall Effect

Advanced Placement Physics

Dr. Timothy Leung

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Olympiads School, Toronto, ON, Canada

Current Through the Conductor

The (instantaneous) electric current through conductor is the rate at which charge carriers pass through a point in the conductor:

$$\boxed{I = \frac{dQ}{dt}} = \frac{dQ}{dt} = \left(\frac{Q}{V}\right)\frac{dV}{dt} = [ne]\,[Av_d]$$

where

- dQ/dV is the total amount of charges *per volume*, which is just the charge carrier density n times the elementary charge e
- dV/dt is the rate the volume of charges moves through the conductor, give by the cross-section area of the conductor A times the **drift velocity** v_d of the charge carrier

For simplicity, we *assume* that the charge carriers are positive. While the opposite is true, the behaviour will be identical.

Current Through the Conductor

Combining the terms:

$$\boxed{I = \frac{dQ}{dt} = neAv_d}$$

Quantity	Symbol	SI Unit
Current	I	A
Charge carrier density (carriers per volume)	n	$/\mathrm{m}^3$
Elementary charge	e	C
Cross-section area of the conductor	A	m^2
Drift velocity of the charge carriers	v_d	m/s

The calculation for the charge carrier density n requires some additional thoughts.

Charge Carrier Density

The charge carrier density in a *metal* conductor involves some physical information about the metal:

1. Divide the metals density ρ by the metal's molar mass M to find the *number of moles of atams per unit volume*
2. Multiply by Avagadro's number N_A to find *number of atoms per unit volume*
3. Multiply by *the number of free electrons per atom* k for that particular metal

Charge Carrier Density

Collecting all the terms from the last slide, we have:

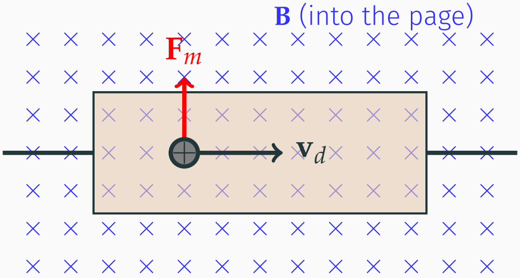
$$\boxed{n = \frac{\rho k N_A}{M}}$$

Quantity	Symbol	SI Unit
Charge carrier density	n	$/\mathrm{m}^3$
Density of material	ρ	kg/m^3
Number of free electrons per atom	k	
Avogadro's number	N_A	$/\mathrm{mol}$
Molar mass	M	kg/mol

For copper, $M = 63.54 \times 10^{-3}\,\mathrm{kg}/\mathrm{mol}$, $\rho = 9.0 \times 10^{3}\,\mathrm{kg}/\mathrm{m}^3$, $k = 1$ and therefore $n = 8.5 \times 10^{28}\,/\mathrm{m}^3$.

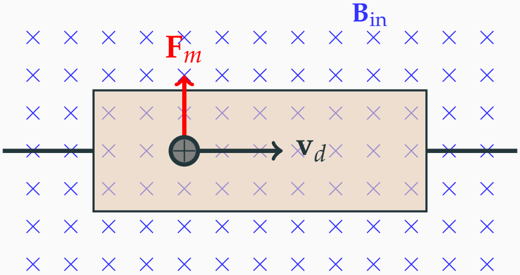
Hall Effect

When a current I flows through a conductor in a magnetic field $\mathbf{B}$, the magnetic field exerts a transverse (i.e. perpendicular to motion) magnetic force $\mathbf{F}_m$ on the moving charges which pushes them toward one side of the conductor. This is called **Hall effect**.



This is most evident in a *thin*, *flat* conductor as illustrated.

Magnetic Force

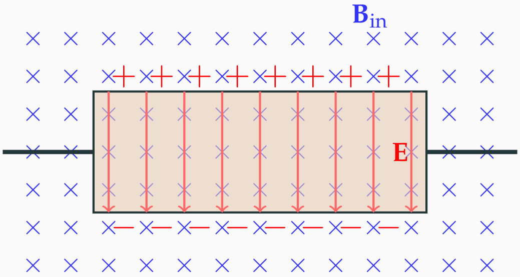


As the charges enter the magnetic field, $\mathbf{F}_m$ is directed toward the top:

$$\mathbf{F}_m = e\mathbf{v}_d \times \mathbf{B} = \frac{e\mathbf{I} \times \mathbf{B}}{neA}$$

leading to a surplus of positive charges on the top edge of the conductor, and negative charges on the bottom.

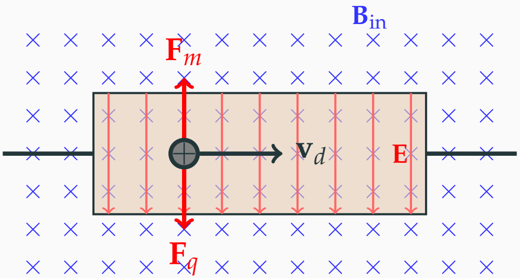
Hall Voltage



The charge imbalance on the conductor creates an electric field $\mathbf{E}$, pointing toward the bottom, and therefore a voltage across two sides of the conductor (width W), called the **Hall voltage**:

$$V_H = EW$$

Balancing Electrostatic & Magnetic Forces



Subsequently, charge carriers entering the magnetic field will experience both a magnetic force and an electrostatic force. At equilibrium, the two forces are balanced:

$$\mathbf{F}_m + \mathbf{F}_q = \mathbf{0}$$

Calculating Hall Voltage

The electrostatic force on the charge carrier can be expressed in terms of the Hall voltage V_H across the two sides of the plate:

$$F_q = eE = \frac{eV_H}{W}$$

Equating the magnitudes of electrostatic and magnetic forces, we can solve for the Hall voltage:

$$F_m = F_q \quad \rightarrow \quad \frac{IB}{nA} = \frac{eV_H}{W}$$

Hall Voltage

Cancelling terms and noting that the thickness of the conductor is

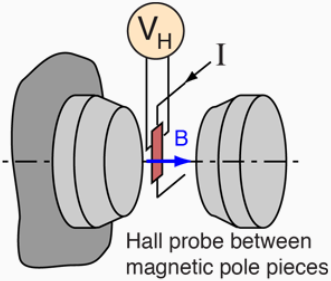
$$d = \frac{A}{W}$$

we find the expression for the Hall voltage V_H:

$$\boxed{V_H = \frac{IB}{ned}}$$

Hall Probe

Large magnetic fields ($\sim 1\,\mathrm{T}$) is often measured using a **Hall probe**. A thin film Hall probe is placed in the magnetic field and the transverse voltage (usually measured in on the order of $10^{-6}\,\mathrm{V}$) is measured.



The polarity of the Hall voltage for a copper probe shows that electrons (negative charge) are the charge carriers.