Class 12: Circuits Analysis

AP Physics

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Olympiads School

February 2018

Notice: Tim Will Be Away Next Week

Tim will be away to do a concert in Ottawa next Saturday (February 17). There will be a supply teacher for next week. Please be kind to him! Tim will return to Olympiads on Sunday, February 18,, and will resume teaching this class on the 24th (class 14).

Files for You to Download

Download from the school website:

1. `12-Circuits_print.pdf`—The print version of this presentation. If you want to print on paper, I recommend printing 4 pages per side.
2. `13-Homework.pdf`—Homework assignment for this class and next class. The file will be posted along Class 13 slides.

Please download/print the PDF file before each class. There is no point copying notes that are already printed out for you. Instead, take notes on things I say that aren't necessarily on the slides.

Resistors

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Resistance of a Conductor

The resistance of a conductor is proportional to the resistivity ρ and its length L, and inversely proportional to the cross-sectional area A:

$$\boxed{R = \rho \frac{L}{A}}$$

Quantity	Symbol	SI Unit
Resistance	R	Ω (ohms)
Resistivity	ρ	Ω m (ohm metres)
Length of conductor	L	m (metres)
Cross-sectional area	A	m^2 (square metres)

Resistance of a Conductor

$$\boxed{R = \rho \frac{L}{A}}$$

Gauge	Diameter (mm)	R/L $(10^{-3}\,\Omega/\text{m})$
0	9.35	0.31
10	2.59	2.20
14	1.63	8.54
18	1.02	21.90
22	0.64	51.70

Material	Resistivity ρ (Ω m)
silver	1.6×10^{-8}
copper	1.7×10^{-8}
aluminum	2.7×10^{-8}
tungsten	5.6×10^{-8}
Nichrome	100×10^{-8}
carbon	3500×10^{-8}
germanium	0.46
glass	10^{10} to 10^{14}

Ohm's Law

The electric potential difference V across a "load" (resistor) equals the product of the current I through the load and the resistance R of the load.

$$\boxed{V = IR}$$

Quantity	Symbol	SI Unit
Potential difference	V	V (volt)
Current	I	A (ampere)
Resistance	R	Ω (ohm)

A resistor is considered "ohmic" if it obeys Ohm's law

Power Dissipated by a Resistor

Power is the rate at which work W is done, and from electrostatics, the change in electric potential energy ΔE_q (i.e. the work done!) is proportional to the amount of charge q and the voltage V. This gives a very simple expression for power through a resistor:

$$P = \frac{dW}{dt} = \frac{d(qV)}{dt} = \left(\frac{dq}{dt}\right) V = \boxed{IV}$$

Quantity	**Symbol**	**SI Unit**
Power through a resistor	P	W (watt)
Current through a resistor	I	A (ampere)
Voltage across the resistor	V	V (volt)

Other Equations for Power

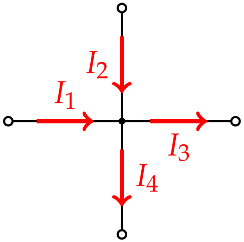
When we combine Ohm's Law ($V = IR$) with power equation, we get two additional expressions for power through a resistor:

$$\boxed{P = \frac{V^2}{R}} \qquad \boxed{P = I^2 R}$$

Quantity	Symbol	SI Unit
Power	P	W (watts)
Voltage	V	V (volts)
Resistance	R	Ω (ohms)
Current	I	A (amperes)

Kirchkoff's Current Law

The electric current that flows into any junction in an electric circuit must be equal to the current which flows out.



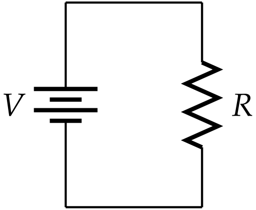
e.g. if there are 4 paths to the junction at the center, with I_1 and I_2 going into the junction, and I_3 and I_4 coming out, then the current law says that

$$I_1 + I_2 - I_3 - I_4 = 0$$

Basically, it means that there cannot be any accumulation of charges anywhere in the circuit. The law is a consequence of conservation of energy.

Kirchkoff's Voltage Law

The voltage changes around any closed loop in the circuit must sum to zero, no matter what path you take through an electric circuit.



Assume that the current flows clockwise and we draw a clockwise loop, we get

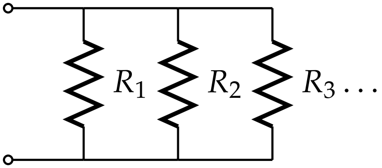
$$V - V_R = 0 \quad \rightarrow \quad V - IR = 0$$

If I incorrectly guess that I flows counterclockwise, I will still have a similar expression

$$-V_R - V = 0 \quad \rightarrow \quad -V - IR = 0$$

When solving for I, we get a negative number, indicating that my guess was in the wrong direction.

Resistors in Parallel



From the current law, we know that the total current is the current through all the resistors, which we can rewrite in terms of voltage and resistance using Ohm's law:

$$I = I_1 + I_2 + I_3 \cdots = \frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3} \cdots$$

But since we also know that $V_1 = V_2 = V_3 = \cdots = V$ from the voltage law, we can re-write as

$$I = \frac{V}{R_{\mathrm{eq}}} = V\left(\frac{1}{R_1} + \frac{1}{R_1} + \frac{1}{R_1} \cdots\right)$$

Resistors in Parallel

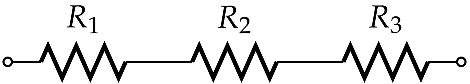
Equivalent Resistance

Through applying Ohm's Law and Kirchkoff's laws, we find the equivalent resistance of a parallel circuit: **The inverse of the equivalent resistance for resistors connected in parallel is the sum of the inverses of the individual resistances.**

$$\boxed{\frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N}}$$

Quantity	Symbol	SI Unit
Equivalent resistance	R_{eq}	Ω (ohm)
Resistance of individual loads	$R_{1,2,3,\cdots,N}$	Ω (ohm)

Resistors in Series



The analysis for resistors in series is similar (but easier). From the current law, the current through each resistor is the same:

$$I_1 = I_2 = I_3 = \cdots = I$$

And the total voltage drop across all resistor is therefore:

$$V = V_1 + V_2 + V_3 + \cdot = I(R_1 + R_2 + R_3 + \cdots)$$

Equivalent Resistance

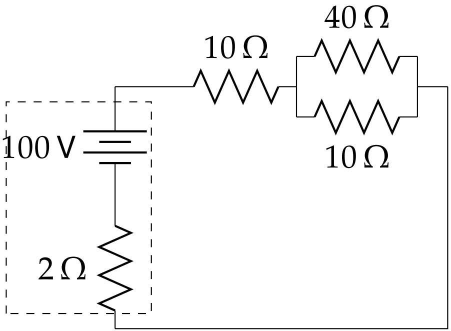
Again, through applying Ohm's Law and Kirchkoff's laws, we find that when resistors are connected in series: **the equivalent resistance of loads is the sum of the resistances of the individual loads.**

$$\boxed{R_{\text{eq}} = R_1 + R_2 + \cdots + R_N}$$

Quantity	Symbol	SI Unit
Equivalent resistance	R_{eq}	Ω (ohm)
Resistance of individual loads	$R_{1,2,3,\cdots,N}$	Ω (ohm)

Example Problem (Simple)

A simple circuit analysis problem will involve one voltage source and resistors connected, some in parallel, and some in series. Below is a typical example:



Two $10\,\Omega$ resistors and a $40\,\Omega$ resistor are connected as shown to a $100\,\mathrm{V}$ emf source with internal resistance $2\,\Omega$. How much power is dissipated by the $40\,\Omega$ resistor?

(A) $160\,\mathrm{W}$

(B) $40\,\mathrm{W}$

(C) $400\,\mathrm{W}$

(D) $5\,\mathrm{W}$

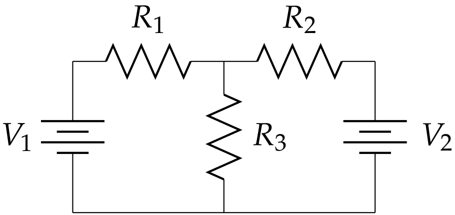
(E) $500\,\mathrm{W}$

Tips for Solving "Simple" Circuit Problems

1. Identify groups of resistors that are in parallel or in series, and find their equivalent resistance.
2. Gradually reduce the entire circuit to one voltage source and one resistor.
3. Using Ohm's law, find the current out of the battery.
4. Uing Kirchkoff's laws, find the current through each of the resistors.

Circuits Aren't Always Simple

Some of these problems require you to solve a system of linear equations. The following is a simple example with two voltage sources:



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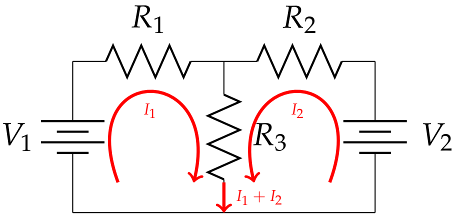
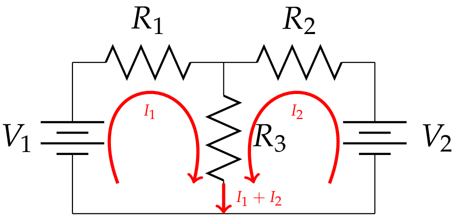


In this case, we have to draw two loops of current.

A More Difficult Example



We split the circuit into two loops, and apply Kirchkoff's voltage in both:

$$V_1 - I_1R_1 - (I_1 + I_2)R_3 = 0$$
$$V_2 - I_2R_2 - (I_1 + I_2)R_3 = 0$$

Two equations, two unknowns (I_1 and I_2). We can subtract (2) from (1), then solve for I_1 and I_2:

$$I_1 = \frac{V_1 - I_2R_3}{R_1 + R_3} \qquad I_2 = \frac{\left[V_2 - \frac{(V_1 - V_2)R_3}{R_1}\right]}{\left[R_2 + \frac{(R_1+R_2)R_3}{R_1}\right]}$$

(Try this at home as an exercise.)

Capacitors in Parallel

$$\boxed{C_{\text{eq}} = C_1 + C_2 + \cdots + C_N}$$

Capacitors in Series

Circuits with Resistors and Capacitors

Discharging a capacitor

Charging a Capacitor