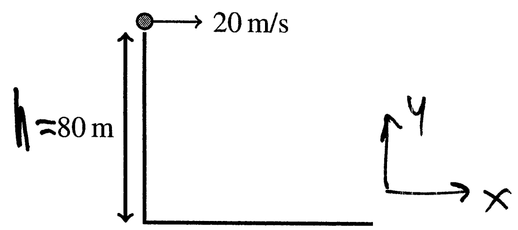
AP PHYSICS 1 & C: KINEMATICS

Directions: Each of the questions or incomplete statements below is followed by five suggested answers or completions. Select the one that is best in each case and place the letter of your choice in the corresponding box on the student answer sheet.

Note: To simplify calculations, you may use $g = 10\,\text{m/s}^2$ in all problems.

Questions 1-2

A ball of mass 0.5 kg is launched horizontally from the top of a cliff 80 m high with a speed of 20 m/s at time $t = 0$.



1. The horizontal distance x traveled by the ball before striking the ground is

   (A) 20 m
   (B) 40 m
   (C) 80 m
   (D) 160 m
   (E) 320 m

   $t=\sqrt{\frac{2y}{g}}=\sqrt{\frac{(2)(80)}{10}}=4$

   $4\times 20=80$

2. The speed of the ball just before striking the ground is

   (A) 4 m/s
   (B) 14 m/s
   (C) 20 m/s
   (D) 44 m/s
   (E) 64 m/s

   $v_x=20$

   $v_y=4\times 10=40$

   $v=\sqrt{20^2+40^2}=44.7$

3. A space explorer throws a tool downward on a planet with an initial velocity of 2.0 m/s from a height of 6 m above the surface. The tool strikes the surface in a time of 2 s. The acceleration due to gravity on the planet is

   (A) $1\,\text{m/s}^2$
   (B) $2\,\text{m/s}^2$
   (C) $3\,\text{m/s}^2$
   (D) $4\,\text{m/s}^2$
   (E) $10\,\text{m/s}^2$

Questions 4–5

A sprinter starting from rest runs a 100-meter race on a straight track. The sprinter covers the first 10 meters with a constant acceleration in 2 seconds. The sprinter runs the remaining 90 m with the same velocity he had at the end of 2 s.

4. The sprinter's velocity at the end of the first 2 s is

   (A) 5 m/s
   (B) 10 m/s
   (C) 20 m/s
   (D) 40 m/s
   (E) 60 m/s

   $d=\frac{1}{2}at^2$

   $a=\frac{2d}{t^2}$

5. The total time it takes for the sprinter to run the full 100 m is

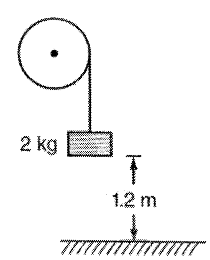
   10m 90m

   2s $t=\frac{d}{v}=\frac{90}{10}=9$

   (A) 2 s
   (B) 9 s
   (C) 10 s
   (D) 11 s
   (E) 12 s

   $9+2=11$

6. A block of mass 2 kg is attached to a string that is wrapped around a pulley of negligible mass and allowed to descend from rest a vertical distance of 1.2 m in a time of 1.5 s. The acceleration of the block is most nearly

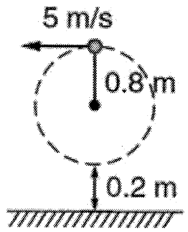


   (A) $0.2\,\text{m/s}^2$
   (B) $0.6\,\text{m/s}^2$
   (C) $1.1\,\text{m/s}^2$
   (D) $1.4\,\text{m/s}^2$
   (E) $1.5\,\text{m/s}^2$

   $y=\frac{1}{2}at^2$

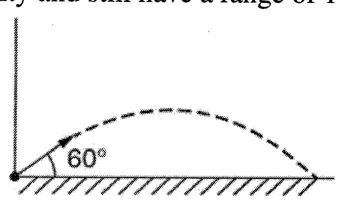
   $a=\frac{2y}{t^2}=\frac{(2)(1.2)}{1.5^2}$

7. A ball is attached to a string of length 0.8 m and is swung in a vertical circle. The bottom of the circle is 0.2 m above the floor. If the string breaks at the top of the circle when the speed of the ball is 5 m/s, the horizontal distance the ball travels before striking the floor is



(A) 0.8 m
(B) 2.3 m
(C) 3.0 m
(D) 5.0 m
(E) 13.2 m

8. A golf ball is hit from level ground and has a horizontal range of 100 m. The ball leaves the golf club at an angle of 60° to the level ground. At what other angle(s) can the ball be struck at the same initial velocity and still have a range of 100 m?



(A) 30°
(B) 20° and 80°
(C) 10° and 120°
(D) 45° and 135°
(E) There is no other angle other than 60° in which the ball will have a range of 100 m.

$90^\circ - 60^\circ = 30^\circ$

Questions 9–10
A particle moves on a horizontal surface with a constant acceleration of 6 m/s^2 in the x-direction and 4 m/s^2 in the y-direction. The initial velocity of the particle is 3 m/s in the x-direction.

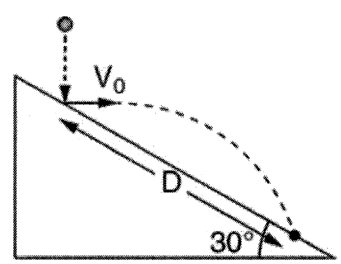
9. The speed of the particle after 4 s is

(A) 16 m/s
(B) 27 m/s
(C) 31 m/s
(D) 44 m/s
(E) 985 m/s

10. The displacement of the particle from its initial position is

(A) 16 m
(B) 32 m
(C) 60 m
(D) 68 m
(E) 92 m

11. A rubber ball is dropped from rest onto a plane angled at 30° to the horizontal floor and bounces off the plane with a horizontal speed v_o. The ball lands on the plane a distance D along the plane, as shown below. In terms of v_o, D, and g, the speed of the ball just before striking the plane is



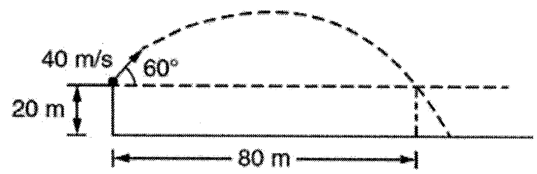
(A) v_o
(B) $\left(v_o^2 + 2D\sin\theta g\right)^{\frac{1}{2}}$
(C) $\left(v_o + \dfrac{D\sin\theta}{g}\right)^{\frac{1}{2}}$
(D) $\left(v_o^2 + \dfrac{D\sin\theta}{g}\right)^{\frac{1}{2}}$
(E) $(2D\sin\theta g)^{\frac{1}{2}}$

$\theta = 30^\circ$

$v^2 = \cancel{v_0^2} + 2ad$

$v^2 = 2g(D\sin\theta)$

12. A projectile is launched from a platform 20 m high above level ground. The projectile is launched with a velocity of 40 m/s at an angle of 60° above the horizontal. The projectile follows a parabolic path and reaches its original height at a horizontal distance of 80 m, but moves past the height of the cliff to strike the ground below. The total time from the launch until it strikes the ground is



(A) 2.0 s
(B) 4.0 s
(C) 5.0 s
(D) 7.5 s
(E) 9.0 s

13. A stack of coffee filters falls from rest through the air. Due to air resistance, the filters fall with an acceleration proportional to the velocity of fall, that is, $a = -kv$, where k is a positive constant. The velocity of the falling filters as a function of time of fall is

(A) $-kv^2$
(B) $-12kv^2$
(C) $-k$
(D) $\ln(kt)$
(E) $v_0 e^{-kt}$

$a = \frac{dv}{dt} = -kv$

$\frac{dv}{v} = -kdt$

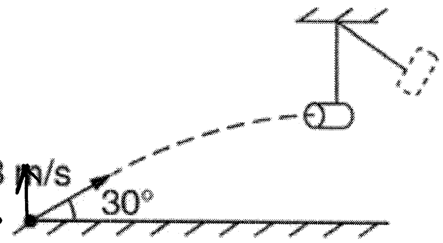
$\ln v = -kt + C$

$v = e^{-kt+C}$

$\to V_0 e^{-kt}$

14. A small ball is launched with a speed of 8 m/s at an angle of 30° from the horizontal. A cup is hung so that it is in position to catch the ball when it reaches its maximum height. How far above the floor should the cup be hung to catch the ball?

$V_y = 4\,m/s$



(A) 2.4 m
(B) 1.6 m
(C) 1.0 m
(D) 0.8 m
(E) 0.4 m

$V_2^2 = V_1^2 + 2ad$

$0 = 4^2 - 20d$

$d = \frac{16}{20}$ (0.8)

Questions 16-17

A car of mass m travels along a straight horizontal road. The car begins with a speed v_0, but accelerates according to the velocity function $v = \left(v_0^2 + \frac{Ct^2}{m}\right)$, where t is time.

always (+) always (+)

15. The speed of the car is zero at a time t of

(A) zero
(B) $2t$
(C) $4t$
(D) $\sqrt{8t}$
(E) The speed of the car is never zero.

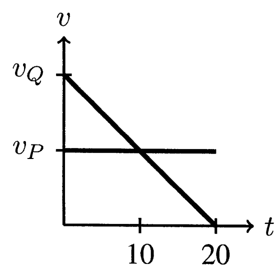
16. The acceleration of the car as a function of time is

(A) $\left(v_0^2 + \frac{Ct^2}{m}\right)$
(B) $\left(v_0^2 + \frac{2Ct^2}{m}\right)$
(C) $\left(v_0 + \frac{Ct}{m}\right)$
(D) $\left(\frac{2Ct}{m}\right)$
(E) $\left(\frac{2Ct^2}{m}\right)$

$a = \frac{dv}{dt}$

Questions 17–18

The graph shown below represents the velocity vs. time graphs for two cars, P and Q. Car P begins with a speed v_P, and Car Q begins with a speed v_Q which is twice the velocity of Car P, that is, $v_Q = 2v_P$.

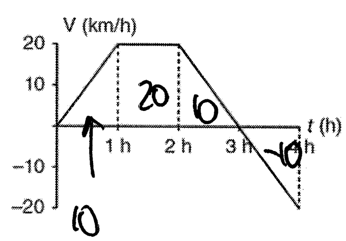


17. Which of the following is true at a time of 10 s?
   (A) The cars occupy the same position.
   (B) Car P is at rest.
   (C) $v_Q > v_P$
   (D) $v_P > v_Q$
   (E) Car Q is ahead of Car P.

area under the curve

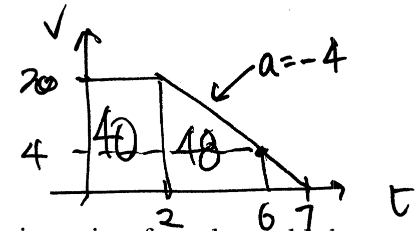
18. Which of the following is true at a time of 20 s?
   (A) The cars occupy the same position.
   (B) Car P is at rest.
   (C) $v_Q > v_P$
   (D) $a_P = a_Q$
   (E) Car P is ahead of Car Q.

19. The velocity vs. time graph below represents the motion of a bicycle rider. The displacement of the rider between 0 and 4 h is

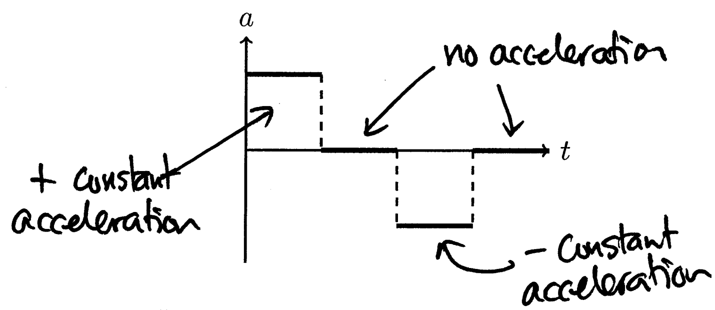


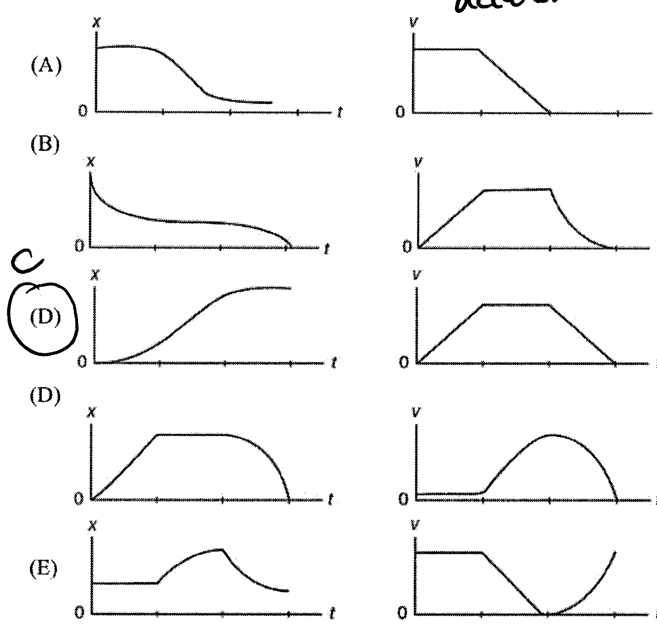
   (A) +10 km
   (B) +20 km
   (C) +30 km
   (D) +40 km
   (E) -10 km

20. A car is initially moving with a positive velocity of 20 m/s when it passes the origin at time $t = 0$. The car continues to move at 20 m/s between $t = 0$ and $t = 2$ s. At $t = 2$ s, the driver presses the brake, giving the car an acceleration of –4 m/s^2. The displacement of the car at $t = 6$ s is
   (A) 40 m
   (B) 32 m
   (C) 48 m
   (D) 64 m
   (E) 88 m



21. Which of the following pairs of graphs could show the position vs. time and velocity vs. time graphs for the acceleration vs. time graph shown above? Assume $v = 0$ and $x = 0$ at $t = 0$.

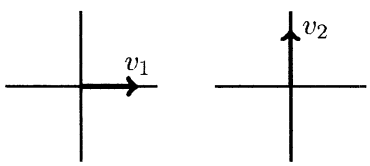




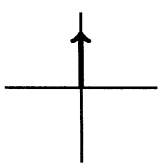
22. A small airplane can fly at 200 km/h with no wind. The pilot of the plane would like to fly to a destination 100 km due north of his present position, but there is a crosswind of 50 km/h east. How much time is required for the plane to fly north to its destination?

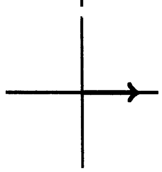
(A) less than 1/2 h
(B) 1/2 h
(C) more than 1/2 h (but less than 1 h)
(D) 1 h
(E) more than 1 h

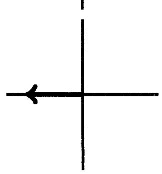
23. Two velocity vectors v_1 and v_2 each have a magnitude of 10 m/s. Graph 1 shows the velocity v_1 at $t = 0$ s, and then the same object has a velocity v_2 at $t = 2$ s, shown in Graph 2. Which of the following vectors best represents the average acceleration vector that causes the object's velocity to change from v_1 to v_2 ?

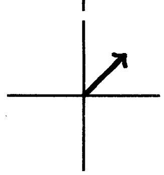


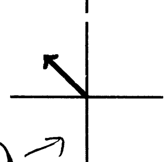
Graph 1 Graph 2

(A)


(B)


(C)


(D)


(E)


$x_0 = 0$ $V_0 = 0$

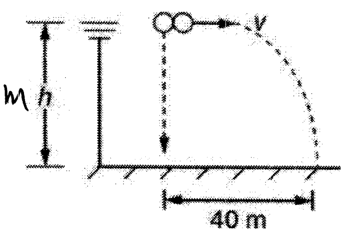
24. An object starts from rest at $t = 0$ and position $x = 0$, then moves in a straight line with an acceleration described by the equation $a = 4t^2$ in m/s^2. What is the position of the object at $t = 3$ s?

(A) 6 m
(B) 1 m
(C) 27 m
(D) 54 m
(E) 108 m

$a = 4t^2$
$v = \int a\,dt = \frac{4}{3}t^3$
$d = \int v\,dt = \frac{1}{3}t^4$
$d(3) = \frac{1}{3}(3)^4 = 3^3 = 27$

25. A ball is dropped from rest from the top of a cliff 80 meters high. At the same time, a rock is thrown horizontally from the top of the same cliff. The rock and ball hit the level ground below a distance of 40 m apart. The horizontal velocity of the rock that was thrown was most nearly

$g = 10\, m/s^2$

80m


(A) 5 m/s
(B) 10 m/s
(C) 20 m/s
(D) 40 m/s
(E) 80 m/s

$h = \frac{1}{2}gt^2$
$80 = \frac{1}{2}(10)t^2$
$t = \sqrt{\frac{80}{5}} = \sqrt{16} = 4$ sec

$V_x = \frac{dx}{t} = \frac{40}{4} = 10$

26. A stone is dropped near the surface of Mars and falls with an acceleration of 3.8 m/s^2. This means that the

(A) distance the stone falls increases 3.8 meters for each second of fall
(B) derivative of the distance fallen with respect to time is 3.8 m/s
(C) derivative of the velocity with respect to time is 3.8 m/s^2
(D) velocity is constant at 3.8 m/s
(E) derivative of the acceleration is 3.8 m/s^2

27. A 600 kg car accelerates uniformly from rest. After 4 s, it reaches a speed of 24 m/s. During the 4 s, the car has traveled a distance of

(A) 12 m
(B) 24 m
(C) 36 m
(D) 48 m
(E) 96 m

$d = V_{ave} t$
$= (12)(4)$
$\frac{12+0}{2}$
if uniform acceleration.

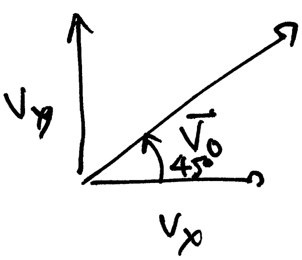
GO ON TO THE NEXT PAGE.

28. A ball is hit straight up into the air with an upward positive velocity. Wich of the following describes the velocity and acceleration of the ball at the instant it reaches the top of its flight?

	Velocity	Acceleration
(A)	0	0
(B)	0	g
(C)	$2v_0$	g
(D)	$\frac{1}{2}v_0$	0
(E)	0	$\frac{1}{2}g$

29. A toy dart gun fires a dart at an angle of 45° to the horizontal and the dart reaches a maximum height of 1 meter. If the dart were fired straight up into the air along the vertical, the dart would reach a height of

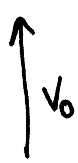
(A) 1 m
(B) 2 m
(C) 3 m
(D) 4 m
(E) 5 m



$V_y = V_0 \sin 45°$
$V_x = V_0 \cos 45°$ } same

vertical
$V^2 = V_y^2 + 2ad$

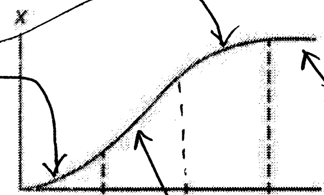
$V_y = \sqrt{(2)(10)(1)}$
$= \sqrt{20} = 4.47$
$V_0 = \frac{V_y}{\sin 45°} = 6.32 m/s$



$V = V_0^2 + 2ad$
$0 = V_0^2 - 2gd$
$d = \frac{V_0^2}{2g} = \frac{40}{20} = 2$

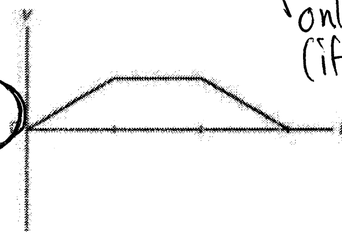
30. The graph below shows the displacement as a function of time for a car moving in a straight line. Which of the following graphs shows the velocity vs. time graph for the same time intervals?

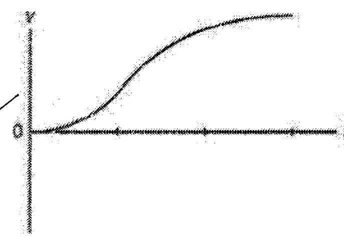
Shape is ≈ quadratic ∴ derivative is ≈ linear

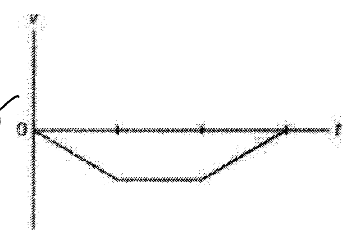


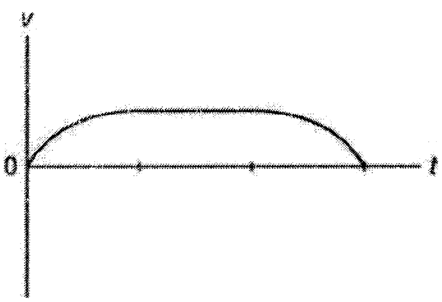
slope = 0 ∴ V = 0 (not B or E)

only slight increase (if at all of slope) ∴ V ≈ constant & positive (not B, C, E)

(A)


(B)


(C)


(D)


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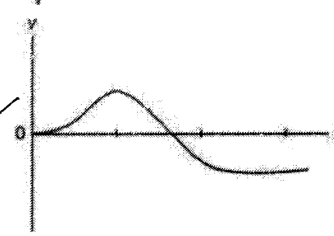

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Questions 31–32

31. An object is released from rest and falls through a resistive medium. The resistance causes the velocity of the object to change according to the equation $v = 16t - \frac{1}{2}t^4$, where v is in m/s and time is in s. Which of the following is a possible equation for the acceleration of the object as a function of time?
    (A) $16 - 2t^2$
    (B) $16 - 2t^3$
    (C) $16 - 2t$
    (D) $8t^3 - 2t^2$
    (E) $32t^3 - 2t^5$

take derivative

32. What is the terminal velocity of the object as it falls?
    (A) 5 m/s
    (B) 10 m/s
    (C) 24 m/s
    (D) 32 m/s
    (E) The object never reaches a terminal velocity.

$a = \frac{dv}{dt} = 0 \quad 16 - 2t^3 = 0$

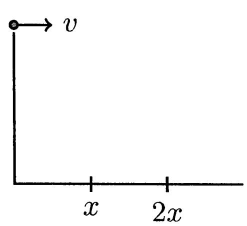
$t^3 = 8 \quad t = 2\,s$

$v(2)$

$= 16(2) - \frac{1}{2}(2)^4$

$= 32 - 8 = 24$

33. A student jumps off a cliff with an initial horizontal velocity v and lands in a lake below at a distance of x from the base of the cliff. In terms of his initial velocity v, how fast would he have had to jump to land a distance $2x$ from the base of the cliff?



(A) $\sqrt{2}v$
(B) $2v$
(C) $4v$
(D) $8v$
(E) $16v$

no change in vertical component of velocity

this question is badly posed because after reaching "terminal velocity", v actually decreases.

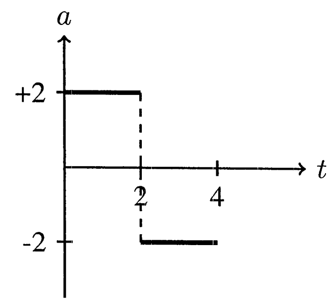
34. An astronaut drops a hammer on a moon with no atmosphere. The hammer falls a distance of 2 meters in the first second. What is the acceleration due to gravity on this moon?
    (A) 1 m/s^2
    (B) 2 m/s^2
    (C) 3 m/s^2
    (D) 4 m/s^2
    (E) 8 m/s^2

$t = 1$ $\quad$ d

$d = \frac{1}{2}at^2 \quad 2 = \frac{1}{2}a(1)$

$a = 4\,m/s^2$

35. The motion of an object is represented by the acceleration vs. time graph below. The object begins from rest. Which of the following statements is true about the motion of the object?



(A) The object returns to its original position.
(B) The velocity of the object is zero at a time of 2 s.
(C) The velocity of the object is zero at a time of 4 s.
(D) The displacement of the object is zero at a time of 4 s.
(E) The acceleration of the object is zero at a time of 2 s.

undefined at $t=2$

AP PHYSICS 1 & C: KINEMATICS
SECTION II
5 Questions

Directions: Answer all questions. The suggested time is about 10 minutes for answering each of the questions. The parts within a question may not have equal weight. All final numerical answers should include appropriate units. Credit depends on the quality of your solutions and explanations, so you should show your work. Credit also depends on demonstrating that you know which physical principles would be appropriate to apply in a particular situation. Therefore, you should clearly indicate which part of a question your work is for.

1. An object has a position vector given by $\mathbf{r} = 30t\hat{\imath} + (40t - 5t^2)\hat{\jmath}$. Find
   (a) its intantaneous velocity and acceleration vectors as functions of time
   (b) its displacement after $t = 3$ s.

$\frac{d\vec{r}}{dt} = \vec{v} = 30\hat{i} + (40-10t)\hat{j}$

$\vec{a} = \frac{d\vec{v}}{dt} = -10\hat{j}$

$\vec{r}(0) = \vec{0}$

$\vec{r}(3) = 90\hat{i} + (120-45)\hat{j}$

$= 90\hat{i} + 75\hat{j}$

$\Delta\vec{r} = \vec{r}(3) - \cancel{\vec{r}(0)}$

$= \vec{r}(3) = \boxed{90\hat{i} + 75\hat{j}}$

2. The position x of an object is described with respect to time t by the following equation: $x = 2t^3 - 15t^2 + 36t - 8$. Answer the following questions.
   (a) Find its displacement between $t = 3$ and 5 s.
   (b) Write out an expression for the velocity of the object with respect to time.
   (c) Write out an expression for the acceleration of the object with respect to time.
   (d) At what point(s) in time is the velocity of the object zero?
   (e) At each of those points (from 2d above), is the acceleration positive, negative, or zero?
   (f) During what intervals of time is the velocity of the object positive?
   (g) During what intervals of time is the acceleration of the object positive?
   (h) On the graph (on the next page), sketch position x, velocity v and acceleration a as functions of time.

a) $\vec{X}(3) = 2(3^3) - 15(3^2) + 36(3) - 8 = 19$

$\vec{X}(5) = 2(5^3) - 15(5^2) + 36(5) - 8 = 47$

$\Delta\vec{X} = \vec{X}(5) - \vec{X}(3) = 47 - 19 = 28$

b) $v = \frac{dx}{dt} = \boxed{6t^2 - 30t + 36}$

c) $a = \frac{dv}{dt} = \boxed{12t - 30}$

(d) $6t^2 - 30t + 36 = 0$

$t^2 - 5t + 6 = 0$

$(t-2)(t-3) = 0$

$t = 2, 3$

(e) $a(2) = 12(2) - 30 < 0$ negative

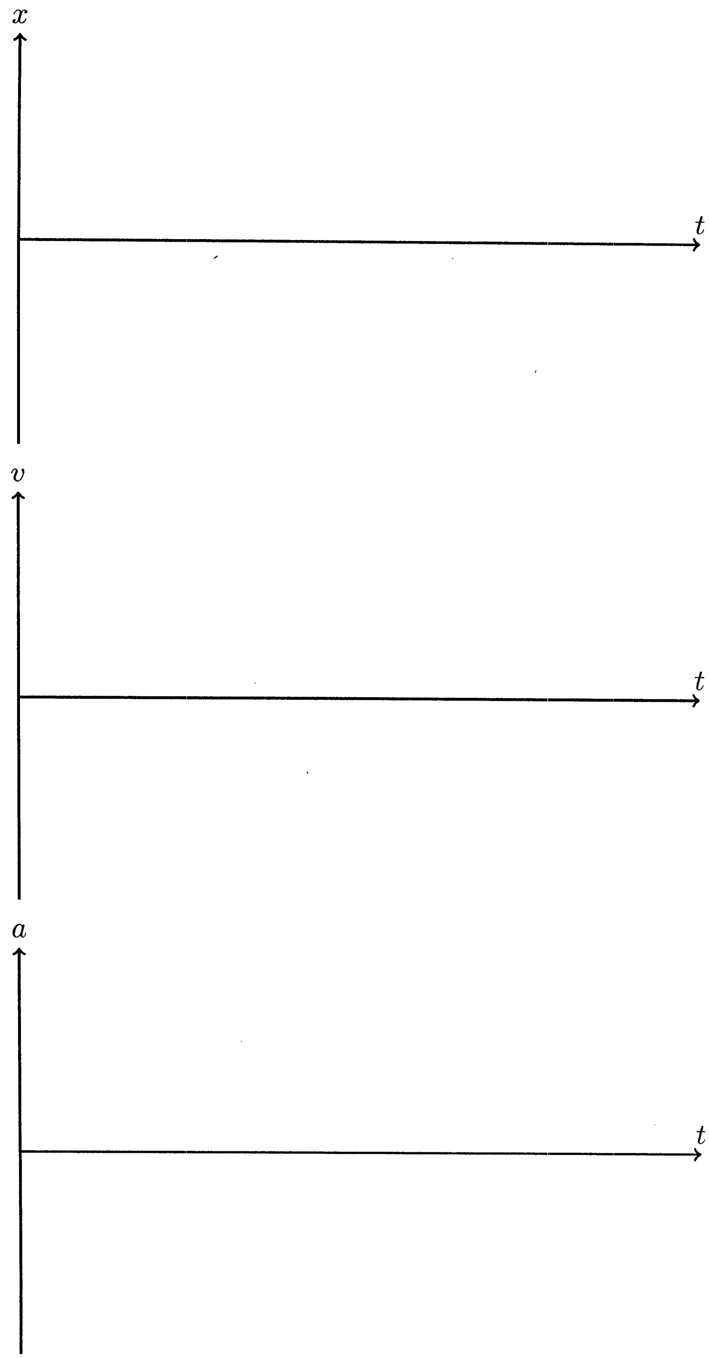
$a(3) = 12(3) - 30 = 0$ ~~zero~~ positive.

(f) $t < 2$, $t > 3$ positive

$2 < t < 3$ negative

(g) $t > 2.5$

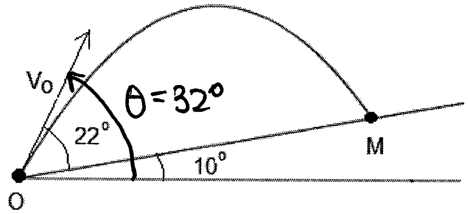
GO ON TO THE NEXT PAGE.


x
t
v
t
a
t

3. A projectile is launched from point O at an angle of 22° with an initial velocity of 15 m/s up an incline plane that makes an angle of 10° with the horizontal. The projectile hits the incline plane at point M.

$V_y = V_0 \sin 32°$

$V_x = V_0 \cos 32°$



$X = V_0 \cos 32° t = OM \cos 10°$

$OM = \frac{V_0 \cos 32 t}{\cos 10} = \boxed{15.0 m}$

(a) Find the time it takes for the projectile to hit the incline plane.
(b) Find the distance OM.

$y = V_0 \sin 32 t - \frac{1}{2} g t^2$

$X = V_0 \cos 32 t$

$\tan 10 = \frac{y}{x} = \frac{\cancel{V_0}\sin 32\cancel{t}}{\cancel{V_0}\cos 32\cancel{t}} - \frac{1}{2}\frac{gt^2}{V_0 \cos 32 t}$

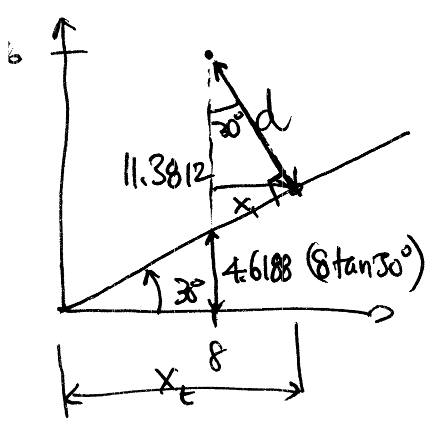
$\tan 10 = \tan 32 - \frac{1}{2}\frac{gt}{V_0 \cos 32}$

$t = \frac{2V_0 \cos 32}{g}(\tan 32 - \tan 10)$

$\boxed{t = 1.16 s}$ (1.14s) with $g = 10 m/s^2$

4. A steel ball is dropped from a point with (x, y) coordinate of (8 m, 16 m). At the same time, another ball is launched from the origin with a speed of 20 m/s at an angle of 30°.
A B
(a) Find the minimum distance of separation occur of the two balls.
(b) At what time does this separation occur?
(c) Give the coordinates of the two balls for the minimum separation.

assume that I "fall" with the steel balls, then



$d = 9.8564$

$\boxed{d = 9.86 m}$

(b) $X_1 = 9.856 \cos 60$
$= 4.928$

$X_t = 8 + X_1 = 12.928 m$

$V_x = 20 \cos 30° = 17.320$

$t = \frac{12.928}{17.320} = \boxed{0.746 s}$

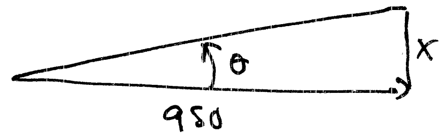
(c) Ball A.
$\vec{X} = 8\hat{i} - (16 - 5t^2)\hat{j}$
$\boxed{= 8\hat{i} + 13.2\hat{j}}$

Ball B
$20 \sin 30° t$
$\vec{X} = 20 \cos 30 t\hat{i} + (10t - 5t^2)\hat{j}$
$\boxed{= \cancel{17.3}\hat{i} + 4.68\hat{j}}$
12.9

5. A high-powered rifle shoots bullets that leave the mizzle at 1.1×10^3 m/s. If a bullet is to hit a target 950 m away at the level, the gun must be aimed at a point above the target. Neglecting air resistance, how far above the target is this point?

Use range equation

$$R = \frac{V_i^2 \sin\theta}{g}$$



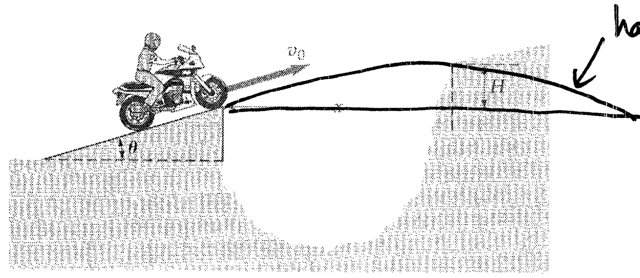
$$\theta = \frac{1}{2}\sin^{-1}\left(\frac{Rg}{V_i^2}\right) = \frac{1}{2}\sin^{-1}\left(\frac{950 \cdot 10}{1100^2}\right) = 0.22^\circ$$

very small!

$\tan\theta = \frac{x}{950}$

$x = 950\tan\theta = \boxed{3.73\text{m}}$

6. A trail bike take off from a ramp with velocity v_0 at angle θ to clear a ditch of width x and land on the other side, which is elevated at a height H.



half of a symmetric trajectory

(a) For a given angle θ and distance x, what is the upper limit for H such that the bike has an chance of making the jump?
(b) For H less than this upper limit, what is the minimum take-off speed v_0 necessary for a successful jump?

Neglect the size of the trail bike, and assume that covering a horizontal distance x and a vertical distance H is sufficient to clear the ditch.

X is ~~half~~ the range $\frac{R}{2} = \frac{V_0^2 \sin 2\theta}{2g} = X \rightarrow V_0^2 = \frac{2gX}{\sin 2\theta}$

H is maximum height $H = \frac{V_0^2 \sin^2\theta}{2g} = \left(\frac{2gX}{\sin\theta}\right)\frac{\sin^2\theta}{2g} = \frac{X\sin^2\theta}{\sin 2\theta} = \frac{X\sin^2\theta}{2\sin\theta\cos\theta} = \boxed{\frac{1}{2}X\tan\theta}$

$= 2\sin\theta\cos\theta$

$$V_0 \geq \sqrt{\frac{2gH}{\sin^2\theta}}$$

(See notes)