Taylor 1.37

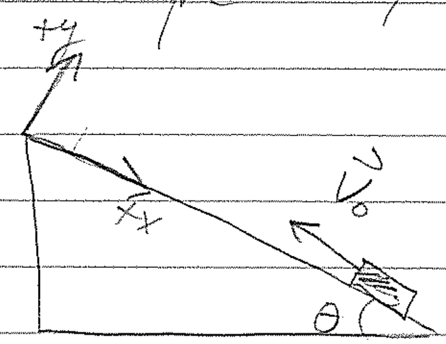
A puck with initial speed v_0 slides straight up a plane that is inclined at angle θ.

(a) Write down Newton's 2^{nd} law for the puck and solve to give its position as a function of time.

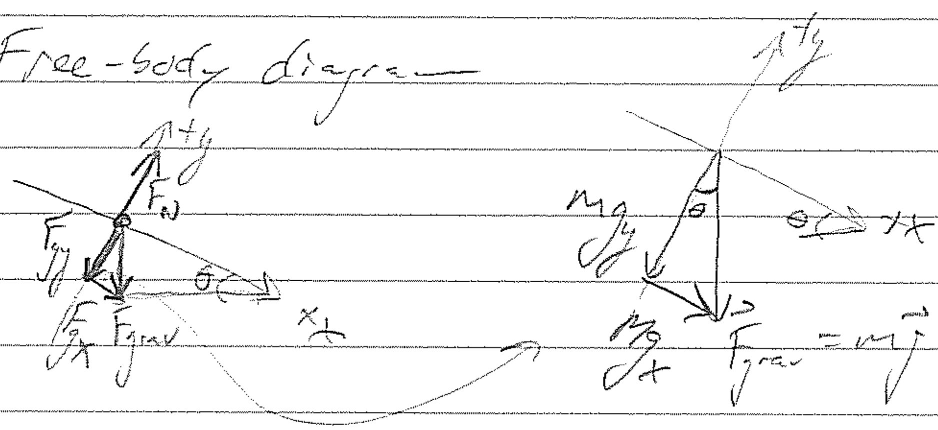
(b) How long will the puck take to return to its starting point?

Example Taylor 1.37

(a)



Free-body diagram



$F_{net_y} = 0$ since $a_y = 0$ so just look at x direction.

$F_{net_x} = F_{grav_x}$

$= mg_x$

$= mg\sin\theta$

Newt. 2nd law.

$$\ddot{x} = \frac{F_{net_x}}{m} = g\sin\theta$$

F_{net_x} is a constant force.

Taylor Section 1.4 derives equations for v_x and $x(t)$ for constant force.

$$\dot{x}(t) = v_{0x} + \frac{F_0}{m} t$$

$$x(t) = x_0 + v_{0x} t + \frac{F_0}{2m} t^2$$

Substitute $F_0 = mg\sin\theta$. Then,

$$\dot{x}(t) = v_{0x} + g\sin\theta\, t$$

$$x(t) = x_0 + v_{0x} t + \frac{1}{2} g\sin\theta\, t^2$$

(b) Upon returning to starting point $x(t) = x_0$.
Also $v_{0x} = -v_0$. Substitute these values.

$$\cancel{x_0} = \cancel{x_0} + (-v_0)t + \frac{1}{2} g\sin\theta\, t^2$$

$$v_0 \cancel{t} = \frac{1}{2} g\sin\theta\, t^{\cancel{2}}$$ $t=0$ is a solution.

$$v_0 = \frac{1}{2} g\sin\theta\, t$$

$$\boxed{t = \frac{2v_0}{g\sin\theta}}$$