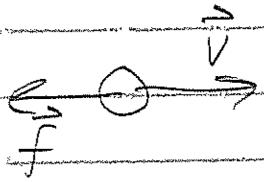
1-D motion with linear drag



$$\vec{f} = -bv\hat{v}$$

Newt. 2nd law: $\ddot{x} = \frac{F_{net,x}}{m}$

In 1-D

$$\ddot{x} = \frac{-bv_x}{m} = -\frac{b}{m}\dot{x}$$

Write as $\frac{d}{dt}(v_x) = -\frac{b}{m} v_x$

$$\frac{dv_x}{v_x} = -\frac{b}{m}\,dt$$

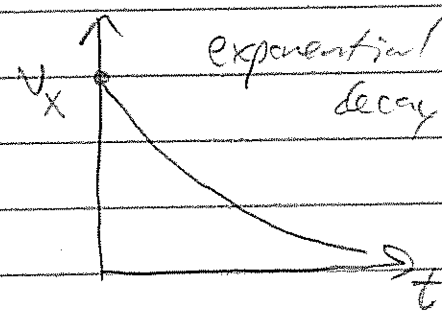
$$\int \frac{dv_x}{v_x} = \int -\frac{b}{m}\,dt$$

$$\ln(v_x)\Big|_{t=0}^{t=t} = -\frac{b}{m}t\Big|_{t=0}^{t=t}$$

$$\ln(v_x) - \ln(v_{x0}) = -\frac{b}{m}t$$

$$\ln\left(\frac{v_x}{v_{x0}}\right) = -\frac{b}{m}t$$

$$\frac{v_x}{v_{x0}} = e^{-\frac{b}{m}t}$$



$$\boxed{v_x = v_{x0}\,e^{-\frac{b}{m}t}}$$

Note that at $t=0$, $v_x = v_{x0}$ as it should be.

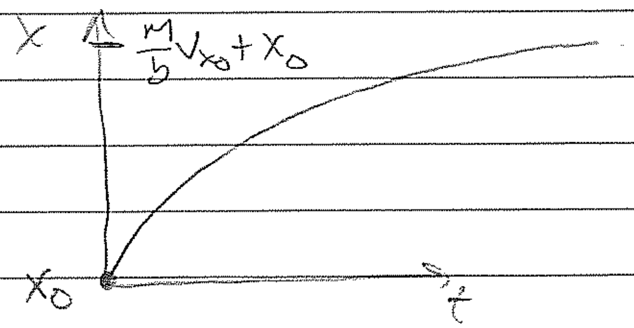
$$x = x_0 + \int v_x \, dt$$

$$= x_0 + \int_{t=0}^{t=t} v_{x0} e^{-\frac{b}{m}t}$$

$$= x_0 + v_{x0} e^{-\frac{b}{m}t} \left(-\frac{m}{b}\right) \Big|_{t=0}^{t=t}$$

$$x = x_0 + -\frac{m}{b} v_{x0} e^{-\frac{b}{m}t} + \frac{m}{b} v_{x0}$$

$$x = x_0 + \frac{m}{b} v_{x0} \left(1 - e^{-\frac{b}{m}t}\right)$$



Asymptotically approaches

$$x = \frac{m}{b} v_{x0} + x_0$$

Thus it comes to rest at this position.

$\Delta x = \frac{m}{b} v_{x0}$ is the displacement.

Note that v_x decays exponentially. The time constant is $\tau = \frac{m}{b}$. Then

$$v_x = v_{x0} e^{-\frac{t}{\tau}}$$

at $t = \tau$, $v_x = 376 \, v_{x0}$