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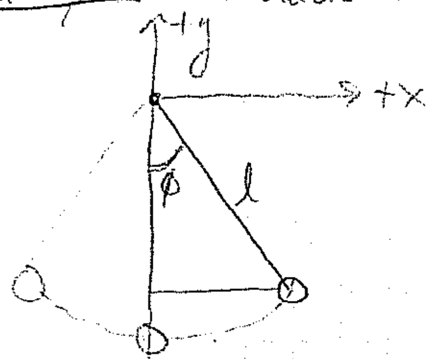
Example Pendulum



Write K
Write U
Write $L = K - U$
Write q_i
Write $\frac{\partial L}{\partial q_i} = \frac{d}{dt}\frac{\partial L}{\partial \dot{q}_i}$ for all q_i

Solve diff eqs for $q(t)$.

$x = l\sin\phi \qquad y = -l\cos\phi$

$\dot{x} = l\cos\phi\,\dot\phi \qquad \dot{y} = l\sin\phi\,\dot\phi$

$$K = \tfrac{1}{2}m(\dot{x}^2+\dot{y}^2) = \tfrac{1}{2}m(l^2\cos^2\phi\,\dot\phi^2 + l^2\sin^2\phi\,\dot\phi^2) = \tfrac{1}{2}ml^2\dot\phi^2$$

$$U_{grav} = mgy = -mgl\cos\phi$$

$$L = K - U = \tfrac{1}{2}ml^2\dot\phi^2 + mgl\cos\phi$$

generalized coordinate is $q = \phi$

$$\frac{\partial L}{\partial q} = \frac{d}{dt}\frac{\partial L}{\partial \dot q}$$

so $\frac{\partial L}{\partial \phi} = \frac{d}{dt}\frac{\partial L}{\partial \dot\phi}$

$$\frac{\partial L}{\partial \phi} = -mgl\sin\phi$$

$$\frac{\partial L}{\partial \dot\phi} = ml^2\dot\phi$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot\phi}\right) = ml^2\ddot\phi$$

$-mgl\sin\phi = ml^2\ddot\phi$

$$\boxed{\ddot\phi = -\frac{g}{l}\sin\phi}$$

This agrees with what we found from Newton's 2nd law in polar coordinates.