Example – Saturn V rocket

$m_0 = 2.8 \times 10^6$ kg

$m_{fuel} = 2.1 \times 10^6$ kg

$thrust = 37 \times 10^6$ N

$v_{exhaust} = 2600 \frac{m}{s}$

What is v_y when fuel runs out?

Launched from Earth

$$\dot{m} = -\frac{thrust}{v_{exhaust}} = \frac{-37 \times 10^6 N}{2600 \frac{m}{s}} = -1.423 \times 10^4 \frac{kg}{s}$$

At burnout, $m = m_{rocket\ body}$

$= m_0 - m_{fuel}$

$= (2.8 - 2.1) \times 10^6$ kg

$= 7 \times 10^5$ kg

$$v_y = v_{exhaust} \ln\left(\frac{m_0}{m}\right) - \frac{1}{\dot{m}}(m - m_0)$$

$$= \left(2600 \frac{m}{s}\right) \ln\left(\frac{2.8 \times 10^6 kg}{7 \times 10^5 kg}\right) - \frac{(9.8 \frac{m}{s^2})}{-1.423 \times 10^4 \frac{kg}{s}} \left(7 \times 10^5 - 2.8 \times 10^6\right)$$

$$= \boxed{2155 \frac{m}{s}}$$

Find t to burnout:

$$\dot{m} = \frac{m - m_0}{t}$$

$$t = \frac{m - m_0}{\dot{m}} = \frac{(7 \times 10^5 - 2.8 \times 10^6)\,kg}{-1.423 \times 10^4 \frac{kg}{s}}$$

$$\boxed{t = 148\ s}$$