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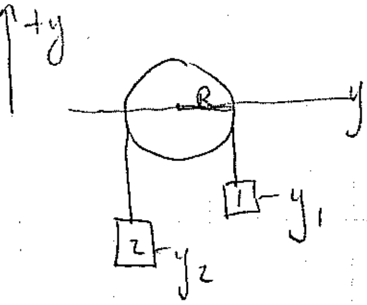
Example: Atwood's Machine



$$-y_1 + y_2 + \frac{1}{2}(2\pi R) = l \equiv \text{length of string}$$

This is a constrained system where

$$\underbrace{y_2 = -l - y_1 + \pi R}_{\dot{y}_2 = -\dot{y}_1}$$

$$K = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2$$
$$= \frac{1}{2}m_1 \dot{y}_1^2 + \frac{1}{2}m_2 \dot{y}_2^2$$
$$= \frac{1}{2}m_1 \dot{y}_1^2 + \frac{1}{2}m_2(-\dot{y}_1)^2$$
$$= \frac{1}{2}(m_1+m_2)\dot{y}_1^2$$

Note that the tension was not required by using the Hamilton's Principle to solve the problem.

$$U_{grav} = m_1 g y_1 + m_2 g y_2$$
$$= m_1 g y_1 + m_2 g\left(-l - y_1 + \frac{\pi}{R}\right)$$
$$U_{grav} = m_1 g y_1 - m_2 g y_1 - m_2 g l + m_2 g \frac{\pi}{R}$$

$L = T - U \rightarrow L = \frac{1}{2}(m_1+m_2)\dot{y}_1^2 - (m_1 - m_2) g y_1 + m_2 g l - m_2 g \frac{\pi}{R}$

The action is a minimum if

q is y_1

$\dot{q}$ is $\dot{y}_1$

$$\frac{\partial L}{\partial y_1} = \frac{d}{dt}\frac{\partial L}{\partial \dot{y}_1}$$

$$\frac{\partial L}{\partial y_1} = -(m_1 - m_2)g$$

$$\frac{\partial L}{\partial \dot{y}_1} = (m_1+m_2)\dot{y}_1$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{y}_1}\right) = (m_1+m_2)\ddot{y}_1$$

$$\boxed{\begin{gathered} -(m_1 - m_2)g = (m_1+m_2)\ddot{y}_1 \\ \ddot{y}_1 = \left(\frac{m_2 - m_1}{m_1+m_2}\right)g \end{gathered}}$$

Newt. 2nd law,

$F_T - m_2 g = m_2 a_{2y}$

$F_T - m_1 g = m_1 a_{1y}$

$$a_{1y} = -a_{2y}$$
$$-m_2 a_{1y} + m_2 g - m_1 g = m_1 a_{1y}$$
$$a_{1y} = \left(\frac{m_2 - m_1}{m_1+m_2}\right)g$$

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