Rocket

Rocket mass is m. Because the rocket ejects the exhaust, the rocket loses mass.

At time t, the rocket's momentum is $P_x = mv_x$.

At time $t+dt$, the rocket's momentum is $P_x = (m+dm)(v_x+dv_x)$

where dm is the change in its mass and dv_x is the change in its velocity. Since the rocket is losing mass, dm is negative. Thus, the mass of the fuel exhaust is $-dm$ and it has a velocity $v_x - v_{exhaust}$ where $v_{exhaust}$ is the exhaust velocity relative to the rocket.

The total momentum is

$$P_x(t+dt) = P_{x\,rocket} + P_{x\,exhaust}$$

$$= (m+dm)(v_x+dv_x) + (-dm)(v_x - v_{exhaust})$$

$$= mv_x + m\,dv_x + \cancel{dm\,v_x} + \underbrace{dm\,dv_x}_{\text{negligibly small}} - \cancel{dm\,v_x} + dm\,v_{exhaust}$$

$$\approx mv_x + m\,dv_x + dm\,v_{exhaust}$$

$$P_{fx} - P_{ix} = \cancel{mv_x} + m\,dv_x + dm\,v_{exhaust} - \cancel{mv_x}$$

$$= m\,dv_x + dm\,v_{exhaust}$$

Project No. ________
Book No. ________ TITLE ________

Page No. ____

For the rocket in space, with negligible external forces

$\vec{F}_{net} = 0$

$\vec{p} \equiv$ constant and $\Delta \vec{p} = 0$

$$m\,dv_x + dm\,V_{exhaust} = 0$$

$$m\,dv_x = -dm\,V_{exhaust}$$

$$\frac{dm}{m} = -\frac{dv_x}{V_{exhaust}}$$

$$\frac{dm/dt}{m} = \frac{-dv_x/dt}{V_{exhaust}}$$

$$\frac{\dot{m}}{m} = \frac{-\dot{v}_x}{V_{exhaust}}$$

$$\dot{m}V_{exhaust} = -m\dot{v}_x$$

$$\underbrace{m\dot{v}_x} = -\dot{m}V_{exhaust}$$

define as thrust so

$$\boxed{\text{thrust} = -\dot{m}V_{exhaust}}$$

Note that $\dot{m}$ is negative so thrust is in $+x$ direction.

$$\int_{m_0}^{m} \frac{dm}{m} = \int -\frac{dv_x}{V_{exhaust}}$$

$$\ln m \Big|_{m_0}^{m} = -\frac{V_x}{V_{exhaust}}\Big|_{V_{x_0}}^{V_x}$$

To Page No. ____

LE ____

Page No. ____

$$\ln(m) - \ln(m_0) = -\frac{(V_x - V_{x0})}{V_{exhaust}}$$

$$\ln(m_0) - \ln(m) = \frac{V_x - V_{x0}}{V_{exhaust}}$$

$$\ln\left(\frac{m_0}{m}\right) = \frac{V_x - V_{x0}}{V_{exhaust}}$$

$$\boxed{V_x = V_{x0} + V_{exhaust} \ln\left(\frac{m_0}{m}\right)}$$

Velocity is in terms of the ratio of initial mass to "current" mass. If you know the rate that mass is lost, you can calculate the velocity at time t.

$$\boxed{\dot{V}_x = -\frac{\dot{m} V_{exhaust}}{m} = \frac{\text{thrust}}{\text{mass}}}$$