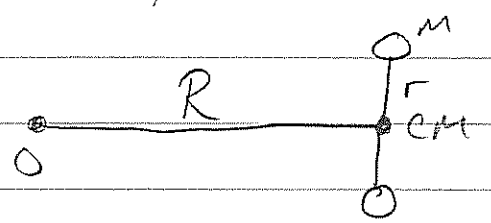
Example

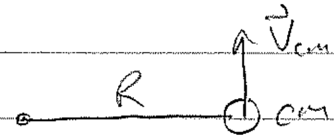
Barbell has two point-like masses, m, that are attached by a rod of negligible mass (i.e. small compared to the point-like masses). The radius is r.

The center of the barbell rotates counter clockwise about an axis at a radius R with angular velocity Ω.



(a) What is L_{trans}?

$$\vec{L}_{trans} = \vec{r}_{cm} \times \vec{p}_{cm}$$



$$L_{trans_z} = R M v_{cm}$$
$$= R(2m)R\Omega$$
$$= 2mR^2\Omega$$

(b) If rod maintains the same orientation relative to this reference frame, what is $\vec{L}_{rot}$? (In this case the axle is frictionless and $\vec{\tau}_{on\ rod} = 0$.)

In this case ω, about cm $= 0$, so $\vec{L}_{rot} = 0$

(c) If rod rotates about cm with angular velocity $\omega = \Omega$, what is $\vec{L}_{rot}$?

$\vec{L}_{rot} = I\vec{\omega}$ $\quad\quad I = mr^2$ for point mass

$$L_{rot_z} = (mr^2)\omega + (mr^2)\omega$$
$$= 2mr^2\omega$$
$$= 2mr^2\Omega \quad \text{since } \omega = \Omega$$

$$L_{tot_z} = L_{trans_z} + L_{rot_z}$$
$$= 2mR^2\Omega + 2mr^2\Omega$$
$$L_{tot_z} = 2m(R^2 + r^2)\Omega$$

(d) Suppose that the barbell rotates counterclockwise $\omega_{cm} = 5\Omega$. What is $\vec{L}_{rot}$ and $\vec{L}_{tot}$?

$$L_{rot_z} = I_{cm}\,\omega_{cm}$$
$$= 2mr^2(5\Omega)$$

$$L_{tot_z} = 2mR^2\Omega + 2mr^2(5\Omega)$$
$$= 2m(R^2 + 5r^2)\Omega$$