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A force $\vec{F}$ is conservative if and only if:

(a) $\vec{F}$ depends only on the particle's position $\vec{r}$.

(b) For any two points, W from 1 to 2 is the same for all paths between 1 and 2.

If $\vec{F}$ is conservative, then define a scalar function U that only depends on position such that

$$\Delta U = -W$$

$$U(\vec{r}) - U(\vec{r}_0) = -\int_{\vec{r}_0}^{\vec{r}} \vec{F} \cdot d\vec{r}$$

Generally $U(\vec{r}_0)$ is defined to be zero, so

$$U(\vec{r}) = -\int_{\vec{r}_0}^{\vec{r}} \vec{F} \cdot d\vec{r}$$

Thus, $\Delta T = W$

$$= -\Delta U$$

and $\underbrace{\Delta T + \Delta U}_{\text{call this } \Delta E_{mech}} = 0$

call this ΔE_{mech} where $\boxed{E_{mech} = T + U}$

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Gradient

$$\Delta U = -\int_{\vec{r}_1}^{\vec{r}_2} \vec{F}\cdot d\vec{r}$$

Likewise, $\vec{F} = -\nabla U$ where $\nabla \equiv$ gradient

In Cartesian coord, $\nabla = -\frac{\partial}{\partial x}\hat{x} - \frac{\partial}{\partial y}\hat{y} - \frac{\partial}{\partial z}\hat{z}$

$$\nabla = \left\langle \frac{-\partial}{\partial x}, \frac{-\partial}{\partial y}, \frac{-\partial}{\partial z} \right\rangle$$

The force is derivable from potential energy if $\vec{F} = \vec{\nabla} U$.

This is the first condition of a conservative force,
F is only a function of position

Curl

The second condition of a conservative force is that
$\int_{\vec{r}_1}^{\vec{r}_2} \vec{F}\cdot d\vec{r}$ is path independent.

This condition is the equivalent of
$\nabla \times \vec{F} = 0$ $\equiv$ "curl of $\vec{F}$"

It is defined as

$$\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_x & F_y & F_z \end{vmatrix} = \hat{x}\left(\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z}\right) - \hat{y}\left(\frac{\partial F_z}{\partial x} - \frac{\partial F_x}{\partial z}\right) + \hat{z}\left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right)$$