Complex Algebra

$z = a + ib$ imaginary number $i = \sqrt{-1}$

$i^2 = -1$

complex number

Real part

Imaginary Part

$a = \mathrm{Re}(z) \quad b = \mathrm{Im}(z)$

Complex conjugate

z^* is z with $-i$ substituted for i.

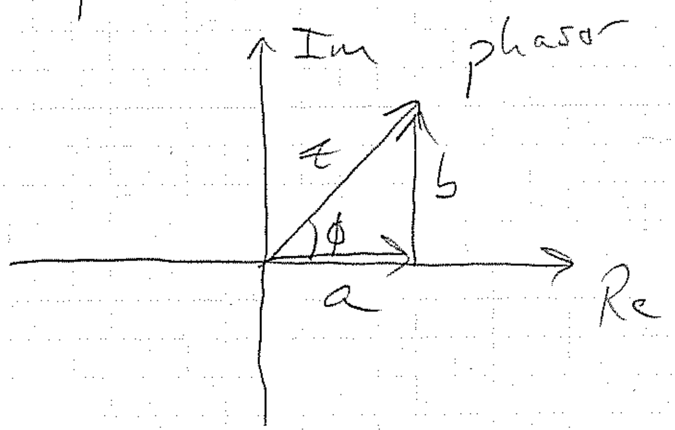
so $z^* = a - ib$ if $z = a + ib$.

Magnitude

$$|z|^2 = z^* z$$
$$= (a - ib)(a + ib)$$
$$|z|^2 = a^2 + b^2$$
$$|z| = (a^2 + b^2)^{1/2}$$

Complex Plane

phasor



$$z = |z|e^{i\phi}$$ magnitude phase

$$e^{i\phi} = \cos\phi + i\sin\phi$$ Euler Formula

Identities

$e^{i\frac{\pi}{2}} = i$

$e^{-i\frac{\pi}{2}} = -i$

can be useful in derivations

$$z = \underbrace{|z|\cos\phi}_{Re} + i\underbrace{|z|\sin\phi}_{Im}$$

$$a = |z|\cos\phi \qquad b = |z|\sin\phi$$

We can represent a sinusoidal function $x = A\cos(\omega t + \delta)$ as the real part of a complex number.

$$x = Re\left(Ae^{i(\omega t+\delta)}\right)$$

Exponentials are easier to deal with. Thus, we can do the math (such as derivatives and integrals) with $e^{i(\omega t+\delta)}$ and take the real part to get the quantity that would actually be measured in a lab.