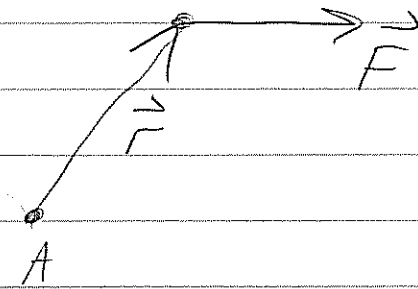
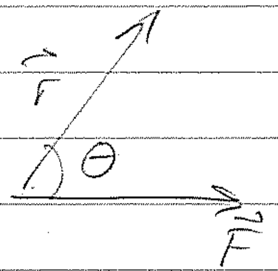
Torque

The torque due to a force at location $\vec{r}$ about a point A is



$$\vec{\tau}_A = \vec{r} \times \vec{F}$$

$$|\vec{\tau}_A| = |\vec{r}||\vec{F}|\sin\theta$$



Dir. is right-hand rule.

$\vec{\tau}$ is $\perp$ to plane of $\vec{r}$ and $\vec{F}$.

In vector component form:

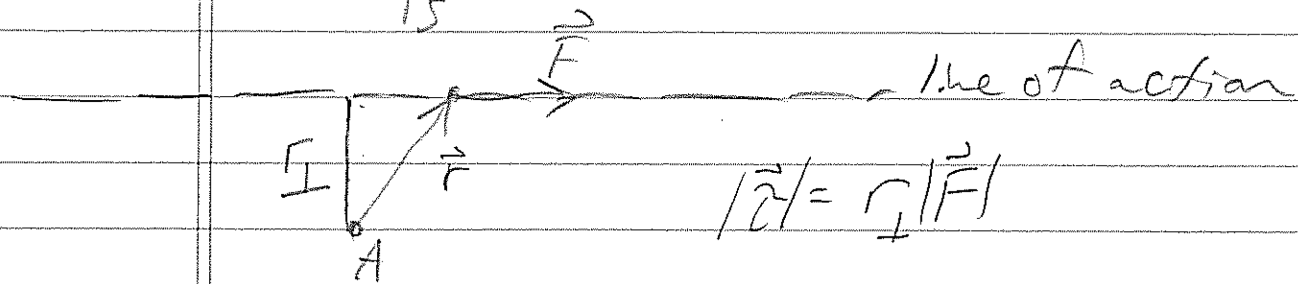
$$\vec{\tau} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

$$= (r_y F_z - r_z F_y)\hat{x} - (r_x F_z - r_z F_x)\hat{y} + (r_x F_y - r_y F_x)\hat{z}$$

$$\vec{\tau} = \langle r_y F_z - r_z F_y,\ r_z F_x - r_x F_z,\ r_x F_y - r_y F_x \rangle$$

$\tau_z = r_x F_y - r_y F_x$ if $\vec{r}$ and $\vec{F}$ are in x-y plane.

Another common way to calculate torque is is



$$|\vec{\tau}| = r_\perp |\vec{F}|$$

$r_\perp$ = moment arm

Angular Momentum Principle

$$\vec{\tau}_{net,A} = \frac{d\vec{L}_A}{dt}$$

$\vec{L}_A$ is the total angular momentum of the system.

If A is the CM, then

$$\vec{\tau}_{net,CM} = \frac{d\vec{L}_{rot}}{dt}$$

$\vec{L}_{rot} = I_{CM} \vec{\omega}_{CM}$

$\vec{L}_{rot}$ is angular momentum about the CM.

For a small time step Δt then

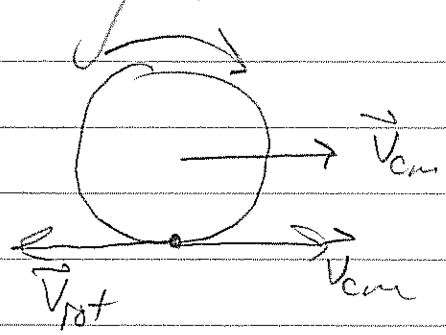
$$\Delta \vec{L}_{tot,A} = \vec{\tau}_{net,A} \Delta t$$

or about the CM,

$$\Delta\vec{\omega}_{cm} = \frac{\vec{\tau}_{cm}}{I_{cm}} \Delta t$$

Application

A ball rolls without slipping. Suppose the rolling friction is negligible.



If it is not slipping, then point in contact with floor is at rest.

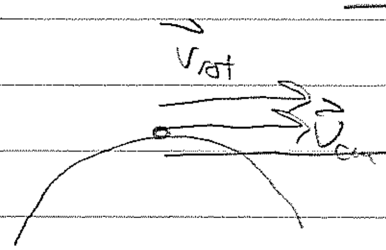
In this case linear velocity due to rotation about CM is equal in magnitude to CM velocity.

Note that $\omega_z = -V_{cm_x} R$ in this case

$V_{rot} = \omega R$ so $V_{rot} = V_{cm}$

and thus $\boxed{V_{cm} = \omega R}$ rolling without slipping!!!

At top:



$V_{top} = 2V_{cm} = 2\omega R$