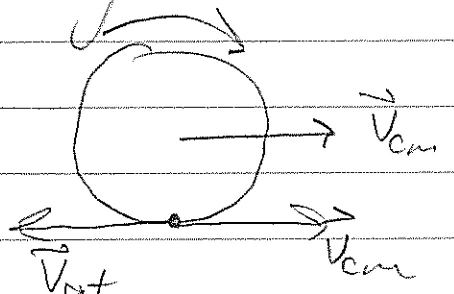
or about the CM,

$$\Delta\vec{\omega}_{cm} = \frac{\vec{\tau}_{cm}}{I_{cm}}\Delta t$$

Application

A ball rolls without slipping. Suppose the rolling friction is negligible.



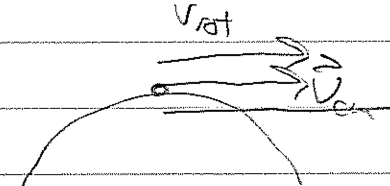
If it is not slipping, then point in contact with floor is at rest.

In this case linear velocity due to rotation about CM is equal in magnitude to CM velocity.

Note that $\omega_z = -V_{cm_x} R$ in this case

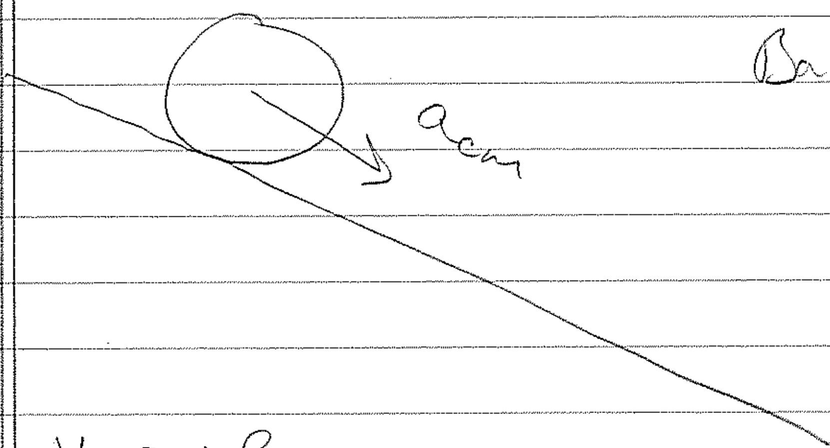
$V_{rot} = \omega R$ so $V_{rot} = V_{cm}$ and thus $\boxed{V_{cm} = \omega R}$ rolling without slipping!!!

At top:



$\rightarrow V_{top} = 2V_{cm} = 2\omega R$

Suppose CM is accelerating:



Ball rolling downhill.

$v_{cm} = \omega R$

$\frac{dv_{cm}}{dt} = \left(\frac{d\omega}{dt}\right) R$

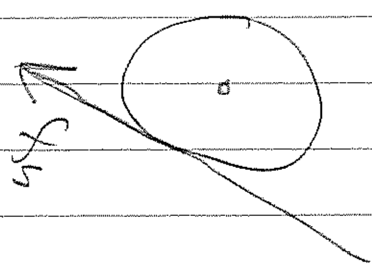
$a_{cm} = \alpha R$ where α is angular accel.

$\alpha = \dot{\omega}$

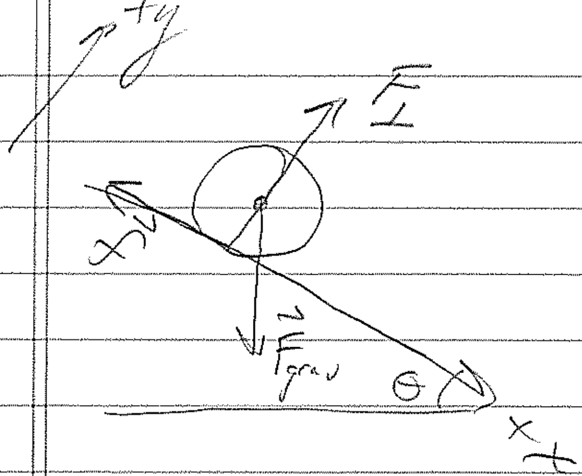
Note that $\vec{L}_{rot}$ changes when $\vec{\omega}$ changes.

Thus $\vec{\tau}_{net} = \frac{\Delta \vec{L}}{\Delta t}$ is <u>not</u> 0. But τ due to F_{grav} is 0 since F_{grav} acts at CM. What exerts the torque?

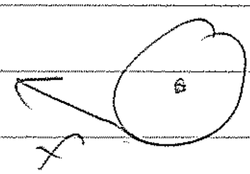
Friction!!



Even if not sliding, there is friction.



$$F_{net_{cm_x}} = mg\sin\theta - f = ma_{cm_x}$$



$$\vec{\tau}_{net,cm} = \frac{d\vec{L}_{rot}}{dt}$$

$$-fR = I_{cm}\frac{d\omega}{dt}$$

$$-fR = I_{cm}\alpha$$

Since it is not slipping, $a_{cm_x} = -\alpha R$ since $+a_{cm_x}$ corresponds to $-\alpha$ in z direction.

$$-fR = I_{cm}\left(-\frac{a_{cm_x}}{R}\right)$$

$$a_{cm_x} = \frac{fR^2}{I_{cm}} \qquad f = mg\sin\theta - ma_{cm_x}$$

$$a_{cm_x} = \frac{(\cancel{m}g\sin\theta - \cancel{m}a_{cm_x})\cancel{R^2}}{\frac{2}{5}\cancel{M}\cancel{R^2}}$$

$$a_{cm_x} = \frac{5}{2}(g\sin\theta - a_{cm_x})$$

$$a_{cm_x} = \frac{5}{2}g\sin\theta - \frac{5}{2}a_{cm_x}$$

$$a_{cm_x}\left(1 + \frac{5}{2}\right) = \frac{5}{2}g\sin\theta$$

$$a_{cm} = \frac{\frac{5}{2} g \sin\theta}{\frac{7}{2}}$$

$$\boxed{a_{cm} = \frac{5}{7} g \sin\theta}$$