# 10 Day 10

**1:**

A fireworks shell is launched from the origin (launch speed $v_0$, initial angle $\theta$) and travels along the usual parabolic path above flat ground. Had it failed to explode, it would have had the usual range

$$R = \frac{2v_0^2}{g}\sin(\theta)\cos(\theta)$$

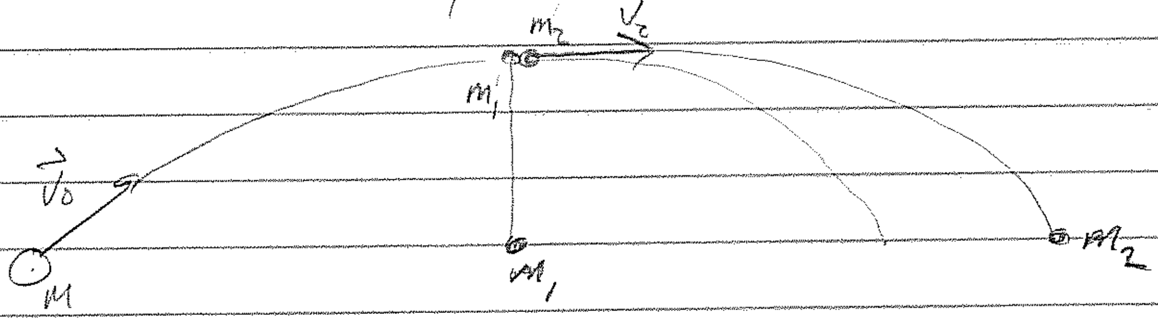
Right at the very peak of its path, it explodes into two pieces of *equal mass*. Just after the explosion, one of those pieces is observed to "stop dead," before falling straight to the ground.

(a) What is the velocity vector of the second piece immediately after the explosion? Answer in terms of only the given quantities: $m$, $g$, $v_0$, and $\theta$) Briefly explain your reasoning.

(b) Where does the second piece land? (Answer in terms of $R$.)

(c) Now, suppose that the first piece does not "stop dead" after the explosion. Instead it shoots straight upward with a speed $v_1$ after the explosion. Now, both pieces will NOT hit the ground at the same time. What is the velocity of the second piece after the explosion and where is the center of mass of the system after both pieces land?

# Day 10




Before $\quad m \quad \vec{v}_i$

After $\quad m_1 \; m_2 \quad \vec{v}_2 \qquad m_1 = m_2 = \frac{1}{2}m$

$v_1 = 0$

$$\vec{P}_i = \vec{P}_f$$

$$m\vec{v}_i = \cancel{m_1\vec{v}_1}^{\,0} + m_2\vec{v}_2$$

$$m\langle v_0\cos\theta, 0, 0\rangle = m_2\vec{v}_2$$

(a) $$v_{2x} = \frac{m v_0 \cos\theta}{m_2} = 2v_0\cos\theta$$

$$v_{2y} = 0$$

peak is at $x = \frac{1}{2}R$

$\Delta x_2 = 2v_0\cos\theta\, t \qquad t$ = time to fall from the peak

$$y = y_0 + v_{0y} t - \frac{1}{2} g t^2$$

$$0 = h - \frac{1}{2} g t^2$$

$$h = \frac{1}{2} g t^2$$

$$t = \sqrt{\frac{2h}{g}}$$ time to fall from peak = time to rise to peak.

$$x = \frac{1}{2} R = v_0 \cos\theta \, t$$

$$\frac{1}{2} R = v_0 \cos\theta \sqrt{\frac{2h}{g}}$$

$$\frac{1}{4} R^2 = v_0^2 \cos^2\theta \left(\frac{2h}{g}\right)$$

$$h = \frac{\frac{1}{4} R^2 g}{v_0^2 \cos^2\theta}$$

$$t = \frac{\frac{1}{2} R}{v_0 \cos\theta}$$

$$\Delta x_2 = 2 \cancel{v_0 \cos\theta} \frac{\frac{1}{2} R}{\cancel{v_0 \cos\theta}}$$

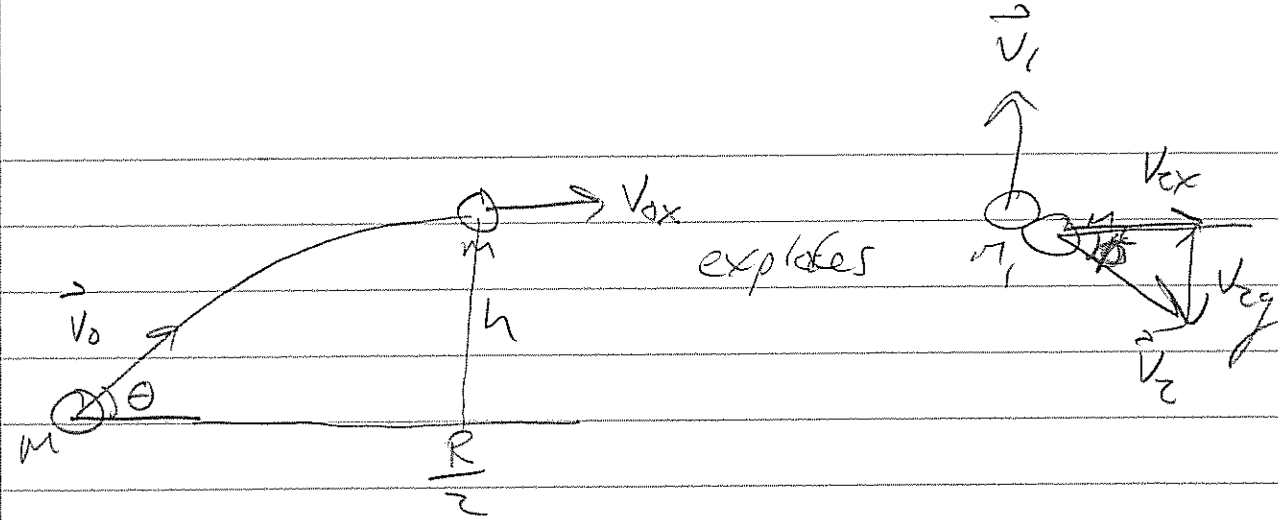
$$\boxed{\Delta x_2 = R}$$ from peak.

from graph: $\boxed{\Delta x_1 = \frac{1}{2} R}$

$$\boxed{\Delta x_2 = \frac{1}{2} R + R = \frac{3}{2} R}$$

CM is at $\boxed{x_{cm} = \frac{\frac{1}{2} R + \frac{3}{2} R}{2} = R.}$

(c)




Apply Cons. of Momentum to the explosion at the peak.

$$\vec{P}_i = \vec{P}_f = \vec{P}_1 + \vec{P}_2$$

$$\langle mV_{0x}, 0, 0\rangle = \langle 0, m_1 V_1, 0\rangle + m_2\langle V_{2x}, V_{2y}, 0\rangle$$

$x$: $mV_{0x} = m_2 V_{2x}$

$$V_{2x} = \frac{mV_{0x}}{m_2} = \frac{mV_0\cos\theta}{\frac{1}{2}m} = 2V_0\cos\theta$$

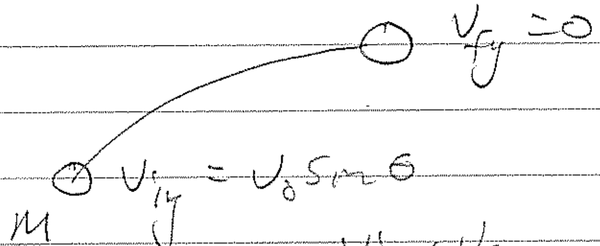
$y$: $0 = m_1 V_1 + m_2 V_{2y}$

$$V_{2y} = \frac{-m_1 V_1}{m_2} = -V_1$$

$$\vec{V}_2 = \langle V_{2x}, V_{2y}, 0\rangle$$

$$\boxed{\vec{V}_2 = \langle 2V_0\cos\theta, -V_1, 0\rangle}$$

find $h$:




$$V_{avg\,y} = \frac{v_{iy} + v_{fy}}{2}$$

$$V_{avg\,y} = \frac{v_0 \sin\theta + 0}{2} = \frac{\Delta y}{\Delta t}$$

$$\Delta x = v_x \Delta t$$

$$\frac{R}{2} = v_0 \cos\theta\, \Delta t$$

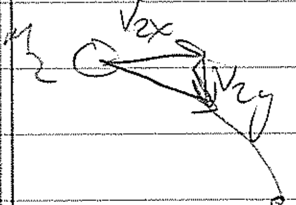
$$\Delta t = \frac{R}{2 v_0 \cos\theta}$$

$$\frac{v_0 \sin\theta}{2} = \frac{h}{\frac{R}{2 v_0 \cos\theta}}$$

$$h = \frac{R v_0 \sin\theta}{4 v_0 \cos\theta}$$

$$\boxed{h = \frac{R}{4} \tan\theta}$$

Find time for piece 2 to fall from peak.




$$V_{2yi} = -V_1$$

$$y = y_i + V_{2yi} t - \frac{1}{2} g t^2$$

$$0 = h - v_1 t - \frac{1}{2} g t^2$$

$$\frac{1}{2} g t^2 + v_1 t - h = 0$$

$$t = \frac{-v_1 \pm \sqrt{v_1^2 - 4\left(\frac{1}{2}g\right)(-h)}}{2\left(\frac{1}{2}g\right)}$$

$$t = \frac{-V_1 \pm \sqrt{V_1^2 + 2gh}}{g}$$

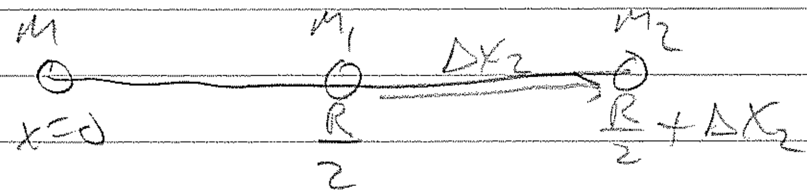
choose + solution because $t$ can not be −.

$$\boxed{t = -\frac{V_1}{g} + \frac{1}{g}\sqrt{V_1^2 + 2gh}}$$

Find $\Delta X_2$ from peak:

$$\Delta X_2 = V_{x2} t$$
$$= (2V_0\cos\theta)\left(-\frac{V_1}{g} + \frac{1}{g}\sqrt{V_1^2+2gh}\right)$$

$$\boxed{\Delta X_2 = -\frac{2V_0V_1\cos\theta}{g} + \frac{2V_0\cos\theta}{g}\sqrt{V_1^2+2gh}}$$




cm is at center between $m_1$ and $m_2$ since $m_1 = m_2$.

$$X_{cm} = \frac{R}{2} + \frac{\Delta X}{2} = \boxed{\frac{R}{2} - \frac{V_0V_1\cos\theta}{g} + \frac{V_0\cos\theta}{g}\sqrt{V_1^2 + \frac{gR}{2}\tan\theta}}$$