Day 26

(a) $m\ddot{\vec{r}} = \cancel{\vec{F}_{net}}^{\,0} + 2m\dot{\vec{r}}\times\vec{\Omega} + m(\vec{\Omega}\times\vec{r})\times\vec{\Omega}$

$m\ddot{\vec{r}} = 2m\langle \dot{y}\Omega, -\dot{x}\Omega, 0\rangle + m\Omega^2\langle x, y, 0\rangle$

$\cancel{m}\ddot{x} = 2\cancel{m}\dot{y}\Omega + \cancel{m}\Omega^2 x$

$$\boxed{\begin{aligned}\ddot{x} &= 2\dot{y}\Omega + \Omega^2 x \\ \ddot{y} &= -2\dot{x}\Omega + \Omega^2 y\end{aligned}}$$

(b) $\eta = x + iy$

$\dot{\eta} = \dot{x} + i\dot{y}$

$\ddot{\eta} = \ddot{x} + i\ddot{y}$

$\ddot{\eta} = 2\dot{y}\Omega + \Omega^2 x + i(-2\dot{x}\Omega + \Omega^2 y)$

$= 2\dot{y}\Omega + \Omega^2 x + -2\dot{x}\Omega i + \Omega^2 y i$

$= 2\Omega\underbrace{(\dot{y} - i\dot{x})}_{-i\dot{\eta}} + \Omega^2\underbrace{(x + iy)}_{\eta}$

$$\boxed{\ddot{\eta} = \Omega^2\eta - i2\Omega\dot{\eta}}$$

Solution $\quad \eta = e^{-i\alpha t}$

$$\dot{\eta} = -i\alpha e^{-i\alpha t}$$

$$\ddot{\eta} = -\alpha^2 e^{-i\alpha t}$$

$$-\alpha^2 \cancel{e^{-i\alpha t}} = \Omega^2 \cancel{e^{-i\alpha t}} - i2\Omega\left(-i\alpha \cancel{e^{-i\alpha t}}\right)$$

$$-\alpha^2 = \Omega^2 - 2\Omega\alpha$$
$$\Omega^2 - 2\Omega\alpha + \alpha^2 = 0$$
$$(\Omega-\alpha)(\Omega-\alpha) = 0$$

$$\boxed{\Omega = \alpha}$$

$te^{-i\alpha t}$ is also a solution

General Solution $\quad \boxed{\eta = C_1 e^{-i\alpha t} + C_2 t e^{-i\alpha t}}$

(c) $\quad t=0: \ \eta_0 = C_1$

$$t=0: \ \dot{\eta} = -i\alpha C_1 e^{-i\alpha t} + C_2\left[(t)(-i\alpha)(e^{-i\alpha t}) + e^{-i\alpha t}\right]$$

$$\dot{\eta}_0 = -i\alpha C_1 + C_2$$

$$\dot{\eta}_0 = -i\alpha \eta_0 + C_2$$

$$C_2 = \dot{\eta}_0 + i\alpha \eta_0$$

$$\eta_0 = x_0 + iy_0 \qquad \dot{\eta}_0 = v_{x0} + iv_{y0}$$

$$C_1 = x_0 + iy_0$$

$$C_2 = v_{x0} + iv_{y0} + i\alpha(x_0 + iy_0)$$

$$C_2 = v_{x0} + iv_{y0} + i\alpha x_0 - \alpha y_0$$

$$= -\alpha y_0 + v_{x0} + i(\alpha x_0 + v_{y0})$$

$$\eta = (x_0 + iy_0)e^{-i\alpha t} + \left((-\alpha y_0 + v_{x0}) + i(\alpha x_0 + v_{y0})\right)te^{-i\alpha t}$$

$$= e^{-i\alpha t}\left(x_0 + (-\alpha y_0 + v_{x0})t + i\left(y_0 + (\alpha x_0 + v_{y0})t\right)\right)$$

$$\alpha = \Omega$$

$$y_0 = 0$$

$$\eta = e^{-i\Omega t}\left((x_0 + v_{x0}t) + i(v_{y0} + \Omega x_0)t\right)$$

$$= (\cos\Omega t - i\sin\Omega t)\left((x_0 + v_{x0}t) + i(v_{y0} + \Omega x_0)t\right)$$

$$= \cos\Omega t(x_0 + v_{x0}t) + \sin\Omega t(v_{y0} + \Omega x_0)t$$

$$+ i\left((v_{y0} + \Omega x_0)t\cos\Omega t - \sin\Omega t(x_0 + v_{x0}t)\right)$$

$$x(t) = \cos\Omega t\,(x_0 + v_{x0} t) + t\,(v_{y0} + \Omega x_0)\sin\Omega t$$

$$y(t) = -\sin\Omega t\,(x_0 + v_{x0} t) + t\,(v_{y0} + \Omega x_0)\cos\Omega t$$