1-D motion with linear drag and a constant force

Consider a falling body on Earth

$\downarrow \vec{v}$ At terminal velocity, $\uparrow b v_{term}$, $\downarrow mg$ so $b v_{term} = mg$

$$V_{term} = \frac{mg}{b}$$

The forces are: $\uparrow \vec{f}_{drag}$, $\downarrow \vec{F}_{grav}$

Newt. 2nd law:

$$F = m\ddot{y}$$

Define +y downward:

$$mg - b v_y = m\ddot{y}$$

$$b v_{term} - b v_y = m \dot{v}_y$$

$$b(v_{term} - v_y) = m\dot{v}_y$$

$$\frac{b}{m}(v_{term} - v_y) = \dot{v}_y$$

$$-\frac{b}{m}(v_y - v_{term}) = \dot{v}_y$$

$$\dot{v}_y = -\frac{b}{m}(v_y - v_{term})$$

ubstitute $u = v_y - v_{term}$ and $\dot{u} = \dot{v}_y$. Then

$$\dot{u} = -\frac{b}{m} u \quad \text{and}$$

$$u = u_0 e^{-b/m\, t}$$

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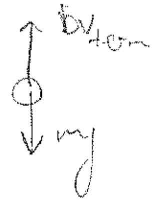
otion with linear drag and a constant force

er a falling body on Earth

At terminal velocity,



so $bv_{term} = mg$

$v_{term} = \frac{mg}{b}$

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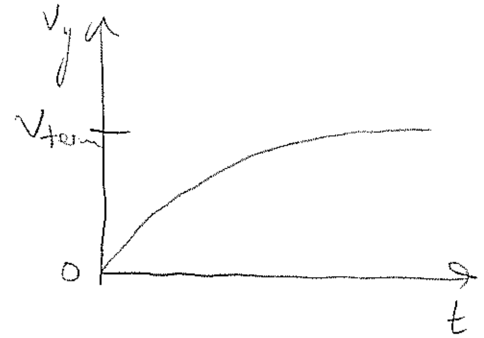
So, $v_y - v_{term} = (v_{y_0} - v_{term})e^{-\frac{b}{m}t}$

$$v_y = v_{term} + v_{y_0}e^{-b/m\, t} - v_{term}e^{-b/m\, t}.$$

$$\boxed{v_y = v_{term}(1 - e^{-b/m\, t}) + v_{y_0}e^{-b/m\, t}}$$

Suppose the object is dropped from rest: $v_{y_0} = 0$

$$v_y = v_{term}(1 - e^{-b/m\, t})$$



$\tau = \frac{m}{b}$.

After 1τ, v_y is $63\%\, v_{term}$
2τ, $86\%\, v_{term}$
3τ, $95\%\, v_{term}$

$$y(t) = \int v_y\, dt$$

$$= \int \left(v_{term} + (v_{y_0} - v_{term})e^{-b/m\, t}\right) dt$$

$$= v_{term}t + (v_{y_0} - v_{term})\left(-\frac{m}{b}\right)e^{-b/m\, t}\Big|_0^t$$

$$= v_{term}t + (v_{y_0} - v_{term})\left(\frac{-m}{b}\right)e^{-b/m\, t} - (v_{y_0} - v_{term})\left(\frac{-m}{b}\right)$$

$$\boxed{y = v_{term}t + (v_{y_0} - v_{term})\left(\frac{m}{b}\right)(1 - e^{-b/m\, t})}$$

Note that as $t \to \infty$, $y(t) = v_{term}t$ since v_y is constant.