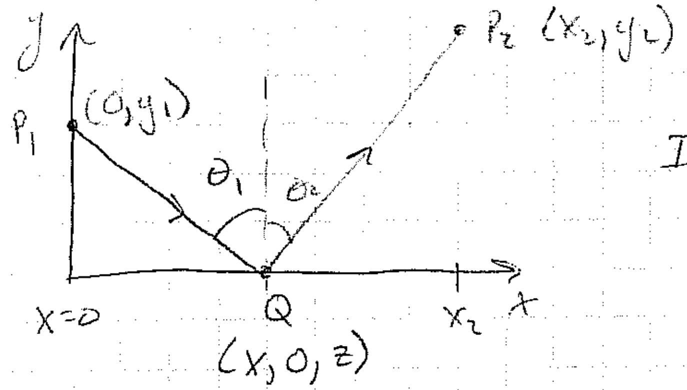
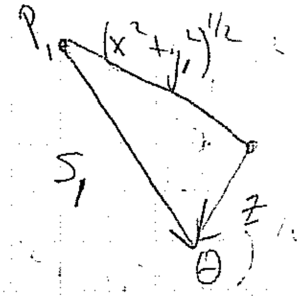
Example 6.3: Reflection from a plane mirror



In 3-D:



$$\Delta t_{total} = \Delta t_{P_1 Q} + \Delta t_{P_2 Q}$$

$$= \frac{S_1}{V} + \frac{S_2}{V}$$

$$= \frac{(x^2+y_1^2+z^2)^{1/2}}{c} + \frac{((x_2-x)^2+y_2^2+z^2)^{1/2}}{c} \Rightarrow t(x,z)$$

We already know the minimum time will be along a straight line, so no need to use calculus of variations.

Minimum time is for

$$\frac{\partial t}{\partial x} = 0 \quad \text{and} \quad \frac{\partial t}{\partial z} = 0$$

$$\frac{\partial t}{\partial z} = \frac{1}{2c}(x^2+y_1^2+z^2)^{-1/2}(2z) + \frac{1}{2c}((x_2-x)^2+y_2^2+z^2)^{-1/2}(2z) = 0$$

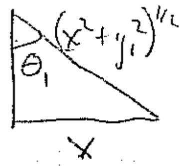
$$\frac{z}{c}\left[\frac{1}{(x^2+y_1^2+z^2)^{-1/2}} + \frac{1}{(x_2-x)^2+y_1^2+z^2}\right] = 0$$

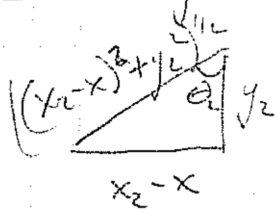
so $z=0$.

Q is in the same plane as P_1 and P_2.

$$\frac{dt}{dx} = \frac{1}{2c}\left(x^2+y_1^2\right)^{-1/2} 2x + \frac{1}{2c}\left((x_2-x)^2+y_2^2\right)^{-1/2}(2)(x_2-x)(-1) = 0$$

$$\frac{x}{(x^2+y_1^2)^{1/2}} = \sin\theta_1 \qquad \frac{(x_2-x)}{\left((x_2-x)^2+y_2^2\right)^{1/2}} = \sin\theta_2$$



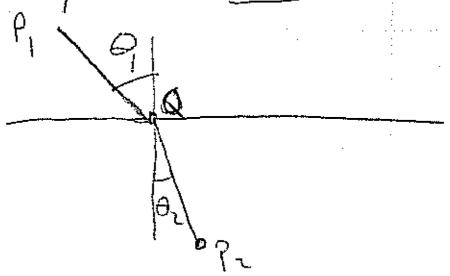


$$\frac{dt}{dx} = \frac{\sin\theta_1}{2c} - \frac{\sin\theta_2}{2c} = 0$$

$$\sin\theta_1 = \sin\theta_2$$

$$\boxed{\theta_1 = \theta_2}$$ – Law of Reflection

Example 6.5 Fermat's principle



P_1 at $(0, h_1, 0)$
Q at $(x, 0, z)$
P_2 at $(x_2, -h_2, 0)$

$v_1 = \frac{c}{n_1} \qquad V_2 = \frac{c}{n_2}$

$$t_{total} = \frac{S_1}{V_1} + \frac{S_2}{V_2}$$
$$= \frac{n_1\left(x^2+h_1^2+z^2\right)^{1/2}}{c} + \frac{n_2\left((x_2-x)^2+h_2^2+z^2\right)^{1/2}}{c} \Rightarrow t = f(x, z)$$

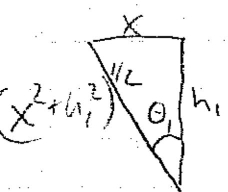
Minimum t is at

$$\frac{\partial t}{\partial x} = 0 \quad \text{and} \quad \frac{\partial t}{\partial z} = 0$$

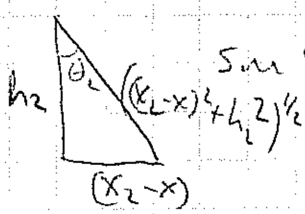
$$\frac{\partial t}{\partial z} = \frac{n_1}{2c}\left(x^2+h_1^2+z^2\right)^{-1/2} 2z + \frac{n_2}{2c}\left((x_2-x)^2+h_1^2+z^2\right)^{-1/2} 2z = 0$$

$z = 0$. So Q is in $z=0$ plane with P_1 and P_2

$$\frac{\partial t}{\partial x} = \frac{n_1}{2c}\left(x^2+h_1^2\right)^{-1/2} 2x + \frac{n_2}{2c}\left((x_2-x)^2+h_2^2\right)^{-1/2}(2)(x_2-x)(-1) = 0$$



$\sin\theta_1 = \frac{x}{h_1}$



$\sin\theta_2 = \frac{x_2-x}{h_2}$

$$\frac{\partial t}{\partial x} = \frac{n_1}{2c}\sin\theta_1 - \frac{n_2}{2c}\sin\theta_2 = 0$$

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

$$\boxed{n_1 \sin\theta_1 = n_2 \sin\theta_2}$$ Law of Refraction

Snell's Law in optics.

Light travels along the path of <u>least time</u>.