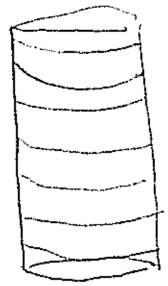
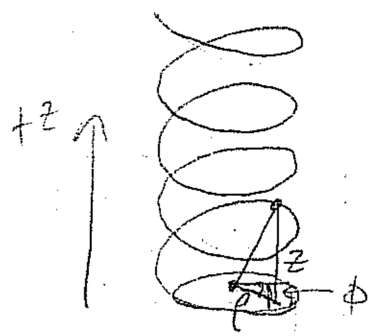
CH07 Homework.
Select 4 problems from CH07.
At least 1 must be a 2-star problem

Example 7.20





$\vec{r} = r(\rho, \phi, z)$

$\rho = R$

$z = \lambda \phi$

so $\dot{z} = \lambda \dot{\phi}$

$\vec{v}$ is tangent to the wire and downward

origin

$\vec{v} = \langle 0, v_\phi, v_z \rangle$

$T = \frac{1}{2} m v^2$ $\qquad U = mgz$

$= \frac{1}{2} m (v_\phi^2 + v_z^2)$

$= \frac{1}{2} m (R^2 \dot{\phi}^2 + \dot{z}^2)$

$= \frac{1}{2} m \left(R^2 \frac{\dot{z}^2}{\lambda^2} + \dot{z}^2\right) = \frac{1}{2} m \left(1 + \frac{R^2}{\lambda^2}\right) \dot{z}^2$

$L = T - U$

$= \frac{1}{2} m \left(1 + \frac{R^2}{\lambda^2}\right) \dot{z}^2 - mgz$

$\frac{\partial L}{\partial \dot{z}} = m\left(1 + \frac{R^2}{\lambda^2}\right) \dot{z}$

$\frac{d}{dt}\frac{\partial L}{\partial \dot{z}} = m\left(1 + \frac{R^2}{\lambda^2}\right) \ddot{z}$

$\frac{\partial L}{\partial z} = -mg$

$\frac{\partial L}{\partial z} = \frac{d}{dt} \frac{\partial L}{\partial \dot{z}}$

$-mg = m\left(1 + \frac{R^2}{\lambda^2}\right) \ddot{z}$

$$\boxed{\ddot{z} = -\frac{1}{\left(1 + \frac{R^2}{\lambda^2}\right)} g}$$

Constant downward acceleration.
If $R = 0$, $\ddot{z} = -g$ as expected