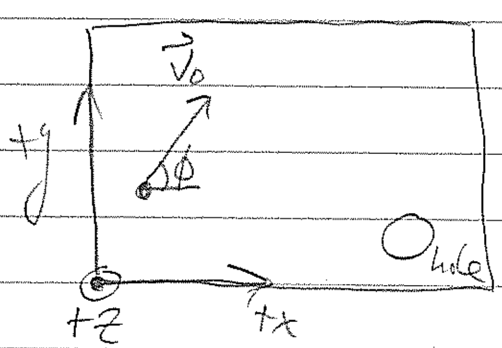
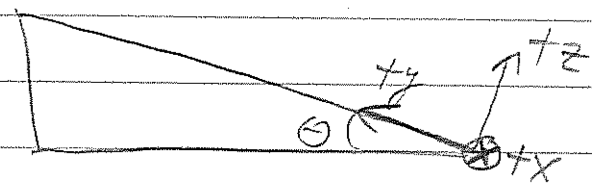
Taylor 1.38

This is basically like a golf ball putted across a sloped green with no friction.

Top View



Side View



$V_{0x} = V_0 \cos\phi$

$V_{0y} = V_0 \sin\phi$

$V_{0z} = 0$

In x-direction, $F_{net_x} = 0$ so V_x is constant.

In y-direction, $F_{net_y} = mg_y = -mg\sin\theta$

In z-dir, $F_{net_z} = 0$ so V_z is constant (0)

We already know the equation of motion for constant net force.

In x-direction $F_{net x} = 0$ so

$$X = X_0 + V_{0x} t$$

In y-direction, $F_{net y} = -mg\sin\theta$

$$V_y = V_{0y} + -g\sin\theta\, t$$

$$y = y_0 + V_{0y} t - \frac{1}{2} g\sin\theta\, t^2$$

How long does it take for the ball to return to the same y_0 value?

$X_0 = 0$

$y_0 = 0$

V_{0x}

V_{0y}

$$0 = 0 + V_{0y} t - \frac{1}{2} g\sin\theta\, t^2$$

$$t = \frac{2V_{0y}}{g\sin\theta}$$

How far is it from origin?

$$X = X_0 + V_{0x} t$$

$$= 0 + V_{0x}\left(\frac{2V_{0y}}{g\sin\theta}\right)$$

$$\boxed{X = \frac{2V_{0x}V_{0y}}{g\sin\theta}}$$

$V_{0x} = V_0 \cos\phi$
$V_{0y} = V_0 \sin\phi$

There are many (infinite) values of V_0 and ϕ that can put the golf ball in the hole. So which one is best?

Suppose the hole is at $x = 8\,m$, $y = 0$ relative to the starting position of the ball.

Find ϕ for which V_0 is a minimum.

Set $x = 8\,m$. Then

$$8 = \frac{2 V_0 \cos\phi \, V_0 \sin\phi}{g \sin\theta} = \frac{2 V_0^2 \cos\phi \sin\phi}{g \sin\theta}$$

$$V_0 = \left(\frac{4 g \sin\theta}{\cos\phi \sin\phi} \right)^{1/2}$$

At a minimum, $\frac{dV_0}{d\phi} = 0$ and $\frac{d^2 V_0}{d\phi^2} = +$

$$\frac{dv_0}{d\phi} = \frac{\sqrt{\frac{g\sin\theta}{\cos^2\phi}} - \sqrt{\frac{g\sin\theta}{\sin^2\phi}}}{\sqrt{\frac{g\sin\theta}{\cos\phi\sin\phi}}}$$

$$= \frac{1 - 2\cos^2\phi}{\sqrt{\sin\phi\cos\phi}}\left(\sqrt{g\sin\theta}\right)$$

$$\frac{dv_0}{d\phi} = 0$$

$$\frac{1-2\cos^2\phi}{\sqrt{\sin\phi\cos\phi}} = 0$$

Note that $\phi = 0$ or $\phi = 90°$ gives an unphysical solution.

Thus $0 < \phi < 90°$ $\quad \phi \neq 0$ and $\phi \neq 90°$.

$$1 - 2\cos^2\phi = 0$$
$$2\cos^2\phi = 1$$
$$\cos^2\phi = \frac{1}{2}$$

$$\cos\phi = \frac{1}{\sqrt{2}}$$
$$\boxed{\phi = 45°}$$

Not surprising! It's just like projectile motion, but with $a = g\sin\theta$.