flip.tanedo@uci.edu

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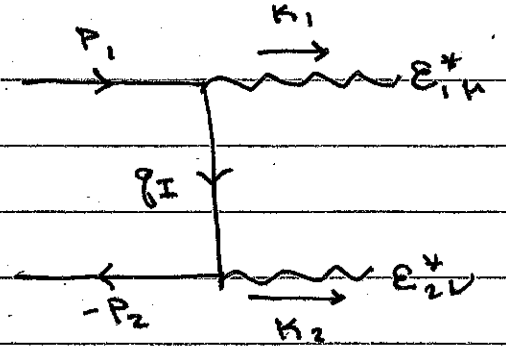
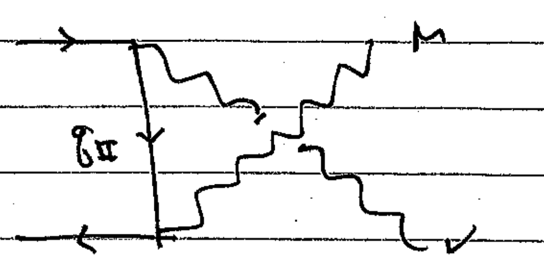
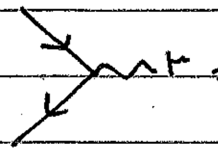
I



II



$$\xrightarrow{\ p\ } = i\Delta(p) = \frac{i(\not{p}+m)}{p^2-m^2} = iD_p\cdot(\not{p}+m)$$



$$= ig\Gamma^\mu \qquad \text{specialize to } \Gamma^\mu = \gamma^\mu$$

$$i\mathcal{M}_I = \bar{v}_2\, ig\gamma^\nu i\Delta(q_I)\, ig\gamma^\mu u_1\, \varepsilon^*_{1\mu}\varepsilon^*_{2\nu}$$

$$i\mathcal{M}_{II} = \bar{v}_2\, ig\gamma^\mu i\Delta(q_{II})\, ig\gamma^\nu u_1\, \varepsilon^*_{1\mu}\varepsilon^*_{2\nu}$$

$$i\mathcal{M} = i\mathcal{M}_I + i\mathcal{M}_{II}$$

$$= -ig^2\,\bar{v}_2\Big[\underbrace{\gamma^\nu\Delta(q_I)\gamma^\mu + \gamma^\mu\Delta(q_{II})\gamma^\nu}_{D_I\gamma^\nu(\not{q}_I+m)\gamma^\mu + D_{II}\gamma^\mu(\not{q}_{II}+m)\gamma^\nu}\Big]u_1\,\varepsilon^*_{1\mu}\varepsilon^*_{2\nu}$$

$$(i\mathcal{M})^\dagger = +ig^2\,\bar{u}_1\Big[D_I\gamma^{\bar{\mu}}(\not{q}_I+m)\gamma^{\bar{\nu}} + D_{II}\gamma^{\bar{\nu}}(\not{q}_{II}+m)\gamma^{\bar{\mu}}\Big]$$
$$\times\, v_2\,\varepsilon_{1\bar{\mu}}\,\varepsilon_{2\bar{\nu}}$$

$$\sum_{\text{SPINS}} |\mathcal{M}|^2 = g^4 \, \mathrm{Tr}\Big\{ [\gamma^\nu \Delta_I \gamma^\mu + \gamma^\mu \Delta_{II} \gamma^\nu] (\not{p}_1 - m) \\ [\gamma_\mu \Delta_I \gamma_\nu + \gamma_\nu \Delta_{II} \gamma_\mu] (\not{p}_2 + m) \Big\}$$

$$\{\dots\} = \Big[\underbrace{(D_1 \gamma^\nu \not{\ell}_1 \gamma^\mu + D_{11} \gamma^\mu \not{\ell}_{11} \gamma^\nu)}_{C^{\mu\nu};\ 3\ \gamma\text{'s}} + \underbrace{(D_1 m \gamma^\nu \gamma^\mu + D_2 m \gamma^\mu \gamma^\nu)}_{B^{\mu\nu};\ 2\ \gamma\text{'s}} \Big]$$

$$\times (\not{p}_1 - m) \times [C_{\nu\mu} + B_{\nu\mu}] \times (\not{p}_2 + m)$$

$$= (C^{\mu\nu} + B^{\mu\nu})(\not{p}_1 - m)(C_{\nu\mu} + B_{\nu\mu})(\not{p}_2 + m)$$

ONLY TERMS W/ EVEN # OF γ's SURVIVE:

$\{\dots\} =$	$C^{\mu\nu} \not{p}_1 C_{\nu\mu} \not{p}_2$	1
	$C^{\mu\nu} (-m) C_{\nu\mu}\, m$	2
	$C^{\mu\nu} \not{p}_1 B_{\nu\mu}\, m$	3
	$C^{\mu\nu} (-m) B_{\nu\mu} \not{p}_2$	4
	$B^{\mu\nu} \not{p}_1 C_{\nu\mu}\, m$	5
	$B^{\mu\nu} (-m) C_{\nu\mu} \not{p}_2$	6
	$B^{\mu\nu} \not{p}_1 B_{\nu\mu} \not{p}_2$	7
	$B^{\mu\nu} (-m) B_{\nu\mu}\, m$	8

$\boxed{\text{PIECE 1}}$: $C^{\mu\nu} \not p_1 C_{\nu\mu} \not p_2$

$$= (D_I \gamma^\nu \not q_I \gamma^\mu + D_{II} \gamma^\mu \not q_{II} \gamma^\nu) \not p_1$$
$$(D_I \gamma_\mu \not q_I \gamma_\nu + D_{II} \gamma_\nu \not q_{II} \gamma_\mu) \not p_2$$

THERE ARE TWO KINDS OF TERMS:

① $\gamma^\alpha \not A \gamma^\beta \not p_1 \gamma_\beta \not B \gamma_\alpha \not p_2$

② $\gamma^\alpha \not A \gamma^\beta \not p_1 \gamma_\alpha \not B \gamma_\beta \not p_2$

† ① $= \gamma^\alpha \not A (2\not p_1 - 4 \not p_1) \not B \gamma_\alpha \not p_2$

$= -2 \not A \not p_1 \not B \gamma_\alpha \not p_2 \gamma^\alpha$ same manipulation

$= 4 \not A \not p_1 \not B \not p_2$

Tr of ≤ 4 γ matrices is easy enough for now.

② = requires more anticommuting...

†: WE HAVE PROVED (LEMMA):

$\mathrm{Tr}(\cdots \gamma^\alpha \not c \gamma_\alpha \cdots) = -2\, \mathrm{Tr}(\cdots \not c \cdots)$

$$\gamma^\alpha \not{A} \gamma^\beta \not{P}_1 \gamma_\alpha \not{B} \gamma_\beta \not{P}_2$$

$$2 = \gamma^\alpha \not{A} \gamma^\beta (2P_{1\alpha} - \gamma_\alpha \not{P}_1) \not{B} \gamma_\beta \not{P}_2$$

$$= 2\not{P}_1 \not{A} \gamma^\beta \not{B} \gamma_\beta \not{P}_2 \qquad (L1)$$
$$- \gamma^\alpha \not{A} \gamma^\beta \gamma_\alpha \not{P}_1 \not{B} \gamma_\beta \not{P}_2$$

$$= (L1)$$
$$- \gamma^\alpha \not{A} (2\delta^\beta_\alpha - \gamma_\alpha \gamma^\beta) \not{P}_1 \not{B} \gamma_\beta \not{P}_2$$

$$= (L1)$$
$$-2\gamma^\alpha \not{A} \not{P}_1 \not{B} \gamma_\alpha \not{P}_2 \qquad (L2)$$
$$+ \gamma^\alpha \not{A} \gamma_\alpha \gamma^\beta \not{P}_1 \not{B} \gamma_\beta \not{P}_2$$

$$= (L1) + (L2)$$
$$+ \gamma^\alpha \not{A} \gamma_\alpha (2P_1^\beta - \not{P}_1 \gamma^\beta) \not{B} \gamma_\beta \not{P}_2$$

$$= (L1) + (L2)$$
$$+2\gamma^\alpha \not{A} \gamma_\alpha \not{B} \not{P}_1 \not{P}_2$$
$$- \gamma^\alpha \not{A} \gamma_\alpha \not{P}_1 \gamma^\beta \not{B} \gamma_\beta \not{P}_2$$

NOW USE LEMMA †: $\mathrm{Tr}(\cdots \gamma^\alpha \not{c} \gamma_\alpha \cdots) = -2\mathrm{Tr}(\cdots \not{c} \cdots)$

(L1) $= -4 \not{A} \not{B} \not{P}_2 \not{P}_1$ simplify!

(L2) $+4 \not{A} \not{P}_1 \not{B} \not{P}_2$

$-4 \not{A} \not{B} \not{P}_1 \not{P}_2$ CANCEL

$-4 \not{A} \not{P}_1 \not{B} \not{P}_2$

$$② = -4 \not{A} \not{B} \{\not{P}_1, \not{P}_2\}$$
$$= -8 (P_1 \cdot P_2) \not{A} \not{B}$$

$$\mathrm{Tr}[②] = -32 (A \cdot B)(P_1 \cdot P_2)$$

LET'S DEFINE THESE COMBINATIONS:

$$F(A,B) \equiv \mathrm{Tr}(①)$$
$$= \mathrm{Tr}(\gamma^\alpha \not{A} \gamma^\beta \not{P}_1 \gamma_\beta \not{B} \gamma_\alpha \not{P}_2)$$
$$= 4 \mathrm{Tr}(\not{A} \not{P}_1 \not{B} \not{P}_2)$$

$$= 16 [(A \cdot P_1)(B \cdot P_2) - (A \cdot B)(P_1 \cdot P_2) + (A \cdot P_2)(B \cdot P_1)]$$

$$G(A,B) \equiv \mathrm{Tr}(②)$$
$$= \mathrm{Tr}(\gamma^\alpha \not{A} \gamma^\beta \not{P}_1 \gamma_\alpha \not{B} \gamma_\beta \not{P}_2)$$
$$= -32 (A \cdot B)(P_1 \cdot P_2)$$

12/12 FIXED THIS SIGN

//

now let's return to $\boxed{\text{PIECE 1}}$

PIECE 1, continued

$$= \mathrm{Tr}\left[(D_1 \gamma^\nu \not{q}_1 \gamma^\mu + D_{11} \gamma^\mu \not{q}_{11} \gamma^\nu) \not{p}_1 \right.$$
$$\left. (D_1 \gamma_\mu \not{q}_1 \gamma_\nu + D_{11} \gamma_\nu \not{q}_{11} \gamma_\mu) \not{p}_2 \right]$$

$$= D_1^2 F(q_1, q_1) + D_1 D_{11} G(q_1, q_2)$$
$$+ D_1 D_{11} G(q_{11}, q_1) + D_{11}^2 F(q_2, q_2)$$

$$= D_1^2 F(q_1, q_1) + D_{11}^2 F(q_2, q_2) + 2 D_1 D_{11} G(q_1, q_2)$$

$$\boxed{\begin{aligned} &= 16 D_1^2 \left[2(q_1 \cdot p_1)(q_1 \cdot p_2) - q_1^2 (p_1 \cdot p_2) \right] \\ &+ 16 D_{11}^2 \left[2(q_2 \cdot p_1)(q_{11} \cdot p_2) - q_{11}^2 (p_1 \cdot p_2) \right] \\ &+ 64 D_1 D_{11} \left[-(q_1 \cdot q_2)(p_1 \cdot p_2) \right] \end{aligned}}$$

$\boxed{\text{PIECE 2}} = -m^2 C^{\mu\nu} C_{\nu\mu}$

$$= -m^2 \left(D_I \gamma^\nu \not{q}_I \gamma^\mu + D_{II} \gamma^\mu \not{q}_{II} \gamma^\nu \right) \times \left(D_I \gamma_\mu \not{q}_I \gamma_\nu + D_{II} \gamma_\nu \not{q}_{II} \gamma_\mu \right)$$

THESE ARE TERMS w/ 6 γ MATRICES.
TWO TYPES:

① $\gamma^\alpha \not{A} \gamma^\beta \gamma_\beta \not{B} \gamma_\alpha$ ← in fact: $A=B$
② $\gamma^\alpha \not{A} \gamma^\beta \gamma_\alpha \not{B} \gamma_\beta$

① $\mathrm{Tr}\left(\gamma^\alpha \not{A} \gamma^\beta \gamma_\beta \not{A} \gamma_\alpha\right)$

$$= 16\ \mathrm{Tr}\left(\not{A}\not{A}\right)$$
$$= 64\ A^2$$

② $\mathrm{Tr}\left(\gamma^\alpha \not{A} \gamma^\beta \gamma_\alpha \not{B} \gamma_\beta\right)$

$$= 2\ \mathrm{Tr}\left(\gamma^\alpha \not{A} \delta^\beta_\alpha \not{B} \gamma_\beta\right) - \mathrm{Tr}\left(\gamma^\alpha \not{A} \gamma_\alpha \gamma^\beta \not{B} \gamma_\beta\right)$$ ← USE LEMMA †

$$= 8\ \mathrm{Tr}\left(\not{A}\not{B}\right) - 4\ \mathrm{Tr}\left(\not{A}\not{B}\right)$$

$$= \cancel{8(A\cdot B)}\ 4\ \mathrm{Tr}\left(\not{A}\not{B}\right) = 16\ (A\cdot B)$$

define: $J(A) \equiv \mathrm{Tr}(\gamma^\alpha \not{A} \gamma^\beta \gamma_\beta \not{A} \gamma_\alpha)$

$$= 64 A^2$$

$$K(A,B) = \mathrm{Tr}(\gamma^\alpha \not{A} \gamma^\beta \gamma_\alpha \not{B} \gamma_\beta)$$

$$= 16 (A \cdot B)$$

$$\boxed{\text{Piece 2}} = -m^2 \left(D_I^2 J(q_I) + D_I D_{II} K(q_I, q_{II}) + D_{II} D_I K(q_{II}, q_I) + D_{II}^2 J(q_{II}) \right)$$

$$= \boxed{-m^2 \left[64 D_I^2 q_I^2 + 64 D_{II}^2 q_{II}^2 + \underbrace{32}_{D_I D_{II}} (q_1 \cdot q_2) \right]}$$

$\boxed{\text{Piece 3}} = C^{\mu\nu} \not{p}_1 B_{\nu\mu} m$

$= \left(D_1 \gamma^\nu \not{q}_1 \gamma^\mu + D_{11} \gamma^\mu \not{q}_{11} \gamma^\nu\right) \not{p}_1 \left(D_1 m \gamma_\mu \gamma_\nu + D_2 m \gamma_\nu \gamma_\mu\right) m$

$= m^2 \left(D_1 \gamma^\nu \not{q}_1 \gamma^\mu + D_{11} \gamma^\mu \not{q}_{11} \gamma^\nu\right)$
$\quad \not{p}_1 \left(D_1 \gamma_\mu \gamma_\nu + D_{11} \gamma_\nu \gamma_\mu\right)$

AGAIN: two types of terms:

① $\mathrm{Tr}\left(\gamma^\alpha \not{q} \gamma^\beta \not{p} \gamma_\beta \gamma_\alpha\right) \equiv L_1(q)$
② $\mathrm{Tr}\left(\gamma^\alpha \not{q} \gamma^\beta \not{p} \gamma_\alpha \gamma_\beta\right) \equiv M_1(q)$

① $=$ ~~[illegible]~~
$\mathrm{Tr}\left(\underbrace{\gamma_\alpha \gamma^\alpha}_{4} \not{q} \underbrace{\gamma^\beta \not{p} \gamma_\beta}_{-2\not{p}}\right)$

$= -8\,\mathrm{Tr}\left(\not{q}\not{p}\right)$
$= -32(p \cdot q)$

② $= 2\,\mathrm{Tr}\left(\gamma^\alpha \not{q} \gamma_\alpha \not{p}\right) - \overbrace{\mathrm{Tr}\left(\gamma^\alpha \not{q} \gamma^\beta \not{p} \gamma_\beta \gamma_\alpha\right)}^{=①}$
$= -4\,\mathrm{Tr}\left(\not{q}\not{p}\right) + 32(p \cdot q)$
$= 16(p \cdot q)$

$\boxed{L_1(q) = -32(p_1 \cdot q)}$ $\boxed{M_1(q) = 16(p_1 \cdot q)}$

Piece 3 $= m^2\Big(D_1^2 L_1(q_1) + D_1 D_2 M_1(q_1) + D_2 D_1 M_1(q_{11}) + D_2^2 L_1(q_{11}) \Big)$

$$= m^2 \cdot 16 \Big(-2(P_1\cdot q_1) D_1^2 + D_1 D_2 (P_1\cdot q_1) + D_1 D_2 (P_1 \cdot q_{11}) - 2(P_1\cdot q_{11}) D_2^2 \Big)$$

$$= \boxed{16 m^2 \Big[-2D_1^2 (P_1\cdot q_1) - 2D_{11}^2 (P_1\cdot q_{11}) + D_1 D_{11} (P_1\cdot q_1) + D_1 D_{11} (P_1\cdot q_{11}) \Big]}$$

$\boxed{\text{Piece 4}}$ $= C^{\mu\nu}(-m) B_{\nu\mu} \not{P}_2$

$$= -m^2 \left(D_1 \gamma^\nu \not{q}_1 \gamma^\mu + D_{11} \gamma^\mu \not{q}_{11} \gamma^\nu \right)\left(D_1 \gamma_\mu \gamma_\nu + D_2 \gamma_\nu \gamma_\mu \right) \not{P}_2$$

once again, 2 types of terms

① $\gamma^\alpha \not{q} \gamma^\beta \gamma_\beta \gamma_\alpha \not{P}_2$

② $\gamma^\alpha \not{q} \gamma^\beta \gamma_\alpha \gamma_\beta \not{P}_2$

$L_2(q) = \mathrm{Tr}$ (①)

$M_2(q) = \mathrm{Tr}$ (②)

$$L_2(q) = 4\,\mathrm{Tr}(\gamma^\alpha \not{q}\, \gamma_\alpha \not{p}_2)$$
$$= -8\,\mathrm{Tr}(\not{q} \not{p}_2)$$
$$= -32\,(p_2 \cdot q)$$

$$M_2(q) = -2\,\mathrm{Tr}(\gamma^\alpha \not{q}\, \gamma_\alpha \not{p}_2)$$
$$= 4\,\mathrm{Tr}(\not{q} \not{p}_2)$$
$$= 16\,(p_2 \cdot q)$$

<u>Piece 4</u> $= -m^2 \big(D_1^2 L_2(q_1) + D_1 D_2 M_2(q_1) + D_1 D_2 M_2(q_2) + D_2^2 L_2(q_2) \big)$

$$= \boxed{16 m^2 \Big[2 D_1^2 (p_2 \cdot q_1) + 2 D_{11}^2 (p_2 \cdot q_1) - D_1 D_2 (p_2 \cdot q_1) - D_1 D_{11} (p_2 \cdot q_2) \Big]}$$

$\boxed{\text{Piece 5}} = B^{\mu\nu}\, \not{p}_1\, C_{\nu\mu}\, M$

$= M^2 (D_1 \gamma^\nu \gamma^\mu + D_2 \gamma^\mu \gamma^\nu)\, \not{p}_1$
$\quad (D_1 \gamma_\mu \not{q}_1 \gamma_\nu + D_{11} \gamma_\nu \not{q}_{11} \gamma_\mu)$

Two types of terms

① $\gamma^\alpha \gamma^\beta \not{p}_1 \gamma_\beta \not{q} \gamma_\alpha$
② $\gamma^\alpha \gamma^\beta \not{p}_1 \gamma_\alpha \not{q} \gamma_\beta$

$L_3(q) \equiv \mathrm{Tr}(①)$
$M_3(q) \equiv \mathrm{Tr}(②)$

$L_3(q) = -2\, \mathrm{Tr}(\gamma^\alpha \not{p}_1 \not{q} \gamma_\alpha)$
$= -8\, \mathrm{Tr}(\not{p}_1 \not{q})$
$= -32\, (p_1 \cdot q)$

$M_3(q) = \mathrm{Tr}[\gamma^\alpha \gamma^\beta \not{p}_1 (2q_\alpha - \not{q}\gamma_\alpha) \gamma_\beta]$
$= 2\,\mathrm{Tr}(\not{q} \gamma^\beta \not{p}_1 \gamma_\beta)$
$\quad - \mathrm{Tr}(\gamma^\alpha \gamma^\beta \not{p}_1 \not{q} \gamma_\alpha \gamma_\beta)$

$= -4\, \mathrm{Tr}(\not{q} \not{p}_1) + 2\, \mathrm{Tr}(\gamma_\alpha \gamma^\alpha \not{p}_1 \not{q})$
$= -16\, (p_1 \cdot q) + 8\, \mathrm{Tr}(\not{p}_1 \not{q})$
$= 16\, (p_1 \cdot q)$

Piece 5 $= m^2 \left[D_1^2 L_3(q_1) + D_1 D_{11} M_3(q_{11}) + D_1 D_{11} M_3(q_1) + D_{11}^2 L_3(q_{11}) \right]$

$$= \boxed{16m^2 \left[-2D_1^2 (p_1 \cdot q_1) - 2D_{11}^2 (p_1 \cdot q_{11}) + D_1 D_{11} (p_1 \cdot q_1) + D_1 D_{11} (p_1 \cdot q_{11}) \right]}$$

$\boxed{\text{Piece 6}}$ $= -m B^{\mu\nu} C_{\nu\mu} \not{p}_2$

$$= -m^2 (D_1 \gamma^\nu \gamma^\mu + D_2 \gamma^\mu \gamma^\nu)(D_1 \gamma_\mu \not{q}_1 \gamma_\nu + D_2 \gamma_\nu \not{q}_{11} \gamma_\mu) \not{p}_2$$

Two types of terms

① $\gamma^\alpha \gamma^\beta \gamma_\beta \not{q} \gamma_\alpha \not{p}_2$

② $\gamma^\alpha \gamma^\beta \gamma_\alpha \not{q} \gamma_\beta \not{p}_2$

$L_4(q) = \mathrm{Tr}$ (①)

$M_4(q) = \mathrm{Tr}$ (②)

$$L_4(q) = 4\,\mathrm{Tr}(\gamma^\alpha \not{q}\, \gamma_\alpha \not{p}_2)$$
$$= -8\,\mathrm{Tr}(\not{q}\, \not{p}_2)$$
$$= -32\,(p_2 \cdot q)$$

$$M_4(q) = -2\,\mathrm{Tr}(\gamma^\beta \not{q}\, \gamma_\beta \not{p}_2)$$
$$= 4\,\mathrm{Tr}(\not{q}\, \not{p}_2)$$
$$= 16\,(p_2 \cdot q)$$

<u>Piece 6</u> $= -m^2\big(D_1^2 L_4(q_1) + D_1 D_2 M_4(q_{11})$
$+ D_1 D_2 M_4(q_1) + D_2^2 L_4(q_{11}) \big)$

$$= \boxed{16m^2\Big[\, 2D_1^2 (p_2 \cdot q_1) + 2D_{11}^2 (p_2 \cdot q_{11}) - D_1 D_2 (p_2 \cdot q_1) - D_1 D_2 (p_2 \cdot q_{11}) \,\Big]}$$

$\boxed{\text{Piece 7}} = B^{\mu\nu} \not{P}_1 B_{\nu\mu} \not{P}_2$

$m^2 (D_1 \gamma^\nu \gamma^\mu + D_2 \gamma^\mu \gamma^\nu) \not{P}_1$
$(D_1 \gamma_\mu \gamma_\nu + D_2 \gamma_\nu \gamma_\mu) \not{P}_2$

same deal: two kinds of terms

① $\gamma^\alpha \gamma^\beta \not{P}_1 \gamma_\beta \gamma_\alpha \not{P}_2$
② $\gamma^\alpha \gamma^\beta \not{P}_1 \gamma_\alpha \gamma_\beta \not{P}_2$

$L_5 \equiv \mathrm{Tr}(①)$
$M_5 \equiv \mathrm{Tr}(②)$

$L_5 = 4\,\mathrm{Tr}(\not{P}_1 \not{P}_2)$
$= 16\,(P_1 \cdot P_2)$

$M_5 = 2\,\mathrm{Tr}(\gamma^\alpha \gamma_\alpha \not{P}_1 \not{P}_2) - \overbrace{\mathrm{Tr}(\gamma^\alpha \gamma^\beta \not{P}_1 \gamma_\beta \gamma_\alpha \not{P}_2)}^{L_5}$
$= 8\,\mathrm{Tr}(\not{P}_1 \cdot \not{P}_2) - L_5$
$= 16\,(P_1 \cdot P_2)$

$\underline{\text{Piece 7}} = \boxed{16 m^2 (P_1 \cdot P_2)\left(D_1^2 + D_2^2 + 2 D_1 D_2^2\right)}$

$\boxed{\text{Piece 8}} = B^{\mu\nu}(-m)\, B_{\nu\mu}\, m$

$= -m^4 (D_1 \gamma^\nu \gamma^\mu + D_{11} \gamma^\mu \gamma^\nu)$
$(D_1 \gamma_\mu \gamma_\nu + D_{11} \gamma_\nu \gamma_\mu)$

Some deal:

① $R = Tr(\gamma^\alpha \gamma^\beta \gamma_\beta \gamma_\alpha)$
$= 64$

② $S = Tr(\gamma^\alpha \gamma^\beta \gamma_\alpha \gamma_\beta)$
$= -2\, Tr(\gamma^\alpha \gamma_\alpha)$
$= -8\, Tr\, \mathbb{1}$
$= -32$

Piece 8 $= -m^4 (D_1^2 R + D_1 D_{11} S + D_{11} D_1 S + D_{11}^2 R)$

$= 32 m^4 (-2D_1^2 - 2D_{11}^2 + 2 D_1 D_{11})$

$= \boxed{64\, m^4 (-D_1^2 - D_{11}^2 + D_1 D_{11})}$

SUMMARY THUS FAR:

$$\sum_{\text{SPINS}} |\mathcal{M}|^2 = g^4 \boxed{\mathrm{Tr}\{\cdots\}}$$

↑ SUM OF EIGHT TERMS

$$① = 16\, D_1^2 \left[2(q_1 \cdot P_1)(q_1 \cdot P_2) - q_1^2 (P_1 \cdot P_2) \right]$$
$$+ 16\, D_{11}^2 \left[2(q_2 \cdot P_1)(q_{11} \cdot P_2) - q_{11}^2 (P_1 \cdot P_2) \right]$$
$$+ 64\, D_1 D_{11} \left[-(q_1 \cdot q_2)(P_1 \cdot P_2) \right]$$

$$② = 16\, m^2 \left[-4 D_1^2 q_1^2 - 4 D_{11}^2 q_{11}^2 - 2 D_1 D_{11} (q_1 \cdot q_{11}) \right]$$

$$③ = 16\, m^2 \left[-2 D_1^2 (q_1 \cdot P_1) - 2 D_{11}^2 (q_{11} \cdot P_1) + D_1 D_{11} (q_1 \cdot P_1) + D_1 D_{11} (q_{11} \cdot P_1) \right]$$

$$④ = 16\, m^2 \left[2 D_1^2 (q_1 \cdot P_2) + 2 D_{11}^2 (q_1 \cdot P_2) - D_1 D_{11} (q_1 \cdot P_2) - D_1 D_{11} (q_{11} \cdot P_1) \right]$$

$$③ = ⑤ = 16\, m^2 \left[-2 D_1^2 (q_1 \cdot P_1) - 2 D_{11}^2 (q_{11} \cdot P_1) + D_1 D_{11} (q_1 \cdot P_1) + D_1 D_{11} (q_{11} \cdot P_1) \right]$$

$$④ = ⑥ = 16\, m^2 \left[2 D_1^2 (q_1 \cdot P_2) + 2 D_{11}^2 (q_{11} \cdot P_2) - D_1 D_{11} (q_1 \cdot P_2) - D_1 D_{11} (q_{11} \cdot P_2) \right]$$

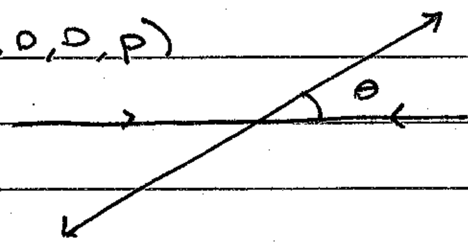
$$⑦ = 16\, m^2 (P_1 \cdot P_2) \left[D_1^2 + D_{11}^2 + 2 D_1 D_{11} \right]$$

$$⑧ = 64\, m^4 \left[-D_1^2 - D_{11}^2 + D_1 D_{11} \right]$$

Kinematics

$K_1 = (\overset{\omega}{\cancel{E}}, 0, \omega S_\theta, \omega C_\theta)$

$P_1 = (E, 0, 0, p)$



$P_2 = (E, 0, 0, -p)$

$K_2 = (\omega, 0, -\omega S_\theta, -\omega C_\theta)$ nb $E = \omega$

$q_I = (P_1 - K_1)$ $q_I^2 = t$

$q_{II} = (P_1 - K_2)$ $q_{II}^2 = u$

$$\begin{aligned}(p_1 \cdot p_2) &= E^2 + p^2 \\ &= m^2 + 2p^2\end{aligned}$$

$$\begin{aligned}(q_I \cdot q_{II}) &= P_1^2 - P_1 \cdot (K_1 + K_2) + K_1 \cdot K_2 \\ &= m^2 - P_1 \cdot (2E, \underline{0}) + 2E^2 \\ &= m^2\end{aligned}$$

$$\begin{aligned}(q_I \cdot P_1) &= (P_1 - K_1) \cdot P_1 \\ &= m^2 - (P_1 \cdot K_1) \\ &= -p^2 + pEC_\theta\end{aligned}$$

$= E^2 - pEC_\theta$
$= m^2 + p^2 - pEC_\theta$

$$\begin{aligned}(q_{II} \cdot P_1) &= (P_1 - K_2) \cdot P_1 \\ &= m^2 - (P_1 \cdot K_2) \\ &= -p^2 - pEC_\theta\end{aligned}$$

$= E^2 + pEC_\theta$

$$(q_I \cdot P_2) = (P_1 - K_1) \cdot P_2$$
$$= (E^2 + p^2) - (K_1 \cdot P_2)$$
$$= m^2 + 2p^2 - (E^2 + EpC_\theta)$$
$$= p^2 - EpC_\theta$$

$$(q_{II} \cdot P_2) = (P_1 - K_2) \cdot P_2$$
$$= (E^2 + p^2) - (K_2 \cdot P_2)$$
$$= E^2 + p^2 - (E^2 - EpC_\theta)$$
$$= p^2 + EpC_\theta$$

$$q_I^2 = (P_1 - K_1)^2 = m^2 - 2P_1 \cdot K_1$$
$$= m^2 - 2(m^2 + p^2 - pEC_\theta)$$
$$= -m^2 - 2p^2 + 2pEC_\theta$$

$$q_{II}^2 = (P_1 - K_2)^2 = m^2 - 2P_1 \cdot K_2$$
$$= m^2 - 2(m^2 + p^2 + pEC_\theta)$$
$$= -m^2 - 2p^2 - 2pEC_\theta$$

KINEMATIC PIECES THAT CONTRIBUTE IN THE $p \to 0$ LIMIT:

$(P_1 \cdot P_2)$, $(q_I \cdot q_{II}) \to m^2$

q_I^2, $q_{II}^2 \to -m^2$

S-WAVE PIECES ONLY

$$\sum_{\text{SPINS}} |\mathcal{M}|^2 = g^4 \, \mathrm{Tr}\{\cdots\}$$

$$① \to 16 D_I^2 \left(-q_I^2 (p_1 \cdot p_2)\right) + 16 D_{II}^2 \left(-q_{II}^2 (p_1 \cdot p_2)\right) - 64 D_I D_{II} (q_I \cdot q_{II})(p_1 \cdot p_2)$$

$$② \to 16 m^2 \left[-4 D_I^2 q_I^2 - 4 D_{II}^2 q_{II}^2 - 2 D_I D_{II} (q_I \cdot q_{II}) \right]$$

$$③, ④, ⑤, ⑥ \to 0$$

$$⑦ \to 16 m^2 (p_1 \cdot p_2) \left[D_I^2 + D_{II}^2 + 2 D_I D_{II} \right]$$

$$⑧ \to 64 m^4 \left[-D_I^2 - D_{II}^2 + D_I D_{II} \right]$$

OBSERVE; in S-wave limit: $D_I, D_{II} \to \boxed{D = \frac{-1}{2m^2}}$

$$① \mapsto 16 D^2 (m^4) \times 2 - 64 D^2 m^4 = -32 \, D^2 m^4$$

$$② \mapsto 16 D^2 m^2 \left[8m^2 - 2m^2 \right] = 6 \times 16 \, D^2 m^4$$

$$⑦ \mapsto 16 m^4 \cdot 4 D^2 = 64 D^2 m^4$$

$$⑧ \mapsto 64 m^4 (-D^2) = -64 D^2 m^4$$

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$$\sum ⓘ \mapsto \boxed{16 D^2 m^4 \times 4} = \boxed{64 \, D^2 m^4}$$