

METHOD of INDUCED REPS

A SKETCH of Weinberg QFT Vol 1. §2.5

what is a particle?

$$\text{state } \psi_0(p) \quad \text{s.t.} \quad \hat{P}^\mu \psi_0(p) = p^\mu \psi_0(p)$$

\uparrow QUANTUM H's WRT "SPIN" (WANT TO UNDERSTAND THIS) \uparrow OPERATOR \uparrow EIG. VAL

Ⓜ NOW WE'RE DEALING W/ POINCARÉ GROUP
 LORENTZ \rightarrow TRANSLATIONS \leftarrow REP: $U(\dots)$

RECALL: $U(\Lambda', a') U(\Lambda, a) = U(\Lambda'\Lambda, \Lambda a + a')$

momentum generates translations

$$U(1, a) \psi(p) = e^{-i p \cdot a} \psi(p)$$

what about Lorentz?

THIS IS THE "WATCH CAREFULLY" MOMENT: ($U(1) = U(1, 0)$)

$$\hat{P}^\mu U(1) \psi(p) = U(1) \underbrace{[U^{-1}(1) \hat{P}^\mu U(1)]}_{1} \psi$$

what is momentum of $U(1) \psi$?

this is a useful (familiar) combination \rightarrow like finite transformation of an Adjoint

(2.4.6)

$$\underbrace{U(\Lambda, a) U(1, \varepsilon) U^{-1}(\Lambda, a)}_{1 - \varepsilon_\rho P^\rho} = \underbrace{U(1, \Lambda \varepsilon)}_{1 - (\Lambda \varepsilon)_\mu P^\mu}$$

$$1 - \varepsilon_\rho P^\rho$$

$$1 - (\Lambda \varepsilon)_\mu P^\mu$$

$$\eta_{\mu\nu} \Lambda^\nu_\rho \varepsilon^\rho = \Lambda_\mu{}^\rho \varepsilon_\rho$$

$$\Rightarrow U(\Lambda, a) P^\rho U^{-1}(\Lambda, a) = \boxed{\Lambda_\mu{}^\rho P^\mu}$$

how to interpret funny position of indices?

REMARK:

$$\eta_{\mu\nu} \Lambda^\mu_\rho \Lambda^\nu_\sigma = \eta_{\rho\sigma}$$

$$\eta^{\sigma\alpha} \eta_{\mu\nu} \Lambda^\mu_\rho \Lambda^\nu_\sigma = \delta^\alpha_\rho$$

$$\Lambda^\mu_\rho \Lambda_\mu{}^\alpha = \delta^\alpha_\rho$$

$$\Rightarrow \boxed{(\Lambda^{-1})^\alpha{}_\mu = \Lambda_\mu{}^\alpha}$$

$$\boxed{U(\Lambda) P^\rho U^{-1}(\Lambda) = (\Lambda^{-1})^\rho{}_\mu P^\mu}$$

ORDER OF INVERSES!

$$U^{-1}(\Lambda) P^\rho U(\Lambda)$$

continuing:

$$P^\rho U(\Lambda) \psi(p) = U(\Lambda) \underbrace{(\Lambda^{-1})^\rho{}_\mu}_{(\Lambda p)^\mu} P^\mu \psi$$

$$= (\Lambda p)^\mu \left[U(\Lambda) \psi(p) \right]$$

now just some #s!

$$= (\Lambda p)^\mu \boxed{U(\Lambda) \psi(p)}$$

so what?

$$\hat{P}^\mu [\underline{U(\Lambda)\psi(p)}] = (\Lambda p)^\mu [\underline{U(\Lambda)\psi(p)}]$$

kind of obvious in retrospect!

the 4-momentum of $U(\Lambda)\psi(p)$ is Λp !

RESULT: $U(\Lambda)\psi(p)$ is some combination
of states w/ 4-momentum Λp .

$$U(\Lambda)\psi_\sigma(p) = \sum_{\sigma'} C_{\sigma'\sigma}(\Lambda, p) \psi_{\sigma'}(\Lambda p)$$

|-----|-----|-----|-----|

not LORENTZ 4-VEE INDICES!

SOME GENERAL INDEX for some
kind of LORENTZ REP.

REM:

ASSUME $C_{\sigma\sigma'}(\Lambda, p)$ is BLOCK-DIAGONAL

↳ EACH BLOCK IS AN IRREP of POINCARÉ
this is a good def for a particle.

~~1~~ ~~2~~ same as ~~3~~ ~~4~~ ~~5~~ ~~6~~ ~~7~~ ~~8~~ ~~9~~ ~~10~~ ~~11~~ ~~12~~ ~~13~~ ~~14~~ ~~15~~ ~~16~~ ~~17~~ ~~18~~ ~~19~~ ~~20~~ ~~21~~ ~~22~~ ~~23~~ ~~24~~ ~~25~~ ~~26~~ ~~27~~ ~~28~~ ~~29~~ ~~30~~ ~~31~~ ~~32~~ ~~33~~ ~~34~~ ~~35~~ ~~36~~ ~~37~~ ~~38~~ ~~39~~ ~~40~~ ~~41~~ ~~42~~ ~~43~~ ~~44~~ ~~45~~ ~~46~~ ~~47~~ ~~48~~ ~~49~~ ~~50~~ ~~51~~ ~~52~~ ~~53~~ ~~54~~ ~~55~~ ~~56~~ ~~57~~ ~~58~~ ~~59~~ ~~60~~ ~~61~~ ~~62~~ ~~63~~ ~~64~~ ~~65~~ ~~66~~ ~~67~~ ~~68~~ ~~69~~ ~~70~~ ~~71~~ ~~72~~ ~~73~~ ~~74~~ ~~75~~ ~~76~~ ~~77~~ ~~78~~ ~~79~~ ~~80~~ ~~81~~ ~~82~~ ~~83~~ ~~84~~ ~~85~~ ~~86~~ ~~87~~ ~~88~~ ~~89~~ ~~90~~ ~~91~~ ~~92~~ ~~93~~ ~~94~~ ~~95~~ ~~96~~ ~~97~~ ~~98~~ ~~99~~ ~~100~~ ~~101~~ ~~102~~ ~~103~~ ~~104~~ ~~105~~ ~~106~~ ~~107~~ ~~108~~ ~~109~~ ~~110~~ ~~111~~ ~~112~~ ~~113~~ ~~114~~ ~~115~~ ~~116~~ ~~117~~ ~~118~~ ~~119~~ ~~120~~ ~~121~~ ~~122~~ ~~123~~ ~~124~~ ~~125~~ ~~126~~ ~~127~~ ~~128~~ ~~129~~ ~~130~~ ~~131~~ ~~132~~ ~~133~~ ~~134~~ ~~135~~ ~~136~~ ~~137~~ 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WE KNOW THAT A GIVEN 4-MOMENTUM p^μ HAS TWO INVARIANTS:

$$p^2 \quad \uparrow \quad \text{sign}(p^0)$$

So: for each choice of these, define a STANDARD 4-momentum

$$\underbrace{(K^\mu)}_{\substack{\uparrow \\ \text{eg } (M, 0) \text{ for massive} \\ (k, 0, 0, k) \text{ for massless}}} \quad \text{s.t.} \quad p^\mu = \underbrace{L^\mu_\nu(p)}_{\substack{\uparrow \\ \text{standard}}} (K^\nu)$$

implicit dep on k .

SAME LORENZ TO TURN STANDARD TO THE SPECIFIC 4-MOMENTUM YOU WANT.

this is just saying: LET'S LOOK @ EACH PARTICLE "in the same way"

an electron moving @ 100 GeV is still an electron 2 might as well talk about it in the rest frame.

thus we can relate $\psi_\sigma(p)$ to $\psi_\sigma(k)$

$$\psi_\sigma(p) \equiv \underbrace{\sqrt{f(p)}}_{\text{NORM.}} \underbrace{U(L(p))}_{\text{state w/ 4-momentum } p=Lk} \psi_\sigma(k)$$

nb: σ labels unaffected.

Now ask: how does Λ affect $\psi(p)$?

$$\begin{aligned} U(\Lambda)\psi_\sigma(p) &= N(p) U(\Lambda L(p)) \psi_\sigma(k) \\ &= N(p) \underbrace{U(L(\Lambda p)) U(L^{-1}(\Lambda p))}_{\uparrow} U(\Lambda L(p)) \psi_\sigma(k) \\ &= N(p) U(L(\Lambda p)) \underbrace{U[L^{-1}(\Lambda p) \Lambda L(p)]}_{\uparrow} \psi_\sigma(k) \end{aligned}$$

this is a Lorentz
transf. that takes
 $k^\mu \rightarrow k^\mu$

\downarrow

$$W^\mu_\nu \equiv L^{-1}(\Lambda p) \Lambda L(p) : k \rightarrow k$$

\uparrow this is part of a SUBGROUP called
the little group. \rightarrow reshuffles σ indices

$$U(W) \psi_\sigma(k) = \sum_{\sigma'} D_{\sigma\sigma'}(W) \psi_{\sigma'}(k)$$

$\underbrace{\hspace{2cm}}$
representation of
the little group

$$D(WW') = D(W)D(W')$$

$$W = L^{-1}(\Lambda p) \Lambda L(p)$$

$$U(\Lambda)\psi_\sigma(p) = N(p) U(L(\Lambda p)) \underbrace{\sum_{\sigma'} D_{\sigma\sigma'}(W)}_{\text{"just #'s"}} \psi_{\sigma'}(k)$$

$$= N(p) \sum_{\sigma'} D_{\sigma\sigma'}(W) \boxed{U(L(\Lambda p)) \psi_{\sigma'}(k)}$$

\uparrow

$$= \frac{1}{N(\Lambda p)} \psi_{\sigma'}(\Lambda p) \quad \text{from p. 4}$$

$$U(\Lambda) \psi_\sigma(p) = \frac{N(p)}{N(\Lambda p)} \sum_{\sigma'} D_{\sigma\sigma'}(\Lambda) \psi_{\sigma'}(\Lambda p)$$

②: $C_{\sigma\sigma'}$ boils down to understanding little group method of induced reps.

Ⓐ $p^2 = m^2$ $K^r = (M, 0)$

LITTLE GROUP

$SO(3)$

Ⓑ $p^2 = 0$ $K^r = (k, 0, 0, k)$

$ISO(2)$

\uparrow 2D transl + rot.

MASSIVE PARTICLE: same transformation as in NR QM

\rightarrow all that 1/2 integer stuff

\rightarrow spherical harmonics, Clebsch-Gordan, etc carry over

\rightarrow this is actually somewhat surprising!

MASSLESS: more subtle.

$$U(W(\theta, \alpha, \beta)) = 1 + i\alpha A + i\beta B + i\theta J_3$$

\nearrow rot. \downarrow trans.

$$A = J_2 + K_1$$

$$B = -J_1 + K_2$$

$\underbrace{\hspace{10em}}$

give continuous spin quantum #

\rightarrow must be zero radius.

\uparrow EIGENVALUE

$$J_3 \psi_{k,\sigma} = \sigma \psi_{k,\sigma}$$

Helicity

why 1/2 integer

$$U(W) = e^{i\theta \cdot \sigma} \psi_\sigma(k) \rightarrow D_{\sigma\sigma'}(W) = e^{i\theta \cdot \sigma} \delta_{\sigma\sigma'}$$