

SU(3) commutators

Physics 262: Group Theory for Physicists
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Define a commutator

```
In[4]:= Com[A_, B_] := MatrixForm[A.B - B.A]
```

Pick a basis

I think these are roughly the basis used in Cahn.

$$\text{In[95]:= } \mathbf{T_p} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix};$$

$$\mathbf{T_m} = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix};$$

$$\mathbf{V_p} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix};$$

$$\mathbf{V_m} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix};$$

$$\mathbf{UUp} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix};$$

$$\mathbf{UUm} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix};$$

$$\mathbf{T_3} = \frac{1}{2} \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix};$$

$$\mathbf{T_8} = \frac{1}{2\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix};$$

Print out relevant commutators

T^\pm

```
In[119]:= Com[Tp, Tm]
Print["T3"]
Com[T3, Tp]
Com[T3, Tm]
Print["T8"]
Com[T8, Tp]
Com[T8, Tm]
```

Out[119]/MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

T3

Out[121]/MatrixForm=

$$\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Out[122]/MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

T8

Out[124]/MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Out[125]/MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

V^\pm

```
In[126]:= Com[Vp, Vm]
Print["T3"]
Com[T3, Vp]
Com[T3, Vm]
Print["T8"]
Com[T8, Vp]
Com[T8, Vm]
```

Out[126]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

T3

Out[128]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Out[129]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -\frac{1}{2} & 0 & 0 \end{pmatrix}$$

T8

Out[131]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & \frac{\sqrt{3}}{2} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Out[132]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -\frac{\sqrt{3}}{2} & 0 & 0 \end{pmatrix}$$

```
In[133]:= Com[1/2 T3 + sqrt(3)/2 T8, Vp]
```

Out[133]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

U^\pm

```
In[141]:= Com[UUp, UUm]
Print["T3"]
Com[T3, UUp]
Com[T3, UUm]
Print["T8"]
Com[T8, UUp]
Com[T8, UUm]
```

Out[141]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

T3

Out[143]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{2} \\ 0 & 0 & 0 \end{pmatrix}$$

Out[144]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 \end{pmatrix}$$

T8

Out[146]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \frac{\sqrt{3}}{2} \\ 0 & 0 & 0 \end{pmatrix}$$

Out[147]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & -\frac{\sqrt{3}}{2} & 0 \end{pmatrix}$$

Check Cross Commutators

```
In[151]:= Com[Tp, Vm]
          Com[Tp, UUp]
          Com[Vp, UUm]
```

Out[151]/MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & -1 & 0 \end{pmatrix}$$

Out[152]/MatrixForm=

$$\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Out[153]/MatrixForm=

$$\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

```
In[154]:= Com[Tm, Vm]
          Com[Tm, UUp]
          Com[Vm, UUm]
```

Out[154]/MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Out[155]/MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Out[156]/MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$