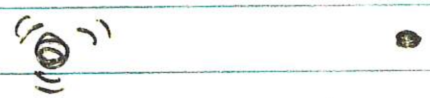
P262 : LEC 17

3/1/2019

Remarks from LAST TIME

$\boxed{\text{Physics Q}}$: why is $n=1,2,\ldots$ HYDROGEN a different particle?



GROUND STATE
$K = (M, \ell)$

BOUND STATE
HIGHER EXCITATION
$K = (M', \ell)$

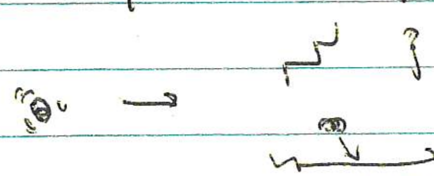
or: $\boxed{p^2 = (M')^2}$ $\qquad$ $\boxed{p^2 = M^2}$

no way to rotate into each other
LORENTZ

DIFF. QUANTUM NUMBERS → DIFF PARTICLE

"but they're both electron – proton!"
THAT'S A STATEMENT ABOUT MICROSCOPIC THY.

"but they're related!"



} EMITS A PHOTON

2 PARTICLE STATE.

WHAT DID THIS INDUCED REP STUFF GIVE US?

LITTLE GROUP

2 common cases: $k = (M, \underline{0}) \longrightarrow SO(3)$

$K = (K, 0, 0, k) \longrightarrow ISO(2)$

$\uparrow$ eff. $SO(2)$

massive particles labelled by mass

massless — " — — " — helicity.

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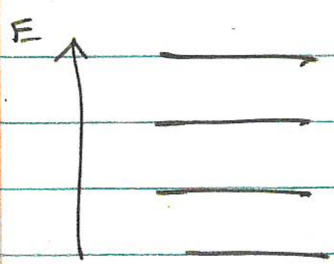
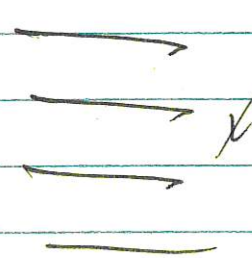
the spirit of this composite/fundamental labelling of states is reminiscent of AdS/CFT (gauge-gravity/holographic principle)

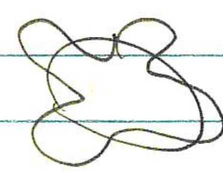
EXPERIMENTAL VERSION:

BOUND STATES:



COMPACT EXTRA DIM





WAVE FUNC. W/ PER BC.

MORE WIGGLES MEANS HIGHER MOMENTUM

SPECTRUM of excitations (typically higher spins...)

"E = M"

$$m^2 = p_0^2 - \Sigma p_i^2$$

$$\underbrace{m^2 + p_{XD}^2}_{\text{EFFECTIVE 4D MASS}} = p_0^2 - p_x^2 - p_y^2 - p_z^2$$

conformal syms are important in th. phys.
ENDPOINTS of RG flows.

$\boxed{\text{AdS/CFT}}$

$AdS_5 \times S^5 \longleftrightarrow \mathcal{N}=4$ SCFT

curved 5D space ("GR")

flat 5D compact space

4 kinds of supersym.

" eg. A FORCE PARTICLE HAS 4 MATTER PARTICLES ASSOCIATED W/ IT "

conformal
→ extend Lorentz to include scale sym.
[not our universe
↳ no particles]

semi CLASSICAL THY ON CURVED SPACETIME

VERY "QUANTUM" THY (RENORMALIZ. GROUP FLOW)

SYM: $\boxed{\text{ISOMETRY GROUP}}$

$AdS_5 : \; SO(4,2) \longleftrightarrow$ 4D conformal sym
(looks like 2 time phys)

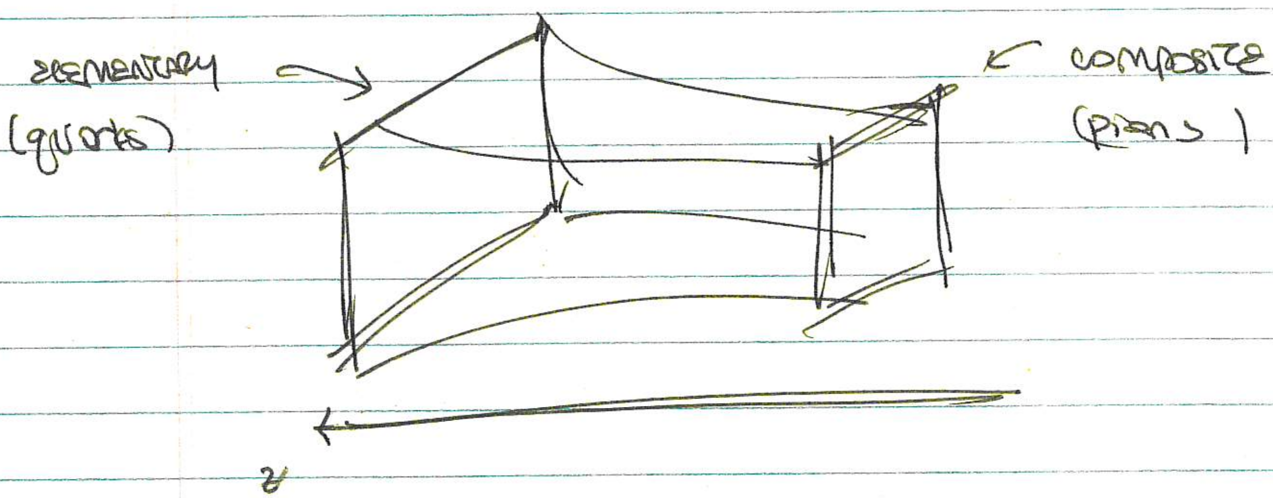
$S^5 : \; SO(6) \cong SU(4) \longleftrightarrow$ R-sym of $\mathcal{N}=4$ susy

"geometerize renormalization group"

PARTICLE PHYSICS TRICK:

RG flow btwn APPROXIMATELY CONFORMAL THEORIES

(Randall-Sundrum scenario)



Solve classical profiles of quantum fields

WRAPPING UP : $\boxed{SL(2,\mathbb{C})}$

REPS OF $SO(3,1)$: not unitary
not compact ⟶ THM: REPS of noncompact groups are ∞ DIM.

inconsistent w/ particles
that have quantum #'s.

ANSWER: needed full POINCARÉ GROUP.
↳ hence induced rep. stuff we did.

$\boxed{SL(2,\mathbb{C})}$ ⟶ universal cover of LORENTZ
Spin $(3,1)$ ← univ. cover of $so(3,1)$

REPRESENTATIONS

L.H. WEYL SPINOR (2 COMP)

$\psi_\alpha \mapsto N_\alpha{}^\beta \psi_\beta$ FUNDAMENTAL

$\chi_{\dot\alpha} \mapsto (N^*)_{\dot\alpha}{}^{\dot\beta} \bar{\chi}_{\dot\beta}$ ANTIFUND/CONJ

R.H. WEYL SPINOR (2 COMP)

DOTTED INDICES.

INVARIANTS

$$\varepsilon^{\alpha\beta} \longrightarrow \varepsilon^{\rho\sigma} N_\rho{}^\alpha N_\sigma{}^\beta = \varepsilon^{\alpha\beta} \det N = \varepsilon^{\alpha\beta}$$

$$= -\varepsilon_{\alpha\beta}$$

mnemonic: $\varepsilon^{\alpha\beta} = i(\sigma^2)^{\alpha\beta}$
(don't take this too seriously)

$$\varepsilon^*_{\alpha\beta} = \varepsilon_{\dot\alpha\dot\beta}$$ ← dots ↔ conjugation

CAN BE USED AS METRIC : $\psi^\alpha = \varepsilon^{\alpha\beta}\psi_\beta$

$$\bar\chi^{\dot\alpha} = \varepsilon^{\dot\alpha\dot\beta}\bar\chi_{\dot\beta}$$

CONTRAVARIANT REPS : or $(N^{*-1})^{\dot\alpha}{}_{\dot\beta}\,\bar\chi^{\dot\beta}$

$$\boxed{\begin{array}{l} \psi^\alpha \longrightarrow \psi^\beta (N^{-1})_\beta{}^\alpha \\ \bar\chi^{\dot\alpha} \longrightarrow \bar\chi^{\dot\beta}(N^{*-1})_{\dot\beta}{}^{\dot\alpha} \end{array}}$$

contravariant
contra-conjugate

eg: $\varepsilon_{\alpha\beta} N^\alpha{}_\gamma N^\beta{}_\delta = \varepsilon_{\gamma\delta} \cancel{\det N}$ 1

$$(N^T)_\gamma{}^\alpha \varepsilon_{\alpha\beta} N^\beta{}_\delta = \varepsilon_{\gamma\delta}$$

$$\Rightarrow \quad \varepsilon_{\alpha\beta} N^\beta{}_\delta = [(N^T)^{-1}]_\alpha{}^\gamma \varepsilon_{\gamma\delta}$$

$$N^\rho{}_\delta = \varepsilon^{\rho\alpha}[(N^T)^{-1}]_\alpha{}^\gamma \varepsilon_{\gamma\delta}$$

UPPER & LOWER INDICES ARE EQUIVALENT REPS. → so : $\boxed{N \;\&\; (N^T)^{-1} \quad \text{EQUIVALENT}}$

similar : $N^* \;\&\; (N^\dagger)^{-1}$ EQUIVALENT convention

INVARIANTS : $\psi\chi \equiv \psi^\alpha \chi_\alpha \quad = \epsilon^{\alpha\beta}\psi_\beta\chi_\alpha$

$= -\psi_\alpha \chi^\alpha$

$= +\chi^\alpha \psi_\alpha$ ← FERMIONS

$= \chi\psi$ ← ORDER INDEP W/ OUR CONV.

$\bar\psi\bar\chi = \bar\psi_{\dot\alpha}\bar\chi^{\dot\alpha}$

$= \bar\chi\bar\psi$ SIMILARLY

VECTOR COMBINATIONS : $(\sigma^\mu)_{\alpha\dot\alpha}$

$\underset{\sim}{x} = x_\mu \sigma^\mu \longrightarrow N_\alpha{}^\beta (x_\nu \sigma^\nu)_{\beta\dot\gamma} N^{*}{}_{\dot\alpha}{}^{\dot\gamma}$

$\equiv (\Lambda_\mu{}^\nu x_\nu)\sigma^\mu{}_{\alpha\dot\alpha}$

invariance : $(\sigma^\mu)_{\alpha\dot\alpha} = N_\alpha{}^\beta (\sigma^\nu)_{\beta\dot\gamma} (\Lambda^{-1})^\mu{}_\nu N^{*}{}_{\dot\alpha}{}^{\dot\gamma}$

RAISED INDEX : $(\bar\sigma^\mu)^{\dot\alpha\alpha} \equiv \epsilon^{\alpha\beta}\epsilon^{\dot\alpha\dot\beta}(\sigma^\mu)_{\beta\dot\beta}$

$= (\mathbb{1}, -\underline{\sigma})$

ORDER OF INDICES

Generators of $SL(2,\mathbb{C})$

$$(\sigma^{\mu\nu})_\alpha{}^\beta = \frac{i}{4}(\sigma^\mu \bar{\sigma}^\nu - \sigma^\nu \bar{\sigma}^\mu)_\alpha{}^\beta$$
$$(\bar{\sigma}^{\mu\nu})^{\dot{\alpha}}{}_{\dot{\beta}} = \frac{i}{4}(\bar{\sigma}^\mu \sigma^\nu - \bar{\sigma}^\nu \sigma^\mu)^{\dot{\alpha}}{}_{\dot{\beta}}$$

HERMITIAN.

analogs of $(M^{\mu\nu})_{\rho\sigma}$

$$\psi_\alpha \longrightarrow e^{\left(-\frac{i}{2}\omega_{\mu\nu}\sigma^{\mu\nu}\right)_\alpha{}^\beta} \psi_\beta$$
$$\bar{\psi}^{\dot{\alpha}} \longrightarrow e^{\left(-\frac{i}{2}\omega_{\mu\nu}\bar{\sigma}^{\mu\nu}\right)^{\dot{\alpha}}{}_{\dot{\beta}}} \bar{\psi}^{\dot{\beta}}$$

FOR LH GENS:

$$J_i = \frac{1}{2}\epsilon_{ijk}\sigma_{jk} = \frac{1}{2}\sigma_i$$
$$K_i = \sigma_{0i} = -\frac{i}{2}\sigma_i$$

$$A_i = \frac{1}{2}(J_i + iK_i) = \frac{1}{2}\sigma_i$$
$$B_i = \frac{1}{2}(J_i - iK_i) = 0$$

SO LH WEYL SPINORS REALIZE $(\frac{1}{2}, 0)$

SIMILARLY, $\bar{\chi}^{\dot{\alpha}}$ ARE $(0, \frac{1}{2})$

DIRAC EQ : $p_\mu \sigma^\mu \psi = m\psi$

$p_\mu \bar{\sigma}^\mu \bar{\chi} = m\bar{\chi}$

for massless : $p^0 = |\vec{p}|$

$$\Rightarrow \left(\frac{\sigma \cdot \vec{p}}{|\vec{p}|}\right)\psi = (+)\psi \quad \leftarrow (p_0\sigma^0 - \vec{p}\cdot\sigma)\psi = 0$$

$$\left(\frac{\sigma \cdot \vec{p}}{|\vec{p}|}\right)\bar{\chi} = (-)\bar{\chi}$$

HELICITY → exactly the LITTLE GROUP QUANTUM # WE SUGGESTED LAST TIME.

not inv't.

MASSLESS PARTICLES w/ Q CHARGE? CANNOT GIVE A NAïVE MASS : $\boxed{m\psi^2}$

↳ need to combine m ψ (anti-ψ)

$$\psi_D = \begin{pmatrix} \psi_\alpha \\ \bar{\chi}^{\dot{\alpha}} \end{pmatrix} \qquad \bar{\psi}_D = (\chi^\alpha, \bar{\psi}_{\dot{\alpha}})$$

DIFF INDICES!

$e^+_R \quad e^-_L \quad e^-_L \quad e^+_L$

$$\text{s.t.} \quad m\bar{\psi}_D\psi_D = m\,\chi\psi + m\,\bar{\chi}\bar{\psi}$$

$$\psi_D \rightarrow e^{-\frac{i}{2}\omega_{\mu\nu}\Sigma^{\mu\nu}}\psi_D$$

$$\Sigma = \begin{pmatrix} \sigma^{\mu\nu} & \\ & \bar{\sigma}^{\mu\nu} \end{pmatrix}$$

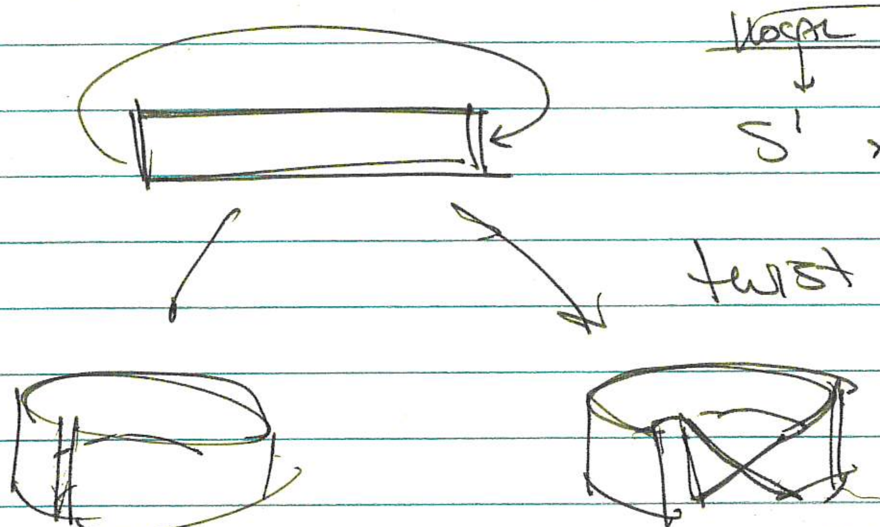
FIBER BUNDLES

hep-ph/0611201 — § 1, 2

$\boxed{\text{LOCAL PRODUCT}}$

$\downarrow$

$S^1 \times L$



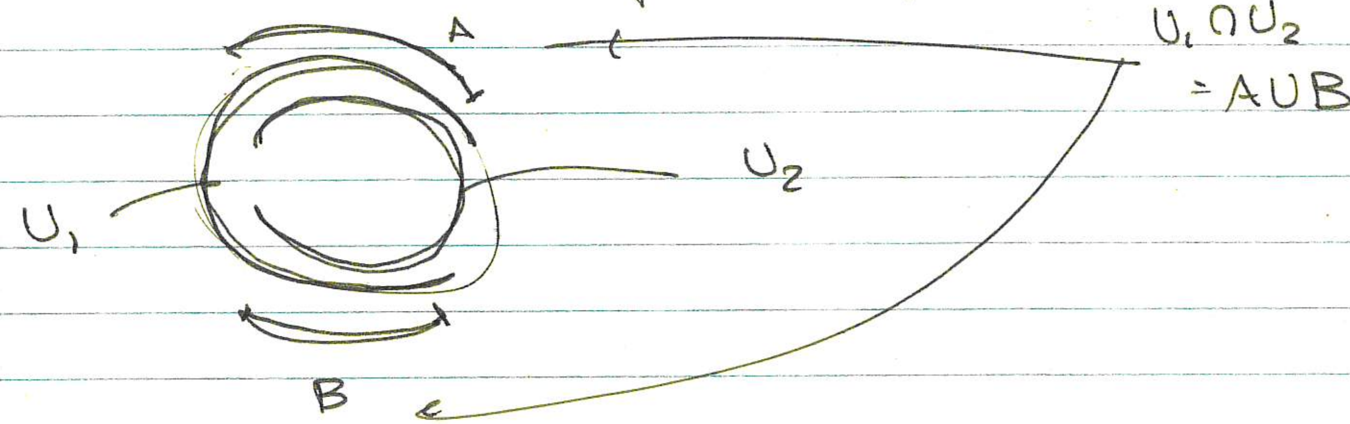
MOBIUS STRIP.

think of this as a <u>BASE SPACE</u> S^1
with a <u>FIBER</u> L

PROJECTION TO BASE: $\pi : M\ddot{o} \to S^1$

<u>LOCAL COORDS</u>: $\phi : \underset{\substack{\uparrow \\ \text{PATCH OF} \\ \text{BASE SPACE}}}{U} \times L \to \underset{\substack{\uparrow \\ \text{PART OF} \\ \text{MOBIUS STRIP}}}{\pi^{-1}(U)}$

nontriviality of Mö is <u>GLOBAL</u> property.
consider two patches



$$\phi_1^{-1} \circ \phi_2 : (A \cup B) \times L \longrightarrow (A \cup B) \times L$$

$B \times L \to$ Mö

$$\phi_1^{-1} \phi_2 (x, t) = (x, g_{12}(t))$$

BASE FIBER

$\in \mathbb{Z}_2$