The detection of single quanta of circulation in liquid helium II

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[Plate 1]

An apparatus is described for detecting single quanta of superfluid circulation round a fine wire in liquid helium II. The wire is stretched down the centre of a cylindrical vessel containing helium, and the circulation may be established by rotating the whole apparatus about the axis of the wire and cooling from above the λ-point. The wire can be set into transverse vibration, and the circulation round it can then be obtained from the rate of precession of the plane of vibration. The technique proves to be sufficiently sensitive for the measurement of circulations of order h/m with an accuracy of about 3 %. The method in its present form measures only an average of the circulation along the length of the wire, and it is found that this average is not quantized. Apparent circulations equal to a fraction of a quantum are attributed to quantized vortices that are attached to only a fraction of the length of the wire, and this interpretation has been confirmed by showing that an apparent circulation of exactly h/m has much greater stability than any other value. In this way the quantization of superfluid circulation in units of h/m has been experimentally verified. Observations made in the course of this work show clearly that superfluid circulations (including free vortex lines) can persist indefinitely even when the rotation of the apparatus is stopped. Values have also been obtained for the circulation round the wire as a function of the angular velocity of rotation, and it is shown from these that the energy of a free vortex line in the helium surrounding the wire may perhaps be considerably smaller than has hitherto been supposed.

1. Introduction

It is now generally believed that the flow of the superfluid component of liquid helium II must be irrotational, and that the circulation $\oint \mathbf{v}_s . d\mathbf{r}$ round any circuit in the liquid must be quantized in units of h/m, where m is the mass of a helium atom (Onsager 1949; Feynman 1955). A non-zero circulation may exist round any multiply connected region, and such a region may be formed either by solid boundaries or by what is in effect a microscopic hole in the liquid; in the latter case there will be a free quantized vortex line. Experimental evidence in favour of these views has been provided by observations on the attenuation of second sound by free vortex lines in uniformly rotating helium (Hall & Vinen 1956) and by studies of the propagation of waves on these lines (Hall 1958). However, the evidence for the quantization of superfluid circulation provided by these experiments was only indirect and applied only to a free vortex line; there was no evidence at all for quantization round a multiply connected region formed entirely by solid boundaries.

The aim of the present experiments was, in the first place, to supply this missing evidence by measuring directly the magnitude of the circulation round a solid cylinder, in the form of a fine wire, situated along the axis of a uniformly rotating vessel of helium II. The apparatus that has been used for this work has, however, also been used for two other studies: first, to make observations from which it was hoped to deduce information about the energy per unit length of a free vortex line; and secondly, to study the creation of free vortex lines in initially undisturbed helium. The work on the evidence for quantization and on the energy of a free vortex line will be described in detail in the present paper; the work on creation of vortex lines will be mentioned only briefly and will be described in detail in a later paper.

An experiment that might have provided direct evidence for quantization of superfluid circulation has already been described by Craig & Pellam (1957). This experiment consisted in measuring the lift on an aerofoil placed in a uniform stream of superfluid. The lift should be proportional to the circulation round the aerofoil, but the measurements showed no evidence for quantization. It seems likely that this apparent absence of quantization was associated with the fact that, if a free vortex line ends at some point along the aerofoil, the number of quanta of circulation round the aerofoil need not be constant along its length; for it is clear that in this case a total lift corresponding to a non-integral number of quanta will be measured. It seems very likely that such partial attachment of vortex lines to the aerofoil could occur, especially under the conditions of Craig & Pellam's experiment, which probably involved turbulent flow of the whole superfluid; and this view is confirmed by the fact that, as will be explained later, an analogous situation has been observed in the present experiments even in the absence of turbulent flow.

An outline of the present experimental method and an indication of the results obtained will be given in §2. Details of the apparatus and of the experimental procedure will be described in §3, and the full experimental results will be presented in §4. Finally, the attempt to deduce information about the energy per unit length of a free vortex line from some of these experimental results will be described in §5.

Preliminary notes on some of this work have already been published (Vinen 1958 *a*, *b*).

2. Outline of the experiments

The apparatus consists essentially of a metal tube, containing helium, down the centre of which is stretched the fine metal wire round which the circulation is to be measured. This measurement is possible through the influence, owing to the Magnus effect, that the circulation has on the modes of transverse vibration of the wire. In the case of an ideal wire, in the absence of a circulation, any given mode of vibration is doubly degenerate owing to the possibility of vibration in two directions at right angles; the Magnus force arising from a circulation κ removes this degeneracy, giving two circularly polarized modes separated in frequency by an amount $\rho_s \kappa / 2\pi w$, where ρ_s is the density of the circulating fluid, and w is the sum of the mass per unit length of the wire and half the mass of fluid displaced by unit length of wire. Thus a measurement of this frequency separation yields a value of the circulation. Now the normal fluid will always flow like an ordinary liquid, so that under ordinary conditions it will contribute to a negligible extent to any circulation round the wire; the measured circulation may therefore be taken to be due entirely to the superfluid.

If a vortex is only partly attached to the wire, so that the circulation κ exists round only the length of wire between, say, the points x_1 and x_2, then there is still a splitting into two circularly polarized modes, but the magnitude of the splitting is reduced and depends on the particular degenerate mode concerned. For the fundamental mode, which has been used throughout the present work, the splitting can be shown to be equal to $\rho_s \kappa_a / 2\pi w$, where

$$\kappa_a = \frac{2\kappa}{l} \int_{x_1}^{x_2} \sin^2 \frac{\pi x}{l} \, \mathrm{d}x,$$

and l is the length of the wire. The generalization to the case where there is more than one voretx on the wire is obvious. The quantity κ_a, which is obtained directly from the experimentally observed splittings, will be called the 'apparent circulation'.

It should be possible to establish a circulation of superfluid round the wire by rotating the whole apparatus uniformly about an axis coincident with the wire. When the system is in equilibrium in the rotating state, there should, as is explained in detail in §5, be a certain number, p, of quanta of circulation round the wire, together with an array of q free vortex lines in the surrounding helium, the values of p and q increasing with increasing angular velocity of rotation, ω. Thus, as ω is increased, the measured circulation round the wire should increase stepwise in units of h/m.

The experimental results, which will be described in detail in §4, show that these predictions are probably essentially correct, but that in practice the situation is made complicated by the fact that it is difficult to achieve equilibrium. Thus, in the first place, it has been found that, at least at the low angular velocities used in the present experiments, it is extremely difficult, if not impossible, to achieve even approximate equilibrium by simply setting the system into rotation below the λ-point; the evidence at present is that under these conditions the superfluid may, at least for long periods, simply remain at rest. This difficulty was in fact anticipated, since it was clear that setting the superfluid into rotation below the λ-point must involve the creation of new free vortex lines, and it seemed likely that this is a difficult process (see, for example, Vinen 1957). The difficulty is of course easily overcome by rotating above the λ-point and then cooling, so that the superfluid is *created* in a state of rotation. This procedure has been used successfully, in that it leads to apparent circulations round the wire that are of the expected order of magnitude. However, it has been found that it does not lead to apparent circulations that are always integral multiples of h/m, so that it must be concluded that even this procedure does not always produce the state of complete equilibrium, but instead a state in which a vortex line is partly attached to the wire, a situation probably analogous to that found by Craig & Pellam.

It is of course necessary to find experimental evidence in favour of this conclusion if the quantization condition is to be verified, and this has been done by studying the behaviour of the apparent circulation after the rotation of the apparatus has been stopped. This behaviour is as follows. As soon as the rotation is stopped, the apparent circulation changes slightly, but, in the absence of disturbances, it soon settles down to a value that appears to be quite constant for an indefinite time. This constant value still corresponds in general to a non-integral multiple of h/m (usually between one and two) and its existence is, incidentally, direct evidence that superfluid currents, and even free vortex lines, can persist indefinitely in helium II contained in a stationary vessel. However, the vital observation is that if the system is now fairly violently disturbed by, for example, exciting a large amplitude of vibration on the wire, then the apparent circulation will generally decrease, *but often it will decrease only as far as a value that is equal, within the experimental error, to exactly* h/m. This is precisely the behaviour to be expected, since the removal of a partly attached vortex line involves only the movement, and, possibly, the stretching, of an existing free vortex and should be comparatively easy, while the removal of a vortex attached to the whole length of the wire involves the difficult creation of a new free line. Thus the quantization condition has been verified.

According to the theory developed in § 5 the number of quanta of circulation round the wire in equilibrium at any given angular velocity depends on the energy per unit length of the free vortex lines that exist in the surrounding helium. Thus measurements of the type made in the present work could in principle lead directly to a value of this energy. Unfortunately, however, the present measurements cannot be used in this simple way, owing to the fact, already stressed, that it has proved impossible to achieve rotating states in exact equilibrium. Nevertheless, as will be shown in § 5, some information about the energy can still be deduced, although the deduction is by no means rigorous.

It should be added finally that a study of the conditions (if any) under which a circulation round the wire may be generated by setting the apparatus into rotation *below* the λ-point should yield information about the problem of the creation of new free vortex lines in initially undisturbed helium. Work in this direction is still in progress, and a description of it must therefore be left for a later paper.

3. Experimental details

Details of the apparatus are shown schematically in figure 1. The wire W round which the circulation is measured consists of about 5 cm of 0·001 in. diameter beryllium-copper, the ends of which are soldered into two cylindrical pistons, A and B. The frequency splitting per quantum of circulation for this wire should be about 0·45 c/s. The pistons slide inside the hollow brass cylinder C, which has an internal diameter of about 4 mm. A tension on the wire is maintained by the spring S and may be adjusted by moving the piston B; this adjustment can be made from outside the cryostat by rotating the copper-nickel tube X. It will be explained later that it is necessary to be able to twist one end of the wire relative to the other, and this can be achieved, again from outside the cryostat, by rotating together the copper-nickel tubes X and Y. The outer cylinder G serves both as a protective shield and to improve the isolation of the helium round the wire from the helium in the main bath. The whole assembly may be rotated by means of a synchronous motor driving through a gearbox on to the tube Z; the axis of rotation is defined by a ball race on the cryostat cap and the brass-in-*PTFE* bearing *P*. Speeds of rotation in the range from 0·1 to 2 rev/min are available.

The lower end of the cryostat is placed between the poles of a magnet so that the wire is situated in a transverse magnetic field of about 3 kG. Transverse vibrations of the wire in a plane perpendicular to the magnetic field can then be observed as a voltage induced between the ends of the wire. (The presence of the Bakelized fabric section *F* ensures that the ends of the wire are not shorted together.) This voltage is amplified, passed through a filter tuned to the natural fundamental frequency of the wire, and displayed on an oscilloscope. The electrical circuit

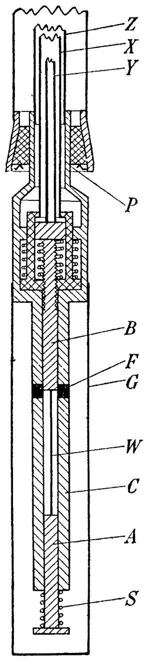


FIGURE 1. Diagram of apparatus.

attached to the wire has to be carefully designed, so that it does not affect appreciably the vibrational characteristics of the wire. In practice, observations are most conveniently made by passing a short electrical pulse through the wire in order to set it vibrating, and then photographing the free decay of the vibrations on the oscilloscope, which is fitted with a slow time base triggered by the pulse. Any splitting of the fundamental mode due to a circulation causes beats to be superimposed on this free decay; the magnitude of the splitting, and hence the magnitude of the circulation, is given by the beat frequency, which is easily measured from the film records. Typical records are shown in figure 2 (plate 1).

The sensitivity of the method is clearly limited by the damping of the vibrations; adequate sensitivity is achieved only if the time constant of the natural decay is not too small compared with the period of the beats caused by the circulation. The damping is caused almost entirely by the viscous drag of the normal fluid, and it is clear from figure 2 that at 1·3 °K (the lowest temperature attainable with the cryostat) this damping is only just small enough to allow the measurement of a circulation of order h/m. At temperatures above about 1·4 °K the damping is no longer small enough, and measurements of such small circulations are no longer possible.

When the present experiments were started, it was felt that it might be necessary to limit the amplitude of vibration of the wire in order to prevent quanta of circulation being thrown off the wire. A possible condition seemed to be that the velocity amplitude should not exceed h/ma, where a is the radius of the wire. It was soon found, however, that this is not a necessary condition, and velocity amplitudes equal to about $3h/ma$ have usually been employed. (The velocity amplitude that is required to throw off a vortex that is completely attached to the wire is still not known, but it certainly exceeds the maximum value that has been used in the present experiments, which is about $30h/ma$.) The frequency to which the wire is tuned is not of critical importance; a value of about 500 c/s has been found convenient.

It is now necessary to refer to a complication that has so far been ignored. It has been assumed in the earlier discussion that in the absence of any circulation the modes of vibration of the wire possess exact double degeneracy. It has been found in practice that this is not true. Owing, presumably, to slight asymmetries in the wire and the mountings, the degeneracy is almost always lifted. The normal modes are always found to be plane polarized, and may be separated in frequency by as much as 5 c/s. Now it can be shown that if a circulation κ is superimposed on this system the normal modes become elliptically polarized and are separated in frequency by an amount $\Delta\nu$ given by

$$(\Delta\nu)^2 = (\Delta\nu_\kappa)^2 + (\Delta\nu_N)^2,$$

where $\Delta\nu_\kappa = \rho_s\kappa/2\pi w$, and $\Delta\nu_N$ is the frequency separation between the original plane polarized normal modes. (The major axes of the two ellipses coincide with the original planes of polarization, and each ellipse has an eccentricity given by

$$e^2 = 2x[(1+x^2)^{\frac{1}{2}} - x],$$

where $x = \Delta\nu_N/\Delta\nu_\kappa$.) Thus the circulation can still be determined if both $\Delta\nu$ and $\Delta\nu_N$ are measured. However, this procedure is satisfactory only if $\Delta\nu_N$ is neither too small nor too large. For if it is too small it cannot be measured, owing to the damping of the wire (and under these circumstances it is not necessarily so small that it can be ignored), while if it is too large $\Delta\nu_\kappa$ has to be found as a small difference between two large quantities. It is found in practice that $\Delta\nu_N$ often turns out to be of an unsatisfactory magnitude, but fortunately it has also been found that it can be adjusted very easily by twisting one end of the wire relative to the other. (By means of this adjustment $\Delta\nu_N$ could presumably be eliminated altogether; but, since frequency differences less than a certain value cannot be measured, there is no way of checking that it has been eliminated.) Thus the following procedure is used. At the beginning of an experimental run the helium is cooled without rotation to the lowest available temperature. (As explained below, the wire is sometimes left after this cooling with a short length of vortex on it;

but the resulting $\Delta\nu_\kappa$ is usually small enough to be neglected in making the following preliminary adjustments.) The vibrating wire assembly is turned so that the normal modes of the undisturbed wire lie at 45° to the magnetic field. The frequency difference can then be easily observed on the oscilloscope as a beat pattern with 100 % modulation, and it is adjusted to a convenient value by twisting the wire. This adjustment usually shifts the normal modes, so that it is necessary continually to turn the assembly to ensure that the modes remain at 45° to the magnetic field. When the adjustment is complete, the positions of the assembly for which the normal modes are at 45° to the magnetic field are recorded, and all subsequent measurements are made when the system is in one of these positions. If observations are made in other positions, the beat pattern exhibits less than 100 % modulation, and measurements are consequently difficult. For example, if one of the undisturbed normal modes lies along the magnetic field, then clearly no beat pattern at all is observed in the absence of a circulation, while the fractional modulation in the presence of a circulation is equal to only $[x-(1+x^2)^{\frac{1}{2}}]^2$.

During the early stages of the present work some difficulty was experienced in measuring $\Delta\nu_N$, since no procedure could be found that could be relied upon to produce a zero value of $\Delta\nu_\kappa$ (the nature of this difficulty will be clear after the detailed experimental results have been presented in § 4). However, it was eventually found that towards the end of an experimental run, just before the helium drains out of the space surrounding the wire, $\Delta\nu$ always decreases to a well-defined minimum value; and this value has been taken to be equal to $\Delta\nu_N$. That this identification is correct has been confirmed by the fact that on *no* occasion during any given experimental run has a value of $\Delta\nu$ less than this minimum ever been observed, and also by the results of experiments described in § 4 in which the system is first cooled from the λ-point rotating in one direction and then rotated in the opposite direction. The reason why any vortex line on the wire invariably disappears at the end of an experimental run is not known with certainty, but it can be shown that a large heat current probably flows through the helium that surrounds the wire just before it drains out (this heat current originating from conduction down the various tubes from the cryostat cap) and it is possible that this is the cause. It might be thought that $\Delta\nu_N$ could be found most easily by taking measurements *after* the helium has drained out of the space surrounding the wire. But it appears that this method is unsatisfactory, since it has been found that the value of the splitting so obtained is not always constant in time and is sometimes greater than values obtained by the method indicated above. The reason for this is again not known with certainty, but it may be associated with a change in the dynamical characteristics of the wire caused either by rapid warming up or by the draining of liquid down the wire.

The three photographs shown in figure 2 are typical of those obtained during a single experimental run, and they correspond to circulations equal to approximately zero, h/m, and $2h/m$. It is found that circulations of the order of or greater than h/m can be measured from such photographs with an accuracy of about 3 %. The mass per unit length of the BeCu wire, which is needed in this measurement, has been found by direct weighing.

Two precautions should be mentioned. First, it is necessary to ensure that the two bearings attached to the cryostat are maintained in good condition, for otherwise rotation is liable to be accompanied by an excessive amount of vibration, leading to excessive noise on the oscilloscope trace and even to unwanted changes in the tension and twist adjustments. Secondly, care must be taken to ensure that the axis of rotation of the assembly remains coincident with the wire, for otherwise the theory of § 5 cannot be expected to apply; checks on this alinement have normally been carried out at room temperature, and it has been checked that the alinement is maintained to liquid nitrogen temperatures.

4. Experimental results

An outline of some of the main experimental results has already been given in § 2, but the details of these results remain to be discussed in the present section. For convenience the results will be presented in roughly the chronological order in which they were obtained.

At the outset of the experimental work the following procedure was followed. The system was first rotated at a temperature above the λ-point for a period of about 20 min, which is long enough for the helium in both the inner cylinder C and the outer protective cylinder G to be brought into complete rotation. The system, still rotating, was then cooled very slowly through the λ-point down to the lowest available temperature (1·3 °K); the duration of the cooling was usually greater than 30 min, and it was hoped that this cooling rate was slow enough to avoid appreciable heat currents that might disturb the equilibrium. Finally, with the system still rotating, photographs of oscillations of the wire were taken at varying intervals of time for about 20 min.

As already indicated in § 2, this procedure gave positive results of the expected order of magnitude. But the apparent circulations, for a given angular velocity, showed no sign of any quantization, and, furthermore, they varied noticeably both from run to run and during the 20 min rotation at the low temperature. It was guessed that this behaviour showed that the system was not in equilibrium, in that one or more vortex lines were only partly attached to the wire.

It should be mentioned at this point that observations have been made to confirm that the observed increases in beat frequency are really due to a circulation round the wire. This has been done by checking the truth of the statement made in § 3 that the fractional modulation observed in the beat pattern should decrease to $[x-(1+x^2)^{\frac{1}{2}}]^2$ when the system is turned through 45° from any of the usual measuring positions corresponding to 100 % modulation. It has also been checked that the results obtained are independent of the *direction* of rotation.

Various attempts were then made to improve the equilibrium. Even slower rates of cooling were tried, and the effects of prolonged rotation at the low temperature studied. It was felt that the ends of the incompletely attached lines might be sticking (in the manner observed by Hall 1958) to either the wire or some part of the surrounding cylinder, and in attempts to free them the apparatus was shaken, the rotation was repeatedly stopped and started, and the wire was given repeated vibrations of very large amplitude. But in no way did it seem possible to produce any consistent and appreciable effect. An example of the results obtained is shown in figure 3, where the apparent circulation is plotted as a function of the time elapsing after the rotating system reached the low temperature. The vertical arrows indicate times at which the wire was vibrated with large amplitude.

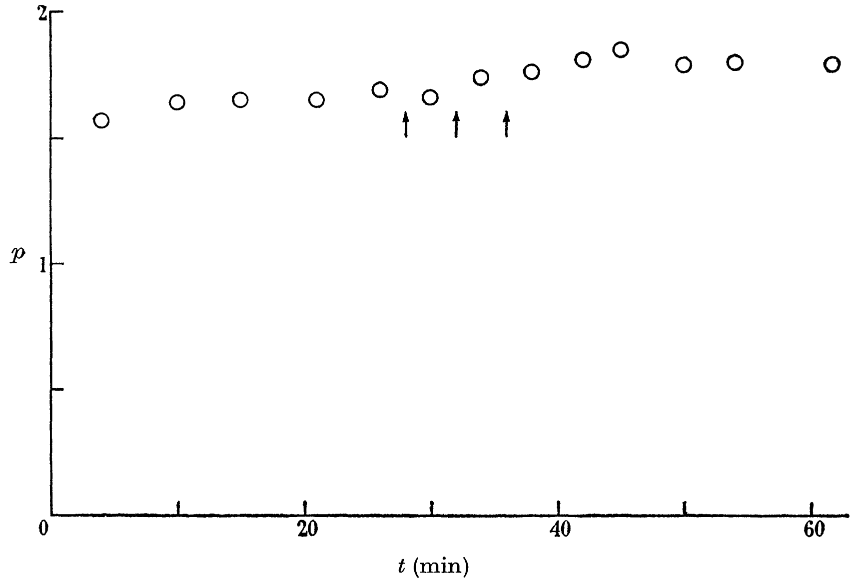


FIGURE 3. Apparent circulation (in units of h/m) in the rotating state ($y = 20$) plotted as a function of the time elapsing since the system reached the lowest temperature (1·3 °K).

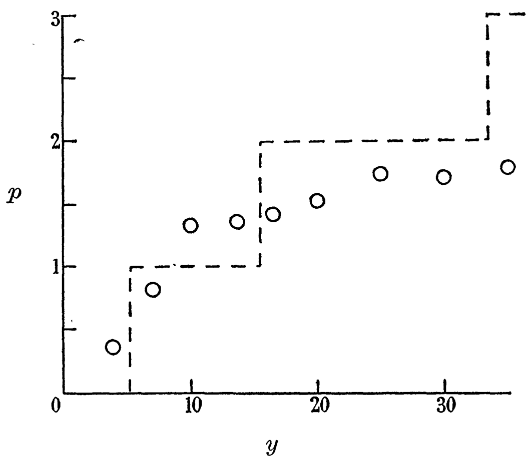
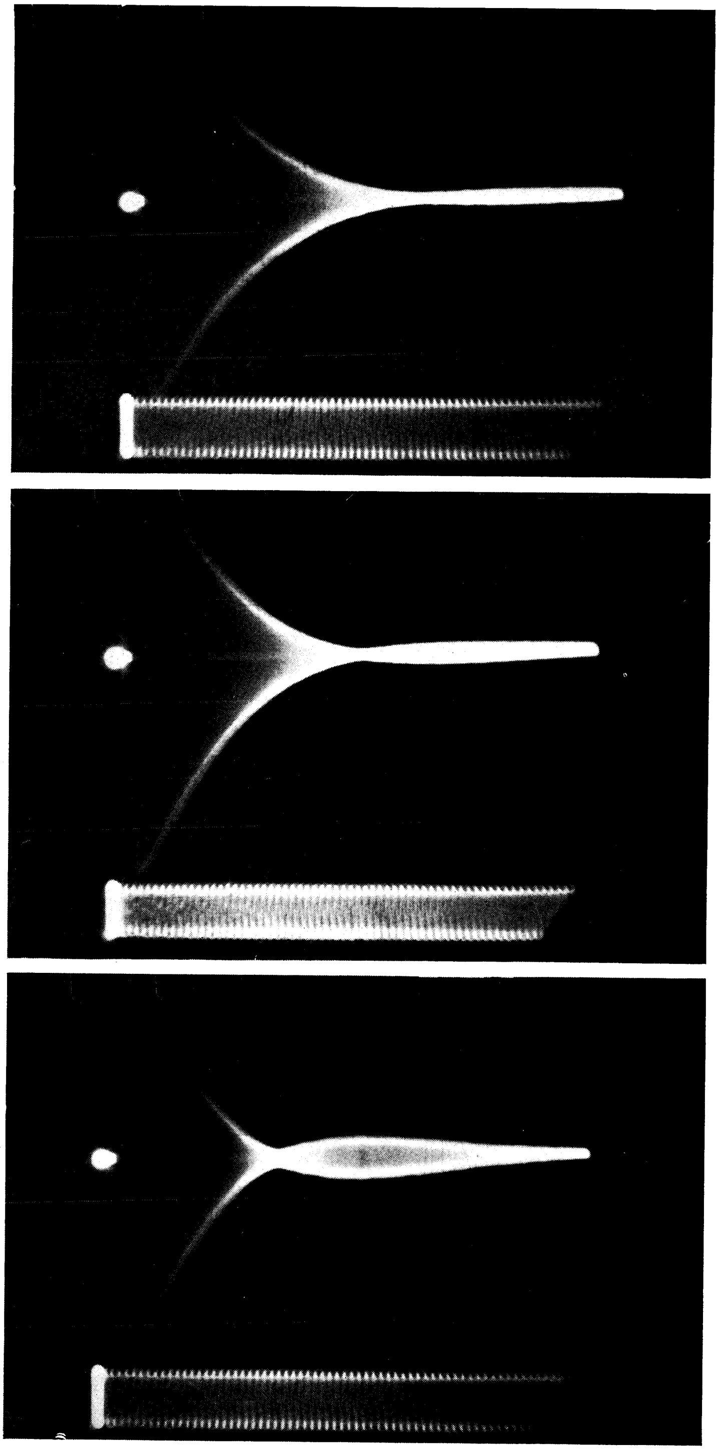


FIGURE 4. Average apparent circulation (in units of h/m) in the rotating state plotted against $y = 2\pi b^2 m\omega/h$, where b is the internal radius of the cylinder surrounding the wire. The theoretical variation for $a_0 = 10^{-8}$ cm (see § 5) is shown by the broken line.

In the course of the experiments described so far, a considerable number of measurements of circulation in the rotating state have been made, and it is interesting to see how the average apparent circulation varies with angular



(*a*)

(*b*)

(*c*)

FIGURE 2. Typical records taken during a single run. The time scale is indicated by the 50 c/s sine wave below the main trace. (*a*) Zero circulation; (*b*) circulation $\approx h/m$; (*c*) circulation $\approx 2h/m$.

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velocity. This variation is shown in figure 4; the quantity y is a convenient dimensionless parameter, and the broken line shows the step-wise variation to be expected from the theory of § 5. Each point in this figure corresponds to an average of about 40 readings, and the standard deviation associated with each point is about 0·2 to 0·3. It can be seen that, although, as emphasized already, the values obtained for a given angular velocity are not exactly reproducible, there is a definite tendency for the average apparent circulation to increase with increasing angular velocity in a manner corresponding roughly to a smoothed version of the theoretical line. This is encouraging, although, as will be explained in § 5, the precise form of this experimental curve remains difficult to understand unless it is assumed that the core of a free vortex line has an effective radius that is much larger than expected. The particularly curious feature of the curve is that it never rises as far as the line corresponding to two quanta; and indeed on only a few isolated occasions have *individual* circulations greater than or equal to $2h/m$ ever been observed.

Attempts to attain better equilibrium in the rotating state were then abandoned, and, as already indicated in § 2, a study was made of the circulation that remains after the rotation is stopped. Two typical kinds of behaviour are shown in figures 5 (a) and (b). In all cases the apparent circulation changes only a little when the rotation is stopped, and, provided the system is not disturbed, this circulation will often quickly settle down to a value which appears to remain unchanged for an indefinite time. On one occasion such a persistent apparent circulation, equal to $1{\cdot}96h/m$, showed no detectable change for $1\frac{3}{4}$ h, and this provides clear evidence that metastable circulations (including free vortex lines) can persist indefinitely in helium II. However, as can be seen most clearly from figure 5 (a), the apparent circulation will in general fall, often by a considerable amount, if the system is disturbed by vibrating the wire with a large amplitude. This may be plausibly interpreted as being due to the freeing of the end of an incompletely attached vortex. If such an interpretation is correct, then apparent circulations equal to exactly integral multiples of h/m should usually be stable against vibration, since it is probable that such circulations will generally be due to completely attached vortices. That this stability is indeed observed for a circulation of one quantum is shown in figure 5 (b), where it can be seen clearly that circulations measured immediately after and immediately before the application of large amplitude vibrations tend to be substantially equal to one another only if the circulation is close to h/m. This tendency has been observed throughout the present experiments, as can be seen from figure 6, which is a histogram of the total number of pairs of observed circulations that have exhibited this type of stability plotted against the initial value of the circulation. (The 64 pairs of observations shown in figure 6 are the only ones, out of a total of 222, to show not more than 4 % difference between the members of each pair.) It is these results that represent the evidence for quantization of circulation. It must be admitted, of course, that the evidence would be more impressive if stable circulations of two or more quanta had also been observed; but this has not been possible, because no way has been discovered of establishing such large circulations.

The results reproduced in figures 5(*a*) and (*b*) reveal another typical feature to which attention should be drawn. This is the difference in the effect of large amplitude vibrations according as the measured circulation is less than or greater

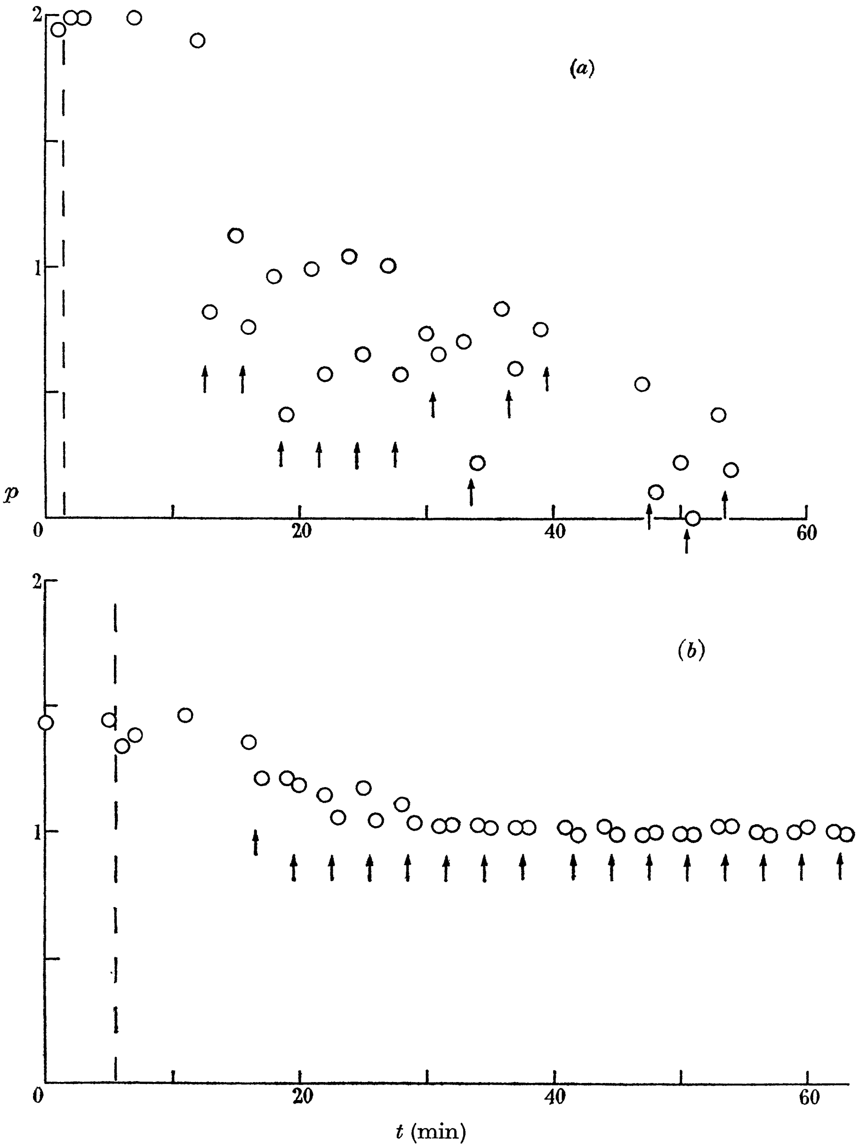

FIGURE 5(*a*) and (*b*). Apparent circulation (in units of h/m) plotted as a function of time after stopping an anticlockwise rotation ($y = 20$). The vertical broken line shows the time at which rotation was stopped, and the vertical arrows indicate times at which the wire was repeatedly vibrated with large amplitude.

than h/m. If the circulation is greater than h/m, these vibrations tend to produce a permanent reduction in its magnitude so that a succession of them causes a steady and rapid fall as far as h/m. If the circulation is less than h/m, however, the vibrations tend to produce only a temporary reduction so that a succession of them tends to produce only oscillations in the circulation with, at most, only a very slow reduction in the average value. This behaviour will be discussed in §5.

The stability associated with a completely attached vortex has been confirmed by another type of experiment, typical results of which are shown in figures 7(*a*) and (*b*). The system is first set into rotation in one direction (say anticlockwise) by the usual technique of rotating above the λ-point and cooling, when a number of free and attached vortices will be created, all presumably with the same (anticlockwise) sense. The rotation is then stopped, and the system allowed to stand for about 10 min. Finally, the rotation is started again, but in the opposite (clockwise) direction. As might be expected, this reverse rotation causes a reduction in the magnitude of the apparent circulation; this is shown clearly in figure 7(*a*),

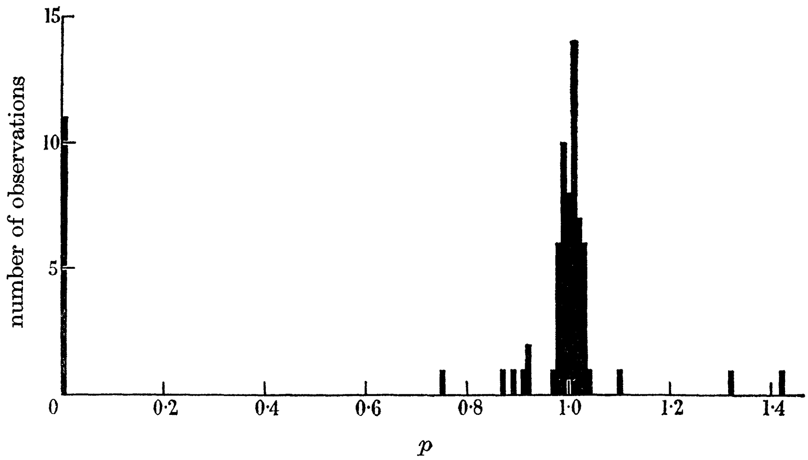

FIGURE 6. Histogram showing the number of measurements of circulation that have been observed to be stable against repeated vibration of the wire with large amplitude.

where the apparent circulation falls steadily to zero in about 5 min. However, in the case shown in figure 7(*b*), the reduction went only as far as one quantum, and it is clear that this demonstrates again the stability of a completely attached vortex, even against a reverse rotation of the whole apparatus. It may be noted that in neither case does the circulation rise again significantly after falling, thus demonstrating that there appears to be no rapid creation of new vortices with the correct sense of rotation. This is to be expected, since, as was emphasized in §2, the creation of new vortices is probably difficult; and it confirms that there is little hope of attaining equilibrium in the rotating system by simply starting the rotation from rest *below* the λ-point.

The final observation to which reference must be made is that, even when the system is cooled from the λ-point in a non-rotating state, there is often, although not always, a single vortex partly attached to the wire. The explanation of this phenomenon is not known, although one interesting possibility is that the vortex is a remnant of the mass of vortex line which, according to a speculation of Feynman (1955), might be present in helium I. Whatever the explanation, the observation itself shows that isolated vortices are very likely to be present in apparently undisturbed helium, and it is possible that this fact is of crucial importance in indicating a mechanism by which the turbulence which is so often observed in superfluid flows could be nucleated.

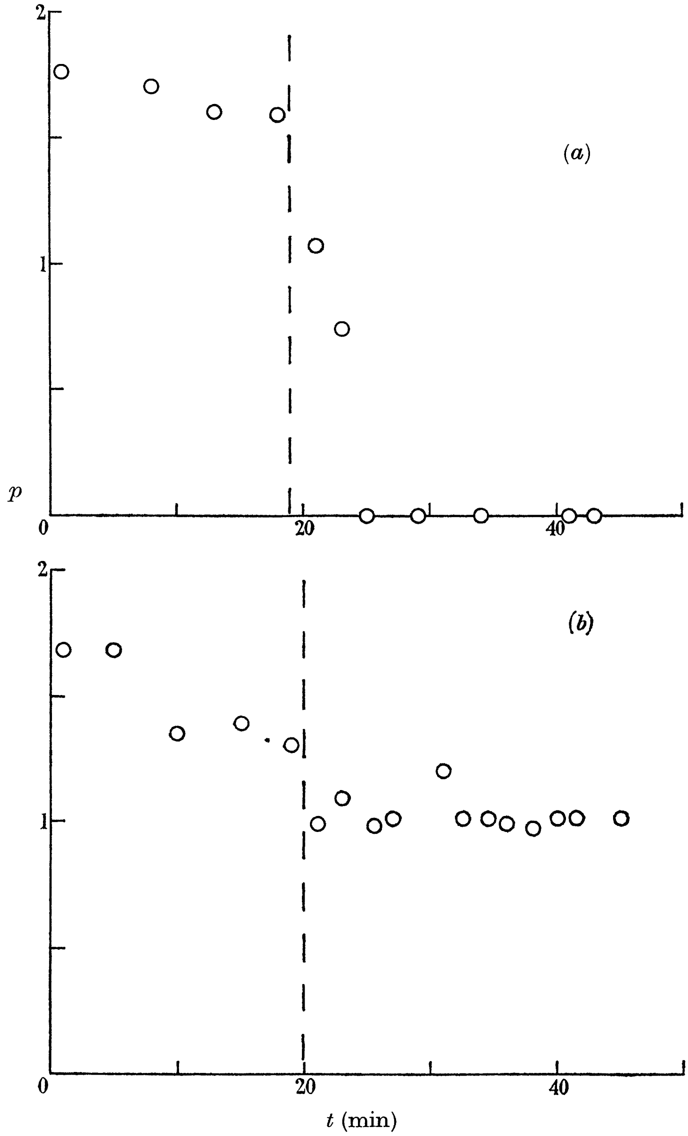

FIGURE 7 (*a*) and (*b*). Apparent circulation (in units of h/m) plotted as a function of time after stopping an anticlockwise rotation ($y = 20$). The rotation is started again, in a clockwise sense, at the time indicated by the vertical broken line.

5. The equilibrium régime in the rotating state; and the energy of a free vortex line

In this final section, the results will first be given of a detailed calculation of the equilibrium state to be expected in the rotating system; and then the observed behaviour of the rotating system will be discussed in the light of these results.

It is clear that the concept of quantized vortices, outlined in the introduction, now has a firm experimental foundation, so that it has been possible to proceed with some confidence to deduce on the basis of this concept the detailed equilibrium state that should obtain in the present apparatus at any given angular velocity. To be precise, it has been assumed that rotation of the superfluid is possible only through the presence of quantized irrotational circulations round the wire and of free quantized vortices in the surrounding liquid; and it has been assumed further that these free vortices behave like vortex lines in a classical incompressible ideal fluid, and that they have an empty cylindrical core of radius a_0. The radius a_0 governs the energy per unit length of the vortex line,* and simple arguments suggest that it should be of order 10^{-8} cm (Feynman 1955). It has also been assumed that no surface energy is associated with a velocity discontinuity at a boundary of the liquid (see Gamtsemlidze 1958; and a forthcoming paper by the author). The present calculations have been carried out only for helium at absolute zero, and it has been assumed that the results apply to the superfluid component at any finite temperature.

The equilibrium state of a fluid in a vessel rotating at an angular velocity ω is obtained by minimizing the quantity $F' = F - M\omega$, where F is the free energy of the liquid (equal to the internal energy at absolute zero) and $M\omega$ is its total angular momentum. The problem therefore has been to calculate F' for all possible arrangements of vortices, and the equilibrium arrangement, at any given ω, is then the one having the smallest value of F'. The calculations are laborious but fairly straightforward. The procedure has been first to consider the various arrangements involving a given circulation (ph/m) round the wire and a given number q of free vortices, and hence to find the particular geometrical arrangement of these free vortices that has the smallest F'; and this minimum F', which will be denoted by $F'(p, q)$, is then evaluated as a function of angular velocity and of the numbers p and q. Some of the results of this type of calculation are shown in figure 8, where the dimensionless parameter $\mathscr{F}'$, equal to $4\pi m^2 F'(p, q)/\rho h^2$, is plotted against the parameter y for various values of p and q. It is clear that the equilibrium values of F', p and q can then be obtained by following the *lowest* line on a plot of this kind. The results depend of course on the particular value of a_0 that is chosen, so that, as mentioned in §2, observations on the equilibrium state should yield information about the magnitude of a_0 and hence about the energy per unit length of a free vortex line. The observable quantity that is of particular interest here is p_e, the number of quanta of circulation round the wire in equilibrium, and the theoretical value of p_e obtained from the present calculations is shown as a function of both y and a_0 in figure 9. It should be added that if either y or a_0 is too large the calculations become prohibitively laborious, and it is for this reason that no calculations have been made for values of a_0 greater than 10^{-4} cm, and

* In the simple case of an isolated vortex line situated at the centre of a circular cylinder of radius b, the total kinetic energy per unit length of line is $(\rho h^2/4\pi m^2) \ln b/a_0$. In general the expression for the energy per unit length of line is more complicated, but it always contains a term of the same form, where b is a length of the order of either the size of the vessel or the spacing between adjacent lines.

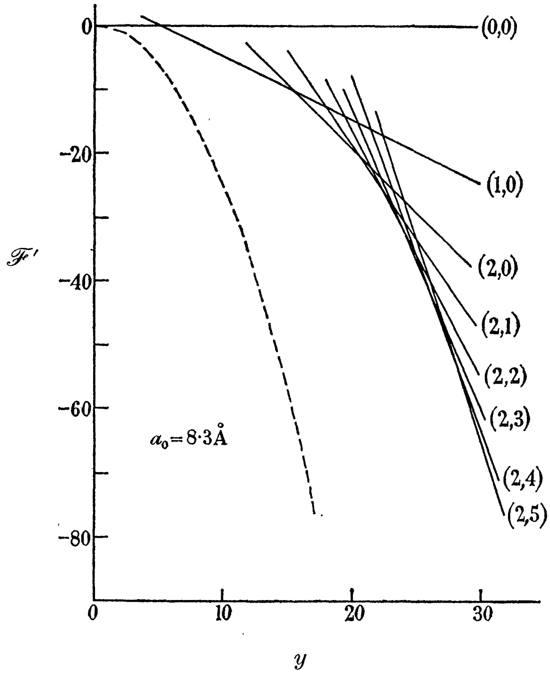

FIGURE 8. $\mathscr{F}'$ [($= \{4\pi F'(p, q)\}/(\rho h^2/m^2)$] plotted as a function of y ($= 2\pi b^2 m\omega/h$) for various values of (p, q) ($a_0 = 8{\cdot}3 \times 10^{-8}$ cm). The broken line corresponds to equilibrium of an ordinary liquid; i.e. one that can rotate like a solid body.

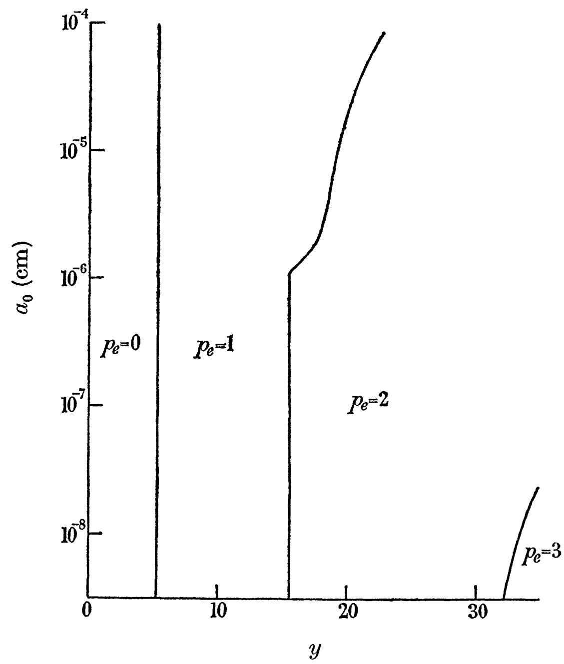

FIGURE 9. Diagram showing the predicted number, p_e, of quanta of circulation round the wire as a function of y and a_0.

neither calculations nor experiments have been carried out for values of y greater than 35.

The experimental results with which the predicted values of p_e can be compared were presented and discussed briefly in § 4. As emphasized earlier, the observation that these results are not strictly reproducible, and that apparent circulations equal to non-integral multiples of h/m are obtained, shows immediately that the system is not always in equilibrium, so that no direct comparison between experiment and theory is possible. But it still remains to discuss why the system is not in equilibrium and whether in spite of this lack of equilibrium any conclusions can be drawn about the value of a_0, particularly from the observed average values of p shown in figure 4. In discussing these problems it is clearly useful to divide the experimental results according as they refer to values of y greater than or less than 15. For if y is less than 15 the predictions displayed in figure 9 are independent of a_0, so that p_e is then known with certainty; it is only if y is greater than 15 that p_e becomes at all dependent on a_0.

Now in the range of values of y up to 15 the experimental results for the average value of p do follow the predicted behaviour to some extent, in that there is a steep rise from values close to zero to values near unity when y is roughly 5. However, the rise takes place to a value that is significantly larger than unity, and moreover, not only are the average values of p larger than unity, but also almost all the individual readings have been found to be larger than unity. Thus it is clear that in this range of low values of y there is a marked tendency for p to exceed the value appropriate to equilibrium.

Now there are two effects that might lead to non-equilibrium values of p, and, since both seem likely to lead to values that are too large, this observed behaviour at low values of y is probably the one to be expected. The first effect arises from the fact that during the reduction in temperature of the rotating system from above the λ-point the cooling of the helium round the wire almost certainly takes place from the outside, either through the brass cylinder C or through the helium in the gaps between this cylinder and the brass pistons A and B. Thus, as the helium is cooled through the λ-point, it is likely that a boundary separating helium II from helium I will move inwards towards the wire. While this boundary is in existence, the helium II is being formed in an annular region in which the inner cylinder is of larger diameter than that of the wire, and this will clearly lead to the appearance of too large a circulation. The second effect is one discussed by Ginsburg & Pitayevskii (1958). Owing to the large kinetic energy associated with the region near the centre of a vortex line and to the surface energy associated with the core, it may be energetically favourable to have a gradual conversion to normal fluid as the centre of the line is approached. The effect becomes important only very close to the λ-point, where it leads to an effective value of a_0 that increases rapidly as the temperature approaches the transition. Thus the vortex line system that is formed on cooling through the λ-point is likely to be that appropriate to a very large value of a_0. Unfortunately, as explained earlier, it has not been possible to calculate the details of equilibrium states for very large values of a_0, but it is fairly clear that these states are likely to involve one quantum round the wire and a *large* number of free vortices in the surrounding helium. As the temperature is lowered, such a state will rapidly cease to be the one with the minimum value of F', so that it will become metastable. However, in practice the cooling process is almost certain to disturb the system, so that there will be a change to an adjacent state with lower F'. This will involve a decrease in the number of free lines, and it seems very likely (although admittedly not certain) that the excess lines that are close to the wire will be attracted on to the wire. Thus the circulation round the wire will probably increase considerably, and may well become larger than that corresponding to the true equilibrium state at a low temperature.

In the range of values of y from 15 to 35 the observed values of p (both the averages and most of the individual readings) are still in the range from one to two, although, as can be seen from figure 4, there is some suggestion of a rise when y is roughly 20. Thus, if it is assumed, as seems reasonable, that the tendency for p to exceed its equilibrium value is maintained in this region of higher angular velocities, then it must be concluded that the equilibrium value of p is still unity even when y is as high as 35 (although it is possible that the state with p equal to 2 is almost stable when y exceeds about 20). It appears therefore from figure 9 that under the conditions of the present experiment the value of a_0 must exceed the maximum value for which it has been possible to carry out calculations, i.e. 10^{-4} cm.

This value is much larger than the value of order 10^{-8} cm suggested by Feynman and the value $6{\cdot}8 \times 10^{-8}$ cm obtained by Hall (1960) from observations of the velocity of vortex waves. It must be emphasized of course that the present argument is not rigorous, since it is quite possible that some unknown factor contributing to the lack of equilibrium could act differently in the two ranges of angular velocity. Further experiments are required; perhaps the present measurements should be extended, but on the whole it is felt that the present method is not inherently very satisfactory, so that it would be better to search for another method. But the fact remains that the present measurements do at least *suggest* that the simplest picture of a free vortex line is in some way inadequate, and that theory behind this picture should be re-examined. It seems possible, for example, that some indeterminacy in the position of a line might appreciably reduce its energy; furthermore, the value of a_0 might vary with the experimental conditions, and the value that governs the vortex wave velocity might differ from the one that governs the energy per unit length of a stationary line. It should perhaps be stressed that a large value of a_0 does not imply the existence of a large hole in the liquid; a_0 is to be regarded simply as a parameter that describes the energy per unit length of line, and its value is determined by the detailed conditions near the centre of the line, which may depart considerably from those in the vortex line of classical hydrodynamics.

Some comments may now be added on the conditions that must be satisfied before the non-equilibrium states that are so frequently encountered in the present work appear able to move towards equilibrium. The forces which are tending to urge the system towards equilibrium are presumably measured by the rate at which the function $\mathscr{F}'$ decreases as the actual existing state moves towards the equilibrium state. Now it is easily shown that if this movement involves the creation of a completely new free vortex line there is a large hump in $\mathscr{F}'$, and it is this hump that is responsible for the difficulty in creating new free vortices. However, if the movement involves only the attachment to or detachment from the wire of a partly attached line, then there are no large humps in $\mathscr{F}'$ to be overcome, but only small humps arising from the sticking of the ends of the free vortex to small protuberances in the apparatus (due, for example, to roughness). The rate at which $\mathscr{F}'$ changes as a line is gradually attached to or detached from the wire is conveniently measured by the difference $\Delta\mathscr{F}'$ in the function $\mathscr{F}'$ between the states corresponding to complete attachment and complete detachment, and it is interesting to consider the minimum values of $\Delta\mathscr{F}'$ that are required to overcome the various humps. The hump involved in the creation of a new vortex will be considered in a forthcoming paper, to which reference was made earlier, so that the present discussion will be confined to a consideration of the minimum value of $\Delta\mathscr{F}'$ that is required to overcome the sticking effects. This minimum value cannot of course be calculated, but some information about it can be obtained from the observations in § 4; furthermore, it is important to compare the observations made in the rotating apparatus with those made in the stationary apparatus in order to check that they yield *consistent* information on this point, since such a check provides additional evidence in favour of the general interpretation of the results. The values of $\Delta\mathscr{F}'$ quoted below follow easily from the calculations described at the beginning of this section, and details of their derivation will not be given.

Consider first the stationary apparatus. The observation of persistent apparent circulations of between one and two quanta shows that in the absence of any disturbance values of $\Delta\mathscr{F}'$ up to 15 are insufficient to overcome sticking effects. However, it will be recalled that, if the stationary apparatus is disturbed by large amplitude vibrations of the wire, any circulation greater than h/m is rapidly and permanently reduced, although circulations less than h/m can only be temporarily reduced. This suggests that with large amplitude vibrations of the wire a value of $\Delta\mathscr{F}'$ of 15 is sufficient to overcome all sticking effects, while a value equal to 5 is sufficient only to overcome sticking effects on the end of the vortex line attached to the wire. In the rotating apparatus the situation is a little more complicated, since the calculated values of $\Delta\mathscr{F}'$ depend on the value of a_0, which is not known with certainty. However, it can be shown that, at least for the values of a_0 for which calculations have been carried out, the values of $\Delta\mathscr{F}'$ for the observed conditions in the rotating apparatus are never likely to exceed 5. Therefore the fact that no continuous and permanent movement towards equilibrium has ever been observed in the rotating apparatus, even when large amplitude vibrations are used, is perfectly consistent with the observations on the stationary apparatus. (It has not been possible to extend measurements of the type shown in figures 5 (*a*) and (*b*) to the rotating apparatus and determine whether large amplitude vibrations lead to a temporary change in circulation that is consistently in one direction.) Finally, the experiments involving reverse rotation should be mentioned, where, as shown in figure 7 (*a*), a single partly attached line is removed quickly and permanently by reverse rotation without aid from large amplitude vibrations. Now it can be shown that the values of $\Delta\mathscr{F}'$ in this case have never been less than 25, so that it may be deduced that this value is sufficient to overcome sticking effects completely, even without the aid of large amplitude vibrations. This result is again consistent with the observations on the stationary apparatus, where it was shown only that values of $\Delta\mathscr{F}'$ up to 15 were not sufficient without the aid of large amplitude vibrations. Thus it can be seen that all the results for the rotating and stationary apparatus are reasonably consistent, and this is an encouraging feature.

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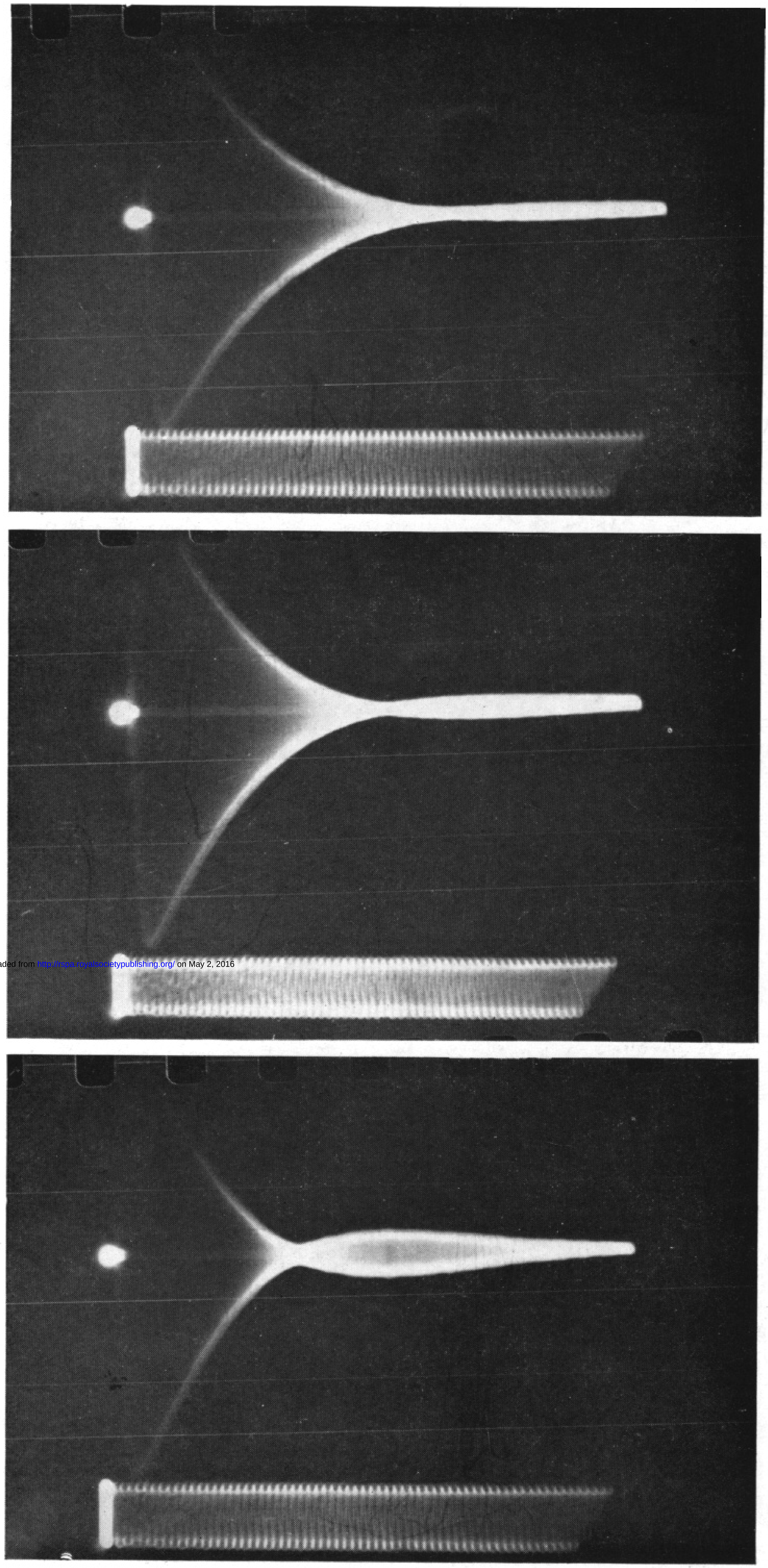

(a)

(b)

(c)

FIGURE 2. Typical records taken during a single run. The time scale is indicated by the 50 c/s sine wave below the main trace. (a) Zero circulation; (b) circulation $\approx h/m$; (c) circulation $\approx 2h/m$.