Solutions to Pathria's Statistical Mechanics
Chapter 1

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Problem 1.1

Problem 1.2

Utilizing the additive characteristic of $S = f(\Omega)$ and get

$$S = S_1 + S_2 = f(\Omega_1) + f(\Omega_2) \tag{1}$$
$$(\frac{dS}{d\Omega_1})_{\Omega_2} = f'(\Omega_1) \tag{2}$$
$$(\frac{dS}{d\Omega_2})_{\Omega_1} = f'(\Omega_2) \tag{3}$$

Inspect a small pertubation near the equilibrium state using the fact that $S = f(\Omega) = f(\Omega_1\Omega_2)$

$$(\frac{dS}{d\Omega_1})_{\Omega_2} = \lim_{\Delta\to 0}\frac{f((\Omega_1+\Delta)\Omega_2) - f(\Omega_1\Omega_2)}{\Delta} \tag{4}$$

Assume that $\delta = \Delta\Omega_2$

$$(\frac{dS}{d\Omega_1})_{\Omega_2} = \lim_{\Delta\to 0}\Omega_2\frac{f(\Omega_1\Omega_2 + \Delta\Omega_2) - f(\Omega_1\Omega_2)}{\Delta\Omega_2} = \lim_{\delta\to 0}\Omega_2\frac{f(\Omega+\delta) - f(\Omega)}{\delta} = \Omega_2 f'(\Omega) \tag{5}$$

Apply to $(\frac{dS}{d\Omega_2})_{\Omega_1}$, we can get similar result.

$$(\frac{dS}{d\Omega_2})_{\Omega_1} = \Omega_1 f'(\Omega) \tag{6}$$

Finally,

$$f'(\Omega_1) = \Omega_2 f'(\Omega) = \frac{\Omega_2}{\Omega_1}f'(\Omega_2) \tag{7}$$
$$\Omega_1 f'(\Omega_1) = \Omega_2 f'(\Omega_2) \tag{8}$$
$$\tag{9}$$

It is obvious that this equation holds for all Ω. Set the value of the equation constant k.

$$\Omega\frac{df(\Omega)}{d\Omega} = k \tag{10}$$
$$f(\Omega) = kln\Omega + C \tag{11}$$
$$\tag{12}$$

Using a special value $\Omega = 1$

$$f(\Omega * 1) = f(\Omega) + f(1) \tag{13}$$
$$C = f(1) = 0 \tag{14}$$
$$\tag{15}$$

And get the result

$$S = f(\Omega) = kln\Omega \tag{16}$$

Problem 1.3

Problem 1.4

Suppose N is the number of particles, v_0 is the volume occupied by one particle and therefore the total number of microstates Ω is

$$\Omega = \frac{1}{N!}(\frac{V}{v_0})\dots(\frac{V}{v_0} - N + 1) \tag{17}$$

Following (1.4.2), we have

$$\frac{P}{T} = k\left(\frac{\partial \ln \Omega}{\partial V}\right)_{N,E} \tag{18}$$
$$= k\frac{\partial \Omega}{\Omega \partial V} \tag{19}$$
$$= k\frac{N}{V}\left(1 + \frac{(N-1)v_0}{2V} + \dots\right) \tag{20}$$

Considering only the first two terms, it corresponds to $P(V - b) = NkT$ with $b = Nv_0/2$.
Notes: I don't know why the problem says $b = 4Nv_0$ since this gas is hard sphere gas. Anyone has an idea?

Problem 1.5

Using equation $(A.11)$, and setting $K = \pi\sqrt{\varepsilon}/L$, it is straight forward to achieve

$$\Sigma_1(\varepsilon) = \frac{\pi}{6}\varepsilon^{3/2} \pm \frac{3\pi}{8}\varepsilon \tag{21}$$

where the first term is the volume term ($V = L^3$) and the next one is the surface correction ($S = 6L^2$).

Problem 1.6

Use the formula for ideal gas $PV = NkT$.

$$Nk \times 300 = 10^5 \times \frac{\pi}{10} \tag{22}$$

Thus $\Delta T = 10^4/Nk \sim 955K$.