Solutions to Pathria's Statistical Mechanics

SM-at-THU

March 5, 2016

Problem 1.1

Problem 1.2

Utilizing the addictive characteristic of $S = f(\Omega)$ and get

$$S = S_1 + S_2 = f(\Omega_1) + f(\Omega_2) \tag{1}$$
$$(\frac{dS}{d\Omega_1})_{\Omega_2} = f'(\Omega_1) \tag{2}$$
$$(\frac{dS}{d\Omega_2})_{\Omega_1} = f'(\Omega_2) \tag{3}$$
$$\tag{4}$$

Inspect a small pertubation near the equilibrium state using the fact that $S = f(\Omega) = f(\Omega_1\Omega_2)$

$$(\frac{dS}{d\Omega_1})_{\Omega_2} = \lim_{\Delta\to 0} \frac{f((\Omega_1 + \Delta)\Omega_2) - f(\Omega_1\Omega_2)}{\Delta} \tag{5}$$

Assume that $\delta = \Delta\Omega_2$

$$(\frac{dS}{d\Omega_1})_{\Omega_2} = \lim_{\Delta\to 0} \Omega_2 \frac{f(\Omega_1\Omega_2 + \Delta\Omega_2) - f(\Omega_1\Omega_2)}{\Delta\Omega_2} = \lim_{\delta\to 0} \Omega_2 \frac{f(\Omega + \delta) - f(\Omega)}{\delta} = \Omega_2 f'(\Omega) \tag{6}$$

Apply to $(\frac{dS}{d\Omega_2})_{\Omega_1}$, we can get similar result.

$$(\frac{dS}{d\Omega_2})_{\Omega_1} = \Omega_1 f'(\Omega) \tag{7}$$

Finally,

$$f'(\Omega_1) = \Omega_2 f'(\Omega) = \frac{\Omega_2}{\Omega_1} f'(\Omega_2) \tag{8}$$
$$\Omega_1 f'(\Omega_1) = \Omega_2 f'(\Omega_2) \tag{9}$$
$$\tag{10}$$

It is obvious that this equation holds for all Ω. Set the value of the equation constant k.

$$\Omega \frac{df(\Omega)}{d\Omega} = k \tag{11}$$
$$f(\Omega) = k ln\Omega + C \tag{12}$$
$$\tag{13}$$

Using a special value $\Omega = 1$

$$f(\Omega * 1) = f(\Omega) + f(1) \tag{14}$$
$$C = f(1) = 0 \tag{15}$$
$$\tag{16}$$

And get the result

$$S = f(\Omega) = kln\Omega \tag{17}$$

Problem 1.3