

Solutions to Pathria's Statistical Mechanics

Chapter 1

SM-at-THU

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Problem 1.1

Problem 1.2

Utilizing the additive characteristic of $S = f(\Omega)$ and get

$$S = S_1 + S_2 = f(\Omega_1) + f(\Omega_2) \quad (1)$$

$$\left(\frac{dS}{d\Omega_1}\right)_{\Omega_2} = f'(\Omega_1) \quad (2)$$

$$\left(\frac{dS}{d\Omega_2}\right)_{\Omega_1} = f'(\Omega_2) \quad (3)$$

$$(4)$$

Inspect a small pertubation near the equilibrium state using the fact that $S = f(\Omega) = f(\Omega_1\Omega_2)$

$$\left(\frac{dS}{d\Omega_1}\right)_{\Omega_2} = \lim_{\Delta \rightarrow 0} \frac{f((\Omega_1 + \Delta)\Omega_2) - f(\Omega_1\Omega_2)}{\Delta} \quad (5)$$

Assume that $\delta = \Delta\Omega_2$

$$\left(\frac{dS}{d\Omega_1}\right)_{\Omega_2} = \lim_{\Delta \rightarrow 0} \Omega_2 \frac{f(\Omega_1\Omega_2 + \Delta\Omega_2) - f(\Omega_1\Omega_2)}{\Delta\Omega_2} = \lim_{\delta \rightarrow 0} \Omega_2 \frac{f(\Omega + \delta) - f(\Omega)}{\delta} = \Omega_2 f'(\Omega) \quad (6)$$

Apply to $\left(\frac{dS}{d\Omega_2}\right)_{\Omega_1}$, we can get similar result.

$$\left(\frac{dS}{d\Omega_2}\right)_{\Omega_1} = \Omega_1 f'(\Omega) \quad (7)$$

Finally,

$$f'(\Omega_1) = \Omega_2 f'(\Omega) = \frac{\Omega_2}{\Omega_1} f'(\Omega_2) \quad (8)$$

$$\Omega_1 f'(\Omega_1) = \Omega_2 f'(\Omega_2) \quad (9)$$

$$(10)$$

It is obvious that this equation holds for all Ω . Set the value of the equation constant k .

$$\Omega \frac{df(\Omega)}{d\Omega} = k \quad (11)$$

$$f(\Omega) = k \ln \Omega + C \quad (12)$$

$$(13)$$

Using a special value $\Omega = 1$

$$f(\Omega * 1) = f(\Omega) + f(1) \tag{14}$$

$$C = f(1) = 0 \tag{15}$$

$$\tag{16}$$

And get the result

$$S = f(\Omega) = k \ln \Omega \tag{17}$$

Problem 1.3