Solutions to Pathria's Statistical Mechanics Chapter 7

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Problem 7.1

Problem 7.4

We can deduce from 7.1.30 and 7.1.26a,when p is a const:

$$g_{\frac{5}{2}} \propto T^{-\frac{5}{2}}$$

So we have:

$$(\frac{\partial}{\partial T} g_{\frac{5}{2}}(z))_p = -\frac{5}{2T} g_{\frac{5}{2}}(z)$$

According to the D.10 from appendix D:

$$z\frac{\partial}{\partial z} g_{\frac{5}{2}}(z) = g_{\frac{3}{2}}(z)$$

Combine the two equation:

$$\frac{1}{z}(\frac{\partial z}{\partial T})_p = -\frac{5}{2T}\frac{g_{\frac{5}{2}}(z)}{g_{\frac{3}{2}}(z)}$$

If you compare with 7.1.36 you can get:

$$\gamma = \frac{5}{3}\frac{g_{\frac{5}{2}}(z) g_{\frac{1}{2}}(z)}{g_{\frac{3}{2}}(z)^2}$$

Problem 7.11

(a)

At the critical point of B-E condensation,$N_0 \ll N$,$z = 1$.

$$\frac{N}{V} = \frac{1}{\lambda^3}\frac{1}{\Gamma(\frac{3}{2})}(\int_0^\infty \frac{x^{\frac{1}{2}}dx}{e^x+1} + \int_0^\infty \frac{x^{\frac{1}{2}}dx}{e^{x+\beta\epsilon_1}+1}) \tag{1}$$

Because$\beta\epsilon_1 >> 1$,so

$$\int_0^\infty \frac{x^{\frac{1}{2}}dx}{e^{x+\beta\epsilon_1}+1} = e^{-\beta\epsilon_1}\int_0^\infty e^{-x}x^{\frac{1}{2}}dx = \Gamma(\frac{3}{2})e^{-\beta\epsilon_1} \ll 1 \tag{2}$$

So, we think $\Delta T = T_c - T_{c0} \ll 1$.

$$\frac{N}{V} = \frac{1}{\lambda^3}(\xi(\frac{3}{2}) + e^{-\beta\epsilon_1}) \approx \frac{1}{\lambda_0^3}(1+\frac{3\Delta T}{2T_{c0}})(\xi(\frac{3}{2}) + e^{\frac{\epsilon_1}{kT_{c0}}}) \tag{3}$$

And we know

$$\frac{N}{V} = \frac{1}{\lambda_0^3}\xi(\frac{3}{2}) \tag{4}$$

So,

$$\frac{3\Delta T}{2T_{c0}}\xi(\frac{3}{2}) + e^{\frac{\epsilon_1}{kT_{c0}}} = 0 \tag{5}$$

$$\frac{T_c}{T_{c0}} = 1 - \frac{\frac{2}{3}}{\xi(\frac{3}{2})}e^{\frac{\epsilon_1}{kT_{c0}}} \tag{6}$$

(b)

$$\frac{N}{V} = \frac{1}{\lambda^3}\frac{1}{\Gamma(\frac{3}{2})}(\int_0^\infty \frac{x^{\frac{1}{2}}dx}{e^x+1} + \int_0^\infty \frac{x^{\frac{1}{2}}dx}{e^{x+\beta\epsilon_1}+1}) = \frac{1}{\lambda^3}(g_{3/2}(1) + g_{3/2}(e^{-\beta\epsilon_1})) \tag{7}$$

As $\beta\epsilon_1 \ll 1$, we can expand $g_{3/2}(e^{-\beta\epsilon_1})$ and keep the first two terms.

$$\frac{N}{V} = \frac{1}{\lambda^3}(2\xi(\frac{3}{2}) + \Gamma(-\frac{1}{2})(\beta\epsilon_1)^{\frac{1}{2}}) = \frac{1}{\lambda_0^3}\xi(\frac{3}{2}) \tag{8}$$

If $\beta\epsilon_1 = 0$,

$$\lambda' = 2^{\frac{1}{3}}\lambda_0 \tag{9}$$

$$T_c' = (\frac{1}{2})^{\frac{2}{3}}T_{c0} \tag{10}$$

And $T_c' - T_c \ll 1$.

$$\frac{N}{V} = \frac{2}{\lambda'^3}(1 + \frac{3\Delta T}{2T_c'})(\xi(\frac{3}{2}) - \sqrt{\pi(\frac{\epsilon_1}{kT_c'})}) = \frac{1}{\lambda_0^3}\xi(\frac{3}{2}) \tag{11}$$

$$\frac{3\Delta T}{2T_c'}\xi(\frac{3}{2}) - \sqrt{\frac{\pi\epsilon_1}{kT_c'}} = 0 \tag{12}$$

$$\frac{\Delta T}{T_c'} = \frac{\frac{2}{3}}{\xi(\frac{3}{2})}\sqrt{\frac{\pi\epsilon_1}{kT_c'}}) = \frac{2^{\frac{4}{3}}}{3\xi(\frac{3}{2})}\sqrt{\frac{\pi\epsilon_1}{kT_{c0}}} \tag{13}$$

$$\frac{T_c}{T_{c0}} = (\frac{1}{2})^{\frac{2}{3}}(1 + \frac{2^{\frac{4}{3}}}{3\xi(\frac{3}{2})}\sqrt{\frac{\pi\epsilon_1}{kT_{c0}}}) \tag{14}$$

Problem 7.14

From the definition of p and U:

$$p = \frac{kT}{\lambda^n}g_{\frac{5}{2}}(z)$$

$$U = \frac{nkTV}{s\lambda^n}g_{\frac{5}{2}}(z)$$

So we get:

$$p = \frac{sU}{nV}$$

And when $T \to \infty$,we could use the ideal gas equation.

$$pV = nRT$$

So

$$C_V = \frac{n}{s}Nk, C_p = (\frac{n}{s}+1)Nk$$

Problem 7.21

$$\frac{N}{V} = \frac{1}{h^3}\int_0^\infty \frac{8\pi\frac{\epsilon^2}{c^3}d\epsilon}{e^{\beta\epsilon}-1} = \frac{8\pi}{\beta^3h^3c^3}\int_0^\infty \frac{x^2dx}{e^x-1} = \frac{8\pi}{\beta^3h^3c^3}\Gamma(3)\xi(3) \tag{15}$$

$$\frac{E}{V} = \frac{1}{h^3}\int_0^\infty \frac{8\pi\frac{\epsilon^3}{c^3}d\epsilon}{e^{\beta\epsilon}-1} = \frac{8\pi}{\beta^4h^3c^3}\int_0^\infty \frac{x^3dx}{e^x-1} = \frac{8\pi}{\beta^4h^3c^3}\Gamma(4)\xi(4) \tag{16}$$

$$\frac{E}{N} = \frac{3\xi(4)}{\xi(3)}kT \approx 2.7kT \tag{17}$$

Problem 7.24

We can deduce from 7.3.12 and 7.3.19 and 7.3.23.

$$u = 4.16 * 10^{-14}$$

$$s = 2.03 * 10^{-14}$$

$$n = 4.09 * 10^8$$

Problem 7.31

The state density for transverse phonon is

$$g(\omega) = \frac{6N}{\omega_{DT}^3}\omega^2 \tag{18}$$

The state density for longitudinal phonon is

$$g(\omega) = \frac{3N}{\omega_{DL}^3}\omega^2 \tag{19}$$

And

$$C_V = C_{VT} + C_{VL} \tag{20}$$

Compared with equation (7.4.17),it is easy to find verify that

$$C_V = Nk(2D(x_{0,T}) + D(x_{0,L}))) \tag{21}$$

The nature of the error in equation (7.4.17) comes from the fact that the longitudinal and the transverse modes of the solid should have their own cutoff frequencies, ω_{DL} and ω_{DT} rather than a common cutoff at ω_D.