Solutions to Pathria's Statistical Mechanics Chapter 5

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Problem 5.1

Problem 5.4

If we take the unsymmetrized wave function

$$\psi_E(\mathbf{q}) = \prod_{i=1}^{N} u_{\epsilon_i}(\mathbf{q}_i)$$

then the density matrix will be:

$$\langle 1, \cdots, N|e^{-\beta H}|1', \cdots, N'\rangle = \sum_E e^{-\beta E} \psi_E(\mathbf{r}_1, \cdots, \mathbf{r}_N)\psi_E^*(\mathbf{r}'_1, \cdots, \mathbf{r}'_N) = \sum_E e^{-\beta E} \prod_{i=1}^{N} u_{\epsilon_i}(\mathbf{r}_i) u_{\epsilon_i}^*(\mathbf{r}_i) \tag{1}$$

Since we are considering about an ideal classical gas, the energy eigenstates are also momentum eigenstates, so we can find that the eigenfunction of momentum will be:

$$u_{\mathbf{k}_i} = \frac{1}{\sqrt{V}} e^{i\mathbf{k}\cdot\mathbf{r}}$$

we can easily find that the density matrix will be:

$$\langle 1, \cdots, N|e^{-\beta H}|1', \cdots, N'\rangle = \sum_{\mathbf{k}_1, \cdots \mathbf{k}_N} e^{-\frac{\beta\hbar^2}{2m}(\mathbf{k}_1^2 + \cdots + \mathbf{k}_N^2)} \prod_{i=1}^{N} \frac{1}{V} e^{i\mathbf{k}_i\cdot(\mathbf{r}_i - \mathbf{r}'_i)} \tag{2}$$

so the result will be

$$\langle 1, \cdots, N|e^{-\beta H}|1', \cdots, N'\rangle = \prod_{i=1}^{N} \frac{1}{(2\pi)^3} \int d^3k \, e^{-\frac{\beta\hbar^2}{2m}\mathbf{k}_i^2 + i\mathbf{k}_i\cdot(\mathbf{r}_i - \mathbf{r}'_i)} = \prod_{i=1}^{N} \left(\frac{2\pi m}{\beta h^2}\right)^{3/2} \exp\left[-\frac{m}{2\beta\hbar^2}(\mathbf{r}_i - \mathbf{r}'_i)^2\right] \tag{3}$$

Obviously there is no correlation term between different particles. Then calculate the partition function:

$$Q_N = \left(\frac{2\pi m}{\beta h^2}\right)^{3N/2} \int \prod_{i=1}^{N} d^3 r_i = V^N \left(\frac{2\pi m}{\beta h^2}\right)^{3N/2} \tag{4}$$

We can find that there is no Gibbs correction factor in the partition function.

Problem 5.5

The partition function of the noninteracting,indistinguishable particles system is:

$$Q_N(V,T) = \frac{1}{N!\lambda^{3N}} \sum_P \delta_P [f(Pr_1 - r_1) \cdots f(Pr_N - r_N)]$$

where $f(r) = e^{-\frac{\pi r^2}{\lambda^2}}$ The first approximation is:

$$\sum_P = 1 \pm \sum_{i<j} f_{ij} f_{ji}$$

So the partition function in first approximation is:

$$Q_N(V,T) = \frac{1}{N!\lambda^{3N}} \int (1 \pm \sum_{i<j} e^{-\frac{\pi r_{ij}^2}{\lambda^2}}) d^{3N} r$$