Solutions to Pathria's Statistical Mechanics

SM-at-THU

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Problem 1.1

Problem 1.2

Utilizing the addictive characteristic of $S = f(\Omega)$ and get

$$S = S_1 + S_2 = f(\Omega_1) + f(\Omega_2) \tag{1}$$
$$(\frac{dS}{d\Omega_1})_{\Omega_2} = f'(\Omega_1) \tag{2}$$
$$(\frac{dS}{d\Omega_2})_{\Omega_1} = f'(\Omega_2) \tag{3}$$
$$\tag{4}$$

Inspect a small pertubation near the equilibrium state using the fact that $S = f(\Omega) = f(\Omega_1\Omega_2)$

$$(\frac{dS}{d\Omega_1})_{\Omega_2} = \lim_{\Delta\to 0}\frac{f((\Omega_1+\Delta)\Omega_2) - f(\Omega_1\Omega_2)}{\Delta} \tag{5}$$

Assume that $\delta = \Delta\Omega_2$

$$(\frac{dS}{d\Omega_1})_{\Omega_2} = \lim_{\Delta\to 0}\Omega_2\frac{f(\Omega_1\Omega_2 + \Delta\Omega_2) - f(\Omega_1\Omega_2)}{\Delta\Omega_2} = \lim_{\delta\to 0}\Omega_2\frac{f(\Omega+\delta) - f(\Omega)}{\delta} = \Omega_2 f'(\Omega) \tag{6}$$

Apply to $(\frac{dS}{d\Omega_2})_{\Omega_1}$, we can get similar result.

$$(\frac{dS}{d\Omega_2})_{\Omega_1} = \Omega_1 f'(\Omega) \tag{7}$$

Finally,

$$f'(\Omega_1) = \Omega_2 f'(\Omega) = \frac{\Omega_2}{\Omega_1} f'(\Omega_2) \tag{8}$$
$$\Omega_1 f'(\Omega_1) = \Omega_2 f'(\Omega_2) \tag{9}$$
$$\tag{10}$$

It is obvious that this equation holds for all Ω. Set the value of the equation constant k.

$$\Omega\frac{df(\Omega)}{d\Omega} = k \tag{11}$$
$$f(\Omega) = kln\Omega + C \tag{12}$$
$$\tag{13}$$

Using a special value $\Omega = 1$

$$
\begin{aligned}
f(\Omega * 1) &= f(\Omega) + f(1) && (14)\\
C = & f(1) = 0 && (15)\\
& && (16)
\end{aligned}
$$

And get the result

$$S = f(\Omega) = k ln \Omega \tag{17}$$

Problem 1.3

Problem 2.3

The Hamiltonian of a rotator in 2 dimension is:

$$H = \frac{L^2}{2I} \tag{18}$$

in which L is the angular momentum and I is the moment of inertia.

Problem 2.4

If we just consider about the orbital angular momentum, it can be written as a function of p_θ and p_ϕ which are the canonical momentum conjugate to the spherical coordinate variables θ and ϕ:

$$L^2 = p_\theta^2 + \frac{p_\phi^2}{\sin^2\theta} \tag{19}$$

thus the phase volume of the region which satisfies $L^2 \leq M^2$ is

$$
\begin{aligned}
\mathbf{\Omega} &= \int_0^\pi d\theta \int_0^{2\pi} d\phi \int_{L^2 \leq M^2} dp_\theta dp_\phi \\
&= \int_0^\pi d\theta \int_0^{2\pi} d\phi \, \pi M^2 \sin\theta \\
&= 4\pi^2 M^2
\end{aligned} \tag{20}
$$

Thus the number of microstates is $\Omega = \mathbf{\Omega}/h^2 = M^2/\hbar^2$. Then let us calculate the number by quantized angular momentum. The number can be written as the following form:

$$\Omega = \sum_{j=0}^{j_{\max}} (2j+1) = (j_{\max}+1)^2 \tag{21}$$

Now we have to determine the number $j_{\max}$. Since we want the absolute value of the angular momentum $< M^2$, we can find that $j_{\max}$ is determined by the following equation:

$$j_{\max} = \left\lfloor \frac{\sqrt{1+\frac{4M^2}{\hbar^2}} - 1}{2} \right\rfloor \tag{22}$$

Problem 2.5

In this problem we need to use the WKB approximation in Quantum Mechanics.