

标架丛 P

元素 $(p, \hat{e}_1, \hat{e}_2, \dots, \hat{e}_n)$

↓
选取坐标系 U

元素 $(u^i, X^{\dot{i}}_{\dot{j}})$

$\{P = (u^1, u^2, \dots, u^n)$

$\{e_i = X^k_i \left(\frac{\partial}{\partial u^k} \right)_p$

一次微分场

$$\begin{cases} \theta^i = X^i_{\dot{i}} du^{\dot{i}} \\ \theta^{\dot{i}}_{\dot{j}} = X^k_{\dot{j}} (dX^k_i + X^e_i \omega^k_e) \end{cases}$$

结构方程

$$\begin{cases} d\theta^{\dot{i}} - \theta^k \wedge \theta^{\dot{k}}_{\dot{j}} = \frac{1}{2} P^{\dot{i}}_{\dot{k}e} \theta^k \wedge \theta^{\dot{k}}_{\dot{j}} \\ d\theta^{\dot{i}}_{\dot{j}} - \theta^k \wedge \theta^{\dot{k}}_{\dot{j}} = \frac{1}{2} S^{\dot{i}}_{\dot{k}e} \theta^k \wedge \theta^{\dot{k}}_{\dot{j}} \end{cases}$$

$$\begin{cases} P^{\dot{i}}_{\dot{k}e} = X^{\dot{i}}_{\dot{r}} X^p_k X^q_e T^{pq}_r \\ S^{\dot{i}}_{\dot{k}e} = X^{\dot{i}}_{\dot{q}} X^p_i X^r_k X^s_e R^q_{prs} \end{cases}$$

黎曼流形

将 θ^i 与 $\theta^{\dot{i}}_{\dot{j}}$ 拉回某局部截面上, 有

$$\begin{cases} d\theta^{\dot{i}} - \theta^k \wedge \theta^{\dot{k}}_{\dot{j}} = 0 \\ dg_{i\dot{j}} = \theta^k_i g_{k\dot{j}} + \theta^{\dot{k}}_{\dot{j}} g_{ik} = \theta_{i\dot{j}} + \theta_{\dot{j}i} \quad g_{i\dot{j}} = G(e_i, e_{\dot{j}}) \end{cases}$$