Summary on QFT

Yuyang Songsheng

September 4, 2016

1 Canonical quantization for particles

1.1 Classical Field Theory

Field:

$$\phi_a(\vec{x},t)$$

Lagrangian density:

$$\mathcal{L}(\phi_a,\dot{\phi}_a,\nabla\phi_a)$$

Action:

$$S=\int d^4x\mathcal{L}$$

Hamilton principle:

$$\delta S=0 \tag{1}$$

Euler-Lagrange equation:

$$\partial_\mu(\frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi_a)})-\frac{\partial\mathcal{L}}{\partial\phi_a}=0 \tag{2}$$

1.1.1 Locality

There are many terms in the Lagrangian coupling $\phi(\vec{x},t)$ directly to $\phi(\vec{y},t)$ with$\vec{x}\neq\vec{y}$.The closet we got for $\vec{x}$ label is coupling between $\phi(\vec{x})$ and $\phi(\vec{x}+\delta\vec{x})$ through the gradient term $(\nabla\phi)^2$.

1.1.2 Lorentz Invariance

Lorentz transformation of fields:

$$\phi'_a(x)=S_{ab}(\Lambda)\phi(\Lambda^{-1}x) \tag{3}$$

Examples:

$$\partial_\mu\phi'(x)=(\Lambda^{-1})^\nu{}_\mu(\partial_\nu\phi)(\Lambda^{-1}x) \tag{4}$$

Lagrangian density is a scalar, or more loosely, action is invariant under Lorentz transformation.

1.1.3 Symmetries

Nother's Theorem Every continuous symmetry of the Lagrangian gives rise to a conserved current $j^\mu(x)$ such that the equation of motion imply $\partial_\mu j^\mu=0$.For $\phi_a\to\phi_a+\delta\phi_a,\mathcal{L}\to\mathcal{L}+\delta\mathcal{L}$,if $\delta\mathcal{L}=\partial_\mu K^\mu$,then

$$j^\mu=\frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi_a)}\delta\phi_a-K^\mu \tag{5}$$

infinitesimal translation $A^\mu \to A^\mu + \epsilon^\mu$

$$j^\mu = \epsilon_\nu T^{\mu\nu} \tag{6}$$

$$T^{\mu\nu} = \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi_a)} \partial^\nu \phi_a - g^{\mu\nu} \mathcal{L} \tag{7}$$

$$\partial_\mu T^{\mu\nu} = 0 \tag{8}$$

$$P^\mu = \int d^3x T^{0\mu} \tag{9}$$