Summary on Classical Mechanics

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1 Lagrangian Mechanics

Lagrangian and Action:

$$S = \int_{t_1}^{t_2} L(q, \dot{q}, t)dt \tag{1}$$

Hamilton Principle:

$$\delta S = 0 \tag{2}$$

Euler-Lagrangian equation:

$$\frac{d}{dt}(\frac{\partial L}{\partial \dot{q}_i}) - \frac{\partial L}{\partial q_i} = 0 \tag{3}$$

The form of Lagrangian for a system of particles in inertial frame:

$$L = \sum_a \frac{1}{2} m_a v_a^2 - U(\vec{r_1}, \vec{r_2}, \cdots,) \tag{4}$$

Notes: To get the form of Lagrangian for a system of interacting particles, we must assume:
(1) Space and time are homogeneous and isotropic in inertial frame;
(2) Galileo's relativity principle and Galilean transformation;
(3) Spontaneous interaction between particles;

2 Symmetry and Conservation Laws

Nother's theorem For $q_i \to q_i + \delta q_i$ and $L \to L + \delta L$, if $\delta L = \frac{df(q,\dot{q},t)}{dt}$,then we get

$$\frac{d}{dt}(p_i \delta q_i - f) = 0 \quad (p_i = \frac{\partial L}{\partial \dot{q}_i}) \tag{5}$$

This can imply the conservation laws of momentum and angular momentum.

Homogeneity of time If $\frac{\partial L}{\partial t} = 0$,then we get

$$\frac{dE}{dt} = 0 \quad (E = \sum_i \dot{q}_i p_i - L) \tag{6}$$

3 Hamilton Mechanics

3.1 Hamilton equation

$$H(q,p,t) = \sum_i p_i \dot{q}_i - L \tag{7}$$

$$\dot{p_i} = -\frac{\partial H}{\partial q_i} \qquad \dot{q_i} = \frac{\partial H}{\partial p_i} \tag{8}$$

3.2 Poisson Brackets

Operation properties:

$$\{f,g\} = -\{g,f\}$$

$$\{\alpha_1 f_1 + \alpha_2 f_2, \beta_1 g_1 + \beta_2 g_2\} = \alpha_1\beta_1 \{f_1, g_1\} + \alpha_1\beta_2 \{f_1, g_2\} + \alpha_2\beta_1 \{f_2, g_1\} + \alpha_2\beta_2 \{f_2, g_2\}$$

$$\{f_1 f_2, g_1 g_2\} = f_1 \{f_2, g_1\} g_2 + f_1 g_1 \{f_2, g_2\} + g_1 \{f_1, g_2\} f_2 + \{f_1, f_2\} g_2 f_2$$

$$\{f,\{g,h\}\} + \{g,\{h,f\}\} + \{h,\{f,g\}\} = 0$$

Here, f, g, h are functions of p_i, q_i, t. If we assume that

$$\{q_i, q_k\} = 0, \{p_i, p_k\} = 0, \{q_i, p_k\} = \delta_{ik}$$

we can deduce that

$$\{f,g\} = \sum_k (\frac{\partial f}{\partial q_k}\frac{\partial g}{\partial p_k} - \frac{\partial f}{\partial p_k}\frac{\partial g}{\partial q_k})$$

The Hamilton equation can be written as

$$\dot{p_i} = \{p_i, H\} \qquad \dot{q_i} = \{q_i, H\} \tag{9}$$