4

$$T[\phi(x_1)\phi(x_2)] = \theta(t_1-t_2)\,\phi(x_1)\,\phi(x_2)+\theta(t_2-t_1)\,\phi(x_2)\phi(x_1).$$

++ -- +- -+

$\langle 0|$ 107

$$\langle 0|T[\phi(x_1)\phi(x_2)]|0\rangle = \theta(t_1-t_2)\,\langle 0|\phi_+(x_1)\phi_-(x_2)|0\rangle+\theta(t_2-t_1)\phi_+(x_2)\phi(x_1$$

$$= \iint \frac{d^3p_1\, d^3p_2}{(2\pi)^6\sqrt{2E_{\vec p_1}2E_{\vec p_2}}}\Big[e^{-ip_1\cdot x_1}\,e^{ip_2\cdot x_2}\,\theta(t_1-t_2)\,\langle 0|a_{\vec p_1}a^{\dagger}_{\vec p_2}|0\rangle + e^{-ip_2x_2}\,e^{ip_1x_1}\,\theta(t_2-t_1)\Big]$$

$$\times\langle 0|a_{p_2}a^{\dagger}_{p_1}|0\rangle$$

haciendo

$$\langle 0|a_{\vec p_i}a^{\dagger}_{\vec p_j}|0\rangle = \langle 0|[a_{\vec p_i},a^{\dagger}_{p_j}]|0\rangle =$$
$$=(2\pi)^3\delta^{(3)}(\vec p_i-\vec p_j)\,\cancel{\langle 0|0\rangle}$$
$$=(2\pi)^3\delta^{(3)}(\vec p_j-\vec p_i).$$

Haremos $\vec p_1=\vec p$ e integraremos en $\vec p_2$

$$\langle 0|T[\phi(x_1)\phi(x_2)]|0\rangle = \int\frac{d^3p}{(2\pi)^3\,2E_{\vec p}}\Big[\theta(t_1-t_2)\,e^{-ip(x_1-x_2)} + \theta(t_2-t_1)\,e^{+ip(x_1-x_2)}\Big] \qquad (3.76)$$

$$=\int\frac{d^3p}{(2\pi)^3\,2E_p}\Big[\theta(t_1-t_2)\,e^{-iE_p(t_1-t_2)+i\vec p\cdot(\vec x_1-\vec x_2)} + e^{+iE_p(t_2-t_1)}\,e^{-i\vec p\cdot(\vec x_2-\vec x_1)}\times\theta(t_2-t_1)\Big]$$

$E_{-\vec p}=+E_{\vec p}$ $\qquad \vec p\to-\vec p$

$$\int_{+\infty}^{-\infty}\int_{+\infty}^{-\infty}\int_{+\infty}^{-\infty}d(-p_x)\,d(-p_y)\,d(-p_z)=\int_{-\infty}^{\infty}d^3p \;!!$$

Cambiando el signo de $+\vec p$ a $-\vec p$ en la segunda integral.

$$=\int\frac{d^3p}{(2\pi)^3}\Big[\theta(t_1-t_2)\,e^{-iE_p(t_1-t_2)+i\vec p\cdot(\vec x_1-\vec x_2)} + \theta(t_2-t_1)\,e^{+iE_p(t_1-t_2)+i\vec p\cdot(\vec x_1-\vec x_2)}$$

(3.79)

$= E_p^2 - \vec{p}^2 = m^2.$

Usando el siguiente resultado

$$\lim_{\varepsilon \to 0} \int_{-\infty}^{+\infty} d\xi \frac{e^{-i\xi t}}{\xi + i\varepsilon} = -2\pi i\,\Theta(t) \qquad (3.72)$$

Tenemos

$$-i\Theta(t_1-t_2) = \int_{-\infty}^{+\infty} \frac{d\xi}{2\pi} \frac{e^{-i\xi(t_1-t_2)}}{\xi + i\varepsilon}$$

$$-i\Theta(t_1-t_2)\, e^{-iE_p(t_1-t_2)} = \int_{-\infty}^{\infty} \frac{d\xi}{2\pi} \frac{e^{-i(\xi+E_p)(t_1-t_2)}}{\xi + i\varepsilon} \qquad (3.73)$$

$$-i\Theta(t_2-t_1)\, e^{-iE_p(t_2-t_1)} = \int_{-\infty}^{\infty} \frac{d\xi}{(2\pi)} \frac{e^{-i(\xi+E_p)(t_2-t_1)}}{\xi + i\varepsilon}$$

$$i\Theta(t_2-t_1)\, e^{-iE_p(t_2-t_1)} = \int_{-\infty}^{\infty} \frac{d\xi}{(2\pi)} \frac{e^{-i(\xi+E_p)(t_2-t_1)}}{-\xi - i\varepsilon} \qquad (3.74)$$

Sea $P_0 = \xi + E_p \Rightarrow E_p = P_0 - \xi$

$$\langle 0 | T[\phi(x_1)\phi(x_2)] | 0 \rangle =$$

$$\int \frac{d^4p}{(2\pi)^4} \frac{e^{-iP_0(t_1-t_2) + i\vec{p}\cdot(\vec{x}_1-\vec{x}_2)}}{(P_0-E_p)+i\varepsilon} + \int \frac{d^4p}{(2\pi)^3} \frac{e^{-iP_0(t_2-t_1) + i\vec{p}\cdot(\vec{x}_1-\vec{x}_2)}}{-(P_0-E_p)-i\varepsilon}$$

$P_0 \to -P_0$

$P_0 \to -P_0$

$$= \int \frac{d^4p}{(2\pi)^4} \frac{e^{-iP_0(x_1-x_2)}}{2E_p} \left[\frac{1}{(P_0-E_p)+i\varepsilon} - \frac{1}{(P_0+E_p)-i\varepsilon} \right] \qquad (3.75)$$

Pero $\Delta_F(p) =$

$$\frac{1}{2E_p}\left[\frac{1}{p_0-(E_p-i\varepsilon)}-\frac{1}{p_0+(E_p-i\varepsilon)}\right] \qquad (3.70)$$

$$= \frac{1}{2E_p}\left[\frac{\cancel{p_0}+E_p-i\varepsilon-\cancel{p_0}+E_p-i\varepsilon}{p_0^2+p_0(\cancel{E_p}-i\varepsilon)-(\cancel{E_p}-i\varepsilon)p_0-(E_p-i\varepsilon)^2}\right]$$

$$= \frac{1}{2E_p}\left[\frac{2E_p-2i\varepsilon}{p_0^2-(E_p-i\varepsilon)^2+2i\varepsilon p_0}\right]$$

$$p_0^2-E_p^2+2E_p i\varepsilon-\varepsilon^2+2i\varepsilon p_0$$

$$\varepsilon = \frac{\varepsilon'}{2E_p}$$

$$\frac{2E_p-\frac{2i\varepsilon'}{2E_p}}{2E_p\left(p_0^2-E_p^2+2\cancel{E_p}i\frac{\varepsilon'}{2\cancel{E_p}}+2i\frac{\varepsilon'}{2E_p}p_0\right.}$$

$$\frac{1}{2E_p}\left[\frac{\cancel{p_0}+(E_p-i\varepsilon)-[\cancel{p_0}-(E_p-i\varepsilon)]}{[p_0-(E_p-i\varepsilon)][p_0+(E_p-i\varepsilon)]}\right]$$

$$= \frac{1}{2E_p}\left[\frac{2E_p-\cancel{2i\varepsilon}^{\,0}}{p_0^2-(E_p-i\varepsilon)^2}\right] \qquad (3.69)$$

Sea ahora. $\quad p^2 = p_0^2 - \vec{p}^2 \qquad m^2 = E_p^2 - \vec{p}^2$

$$p^2 - m^2 \cancel{+ i\varepsilon = p_0^2 - E_p^2 + i\varepsilon}$$

$$= p_0^2 - E_p^2$$

Si $E_p \to E_p - i\varepsilon \Rightarrow p^2 - m^2 = p_0^2 - (E$

Entonces

$$p_0^2 - (E_p - i\varepsilon)^2 = p_0^2 - E_p^2 + \varepsilon^2 + 2iE_p\varepsilon$$

$$= p_0^2 - E_p^2 + \underbrace{2i\varepsilon E_p}_{\varepsilon'} \cancel{- \varepsilon^2}$$

$$= p_0^2 - E_p^2 + i\varepsilon'$$

Entonces

$$\left.\begin{array}{l} \cancel{\frac{1}{2E_p}}\left[\frac{\cancel{2E_p}}{p_0^2 - (E_p - i\varepsilon)^2}\right] = \frac{1}{p_0^2 - (E_p - i\varepsilon)^2} \\ \Delta_F(p) \approx \frac{1}{p_0^2 - E_p^2 + i\varepsilon} \end{array}\right\} \quad (3.69)$$

$$\Delta_F(x - x') = \int \frac{d^4p}{(2\pi)^4} e^{-ip\cdot(x - x')} \frac{1}{p_0^2 - E_p^2 - i\varepsilon}$$

$$= \int \frac{d^4p}{(2\pi)^4} \frac{e^{-ip\cdot(x - x')}}{(p_0^2 - \vec{p}^2) - (E_p^2 - \vec{p}^2) - i\varepsilon}$$

$$= \int \frac{d^4p}{(2\pi)^4} \frac{e^{-ip\cdot(x - x')}}{p^2 - m^2 \quad i\varepsilon}$$

En el límite de $\varepsilon \to 0$
definimos la función de Green

$$G(x-x') = \int \frac{d^4p}{(2\pi)^4} \frac{e^{-ip(x-x')}}{p^2-m^2}$$

t.q que es sln de

$$(\Box_x + m^2)\, G(x-x') =$$

$$\int d^4p \,\frac{(-p^2+m^2)}{p^2-m^2}\, e^{-ip(x-x')}$$

$$= -\int d^4p \; e^{-ip(x-x')}$$

$$= -\delta^{(4)}(x-x')$$

Por lo tanto si

$$\phi(x) = \phi_0(x) - \int d^4x' \, G(x-x')\, J(x')$$

A.q $(\Box + m^2)\phi_0 = 0$

entonce:

$$(\Box + m^2)\,\phi = 0 - (-J(x)) = J(x)$$

Correspondiente a la solución de la ecuación de Klein-Gordon con una fuente.