Peeter Joot
peeterjoot@protonmail.com

ECE1228H Electromagnetic Theory. Lecture 10: Fresnel relations. Taught by Prof. M. Mojahedi

Two interface problem. It turns out that light passing through two interfaces ends up with reflected and transmitted components, but no mode that goes through the slab in a wave guide fashion. Such a mode is possible when the light is incident in an end-fire configuration (i.e. a waveguide). It's interesting that such a wave guide like mode isn't possible.

Group delay Under matched conditions where $t^{\text{TE}} = e^{j\phi} = e^{jk_{2z}d}$, we can write

$$\begin{aligned}
-\frac{\partial \omega}{\partial \phi} &= \frac{\partial}{\partial \omega}\left(k_{2z}d\right) \\
&= d\frac{\partial k_{2z}}{\partial \omega} \\
&= d/\frac{\partial \omega}{\partial k_{2z}} \\
&= d/v_g,
\end{aligned} \tag{1.1}$$

so

$$v_g = -\frac{d}{\frac{\partial \omega}{\partial \phi}}. \tag{1.2}$$

This is called the group delay, and is essentially the velocity of the peak of the wave form, as sketched in

F5.

This is different than the phase velocity $v_p = \omega/k$, as illustrated in the sketch of

F2

If (FIXME: why?)

$$\begin{aligned}
v_g &= \frac{\partial \omega}{\partial k} \\
&= 1/\frac{\partial k}{\partial \omega},
\end{aligned} \tag{1.3}$$

and in an unbounded medium

$$k = \frac{\omega}{c} n(\omega), \tag{1.4}$$

so

$$\frac{\partial k}{\partial \omega} = \frac{1}{c}\left(n(\omega) + \omega \frac{\partial n}{\partial \omega}\right), \tag{1.5}$$

so

$$v_g = \frac{c}{n(\omega) + \omega \frac{\partial n}{\partial \omega}} = \frac{c}{n_g} \tag{1.6}$$

Since the phase is

$$\phi = -\frac{\omega}{c} n(\omega) d \tag{1.7}$$

we can show that

$$\begin{aligned} v_g &= \frac{d}{-\frac{\partial \phi}{\partial \omega}} \\ &\equiv \frac{d}{\tau_g}. \end{aligned} \tag{1.8}$$

By bounded medium, it is meant that the medium is not matched.

Brewster's angle Self study (examinable!) Brewster's angle for a single interface: The angle for which there is no reflection.

Does it exist for a TE mode, and also for a TM mode, and if not, for which polarization?

Should find that there is no Brewster angle for one of the polarizations when $\mu_1 = \mu_2$. It may be that it's TE that always has some reflection.

Critical angle Self study (examinable!) critical angle, the angle for which $|r| = 1$

Does it exist for a TE mode, and also for a TM mode, and if not, for which polarization?

We should find that this only occurs when we go from a medium with a low index of refraction to one where with a higher index of refraction (i.e. $\theta_i \geq \theta_c$.

For such an angle, what is the value of the phase?

We should find that there is an electric and magnetic field on the other side of the medium, but that there is no transmitted power on that side of the interface (i.e. Poynting vector is zero (no power transfer) when the angle of incidance is greater than θ_c).

Can have photon tunnelling through the space between two prisms separated by an air gap

F4

where, despite the critical angle, the time for the photon to "tunnel" through the space between the prisms is less than the time for the photon to go through just the first prism.

1.1 Vector and electrostatic (scalar) potentials

In electrostatics where

$$\nabla \times \mathbf{E} = 0, \tag{1.9}$$

the electric field must be the gradient of some function

$$\mathbf{E} = \pm \nabla V, \tag{1.10}$$

We pick negative to have things consistent with the notion of force.
In electrodynamics we have

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \tag{1.11}$$

however, we also have

$$\nabla \cdot \mathbf{B} = 0, \tag{1.12}$$

so

$$\mathbf{B} = \nabla \times \mathbf{A}. \tag{1.13}$$

This gives

$$\nabla \times \mathbf{E} = -\frac{\partial}{\partial t} \left(\nabla \times \mathbf{A} \right), \tag{1.14}$$

or

$$0 = \nabla \times \left(\mathbf{E} + \frac{\partial \mathbf{A}}{\partial t} \right), \tag{1.15}$$

so this curled quantity can be the gradient of something

$$\mathbf{E} + \frac{\partial \mathbf{A}}{\partial t} = \pm \nabla V. \tag{1.16}$$

We pick negative again, so

$$\mathbf{E} = -\nabla V - \frac{\partial \mathbf{A}}{\partial t}. \tag{1.17}$$

Observe that when the fields are electrostatic with no time dependence, we would have $\mathbf{E} = -\nabla V$.