Worksheet 5: Monte-Carlo

April Cooper, Patrick Kreissl und Sebastian Weber

January 22, 2013

University of Stuttgart

Contents

1 Simple Sampling- Intrgration

First we were asked to program a function runge(x) that computes the runge function $f(x) = \frac{1}{1+x^2}$ and to plot it on the interval $[-5, 5]$.

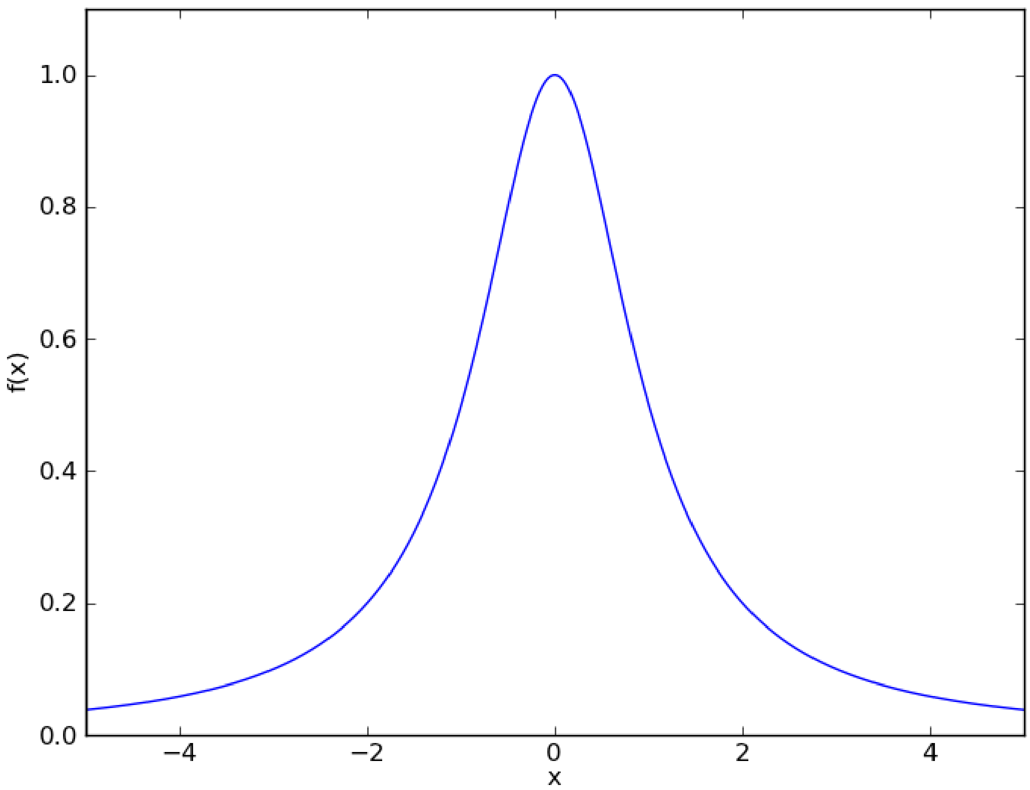

Figure 1: Runge function f(x) on the interval [-5,5]

Then we had to write a function that computes the exact integral of the runge function. The exact result is:

$$\int_{-5}^{5} \frac{1}{1+x^2} = arctan(5) - arctan(-5) \approx 2.7468$$

After that we were asked to program a function simple_sampling(f,a,b,N) that performs N steps of a simple sampling Monte-Carlo integration and to use this function to compute the Integral for $N = 2^i$ with $2 \leq i \leq 20$. Furthermore we were asked to determine the actual and statistical error and to plot them against the number of integration steps N. In the following are two plots of the error from two different runs.

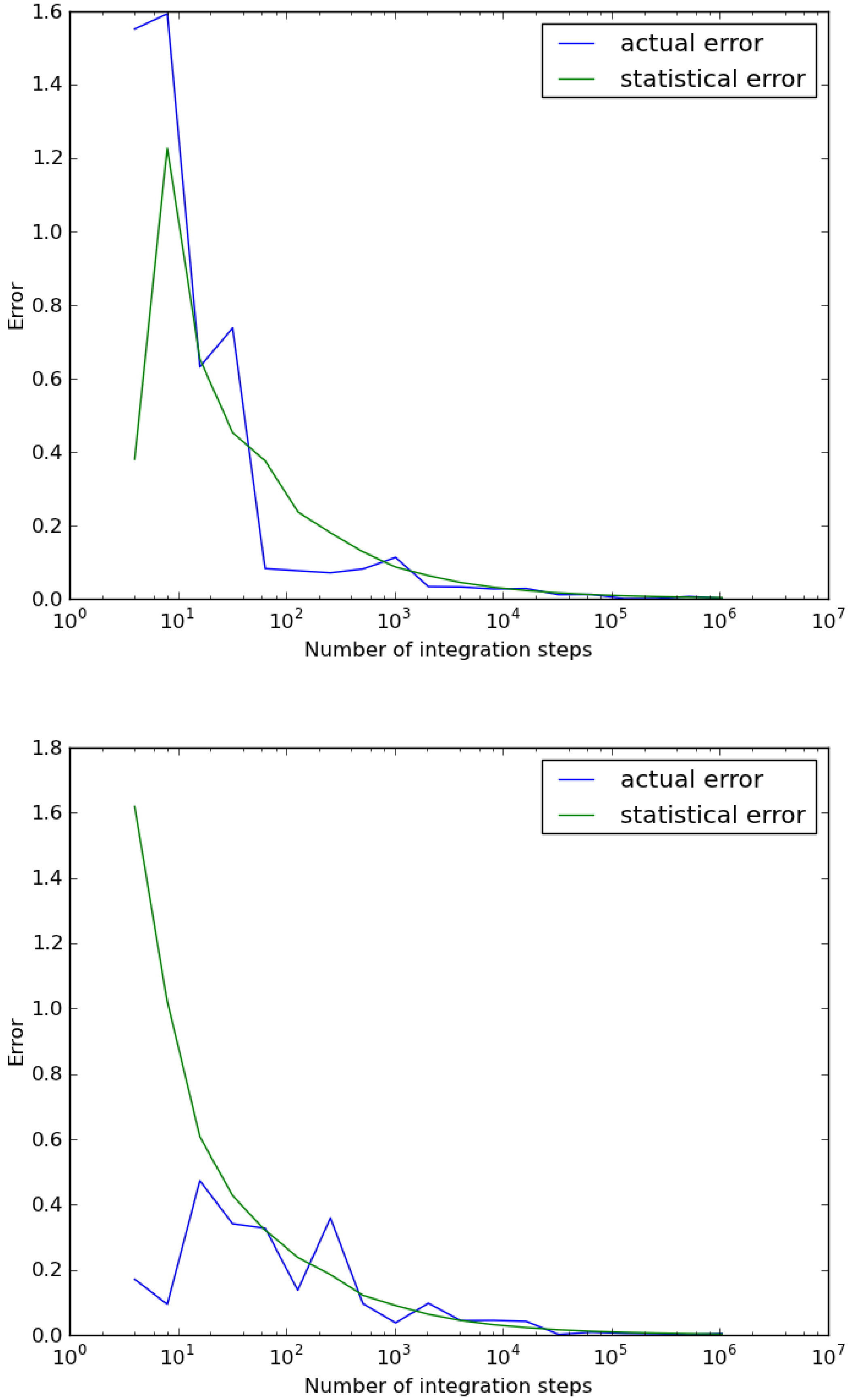

Figure 2: Statistical and actual error of the Integration with the simple sampling method

As it can be seen due to the randomness of the points in which f is evaluated the actual error behaves quite different from run to tun. Furthermore it also varies for small numbers of integration steps quite a lot from the statistical error. In contrast to this the statistical error has a much more comparable behavior in different runs.

2 Importance Sampling- Metropolis-Hastings-Algorithm

In this task we were asked to implement the Metropolis-Hastings-algorithm and to use that function to generate a given number of samples for several Δx being used in the trial move. The resulting histograms can be found in the following plots

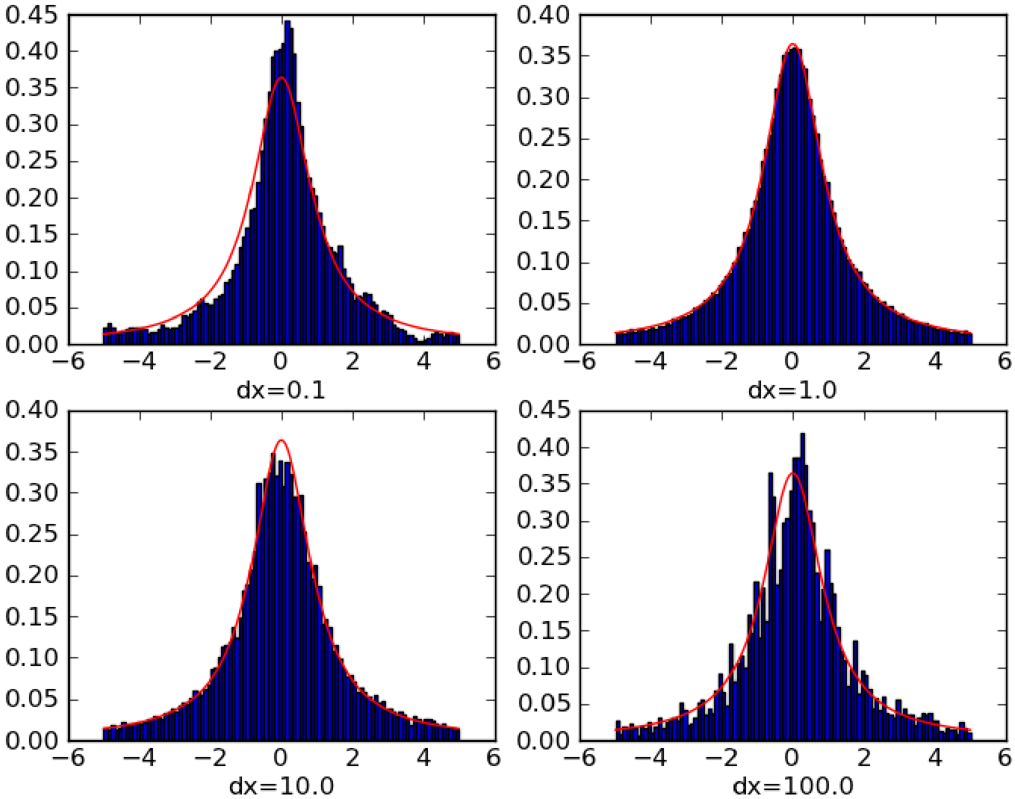

Figure 3: Distribution histograms of the Metropolis samples with 100 bins compared to the normalized runge function (red)

By eye it is hard to say wether $dx = 1.0$ or $dx = 1.0$ is the better choice. Having a look at the acceptance rates - $\approx 84.8\%$ for $dx = 1.0$, $\approx 32.5\%$ for $dx = 10.0$ -and taking into account that it was stated on the worksheet that an acceptance rate of about 30% is optimal, it can be said that $dx = 10.0$ is a better choice than $dx = 1.0$.