Worksheet 6: Finite-Size Scaling and the Ising Model

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February 5, 2013

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Contents

1 Speeding up the Simulation

The simulation was written in c++ because many loops were needed that slowed down python. The data analysis part was implemented in python, so that plotting was very easy. The c++ part was realized as command line tool. It was included in python by the use of the library *subprocess*. We ducked out of *cython* ;-).

2 Values and Errors

Mean energy

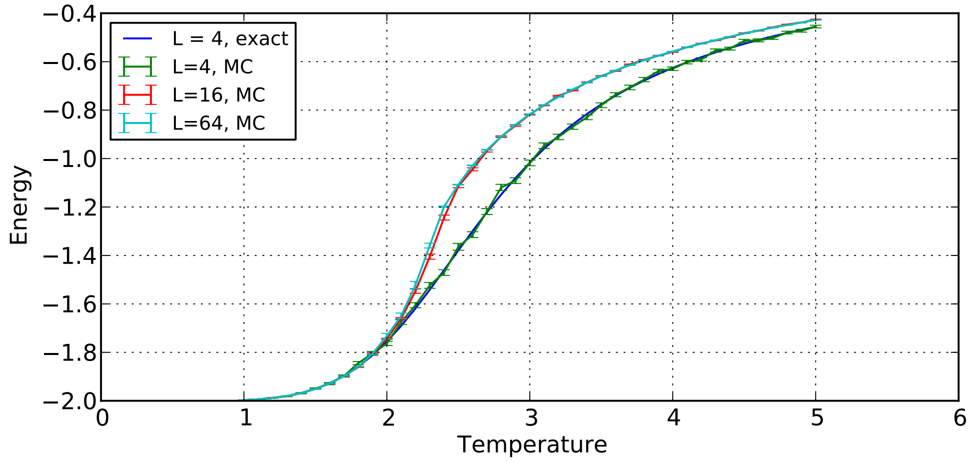

Figure 1: Plot of the mean energy and its error (binning was used). The exact result for $L = 4$ is in good conformity with the Monte Carlo simulation.

Mean absolute magnetization

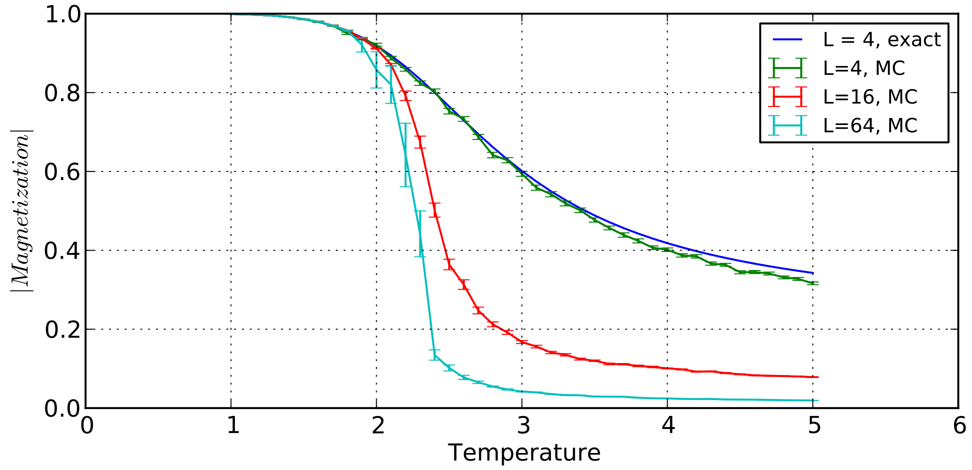

Figure 2: Plot of the mean absolute magnetization and its error (binning was used).

Dependence on L

The higher L is, the steeper the curves are near the critical temperature. The energy and magnetization mutate to step functions for infinite temperature.

2.1 Frequency distributions

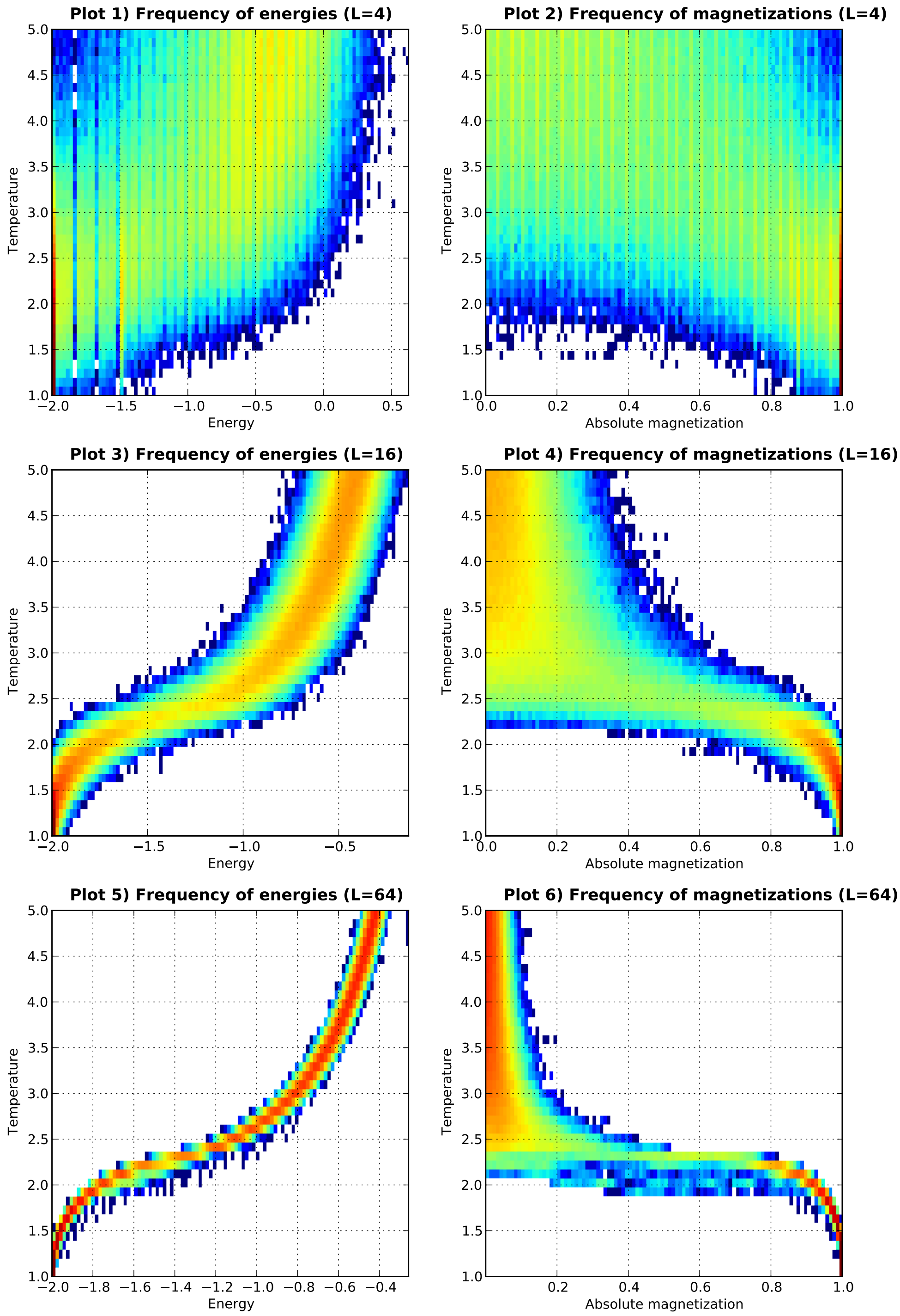

Figure 3: With an increase in L the distributions get sharper.

3 Finite Size Scaling

3.1 Determinig T_C

In this task the Binder parameter $U = 1 - \frac{1}{3} < \mu^4 > / < \mu^2 >^2$ was implemented. The resulting plot of the Binder parameter over the Temperature for different L can be seen below:

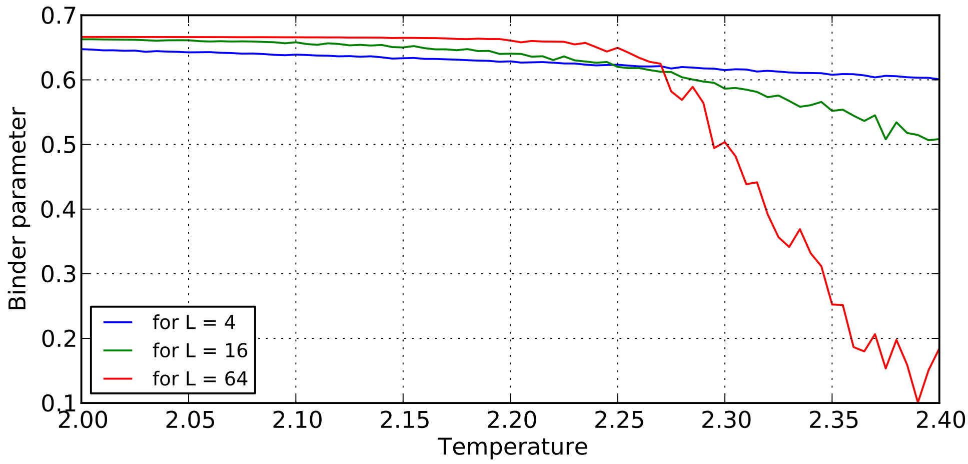

Figure 4: Plot of the Binder parameter for different lattice sizes L. From the intersection point of the different curves the critical Temperature can be determined as $T_C = 2.27K$. In order to get a good result a 100000 sweeps and a temperature step size of $\Delta t = 0.005$ K were used.

3.2 Estimating β_m

Here we performed different simulations at $T_C = 2.27K$ were performed for $L \in \{8, 16, 32, 64, 128\}$. The resulting plot of the magnetization M over L is as follows:

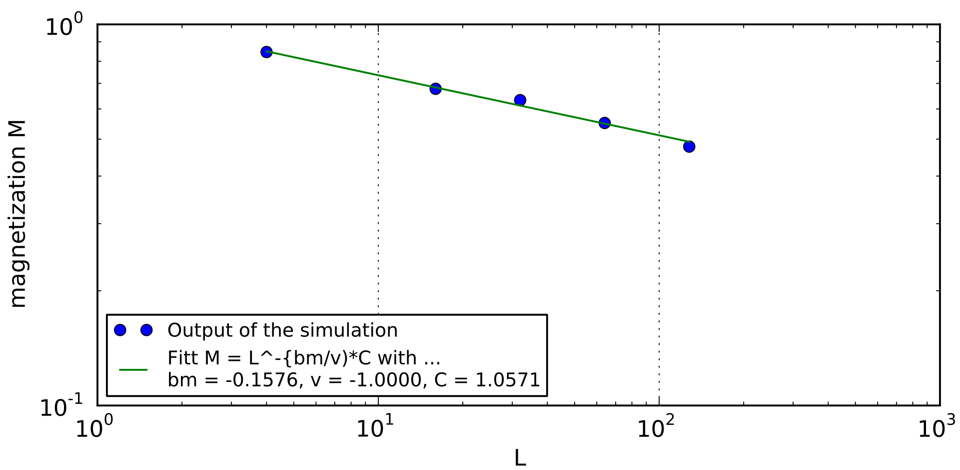

Figure 5: Plot of the magnetization M over L with double logarithmic scale. From the linear fit through the computet magnetizations β_m can be determined as -0.1576.

The formula which connects the magnetization M, lattice size L and β_{m} is the following (for $T = T_C \Rightarrow t = 0$):

$$M = L^{-\beta_{\mathrm{m}}/\nu} \cdot const$$

It was also used in order to get a value for β_{m} from the plot.

3.3 The Master Curve

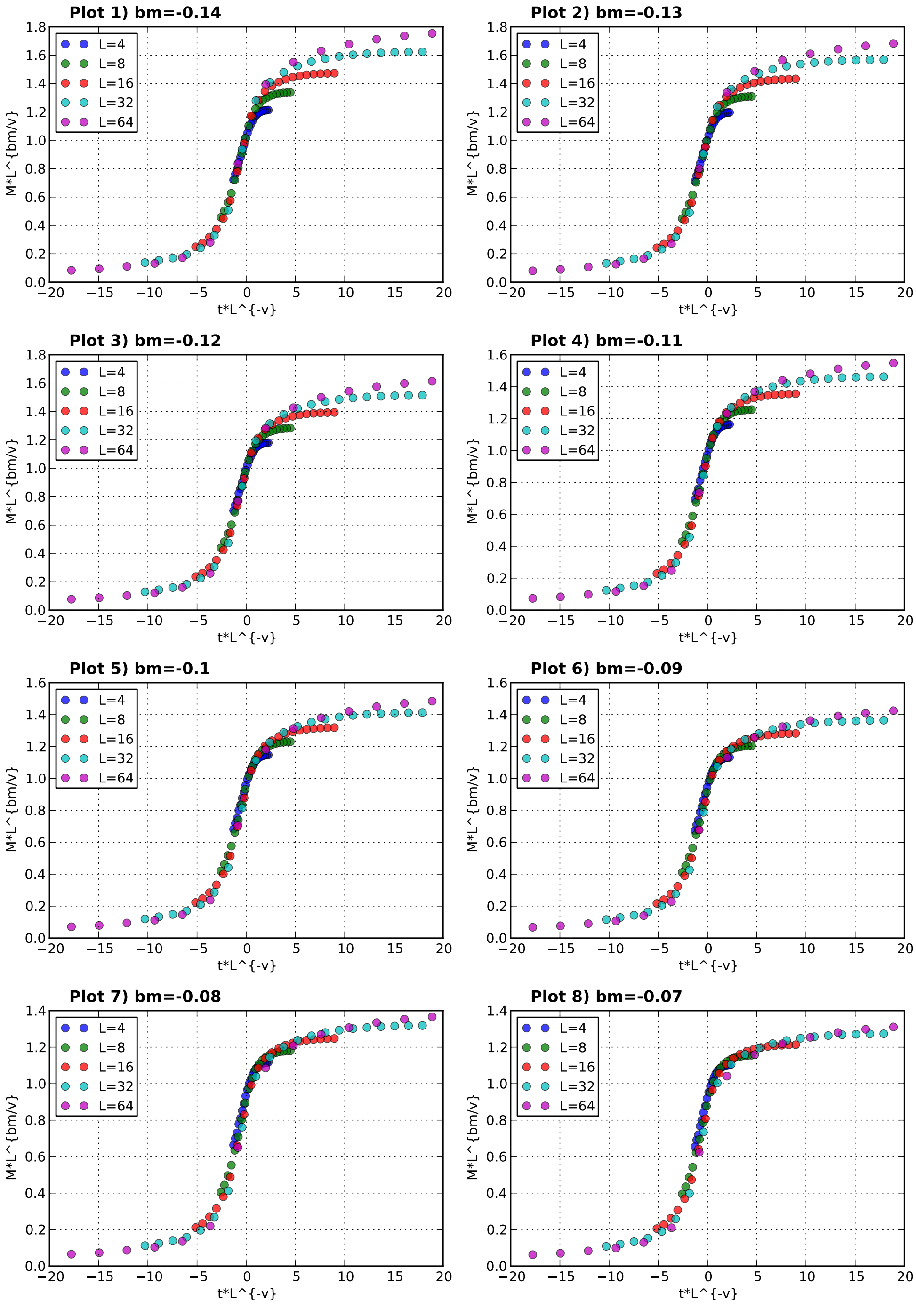

Figure 6: The fit is bes in the core part of the curve at plot 3, so that $\beta_{\mathrm{m}} \approx -0.12$