Worksheet 5: Monte-Carlo

April Cooper, Patrick Kreissl und Sebastian Weber

January 24, 2013
University of Stuttgart

Contents

1 Simple Sampling- Intrgration

First we were asked to program a function runge(x) that computes the runge function $f(x) = \frac{1}{1+x^2}$ and to plot it on the interval $[-5, 5]$.
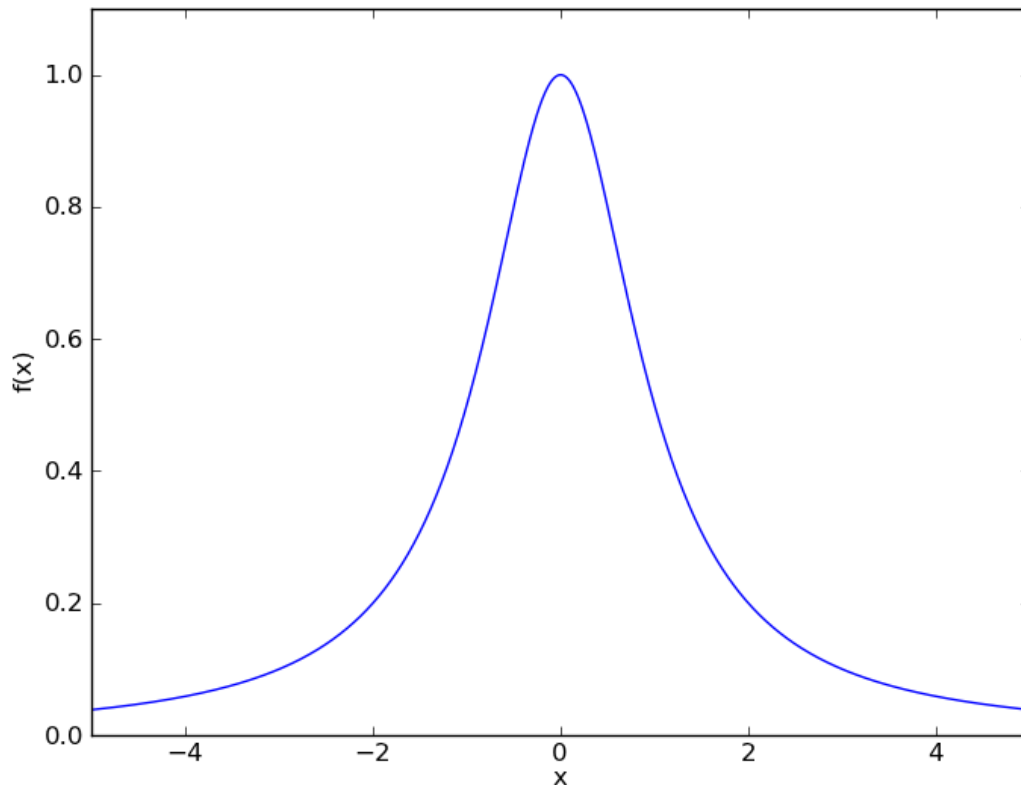

Figure 1: Runge function f(x) on the interval [-5,5]

Then we had to write a function that computes the exact integral of the runge function. The exact result is:

$$\int_{-5}^{5} \frac{1}{1 + x^2} = arctan(5) - arctan(-5) \approx 2.7468$$

After that we were asked to program a function simple_sampling(f,a,b,N) that performs N steps of a simple sampling Monte-Carlo integration and to use this function to compute the Integral for $N = 2^i$ with $2 \leq i \leq 20$. Furthermore we were asked to determine the actual and statistical error and to plot them against the number of integration steps N. In the following are two plots of the error from two different runs.
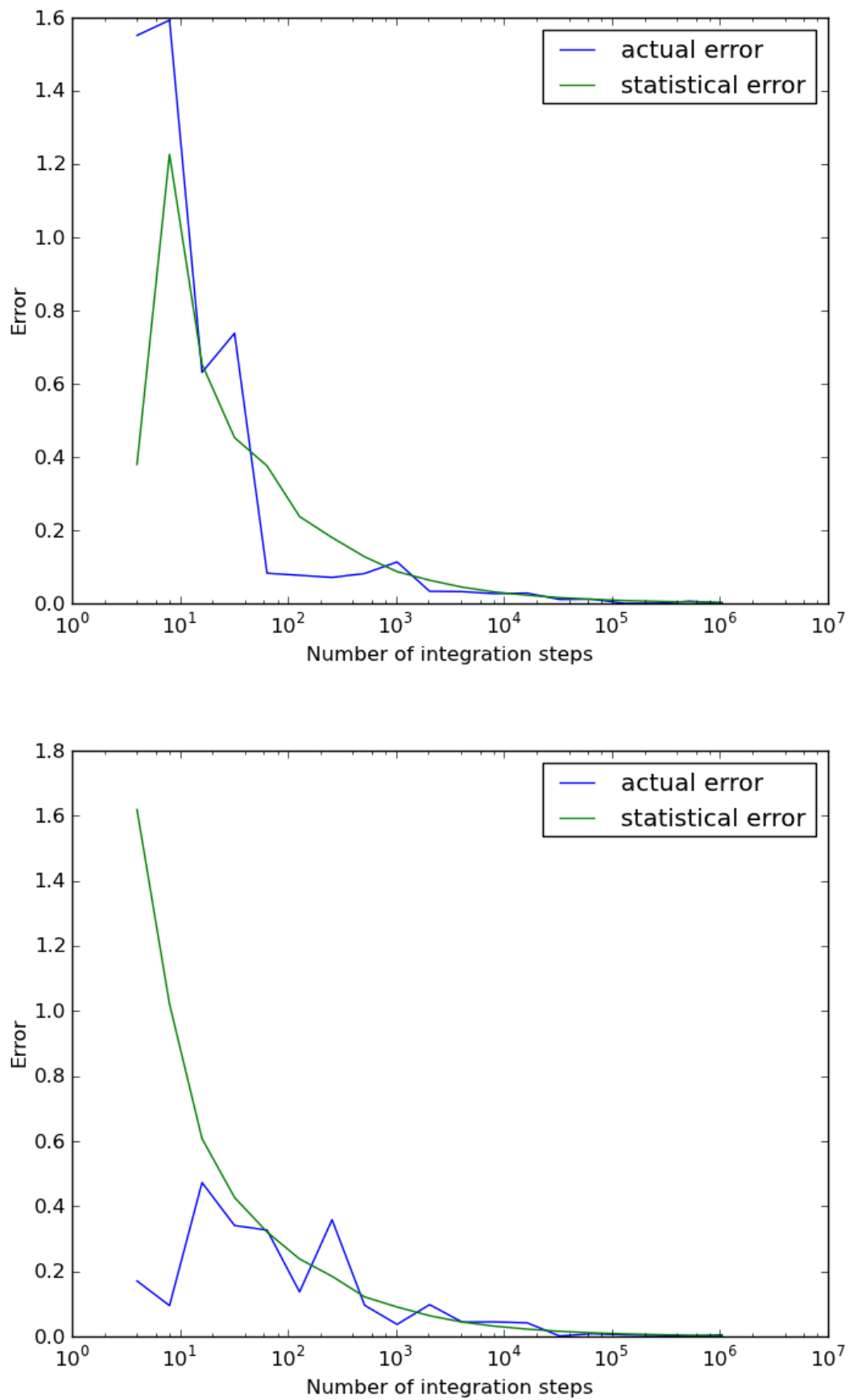
Figure 2: Statistical and actual error of the Integration with the simple sampling method

As it can be seen due to the randomness of the points in which f is evaluated the actual error behaves quite different from run to tun. Furthermore it also varies for small numbers of integration steps quite a lot from the statistical error. In contrast to this the statistical error has a much more comparable behavior in different runs.

2 Importance Sampling- Metropolis-Hastings-Algorithm

In this task we were asked to implement the Metropolis-Hastings-algorithm and to use that function to generate a given number of samples for several Δx being used in the trial move. The resulting histograms can be found in the following plots
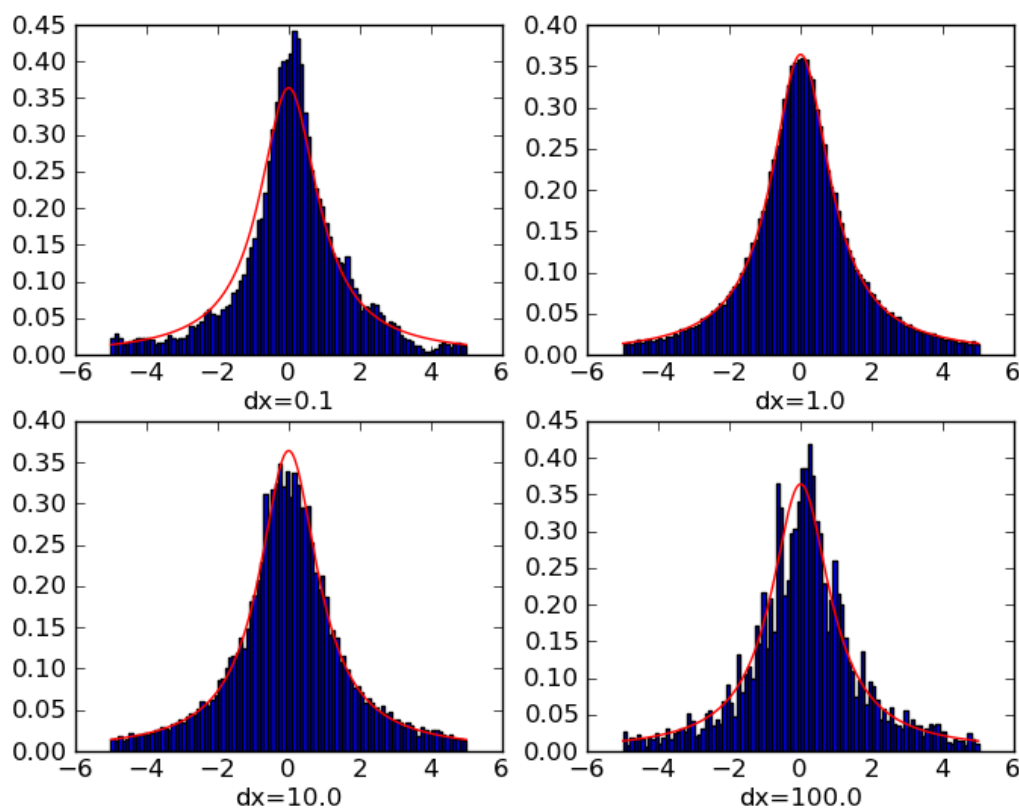

Figure 3: Distribution histograms of the Metropolis samples with 100 bins compared to the normalized runge function (red)

By eye it is hard to say wether $dx = 1.0$ or $dx = 1.0$ is the better choice. Having a look at the acceptance rates - $\approx 84.8\%$ for $dx = 1.0$, $\approx 32.5\%$ for $dx = 10.0$ -and taking into account that it was stated on the worksheet that an acceptance rate of about 30% is optimal, it can be said that $dx = 10.0$ is a better choice than $dx = 1.0$.

3 Simulating the Ising Model

The exact solution of the Ising Model was obtained by using the canonical partition function. The metropolis Monte-Carlo algorithm was implemented according to worksheet 5. To determine the statistical errors Binning errors was used. However this approach doesn't seem to be ideal. Binning errors gave to little error bars independently from the block size. Maybe this was because energy and magnetization of a fixed configuration of the Ising Model can only have discrete values.
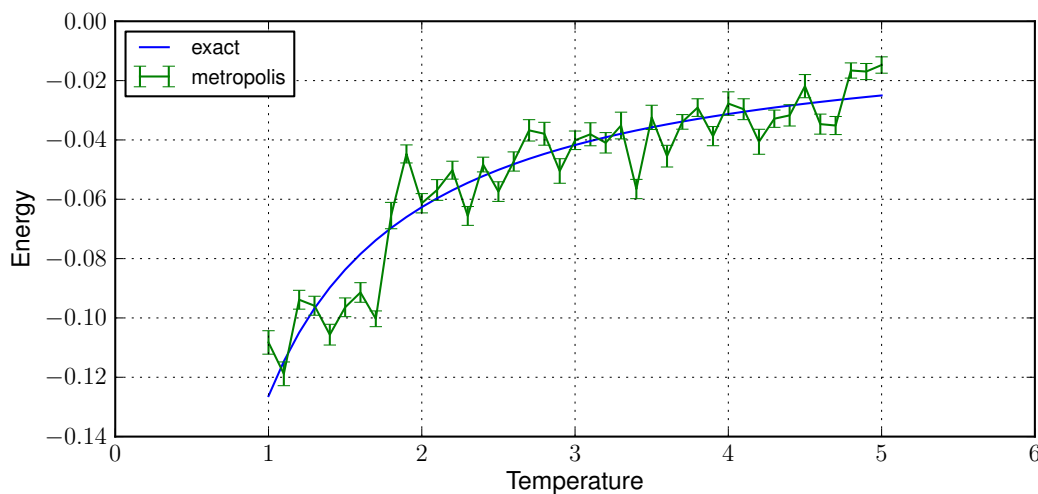
3.0.1 Mean energy

Figure 4: The energy becomes higher, when the temperature raises. The metropolis Monte-Carlo curve is in acceptable conformity to the exact one.
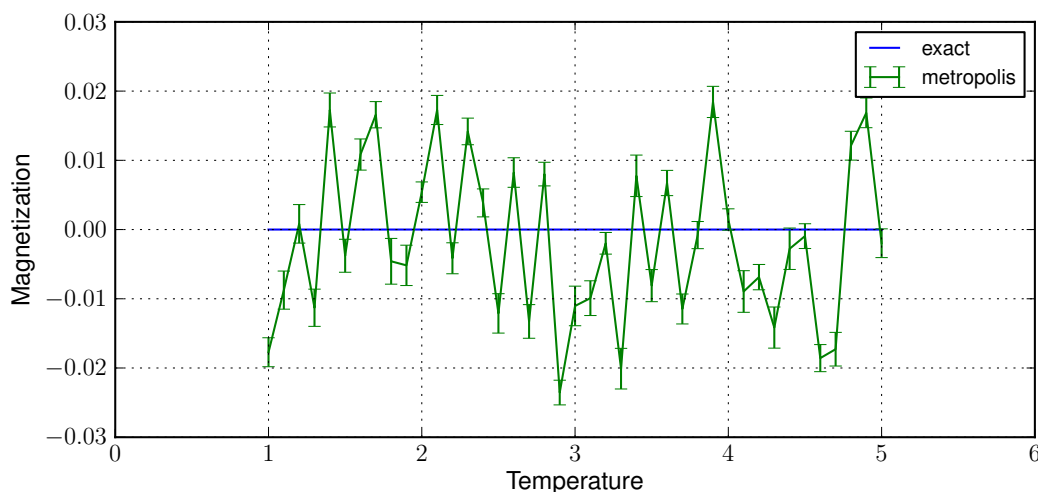
3.0.2 Mean magnetization

Figure 5: The mean magnetization is around zero. This full fills our expectations because for every magnetization below zeros there's a equiprobable magnetization above zero.
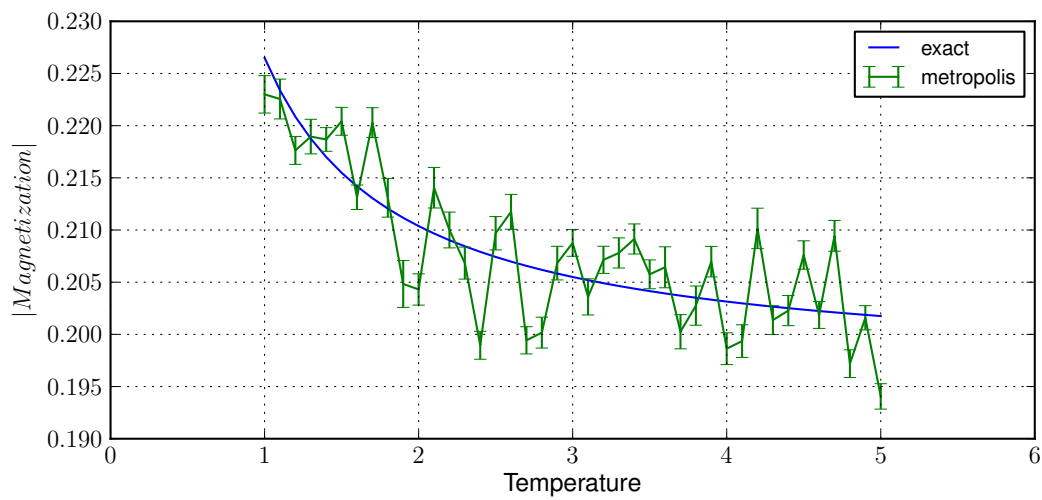
3.0.3 Mean absolute magnetization

Figure 6: The curve of the absolute magnetization is interesting. It clearly shows that a high temperature destroys regimes of equal orientated spins.