# Pre-U Physics Revision Guide

Westminster School

February 2, 2017

# Contents

# Introduction

## Structure of Assessment

| Components | Weighting |
|---|---|
| **Paper 1 Multiple Choice** **1 hour 30 minutes**<br>Candidates answer 40 multiple-choice questions based on Parts A and B of the syllabus content.<br>40 marks | 20% |
| **Paper 2 Written Paper** **2 hours**<br>Section 1: Candidates answer structured questions based on Part A of the syllabus content.<br>Section 2: Candidates answer structured questions related to pre-released material.<br>100 marks | 30% |
| **Paper 3 Written Paper** **3 hours**<br>Section 1: Candidates answer structured questions requiring short answers or calculations and some longer answers. The questions are focused on Part B of the syllabus content, but may also draw on Part A.<br>Section 2: Candidates answer three questions from a choice of six. Three questions will have a strong mathematical focus and three questions will focus on philosophical issues and/or physics concepts. Learning outcomes marked with an asterisk (*) will only be assessed in this section.<br>140 marks | 35% |
| **Practical Investigation** **20 hours** | 15% |

# Part A

# 1 Mechanics

## Scalars and Vectors

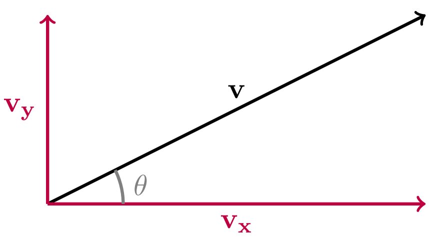
*(a) distinguish between scalar and vector quantities and give examples of each*

A scalar quantity[1] is one which has only a magnitude whereas a vector has *both* magnitude and direction. We often use positive and negative values to indicate direction (e.g. $v = -2\,ms^{-1}$) but this does not mean that all negative values are vectors!

Note that there are different ways of multiplying vectors and scalars. Two vectors can be multiplied to give a scalar *or* a vector. For example, word done is the (scalar) product of force and displacement, both vectors.

*(b) resolve a vector into two components at right angles to each other by drawing and by calculation*

Vectors can be split into two components using trigonometry. The diagram below shows a velocity vector being split into horizontal and vertical components $v_x$ and $v_y$.

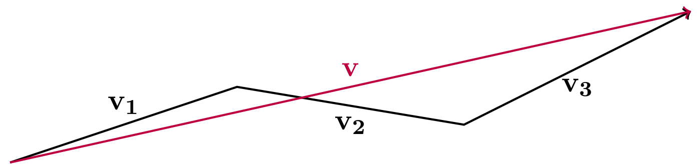



$$\mathbf{v} = \mathbf{v_x} + \mathbf{v_y}$$
$$v_x = v\cos\theta$$
$$v_y = v\sin\theta$$

[1] strictly we are modelling a physical quantity as a mathematical object

*(c) combine any number of coplanar vectors at any angle to each other by drawing*

Vectors can be added by placing them end to end. The resultant vector is the one joining the start of the first vector to the end of the final vector. Its magnitude and direction can be calculated by trigonometry or scale drawing.

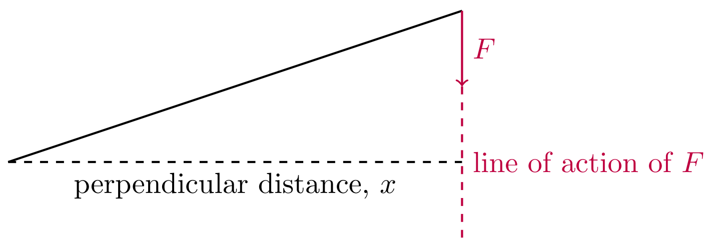



$$\mathbf{v} = \mathbf{v_1} + \mathbf{v_2} + \mathbf{v_3}$$

## Static Equillibrium

*(d) calculate the moment of a force and use the conditions for equilibrium to solve problems (restricted to coplanar forces)*

The moment of a force is calculated by multiplying its magnitude by the perpendicular distance of the force's line of action to the pivot point. This is mathematically equivalent to multiplying the distance from the pivot by the component of the force perpendicular to that distance.




$$\text{moment} = Fx$$

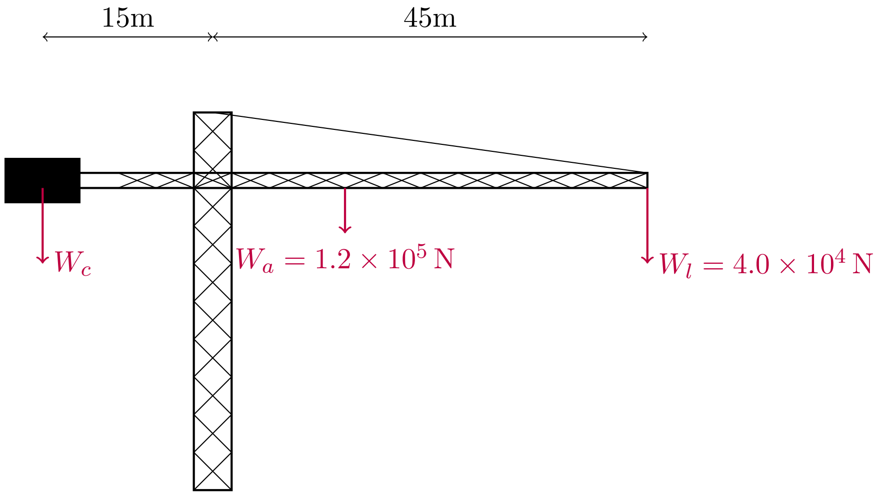
The conditions for equilibrium are:

1. The sum of all the forces acting on the object must be zero.

2. The sum of all the moments on an object must be zero.

**Example Question**

A Tower Crane lifts a load into position. The load has a weight of $4.0 \times 10^4$ N and the arm of the crane has a weight of $1.2 \times 10^5$ N. Calculate the required weight of the counterweight and the force the tower must support. Assume the centre of mass of the arm is at its centre.




**Answer**

We begin by taking moments around the tower of the crane. The weight of the arm, $W_a$, acts 15 m from the tower so solving for moments gives:

$$15W_c = 15W_a + 25W_l$$

$$W_c = 4.2 \times 10^5 \text{ N}$$

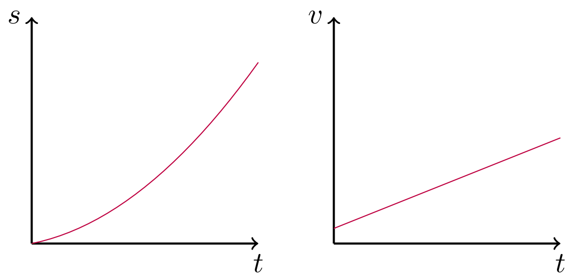
The sum of the downward forces must equal the reaction force of the tower so:

$$R = 4.0 \times 10^5 \text{ N}$$

# Kinematics

*(e) construct displacement-time and velocity-time graphs for uniformly accelerated motion*

For uniform acceleration, a graph of velocity against time will be linear, with the formula $v = u + at$, and a graph of displacement against time will be parabolic, with the formula $s = ut + \frac{1}{2}at^2$.




*(f) identify and use the physical quantities derived from the gradients of displacement–time and areas and gradients of velocity–time graphs, including cases of non-uniform acceleration*

The quantities are given in the table below:

| | gradient | area |
|---|---|---|
| displacement-time | velocity | – |
| velocity-time | acceleration | displacement |

If the graph is non-linear then the gradient of a tangent must be taken. Note that areas below the axis in a velocity-time graph represent *negative* displacement.

*(g) recall and use:*

$$v = \frac{\Delta x}{\Delta t}$$
$$a = \frac{\Delta v}{\Delta t}$$

*(h) recognise and use the kinematic equations for motion in one dimension with constant acceleration:*

$$s = ut + \frac{1}{2}at^2$$
$$v^2 = u^2 + 2as$$
$$s = \left(\frac{u+v}{2}\right)t$$

*(i) recognise and make use of the independence of vertical and horizontal motion of a projectile moving freely under gravity*

When an object moves in a uniform gravitational field it motion can be modeled by considering the horizontal and vertical components of motion separately. The horizontal component has a constant velocity and the vertical has a constant acceleration.

**Example Question**
A ball is thrown with a velocity of $5\,\mathrm{m\,s^{-1}}$ from a height of $1.2\,\mathrm{m}$. If its initial angle to the horizontal is $50°$ calculate the distance it travels before it hits the ground.
**Answer**
The first step is to split the velocity into horizontal and vertical components:

$$v_x = 5\cos 50$$
$$v_y = 5\sin 50$$

The time for the ball to reach the ground can now be calculated using the vertical motion and the equation $s = ut + \frac{1}{2}at^2$, setting $s = -1.2\,\mathrm{m}$. This gives $t = 1.02\,\mathrm{s}$.
Finally, the horizontal distance is calculated using the simple constant velocity formula to give $x = 3.28\,\mathrm{m}$.

## Forces

*(j) recognise that internal forces on a collection of objects sum to zero vectorially*

This is as a result of Newton's Third Law.

*(k) recall and interpret statements of Newtons laws of motion*

1. An object will remain at rest, or continue at a constant velocity, unless a resultant force acts upon it.

2. $F = ma$, where $F$ is the vector sum of the forces acting on the body. Or, alternatively $F = \frac{dp}{dt}$ (see below).

3. For every force of object A acting on object B there exists a force of the same type, of equal magnitude and opposite direction of object B acting on object A.

   *It is important to be able to distinguish the 'equal and opposite forces which may act on a single object in equilibrium from a Newton's Third Law pair of forces.*

*(l) recall and use $F = ma$ in situations where mass is constant*

Remember that $F$ is the *resultant* force acting on the body.

*(m) understand the effect of kinetic friction and static friction*

*(n) use $F_k = \mu_k N$ and $F_s = \mu_s N$, where $N$ is the normal contact force and $\mu_k$ and $\mu_s$ are the coefficients of kinetic friction and static friction, respectively*

Friction occurs between two objects when they are pushed together by a normal force. A useful model is that the maximum size of the frictional force is proportional to the normal force. There is usually a difference between the constant of proportionality when the two surfaces are stationary compared to each other (static friction) compared to when they are sliding past each other (kinetic friction). It is usually the case that $\mu_k < \mu_s$.

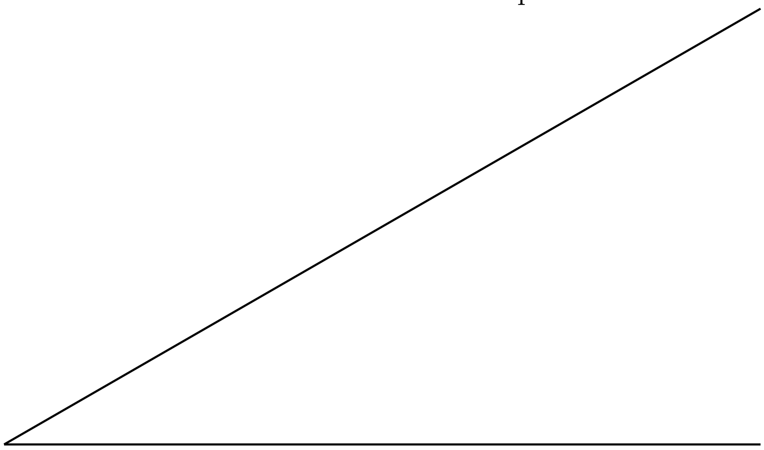
An interesting result is that blocks of different masses should take the same distance to slide to a halt:

$$s = \frac{\frac{1}{2}mv^2}{F} = \frac{\frac{1}{2}mv^2}{\mu_k N} = \frac{\frac{1}{2}mv^2}{\mu_k mg} = \frac{v^2}{2\mu_k g}$$

*(o) recall and use the independent effects of perpendicular components of a force*

As with velocities, forces can be split into two perpendicular components and their effects considered independently.

**Example Question**
A block of mass 4 kg is on a frictionless slope of 30°. Calculate the rate at which it accelerates down the slope.



**Answer**
The weight should be split into components along the slope and perpendicular to the slope (shown in green). Only the component along the slope contributes to the acceleration.

$$a = \frac{F}{m} = \frac{mg\cos 30}{m} = 8.5\,\mathrm{ms}^{-2}$$

This question could be extended to include friction by calculating the normal force, the frictional force and hence a new acceleration. If $\mu_k = 0.4$ then the answers are 19.62 N, 7.85 N and 6.5 $\mathrm{ms}^{-2}$ respectively (Try it!)

*(p) recall and use $p = mv$ and apply the principle of conservation of linear momentum to problems in one dimension*

Momentum is a conserved quantity (along with energy and charge). It can be calculated using the formula $p = mv$ where $p$ is the momentum. In any closed system the total momentum of the particules must remain constant. This can be used to predict the outcomes of collisions in certain cases.

*(q) distinguish between elastic and inelastic collisions*

An elastic collision is one in which *kinetic energy* is conserved. An inelastic collision is one in which it is not. In general, a collision in which two objects adhere will not conserve kinetic energy as the final velocity will be given by:

$$v = \frac{m_1u_1 + m_2u_2}{m_1 + m_2}$$

and therefore the final kinetic energy will be given by:

$$\text{KE} = \frac{1}{2}(m_1 + m_2)v^2 = \frac{1}{2}\frac{(m_1u_1 + m_2u_2)^2}{m_1 + m_2}$$

which cannot be equal to the original kinetic energy.

*(r) relate resultant force to rate of change of momentum in situations where mass is constant and recall and use $F = \frac{\Delta P}{\Delta t}$*

Newton's second law is more properly given by:

$$F = \frac{dp}{dt}$$

This simplifies to the GCSE formulation for constant mass:

$$F = \frac{dp}{dt} = m\frac{dv}{dt} = ma$$

A simplified version is:

$$F = \frac{\Delta P}{\Delta t}$$

This will give the correct result for a constant force or otherwise give the average force.

*(s) recall and use the relationship impulse = change in momentum*

Multiplying both sides of the equation above by time gives:

$$F\Delta t = \Delta P$$

The quantity on the left hand side is the impulse.

*(t) recall and use the fact that the area under a force-time graph is equal to the impulse*

Using calculus to solve differential version of Newton's Second Law above gives:

$$\Delta P = \int_{t_0}^{t_1} F dt$$

The right-hand side of this equation represents the area under a force-time graph.

*(u) apply the principle of conservation of linear momentum to problems in two dimensions*

When objects are free to move in two dimensions then momentum must be conserved along two axis.

**Example Question**
Two objects are able to slide frictionlessly over a horizontal surface. The first object, $m_1 = 3\,\mathrm{kg}$ is propelled with an initial speed $u_1 = 5\,\mathrm{m\,s^{-1}}$ towards a second mass, $m_2 = 1.5\,\mathrm{kg}$, which is initially at rest. After the collision both objects move at 30° on either side of the line of the original motion. What are the final speeds of the two objects? Is the collision elastic?

**Answer**
Conservation of momentum along the x-axis gives

$$m_1 u_1 = m_1 v_1 \cos\theta + m_2 v_2 \cos\theta$$

Conservation of momentum along the y-axis gives

$$m_1 v_1 \sin\theta = m_2 v_2 \sin\theta$$

These equations can be combined to give

$$v_1 = \frac{u_1}{2\cos\theta} = 2.887\,\mathrm{m\,s^{-1}}$$

and

$$v_2 = \frac{m_1}{m_2} v_1 = 5.773\,\mathrm{m\,s^{-1}}$$

The initial KE of the system is

$$K_i = \frac{1}{2} m_1 u_1^2 = 37.5\,\mathrm{J}$$

and the final KE of the system is

$$K_f = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 u_2^2 = 37.5\,\mathrm{J}$$

since $K_i = K_f$, the collision is elastic

*(v) recall and use density = mass / volume*

*(w) recall and use pressure = normal force / area*

*(x) recall and use* $p = \rho g h$ *for pressure due to a liquid.*

These are GCSE equations and should present no problems.

# 2 Gravitational Fields

*(a) recall and use the fact that the gravitational field strength g is equal to the force per unit mass and hence that weight $W = mg$*

A **field** is a region where a particle experiences a force. If this is applied to gravitation, then we can say that a **gravitational** field is a region where a **mass** experiences a force.

You can only tell if a field exists when it exerts a force on something. It is a way of envisaging (seeing in your mind's eye) the size and the direction of the force that would be exerted on a particle when placed in that field.

A gravitational field is produced by anything with mass.

Therefore, a gravitational field is a way of envisaging what would happen to a mass if it were placed in the field due to another mass.

The field is usually represented by lines which show both the **direction** and **strength** of the field.

The **strength** of a gravitational field (the field strength) at any point is the force felt **per unit mass** at that point. This is a **definition.**

It can be written as a word equation:

Gravitational field strength at a point (N/kg)= Force felt by mass (measured in Newtons)/Size of mass (measured in kilograms)

Or in symbols:

$$g = \frac{F}{m}$$

The force, F, felt by any object on the surface of the Earth due to the gravitational field strength of the Earth is known as its **weight.** It is given the symbol **W**.

This means that we can re-write equation above for the field strength at the surface of the Earth by putting W instead of F.

$$g = \frac{W}{m}$$

This then rearranges to an equation that you have all seen before:

$$W = mg$$

Thus the weight of an object on the surface of the Earth is its mass multiplied by the gravitational field strength g.

*(b) recall that the weight of a body appears to act from its centre of gravity*

The centre of gravity of an object is the point where the weight acts or appears to act.

Thus, when you draw a free-body force diagram for any object in a gravitational field, you draw **one** arrow from the centre of gravity of the object to represent the force due to the field. On the Earth this is, of course, the weight and the arrow points vertically downwards.

*(c) sketch the field lines for a uniform gravitational field (such as near the surface of the Earth)*

A uniform field is a field where the field strength is the same at all points in the field.

This means that for a gravitational field the force felt per unit mass (see definition) is the same at all points.

The surface of the Earth is a very good approximation to a uniform field.

Therefore if you draw a diagram of the Earth's gravitational field at the Earth's surface over a small area, it will look like this:

As you can see, the field lines are **parallel** and **evenly-spaced.** This is always the case for a uniform field.

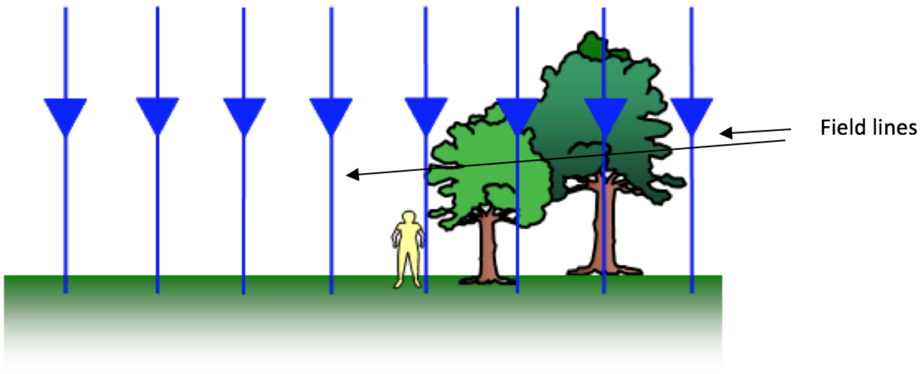



Figure 2.1: Uniform Field

*(d) explain the distinction between gravitational field strength and force and explain the concept that a field has independent properties.*

There is a very important distinction to make between **gravitational field strength** and **force** at this point: The field strength at any point is the same for all bodies in the field and is the force felt per kilogram, but the force is different and depends on the size of the mass there.

This is best illustrated with an example: If a mass of 60kg is in the Earth's gravitational field at the surface of the Earth, then we can calculate the force acting on it, its weight, using equation (3):

$$W = mg = 60\,\mathrm{kg} \times 9.8\,\mathrm{N\,kg^{-1}} = 590\,\mathrm{N}$$

So the force felt by the 60kg mass is 590N but the field strength for the mass **and for any other mass** is $9.8\mathrm{Nkg^{-1}}$. So the field strength is fixed by your position in the field and the size of the mass that is exerting the field, and nothing else. The force depends on the mass in the field as well.

# 3 Deformation of Solids

# 4 Energy Concepts

# 5 Electricity

# 6 Waves

# 7 Superposition

# 8 Atomic and Nuclear Processes

# 9 Quantum Ideas

# Part B

# 10 Rotational Mechanics

This chapter contains revision on the topics of:

- kinematics of uniform circular motion
- centripetal acceleration
- moments of inertia
- kinematics of rotational motion

**Candidates should be able to:**

*(a) Define and use the radian*

An angle in radians is defined by the length of the arc of circle it subtends divided by the radius of the circle. Numerically $2\pi$ radians is equivalent to 360 degrees.

*(b) Understand the concept of angular velocity, and recall and use the equations $v = r\omega$ and $T = \frac{2\pi}{\omega}$*

These equations are valid for an object travelling in a circle in a uniform manner.

Angular velocity is defined as the rate of change of angle, $\omega = \frac{d\theta}{dt}$ ie how many radians per second the rotating object passes through. Hence the time for one complete rotation will be $T = \frac{2\pi}{\omega}$

From the definition of the radian, in a small time interval $\delta t$ we can say that the displacement of the rotating object is $\delta s \approx r\delta\theta$ (see figure 10.1), and hence in the limit as we make the time interval smaller that means $v = \frac{ds}{dt} = r\frac{d\theta}{dt} = r\omega$

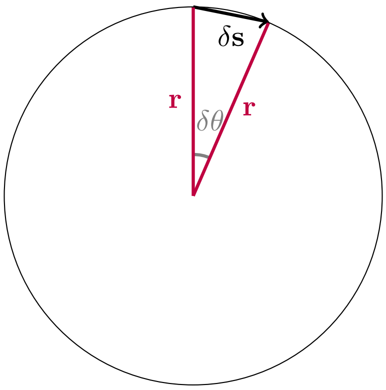



Figure 10.1:

**Example Question**
Calculate the linear velocity of the Earth relative to the sun, given the Earth-Sun distance is 1.5 x $10^{11}$m
**Answer**
First calculate the angular velocity of the Earth. It performs a complete orbit (ie $2\pi$ radians) in 1 year, so $\omega = \frac{2\pi}{365 \times 24 \times 3600}$
Then $v = r\omega = 30000\ ms^{-1}$

*(c) Derive, recall and use the equations for centripetal acceleration $a = \frac{v^2}{r}$*

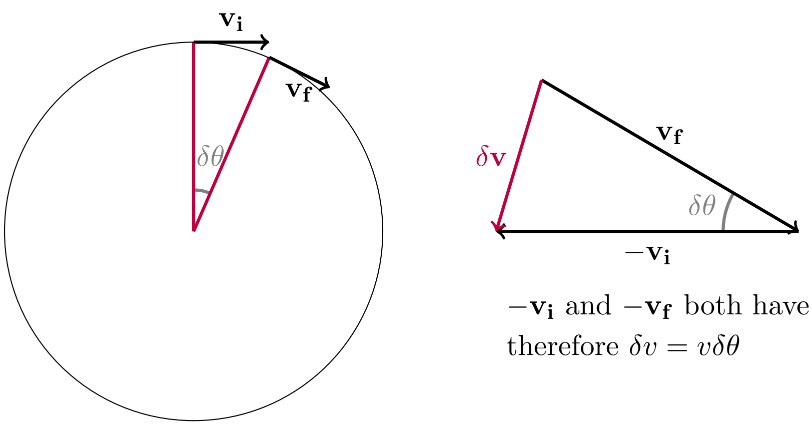
and $a = r\omega^2$

Since acceleration is defined as change in velocity, we can see from the following diagram (figure 10.2) that the velocity change when a uniformly rotating object moves through a small angle $\delta\theta$ can be written as $\delta v = v\delta\theta$ since the magnitudes of the initial and final velocity are both equal to v.

The acceleration is therefore $a \approx \frac{v\delta\theta}{\delta t}$ and as we let $\delta t \to 0$ we get $a = v\frac{d\theta}{dt} = v\omega$. since $v = r\omega$ we also get $a = \frac{v^2}{r} = r\omega^2$

*(d) Recall that $F = ma$ applied to circular motion gives resultant $F = \frac{mv^2}{r}$*

Since $a = \frac{v^2}{r}$ and $F = ma$ we can combine these to give $F = \frac{mv^2}{r}$. This can be very useful for example when combined with Newton's law of Gravity to explain planetary orbits etc.




$-\mathbf{v_i}$ and $-\mathbf{v_f}$ both have magnitude $v$, therefore $\delta v = v\delta\theta$

Figure 10.2:

**Example Question**
Show that, for a circular orbit, the time period squared is proportional to the radius cubed.
**Answer**
Start by equating $F = mr\omega^2$ with $F = \frac{GMm}{r^2}$ (we can ignore the minus sign)

$mr\omega^2 = \frac{GMm}{r^2}$

Then rearrange and cancel $m$ to find

$r^3 = \frac{GM}{\omega^2}$

then using $\omega = \frac{2\pi}{T}$ we get

$r^3 = \frac{GM}{4\pi^2}T^2$

*(e) Describe qualitatively the motion of a rigid solid object under the influence of a single force in terms of linear acceleration and rotational acceleration.*

When a rigid solid object has a single force applied to it, it may just result in linear acceleration, if the force acts through the centre of mass of the object. However it is more likely that the force will not act through this point, and therefore also cause some rotational acceleration, causing the object's angular velocity to change.

The remaining sections in this chapter deal with how we describe this rotational acceleration, but it should be noted that they are asterisked sections so will only form part of section 2 of paper 3.

### *(f) *Recall and use $I = \Sigma mr^2$ to calculate the moment of inertia of a body consisting of three or fewer point particles fixed together*

The moment of inertia of a body is the rotational analogue of mass, and basically describes how resistant an object is to angular acceleration when a torque is applied (in the same way that mass describes how resistant an object is to accelerating linearly when a force is applied...)

A point mass $m$ at a distance $r$ from an axis of rotation will have Moment of Inertia $I = mr^2$

More generally the moment of inertia is the sum of all the $mr^2$ values for all the point masses that make up an object. For a small number of particles this can just be found by adding the values of individual moments of inertia.

### *(g) *Use integration to calculate the moment of inertia of a ring, a disk and a rod*

For more complex bodies, the summing of moments of inertia needs to be done by setting up an integration. This is acheived by summing moments of inertia for a series of small sections of the object, strips or rings of thickness $\delta x$ say, and using the fact that as $\delta x \to 0$ the sum $\Sigma \delta x \equiv \int dx$

As an example, consider a disk of radius r , thickness t and density $\rho$. The moment of inertia about its central axis could be found by splitting it up into rings of radius $x$ and thickness $\delta x$. See figure 10.3.

Each ring has moment of inertia equal to its mass multiplied by $x^2$, as all the particles in it have (approximately) the same distance from the axis.[1]

Hence $I_{ring} = (2\pi x t \rho \delta x)x^2 = 2\pi\rho x^3 \delta x$

To find the moment of inertia of the disk, we sum all the rings and let their size $\delta x$ tend to 0.

[1] This can be proved by another, much simpler, sum. If you imagine the ring as the sum of many small points of mass $\delta m$ each at the same radius $r$, the moment of inertia becomes $\Sigma r^2 \delta m$ which is the same as $r^2 \Sigma \delta m$ and hence $I = mr^2$

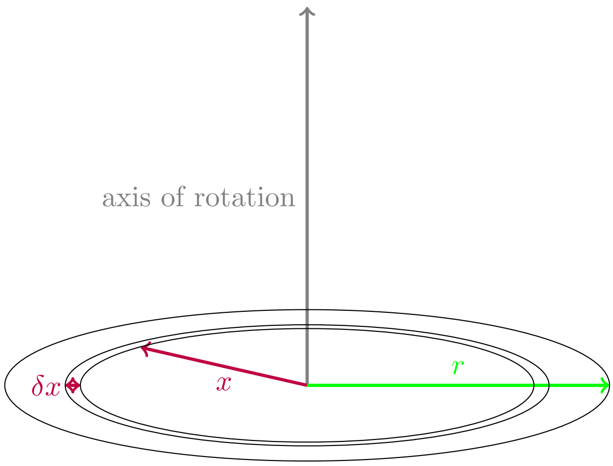



Figure 10.3:

$I_{disk} = \Sigma 2\pi\rho x^3 \delta x$

$I_{disk} = \int_0^r 2\pi\rho t x^3 \delta x$

giving

$I_{disk} = \frac{2\pi\rho t r^4}{4}$

which is equal to

$I_{disk} = \frac{mr^2}{2}$

Other objects would be derived in a similar way. You should get $I = \frac{mr^2}{3}$ for a rod rotating about one end, and $I = \frac{mr^2}{12}$ for a rod rotating about its centre of mass.

*(h) *Deduce equations for rotational motion by analogy with Newtons laws for linear motion, including $E = \frac{1}{2}I\omega^2$, $L = I\omega$ and $\Gamma = I\frac{d\omega}{dt}$*

Here is a table outlining the analogies between linear and rotational moton:

| **Linear Motion** | **Rotational Motion** |
|---|---|
| Mass m | Moment of Inertia I |
| linear velocity v | Angular velocity $\omega$ |
| Force F | Torque $\Gamma$ |
| Linear Momentum p | Angular Momentum L |
| $p = mv$ | $L = I\omega$ |
| $F = ma = m\frac{dv}{dt}$ | $\Gamma = I\frac{d\omega}{dt}$ |
| $KE_{linear} = \frac{1}{2}mv^2$ | $KE_{rotational} = \frac{1}{2}I\omega^2$ |

*(i) *Apply the laws of rotational motion to perform kinematic calculations regarding a rotating object when the moment of inertia is given.*

**Example Question**
The moment of inertia of a large flywheel in a factory is 60 kgm$^2$. Calculate how long it would take the flywheel to obtain an angular velocity of 6.0 rad s$^{-1}$ when a torque of 24 Nm was applied.
**Answer**
First use $\Gamma = I\frac{d\omega}{dt}$ to find the angular acceleration:
$\frac{d\omega}{dt} = \frac{\Gamma}{I} = \frac{24}{60} = 0.4$ rad s$^{-2}$
Then the time taken will be
$T = \frac{6}{0.4} = 15$ seconds

# 11 Oscillations

## Simple Harmonic Motion

*(a) Recall the condition for simple harmonic motion and hence identify situations in which simple harmonic motion will occur*

Any oscillation where the acceleration is proportional to the displacement from an equilibrium position and in the opposite direction to the displacement is described as Simple Harmonic Motion (SHM).

These two conditions can be expressed in equation form as:

$$a \propto -x$$

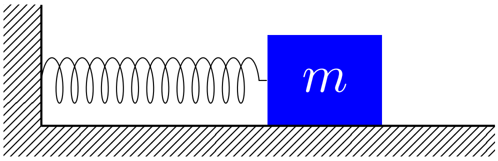
Notice the minus sign to signify that the acceleration is in the opposite direction to the displacement.

We will mainly be looking at idealised springs and pendulums to understand the maths behind it but the real reason for studying SHM is that it is an excellent approximation for many of the oscillations that we come across in the natural world.

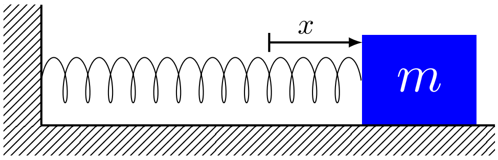
So while the topic is introduced with some rather prosaic examples this provides the building blocks to understanding earthquakes or how atoms vibrate in lattices as well as any musical instrument you can think of.

So without further ado let's look at our first and perhaps simplest example.

A mass on a spring on a smooth horizontal surface.




What happens if we move the mass by a distance x and then let it go?




Before going into any mathematical detail we can think about what will happen to the mass.

- We know that there is now a force from the stretched string which is pulling the mass back to its original position.
- This will cause the mass to accelerate to the left.
- Once the mass reaches its starting point it will overshoot and start compressing the spring.
- This creates a resultant force which will again restore the mass to its original position.
- This will cause the mass to accelerate to the right.
- The mass will overshoot again and the cycle will continue.

Note that the direction of the acceleration is always opposite to the displacement.

The mathematical treatment for this starts very simply with a basic knowledge of Hooke's law.

The force on a stretched or compressed spring is given by

$$F = -kx$$

where k is the spring constant.

Newton's second law tells us that

$$F = ma$$

Putting these together we get

$$a = \frac{F}{m} = -\frac{k}{m}x$$

Because k and m are constant this satisfies the condition for SHM,

$$a \propto -x$$

**Example Question**
A 2kg mass attached to a horizontal spring of spring constant $0.3\text{Nm}^{-1}$ is stretched by 10cm and then released.
Find the maximum acceleration of the mass

**Answer**
Using the formula $a = -\frac{k}{m}x$ The maximum acceleration will occur when x is a maximum so

$$acceleration_{max} = -\frac{0.3}{2} \times 0.1$$

$$acceleration_{max} = -0.015ms^{-2}$$

*(b) * show that the condition for simple harmonic motion leads to a differential equation of the form*

$$\frac{d^2x}{dt^2} = -\omega^2 x$$

*and that*

$$x = Acos\omega t$$

*is a solution to this equation*

Acceleration is the rate of change of velocity

$$a = \frac{dv}{dt}$$

and velocity is the rate of change of displacement

$$v = \frac{dx}{dt}$$

Putting these two together gives the condition for simple harmonic motion as.

$$\frac{d^2x}{dt^2} = -\omega^2 x$$

where $\omega^2$ is a (strange choice) of constant.

This is a second order differential equation and to solve it we need to find a function which when differentiated twice gives us the negative of the original function.

By inspection we can see that functions with $\sin\omega t$ and $\cos\omega t$ are both possible solutions.

e.g.

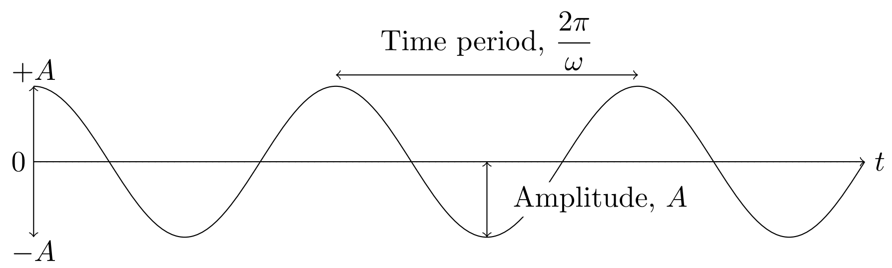
$$\begin{aligned} x &= Acos\omega t \\ \frac{dx}{dt} &= -A\omega sin\omega t \\ \frac{d^2x}{dt^2} &= -A\omega^2 cos\omega t = -\omega^2 x \end{aligned}$$

So $x = Acos\omega t$ is a solution (using sin is equivalent but with a phase offset.)

Looking at this function we can see that it will give us a cosine wave with an Amplitude of A and a time period of $\frac{2\pi}{\omega}$.

This gives us a frequency on $\frac{1}{T} = \frac{\omega}{2\pi}$

So $\omega$ is the angular frequency (which explains our strange choice of constant).




We now have a general expression for the displacement x of the object after a time t.

The value of $\omega$ will be given by the physical properties of the system.

e.g. For the mass on a spring we looked at earlier

$$
\begin{aligned}
F &= -kx && \text{Force on mass by Hooke's Law} \\
a &= -\frac{k}{m}x && \text{gives acceleration} \\
\frac{d^2x}{dt^2} &= -\omega^2 x && \text{comparing with condition for SHM} \\
\omega^2 &= \frac{k}{m} && \text{gives value for constant } \omega \\
\omega &= \sqrt{\frac{k}{m}}
\end{aligned}
$$

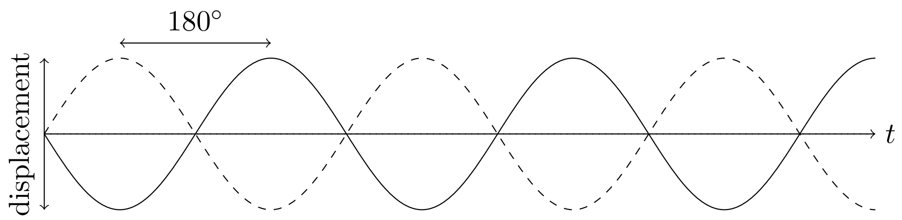
*(c) * use differential calculus to derive the expressions*

$$v = -A sin\omega t$$

*and*

$$a = -A\omega^2 cos\omega t$$

*for simple harmonic motion.*



# 12 Electric Fields

# 13 Gravitation

*(a) state Keplers laws of planetary motion*

Before you learn Kepler's laws (which you MUST learn) you should spend some time looking through the chapter on rotational mechanics and making absolutely sure that you know how to apply Newton's laws of motion to an orbiting body. This is vital or you won't get the maths in this chapter.

The specification dictates that you need to be able to state Kepler's Laws. They are as follows:

1. Planets move in elliptical orbits with the Sun at one focus. *(motion in ellipses is not part of the specification, so don't worry about the mathematics of this – we approximate to a circle for pre-U)*

2. The Sun-planet line sweeps out equal areas in equal times.

3. The orbital period squared of a planet is proportional to its mean distance from the Sun cubed.

### First Law

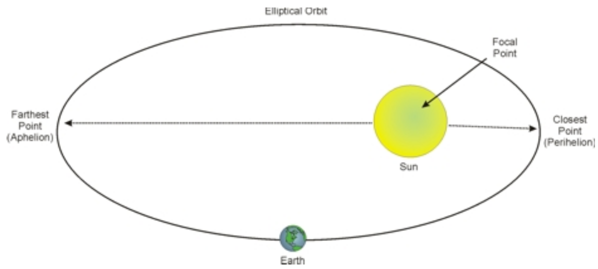



Figure 13.1: Kepler's First Law

Of course, it looks nothing like this – this is MASSIVELY exaggerated for the sake of seeing what is going on. The Earth's orbit around the Sun is very nearly circular, which is why it took so long for astronomers to realise that it wasn't.

### Second Law

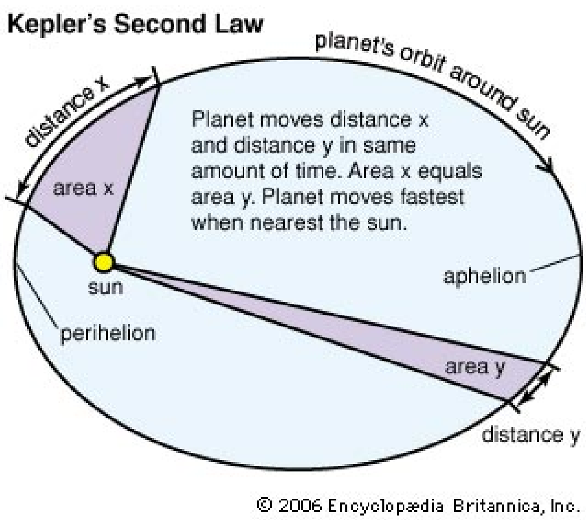



Figure 13.2: Kepler's Second Law

As you can see from the diagram, areas x and y are the same. The reason for this, put simply, is that the objects move more quickly when they are closer to the Sun and more slowly when they are further away.

### Third Law

This will be described in more detail below.

*(b) recognise and use $F = -\frac{Gm_1m_2}{r^2}$*

"The gravitational force between two objects is proportional to the product of their masses and inversely proportional to the square of the distance between their centres."

That is a lot of words and it is much more easily explained with an equation:

$$F = -\frac{Gm_1m_2}{r^2}$$

Where: F is the force, measured in newtons (N)

G is the universal gravitational constant, which has a value of $6.67 \times 10^{-11}$ $Nm^2kg^{-2}$

$m_1$ and $m_2$ are the two masses measured in kilograms (kg)

r is the distance between their centres, measured in metres (m)

Important things to note:

1. The force is negative. This is because it is always attractive and attractive forces are always negative, by definition.

2. The force exerted by mass 1 on mass 2 is the same magnitude but opposite in direction to the force exerted by mass 2 on mass 1. This is a direct consequence of Newton's third law. In most cases that we study the **effect** of the force on the smaller mass is much greater than it is on the larger mass and we ignore the effects of the force on the larger mass.

*(c) use Newtons law of gravity and centripetal force to derive $r^3 \propto T^2$for a circular orbit*

Now that we know the size of the force acting on a body moving in circular motion due to the gravitational force acting on it, we can prove Kepler's third law:

For a body orbiting another body, the centripetal force is provided by the gravitational force.

$$F = -\frac{Gm_1m_2}{r^2} \quad (1)$$

As the body is moving in circular motion, the gravitational force causes the body to accelerate towards the centre of the circle, as in the diagram:

Thus we apply F=ma, with F being the gravitational force, as on the diagram, and the acceleration being equal to r$^2$.

$$F = ma$$

So

$$G\frac{m_1m_2}{r^2} = m_1r\omega^2$$

m$_1$ is the object that is orbiting, so it is the mass of m$_1$ that is feeling the acceleration and thus m$_1$ goes into the right hand side of the equation.

Thus we can cancel m$_1$ and collect the terms in r to give:

$$Gm_2 = r^3\omega^2 \quad (2)$$

But we know from our revision of circular motion that the period of the orbit is related to the angular velocity by:

$$\omega = \frac{2\pi}{T} \quad (3)$$

Therefore we substitute equation (3) into equation (2) to give:

$$Gm_2 = \frac{4\pi^2r^3}{T^2}$$

Now re-arrange to make r the subject of the formula:

$$r^3 = \frac{Gm_2}{4\pi^2}T^2 \quad (4)$$

G, m$_2$ and  are all constants, so we can finally write:

$$r^3 \propto T^2$$

You need to be able to do this for your examination, so make sure that you learn this proof.

*(d) understand energy transfer by analysis of the area under a gravitational force-distance graph*

Later you learned that in fact it was the area under the Force-distance graph.

*(Of course, it's a little bit more difficult than that. It is actually given by:*

$$Work\ done = \int_r^\infty F dx \qquad (8)$$

*You use this integral in other parts of the specification, but not this part.)*

Therefore if we want to know the gravitational potential energy gained or lost by an object in a gravitational field, we look at the area under the force-distance graph.

The zero of gravitational potential energy is taken as being at infinity, which makes sense. If the object isn't in a field, then it isn't experiencing any force, so it doesn't have any GPE.

This does mean, however, that all gravitational potential energies are negative as they lose GPE as they fall towards a mass.

For example, if you want to know the GPE gained/lost by an object as it moves from point A to point B in the field you look at the area under the force-distance graph:

*(Generally, we tend to look at the gravitational potential rather than the GPE, and use the field strength-distance graph, but the specification asks for GPE and a force-distance graph, so that is what we are looking at!)*

*(e) derive and use $g = -\frac{Gm}{r^2}$ for the magnitude of the gravitational field strength due to a point mass*

If you look back to chapter 2 on gravitational fields, you will know the definition of the gravitational field strength. It is given by the equation:

$$g = \frac{F}{m} \quad (5)$$

But we now know how the force is provided by a spherical mass from Newton's Law of Gravitation. It is given by equation (1) as:

$$F = \frac{Gm_1m_2}{r^2} \quad (6)$$

So we can substitute equation (6) into equation (5) to give us:

$$g = \frac{\text{Gm}}{r^2} \quad (7)$$

This is the gravitational field strength due to a spherical mass m at a distance r from its centre and you need to be able to derive this equation.

A graph of the field strength due to a spherical body against distance looks like this:

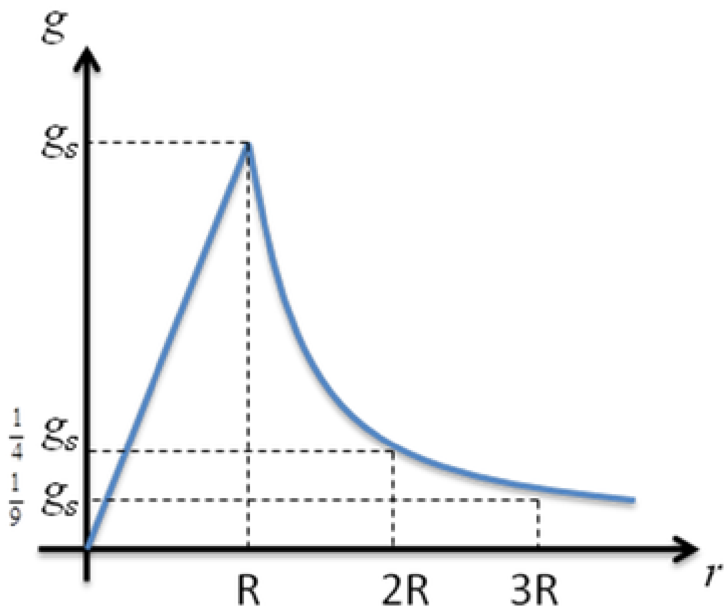



Figure 13.3: Field strength due to a sphere

From the centre of the object up to its radius (0 to R) the variation with field strength inside the body varies linearly **if the density is constant.** R is the radius of the body, so therefore the distance from the centre. After R it drops off as $1/r^2$.

*(f) recall similarities and differences between electric and gravitational fields*

| | Gravitational Fields | Electric Fields |
|---|---|---|
| Similarities | Both obey an inverse square law for the field at a distance r from a point mass/charge. | |
| | Both have a potential that drops off as 1/r from a point mass/charge. | |
| | Field strength is defined as the force per unit mass/charge | |
| | | |
| Differences | Only radial. All uniform gravitational fields are approximations. | Can have both radial and uniform fields |
| | Only ever attractive | Can be both attractive and repulsive |
| | Very weak | Very strong |

Figure 13.4: Similarities and Differences

*(g) recognise and use the equation for gravitational potential energy for point masses $E = -\frac{Gm_1m_2}{r^2}$*

There is an equation for the GPE which you need to be able to use. It is found from integrating the expression for the force as mentioned earlier, and it is given by:

$$E = -\frac{Gm_1m_2}{r} \qquad (9)$$

*(h) calculate escape velocity using the ideas of gravitational potential energy (or area under a force-distance graph) and energy transfer*

We can now use these methods of working out the GPE gained by an object in a gravitational field to calculate a quantity called the **escape velocity.**

NB. A lot of people get escape velocity wrong. It is the velocity needed to be given to an object **on the surface of the planet** in order for it to escape the gravitational field of the planet and have zero KE at that time. Once it has been given this velocity (by, for example, a cannon) **no more energy is put into the system**. From that moment on it is a projectile and is constantly losing KE as it gains GPE.

Of course, this means that when it finally escapes the gravitational field it has no energy at all, which also means that it has zero overall energy to start with as well!

Therefore we use the law of conservation of energy to work out what the escape velocity must be:

Total energy before = Total energy after = 0

Therefore Initial KE + Initial GPE = 0

$$\frac{1}{2}mv_e^2 - \frac{\text{GMm}}{R} = 0 \qquad (10)$$

Where M is the mass of the planet in kg

R is the radius of the planet in m

m is the mass of the projectile in kg

$v_e$ is the escape velocity in $ms^{-1}$

You can cancel the mass of the projectile and re-arrange for $v_e$ from equation (10) to give:

$$v_e = \sqrt{\frac{2GM}{R}} \qquad (11)$$

This is the escape velocity and you can work it out for the Earth. You should get about 12 $kms^{-1}$.

There is a graphical way of looking at this as well. The GPE gained by the body as it moves from the surface of the planet, radius R, is given by the area under the force-distance graph from the surface of the planet to infinity. If it is launched from the surface of the planet then that also equals its initial KE.

*(i) calculate the distance from the centre of the Earth and the height above its surface required for a geostationary orbit.*

A geostationary orbit is one which stays above the same position on the Earth's equator at all times.

This means, therefore, that it has a period of 24 hours.

It is therefore possible to calculate, using equation (4), r for a geostationary orbit.

N.B. A very common mistake is to say that r is the height of the orbit. This isn't the case – it is the radius of the orbit, so it is the distance of the satellite from the **centre** of the Earth, not the surface of the Earth.

You should make sure that you can do this. Have a go at working out r, given the following data:

$G = 6.67 \times 10^{-11}$ $Nm^2kg^{-2}$

$M_E = 5.98 \times 10^{24}$ kg

and remembering that T = 24 hours (but don't forget to convert to seconds!)

You should have got an answer of $4.23 \times 10^7$ m.

If I now tell you that the radius of the Earth is $6.36 \times 10^6$ m, you can also write down the height of the satellite above the surface of the Earth, and this comes to $3.6 \times 10^6$ m.

So a geostationary satellite orbits at a height that is about 6 times greater than the radius of the Earth.

# 14 Electromagnetism

# 15 Special Relativity

# 16 Molecular Kinetic Theory

# 17 Nuclear Physics

# 18 The Quantum Atom

# 19 Interpreting Quantum Theory

# 20 Astronomy and cosmology