## **library** libraryBag

```
from Basic/Numbers get Int
spec Bag0[sort Elem] given Int =
     generated type Bag[Elem] ::= bagVide | ajouter(Bag[Elem]; Elem)
            frequence: Bag[Elem] * Elem -> Int
     forall x, y : Elem; M, N : Bag[Elem]
     . frequence(bagVide, y) = 0
     . frequence(ajouter(M, x), y)
       = 1 + frequence(M, y) when x = y else frequence(M, y)
     . M = N
        <=> forall x : Elem . frequence(M, x) = frequence(N, x)
end
spec Bag[sort Elem] given Int =
     Bag0[sort Elem]
then preds estVide : Bag[Elem];
            appartient : Elem * Bag[Elem];
            inclut : Bag[Elem] * Bag[Elem]
            enlever : Bag[Elem] * Elem -> Bag[Elem]
     forall x, y : Elem; M, N : Bag[Elem]
     . estVide(M) <=> M = bagVide
     . appartient(x, M) \ll frequence(M, x) > 0
     . inclut(M, N)
       <=> forall x : Elem . frequence(M, x) <= frequence(N, x)
     . enlever(M, x) = N
       <=> forall y : Elem
                   . (frequence(N, y) = frequence(M, x) - 1 if x = y)
                     \ (frequence(N, y) = frequence(M, y) if not x = y)
end
```