CSC343 A4 Part2

Q1

(a)

Closure of All FDs:

$L^+ = NQLSOPRM$

$MNR^+ = OMNR$

$O^+ = MO$

$NQ^+ = LSNQOPRM$

$S^+ = SOPRM$

FDs that violates BCNF:

$MNR^+ = OMNR$

$O^+ = OM$

$S^+ = SOPRM$

(b)

Decompose R using FD $MNR \rightarrow O$. $MNR^+ = OMNR$, so this yields two relations: R1 = LSQPMNR and R2 = OMNR.

Project the FDs onto R1 = LSQPMNR.

L	S	Q	P	M	N	R	closure	FDs
✓							$L^+ = LNQSPROM$ (since L is a superkey then we don't need to consider any combination that contains L)	$L \rightarrow NQ$ (L is superkey of R1)
	✓						$S^+ = SPRM$	$S \rightarrow MOPR$ (violate BCNF)
		✓					$Q^+ = Q$	nothing
			✓				$P^+ = P$	nothing
				✓			$M^+ = M$	nothing
					✓		$N^+ = N$	nothing

						✓	$R^+ = R$	nothing
		✓			✓		$NQ^+ = LNQSPROM$	NQ→LS (NQ is superkey of R1)

We must decompose R1 further.

Decompose R1 using FD S→OPR. This yields two relations: R3 = LQNS and R4 = SPRM.

Project the FDs onto R3 = LMNS

L	S	Q	N	closure	FDs
✓				$L^+ = LNQSPROM$ (since L is a superkey then we don't need to consider any combination that contains L)	L→NQ (L is superkey of R3)
	✓			$S^+ = S$	nothing
		✓		$Q^+ = M$	nothing
			✓	$N^+ = N$	nothing
		✓	✓	$NQ^+= LNQSPROM$	NQ→LS (NQ is superkey of R3)

This relation satisfies BCNF.

Project the FDs onto R4 = SPRM

S	P	R	M	closure	FDs
✓				$S^+ = SRPM$ (since S is a superkey then we don't need to consider any combination that contains S)	S→ORP (S is superkey of R4)
	✓			$P^+ = P$	nothing
		✓		$R^+ = R$	nothing
			✓	$M^+= M$	nothing

This relation satisfies BCNF.

Return to R2 = OMNR and project the FDs onto it.

O	M	N	R	closure	FDs
✓				$O^+ = OM$	O→M (violate BCNF)
	✓			$M^+ = M$	nothing
		✓		$N^+ = N$	nothing
			✓	$R^+ = R$	nothing
	✓	✓	✓	$MNR^+ \rightarrow OMNR$	MNR→O (MNR is superkey of R2)

We must decompose R2 further.

Decompose R2 using FD O→M. This yields two relations: R5 = OM and R6 = ONR.

Project the FDs onto R5 = OM

O	M	closure	FDs
✓		$O^+ = OM$ (since L is a superkey then we don't need to consider any combination that contains L)	O→M (O is superkey of R5)
	✓	$M^+ = M$	nothing

This relation satisfies BCNF.

Project the FDs onto R6 = ONR

O	N	R	closure	FDs
✓			$O^+ = O$	nothing
	✓		$N^+ = N$	nothing
		✓	$R^+ = R$	nothing
✓	✓		$ON^+ = ON$	nothing
	✓	✓	$NR^+ = NR$	nothing

✓		✓	$OR^+ = OR$	nothing
✓	✓	✓	$ONR^+ = ONR$	nothing

This relation satisfies BCNF.

Final decomposition:
(a) R3 = LNQS with FD L→NQ, NQ→LS,
(b) R4 = MPRS with FD S→ORP,
(c) R5 = MO with FD O→M,
(d) R6 = NOR with no FDs.

Q2
(a)
Step 1: split RHS

$AB \to C$
$C \to A$
$C \to B$
$C \to D$
$CFD \to E$
$E \to B$
$BF \to E$
$BF \to C$
$B \to D$
$B \to A$

Step2: reduce the LHS

1) $A^+ = A,\ B^+ = BDAC.$ So we can reduce this FD to $B \to C$
2) Keep the same. Because there is only one attribute on LHS
3) Same as 2)
4) Same as 2)
5) $C^+ = CABD,\ F^+ = F,\ D^+ = D,\ CF^+ = CFABDE$. So we can reduce this FD to $CF \to E$
6) Same as 2)
7) $B^+ = BDAC,\ F^+ = F$. No simplification
8) $B^+ = BDAC$. So we can reduce this FD to $B \to C$
9) Same as 2)
10) Same as 2)

Our new set of FDs, let's call it S_2, is

1) $B \rightarrow C$
2) $C \rightarrow A$
3) $C \rightarrow B$
4) $C \rightarrow D$
5) $CF \rightarrow E$
6) $E \rightarrow B$
7) $BF \rightarrow E$
8) $B \rightarrow C$
9) $B \rightarrow D$
10) $B \rightarrow A$

Step3: try to eliminate each FD

1) $B^+_{s_2-(1)} = BCDA,\ we\ can\ remove\ this$
2) $C^+_{s_2-(1)-(2)} = CBDA,\ we\ can\ remove\ this$
3) $C^+_{s_2-(1)-(2)-(3)} = CD,\ keep\ this$
4) $C^+_{s_2-(1)-(2)-(4)} = CBDA,\ we\ can\ remove\ this$
5) $CF^+_{s_2-(1)-(2)-(4)-(5)} = CFBEDA,\ we\ can\ remove\ this$
6) $E^+_{s_2-(1)-(2)-(4)-(5)-(6)} = E,\ keep\ this$
7) $BF^+_{s_2-(1)-(2)-(4)-(5)-(7)} = BFCDA,\ keep\ this$
8) $B^+_{s_2-(1)-(2)-(4)-(5)-(8)} = BDA,\ keep\ this$
9) $B^+_{s_2-(1)-(2)-(4)-(5)-(9)} = BCA,\ keep\ this$
10) $B^+_{s_2-(1)-(2)-(4)-(5)-(10)} = BCD,\ keep\ this$

Our final set of FDs are:

1) $B \rightarrow ACD$
2) $BF \rightarrow E$
3) $C \rightarrow B$
4) $E \rightarrow B$

(b)

Given the minimal basis as above, we have the following table:

Attribute	LHS	RHS	Conclusion
G,H	X	X	Must be in every

			key
F	✓	X	Must be in every key
A,D	X	✓	Is not in any key
YB,C,E	✓	✓	Check

$GHFB^+ = GHFBACDE$, so BFGH is a key
$GHFC^+ = GHFCBADE$, so CFGH is a key
$GHFE^+ = GHFEBACD$, so EFGH is a key
All other possibilities include GHFB, GHFC or GHFE, so we are done.

(c)
Given the minimal basis in part(a)
For each FD $X \to Y$ in minimal basis, define a new relation with schema $X \cup Y$:
$BACD,\ BFE,\ CB,\ EB$. And we can eliminate CB, EB, because they are included in BACD, BFE.
Add a relation whose schema is some key, because no relation is a superkey of relation P:
We choose BFGH

Finally, the 3NF decomposition of relation P is ABCD, BEF, BFGH

(d)
BCNF check:
$E \to B$ holds in BEF
But $E^+ = EBACD$, given the FD sets of relation P. So E is not a superkey of BEF.
This violates BCNF.
So our schema allows redundancy.