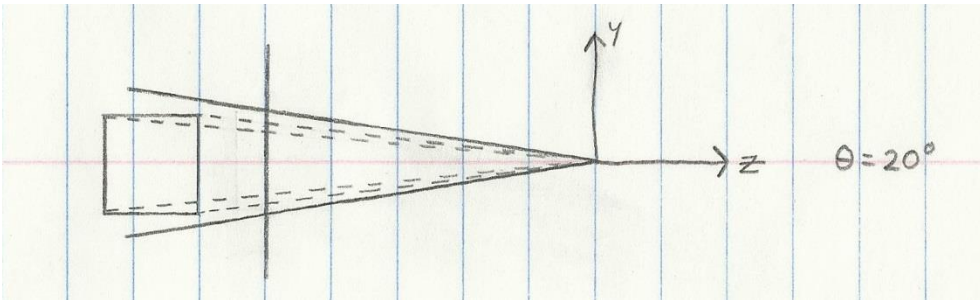
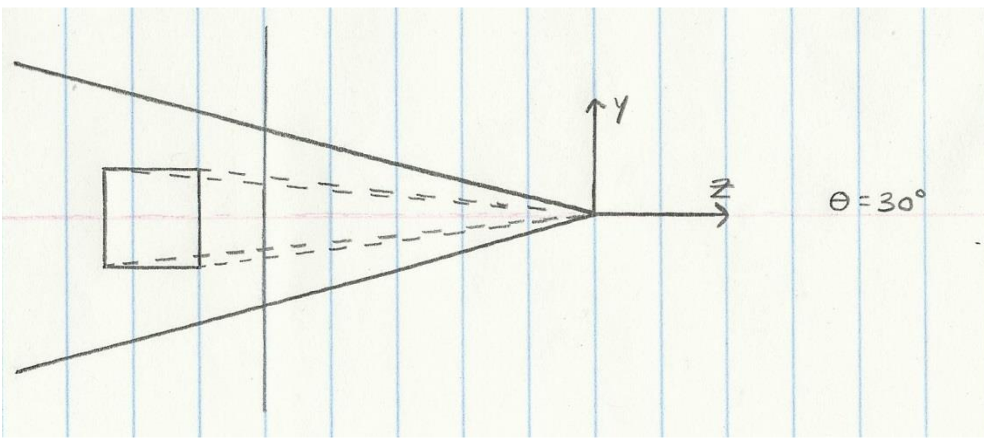
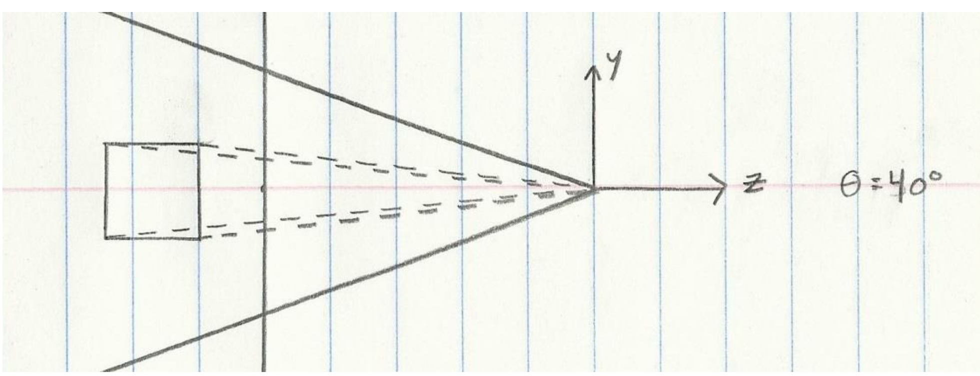
1.

All camera parameters except for the field of view are held constant. This means that the cube is the same distance away from the camera.







As seen from the three drawings (using the view that avoids having to flip the image), the margin between the front face and the rear face remains constant in the diagram. However, when the final image is actually produced, the margin is different because the fields of view determine the cube's size. In the 40° diagram, the cube will take up relatively less space in the final image. This is consistent with A, B, and C. Its margin between the front and rear face will also appear smaller because the cube itself is smaller. This rules out C because in C, as the field of view narrows, the margins also narrow, when they should actually get wider. To decide between A and B, we can look at the size of the rear face. In A, the rear face's size remains fixed as the field of view changes. This is incorrect, as the increase in the non-cube area in the image also will decrease the rear face's size. This leaves us with B, which is correct because all cube components decrease in size as the field of view increases. Another indicator is that the margin between the rear and front face gradually decreases as the field of view increases.

2.

Ex 11.3

$$P = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix} \qquad S = \begin{bmatrix} 3 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix}$$

$$PS = \begin{bmatrix} 3 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & -3 & 0 \end{bmatrix}$$

$$P \begin{bmatrix} x_e \\ y_e \\ z_e \\ 1 \end{bmatrix} = \begin{bmatrix} x_e \\ y_e \\ 1 \\ -z_e \end{bmatrix} \qquad PS \begin{bmatrix} x_e \\ y_e \\ z_e \\ 1 \end{bmatrix} = \begin{bmatrix} 3x_e \\ 3y_e \\ 3 \\ -3z_e \end{bmatrix}$$

$$x_n = \frac{x_e}{-z_e} \quad y_n = \frac{y_e}{-z_e} \qquad x_n = \frac{3x_e}{-3z_e} = \frac{x_e}{-z_e} \quad y_n = \frac{3y_e}{-3z_e} = \frac{y_e}{-z_e}$$

$$z_n = \frac{1}{-z_e} \qquad z_n = \frac{3}{-3z_e} = \frac{1}{-z_e}$$

There is no effect. PS does not change the resulting $[x_n, y_n, z_n]$ coordinates that affect what is drawn

3.

Ex

11.2

$$P = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix} \qquad PQ = \begin{bmatrix} 3 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -3 & 0 \end{bmatrix}$$

$$P \begin{bmatrix} x_e \\ y_e \\ z_e \\ 1 \end{bmatrix} = \begin{bmatrix} x_e \\ y_e \\ 1 \\ -z_e \end{bmatrix} \qquad PQ \begin{bmatrix} x_e \\ y_e \\ z_e \\ 1 \end{bmatrix} = \begin{bmatrix} 3x_e \\ 3y_e \\ 1 \\ -3z_e \end{bmatrix}$$

$x_n = x_e/-z_e$, $y_n = y_e/-z_e$ $\qquad$ $x_n = x_e/-z_e$, $y_n = y_e/-z_e$

$z_n = 1/-z_e$ $\qquad$ $z_n = 1/-3z_e$

This changes the calculation of z_n, which may be important when doing visibility calculations that rely on a concept of relative closeness. Things mapped with PQ are "closer" because there's a division by 3.

This would affect clipping in that something with $z_e <= 1/3$ wouldn't be mapped with P but would with PQ ($z_n = \frac{1}{-1/3} = -3$ vs. $z_n = \frac{1}{-3 \cdot \frac{1}{3}} = -1$)

4.

Ex 11.4

$$P = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix} \qquad QP = \begin{bmatrix} 3 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

$$P \begin{bmatrix} x_e \\ y_e \\ z_e \\ 1 \end{bmatrix} = \begin{bmatrix} x_e \\ y_e \\ 1 \\ -z_e \end{bmatrix} \qquad QP \begin{bmatrix} x_e \\ y_e \\ z_e \\ 1 \end{bmatrix} = \begin{bmatrix} 3x_e \\ 3y_e \\ 3 \\ -z_e \end{bmatrix}$$

$$x_n = x_e / -z_e, \; y_n = y_e / -z_e \qquad x_n = \frac{3x_e}{-z_e} \quad y_n = \frac{3y_e}{-z_e}$$

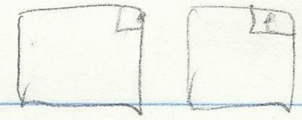
$$z_n = 1 / -z_e \qquad z_n = \frac{3}{-z_e}$$

Using QP causes the image to look zoomed in by a factor of 3. In the z direction, there is also ~~t~~less flexibility in what is included in the image. (e.g. $z_e = 1$ gives $1/-1 = -1$ vs. $3/-1 = -3$)

not included in clipping stage

5.

$$z_n = \frac{1}{-z_e} \qquad z_n = \frac{1}{-3z_e}$$



Ex:

12.3

$$V = \begin{bmatrix} W/2 & 0 & 0 & (W-1)/2 \\ 0 & H/2 & 0 & (H-1)/2 \\ 0 & 0 & 1/2 & 1/2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad Q = \begin{bmatrix} 3 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} W/2 & 0 & 0 & (W-1)/2 \\ 0 & H/2 & 0 & (H-1)/2 \\ 0 & 0 & 1/2 & 1/2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3\frac{W}{2} & 0 & 0 & 3\frac{(W-1)}{2} \\ 0 & 3\frac{H}{2} & 0 & 3(H-1)/2 \\ 0 & 0 & 1/2 & 1/2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

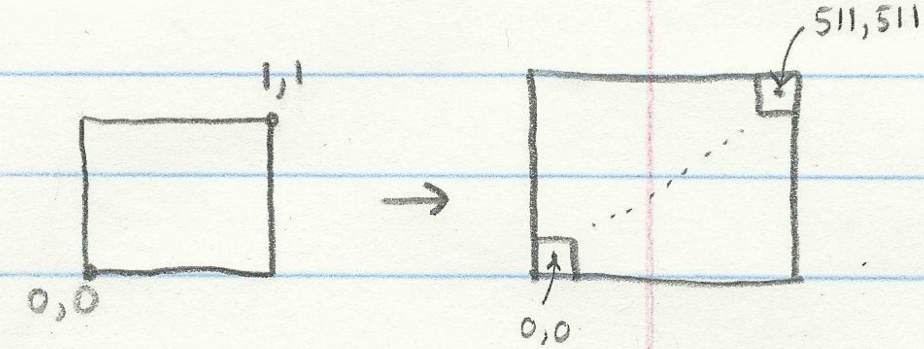
$$V\begin{bmatrix} x_n \\ y_n \\ z_n \\ 1 \end{bmatrix} = \begin{bmatrix} x_n \cdot \frac{W}{2} + x_n \cdot \frac{W-1}{2} \\ y_n \cdot \frac{H}{2} + y_n \cdot \frac{H-1}{2} \\ (1/2)z_n + 1/2 \\ 1 \end{bmatrix} = \begin{bmatrix} x_n \cdot W - .5x_n \\ y_n \cdot H - .5y_n \\ .5(z_n+1) \\ 1 \end{bmatrix}$$

$$QV\begin{bmatrix} x_n \\ y_n \\ z_n \\ 1 \end{bmatrix} = \begin{bmatrix} x_n \cdot \frac{3W}{2} + x_n \cdot \frac{3(W-1)}{2} \\ y_n \cdot \frac{3H}{2} + y_n \cdot \frac{3(H-1)}{2} \\ (1/2)z_n + 1/2 \\ 1 \end{bmatrix} = \begin{bmatrix} 3x_n \cdot W - 1.5x_n \\ 3y_n \cdot H - 1.5y_n \\ .5(z_n+1) \\ 1 \end{bmatrix}$$

The z_w is the same. The x_w and y_w coordinates are scaled by 3 now.

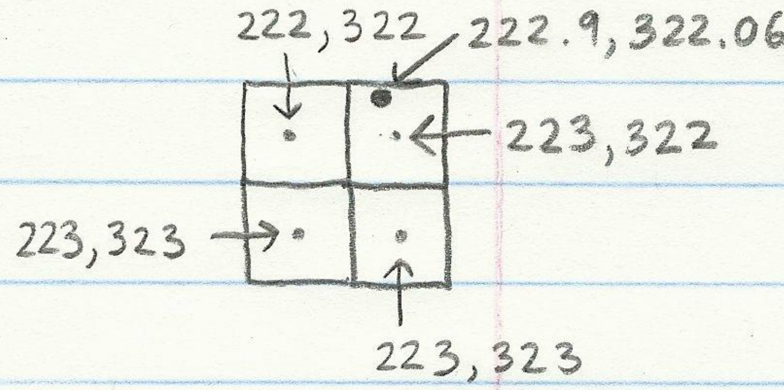
In 11.4, scaling is done before applying the viewport matrix; in this one, scaling is done afterwards. The one ~~in~~ 11.4 scales from the center out, whereas this one scales from the bottom left corner, as the coordinate system has 0,0 at the bottom left.

6)



$$\begin{bmatrix}.45\\.63\\-\\1\end{bmatrix}$$

$$\begin{bmatrix}512&0&0&-.5\\0&512&0&-.5\\-&-&-&-\\0&0&0&1\end{bmatrix}\begin{bmatrix}.45\cdot512-.5\\.63\cdot512-.5\\-\\1\end{bmatrix}=\begin{bmatrix}222.9\\322.06\\-\\1\end{bmatrix}$$



The point is closest to 223,322

7)

$$v = \begin{bmatrix} a & b & c \end{bmatrix} \begin{bmatrix} x_n \\ y_n \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x_n w_n \\ y_n w_n \\ w_n \end{bmatrix} = p' \begin{bmatrix} x_e \\ y_e \\ z_e \\ 1 \end{bmatrix}$$

$$w_n \begin{bmatrix} x_n \\ y_n \\ 1 \end{bmatrix} = p' \begin{bmatrix} x_e \\ y_e \\ z_e \\ 1 \end{bmatrix}$$

factor out w_n

$$w_n \begin{bmatrix} v \\ 1 \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} \begin{bmatrix} x_n \\ y_n \\ 1 \end{bmatrix} \cdot w_n$$

Include the function that is affine w.r.t. NDCs

$$\begin{bmatrix} v \cdot w_n \\ w_n \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} p' \begin{bmatrix} x_e \\ y_e \\ z_e \\ 1 \end{bmatrix}$$

substitute in for $w_n \begin{bmatrix} x_n \\ y_n \\ 1 \end{bmatrix}$

To get v, we now divide $\frac{v \cdot w_n}{w_n}$ and can interpolate w.r.t. the eye coordinates x_e, y_e, z_e.