These are just two different interpretations of the same final composed transformations. (1) Translate with respect to $\vec{\mathbf{f}}^t$ then rotate with respect to the intermediate frame. (2) Rotate with respect to $\vec{\mathbf{f}}^t$ then translate with respect to the original frame $\vec{\mathbf{f}}^t$. At times, it will be more convenient to use the first interpretation, and at other times it may be more convenient to use the second one.

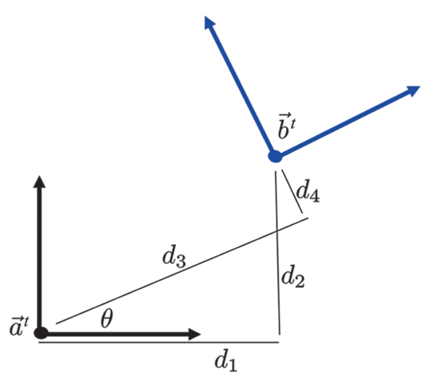
These types of interpretations are often summarized as follows: If we read transformations from left to right, then each transform is done with respect to a newly created "local" frame. If we read the transformations from right to left, then each transform is done with respect to the original "global" frame.

Exercises

4.1 Using the definitions of section 4.2, draw two different sketches illustrating the transformation: $\vec{\mathbf{f}}^t \Rightarrow \vec{\mathbf{f}}^t RT$ (compare with Figure 4.3).

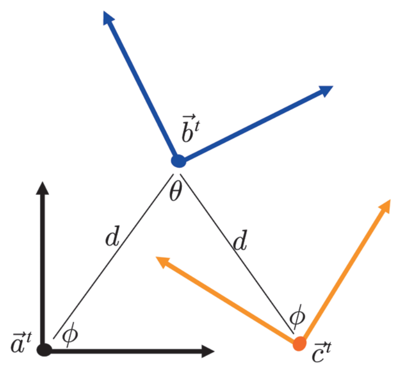
4.2 Suppose $\vec{\mathbf{f}}^t$ is an orthonormal frame, and we apply the transform $\vec{\mathbf{f}}^t \Rightarrow \vec{\mathbf{f}}^t ST$, where S is a matrix that applies a uniform scale by a factor of 2, and T translates by 1 along the x axis. How far does the frame's origin move, measured in the original units of $\vec{\mathbf{f}}^t$?

4.3 Given the following two orthonormal frames $\vec{\mathbf{a}}^t$ and $\vec{\mathbf{b}}^t$



with distances given by the positive quantities d_i. What are the matrices R and T such that $\vec{\mathbf{b}}^t = \vec{\mathbf{a}}^t TR$? What are the matrices R and T such that $\vec{\mathbf{b}}^t = \vec{\mathbf{a}}^t RT$? (Note: Do this without using trigonometric terms in the matrix T.)

4.4 Given the following three frames



Suppose that $\vec{\mathbf{b}}^t = \vec{\mathbf{a}}^t N$ and $\vec{\mathbf{c}}^t = \vec{\mathbf{a}}^t M$. Express the matrix M using only the symbols N and θ.