CITM

Mock Exam, December 15, 2015

MATVJII

Surnames and name: ______________________________

ID number: ______________________________

CITM
Final Exam, January 22, 2016
MATVJII

Exercise 1

If it is known that

$$\mathbf{R} = \begin{pmatrix} 0.7972 & -0.5875 & ? \\ 0.5722 & ? & ? \\ 0.1925 & 0.0293 & 0.9809 \end{pmatrix} \tag{1}$$

represents a valid rotation matrix.

0 Point 1. Find the entries marked as ?.

Exercise 2

Let the orientation in space of a 3D body with respect to a world frame, to be described by the matrix

$$\mathbf{R}_i = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{2}$$

After some time a new estimation of the orientation arises as:

$$\mathbf{R}_f = \begin{pmatrix} 0 & 0 & 1 \\ 0 & -1 & 0 \\ 1 & 0 & 0 \end{pmatrix}. \tag{3}$$

0 Point 1. Find the axis of rotation about which the initial object has to be rotated to achieve the second orientation.

0 Point 2. Find the angle rotated to achieve the second orientation.

To avoid jumps in the transition between both orientations is desired to interpolate the rotation matrix. Using the parameter u, which is proportional to the time, for which $\mathbf{R}(u = 0) = \mathbf{R}_0$ and $\mathbf{R}(u = 1) = \mathbf{R}_f$. Calculate:

0 Point 1. $\mathbf{R}(0.25)$.

0 Point 2. $\mathbf{R}(0.5)$.

0 Point 3. $\mathbf{R}(7/8)$.

The exam continues on next page ...

Exercise 3

Let the quaternion

$$\mathring{q}_1 = \begin{pmatrix} \cos(\alpha) & -\frac{1}{\sqrt{3}}\sin(\alpha) & \frac{2}{\sqrt{3}}\sin(\alpha) & 0 \end{pmatrix}^\intercal . \tag{4}$$

with $\alpha = 20\,\text{deg}$ to represent the attitude of a frame $\{B\}$ with respect to $\{A\}$. I.e. a vector defined in $\{B\}$ can be expressed in $\{A\}$ by performing the next operation

$$^{A}\mathring{v} = \mathring{q}_1 \, ^{B}\mathring{v} \, \bar{\mathring{q}}_1 \tag{5}$$

Following the same lines

$$\mathring{q}_2 = \begin{pmatrix} \cos(\gamma) & -\frac{3}{\sqrt{13}}\sin(\gamma) & 0 & \frac{2}{\sqrt{13}}\sin(\gamma) \end{pmatrix}^\intercal . \tag{6}$$

with $\gamma = -35\,\text{deg}$ allow to pass from base $\{C\}$ to $\{A\}$,

$$^{A}\mathring{v} = \mathring{q}_2 \, ^{C}\mathring{v} \, \bar{\mathring{q}}_2 \tag{7}$$

0 Point 1. Find the quaternion $\mathring{q}_3$ that allows to transform a vector defined in $\{C\}$ to the frame $\{B\}$.

0 Point 2. Find the quaternion $\mathring{q}_4$ that allows to transform a vector defined in $\{B\}$ to the frame $\{C\}$.

Exercise 4

A triangle is represented by three points in space. Of this 3 points, two of them lets say

$$\begin{aligned} ^{f1}\boldsymbol{p}_1 &= (0,\, -0.5977,\, 1.2817)^\intercal \text{ m} \\ ^{f1}\boldsymbol{p}_2 &= (-1,\, 1.0261,\, 2.8191)^\intercal \text{ m} \end{aligned} \tag{8}$$

are known in frame $\{F_1\}$. The coordinates of the third point are known in the frame $\{F_2\}$ as

$$^{f2}\boldsymbol{p}_3 = (0,\, -0.2724,\, -1.7821)^\intercal \text{ m} \tag{9}$$

Knowing that:

- The origin of $\{F_1\}$ is at coordinates

$$^{w}\boldsymbol{o}_{f1} = (0,\, 4,\, 1)^\intercal \text{ m} \tag{10}$$

  with respect a world frame and that its orientation is achieved by rotating the world frame 30 deg about the world x axis.

- The origin of $\{F_2\}$ is at coordinates

$$^{w}\boldsymbol{o}_{f2} = (1,\ 7,\ 4)^{\intercal}\ \mathrm{m} \tag{11}$$

  with respect the world frame and the orientation of $\{F_2\}$ is achieved by rotating the world frame $-25\,\mathrm{deg}$ about the world x axis.

If a camera with focal length $f = 1/55\,\mathrm{m}$, with origin at

$$^{w}\boldsymbol{o}_{c} = (6,\ 3,\ 0)^{\intercal}\ \mathrm{m}, \tag{12}$$

and which orientation is achieved by consecutively apply the next rotations to the world reference system

1. A rotation defined by the euler angles $(\psi = \pi/2, \theta = 0, \phi = -pi/2)\,\mathrm{rad}$.
2. Followed by a rotation of $-\pi/20\,\mathrm{rad}$ about the z axis
3. Followed by a rotation of $0.3\,\mathrm{rad}$ about the y axis.

Calculate

$\boxed{\text{0 Point}}$ 1. The transformation needed to go from frames $\{F_1\}$ and $\{F_2\}$ to the world frame.

$\boxed{\text{0 Point}}$ 2. The transformation needed to go from world frame to the camera frame.

$\boxed{\text{0 Point}}$ 3. The position of points $\boldsymbol{p}_1$, $\boldsymbol{p}_2$ and $\boldsymbol{p}_3$ projected on the camera plane.