CITM

Midterm Exam, April 22nd 2016

MATVJII

Surnames and name: ______________________________

ID number: ______________________________

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Exercise 1

Given the matrices

$$\mathbf{A} = \begin{pmatrix} 1 & -2 & 1 \\ 2 & 7 & 4 \\ 3 & 1 & -1 \end{pmatrix} \quad \mathbf{B} = \begin{pmatrix} -1 & -1 & -1 \\ -4 & -1 & 2 \\ 8 & 4 & 5 \end{pmatrix} \quad \mathbf{C} = \begin{pmatrix} 2 & 1 & 1 \\ 4 & -1 & 1 \\ -3 & 2 & 1 \end{pmatrix}$$

1. Expand the next equation and find the value of the matrix $\mathbf{X}$

$$\mathbf{A} + (\mathbf{CB})^{\intercal} = \mathbf{B}^{\intercal}\mathbf{X}$$

Exercise 2

Describe the solution of the linear system $\mathbf{A}\boldsymbol{x} = \boldsymbol{b}$ in terms of the parameter λ, when

$$\mathbf{A} = \begin{bmatrix} 1 & -1 & 4 & 2 \\ 1 & 1 & 6-\lambda & 4 \\ 0 & \frac{\lambda}{2} & -2 & -1 \\ -1 & 3 & -\lambda-2 & 0 \end{bmatrix}$$

1. Discuss the dimension of the sub/space spanned for the columns vectors of $\mathbf{A}$
2. If there exist any value of λ for which the dimension of the column space of $\mathbf{A}$ is less than 4, it is still possible to solve the system? Give at least 1 example for every case of λ.

Exercise 3

We want to model the temperature t of a house in function of the next measurable parameters: h, the heat sources power; H the humidity; r the sun radiation; and w the quantity of re-circulation air coming form outside. The linear function that we are expecting to obtain is

$$t = ah + bH + cr + dw$$

We have measure those quantities on some experiments and the results are provided in the array below

1. Calculate the value of the model parameters
2. Which temperature do you expect for $a = 5$, $b = 5$, $c = 5$ and $d = 1$?

 The exam continues on next page ...

	t	w	r	H	h
exp 1	38	0	20	40	10
exp 2	20	2	30	20	0
exp 3	11	2	20	10	0
exp 4	0	4	10	0	0

Exercise 4

Two different Basis are

$$\mathfrak{B}_0 = \left\{ \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ -2 \\ -1 \end{pmatrix} \right\}$$

$$\mathfrak{B}_1 = \left\{ \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 2 \\ 2 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} \right\}$$

1. Demonstrate that the given basis are basis of $\mathbf{R}^3$.
2. Find the components of the vector $\boldsymbol{u}$ defined in the basis $\mathfrak{B}_0$ as

   $$\boldsymbol{u}_{\mathfrak{B}_0} = (1, -1, 2)^{\mathsf{T}}$$

   in the basis $\mathfrak{B}_1$
3. Find the components of the vector $\boldsymbol{u}$ defined in the basis $\mathfrak{B}_1$ as

   $$\boldsymbol{u}_{\mathfrak{B}_1} = (2, -2, 1)^{\mathsf{T}}$$

   in the basis $\mathfrak{B}_0$

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