Repaso tiempo de ejecución

miércoles, 21 de junio de 2023 15:29

7.

```
for(int i = 0; i< n; i++)
    sum++;
```

$T(n) = \sum_{i=0}^{n-1} c \rightarrow (n-1+1)c = n \cdot c \rightarrow O(n)$

```
for(int i = 0; i< n; i+=2)
    sum++;
```

$T(n) = \sum_{i=0}^{(n-1)/2} c \rightarrow \left(\frac{n-1}{2}+1\right) \cdot c = \left(\frac{n-1+2}{2}\right) \cdot c = \left(\frac{n+1}{2}\right) \cdot c \rightarrow O(n)$

```
for(int i = 0; i< n; i++)
    for(int j = 0; j< n; j++)
        sum++;
```

$\sum_{i=0}^{n-1} \sum_{j=0}^{n-1} c$

$\sum_{i=0}^{n-1} (n-\cancel{1+1}) \cdot c$

$(n-\cancel{1+1}) \cdot (nc)$

$n^2 \cdot c \rightarrow O(n^2)$

```
for(int i = 0; i< n; i++)
    for(int j = 0; j< n*n; j++)
        sum++;
```

$\sum_{i=0}^{n-1} \sum_{j=0}^{n^2-1} c$

$\sum_{i=0}^{n-1} (n^2-\cancel{1+1}) \cdot c$

$(n-\cancel{1+1})^3 c$

$n^3 \cdot c \rightarrow O(n^3)$

```
for(int i = 0; i< n; i++)
    for(int j = 0; j< n; j++)

        sum++;

for(int i = 0; i< n; i++)

    sum++
```

$\sum_{i=0}^{n-1} \sum_{j=0}^{n-1} c + \sum_{i=0}^{n-1} c$

$n-1$

$$\sum_{i=0}^{n} (n-1+1)\cdot c + (n-1+1)\cdot c$$

$$(n-1+1)\cdot nc + nc$$

$$n^2 c + nc \Rightarrow \text{orden}(n^2)$$

```
for(int i = 0; i< n/2; i++)
    for(int j = 0; j< n/2; j++)
        sum++;
```

$$\sum_{i=0}^{\frac{n}{2}-1} \sum_{j=0}^{\frac{n}{2}-1} c$$

$$\sum_{i=0}^{\frac{n}{2}-1} \left(\frac{n}{2}-1+1\right) c$$

$$\sum_{i=0}^{\frac{n}{2}-1} \frac{n}{2} c$$

$$\left(\frac{n}{2} - \cancel{1+1}\right)\frac{n}{2} c$$

$$\left[\frac{n}{2}\right]^2 c$$

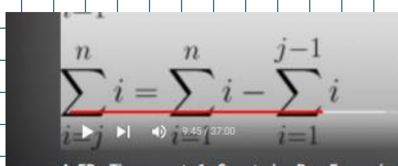
$$\frac{n^2}{4} c \Rightarrow O(n^2)$$

```
public static void ejercicio2 (int n) {

    int i, j;

    for (i = 1; i <= n; i++) {
        j = i;
        while (j <= Math.pow (n,3)){ // esto significa: while (j <= n^3)
                j++;
        }
        System.out.println ("El indice j es: " + j);
    }
}
```

c, c



$$\sum_{i=1}^{n} \left(c_1 + \sum_{j=i}^{n^3} c_2\right)$$

$$\sum_{i=1}^{n} \left(c_1 + \sum_{j=1}^{n^3} c_2 - \sum_{j=1}^{i-1} c_2\right)$$

$$\sum_{i=1}^{n} \left(c_1 + n^3 c_2 - (i-1)\cdot c_2\right)$$

$$\sum_{i=1}^{n} \left(c_1 + n^3 c_2 - i c_2 + c_2\right)$$

$$\sum^{n} c_1 + \sum^{n} n^3 c_2 - \sum^{n} i c_2 + \sum^{n} c_2$$

$\sum_{i=1}^{n} c_1 + \sum_{i=1}^{n} n^3 c_2 - \sum_{i=1}^{n} i c_2 + \sum_{i=1}^{n} c_2$

$n \cdot c_1 + n \cdot (n^3 c_2) - c_2 \sum_{i=1}^{n} i + n c_2$

$n c_1 + n^4 c_2 - c_2 \frac{n(n+1)}{2} + n c_2$

$n c_1 + n^4 c_2 - c_2 \left(\frac{n^2+n}{2}\right) + n c_2$

$n c_1 + n^4 c_2 - \frac{c_2 n^2 + c_2 n}{2} + n c_2 \to O(n^4)$

siguiente:

$T(n) = 5n + 3n^2 + 2n^2 \log_2(n) \le O(\boxed{n^2 \log_2(n)})$

$5n \le C \cdot n^2 \log_2 n$, $3n^2 \le c \cdot n^2 \log n$, $2n^2 \log_2(n)$

$5n \le c \cdot n^2 \log_2 n$

$C_1 = 5$

$15 \le 5 \cdot 9 \log_2 3$

$n_0 = 3$

$3n^2 \le c \cdot n^2 \log n^2$

$3n^2 \le C_2 \cdot n^2 \log n^2$

$C_2 = 3$

$3 \cdot 2^2 \le 3 \cdot 2^2 \log_2 2$

$n_0 = 2$

$12 \le 3 \cdot 4$

$2n^2 \log_2(n) \le c \cdot n^2 \log n^2$

$C_3 = 2$

$2 \cdot 2^2 \log_2(n) \le 2 \cdot 2^2 \log_2 2$

$n_0 = 2$

$5n + 3n^2 + 2n^2 \log_2(n) \le c_1 n^2 \log n + c_2 \cdot n^2 \log n + c_3 \cdot n^2 \log n$

$T(n) \le (c_1 + c_2 + c_3) \cdot n^2 \log n$

$T(n) \le 10\, n^2 \log n$

$T(n) \le c\, n^2 \log n$

$T(n) \le n^2 \log_2 n$, con $c = 10$ para todo $n \ge n_0$, $n_0 = 3$

1.

```
public static void uno (int n)  {
      int  i, j, k ;
      int [] [] a, b, c;
      a = new int [n] [n];
      b = new int [n] [n];
      c = new int [n] [n];
      for ( i=1; i<=n-1; i++) {
            for ( j=i+1; j<=n; j++) {
                  for ( k=1; k<=j; k++)  {
                        c[i][j] = c[i][j]+ a[i][j]*b[i][j];
                  }
            }
      }
}
```

C

$C_1 + \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \sum_{k=1}^{j} c_2$

$$C_1 + \sum_{i=1}^{n-1}\sum_{j=1+i}^{n}\sum_{k=1}^{j} c_2$$

$$C_1 + \sum_{i=1}^{n-1}\sum_{j=1+i}^{n} J\cdot c_2$$

$$C_1 + \sum_{i=1}^{n-1}\sum_{j=1}^{n} J\cdot c_2 - \sum_{j=1}^{i} J\cdot c_2$$

$$C_1 + \sum_{i=1}^{n-1}\left(c_2\,\frac{n(n+1)}{2} - c_2\,\frac{i\cdot(i+1)}{2}\right)$$

$$C_1 + \sum_{i=1}^{n-1} c_2\cdot\frac{n^2+n}{2} - c_2\cdot\frac{i^2+i}{2}$$

$$C_1 + \sum_{i=1}^{n-1}\left(c_2\,\frac{n^2+n}{2} - c_2\,\frac{i^2+i}{2}\right)$$

$$C_1 + \sum_{i=1}^{n-1} c_2\cdot\frac{n^2+n}{2} - c_2\sum_{i=1}^{n-1}\frac{i^2+i}{2}$$

$$C_1 + (n-1)\cdot\frac{c_2 n^2 + c_2 n}{2} -$$

$$C_1 + \frac{c_2 n^3 + \cancel{c_2 n^2} - \cancel{c_2 n^2} - c_2 n}{2} -$$

1.

```java
public static void uno (int n)  {
    int  i, j, k ;
    int [] [] a, b, c;
    a = new int [n] [n];
    b = new int [n] [n];
    c = new int [n] [n];
    for ( i=1; i<=n-1; i++) {
        for ( j=i+1; j<=n; j++) {
            for ( k=1; k<=j; k++)  {
                c[i][j] = c[i][j]+ a[i][j]*b[i][j];
            }
        }
    }
}
```

$$C_1 + \sum_{i=1}^{n-1}\sum_{j=I+1}^{n}\sum_{k=1}^{J} c_2$$

$$C_1 + \sum_{i=1}^{n-1}\sum_{j=I+1}^{n}\left(J c_2\right)$$

$$C_1 + \sum_{i=1}^{n-1}\sum_{j=1}^{n} J\cdot c_2 - \sum_{j=1}^{I} J c_2$$

$$C_1 + \sum_{i=1}^{n-1} c_2\sum_{j=1}^{n} J - c_2\,\frac{I\cdot(I+1)}{2}$$

$$C_1 + \sum_{i=1}^{n-1} c_2\,\frac{n\cdot(n+1)}{2} - \frac{c_2\cdot i^2 + i}{2}$$

$$C_1 + \sum_{i=1}^{n-1}\frac{c_2 n^2 + c_2 n}{2} - \frac{c_2 i^2 + c_2 i}{2}$$

$$C_1 + \sum^{n-1} c_2 n^2 + c_2 - \sum^{n-1} c_2 i^2 + c_2 i$$

$$c_1 + \sum_{i=1}^{n-1} c_2 n^2 + c_2 - \sum_{i=1}^{n-1} \frac{c_2 i^2 + c_2 i}{2}$$

$$c_1 + (n-1) \cdot c_2 n^2 + c_2 - \frac{c_2}{2} \sum_{i=1}^{n-1} i^2 + \frac{c_2}{2} \sum_{i=1}^{n-1} i$$

$$c_1 + c_2 n^3 + c_2 n - c_2 n^2 - c_2 - \frac{c_2}{2} \cdot \frac{(n-1) \cdot n \cdot (2(n-1)+1)}{6} + \frac{c_2}{2} \cdot \frac{(n-1) \cdot (n-1+1)}{2}$$

$$c_1 + c_2 n^3 + c_2 n - c_2 n^2 - c_2 - \frac{c_2}{2} \cdot \frac{(n^2-n) \cdot (2n-1)}{6} + \frac{c_2}{2} \cdot \frac{n^2-n}{2}$$

$$c_1 + c_2 n^3 + c_2 n - c_2 n^2 - c_2 - \frac{c_2}{2} \cdot \frac{2n^3 - n^2 - 2n + 1}{6} + \frac{c_2 n^2 - c_2 n}{4}$$

$$c_1 + c_2 n^3 + c_2 n - c_2 n^2 - c_2 - \frac{c_2 (2n^3 - n^2 - 2n + 1)}{12} + \frac{c_2 n^2 - c_2 n}{4} \rightarrow O(n^3)$$:)

```
int recursivo(int n){
if (n <= 1)
        return  1;
else
        return (recursivo (n-1));
}
```

$$T(n) = \begin{cases} 1, n \leq 1 \\ T(n-1) + c, n \geq 2 \end{cases}$$

$$T(n) \begin{cases} 1 & n \leq 1 \\ T(n-1) + c & n \geq 2 \end{cases}$$

paso 1 $= T(n) = T(n-1) + c$

paso 2 $= T(n-1) = T(n-2) + 2c$

paso 3 $= T(n-2) = T(n-3) + 3c$

paso 4 $= T(n-3) = T(n-4) + 4c$

paso i $= T(n) = T(n-i) + ci$

$T(n) = T(n-i) + ci$

$T(n) = T(n-(n-1)) + c(n-1)$

$n - i = 1$ $\quad$ $T(n) = T(1) + c(n-1)$

$n - 1 = i$

$T(n) = \cancel{1} + cn - \cancel{c}$

$T(n) = cn \rightarrow O(n)$

```
int recursivo(int n){
if (n = 1)
        return  1;
else
        return (recursivo (n/2));
}
```

$$T(n)=\begin{cases}1, n=1\\ T(n/2)+c, n\geq 2\end{cases}$$

$$T(n)=\begin{cases}1 & n=1\\ T(n/2)+c, & n\geq 2\end{cases}$$

Paso 1 $\rightarrow T(n)=T(n/2)+c$

$2 \rightarrow T(n/2)=T\left(\frac{n}{4}\right)+2c$

$3 \rightarrow T\left(\frac{n}{4}\right)=T\left(\frac{n}{8}\right)+3c$

$i \rightarrow T(n)=T\left(\frac{n}{2^i}\right)+ic$

$\frac{n}{2^i}=1$

$n=2^i$

$\log_2 n = i$

$T(n)=T\left(\frac{n}{2^{\log_2 n}}\right)+\log_2 n \cdot c$

$T(n)=T\left(\frac{n}{n}\right)+\log_2 n \cdot c$

$T(n)=T(1)+\log_2 n \cdot c$

$T(n)=1+\log_2 n \cdot c \rightarrow \log_2 n$

```
int recur (int n){
if (n = 1)
        return  1;
else
        return(recur(n/2)+recur(n/2));
```

$$T(n)\begin{cases}1 & n=1\\ 2T\left(\frac{n}{2}\right)+c & n\geq 2\end{cases}$$

Paso 1 $\rightarrow T(n)=2T\left(\frac{n}{2}\right)+c$

$2 \rightarrow T\left(\frac{n}{2}\right)=2\left[2T\left(\frac{n}{4}\right)+c\right]+c \rightarrow 4T\left(\frac{n}{4}\right)+2c+c$

$3 \rightarrow T\left(\frac{n}{4}\right)=2\left[4T\left(\frac{n}{8}\right)+2c+c\right]+c \rightarrow 8T\left(\frac{n}{8}\right)+4c+2c+c$

$i \rightarrow T(i)=2^i T\left(\frac{n}{2^i}\right)+\sum_{j=1}^{i} c \cdot j$

$\frac{n}{2^i}=1$ $\qquad 2^i T\left(\frac{n}{2^i}\right)+c \cdot \frac{i(i+1)}{2}$

$n \, T(1)+c \cdot \frac{i^2+i}{2}$

$n=2^i$

$\log_2 n = i$ $\qquad nc+c \cdot \frac{n+\log_2 n}{2} \rightarrow O(n)$

```
int recursivo(int n){
if (n <= 5)
        return  1;
else
        return (recursivo (n-5));
}
```

$$T(n)=\begin{cases}1 & n\leq 5\\ T(n-5)+c & n\geq 6\end{cases}$$

Paso 1 $\to T(n)=T(n-5)+c$

2 $\to T(n-5)=T(n-10)+2c$

3 $\to T(n-10)=T(n-15)+3c$

i $\to T(n-i)=T(n-5.i)+ic$

$T(n)=T\left(n-5.\left(\frac{n}{5}-1\right)\right)+\left(\frac{n}{5}-1\right).c$

$n-5i=5$

$T(n)=T(n-(n-1))+\frac{n}{5}c-c$

$n-5=5i$

$T(n)=T(1)+\frac{n}{5}c-c$

$\frac{n-5}{5}=i$

$T(n)=c+\frac{n}{5}c-c\to O(n)$

$\frac{n}{5}-1=i$

```
int recur (int n){
if (n = 1)
        return  1;
else
        return(recur(n-1)+recur(n-1));
}
```

$$T(n)=\begin{cases}1, n=1\\ 2T(n-1)+c, n\geq 2\end{cases}$$

Paso 1 $\to T(n)=2T(n-1)+c$

2 $\to T(n-1)=2\left[2T(n-2)+c\right]+c\to 4T(n-2)+2c+c$

3 $\to T(n-2)=2\left[4T(n-3)+2c+c\right]+c\to 8T(n-3)+\boxed{4c+2c+c}$

4 $\to T(n-3)=2\left[8T(n-4)+4c+2c+c\right]+c\to 16T(n-4)+8c+4c+2c+c$

i $\to T(i)=2^iT(n-i)+\sum$

$$\sum_{i=0}^{3-1}2^ic$$

$c+2c+4c$

Paso 1 $\to T(n)=T(n-1)+n$

2 $\to T(n-1)=T(n-2)+n-1+n$

3 $\to T(n-2)=T(n-3)+n-2+n-1+n$

$2 \rightarrow T(n-1) = T(n-2) + n-1 + n$

$3 \rightarrow T(n-2) = T(n-3) + n-2 + n-1 + n$

$4 \rightarrow T(n-3) = T(n-4) + n-3 + n-2 + n-1 + n$

$i \rightarrow T(i) = T(n-i) + \sum_{j=0}^{i-1} n-j$ (4)

$\overset{0}{n-0} + \overset{1}{n-1} + \overset{2}{n-2} + \overset{3}{n-3}$

$T(i) = T(n-i) + \sum_{j=0}^{i-1} n - \sum_{j=1}^{i-1} j$

$T(i) = T(n-i) + in - \frac{(i-1)\cdot((i-1)+1)}{2}$ $\qquad n-i=1$

$T(i) = T(n-(n-1)) + (n-1)n - \frac{(n-2)\cdot(n-1)}{2}$ $\qquad n-1=i$

$T(i) = T(1) + n^2 - n - \frac{n^2-n-2n+2}{2}$

$T(i) = 2 + n^2 - n - \frac{n^2-3n+2}{2}$

$T(i) = 2 + n^2 - n - \frac{1}{2}(n^2-3n+2)$

$T(i) = 2 + n^2 - n - \frac{n^2}{2} + \frac{3n}{2} - 1$

$T(i) = 1 + \frac{n^2}{2} - \frac{n}{2}$

```
int x=1;
for (int i = 1; i <= n-1; i++)  {
      for (int j = 1; j <= i, j++) {
            int k=1;
            while (k<=n) {
              x = x+1;
              k=k+1;
            }
      }
}
```

$\sum_{i=1}^{n-1} \sum_{j=1}^{i} \sum_{k=1}^{N} c$

$\sum_{i=1}^{n-1} \sum_{j=1}^{i} Nc$

$\sum_{i=1}^{n-1} Nci$

$Nc \sum_{i=1}^{n-1} i$

$Nc \cdot \frac{n-1 \cdot (n-1+1)}{2}$

$Nc \cdot \frac{(n-1)n}{2}$

$Nc \cdot \frac{n^2-n}{2}$

$Nc \cdot \left(\frac{n^2}{2} - \frac{n}{2}\right)$

$$nc \cdot \left(\frac{n^2}{2} - \frac{n}{2}\right)$$

$$\frac{n^3 c}{2} - \frac{n^2}{2} + \frac{cn^2}{2} - \frac{cn}{2} \to O(n^3)?$$

2.b. Calcule el O(N) por definición (1 punto)
$T(n) = 200n^3 + 4n^4 + 5n$ → orden n^4

$200n^3 \leq c_1 n^4$

$200n^3 \leq 200\, n^4$ $\quad c_1 = 200$

$200n^3 \leq 200\, n^4$ $\quad n_0 = 0$

$0 \leq 0$

$4n^4 \leq c_2 \cdot n^4$ $\quad c_2 = 4$

$4n^4 \leq 4n^4$ $\quad n_0 = 0$

$0 \leq 0$

$5n \leq c_3 n^4$ $\quad c_3 = 1$

$5n \leq 5 \cdot n^4$ $\quad n_0 = 0$

$0 \leq 0$

$200n^3 + 4n^4 + 5n \leq c_1 n^4 + c_2 n^4 + c_3 n^4$

$T(n) \leq (c_1 + c_2 + c_3) \cdot n^4$

$T(n) \leq 205\, n^4$

$T(n) \leq n^4 c$, con $c = 205$ y $n \geq n^0$ siendo $n^0 = 0$

2.c. Responda a las siguientes preguntas (0,5 puntos)

i.- Considerando que un algoritmo requiere $f(n)$ operaciones para resolver un problema y la computadora procesa 10.000 operaciones por segundo. Si $f(n) = n^2$, determine el tiempo en segundos requerido por el algoritmo para resolver un problema de tamaño n=2.000.

10000 op/s

$f(2000) = 4.000000 / 10.000 = \underline{400 \text{ seg}}$

ii.- ¿De qué orden es, en el peor caso, la operación de inserción de un elemento en un arreglo ordenado, teniendo en cuenta que la operación lo mantiene ordenado? Justifique su respuesta.

Si está al final recorro todo y lo meto.

si está al principio, tengo q' correr todo lo q' está después

En todo caso, siempre recorro el arreglo como mucho una vez.

$\boxed{n?}$

```
private int cuenta(ArbolBinario<Integer> a){
    int cuenta = 0;
    if (!a.esVacio()) {
        if (a.esHoja()) cuenta = 1;
        else
            cuenta = 1 + cuenta(a.getHijoIzquierdo())+ cuenta(a.getHijoDerecho());
    }
    return cuenta;
}
```

c

1 Hoja

$2T(n) + c$ Hoja no

b.- Determinar si la siguiente sentencia es verdadera o falsa. Justificar usando Big OH.

$2^{2 \cdot n}$ es del O(2^n)

$2^{2n} \leq 2^n$

$2^n \leq 2^{2n} \cdot c$ $\quad$ Falso

$2^1 \leq 2^{2 \cdot 1}$ 1 $\quad c = 1$

$2^n \leq 2^{2n} \cdot c$ falso

$2^1 \leq 2^{2 \cdot 1} \cdot 1$ $\quad c = 1$

$2 \leq 4$ $\quad n_0 = 0$

para que la afirmación sea verdadera, deben existir un N_0 y una c tal que para todo $n \geq N_0$ $\quad 2^n \geq c \cdot 2^{n \cdot 2}$

Esto es incorrecto y, de hecho, se da exactamente lo contrario.

La f $c \cdot 2^{2n}$ crecerá más rápido que 2^n con $c=1$ para todo $n \geq n_0$, ya que la primera tiene un 2 multiplicando el exponente. Por tanto, la sentencia es falsa.

C.- Plantear y resolver la función de T(n) para el siguiente método:

```
private int funcion (int x, int y){
    if (y<=1) return x;
    else {
     int calc=0;
     for (int i=1; i <=100; i++)
          calc = 1 + (funcion(x-1, y/2) * funcion(x-2, y/2));
    return calc;
    }
}
```

$x \quad y \leq 1$

$$T(n)\begin{cases} c & n = 0 \\ d & n = 1 \\ e + T(n-2) + n - 2 & n > 1 \end{cases}$$

$n-4$

caso 1 $\rightarrow T(n) = e + T(n-2) + n - 2$

2 $\quad T(n-2) = e + e + T(n-4) + n-2-2 + n - 2$

3 $\quad T(n-6) = e + e + e + T(n-6) + n-6 + n-4 + n-2$

i $\quad T(i) = e \cdot i + T(n - 2 \cdot i) + \sum_{k=1}^{i} n - k \cdot 2$

$e \cdot i + T(n-2i) + \sum_{k=1}^{i} n - \left(2 \sum_{k=1}^{i} k\right)$

$n - 2i = 1$ $\quad e \cdot i + T(n-2i) + ni - 2 \cdot \frac{i(i+1)}{2}$

$n = 1 + 2i$

$n - 1 = 2i$ $\quad e \cdot i + T(n-2i) + ni - \frac{\cancel{2} \cdot i^2 + i}{\cancel{2}}$

$\frac{n-1}{2} = i$ $\quad e \cdot i + T(n-2i) + ni - (i^2 + i)$

$n - 2i = 0$ $\quad e \cdot \frac{n}{2} + T\left(n - 2 \cdot \frac{n}{2}\right) + n \cdot \frac{n}{2} - \left(\left(\frac{n}{2}\right)^2 + \frac{n}{2}\right)$

$n = 2i$

$\frac{n}{2} = i$ $\quad \frac{en}{2} + T(0) + \frac{n^2}{2} - \frac{n^2}{4} - \frac{n}{2}$

$e \frac{n}{2} + c + \frac{2n^2 - n^2}{4} - \frac{n}{2}$

$e \frac{n}{2} + c + \frac{n^2}{4} - \frac{n}{2} \longrightarrow O(n^2)$