Topological Modular Forms and the Adams Spectral Sequence

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Abstract

Topological modular forms are…

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Introduction

Desar's chosen field in mathematics was so esoteric that nobody in the institute or the Maths Federative could really check his progress |
The Dispossessed, Ursula K. Le Guin

THE ADAMS SPECTRAL SEQUENCE

This section is a short introduction to the Adams spectral sequence, which is a tool that computes the homotopy groups $\pi_n(X)$ of a spectrum X from its $\mathbb{Z}/p\mathbb{Z}$-homology for some prime p (usually $p = 2$). The first step is to give an Adam's filtration of our spectrum X, i.e. a sequence

$$X_n \to X_{n-1} \to \cdots \to X_1 \to X_0 := X$$

such that X_n maps onto a space K_n given by the wedge of suspensions of the Eilenberg-MacLane spectrum of the $\mathbb{F}_p := \mathbb{Z}/p\mathbb{Z}$-homology, inducing a surjection on $H\mathbb{F}_p$-homolgy and with $X_{n+1} = \mathrm{fib}(X_n \to K_n)$. We will work more generally and replace $H\mathbb{F}_p$ with an arbitrary spectrum $\mathcal{E}$.