Term-Indexing for the Beagle Theorem Prover

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Except where otherwise indicated, this thesis is my own original work.

Timothy Clarence Richard Cosgrove
15 September 2013

For Dana.

Acknowledgements

Thank you to my Supervisor and all. . .

Abstract

This should be the abstract to your thesis. . .

Contents

Chapter 1

Introduction

1.1 Motivation

- Describe beagle
- Advantages of beagle
- drawbacks
- some intstrumentation

1.1.1 A Theoretical Framework

Chapter 2

Background

2.1 First-Order Logic Terms and Notation

This thesis focuses around the extension of *beagle*, a *first-order logic* (FOL) theorem prover. In order to understand beagle's purpose and functions a basic understanding of the FOL logical system is required. This section provides a rudimentary overview of FOL syntax and uses; but also includes an explanation of any specialised terms and notation used throughout the paper.

2.1.1 FOL basics

2.1.2 Calculi and FOL problems

2.1.3 The Superposition Calculus

Should contain

- Variables
- Symbols
- Predicates
- Quantifiers
- Notion of soundness and completeness
- Description of a 'calculus'

2.1.4 Specialised Syntax in this Paper

- Positions

2.2 Automated Reasoning and Theorem Proving

Automated Reasoning is a rapidly growing field of research where computer programs are used to solve problems stated in first order logic statments or other formal logics.

Some existing theorem provers include:

SPASS

[Weidenbach et al. 1999]

Vampire

[Riazanov and Voronkov 1999]

E

[Schulz 2002]

Should contain

- Theorem prover examples

2.3 Term Indexing

Term indexing is a technique used to better locate logical terms which match rules in a prover's calculus.

Top Symbol Hashing

Discriminant Trees

2.4 Fingerprint Indexing

Fingerprint Indexing is a recent technique developed by Schulz [2012], the creator of the E prover.

Table 2.1: Fingerprint matches for Unification [Schulz 2012, p6]

	f_1	f_2	**A**	**B**	**N**
f_1	Y	N	Y	Y	N
f_2	N	Y	Y	Y	N
A	Y	Y	Y	Y	N
B	Y	Y	Y	Y	Y
N	N	N	N	Y	Y

Table 2.2: Fingerprint matches for Matching [Schulz 2012, p6]

	f_1	f_2	**A**	**B**	**N**
f_1	Y	N	N	N	N
f_2	N	Y	N	N	N
A	Y	Y	Y	N	N
B	Y	Y	Y	Y	Y
N	N	N	N	N	Y

2.5 The Beagle Theorem Prover

The core implementation of Beagle was developed by Peter Baumgartner et al. of NICTA. Its purpose was to demonstrate the capabilities of the *Weak Abstraction with Heirachic Superposition Calculus*; which allows the incorporation of prior knowledge via a 'background reasoning' modules.

2.5.1 The Heirachic Superposition with Weak Abstraction Calculus

2.5.2 Beagle's Shortcomings

2.6 Tools Used

2.6.1 Scala

As mentioned above *beagle*is written in *Scala*, the Scalable Language. Scala is a functional language and may be confusing to those who are not familiar with the functional programming paradigm. This thesis will contain occasional snippets of Scala code; but note that any snippets used will be accompanied by an explanation and in general an understanding of Scala/funcitonal programming is not required.

2.6.2 VisualVM

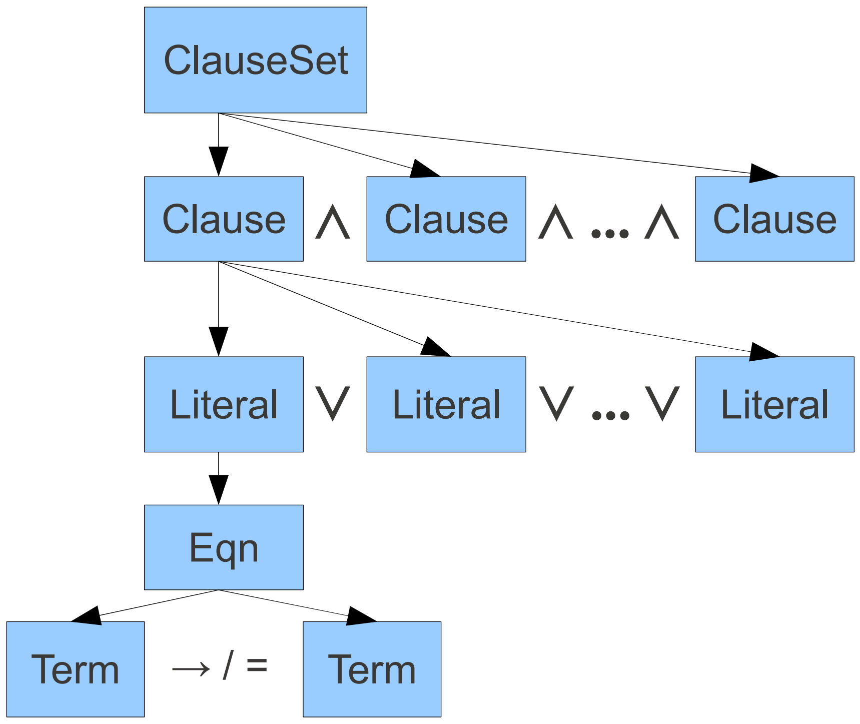
2.6.3 Eclipse

Chapter 3

Implementing Fingerprint Indexing

3.1 Structure of *beagle*

To be able to make any significant contribution to the *beagle* project, I first had to gain a solid understanding of the existing Scala codebase.


ClauseSet
Clause ∧ Clause ∧ ... ∧ Clause
Literal ∨ Literal ∨ ... ∨ Literal
Eqn
Term → / = Term

Describe main inference loop

Refer to results for instrumentation; showing what may be improved with indexing.

3.2 Implementing Fingerprint Indexing

3.2.1 Adding terms to the index

The first step in adding Fingerprint Indexing to *beagle* is creating the indexer itself; a Scala class which will manage the index and provide functions for adding to/retreiving from the Index.

The following block of Scala code extracts a single Fingerprint Feature from a Term at the given position.

```scala
/** Extract the operator at position pos. Note that matching Var
  * and Funterm is an exhaustive pattern for Term. */
 def extractFeature(term: Term, pos: Position) : FPFeature = pos match {
   case Nil     => term match {
     case t:FunTerm => FPF(t.op) // Found function symbol, return it
     case t:Var     => FPA       // Found variable, return A
   }
   case p :: ps => term match {
     case t:FunTerm => try   {extractFeature(t.args(p), ps) }
                       //Attempted to index non-existent position, return N
                       catch {case e:IndexOutOfBoundsException => FPN}
     // Found variable BEFORE end of position, return B
     case t:Var     => FPB
   }
 }
```

Listing 1: Scala code to extract fingerprint features for matching.

3.2.2 Retrieving Compatible terms

To compare two fingerprints with each other we look at them side-by-side and check that each position shows a Y in the Fingerprint unifcation table.

```
/** Check two Fingerprint features for compatibility based
  * on the unification table (See page 6 of [Shulz 2012]).
  * This table is reduced to 4 cases:
  *  - True if Features are equal,
  *  - True if at least one Feature is B,
  *  - True if at least one Feature is A; but no Ns,
  *  - False otherwise  */
 def compareFeaturesForUnification
         (a:FPFeature, b:FPFeature) : Boolean =
 (a == b) ||
 (Set(a,b) match {
   case x if (x contains FPB) => true
   case x if (x contains FPA) => !(x contains FPN)
   case _ => false})
```

Listing 2: Scala implementation of the Fingerprint unification table. [Schulz 2012, p6]

3.3 Adding Indexing to *Beagle*

3.3.1 Refactoring Current Implementation

Actually making use of our indexer class will require significant modification to *beagle*'s structure and proving sequence.

Refer to class and flow diagrams from 3.2

3.3.2 Initial Problems

3.3.3 Matching with Subterms

dicsuss how we are required to match against subterms. Requires significant modification to the fingerprint indexer

3.4 Extending the Indexer

3.4.1 Matching and Simplification in *Beagle*

3.4.2 Generalising our FingerprintIndex

3.4.3 Applying new Indices to Simplification

3.5 Tailoring to *Beagle*'s Heirachic Superposition with Weak Abstraction Calculus

In this section we discuss the thought process in developing and implementing extensions to Fingerprint Indexing in order to better tailor it to *beagle*'s rather unique logical calculus.

3.5.1 Foreground and Background Terms

In the Heirachic Superposition with Weak Abstraction Calculus all terms have a concept of being 'Foreground' or 'Background'. In Section 2.5 we discussed this concept; referring to it as the *layer* of a term. It is worht noting at this stage that computing the layer of a term is cheap (or rather, zero, as it is computed on the fly during term generation and stored for later use).

Recall the four orignal fingerprint feature symbols from 2.3:

- f: arbitrary constant function symbols.
- **A**: Variable at the exact position.
- **B**: A variable could be expanded to meet the position.
- **N**: Position can never exist regardless of variable assignment.

We introduce two new fingerprint features: **A**+ and **B**+. These symbols will be used for the same purpose as the original **A** and **B**, but only for '*background*' or '*abstraction*' variables. These variables can only be used for pure background terms; a fact we may use to restrict the possible matches for unification.

The layeredness of function symbols is also relevant to our comparison. $f+$ in the following table signifies a position where the entire subterm from this position downwards is 'pure background'. Keep in mind that this definition is slightly different to the definition for **A**+ and **B**+; as we must consider all function symbols below f itself.

At this point it is important to note that these added fingerprint features slightly modify the original **A**, **B** and f features. These features will now only represent the foreground layered positions.

We present here the (rather large) extended unification table with no justification. Refer to the original unification table (Table 2.1) for an in-depth explanation of how this table should be interpreted [Schulz 2012].

Table 3.1: Fingerprint matches for unification; extended by considering term layers.

	f_1	f_2	**A**	**B**	**N**	f_1+	f_2+	**A+**	**B+**
f_1	Y	N	Y	Y	N	N	N	N	N
f_2	N	Y	Y	Y	N	N	N	N	N
A	Y	Y	Y	Y	N	Y	Y	Y	Y
B	Y	Y	Y	Y	Y	Y	Y	Y	Y
N	N	N	N	Y	Y	N	N	N	Y
f_1+	N	N	Y	Y	N	Y	N	Y	Y
f_2+	N	N	Y	Y	N	N	Y	Y	Y
A+	N	N	Y	Y	N	Y	Y	Y	Y
B+	N	N	Y	Y	Y	Y	Y	Y	Y

We now present brief reasoning for all our new entries in the table. Note that as this table is for unification it is symmetric along the leading diagonal (as in the original unification table); so we need only discuss the lower triangle of the matrix. Furthermore, notice that the bottom right segment of the table is actually identical to the original unification table. This is expected as when we compare two pure background features the comparison behaves normally.

We will justify the new section of the table line by line:

- Background function symbols ($f+$): Recall that this feature is only applicable if the entire subterm below f is pure background. Therefore it does not match the foreground version of the same symbol. It does however match both **A** and **B**. This is required since these symbols still match '*impure*' background variables; which may be expanded to either foreground or pure background terms.

- Abstraction variables (**A**+): Similarly to the pure background function symbol feature, this feature cannot match any terms which sit in the foreground. It can however match both **A** and **B** as they may represent either foreground or background expressions.

- Potential expansion of an abstraction variable **B**+: Same as for **A**+ but can also match **N**.

To go with this table we present its corresponding Scala matching code in Listing 3. Unfortunately the steep increase in table size results in the amount of code required exploding. It also becomes impossible to use our earlier trick of Set matching; due to the need for parametrised Fingerprint symbols (i.e. **A**+ and **B**+ represented as FPA(true) and FPB(true)).

```
/** Check two Fingerprint features for compatibility based
  * on the *extended* unification table (See table in report).*/
 def compareFeaturesForUnification
      (a:FPFeature, b:FPFeature) : Boolean =
  (a,b) match {
    case (FPF(f1), FPF(f2))     => (f1.op == f2.op) &&
                                   (if (f1.isFG || f2.isFG)
                                         (!f1.isPureBG && !f2.isPureBG)
                                    else true)
    case (FPF(f), FPB(true)) => f.isPureBG
    case (FPB(true), FPF(f)) => f.isPureBG
    case (_, FPB(_))         => true
    case (FPB(_), _)         => true
    case (FPF(f), FPA(true)) => f.isPureBG
    case (FPA(true), FPF(f)) => f.isPureBG
    case (FPN, FPA(_))       => false
    case (FPA(_), FPN)       => false
    case (_, FPA(_))         => true
    case (FPA(_), _)         => true
    case (FPN, FPN)          => true
    case _                   => false
  }
```

Listing 3: Scala code to extract fingerprint features for extended layer matching.

Chapter 4

Results

4.1 Beagle Before Indexing

4.2 Indexing Subsumption

4.2.1 False Positives

We observe many, even after a myriad of optimisations. Notice that this is due to the structure of beagle; many retrieved terms are cheaply thrown out due to other conditions (such as being a parent term, being ordered, etc.) and do not significantly impact performance.

4.2.2 Speed

4.2.3 Comparison

4.3 Indexing Simplification and Matching

4.3.1 Further Intstrumentation

4.3.2 *Beagle*with Simplification Improvements

4.4 Tailored Improvements

4.4.1 Layer Checking

Chapter 5

Conclusion

5.1 Why this is a Very Clever Thesis

Appendix A

Some Other Stuff

A.1 Why I Did It

Appendix B

More Stuff

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