Term Indexing for the Beagle Theorem Prover

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The Beagle Theorem Prover

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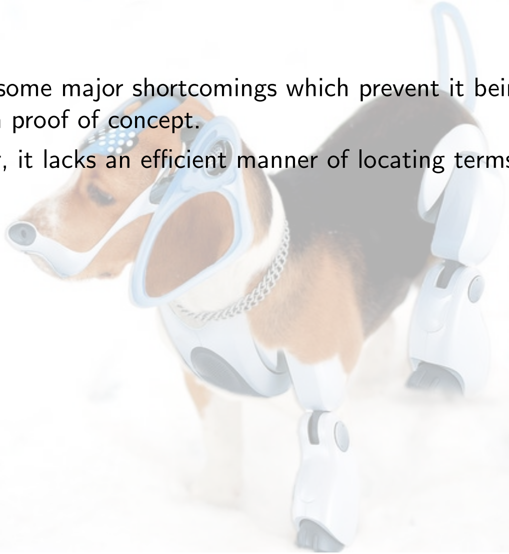
- This calculus is capable of *hierarchic reasoning* by incorporating a *background prover*.
- Background provers act as a black box which can instantly prove known facts. For example integer arithmetic.
- The calculus is carefully constructed with a process known as *weak abstraction* in order to ensure consistency and completeness.

Extending Beagle



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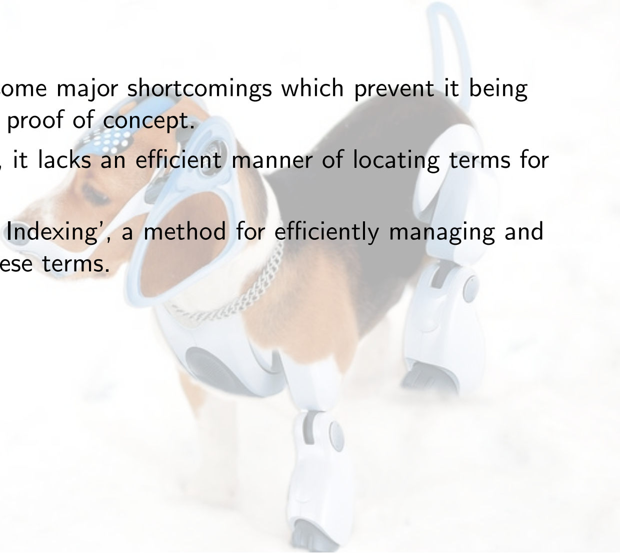

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- Beagle has some major shortcomings which prevent it being more than a proof of concept.
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- Enter 'Term Indexing', a method for efficiently managing and collecting these terms.
- Research Questions:
    - How can Term Indexing (in particular Fingerprint Indexing) be implemented and applied to Beagle?
    - How can Fingerprint Indexing be improved; generally and with respect to Beagle's specific calculus?
    - What sort of improvement will this implementation yield in the prover?

Terminology Used in this Presentation

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- Substitutions:
  - s is '*unifiable*' with t : $\sigma s = \sigma t$
  - s '*subsumes*' t : $\sigma s = t$

The Superposition Calculus

- Normal Superposition rule

Positive Superposition $$\frac{l \approx r \vee C \qquad s[u] \approx t \vee D}{(s[r] \approx t \vee C \vee D)\sigma}$$

Where (i) $\sigma =$ simple mgu (l, u),

and (ii) u is not a variable.

The Hierarchic Superposition with Weak Abstraction Calculus

- Extension of the Superposition Calculus to accommodate hierarchic reasoning.

Positive Superposition $\dfrac{l \approx r \vee C \qquad s[u] \approx t \vee D}{\text{abstr}((s[r] \approx t \vee C \vee D)\sigma)}$

Where (i) $\sigma = \text{simple mgu } (l, u)$,
(ii) u is not a variable,
(iii) $r\sigma \not\succeq l\sigma$,
(iv) $t\sigma \not\succeq s\sigma$,
(v) l and u are not pure background terms,
(vi) $(l \approx r)\sigma$ is strictly maximal in $(l \approx r \vee C)\sigma$,
and (vii) $(s \approx t)\sigma$ is strictly maximal in $(s \approx t \vee D)\sigma$.

Term Indexing Techniques

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- Top-Symbol Hashing.
- Discrimination Trees.
- Path Indexing.

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Table: Fingerprint Feature comparison tables for *unification* (left) and *subsumption* (right)

	f_1	f_2	**A**	**B**	**N**
f_1	Y	N	Y	Y	N
f_2	N	Y	Y	Y	N
A	Y	Y	Y	Y	N
B	Y	Y	Y	Y	Y
N	N	N	N	Y	Y

	f_1	f_2	**A**	**B**	**N**
f_1	Y	N	N	N	N
f_2	N	Y	N	N	N
A	Y	Y	Y	N	N
B	Y	Y	Y	Y	Y
N	N	N	N	N	Y

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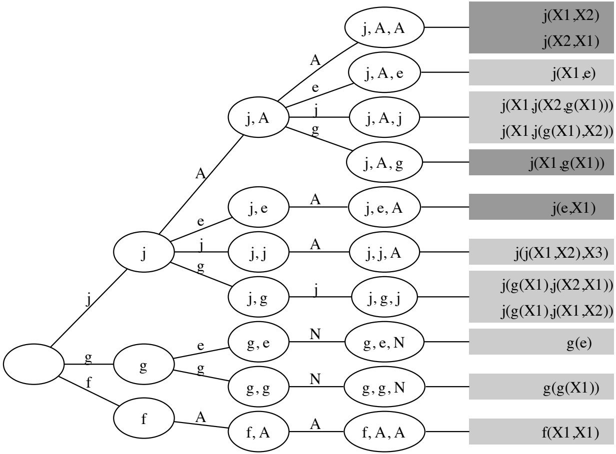
A	Y	Y	Y	N	N
B	Y	Y	Y	Y	Y
N	N	N	N	N	Y

- Schulz, Stephan: Fingerprint Indexing for Paramodulation and Rewriting. In: Lecture Notes in Computer Science volume 7364 pp. 447–483 (2012).

Fingerprint Indexing – Example
Fingerprint Index



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- Retrieval of terms.
- Requires implementation of the comparison tables and a more complex Index traversal algorithm.

Beagle's Main Procedure

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  Remove Clauses from *new* one at a time, simplify them and then attempt inference rules.
- Two key areas of improvement ($O(n)$ operations):
  - Inferences via the Superposition rules.
  - Simplifying Clauses.

Indexing Superposition

Positive Superposition $\dfrac{l \approx r \vee C \qquad s[u] \approx t \vee D}{\text{abstr}((s[r] \approx t \vee C \vee D)\sigma)}$

Where (i) $\sigma =$ simple mgu (l, u),

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- Requires that we index all subterms. Furthermore we must implement two separate cases for *from* and *into*.

Indexing Simplification

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- These special cases allow redundant Clauses to be removed; preventing clutter in the inference process.
- The two main simplification rules used by Beagle are *Negative Unit Simplification* and *Demodulation*
- These rules operate only on *unit Clauses*, so using our current index clogged with subterms is inefficient.
- The rules also require checking for *subsumption*, which requires implementing a new comparison table for Fingerprint Indexing.

Indexing Negative Unit Simplification

Negative Unit Simplification $\dfrac{l \not\approx r \qquad s \approx t \vee C}{C}$

Where (i) $\exists\sigma \;\; s.t. \;\; (l \approx r)\sigma \equiv s \approx t$.

The clause $s \approx t \vee C$ may be removed.

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- Searching for valid subsuming Literals is extremely time consuming.
- Requires an index capable of matching Equations rather than Terms.

Indexing Demodulation

Demodulation $$\frac{l \rightarrow r \qquad s[u] \approx t \vee D}{s[r\sigma] \approx t \vee D}$$

Where $l\sigma = u$

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- For a simple example, with a Literal $X \to f(a)$ we may replace all occurrences of X with $f(a)$.
- Like in Negative Unit Simplification, the most costly operation is searching for subsuming l Terms.
- We must perform this search for every possible subterm u of s.

Fingerprint Indexing for the Hierarchic Superposition with Weak Abstraction Calculus

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Table: Fingerprint comparison table for unification; extended by considering the term hierarchy.

	f_1	f_2	**A**	**B**	**N**	f_1+	f_2+	**A+**	**B+**
f_1	Y	N	Y	Y	N	N	N	N	N
f_2	N	Y	Y	Y	N	N	N	N	N
A	Y	Y	Y	Y	N	Y	Y	Y	Y
B	Y	Y	Y	Y	Y	Y	Y	Y	Y
N	N	N	N	Y	Y	N	N	N	Y
f_1+	N	N	Y	Y	N	Y	N	Y	Y
f_2+	N	N	Y	Y	N	N	Y	Y	Y
A+	N	N	Y	Y	N	Y	Y	Y	Y
B+	N	N	Y	Y	Y	Y	Y	Y	Y

Metrics for Analysing Indexing Performance

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- False Positives - Relevant, but can be misleading depending on number of positions being sampled.
- Run time *per Inference* - Most accurate measure of performance. Must still take care when interpreting.

Comparing Varieties of Beagle

Table: Totalled inference counts and indexing statistics for various versions of beagle.

	Inference Counts			Indexing Results		
Version	Sup	Demod	NegUnit	TotalFound	SupFP	SimpFP
Unmodified [1]	414216	29097	1826	0	0	0
Standard	162881	41414	2452	61884768	15525	39778148
Enhanced	146861	35326	1960	58119897	17641	39916687

Table: Totalled timing results for various versions of beagle.

	Time Spent (seconds)					
Version	Indexing	Retrieving	Sup	Demod	NegUnit	Total
Unmodified [1]	0	0	730.44	9.44	31.99	5623.21
Standard	28.4	38.73	254.17	41.66	3.18	381.36
Enhanced	18.74	17.58	168.79	30.56	2.12	259.02

[1] This version failed to solve two out of the fifty problems within 8 hours.

Time Spent Per Inference

Table: Superposition time for the 6 most extreme problem examples.

Version	Superposition	Demodulation	NegUnit Simplification
Unmodified	1.7ms	0.3ms	17.5ms
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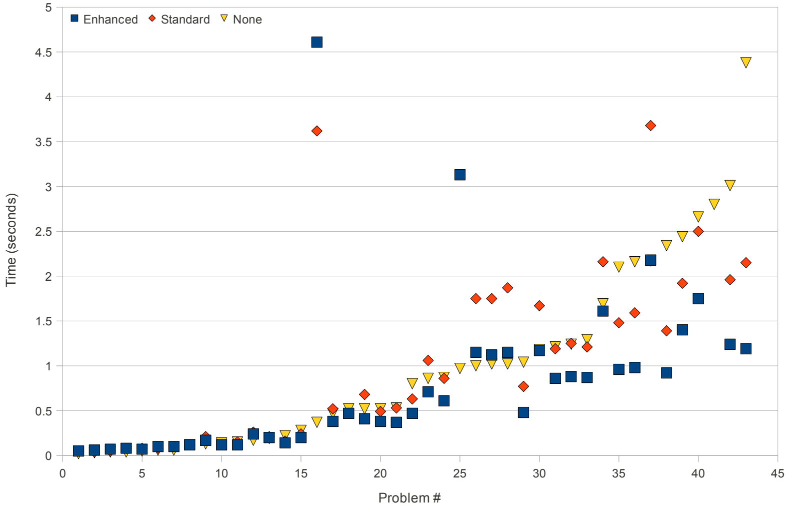
- When excluding PUZ037-1.p we have 0.29, 0.39 and 0.31 milliseconds per Demodulation.

Runtimes under 5 seconds



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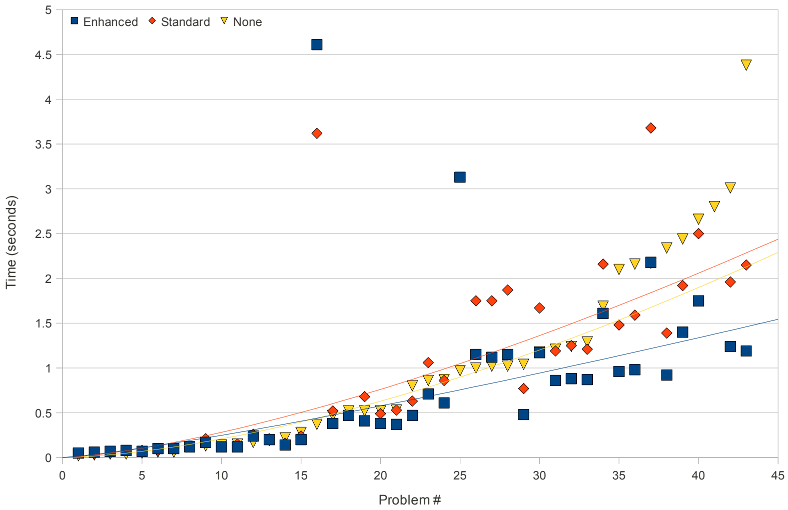

Results Analysis

Table: Superposition time for the 6 most extreme problem examples.

Problem	Enhanced	Standard	Unmodified
DAT050=1.p	17.53	31.54	48.62
DAT039=1.p	13.2	22.51	130.77
DAT040=1.p	14.49	21.29	190.71
DAT038=1.p	12.53	24.04	294.86
DAT043=1.p	18.67	26.08	**N/A**
DAT048=1.p	17.65	35.77	**N/A**

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- When taking number of inferences into account for DAT038=1.p we observe 1.2 milliseconds per superposition for the full implementation versus 2.2 milliseconds per superposition for unindexed beagle.

Fingerprint Sampling Varieties

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- Different position samples

Fingerprint Sampling Varieties

Table: Totalled inference counts and indexing statistics for various Fingerprint sampling sets.

	Inference Counts			Indexing Results		
Sample Set	Sup	Demod	NegUnit	TotalFound	SupFP	SimpFP
FP3W	162218	42402	2472	13913606	69429	1815992
FP4M	147798	35709	1963	13469779	26847	1851515
FP6M	144505	35326	1959	12601762	16406	1694731
FP7	159055	41005	2440	13011130	21281	1596575
FP8X2	159385	40876	2438	12819184	11229	1602033

Table: Totalled timing results for various Fingerprint sampling sets.

	Time Spent (seconds)					
Sample Set	Indexing	Retrieving	Sup	Demod	NegUnit	Total
FP3W	11.52	14.02	170.37	9.26	1.78	237.75
FP4M	13.09	14.12	164.95	9.51	1.82	230.68
FP6M	16.82	16.5	159.93	10.78	2.11	229.59
FP7	19.98	18.74	170.83	12.37	2.37	249.22
FP8X2	45.56	32.59	181.43	21.45	4.06	294.8

Note that for more relevant comparisons these results exclude PUZ037-1.p.

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