Term Indexing for the Beagle Theorem Prover

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The Beagle Theorem Prover

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- Makes use of modular 'Background Theories' to make efficient use of known facts.
- This requires the carefully constructed 'Hierarchic Superposition with Weak Abstraction Calculus' in order to ensure consistency and completeness.
- Has been extended with term indexing to efficiently locate clauses which match the calculus resolution rules.

Term Indexing Techniques

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  - 'Generalises': $\sigma s = t$
- Top-Symbol Hashing.
- Discriminant Trees.

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Unification

	f_1	f_2	**A**	**B**	**N**
f_1	**Y**	**N**	**Y**	**Y**	**N**
f_2	**N**	**Y**	**Y**	**Y**	**N**
A	**Y**	**Y**	**Y**	**Y**	**N**
B	**Y**	**Y**	**Y**	**Y**	**Y**
N	**N**	**N**	**N**	**Y**	**Y**

Matching

	f_1	f_2	**A**	**B**	**N**
f_1	**Y**	**N**	**N**	**N**	**N**
f_2	**N**	**Y**	**N**	**N**	**N**
A	**Y**	**Y**	**Y**	**N**	**N**
B	**Y**	**Y**	**Y**	**Y**	**Y**
N	**N**	**N**	**N**	**N**	**Y**

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N	**N**	**N**	**N**	**Y**	**Y**

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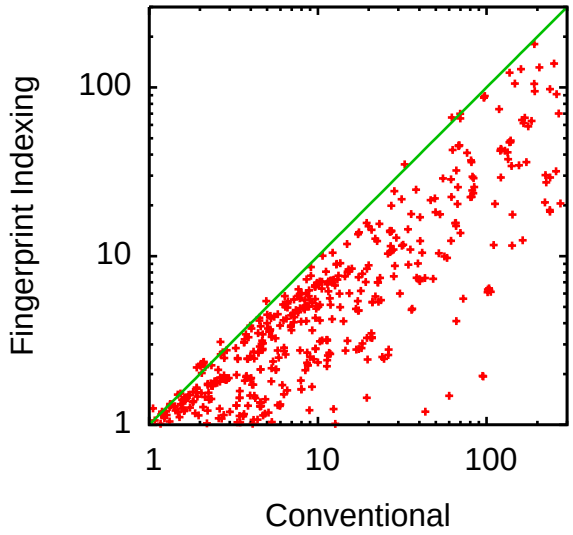
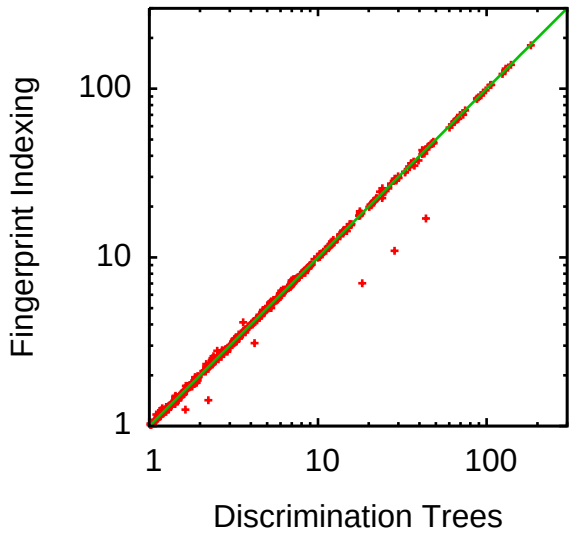
f_2	**N**	**Y**	**N**	**N**	**N**
A	**Y**	**Y**	**Y**	**N**	**N**
B	**Y**	**Y**	**Y**	**Y**	**Y**
N	**N**	**N**	**N**	**N**	**Y**

- Schulz, Stephan: Fingerprint Indexing for Paramodulation and Rewriting. In: Lecture Notes in Computer Science volume 7364 pp. 447–483 (2012).

Fingerprint Indexing – Potential Performance

Index	Run time	Sat time	PM time	PMI time	MGU time	BR time	BRI time
NoIdx	16062.392	14078.300	8980.320	0.000	2545.080	2280.250	0.000
FP1	7006.758	6145.870	1816.100	25.710	450.760	379.570	40.150
FP6M	6000.177	5385.810	1181.710	38.240	99.110	39.010	55.660
NPDT	6082.246	5434.760	1184.750	64.910	83.110	33.200	79.910





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- Operates on Beagle's most costly inference rule, superposition:

$$\frac{l \approx r \vee C \qquad s[u] \approx t \vee D}{\mathrm{abstr}((s[r] \approx t \vee C \vee D)\sigma)}$$

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- Furthermore indexing may be applied to more of HSWA's inference rules; in particular simplification.

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- Fingerprint indexing could be greatly improved by tailoring it specifically to Beagle's FOL calculus.
- Main improvement is to consider Beagle's *foreground* and *background* terms.
- Furthermore indexing may be applied to more of HSWA's inference rules; in particular simplification.
- These extensions will not require so much modification; as the fingerprint indexing framework is already built.

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- The main area to consider here is the sampling positions. Sampling many positions reduces the returned sets, but increases indexing overhead.
- Large problems better suit indexing; but it is difficult to know ahead of time what a ‘large’ problem is.

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- Time Spent *per Inference* - Booyah

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- Indexing with Optimisations.

Comparing Varieties of Beagle

- Results table

Fingerprint Sampling Varieties

- Reasoning. Cite shulz and FP/Speed balance

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- Reasoning. Cite shulz and FP/Speed balance
- Different position samples

Fingerprint Sampling Varieties

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