Tim Cosgrove

Outlin

ect Overv

Motivation

Backgroui

The Beagle Theorem Prover Term Indexing Fingerprint Indexing

Implementatio

Implementing
Fingerprint
Indexing
Indexing
Applications
Tailoring to

Other

Improvemen

Evaluation Metrics Beagle

Sample Position

## Term Indexing for the Beagle Theorem Prover

#### Tim Cosgrove

COMP4006 Honours Research Project

Research School of Computer Science, Australian National University

u4843619@anu.edu.au

Supervisor: Peter Baumgartner

October 12, 2013

Tim Cosgrove

#### Outline

2 Background

The Beagle Theorem Prover Term Indexing Theorem Prover Fingerprint Indexing

Term Indexing

3 Implementation

1 Project Overview

Motivation

Implementing Fingerprint Indexing Indexing Applications Tailoring to Beagle Other Improvements

A Results

**Evaluation Metrics** Beagle Comparisons Sample Position Comparisons

Tim Cosgrove

Outline

net Overvi

Motivation

Backgroun

The Beagle Theorem Prover

Term Indexing Fingerprint Indexing

Implementati

Implementin Fingerprint Indexing Indexing Applications Tailoring to Beagle

Other Improvement

Evaluation Metrics

Comparisons
Sample Position

### The Beagle Theorem Prover

 Beagle is a First-Order-Logic resolution theorem prover with equality.

Tim Cosgrove

Outline

Project Over

Motivation

Backgroui

The Beagle Theorem Prover Term Indexing

Term Indexin Fingerprint Indexing

Implementation

Implementing Fingerprint Indexing Indexing Applications Tailoring to Beagle

Other Improvemen

Improvemen

Evaluatio Metrics

Sample Positio

### The Beagle Theorem Prover

- Beagle is a First-Order-Logic resolution theorem prover with equality.
- Makes use of modular 'Background Theories' to make efficient use of known facts.

Tim Cosgrove

The Beagle Theorem Prover Term Indexing

Indexing

### The Beagle Theorem Prover

- Beagle is a First-Order-Logic resolution theorem prover with equality.
- Makes use of modular 'Background Theories' to make efficient use of known facts.
- This requires the carefully constructed 'Hierarchic Superposition' with Weak Abstraction Calculus' in order to ensure consistency and completeness.

Tim Cosgrove

Outline

ct Overvie

Motivation

Backgrour

The Beagle Theorem Prover

Term Indexing Fingerprint

Indexing

nplementati

Implementing Fingerprint Indexing Indexing Applications Tailoring to Beagle

Other

Result

Evaluation Metrics

Beagle Comparisons

Sample Positio

## Superposition Calculus

Normal Superposition rule

Tim Cosgrove

Motivation

The Beagle Theorem Prover

Term Indexing Indexing

Indexing

Evaluation

## Hierarchic Superposition with Weak Abstraction Calculus

• Extension of Superposition Calculus to accommodate hierarchic reasoning.

Tim Cosgrove

Theorem Prover Term Indexing

Indexing

Indexing

Other

### Term Indexing Techniques

- Term indexers aim to collect all FOL terms which potentially match a 'query' term.
- Three important relations:

Tim Cosgrove

Theorem Prover Term Indexing

Indexing

Indexing

Other

Evaluation

### Term Indexing Techniques

- Term indexers aim to collect all FOL terms which potentially match a 'query' term.
- Three important relations:

• 'Unifiable':  $\sigma s = \sigma t$ 

• 'Instance Of':  $s = \sigma t$ 

• 'Generalises':  $\sigma s = t$ 

Tim Cosgrove

Outlin

Project Overvie

Motivation

Backgroui

Theorem Prover
Term Indexing

Fingerprint Indexing

Implementation

Implementing Fingerprint Indexing Indexing Applications

Applications
Tailoring to
Beagle
Other

Improvemer

Evaluation

Beagle Comparisons Sample Position Comparisons

### Term Indexing Techniques

- Term indexers aim to collect all FOL terms which potentially match a 'query' term.
- Three important relations:

• 'Unifiable':  $\sigma s = \sigma t$ • 'Instance Of':  $s = \sigma t$ 

• 'Generalises':  $\sigma s = t$ 

Top-Symbol Hashing.

- Discrimination Trees.
- Path Indexing.

Tim Cosgrove

Outline

roiect Overvie

Motivation

Backgroun

The Beagle Theorem Prover

Term Indexing Fingerprint

Indexing

Implementatio

Implementing Fingerprint Indexing Indexing Applications Tailoring to Beagle Other

....

Evaluation Metrics Beagle Comparisons

## Fingerprint Indexing

• Maintain a collection of *fingerprints* for terms.

Tim Cosgrove

Outline

Project Overvi

Motivation

The Beagle Theorem Prover

Term Indexing

Fingerprint Indexing

Implementat

Implementing
Fingerprint
Indexing
Indexing
Applications
Tailoring to
Beagle

Beagle Other

Reculte

Evaluation Metrics Beagle

Sample Positio Comparisons

## Fingerprint Indexing

- Maintain a collection of *fingerprints* for terms.
- A term fingerprint is an array over  $F \cup \{A, B, N\}$ .

Tim Cosgrove

Theorem Prover

Term Indexing Fingerprint

Indexing

Indexing

Other

## Fingerprint Indexing

- Maintain a collection of fingerprints for terms.
- A term fingerprint is an array over  $F \cup \{A, B, N\}$ .

U	Unification						
	$f_1$	$f_2$	Α	В	N		
$f_1$	Y	Ν	Y	Y	N		
$f_2$	N	Y	Y	Y	N		
A	Y	Y	Y	Y	N		
В	Y	Y	Y	Y	Y		
N	N	Ν	Ν	Y	Y		

Matching							
	$f_1$	$f_2$	A	В	Ν		
$f_1$	Y	N	N	N	Ν		
$f_2$	N	Y	Ν	Ν	Ν		
A	Y	Y	Y	Ν	Ν		
В	Y	$\mathbf{Y}$	Y	Y	Y		
N	N	Ν	Ν	Ν	Y		

Tim Cosgrove

Outline

Project Overvi

Backgroun

The Beagle Theorem Prover

Term Indexing Fingerprint

Indexing

Implementing Fingerprint Indexing

Indexing Applications Tailoring to

Other

D. H.

Evaluation Metrics Beagle

Comparisons
Sample Position
Comparisons

## Fingerprint Indexing

- Maintain a collection of *fingerprints* for terms.
- A term fingerprint is an array over  $F \cup \{A, B, N\}$ .

Unification						
	$f_1$	$f_2$	Α	В	N	
$f_1$	Y	Ν	Y	Y	N	
$f_2$	N	Y	Y	Y	N	
A	Y	Y	Y	Y	N	
В	Y	$\mathbf{Y}$	Y	Y	$\mathbf{Y}$	
N	N	N	Ν	Y	$\mathbf{Y}$	



Schulz, Stephan: Fingerprint Indexing for Paramodulation and Rewriting.
 In: Lecture Notes in Computer Science volume 7364 pp. 447–483 (2012).

Tim Cosgrove

Motivation

Theorem Prover

Term Indexing Fingerprint

Indexing

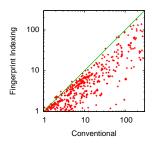
Indexing

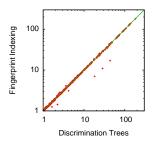
Other

Evaluation

## Fingerprint Indexing - Potential Performance

Index	Run time	Sat time	PM time	PMI time	MGU time	BR time	BRI time
Noldx	16062.392	14078.300	8980.320	0.000	2545.080	2280.250	0.000
FP1	7006.758	6145.870	1816.100	25.710	450.760	379.570	40.150
FP6M	6000.177	5385.810	1181.710	38.240	99.110	39.010	55.660
NPDT	6082.246	5434.760	1184.750	64.910	83.110	33.200	79.910





Tim Cosgrove

Outlin

. . .

Motivation

Backgroun

The Beagle Theorem Prover Term Indexing

Indexing

mplementati

#### Implementing Fingerprint Indexing

Applications
Tailoring to
Beagle
Other

Improv

Result

Evaluation

Comparisons

Sample Positio

## Base Fingerprint Indexing

• Analysis of program indicated two key areas of improvement:

Tim Cosgrove

Outlin

Project Overvi

iviotivation

Backgrour

The Beagle Theorem Prover Term Indexing Fingerprint

Indexing

Implementati

Implementing Fingerprint Indexing

Indexing Applications Tailoring to Beagle Other

Improvemen

Evaluatio

Comparisons
Sample Position

Base Fingerprint Indexing

- Analysis of program indicated two key areas of improvement:
- Inferences via the Superposition rules.
- Simplifying Clauses.

Tim Cosgrove

Outline

oject Overvi

Motivation

Backgroun

The Beagle Theorem Prover Term Indexing

Fingerpri Indexing

Implementati

#### Implementing Fingerprint Indexing

Applications
Tailoring to
Beagle
Other

Other

Result

Evaluation

Metrics

Sample Position

## Creating the Fingerprint Index

· Addition of terms.

Tim Cosgrove

Outline

iect Ovenzi

Motivation

Backgroui

The Beagle Theorem Prover Term Indexing

Indexing

mplementati

#### Implementing Fingerprint Indexing

Applications
Tailoring to
Beagle
Other

Improvemen

Evaluation

Metrics
Beagle
Comparisons
Sample Position

Creating the Fingerprint Index

- Addition of terms.
- Retrieval of terms.

Tim Cosgrove

Outline

ct Overvie

Motivation

Backgroui

The Beagle Theorem Prover Term Indexing

Fingerprint

Indexing

mplementation

Implementing Fingerprint Indexing

Indexing Indexing

Applications

Tailoring to Beagle

Other

Result

Evaluation

Metrics

Sample Position

Indexing Superposition

• Refer to rule. Requires...

Tim Cosgrove

Motivation

Theorem Prover Term Indexing

Indexing

Implementing Indexing

Indexing

Applications

Other

Evaluation

## Indexing Simplification

• Refer to rule. Requires...

Tim Cosgrove

Outlin

Project Overvie

Motivation

Басквтопт

The Beagle Theorem Prover Term Indexing

Indexing

Implementati

Implementin Fingerprint Indexing

Application

Tailoring to Beagle

Other

Result

Evaluation

Beagle Comparisons

# Fingerprint Indexing for the Hierarchic Superposition with Weak Abstraction Calculus

• As mentioned, current implementation is somewhat 'naïve'.

Tim Cosgrove

Outlin

Project Overvi

Background

TL- D--

The Beagle
Theorem Prover
Term Indexing

Indexing

1 1 .....

Implementing Fingerprint Indexing

Applications
Tailoring to
Beagle

Other

Improvemen

Evaluatio

Comparisons

Sample Position Comparisons

# Fingerprint Indexing for the Hierarchic Superposition with Weak Abstraction Calculus

- As mentioned, current implementation is somewhat 'naïve'.
- Fingerprint indexing could be greatly improved by tailoring it specifically to Beagle's FOL calculus.
- Main improvement is to consider Beagle's foreground and background terms.

Tim Cosgrove

Theorem Prover Term Indexing

Indexing

Tailoring to Beagle

## Fingerprint Indexing for the Hierarchic Superposition with Weak Abstraction Calculus

- As mentioned, current implementation is somewhat 'naïve'.
- Fingerprint indexing could be greatly improved by tailoring it specifically to Beagle's FOL calculus.
- Main improvement is to consider Beagle's foreground and background terms.
- Furthermore indexing may be applied to more of HSWA's inference rules; in particular simplification.

Tim Cosgrove

Outline

Project Overvi

Backgroun

The Beagle Theorem Prover Term Indexing Fingerprint

Implementing

Fingerprint
Indexing
Indexing

Tailoring to Beagle

Improvemen

Evaluatio Metrics

Beagle Comparisons Sample Position Comparisons

# Fingerprint Indexing for the Hierarchic Superposition with Weak Abstraction Calculus

- As mentioned, current implementation is somewhat 'naïve'.
- Fingerprint indexing could be greatly improved by tailoring it specifically to Beagle's FOL calculus.
- Main improvement is to consider Beagle's *foreground* and *background* terms.
- Furthermore indexing may be applied to more of HSWA's inference rules; in particular simplification.
- These extensions will not require so much modification; as the fingerprint indexing framework is already built.

Tim Cosgrove

Outline

Project Overv

iviotivation

Басквтоп

The Beagle Theorem Prover Term Indexing Fingerprint Indexing

Implementation

Implementin Fingerprint Indexing Indexing Applications Tailoring to Beagle

Beagle Other Improvements

\_ \_\_\_\_\_

Evaluation Metrics

Sample Positio

## Other Potential Indexing Improvements

 An additional goal of the project is to consider how Fingerprint Indexing could be improved upon more generally.

Tim Cosgrove

Outlin

Project Overv

Backgrou

The Beagle Theorem Prover Term Indexing Fingerprint Indexing

Implementati

Implementing Fingerprint Indexing Indexing Applications Tailoring to Beagle

Other Improvements

Result

Evaluatio Metrics

Sample Positio

## Other Potential Indexing Improvements

- An additional goal of the project is to consider how Fingerprint Indexing could be improved upon more generally.
- The main area to consider here is the sampling positions.
   Sampling many positions reduces the returned sets, but increases indexing overhead.

Tim Cosgrove

Outlin

Project Overv

Backgrou

The Beagle Theorem Prover Term Indexing Fingerprint Indexing

Implementati

Implementing
Fingerprint
Indexing
Indexing
Applications
Tailoring to
Beagle
Other

Improvements

Results

Metrics
Beagle

Sample Positio Comparisons

## Other Potential Indexing Improvements

- An additional goal of the project is to consider how Fingerprint Indexing could be improved upon more generally.
- The main area to consider here is the sampling positions.
   Sampling many positions reduces the returned sets, but increases indexing overhead.
- Large problems better suit indexing; but it is difficult to know ahead of time what a 'large' problem is.

Tim Cosgrove

Outline

Project Overv

Motivation

Backgroui

The Beagle Theorem Prover Term Indexing Fingerprint Indexing

molementati

Implementing
Fingerprint
Indexing
Indexing
Applications
Tailoring to
Beagle

Other Improvement

Result

Evaluation Metrics Beagle Comparisons

## Metrics for Analysing Indexing Performance

• Speed - Not necessarily relevant

Tim Cosgrove

Outlin

Project Overvi

Motivation

Васкдоси

The Beagle
Theorem Prover
Term Indexing
Fingerprint
Indexing

Implementati

Implementing Fingerprint Indexing Indexing Applications Tailoring to Beagle

Other Improvemen

Resul

Evaluation Metrics

Comparisons
Sample Positio

## Metrics for Analysing Indexing Performance

- Speed Not necessarily relevant
- False Positives Relevant, but can be misleading depending on number of positions being sampled.

Tim Cosgrove

Outlin

Project Overvi

Backgroun

The Beagle Theorem Prover Term Indexing Fingerprint

Indexing

Implementing
Fingerprint
Indexing
Indexing
Applications
Tailoring to
Beagle
Other

Improvemen

Resun

Evaluation Metrics Beagle Comparisons Sample Position

## Metrics for Analysing Indexing Performance

- Speed Not necessarily relevant
- False Positives Relevant, but can be misleading depending on number of positions being sampled.
- Time Spent *per Inference* Booyah

Tim Cosgrove

Outlin

Project Ovenije

Motivation

Backgroui

The Beagle Theorem Prover Term Indexing Fingerprint Indexing

mplementat

Implementing Fingerprint Indexing Indexing Applications Tailoring to Beagle Other

Result

Evaluation Metrics

Beagle Comparisons

Sample Positio

## Comparing Varieties of Beagle

• Un-indexed beagle.

Tim Cosgrove

Motivation

Theorem Prover Term Indexing Indexing

Indexing Indexing Beagle Other

Evaluation Beagle

Comparisons

## Comparing Varieties of Beagle

- Un-indexed beagle.
- Minimal Indexing.

Tim Cosgrove

Outline

Project Overvi

Motivation

Backgroui

The Beagle Theorem Prover Term Indexing Fingerprint Indexing

Implementation

Implementin Fingerprint Indexing Indexing Applications Tailoring to Beagle

Other Improvement

Evaluation

Beagle Comparisons

Sample Positio

## Comparing Varieties of Beagle

- Un-indexed beagle.
- Minimal Indexing.
- Full Indexing.

Tim Cosgrove

Outline

Project Overv

Darel .....

The Beagle Theorem Prover Term Indexing

Indexing

Implementing Fingerprint Indexing Indexing Applications

Tailoring to Beagle Other

Improvemen

Evaluation

Beagle Comparisons

Sample Position Comparisons

## Comparing Varieties of Beagle

- Un-indexed beagle.
- Minimal Indexing.
- Full Indexing.
- Indexing with Optimisations.

Tim Cosgrove

Outline

iect Ovenie

Motivation

Backgrou

The Beagle Theorem Prover Term Indexing

Fingerpri Indexing

mplementati

Implementing Fingerprint Indexing Indexing Applications Tailoring to Beagle

Other

Results

Evaluation Metrics

Beagle

Comparisons
Sample Position

## Comparing Varieties of Beagle

Results table

Tim Cosgrove

Outline

Project Overvi

Motivation

Backgrour

The Beagle Theorem Prover Term Indexing Fingerprint

Indexing

Implementing Fingerprint Indexing Indexing Applications Tailoring to

Beagle Other

Result

Evaluation Metrics

Comparisons

Sample Position Comparisons

## Fingerprint Sampling Varieties

Reasoning. Cite shulz and FP/Speed balance

Tim Cosgrove

Outline

Project Overvie

Motivation

Backgrour

The Beagle Theorem Prover Term Indexing Fingerprint Indexing

Implementati

Implementin Fingerprint Indexing Indexing Applications Tailoring to Beagle

Other Improvement

Evaluation Metrics

Comparisons
Sample Position

Sample Position Comparisons

## Fingerprint Sampling Varieties

- Reasoning. Cite shulz and FP/Speed balance
- Different position samples

Tim Cosgrove

Outline

niect Ovenvie

Motivation

Backgrou

The Beagle Theorem Prover Term Indexing

Indexing

mplementation

Implementing Fingerprint Indexing Indexing Applications Tailoring to Beagle

Other

Result

Evaluation Metrics

Beagle Compariso

Sample Position Comparisons

## Fingerprint Sampling Varieties

Results table