Term Indexing for the Beagle Theorem Prover

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The Beagle Theorem Prover

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- Makes use of modular 'Background Theories' to make efficient use of known facts.
- This requires the carefully constructed 'Hierarchic Superposition with Weak Abstraction Calculus' in order to ensure consistency and completeness.

Extensions to Beagle

- Beagle has some major shortcomings which prevent it being more than a proof of concept.

Terminology Used in this Presentation

- First Order Logic

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- Substitutions:
  - s is '*unifiable*' with t : $\sigma s = \sigma t$
  - s '*subsumes*' t : $\sigma s = t$

The Superposition Calculus

- Normal Superposition rule

Positive Superposition $$\frac{l \approx r \vee C \qquad s[u] \approx t \vee D}{(s[r] \approx t \vee C \vee D)\sigma}$$

Where (i) $\sigma =$ simple mgu (l, u),

and (ii) u is not a variable.

The Hierarchic Superposition with Weak Abstraction Calculus

- Extension of the Superposition Calculus to accommodate hierarchic reasoning.

Positive Superposition $\dfrac{l \approx r \vee C \qquad s[u] \approx t \vee D}{\mathrm{abstr}((s[r] \approx t \vee C \vee D)\sigma)}$

Where (i) $\sigma =$ simple mgu (l, u),

(ii) u is not a variable,

(iii) $r\sigma \not\succeq l\sigma$,

(iv) $t\sigma \not\succeq s\sigma$,

(v) l and u are not pure background terms,

(vi) $(l \approx r)\sigma$ is strictly maximal in $(l \approx r \vee C)\sigma$,

and (vii) $(s \approx t)\sigma$ is strictly maximal in $(s \approx t \vee D)\sigma$.

Term Indexing Techniques

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- Three important relations:
- Top-Symbol Hashing.
- Discrimination Trees.
- Path Indexing.

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Table: Fingerprint Feature comparison tables for *unification* (left) and *subsumption* (right)

	f_1	f_2	**A**	**B**	**N**
f_1	Y	N	Y	Y	N
f_2	N	Y	Y	Y	N
A	Y	Y	Y	Y	N
B	Y	Y	Y	Y	Y
N	N	N	N	Y	Y

	f_1	f_2	**A**	**B**	**N**
f_1	Y	N	N	N	N
f_2	N	Y	N	N	N
A	Y	Y	Y	N	N
B	Y	Y	Y	Y	Y
N	N	N	N	N	Y

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A	Y	Y	Y	Y	N
B	Y	Y	Y	Y	Y
N	N	N	N	Y	Y

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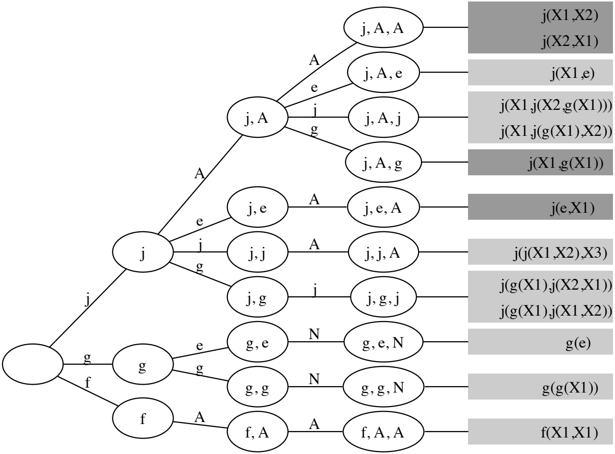
f_1	Y	N	N	N	N
f_2	N	Y	N	N	N
A	Y	Y	Y	N	N
B	Y	Y	Y	Y	Y
N	N	N	N	N	Y

- Schulz, Stephan: Fingerprint Indexing for Paramodulation and Rewriting. In: Lecture Notes in Computer Science volume 7364 pp. 447–483 (2012).

Fingerprint Indexing – Example Fingerprint Index



Base Fingerprint Indexing

- Analysis of program indicated two key areas of improvement:

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- Inferences via the Superposition rules.
- Simplifying Clauses.

Creating the Fingerprint Index

- Addition of terms.

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- Addition of terms.
- Retrieval of terms.

Indexing Superposition

- Refer to rule. Requires...

Indexing Simplification

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Negative Unit Simplification $\dfrac{l \not\approx r \qquad s \approx t \vee C}{C}$

Where (*i*) $\exists\sigma \;\; s.t. \;\; (l \approx r)\sigma \equiv s \approx t$.

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- This requires an index capable of matching converted Equation objects.

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- Fingerprint indexing could be greatly improved by tailoring it specifically to Beagle's FOL calculus.
- Main improvement is to consider Beagle's *foreground* and *background* terms.

Fingerprint Indexing for the Hierarchic Superposition with Weak Abstraction Calculus

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- Fingerprint indexing could be greatly improved by tailoring it specifically to Beagle's FOL calculus.
- Main improvement is to consider Beagle's *foreground* and *background* terms.
- Furthermore indexing may be applied to more of HSWA's inference rules; in particular simplification.

Fingerprint Indexing for the Hierarchic Superposition with Weak Abstraction Calculus

- As mentioned, current implementation is somewhat ‘naïve’.
- Fingerprint indexing could be greatly improved by tailoring it specifically to Beagle’s FOL calculus.
- Main improvement is to consider Beagle’s *foreground* and *background* terms.
- Furthermore indexing may be applied to more of HSWA’s inference rules; in particular simplification.
- These extensions will not require so much modification; as the fingerprint indexing framework is already built.

Other Potential Indexing Improvements

- An additional goal of the project is to consider how Fingerprint Indexing could be improved upon more generally.

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- The main area to consider here is the sampling positions. Sampling many positions reduces the returned sets, but increases indexing overhead.
- Large problems better suit indexing; but it is difficult to know ahead of time what a 'large' problem is.

Metrics for Analysing Indexing Performance

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- Time Spent *per Inference* - Booyah

Comparing Varieties of Beagle

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- Un-indexed beagle.
- Minimal Indexing.
- Full Indexing.
- Indexing with Optimisations.

Comparing Varieties of Beagle

Table: Totalled inference counts and indexing statistics for various versions of beagle.

	Inference Counts			Indexing Results		
Version	Sup	Demod	NegUnit	TotalFound	SupFP	SimpFP
Unmodified [1]	414216	29097	1826	0	0	0
Standard	162881	41414	2452	61884768	15525	39778148
Enhanced	162997	41435	2454	58535681	15401	39779224

Table: Totalled timing results for various versions of beagle.

	Time Spent (seconds)					
Version	Indexing	Retrieving	Sup	Demod	NegUnit	Total
Unmodified [1]	0	0	730.44	9.44	31.99	5623.21
Standard	28.4	38.73	254.17	41.66	3.18	381.36
Enhanced	22.91	20.29	180.54	32.6	2.51	281.38

[1] This version failed to solve two out of the fifty problems within 8 hours.

Results Analysis

- Times per inference

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- Extreme examples

Fingerprint Sampling Varieties

- Reasoning. Cite shulz and FP/Speed balance

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- Reasoning. Cite shulz and FP/Speed balance
- Different position samples

Fingerprint Sampling Varieties

Table: Totalled inference counts and indexing statistics for various Fingerprint sampling sets.

	Inference Counts			Indexing Results		
Sample Set	Sup	Demod	NegUnit	TotalFound	SupFP	SimpFP
FP3W	164574	42402	2473	59433145	72051	40037948
FP4M	150154	35709	1964	58989318	29469	40073471
FP6M	146861	35326	1960	58119897	17641	39916687
FP7	161411	41005	2441	58530669	23903	39818531
FP8X2	161741	40876	2439	58336597	11754	39823989

Table: Totalled timing results for various Fingerprint sampling sets.

	Time Spent (seconds)					
Sample Set	Indexing	Retrieving	Sup	Demod	NegUnit	Total
FP3W	12.48	14.86	177.82	28.98	1.79	265.65
FP4M	14.37	15.02	173.32	31.53	1.83	261.84
FP6M	18.74	17.58	168.79	30.56	2.12	259.02
FP7	22.26	19.82	180.13	35.23	2.38	282.22
FP8X2	51.73	34.23	195.18	42.75	4.07	331.01