Term Indexing for the Beagle Theorem Prover

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The Beagle Theorem Prover

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- Makes use of modular 'Background Theories' to make efficient use of known facts.
- This requires the carefully constructed 'Hierarchic Superposition with Weak Abstraction Calculus' in order to ensure consistency and completeness.

Superposition Calculus

- Normal Superposition rule

Hierarchic Superposition with Weak Abstraction Calculus

- Extension of Superposition Calculus to accommodate hierarchic reasoning.

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- Top-Symbol Hashing.
- Discrimination Trees.
- Path Indexing.

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Unification

	f_1	f_2	**A**	**B**	**N**
f_1	**Y**	**N**	**Y**	**Y**	**N**
f_2	**N**	**Y**	**Y**	**Y**	**N**
A	**Y**	**Y**	**Y**	**Y**	**N**
B	**Y**	**Y**	**Y**	**Y**	**Y**
N	**N**	**N**	**N**	**Y**	**Y**

Matching

	f_1	f_2	**A**	**B**	**N**
f_1	**Y**	**N**	**N**	**N**	**N**
f_2	**N**	**Y**	**N**	**N**	**N**
A	**Y**	**Y**	**Y**	**N**	**N**
B	**Y**	**Y**	**Y**	**Y**	**Y**
N	**N**	**N**	**N**	**N**	**Y**

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A	**Y**	**Y**	**Y**	**Y**	**N**
B	**Y**	**Y**	**Y**	**Y**	**Y**
N	**N**	**N**	**N**	**Y**	**Y**

Matching

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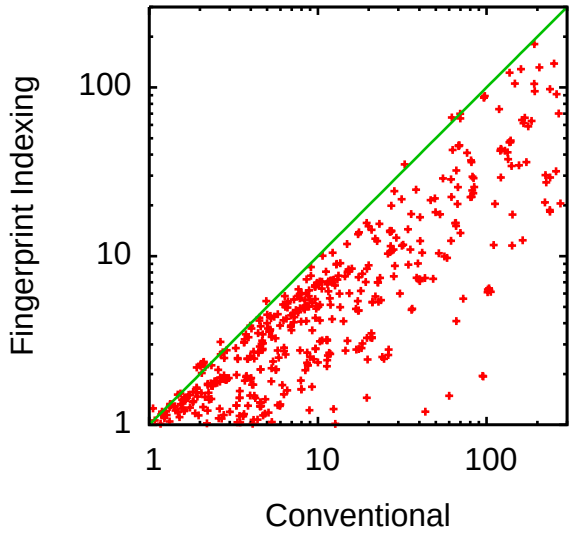
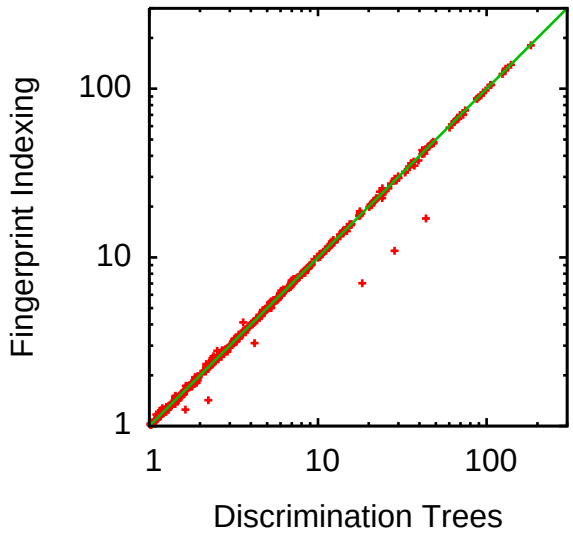
f_1	**Y**	**N**	**N**	**N**	**N**
f_2	**N**	**Y**	**N**	**N**	**N**
A	**Y**	**Y**	**Y**	**N**	**N**
B	**Y**	**Y**	**Y**	**Y**	**Y**
N	**N**	**N**	**N**	**N**	**Y**

- Schulz, Stephan: Fingerprint Indexing for Paramodulation and Rewriting. In: Lecture Notes in Computer Science volume 7364 pp. 447–483 (2012).

Fingerprint Indexing – Potential Performance

Index	Run time	Sat time	PM time	PMI time	MGU time	BR time	BRI time
NoIdx	16062.392	14078.300	8980.320	0.000	2545.080	2280.250	0.000
FP1	7006.758	6145.870	1816.100	25.710	450.760	379.570	40.150
FP6M	6000.177	5385.810	1181.710	38.240	99.110	39.010	55.660
NPDT	6082.246	5434.760	1184.750	64.910	83.110	33.200	79.910





Base Fingerprint Indexing

- Analysis of program indicated two key areas of improvement:

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- Inferences via the Superposition rules.
- Simplifying Clauses.

Creating the Fingerprint Index

- Addition of terms.

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- Addition of terms.
- Retrieval of terms.

Indexing Superposition

- Refer to rule. Requires...

Indexing Simplification

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- Furthermore indexing may be applied to more of HSWA's inference rules; in particular simplification.

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- Fingerprint indexing could be greatly improved by tailoring it specifically to Beagle's FOL calculus.
- Main improvement is to consider Beagle's *foreground* and *background* terms.
- Furthermore indexing may be applied to more of HSWA's inference rules; in particular simplification.
- These extensions will not require so much modification; as the fingerprint indexing framework is already built.

Other Potential Indexing Improvements

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- Large problems better suit indexing; but it is difficult to know ahead of time what a ‘large’ problem is.

Metrics for Analysing Indexing Performance

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- Time Spent *per Inference* - Booyah

Comparing Varieties of Beagle

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- Un-indexed beagle.
- Minimal Indexing.
- Full Indexing.
- Indexing with Optimisations.

Comparing Varieties of Beagle

- Results table

Fingerprint Sampling Varieties

- Reasoning. Cite shulz and FP/Speed balance

Fingerprint Sampling Varieties

- Reasoning. Cite shulz and FP/Speed balance
- Different position samples

Fingerprint Sampling Varieties

- Results table