35A – Two Beam Interference – Path Length

When two electromagnetic waves (A and B) with the same intensity (I_0), frequency (ω) and polarization, but different phase, are coincident, the resultant intensity is:

$$I = 4I_0 \cos^2\left(\frac{\Delta\phi}{2}\right) \qquad \text{Eq. 35a}$$

where $\Delta\phi$ is the phase difference between the two waves. Each wave has a phase which depends on various effects, including optical path length (distance and index), reflection, and original phase difference; for instance, for wave A,

$$\phi_A = \phi_{originalA} + \frac{2\pi}{\lambda_v}\Delta(nL)_A + \Delta\phi_{reflectionA} \qquad \text{Eq. 35b}$$

We focus here on phase difference due to path difference $\Delta(nL)$.

1) Two parallel beams of light are brought together on a surface. Each beam has wave length $\lambda=500$ nm, index of refraction $n = 1.0000$, and intensity 1 W/m^2 at the surface. Also assume that there are no reflections, and that both beams have the same $\phi_{original}$. What is the resultant intensity for each of the following path lengths or phase differences between the beams?

Phase or path length difference	Intensity (W/m^2)
0 radians (phase)	
$\pi/2$ radians (phase)	
60 degrees (phase)	
3π radians (phase)	
300 nm (path length)	
1000 nm (path length)	
5.5 λ (path length)	

2) A wave of wavelength λ travels along a straight line between two points in space separated by distance L.
   a) Find the phase difference in the wave at these two points.

_____radians

b) A wave (A) with vacuum wavelength of λ_v travels a distance of L_1 in a medium with index of refraction n_1, then distance L_2 in a medium with index n_1, then a distance L_3 in a medium of index n_1. Fill in the table to the right with the change in phase for the wave traveling through each of these regions.
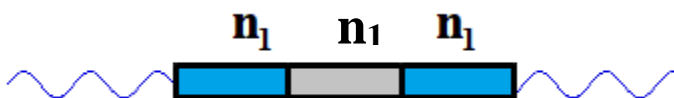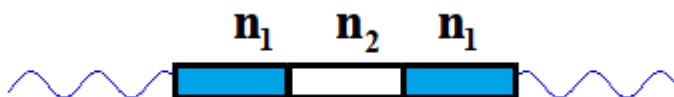
Distance for wave A	Index for A	Change in phase region for A
L_1	n_1	
L_2	n_1	
L_3	n_1	

n_1 n_1 n_1

c) For wave A, what is the total change in phase from the start to the end of this path (in terms of distances, vacuum wavelength, and indexes of refraction)?

_____radians

d) Light wave B travels the same physical distances as beam A, but the index in one of the regions is different (L_2, n_2). Fill in the change in phase in the table to the right for this case.

Distance for B	Index for B	Change in phase in region for B
L_1	n_1	
L_2	n_2	
L_3	n_1	

n_1 n_2 n_1

e) In many devices (such as an interferometer), a single beam can be split so that part of it travels path A as above and the other travels path B. What is the difference in phase between these two beams in terms of (L_i, n_i, and λ_v)?

f) On which of the following variables does the phase difference between the two beams depend? (Circle all that apply.)

L_1	L_2	L_3	n_1	n_2	λ_v

2

g) Some of the variables above don't matter to the phase difference. Explain which ones and why you think this is so.

L_1	L_2	L_3	n_1	n_2	λ_v

3) A beam of light of vacuum wavelength λ_v = 600.00 nm is split into two beams.
   - Beam I travels the following lengths through the respective mediums:

AIR	GLASS	AIR	GLASS	AIR
L_{air} = 10 cm	L_{glass} = 1 mm	L_{air} = 1 mm	L_{glass} = 1 mm	L_{air} = 10 cm

   - Beam II travels the following lengths through the respective mediums:

| AIR	GLASS	VACUUM	GLASS	AIR
L_{air} = 10 cm	L_{glass} = 1 mm	L_{vac} = 1 mm	L_{glass} = 1 mm	L_{air} = 10 cm

The index of refraction for each medium is:

n_{air} = 1.0003, n_{glass} = 1.5000, n_{vac} = 1.0000

(Take each of the indexes and thicknesses to be exact to 8 decimal places.)

a) Find the phase difference between these two beams at the end of their paths.

b) Assuming each of the beams has an intensity of 1W/m², find the intensity of the combined beams.

One type of device that uses phase differences between waves traveling on different paths for measurement purposes is an interferometer. At right is a schematic diagram of one type of interferometer known as a "Michelson interferometer."
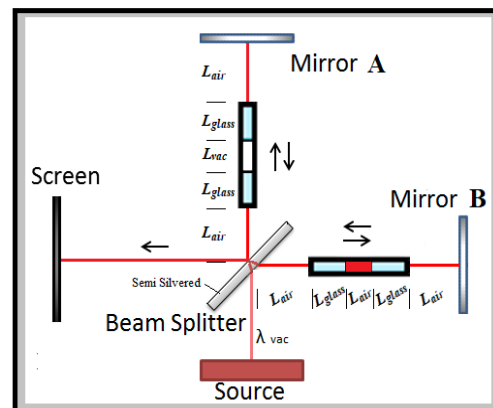
INTERFEROMETER DIAGRAM

35B – Experiment: Two Slit Interference

Equipment: Pasco magnetic optical rail; Red semiconductor laser diode; diode power
supply; Pasco Multiple Slit mask; a white screen to view the interference pattern; a small
ruler to measure distance on the interference image.

Background: If waves with the same wavelength emanate from two points and travel in straight paths (r_1
and r_2) through a uniform material (index n) to a common point as shown to the right, they will be out of
phase according to:

$\Delta\phi = \dfrac{2\pi n(r_1 - r_2)}{\lambda_v} + (\phi_{i1} - \phi_{i2})$. If r_1 and r_2 are very large compared to the distance between the sources d,

then $(r_1 - r_2) \cong d\sin\theta$ where θ is the angle between the normal to a line from one source to the other and
a line from sources to measurement point. This is shown schematically in the lower sketch above. If also

the original phases $(\phi_{i1}$ and $\phi_{i2})$ for the two sources are the same, $\Delta\phi = \dfrac{2\pi nd\sin\theta}{\lambda_v}$, which exhibits a

maximum in intensity whenever $\Delta\phi$ is an integer (m) multiple of 2π , or when $\underline{m\lambda_v = nd\sin\theta_m}$.
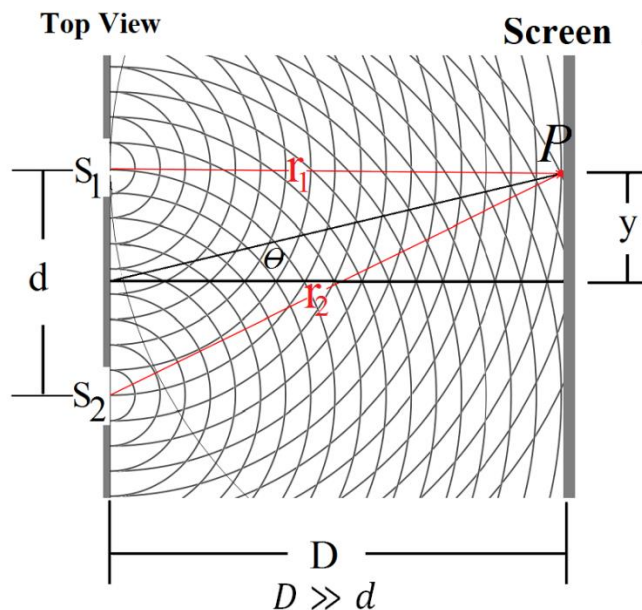

1) We made a point of labeling the wavelength with a subscript v.
    a) What do you think λ with this subscript denotes?

    b) Describe the meaning of $\dfrac{\lambda_v}{n}$.

If the angle θ_m at which a particular maximum intensity is small, then $\sin\theta_m \cong \dfrac{y_m}{D}$ where y_m is the distance y that the bright spot on the screen is displaced from the central maximum and D is the distance from the sources to the view screen. Putting it all together:

$m\lambda_v = nd\dfrac{y_m}{D}$ so the distance between the *m*th and the *m*'th bright spots is $\Delta y = y_m - y_{m'} = \dfrac{(m-m')D\lambda_v}{nd}$.

Order of Interference
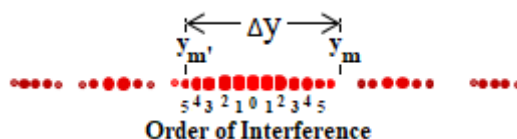
SAFETY NOTE: Avoid shining the laser directly into anyone's eyes. Prolonged direct viewing of the beam can lead to permanent eye damage.

In this experiment, you will observe the two slit interference pattern and estimate the wavelength of red light from the laser by measuring the distance between interference maxima.
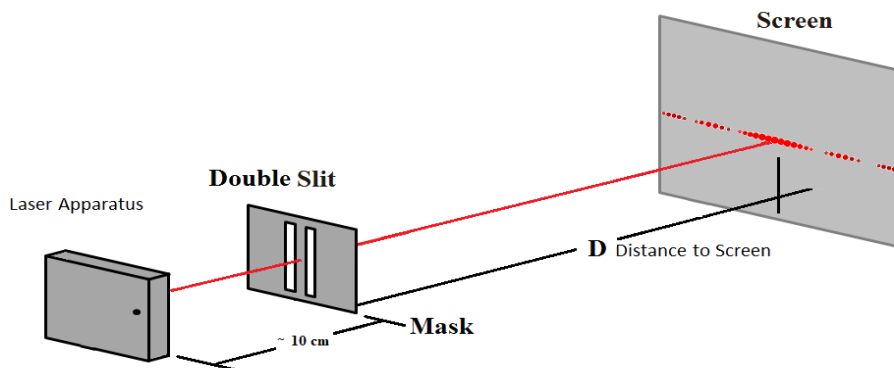
- Set up the diode laser to point through the apertures in the Multiple Slit mask and project onto the viewing screen. The laser and mask should be only 1-10 cm apart while the viewing screen should be $D = 70$ -100 cm away (more if you can).
- Set the Multiple Slit mask so that the laser passes through the slit width $a = 0.04$ mm and the slit separation $d = 0.25$ mm slits simultaneously. A double slit pattern should be observed on the viewing screen. Pick the fourth maximum located to the left of center and a fourth maxima located to the right of the central maximum (SHOULD BE nine maxima including the central maxima ($\Delta m = 8$).) Measure the distance between them and record in the table below. Repeat three times with different Δm or D values.
- Set the Multiple Slit mask so that the beam passes through the slit width $a = 0.04$ mm and the slit separation $d = 0.50$ mm. Make three measurements of $\Delta y = y_{right}$ - y_{left} for three sets of $\Delta m, D$ values and record below.

2) Record the information for the pattern you observe in the table below.

d (mm) (from the mask)	D (mm) (measured using meter stick)	Δm (chosen by you)	$\Delta y = y_{right} - y_{left}$ (observed on the screen)	λ (nm) (Calculated from the other information in the table)
0.25				
0.25				
0.25				
0.50				
0.50				
0.50				

3) Calculate the wavelength of the laser (in nm) using the information in your table and record it in the table. Calculate the average wavelength from your measurements.

Average Wavelength:_____

4) Compare your measurement to the typical wavelength range for red light? (You can check the text or Bing™ or Google ™ it, but indicate your source.)

5) You should have noticed that the width of the region on the screen that you could see bright spots on was relatively narrow. The intensity pattern looked something like this. Why do you think the overall pattern drops in intensity to the sides?
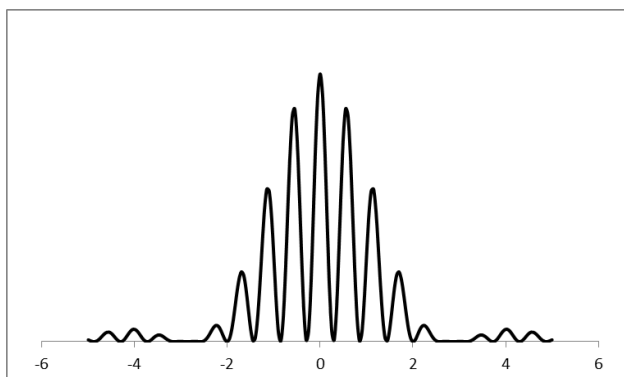
35D – The Diffraction (Interference) Grating - 3 Points EXTRA CREDIT
Objective: Determine the spacing between lines of a diffraction grating by observing interference peaks for light of a specific wavelength.
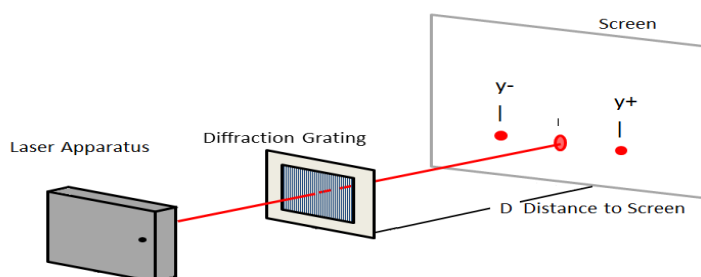
> **Equipment**: Low power laser. Diffraction grating, PASCO magnetic optics rail, PASCO magnetic optics carrier, Black metal safety screen, Ruler.

> **LASER SAFETY**: Avoid shining the laser in these laboratories directly into anyone's eyes. Prolonged direct viewing of the beam can lead to permanent eye damage.

> *Background*: A diffraction grating consists of a reflective or transmitting surface with a periodic array of lines. In the simplest theoretical model, we assume that each line on the grating acts as a infinitesimally thin emission source with all of the sources emitting at the same wavelength and in set phase to one another. For a beam of wavelength λ at normal incidence to the plane of the grating, interference peaks will be observed at angles $\theta_m = \arcsin\left(\dfrac{m\lambda}{d}\right)$, where m is an integer ("Order of Interference") and d is the distance between the grating emitting lines.

Procedure: Set up the laser so that it shines through the middle of a grating held in place on a magnetic carrier. Measure the lateral displacement between the central maximum peak and the two interference maxima on either side of it. Record this measurement and the distance D of the ruler from the grating to the screen to compute the angle. (*The angle may be large – greater than 20 degrees – therefore the viewing ruler may have to be close to the grating*.)
Make at least 4 measurements with different D's. Use a laser wavelength λ = 650 nm and compute the d-spacing for each measurement.

Note:
θ+ = *angle of bright spot located at* y+.
d+ = *calculated spacing between slits for bright spot at angle* θ+.
θ- = *angle of bright spot located at* y-.
d- = *calculated spacing between slits for bright spot at angle* θ-.

TRIALS	D (mm)	y+	y-	θ+	θ-	d+	d-
1							
2							
3							
4							

Average d-spacing = _____ +/- _____ (from standard deviation of your 8 measurements d+, d-)

Gratings are often described in terms of the number of lines per centimeter, n_{lines}.
Convert your d-spacing into lines per centimeter.   _____