Notice: Class Section Numbers

- The number of students who enter the wrong section class section number on their exams and go to the wrong exam room has become disconcertingly high.
- To promote better mental health and discipline amongst our students, we are implementing the following policy.
- **Students must write their PHYS 1200 class section number on the front page of their submitted labs.** Labs that are missing their section number, or are incorrect, will have three (3) points deducted from their total lab score.
- This policy starts with Class 20 (Thursday Mar. 28/Friday Mar. 29) and will remain in effect until the end of the semester.

Physics 1200
Lecture 20
Spring 2024

Superposition of Waves and Wave Interference of Coherent Light Beams, Young's Double-Slit Experiment, Intensity in Two-Beam Interference, Diffraction Grating, Michelson Interferometer

Addition of Light Waves: Superposition

- Interference phenomena between two or more light waves can occur when the waves are coherent and overlap in a certain region of space.

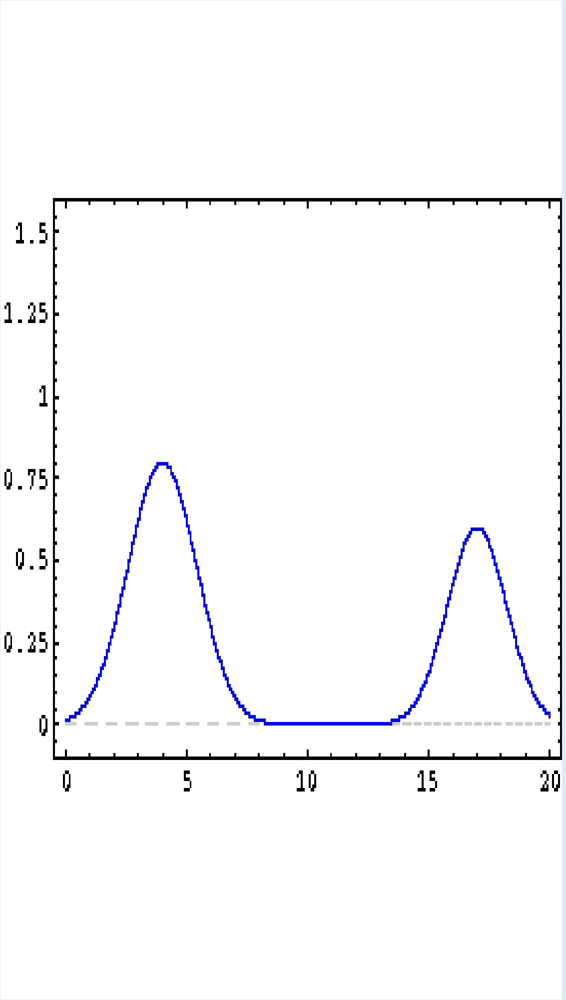
  - “Coherent” means that the waves have a phase relationship that is constant in time.

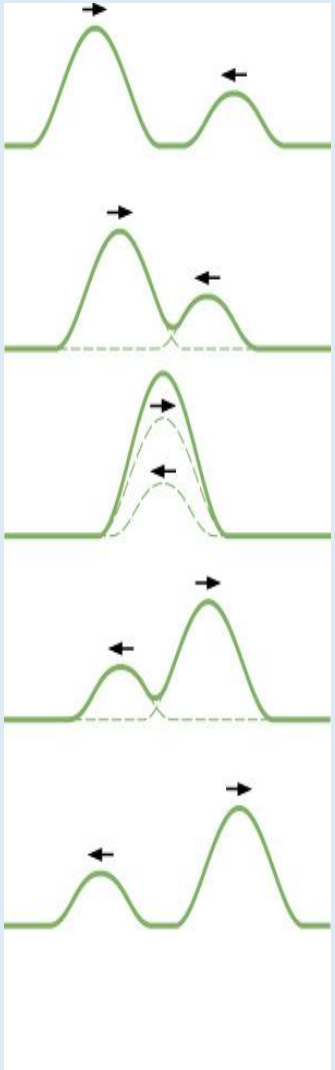
  - When this occurs, the resultant wave from the interactions of the waves is obtained by the principle of superposition: when two or more waves overlap, the resultant displacement at any point and at any instant is found by adding the instantaneous displacements that would be produced at the point by the individual waves if each were present alone.

  - In a nutshell: sum the waves at the point of interest to get net wave.

Addition of Light Waves: Superposition (2)

- $y(x,t) = y_1(x,t) + y_2(x,t)$





Interference of Waves

- Interference relates the difference in the phases of waves that are being superposed at a point.
- Consider two waves of <u>equal amplitude and the same frequency</u> being superposed at same point in space:

$$E_1 = E_0 \cos(-\omega t), \;\; E_2 = E_0 \cos(-\omega t + \Delta\phi),$$

Resultant wave:

$$E_{tot} = E_1 + E_2 = 2E_0 \cos\left(\frac{\Delta\phi}{2}\right) \cos\left(-\omega t + \frac{\Delta\phi}{2}\right),$$

used well-known trigonometric relation

$$\cos A + \cos B = 2\cos\frac{1}{2}(A - B)\cos\frac{1}{2}(A + B)\ .$$

Note: "kx" terms of the waves have been absorbed into phase value difference, $\Delta\phi$.

Interference of Waves (2)

- Resultant wave has form

$$\boxed{E_{tot} = E_m \cos\left(-\omega t + \frac{\Delta\phi}{2}\right)},$$

where $\boxed{E_m = 2E_0 \cos\left(\frac{\Delta\phi}{2}\right)}$ = amplitude of the resultant wave,

and $\boxed{\Delta\phi = \phi_1 - \phi_2}$ is net phase difference between the waves.

- To get the intensity of the resultant wave, recall that

$$I = S_{\text{av}} = \frac{1}{2}\epsilon_0 c E_m^2 = \frac{1}{2}\epsilon_0 c \left(2E_0 \cos\left(\frac{\Delta\phi}{2}\right)\right)^2$$

$$\Rightarrow \boxed{I = 4I_0 \cos^2\left(\frac{\Delta\phi}{2}\right)},$$

where $\boxed{I_0 = \frac{1}{2}\epsilon_0 c E_0^2}$ is the intensity of just <u>one</u> of the waves 1, 2.

- Note: could have also found expression for E_m using <u>phasors</u>, which are designed specifically for the case of calculating amplitudes by adding waves of differing phases.

Lecture Question 20.1

- For which of the phase differences is the amplitude for two-wave (equal amplitude and frequency) interference a minimum?

A. $\Delta\phi = 0$ radians.

B. $\Delta\phi = \frac{\pi}{3}$ radians.

C. $\Delta\phi = \frac{\pi}{2}$ radians.

D. $\Delta\phi = \pi$ radians

E. None of the Above.

Solution

- Have intensity relation

  $I = 4I_0 \cos^2\left(\frac{\Delta\phi}{2}\right).$

  Intensity minimum occurs when $\cos^2\left(\frac{\Delta\phi}{2}\right) = 0.$

  For this to be true, require $\frac{\Delta\phi}{2} = \frac{m\pi}{2}$, where $m = \pm(odd\ integer)$.

  $\therefore \Delta\phi = \pm\pi, \pm 3\pi, \pm 5\pi, \ldots$, for there to be a minimum.

  From the list of choices, the only answer that satisfies this is

  $\Delta\phi = \pi$ radians.

  Answer D.

Interference Effects: Phase Differences.

- Result for two-wave interference shows that intensity of the resultant wave depends on the phase difference between the two waves that are being superposed (i.e., added).
- Phase differences can arise for the following reasons:
  - ➢ Phase differences generated from sources of the waves. “Intrinsic (or, source)” phase shifts.
  - ➢ Differences in distances traveled (path length) to get to overlap point.
  - ➢ Path length through regions of differing index of refraction.
  - ➢ Reflection.

Interference Effects: Phase Differences

- Above listed wave phase differences can be written as the sum

$$\boxed{\Delta\phi = \Delta\phi_{\text{path}} + \Delta\phi_{\text{source}} + \Delta\phi_{\text{ref}}},$$

$\Delta\phi_{\text{path}}$ = phase difference between waves 1 and 2 due to differences in path lengths traveled by the waves, including differences in refractive index n along paths,

$\Delta\phi_{\text{source}}$ = intrinsic phase difference introduced by wave sources,

$\Delta\phi_{\text{ref}}$ = the phase difference caused by reflection of the waves at an interface between regions of different index of refraction n.

Phase Differences: Path Length in Vacuum

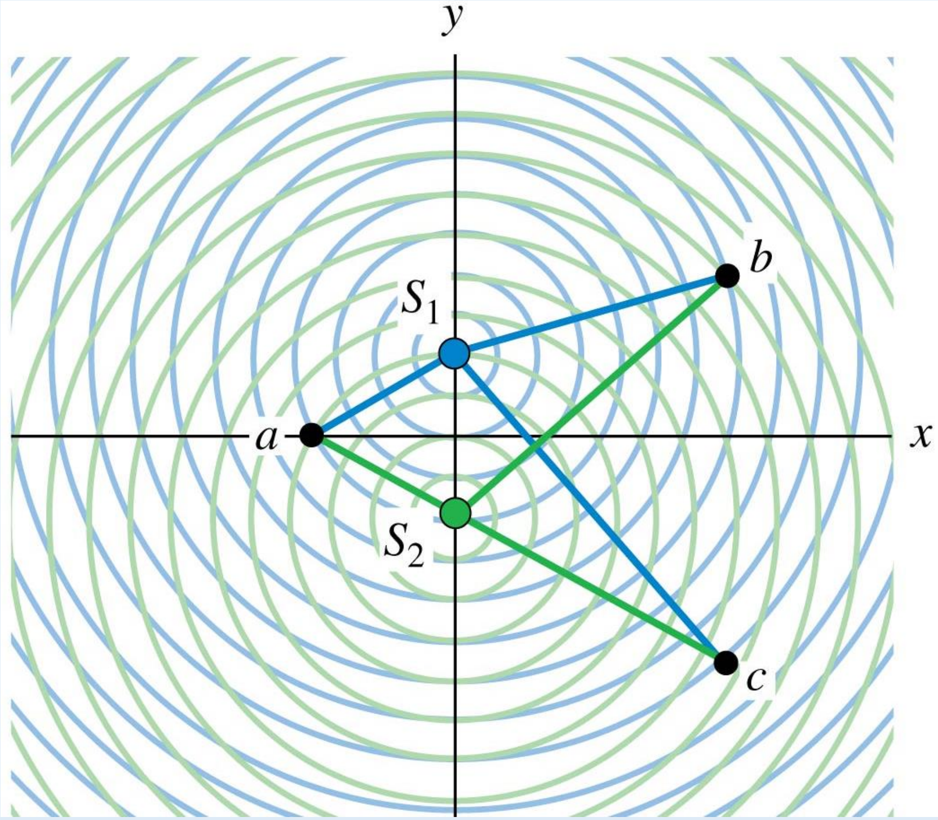
- In vacuum, if the waves have same frequency, they have same wavelength.
- If wave 1 travels distance L_1 to get to overlap point, the travel phase for that path is $\phi_1 = kL_1 = 2\pi\frac{L_1}{\lambda}$ (used $k = 2\pi/\lambda$).
- Likewise, if wave 2 travels distance L_2 to get to overlap point, travel phase for that path taken is $\phi_2 = kL_2 = 2\pi\frac{L_2}{\lambda}$.
- Phase difference between the two waves for travel to overlap point in this case is

$$\boxed{\Delta\phi = \phi_1 - \phi_2 = 2\pi\left(\frac{L_1 - L_2}{\lambda}\right)}\ .$$

  - Constructive interference when $\frac{(L_1 - L_2)}{\lambda} = m, \quad m = 0, \pm 1, \pm 2, \pm 3, \ldots$
  - Destructive interference when $\frac{(L_1 - L_2)}{\lambda} = \frac{m'}{2}, \quad m' = \pm 1, \pm 3, \pm 5, \ldots$

Phase Differences: Path Length in Vacuum (2)

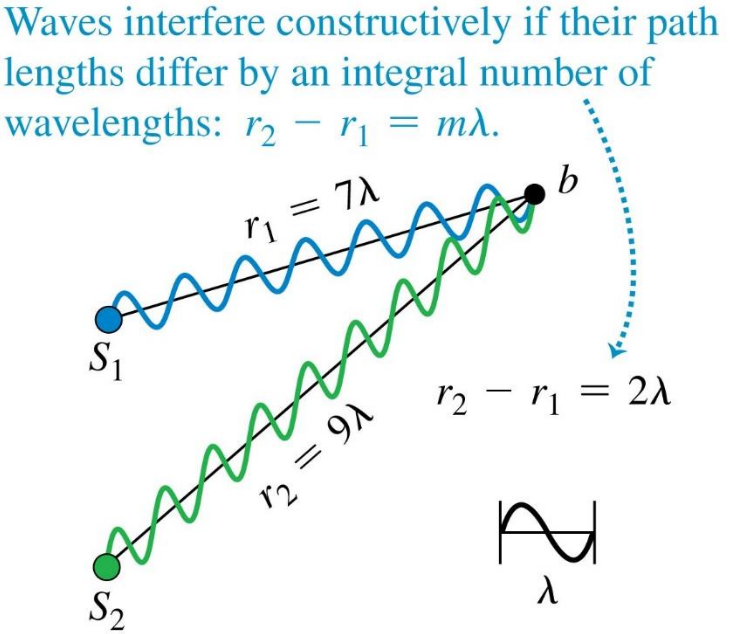
- Vacuum example, continued: shown are two identical sources of monochromatic waves, S_1 and S_2.
- The two sources are coherent (i.e., they permanently maintain a <u>constant intrinsic phase difference</u>); they vibrate in unison.
- Constructive interference occurs at point *a* (equidistant from the two sources).



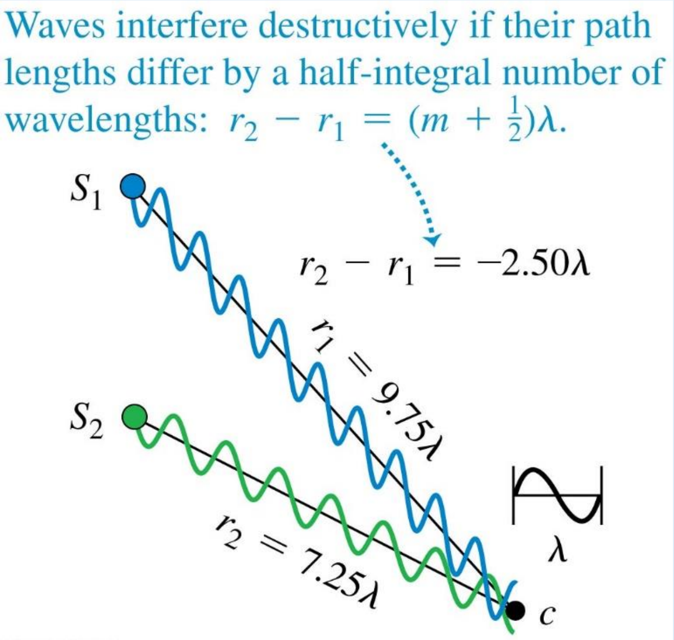
Phase Differences: Path Length in Vacuum (3)

Vacuum example, continued:

- The distance from S_2 to point *b* is exactly two wavelengths greater than the distance from S_1 to *b*.
- The two waves arrive in phase, and they reinforce each other. Constructive interference.



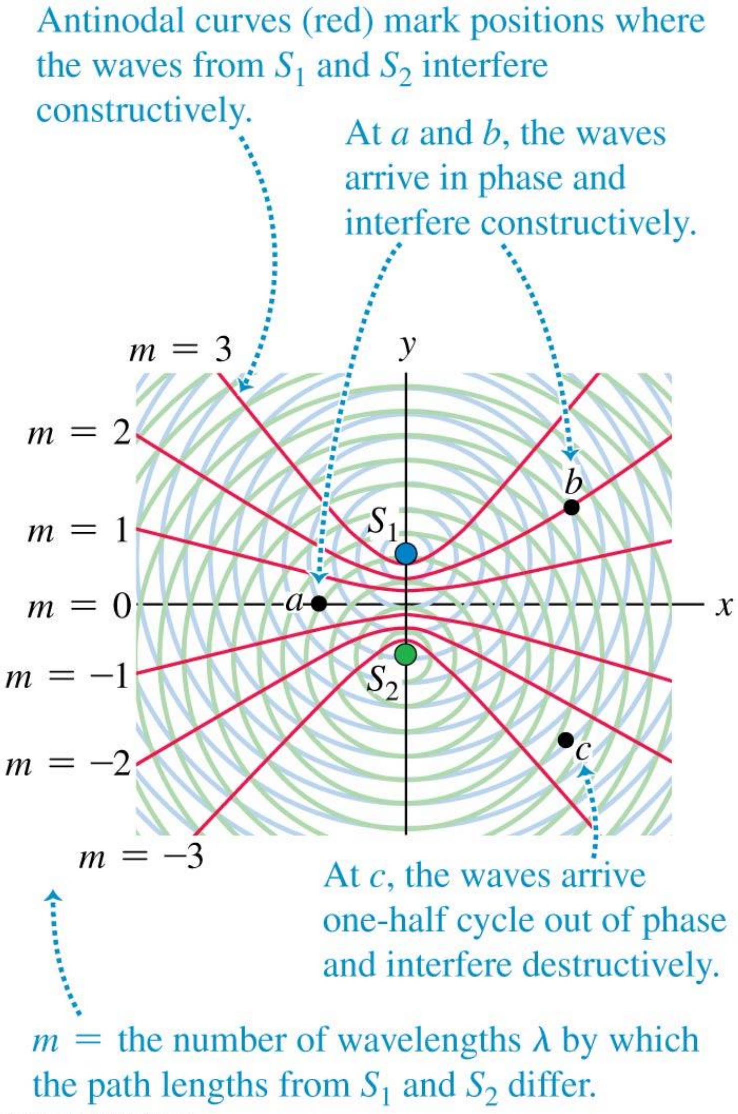
- The distance from S_1 to point *c* is a half-integral number of wavelengths greater than the distance from S_2 to *c*.
- The two waves cancel or partly cancel each other. Destructive interference.



Phase Differences: Path Length in Vacuum (4)

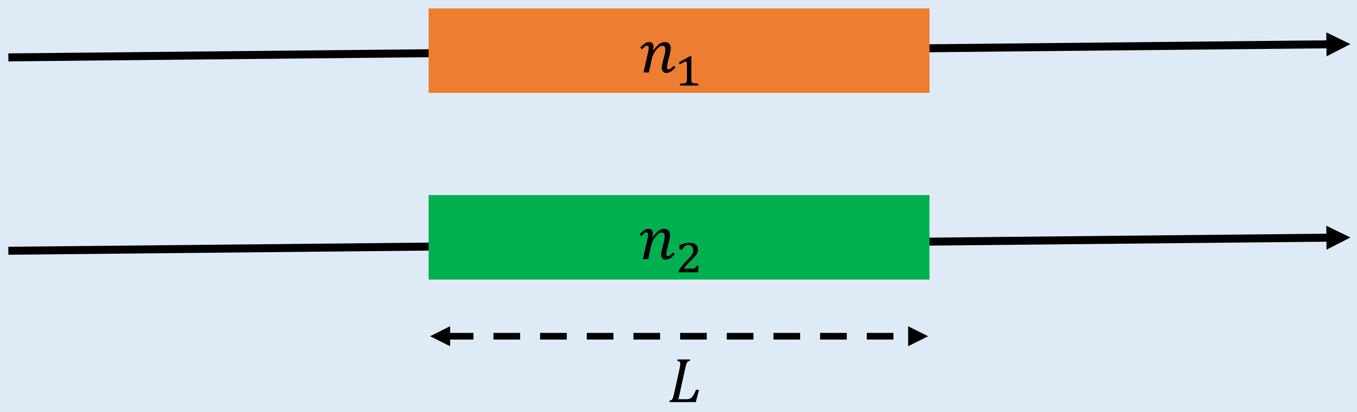
Vacuum example, continued:

- The red curves show all positions where constructive interference occurs; these curves are called antinodal curves.
- Not shown are the nodal curves, which are the curves that show where destructive interference occurs.



Phase Change Due to Paths with Different Refractive Index

- Example: two different waves, both through a path segment of equal length but different indices of refraction:



Phase Differences: Path Length With Differing Index of Refraction

- If the two waves travel in regions of different index of refraction, the effective (or, optical) path length can be different even if they travel the same physical distance L, due to the fact that the wavelength of the waves will be different in regions of different refractive index.
- Consider case where waves 1 and 2 both travel a distance L to get to overlap point. Suppose wave 1 travels in a region of refractive index n_1, and wave 2 travels in a region of refractive index n_2.
- Phase of travel for wave 1 will be $\phi_1 = k_1 L = 2\pi\, L/\lambda_1 = 2\pi\, L n_1/\lambda$, where we used the relation $\lambda_1 = \lambda/n_1$, λ = vacuum wavelength of the wave.
- Similarly, for wave 2 the travel phase $\phi_2 = k_2 L = 2\pi\, L/\lambda_2 = 2\pi L\, n_2/\lambda$.
- Net phase change between the two waves is

$$\boxed{\Delta\phi = \phi_1 - \phi_2 = 2\pi\left(\frac{L}{\lambda}\right)(n_1 - n_2)}\ .$$

Phase Differences: Combined Path and Refraction Effects

- The effects of differing path lengths <u>and</u> refractive indexes for the two-wave interference being considered can be combined into more general phase difference result:

$$\boxed{\Delta\phi = 2\,\pi\,\left(\frac{n_1 L_1 - n_2 L_2}{\lambda}\right)}\,,$$

where λ is the wavelength in vacuum.

Phase Differences: General Multi-Step Path Length

- Including the effect of index of refraction on wavelength in a medium, the phase difference between two interfering waves has the general expression

$$\boxed{\begin{aligned}\Delta\phi_{\text{path}} &= \sum_{j=1}^{N_1} k_{1j}x_{1j} - \sum_{j=1}^{N_2} k_{2j}x_{2j} \\ &= \frac{2\pi}{\lambda_{\text{vac}}}\left(\sum_{j=1}^{N_1} n_{1j}x_{1j} - \sum_{j=1}^{N_2} n_{2j}x_{2j}\right)\end{aligned}} \;,$$

λ_{vac} = vacuum wavelength of waves 1 and 2,

N_i = number of path segments wave i $(= 1, 2)$ to get from starting point to position where the waves overlap,

n_{ij} = index of refraction wave i propagates in on j-th segment of its travel path,

x_{ij} = distance wave i travels across segment of path j.

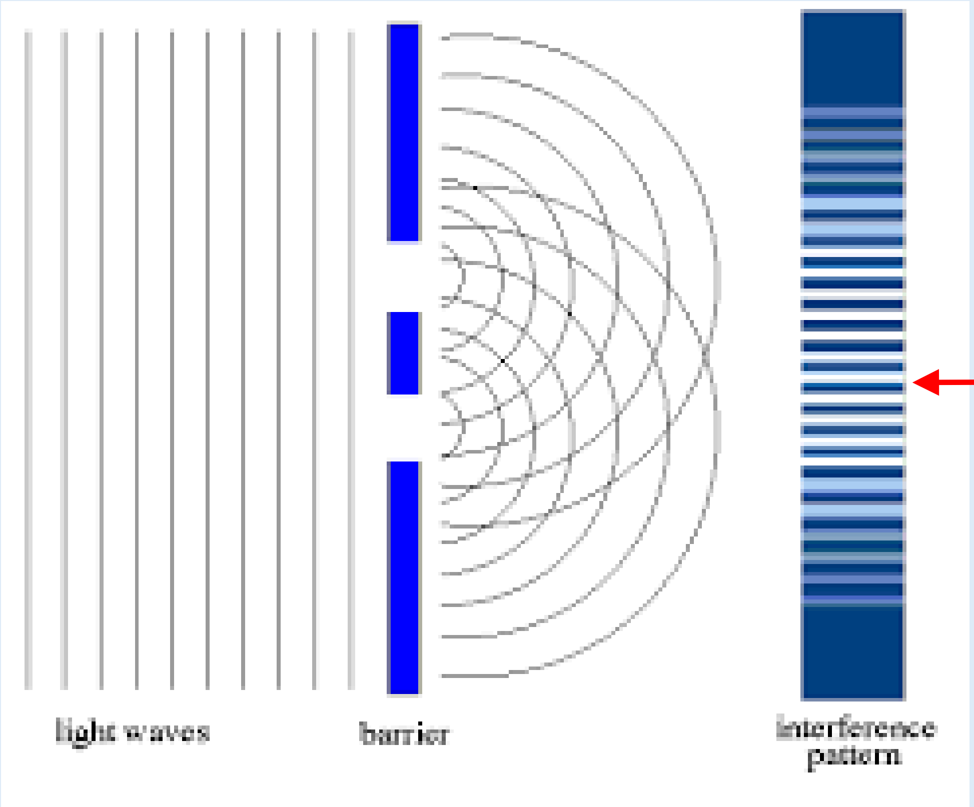
- Used the relations $k = 2\pi/\lambda$, and $\lambda = \lambda_{\text{vac}}/n$ in the path expression.
- Quantity nx is often referred to as the "<u>optical path length</u>."

Path Difference in Vacuum: Young's 2-Slit Interference Experiment

- Experiment performed by Young (1803). Demonstrated interference of light waves, establishing wave-like behavior of light.
- It uses two slits on a barrier: incident light on the barrier is blocked everywhere by the barrier, except at the slits, where the light can pass through. From Huygens' principle, the two individual slits act as independent sources of light waves.
- Additionally, the light passing through the slits comes from same original wave front and is therefore coherent with each slit.
- Light from slit-sources propagate to a distant screen (relative to the slit size and separation, and the wavelength of the interfering light).
- You will be performing this experiment using lasers in your lab today.

Path Difference in Vacuum: Young's 2-Slit Interference Experiment (2)

- Schematic:

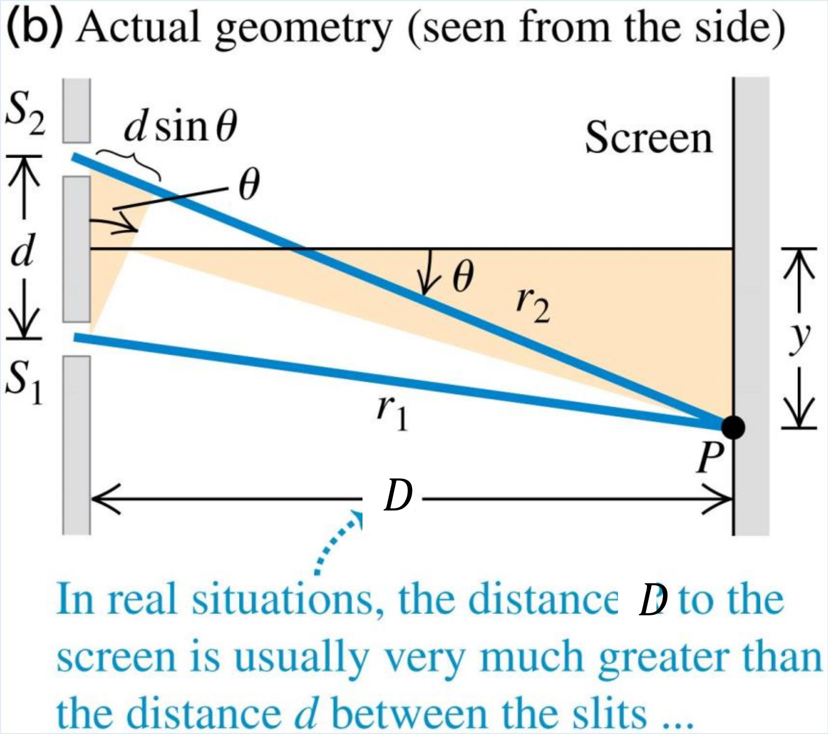


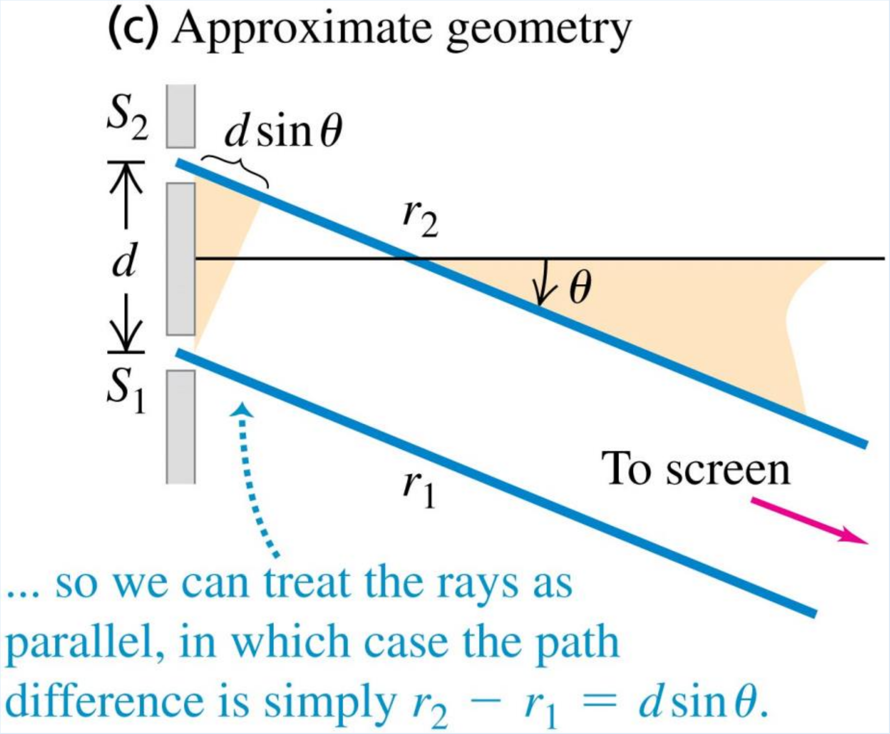
This is what is seen in the experiment.

The interference pattern on the screen shows bright bands (locations of intensity maxima, i.e., constructive interference) and dark bands (locations of intensity minima, i.e., destructive Interference) on the far-away screen.

Path Difference in Vacuum: Young's 2-Slit Interference Experiment (3)

- Geometry and path lengths: figure (b) shows actual geometry of Young's experiment. If distance D to screen is much greater than distance d between the slits, we can use approximate geometry of figure (c).





Path Difference in Vacuum: Young's 2-Slit Interference Experiment (4)

- Constructive interference (reinforcement) occurs at points where path difference is an integral number of wavelengths, $m\lambda$.
- Bright regions on the screen occur at angles θ for which:

Constructive interference, two slits:

Distance between slits   Wavelength

$$d\sin\theta = m\lambda \qquad (m = 0, \pm 1, \pm 2, \ldots)$$

Angle of line from slits to *m*th bright region on screen

- Destructive interference (cancellation) occurs, forming dark regions on the screen, at points for which the path difference is a half-integral number of wavelengths.

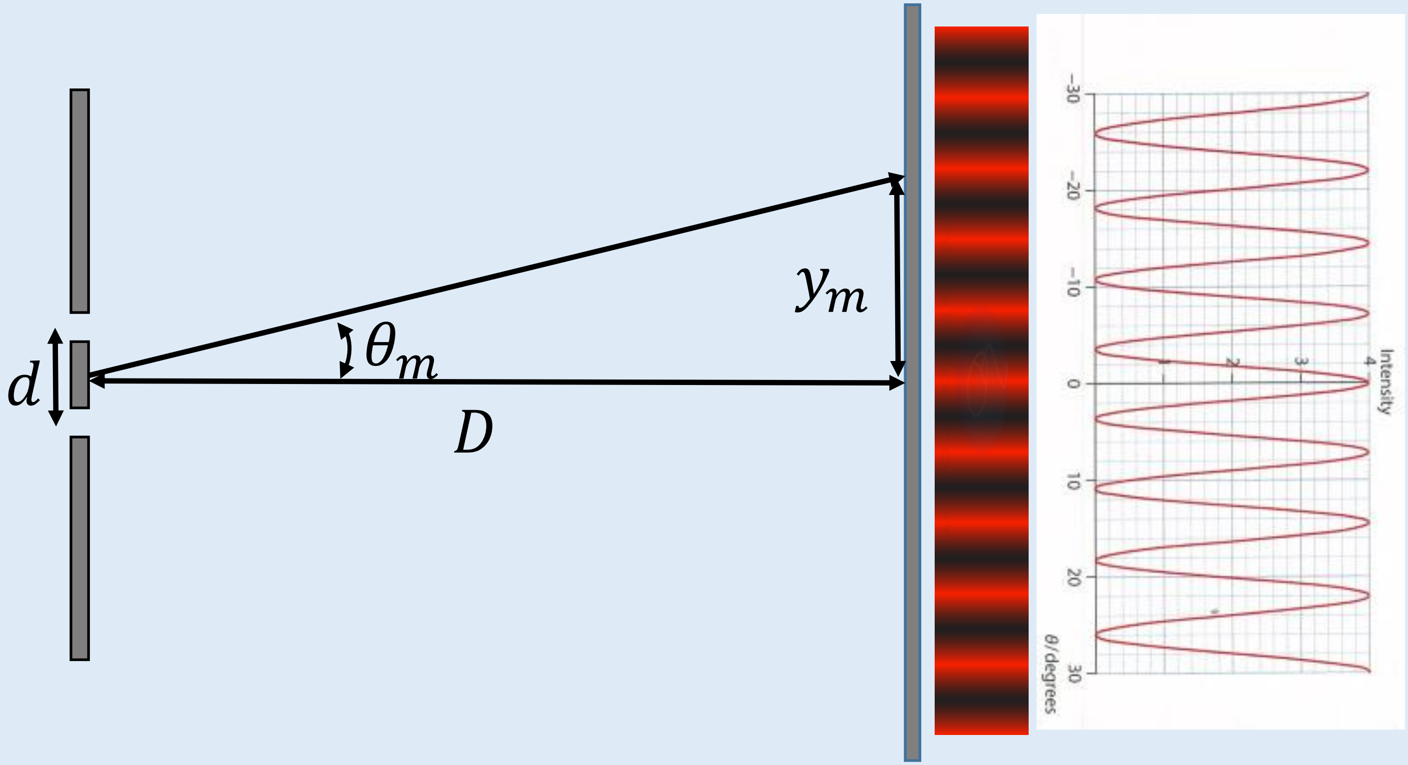
Destructive interference, two slits:

Distance between slits   Wavelength

$$d\sin\theta = \left(m + \tfrac{1}{2}\right)\lambda \qquad (m = 0, \pm 1, \pm 2, \ldots)$$

Angle of line from slits to *m*th dark region on screen

Path Difference in Vacuum: Young's 2-Slit Interference Experiment (4)



- For small angles, can make approximation $\sin\theta_m \simeq \tan\theta_m = y_m/D$.

  $\therefore$ For interference maxima (at small angles) in Young's experiment,

$$\frac{y_m d}{D} = m\lambda \quad \Rightarrow \boxed{y_m = \frac{m\lambda D}{d}}, m = 0, \pm 1, \pm 2, \ldots$$

Interference Effects: Animation

- A great simulator for two-wave two-dimensional interference can be found at http://www.falstad.com/ripple/ .
- Simulator can also be used to investigate all the phenomena discussed in this class, as well as diffraction phenomena (forthcoming), interference between multiple sources, scattering, etc.

The Diffraction Grating

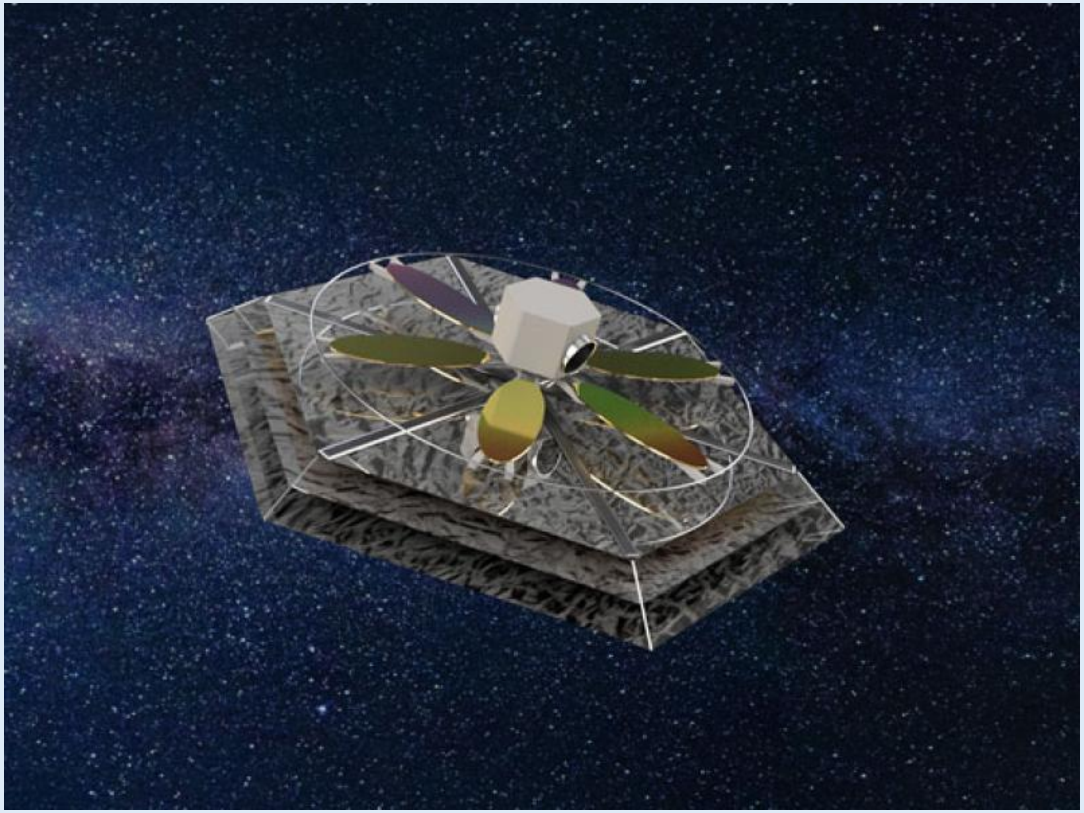
- A diffraction grating is an instrument that can be used in spectroscopy and spectrometry to measure the spectra of light emitted by a source.
  - ➢ They are used regularly in astronomy to determine the spectral properties of stars and astrophysical phenomena. They are used to separate the spectral lines of a particular astrophysical emitter (e.g., stars, quasars, planets and their atmospheres, etc.).
  - ➢ Diffraction gratings are also used for DNA spectrophotometry, and other studies of light absorption by biological molecules.

The Diffraction Grating – Henry Rowland

- Henry Rowland (RPI, class of 1870) was a pioneer in the field of creating the best-ruled diffraction gratings of his time, crucial to the foundation and development of astrophysical spectroscopy.
  - Maxwell held Rowland in high esteem.
  - Rowland was also a professor in the RPI Physics Dept. for a while.
  - But RPI blew it (didn't support him and his work enough). He moved on to Johns Hopkins, later becoming chair of their Physics Dept.
  - RPI's Jonsson-Rowland Science Center (J-Rowl SC) is named in part after him.

The Diffraction Grating: DICER Space Telescope

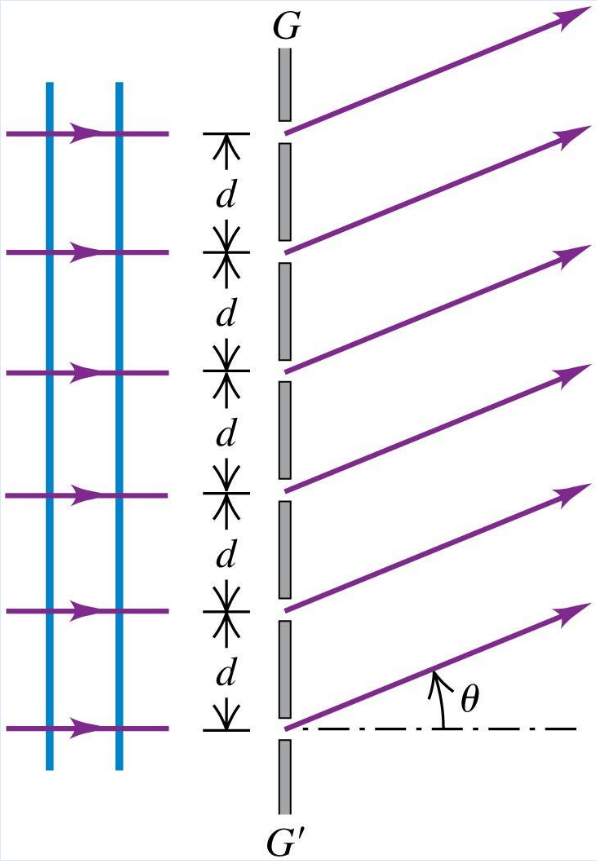
- Diffractive Interfero Coronagraph Exoplanet Resolver (DICER)
- New telescope concept design project by Prof. Heidi Newberg (Dept. of Physics, RPI) and coworkers, recently awarded large NASA grant.
- Uses 10 m diffraction gratings. Will look for oxygen spectral lines from atmospheres of Earth-like exoplanets in "habitable zones" (where water can be liquid) within 10 pc from the Sun (1 pc = 3.26 light years).
- Would orbit Sun in Sun-Earth Lagrange Point 2 (L2).



https://news.rpi.edu/content/2023/01/26/team-aims-find-%E2%80%98earth-20%E2%80%99

The Diffraction Grating – Fundamentals

- Diffraction gratings consist of either a large series of slits that pass light (a transmission grating), or grooves that reflect light (a reflection grating).
- The slits or grooves act as scatterers, that is, coherent multiple sources of light that can interfere constructively or destructively depending on the angle of the scattered light rays with respect to their incident direction.
- In the figure, GG' is a cross section of a transmission grating. The slits are perpendicular to the plane of the page. The diagram shows only six slits; an actual grating may contain several thousand. (You will experimentally determine the number for a diffraction grating in your lab today.)



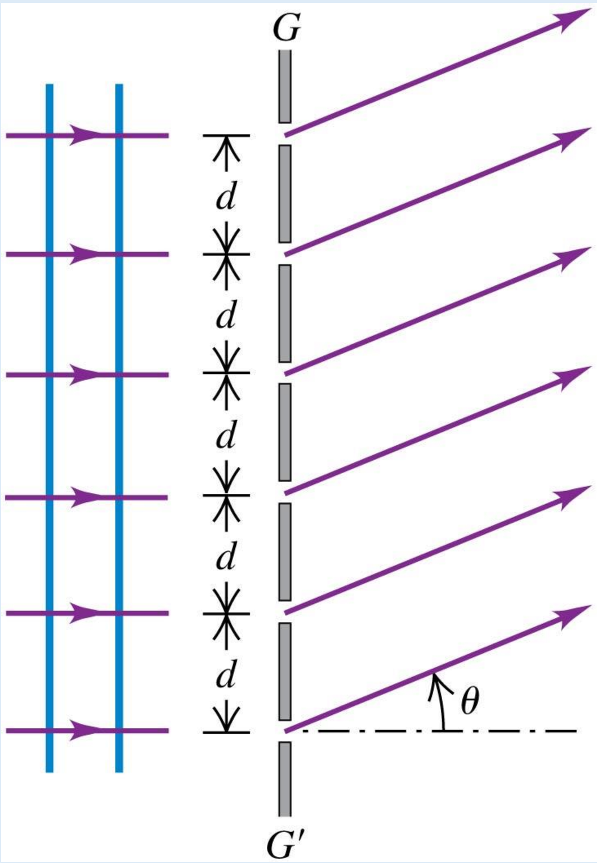
The Diffraction Grating – Fundamentals (2)

- Like the analysis for the two-slit interference problem (Young's experiment), the condition for constructive interference (= intensity maxima) for the multiple-slit diffraction grating for normally incident light (as shown in the figure) is

  $$\boxed{d \sin\theta = m\lambda}\ , \quad m = 0, \pm 1, \pm 2, \pm 3, \ldots$$

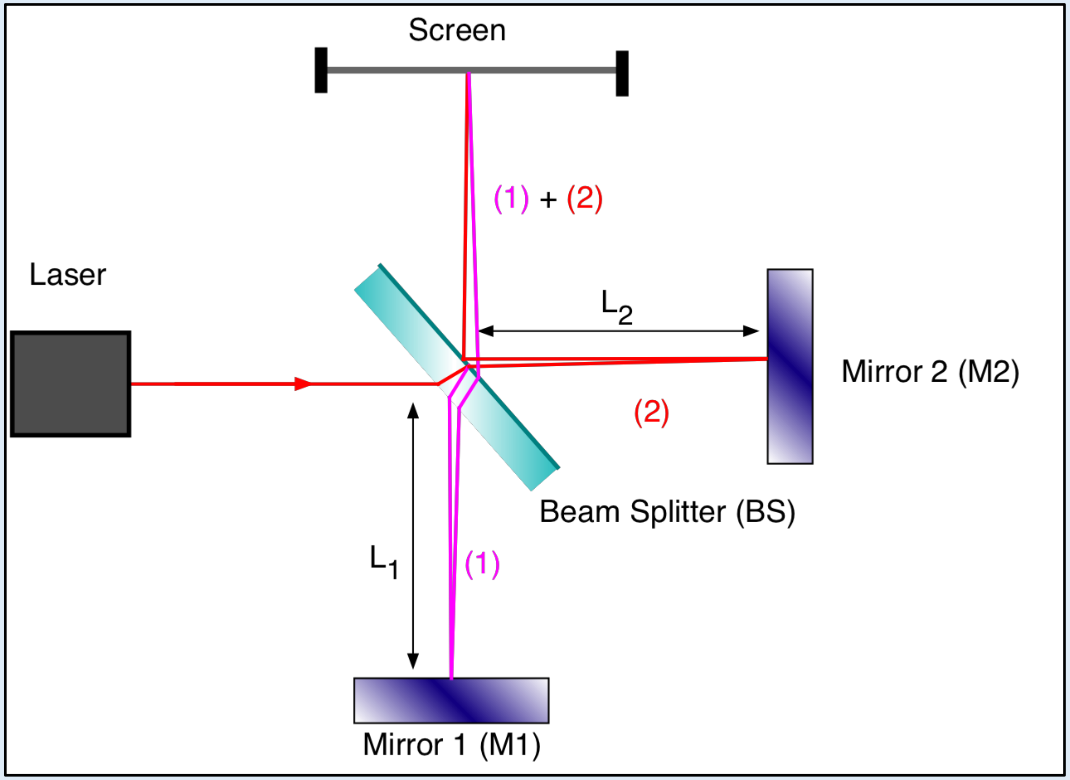
  where d is the spacing between slits and θ is the angle with respect to the grating normal.

- The magnitude of the integer m is usually referred to as the 'order' of the maximum. Hence $|m| = 1$ would be a 'first-order' maximum.
- For large number of slits N, the spacing d $\left(\propto \frac{1}{N}\right)$ can be very small. As a result, the angles θ where maxima are located can be quite spread out. Gives good resolution for spectral work.
- Note that θ in the relation above depends on the value of λ, hence, different colors can also be quite spread out.



The Michelson Interferometer

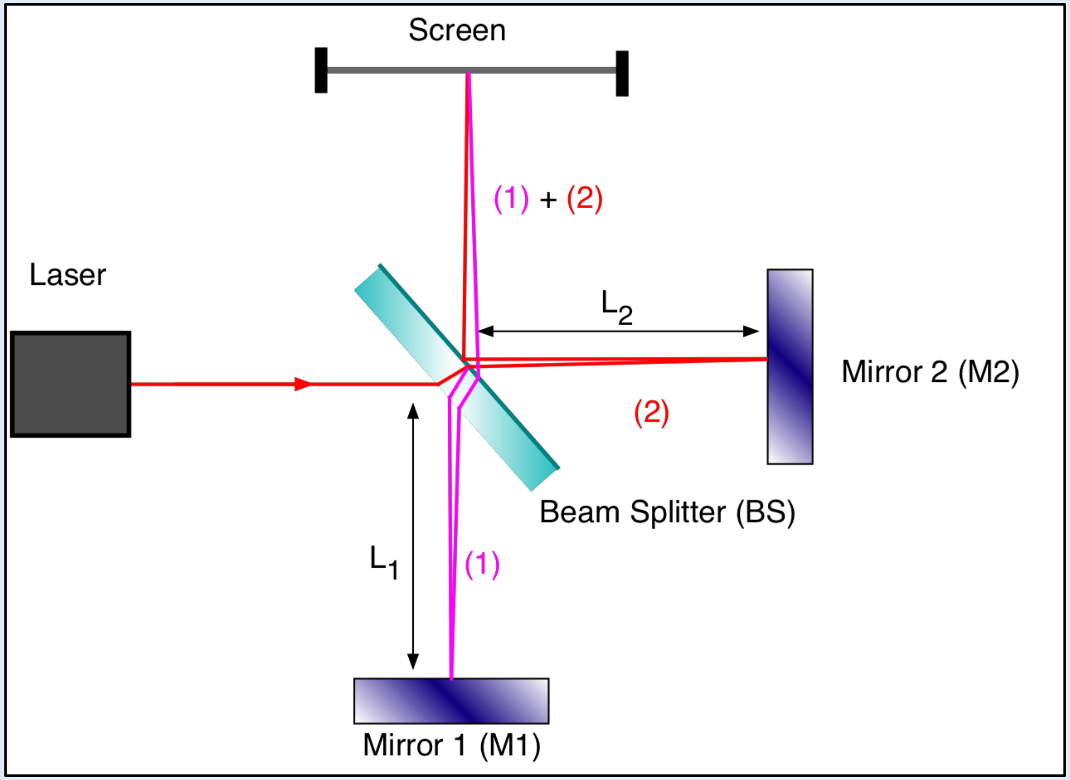
- This is a device that also involves interference between two EM waves.
- Light from a source is split up by a half-silvered mirror, which passes half the light through and onto to a second mirror (M_2)a distance L_2 away from the splitter, and the other half is reflected toward another mirror (M_1) a distance L_1 away.



- The split beams bounce off the other mirrors and return to the beam splitter, which then directs the beams to a screen where they can interfere.
- As in our earlier finding for two-wave interference, constructive interference occurs when the phase difference between the two waves for the paths they take is either zero or an even multiple of π, or equivalently, the optical path lengths of the two legs of the trip differ by zero or an integral number of wavelengths λ.

The Michelson Interferometer (2)

- If a sample of medium with an unknown index of refraction is placed along one of the legs of the path, by counting the number of fringe (bright or dark bands) shifts that occur in the interference patter you can figure out the change in the optical path length, and then the index of refraction of the unknown sample.



1. Michelson interferometer can also be used to determine if either of the mirrors moves a certain distance, even for extremely small changes in lengths L_1 or L_2, by again watching the change in the interference fringe pattern.

2. <u>This sensitivity to minute changes in distance makes the Michelson interferometer the main instrument for the detection of gravitational waves</u>.

LIGO

- LIGO = Laser Interferometer Gravitational-wave Observatory.
- Successful detection of gravitational wave emission due to colliding black holes and neutron stars.
- Sites in Hanford Washington and Livingston Louisiana.
- Europeans (French-Italian) have one called Virgo.
- Proposed follow-up is LISA: an orbiting space platform interferometer.