Physics 1200
Lecture 22
Spring 2024

Relativity, Frames of Reference, Lorentz Factor, Time Dilation, and Length Contraction

The Special Theory of Relativity

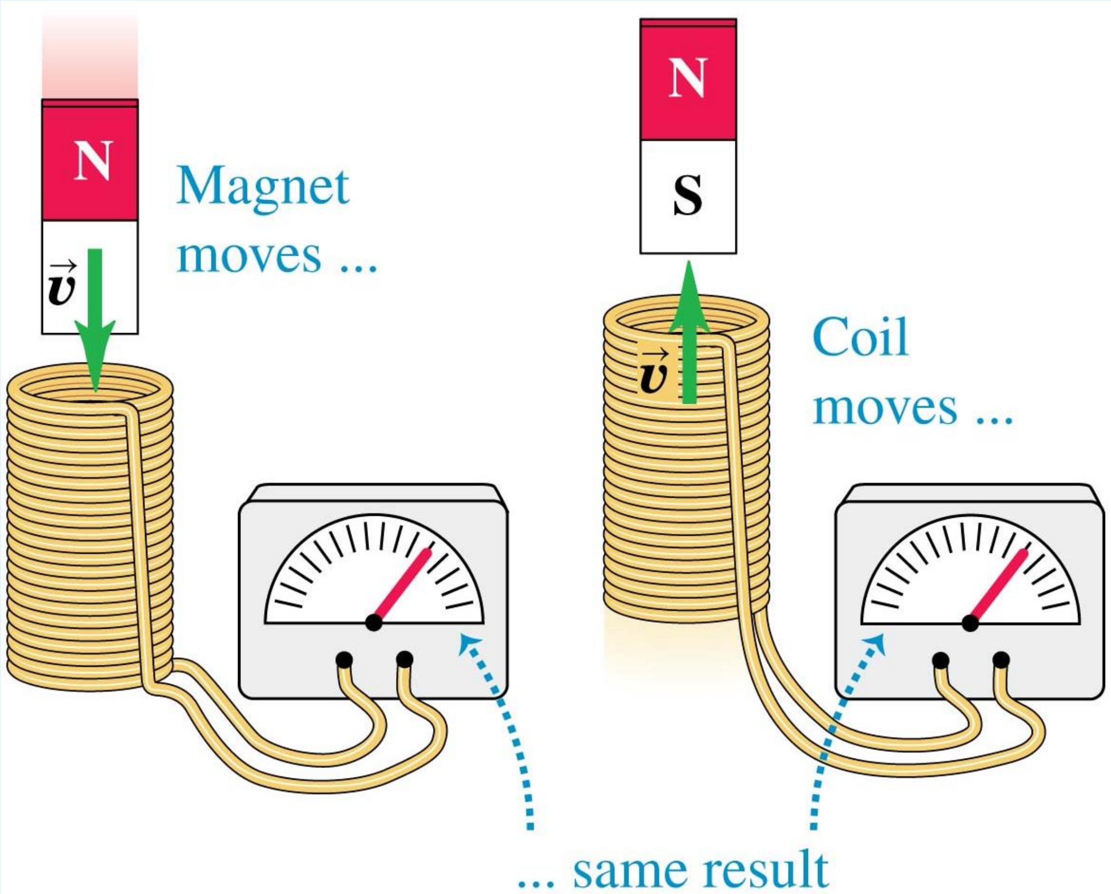
- In 1905 Einstein proposed what is now known as the special theory of relativity.
- Theory founded on two postulates:
  I. *Relativity principle*: The laws of physics are the same in all inertial reference frames, despite the fact that these frames may be in uniform translation with respect to each other. Consequently, all inertial frames are completely equivalent. There are no preferred inertial reference frames.
  II. *Constancy of the speed of light*: The velocity of light in vacuum is the same (c) in all inertial reference frames and is independent of the motion of its source.

Relativity Postulate I

- Reference frame: a state (such as a coordinate system) or set of criteria in which one measures physical quantities, such as time and position.
- Inertial frame of reference: a reference frame in which Newton's 1st law (law of inertia) is observed to be valid. Basically, a non-accelerating, non-rotating reference frame.
- Postulate is a statement of the invariance of physical laws in inertial reference frames.
  - Doesn't matter whether moving at constant velocity with respect to some reference frame, or at rest. Any experiment performed in any of those frames has the same outcome.
  - There is no physical experiment that can be performed to detect whether you are moving at constant velocity with respect to some other reference frame, or whether it is moving with respect to you. Relative motion can be determined only. There is no absolute (i.e., preferred) frame of reference.

Relativity Postulate I (2)

- Example: Faraday's law experiment.
  - Motion of a magnet relative to a solenoid results in same value for the induced emf whether magnet is moving toward solenoid, or if solenoid is moving toward magnet.
  - Meter reading can't tell you which situation is occurring.



Relativity Postulate II

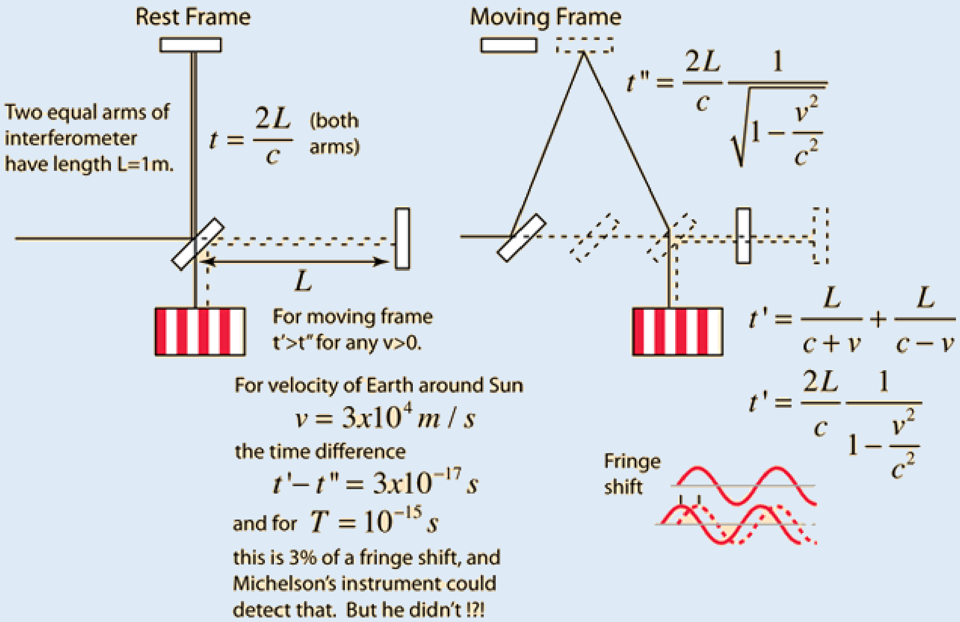
- Postulate is consistent with prediction from vacuum electromagnetic wave equation derived from Maxwell's equations.
  - Found from that speed of EM waves in vacuum is $c = 1/\sqrt{\epsilon_0 \mu_0}$ $(= 2.998 \times 10^8$ m/s in SI units). This result involves universal constants and is independent of state of motion of any reference frame.
  - Some people proposed in the late 1800's that light propagated in a medium called the "ether" ("luminiferous aether"), and that the values of c (and ϵ_0 and μ_0) were valid only in the ether. If true, one should be able to detect a change in the EM vacuum wave speed in laboratories on Earth depending on how the Earth moved with respect to the "ether". (Idea like acoustic waves emitted by a moving source in a stationary fluid.)
  - If true, would mean that the "ether" would be an absolute frame of reference for EM waves, and contrary to postulate 1.

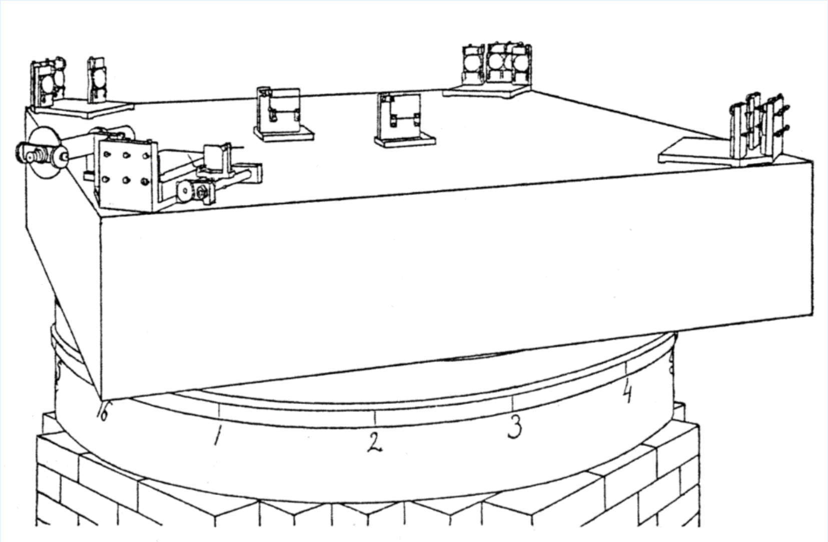
Relativity Postulate II (2)

- Michelson and Morley (1887) used a Michelson Interferometer to try to find changes in the speed of light. By changing the orientation of the interferometer, they looked for variation in the speed of light with respect to the Earth's motion through the "ether".
    - No variation in speed found, no matter what the orientation was.
    - Conclusion: no "ether." Speed of light constant in all directions, consistent with Einstein's Second Postulate.
    - Einstein seemed to be unaware of the Michelson – Morley experiment.
- Einstein's second postulate states that the speed of light is measured by all observers to be c, in all directions and no matter what their state of motion is.

The Michelson-Morley Experiment

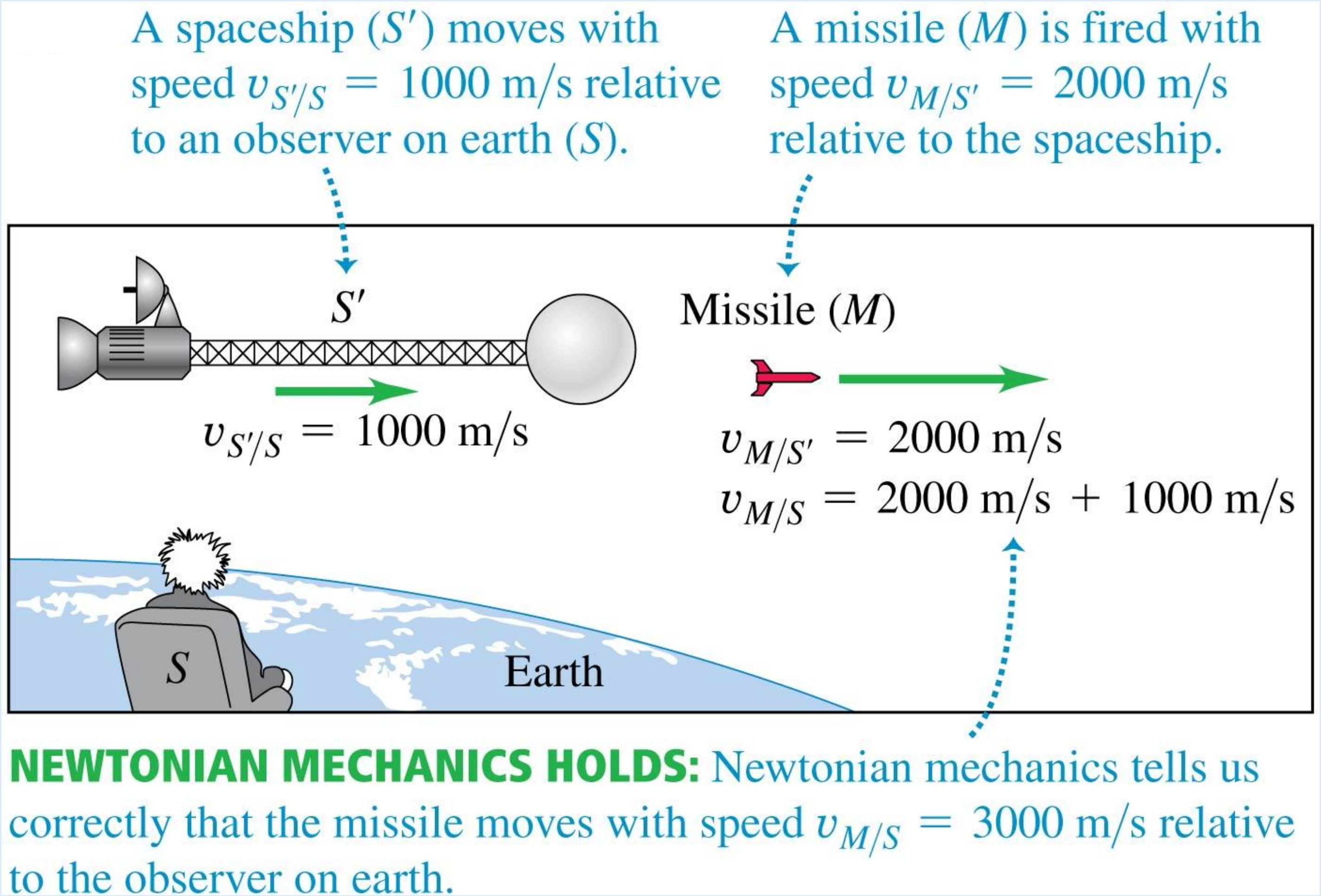
- Michelson-Morley experiment result was a null result: no fringe shift observed in their interferometer, and therefore, no detection of motion of Earth with respect to an “ether wind”. (The “ether” being a postulated “preferred” absolute reference frame and medium for light.)
    - Null result is consistent with Second Postulate of Relativity.
    - Michelson believed in the “ether.” Believed the null result a “failed experiment”, and that the null meant that the “ether” was dragged along with the Earth (“ether drag”).
    - Michelson won the Nobel Prize in 1907, partly for the M-M result. First American to win a Nobel prize in a scientific field.





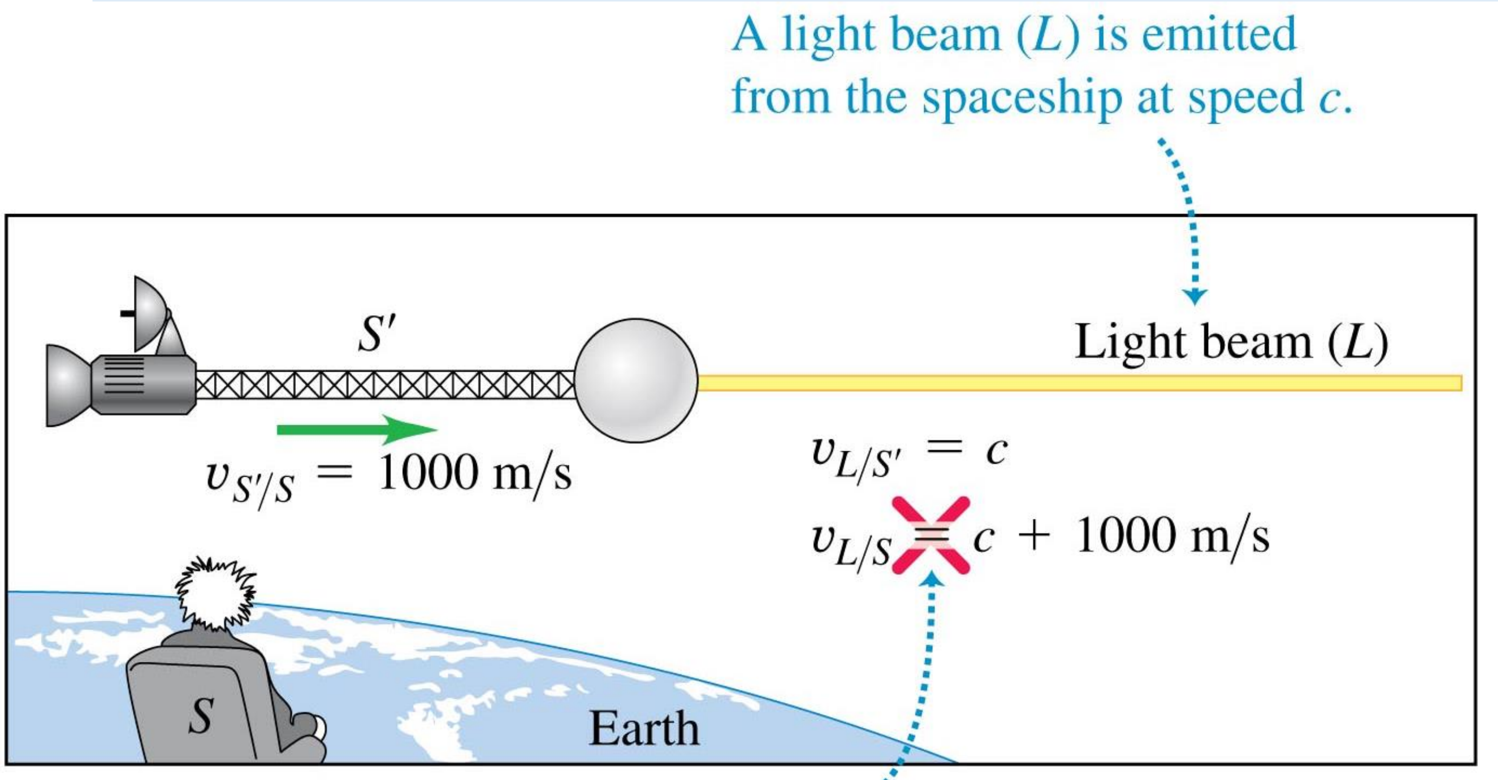
Consequences of Relativity Postulate 2

- Einstein's Second Postulate affects classical (Newtonian) idea of addition of velocities between reference frames:



Consequences of Relativity Postulate 2 (4)

- Einstein's Second Postulate says that simple velocity addition between inertial reference frames is incorrect:



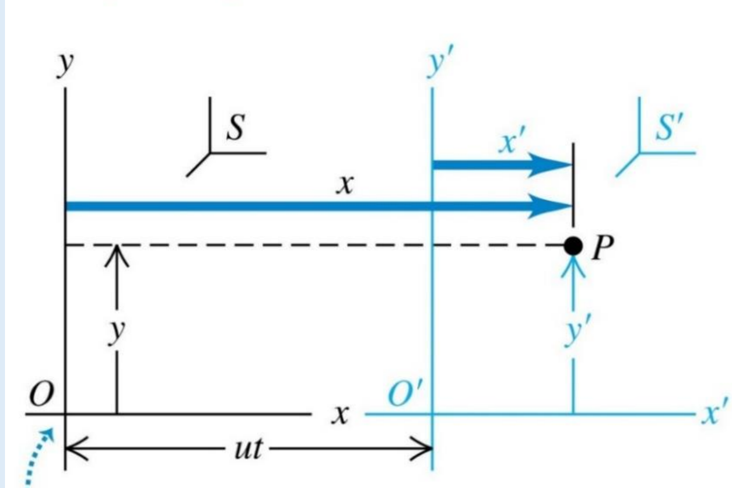
NEWTONIAN MECHANICS FAILS: Newtonian mechanics tells us *incorrectly* that the light moves at a speed greater than c relative to the observer on earth ... which would contradict Einstein's second postulate.

Galilean Transformation

- Before Einstein, there was a relativity principle (or, ‘transformation’ principle) in physics, espoused by Galileo, who also argued that it is not possible to discern absolute motion between reference frames differing by a constant velocity.
- Classical Newtonian mechanics (mechanics we’ve taught you up until now) is completely compatible with a Galilean transformation. However, Maxwell’s equations are not compatible with a Galilean transformation.
- To derive the Galilean transformation, consider two reference frames S and S' that are in relative motion at constant speed u along the direction of their x and x' axes.
- Assuming origins of both reference frames coincide at initial common time $t_i = t_i' = 0$, coordinates of an arbitrary point P viewed in both frames are linked by relations:

$$x = x' + ut \, , y = y' \, , z = z' \quad , \text{ and}$$
$$t = t' \, .$$

Frame S' moves relative to frame S with constant velocity u along the common x-x'-axis.



Origins O and O' coincide at time $t = 0 = t'$.

Galilean Transformation (2)

- From definition of velocity, it follows from Galilean transformation equations that components of velocity of an object at the arbitrary point P as measured in the S-reference frame ($\vec{\boldsymbol{v}}_P = v_x\hat{\boldsymbol{\imath}} + v_y\hat{\boldsymbol{\jmath}} + v_z\widehat{\boldsymbol{k}}$) and in the S'-reference frame ($\vec{\boldsymbol{v}}_P{}' = v_{x'}\hat{\boldsymbol{\imath}} + v_{y'}\hat{\boldsymbol{\jmath}} + v_{z'}\widehat{\boldsymbol{k}}$) are related by the expressions

Galilean velocity addition

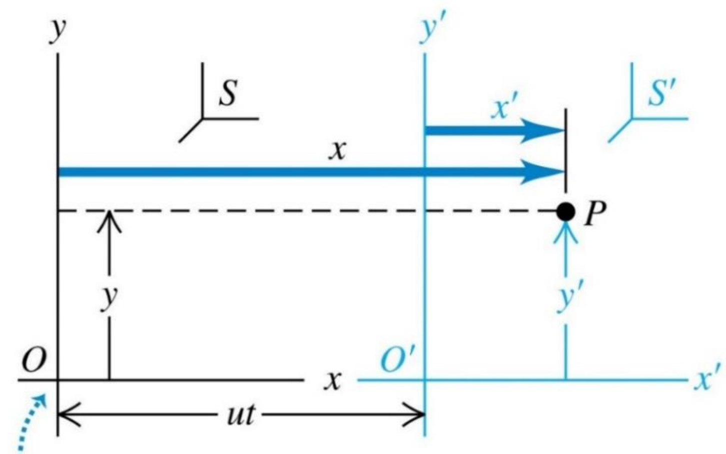
$$v_x = \frac{dx}{dt} = \frac{d}{dt}(x' + ut) = \frac{dx'}{dt} + u \Rightarrow \boxed{v_x = \frac{dx'}{dt'} + u = v_{x'} + u}\ .$$

$$\boxed{v_y = \frac{dy}{dt} = \frac{dy'}{dt} = \frac{dy'}{dt'} = v_{y'}}\ , \text{ and } \boxed{v_z = \frac{dz}{dt} = \frac{dz'}{dt} = \frac{dz'}{dt'} = v_{z'}}\ .$$

- Suppose an EM (light) wave pulse was launched from the origin at time $t_0 = t_0' = 0$ and travels along x, x'- axes.
  - Einstein's Second Postulate: the pulse has $v_x = c$, $v_y = v_z = 0$ in the S-frame, and $v_{x'} = c$, and $v_{y'} = v_{z'} = 0$ in S'.
  - Inserting those into the Galilean transformation equation for velocity addition gives $v_x = c = v_{x'} + u = c + u$.

$\Rightarrow \boxed{c = c + u}$!!!!!!!!!????????

Frame S' moves relative to frame S with constant velocity u along the common x-x'-axis.



Origins O and O' coincide at time $t = 0 = t'$.

Galilean Transformation (3)

- Nonsensical result from combination of Galilean transformation and Einstein's Second Relativity Postulate shows that the two are incompatible. An error must have occurred in the derivation.

- Einstein's response: Galilean transformation links spatial coordinates in each frame, with implicit assumption that $t' = t$. That is, clocks are running at the same rate in both reference frames. That is an unsupported and unnecessary assumption.

  - If, because of the relative motion between reference frames, $t' \neq t$, the velocity measured along the x'-axis in the S' frame,

$$v_{x'} = \frac{dx'}{dt'} \neq \frac{dx'}{dt} .$$

- ∴ A crucial step in the derivation of the Galilean transformation law is unphysical and fundamentally flawed. Transformation limited to only spatial coordinates of the two references frames isn't enough.

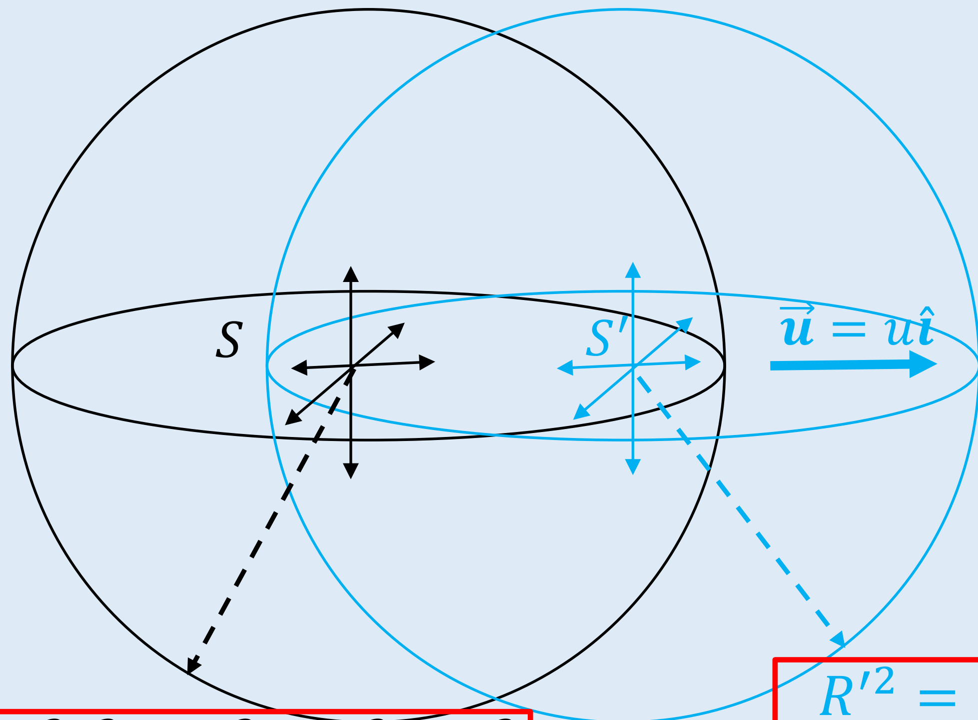
  - Einstein's solution: use Second Postulate of Relativity (constancy of the speed of light) to find a spacetime transformation (a transformation of space and time coordinates in the two frames) satisfying the physically supported condition that a light pulse travels at speed c in both the S and S' inertial frames of reference.

  - Einstein's result: a transformation that links the space and time coordinates in both reference frames – a spacetime continuum.

Einstein's Spacetime Transformations

- To determine how space and time coordinates of inertial reference frames depend on relative motion, Einstein considered the following situation:
  - Again, use frames S and S', with relative speed u between them along the x, and x'-axes, and that their origins coincide at the initial times $t_i = 0 = t_i'$.
  - Imagine a burst of light is emitted from the origin when their origins overlap at the times $t_i = 0 = t_i'$.
  - By both Postulates of Relativity, observers in each frame see a spherical wave front of light propagating radially outward from the burst point (i.e., the origin) in each of their reference frames, traveling at speed of light c, which is the same in both reference frames.
  - In S-frame the radius of the sphere at a later time t will be $R = ct$, and in S'-frame the radius of the sphere at corresponding time t' will be $R' = ct'$.
  - Coordinate points (x, y, z) that lie on the sphere in S satisfy the relation $R^2 = c^2t^2 = x^2 + y^2 + z^2$, and likewise, points (x', y', z') that lie on the sphere in S' satisfy the relation $R'^2 = c^2t'^2 = x'^2 + y'^2 + z'^2$.

Einstein's Spacetime Transformations (2)



$$\boxed{R^2 = c^2t^2 = x^2 + y^2 + z^2}$$

$$\boxed{R'^2 = c^2t'^2 = x'^2 + y'^2 + z'^2}$$

Einstein's Spacetime Transformations (3)

- Einstein assumed that solutions for the space-time coordinates in S' had the form:

$$\boxed{x' = \gamma(x - ut)\,,\ y' = y\,,\ z' = z, \text{ and } t' = \gamma(t - \beta x/c)},$$

where γ and β are constants.

  - To find γ and β, Einstein inserted the expressions above into the expanding light-sphere equation $R'^2 = c^2t'^2$, and then, using that new equation, along with the other light-sphere equation $R^2 = c^2t^2$, he canceled common terms, leaving two independent equations in two unknowns (γ and β). In other words: he only had to do some algebra. (A neat thing about special relativity is that you can do a lot of calculations using only algebra!)

  - Found the solutions:

$$\boxed{\gamma = \frac{1}{\sqrt{1-\left(\frac{u}{c}\right)^2}}}, \text{ and } \boxed{\beta = \frac{u}{c}}. \text{ Note: can also write } \boxed{\gamma = \frac{1}{\sqrt{1-\beta^2}}}.$$

The Lorentz Transformations

- Einstein spacetime coordinate transformations for a reference frame S' that has a relative velocity $\vec{\boldsymbol{u}} = u\hat{\boldsymbol{\imath}}$ with respect to a reference frame S, are:

$$\boxed{\begin{aligned} x' &= \gamma(x - ut)\,, \\ y' &= y\,, \\ z' &= z, \\ t' &= \gamma(t - ux/c^2) \end{aligned}}\;,$$

where $$\boxed{\gamma = \frac{1}{\sqrt{1-\left(\frac{u}{c}\right)^2}}}\;.$$

- The set of four spacetime equations above are commonly referred to as a <u>Lorentz coordinate transformation</u>. The dimensionless factor γ is called the <u>Lorentz factor</u>. These are all named after Hendrick A. Lorentz, who was an older contemporary of Einstein's. The quantity γ first appeared in work by Lorentz, as well as a subset of the transformation equations, although he didn't have the insight to posit the theory of relativity, as brought forth by Einstein.

The Lorentz Transformations (2)

- For speeds much than the speed of light, $u/c \ll 1$, $\gamma \approx 1$, and the Lorentz transformation equations linking the two reference frames are,

$$x' \approx x - ut,\ y' = y,\ \ z' = z, \text{ and } t' \approx t.$$

∴ In the slow-speed limit, a Lorentz transformation is essentially the same as a Galilean transformation. This is the reason why velocity addition works in "everyday" mechanics: the speeds in those situations are usually $\ll c$.

- Lorentz transformations are reversible. That is, considering case of reference frame S moving (in the opposite direction) at relative velocity $\vec{\boldsymbol{u}}' = -\vec{\boldsymbol{u}} = -u\hat{\boldsymbol{\imath}}$ with respect to reference frame S', the transformation equations are modified by making the change $u \to -u$, and interchanging primed coordinates with unprimed coordinates:

$$\boxed{\begin{aligned} x &= \gamma(x' + ut')\ , \\ y &= y', \\ z &= z', \\ t &= \gamma(t' + ux'/c^2) \end{aligned}}\ .$$

Relativistic Time

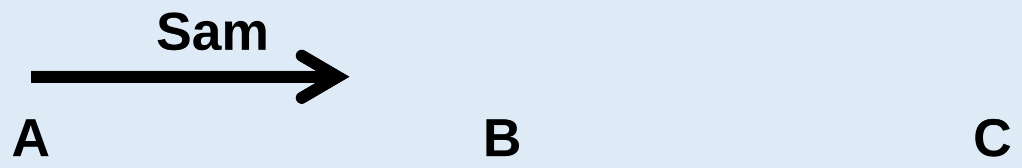
- The theory of special relativity is a theory of spacetime. One consequence of the Lorentz transformation equations is that time intervals between events measured by observers in two different reference frames depends on their state of relative motion. Gives rise to a phenomenon known as <u>time dilation</u>.

- Define key terms <u>rest frame</u> and <u>proper time</u>.

    - <u>The rest frame for a sequence of events in an inertial reference frame is the frame for which they are observed to occur at the same place</u>. That is, the events take place at the same spatial coordinates in a particular inertial reference frame.

    - <u>Proper time is defined as being the time interval between events as measured in the rest frame for which they occurred</u>. That is, proper time is the time interval that occurs between events at the same location.

Lecture Question 1: Concept Check



- Sam leaves point A and travels at constant velocity past point B to point C. Beatrice stays at home at point B.

Each measures the travel time from point A to C. Who measures the proper time for the trip?

A. Alice.

B. Beatrice.

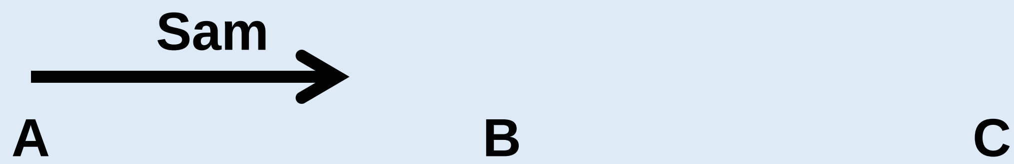
C. Sam.

D. Both.

E. None of them.

F. Who is Alice???

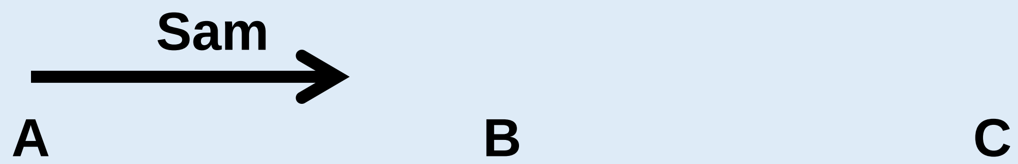
Lecture Question 1: Concept Check



- Proper time corresponds to reference frame where the events occur at the same place. The time between events in that frame will be the proper time.
- The events in this situation are: Sam leaves A, goes past B, and then to C. Question is then, is there a reference frame in which those events occur in the same place?
    - Beatrice sees Sam moving. ∴ Beatrice does not see the events happen in the same place, and she cannot measure proper time for this situation.
    - From Sam’s perspective, he is at rest, and the points A, B, and C move past him while he is stationary. In his reference then, the events all occur where he is located. Therefore, he would measure proper time for these events.
    - Alice – whoever she is – if she isn’t moving with Sam, then she would say that the events occur at difference place. ∴ She can’t measure proper time.

Anwer: C. Sam measures proper time here.

Lecture Question 2: Another Concept Check



- Sam leaves Alice at point A, and travels at constant velocity past point B and to point C. Beatrice stays at point B. Along the way, when at the point B, Sam sends a light pulse from his vehicle to point C.

Who measures the proper time for the travel of the pulse?

A. Alice.

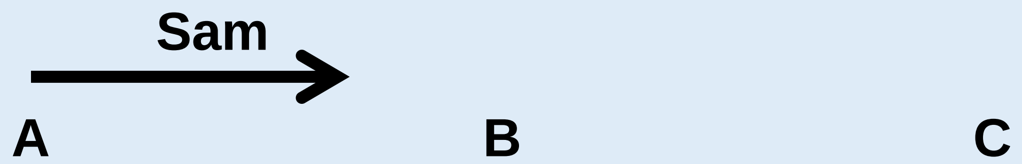
B. Beatrice.

C. Sam.

D. All of them.

E. None of them.

Lecture Question 2: Another Concept Check



- Question same as before: who sees the events occur in the same place? The events are the light pulse moves past B and C.
- Beatrice and Alice see the pulse move past B and C, different places. ∴ Neither of them can measure proper time between the events.
- To Sam, the light pulse leaves him behind, because the pulse races ahead at the speed of light relative to him. ∴ The passing of the light pulse past B and C also occur at different places for him as well. This means that he also cannot measure the proper time, either.

Anwer: E. None of them measure proper time here.

Relativistic Time Dilation

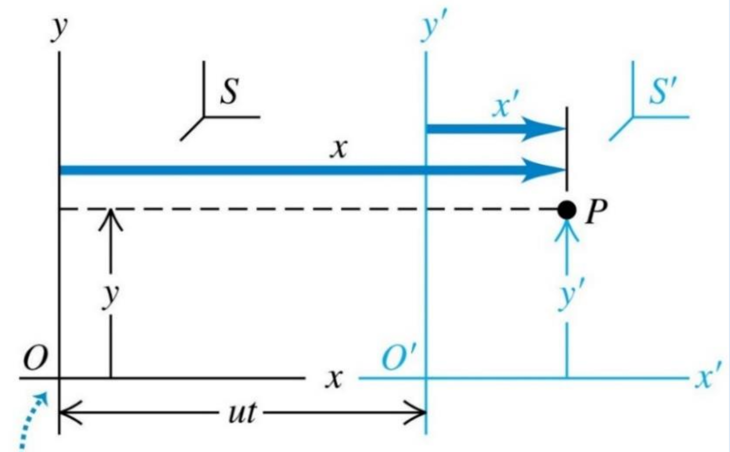
- Consider a clock at rest and located at position x' in reference frame S'. The time interval between two events A and B that occur at the clock location at times t'_A and t'_B (recorded by an observer in S' who also stays located at x') is the proper time interval:

$$\Delta t_0 = t'_B - t'_A \; .$$

- For an observer in reference frame S, the events A and B occur at different locations x_A and x_B , recorded at times t_A and t_B by their own clock. Using the Lorentz transformation equations, including the fact that location x' of the events is unchanging in the S' frame, yields:

$$\boxed{t_B - t_A = \Delta t = \gamma \Delta t_0} \; .$$

Frame S' moves relative to frame S with constant velocity u along the common x-x'-axis.



Origins O and O' coincide at time $t = 0 = t'$.

Relativistic Time Dilation

- The relation between time intervals we derived is a statement of relativistic time dilation.
- Because the Lorentz factor

$$\gamma = \frac{1}{\sqrt{1-\left(\frac{u}{c}\right)^2}} > 1 \text{ for } u > 0,$$

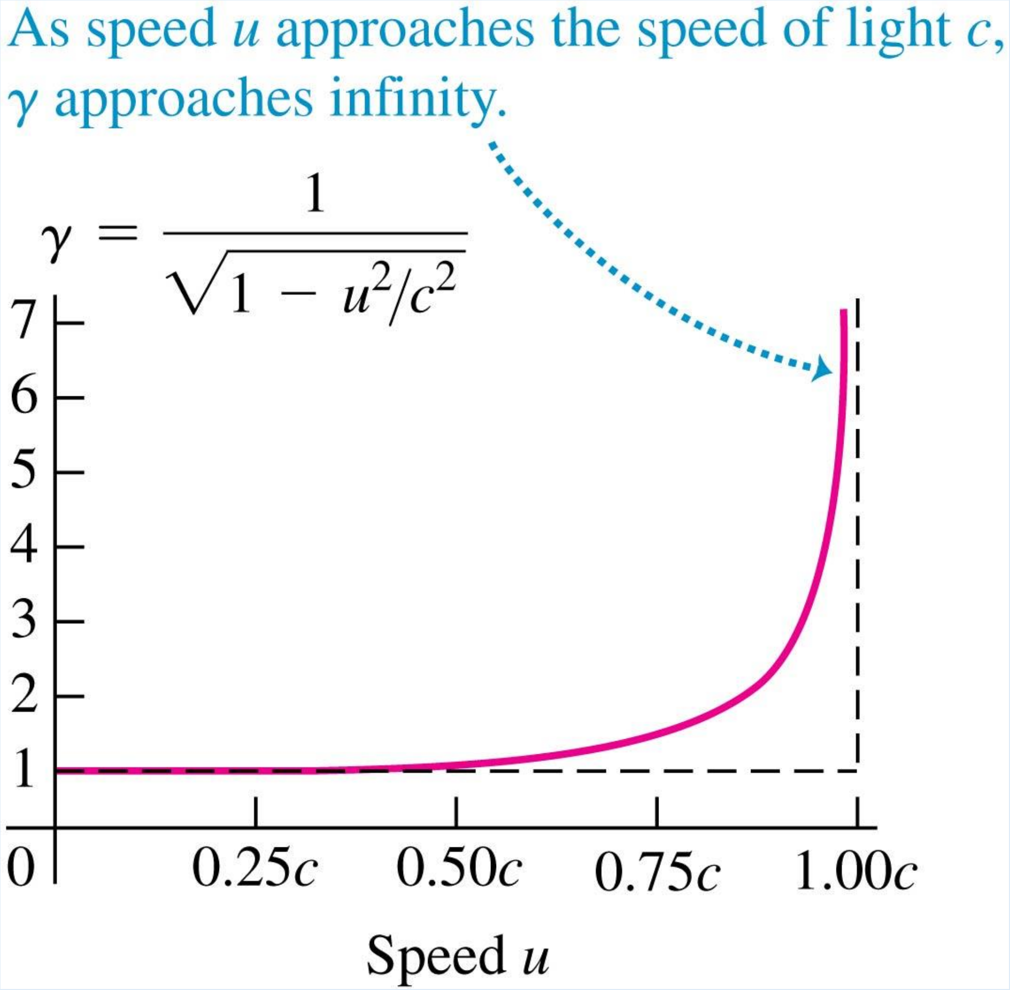
it follows that

$$\frac{\Delta t}{\Delta t_0} = \gamma > 1.$$

> ➢ ∴ The smallest time interval between events will always be the proper time for those events. That is, the time measured in the rest frame for which the events occur in the same place. Measurements of the same event interval Δt in any other inertial reference frame is always $> \Delta t_0$.
>
> ➢ An observer in S would say that the clock in S', which is perceived as moving relative to S, is running slow compared to their own clock. Clocks (and time between events) run more slowly when moving relative to an observer.

The Lorentz Factor

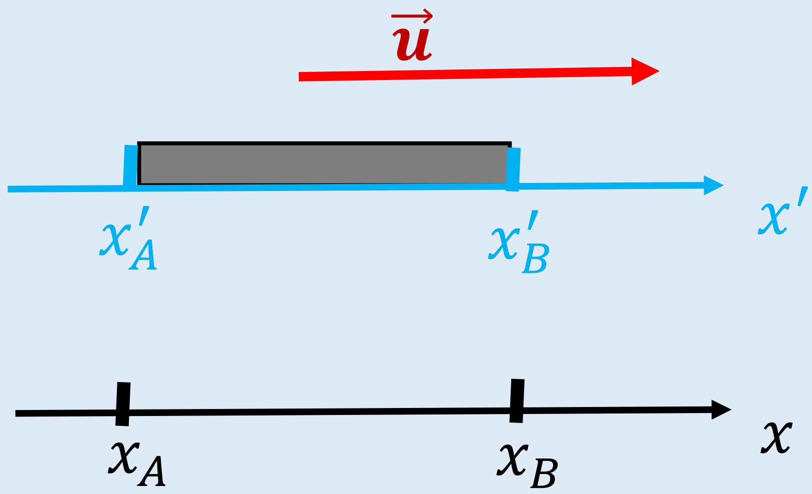
1. For relative speed between reference frames $u \ll c, \gamma \simeq 1$.
2. If u is great enough, such that $\gamma > 1$, the speed is said to be <u>relativistic</u>.
3. At $u = c, \gamma$ diverges. The speed of light is therefore a <u>universal limiting speed</u>.



Relativistic Length – Proper Length

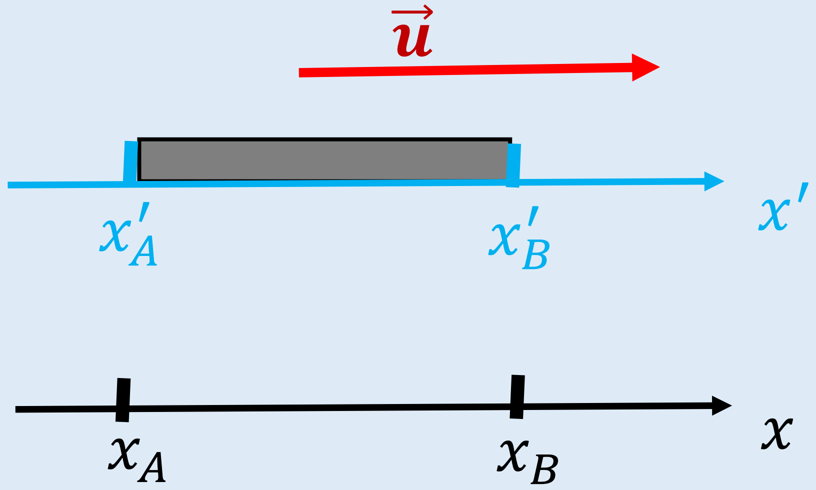
- To consider the effect of relativity on the measurement of distances, we define proper length as being the length of an object or a distance interval along the x-direction (that is, parallel to the direction of the relative motion of the reference frames) as measured in an observer's rest frame.

Relativistic Length – Length Contraction



- Consider rod at rest and parallel to x' axis with respect to an observer in frame S', moving with respect to an observer in S. Length of the rod measured by observer in S' is proper length $x'_B - x'_A = L_0$, where x'_B and x'_A are endpoint locations of the rod in S' -coordinates.
- Observer in S measures length of the rod by taking the difference of the rod's endpoint coordinates x_A and x_B, as measured <u>simultaneously</u> in their coordinate system (i.e., measurements of x_A and x_B are made <u>at the same time t by the observer in S</u>).

Relativistic Length – Length Contraction (2)



- Using the Lorentz transform equations and conditions mentioned above, gives:

  $x'_B - x'_A = \gamma(x_B - x_A) \quad \Rightarrow \; L_0 = \gamma L$, where $L = x_B - x_A$ is the length of the rod as measured in S -frame (the frame in which the rod is seen as moving).

- Rewriting the equation: $\boxed{L = \frac{1}{\gamma}\, L_0}$.

  This expresses the effect of <u>relativistic length contraction</u>.

Relativistic Length – Length Contraction (3)

- Because $\frac{1}{\gamma} = \sqrt{1 - \left(\frac{u}{c}\right)^2} < 1$ for $u > 0$, it follows that $L < L_0$.

  - The proper length of an object (such as a rod or meter stick) aligned with the axis of relative motion (between reference frames) is the maximum length along that axis that can be measured in any inertial reference frame. Measurement of the length from any other reference frame that is in motion parallel to the length is always less than the proper length.

  - In a nutshell: moving meter sticks (or any other object) aligned with the direction of motion always appear longer when they are at rest. (Or, alternatively, they appear shorter when they are in motion.)

Example Problem 1

- Jill claims that her new rocket is 100 m long. As she flies past your house, you measure the rocket's length and find that it is only 80 m. Should Jill be cited for exceeding the 0.5c speed limit?

Problem 1 Solution

- To Jill, the rocket is at rest. Hence, the length she measures is the proper length L_0.
- As the observer, you will see the moving rocket as being length-contracted along the direction of relative motion, with length

$$L = \frac{1}{\gamma} L_0 = L_0 \sqrt{1 - \left(\frac{u}{c}\right)^2} \ ,$$

where u is the relative speed between you, Jill, and her rocket.

$$\left(\frac{L}{L_0}\right)^2 = 1 - \left(\frac{u}{c}\right)^2 \quad \Rightarrow \left(\frac{u}{c}\right)^2 = 1 - \left(\frac{L}{L_0}\right)^2$$

$$\Rightarrow \frac{u}{c} = \sqrt{1 - \left(\frac{L}{L_0}\right)^2} = \sqrt{1 - \left(\frac{80 \text{ m}}{100 \text{ m}}\right)^2} = 0.6$$

$\therefore u = 0.6c > 0.5c$. Jill is in excess of the speed limit!

Example Problem 2

- A cosmic ray travels 60 km through the earth's atmosphere in 500 microseconds, as measured by experimenters on the ground. How long does the journey take according to the cosmic ray?

Problem 2 Solution

- The speed that the Earth-based observers would measure for the cosmic-ray speed would be

$$u = \frac{L_0}{\Delta t} = \frac{60 \times 10^3 \text{ m}}{500 \times 10^{-6} \text{ s}} = 1.2 \times 10^8 \frac{\text{m}}{\text{s}} = 0.4\,c.$$

This is the relative speed between the cosmic ray and the Earth observers.

The time for the cosmic ray to traverse the column of atmosphere would be the proper time Δt_0, as it "thinks" it is at rest and the atmosphere passes it at the relative speed u. Hence, the events happen in the same place for the cosmic ray

Recalling the relation between proper time and time measured in other reference frames:

$$\Delta t = \gamma \Delta t_0 \;\Rightarrow\; \Delta t_0 = \frac{1}{\gamma}\,\Delta t = \Delta t\sqrt{1 - \left(\frac{u}{c}\right)^2}$$

$$\Delta t_0 = (500 \times 10^{-6} \text{ s})\sqrt{1 - \left(\frac{0.4\,c}{c}\right)^2} = 4.58 \times 10^{-4} \text{ s} = 458\ \mu\text{s}.$$