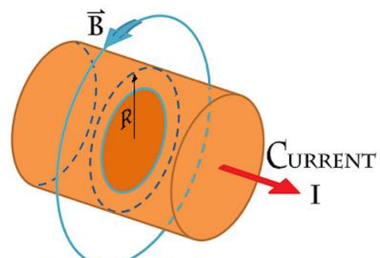
28E – Ampere's Law

Ampere's Law can be derived from the Law of Biot and Savart (but we won't make you do that here.) Like Gauss' Law, Ampere's Law can be used to calculate the magnetic field from the current in certain situations of special symmetry. (Planar and Cylindrical for this course.)
Ampere's Law relates the line integral of the magnetic field around a closed loop to the total current passing through an area bounded by that loop:

$$\oint \vec{\boldsymbol{B}} \cdot d\vec{\boldsymbol{s}} = \mu_0 I_{enclosed}.$$

Cylindrical Symmetry – Many practical applications can be treated as though they have simple cylindrical symmetry. As long as the diameter of a wire and the distance of observation from the axis are small compared to the length of a straight wire segment, the approximation of cylindrical symmetry is useful. Real physical systems include simple wires and coaxial wires.



Section of Conductor

Example 1: Consider an infinitely long, cylindrical conductor of radius R carrying a non-uniform current density $J(r)=\alpha r^n$, where n is a positive constant, α is a positive constant and r is the distance from the center of the cylinder. The following will step you through the logic to find the field inside and outside the wire.

1) The path for the integral can be chosen to be a circle parallel to the magnetic field $\vec{B}$, which circulates the wire.
   a) Rewrite the left hand integral with this simplification. (Do the dot product, but do not solve the integral yet.)

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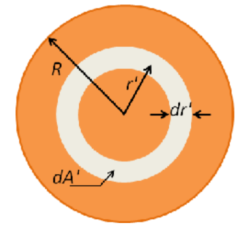
   b) The fact that $|\vec{\boldsymbol{B}}(r)|$ is a constant for any value of r allows a further simplification of the integral to $B\oint ds$. What is the value of $\oint ds$ for a circular path of radius r?

   $\oint ds$ = __________

Rewrite the left-hand side (LHS) of Ampere's Law using this simplification. LHS=____________

   c) Now we will focus on the right-hand side (RHS) of Ampere's Law. The current passing through a small area dA is $\int \vec{\boldsymbol{J}} \cdot d\vec{\boldsymbol{A}}$, where $\vec{\boldsymbol{J}}$ is the current density. This simplifies to $\int J\, dA$ if the surface normal and J are parallel. The area of a thin annular ring of radius r' and thickness dr' as sketched to the right is: $2\pi r' dr'$, so the total current passing through an area bounded by a circle of radius r is:



$$I_{total}(r) = \int_0^r J(r')2\pi r' dr'.$$

i. Find the total current $I_{enclosed}(r)$ enclosed for any value of $r < R$ in terms of α and r for $J(r') = \alpha r'^3$.

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ii. Find the total current enclosed for $r > R$. Note that, for $r' < R$, $J(r')$ is given in part i above, and, for $r' > R$, $J(r') = 0$.

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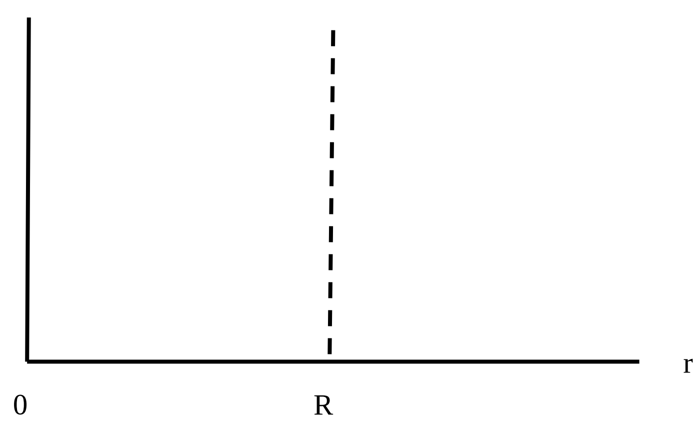
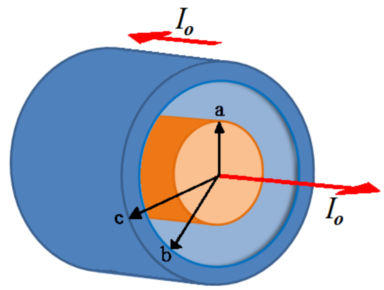
d) Solve for $B(r)$ in terms of constants, r, α, and R for each of the cases i and ii above.

i) ______________

ii) ______________

e) Sketch the magnitude of the magnetic field as a function of r. Include axis labels and scales.





Example 2: A coaxial cable consists of a solid inner conductor of radius a, surrounded by a concentric cylindrical tube of inner radius b and outer radius c. The conductors carry equal and opposite currents I_0 distributed uniformly across their cross-sections. The current on the inner conductor points out of the paper and on the outer points into the paper.

2) Determine the magnitude and direction of the magnetic field in terms of constants, r, I_0, a, b, and c at a distance r from the axis for the following situations. (Note that the case for uniform current density in a cylindrical conductor is covered in the textbook example 28.08.)

a) $r < a$

$B(r)$ =____________

b) $r = a$

$B(r)$ =____________

c) $a < r < b$

$B(r)$ =____________

d) $r = b$

$B(r)$ =____________

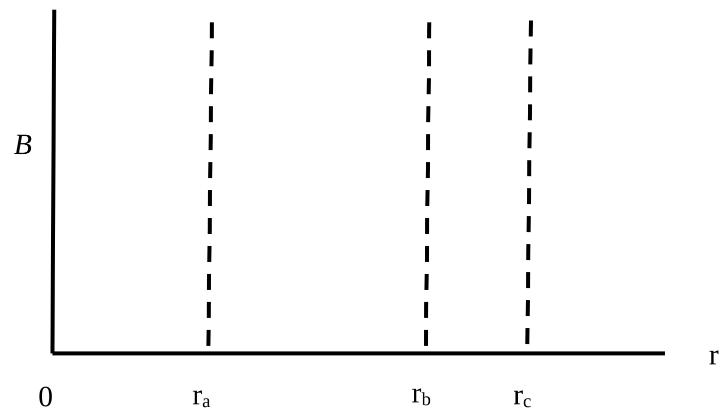
e) $b < r < c$

$B(r)$ =____________

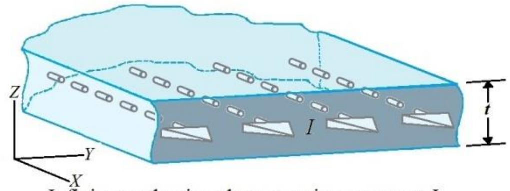
f) $r > c$.

$B(r)$ =____________

g) Make a graph of the magnitude of the magnetic field as a function of the distance *r* from the axis. Include values at key values of *r*.



<u>Example 3</u>: Magnetic field due to an infinite sheet of current. We wish to find the field for an infinite conducting sheet of thickness t carrying a constant current density of $J_0\hat{i}$. A section of the infinite sheet is sketched to the right in perspective and edge-on).



Infinite conducting sheet carrying a current *I*.

3) We wish to find the field for an infinite conducting sheet. But first some preliminary steps
   a) In what direction does the magnetic field point at point P, above the sheet (as sketched for the edge-on image)? (left, right, up down…?)

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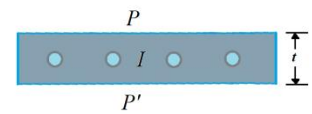
   b) In what direction does the magnetic field point at point P’, below the sheet (as sketched for the edge-on image)? (left, right, up down…?)



Edge view of infinite conductive sheet. Current *I* out of sheet.

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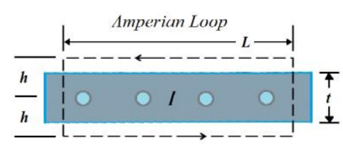
   c) Consider the Amperian loop shown as the dashed rectangle to the right. Take the loop to have length L and height $2h$ (h above and h below the center). Find the integral $\oint \vec{\boldsymbol{B}} \cdot d\vec{\boldsymbol{s}}$ around this loop assuming that the field above the loop points to the left and has magnitude $B(h)$ and the field below points to the right and has the same magnitude.



Edge view of infinite conductive sheet. Current *I* out of sheet.

$\oint \vec{\boldsymbol{B}} \cdot d\vec{\boldsymbol{s}}$ =________________

   d) What is the current $I_{enclosed}$ enclosed by this loop in terms of constants, sheet thickness t, length of loop L, and current density J_0.?

$I_{enclosed}$ =_______________

   e) Find the magnetic field $B(h)$ in terms of constants, length of loop L, sheet thickness t, and current density J_0.

$B(h)$ =_______________

28F – ALICE Software Installation

Install the Analog Devices interface for running the ADALM1000 ("M1K") Electronics Board using the instructions in the RPI ALICE M1K install guide, which can be found in the LMS folder 'Lab Information and Software.' Here are two other sources that give instructions on how to install the ALICE software for the M1K:

Windows OS: https://www.youtube.com/watch?v=rT6r_AlPpbk

MacOS: https://github.com/belsten/ALICE-OSX_Installation

Run ALICE and take a screenshot (CTRL+ALT+PrtSc) of the running Alice M1K Desktop interface. Post it (CTRL V) here.