Physics 1200
Lecture 01
Spring 2024

Electric Charge, Coulomb's Law,
Conductors, Insulators, & Polarization

Observational Data

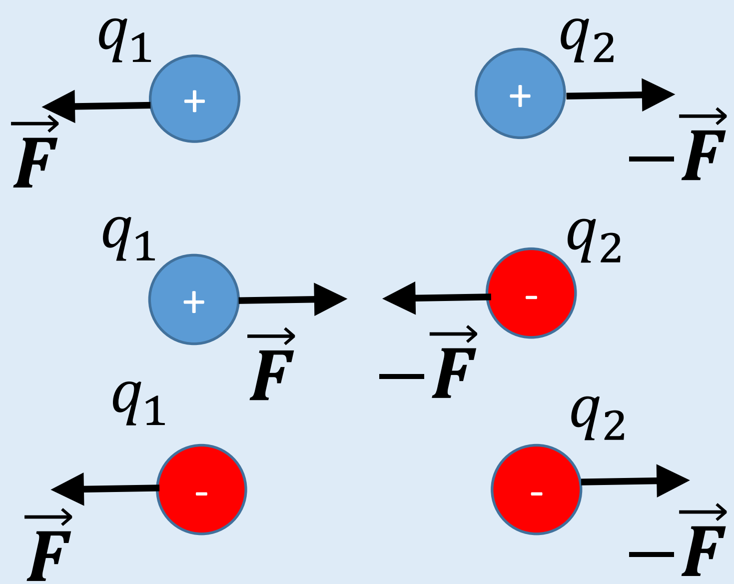
- There is a unique force between some objects that acts along the line that passes between them.
  - Force can be generated just by rubbing two objects of right types against one another.
  - Sometimes there is a spark or a “zap!” and the force goes away, or changes sign.
  - Force falls off with distance r like $1/r^2$, just like Newton’s gravitational force law.

- Unlike gravity (always attractive force between objects), this force can be attractive, repulsive, or zero.
  - For instance, if object A attracts object B, but repels object C, find that B and C also attract one another. And, if same object A feels no force from an object D, objects B and C also feel no force from D.

Electric Charge

- Quantity associated with this force is called ‘electric charge’, or just ‘charge’.
- Data for the preceding described situations are consistent with the idea that there are two types of charge: one type known as ‘positive’, and the other type as ‘negative’.
    - *Fact: Ben Franklin coined these terms.*
- Observational data is consistent with law: **like charges repel, unlike charges attract.**
- Another observational fact, and a fundamental law: **charge is conserved in a closed system.**
- Experiment (Millikan oil drop experiment) determined that charge is **quantized**. It exists in definite, discrete amounts.

Charge and Electric Forces

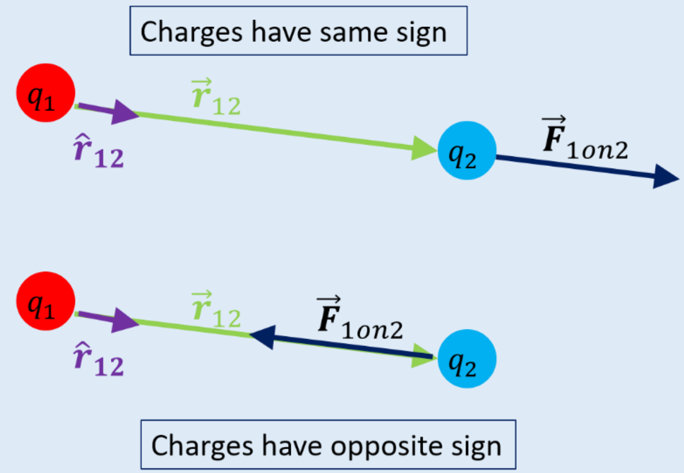


Because there are both attractive and repulsive forces, conclude that there are two types of charge: positive (+) and negative (-). Like charges repel, unlike charges attract.

Coulomb's Law

Coulomb's Law (Charles Augustin de Coulomb, 1784) between two point charges:

$$\boxed{\vec{\boldsymbol{F}}_{12} = k\frac{q_1 q_2}{r_{12}^2}\frac{\vec{\boldsymbol{r}}_{12}}{r_{12}} = k\frac{q_1 q_2}{r_{12}^2}\hat{\boldsymbol{r}}_{12}},$$



$\vec{\boldsymbol{F}}_{12}$ = electric force charge q_1 exerts on charge q_2,

$\vec{\boldsymbol{r}}_{12}$ = displacement (i.e., distance) vector pointing from charge q_1 to q_2,

$r_{12} = |\vec{\boldsymbol{r}}_{12}|$ = distance between q_1 and q_2,

$\boxed{\hat{\boldsymbol{r}}_{12} = \vec{\boldsymbol{r}}_{12}/r_{12}}$ = unit vector directed along $\vec{\boldsymbol{r}}_{12}$,

$k = \frac{1}{4\,\pi\epsilon_0} \cong 9.0 \times 10^9 \mathrm{N}\frac{\mathrm{m}^2}{\mathrm{C}^2}$ is a physical constant.

'C' stands for Coulomb (SI unit of electrical charge), and $\boxed{\epsilon_0 = 8.85 \times 10^{-12}\mathrm{F/m}}$ is the 'permittivity constant' (textbook calls it the 'electric constant'). The 'F' is for 'Farad', which is the SI unit for capacitance. (Topic later on.)

Class Discussion Question

What can you tell about two charges if they are attracted to one another?

A. They have the same sign, but you can’t determine the magnitude of either one.
B. They have opposite sign, but you can’t determine the magnitude of either one.
C. They have the same sign and magnitude.
D. They have opposite sign and the same magnitude.
E. None of the above.

Fundamental Physical Properties

- SI unit of charge is the coulomb, C.
- Charge on one electron = -1.602×10^{-19}C.

Object	Charge (C)	Mass (kg)	Charge/Mass (C/kg)
Electron	-1.60×10^{-19}	9.11×10^{-31}	1.76×10^{11}
Proton	$+1.60 \times 10^{-19}$	1.67×10^{-27}	9.58×10^{7}
Neutron	0	1.67×10^{-27}	0
Carbon atom	0	20.0×10^{-26}	0
Person shuffling on a carpet	$\sim 10^{-10}$C	$\sim 10^{2}$kg	$\sim 10^{-12}$

- Charge on one electron is written as $-e$, and the charge on one proton as $+e$. The quantity e is "the magnitude of the elementary or fundamental charge". It is the fundamental because every charge ever observed can be written as an integer multiple of e.

Charge: Scaling

- A Coulomb (1C) is a large amount of charge.
    - For an ‘every day’ situation, such as shuffling on a floor in your stocking feet, might accumulate a net charge of a nano-Coulomb (1 nC = 10^{-9}C).
    - The magnitude of the charge on an electron is of the order of one-tenth of a nano-nano C ($= 10^{-19}$ C).
- There are situations, though, where large charges can occur. The amount of charge in a chemical cell (such as a car battery) can be $\sim 10^5$C.

Review: Vector Forces

- Total (net) electric force acting on a charge is the sum of the individual electric forces exerted by all other charges acting on it:

$$\vec{\boldsymbol{F}}_1 = \vec{\boldsymbol{F}}_{21} + \vec{\boldsymbol{F}}_{31} + \vec{\boldsymbol{F}}_{41} + \vec{\boldsymbol{F}}_{51} + \cdots$$

Remember: this is a <u>vector</u> equation.

Resolve vectors into components (e.g., x-, y-, and z-components), and then sum all of the like component terms:

$$\vec{\boldsymbol{F}}_1 = F_{1x}\hat{\boldsymbol{\imath}} + F_{1y}\hat{\boldsymbol{\jmath}} + F_{1z}\widehat{\boldsymbol{k}}\ ,$$

$$F_{1x} = F_{21x} + F_{31x} + F_{41x} + F_{51x} + \cdots$$

$$F_{1y} = F_{21y} + F_{31y} + F_{41y} + F_{51y} + \cdots$$

$$F_{1z} = F_{21z} + F_{31z} + F_{41z} + F_{51z} + \cdots$$

Review: Unit Vectors

- The unit vector in Coulomb's law defines the direction of the force vector in space. A unit vector by definition has unit magnitude and no dimensions:

$$|\hat{\boldsymbol{r}}| = 1.$$

- A unit vector can be constructed from any non-zero vector. For example, $\hat{\boldsymbol{r}} = \vec{\boldsymbol{r}}/r$, because $r = |\vec{\boldsymbol{r}}|$.

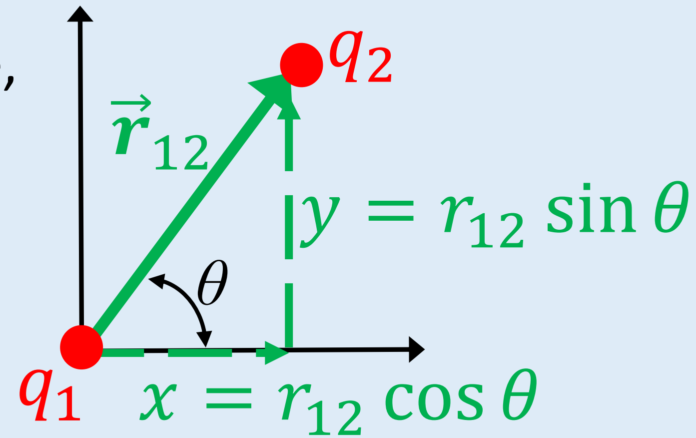
- Any non-zero vector can be written in terms of a unit vector. For example, $\vec{\boldsymbol{r}} = r\hat{\boldsymbol{r}}$.

Coulomb's Vector Force Law in a Cartesian Plane

- Consider q_1 and q_2 in a 2-dimensional plane, with distance vector

  $\vec{\boldsymbol{r}}_{12} = \vec{\boldsymbol{r}}_2 - \vec{\boldsymbol{r}}_1 = x\hat{\boldsymbol{\imath}} + y\hat{\boldsymbol{\jmath}}$

  as shown in the figure. Solve for the force $\vec{\boldsymbol{F}}_{12}$ exerted on q_2 by q_1.



$$\hat{\boldsymbol{r}}_{12} = \frac{\vec{\boldsymbol{r}}_{12}}{r_{12}} = \frac{x\hat{\boldsymbol{\imath}}+y\hat{\boldsymbol{\jmath}}}{r_{12}} = \frac{r_{12}\cos\theta\hat{\boldsymbol{\imath}}+r_{12}\sin\theta\hat{\boldsymbol{\jmath}}}{r_{12}} = \cos\theta\,\hat{\boldsymbol{\imath}} + \sin\theta\,\hat{\boldsymbol{\jmath}}\ .$$

Coulomb's law:

$$\vec{\boldsymbol{F}}_{12} = k\frac{q_1 q_2}{r_{12}^2}\,\hat{\boldsymbol{r}}_{12} = k\frac{q_1 q_2}{r_{12}^2}\,(\cos\theta\,\hat{\boldsymbol{\imath}} + \sin\theta\,\hat{\boldsymbol{\jmath}})$$
$$= F_{12}(\cos\theta\,\hat{\boldsymbol{\imath}} + \sin\theta\,\hat{\boldsymbol{\jmath}}),$$

where $F_{12} = k\frac{q_1 q_2}{r_{12}^2}$.

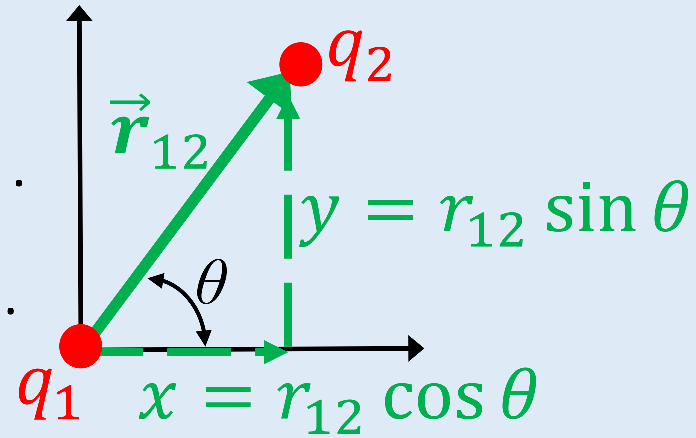
Note: q_1 and q_2 have algebraic signs (+ or -) in this expression.

Coulomb's Vector Force Law in a Cartesian Plane (2)

- The x- and y-components of the force are:

$$F_{12x} = \vec{\boldsymbol{F}}_{12} \cdot \hat{\boldsymbol{\imath}} = k\frac{q_1 q_2}{r_{12}^2}\cos\theta = k\frac{q_1 q_2}{r_{12}^2}\left(\frac{x}{r_{12}}\right).$$

$$F_{12y} = \vec{\boldsymbol{F}}_{12} \cdot \hat{\boldsymbol{\jmath}} = k\frac{q_1 q_2}{r_{12}^2}\sin\theta = k\frac{q_1 q_2}{r_{12}^2}\left(\frac{y}{r_{12}}\right).$$



To find instead the force of q_2 on q_1, proceed in same fashion. For that case,

$$\vec{\boldsymbol{r}}_{21} = \vec{\boldsymbol{r}}_1 - \vec{\boldsymbol{r}}_2 = -x\hat{\boldsymbol{\imath}} - y\hat{\boldsymbol{\jmath}} = -\vec{\boldsymbol{r}}_{12}, \quad \text{and} \quad \hat{\boldsymbol{r}}_{21} = -\cos\theta\,\hat{\boldsymbol{\imath}} - \sin\theta\,\hat{\boldsymbol{\jmath}}$$

(reversed direction from the first case).

Result: $$\vec{\boldsymbol{F}}_{21} = -k\frac{q_1 q_2}{r_{12}^2}(\cos\theta\,\hat{\boldsymbol{\imath}} + \sin\theta\,\hat{\boldsymbol{\jmath}}) = -\vec{\boldsymbol{F}}_{12},$$

with components $F_{21x} = -F_{12x}$, and $F_{21y} = -F_{12y}$.

This is consistent with Newton's 3rd Law!

Practice Problem with 2-D Vector Forces

$q_1 = +1\ \mathrm{nC}$

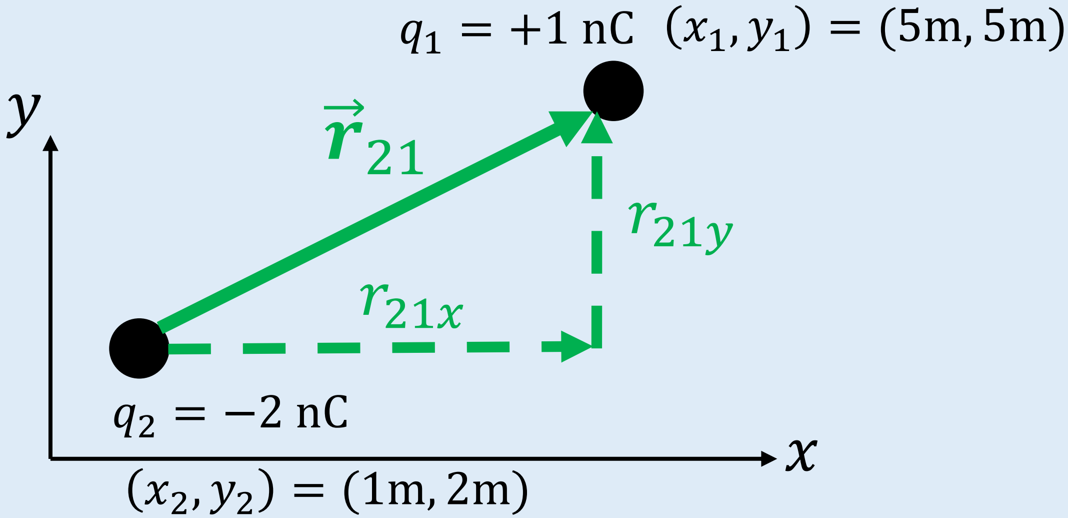
$(x_1, y_1) = (5\mathrm{m}, 5\mathrm{m})$

$q_2 = -2\ \mathrm{nC}$

$(x_2, y_2) = (1\mathrm{m}, 2\mathrm{m})$

1. Find the magnitude of the force q_2 exerts on q_1.
2. Find the unit vector for that force.
3. Find the x- and y-components of the force.

Practice Problem with 2-D Vector Forces (2)



$$\vec{\boldsymbol{r}}_1 = 5\mathrm{m}\hat{\boldsymbol{\imath}} + 5\mathrm{m}\hat{\boldsymbol{\jmath}}$$

$$\vec{\boldsymbol{r}}_2 = 1\mathrm{m}\hat{\boldsymbol{\imath}} + 2\mathrm{m}\hat{\boldsymbol{\jmath}}$$

$$\vec{\boldsymbol{r}}_{21} = \vec{\boldsymbol{r}}_2 - \vec{\boldsymbol{r}}_1$$

$$\vec{\boldsymbol{r}}_{21} = (5\mathrm{m} - 1\mathrm{m})\hat{\boldsymbol{\imath}} + (5\mathrm{m} - 2\mathrm{m})\hat{\boldsymbol{\jmath}} = 4\mathrm{m}\hat{\boldsymbol{\imath}} + 3\mathrm{m}\hat{\boldsymbol{\jmath}}$$

$$r_{21} = \sqrt{\vec{\boldsymbol{r}}_{21} \cdot \vec{\boldsymbol{r}}_{21}} = \sqrt{(4\mathrm{m})^2 + (3\mathrm{m})^2} = 5\ \mathrm{m}.$$

Practice Problem with 2-D Vector Forces (2)

$$\left|\vec{\boldsymbol{F}}_{21}\right| = k\,\frac{|q_1||q_2|}{r_{21}^2} = \left(9\times10^9\mathrm{N}\,\frac{\mathrm{m}^2}{\mathrm{C}^2}\right)\frac{\left|10^{-9}\mathrm{C}\right|\left|-2\times10^{-9}\mathrm{C}\right|}{(25\ \mathrm{m}^2)} = 7.2\times10^{-10}N.$$

$$\hat{\boldsymbol{r}}_{21} = \frac{\vec{\boldsymbol{r}}_{21}}{r_{21}} = \frac{(4\mathrm{m})\hat{\boldsymbol{\imath}}+(3\mathrm{m})\hat{\boldsymbol{\jmath}}}{5\ \mathrm{m}} = \frac{4}{5}\hat{\boldsymbol{\imath}} + \frac{3}{5}\hat{\boldsymbol{\jmath}}.$$

$$F_{21x} = \vec{\boldsymbol{F}}_{21}\cdot\hat{\boldsymbol{\imath}} = k\,\frac{q_1q_2}{r_{21}^2}\,\hat{\boldsymbol{r}}_{21}\cdot\hat{\boldsymbol{\imath}}$$

$$= \left(9\times10^9\mathrm{N}\,\frac{\mathrm{m}^2}{\mathrm{C}^2}\right)\frac{(10^{-9}\mathrm{C})(-2\times10^{-9}\mathrm{C})}{25\ \mathrm{m}^2}\left(\frac{4}{5}\right) = -5.76\times10^{-10}\mathrm{N}.$$

$$F_{21y} = \vec{\boldsymbol{F}}_{21}\cdot\hat{\boldsymbol{\jmath}} = k\,\frac{q_1q_2}{r_{21}^2}\,\hat{\boldsymbol{r}}_{21}\cdot\hat{\boldsymbol{\jmath}}$$

$$= \left(9\times10^9\mathrm{N}\,\frac{\mathrm{m}^2}{\mathrm{C}^2}\right)\frac{(10^{-9}\mathrm{C})(-2\times10^{-9}\mathrm{C})}{25\ \mathrm{m}^2}\left(\frac{3}{5}\right) = -4.32\times10^{-10}\mathrm{N}.$$

$$\vec{\boldsymbol{F}}_{21} = F_{21x}\,\hat{\boldsymbol{\imath}} + F_{21y}\,\hat{\boldsymbol{\jmath}} = -5.76\times10^{-10}\mathrm{N}\,\hat{\boldsymbol{\imath}} - 4.32\times10^{-10}\mathrm{N}\,\hat{\boldsymbol{\jmath}}.$$

Force components are directed toward charge q_2.

$\therefore$ Force is attractive, as expected for opposite sign charges.

Algorithm for Solving Vector Addition Problems

1. Draw a diagram with the given vectors.
2. Choose axes to simplify work. (Try to eliminate calculations using symmetry.)
3. For 2-D planar situations, resolve each vector $\vec{\boldsymbol{A}}_i$ into x- and y-components,

$$A_{ix} = A_i \cos\theta_i \text{ , and } A_{iy} = A_i \sin\theta_i \text{ ,}$$

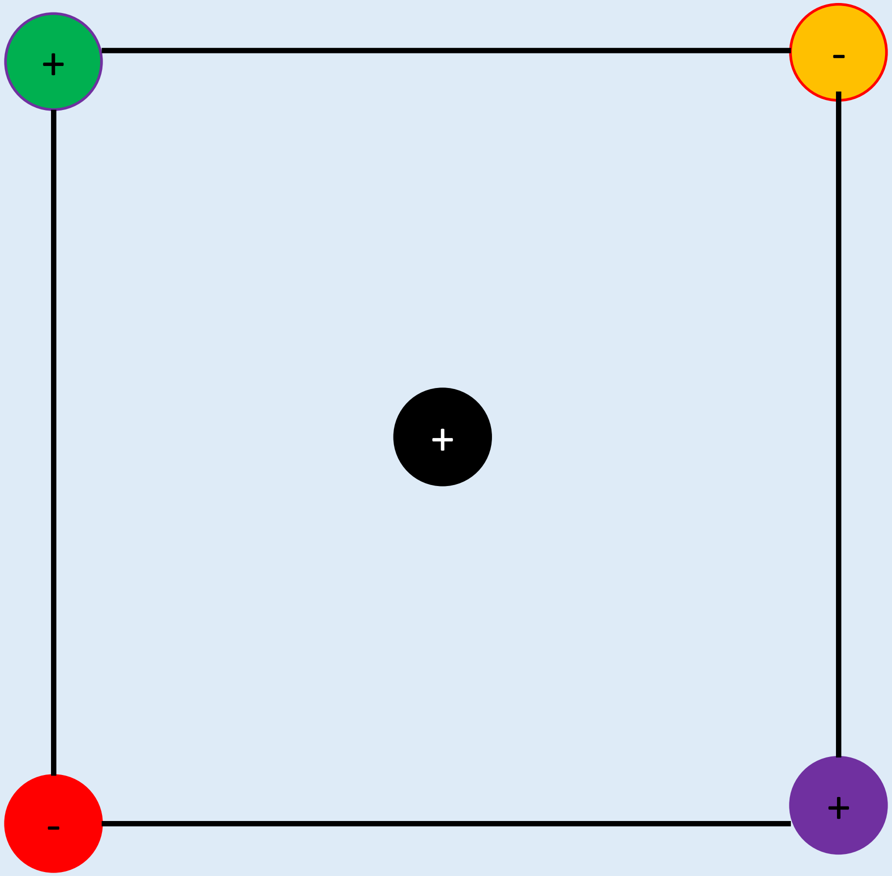
$\theta_i =$ angle that each vector makes with respect to the x-axis.

4. Add components to get sums: $A_x = \sum_i A_{ix}$, and $A_y = \sum_i A_{iy}$.
5. Calculate the angle of the resultant vector $\vec{\boldsymbol{A}} = \sum_i \vec{\boldsymbol{A}}_i = A_x \hat{\boldsymbol{\imath}} + A_y \hat{\boldsymbol{\jmath}}$ with respect to the x-axis by using $\tan\theta = \frac{A_y}{A_x}$.
6. Extension to 3-dimensional geometry follows in similar fashion.

Example Problem 1.1

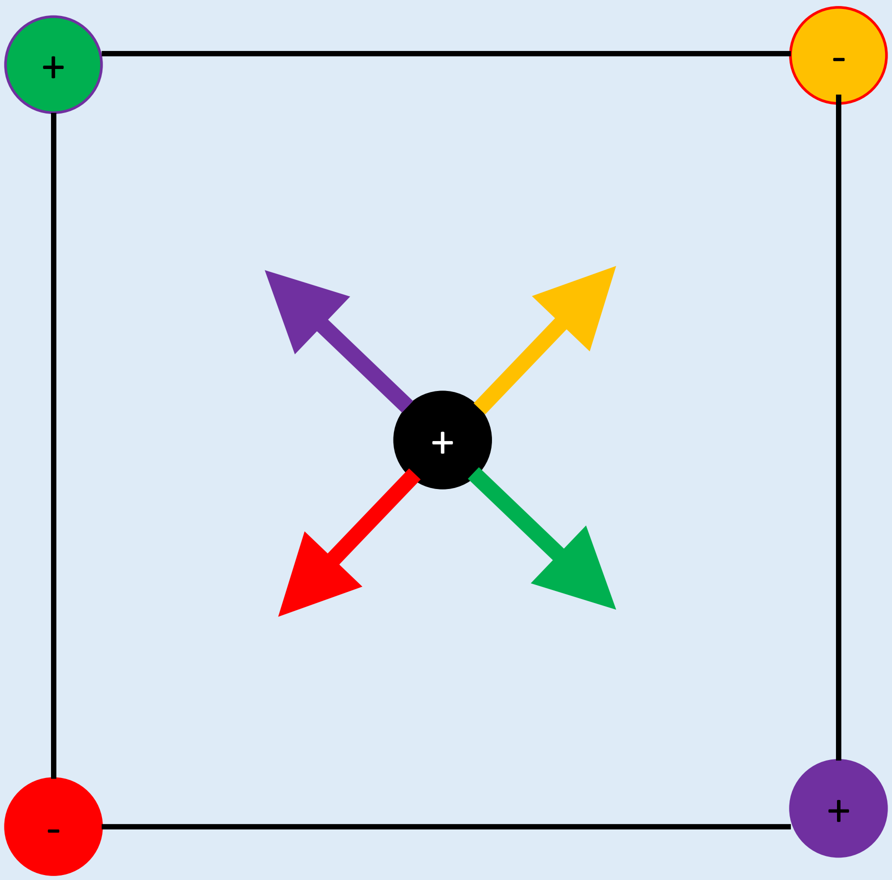
- Four charges each with charge $|q| = 10^{-5}\mathrm{C}$ are placed on the corners of a square 1 m on a side.

- Find the force on a $+10^{-7}\mathrm{C}$ charge at the center of the square.



Example Problem 1.1 (2)

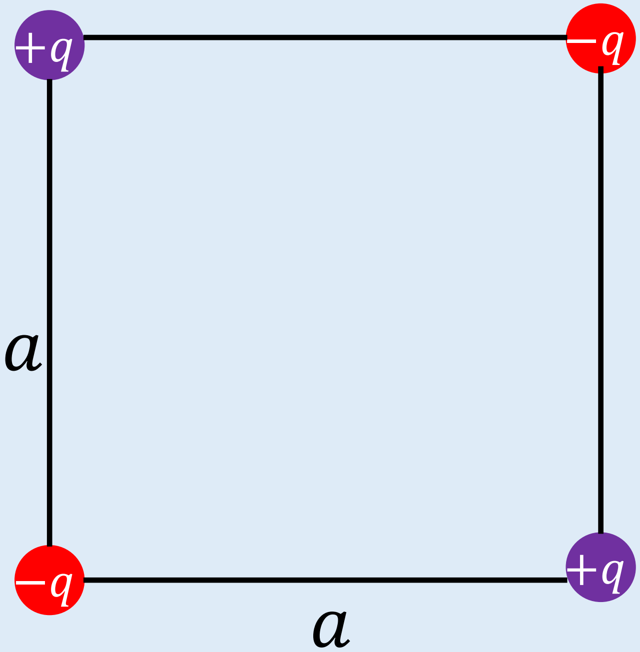
- Four charges each with charge $|q| = 10^{-5}\text{C}$ are placed on the corners of a square 1 m on a side.
- Find the force on a $+10^{-7}\text{C}$ charge at the center of the square.



Example Problem 1.2

- Four charges, each with a charge $|q|$, are on the corners of a square of side a.

- Calculate the total force on the charge in the upper right corner of the square



Example Problem 1.2 (2)

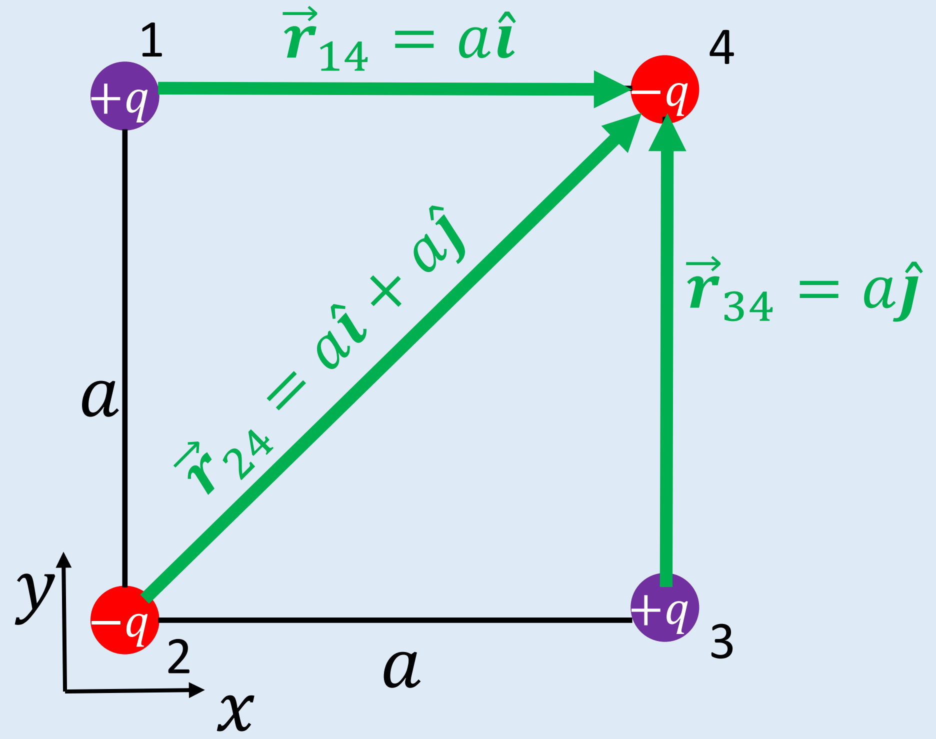
- Calculate the individual vector forces on the charge, and then use vector addition to get the net force on the corner charge.

  - From the diagram,

$$\hat{\boldsymbol{r}}_{14} = \frac{\vec{\boldsymbol{r}}_{14}}{r_{14}} = \frac{a\hat{\boldsymbol{\imath}}}{a} = \hat{\boldsymbol{\imath}}$$

$$\hat{\boldsymbol{r}}_{34} = \frac{\vec{\boldsymbol{r}}_{34}}{r_{34}} = \frac{a\hat{\boldsymbol{\jmath}}}{a} = \hat{\boldsymbol{\jmath}}$$

$$\hat{\boldsymbol{r}}_{24} = \frac{\vec{\boldsymbol{r}}_{24}}{r_{24}} = \frac{a\hat{\boldsymbol{\imath}}+a\hat{\boldsymbol{\jmath}}}{\sqrt{2}a} = \frac{1}{\sqrt{2}}\hat{\boldsymbol{\imath}} + \frac{1}{\sqrt{2}}\hat{\boldsymbol{\jmath}}$$



Example Problem 1.2 (3)

- Use Coulomb's law to get individual force vectors acting on upper right corner charge:

$$\vec{\boldsymbol{F}}_{14} = k\frac{(+q)(-q)}{r_{14}^2}\,\hat{\boldsymbol{r}}_{14} = -k\frac{q^2}{a^2}\hat{\boldsymbol{\imath}}$$ Attractive force

$$\vec{\boldsymbol{F}}_{24} = k\frac{(-q)(-q)}{r_{24}^2}\hat{\boldsymbol{r}}_{24} = k\frac{q^2}{2a^2}\left(\frac{1}{\sqrt{2}}\hat{\boldsymbol{\imath}} + \frac{1}{\sqrt{2}}\hat{\boldsymbol{\jmath}}\right) = k\frac{q^2}{a^2}\left(\frac{1}{2\sqrt{2}}\hat{\boldsymbol{\imath}} + \frac{1}{2\sqrt{2}}\hat{\boldsymbol{\jmath}}\right)$$ Repulsive force

$$\vec{\boldsymbol{F}}_{34} = k\frac{(+q)(-q)}{r_{34}^2}\hat{\boldsymbol{r}}_{34} = -k\frac{q^2}{a^2}\hat{\boldsymbol{\jmath}}$$ Attractive force

- Add components to get net force vector:

$$\vec{\boldsymbol{F}}_{4} = \vec{\boldsymbol{F}}_{14} + \vec{\boldsymbol{F}}_{24} + \vec{\boldsymbol{F}}_{34} = -k\frac{q^2}{a^2}\left(1 - \frac{1}{2\sqrt{2}}\right)\hat{\boldsymbol{\imath}} - k\frac{q^2}{a^2}\left(1 - \frac{1}{2\sqrt{2}}\right)\hat{\boldsymbol{\jmath}}$$

$$= -0.65\,k\frac{q^2}{a^2}\hat{\boldsymbol{\imath}} - 0.65\,k\frac{q^2}{a^2}\hat{\boldsymbol{\jmath}}$$

Classes of Materials: Conductors and Insulators

- Behave differently when an electric field is present. Or, equivalently, when acted upon by electric forces.
  - Charge on an insulator remains fixed in position once placed on it.
    - Charge must be placed directly on an insulator.
  - Charge on a conductor moves around in order to make the electric field (next class topic) inside the conductor zero.
    - Charge can be directly placed on a conductor by touching it.
    - Charge can also be transferred to a conductor indirectly, by induction, without directly touching charge to the conductor.
    - Charge on a conductor can be ‘run to ground.’ Attaching a metal wire attached to ground at one end, and to a conductor at the other end, provides an ‘escape path’ for excess charge on the conductor. Since ground is also a very good conductor, it acts as a ‘sink’ and ‘drains away’ any excess charge on the conductor.

Polarization and Attraction.

- Metals are very good conductors.
  - ➢ Polarization – the rearrangement of charges within a metal in response to an external charge – is very important.
  - ➢ Opposite-sign charge in the metal is attracted closer toward a nearby external charge and feels an attractive force toward that charge.
- Insulators
  - ➢ Don’t have much internal charge motion.
  - ➢ Polarization forces and response are very weak.

Example: Induction and Polarization

1. Start with an uncharged metal ball (net charge of ball is zero) supported by an insulating stand.
2. Bring a negatively charged rod near it, without touching it. Free electrons in the metal ball are repelled by the negative charges on the rod, and they shift toward the right side, away from the rod. Positive charges remain behind on the left side of the ball. The charge separation is an example of polarization.
3. Because the positive charges in the ball are closer to the negatively charged rod, the rod feels a net attractive force toward the metal ball.