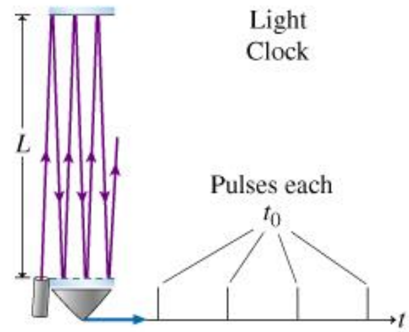
37A– Special Relativity: Lorentz Transformations of Space and Time

When the ticking of a clock is observed in the rest frame of the clock, the time between ticks t_0 is called the proper time. When the same clock is observed by an observer who is moving at speed v with respect to the clock, the time between ticks is measured to be $t' = \gamma t_0$. In this section you will derive γ in terms of v/c. Assume that the speed of light is $c = 3\text{x}10^8$ m/s independent of the inertial frame in which it is observed.



Consider a clock that ticks every time light makes a round trip between two mirrors separated by a distance L. Each time a pulse hits the lower mirror a "tick" is recorded.

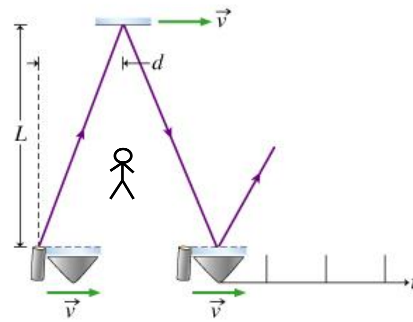
1a) What is the time between ticks, in the frame of the clock (S) as shown to the right, expressed in terms of L and the speed of light c?

t_0 =_____________________

Now imagine that the clock is moving with velocity v relative to a stationary observer ( )as shown in the figure to the right. (We will call the observer's frame the S frame.) To calculate the tick rate of the clock observed by ( ) we calculate the distance between mirror hits as observed by ( ).

Consider the length of the path the light traverses in the clock from the point of view of an observer in reference frame S.

(NOTE: There is no length contraction along the direction perpendicular to relative motion, so L is still L.)



b) Express the distance of the round trip in terms of d and L. ________________________

c) Now solve for the time t_S between the ticks of the light clock as viewed from reference frame S in terms of c, d and L.

t_S=_______________________

d) The bottom mirror moves $2d$ between clicks. Rewrite this distance in terms of t_S and v.

$2d$=_______________________

e) Now solve for the time t_S in terms of t_0, v, and c.

t_S=_______________________

You have now solved for the Lorentz factor, γ! (Good job!)

RELATIVISTIC TIME CALCULATION 1 – "HIGH" SPEED

2) Adam holds a clock (A) that ticks once per second in his rest frame. Eve flies by at $v = 0.8c$ carrying her own clock (E) that ticks once per second in her frame.
   a) According to Adam, which clock ticks slower? _____________

   b) According to Eve, which clock ticks slower? _____________

   c) What is the numerical value of the Lorentz factor γ? _____________

   d) According to Eve, how much times passes on her clock while Adam's clock ticks once?

   ____

RELATIVISTIC TIME CALCULATION 2 – "LOW" SPEED

3) Two atomic clocks are synchronized on the ground. One is placed on a jet and sent around the Earth above the equator at an altitude of 30,000 m and speed of 700 m/s (Mach 2).

   a) Approximately how long does it take the jet to circle the Earth once? (Show work)

   ____

   b) What is the numerical value of the Lorentz factor γ for the velocity of the jet? (include at least 15 significant digits!) Note that your calculator probably does not have 15 digits, so use the trick given in Example 37.2 in your text book.

   ____

   c) According to Special Relativity and assuming the jet moves while the ground clock is stationary, what is the difference in time between the two clocks when the jet arrives back at the take off point?

   ________________s

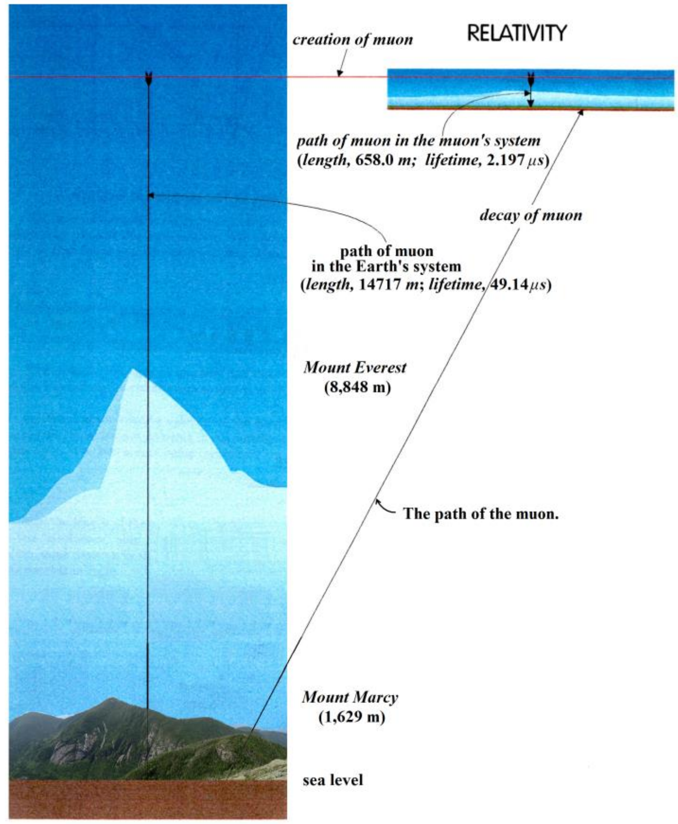
   d) Is there a significant difference between the watches for a globe-trotting politician and her spouse who stays behind after she has circled the globe 20 times at Mach 2? (Consider anything less than 1 second to be negligible.)

   ____

NOTE: There is a paradox here as to which clock actually ticks slower which is settled by the fact that only one clock accelerates. (The "Twin Paradox" is nicely discussed at http://www.einstein-online.info.) This requires General Relativity to fully understand.

A COSMIC RAY EXAMPLE

The muon is a sub-atomic particle that decays into an electron and two very light neutrinos in a characteristic time $\tau = 2.2 \times 10^{-6}$ s in its own rest frame. Muons are created by very high energy cosmic ray protons hitting the upper atmosphere of the Earth. They have a mass equal to about 200 times that of the electron and 1/10 of that of a proton.



1) Consider a typical cosmic ray muon created 14 km above the surface of the Earth which just reaches the surface before decaying.
   a) Muons, in a cosmic shower, travel very close to the speed of light. What would the muons lifetime have to be in the <u>Earth observer's frame</u> to have traveled 14 km after it was created?

   t_{Earth} = ________________

   b) By what factor is this larger or smaller than the rest lifetime?

   ____

   c) Find the Lorentz factor for this muon from the ratio of lifetimes you just found.

   γ =________________

2) Now shift to the muon observer frame.
   a) First from part (c) above, find the speed of the muon as a fraction of the speed of light, c, from the Lorentz factor.

   v_{muon} =________________

   b) Next, recalculate the time for the muon to travel 14 km in the earth's frame. (Does this differ significantly from the time you found in part (1 a)?)

   t_{muon} =________________

   c) How far does an observer traveling with the muon think he/she has traveled from creation to decay of the muon?

   d_{μ}=________________

* This example is similar to the real quandary posed to scientists who observed cosmic ray flux as a function of altitude and tried to figure out what it meant about the lifetime of the particles being observed.