39B - Probability Distributions

1) Consider the situation of a particle for which the probability density for finding a particle in the region $0 < x < L$ is given by $P_{density}(x,t) = \frac{2}{L}\sin^2\left(\frac{\pi x}{L}\right)$. The probability of finding the particle outside this region is zero.

a) Plot or sketch this distribution between $x = 0$ and $x = L$.

b) What is the probability of finding the particle between $x = 0$ and $x = L$? (You need to explicitly do the integral or make a solid mathematical argument. Show your work.)

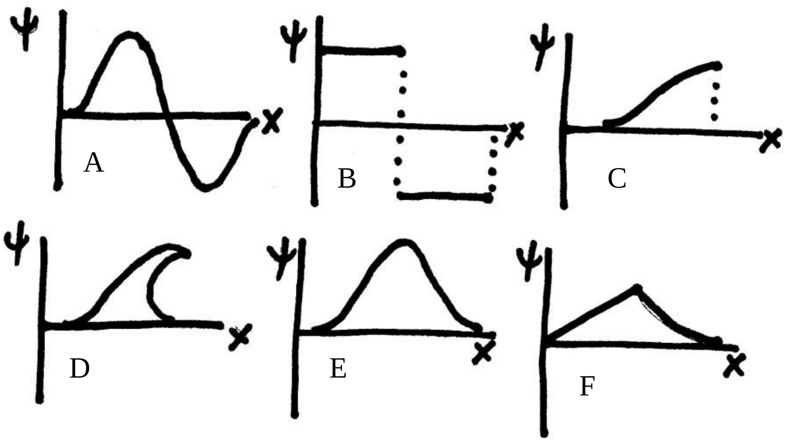
c) What is the probability of finding the particle between $x = 0$ and $x = L/2$? (Hint: You might be able to do this without doing the integral. Think about symmetry.)

d) What is the probability of finding the particle between $\frac{L}{2} - \frac{\delta}{2}$ and $\frac{L}{2} + \frac{\delta}{2}$ where $\delta = 10^{-20}L$.

Your calculator will probably fail you here because we chose the region to be extremely narrow. (Hint: You might be able to do this without doing the integral in a formal way. Think about the idea of the Riemann sum in Calculus.)

40A - Wavefunctions

2) Several possible forms for the spatial form of the wavefunction for a particle are sketched below. List the ones that are not physical, and give a reason for why each one is not. Dashed lines indicate that the wavefunction jumps from one value to the next at a single point.

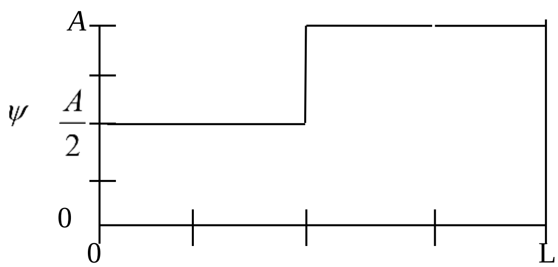


2) Consider the wavefunction $\Psi(x,t) = \psi(x)(\cos\omega t + i\sin\omega t)$ where $\psi(x)$ is a real function. Calculate the probability density for this wavefunction in terms of the variables given in the equation.

3) a) Show that $\psi(x) = A\sin kx$ is a solution to the time-independent Schrodinger Equation with potential equal to zero: $-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} = E\psi$ and find the relation between $k\ and\ E$ that solves the equation.

b) Note that $k = \frac{2\pi}{\lambda}$ and find the relation between momentum p and energy E implied by the solution above. Does this make physical sense?

4) An electron is in the n=10^9 energy level of a strangely shaped quantum well so that the wavefunction $\psi(x)$ has the approximate form shown below. (The value of the wavefunction is zero outside of the limits of the graph.) <u>Note that the function at $x = L/2$ is steep, but it is not discontinuous (the same is true for it's derivative as well).</u>



a) Sketch the probability density that is consistent with the wavefunction shown. Include a scale in terms of A.

b) Assuming that the probability of finding the particle in the space between 0 and L is unity, find the value of A.

c) What is the probability that the electron will be found in the left-hand side of the box? (Your answer must be consistent with your sketch in part b. It may either be in terms of *A* and *L*, or it may be a number.) Explain your logic.

40B – Heisenberg Uncertainty Principle - Again

The Heisenberg Uncertainty Principle states that it is not possible to simultaneously measure the position and momentum of a particle with absolute certainty. The mathematical statement of the principle in the textbook is:

$$\Delta x \Delta p = \sigma_x \sigma_p \geq h/4\pi$$

One of the approaches to understanding the uncertainty principle is to think about how to add waves of different wavelengths to one another to create a peaked waveform.

1) Consider a particle for which the spatial part of a wavefunction is $\psi(x) = Ae^{-ax^2}$ where A and a are real, positive constants.

a) If the value of a is increased, what effect does this have on the uncertainty in the position of the particle? Explain.

b) If the value of a is increased, what effect would this have on the uncertainty in momentum of the particle? Explain.

2) Electrons of kinetic energy K_0 are shot through a very narrow slit of width L and are detected when they hit a phosphor screen a few meters away. A diffraction pattern is observed in the probability distribution of arriving electrons with a central maximum of width y_{width}. If the kinetic energy of the electrons is reduced to one-half its initial value, how does the width of the diffraction pattern change?