Note

There are no classes on campus on Monday, Jan. 15 (holiday).

There are no PHYS 1200 classes on Tuesday, Jan. 16.

Physics 1200
Lecture 02
Spring 2024

Electric Fields

Review: Ideas from Class 01

- Coulomb's law (force of q_1 acting on q_2):

$$\vec{\boldsymbol{F}}_{12} = k\frac{q_1 q_2}{r_{12}^2}\frac{\vec{\boldsymbol{r}}_{12}}{r_{12}} = k\frac{q_1 q_2}{r_{12}^2}\hat{\boldsymbol{r}}_{12}$$

$$\vec{\boldsymbol{r}}_{12} = \vec{\boldsymbol{r}}_2 - \vec{\boldsymbol{r}}_1$$

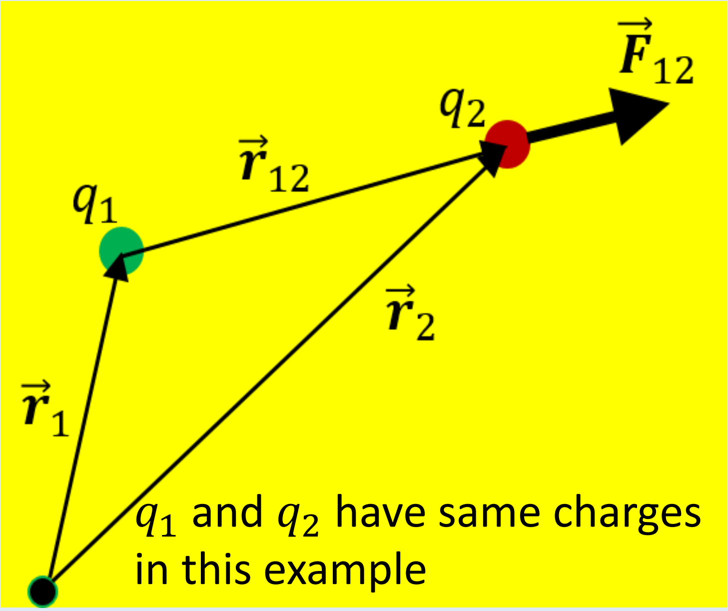
  - Algorithmic, always works, easy to program.
  - Alternative intuitive method that works:

$\left|\vec{\boldsymbol{F}}_{12}\right| = k\frac{|q_1||q_2|}{r_{12}^2}$, for magnitude,

then use like charges repel / unlike charges attract to get force vector direction.

  - Can use one method to get solution, other to check result.

- Polarization of conductors
  - Total charge does not change.
  - Charge does not cross insulators.
  - Charge can be sent to ground or drawn from ground.



The Electric Field

- Can rewrite Coulomb's law: $\boxed{\vec{\boldsymbol{F}}_{1on2} = q_2 \vec{\boldsymbol{E}}_1}$

  where $\boxed{\vec{\boldsymbol{E}}_1 \equiv k \frac{q_1}{r_{12}^2} \frac{\vec{\boldsymbol{r}}_{12}}{r_{12}} = k \frac{q_1}{r_{12}^2} \hat{\boldsymbol{r}}_{12}}$

  is the <u>electric field of point charge q_1 at the location of charge q_2</u>.

  - q_1 is 'source' of the field. Position of q_2 is 'target' or 'field' location.
  - Distance vector $\vec{\boldsymbol{r}}_{12}$ <u>always points from source to target</u>. Same for $\hat{\boldsymbol{r}}_{12}$.
  - Sign (+, –) of q_1 must be included in equation above.
    - <u>Electric field vectors point radially outward for positive point charges, and radially inward for negative point charges.</u>
  - Field $\vec{\boldsymbol{E}}_1$ from q_1 is <u>independent</u> of whether a charge q_2 (or any other charge) is at target location or not.
  - Removes 'action-at-a-distance' in Coulomb's law: q_2 responds to <u>local</u> value of $\vec{\boldsymbol{E}}_1$.
  - Changes in $\vec{\boldsymbol{E}}_1$ (e.g., due to change in position of q_1) propagate outward along field at the speed of light (electromagnetic waves – later classes).

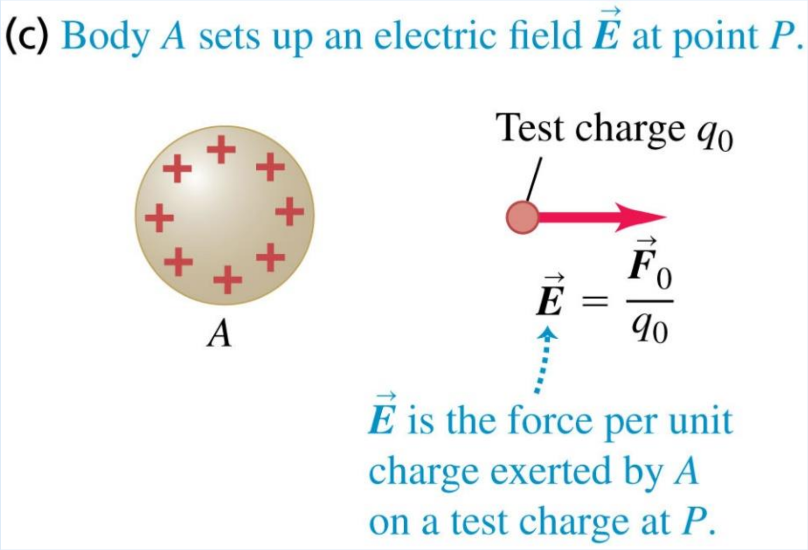
The Electric Field (2)

- For <u>positive charges</u>, expression

$$\vec{\boldsymbol{F}}_{1on2} = q_2\vec{\boldsymbol{E}}_1$$

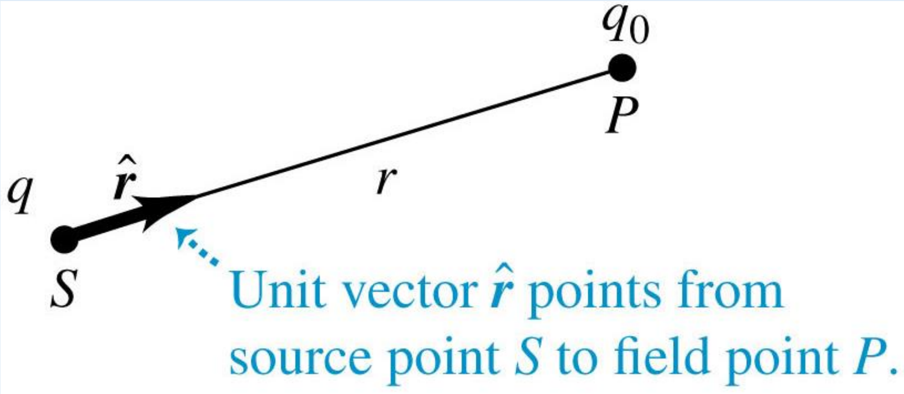
gives directions of $\vec{\boldsymbol{F}}_{1on2}$ and $\vec{\boldsymbol{E}}_1$ <u>as being the same</u>.

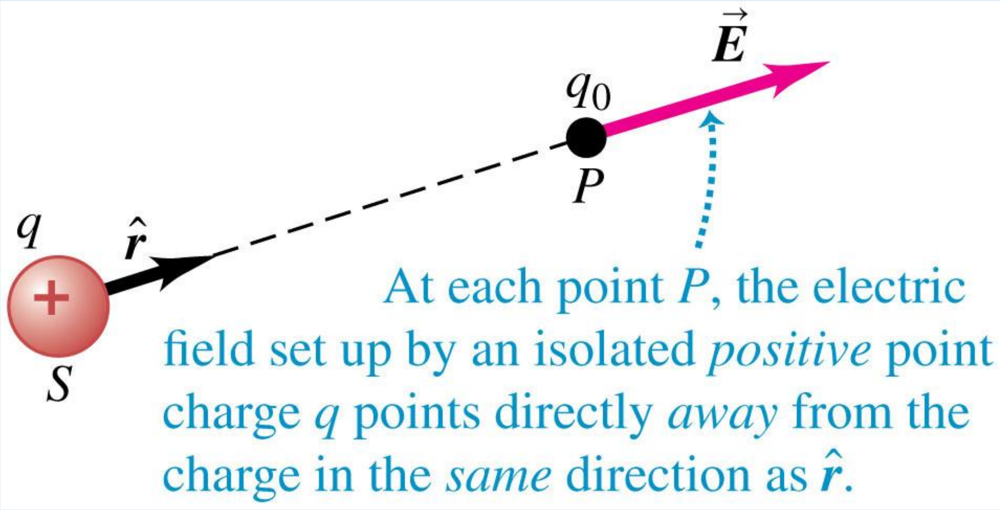
∴ Can always find direction of $\vec{\boldsymbol{E}}$ by finding the direction electric force acting on a <u>positive 'test charge'</u> placed at that location:



<u>Note: by definition, test charges q_0 are always positive (+).</u>

The Electric Field (3)





The Electric Field (4)

- From

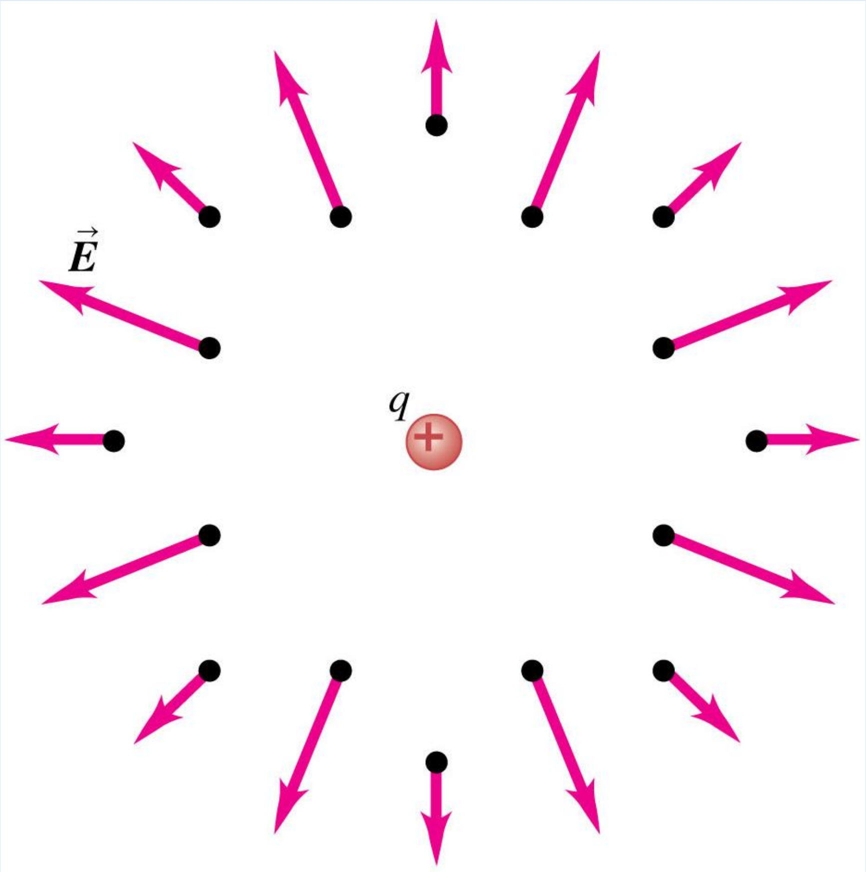
$$\vec{\boldsymbol{F}}_{1on2} = q_2\vec{\boldsymbol{E}}_1 \ ,$$

follows that SI unit of electric field is a N/C. Another acceptable SI unit is a V/m, where ‘V’ = volt (SI unit of electric potential – discussed next week).

Visualizing the Electric Field

$$\vec{\boldsymbol{E}} = \frac{kq}{r^2}\frac{\vec{\boldsymbol{r}}}{r} = \frac{kq}{r^2}\hat{\boldsymbol{r}}$$

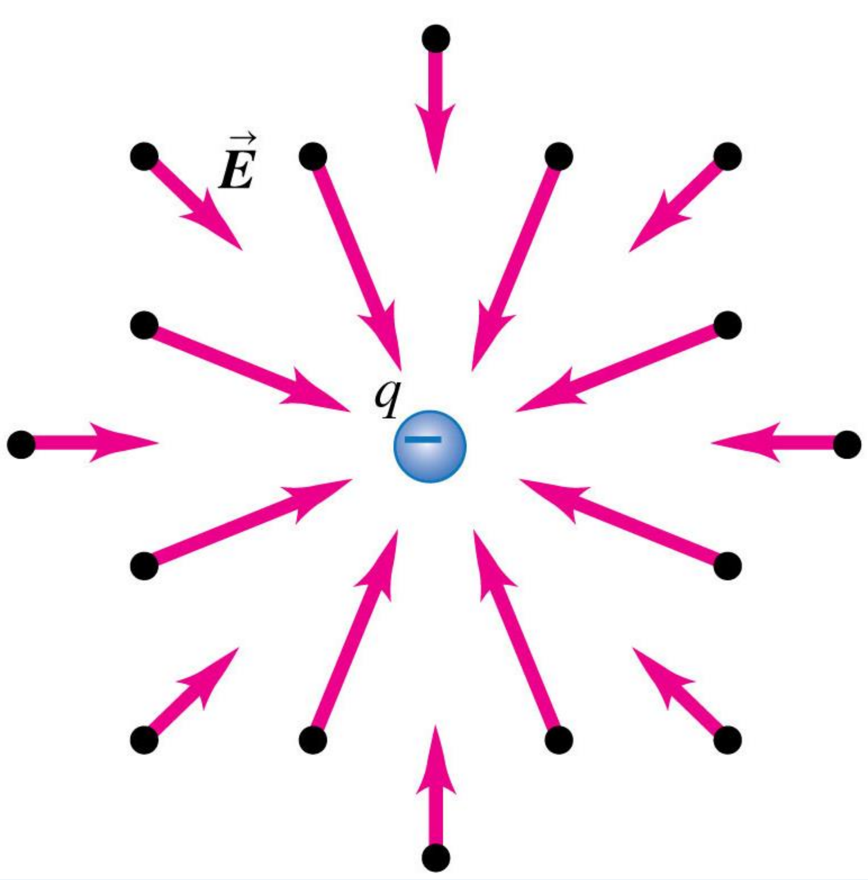
- Point charge *q* produces electric field at *all* points in space.
- <u>Field produced by a positive point charge points *away from* charge.</u>
- Tail of the vector is where the field has magnitude value E.
- Length of the vector arrow indicates strength of the field.
- Field strength magnitude decreases with greater distance.



Visualizing the Electric Field (2)

$$\vec{\boldsymbol{E}} = \frac{kq}{r^2}\frac{\vec{\boldsymbol{r}}}{r} = \frac{kq}{r^2}\hat{\boldsymbol{r}}$$

- Point charge q produces an electric field at *all* points in space.
- <u>Field produced by a negative point charge points *toward* charge.</u>
- Tail of the vector is where the field has magnitude value E.
- Length of the vector arrow indicates strength of the field.
- Field strength magnitude decreases with greater distance.



Superposition: Total Field

- Net force vector acting on a charge is vector sum (superposition) of all the individual force vectors acting on the charge ⇒ net electric field is the vector sum (superposition) of the individual electric field vectors at any location:

$$\begin{aligned}\vec{\boldsymbol{F}}_0 &= \vec{\boldsymbol{F}}_{10} + \vec{\boldsymbol{F}}_{20} + \vec{\boldsymbol{F}}_{30} + \vec{\boldsymbol{F}}_{40} + \cdots \\ &= q_0\vec{\boldsymbol{E}}_1 + q_0\vec{\boldsymbol{E}}_2 + q_0\vec{\boldsymbol{E}}_3 + q_0\vec{\boldsymbol{E}}_4 + \cdots = q_0\vec{\boldsymbol{E}}_{tot}\end{aligned}$$

∴ Total force on q_0 at $\vec{\boldsymbol{r}}_0$ due to a system of charges q_i located at positions $\vec{\boldsymbol{r}}_i$ $(i = 1, 2, 3, \cdots, N)$ is

$$\vec{\boldsymbol{F}}_{tot} = q_0\vec{\boldsymbol{E}}_{tot}\ ,$$

$$\vec{\boldsymbol{E}}_{tot} = \vec{\boldsymbol{E}}_1 + \vec{\boldsymbol{E}}_2 + \vec{\boldsymbol{E}}_3 + \vec{\boldsymbol{E}}_4 + \cdots = \sum_{i=1}^{N}\vec{\boldsymbol{E}}_i\ ,$$

$$\vec{\boldsymbol{E}}_i = \frac{kq_i}{r_{i0}^2}\frac{\vec{\boldsymbol{r}}_{i0}}{r_{i0}} = \frac{kq_i}{r_{i0}^2}\hat{\boldsymbol{r}}_{i0}\ ,$$

$$\vec{\boldsymbol{r}}_{i0} = \vec{\boldsymbol{r}}_0 - \vec{\boldsymbol{r}}_i = (x_0 - x_i)\hat{\boldsymbol{\imath}} + (y_0 - y_i)\hat{\boldsymbol{\jmath}} + (z_0 - z_i)\hat{\boldsymbol{k}}\ .$$

Question 2.1

- Two equal charges lie at two vertices of an equilateral triangle. The electric field at point P (the third vertex) points:

A. Up the page.

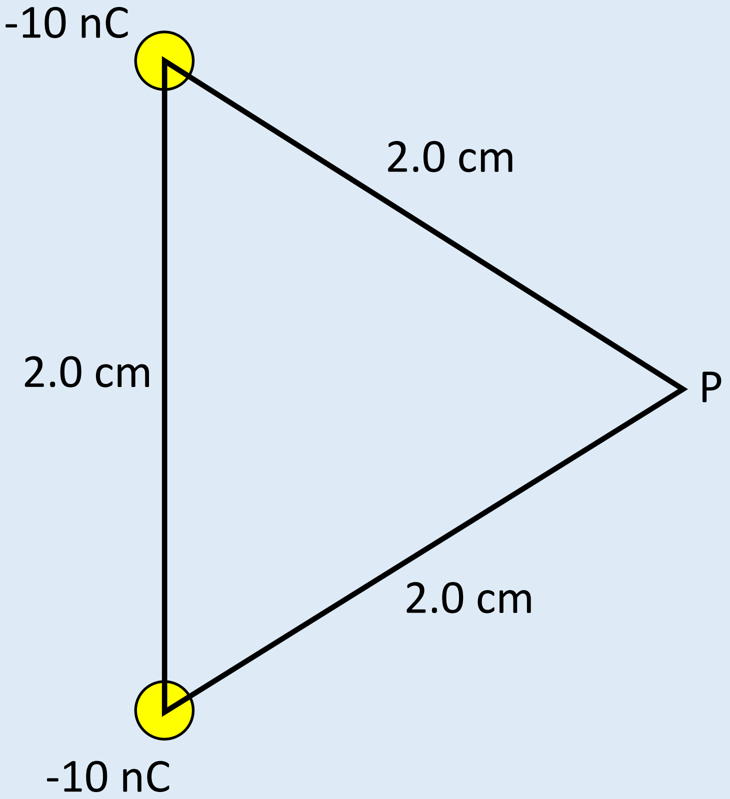
B. Down the page.

C. To the right.

D. To the left.

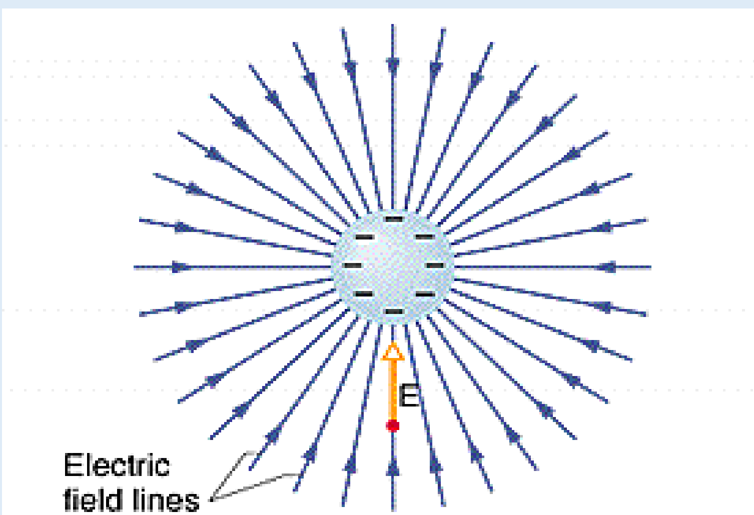
E. Out of the page.

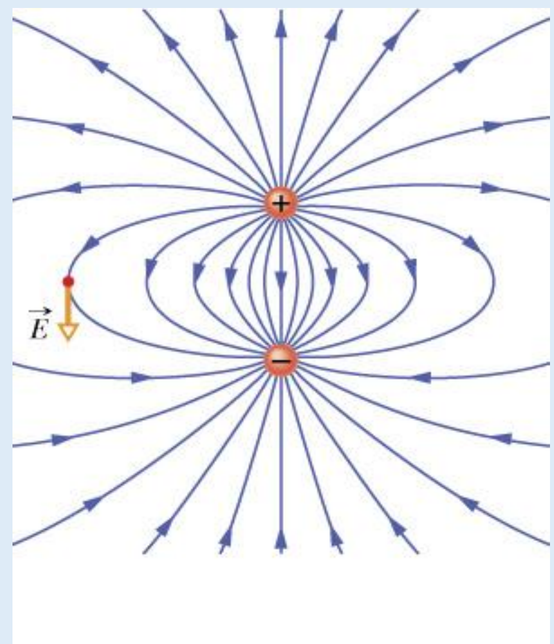
F. Into the page.



Electric Field Lines

- Electric field lines show the direction of $\vec{\boldsymbol{E}}$ at any point.
- Magnitude of $\vec{\boldsymbol{E}}$ proportional to the density (spacing) of field lines.
- Electric field lines originate from positive charges and terminate on negative charges, unless they extend to infinity.

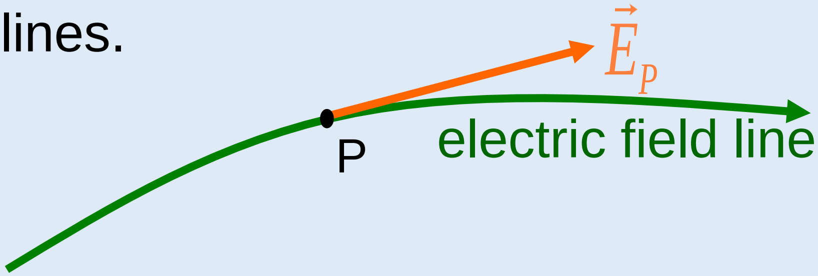


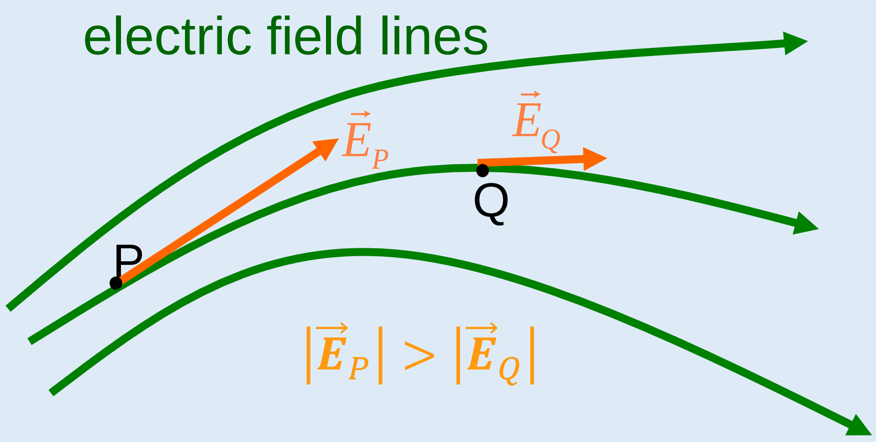


Electric Field Lines (2)

The concept of field lines was introduced by Faraday to understand fields on a more intuitive, rather than mathematical, basis.

1. Field vectors are tangent to field lines.





2. Strength of the field is reflected by density (spacing) of field lines. Closer spacing = stronger field

Calculating the Electric Field for a Continuous Charge Distribution

- For a continuous distribution of charge (instead of discrete point charges), the summation to calculate the electric field at position $\vec{\boldsymbol{r}}_0 = x_0\hat{\boldsymbol{\imath}} + y_0\hat{\boldsymbol{\jmath}} + z_0\widehat{\boldsymbol{k}}$, instead becomes an integration, with

$$\vec{\boldsymbol{E}}_{tot}(x_0, y_0, z_0) = \int \overrightarrow{\boldsymbol{dE}}\,(x', y', z'),$$

$$\overrightarrow{\boldsymbol{dE}}(x', y', z') = k\,\frac{dq'(x', y', z')}{r_0'^2}\frac{\vec{\boldsymbol{r}}_0'}{r_0'}\ ,$$

$$\vec{\boldsymbol{r}}_0' = \vec{\boldsymbol{r}}_0 - \vec{\boldsymbol{r}}' = (x_0 - x')\hat{\boldsymbol{\imath}} + (y_0 - y')\hat{\boldsymbol{\jmath}} + (z_0 - z')\widehat{\boldsymbol{k}}\ ,$$

where

$$\vec{\boldsymbol{r}}' = x'\hat{\boldsymbol{\imath}} + y'\hat{\boldsymbol{\jmath}} + z'\widehat{\boldsymbol{k}}$$

is the location of a small (infinitesimal) amount of charge dq'.

Calculating the Electric Field (2)

- The charge element dq' is usually calculated by using an appropriate "charge density":

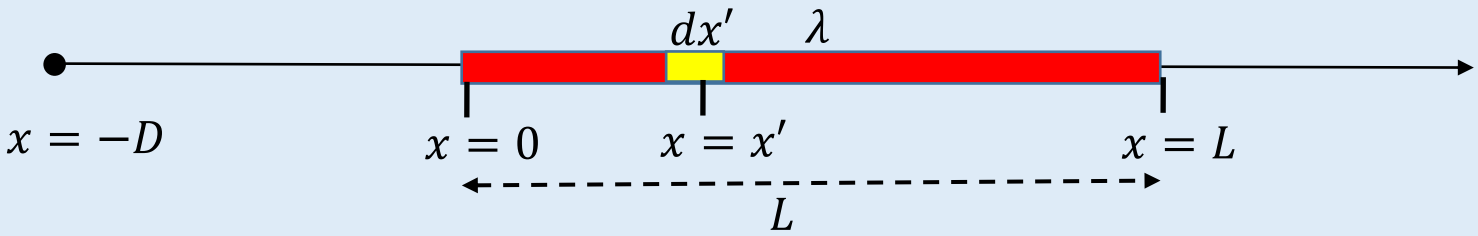
Name	Symbol	Meaning	SI Unit
Point Charge	q	Single point charge	C
Linear Charge Density	λ	dq/dx, charge per unit length	C/m
Surface Charge Density	σ	dq/dA, charge per unit area	C/m^2
Volume Charge Density	ρ	dq/dV, charge per unit volume	C/m^3

For instance, if given a volume charge density, $dq' = \rho\, dV' = \rho\, dx'dy'dz'$, and inserting into the continuous electric field expression gives

$$\vec{E}(x,y,z) = k \iiint_{\text{charge}} \frac{\rho(x',y',z')\left[(x-x')\hat{i} + (y-y')\hat{j} + (z-z')\hat{k}\right]}{\left((x-x')^2 + (y-y')^2 + (z-z')^2\right)^{3/2}} dx'dy'dz'$$

Example: field at a point a distance D away from a line charge

- Choose a coordinate system.
  - Exploit any symmetries that simplify calculation.
- Write down field due to a small charge element, then sum up (i.e., integrate) contributions from all elements.



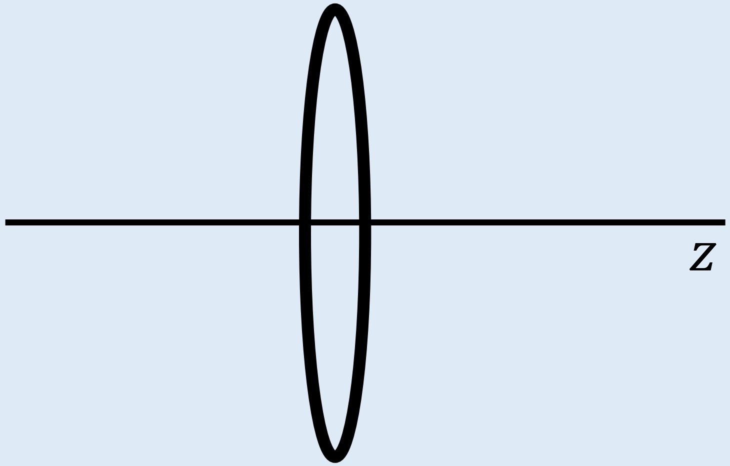
Amount of charge in dx': $dq' = \lambda\, dx'$.

Field at $x = -D$ due to dq' is:

$$\overrightarrow{\boldsymbol{dE}} = k\,\frac{\lambda dx'}{(D+x')^2}\left(\frac{[D+x'][-\hat{\boldsymbol{\imath}}]}{D+x'}\right) = -k\,\frac{\lambda dx'}{(D+x')^2}\,\hat{\boldsymbol{\imath}}.$$

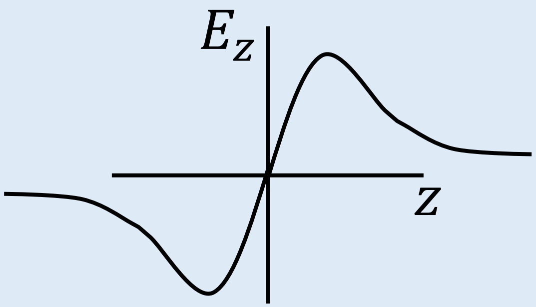
$\therefore$ At $x = -D$, total field $\vec{\boldsymbol{E}} = \int \overrightarrow{\boldsymbol{dE}} = -k \int_0^L \frac{\lambda dx'}{(D+x')^2}\,\hat{\boldsymbol{\imath}}$

Question 2.2: Using Symmetry

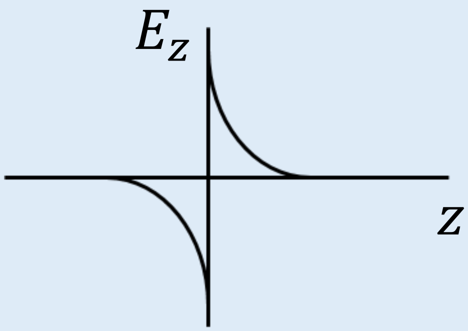


The field along the axis of symmetry of a uniform circle (i.e., hoop) of positive charge looks like:

A)



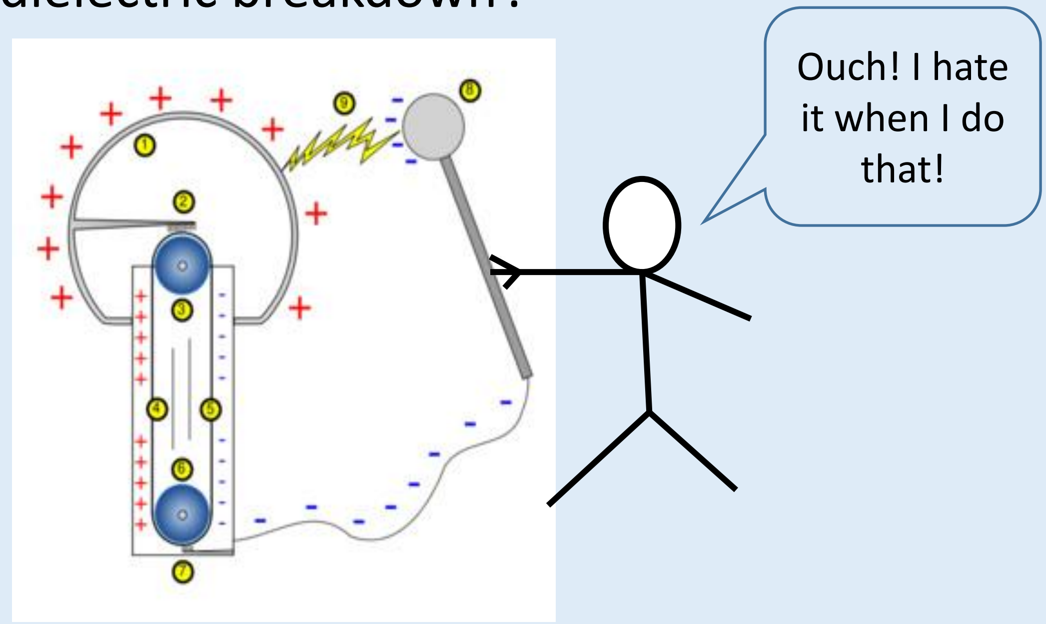
B)



C) The field along the axis is zero everywhere.

Electric field strength: Breakdown

- At sufficiently high electric fields, electrons in insulators can be pulled off the atoms they are associated with.
- In air, occurs at $\sim 10^8$ N/C (or V/m).
- When this occurs, the electron that is freed accelerates in the field, bashing into nearby atoms and ionizing them, freeing more electrons. This is called ‘dielectric breakdown’.



Demo at: https://www.youtube.com/watch?app=desktop&v=rNEY3Yv9kC8