23A – Electric Potential and Electric Potential Energy

Equipment: Pencil, paper, and brain.
Key Theoretical Ideas:
Reading: *Young and Freedman* Chapter 23.1-23.5

Relationships Between Electric Field, Potential Energy, and Electric Potential

We know from PHYS-1100 that moving an object with a conservative force gives rise to a change in potential energy,

$$U_f - U_i = -\int_{initial}^{final} \vec{\boldsymbol{F}} \cdot d\vec{\boldsymbol{s}} \qquad \text{Eq. 23a}$$

(The change in potential energy is equal to the negative of the work done on the object.) You studied this for forces due to springs and gravity. This idea can be applied to electric forces as well. Note that Eq. 23a involves a dot product between the force and the steps (ds) along the path of the integral.

1) Write the relationship for the force on a point charge q_{test} due to an electric field $E(x)$:

$\vec{\boldsymbol{F}}$___________

2) Find the dot product $\vec{\boldsymbol{F}} \cdot \boldsymbol{d\vec{s}}$ for $\vec{\boldsymbol{E}} = (3\hat{\boldsymbol{\imath}} + 2\hat{\boldsymbol{\jmath}})\ {}^{N}/_{C}$ and q_{test} = +1nC and $d\vec{\boldsymbol{s}} = dx\ \hat{\boldsymbol{\imath}}$.

$\vec{\boldsymbol{F}} \cdot d\vec{\boldsymbol{s}}$`=_______________(units)_______

3) Find the change in potential energy for the particle above if it is moved from x = 2 to x = 7 m.

ΔU =__________________(units)_______

(Aside: If the path and/or the field are curved, then we need to perform a more careful integration, which we won't address in this course.)

4) Assume that the electric field in a region of space is $\vec{\boldsymbol{E}} = \left(\frac{3}{1\text{m}}\right) x\ \hat{\boldsymbol{\imath}}\ \ {}^{N}/_{C}$ and find the difference in potential energy if a charge q_{test} = +1 nC is moved from x = 2 m to x = 7 m with $d\vec{\boldsymbol{s}} = dx\hat{\boldsymbol{\imath}}$. Write your answer on the line but include sufficient steps in your solution so the grader can follow what you did.

$\Delta U =$ ______________units

For the situation above, we separately specified the field, the test charge, and the position. It will be very useful to us later if we create a new quantity, called "electric potential," that separates the effects of the choice of the test charge from the field and position.

$$\Delta V \equiv \frac{\Delta U}{q_{test}} = -\frac{1}{q_{test}} \int_{initial}^{final} q_{test} \vec{\boldsymbol{E}} \cdot d\vec{\boldsymbol{s}} = -\int_{initial}^{final} \vec{\boldsymbol{E}} \cdot d\vec{\boldsymbol{s}} \qquad \text{Eq 23b}$$

<u>The difference in electric potential between two points is what we measure when we use a voltmeter. Unfortunately the symbol for the MKS unit of voltage is</u> **<u>V</u>** ***<u>(volts) and the symbol for electric potential is also V. Check for context.</u>***

If we know the electric potential ($V(x, y, z)$)as a function of position, then we can find the electric field components E_x, E_y, E_z by taking derivatives:

$$E_x(x,y,z) = -\frac{\partial V(x,y,z)}{\partial x} \; ; \; E_y(x,y,z) = -\frac{\partial V(x,y,z)}{\partial y} \; ; \; E_z(x,y,z) = -\frac{\partial V(x,y,z)}{\partial z} \qquad \text{Eq.23c}$$

5) Assume that the electric potential in a region of space is given by $V = \left(\left[\frac{3}{\mathrm{m}}\right]x + \left[\frac{2}{\mathrm{m}^2}\right]y^2 + 1000\right)$ V.

a) Find the x-component of the electric field at the point (4,4,0): $E_x(4, 4, 0)$ =______________.

b) Find the y-component of the electric field at the point (4,4,0): $E_y(4, 4, 0)$ =______________.

c) Find the z-component of the electric field at the point (4,4,0): $E_z(4, 4, 0)$ =______________.

The electric potential at point $P = (x, y, z)$ due to a charge q at point $P' = (x', y', z')$ is:

$$V(x,y,z) - V(\infty) = \frac{1}{4\pi\varepsilon_0} \frac{q}{\left((x-x')^2 + (y-y')^2 + (z-z')^2\right)^{1/2}} \qquad \text{Eq. 23d}$$

The potential at a point due to a collection of charges is the sum of the potentials from each of the charges. (equations 23.15 and 23.16 in Young and Freedman).

Note: *For finite charge distributions we usually take the potential at infinite distance from the charge to be zero. Potentials are usually measured with respect to this potential.*

Calculating the Electric Potential from Known Charges

6) For a given situation, calculate a few observational points for electric potential to determine the geometry of the configuration.

a) Calculate the potential at the point P = (0,3,0) due to a +1 nC point charge at P'= (0,1,0).

V =____________V(volts)

b) Calculate the potential at the point P = (2, 1, 0) due to a +1 nC point charge at P'= (0,1,0).

V =____________V(volts)

c) How does the potential depend on the direction of the measuring point from a point source?

d) Describe the 3-dimensional surface on which the electric potential for this point charge is equal to the potential you found for part 6a above.

23B – Exploration of Electric Potential and Field by PhET Simulation

Equipment: Download and run the PhET simulation, "Charges and Fields".
https://phet.colorado.edu/en/simulation/charges-and-fields

- Click/Check "Electric Field", "Values", and "Grid" in upper right box.
- Drag a +1 nC charge to a grid point near the center of the grid. The arrows that appear indicate the direction of the field at the points indicated.
- Drag the potential tool ( ) from the toolbox on the right side so that the cross hair is 1 m to the right of your charge.
- Click on the pencil icon in the potential tool. It should draw an equipotential surface for you.

1) What is the shape of this surface and is it consistent with your answer to part 23A6d above? If not, why not?

2) What is the value of the potential found in this simulation for a point 2 m from the source? Is it consistent with your answer to part 23A6a above? If not, why not?

V = ____________units______

- Find points at which the potential V is equal to 20 V, 16 V, 12 V, 8 V, and 4 V. Click the pencil for each to create the equipotential surface.

3) Is the difference between the radii for the 20 and 16 V equipotentials greater than, the same as, or smaller than the distance between the radii for the 8 and 4 V surfaces?

4) Does this difference indicate that magnitude of the average field between the 16 and 20 V surfaces is greater than, equal to, or less than the field between the 8 and 4 V surfaces? (The magnitude of the average electric field in a region is $|E_{x,avg}| = \Delta V/\Delta x$.)

- Drag a second positive charge to a point 2 meters to the right of the first charge.

5) Use the potential tool to find the electric potential close to midway between the two charges.

V(midway)=____________units______

- Click on the pencil to draw an equipotential that is slightly to the right of midway.
- Click on the pencil again to draw an equipotential that is slightly to the left of midway.
- There are two separated equipotential surfaces with the value 25V. Find and click on the pencil to draw them on the screen.
- Find the equipotential(s) for 15V and click the pencil to draw them on the screen.

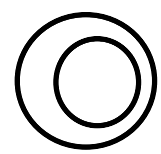
6) Make a sketch below of the charges and equipotentials (including the surfaces above) for the two positive charges.

Note that the electric field on an equipotential surface is always perpendicular to that surface.

- Drag the second positive charge back to the box at the bottom of the screen. Drag a -1 nC charge to a point 2 meters to the right of the first charge.

7) Sketch the charges and equipotentials for several equipotentials including +5 V, +2 V, +1 V, and 0 V.

8) A top view of two equipotentials in a certain region of space are sketched to the right. Describe the region(s) in which the electric field has the greatest magnitude. (e.g.- "At the center of the smaller circle." "To the left of center and between the two circles.") Explain your answer.



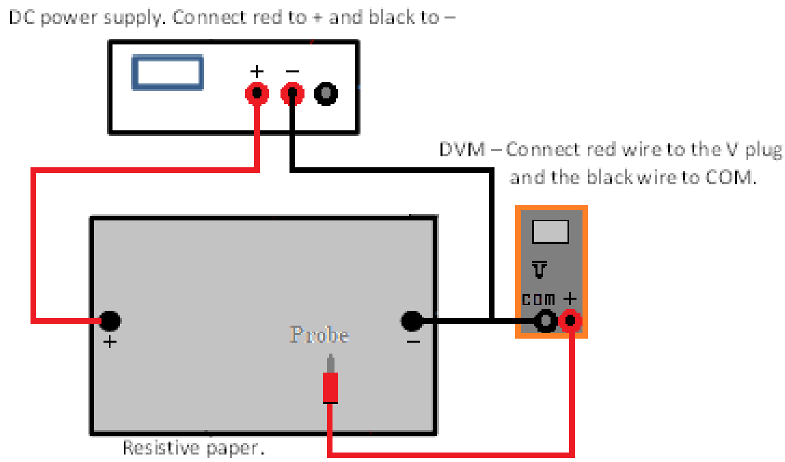
9) The electric field vector is always perpendicular to any equipotential surface. Explain why.

10) Under electrostatic equilibrium (no charge flow) the interior of a conductor has zero electric field. Explain how this leads to the conclusion that the potential within a conductor is a constant.

23C – Experiment: Electric Potential and Electric Field

Equipment:
1 rectangular conducting black paper, 1 DC power supply w/power cable, 2 red banana cables, 2 black banana cables, 2 brass paperclips, 2 alligator clips, 1 digital multimeter, 1 ruler for measuring position.

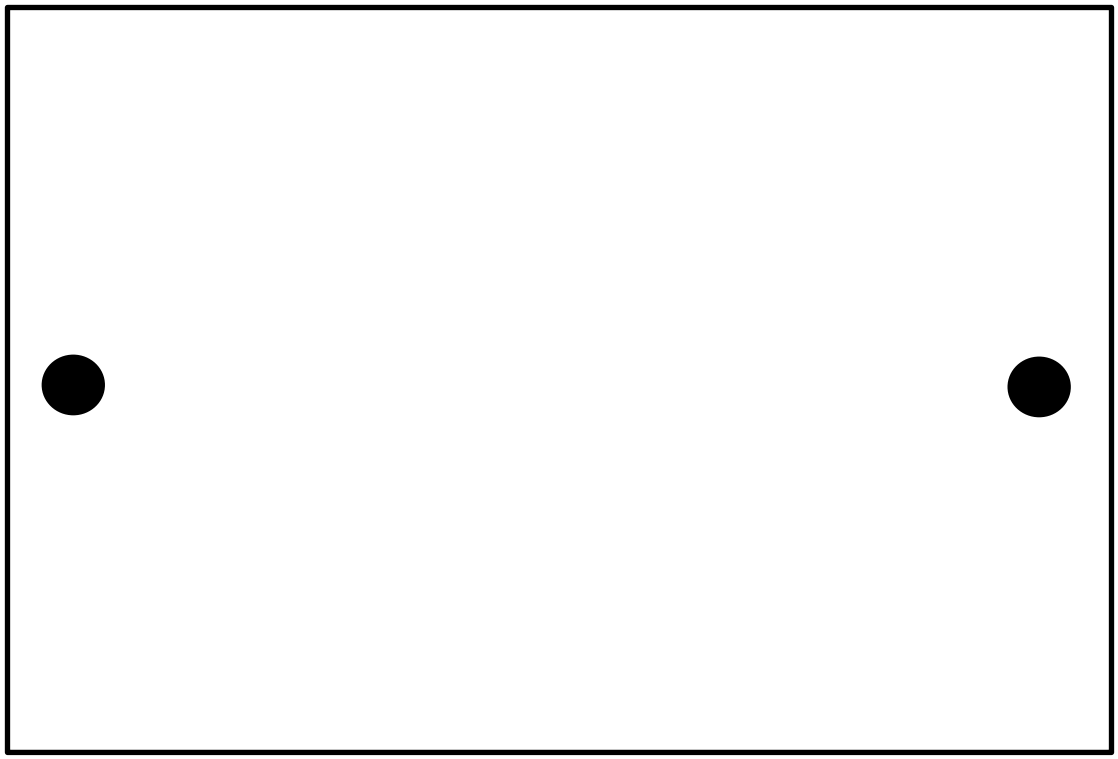
- You will use the multimeter to measure the difference in electric potential between various points on the conducting paper.
- Set up the measurement system as indicated below. Solid lines are wires.



- Set the power supply voltage to 10 V. Connect the two black wires to the same silvered point on the paper. The brass paperclips should be used with the alligator clips to prevent them from ripping the paper. (Banana wires can be piggybacked into one another.) Connect the red wire from the power supply to the opposite dot on the paper. The red wire from the DMM will be used as a variable position *probe*.
- Set the multimeter to measure DC Voltage. (Indicated by a $\overline{\text{V}}$.)
- Drag the probe tip gently around the paper to find the shape of the equipotential for 2V, 4V, 6V, and 8V.

Your voltmeter measures the difference in potential between one probe tip and the other. Discuss what this means with your classmates and facilitators.

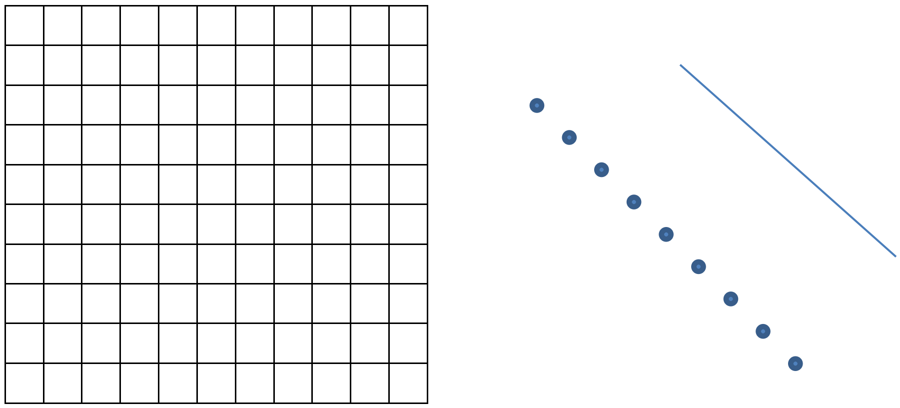
1. Sketch the four equipotentials here.



2. Measure the potential difference between the left hand source dot and the probe tip as a function of probe position along a straight line from the center of one contact to the other and enter it in the second column. *Measure in a direction such that the electrical potential (V) decreases from* x = 3 cm to x = 20cm.

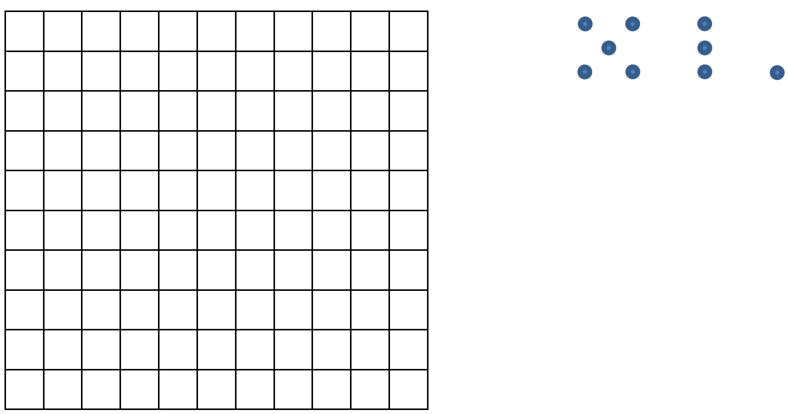
Position (cm)	Measured Potential (V)	Midpoint (cm)	Δx (m)	ΔV (V)	Field $-\Delta V/\Delta x$ (V/m)
3					
		4.5			
6					
		7.5			
9					
		10.5			
12					
		13.5			
15					
		16.5			
18					
		19.5			
21					
		22.5			
24					
		25.5			
27					

3. Plot the potential as a function of position on the grid below. If you wish, you can do this in the word version of this file by dragging the points to appropriate positions on the grid. Don't forget to add a scale. Test if the behavior is linear by dragging the line to make an approximate "best fit".



4. Calculate the electric field between adjacent measured points. Add it to the last column of the table.

5. Plot the average field as a function of position on the grid below.



a) Is the field constant across the paper? _____________

b) Explain why you think the field is constant (or not) using the concepts of the potential near point charges.

It is possible to measure the average field directly by using the two tips of a coaxial/dual banana connector to provide a consistent spacing between measured points. This provides an easier way to measure the field in a particular direction and avoiding having to make a 2D map of potential.

- Hold the plastic ends of the banana wire together to make a “field probe”. (You may find the using a rubber band to hold them helps to keep them at a fixed distance from one another.)

6) Move your “field probe” across the paper.
   a) Is the field constant? ________________

   b) Measure the potential difference when the dual probe is centered between the sources and is aligned (parallel) along the straight line between the source points ($\leftrightarrow$). Calculate the field in V/m using this value and the distance between the probe tips.

      E(center, parallel) =________________V/m

   c) How does the value you found here compare with the field you calculated in the center previously above?

7) Measure the potential difference when the dual probe is centered between the sources and is aligned perpendicular ($\updownarrow$)to the straight line between the source points. Calculate the field in V/m using this value and the distance between the probe tips.

   E(center, perpendicular)=________________V/m

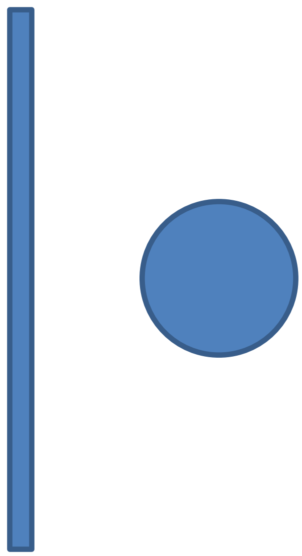
8) Discuss the relationship and differences between Electric Potential and Electric Field.

Equipotentials and Conductors

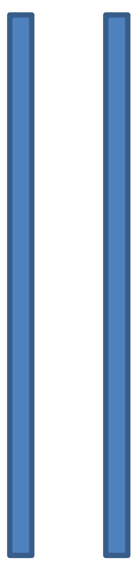
Important Conclusions about Electric Potential:
When all charges in a system are at rest, the entire volume of a conductor must be an equipotential. This is because if there is a non-zero electric field in the conductor, then charge in the conductor will be accelerated and flow. There is no potential difference between two points between which the field is zero.
When all charges are at rest, the electric field outside of a conductor must be perpendicular to the surface of the conductor. This is because the surface of the conductor is an equipotential by the argument above.

9) For the cases sketched below, indicate the directions of the electric field near each of the surfaces. Shaded regions are conducting. In each case, the left conductor is at positive potential with respect to the right one.

Case a)

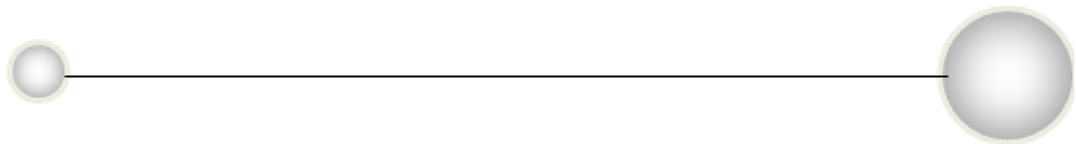


Case b)



EXTRA CREDIT (3 pts)

10) Consider a situation in which there are two conducting spheres that are far apart but connected by a long thin conducting wire. One sphere of radius 1.0 mm has charge +3.0 nC. The other sphere has radius R = 3.0 mm, but the charge is not given. You are asked to find the charge on the R= 3.0 mm sphere theoretically. The questions below may help you to find the solution.



a) Give an argument for why the surfaces of the small and large spheres are at the same potential.

b) Given the charge on the small sphere find the potential at the surface of the small sphere.

c) Given the potential, find the charge on the large sphere.