Physics 1200
Lecture 17
Spring 2024

Maxwell's Equations, Electromagnetic Traveling Waves, EM Energy and Momentum, EM Waves & Light

Light as an Electromagnetic Phenomenon

- James Clerk Maxwell (1831–1879) was the first person to truly understand one of the most fundamental aspects of the nature of light.
  - Proved in 1865 that electromagnetic disturbances propagate (travel) in free space with a speed equal to that of light.
  - Correctly deduced that light propagates as an electromagnetic wave.

Light as an Electromagnetic Phenomenon (2)

- Maxwell's equations: an accelerating electric charge must produce electromagnetic waves.
  - Example: power lines carry a strong alternating current, which means that a substantial amount of charge is accelerating back and forth and generating electromagnetic waves.
  - These waves produce a buzzing sound from your car radio when you drive near the lines.

Maxwell's Equations in Vacuum

- Electromagnetic wave relations most easily derived for case of there being no sources – that is, no charges or moving charges (convective current) present. Corresponds to case of waves propagating (traveling) in vacuum.
- In vacuum, Maxwell's equations are:

$$\Phi_E = \oint \vec{\boldsymbol{E}} \cdot d\vec{\boldsymbol{A}} = 0$$

Gauss's law for electric flux.

$$\Phi_B = \oint \vec{\boldsymbol{B}} \cdot d\vec{\boldsymbol{A}} = 0$$

Gauss's law for magnetic flux.

$$\oint \vec{\boldsymbol{B}} \cdot d\vec{\boldsymbol{l}} = \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$$

Ampere-Maxwell law.

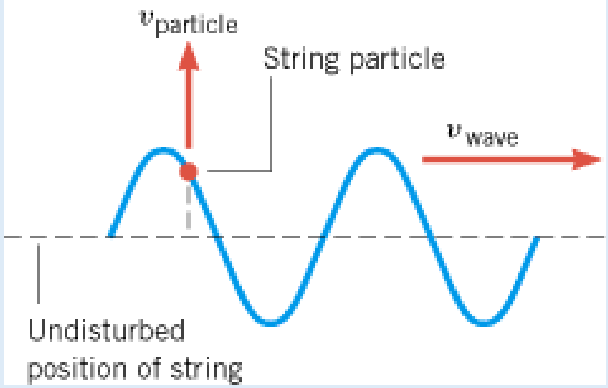
$$\oint \vec{\boldsymbol{E}} \cdot d\vec{\boldsymbol{l}} = -\frac{d\Phi_B}{dt}$$

Faraday's law.

- Expressions have terms involving temporal (time) and spatial changes of the fields. Both types of change are needed for a wave – which is an oscillation of a physical quantity in time <u>and</u> space.
- Note: Maxwell's displacement current links time changes in electric fields to spatial changes in magnetic fields. Wouldn't be possible if Maxwell hadn't thought to include this term in Ampere's law (now better known as the Ampere-Maxwell law).

Transverse Electromagnetic Waves

- Electromagnetic (EM) waves in vacuum are <u>transverse waves</u>.
  - For these waves, physical variables oscillate in a direction perpendicular to the direction of wave propagation (= direction of the wave travel).
  - Classical mechanical analog is a transverse wave on a string:



- EM waves: transverse quantities are the electric and magnetic fields.
- The two Gauss's laws force the electric and magnetic field components of the vacuum EM wave to be transverse to the propagation direction.

Transverse Electromagnetic Waves (2)

- Use Maxwell's equations to derive EM wave equation.
- In our example, the wave will be traveling (propagating) in the $+x$ –direction.
- Electric field component of the wave will be aligned with the $+y$ –axis:

$$\vec{\boldsymbol{E}}(x,t) = E_y(x,t)\hat{\boldsymbol{j}} \ .$$

- Magnetic field component of the wave will be aligned with the $+z$ –axis :

$$\vec{\boldsymbol{B}}(x,t) = B_z(x,t)\hat{\boldsymbol{k}}.$$

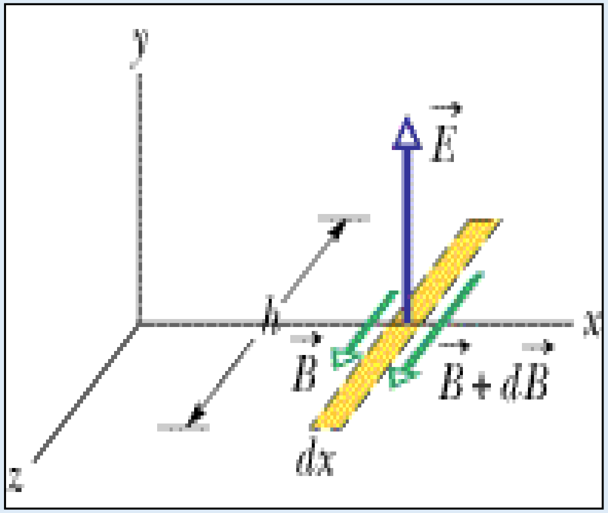
  - Electric and magnetic fields transverse to each other, as well as being transverse with respect to the direction of travel.

- <u>We derive the wave equation and its solution for propagation of transverse EM waves in vacuum</u>. It will allow us to calculate the wave's electric and magnetic field oscillations in time (t) and space (x).

Electric Field Changes in Time Cause Magnetic Field Changes in Space

- Consider: thin rectangular strip in (x, z) plane with perpendicular (transverse) electric and magnetic field vectors. Use the Ampere-Maxwell law to relate time change of the electric field to spatial change in the magnetic field:

- Thin strip of area $h\,dx$ located about position x.



- Line-integral of $\vec{\boldsymbol{B}}$ around the strip's bounding curve:

$$\oint \vec{\boldsymbol{B}} \cdot d\vec{\boldsymbol{l}} = B_z(x+dx)h - B_z(x)h$$

$$= \left[B_z(x)h + \frac{\partial B_z}{\partial x} h dx\right] - B_z(x)h = \frac{\partial B_z}{\partial x} h dx$$

- Time-derivative of electric flux through strip:

$$\frac{d\Phi_E}{dt} = \frac{d}{dt}\int \vec{\boldsymbol{E}} \cdot d\vec{\boldsymbol{A}} = \frac{d}{dt}\int E_y \hat{\boldsymbol{j}} \cdot (-dA\hat{\boldsymbol{j}}) = -h\,dx\,\frac{\partial E_y}{\partial t}$$

- Ampere-Maxwell law in vacuum: $\oint \vec{\boldsymbol{B}} \cdot d\vec{\mathrm{l}} = \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$.

$$\therefore \quad \boxed{\frac{\partial B_z}{\partial x} = -\epsilon_0 \mu_0 \frac{\partial E_y}{\partial t}} \; .$$

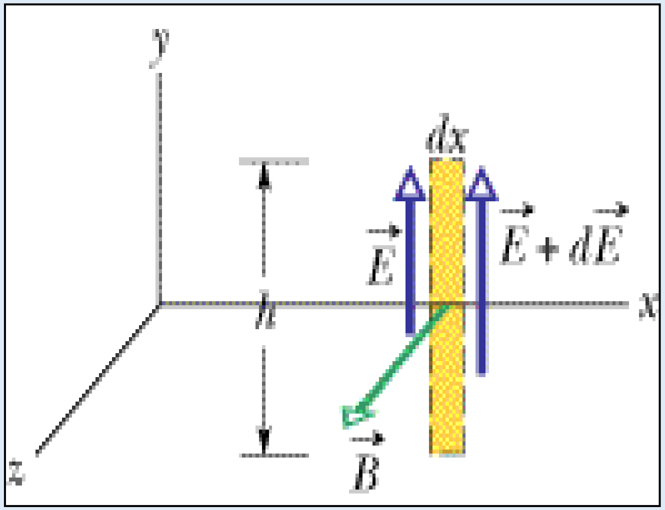
Magnetic Field Changes in Time Cause Electric Field Changes in Space

- Consider: thin strip in the same neighborhood of x, but in (x, y) plane.
- Line-integral of $\vec{\boldsymbol{E}}$ around strip's bounding curve:

$$\oint \vec{\boldsymbol{E}} \cdot d\vec{\boldsymbol{l}} = E_y(x + dx)h - E_y(x)h$$

$$= \left[E_y(x)h + \frac{\partial E_y}{\partial x} h dx\right] - E_y(x)h$$

$$= \frac{\partial E_y}{\partial x} h dx \,.$$



- Time-derivative of magnetic flux through strip:

$$\frac{d\Phi_B}{dt} = \frac{d}{dt} \int \vec{\boldsymbol{B}} \cdot d\vec{\boldsymbol{A}} = \frac{d}{dt} \int B\hat{\boldsymbol{k}} \cdot dA\hat{\boldsymbol{k}}$$

$$= h\, dx \frac{\partial B_z}{\partial t} \,.$$

- Faraday's law: $\oint \vec{\boldsymbol{E}} \cdot d\vec{\boldsymbol{l}} = -\frac{d\Phi_B}{dt}$

$$\therefore \quad \boxed{\frac{\partial B_z}{\partial t} = -\frac{\partial E_y}{\partial x}} \,.$$

Putting Things Together: Unification of Fields

- Collecting expressions:

$$\frac{\partial B_z}{\partial x} = -\epsilon_0\mu_0 \frac{\partial E_y}{\partial t} \quad \text{and} \quad \frac{\partial B_z}{\partial t} = -\frac{\partial E_y}{\partial x}$$

- Take partial time-derivative of first equation, and partial x-derivative of second:

$$\frac{\partial^2 B_z}{\partial t \partial x} = -\epsilon_0\mu_0 \frac{\partial^2 E_y}{\partial t^2} \quad \text{and} \quad \frac{\partial^2 B_z}{\partial x \partial t} = -\frac{\partial^2 E_y}{\partial x^2} \; .$$

- Using fact that $\frac{\partial^2 B_z}{\partial t \partial x} = \frac{\partial^2 B_z}{\partial x \partial t}$ yields:

$$\boxed{\epsilon_0\mu_0 \frac{\partial^2 E_y}{\partial t^2} = \frac{\partial^2 E_y}{\partial x^2}} \, .$$

  This is a form of the <u>electromagnetic wave equation</u>.

- Could also take different sequence of derivatives of change-equations between electric and magnetic fields above. Doing so yields:

$$\boxed{\epsilon_0\mu_0 \frac{\partial^2 B_z}{\partial t^2} = \frac{\partial^2 B_z}{\partial x^2}} \; .$$

  This is also a form of the EM wave equation.

Putting Things Together: Unification of Fields (2)

- Maxwell's triumph: dual wave equations for electric and magnetic fields. Both unified into a single electromagnetic field. Waves are therefore called electromagnetic (EM) waves.

- Solutions of the wave equations have form:

$$\vec{\boldsymbol{E}}(x,t) = E_m \cos(kx - \omega t)\,\hat{\boldsymbol{j}}\text{ , and }\vec{\boldsymbol{B}} = -B_m \cos(kx - \omega t)\,\hat{\boldsymbol{k}}\text{ ,}$$

  where E_m and B_m are amplitudes of the waves, $\omega = 2\pi f$ is angular frequency of the wave (f is the linear frequency of the wave), and $k = 2\pi/\lambda$ is the wave number of the wave (λ is the wavelength of the wave). Additionally, waves have speed ("phase speed")

$$\boxed{v = \frac{\omega}{k} = f\lambda}\text{ .}$$

- Solutions are derived using techniques of standard calculus. You can verify that they are correct by inserting them into the wave equations and showing that they yield an equality; i.e., putting solutions given above in the wave equations results in the left- and right-hand sides of the equations being identically equal.

Putting Things Together: Unification of Fields (3)

- . From wave equations and their solutions, find that the speed for EM waves is

$$\boxed{v = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = c},$$

  where $c = 2.998 \times 10^8 \mathrm{m/s}$ is the speed of light!
- Jackpot! Not only did Maxwell unify electric and magnetic fields into the EM field, he also got the bonus of showing that light propagates as an EM wave.
- After studying Maxwell's equations, H. Hertz was able to show that waves generated by electrical circuit phenomena were traveling at the speed of light, experimentally confirming that EM disturbances (oscillations) travel at speed c.
- This was later developed for technological and economic purposes by Marconi (pioneer of radio) and others. The world was irreversibly changed by Maxwell's work.

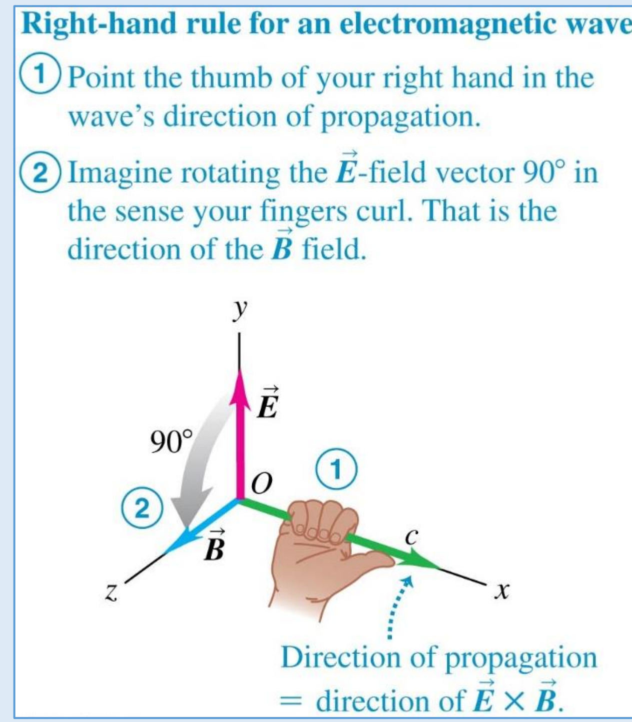
Vacuum EM Field Relations

- Inserting vacuum wave solutions into the equations relating space derivatives of one field being equal to time derivatives of the other yields relation between wave amplitudes:

$$\boxed{E_m = cB_m}\ .$$

- Transverse nature of EM wave holds between its electric and magnetic field components, and the direction of the propagation of the wave. In our example derivation, direction that wave is traveling can be found by using a right-hand rule between the electric field vector and the magnetic field vector:

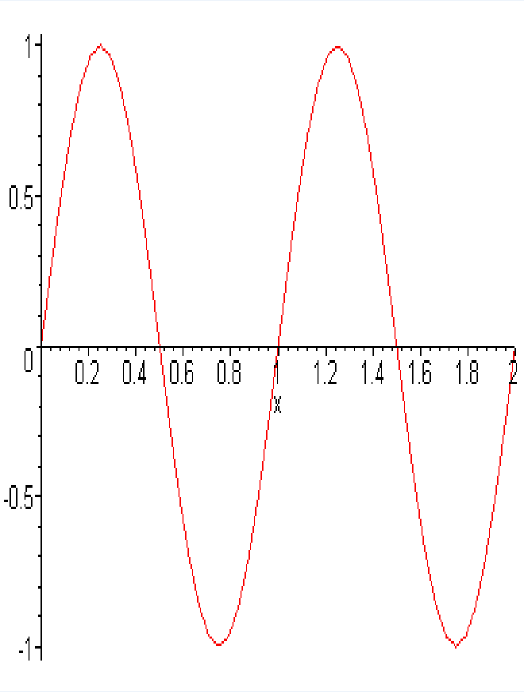
- Direction of propagation of an electromagnetic wave is the direction of the vector product of the electric and magnetic fields.



Review: A travelling sine wave

Wave solutions can also be written in a slightly different fashion, by instead writing the <u>phase</u> of the sinusoidal function as

$$(kx - \omega t) = k\left(x - \frac{\omega}{k}\,t\right) = \frac{2\pi}{\lambda}(x - vt).$$

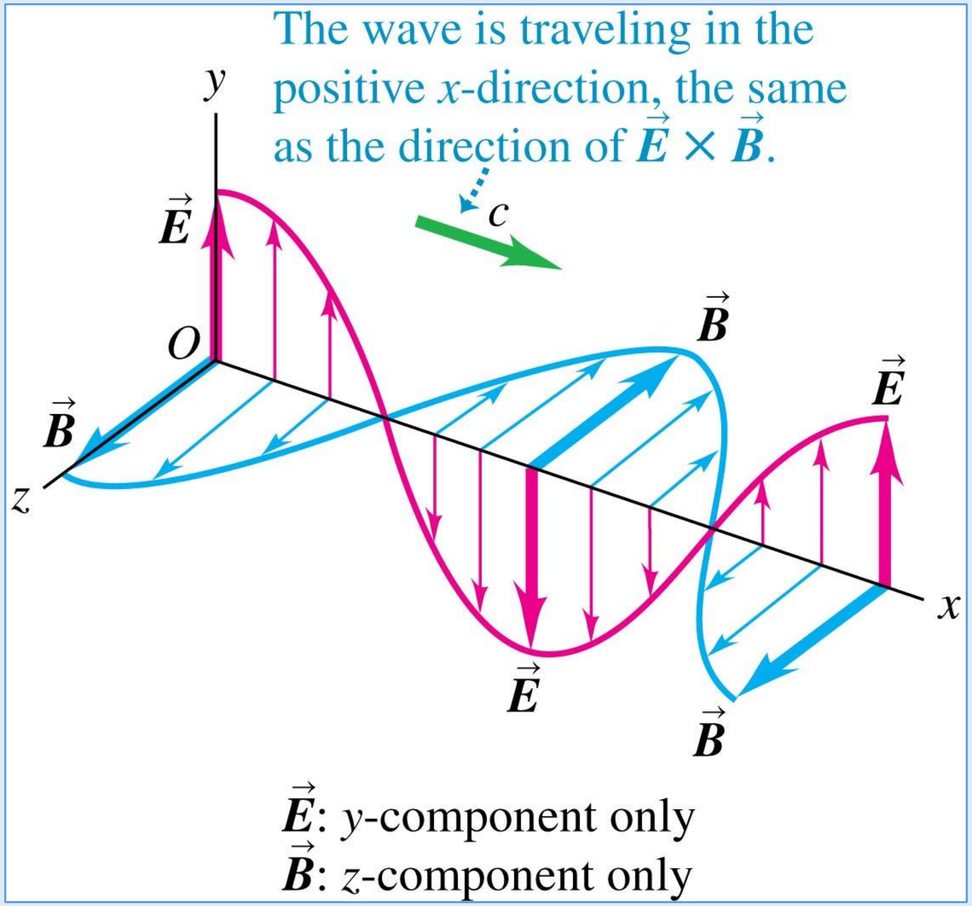


$$y(t) = y_{\max}\sin\Bigg[\overbrace{\frac{2\pi}{\lambda}(x - vt) + \phi_0}^{\text{phase}}\Bigg]$$

A point with a specific phase moves to the right at speed v.

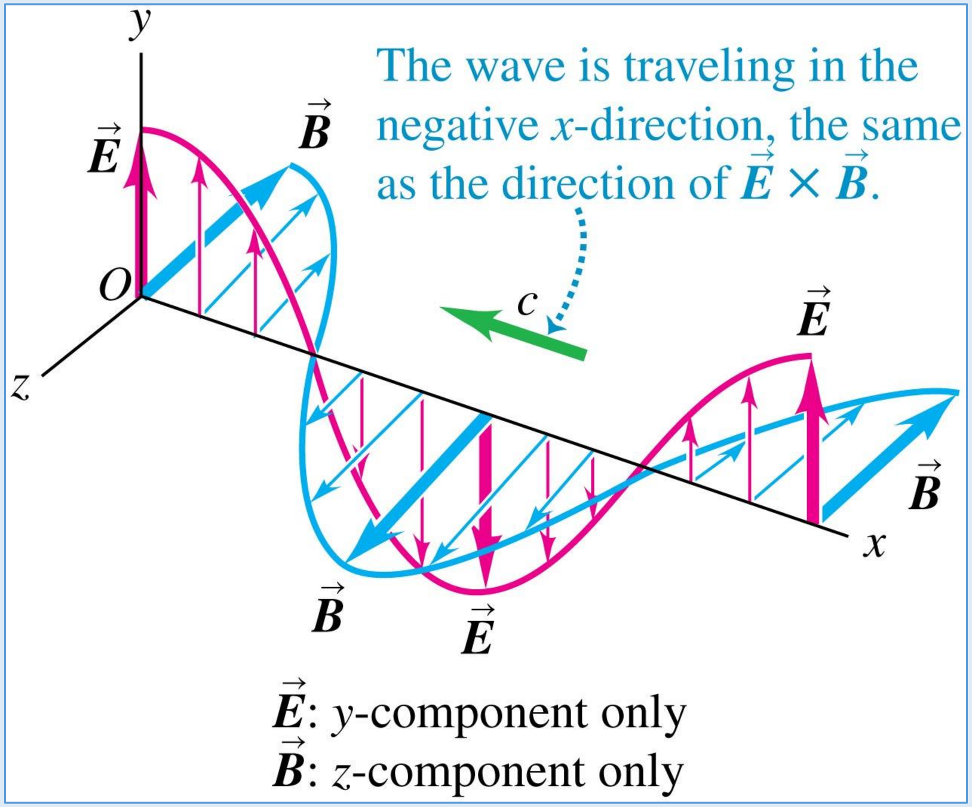
Field of a Sinusoidal Wave (1)

- Shown is a linearly polarized sinusoidal EM wave traveling in the $+x$-direction.
- One wavelength of the wave is shown at time $t = 0$.
- Fields are shown for only a few points along the x-axis.



Field of a Sinusoidal Wave (2)

- Shown is a linearly polarized sinusoidal EM wave traveling in the $-x$-direction.
- One wavelength of the wave is shown at time $t = 0$.
- Fields are shown for only a few points along the x-axis.



Energy Transport in an EM Wave

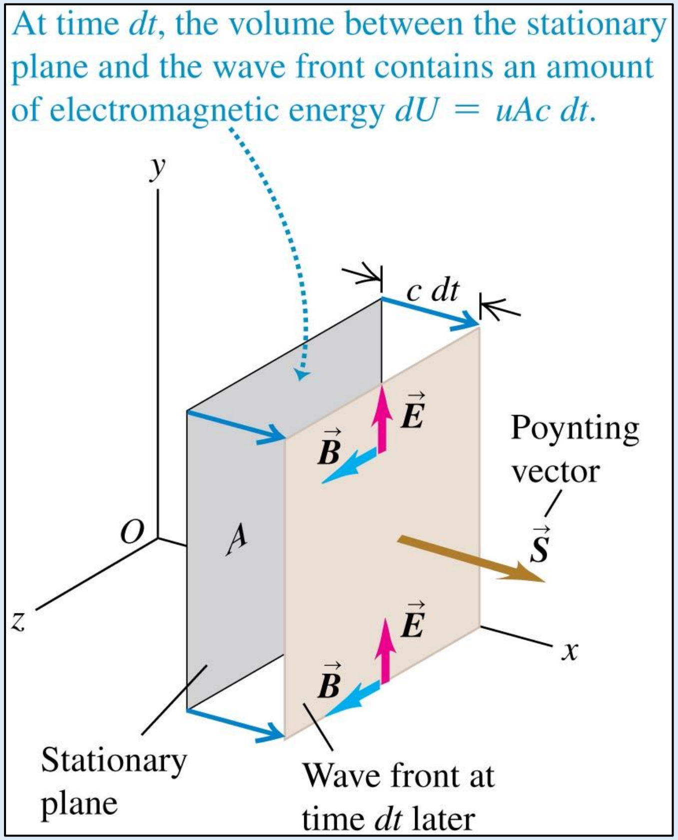
- Found in previous classes energy densities of vacuum electric and magnetic fields:

$$u_E = \frac{1}{2}\,\epsilon_0 E^2\text{ , and } u_B = \frac{B^2}{2\mu_0}\,.$$

- EM waves have propagating electric and magnetic field components. $\therefore$ Reasonable to expect that EM energy will be transported by an EM wave.
- John Poynting introduced what later became known as the <u>Poynting vector</u>,

$$\boxed{\vec{\boldsymbol{S}} = \frac{1}{\mu_0}\,\vec{\boldsymbol{E}} \times \vec{\boldsymbol{B}}}\quad .$$

- The Poynting vector points in the wave's direction of propagation; its magnitude is the power per unit area carried by in the wave.



Energy Transport in an EM Wave (2)

- Magnitude of the average value of $\vec{S}$ is called the <u>intensity</u>. SI unit of intensity is 1 W/m^2.
  - Intensity is the average power per unit area delivered by an EM wave.
- Because of the high frequencies (rapid oscillations) of EM waves, typically use average power and Poynting flux in definition of intensity:

$$I = S_{\text{av}} = \frac{1}{2}\frac{E_m B_m}{\mu_0} = \frac{1}{2}\frac{E_m\left(\frac{E_m}{c}\right)}{\mu_0} = \frac{1}{2}\frac{E_m^2}{\mu_0 c} = \frac{1}{2}\epsilon_0\, c^2\frac{E_m^2}{c} = \frac{1}{2}c\epsilon_0 E_m^2$$

$$\Rightarrow \boxed{I = c\left(\frac{1}{2}\epsilon_0 E_m^2\right)}\ .$$

(Used EM wave relations: $E_m = cB_m$, and $c^2 = \frac{1}{\epsilon_0\mu_0}$.)

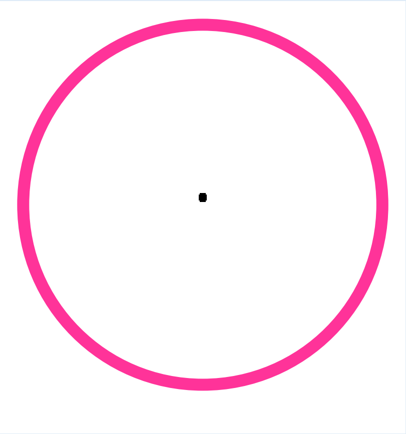
- Intensity can also be written as

$$\boxed{I = c\left(\frac{B_m^2}{2\mu_0}\right)}.$$

- Both expressions show the power flow to be transport of field energy density at wave speed c.

Isotropic flux: A Point Source

Net power transmitted through a closed surface is the power emitted by all sources inside the surface.



Imagine a spherical surface of radius r centered on the point source. The intensity will be the same at any point on that surface.

$$I = \frac{P}{A} = \frac{P}{4\pi r^2}$$

P = Power emitted.

A = surface area = integral of dA.

Notice that I is proportional to $1/r^2$.

Momentum Transport in an EM Wave

- EM waves also carry momentum. If the EM waves interact with a surface (impact the surface), they deliver an impulse to the surface ⇒ exert <u>radiation pressure</u> on a surface:

$$p_{\mathrm{rad}} = \frac{S_{\mathrm{av}}}{c} = \frac{I}{c} \qquad \text{(radiation pressure, wave totally absorbed)}$$

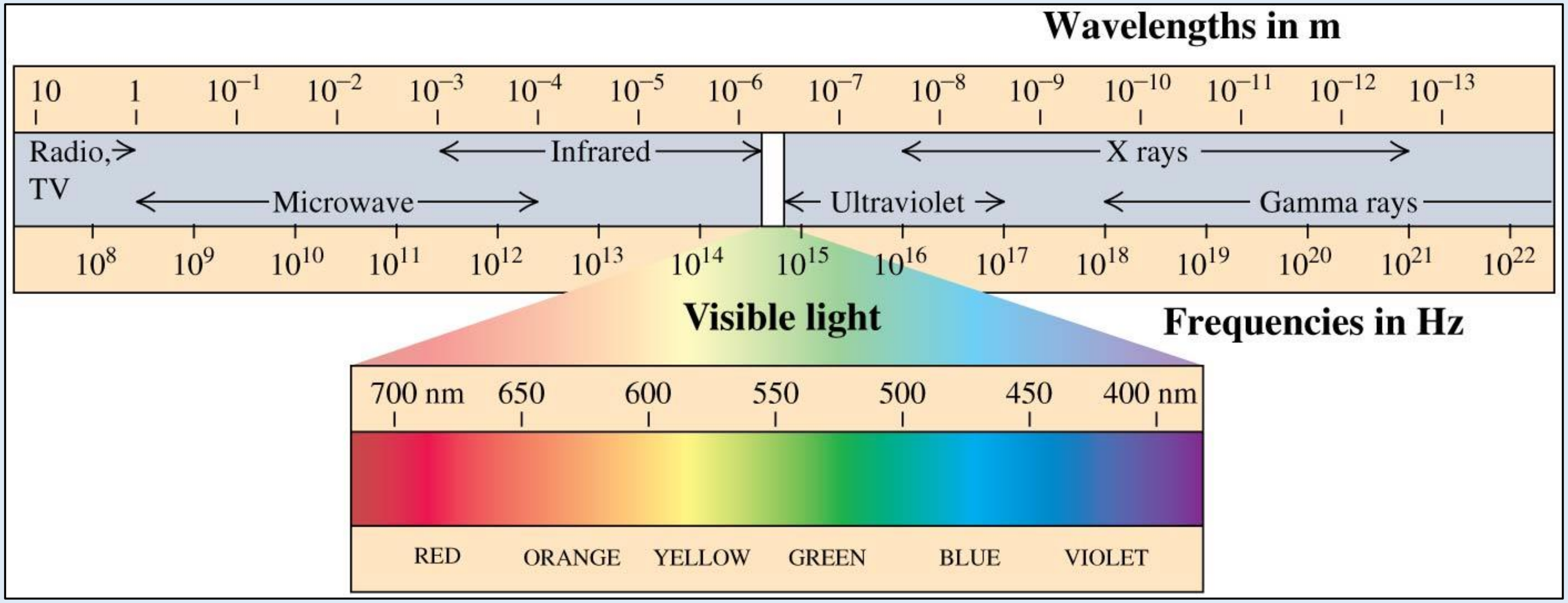
$$p_{\mathrm{rad}} = \frac{2S_{\mathrm{av}}}{c} = \frac{2I}{c} \qquad \text{(radiation pressure, wave totally reflected)}$$

- Radiation pressure is important in some physical environments:
  - Radiation pressure can be a significant source of support for stars. If a star has too much radiation pressure, it can blow itself apart! (Called the “Eddington limit” in astrophysics.)
  - Radiation pressure is being investigated as a driver for fuel compression in laser fusion devices. (Called “inertial confinement fusion.”)
  - Radiation pressure has been suggested as a source of space propulsion, by using “light sails” to use the Sun’s light to drive spacecraft.

EM Waves and Light

- Maxwell's EM wave solutions describe propagation of light. That is, how EM/light waves travel from one place to another.
- Wave solutions also predict interference effects, which is how light waves interact with each other. This topic (and related subjects) are discussed in upcoming classes.
- However, the wave solutions do not well describe certain interactions that light has with matter. These interactions can be understood if one instead uses a model where light consists of particle-like entities. This aspect of light – the photon model – will also be discussed in later classes.
  - In his studies on light, Isaac Newton advocated the "corpuscular" theory of light: light consisted of small, discreet corpuscles. He described his theory in his book, Opticks.

Properties of EM/Light Waves: Spectrum



- All EM waves have the universal characteristic:

$$\boxed{\text{EM vacuum wave speed} = c = \frac{\omega}{k} = \lambda f = \frac{\lambda}{T}}\ .$$

- Frequencies and wavelengths of EM waves found in nature extend over a wide range. Must use a logarithmic scale to show all the important bands.
- Boundaries between bands decided by conventional use.

Properties of EM/Light Waves: Visible Light

- Visible light is the segment of the electromagnetic spectrum that we can see.
- Visible light extends from the violet end (~ 400 nm) to the red end (~ 700 nm).

TABLE 32.1 Wavelengths of Visible Light

380–450 nm	Violet
450–495 nm	Blue
495–570 nm	Green
570–590 nm	Yellow
590–620 nm	Orange
620–750 nm	Red