25A – Current and Kirchhoff's Junction Rule

Background: In earlier classes we used the fact that the electric field in a conductor is zero, but we remind you that this is under the specific situation where there is no charge flow. If a field is present in a conductor, it causes charge to flow. The macroscopic flow of charge is called current.
In high school, you learned a basic relationship between current flow and potential difference,

$$I = \Delta V/R, \qquad \text{Eq. 25a}$$

where I is the rate at which charge flows through a device in Coulombs/second and ΔV is the potential difference between the two ends of the device. The proportionality constant between these quantities is called the resistance R, the MKS unit of resistance is the ohm=1 Volt/1 Ampere. The resistance is a property of the combination of both material and geometry of the device under the measurement conditions. (***Ohm's Law*** is a special case of equation 25a in which R is a constant, independent of current I or potential difference ΔV.)

A fundamental physical fact is that net charge can neither be created nor destroyed. This means that when charge flows into a point in space, it must flow out. This is stated in **Kirchhoff's Junction Rule**:

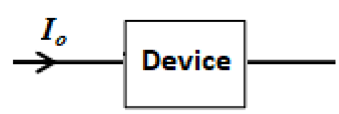
The algebraic sum of all currents at a junction (a point in a circuit) must be equal to zero.

$$\sum_{1}^{n} I_n = 0$$

We will apply this concept in solving for current and potentials in circuits later. For now we will just do a few practice activities to secure the knowledge.

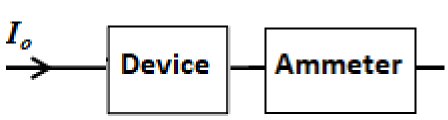
An ammeter is a device for measuring the flow of charge (current). An ammeter is always connected in **series** in a circuit! An *ideal* ammeter has no potential drop across it as current passes through it, so that when wired in series it does not change the circuit being measured.

1) Current I_0 flows into the device shown in the sketch to the right. The magnitude and direction of the current that flows out of the device is:
A) Equal to I_0 and to the left. B) Equal to I_0 and to the right.
C) Less then I_0 and to the left. D) Less than I_0 and to the right.
Ans.______________

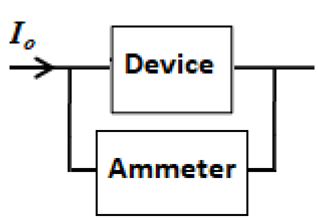


2) How would you wire an ammeter to measure the current through the device above?

A)



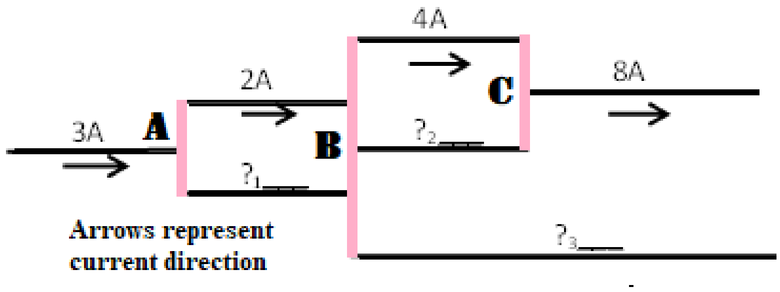
B)



Explain why you chose the circuit you did.

Ans_______________

3) The currents flowing through certain segments of a circuit are shown below.



The junction points for the above segments are labeled **A, B,** and **C**. For the above segments one will use *Kirchhoff's junction rule*

$$\sum_{1}^{n} I_n = 0$$

a) At junction **A** write *Kirchhoff's junction rule* in terms of the currents and $?_1$ (Consider $?_1$ as a current which will be found later).

__________________________________ = 0

b) At junction **B** write *Kirchhoff's junction rule* in terms of the given currents and $?_1$, $?_2$, and $?_3$ (Consider $?_1$,$?_2$ and $?_3$ as currents).

__________________________________ = 0

c) At junction **C** write *Kirchhoff's junction rule* in terms of the given currents and $?_2$ (Consider $?_2$ as a currents).

__________________________________ = 0

d) Fill in the unknown [?'s__] current magnitudes and directions if they can be determined.

$?_1$ = ____________________Direction_________________________
$?_2$ = ____________________Direction_________________________
$?_3$ = ____________________Direction_________________________

4) The current in a wire moves from left to right. In what direction does the average velocity of electrons point?
A) To the right. B) In no particular direction. C) It varies rapidly in time. D) To the left.
Ans______________

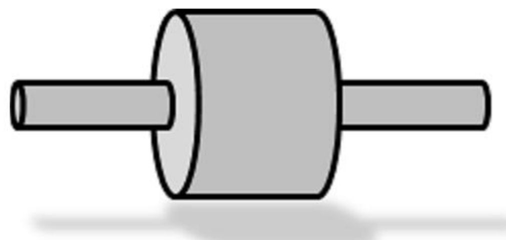
Explain your answer:

25B – Current Density and Electric Field: Background Concepts

The microscopic behavior of charge flow can be described using the concept of current density. Current density is the average rate of charge flow per unit cross-sectional area, $i = \vec{J} \cdot \vec{A}$. The current density in some region is related to the properties of the material in which the current flows and the electric field $\vec{E}$ in that region: $\vec{J} = \vec{E}/\rho$ Eq. 25b

The resistivity ρ is a property of the material that scales the current density for a given field.

1) A conductor is shaped as shown to the right with a narrow cylindrical wire segment of radius R = 0.2 mm connected to a middle segment of radius 2.5R and then another narrow segment of radius R. Current I_0 = 5A flows into the left hand face of the narrow wire. The three segments have the same length L = 3 cm. (Clearly, they are not drawn to scale!) The electric field in the left-hand narrow segment is E = 0.01 V/m. The composition of all the wires is the same.



a) What is the magnitude of the current in the middle wire segment? ________________A

b) What is the current density in the left-hand narrow wire segment? ______________A/m^2

c) What is the resistivity of the narrow wire? _____________ ohm-m

d) What is the current density in the middle wire? _____________ A/m^2

e) What is the field strength in the middle wire segment? _______________V/m

f) What is the field strength in the right-hand narrow wire segment? ______________V/m

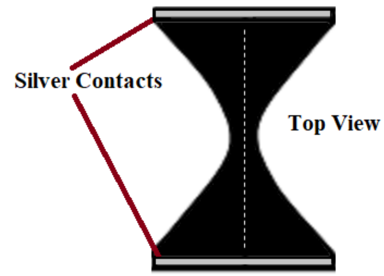
g) What is the potential difference across the entire conductor (left to right)? _______________V

h) What is the resistance of the entire conductor? _______________Ω

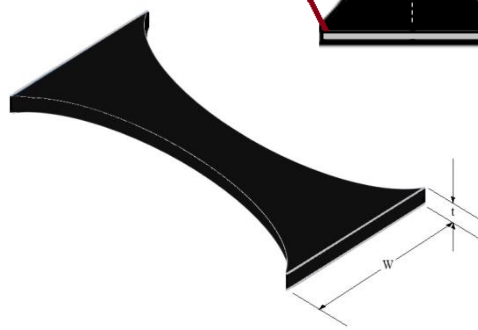
25C – Experiment: Current Density and Field in a Non-Uniform Conductor

Equipment: DC Power Supply; Digital Multimeter (DMM); 4 banana wires (2 short, 2 long); 2 brass paper clips; Shaped conducting paper; 2 Alligator clips.

You will measure and map the electric potential as a function of position as a demonstration of the concepts from the previous section.
The resistor in this experiment consists of a thin sheet of resistive paper that has been cut into a shape approximated by the sketch to the right. Conducting contacts are painted in lines on the two straight sides. The white dashed line in the sketch will be referred to as the axis of the shape.



You can measure the electric field by measuring the potential difference between two wires that are held a fixed distance apart. This can be done as you did before, by holding two banana jacks together.



Find the current density using the following method.

- Using the Resistance (Ω) setting on your multimeter, measure the resistance of your sheet from contact to contact. (It should be a few thousand ohms.)

  Total Resistance: _________Ω

- For an applied voltage of $V_{applied}$ = 10 V, calculate the current flow through the sheet using $I = V/R$.

  I =__________A

- Measure the width w of the conducting sheet as a function of position across the sheet for the positions listed in the table on the next page and enter it into the table.
- Calculate the current density for each position, assuming that **the sheet is 2×10^{-4} m thick** and that the current is evenly distributed over the vertical width of the sheet. Enter the current density into the table on the next page.

1) Plot the current density as a function of position in the left hand grid on the next page. Label axes appropriately.

Find the field using the following method.

- Measure the distance between your banana jacks tips when they are held together.

  __________m

- Connect the + and – outputs of the power supply to the two ends of the conducting paper using two banana wires, the brass paper clips, and the alligator clips. Set the power supply to 10 V.
- Set the DMM to measure DC voltage ($\overline{V}$) and use the DMM to measure the potential difference between the tips as a function of position from one end of the sheet to the other by holding the two banana jacks together. Align the jacks along the center axis (dashed line above) and straddle the position points listed in the table i.e., take the *Position setting* at 6 cm (0.06 m in the table) align the probe along the dashed white line so that the *position point* is between the probe tips.
- Calculate the field as a function of position and enter it into the table.

2) Plot the measured field as a function of calculated current density on the right-hand grid below. Label axes appropriately. (If you use the Word version of this activity, you can create an Excel spreadsheet and paste the data and plots here instead of using the templates here.)

Position (m)	Width (m)	Current density (A/m^2)	A blank column	Potential difference (V)	Field (V/m)
0.03					
0.06					
0.09					
0.12					
0.15					
0.18					
0.21					
0.24					
0.27					

a) Is the current density – field plot consistent with Equation 25b (Module 25B)? ___________

b) Estimate the resistivity of the black conductive material from the slope of the current density – field plot.

ρ =___________units_______

c) Compare the resistivity you measured with the typical values for metals and insulators. Is your material consistent with either one?

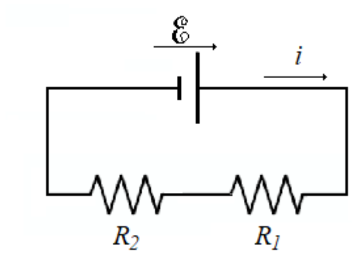
25D – Power Dissipation in Resistors

When charge flows through a circuit element, it can gain or lose potential energy and it can convert electrical energy into other forms. The potential energy change as a charge passes through a resistor is $\Delta U = q\Delta V$. The rate at which power is exchanged is given as power $P = \Delta U/\Delta t$: Therefore, the power dissipated by the resistor is expressed as: $P = \frac{\Delta}{\Delta t}(q\Delta V) = (\Delta q/\Delta t)\Delta V = I\,\Delta V$. This relationship can be manipulated using $\Delta V = I\,R$ to find the rate at which electrical energy is converted into heat as charge moves through a resistor.

1) When charges pass through a resistor, they give up kinetic energy to the environment in the form of heat. Consider a circuit in which the wires on either side of a resistor are exactly the same diameter. The average *drift speed* for charges entering the circuit is v_{enter}. The drift speed for charge exiting the resistor is v_{exit}. Is v_{exit} greater than, less than, or equal to v_{enter}? Explain your answer.

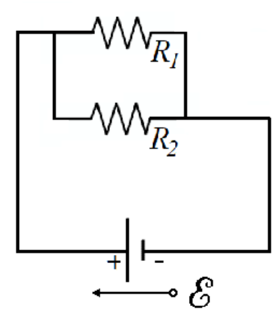
Ans__________________

2) For the circuit shown to the right, R_1 = 20 ohms and R_2 = 60 ohms. What is the ratio of power dissipated in the two resistors (P_2 / P_1)? Explain.



Ans.________________

3) For the circuit shown to the right, R_1 = 20 ohms, and R_2 = 60 ohms. What is the ratio of power dissipated in the two resistors (P_2 / P_1)?



Ans.________________

4) The voltage supplied by wall sockets in the US is given by $V(t) = V_{Max}\cos(\omega t)$ where $\omega = 120\pi$ radians/second and $V_{\max} \cong 170$ V . The average power dissipated in a device plugged into the wall is $P_{average} = \frac{1}{T}\int_0^T P(t)dt$. For an ohmic resistor, $\Delta V = IR$. Find the average power dissipated in a resistor plugged into the wall in terms of $V_{\max}$ and R. (Do the integral.)

$P_{average}$ = ______________________