Make-up Lab for Chinese New Year Celebrants

An alternate make-up lab is being offered to students who have a conflict with their labs on Friday Feb. 9 and Chinese New Year celebrations. The alternate make-up lab will be from 4 pm - 5:50 pm on Thursday, Feb. 8 in room 2C14 of the J-Rowl Science Center.

Physics 1200
Lecture 08
Spring 2024

Reduction of Resistor Networks, Applying Kirchoff's Rules, Solving Circuit Systems

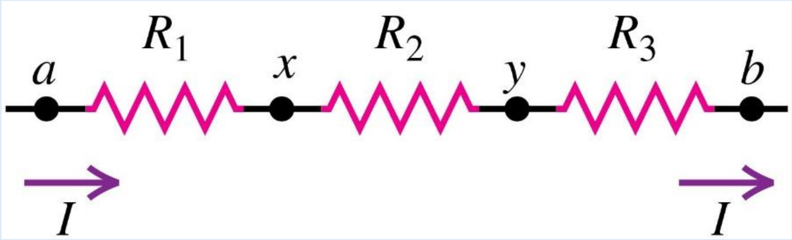
DC and AC Currents

- Circuits we have been dealing with thus far have currents that do not change their direction with time: Once the circuit is set up and starts running (perhaps by closing a switch) it continues in the same direction for the duration of the circuit being closed and operational. This is referred to as a DC (direct current) circuit.
  - Examples: flashlights, automobile wiring circuits, any portable electronic system that runs by action of a battery.
- Circuits that have current alternating in direction over time are called AC (alternating current) circuits.
  - Example: any household power supply (i.e., wall-sockets). Currents in those systems oscillate sinusoidally in time.
- AC circuits studied later this semester.
- The methods we use to analyze DC circuits – Kirchoff's rules – are also valid for the study of AC circuits.

Series and Parallel Resistors

- Last class: derived a rule for the effective resistance of a network of series resistors:

  $$\boxed{R_{eff,s} = \sum_{i=1}^{N} R_i} \; .$$

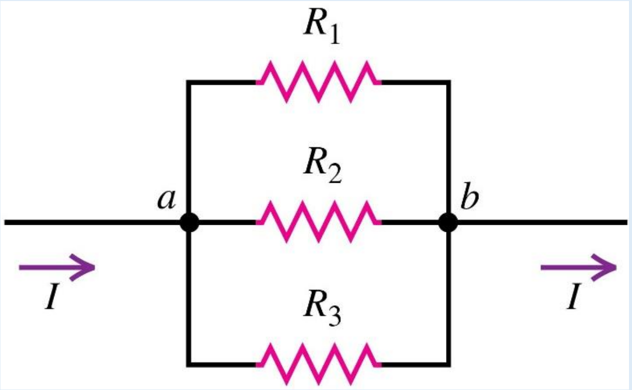
  

  - ➢ Note: $R_{eff,s} > R_i$.
  - ➢ Key feature of series resistors: current through resistors is the same.

- Resistors in parallel have effective resistance:

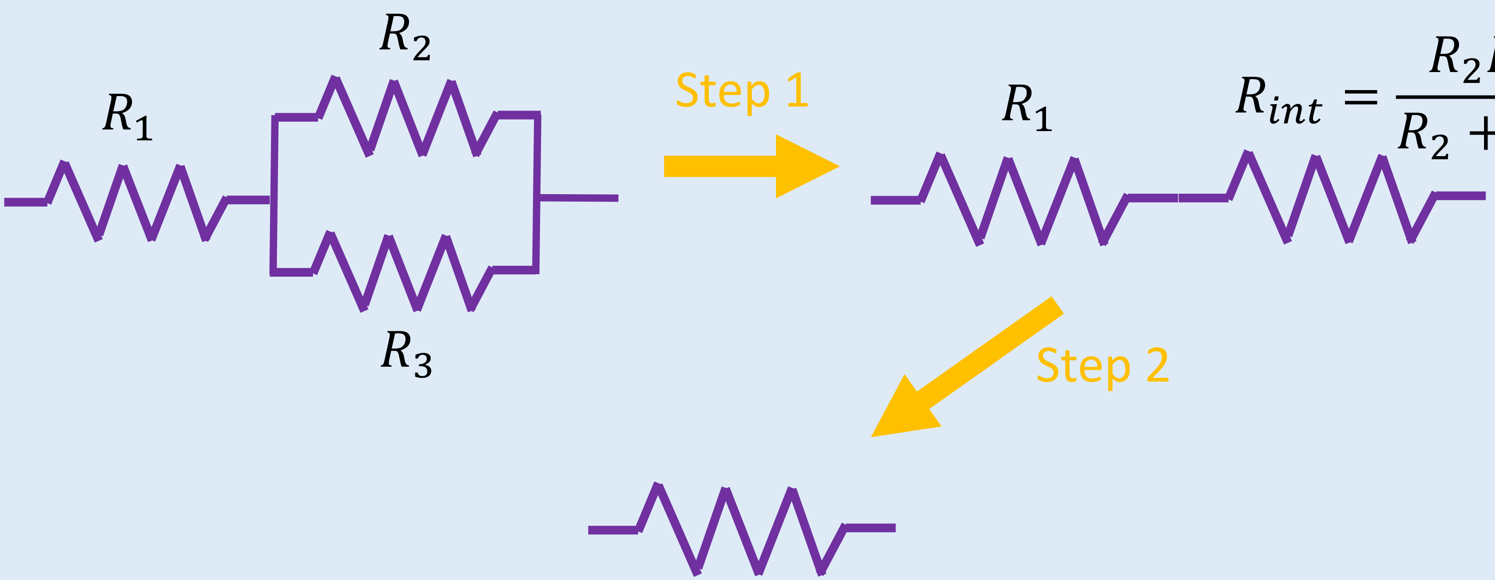
  $$\boxed{\frac{1}{R_{eff,p}} = \sum_{i=1}^{N} \frac{1}{R_i}} \; .$$

  - ➢ Note: $R_{eff,p} < R_i$.
  - ➢ Key feature of parallel resistors:
    voltage across each resistor is the same.



Reduction of Combined Series and Parallel Resistors: Example 1

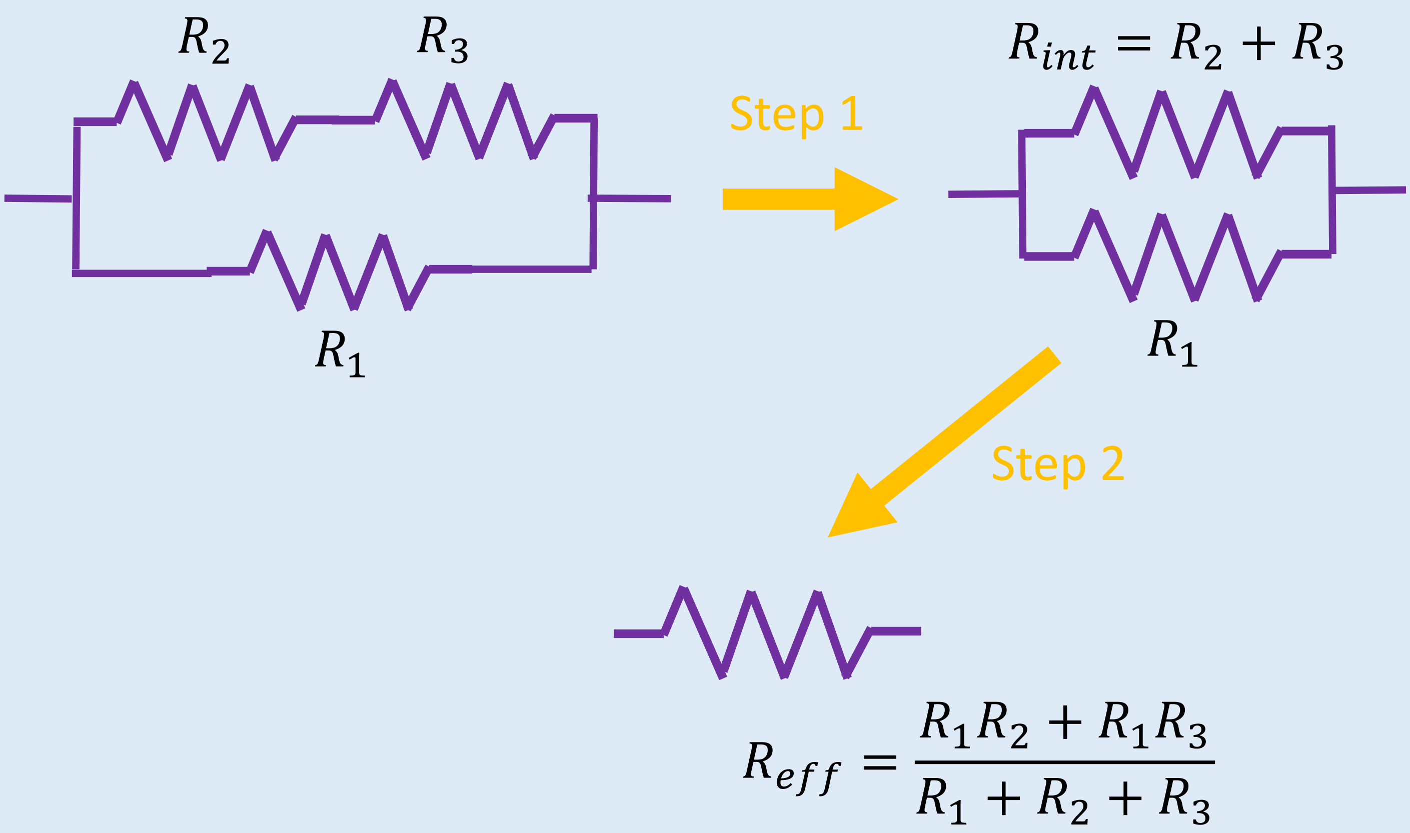
- Combinations of series and parallel resistors can be reduced to a simpler system. A methodical way is to consider intermediate steps:



$$R_{int} = \frac{R_2 R_3}{R_2 + R_3}$$

$$R_{eff} = \frac{R_1 R_2 + R_1 R_3 + R_2 R_3}{R_2 + R_3}$$

Reduction of Combined Series and Parallel Resistors: Example 2



Question 8.1

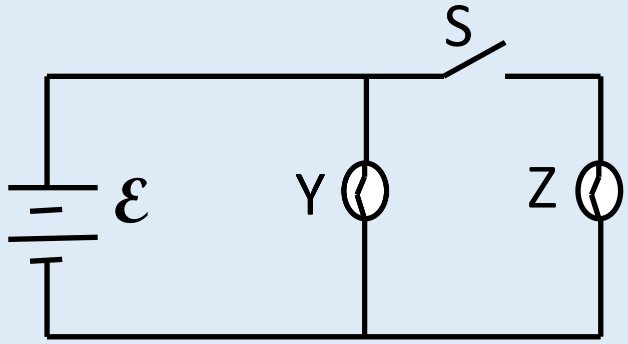
- Light bulbs Y and Z are connected in a circuit with an ideal battery $\mathcal{E}$ as shown in the diagram. Initially the switch S is open, and bulb Y shines with a certain brightness. When S is closed, the brightness of bulb Y

A. decreases.

B. increases.

C. remains the same.

D. is zero.



Question 8.2

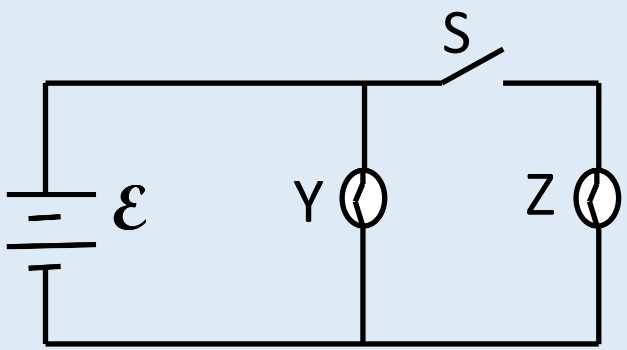
- What happens to the current running through $\mathcal{E}$ when switch S is closed?

A. It decreases.

B. It increases.

C. It remains the same.

D. It becomes zero.



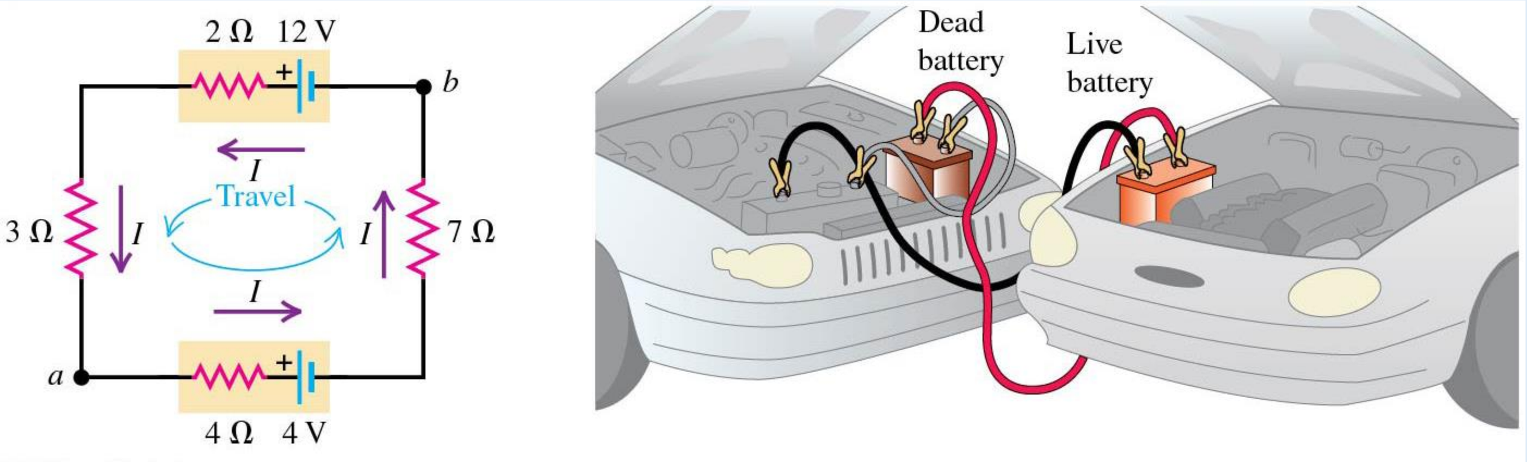
Systematic Circuit Solution

- Determine what quantity you are trying to find in the posed problem
  - Write it down clearly so that you can stay focused on your goal.
- Create a (or annotate a given) circuit diagram.
  - Write down and label each emf, current, and component value that you know.
- Group components to reduce and simplify circuits.
  - Resistors in series and in parallel.
  - Capacitors in series and in parallel.
- Isolate parts of the circuit using Kirchoff's loop and junction rules.

Systematic Circuit Solution (2)

- Write loop and current equations.
  - Count the number of unknowns that appear in your system of equations.
  - To obtain a unique solution, you must have as many **independent equations** as you do unknowns! If there's a mismatch between the number of equations and unknowns, it means you need to find additional equations or unknowns to make the numbers equal.
  - The equations must be independent – that is, one equation should not be derivable from a combination of the others. If they aren't independent, you'll be short the number of equations needed to uniquely solve the system. The problem will not be properly posed in that case, and no solution is possible.
- Solve the system of equations for the quantity you seek to find. Use any valid mathematical technique (e.g., substitution, Cramer's rule, matrix reduction, etc.) that you prefer. If you use the technique correctly, you will get the correct answer.
- You get better with practice. So, practice, practice, practice!

Single Loop System Example

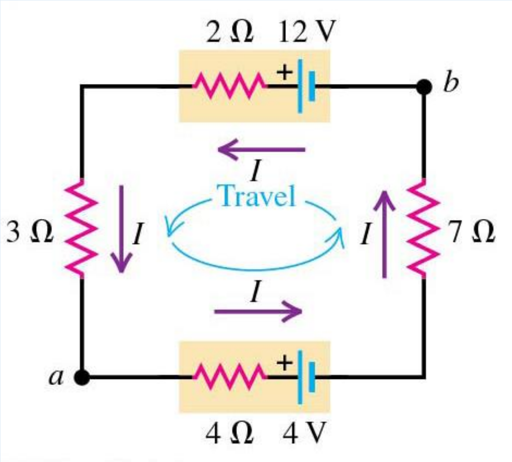


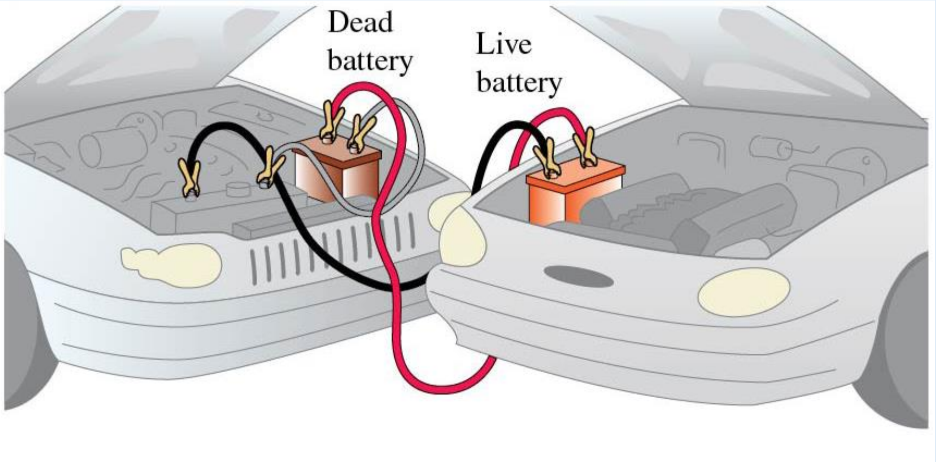
- Circuit is for recharging a car battery by another. Includes internal resistance of the batteries.
- Want to find the current I. Once found, can find the power supplied to the circuit by the emf of the live battery, and the power received by the emf of the dead (depleted) battery.
- Traverse the circuit in counterclockwise direction shown in the figure. Kirchoff's loop rule gives (starting and ending at point b):

$$+12\text{ V} - I(2\Omega + 3\Omega + 4\Omega) - 4\text{V} - I(7\Omega) = 0 ,$$

$$\Rightarrow 8V - I(16\Omega) = 0 \quad \Rightarrow \quad I = 0.5\text{ A.}$$

Single Loop System Example (2)





- Power from live battery emf is $P = IV$:

$$P_{live} = (0.5\ \mathrm{A})(+12\ \mathrm{V}) = 6\ \mathrm{W}.$$

  - $P_{live} > 0$ ⇒ Live emf is <u>supplying</u> power to the circuit.

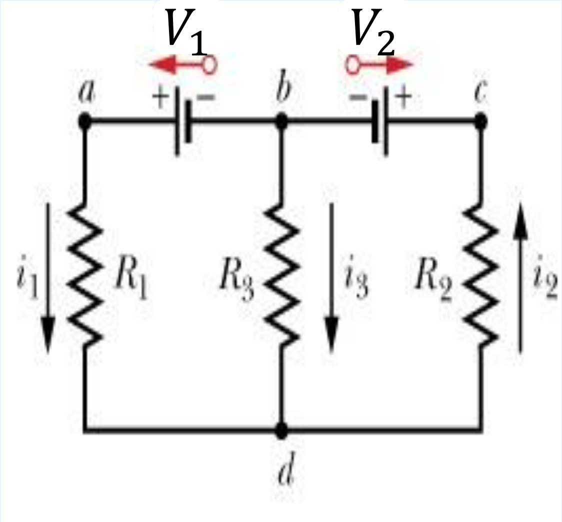
- The power from the dead battery emf is

$$P_{dead} = (0.5\ \mathrm{A})(-4\ \mathrm{V}) = -2\ \mathrm{W}.$$

  - $P_{dead} < 0$ ⇒ Dead emf is <u>absorbing</u> power. Being recharged!

Multi-loop Circuits: Example

- Assign directions to currents and use them consistently in your junction and loop equations.
- Write down loop equations and junction equations.
- Need as many independent equations as you have unknowns.
- Solve the system of equations for the unknowns.



- Example problem: in the circuit shown, what are the currents i_1, i_2, and i_3?
- Loop equation for left loop (start and end at b, traveling CCW):

$$+V_1 - i_1R_1 + i_3R_3 = 0.$$

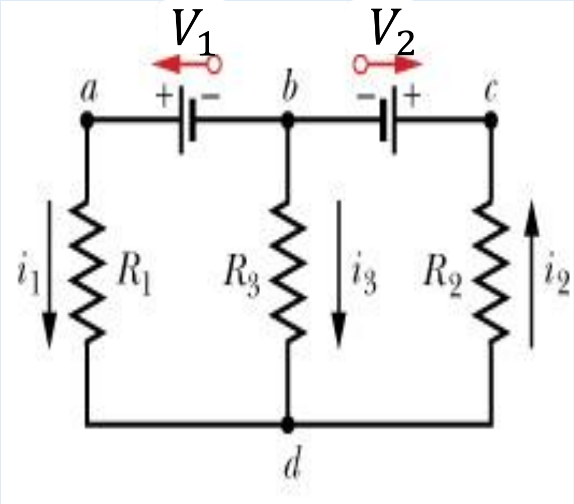
- Loop equation for right loop (start and end at b, traveling CW):

$$+V_2 + i_2R_2 + i_3R_3 = 0.$$

- Junction equation at b:

$$i_2 = i_1 + i_3\ .$$

Multi-loop Circuits: Example (2)



- 3 Unknowns: i_1, i_2, and i_3. (All V's and R's presumably given.)
- 3 independent equations.
- Numbers match. ∴ Can solve.
- Solve system for i_1, i_2, and i_3 any way you can.
- Example: substitute junction equation $i_2 = i_1 + i_3$ into second loop equation to get reduced system of equations:

$$+V_1 - i_1R_1 + i_3R_3 = 0 \qquad (1)$$
$$+V_2 + (i_1 + i_3)R_2 + i_3R_3 = 0 \qquad (2)$$

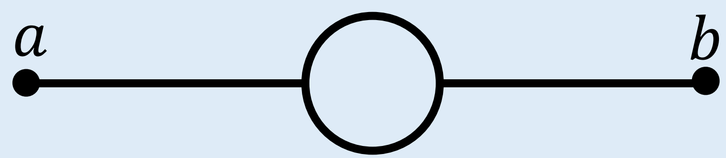
Multiply (1) by R_2 $\Rightarrow V_1R_2 - i_1R_1R_2 + i_3R_3R_2 = 0$ (4)
Multiply (2) by R_1 $\Rightarrow V_2R_1 + (i_1 + i_3)R_2R_1 + i_3R_1R_3 = 0$ (5)

Add (4) and (5) $\Rightarrow V_1R_2 + V_2R_1 + i_3(R_1R_2 + R_1R_3 + R_2R_3) = 0.$ (6)

Solve eqn. (6) for i_3, then insert result into (1) to get i_1. Then insert i_1 and i_3 expressions into junction equation to get $i_2 = i_1 + i_3$.

Practice Problem 1

- Young & Freedman 26.1 (p. 873)
  A uniform wire of resistance R is cut into three equal lengths. One of these lengths is formed into a circle and connected between the other two. What is the resistance between the opposite ends a and b?



Practice Problem 2

- Young & Freedman 26.23:
  Find the current through the resistor R, the resistance R, and the emf $\mathcal{E}$.