29A – Experiment: Faraday's Law – Measuring B Field Strength

Faraday's Law of induction is one of the building blocks for Maxwell's equations. It relates the instantaneous induced *EMF* to time rate of change of magnetic flux in the circuit. In mathematical form this can be stated as:

$$\mathcal{E} = \oint \vec{\boldsymbol{E}} \cdot d\vec{\boldsymbol{l}} = -\frac{d\Phi_B}{dt}$$

The magnetic flux is given by:

$$\Phi_B = \oint \vec{\boldsymbol{B}} \cdot \hat{\boldsymbol{n}} d\vec{A}$$

1) State Faraday's Law in your own words in a sentence or two.

In this set of experiments, we will explore Faraday's Law and Lenz's Law. In the first part you will use Faraday's Law to measure the strength of a magnet using a coil.

Part 1 – Magnetic field observations

Equipment: Large red and blue magnet, mounted pick up coil, Tektronix or Hewlett Packard oscilloscope, power cord.

Theoretical Background

This is a practical example of using Faraday's Law to measure the strength of small magnets.
Consider a situation where a coil of wire is pulled across the face of a magnet so that it moves from a region of high field to a region of no field.

Some manipulation of Faraday's Law yields: $\mathcal{E}(t)dt = -Nd\Phi_B = Nd(\vec{\boldsymbol{B}} \cdot \vec{\boldsymbol{A}})$

where $\mathcal{E}(t)$ is the *induced emf* (Electromotive Force), and *N* is the number of turns in a coil. Integrating both sides, assuming that the area *A* is constant, yields the result:

$$\int_{t_i}^{t_f} \mathcal{E}(t)\, dt = -NA \int_{B_i}^{B_f} dB = -NA(B_f - B_i)$$

The integral $\int_{t_i}^{t_f} \mathcal{E}(t)\, dt$ is just the area under a measurement of $\mathcal{E}(t)$.

Experiment

- Connect the coaxial cable from the coil to the Channel 1 input of your oscilloscope. You can use an oscilloscope to capture a single pulse of voltage as you withdraw a coil from a magnetic field and then estimate the area under that curve. ***Detailed instructions for setting up the oscilloscope can be found at the end of this description write-up (p. 9).*** Choose a suitable sweep time (100 ms/division is good for a start). Choose a suitable voltage scale (1 V/div is also good for a start). Set the trigger to “Auto”.

- Try moving the coil in and out of the magnetic field and observe the results on the oscilloscope.
- Hold the coil at the center of the magnetic pole face and then pull it out at a medium speed. Observe the voltage pulse. (Is it positive or negative? Is it narrow or wide? Tall or short?)

2) For a given change in motion record in the table how the pulse compares with the first pulse.

Motion	Amplitude (larger, smaller, same)	Width (larger, smaller, same)	Sign (changes remains the same)
The coil is pushed into the magnet at about the same speed as it was pulled out.			
The coil is flipped 180^o, so it faces the other pole, but pulled out at about the same speed.			
The coil is pulled more rapidly from the magnet.			

In preparation for the next part of the experiment measure the radius and calculate the area of the coil. Also record the number of turns of the coil.

a) What is the radius *r* (m) of your coil? _______________

b) What is the area *A* of your coil? _____________m^2

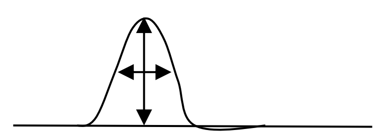
c) How many turns were you told this coil has? _______________

Part 2 – Magnetic field measurement

- Now set up the oscilloscope to capture a single pulse. (The following instructions are sufficient if you are familiar with oscilloscope functions. More detailed instructions for each type of oscilloscope are given in the Appendix on page 9.)
    - Set the voltage scale and time base to fill the screen with a typical positive pulse
        - (1V/div), and
        - (10 ms/div) are usually good, respectively.
    - Set the Trigger mode to "Normal" with positive slope. Set the Trigger level so that it is smaller than ½ the height of a pulse, but large enough to avoid triggering on noise.
    - Hit "stop" when you have a pulse you like on the screen.
- Use the cursors and the "Measure" controls on the oscilloscope to measure the height and half width of the pulse as discussed above. Fill out the table below for five separate measurements.

Perform five trials and record the measurements in the table below.

You can make an estimate of the area from the height and width of the pulse. It is usually best to take the full height (as shown in the figure) and the width of the pulse at half of the full height (full width at half maximum - FWHM). (This technique would be very accurate if the pulse were a perfect triangle.)



Trials	Height (V)	FWHM (s)	Height times Width
1			
2			
3			
4			
5			

3) Based on the above measurements, perform the following calculations.
   a) *Average* of the Height times Width column of your measurements: _______________V · s

   b) *Average* magnetic *B* field from your measurements: _____________ T

   c) Is this field consistent with what you were told the range of experimental fields might be, i.e., from 0.1 to 0.3 T?

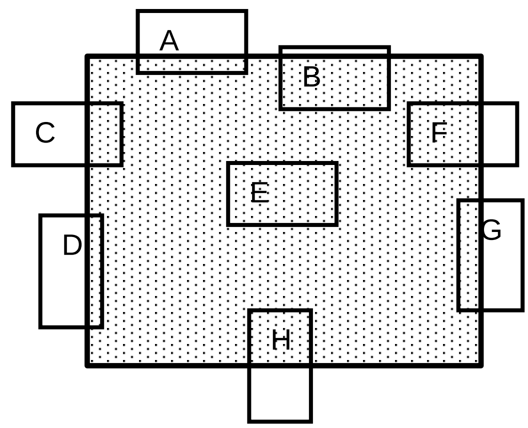
   d) How does this field compare with the strength of the Earth's magnetic field? (Much larger. Slightly larger. About the same. Slightly smaller. Much smaller.)

29B – Lenz's Law

When the magnetic flux through a conducting loop changes, Faraday's Law states that there is an EMF generated around the loop, $\mathcal{E} = -\frac{d\Phi_B}{dt}$. This *EMF* induces a current in the loop, $I = \mathcal{E}/R$. The current induces a magnetic field that opposes the change in the field. The induced current also interacts with the magnetic field causing a force on the loop $\vec{\boldsymbol{F}} = I\,\vec{\boldsymbol{l}} \times \vec{\boldsymbol{B}}$ that opposes the motion of the loop. In addition, if the loop is turning in a magnetic field there is a torque applied to the loop to oppose the rotation. This torque is expressed as $\vec{\boldsymbol{\tau}} = \vec{\boldsymbol{\mu}} \times \vec{\boldsymbol{B}}$.

1) State Lenz' Law in a sentence or two.

2) The shaded region shown in the drawing to the right has a uniform magnetic field pointing out of the paper in the shaded region. All of the rectangular loops shown in the figure have the same shape and size, are made of the same conducting material, and have the same resistance. All of the loops are moving to the right at the same speed.
   a) Rank the magnitude of the current induced in the loops from largest to smallest (smallest being zero.) If the current in two loops is the same, group them together in a set of parentheses (e.g.- T, U,(X, Y, Z)).



Largest ___________________________________ Smallest

   b) Classify the loops with respect to the current direction (sunwise (S), widdershins (W), none (N)).

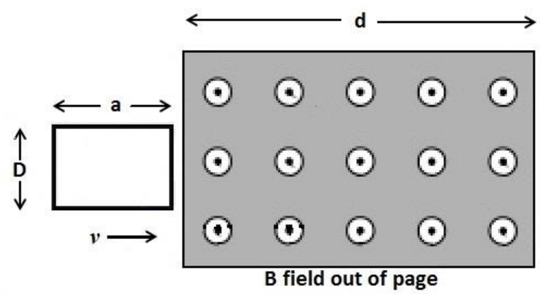
Clockwise_____________________

Counterclockwise_______________

No current_____________________

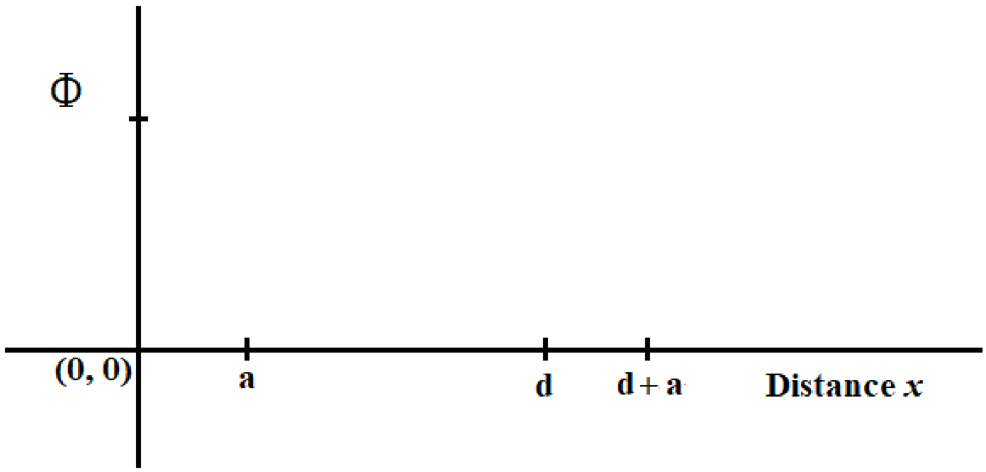
29C – Applications of Faraday's and Lenz's Laws

1) The figure shows a conductive rectangular loop of resistance R, width D, and a length a moving at a constant speed v in the positive X-direction into and through a region of uniform magnetic field $\boldsymbol{B}$, pointing out of the page.

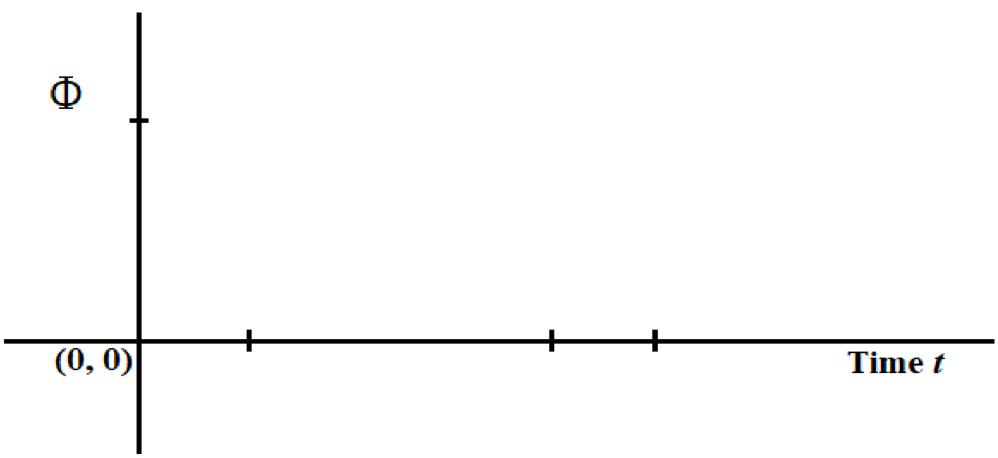


Use the following values:
D = 8.0 cm, a = 12 cm, d = 21 cm, R = 24 Ω, and B = 3.0 T.

a) Plot the flux Φ through the loop as a function of the position of the right hand edge of the loop,x. Take the left-hand edge of the field region to be x = 0. Include vertical scale.



b) Plot the flux through the loop as a function of <u>*time*</u> using the speed v = 3.0 m/s and x(0)=0. Include numerical values – horizontal and vertical.



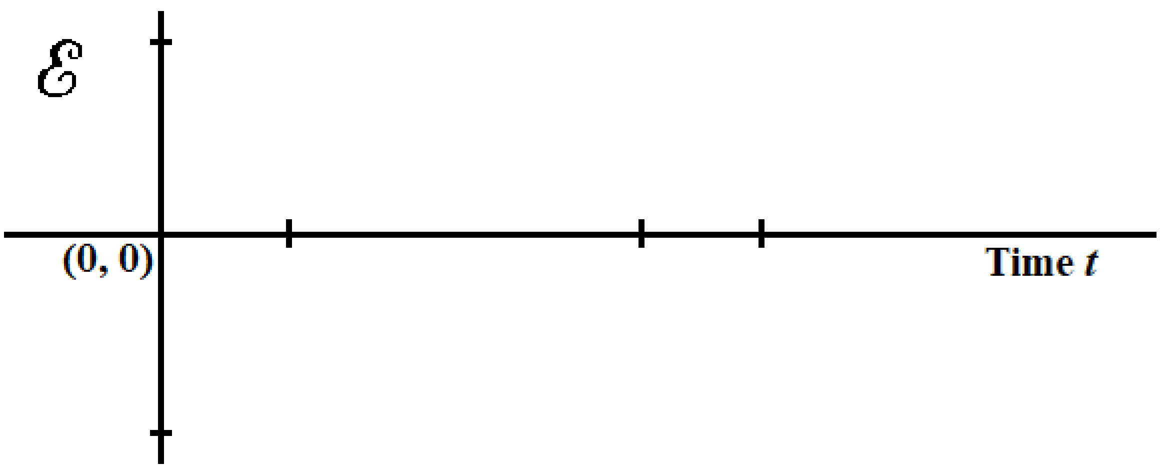
c) What is the direction of the current in the loop as the leading edge of the loop enters the field region?
(clockwise, counterclockwise, no current) Explain your reasoning.

d) What is the direction of the net magnetic force on the loop as the leading edge of the loop enters the field region?
(left, right, up, down, into paper, out of paper, no force) Explain.

e) What is the direction of the current in the loop as the leading edge of the loop leaves the field region?
(clockwise, counterclockwise, no current) Explain your reasoning.

f) What is the direction of the net magnetic force on the loop as the leading edge of the loop leaves the field region?
(left, right, up, down, into paper, out of paper, There is no force…) Explain.

g) Plot the *EMF* around the loop as a function of time. Include a voltage scale.

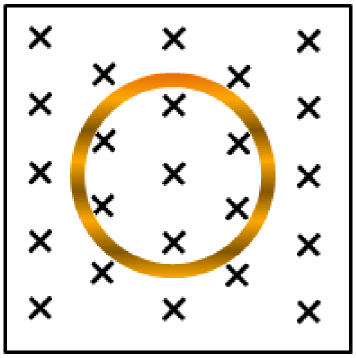


h) What is the magnitude of the force F_{max} on the loop as it enters the magnetic field region?

i) What is the magnitude of the force F_{max} on the loop as it leaves the field region?

j) What is the rate at which work must be done (Power - J/s) on the loop during the time that it is entering the field?

k) The loop is clipped so that it is no longer continuous. What is the direction of the force as the loop leaves the field region? (left, right, up, down, into paper, out of paper, There is no force…) Explain.



2) A loop of wire of resistance R sits in a spatially uniform magnetic field B as shown schematically. The loop has an area A_L and the square field region has area A_S. Suppose that the B field is changing inside the square with time t according to

$$B(t) = k\,t^2 + B_{O.}$$

Here k and B_0 are positive constants.

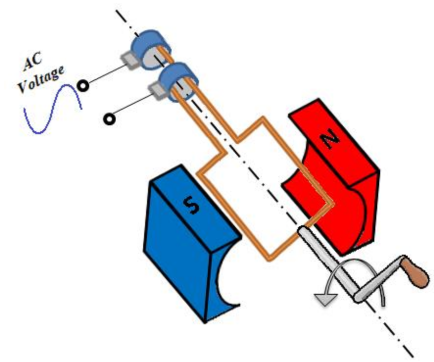
a) What is the direction of the induced B field (B_{ind})? (e.g.- Into paper, Out of paper, Clockwise, etc.) Explain.

b) What is the direction of the induced current i_{ind} in the loop? Explain.

c) For R = 0.2 ohms, k = +0.3 T/s^2, B_0 = 1.8 T, A_L = 0.75 m^2, and A_S = 4.0 m^2, find the current induced i_{ind} in the loop at t = 5.0 s.

d) Find the magnitude of the B field induced by the induced current, i_{ind}, in the loop.

3) In its simplest form an AC generator consists of a rectangular coil mounted on a rotor shaft. The coil is also referred to as the armature. The coil is placed in a uniform magnetic field. The shaft is normal to the direction of the magnetic field. The coil (armature) is mechanically rotated, with an angular frequency of ω, in the uniform magnetic field by some external means. Let the area of the coil be $\boldsymbol{A}$**,** and the number of turns in the coil be *N***.**
Assume the following numbers for a somewhat realistic generator: B = 0.35 T, Coil area A_{coil} = 5.0 cm^2, Coil resistance R_{coil} = 2.5 ohms and the coil is in the shape of a *square*.



a) The coil rotates at angular frequency ω. Write an expression for the flux through a single loop as a function of time in terms of magnet area, coil area, and field strength. (It should include a sinusoidal function.)

b) The coil can be rotated by a hand crank at 350 revolutions per *minute*. Find the amplitude of the *EMF* **-** $\mathcal{E}$ that can be generated with a single turn in the coil.

c) How many turns should the coil have to generate an amplitude (peak voltage), $\boldsymbol{\mathcal{E}}_{\text{total}}$ = 10 V at the rotation speed derived in part b?

d) For a voltage amplitude $\mathcal{E}_{\text{total}}$ = 10 V, what is the current in the coil, i_{coil}?

e) Using your current amplitude from part d, $\mathcal{E}_{\text{total}}$ = 10 V, what is the maximum value of the torque $\boldsymbol{\tau}_{\text{max}}$ required to turn the magnet? (The torque is due to two sides of the coil moving at right angles through the field when the magnet face normal is at right angles to the coil normal.) The torque is defined as:

$$\vec{\boldsymbol{\tau}} = N\vec{\boldsymbol{\mu}} \times \vec{\boldsymbol{B}}$$

Where $\vec{\boldsymbol{\mu}}$ is the magnetic moment of a single coil and N is the number of turns of the coil.

APPENDIX: Setting up your oscilloscope

These are general instructions – we have a few different models of oscilloscope in the labs so details might be different.

1) Plug your signal into the appropriate input channel. (Channel 1 for this lab.)
2) Set the scale for each channel by rotating the Volts/DIV knob. You can read the scale at the bottom of the display. One DIV is usually about 1 cm on the screen. (It will usually read just 1.00 V for the 1V/DIV scale.) A good starting scale is 1.0 V/DIV.
3) Set the time scale by rotating the SEC/DIV knob (usually the third one from the left). You can read the scale at the bottom of the display. A good starting scale is 10 ms/DIV.
4) Select the way that the oscilloscope couples to the signal source. You will usually choose DC Coupling on the Channel menu.
5) Select the mode that controls when the scan starts for each sweep of the signal on the screen. This is done using the TRIGGER menu.
The Trigger mode selects whether the sweep runs continuously or is started on each sweep by a particular signal.
AUTO mode – Runs continuously and is good for looking at how a signal is affected by experimental changes.
NORMAL mode – The sweep is triggered by a certain condition and is good for capturing a single pulse.
For NORMAL mode:
a) Choose the source for the trigger signal (usually Channel 1 or 2).
b) In the trigger menu choose EDGE trigger and SLOPE=Positive. You must also select a voltage that will trigger the start. Usually this is large enough to exceed noise on the signal. (0.1 V for this experiment is good.)
6) When you have a signal you want to analyze, press the RUN/STOP button to freeze it. On the LCD (newer) oscilloscopes you can change the settings after you have frozen the screen. On the CRT (older) oscilloscopes the signal is erased whenever you change a setting.