Helio Campus/AEFIS Course Survey Participation Incentive Program

- The HelioCampus/AEFIS end-of-course survey is now open, and will remain so until Thursday, April 25.
- We value your participation in the course survey, and any comments or suggestions that you may make to help us improve this class.
- To encourage your participation in the survey we are offering the following incentive program:
    - ✓ If the overall class response to the survey reaches the 70% participation mark by 6 pm, Thursday April 25, the entire class will start the Final Exam with a score of 10 exam points.
    - ✓ If the overall class response to the survey reaches the 80% participation mark by 6 pm, Thursday April 25, the entire class will start the Final Exam with a score of 20 exam points.
    - ✓ If the overall class response to the survey reaches the 90% participation mark by 6 pm, Thursday April 25, the entire class will start the Final Exam with a score of 30 exam points.

Physics 1200
Lecture 26
Spring 2024

Light Wave-Particle Duality; Probability;
Light Uncertainty Principle;
Matter Waves: deBroglie Hypothesis

Recap: Light: Waves or Particles? Wave-Particle Duality

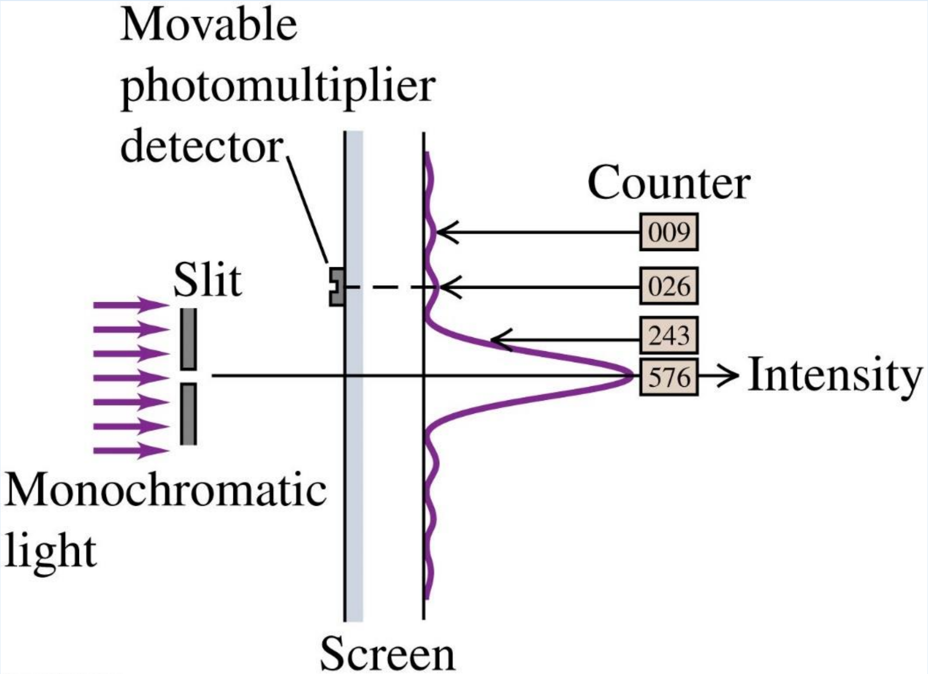
- No single model or interpretation (EM wave model, photon [particle-like] model) suffices in describing <u>all</u> physically observed aspects of light (propagation, interaction with matter, etc.).
- It comes down to: what sort of experiment is being performed?
  - An experiment designed to detect wave-like properties of light <u>will</u> detect the wave-like properties of light, e.g.,
    - Two-slit interference.
    - Diffraction.
  - An experiment designed to detect particle-like properties of light <u>will</u> detect the particle-like aspects of light, e.g.,
    - Photoelectric effect.
    - Compton scattering.
- Physical experiments thus reveal light has a <u>dual</u> nature: a wave-particle duality.

The Principle of Complementarity

- First proposed by N. Bohr, 1928, often referred to as "Bohr's Principle of Complementarity":
  - *"The wave and the particle aspect of a quantum entity are both necessary for a complete description. However, the two aspects cannot be revealed simultaneously in a single experiment. The aspect that is revealed is determined by the nature of the experiment being done."*
- Basic idea: the wave and particle descriptions of light are complementary. Both are needed to accurately provide a complete model of nature.
- This principle applies to material particles as well, because, as we will see, they also exhibit a wave-particle duality.

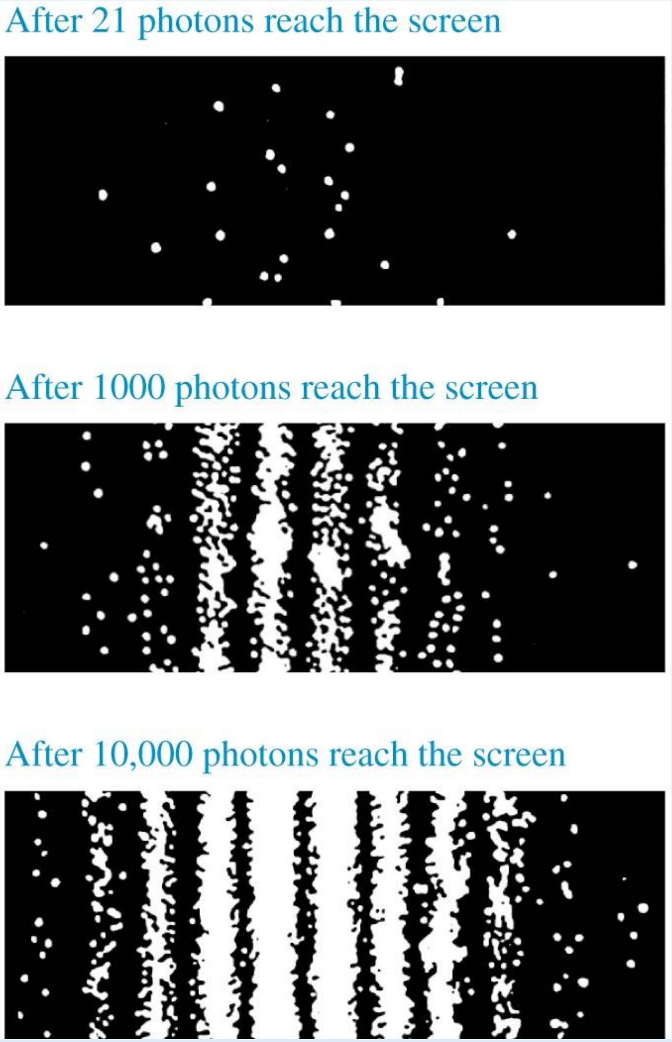
Diffraction & Two-Slit Interference: Perspective from Single Photons

- Carry out a single-slit experiment with light, using an intensity so low that photons pass through the slit one at a time.
- Photon passes through slit, goes to analyzer screen, where a photodetector records where a photon impacts the screen.
- Intensity at a certain location on the screen will be proportional to the number of photons detected (over time) at that location.
- Experiment shows that resulting built-up intensity pattern over time is <u>identical</u> to standard single-slit diffraction pattern!



Diffraction & Two-Slit Interference: Perspective from Single Photons (2)

- Can also carry out double-slit experiment with light, with one photon at a time incident upon the slits.
- Experiment shows that resulting built-up intensity pattern over time is <u>identical</u> to the standard double-slit diffraction pattern!
  - ➢ This is even though when a single photons passes through slit, there's no other photon to interfere with … except itself.
- Dark regions on screen indicate regions places where the photons did not strike. Likewise, brighter regions show where they did (and with greater frequency).
  - ➢ Both situations related to idea of where an individual photon is <u>likely</u> to hit screen. That is, the <u>probability</u> of where the photon will be located when it gets to the screen.

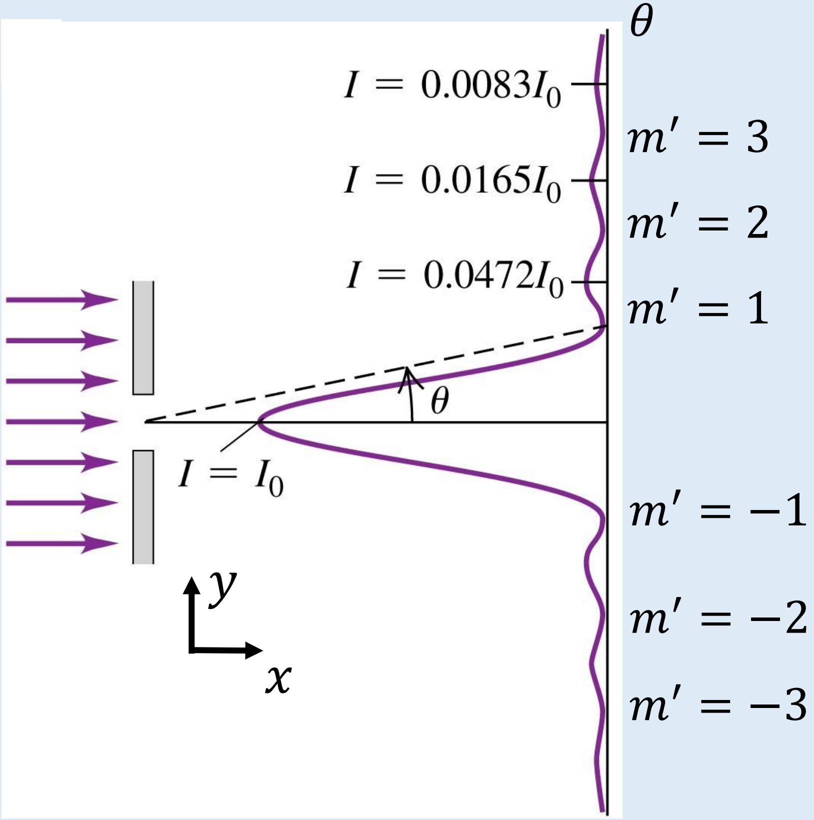


Photons: Probability, Statistics, and Uncertainty

- Re-interpreting diffraction and interference experiments in terms of the photon (particle-like) picture can be done by considering intensity patterns being related to the <u>probability</u> of where a photon could be found.
- In this re-interpretation, built-up intensity pattern over time is a result of statistical/probabilistic behavior of photons.
- Statistics and probability are fundamentally related. These concepts come into play when studying systems having <u>imperfect or imprecise knowledge with respect to the state of the system</u>. That is, for systems in which there is some <u>uncertainty</u> regarding the properties of the system.
- The standard deviation of a set of a data from a physical experiment is one example of how uncertainty can be quantified, when dealing with unknown or poorly understood sources of error when making measurements.

Photons: Uncertainty and the Diffraction Intensity Pattern

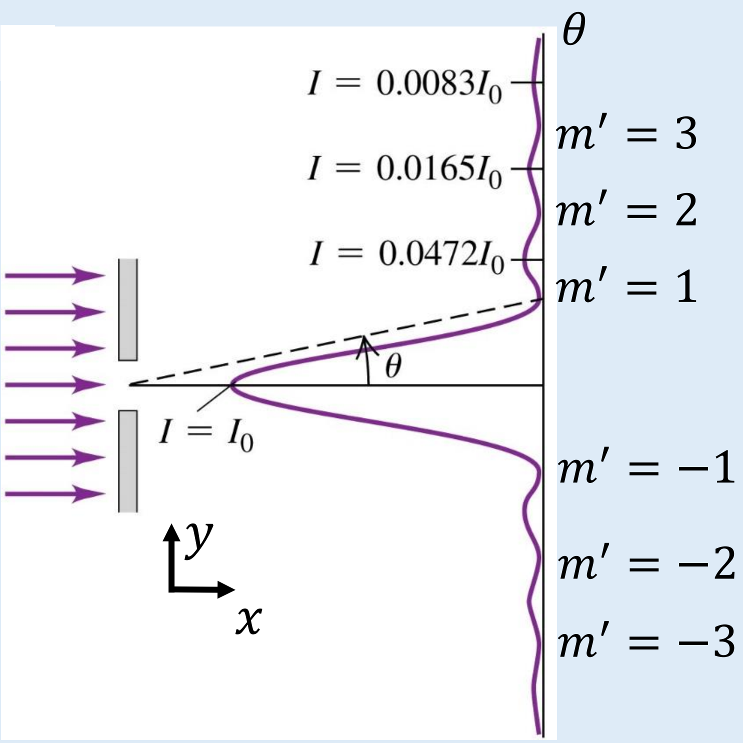
- Using knowledge of the intensity pattern for single-slit diffraction of light, it can be shown that it is related to an uncertainty relation regarding certain physical properties of a photon.
- Fundamental feature of single-slit diffraction pattern is that there is a central maximum of characteristic angular size θ, where θ is the angle of the first diffraction minimum.
- Note that the diffraction pattern occurs for light that is normally incident on the slit. That is, for light directed entirely along the x-axis for the coordinate system shown.



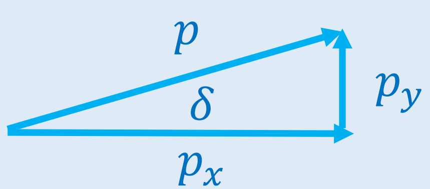
- A fundamental physical feature in photon picture is that incident light would be a stream of photons directed entirely along the x-axis. That is, incident light would consist only of photons with momentum x-component $p_x \neq 0$, and y-component $p_y = 0$.

Photons: Uncertainty and the Diffraction Intensity Pattern (2)

- Fact that the light is spread out from the central maximum by the angle $\boxed{\theta \simeq \sin\theta = \lambda/a}$ (for small angles) – where a is the slit width – tells us that $p_y \neq 0$. That is, there is <u>an inherent uncertainty in the measurement of the y-momentum of the photon as it passes through the slit.</u>



- For there to be a spread in angle, the photon would have to be on a ballistic path with geometry:



where p_x and p_y are the non-zero momentum components.

- For small angles, $p_x \simeq p$, and $\delta \simeq \tan\delta = p_y/p$.

Photons: Uncertainty and the Diffraction Intensity Pattern (3)

- The extent of the angular spread of the photons for the size of the central maximum of the diffraction pattern corresponds to the range of possible momenta in the y-direction, Δp_y (the uncertainty in the p_y) up to the diffraction angle, $\delta = \theta$. That is,

$$\theta \simeq \frac{\Delta p_y}{p} \;\Rightarrow\; \frac{\lambda}{a} = \frac{\Delta p_y}{p} \Rightarrow \; \Delta p_y a = \lambda p = \lambda\left(\frac{h}{\lambda}\right) = h.$$

- While it is known that the photon passes through the slit, it's not known <u>exactly where</u> the photon is in the slit when it passes through it. Hence, there is a <u>range</u> of possible values $\Delta y = a$ for the location of the photon when it's going through the slit.

- Above relation can be rewritten as

$$\Delta p_y \Delta y = h.$$

The Uncertainty Principle for Photons

- If we take more carefully into account the fact that the intensity pattern also includes further side lobes to the diffraction pattern, we find that the uncertainty relation found on the preceding slide should be

$$\boxed{\Delta p_y \Delta y \geq \hbar/2} \text{ , where}$$

$$\boxed{\hbar \equiv h/2\pi} \text{ (pronounced “h-bar”).}$$

- This an example of the Uncertainty Principle, which is found to be a universal characteristic property of quantum objects that have a wave-particle duality, as shown by W. Heisenberg, one of the pivotal figures in the development of Quantum Mechanics.

- In summary: the Uncertainty Principle says that we cannot simultaneously know both the y-momentum and the y-position of photon to arbitrary precision. If one of these quantities is very well known, the other is poorly known.

The Uncertainty Principle for Photons (2)

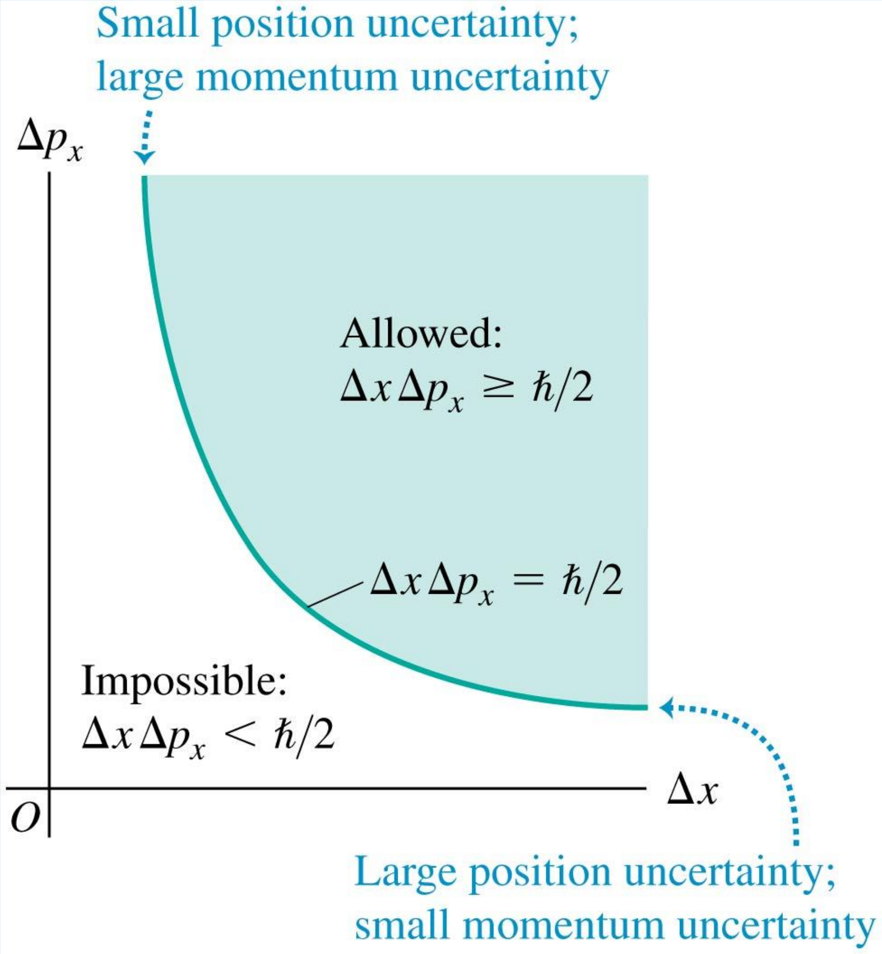
- It's not too hard to show similar relations hold for the other coordinate axes (isotropy of space). Therefore, there is a set of uncertainty relations:

$$\boxed{\Delta p_x \Delta x \geq \hbar/2 \quad , \quad \Delta p_y \Delta y \geq \hbar/2 \quad , \quad \Delta p_z \Delta z \geq \hbar/2}$$

- While the product of the uncertainties in a certain direction are constrained by the above relations, there's nothing preventing one from completely reducing the momentum uncertainty along one coordinate axis while simultaneously knowing the position perfectly along another axis. For instance, it is possible to have $\Delta p_x \Delta y = 0$, etc.

The Uncertainty Principle for Photons (3)

- Shown is a graphical representation of the Heisenberg uncertainty principle.
- A measurement with uncertainties whose product puts them to the left of or below the blue line is not possible to make.



The Uncertainty Principle for Photons (4)

- By considering the spread in the frequencies Δf of a photon that is created during a short pulse of radiation of duration Δt, by using the fact that any uncertainty in Δf is related to an equivalent uncertainty in the value of a photon's energy $\Delta E = h\Delta f$, one finds an Energy-Time Uncertainty Principle for photons:

$$\boxed{\Delta E \Delta t \geq \hbar/2}.$$

- Therefore, to measure the energy of a photon to infinite precision (i.e., zero uncertainty), the measurement would have to done over an infinite amount of time.
- This relation was also determined by Heisenberg, and it applies to other objects having a wave-particle duality.

Introduction to Matter Waves

- In 1924, Louis de Broglie turned Physics (and the rest of the world) upside down by simply rearranging a known equation.
- While simple to perform, the act of doing so was radical then, and can still be considered to be so.
- The known equation is the momentum-wavelength relation for photons:

$$p = h/\lambda \, .$$

  In words: a wave-like entity (the photon) can have a discreet momentum (a particle-like quantity).

- de Broglie's radical hypothesis: preserve the symmetry! In his doctoral thesis he postulated that material particles can also have wave-like properties, with a wavelength given by the relation:

$$\boxed{\lambda = h/p} \, .$$

- The stunning prediction: particles can act as though they were "<u>matter waves</u>."
- de Broglie's hypothesis became an impetus to the later development of quantum mechanics (1925 -1926), which is often referred to as "wave mechanics".

Matter Waves

- In the non-relativistic limit, the matter wavelength of a particle can be written as

$$\boxed{\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{h}{\sqrt{2mK}}},$$

where $K = \frac{1}{2}mv^2 = \frac{p^2}{2m}$ is the kinetic energy of a particle in that limit (i.e., the classical Newtonian expressions).

- de Broglie also proposed that the frequency of a matter wave is (in analogy to the Einstein photon relation) is

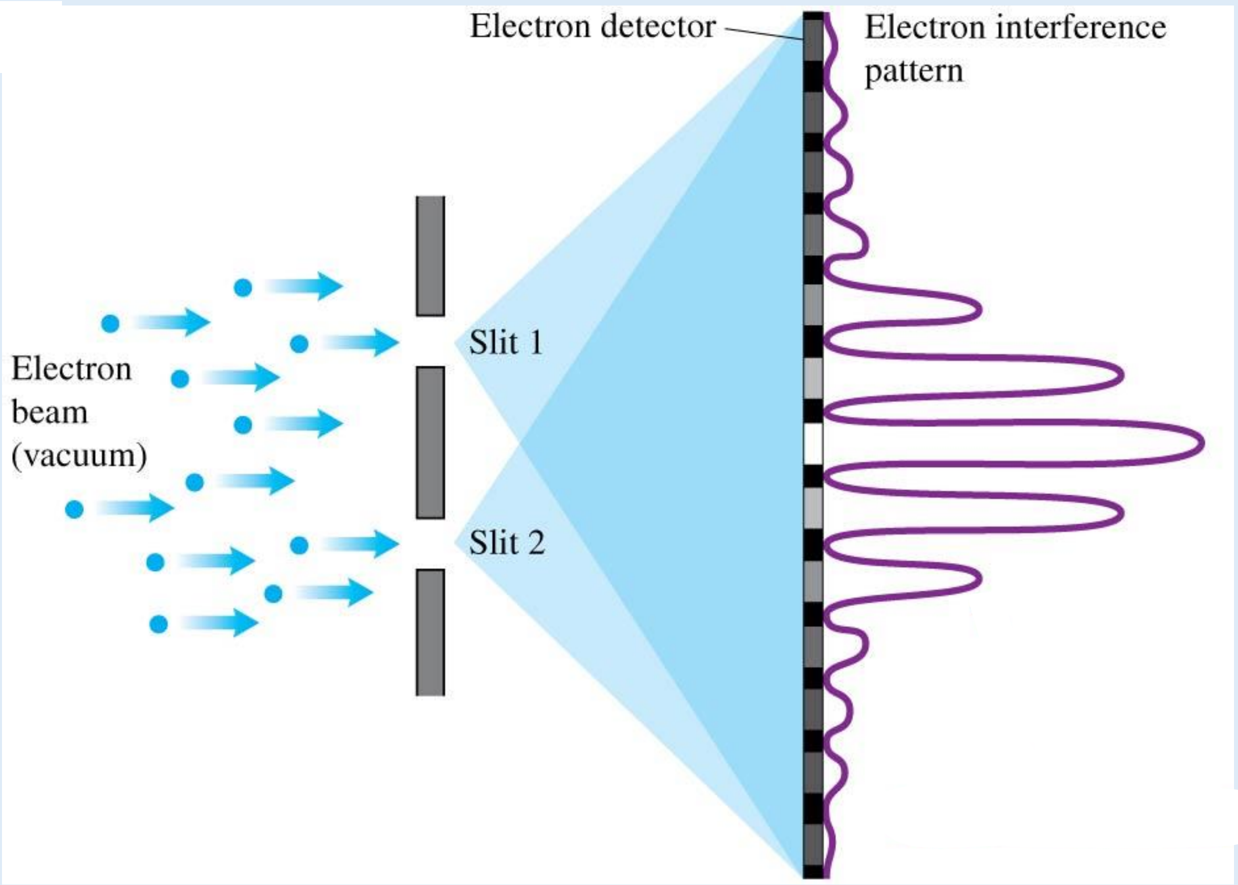
$$\boxed{f = E/h}.$$

Matter Waves (2)

- de Broglie’s hypothesis got noticed fast. And confirmed fast!
  - Davisson & Germer (1927) diffracted electrons from a crystal lattice, with a pattern that is indistinguishable from what is obtained by diffracting X rays from a crystal. Davisson won the Nobel Prize for his work in 1937.
- Nowadays experimenters and other workers routinely perform matter interference and diffraction experiments. Again, results very similar to those obtained from experiments with light.

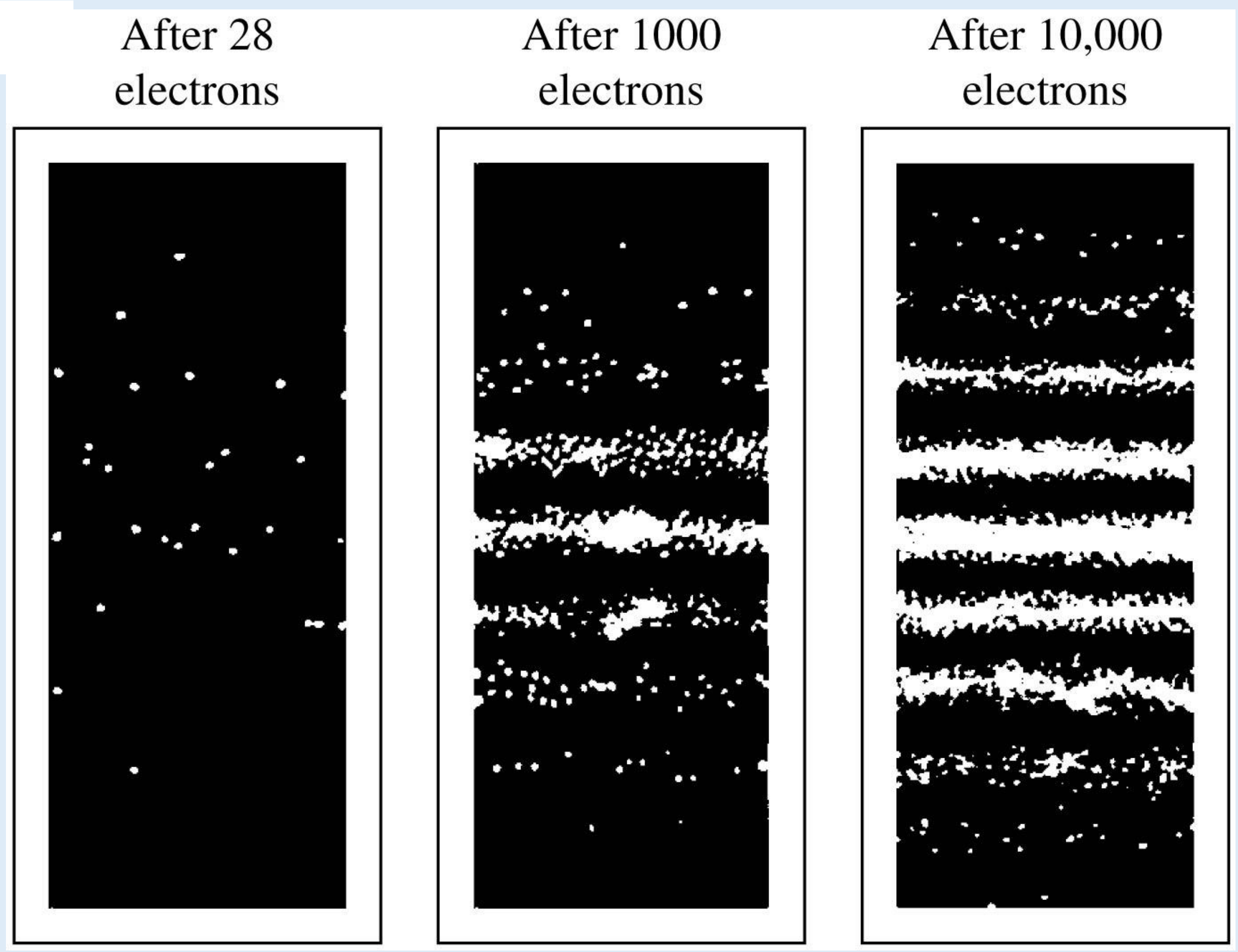
Matter Waves (3)

- Example: electron double-slit diffraction. Set-up:



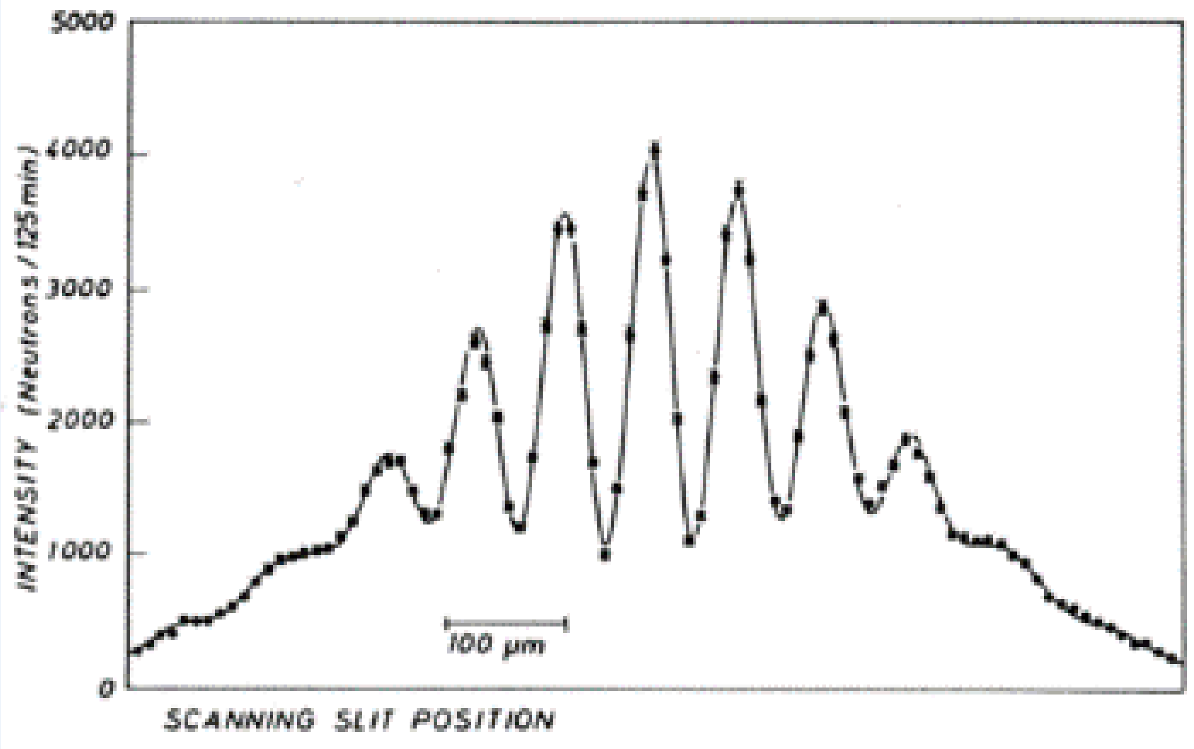
Matter Waves (4)

- Example: electron double-slit diffraction. Results:



Matter Waves (5)

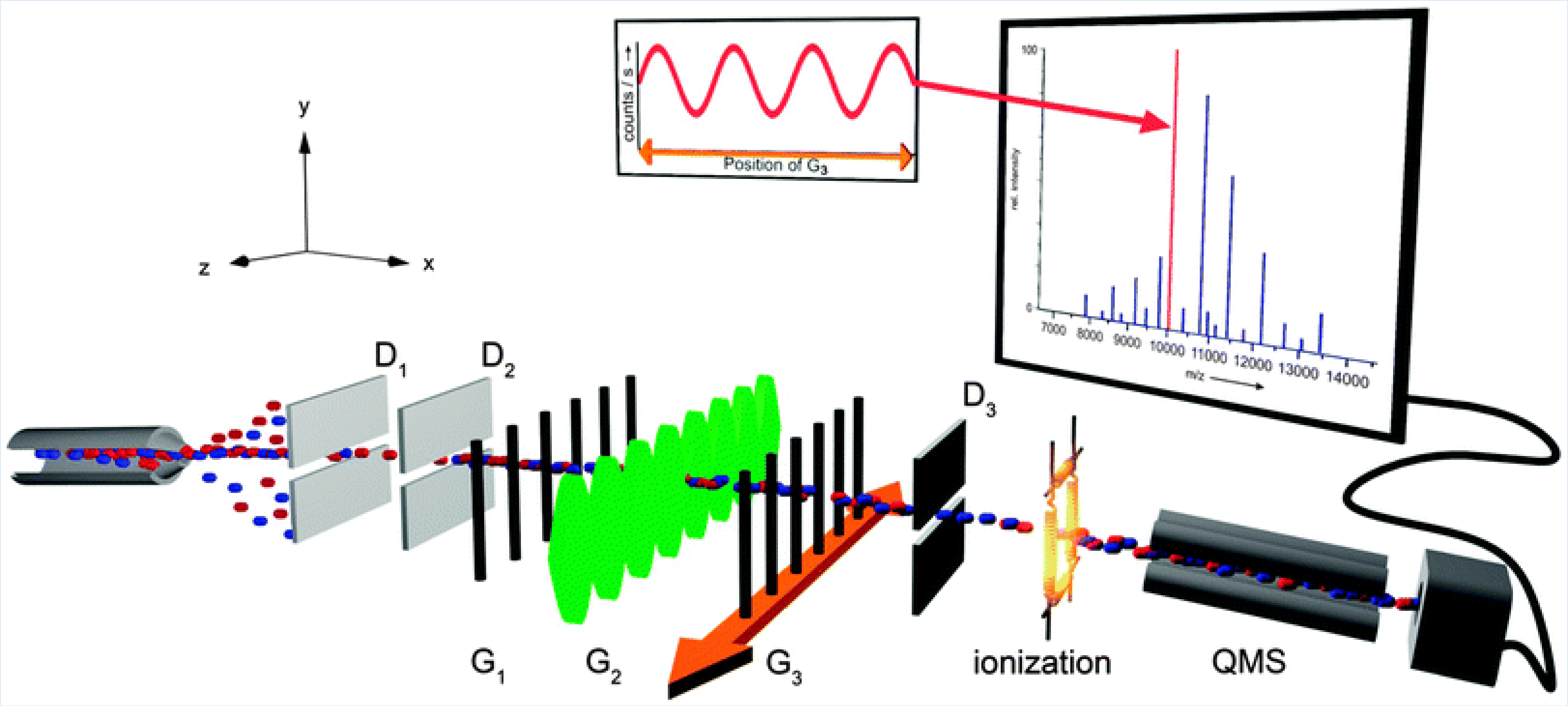
- Example: double-slit interference & diffraction of cold neutrons.
  Results:



(Zellinger et al., Rev. Mod. Phys. 1988, **60**, 4, 1067)

Matter Waves (6)

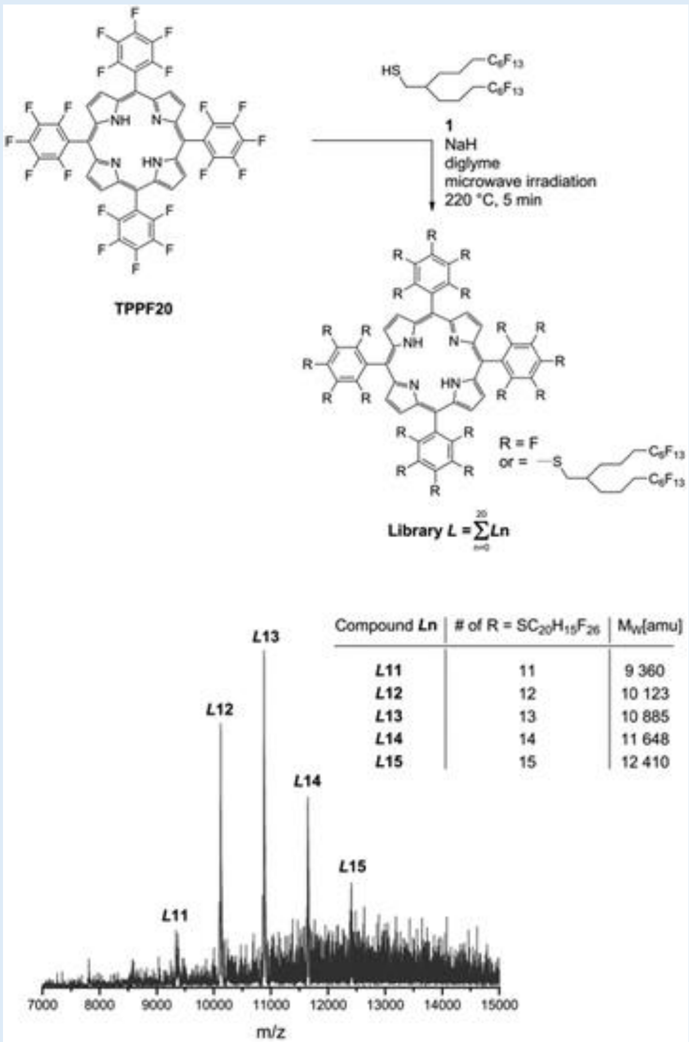
- Large molecule "L12" (from the fluorous porphyrin L-library).
- Mass = 10,123 u! (810 atoms in a single particle.)

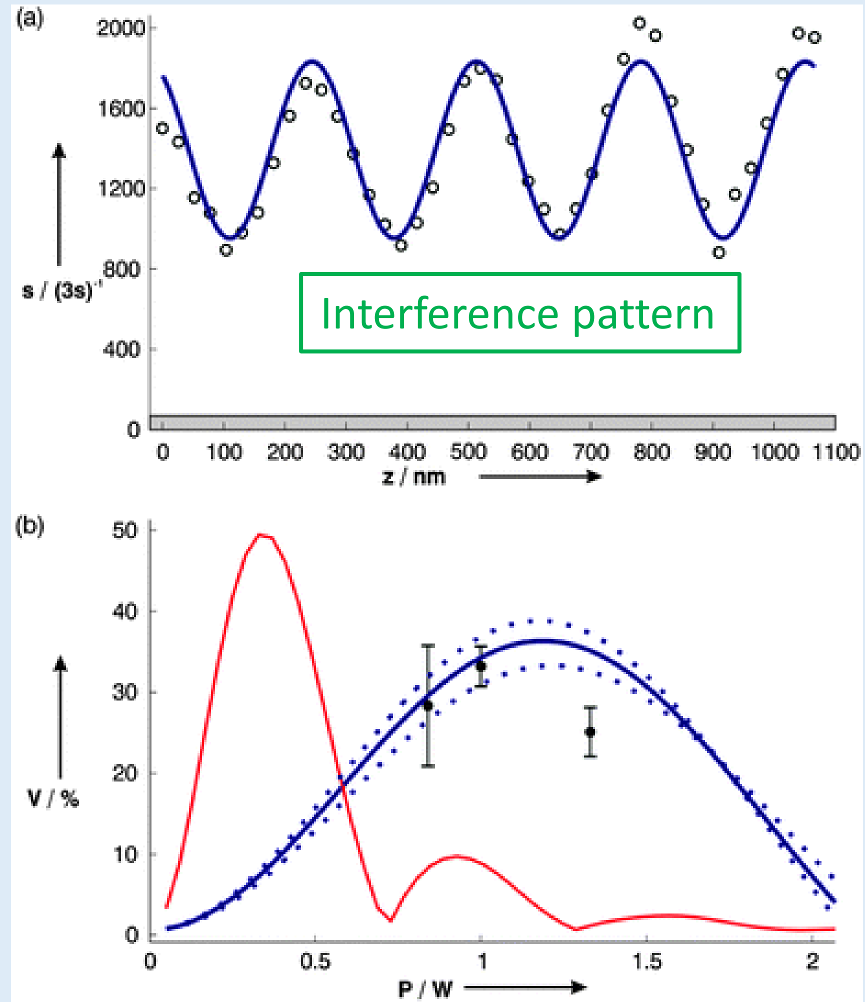


(Eilbenberger et al. 2013, *Phys. Chem. Chem. Phys.*, **15**, 14696-14700)

Matter Waves (7)

- Large molecule “L12”. Interference results.





Application: Electron Microscopy

- Electron microscope. Works just like a light microscope, only here it exploits the wave properties of electrons.
- The wave aspect of electrons means that they can be used to form images, just as light waves can.
- This is the basic idea of the transmission electron microscope (TEM), shown.
- The "lenses" are coils that use magnetic fields to focus the electrons.