MasteringPhysics: Extra Study Aids

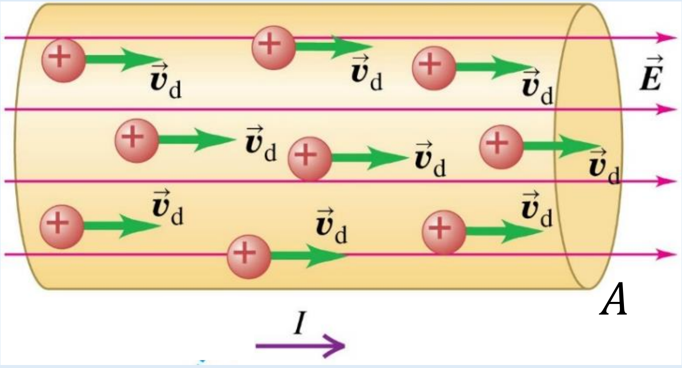
- Some students have asked for more practice or help with concepts of electric fields, Gauss's law, etc.
- Our MasteringPhysics course offers the use of Dynamic Study Modules ("Course Home" page).
  - Based on "brain science" (i.e., cognitive and education studies) to interact with users and enhance understanding of ideas and concepts.
  - Dynamic Study modules can be accessed by all students registered in our MasteringPhysics sections.
  - The "Electrostatics" Dynamic Study Module has modules that covers the topics already talked about in class, and the topics that we will cover prior to the first class exam (module numbers correspond to the chapter numbers in our textbook).
- Our MasteringPhysics course also has practice quizzes, which can be found in the "Study Area."

Physics 1200
Lecture 06
Spring 2024

Current, Current Density, Resistivity, Resistance, & Power Dissipation

Current

- Until now we have confined ourselves to static situations. We now expand to cases with sustained motion of charges.
- Electrical current is the motion of charge.



- For area A shown in the figure, the current passing through it is defined to be the rate at which charge passes through A:

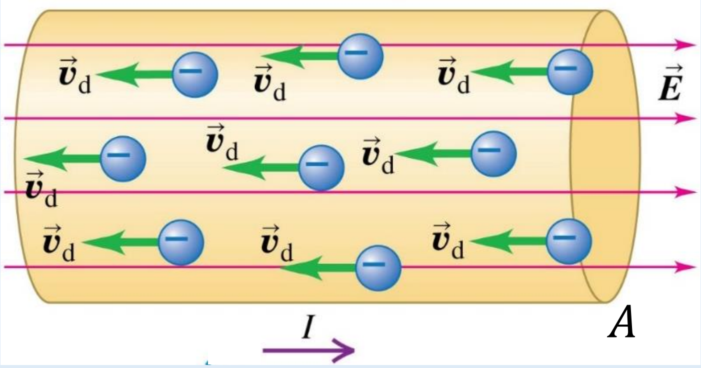
$$\boxed{I = \frac{dQ}{dt}}$$

(Sometimes we may write as $i = \frac{dq}{dt}$. We're flexible.)

- SI unit of current is the Ampere, 1 A = 1 C/s.

Current (2)

- By convention, flow of positive current corresponds to the direction of motion of positive charges (opposite to the direction of motion of negative charges).

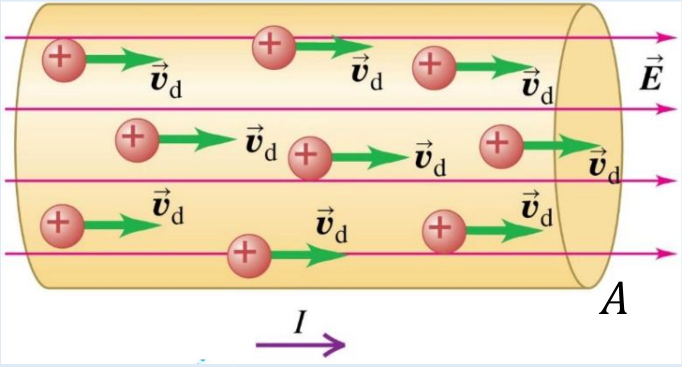


- Current is a scalar quantity.
- The amount of charge carried by the current through area A over time interval t_1 to t_2 is

$$Q = \int_{t_1}^{t_2} I \, dt \, .$$

Current in Conductors

- If an external agent (such as a battery) produces an electric field within a conductor – by creating a potential difference across the conductor – a current flows inside the conductor.



- Current will flow in the direction of the applied electric field, from high potential to low potential.

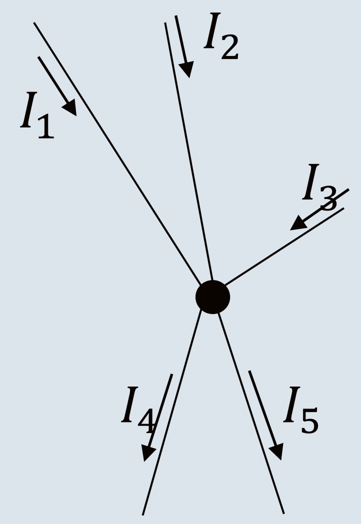
Kirchoff's Junction Rule

- Charge is a conserved quantity: net charge is neither created nor destroyed. Kirchoff (1824 – 1887) expressed continuity of charge within conductors and electrical circuits with the junction rule: for cases when the charge density is not changing in time, the sum of the currents into any point inside a conductor is zero.
- For a junction (i.e., a point where conductors are joined), this is expressed as

$$\boxed{\sum_1^N I_i = 0},$$

where N is the number of conductors meeting at the junction.

- Rule basically says: in a steady flow, whatever net current flows into a junction must also flow out.



$$I_1 + I_2 + I_3 = I_4 + I_5$$

Question 6.1

- Current flows in the directions of the arrows shown in the figure. The current I_3 is

A. 0 A.

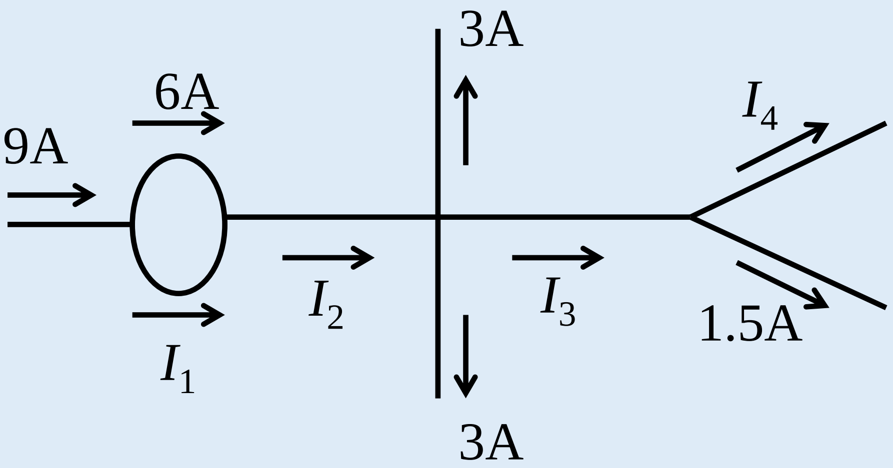
B. 1 A.

C. 2 A.

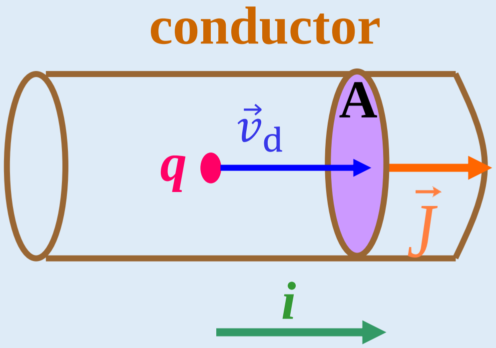
D. 3 A.

E. 4 A.

F. Cannot be determined.



Current Density



- Define current density vector $\vec{\boldsymbol{J}}$ via relation:

$$\boxed{I = \int \vec{\boldsymbol{J}} \cdot d\vec{\boldsymbol{A}}}\,,$$

  $d\vec{\boldsymbol{A}}$ = element of cross-sectional area charge flows through.

- Current is the <u>flux</u> of the current density. SI units of current density are: $\mathrm{A\,m^{-2}} = \mathrm{C\,m^{-2}\,s^{-1}}$.

- For uniform flows and areas,

$$I = \int J\,dA\cos\theta = J\,A\,\cos\theta\,,$$ where θ is the angle between $\vec{\boldsymbol{J}}$ and $d\vec{\boldsymbol{A}}$.

- For $\theta = 0°$ (area vector and current density aligned),

$$I = JA \quad \Rightarrow J = \frac{I}{A}\,.$$

Current Density (2)

- From physical considerations, and/or dimensional analysis, can show

$$\boxed{\vec{\boldsymbol{J}} = qn\vec{\boldsymbol{v}}_{\mathrm{d}}},$$

q = charge (including sign) per particle,

n = number of charges/m^3 (also called 'concentration')

$\vec{\boldsymbol{v}}_{\mathrm{d}}$ = 'drift velocity' of charges.

- Drift velocity $\vec{\boldsymbol{v}}_{\mathrm{d}}$ is the mean velocity of charges being driven by an externally-imposed electric field $\vec{\boldsymbol{E}}$ within a conductor.
  - Positive charges: $\vec{\boldsymbol{v}}_{\mathrm{d}}$ and $\vec{\boldsymbol{E}}$ are in same direction ($\vec{\boldsymbol{v}}_{\mathrm{d}} \propto +\vec{\boldsymbol{E}}$).
  - Negative charges: $\vec{\boldsymbol{v}}_{\mathrm{d}}$ and $\vec{\boldsymbol{E}}$ are in opposite directions ($\vec{\boldsymbol{v}}_{\mathrm{d}} \propto -\vec{\boldsymbol{E}}$).
- Follows that $\vec{\boldsymbol{J}}$ (and I) are in same direction, <u>independent of the sign of the moving charges</u>.
- Magnitude for uniform aligned flow: $J = \frac{I}{A} = |q|n|\vec{\boldsymbol{v}}_d|$.

Question 6.2

- Through which line in the figure is the current the <u>greatest</u>?

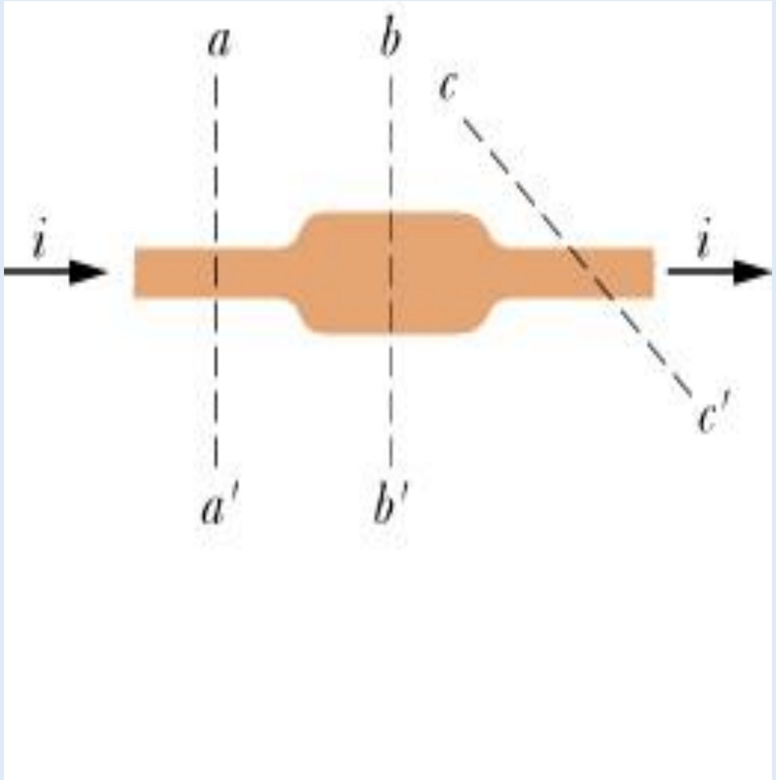
A. Line aa'.

B. Line bb'.

C. Line cc'.

D. They are all the same.

E. It's classified. You'll never make me talk! Never!!!



Question 6.3

- Through which line in the figure is the current density the smallest?

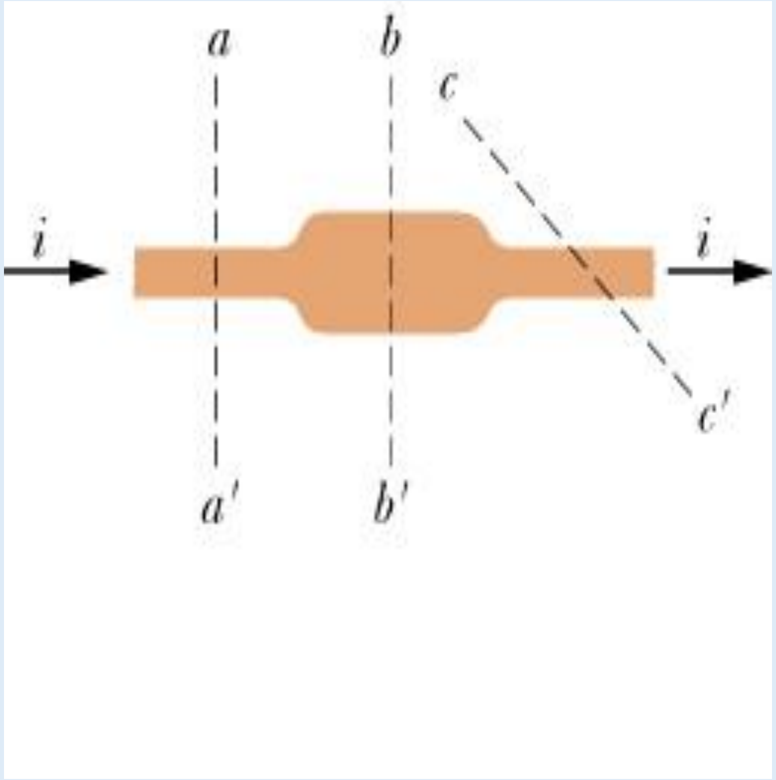
A. Line aa'.

B. Line bb'.

C. Line cc'.

D. They are all the same.

E. Didn't you hear me earlier? It's still classified. You'll never make me talk! Never!!!



Resistivity

- Electric field $\vec{\boldsymbol{E}}$ inside a conductor exerts a force $\vec{\boldsymbol{F}}_E$ on charges and accelerates them.
- Charges collide with impurities in conductor, randomly scattering charges in different directions.
- Eventual balance between $\vec{\boldsymbol{F}}_E$ and average collisional drag force $\vec{\boldsymbol{F}}_{\mathrm{coll}}$ leads to a mean drift velocity $\vec{\boldsymbol{v}}_{\mathrm{d}}$ of charged particles:
    - $\vec{\boldsymbol{F}}_E = q\vec{\boldsymbol{E}}$,
    - $\vec{\boldsymbol{F}}_{\mathrm{coll}} = -\dfrac{m}{\tau_{\mathrm{coll}}}\,\vec{\boldsymbol{v}}_{\mathrm{d}}$, m = mass of charged particle,

      τ_{coll} = mean time between collisions.

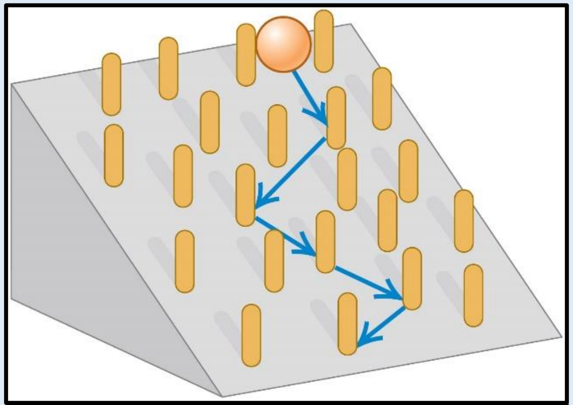
  Note: minus sign in the collisional force expression because it <u>opposes</u> the motion of a charged particle. It’s a drag (friction-like) force.

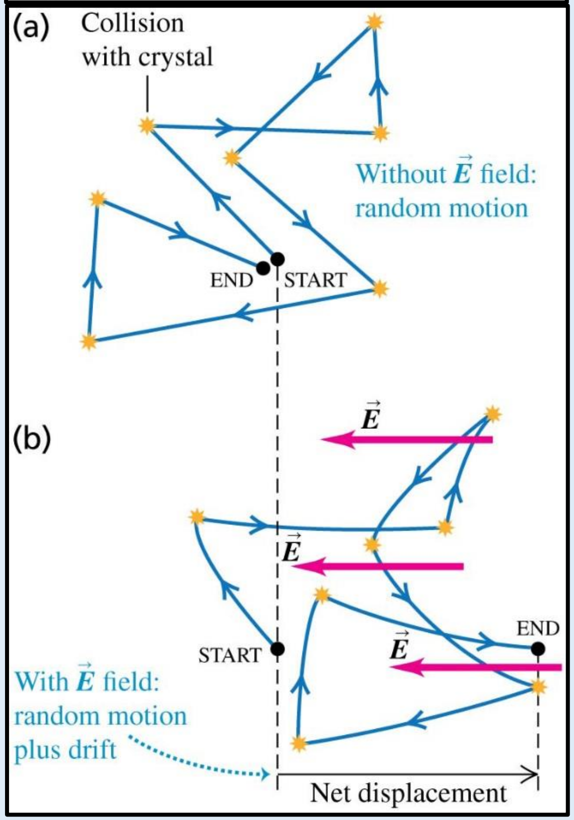
    - When forces balance (= equilibrium) on charged particle,

      $$\vec{\boldsymbol{F}}_{\mathrm{net}} = \vec{\boldsymbol{F}}_E + \vec{\boldsymbol{F}}_{\mathrm{coll}} = 0,$$

      yielding:

      $$\vec{\boldsymbol{F}}_{\mathrm{coll}} = -\vec{\boldsymbol{F}}_E \qquad \Rightarrow \qquad \vec{\boldsymbol{v}}_{\mathrm{d}} = \frac{q\tau_{\mathrm{coll}}}{m}\,\vec{\boldsymbol{E}}$$





Resistivity (2)

- Inserting drift speed expression into relation for current density gives:

$$\vec{\boldsymbol{J}} = nq\vec{\boldsymbol{v}}_{\boldsymbol{d}} = \frac{nq^2\tau_{\text{coll}}}{m}\,\vec{\boldsymbol{E}}\,.$$

Can be rewritten as

$$\boxed{\vec{\boldsymbol{E}} = \rho\vec{\boldsymbol{J}}}\,,$$

$\rho = \dfrac{m}{nq^2\tau_{\text{coll}}}$ is the resistivity of the conductor.

- Note: 'ρ' here is the symbol for resistivity, not charge density! It has a completely different meaning.
- SI unit for resistivity is Ω·m ('ohm-meter').
- A related quantity is conductivity, which is the inverse of resistivity.

Resistivity (3)

- Resistivity is an intrinsic property of a material.
- Objects can be characterized by their resistivity (or, their conductivities). There are three general categories of classification:
    - Conductors: Very small resistivities.
    - Semiconductors: Intermediate resistivities.
    - Insulators: Large resistivities.

Resistivity (4)

Conductors

Semiconductor:

Insulators

Substance	$\rho\ (\Omega \cdot \mathrm{m})$
Copper	1.72×10^{-8}
Gold	2.44×10^{-8}
Lead	22×10^{-8}
Pure carbon (graphite)	3.5×10^{-5}
Glass	$10^{10} - 10^{14}$
Teflon	$>10^{13}$
Wood	$10^{8} - 10^{11}$

Resistance

- Conductor with uniform electric field and resistivity: magnitude of potential difference along a distance L is

$$|\Delta V| = \left|-\int_0^L \vec{\boldsymbol{E}} \cdot d\vec{\boldsymbol{s}}\right| = EL\ .$$

- Inserting relation between current density, resistivity, and electric field gives (taking just magnitudes)

$$\boxed{\Delta V = (\rho J)L = \rho\left(\frac{I}{A}\right)L = I\left(\frac{\rho L}{A}\right) = IR}\ .$$

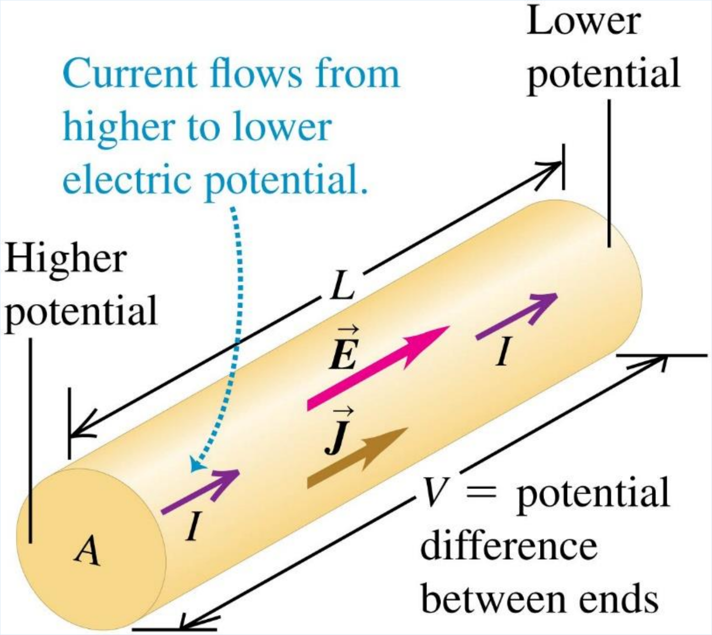
- Quantity

$$\boxed{R \equiv \frac{\rho L}{A}}$$

is the <u>resistance</u> of region of size L,

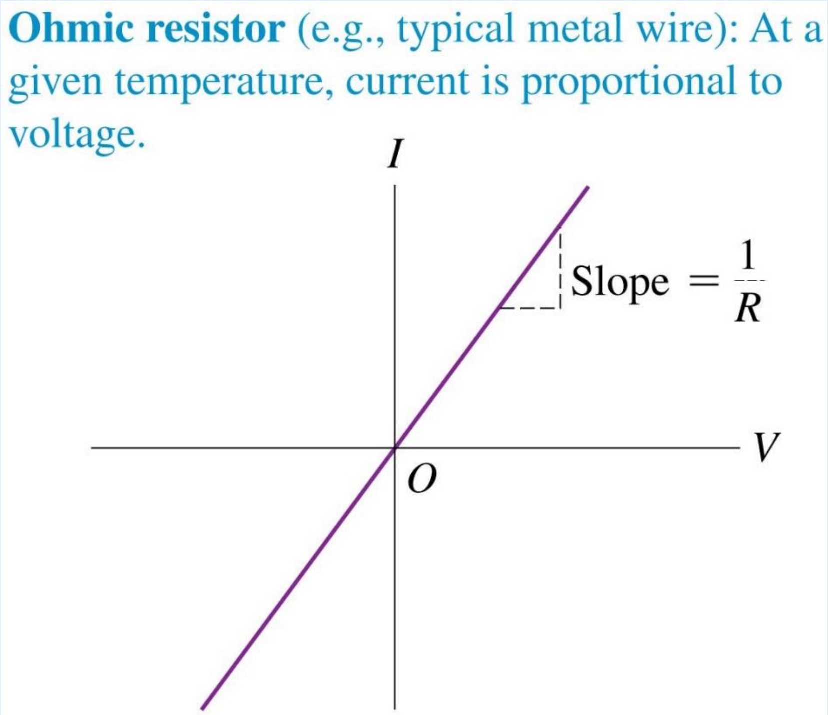
cross-sectional area A, and resistivity ρ.

- SI unit for resistance is the ohm (Ω), where $1\ \Omega = 1\ \mathrm{V/A}$.



Resistance (2)

- $\Delta V = IR$ is a general definition for resistance.
- Sometimes it is referred to as "Ohm's law", but that is not correct. Only proper to refer to it as Ohm's law when the resistance R is constant, independent of the potential difference ΔV and current I.
  - In those cases, the resistor is referred to as being "ohmic".
- Many devices are non-ohmic: e.g., light bulb filaments, transistors, diodes, etc.



Question 6.4

- A wire 100 m long and 1 mm in diameter has resistance of 1000 Ω. What is the resistance of a wire of the same material if we double the length, keeping the diameter the same?

A. 40,000 Ω

B. 2000 Ω

C. 1000 Ω

D. 250 Ω

E. Not enough information. Can't be found.

Resistance and Resistivity

- Although related, remember they are not the same thing.
- Resistivity is an intrinsic property of a material.
- Resistance of a device depends on its shape, as well as the resistivity of the material from which it is made. Two different devices may be made of the same material but vary significantly in resistance because of the way they are shaped.

Power Dissipation

- Incremental amount of energy added to a system by an infinitesimal charge dq moved through potential difference V is:

$$dU_E = dq\,V = I\,dt\,V.$$

- Rate that energy is added to a system:

$$\frac{dU_E}{dt} = IV$$

- Source of energy change is the battery (or other source) supplying the power (= time rate of energy change) driving current in the conductor.

- Equating source power to rate of energy change of current traveling through a potential difference gives the rate that power is delivered to a device:

$$P = IV\ .$$

Power Dissipation (2)

- If power is being delivered to a device with resistance R, current I, and potential difference V, the power dissipated in the resistor is

$$\boxed{P = IV = I(IR) = I^2R},$$

or, equivalently,

$$\boxed{P = IV = \left(\frac{V}{R}\right)V = \frac{V^2}{R}}.$$

- SI unit of power is the watt, 1 W = 1 J/s.

Question 6.5

- A current of 2 A flows through a resistance of 5 ohms. What is the power dissipated in the resistor?

A. 2 W.
B. 5 W.
C. 10 W.
D. 20 W.

Resistivity: Temperature Dependence

- Resistivity of a material can be affected by temperature; we can use the expression found earlier to estimate how resistivity changes with temperature:

$$\rho = \frac{m}{nq^2\tau_{\text{coll}}}$$

- For conductors:
  - ➢ Number density n of charge carriers (electrons in the "conduction band") does not change much with changing temperature.
  - ➢ However, for increasing temperature, impurities (which cause scattering) in the conductor fluctuate more rapidly and with a greater extent, increasing the collision cross-section and the probability that collisions occur. ∴ Collisions become more frequent, and the time between collisions, τ_{coll}, decreases. This increases ρ.
  - ➢ Temperature dependence for conductors found to behave as

$$\boxed{\rho(T) = \rho_0 + \rho_0\alpha(T - T_0)},$$

where T is temperature in degrees Celsius (°C), α is the temperature coefficient of resistivity, and ρ_0 is a reference resistivity at a reference temperature T_0.

Resistivity: Temperature Dependence (2)

- Semiconductors: increasing temperature can dramatically (exponentially) increase the number of charge carriers n, without much change in the mean collision time with impurities, τ_{coll}.

- Leads to a significant decrease in the resistivity of semiconductors. Temperature dependence of resistivity for semiconductors behaves as

$$\boxed{\rho(T) = \rho_0 \, e^{\left(\frac{E_a}{kT}\right)}},$$

where temperature T is in kelvins (K), ρ_0 is a reference resistivity, k is "Boltzmann's constant" ($= 1.38 \times 10^{-23}\,\mathrm{J/K}$), and E_a is an "activation energy".