Notice

- There are no classes on Monday, Feb. 19 (campus holiday).
- There are no PHYS 1200 classes Tuesday, Feb. 20. The campus is on a Monday schedule, but PHYS 1200 classes are cancelled that day.

Physics 1200
Lecture 11
Spring 2024

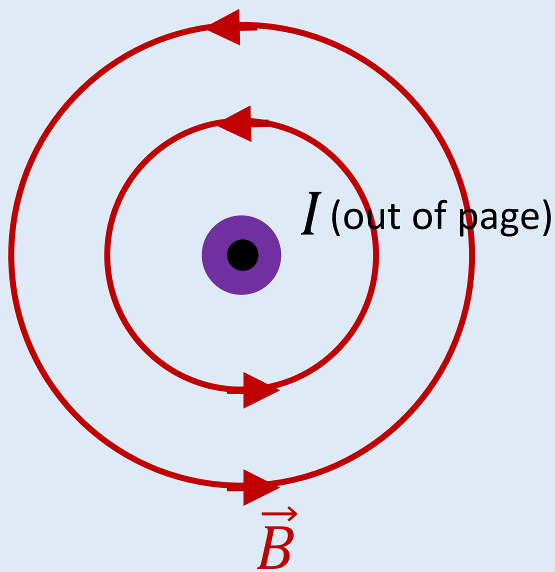
Sources of Magnetic Fields, Law of Biot-Savart, Magnetic Fields of Current-Carrying Wires & Loops, Magnetic Forces Between Conductors

Moving Charges & Magnetic Fields: Oersted's Discovery

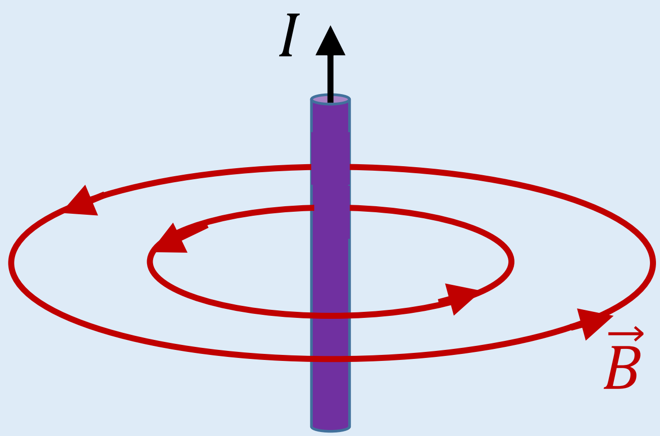
- Giving a lecture demonstration on electricity in 1820, Oersted (1777 – 1851) noticed that a nearby compass needle was deflected by an electrical current passing through a wire.
- Follow-up experiments by him confirmed that there was a connection between electrical currents and magnetic fields.
- We now know that electrical currents (= moving charges) create magnetic fields. They are a source of magnetic field.

Demonstration - Observation

- Experimental observation: a straight, current carrying wire creates a circular magnetic field about the wire.



Top View



Perspective View

Magnetic Field Generation by a Moving Charge

- Can quantify the relation between motion of a charge and the magnetic field that it is observed to create.
- Need to account for the following experimental observations:
  - Field is proportional to velocity of the charge.
  - Field is proportional to the charge.
  - Field changes direction if either sign of charge or direction of velocity is reversed. But not if both are simultaneously reversed.
  - Field is proportional to the inverse of the square of the distance from the charge to the point where the field is being measured.
  - Direction of field given by a right-hand rule with regard to the velocity, charge, and position of the field point with respect to the location of the moving charge.
    - Tells us that there must be a vector multiplication (i.e., vector cross product) relation between those three physical variables!

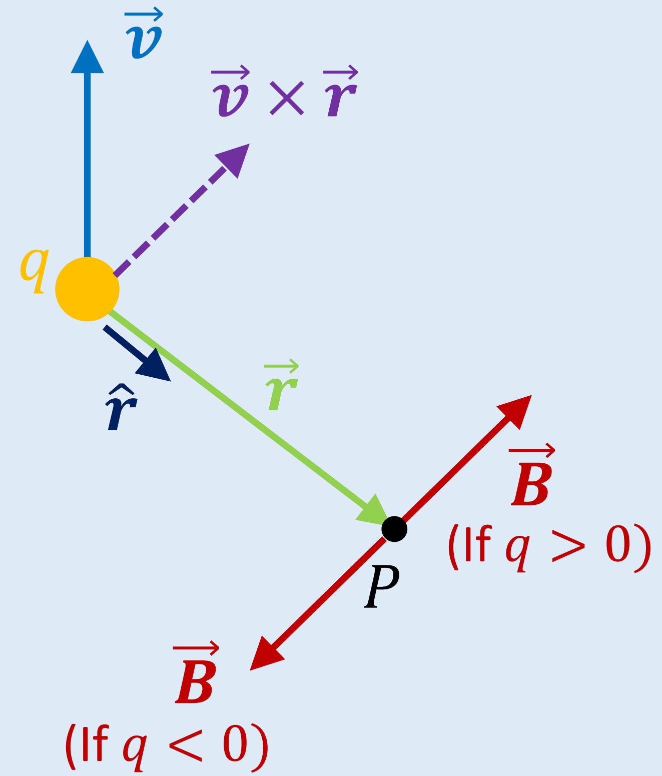
Magnetic Field Generation by a Moving Charge (2)

- Field due to uniform (constant-velocity) motion of a point charge:

$$\boxed{\vec{\boldsymbol{B}} = \frac{\mu_0}{4\pi}\frac{q\vec{\boldsymbol{v}}\times\vec{\boldsymbol{r}}}{r^3} = \frac{\mu_0}{4\pi}\frac{q\vec{\boldsymbol{v}}\times\hat{\boldsymbol{r}}}{r^2}},$$

  - $\mu_0 =$ "permeability of free space" is a physical constant (text calls it the "magnetic constant").
  - $q =$ charge of point charge (sign included),
  - $\vec{\boldsymbol{v}} =$ velocity of charge,
  - $\vec{\boldsymbol{r}} =$ position vector of field point P, relative to location of charge. Points from q to P.
  - $\hat{\boldsymbol{r}} =$ unit vector along direction of $\vec{\boldsymbol{r}}$.

Known as the Law of Biot-Savart for the magnetic field of a moving point charge.



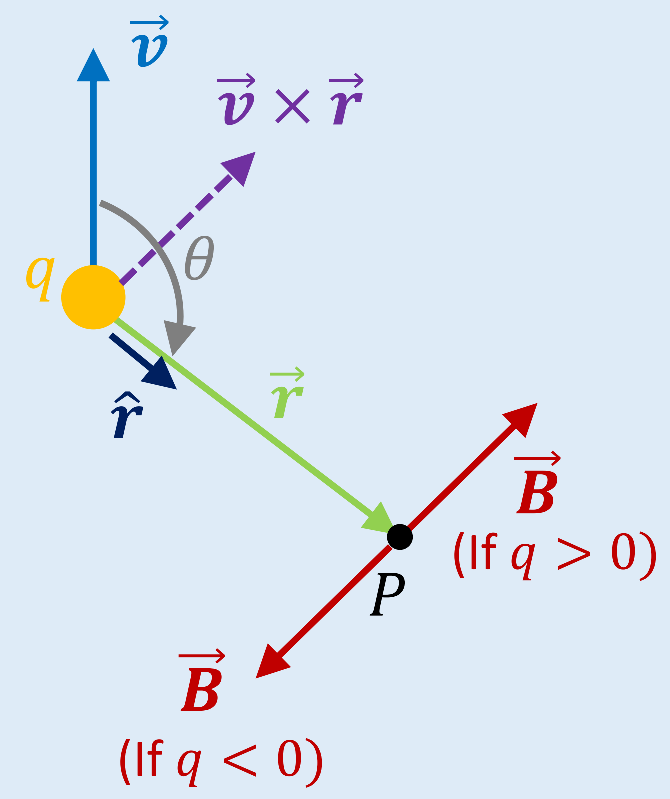
Magnetic Field Generation by a Moving Charge (3)

- From vector cross-product rule, the magnitude of the field is

$$\boxed{B = \frac{\mu_0}{4\pi} \frac{|q| v \sin\theta}{r^2}} \quad ,$$

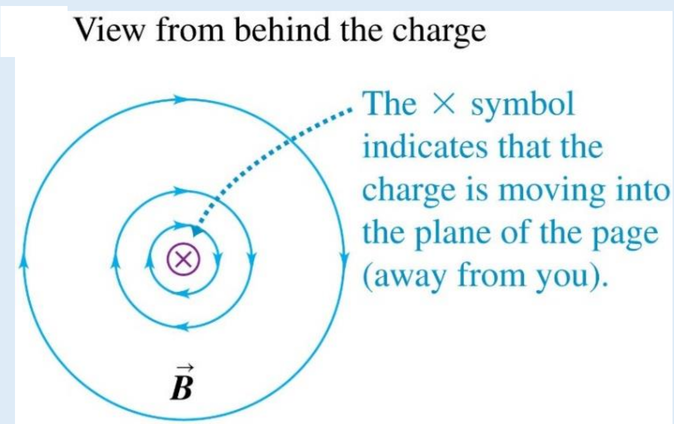
θ is the interior angle between $\vec{\boldsymbol{v}}$ and $\vec{\boldsymbol{r}}$.

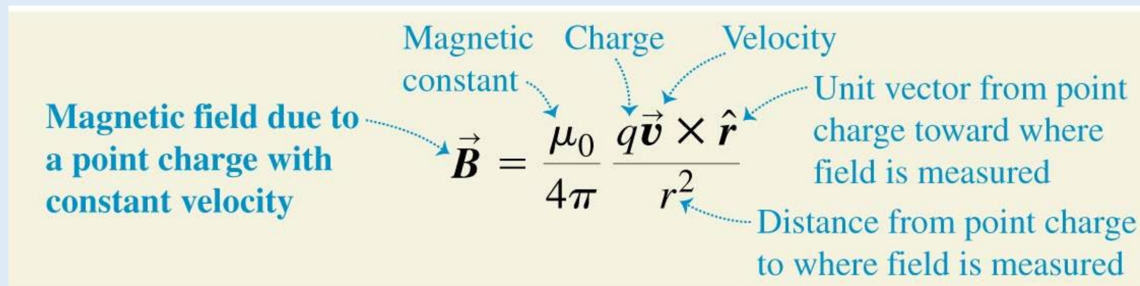
- SI value: $\mu_0 = 4\pi \times 10^{-7} \frac{\mathrm{T\,m}}{\mathrm{A}}$.



Magnetic Field Generation by a Moving Charge (4)

- Field direction from Biot-Savart law behaves in the correct way.
- Example: positive point charge q moving into page with velocity $\vec{\boldsymbol{v}}$.





- Note: a negative point charge, $-q$, moving with velocity $-\vec{\boldsymbol{v}}$ (i.e., out of page) would give the same magnetic field direction! Consistent with experiment.
  - ➢ Follows from fact that charge and velocity appear as a product ($q\vec{\boldsymbol{v}}$) in the Biot-Savart law.

Lecture Question 11.1

A positive charge moves in the upward direction, as shown in the figure. The direction of the magnetic field at the point P is

A. into the screen.

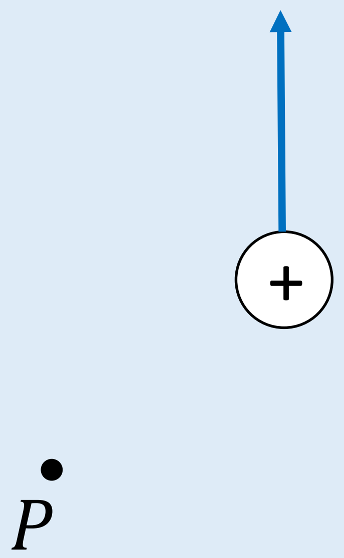
B. out of the screen.

C. up the screen.

D. down the screen.

E. to the left.

F. to the right.



Contrast: Biot-Savart Field Law and Coulomb's Law

- Biot-Savart:

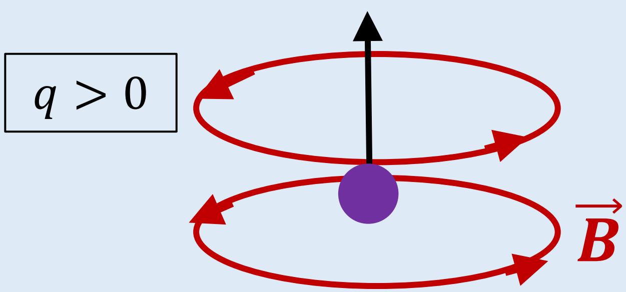
$$\vec{\boldsymbol{B}} = \frac{\mu_0}{4\pi} \frac{q\vec{\boldsymbol{v}} \times \hat{\boldsymbol{r}}}{r^2}$$

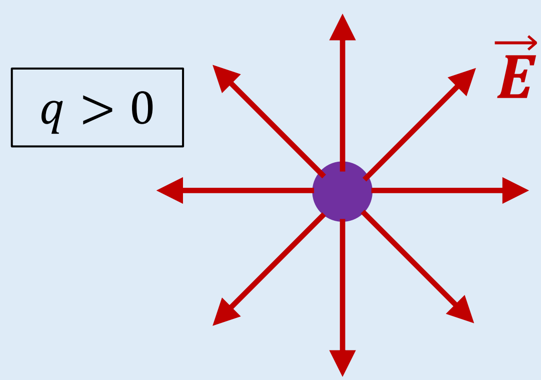
- Field depends on charge moving.
- Field perpendicular to velocity of charge and line between charge location and point of observation.
- Field lines form closed circles.

- Coulomb:

$$\vec{\boldsymbol{E}} = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2}\, \hat{\boldsymbol{r}}$$

- Field independent of charge motion. Only needs charge.
- Field along line between charge and point of observation.
- Field lines radially outward for positive charges, inward for negative charges.



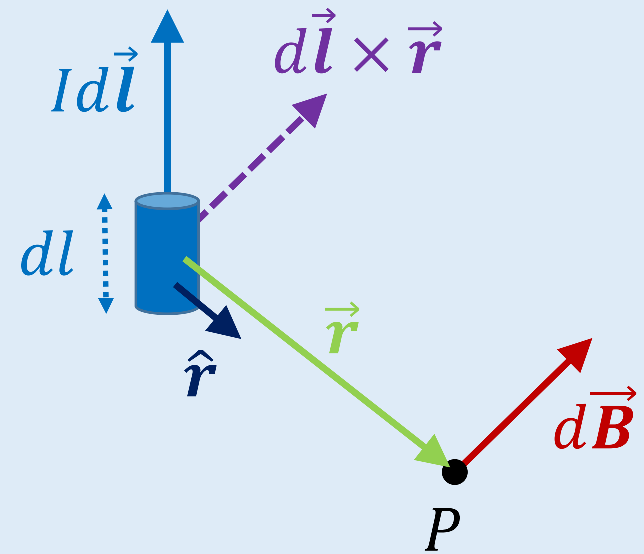


Magnetic Field Generation by a Current in a Conductor

- Historically, source-magnetic field equation first deduced by examining creation of magnetic fields due to currents in conductors (wires, etc.), following Oersted's discovery.
- Biot and Savart (1820) found that generation of a magnetic field increment ($d\vec{\boldsymbol{B}}$) at a point P due to an element of conducting wire (dl) carrying a current I can be expressed as

$$\boxed{d\vec{\boldsymbol{B}} = \frac{\mu_0}{4\pi}\frac{Id\vec{\boldsymbol{l}}\times\vec{\boldsymbol{r}}}{r^3} = \frac{\mu_0}{4\pi}\frac{Id\vec{\boldsymbol{l}}\times\hat{\boldsymbol{r}}}{r^2}}$$

(also known as the Biot-Savart law), where $d\vec{\boldsymbol{l}}$ is direction of the current and all the other symbols have the same meaning as in the point-charge form of the Biot-Savart law.



Magnetic Field Generation by a Current in a Conductor (2)

- Magnitude of the increment of field is given by

$$\boxed{dB = \frac{\mu_0}{4\pi} \frac{Idl \sin\theta}{r^2}},$$

θ is the interior angle between $d\vec{\boldsymbol{l}}$ and $\vec{\boldsymbol{r}}$.

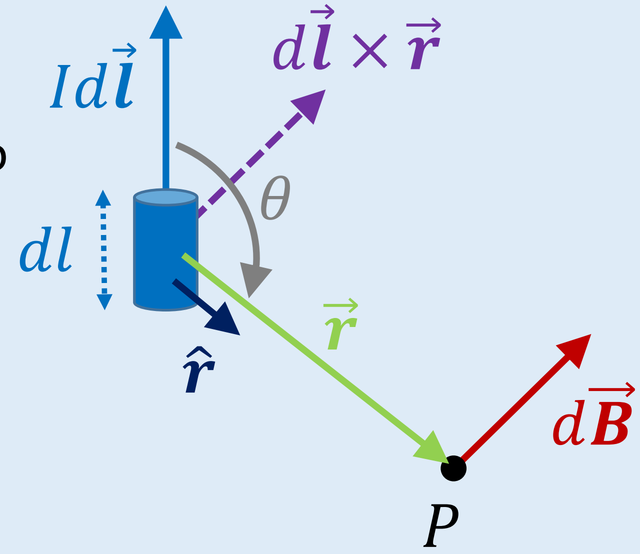
- Current-element and point-charge forms of the two laws seen as equivalent by using correspondence:

$$Idl = \frac{dq}{dt} dl = dq \frac{dl}{dt} = dq\ v$$

for charge increment dq.

- Total magnetic field from a wire obtained by integrating field contribution from each element dl:

$$\boxed{\vec{\boldsymbol{B}} = \int d\vec{\boldsymbol{B}} = \int \frac{\mu_0}{4\pi} \frac{Id\vec{\boldsymbol{l}} \times \hat{\boldsymbol{r}}}{r^2}}\ .$$



Direction of the Magnetic Field From a Current-Carrying Wire or a Moving Positive Charge: Another Right-Hand Rule (RHR)

- Point your right thumb along the direction of the current, or the direction of the velocity vector for a moving positive point charge.
- Direction that the fingers of your right hand wrap around give the direction of the magnetic field loops/lines.
- If considering motion of negative charge, point your right thumb in the <u>opposite</u> direction of the charge's velocity vector. Direction that your fingers wrap around will be the direction of the magnetic field lines.
- RHR short-cut verified by using Biot-Savart laws and normal RHRs for vector cross-products.

Example:
Magnetic Field For a Straight Current-Carrying Wire

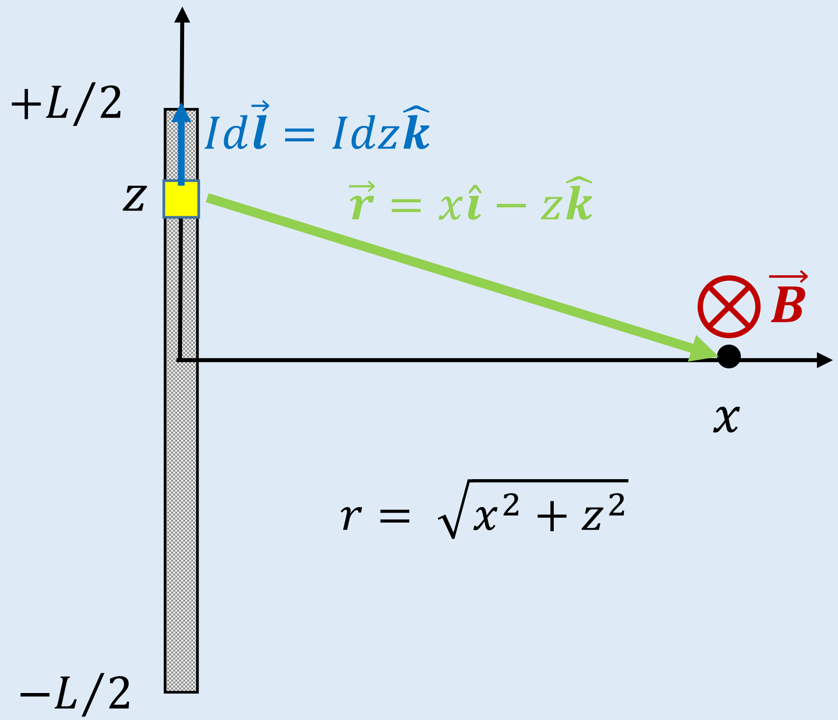
- Wire of length L with upward current I as shown. Find the total field at the point P a distance x away along the line bisecting the wire. Take the wire to be along the z-direction.

- From figure,

$$d\vec{\boldsymbol{B}} = \frac{\mu_0}{4\pi}\frac{Id\vec{\boldsymbol{l}}\times\vec{\boldsymbol{r}}}{r^3}$$

$$= \frac{\mu_0}{4\pi}\frac{Idz\widehat{\boldsymbol{k}}\times\left(x\hat{\boldsymbol{\imath}}-z\widehat{\boldsymbol{k}}\right)}{(x^2+z^2)^{3/2}}$$

$$\Rightarrow d\vec{\boldsymbol{B}} = \frac{\mu_0}{4\pi}\frac{Ix\,dz}{(x^2+z^2)^{3/2}}\hat{\boldsymbol{\jmath}}$$



Example:
Magnetic Field For a Straight Current-Carrying Wire (2)

- Integrate along wire length to get total field at P :

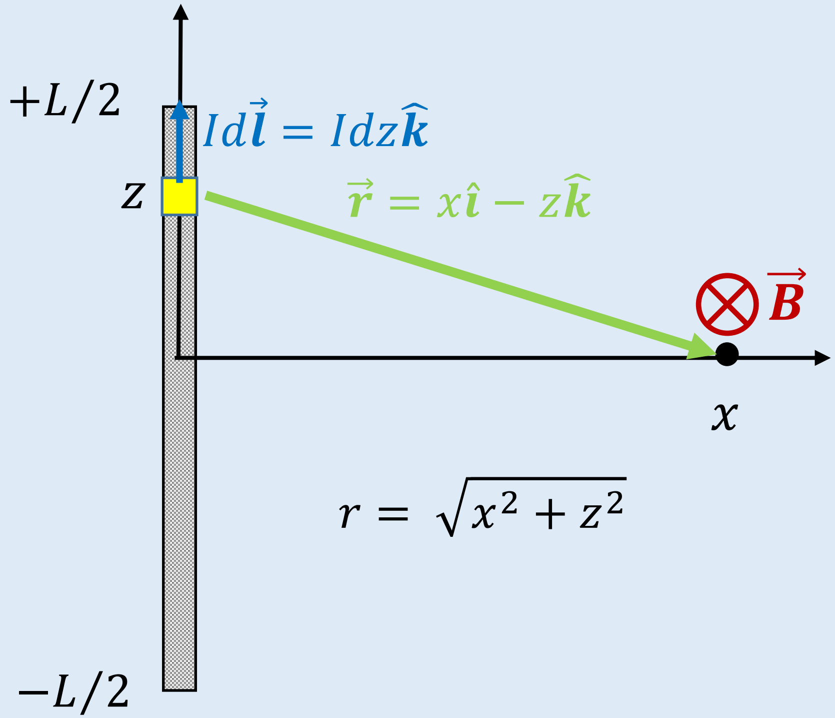
$$\vec{\boldsymbol{B}} = \int d\vec{\boldsymbol{B}}$$
$$= \int_{-L/2}^{L/2} \frac{\mu_0}{4\pi} Ix \frac{dz}{(x^2+z^2)^{3/2}} \hat{\boldsymbol{j}}$$

$$\Rightarrow \boxed{\vec{\boldsymbol{B}} = \frac{\mu_0 I}{4\pi} \frac{L}{x\sqrt{x^2+(L/2)^2}} \hat{\boldsymbol{j}}}.$$

- In the limit $L \to \infty$ (i.e., a very long wire), magnetic field magnitude becomes:

$$\boxed{B = \frac{\mu_0 I}{2\pi r}},$$

where we set $x = r =$ radial distance from the wire.



Magnetic Fields from Computer Cables & Other Devices

- Computer cables, cables for audio-video equipment, and other every-day devices create little or no magnetic field.

- This is because within each cable, closely spaced wires carry current in both directions along the length of the cable.

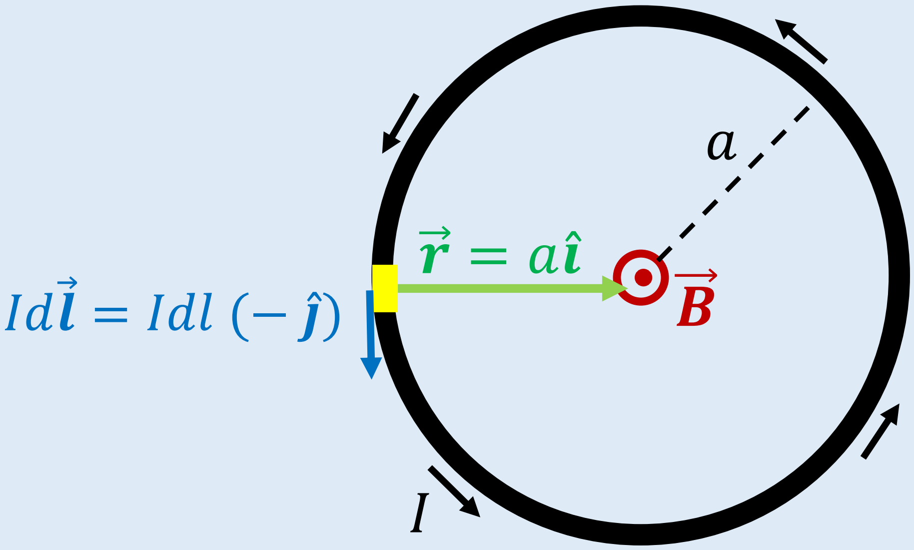
- Result of close spacing of wires: magnetic fields from the opposing currents <u>cancel</u> each other.

Example:
Magnetic Field at the Center of a Current-Carrying Wire Loop

- Want field at center of circular current-carrying wire of radius a.
- Right-hand rule and Biot-Savart reveal the magnetic field at the center is directed outward from the page (along axis of symmetry) which we take to be the z-axis.
- Example: current element shown in figure yields

$$d\vec{\boldsymbol{B}} = \frac{\mu_0}{4\pi}\frac{Id\vec{\boldsymbol{l}}\times\vec{\boldsymbol{r}}}{r^3} = \frac{\mu_0}{4\pi}\frac{(-Idl\hat{\boldsymbol{j}}\times a\hat{\boldsymbol{i}})}{a^3}$$

$$\Rightarrow d\vec{\boldsymbol{B}} = \frac{\mu_0 Idl}{4\,\pi a^2}\hat{\boldsymbol{k}}$$



Example:
Magnetic Field at the Center of a Current-Carrying Wire Loop (2)

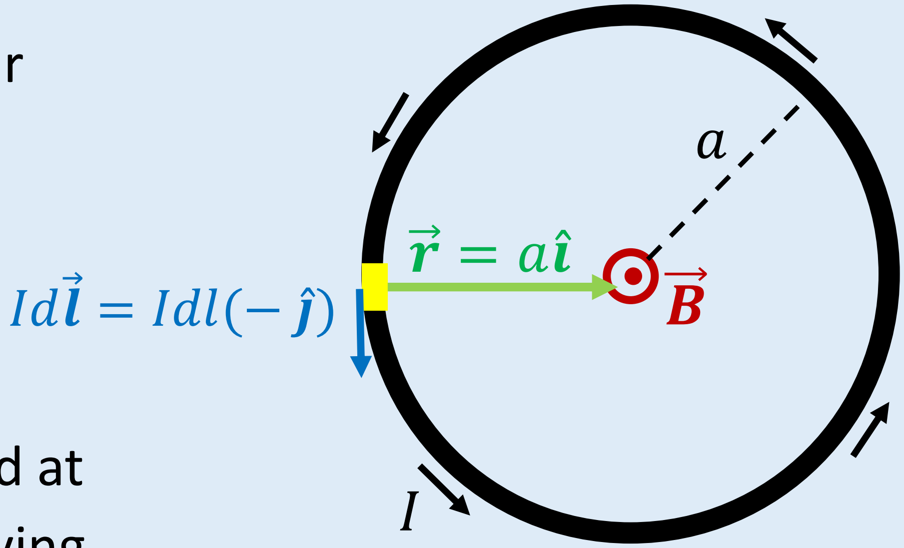
- The other elements of the loop give the same result.
- Integrating around the entire circular wire for the total magnetic field:

$$\vec{\boldsymbol{B}} = \int d\vec{\boldsymbol{B}} = \int_0^{2\pi a} \frac{\mu_0 I}{4\pi a^2}\, dl\, \widehat{\boldsymbol{k}}$$

$$= \frac{\mu_0 I}{4\pi a^2}\,(2\pi a)\widehat{\boldsymbol{k}}\,.$$

∴ The magnitude of the magnetic field at the center of a circular current-carrying wire is

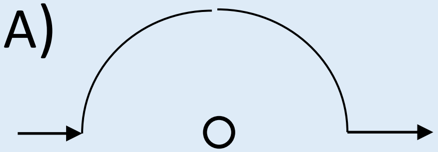
$$\boxed{B = \frac{\mu_0 I}{2a}}\,.$$



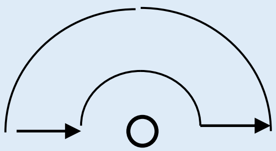
Lecture Question 11.2

- The same current flows through each of the wires sketched below. For which case is the magnetic field at point O the largest? (All segments are circular or straight. Radii are *d* or *d/2*. When wires point off along straight lines, they continue to infinite distance.)

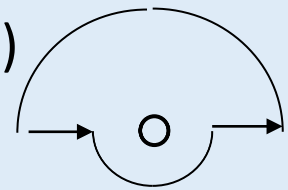
A)



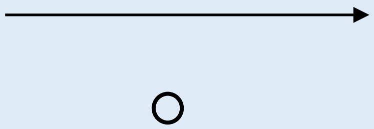
B)



C)

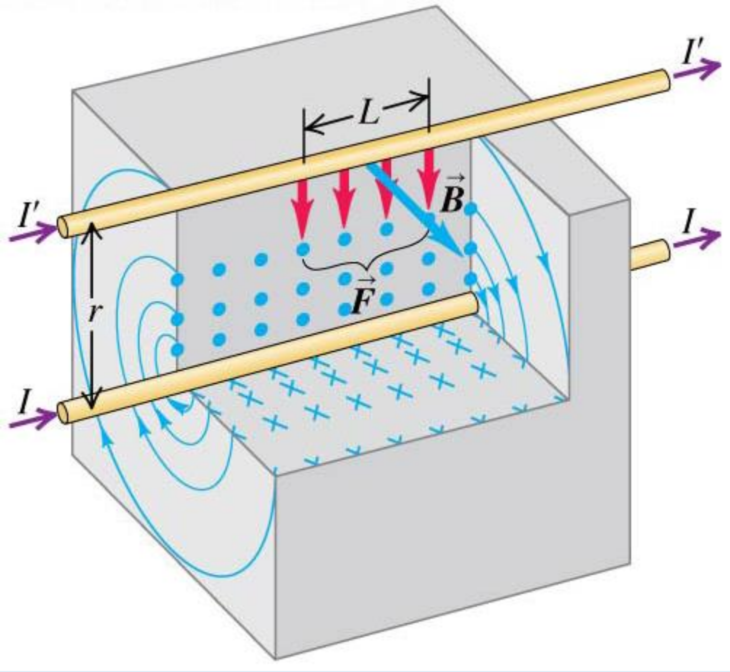


D)



Magnetic Forces
Between Current-Carrying Conductors

- Current-carrying conductors (e.g., wires) act as sources of magnetic field. They can also experience magnetic forces due to the fields from *other* current-carrying conductors.
- The magnetic field of the lower wire exerts an *attractive* force on the upper wire.
- If the wires had currents in *opposite* directions, they would *repel* each other. Rule for magnetic forces between current-carrying conductors: "*Like currents attract, unlike currents repel*."



Magnetic Forces Between Current-Carrying Conductors (2)

- Magnitude of force on I' due to the magnetic field B_I from current I is

$$F_{II'} = I'LB_I \sin 90° \,,$$
$$= I'L\left(\frac{\mu_0 I}{2\pi r}\right).$$

  - Can show that magnitude of force on I due to field from I' is the same (consistent with Newton's 3rd law).

- Magnitude of the force per unit length on I' due to I is

$$\boxed{\frac{F_{II'}}{L} = \frac{\mu_0 I I'}{2\pi r}}\,.$$