Physics 1200
Lecture 21
Spring 2024

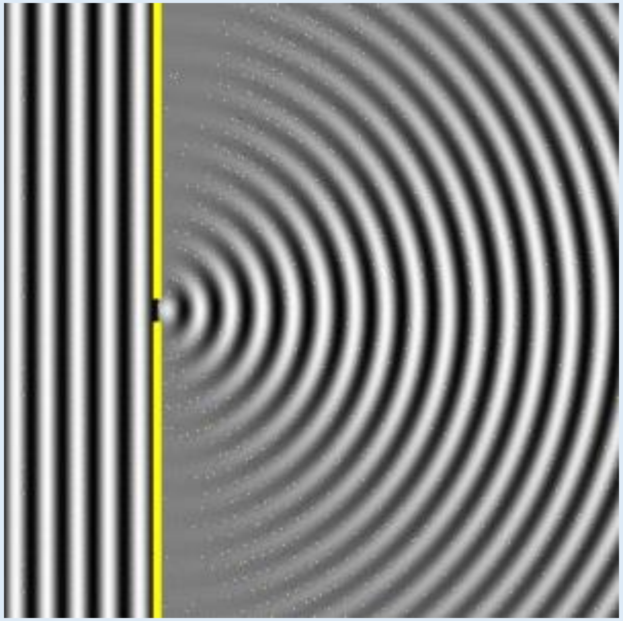
Diffraction, Single-Slit Diffraction Pattern and Intensity, Circular Apertures, Resolution, Two-Slit Interference with Diffraction

Diffraction

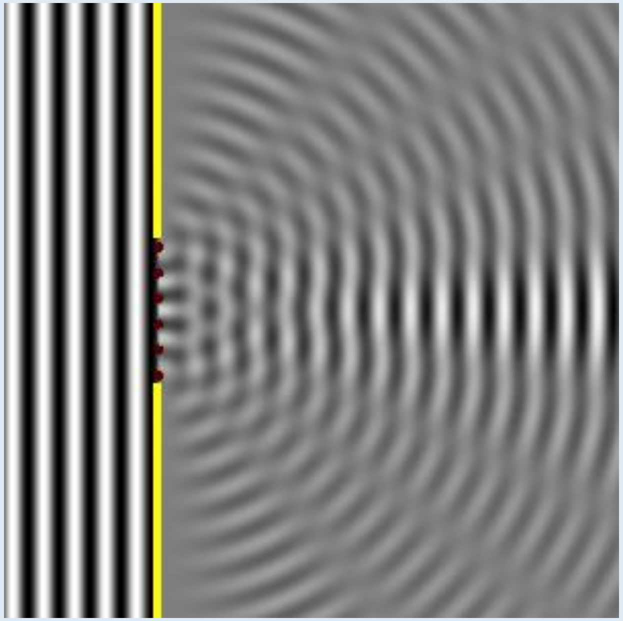
- Diffraction is a phenomenon closely related to interference.
- Until now, we've considered interference only for a finite number of sources. Examples:
    - ➢ Young's two-slit experiment.
    - ➢ Gratings.
- Implicit in earlier discussion was that the size of the slits (apertures) were infinitesimally thin (small), so that there was only one point source of Huygens wavelets in a slit (aperture).
- However, slits and apertures have <u>finite</u> widths. According to Huygens's principle, each point of a wave front acts as a source of new waves.
    - ➢ Even for a finite width, there is an infinite number of point sources within the aperture.
    - ➢ Expectation: there should be interference between wavelets created in a single slit: a slit <u>self-interference effect</u>. This effect – even for a single slit (or aperture) – creates what is referred to as a <u>diffraction</u> pattern.

Diffraction (2)

- Waves impinging on a barrier with a slit.



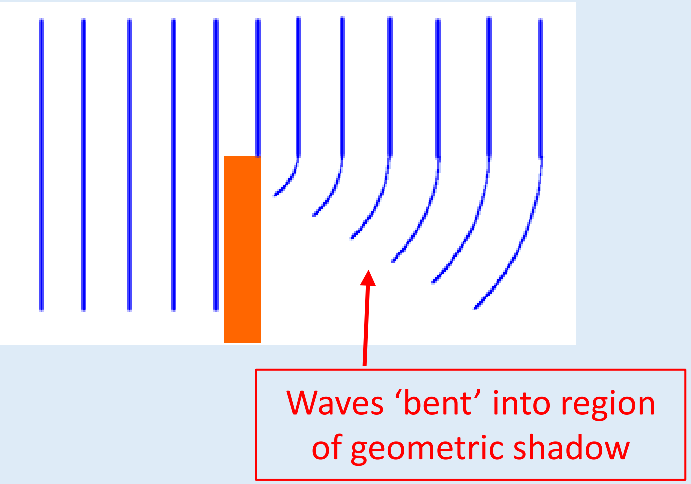
Very narrow slit. Single wavelet emerges from the slit.

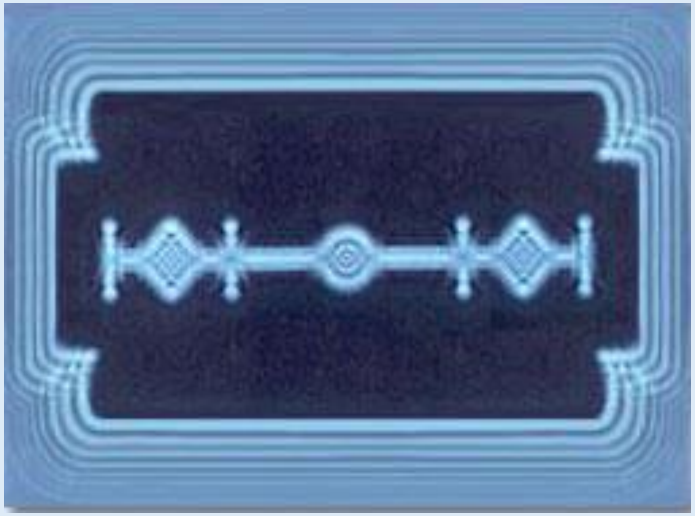


Wider slit. Multiple wavelets emerge from the slit.
Diffraction/interference pattern in the emerging waves visible.

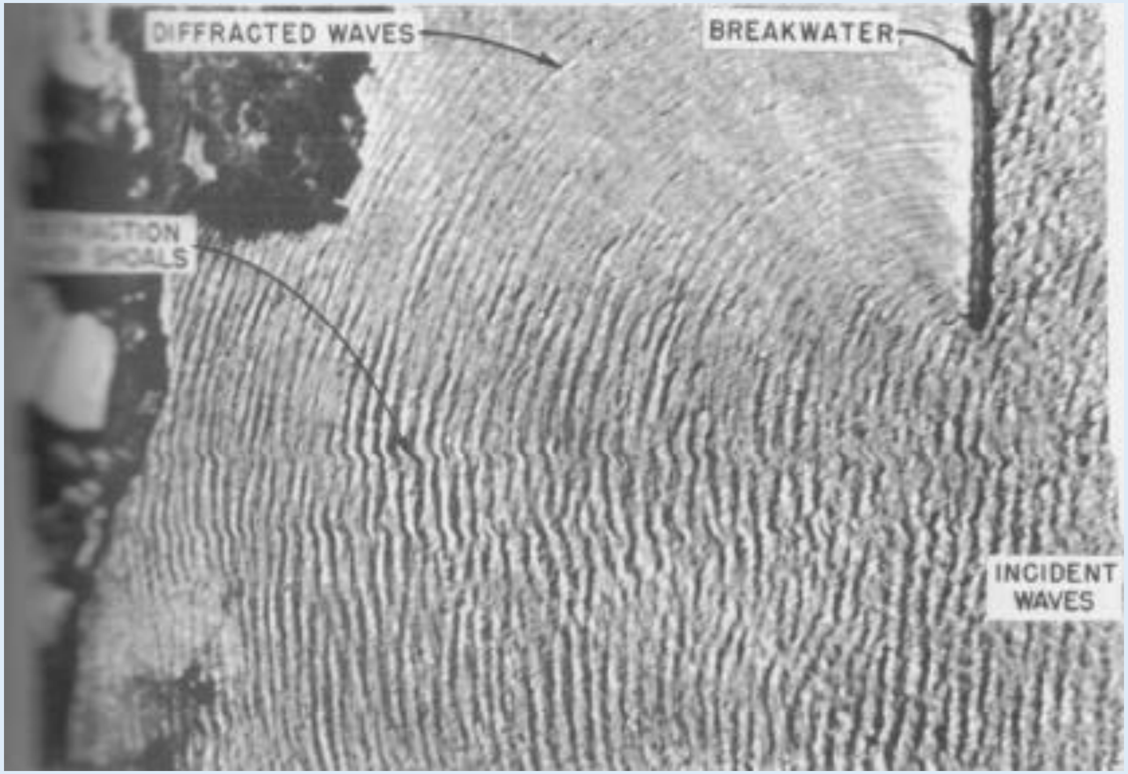
Diffraction (3)

- A ‘slit’ (or ‘aperture’) can be extremely large. For instance, the edge of a barrier of finite extent can be considered as a semi-infinite slit.
  - Huygens wavelets from unblocked portion of an impinging wave front explains ‘bending’ of a wave around a barrier’s edge. The shadow from the obstruction is reduced from that which would be predicted purely by un-deviated propagation of rays.
  - Diffraction pattern (for coherent waves) seen near edge of the obstacle.
  - Similar results for viewing coherent light diffraction on other finite-extent objects (e.g., image of shadow of a razor blade).



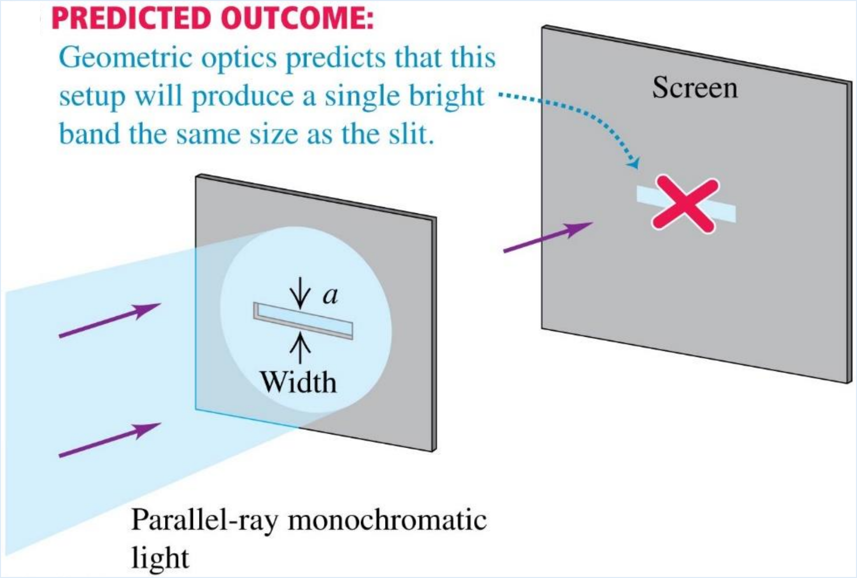


Diffraction (4)

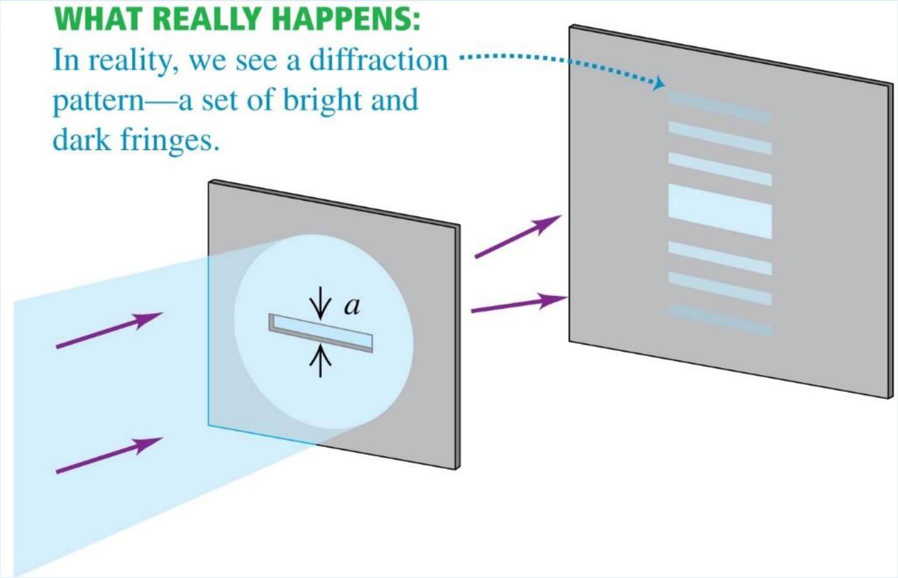


Diffraction/wave-bending in ocean waves.

Geometric Optics vs. Diffraction



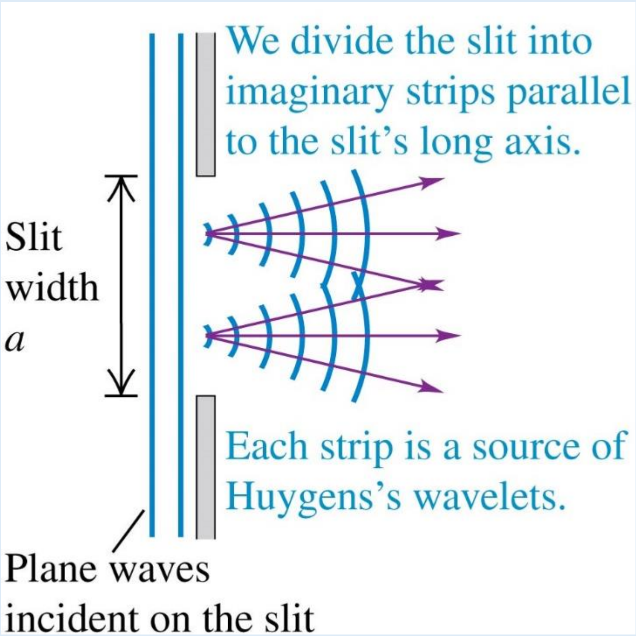
Coherent light assumed for both cases.

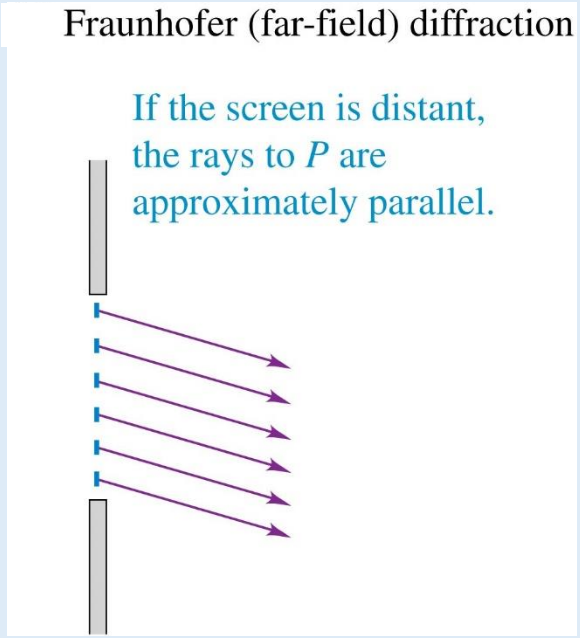


Types of Diffraction

- Diffraction intensity pattern observed from an object often broken up into two different types, categorized by distance of the source from the observation point. (Simplifying assumptions used to solve the EM wave/light propagation equations for the intensity at some location are based on the observation distance.)
  - If distance of observation is relatively close to the source, the type of diffraction is referred to as 'Fresnel', or near-field diffraction.
  - If distance between the observation point and the source is relatively large, the diffraction is referred to as 'Fraunhofer', or far-field diffraction.
- For either type, location of intensity maxima and minima follow from conditions of constructive and destructive interference studied last class.
- Because the math is less involved for Fraunhofer diffraction, we focus on that type of diffraction in this class.

Single-Slit Diffraction

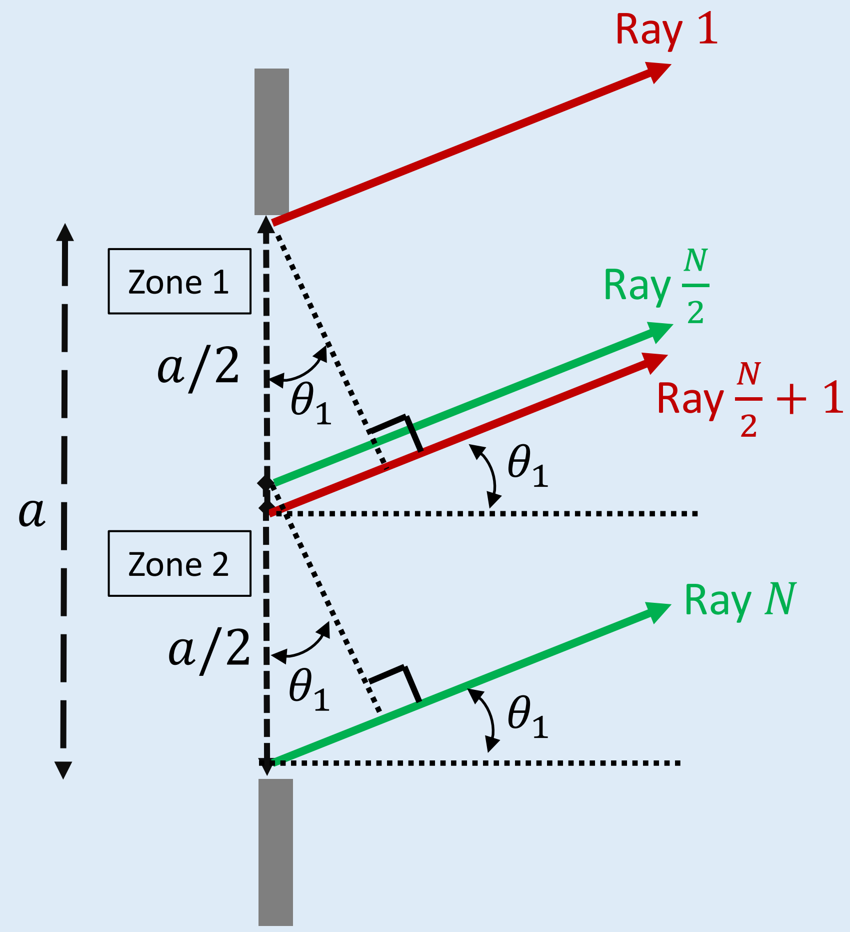




- Single-slit Fraunhofer diffraction: consider single slit of width a to be a source of a large number N wavelets.
- Adopt parallel-ray approximation again, like that was done in the interference study.
- We consider only the case for which the light is normally incident to the plane of the slit.

Single-Slit Diffraction: Location of Minima

- To locate angular position of first intensity minimum, imagine breaking up the slit into an even number of N sources.
  - N sources emit N parallel rays at angle θ with respect to the line going from the slit to the viewing screen.
  - For angle $\theta = 0$, all the rays travel the same distance to the screen and will interfere constructively. $\therefore$ The angle $\theta = 0$ is a central intensity maximum.
  - Angle of the first intensity minimum will be some angle θ_1.
  - Then divide sources into two equally-sized zones of width $a/2$. Rays 1 through $N/2$ are in the first zone, and rays $(N/2) + 1$ through N are in the second zone.



Single-Slit Diffraction: Location of Minima (2)

- Angle θ_1 of the first intensity minimum occurs when all Zone 1 rays destructively interfere with all Zone 2 rays.

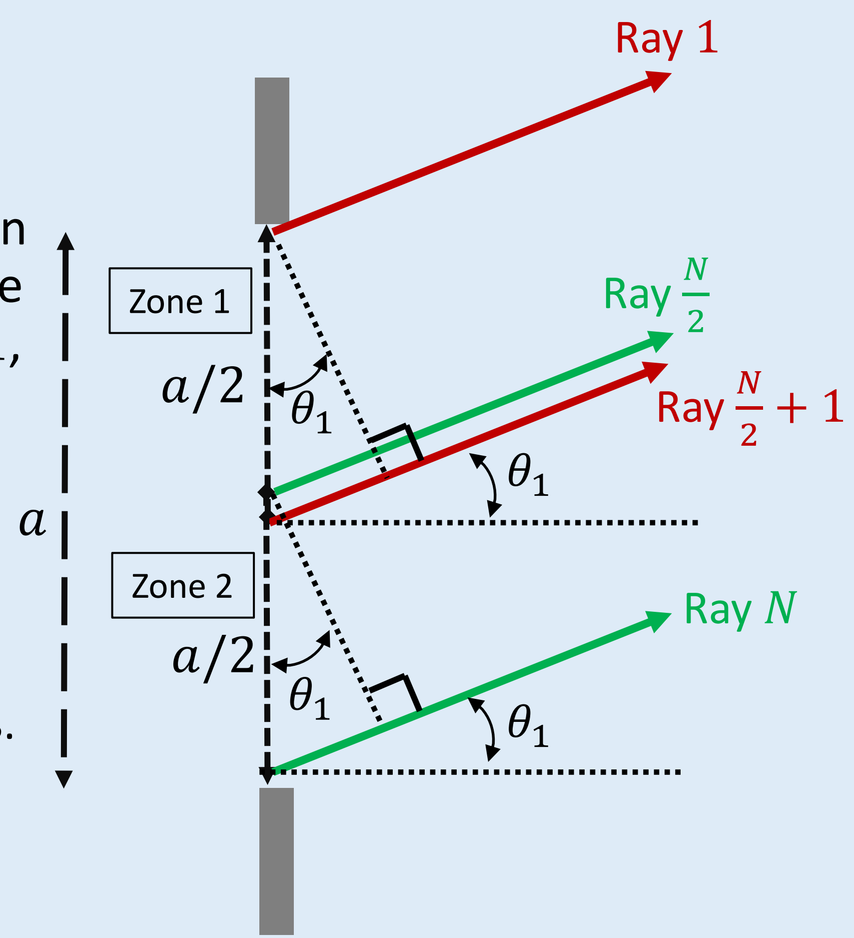
- Pairwise destructive interference of the rays occurs when path difference between a ray in Zone 1 and its counterpart in Zone 2 [e.g., ray 1 in Zone 1 and ray $(N/2)+1$, ray $N/2$ in Zone 1 and ray N in Zone 2, and all of the other rays in between] satisfy the criterion:

$$\frac{a}{2}\sin\theta_1 = \frac{\lambda}{2},$$

λ is the wavelength of the incident waves.

  - Equivalent to the condition

$$a\sin\theta_1 = \lambda\,.$$



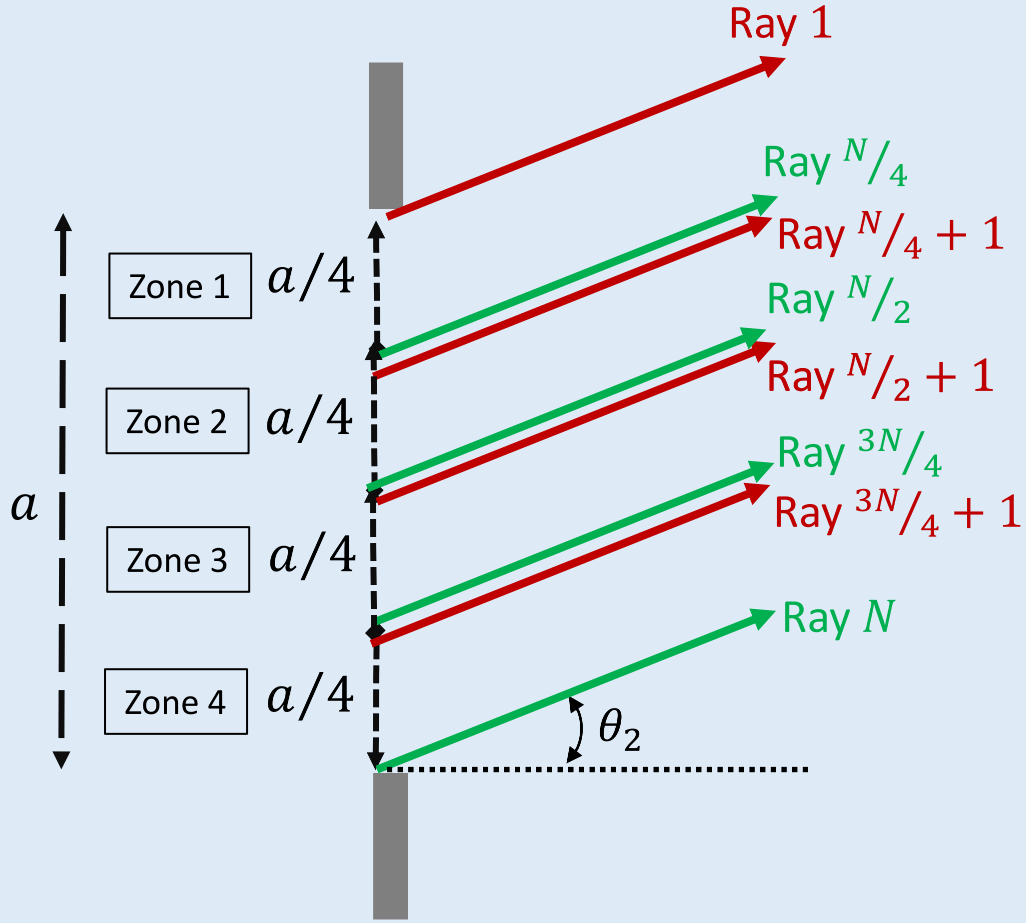
Single-Slit Diffraction: Location of Minima (3)

- Angle of second diffraction minimum θ_2 found in a way completely analogous to how first minimum θ_1 was found:
  - Imagine breaking slit up into four equal zones of width $a/4$. Pairwise destructive interference of rays in Zones 1 and 2, and those in Zones 3 and 4, occurs if angle θ_2 satisfies the condition

$$\frac{a}{4}\sin\theta_2 = \frac{\lambda}{2} \quad ,$$

or, equivalently,

$$a\sin\theta_2 = 2\lambda \, .$$



Single-Slit Diffraction: Location of Minima (4)

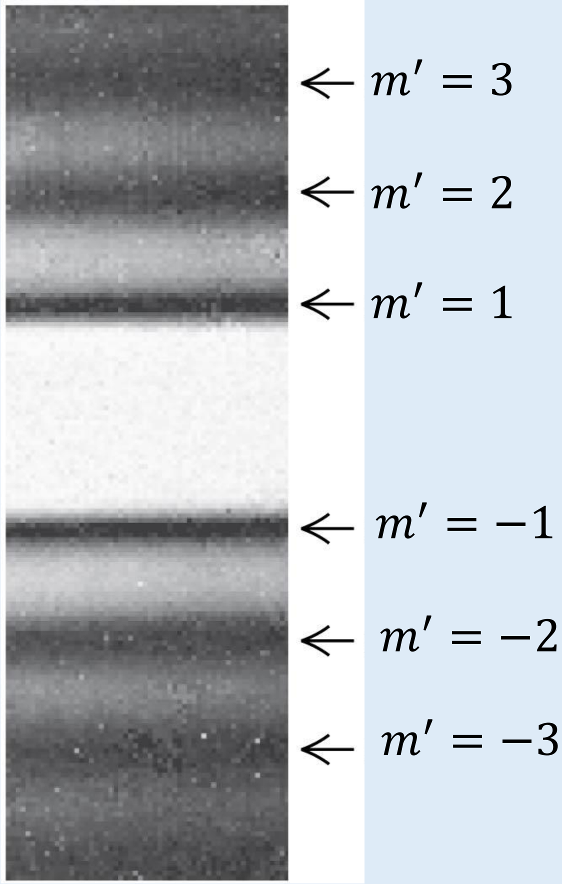
- The angle of other minima can be found in the same way, by imagining the slit divided up into 6 zones, 8 zones, etc.
- By induction, the resulting general relation one finds for the angle at which there is an intensity minimum for waves of wavelength λ diffracting from a single slit of width a is

$$\boxed{a\ \sin\theta_{m'} = m'\lambda}\ ,$$

where $m' = \pm 1, \pm 2, \pm 3, \ldots$

Single-Slit Diffraction: Diffraction Pattern

- Shown here is the intensity pattern for Fraunhofer diffraction from a single slit, as viewed on a screen a distance D away from the slit.
- It is characterized by a central bright fringe centered at $\theta = 0$ (this is just the image of the slit), surrounded by a series of dark fringes.
- The central bright fringe is twice as wide as the other bright fringes.



Single-Slit Diffraction: Intensity

- Intensity for single-slit diffraction can be obtained by adding waves from all the sources in the slit to get total resultant wave and its amplitude.

- The waves will have different phases. Can perform addition using phasors.

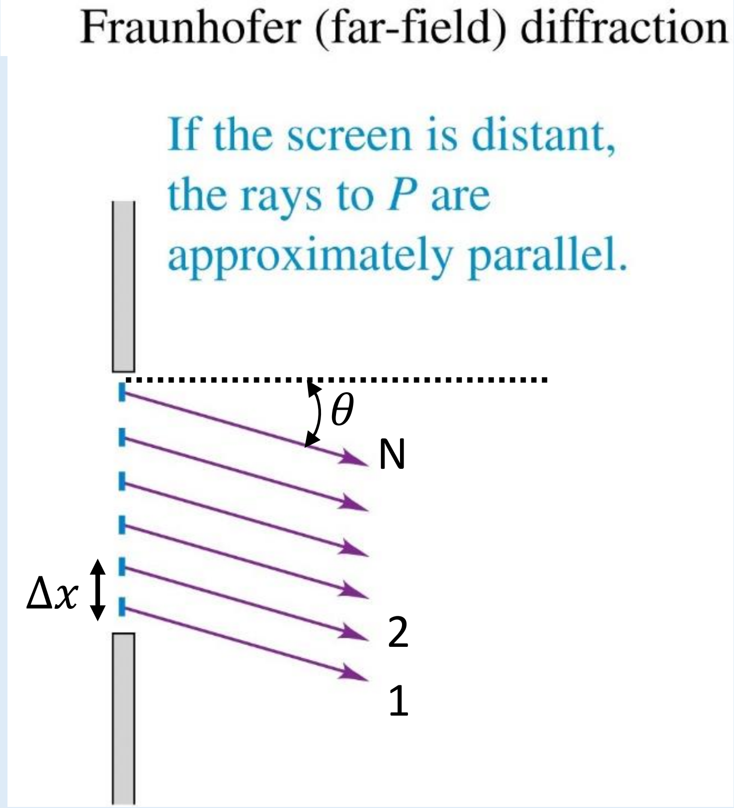
  - Assume there are N sources in a slit, creating waves $1, 2, 3, \ldots, N$, having equal spacing $\Delta x = \frac{a}{N}$ between sources.

  - From slit and parallel-ray geometry, wave 2 has phase difference with respect to wave 1,

$$\Delta\phi = k(\Delta x \sin\theta) = \frac{2\pi}{\lambda}(\Delta x \sin\theta)$$

$$\Rightarrow \boxed{\Delta\phi = \frac{2\pi a}{\lambda N}\sin\theta},$$

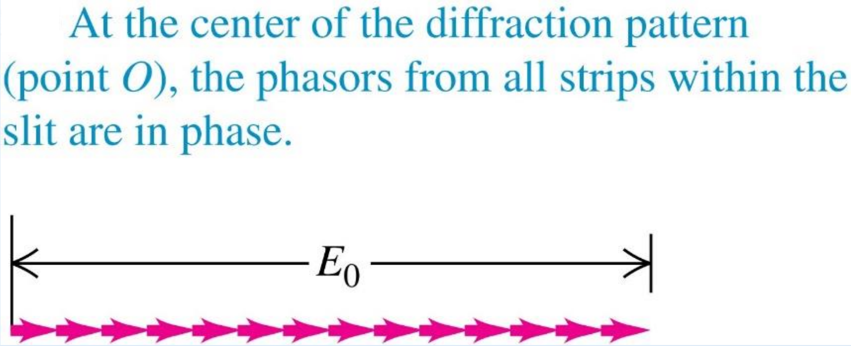
where θ is the angle each ray makes with respect to the line from the slit to the screen where the intensity pattern is observed (see the figure).



Single-Slit Diffraction: Intensity (2)

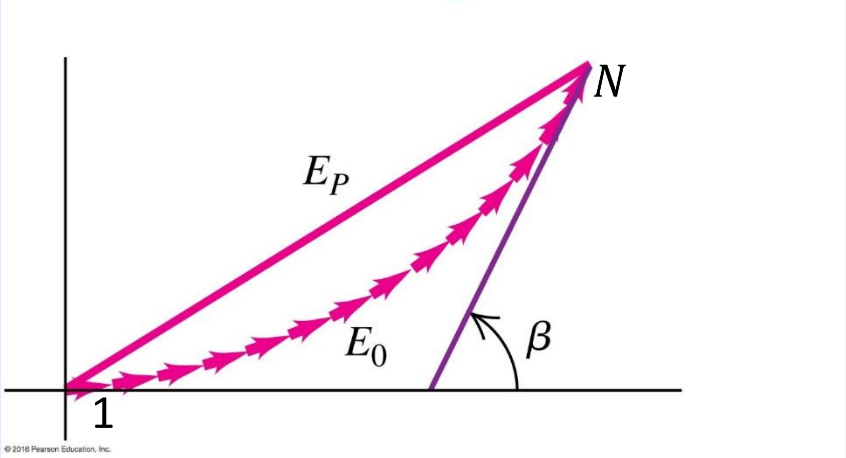
- For assumed parallel-wave geometry, phase difference between waves 2 and 3 is the same as between 1 and 2, and so on. ∴ Phase difference between all adjacent waves is $\Delta\phi$.
  - Phase difference between each phasor (representing each wave) will also be $\Delta\phi$.
- To get total wave amplitude for interference of waves 1 through N, add all phasors for waves 1 through N using standard rules of vector addition.
- Assume that the electric field amplitude of each wave (and the phasor for each wave) is some value E_0/N.
- For angle $\theta = 0$, phase difference between phasors is $\Delta\phi = \frac{2\pi a}{\lambda N}\sin(0) = 0$. That is, all are in phase: all phasors line up in the same direction:
  - Net field amplitude in the center of the diffraction pattern ($\theta = 0$) is

$$N\left(\frac{E_0}{N}\right) = E_0.$$



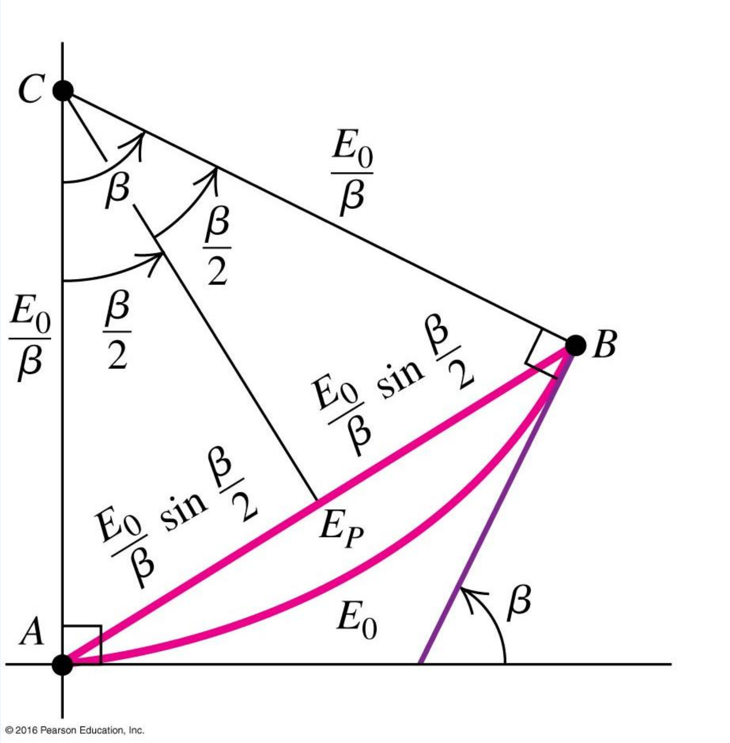
Single-Slit Diffraction: Intensity (3)

- For angles $\theta \neq 0$, the amplitude (E_P) of the net electric field at a point P on an analyzer screen is also found by adding all the phasors from the N waves generated in the slit.
  - Sum of the phasors along the arc is still
  $$N\left(\frac{E_0}{N}\right) = E_0.$$
  - Angle the last phasor (for wave N) makes with respect to the horizontal axis (which is the direction of wave 1) is
  $$\boxed{\beta = N\Delta\phi = N\left(\frac{2\pi a}{\lambda N}\sin\theta\right) = \frac{2\pi a}{\lambda}\sin\theta}\ .$$
  - Amplitude of the electric field at point P on the screen is the resultant phasor vector E_P shown in the diagram.



Single-Slit Diffraction: Intensity (4)

- Approximating resultant of vector-added phasors as a smooth, continuous arc (equivalent to taking number of phasors $N \to \infty$), the geometry of the problem becomes that shown in the figure.
  - Yields result: $\boxed{E_p = \frac{2E_0}{\beta} \sin\left(\frac{\beta}{2}\right)}$.



- Because intensity of a wave is proportional to the square of the electric field amplitude, can write single-slit intensity ($\propto E_P^2$) as

$$\boxed{I = I_0 \left[\frac{\sin\left(\frac{\beta}{2}\right)}{\left(\frac{\beta}{2}\right)} \right]^2},$$

$$\boxed{\beta = \frac{2\pi a}{\lambda} \sin\theta},$$

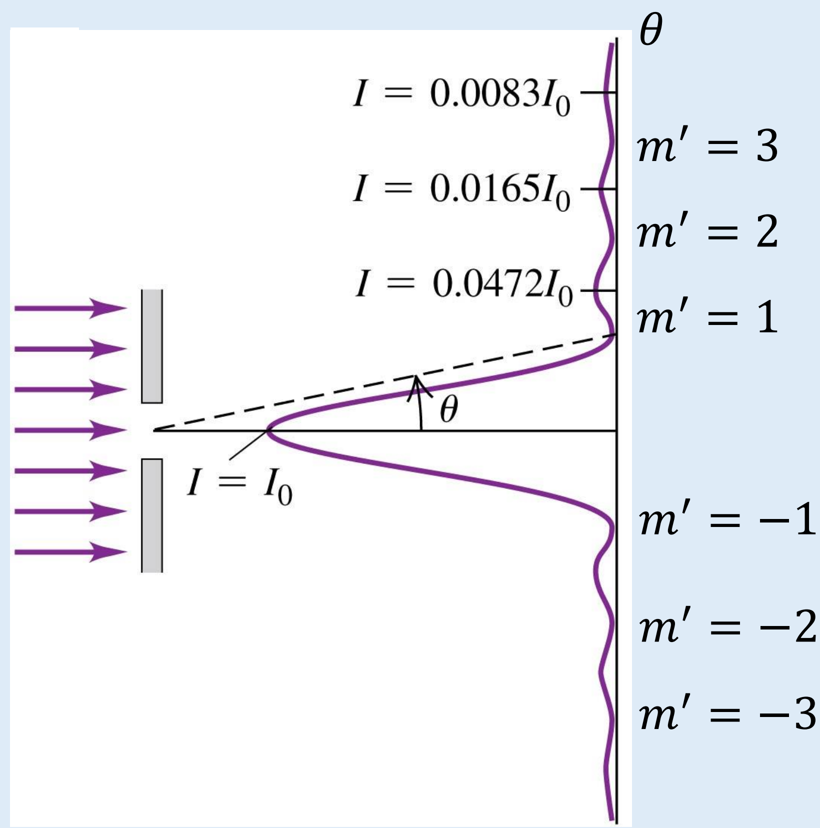
and I_0 is the intensity at the central maximum.

- Note: intensity equation yields minima for $\beta/2 = m'\pi, \quad m' = \pm 1, \pm 2, \pm 3, \ldots$

  $\Rightarrow \quad a \sin\theta = m'\lambda$. In agreement with diffraction result found earlier!

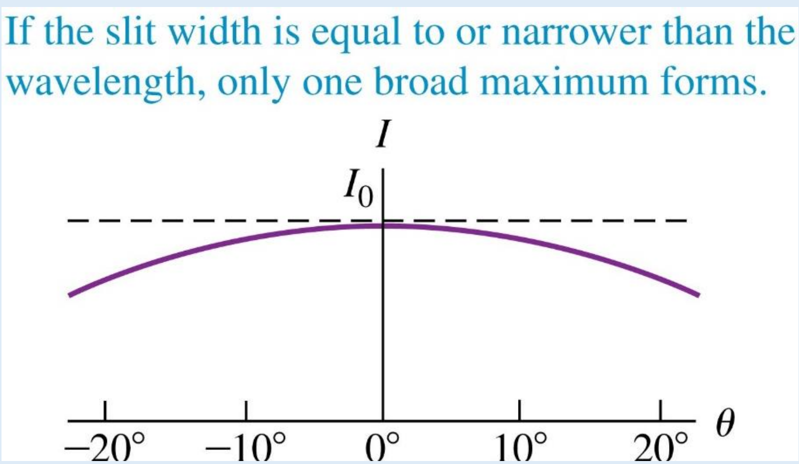
Single-Slit Diffraction: Intensity Pattern

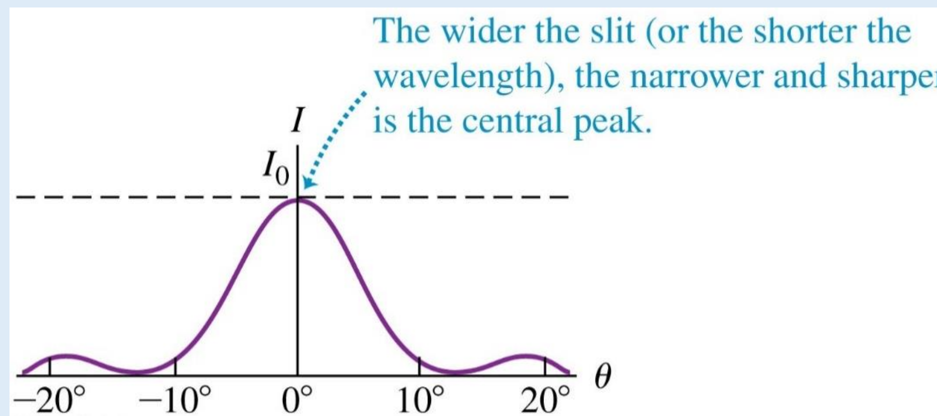
- Shown is a plot of intensity versus angle in a single-slit diffraction pattern.
- Marked are locations of intensity minima ($I = 0$) for each integer (order) m'.
- Also shown, between minima, are locations of maximum intensity, with their intensity values.
- Most of wave power goes into the central intensity peak, between $m' = 1$ and $m' = -1$ intensity minima.



Single-Slit Diffraction: Intensity Pattern

- Single-slit diffraction pattern depends on the ratio of slit width a to wavelength λ.
- Below are intensity patterns for $a = 5\lambda$ (left) and $a = 8\lambda$ (right).





Two-Slit Interference: Identical Slits with Finite Width

- For finite-width slits, diffraction effects from the slits must be considered.
- For two-slit interference, including effect of diffraction for each slit, find from phasor calculation that the resultant intensity is the product of a diffraction pattern of a single slit multiplied by the pattern for two-slit interference in the absence of diffraction :

$$\boxed{I(\theta) = I_0 \underbrace{\left[\frac{\sin\left(\frac{\beta}{2}\right)}{\frac{\beta}{2}}\right]^2}_{\text{Single-slit diffraction effect}} \underbrace{\cos^2\left(\frac{\alpha}{2}\right)}_{\text{Two-slit interference effect}}},$$

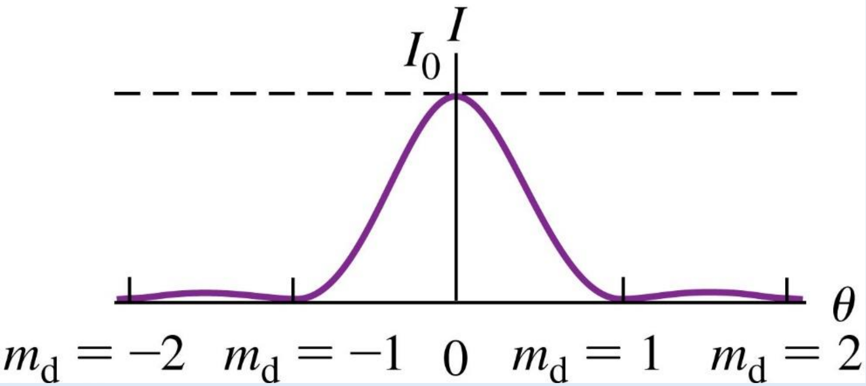
$$\boxed{\beta = \frac{2\pi a}{\lambda}\sin\theta}, \quad \boxed{\alpha = \frac{2\pi d}{\lambda}\sin\theta},$$

θ is angle with respect to line from the slits to the viewing screen, $a =$ slit width, $d =$ distance between slits, and $I_0 =$ intensity of central maximum.

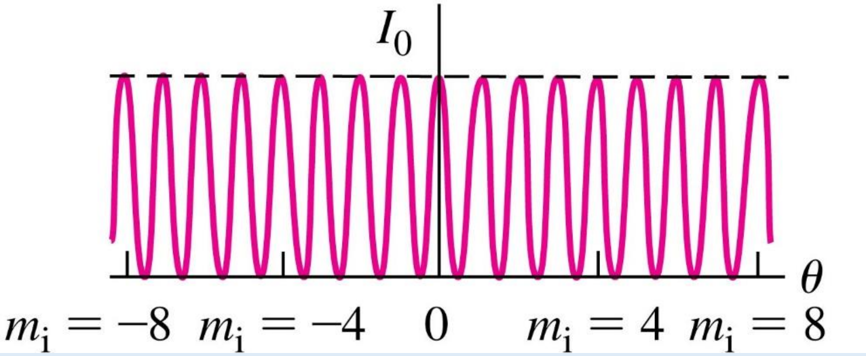
Two-Slit Interference: Identical Slits with Finite Width (2)

- Figure (a) shows intensity pattern for single-slit diffraction from a slit of width a.
  - ➢ Diffraction minima are labeled by integer $m_{\mathrm{d}} = \pm 1, \pm 2, \ldots$ ("d" is for "diffraction").
- Figure (b) shows pattern formed by two very narrow slits with distance d between slits, where d is four times as great as the single-slit width a.
  - ➢ Interference maxima labeled by integer $m_{\mathrm{i}} = 0, \pm 1, \pm 2, \ldots$ ("i" is for "interference").

(a) Single-slit diffraction pattern for a slit width a



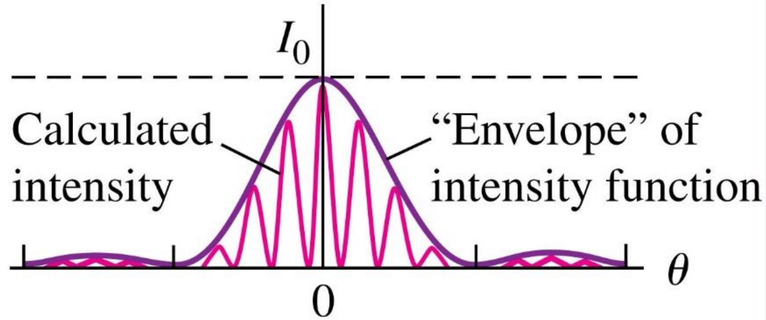
(b) Two-slit interference pattern for narrow slits whose separation d is four times the width of the slit in **(a)**



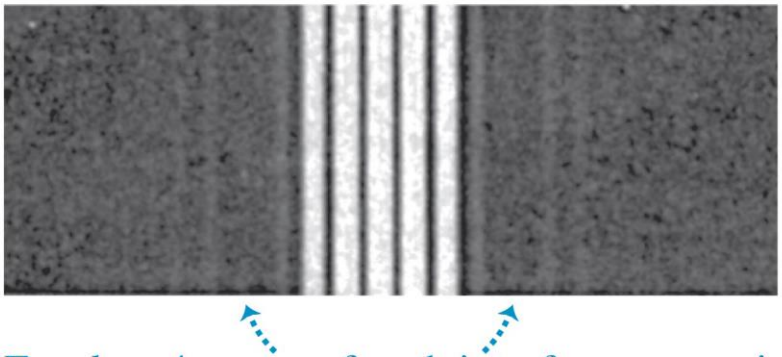
Two-Slit Interference: Identical Slits with Finite Width (3)

- Figure (c) shows intensity pattern for two slits with width a, separated by distance (between centers) $d = 4a$.
  - ➢ Two-slit peaks are in the same positions as before, but their intensities are modulated by the single-slit pattern, which acts as an “envelope” for the intensity function. That is, the overall pattern is an overlap (“convolution”) of the diffraction and two-slit interference effects.
  - ➢ Figure (d) shows screen photograph, displaying combined effects of diffraction <u>with</u> interference. Bright bands are diffracted images of the slit.

(c) Calculated intensity pattern for two slits of width a and separation $d = 4a$, including both interference and diffraction effects



(d) Photograph of the pattern calculated in **(c)**



For $d = 4a$, every fourth interference maximum at the sides ($m_i = \pm 4, \pm 8, ...$) is missing.

Two-Slit Interference: Identical Slits with Finite Width (4)

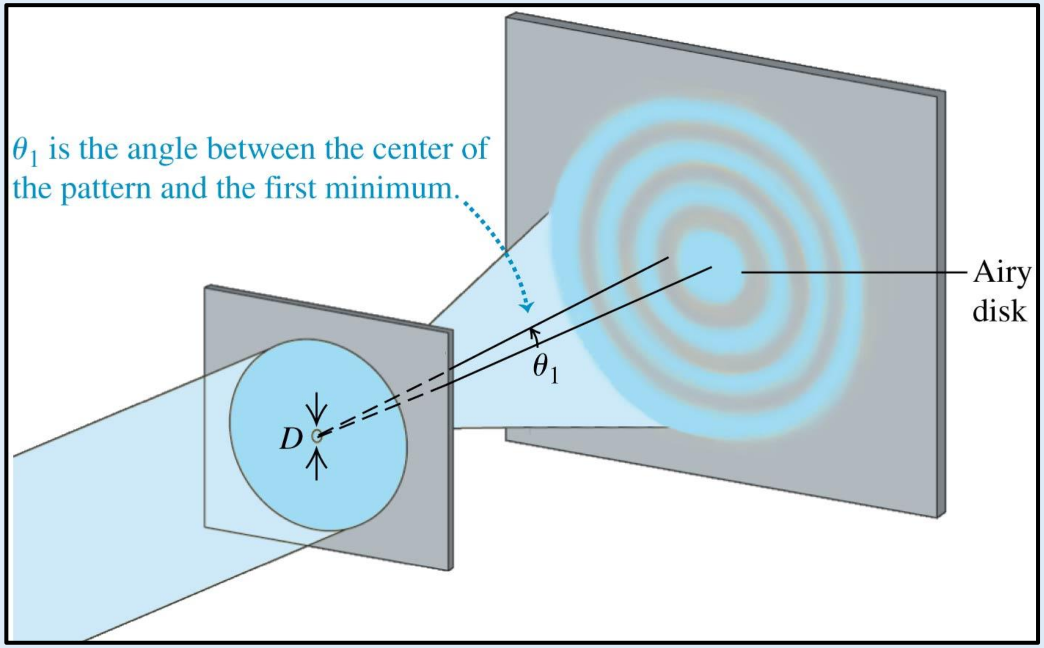
- An online slit diffraction calculator (with one- and multi-slit options) can be found at

https://demonstrations.wolfram.com/MultipleSlitDiffractionPattern/

- Can use to investigate how intensity pattern changes with slit spacing and width.

Diffraction from Circular Apertures

- For all the same reasons that diffraction occurs in barriers with long slits, diffraction also occurs for other apertures, including circular apertures.
- Has direct applications to lenses, including the human eye.
- Diffraction pattern formed by a circular aperture has a central bright spot encircled by alternating series of dark and bright rings.



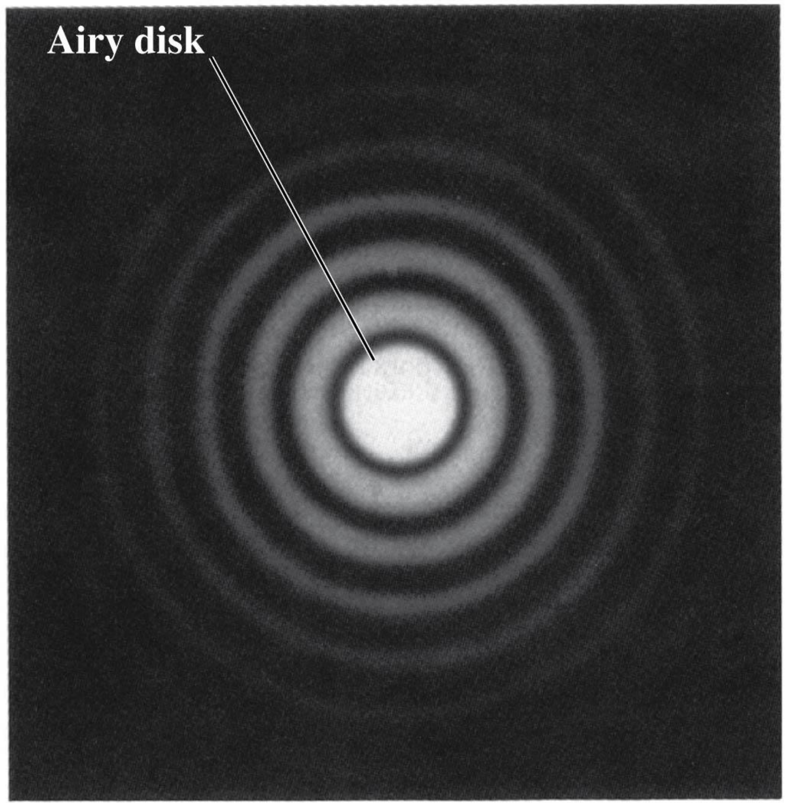
Diffraction from Circular Apertures (2)

- Central bright spot in the diffraction pattern of a circular aperture is called the "Airy disk", after the astronomer/mathematician G. Airy
- Airy disk is bound by the first dark ring (i.e., first diffraction minimum), which has angular radius θ_1 given by :

$$\boxed{\sin\theta_1 = 1.22\frac{\lambda}{D}} ,$$

$D =$ diameter of circular aperture.

- Factor of 1.22 in above expression is there because the minimum intensity occurs at the zero of a "Bessel function", which is a periodic function in cylindrical geometry. (Akin to sinusoidal functions in cartesian geometry.)



Diffraction and Image Resolution

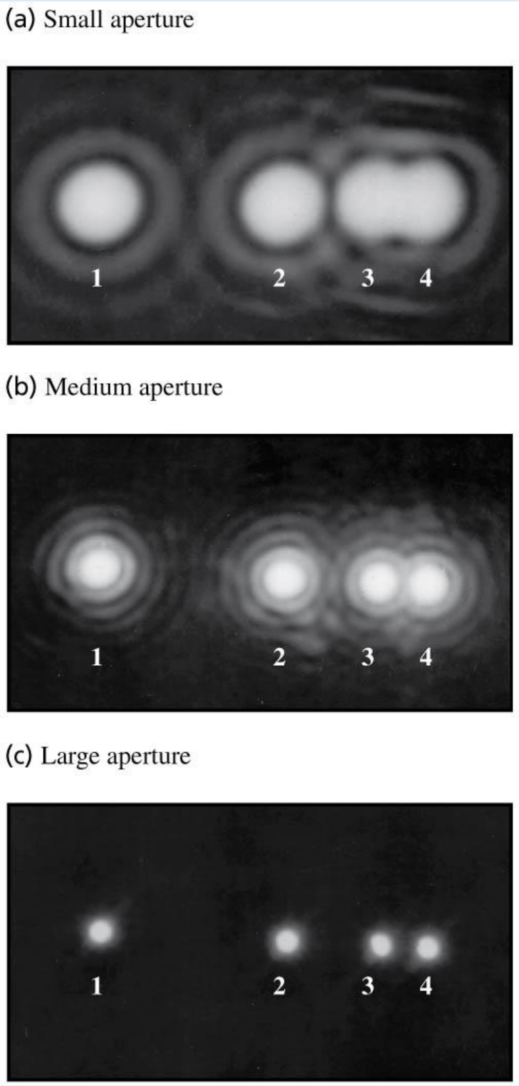
- Diffraction limits resolution of optical equipment, such as telescopes, human eyes, etc.
  - Larger the aperture, the better the resolution.
- Criterion for resolution of two point objects is Rayleigh's criterion:
  - Two objects are just barely resolved (i.e., distinguishable) if the center of one diffraction pattern coincides with the first minimum of the other.
  - The angle for the Rayleigh criterion is

$$\boxed{\theta_R = \sin^{-1}\left(1.22\frac{\lambda}{D}\right)} \ .$$

  - For small angles, $\sin\theta \approx \theta$, and

$$\boxed{\theta_R = 1.22\frac{\lambda}{D}}$$

(angle in radians).

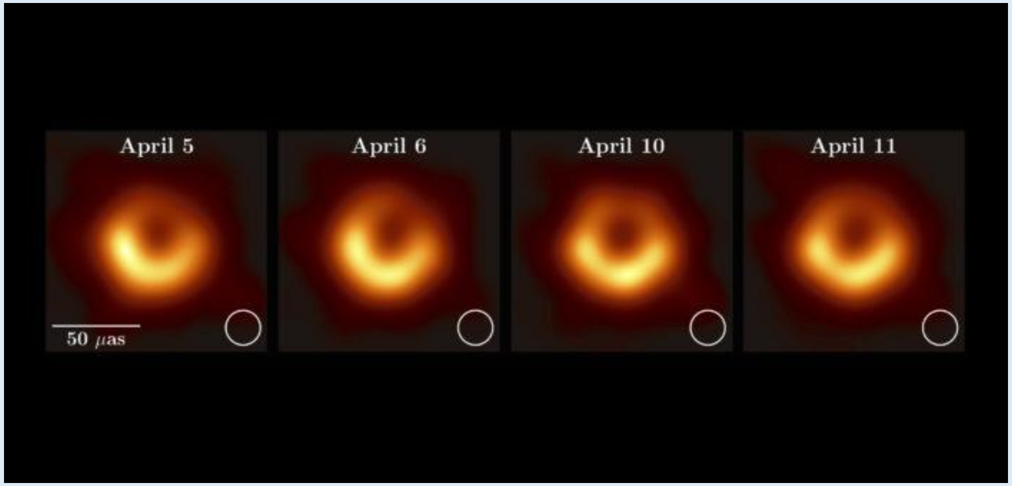


Telescopes: Better Resolution with Larger Apertures

- Because of diffraction, large-diameter telescopes – such as the VLA (Very Large Array) radio telescope network shown below – give sharper images than small ones.
    - ➢ The effective aperture of the system has a diameter much greater than the aperture/diameter of a single radio telescope dish making up the array.

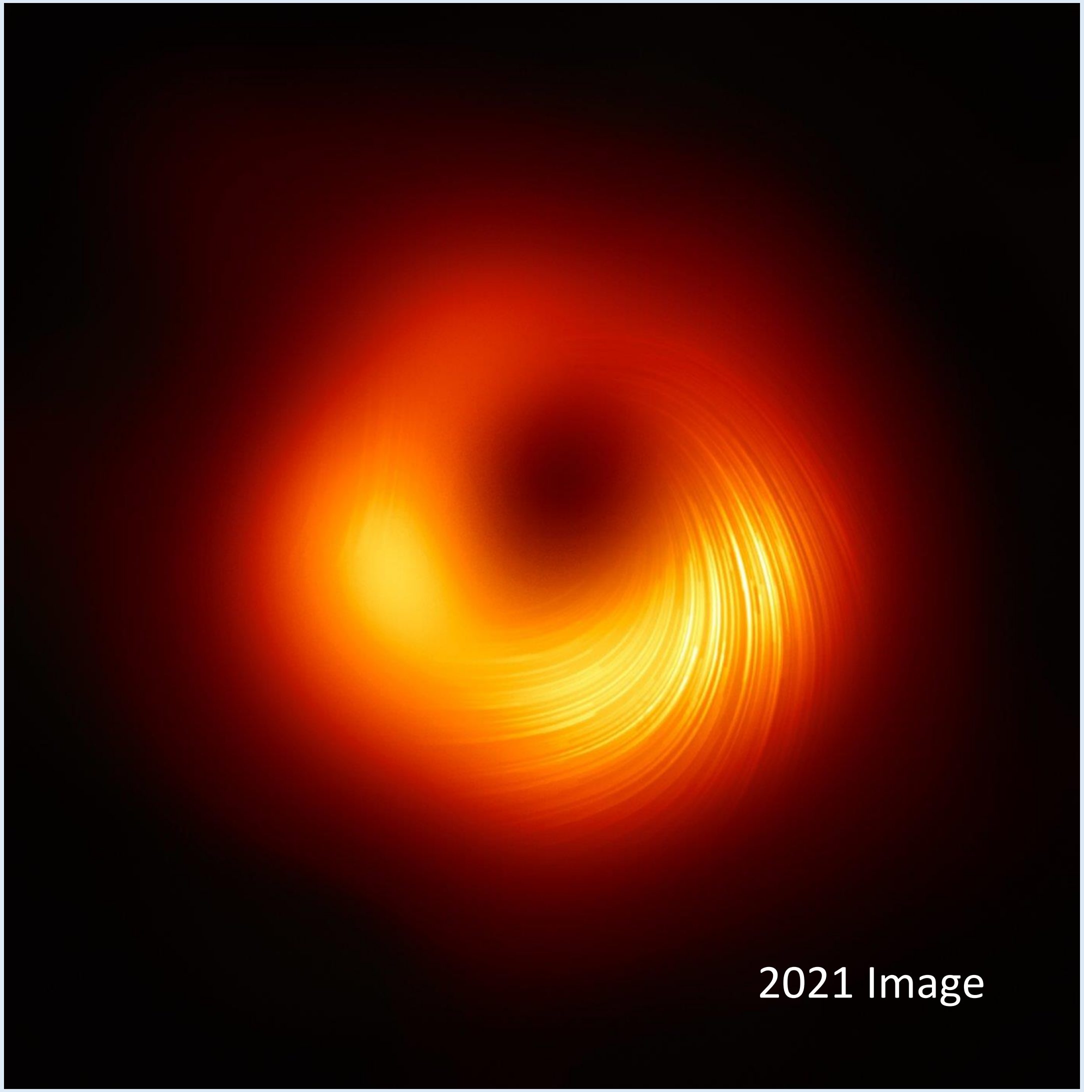
Long-Baseline Telescope: Direct Imaging of the Black Hole in M87

- Various telescopes across the world are network-linked together to form an effectively Earth-sized large-aperture radio telescope called the Event Horizon Telescope (EHT).
- Resolution is sufficient to directly image the supermassive black hole (SMBH) in the center of the M87 galaxy (~ 50 Mly away).



2019 Images

Long-Baseline Telescope: Black Hole M87* in the News



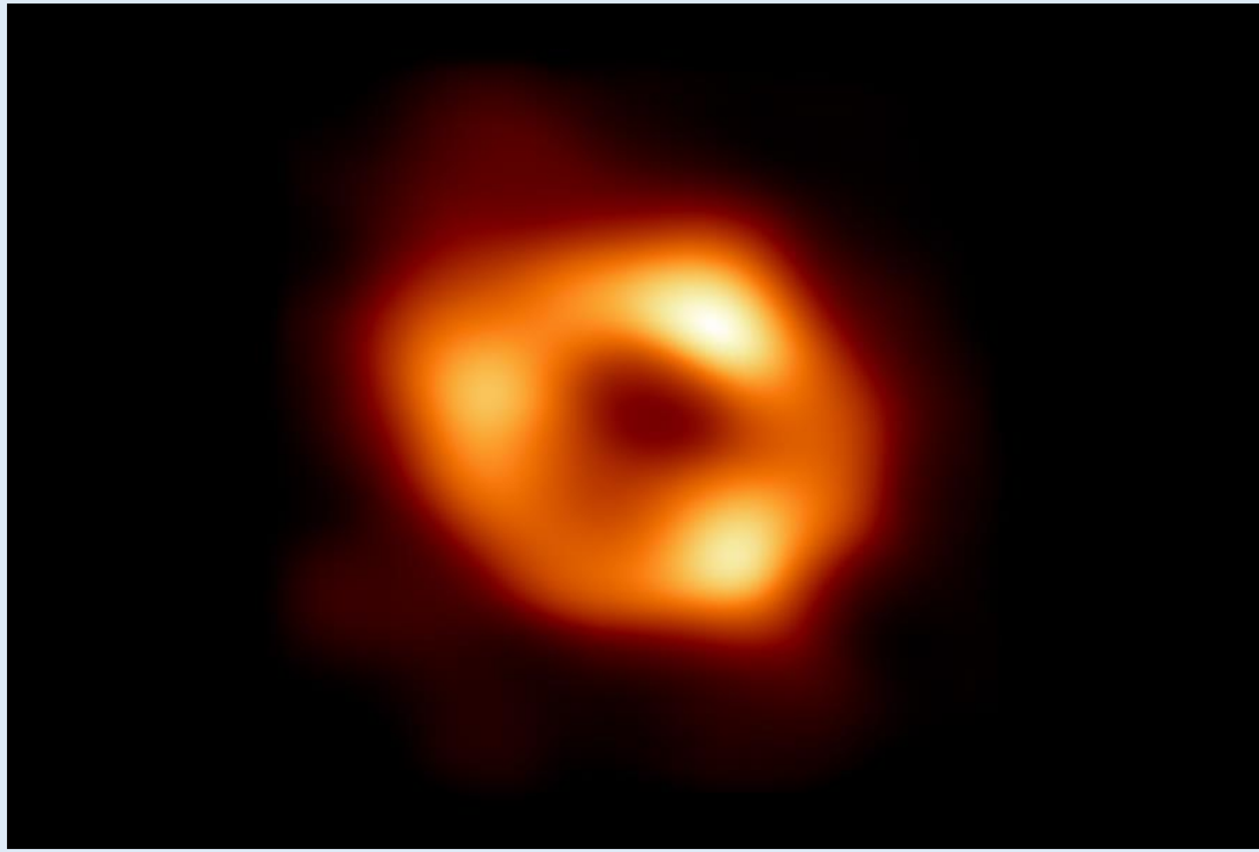
- EHT map of polarized light from M87*. Direction of polarization shows the direction of the magnetic field in the vicinity of the supermassive black hole.

Long-Baseline Telescope: Black Hole M87* in the News

- Enhanced resolution image (right) of original image (left) using machine learning methods (PRIMO algorithm from NOIRlab).

Long-Baseline Telescope: Milky Way's Supermassive Black Hole Sgr A*

- 2022: EHT first radio images of the supermassive black hole at the center of the Milky Way Galaxy.
- About 8 kpc (1 pc = 3.26 light-years) away from us.
- "Monster" at the heart of our galaxy (~ 10^6 solar masses, but only ~ 10^{-3} the mass of M87*).



Extension to Visible Light – Optical Wavelengths

- Long baseline telescopes can be done for radio waves, because the phase differences are readily determined for those wavelengths/frequencies.
- Much more difficult to do this with visible wavelengths – the frequencies are too high in the optical region for methods that worked for radio/millimeter telescopes. Can't be ported over to the visible region.
- Quantum technologies are being developed to make effectively large combined optical interferometer telescopes:

https://www.science.org/content/article/can-quantum-tech-give-telescopes-sharper-vision

- If this can be done, combined, effectively large optical telescopes will have much greater resolution (Rayleigh criterion) than currently available at those wavelengths.