Note: Instructor Change

For the remainder of this semester, Prof. Peter Persans will be the section instructor for Section 5 of PHYS 1200.

(Somebody in the Physics Dept. Office must have realized that Prof. Persans was getting too much time off.)

Physics 1200
Class Lecture 03
Spring 2024

Electric Flux, Gauss's Law, Conductors

Ways to Calculate Electric Field

1. <u>Measure</u> the electric force on a known charge. Use $\vec{\boldsymbol{E}} = \frac{\vec{\boldsymbol{F}}}{q}$.
2. Add up the vector electric fields due to every point charge.
3. Use Gauss's Law.
4. Use $E_x = -\frac{\partial V}{\partial x},\ E_y = -\frac{\partial V}{\partial y}, E_z = -\frac{\partial V}{\partial z}$. (Discussed next class.)

Today we'll show how (3) can be used to find $\vec{\boldsymbol{E}}$ for some situations.

Preliminary: Electric Flux

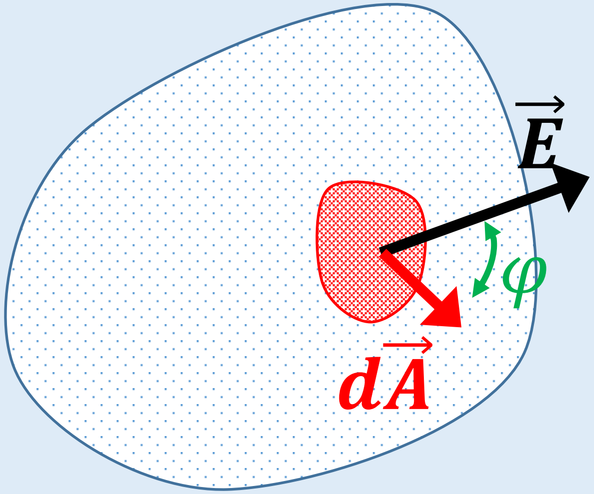
- Electric flux through a surface is defined as

$$\boxed{\Phi_E \equiv \int \vec{\boldsymbol{E}} \cdot d\vec{\boldsymbol{A}} = \int E\, dA\, \cos\varphi\ .}$$

$\vec{\boldsymbol{E}}$ is the electric field at the surface,

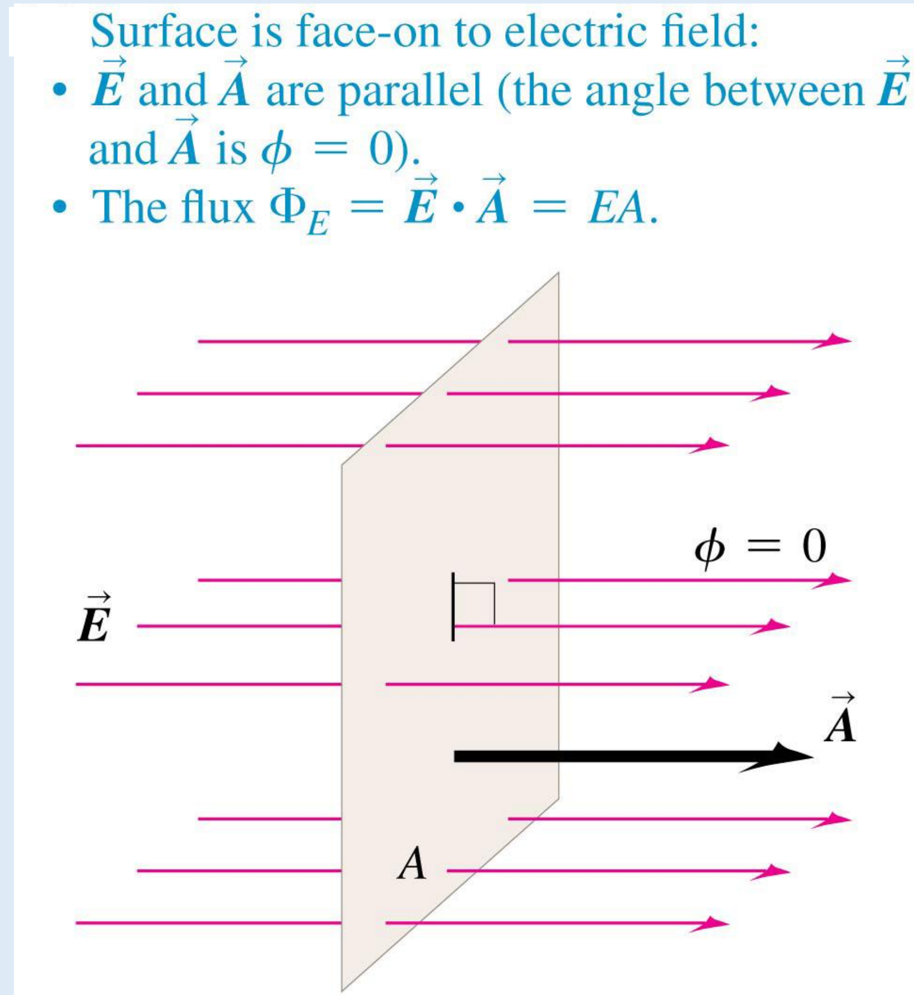
$$\boxed{d\vec{\boldsymbol{A}} = dA\, \hat{\boldsymbol{n}}}$$

is a vector element of area directed along the unit vector $\hat{\boldsymbol{n}}$ normal (i.e., perpendicular) to the surface, and φ is the angle between the vectors:



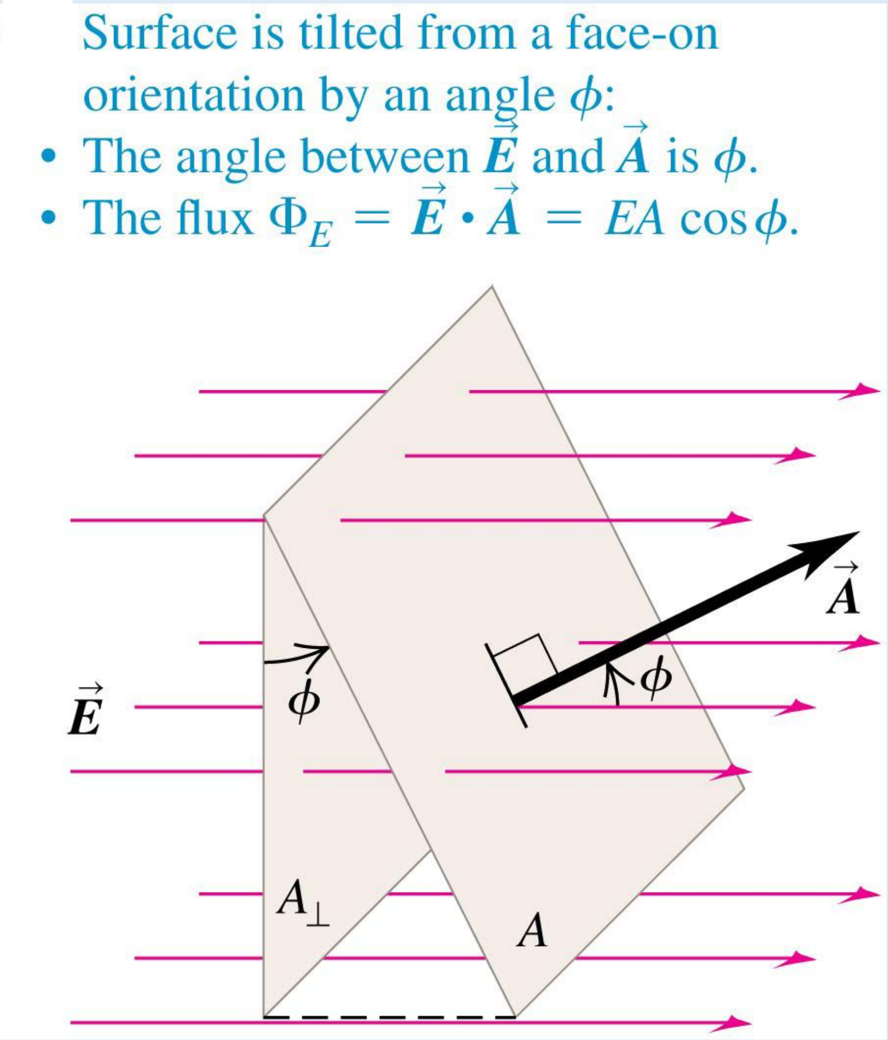
Calculating Electric Flux

- Electric flux is proportional to number of electric field lines (or field vectors) passing through an area in a certain direction.
- Consider flat area perpendicular to a uniform electric field:
  - ➢ Increasing area means more electric field lines pass through area, increasing the flux.
  - ➢ Stronger field means more closely spaced lines, and therefore more flux.



Calculating Electric Flux (2)

- If area is not perpendicular to field, fewer field lines pass through it.
- Area in flux calculation is the projected silhouette area $A_{\perp} = A\cos\phi$ perpendicular to the direction of the field.

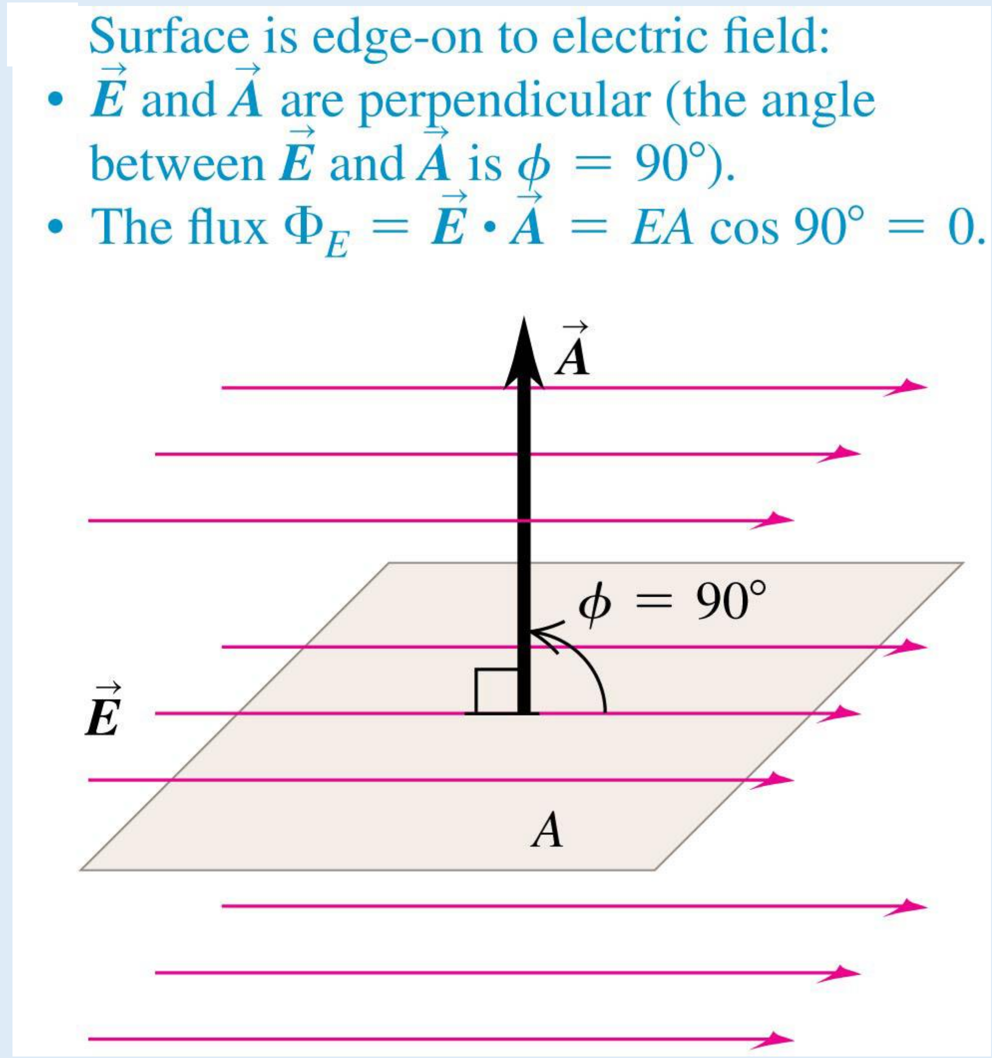


Calculating Electric Flux (3)

- Area edge-on to field: area vector perpendicular to field, flux = 0.

- Flux can be positive, zero, or negative. If $\vec{\boldsymbol{E}}$ and $\vec{\boldsymbol{A}}$ are in opposite directions (i.e., $\varphi = 180°$):

$$\Phi_E = EA\ \cos 180° = -EA.$$



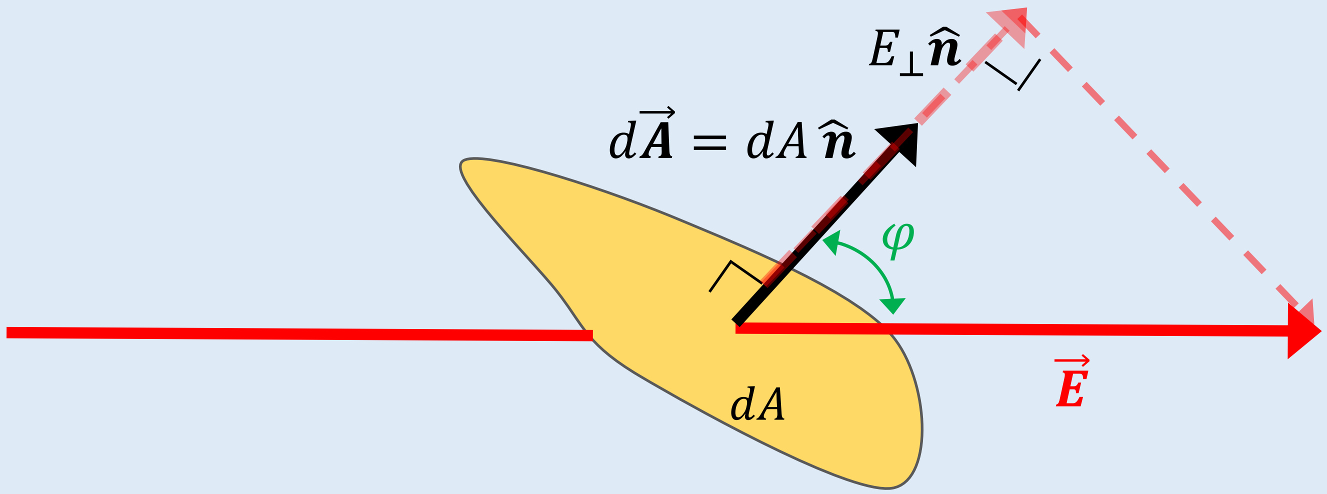
Calculating Electric Flux (4)

- For nonuniform electric field:

$$\Phi_E = \int \vec{\boldsymbol{E}} \cdot d\vec{\boldsymbol{A}} = \int \vec{\boldsymbol{E}} \cdot dA\,\hat{\boldsymbol{n}} = \int E dA \cos\varphi = \int E_\perp \, dA.$$

$$E_\perp = E\cos\phi$$

is the component of field perpendicular to surface (i.e., parallel to $d\vec{\boldsymbol{A}} = dA\,\hat{\boldsymbol{n}}$).



Question 3.1

- The electric flux through the shaded surface is closest to:

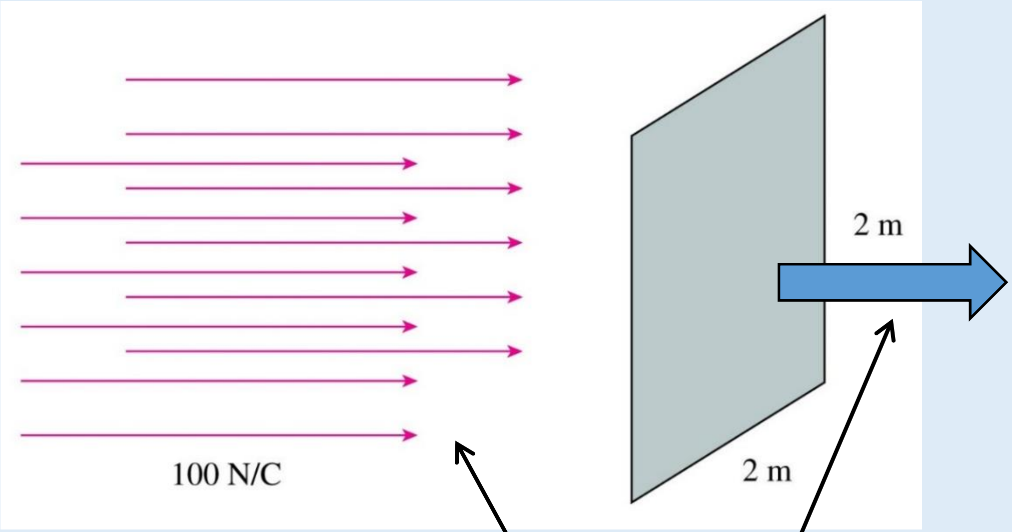
A. $0 \text{ N m}^2/\text{C}$.

B. $200 \text{ N m}^2/\text{C}$.

C. $400 \text{ N m}^2/\text{C}$.

D. $-200 \text{ N m}^2/\text{C}$.

E. None of these.



The field and normal vectors are parallel

Gauss's Law

- "The net electric flux emerging out of a closed surface is proportional to the total charge enclosed by the surface."
- Mathematically:

Gauss's law:

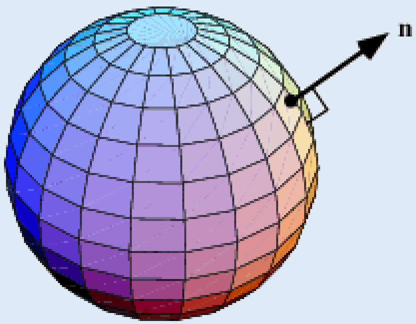
$$\Phi_E = \underbrace{\oint \vec{E} \cdot d\vec{A}}_{} = \frac{Q_{\text{encl}}}{\epsilon_0}$$

Total charge enclosed by surface

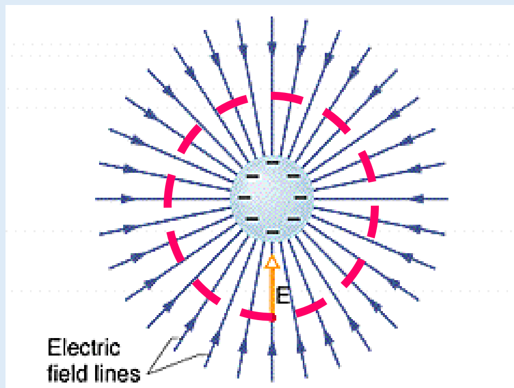
Electric flux through a closed surface of area A = surface integral of $\vec{E}$

Electric constant

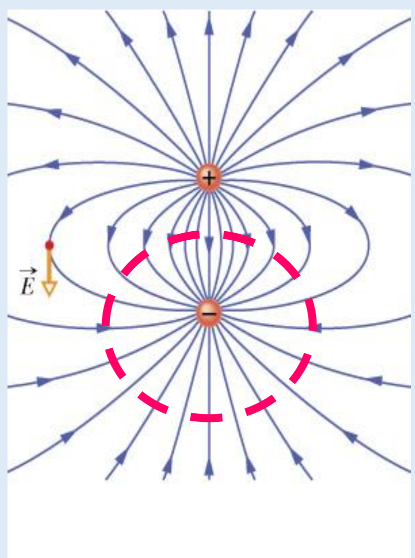
- Note 1: Q_{encl} can be positive, zero, or negative.
- Note 2: direction of area unit vector $\hat{\boldsymbol{n}}$ (needed for direction of $d\vec{\boldsymbol{A}} = dA\,\hat{\boldsymbol{n}}$) for a surface enclosing volume always taken outward from closed bounding surface.
  - Example: normal vector to a closed spherical surface would be



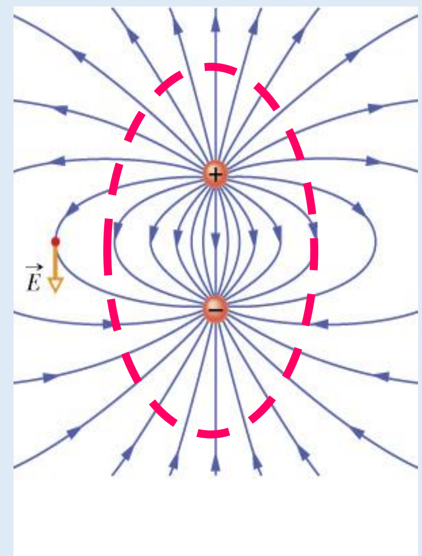
Gauss's Law (2)



Net Charge
Contained:
Net Flux



No Net Charge
Contained:
No Net Flux



Calculating charge enclosed from flux

In some cases, you may be given the field and then asked to find enclosed charge. Will need to calculate the flux over all surfaces.

- Pay attention to direction of field. Some surfaces may have zero flux.
- Take advantage of symmetry between pairs of surfaces.
- If field is constant in a region, there will be no net flux in that region: Just as much flux comes in one place as goes out another.

Question 3.2

A cubical box with sides of length a has electric field of magnitude E pointing straight out of each face. What must be the charge enclosed by the walls of the box?

A. $0\ \mathrm{C}$.

B. Ea^2/ϵ_0.

C. $\epsilon_0 E a^2$.

D. $6\epsilon_0 E a^2$.

E. $3\ \epsilon_0 E a^3$.

F. $6Ea^2/\epsilon_0$.

G. Not enough information to determine.

Recipe for applying Gauss's law to find the field

$$\oint \vec{\boldsymbol{E}} \cdot d\vec{\boldsymbol{A}} = \Phi_E = Q_{enc}/\epsilon_0$$

1. Make a sketch of charge distribution

2. Check symmetry of distribution and its effect on the electric field. Only apply Gauss's law when it helps to solve the problem.

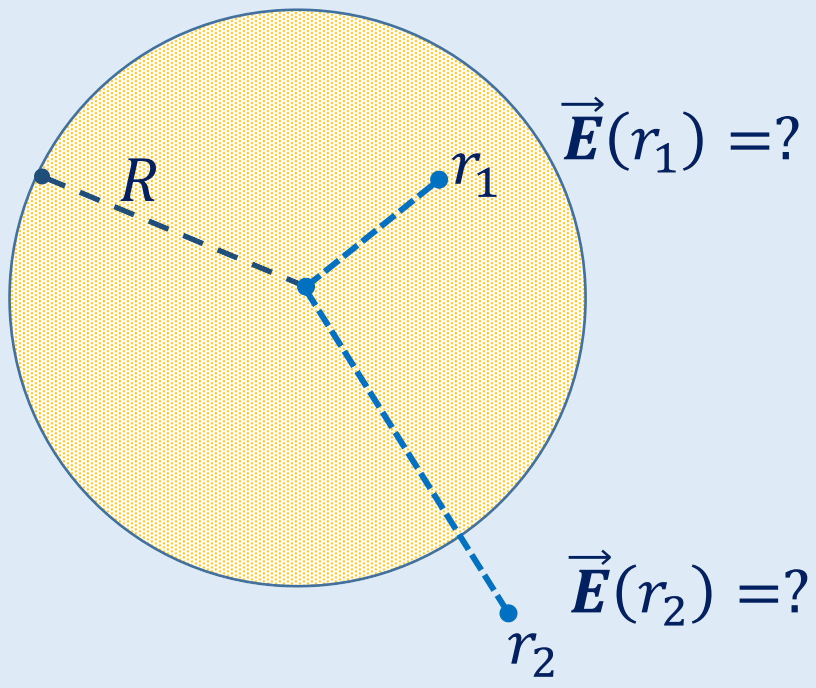
3. Gauss's law is true for **any** closed surface S. Choose one on which the field is constant, or zero. Allows removing electric field E from integral. Leaves just integral for surface area = total surface area.

4. Find total flux.

5. Divide by area (on which the field is constant) to find field.

Spherically Symmetric Charge

- Gauss's law can simplify calculating electric field for various symmetric situations, such as a spherical charge distribution.



Spherically Symmetric Charge (2)

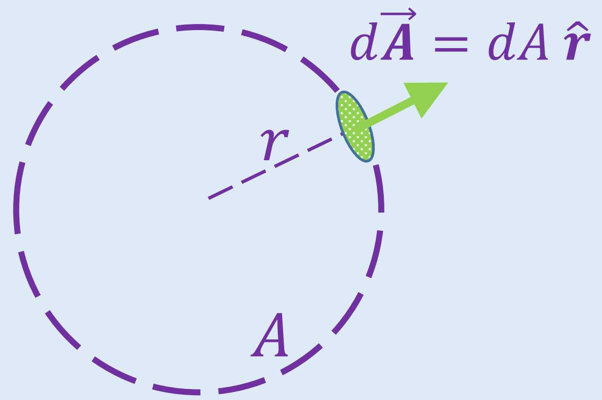
- Spherical charge distribution: ball of charge looks like 'big point charge'. For point charges $\vec{\boldsymbol{E}}(r) = E(r)\hat{\boldsymbol{r}}$, with $\hat{\boldsymbol{r}}$ directed radially outward from origin (= center of sphere of charge).
- Imagine/create a Gaussian surface that has same symmetry as the charge distribution: <u>use a spherical surface of constant radius r about center of charged sphere</u>. For this choice, $d\vec{\boldsymbol{A}} = dA\,\hat{\boldsymbol{r}}$.

$$\therefore \oint \vec{\boldsymbol{E}}(r) \cdot d\vec{\boldsymbol{A}} = \oint E(r)\,\hat{\boldsymbol{r}} \cdot dA\,\hat{\boldsymbol{r}} = \oint E(r)\,dA\,\hat{\boldsymbol{r}} \cdot \hat{\boldsymbol{r}} = E(r)\oint dA = E(r)4\pi r^2.$$

Gauss's law gives

$$E(r)4\pi r^2 = \Phi_E = Q_{\text{enc}}(r)/\epsilon_0$$

$$\Rightarrow E(r) = \frac{Q_{\text{enc}}(r)}{4\,\pi\epsilon_0 r^2} = \frac{kQ_{\text{enc}}(r)}{r^2},$$

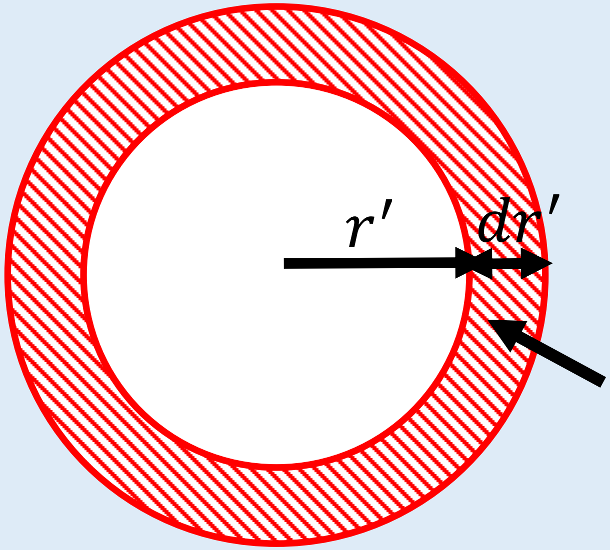


where $Q_{\text{enc}}(r)$ is the total charge enclosed by the spherical Gaussian surface of radius r.

Now need to calculate $Q_{\text{enc}}(r)$!

Spherically Symmetric Charge (3)

- To find total enclosed charge, calculate increment of charge dq for a thin spherical shell of radius r', infinitesimal thickness dr', and charge density $\rho(r')$:



$$dq = \rho(r')dV' = \rho(r')4\pi r'^2 dr',$$

$dV' = 4\pi r'^2 dr'$ = Volume of thin spherical shell

- Total enclosed charge is sum of charge contained in all shells interior ($0 \leq r' \leq r$) to Gaussian surface of radius r:

$$Q_{\mathrm{enc}}(r) = \int_0^r \rho(r')4\pi r'^2 dr' = 4\pi \int_0^r \rho(r')r'^2 dr'.$$

Example: Uniform Spherical Charge

- Consider case $r \leq R$ for sphere of uniform charge, where R is radius of sphere. For this case, charge volume density

$$\rho(r) = constant = \frac{Q}{V} = \frac{Q}{\frac{4\pi}{3}R^3} = \frac{3Q}{4\,\pi R^3}\,.$$

  $Q =$ total charge of sphere.

  From preceding slide, get

$$Q_{\text{enc}}(r) = \frac{4\pi r^3}{3}\rho = Q\,\frac{r^3}{R^3}.$$

  This gives, for $r \leq R$,

$$E(r) = \frac{kQr}{R^3} = \frac{1}{4\,\pi\epsilon_0 R^3}\frac{4\pi R^3}{3}\,\rho r = \frac{\rho r}{3\epsilon_0}.$$

- What is the field for $r > R$? In this case, $Q_{\text{enc}}(r) = Q_{\text{enc}}(R) = Q$,

$$E(r) = kQ/r^2$$ ← Acts like a “big point charge.”

Example Problem 3.1

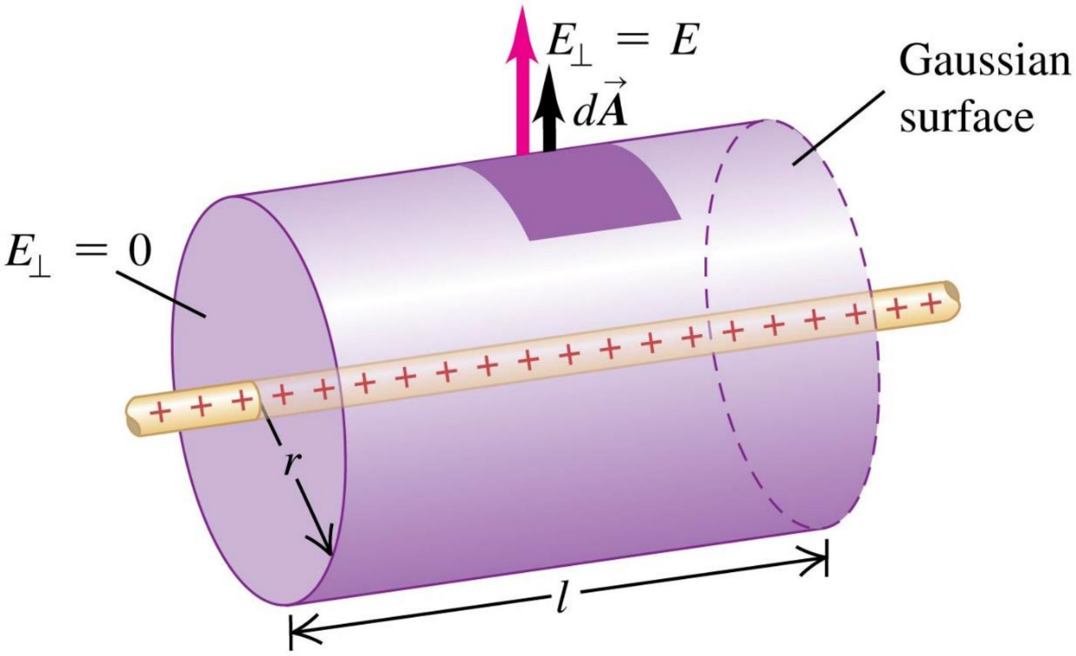
A charged paint is spread in a very thin uniform layer over the surface of a plastic sphere of diameter 15.0 cm, giving it a charge of -59.0 μC .

A) Find the electric field just inside the paint layer.
B) Find the electric field just outside the paint layer.
C) Find the electric field 7.00 cm outside the surface of the paint layer.

Field of a Uniform Line Charge

- Electric charge is distributed uniformly along an infinitely long, thin wire. Charge per unit length is λ (positive here).
- Using Gauss's law, find:

$$E = \frac{1}{2\pi\epsilon_0}\frac{\lambda}{r}$$



Example Problem 3.2

The electric field due to an infinite line of charge is perpendicular to the line and has magnitude $E = \lambda/2\pi\epsilon_0 r$. Consider an imaginary cylinder with a radius of r = 0.170 m and a length of l = 0.405 m that has an infinite line of positive charge running along its axis. The charge per unit length on the line is λ = 4.75 μC/m .

A) What is the electric flux through the cylinder due to this infinite line of charge?

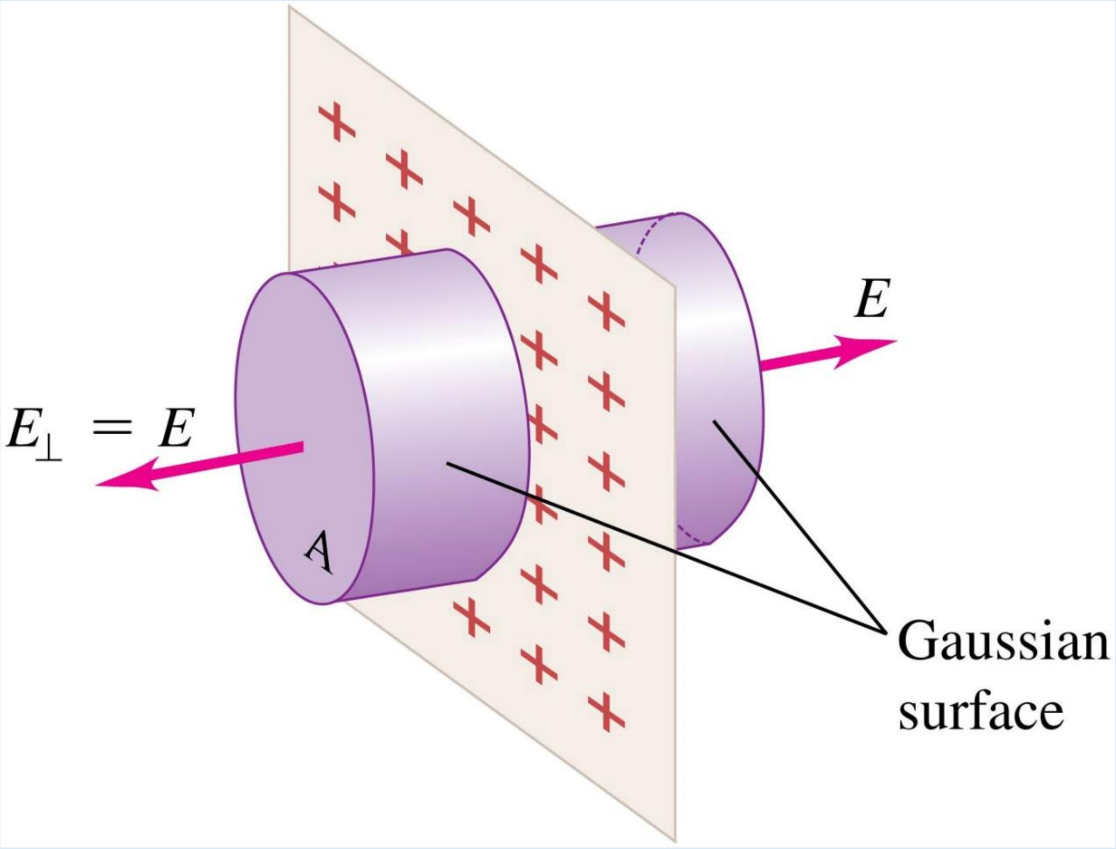
B) What is the flux through the cylinder if its radius is increased to r = 0.550 m ?

C) What is the flux through the cylinder if its length is increased to l = 0.915 m ?

Field of an infinite plane sheet of charge

- Gauss's law can be used to find the electric field from a thin, flat, infinite sheet with a uniform positive surface charge density σ.

$$E = \frac{\sigma}{2\epsilon_0}$$



Electrostatics: Electric Field in Conductors

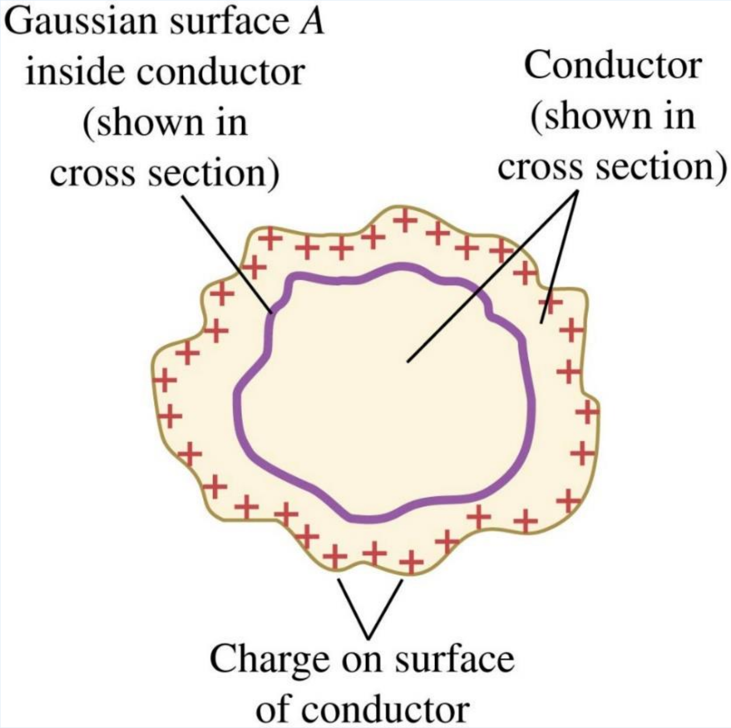
- As discussed in class 1, conductors are materials in which charges are free to move about within the material medium.
- No motion of charges in <u>electrostatic</u> situations. For that case, can't be a net electric field within a conductor, because electric forces would cause flow of charge in the conductor --- which would be a <u>non-static</u> situation. Opposite to what we are considering!
- Therefore, for electrostatic conditions, net electric field

$$\boxed{\vec{\boldsymbol{E}} = 0}$$

within a conductor.

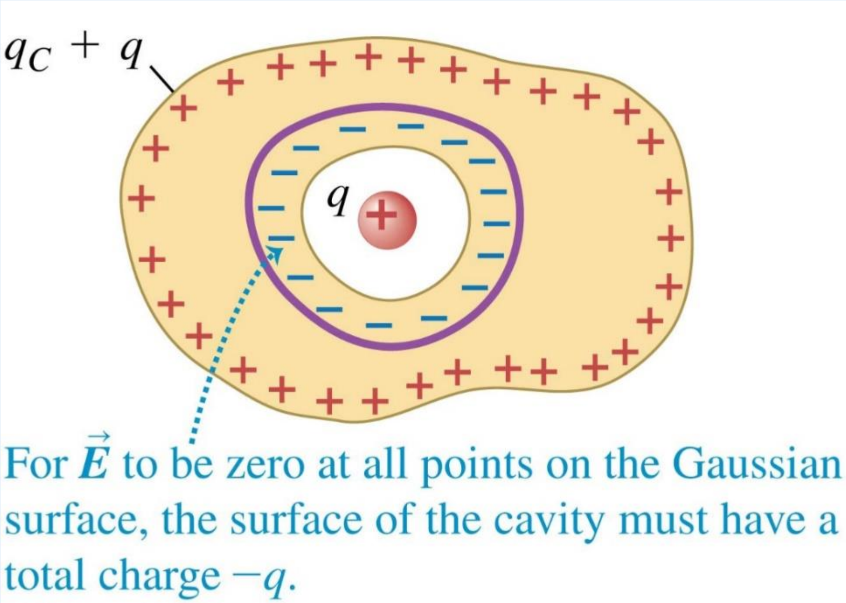
Gauss's Law Application - Conductors

- Construct a Gaussian surface inside a static conductor.
- Because $\vec{\boldsymbol{E}} = 0$ everywhere on this surface, Gauss's law requires net charge inside the surface to be zero.
- Under these conditions, any excess charge on a solid conductor must <u>reside entirely on conductor's outer surface.</u>



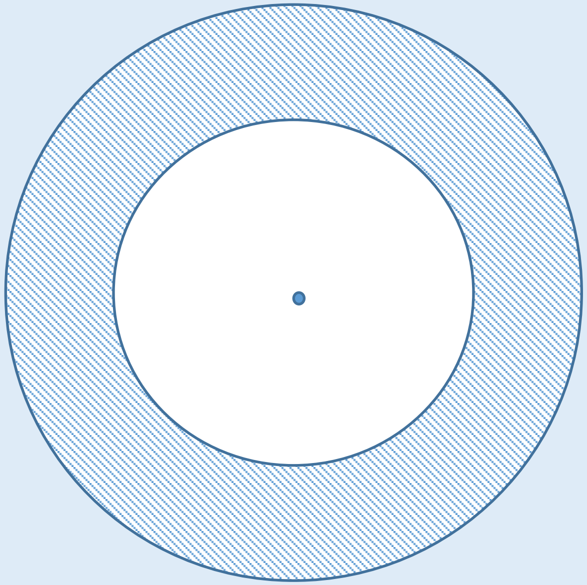
Gauss's Law Application – Conductors (2)

- Place a charge *q* inside a cavity within a conductor. Conductor has a charge q_c and is insulated from the charge *q*.
- Gauss's law: must be a charge −*q* distributed on the inner surface of the cavity, drawn there by the cavity charge *q*.
- Total charge on the conductor must remain q_c, so an additional charge +*q* must appear on its outer surface.



Spherical charged metal shell

- Charge at center = Q_C
- Charge on shell=Q_S



$$E_{metal} = 0$$

$$E_{INSIDE} = k\frac{Q_C}{r^2}$$

$$q_{INSIDESURFACE} = -Q_C$$

$$E_{OUTSIDE} = k\frac{Q_C + Q_S}{r^2}$$

$$q_{OUTSIDESURFACE} = Q_C + Q_S$$

Spherical metal shell w/inner charge off-center

- Charge inside off-center = Q_C
- Charge on shell=Q_S

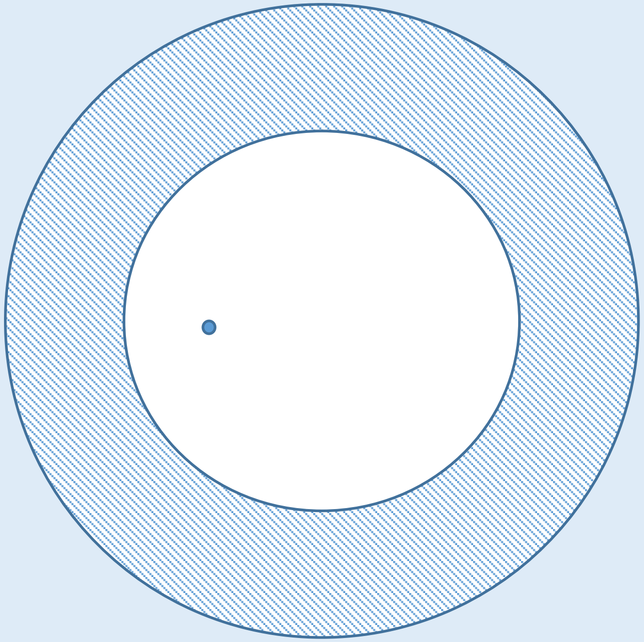
$$E_{metal} = 0$$

$$E_{INSIDE} = \text{not easily found}$$

$$q_{INSIDESURFACE} = -Q_C$$

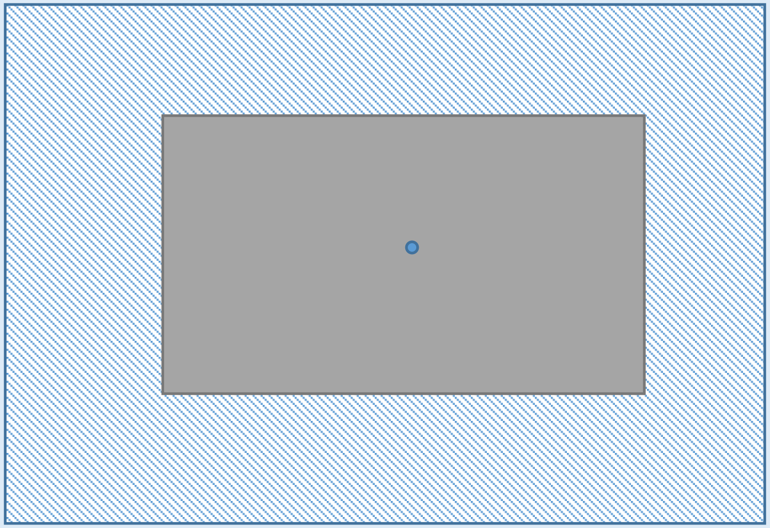
$$E_{OUTSIDE} = k\frac{Q_C + Q_S}{r^2}$$

$$q_{OUTSIDESURFACE} = Q_C + Q_S$$



Non-spherical metal shell with charge at center

- Charge at center = Q_C
- Charge on shell=Q_S



$$E_{metal} = 0$$

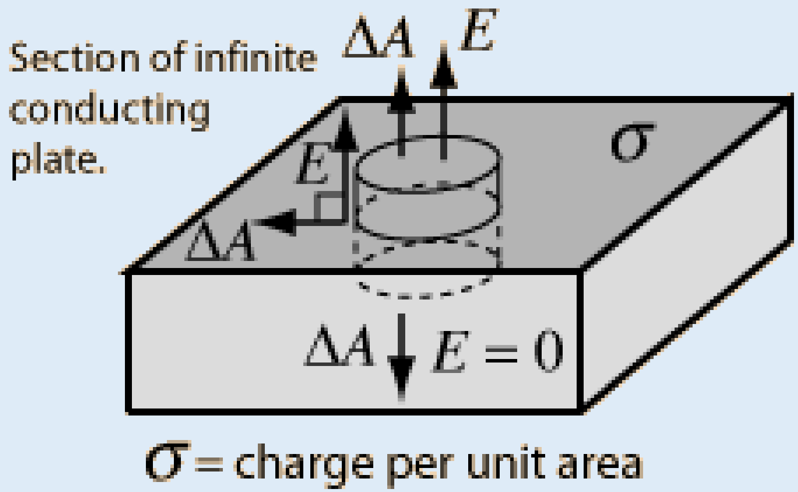
$$E_{INSIDE} = \text{not easily found}$$

$$q_{INSIDESURFACE} = -Q_C$$

$$E_{OUTSIDE} = \text{not easily found}$$

$$q_{OUTSIDESURFACE} = Q_C + Q_S$$

Electric Field of a Uniform Charge on a Flat Metal Sheet



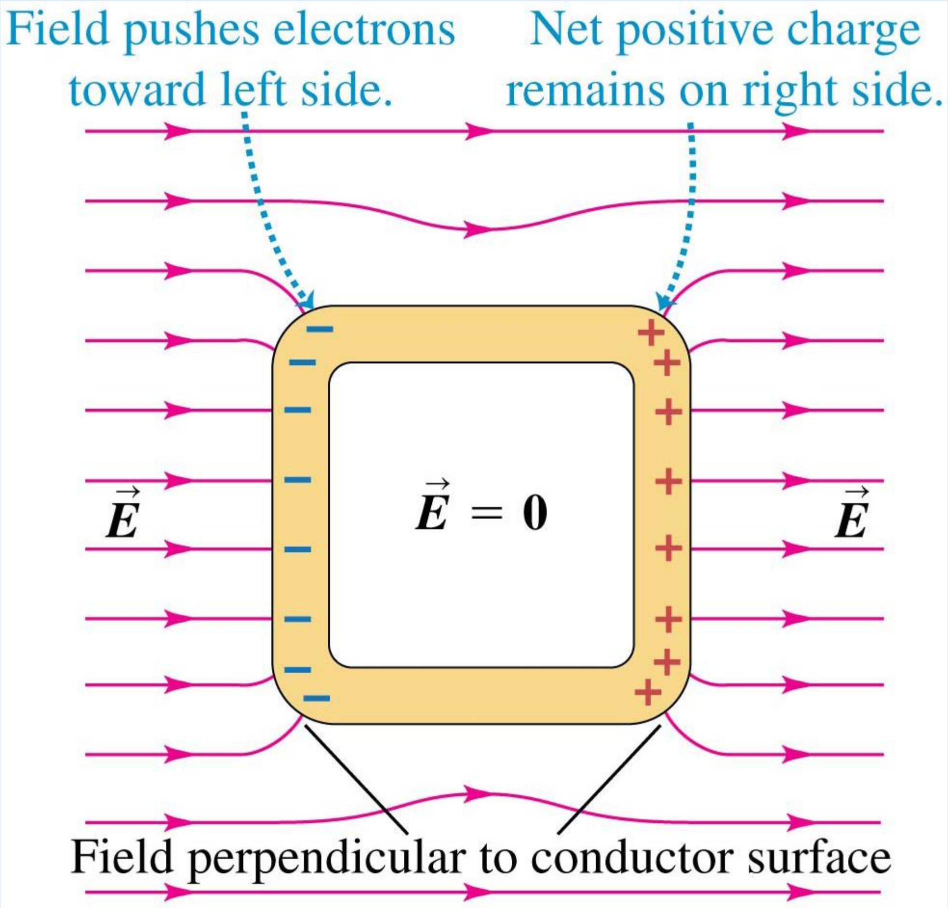
$$\oint \vec{\boldsymbol{E}} \cdot d\vec{\boldsymbol{A}} = \frac{Q_{enc}}{\epsilon_0}$$

$$\rightarrow E\Delta A \cos 0° = \sigma \Delta A / \epsilon_0$$

$$E = \frac{\sigma}{\epsilon_0}$$

Application: Electrostatic Shielding

- Conducting box immersed in uniform electric field.
- Field of induced charges on outer box surface combines with the uniform field to give <u>*zero*</u> net field inside box.



Application: Electrostatic Shielding (2)

- Suppose we have an object that we want to protect from electric fields.
- Surround the object with a conducting box, called a Faraday cage.
- Little to no electric field penetrates the box.
- Person in photograph is protected from the powerful electric discharge.

Video demonstration of shielding ("Faraday cage") on MasteringPhysics:
https://mediaplayer.pearsoncmg.com/assets/secs-vtd33_electroscope