

# Quantum Computing & Applications for Engineering



9/17/2024

## Lecture 2: Making, Using, and Measuring Qubits

# Today's Goals

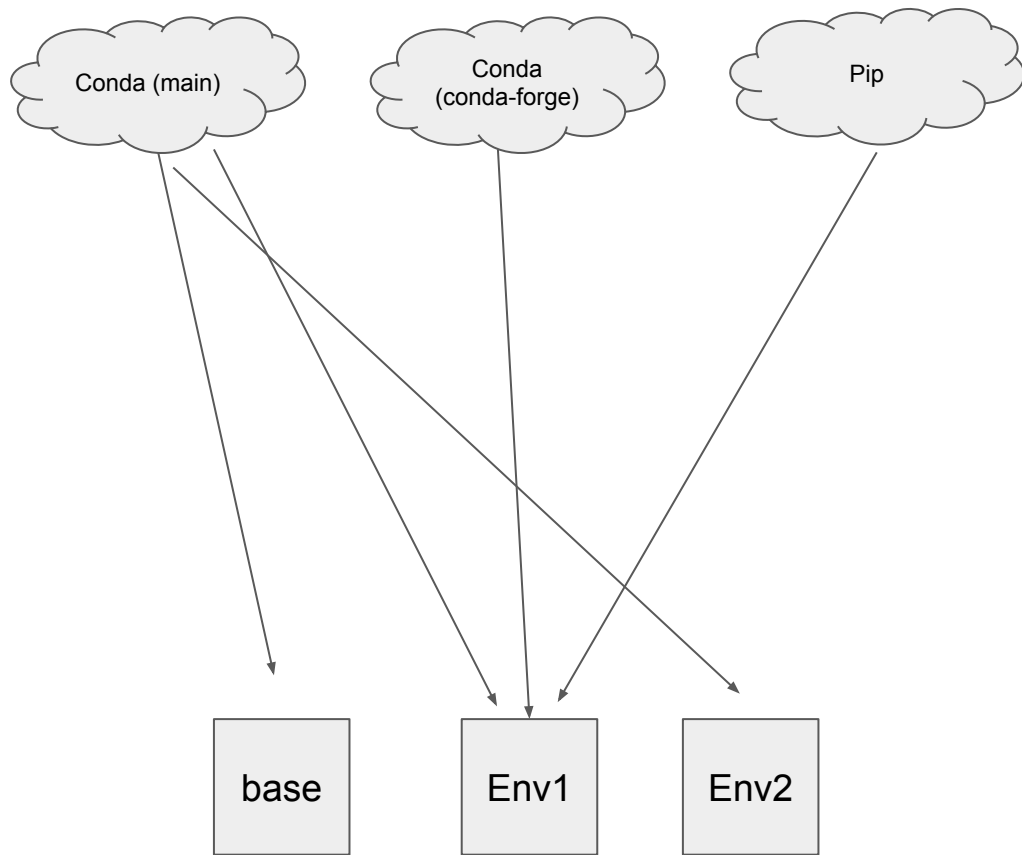
- Learn:
  - What makes a qubit a qubit?
  - How do we use them?
  - How can we measure their properties?
- Do:
  - Go over common issues
  - Some simple Qiskit patterns
  - Basic qubit experiments

# Announcements

- Office Hours
  - Thanks to everyone who came!
  - Helps me understand where the gaps are, what to focus on.
- Common Issues
  - Conda and Jupyter Usage
  - Command Prompt/Terminal Usage
  - Programming practices and style
  - Outputting lists, printing to PDF
  - What the heck did I just run?

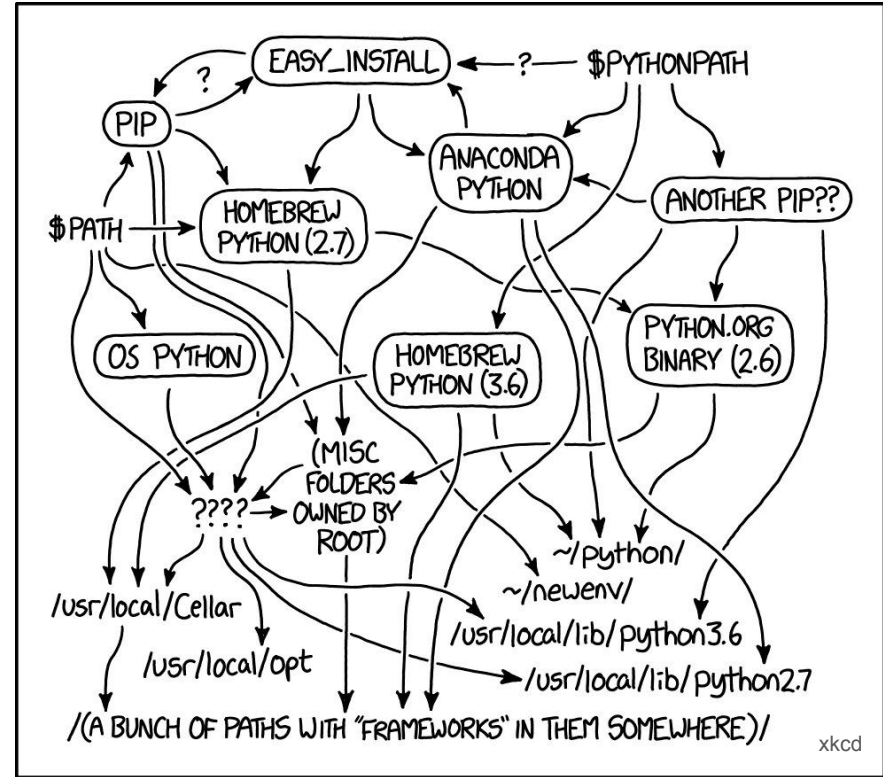
# Anaconda Python

- Anaconda is a Python *distribution*.
  - Python itself is open source
  - Anaconda bundles it up with some common stuff to make life easier
- Main purpose is to manage environments.
  - Installing packages (`conda/pip`)
  - Maintaining consistency
- Possible to use different versions in different environments



# Anaconda Python

- Each environment is a “box” with its own version of python, pip, and any other packages you want to install.
  - Environments are completely independent.
  - Package versions are cached/linked to avoid filling your hard drive with duplicates
- Bad idea to install in the base environment
  - Some packages are incompatible with each other and with certain versions of Python
  - Leave base alone, always try to make a new environment for different lines of work.
- For this class, we should only need the environment we created last time.

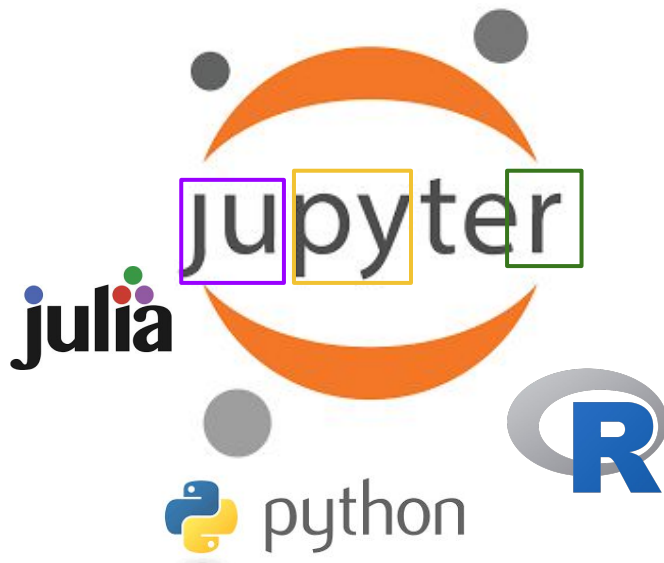


# Command Line Usage

- On Windows, Anaconda gives you a “navigator” interface.
  - On Mac/Linux too, but it’s less visible.
  - Let’s you do everything graphically.
- Use what you prefer.
  - Powershell (Windows)
  - Bash/zsh/fish (Mac/Linux/WSL)
  - None! (although I recommend learning a bit)
- Using the command line can be simpler.
  - Navigator can hang or crash.
  - Activating environments is more intuitive.
  - More control and history tracking.
- You can send outputs to a file:
  - Use the “>” operator.
  - Fastest way to do problem 1 on the HW.

# Jupyter

- Jupyter is a program (written in Python) that runs within a Python environment.
  - Starting Jupyter starts a server that runs locally in the background.
  - You launch “kernels” in Python or other languages.
- Need to start Jupyter before you can open notebooks.
- Each notebook is a completely isolated session.
- Can navigate deeper into folders, but you can't go higher than the one you started in.





# Using Jupyter

- Each notebook consists of runnable “cells.”
  - Code, Markdown and Raw cells
  - Can move, split, copy, paste cells
- Cells can execute out of order.
  - Look at the number along the side to see what ran last.
  - Variables you delete will still be in memory.
- Exporting notebooks
  - The raw notebook format is “readable” but unwieldy JSON.
  - Can save as HTML or PDF.
  - PDF export requires LaTeX
  - Can try printing to PDF.

# Python Style

- Imports

- Best practice is to put them at the top of the code/notebook.
- Use aliases or import individual objects to avoid retyping module names over and over.
- Don't pollute the namespace!

- Reusing code

- If you are copying/pasting more than 2-3x, consider using functions, classes and loops

- Formatting

- Python standard (PEP8) recommends 80-character line lengths

- Comments

- Comment your code or use Markdown cells
- Helps with understanding and grading.

- I don't grade on style, but it will help with partial credit if I can understand what you're doing!

```
import qiskit
import qiskit as qs
```

One blank line →

```
from qiskit import QuantumCircuit
from qiskit import *
```

**Danger!**

```
QiskitRuntimeService(var1, var2,
```

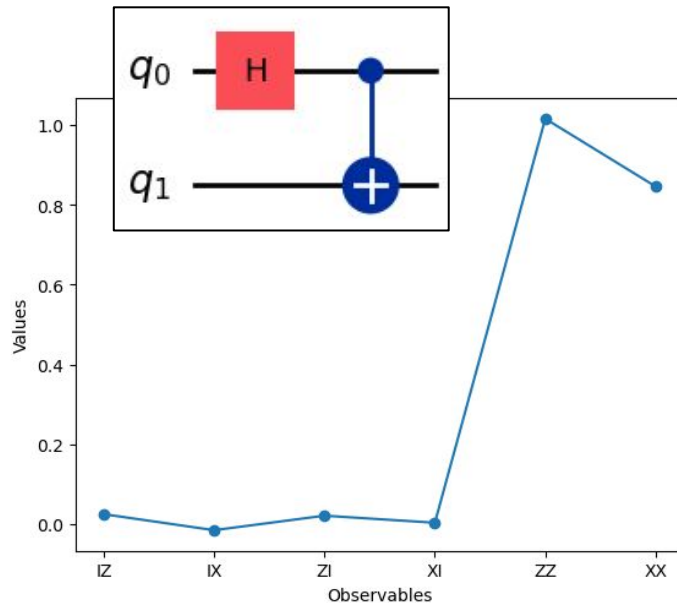
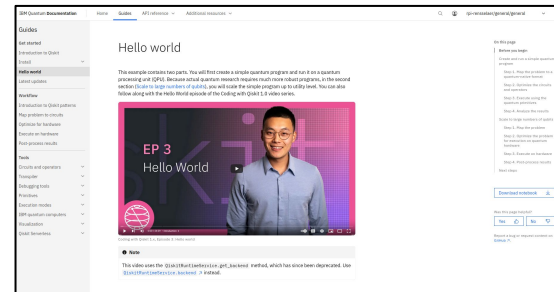
Needs to side-scroll on small monitors.

```
QiskitRuntimeService(var1,
                      var2,
                      var3,
                      var4)
```

Line breaks make it easier to read.

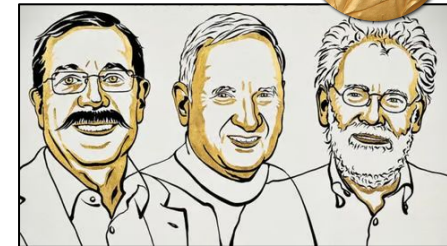
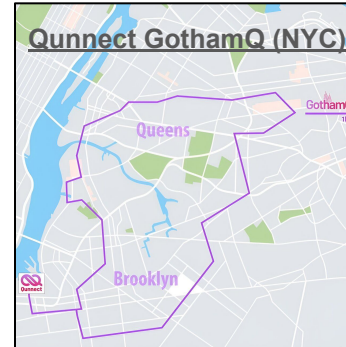
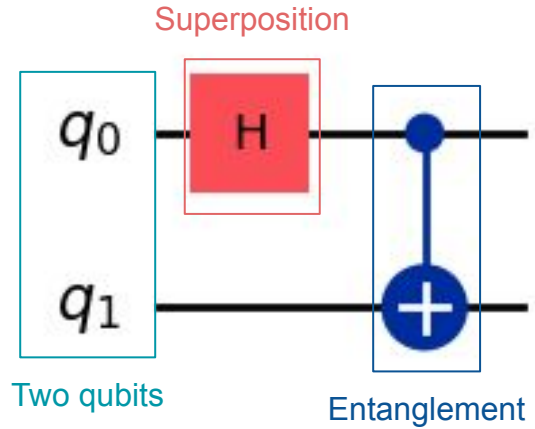
# What the heck did I run?

- The Qiskit “Hello World” example:
  - Simple 2-qubit test.
  - Meant to be a sanity check on our computing environment.
- We blindly copied/pasted a bunch of magic and got a plot.
- What did it actually do?
  - Something very important!
  - We’ll spend this class and next working up to it.



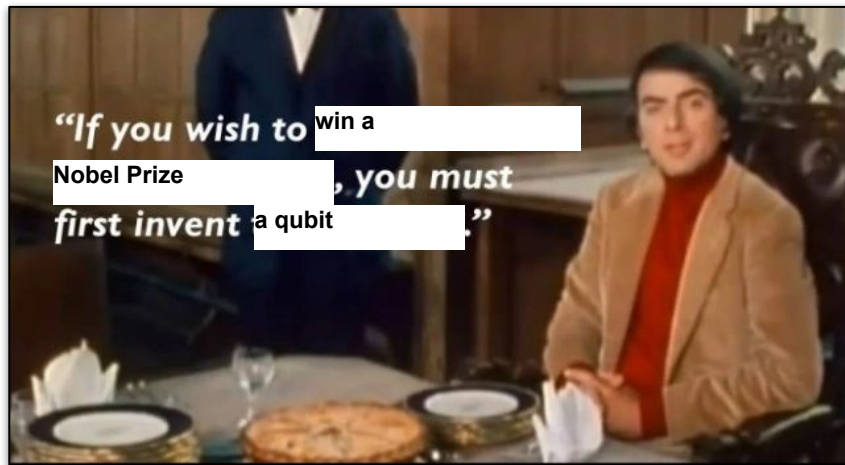
# The Bell State

- Theory developed by John Stewart Bell in 1964.
- The simplest demonstration of quantum phenomena:
  - Superposition
  - Entanglement
  - Ruling out “hidden variables”
- Won the 2022 Nobel Prize
  - Alain Aspect, John Clauser, Anton Zeilinger.
  - Experimental validation and theoretical development.
- This is the building block of all quantum technologies.



# But first...

- ...you need some qubits.
  - ...and ways to operate on them.
  - ...and ways to entangle them.
- ...preferably stable ones.
- ...preferably with a Python API.
  - We want to make a computer after all.
  - ...and Python is the second best programming language for everything.



# The DiVincenzo Criteria

We need a quantum system that:

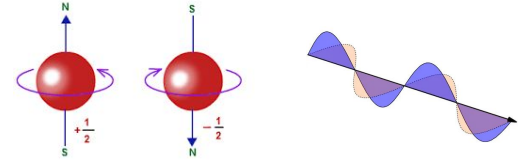
1. Have two (or more) well-characterized quantum states.
2. Can be reliably initialized into a known state.
3. Can be controlled through a set of universal operations.
4. Can be measured through a controllable readout process.
5. Have long coherence times.

For networking, we also need:

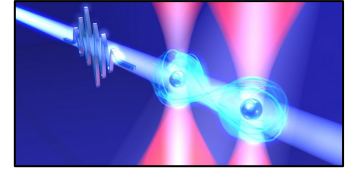
6. The ability to convert stationary and flying qubits.
7. The ability to transmit flying qubits between locations.



D. DiVincenzo. (2000) "The Physical Implementation of Quantum Computation",  
[Fortschritte der Physik 48 \(9–11\): 771–783. \(arXiv\)](#)



Two-Level Systems

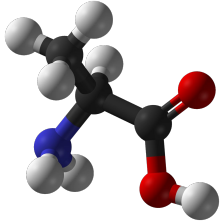


Initialization, Control, Readout

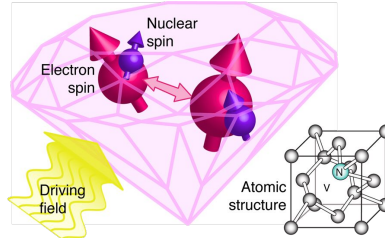


Isolation from environmental noise

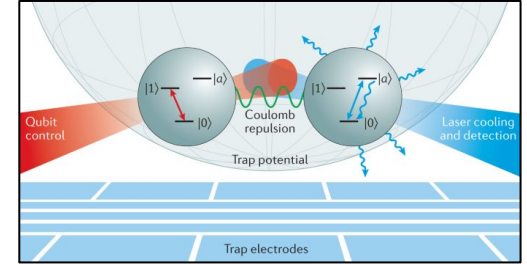
# Criterion 1: Two-Level Quantum Systems



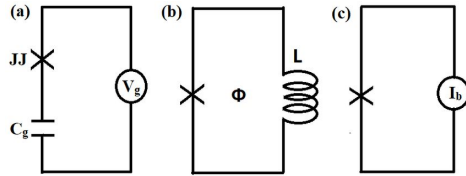
Nuclear Magnetic Resonance



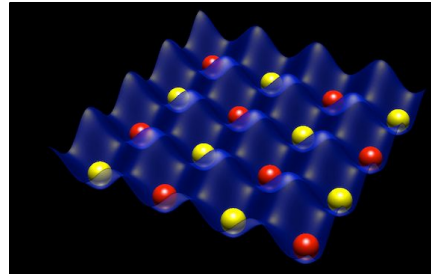
Crystal Spin Defects



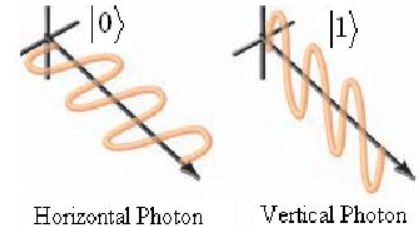
Trapped Ions



Superconducting Circuits



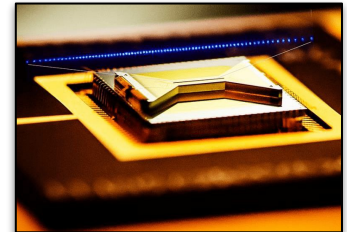
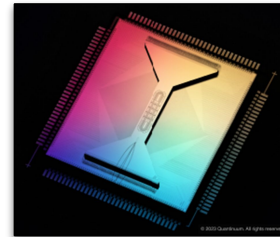
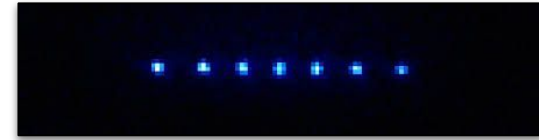
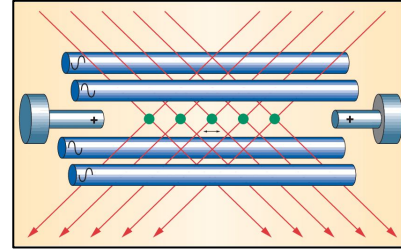
Neutral Atoms



Single Photons

# Trapped Ions

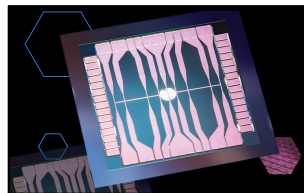
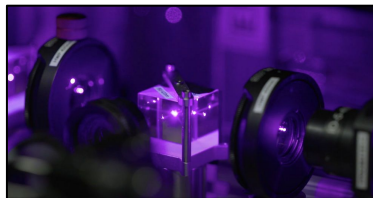
- Individual ions of  $\text{Yb}^+$  or  $\text{Ba}^+$ 
  - Trapped by electrostatic fields
  - Addressed and readout by lasers
- Scales to 100s of qubits per chip
  - Long coherence time (seconds to minutes)
  - All-to-all connections
  - Networking needed
  - Error correction has been demonstrated
- One of the first modalities tested
  - Spinoff of atomic clock technologies
  - Developed at NIST and Sandia





# Neutral Atoms

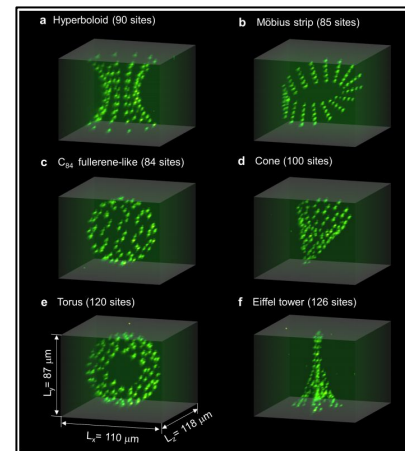
- Atoms are cooled to nK temperatures with lasers
- Magnetic fields and lasers are used to trap atoms in a grid
  - Nearest-neighbor connectivity
  - Atoms can be physically moved
- Very long coherence times
  - Several seconds
  - Error correction has been demonstrated
- Can also be used as quantum memory.



Infleqtion



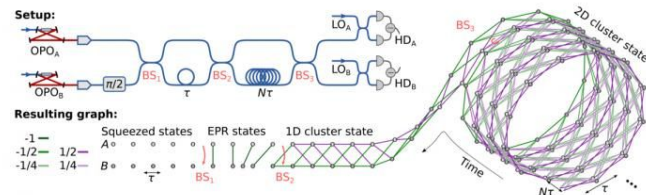
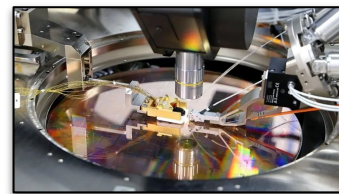
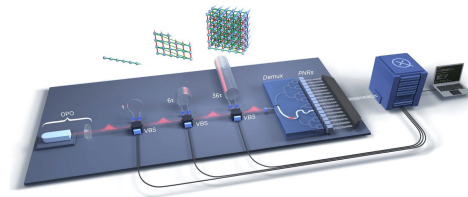
atom  
computing



QERA  
Computing Inc.

# Photons

- Photons can be used as qubits
  - Polarization
  - Resonant states in loops and cavities
- Integrated photonics can yield a high density of qubits
  - Millions of physical qubits
- Noise-resistance is a double-edged sword.
  - Reliable computations
  - Challenging to address and manipulate qubits.
- Necessary for networking.

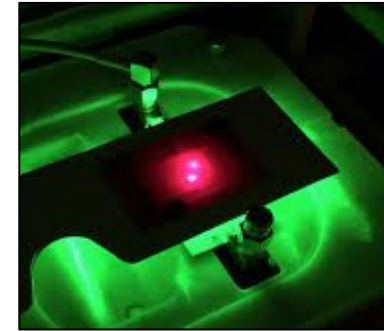
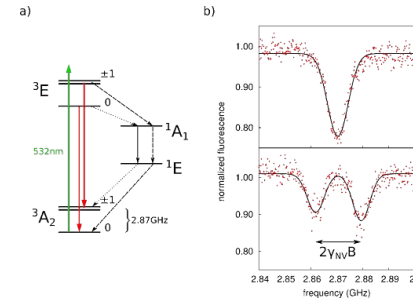
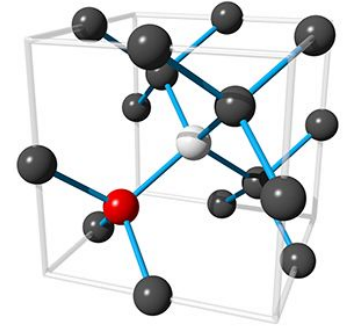
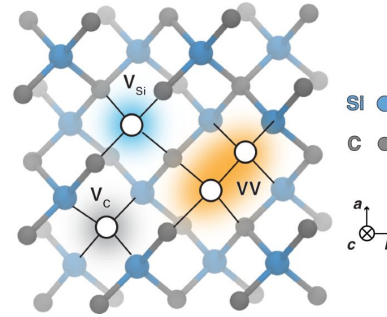


XANADU

Ψ PsiQuantum

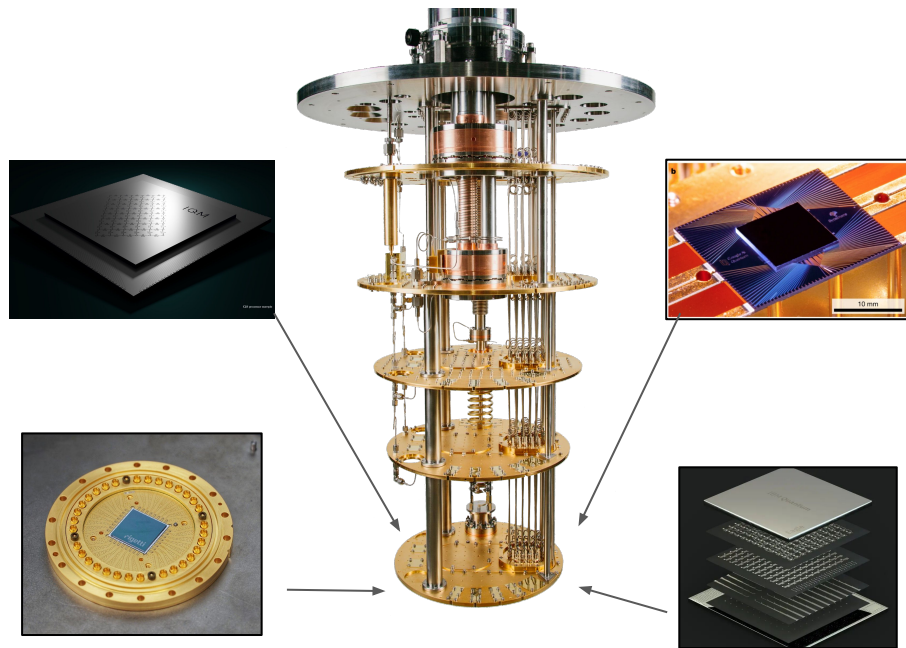
# Crystal Defects

- Defects in crystals can create spin and energy structures in crystals
  - Wide-bandgap semiconductors
  - Diamond, SiC, hBN, ...
- High temperature operation
  - Up to 800K for some devices!
  - Low temperature improves resolution.
- Mainly of interest for quantum sensing & networking.
  - Can make very tiny sensors
  - Can transduce light to RF and back



# Superconducting Circuits

- Superconducting Josephson junctions coupled to a capacitor or inductor.
  - ~10mK temperatures
  - Resonant oscillation modes at microwave frequencies
- Well-rounded performance
  - Fast gate times (~100s of ns)
  - Moderate coherence times (~100s of  $\mu$ s)
  - 100s of qubits, various connectivity



IQM



Google AI  
Quantum

rigetti

IBM Quantum

# What kind should I pick?

- Roughly a dozen competing methods
  - Different benefits and tradeoffs
  - Everyone thinks theirs is the “one true way.”
- It’s not clear yet which hardware will “win.”
  - It took 20 years for silicon-based CMOS\* devices to dominate classical hardware
  - It took another 20 years for x86 to become the dominant CPU architecture
- There may never be a winner.
  - Superconductors are fast, but require cryogenics and have finite connectivity.
  - Ions are stable, but slow and require networking to scale.
  - Atoms can scale and are stable, but are very slow.
  - Photons are so stable they’re challenging to control.
  - Crystal defects are moderately fast, but hard to manufacture.
  - ...

\*Complementary Metal-Oxide-Semiconductor - basically all microelectronics today

# All different, yet all (mostly) the same

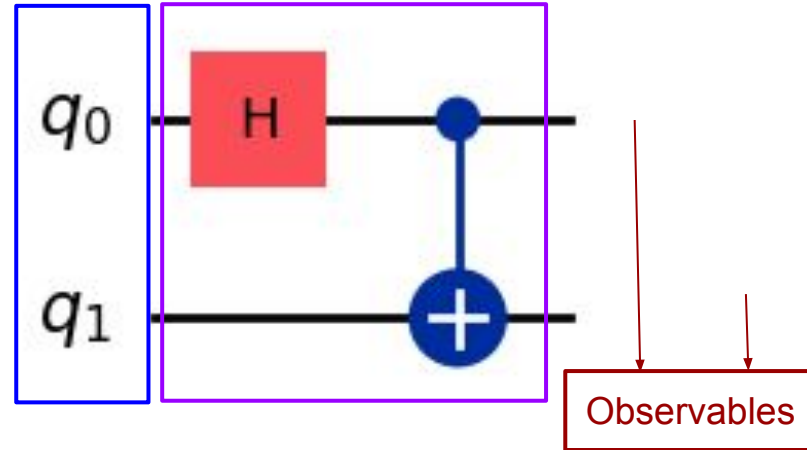
- Regardless of the hardware implementation, the theory is the same.
- Schrodinger's equation
  - Apply a Hamiltonian to a quantum state
  - The energy values contain information on the state.
- The exact values for these will vary.

$$E\Psi = H\Psi$$

Energy (scalar)

Hamiltonian (matrix)

Qubits' State (vector)

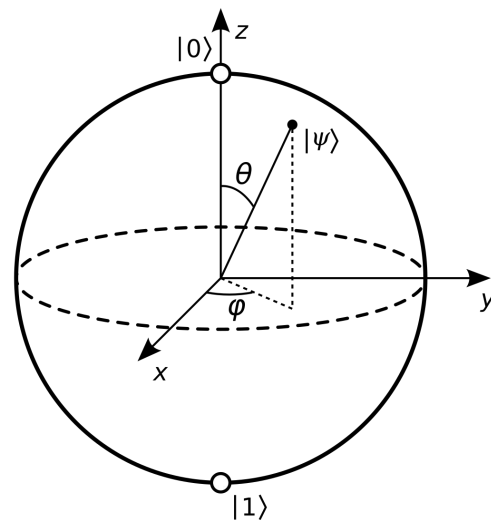


# Quantum States

- The state of any qubit (or group of qubits) is represented by a vector.
  - The elements of the vector are complex numbers.
  - The length of the vector is always 1.
- Single qubit states are plotted on the Bloch Sphere (unit sphere).
- The Z-axis is taken to be the computational basis.
  - The  $|0\rangle$  and  $|1\rangle$  values are the basis states
  - Qubits are measured with respect to the computational basis.

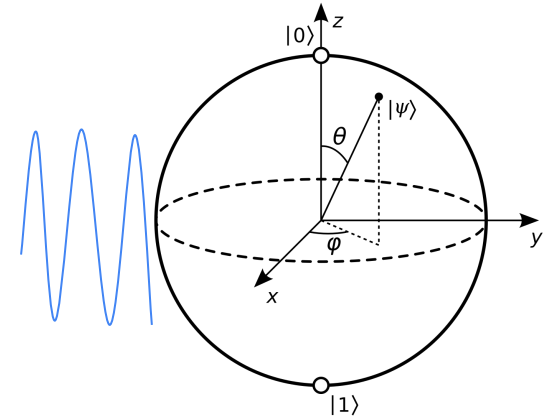
$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle \quad |\alpha^2 + \beta^2| = 1$$

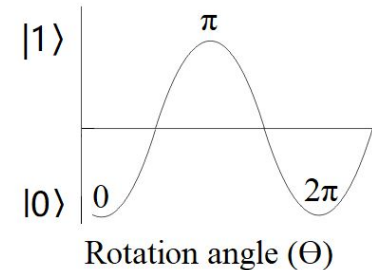


# Criteria 2-4: Initialization, Control and Readout

- We use photons to interact with qubits
  - Radio/Microwave (RF) - Superconducting, spins
  - IR/Visible/UV (Laser) - Ions, neutral atoms, crystal defects
- Carefully timed RF/Laser pulses cause qubits to change state.
  - Align a signal generator to the qubit's resonant frequency
  - Send the signal to a modulated RF or laser source
- Duration and phase of the pulses determine the resulting qubit state.



Pulses cause the qubit statevector to rotate



The pulse duration determines the new state



# Quantum Gates

- Pulses act as single quantum computational instructions, or gates.
- A square  $2^n \times 2^n$  matrix
  - Sometime called an operator
  - Defines how probability amplitudes are exchanged.
- Unitary matrix
  - Measurement probability among all basis states is conserved.
  - Measurement is always a real number.
  - Self-adjoint.

$$\mathbf{X} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\mathbf{Y} = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$\mathbf{Z} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\mathbf{H} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$A = \begin{pmatrix} 1+i & 2-i \\ 3i & 4 \end{pmatrix}$$

$$\mathbf{CX} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$$\mathbf{SWAP} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$U^\dagger U = I$$
$$U U^\dagger = I$$

$$A^\dagger = \begin{pmatrix} 1-i & -3i \\ 2+i & 4 \end{pmatrix}$$

Note: The above oversimplifies a semester-long math/physics course into a single slide.

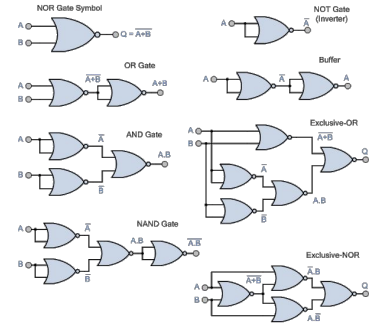
# Quantum Gates

- It can be shown that there is a universal gate set.
  - Any other gates can be built from a handful of single and two-qubit gates.
  - Classical analogy - NAND gate
- Two-qubit gates that create entanglement cannot be decomposed into separate single-qubit operations.

$$U(\theta, \phi, \lambda) = \begin{pmatrix} \cos\left(\frac{\theta}{2}\right) & -e^{-i\lambda} \sin\left(\frac{\theta}{2}\right) \\ e^{i\phi} \sin\left(\frac{\theta}{2}\right) & e^{i(\phi+\lambda)} \cos\left(\frac{\theta}{2}\right) \end{pmatrix}$$

A general 1-qubit gate

$$\mathbf{T} = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\frac{pi}{4}} \end{pmatrix}$$



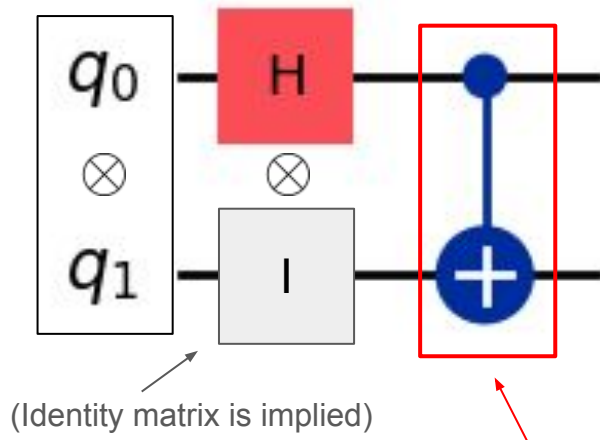
Classical Universal Gates

# Composing Things

- We (mathematically) assemble individual gates and qubits using the tensor product.
- Mechanically, you “tile” the elements onto each other.
- Entangled things cannot be decomposed.
  - Tensor product states need  $2N$  storage elements
  - Entangled states require  $2^N$  storage elements.
  - This is why classical representations of quantum states scale exponentially.

$$|0\rangle \otimes |0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ 0 \begin{pmatrix} 1 \\ 0 \end{pmatrix} \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} = |00\rangle$$

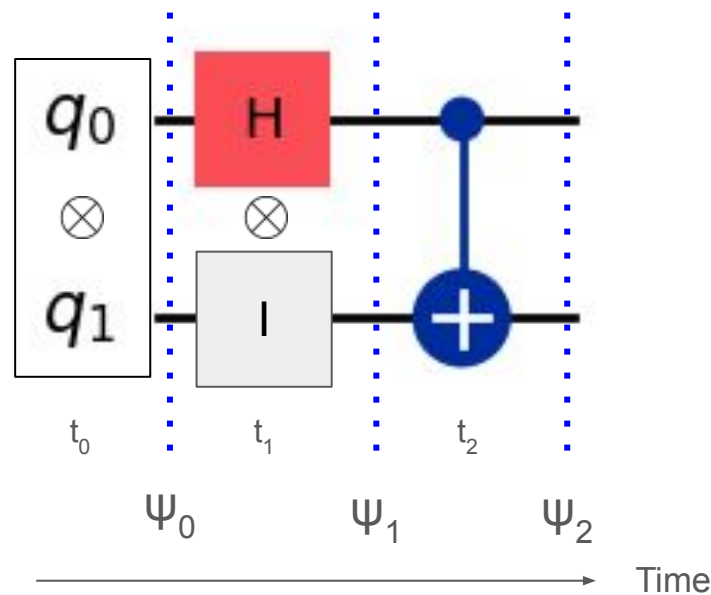
$$|0\rangle^{\otimes 2} = |00\rangle$$



This is a 4x4 that cannot be decomposed into two 2x2's!

# Composing Things

- Recall matrix-vector multiplication
  - Tip the vector on its side.
  - Drop it through the matrix, multiplying the aligned elements.
  - Sum the results.
- Quantum circuits read left to right
  - Each qubit has a “wire” or world-line.
  - Time flows to the right.
  - Gates are applied at each step
- Equivalent math goes right to left



$$|\psi_0\rangle = |0\rangle \otimes |0\rangle$$

$$|\psi_1\rangle = (\mathbf{H} \otimes \mathbf{I})|\psi_0\rangle$$

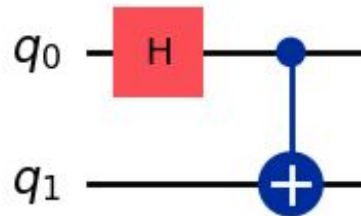
$$|\psi_2\rangle = \mathbf{CX}|\psi_1\rangle$$

# Generating Entanglement

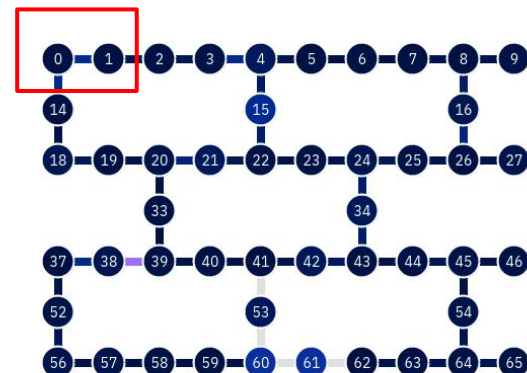
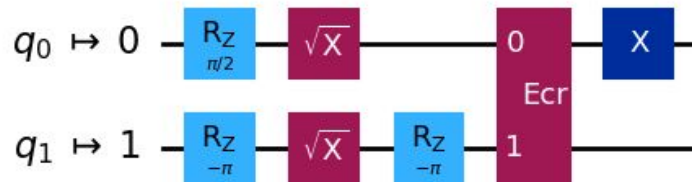
- Entanglement is how we compose qubits into collective systems.
  - The multi qubit system acts as a collective, regardless of physical separation.
  - Individual qubits participating in entanglement are correlated with their partners.
- Physically, we generate entanglement with a coupled pulse that acts on both qubits simultaneously.
- Mathematically, we represent entangling gates with “controlled gates”
- Entanglement is what enables us to represent more information

# Transpilation

- The signal generators for quantum computers typically only support a few “native” operations.
- Entangling gates can only be applied on qubits that are physically near each other.
- The transpiler tries to find an optimum, mathematically-equivalent configuration.



Global Phase:  $3\pi/4$



# Measurement

- Measurement is performed with respect to an “observable.”
  - Observable is a unitary matrix
  - Measurement projects the qubit statevector onto the observable axis according to its probability amplitudes.
  - Matrix eigenvectors are the possible outcomes
- A qubit is a “3D-bit” so we may need to look at it a few different ways.
- Pauli Z, X operators are the most common observables.
  - Z-axis: Standard 0/1 basis
  - X-axis: “Hadamard basis”
- Many repeated measurements give us the expectation value of the observable.
- Measurement ends the quantum part of the computation.
  - The measured qubits can no longer participate.
  - They can be re-initialized and used for other things

# Doing all of this on a real machine

- IBM launched their cloud devices in 2016
  - 5 qubits was a lot!
  - 14 was “premium”
  - Other vendors followed this model
- Composer - Draw circuit diagrams by hand
  - Good for playing around and understanding
  - Cumbersome for big circuits
- Qiskit - Write circuits in Python
  - Define qubits and gates using quantum circuits
  - Define measurement and experiment types using Primitives
  - Connect to hardware or backend simulators



Break

Activity: Making things in Qiskit

# Services & Backends

- The *service* runs locally and manages interactions with *backends*
- A backend is a quantum device or a simulator
- Demo: Setup a service and a backend

# Quantum Circuits

- Quantum circuits are the programs we run on the machine.
- Start by creating an empty circuit
  - Append Gates
  - Draw it (optional)
- Use the Statevector object for debugging small circuits.
- Demo

# Transpilation

- We need to transpile the circuits we made so they can run on the native gates.
- Qiskit provides a “pass manager” that performs this step automatically.
  - You can write your own pass managers too

# Running Stuff

- Qiskit IBM Runtime provides “Primitives” for running quantum circuits on the hardware.
  - Sampler - Run the circuit and measure the qubits. Repeat for a predefined number of “shots”
  - Estimator - Compute the expectation value of an observable in a quantum circuit. Runs on top of a sampler and implements its own post-processing to save you some steps.
- We will focus on samplers this week

# Post-Processing Results

- After running a job, we need to do something with the results
  - Statistics
  - Plotting
  - ...
- In this example, we plot a histogram of the counts of each state we measured.