Quantum Computing & Applications for Engineering

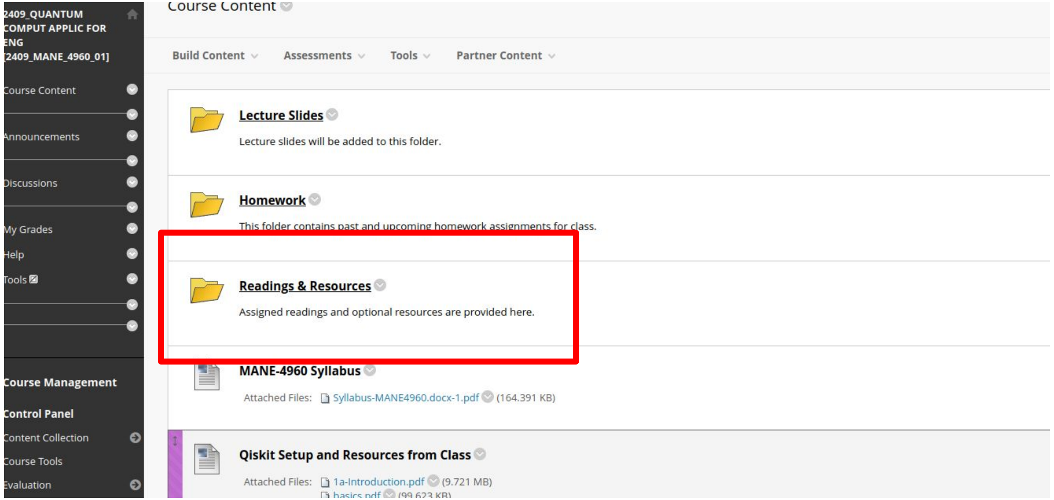
10/1/2024

Today's goals

- Qubit Errors and Noise
- Measuring Decoherence Processes
- Noise models
- Example quantum algorithm workflow
- Discuss HW

Announcements

- New resources on LMS
  - Textbook by Tom Wong
  - Recommended Reading, Websites
- Bonus quantum computing survey
  - Complete by 10/11 for +10 points

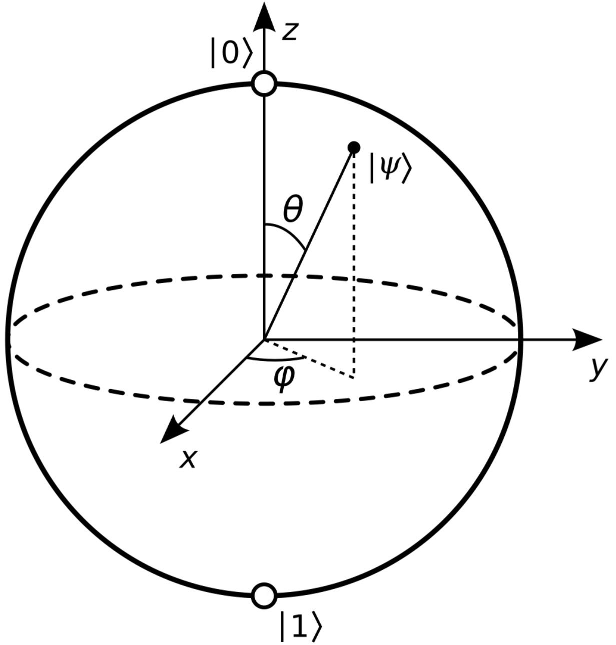


HW2 Questions?

- Phase troubles?
- Flipping a sign vs. flipping a qubit

Pure States

- So far, we've been talking in terms of *statevectors*.
- A statevector represents what is known as a *pure state*
- Each element tells of how much of each basis state we have in the superposition.
- **What happens if qubits aren't perfect?**



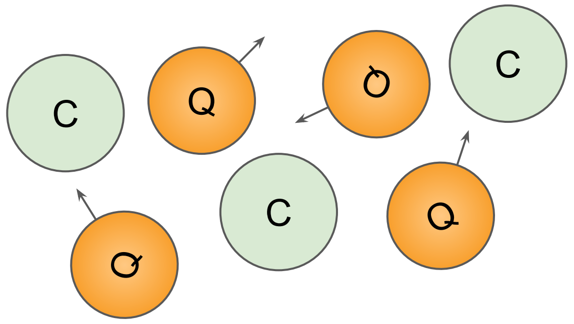
The Bloch sphere represents a pure state

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

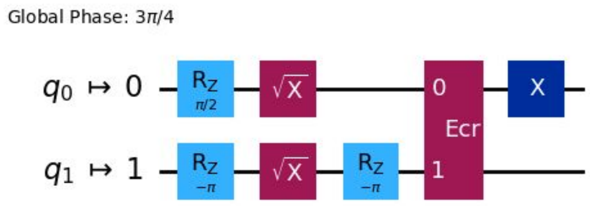
$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle \qquad |\alpha^2 + \beta^2| = 1$$

Density Matrices

- Consider the case where we have a bunch of qubits in the same system…
  - Clouds of atoms
  - Liquid NMR
  - NV Centers
- …or a single state prepared and measured over and over.
  - Quantum circuits and algorithms
  - Quantum communication protocols
- The density matrix gives us a unified representation of quantum coherence and classical mixtures.



A mixture of quantum and classical states



Running a circuit over and over

Density Matrices

- Last time (and on HW3) we showed that the outer product of two statevectors gives us a matrix.
    - Projectors, if we're talking about the basis states.
- In general, the outer product of two quantum statevectors gives us a *density matrix*.
- Density matrices represent a statistical mixture of quantum *and* classical states.

$$P_0 = |0\rangle\langle 0| \quad P_0 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

$$P_1 = |1\rangle\langle 1| \quad P_1 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\rho = \begin{pmatrix} \rho_{00} & \rho_{01} \\ \rho_{11} & \rho_{11} \end{pmatrix}$$

Density Matrices & Mixed States

- Consider the Hadamard state.
- The statevector shows we have a 50-50 superposition.
- The density matrix shows we also have a 50-50 classical mixture!
  - The off-diagonal elements are the quantum, or “coherent” parts.
  - The diagonal elements are the classical parts.
- How do we know if we have a quantum state?

$$|+\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}$$

$$\rho_+ = |+\rangle\langle+|$$

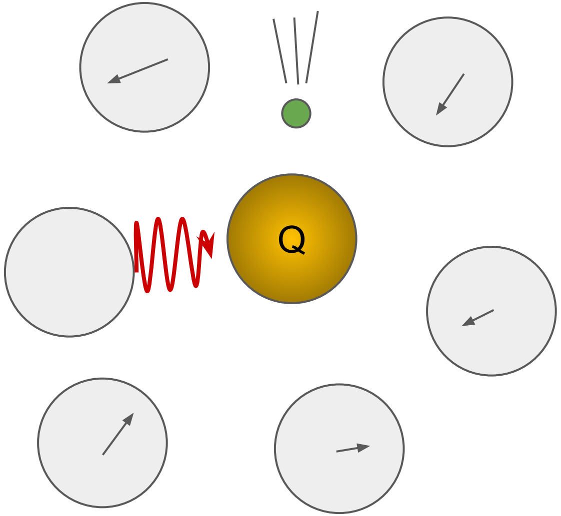
$$\rho_+ = \begin{pmatrix}\frac{1}{2} & \frac{1}{2}\\ \frac{1}{2} & \frac{1}{2}\end{pmatrix}$$

Mixed States

- The quantum, or coherent, part we can detect and estimate by measuring in different bases.
- Mixed states can only be described with density matrices.
- Effects in the environment transfer information from the coherent elements into the classical elements.
- This leakage of quantum information is termed Decoherence.
    - A measurement is intentional decoherence.
    - Unintentional decoherence occurs when:
        - Our qubits get entangled with something else in the environment.
        - Our qubits get measured by something else in the environment.

Physical Decoherence

- Schrodinger’s equation assumes a quantum system that is perfectly isolated.
  - Real systems lose energy to the environment
  - Formally, the environment is called a “bath”
- Open quantum systems are governed by the *Lindblad Master Equation*.
  - The real world is non-unitary.
  - Quantum term plus a dissipative term.
  - In the limit of zero dissipation, the Schrodinger equation is recovered.
- Lindblad simulations are used in qubit and quantum device design.



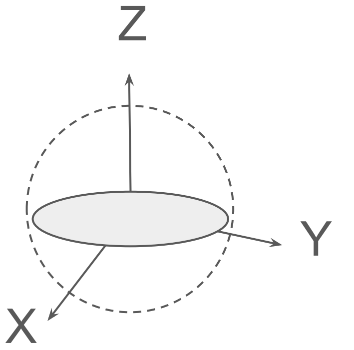
$$\frac{d\rho}{dt} = \boxed{-i[H,\rho]} + \boxed{\sum_k \left(L_k \rho L_k^\dagger - \frac{1}{2}\left\{ L_k^\dagger L_k, \rho \right\} \right)}$$

Coherent Evolution | Decoherence

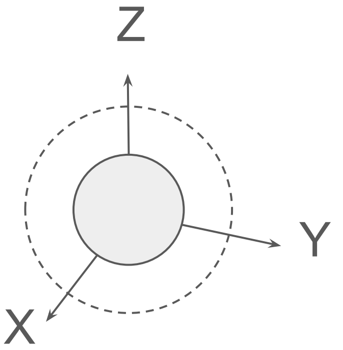
QuTiP - Quantum Toolbox in Python

Decoherence of Qubits

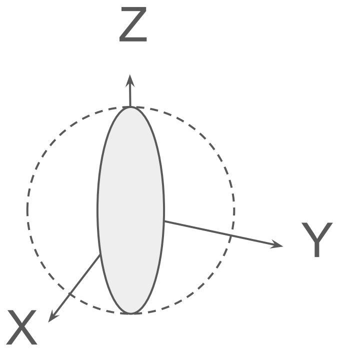
- The quantum state in a qubit (or set of qubits) is very fragile
  - Thermal vibration
  - Stray E/M fields
  - Radiation
- Decoherence is the process that returns quantum states back to classical ones.
  - Measurement is a “controlled” decoherence process.
  - Noise is measurement done by the environment.
- There are several “channels” for decoherence.
  - Amplitude damping
  - Phase damping
  - Depolarization (combination of the first 2)
- Decoherence channels “squish” the reachable parts of the Bloch sphere



Amplitude Damping



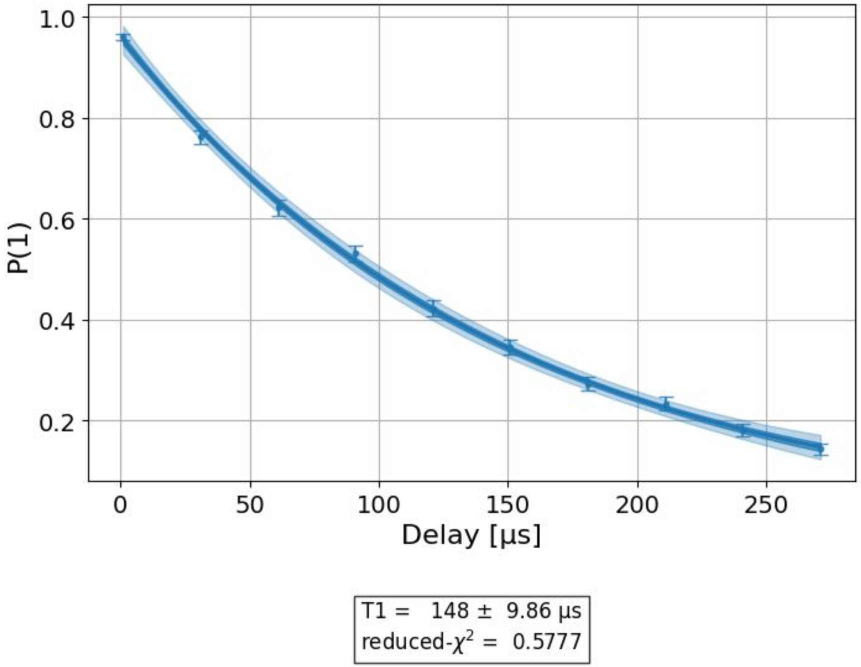
Depolarization



Dephasing

Amplitude Damping

- A qubit placed in an excited state only stays there temporarily before losing energy to the environment.
    - De-excitation
    - Spin relaxation
- This decoherence process is called *amplitude damping*.
- The “decay constant” that represents its de-excitation back to the 0 state is denoted $\mathbf{T_1}$.



For superconducting qubits, T_1 values of ~150 μs are typical

Amplitude Damping

- We can measure T_1 using individual qubits, or many in parallel.
- Experiment Algorithm:
  - Start with a qubit in the 0 state.
  - Apply an X gate to put it in the 1 state.
  - Wait for a predefined delay time, t_d.
  - Measure the qubit.
  - Tally the expectation value with the (I-Z)/2 operator.
  - Repeat for N_d delay times.
- Fitting an exponential function to the data points gives us T_1.

Phase Damping

- When we place a qubit in superposition, it has a well-defined phase.
    - Consider the + and - Hadamard states.
- As it undergoes interactions with the environment, the phase drifts.
- Coherent phase information leaks into the classical elements of the density matrix.
- The time constant for this phase coherence loss is called $\mathbf{T_2}$ or $\mathbf{T_2}^*$, depending on how it is measured.

Phase Damping

- T_2 is measured using a *Hahn Echo* sequence
  - Apply a Hadamard gate
  - Wait for a delay time
  - Apply an X gate
  - Wait another delay time
  - Apply a Hadamard
  - Measure
- During the delay the qubit precesses around the Z-axis due to its natural frequency.
  - Applying an X gate flips it 180 degrees.
  - Applying the last Hadamard should bring us back to 0, if the coherent phase is well defined.

Phase Damping

- T_2^* is an alternate measurement that can also provide an estimate of the qubit's intrinsic frequency.
- Measurement protocol:
  - Apply a Hadamard gate
  - Wait a delay time
  - Apply an R_z rotation around the Z-axis.
  - Apply another Hadamard gate
  - Measure
- If the R_z gate and the natural precession offset each other perfectly, then the qubit will return to zero.
- Offsets from zero occur as the phase coherence decays.

Qubit Noise

- The combination of the T_1 and T_2 processes lead to single-qubit errors.
- Errors occur when the statevector is not aligned with where you'd expect it to be, based on the operations you've performed.
  - Bit flip - T_1 errors
  - Phase flip - T_2 errors
- If your circuit takes substantially longer than T_1 or T_2 to run, you have a higher probability of errors.
- Many other noise sources too.

Gate Noise

- Applying gates isn't always a perfect process.
  - Frequency drift in lasers or RF
  - Qubit manufacturing variations
- Two-qubit gates are more challenging than single qubit gates.
- We can measure gate fidelity to estimate how many gates we can run before the noise adds up.

Noise Models

- Quantum simulators can model the noise behavior of quantum computers.
    - Useful when you want to check if something is even worth running.
    - Essential when you don't have hardware access.
- Most quantum computing research is done with noise models.
- "All models are wrong, some are useful!"

Noise Models

- A simulator with noise models the evolution of the density matrix with each gate application.
- The effects of decoherence are applied using *Kraus operators*.
- Qiskit Aer lets you define your own Kraus operators, or pick from a library of common error channels.
- Once we have T_1 and T_2, we can define Kraus operators for these processes.

$$\rho' = \sum_k \boxed{E_k} \rho E_k^\dagger$$

The new density matrix after the application of *k* different noise channels with Kraus operators

$$\gamma = 1 - e^{t/T_1}$$
$$E_0 = \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{1-\gamma} \end{pmatrix}$$
$$E_1 = \begin{pmatrix} 0 & \sqrt{\gamma} \\ 0 & 0 \end{pmatrix}$$

Amplitude Damping

$$\lambda = 1 - e^{t/T_2}$$
$$E_0 = \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{1-\lambda} \end{pmatrix}$$
$$E_1 = \begin{pmatrix} 0 & 0 \\ 0 & \sqrt{\lambda} \end{pmatrix}$$

Phase Damping

Using T_1 and T_2, we define *damping parameters* that we use to make the Kraus operators

Kraus Operators - Example

- Let's look at how T_1 and T_2 processes affect a Hadamard gate.
  - Let T_1 = 150 μs
  - Let T_2 = 50 μs
- Assume the gate time T_g is 60 ns
- Demo Notebook

Noise & Performance Benchmarks

- There are so many noise sources, a zoo of benchmarking techniques has been developed to try to characterize them.
- There’s a hierarchy of benchmarks:
  - Qubit and gate-level
  - Circuit-level
  - Application-level
- Not all benchmarks are created equal.
  - Physical timings and fidelity of operations are relatively objective.
  - Integrated measures at the circuit and gate level can vary widely.
  - Application-oriented benchmarks
- Pay attention to which benchmarks are preferred by different hardware vendors!

Break

What to do with noise?

- The maximum circuit length is constrained by the coherence time and the gate error (especially the two-qubit gate error); circuit depth
- On superconducting systems, circuit depths greater than ~100 start to become very challenging to run.
- On trapped ion systems, circuit depths of 500-2000 are possible.
- No qubit modality has currently gotten better than a 10^{-4} total error rate (all error sources combined)
- For comparison, a typical CPU has error rates below 10^{-14}
- We need something to make use of the qubits we have.

The Fault Tolerance Theorem

- In the 1990's researchers demonstrated that extra qubits can be used to catch and correct errors. Quantum Error Correction
- This means that reliable quantum computations are possible even with imperfect qubits.
- There is a threshold at which error-corrected qubits can be stacked up to recover all lost information.
- That threshold is not known exactly, but is around 10^{-2}-10^{-4}.
- If we can make qubits better than this (we can), then we can achieve fault tolerance.