Quantum Computing & Applications for Engineering

9/24/2024

Measurements, Estimators, and Circuits

Today's goals

- More on measurements and measuring in different bases.
- Using simulators instead of the hardware.
- Using estimators to get expectation values.
- Discussion of industry trends and careers

Points on HW1

- If you are still having technical difficulties, please send me an email.
- If you do your work in a Google doc or other online platform, make sure both Chris and I can access it!
- Come to office hours if you need help understanding Python, Statevectors, Qiskit, or want to discuss anything in more detail
  - We have LOW 4040 10-12 Fridays for most of the semester
  - Make an appointment if you need a different time.
- Corrections will be accepted.

Questions on HW2?

- Bell states?
- Reading?

Recap on Statevectors

- The statevector gives the values of the probability amplitudes *before* we measure the qubit.
- Squaring the amplitude gives us the probability of seeing that particular state when we measure.
- We can simulate how the statevector changes in a quantum circuit, for small numbers of qubits.

$$\alpha|0\rangle + \beta|1\rangle$$

$$\begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

$$\alpha|00\rangle + \beta|01\rangle + \gamma|10\rangle + \delta|11\rangle$$

$$\begin{pmatrix} \alpha \\ \beta \\ \gamma \\ \delta \end{pmatrix}$$

Quantum Circuits and Statevectors

- Every "layer" of a quantum circuit causes the statevector to evolve.
  - Always a matrix-vector multiply
  - Matrix is always 2^n x 2^n (more advanced simulators can take shortcuts)
- Measurement causes the statevector to snap to 0 or 1 with some probability.
- For small numbers of qubits, a Statevector simulator can help with debugging.

Local Simulators

- Qiskit provides the AerSimulator if we want to run things locally.
  - Testing and debugging
  - Different noise models (or no noise)
- Define Sampler or Estimator primates as usual.
- Works up to moderate numbers of qubits (~28-30 on a PC, ~50 on a cluster)
  - Numbers can vary depending on how much entanglement you have
  - Larger qubit counts and circuits can be slow.
- Good for fast turnaround tests.

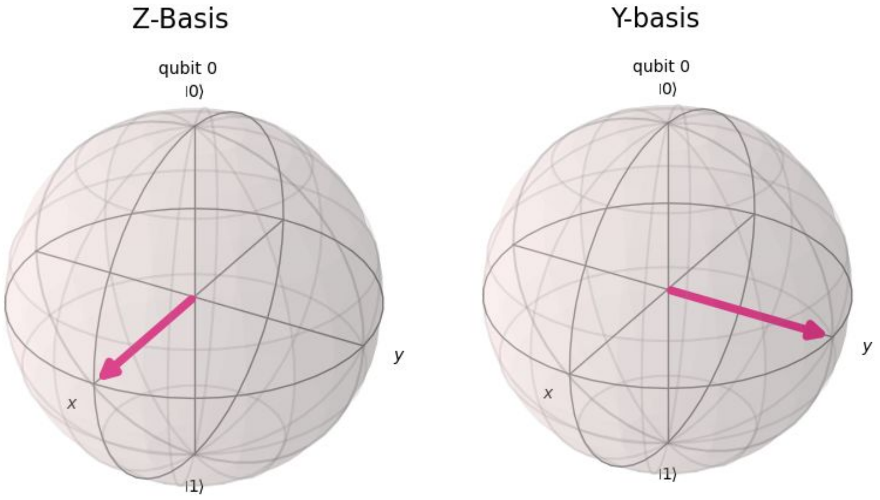
Samplers Recap

- When running a quantum circuit, we measure the final state of the qubits in the computational basis (Z-axis).
- For each shot, we tally the qubit state that we measured and build a histogram.
- This sampling process gives us an estimate of the probability distribution.

Demo - Simulators

Measurements & Bases

- If we only measure in one basis, we don't get all the information!
    - Qubits are "3D bits"
    - Cannot distinguish phase differences
    - We need to look from a few different angles to draw conclusions.
- Physical hardware can only measure in the Z basis.
- We can "fake" measuring in other bases with rotations that change the coordinate system.



How do you tell these apart?

How Measurement Works

- In quantum computing, measurement operations are *projective.*
  - We're protecting the statevector onto the measurement basis.
  - Like casting a shadow.
- Mathematically, we create *projectors*.
  - These are matrix operators.
  - Extracts the probability amplitude of a basis state.

$$P_0 = |0\rangle\langle 0|$$

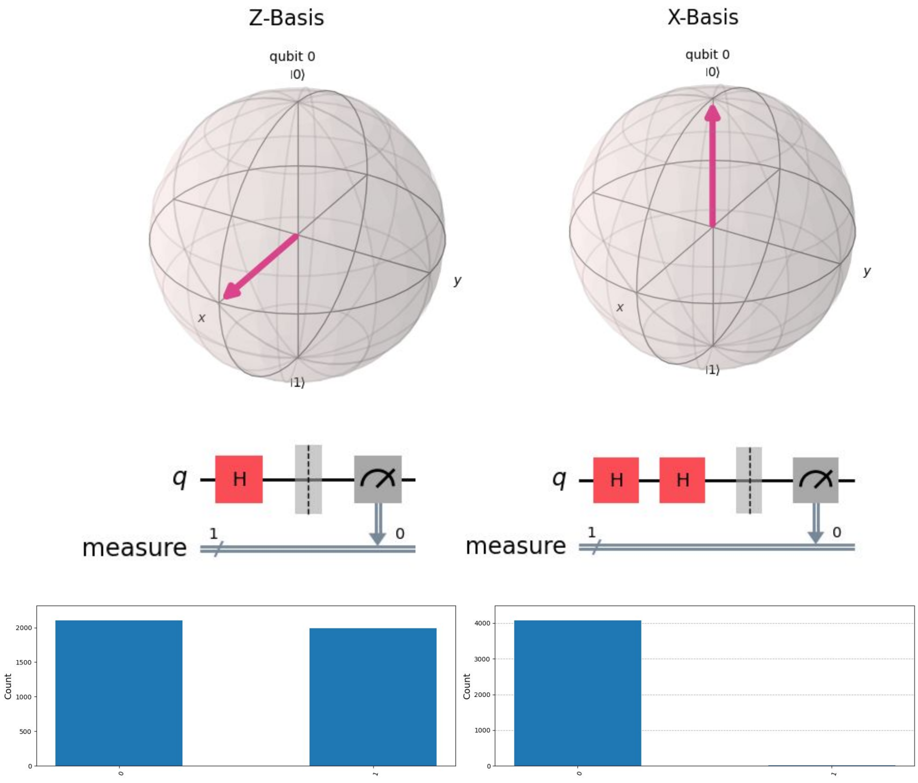
$$P_1 = |1\rangle\langle 1|$$

$$P_0 = \begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

$$P_1 = \begin{pmatrix} 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

Measuring in other bases

- With a coordinate transformation, we can get measurements in other bases:
  - Still a Z measurement physically.
  - Results are the same as if we measured in the other basis.
- Example: Hadamard coin flip
  - Measuring in the Z basis gives us a 50-50 probability of 0 or 1
  - Measuring in the X basis (trivially) gives us 0.



Z **X**

Demo: Measuring in Other Bases

Expectation Values

- So far, we've been *sampling* the quantum states' probability distribution.
    - Use a Qiskit Sampler
    - Tally the occurrence of each basis state bitstring when we measure
- What if we want an average?
    - Minimizing the energy of a system
    - Training a machine learning model
    - Detecting changes in state (e.g., quantum sensing)
- We compute the *expectation value*.

Expectation Values

- Expectation values are variant on projective measurements
  - Measurement is with respect to an *observable*.
  - Observables are any 2^n x 2^n self-adjoint matrix.
- In Qiskit, we use an Estimator to compute expectation values.
  - Estimator is a convenience function that edits your circuit and output.
  - Can use a Sampler too, with extra code and post-processing.

$$E = \langle \psi | \hat{O} | \psi \rangle$$

$$E_0 = \langle \psi | P_0 | \psi \rangle$$

$$E_1 = \langle \psi | P_1 | \psi \rangle$$

Expectation values of measuring 0 or 1

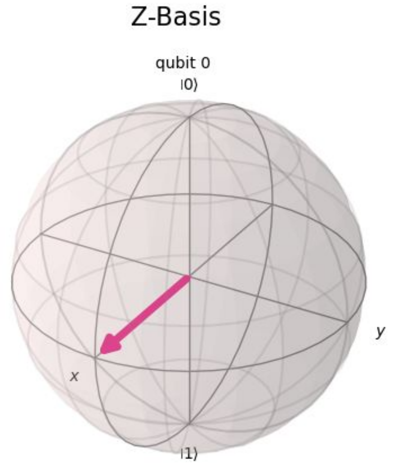
Using Observables

- Can be thought of as the "projection" axis of a measurement
- In Qiskit, Observables are defined in terms of the Pauli matrices.
- Example - A Z-observable is a standard basis measurement, and gives an expectation value in the range [-1, 1]
- Question: What if we wanted an answer in the range [0, 1]?

$$\langle + | \mathbf{Z} | + \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$= \frac{1}{2} \begin{pmatrix} 1 & -1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$= \boxed{0}$$



Sweeping Parameters

- Sometimes, we want to vary how much we're rotating the statevector.
- Qiskit lets us do this in a cleaner way by providing a Parameter object that we can attach to circuits.
- We can create a grid of parameters, or determine them on-the-fly using a classical computation.
    - Grids are for when we want to get an idea of the state.
    - On-the-fly is used for problems in optimization, machine learning, and simulation.

Demo: Expectation Values & Parameters

Break

Recap on Entanglement

- When we have multiple qubits, we use the *tensor product* to compose them.
- Entanglement occurs when we have a state that cannot be expressed as a tensor product of two single-qubit states.

The Bell States

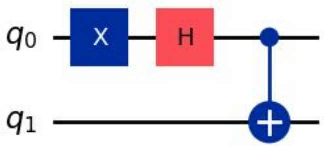
- These are the simplest entangled states.
- In HW2, you are asked to make a circuit that gets you the statevector of all 4 Bell states.
  - “How do I get that sign to flip?”
  - Depends on if you want it to be flipped before or after entangling it.
- Remember that if something is entangled, you need a 2-qubit gate.

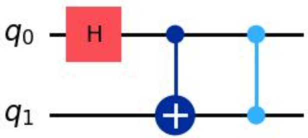
$$|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

$$|\Phi^-\rangle = \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)$$

$$|\Psi^+\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$$

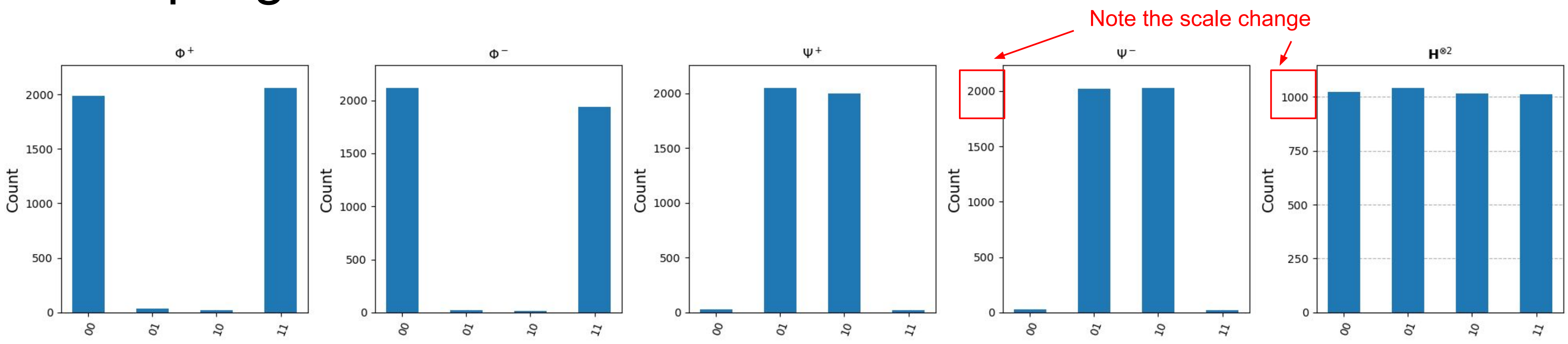
$$|\Psi^-\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$$





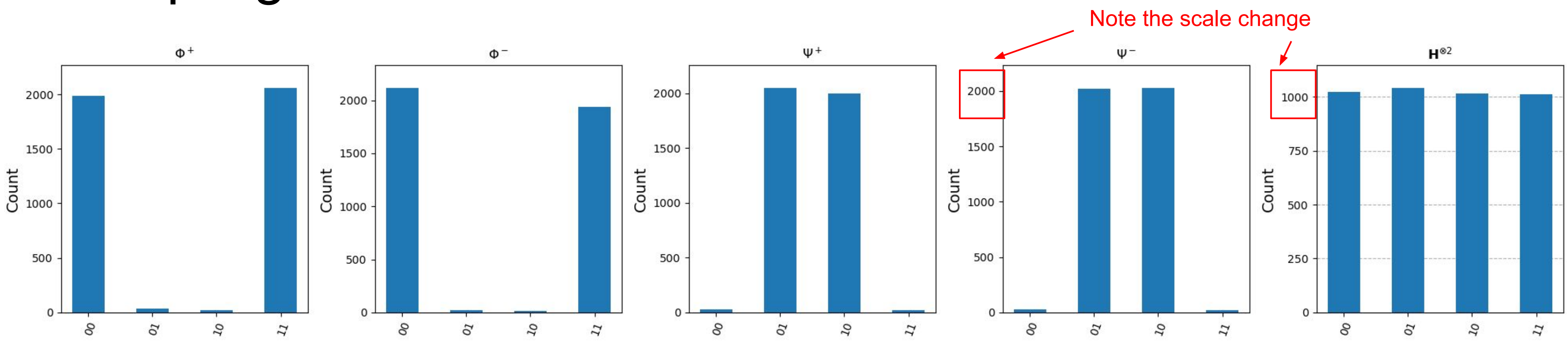
<u>Bonus: Show that these are equal!</u>

Sampling the Bell States



- In the send part of HW2 programming exercise, you sample all 4 Bell states (and an uncorrelated one).
- Can you tell the difference?

Sampling the Bell States

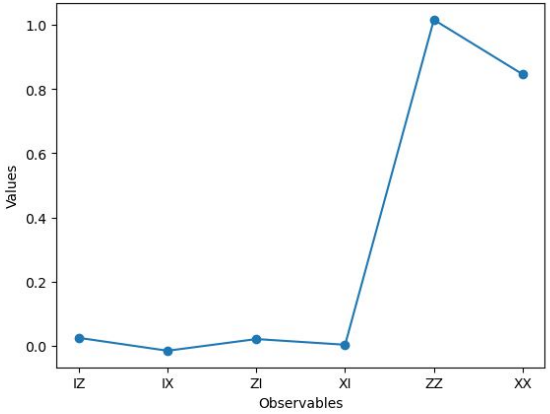


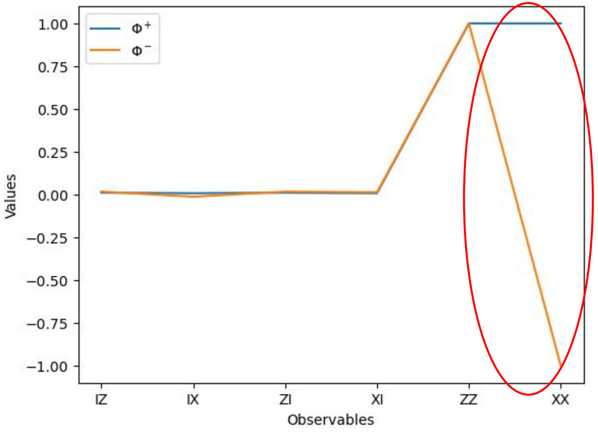
- In the HW2 programming exercise, you sample all 4 Bell states (and an uncorrelated one)
- Can you tell the difference?

No!

Expectation Values of Multi-Qubit Systems

- In the Hello World example in HW1, we ran an estimator problem on one of the Bell States.
- We can tensor-product observables together to get multi-qubit observables
- Looking at the different bases, we can identify the phase flip!



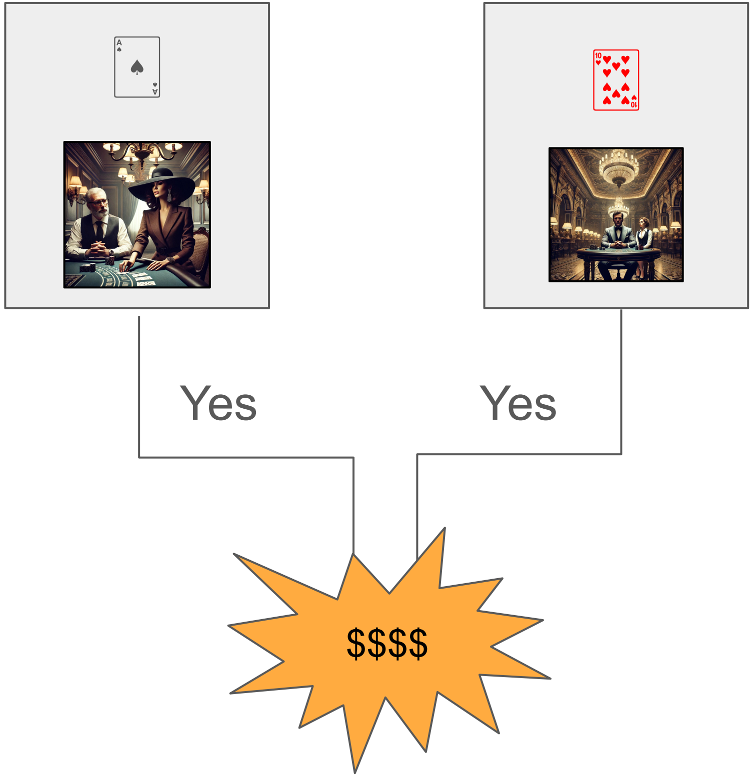


Bringing Down the House - The CHSH Game

- Alice and Bob are visiting a Las Vegas casino that has just unveiled a new two-person game.
- They happen to secretly be quantum physicists and think they can beat the house.
- The Nevada Gaming Commission doesn’t (yet) have rules against using quantum entanglement for advantage.
- This is called the Clauser-Horne-Shimony-Holt (or CHSH) game.

Rules of the Game

- The players go to isolated, soundproof rooms, each with a house dealer.
- They have a yes button and a no button on the table in front of them.
- They are dealt cards simultaneously.
  - If any player's card comes up red, they must both push the same button to win.
  - If both cards comes up black, they must push opposite buttons to win.
- They may discuss their strategy beforehand.



Developing a Strategy

- Casinos make profit by ensuring their expectation value is higher than that of the guests.
  - As long as the house’s expectation value exceeds that of the players, they will always win (in aggregate).
  - An individual player (or group of players) can develop strategies to maximize their expectation value.
- In the CHSH game, the classical strategy with the highest expectation value is to press “No” every time, regardless of the color.
- The maximum win probability is 75% using this strategy.

The Quantum Advantage

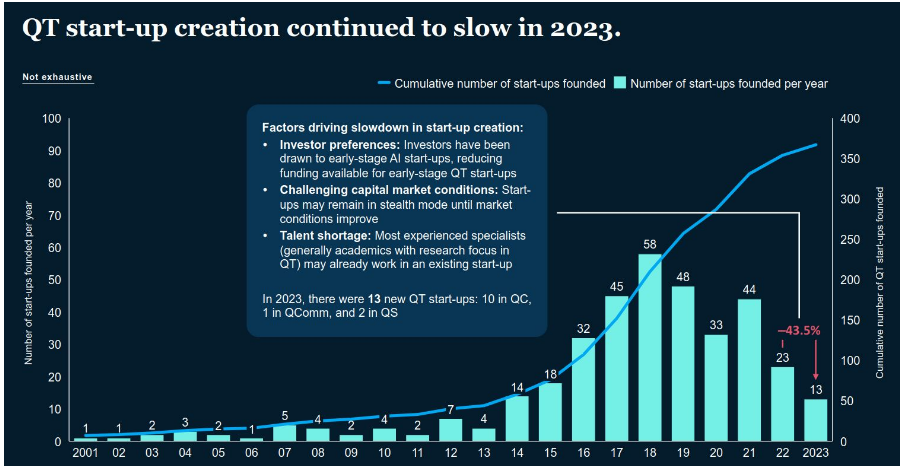
- Alice and Bob each have a qubit that they've pre-entangled before arriving at the casino.
  - The qubit was entangled with a Bell state.
  - A diamond full of NV centers, a gas cell of entangled atoms, …
- They agree on a new strategy:
  - When the card is displayed, the choose a basis and measure their entangled qubits.
  - If Alice and Bob choose the right measurement bases, they can increase their expectation value.
- Q1: What bases should Alice and Bob pick?
- Q2: Can they beat the house?
- Q3: If so, by how much? (What is the maximum winning probability?)

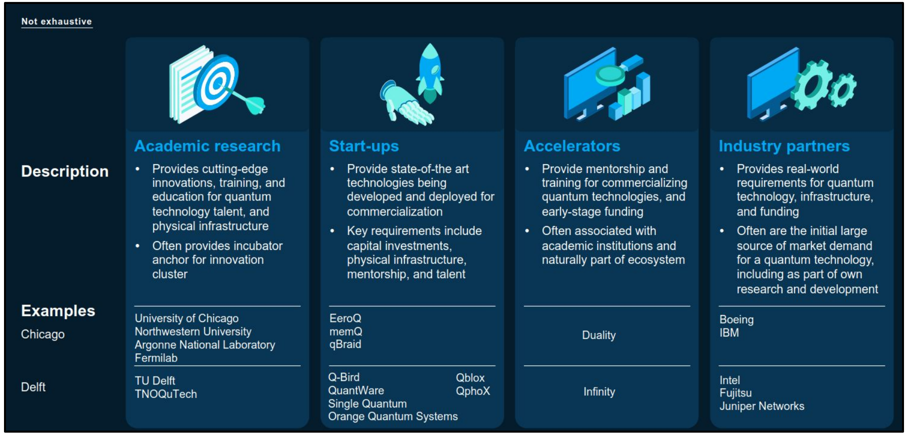
The Quantum Advantage

- Answers:
  - A1: This is your task for HW3. (Using Estimators and Parameters)
  - A2: Yes!
  - A3: They now have a ~13% advantage (83% winning probability)
- Entanglement amplifies the correlations between individual elements (e.g., qubits) over what we can achieve classically.
  - Look back at the sampler plots of the Bell and multi-Hadamard states.
  - Entangling operations are the only way to shape the distribution such that we get sharpened probabilities on the multi-qubit states.

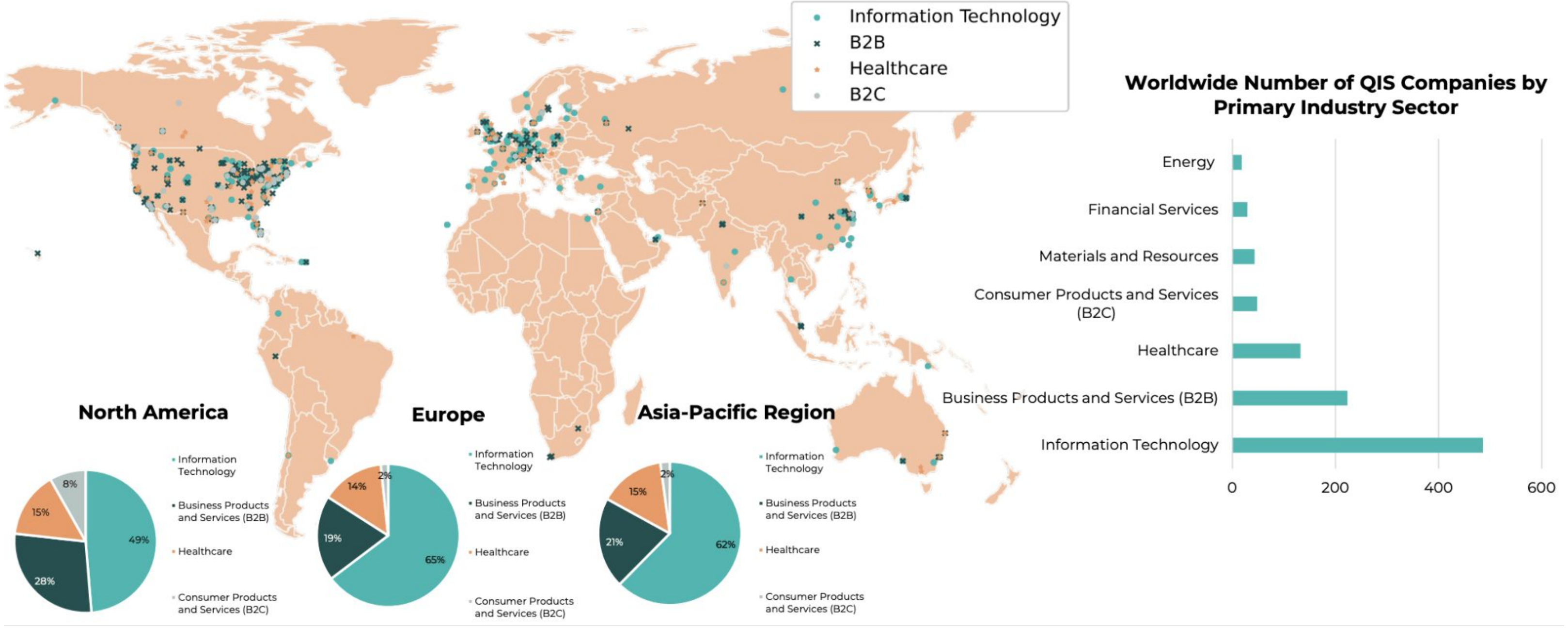
The Industry Landscape

- In HW1, you read the Quantum Technology Monitor
  - Published by McKinsey in April 2024
- Discussion:
  - What did you think?
  - Are there technologies or trends that surprised you?
  - Will there be a quantum winter?
- My Answers:
  - The investment drop?
  - Industry Consolidation?
  - Probably not a winter, but a shift.



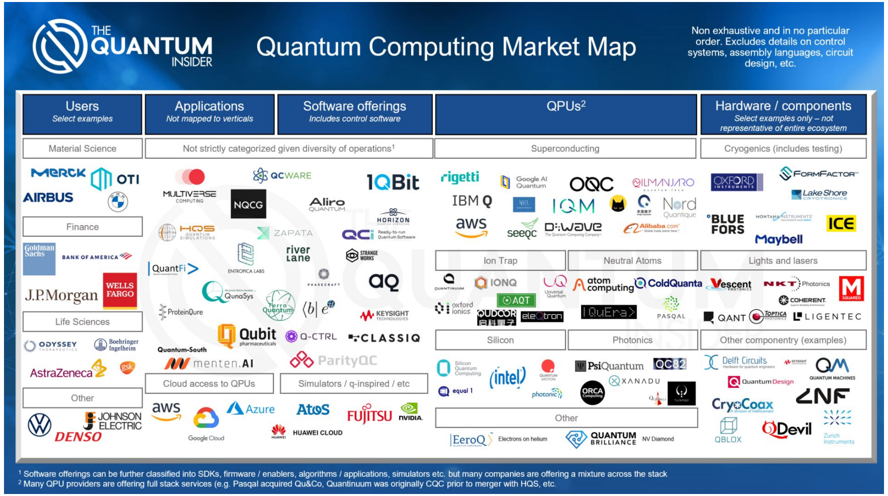


Where is Quantum Happening?



What jobs are there for me?

- Quantum is a small but growing field.
    - Startup companies are small and risk-leveraged.
    - Large companies are less risky, but highly competitve.
    - Networking and connections can be more important than paper resumes.
- Quantum is capital-intensive.
    - Equipment is expensive, often specialized.
    - High skill is required to engineer and operate quantum devices.
- Quantum has dozens of supporting technologies.
    - Optics, Photonics - Applied Physics, Engineering
    - RF, Signals - EE, Compsci
    - Cryogenics - Applied Physics, Engineering
- Some companies don't even exist yet!



What should I focus on?

- Quantum computing
  - Hardware improvements
  - Error correction methods
  - Algorithm R&D
- Quantum Sensing
  - Packaging & miniaturization
  - Cost reductions (electronics)
  - AI/ML and readout methods for improving performance
- Quantum Networks
  - Improving fidelities
  - Memories and repeaters
  - NOT BB84 QKD!

My opinions only, always get multiple perspectives before making major decisions!

Where do I start?

- Quantum Economic Development Consortium (QED-C)
  - Jobs Board
  - Office Hours
  - Research Symposium
- Events and conferences
  - $$$, but look for student discounts
  - Industry conferences: Quantum.Tech, Quantum World Congress, Inside Quantum Technologies, Q2B Silicon Valley
  - Technical Conferences: American Physical Society (March), SPIE Photonics West, IEEE Quantum Week, DEFCON, Many smaller focus areas and symposia.
- Quantum Computing Club
- RPI Faculty for Research

Next Time

- Circuits with quantum advantage.
- Qubit errors.
- 2 weeks between HWs now.
- Be on the lookout for Project 1.
  - Find team members if you want to work in a group
  - Next class
- Start thinking about Project 2.
  - Come talk to me about ideas.
  - HW4 will have a question asking for a proposal.

Thank You!