Quantum Physics 1
Spring 2025

Class 01B

The Need for Quantum Mechanics

Compelling Experiments

Topic Overview

1. Why and when do we use quantum mechanics?
2. Probability waves and the Quantum Mechanical State function; Wave packets and particles
3. Observables and Operators
4. The Schrodinger Equation and some special problems (square well, step barrier, harmonic oscillator, tunneling)
5. More formal use of operators
6. The single electron atom/ angular momentum

Intellectual overview

- The study of quantum physics includes several key parts:
  - Learning about experimental observations of quantum phenomena.
  - Understanding the meaning and consequences of a probabilistic description of physical systems
  - Understanding the consequences of uncertainty
  - Learning about the behavior of waves and applying these ideas to state functions
  - Solving the Schrodinger equation and/or carrying out the appropriate mathematical manipulations to calculate energies and probabilities.

Classical Mechanics

- A **particle** is an indivisible point of mass.
- A **system** is a collection of particles with defined forces acting on them.
- A **trajectory** is the position and momentum (**r**(t), **p**(t))of a particle as a function of time.
- If we know the trajectory and forces on a particle at a given time, we can calculate the trajectory at a later time. By integrating through time, we can determine the trajectory of a particle at all times.

$$m\frac{d^2\vec{r}}{dt^2} = -\nabla V(\vec{r},t); \quad \vec{p}(t) = m\frac{d\vec{r}}{dt}$$

Some compelling experiments: The particlelike behavior of electromagnetic radiation

- Simple experiments that tell us that light has both wavelike and particle-like behavior
  - The photoelectric effect (particle)
  - Double slit interference (wave)
  - X-ray diffraction (wave)
  - The Compton effect (particle)
  - Photon counting experiments (particle)

The photoelectric effect

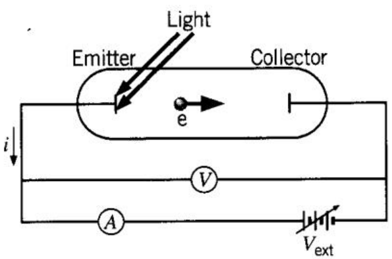


FIGURE 3.10 Apparatus for observing the photoelectric effect. The flow of electrons from the emitter to the collector is measured by the ammeter A as a current i in the external circuit. A variable voltage source V_{ext} establishes a potential difference between the emitter and collector, which is measured by the voltmeter V.

Krane

Krane

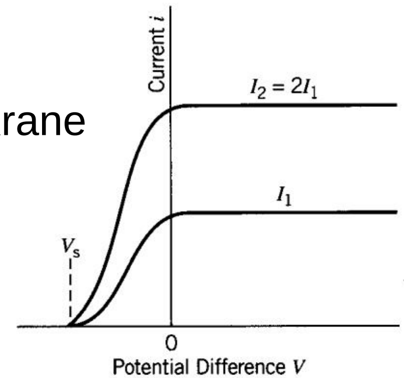


FIGURE 3.11 The photoelectric current i as a function of the potential difference V for two different values of the intensity of the light. When the intensity is doubled, the current is doubled (twice as many photoelectrons are emitted), but the stopping potential V_s remains the same.

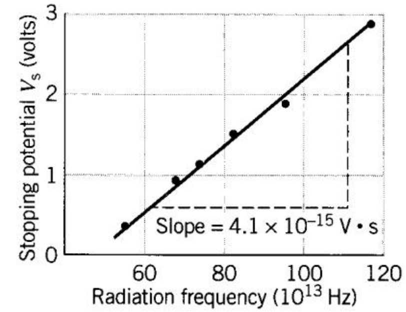


FIGURE 3.12 Millikan's results for the photoelectric effect in sodium. The slope of the line is h/e; the experimental determination of the slope gives a way of determining Planck's constant. The intercept should give the cutoff frequency; however, in Mil-

Krane

- In a photoelectric experiment, we measure the voltage necessary to stop an electron ejected from a surface by incident light of known wavelength.

Interpretation of the photoelectric effect experiment

Einstein introduced the idea that light carries energy in quantized bundles – photons.
The energy in a quantum of light is related to the frequency of the electromagnetic wave that characterizes the light.
The scaling constant can be found from the slope of eV_{stop} vs wave frequency, ν. It is found that:

$$\boldsymbol{eV_{stop} = h\nu - \Phi_{material} = E_{photon} - \Phi}$$

where $h = 6.626x10^{-34}$ joule–sec
For light waves in vacuum, $c = \lambda\nu = 3x10^{8}$ m/s,
so we can also write:

$$eV_{stop} = \frac{hc}{\lambda} - \Phi_{material}$$

A convenient (non–SI) substitution is $hc = 1240$ eV–nm.

from Millikan's 1916 paper

1. *That there exists for each exciting frequency* v, *above* a certain *critical value, a definitely determinable maximum velocity of emission of corpuscles.*

2. *That there is a linear relation between V and v.*

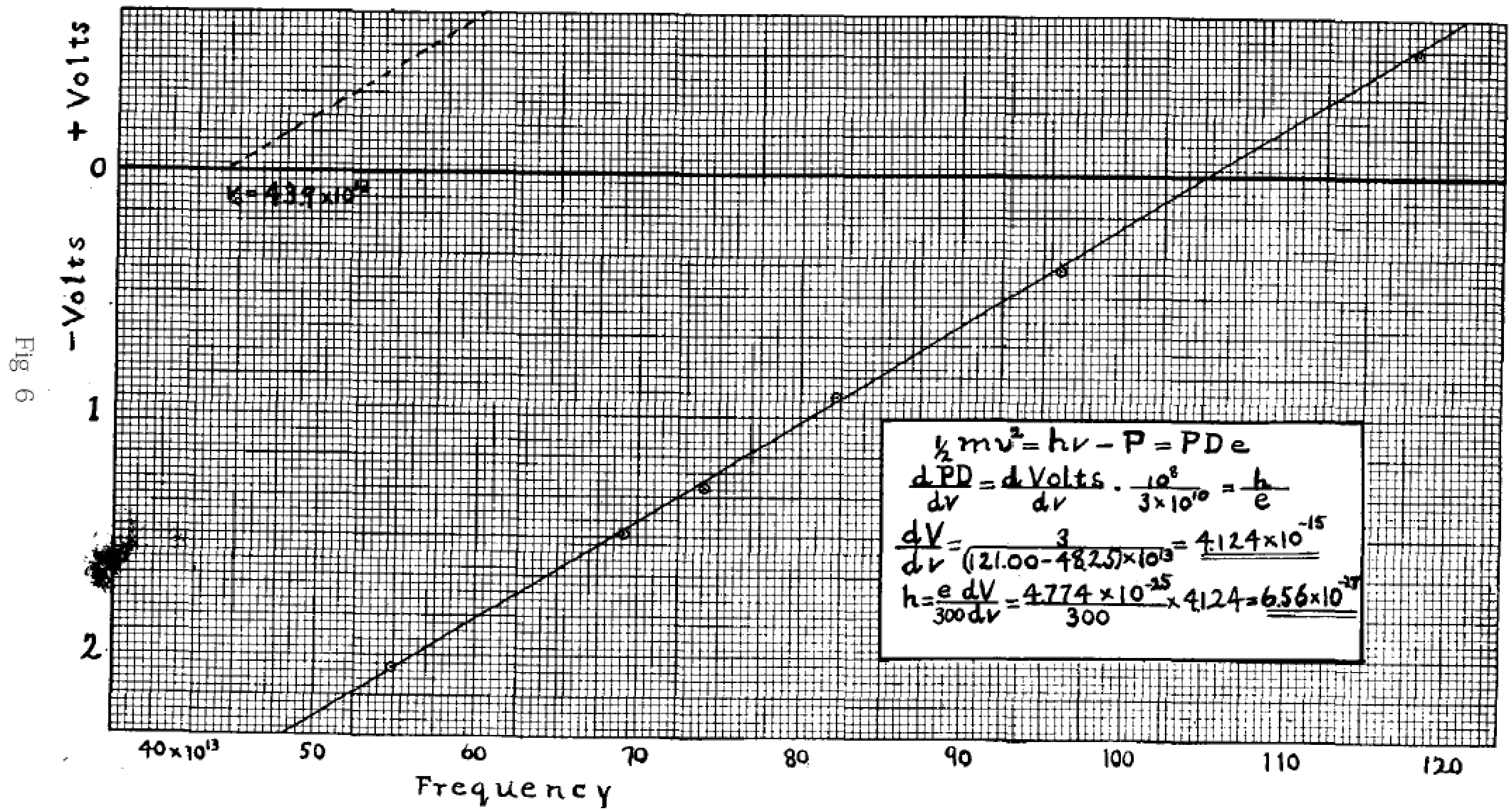
3. *That* $\frac{dV}{d\nu}$ *or the slope of the V v line is numerically equal to* h/e.

4. *That at the critical* frequency ν_0 *at which v = o,* $p = h\nu_0$, *i. e., that the intercept* of *the Vv* line on *the* v axis is *the lowest* frequency *at which the metal* in *question can be photoelectrically active.*

5. *That the contact* $E.M.F.$ *between* any *two* conductors is given *by the equation*

$$\text{Contact E.M.F.} = h/e(\nu_0 - \nu_0') - (V_0 - V_0').$$

R. A. Millikan, Phys Rev 7, 355, (1916)



Photon Momentum: Compton scattering

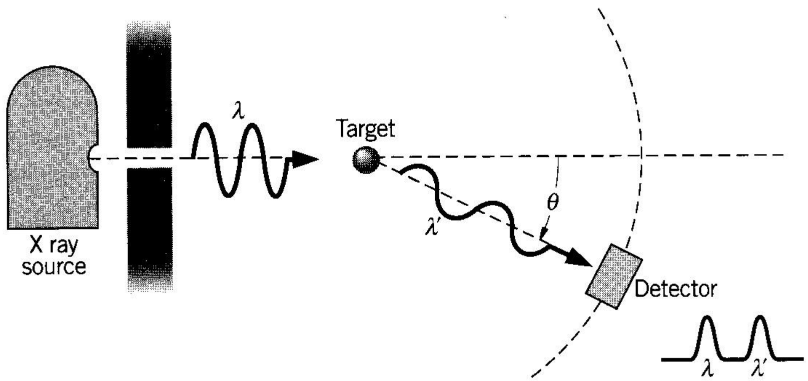


FIGURE 3.19 Schematic diagram of Compton scattering apparatus. The wavelength λ' of the scattered X rays is measured by the detector, which can be moved to different positions θ. The wavelength difference $\lambda' - \lambda$ varies with θ.

(from Krane)

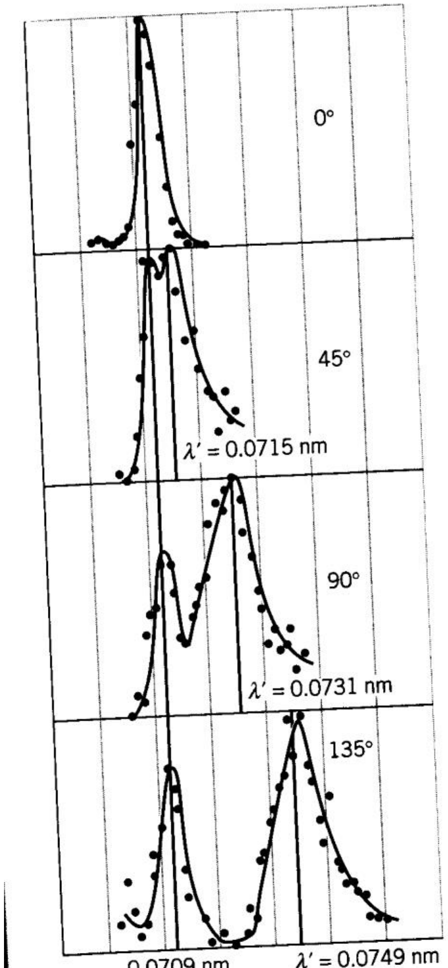


FIGURE 3.20 Compton's original results for X-ray scattering.

- The Compton effect involves scattering of electromagnetic radiation from electrons.
- The scattered x-ray has a shifted wavelength (energy) that depends on scattered direction

Compton effect

We use energy and momentum conservation laws:

Energy

$hv' = hv - K_{electron}$: K = kinetic energy

Momentum

x component: $\frac{h}{\lambda} = \frac{h}{\lambda'}\cos\theta + \gamma m v \cos\phi$

y component: $0 = \frac{h}{\lambda'}\sin\theta + \gamma m v cos\ \phi$

$\lambda' - \lambda = \Delta\lambda = \frac{h}{mc}(1 - \cos\theta)$

m = mass of electron;

$\frac{h}{mc} = 2.4 \times 10^{-12} m = "Compton\ wavelength"$

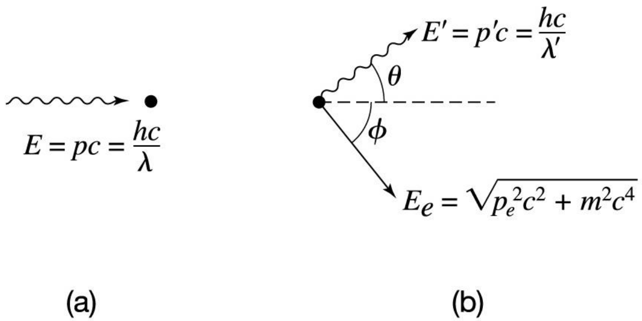


Figure 1.16 copyright 2009 University Science Books

Conclusions from the Compton Effect

X−ray quanta of wavelength λ have:

Kinetic energy: $K = \dfrac{hc}{\lambda}$

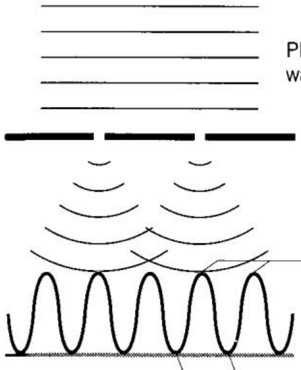
Momentum: $p = \dfrac{h}{\lambda}$

And

$$\lambda' - \lambda = \Delta\lambda = \frac{h}{mc}(1 - \cos\theta)$$

$$\frac{h}{mc} = 2.4 \times 10^{-12} m = \text{"}Compton\ wavelength\text{"}$$

Double slit interference



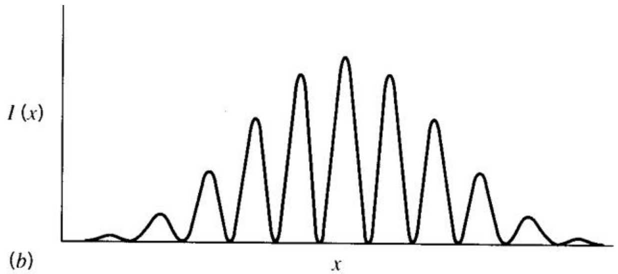


FIGURE 5-10 Results from the two-slit experiment using photons. (a) Measured photon intensity as a function of position. From M. Cagnet, *Atlas of Optical Phenomena*, Springer-Verlag (1962). (b) Calculated photon intensity.

The intensity maxima in a double slit wave interference experiment occur at:

$d \sin\theta = n\lambda$

where d is the distance between the slits.

(The width of the overall pattern depends on the width of the slits.)

X-ray diffraction

- In x-ray diffraction, x-ray waves diffracted from electrons on one atom interfere with waves diffracted from nearby atoms.
- Such interference is most pronounced when atoms are arranged in a crystalline lattice.

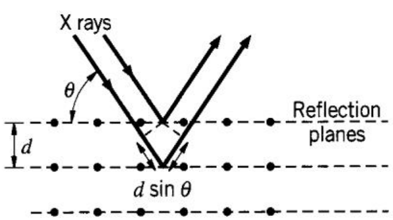


FIGURE 3.5 A beam of X rays reflected from a set of crystal planes of spacing *d*. The beam reflected from the second plane travels a distance $2d \sin\theta$ greater than the beam reflected from the first plane.

Bragg diffraction maxima
are observed when:
$2d \sin\theta = n\lambda$
$d =$ the distance between adjacent planes of atoms in the crystal.

Laue X-ray Diffraction Pattern

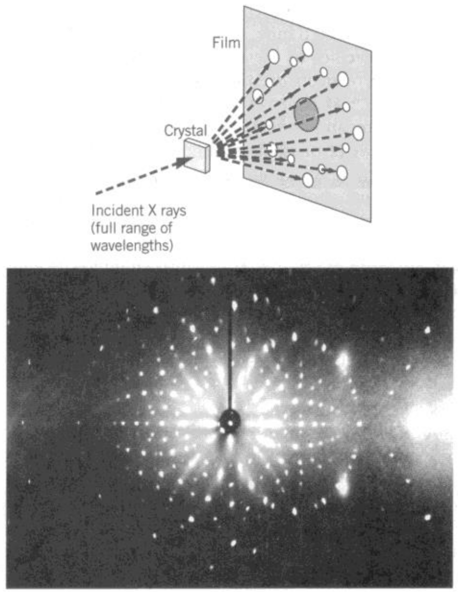


FIGURE 3.7 (Top) Apparatus for observing X-ray scattering by a crystal. An interference maximum (dot) appears on the film whenever a set of crystal planes happens to satisfy the Bragg condition for a particular wavelength. (Bottom) Laue pattern of NaCl crystal.

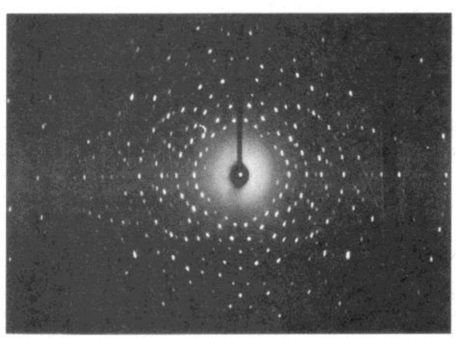


FIGURE 3.8 Laue pattern of a quartz crystal. The difference in crystal structure and spacing between quartz and NaCl makes this pattern look different from Figure 3.7.

- A Laue diffraction pattern is observed when x-rays of many wavelengths are incident on a crystal and diffraction can therefore occur from many planes simultaneously.

…pretty

from Krane

Photon Coincidence Experiments

A more direct proof of the particle nature of light comes from an experiment in which a set of optical transitions (shown on the left) is exploited*.

When a calcium atom is excited, the electron decays in two steps from the 4p excited state to the 4s ground state.

The two decays occur within nanoseconds of each other, so a detector can be set up to detect only events co-incident with the first (551 nm) event.

The intensity is set so that the system only observes one event at a time.

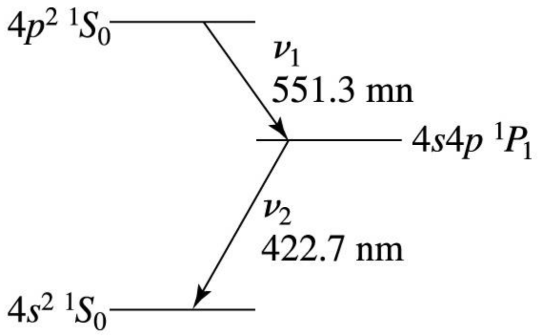


Figure 1.19 copyright 2009 University Science Books

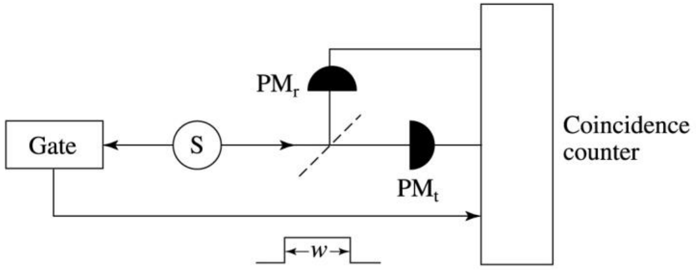


Figure 1.20 copyright 2009 University Science Books

*Grainger et al., Europhys Lett 1986

Photon Coincidence Experiments

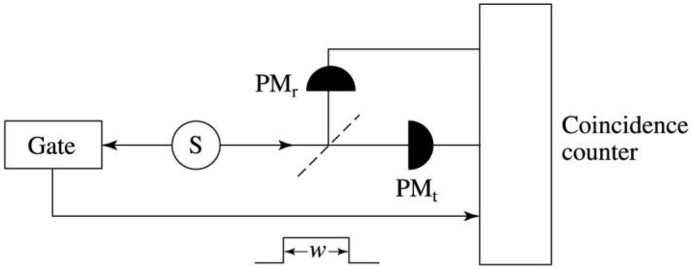


Figure 1.20 copyright 2009 University Science Books

Grainger et al., Europhys Lett 1986

- A 50% beam splitter is placed in the beam arm set up to detect 423 nm waves.
- Two photomultipliers (to detect single events) are set up on the two arms out of the beam-splitter.
- It is observed that each detected event in PM1 and/or PM2 occurs only in PM! OR PM2, never both.
- This provides evidence that photons are indivisible quanta.

Single Photon Experiments

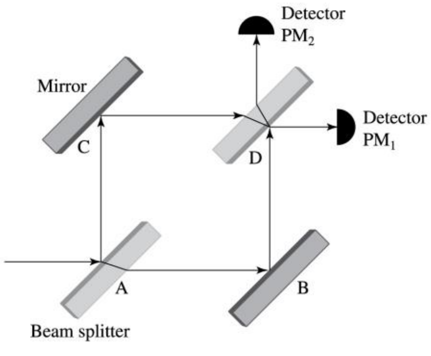


Figure 1.23 copyright 2009 University Science Books

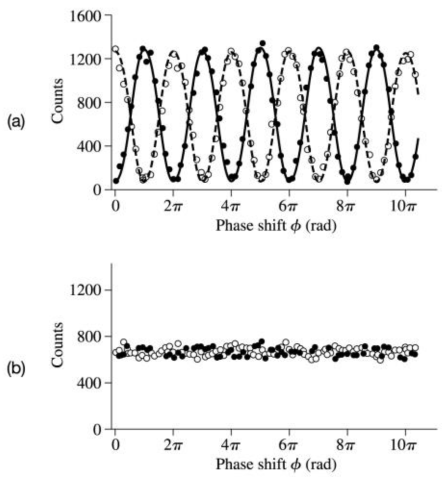


Figure 1.26 copyright 2009 University Science Books

In this experiment, the interferometer is large enough that it is possible to move the second beam splitter D after a photon has entered the system (A) and before it arrives at D.

For the upper curve (a), arrival of single photons is detected at either PM1 or PM2, but never both simultaneously. The relative path lengths are varied after the photon enters the system. In case (a) we don't know whether the photon traveled ABD or ACD, therefore the amplitudes interfere.

For curve (b) the beam splitter D is rapidly removed after the photon enters the system and before it arrives at point D. We therefore know which path the photon took to a given detector, and the interference pattern disappears.

Jaques et al, Science, 2007

The bottom line on light

- In many experiments, light behaves like a wave (c=phase velocity, ν=frequency, λ=wavelength).
- In many other experiments, light behaves like a quantum particle (photon) with properties:

energy: $E_{photon} = h\nu = \dfrac{hc}{\lambda}$

momentum: $p = \dfrac{h}{\lambda}$

and thus: $E = pc$

Some compelling experiments: The wavelike behavior of particles

- Experiments that tell us that electrons have wave-like properties
  - electron diffraction from crystals (waves)
- Other particles
  - proton diffraction from nuclei
  - neutron diffraction from crystals

Electron diffraction from crystals

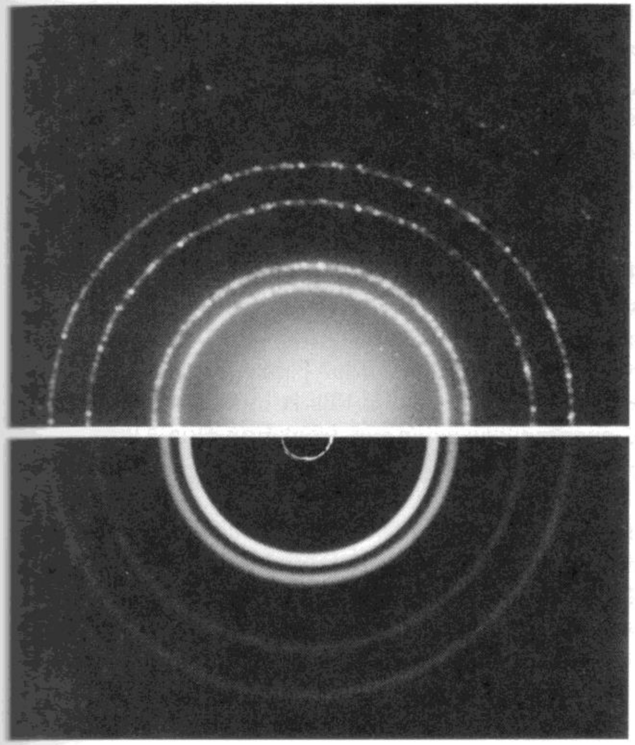
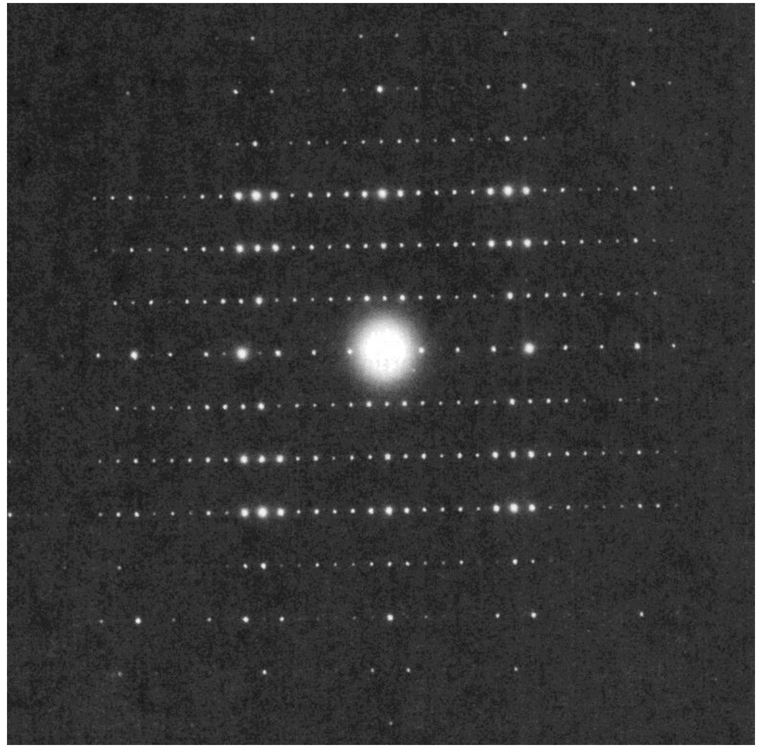


FIGURE 4.3 Comparison of X-ray diffraction and electron diffraction. The upper half of the figure shows the result of scattering of 0.071 nm X rays by an aluminum foil, and the lower half shows the result of scattering of 600 eV electrons by aluminum. (The wavelengths are different so the scales of the two halves have been adjusted.)



- electron diffraction patterns from single crystal (above) and polycrystals (left) [from Krane]

Proton diffraction from nuclei

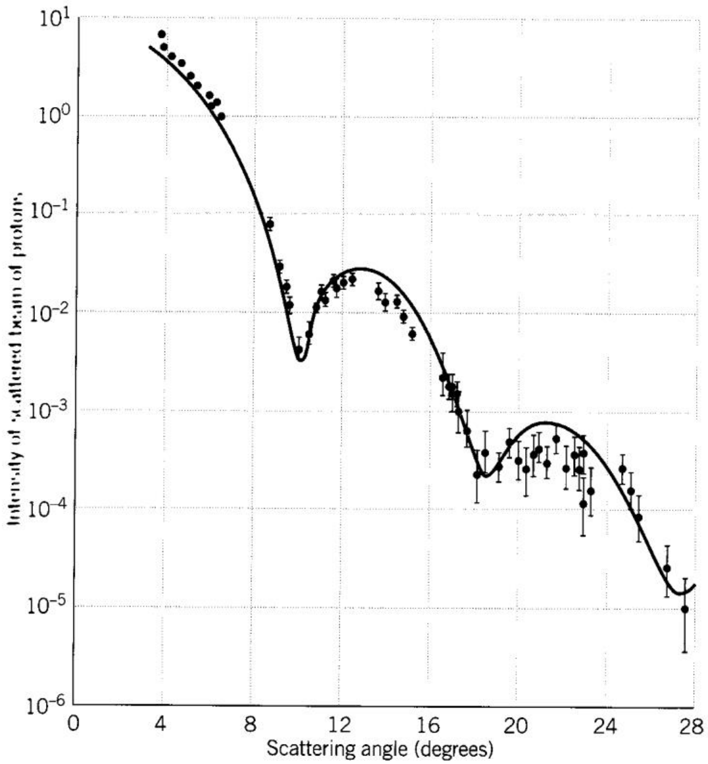


FIGURE 4.8 Diffraction of 1 GeV protons by oxygen nuclei. The pattern of maxima and minima is similar to that of single-slit diffraction of light waves.

from Krane

Neutron diffraction from a crystal

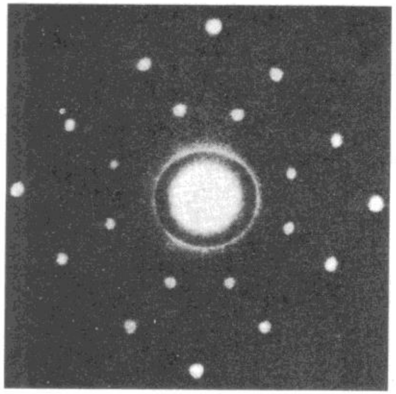
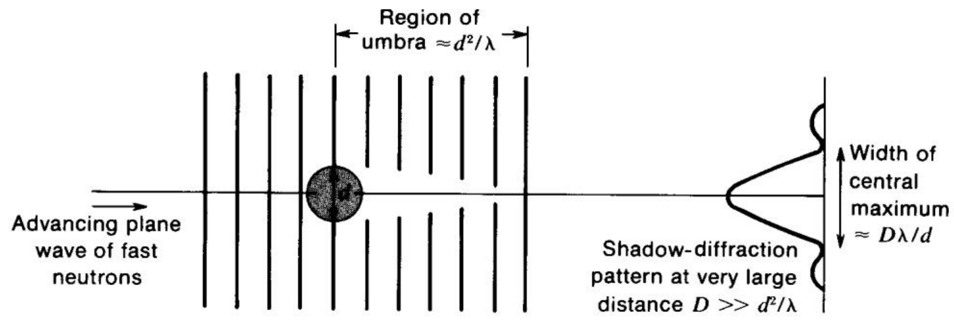
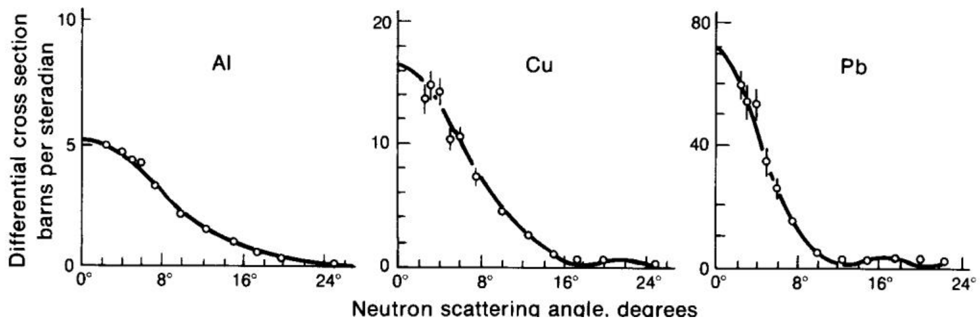


FIGURE 4.7 Diffraction of neutrons by a sodium chloride crystal.

from Krane



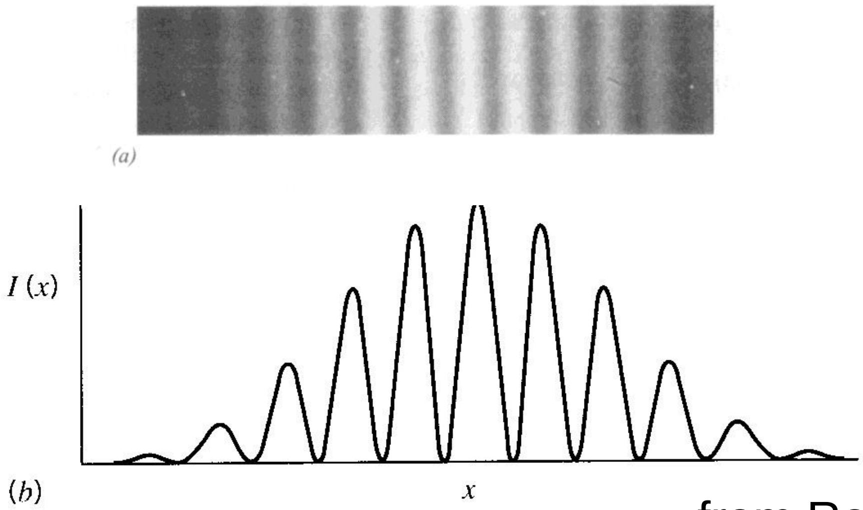
(a)



Diffraction of fast neutrons from Al, Cu, and Pb nuclei. [from French, after A Bratenahl, Phys Rev 77, 597 (1950)]

Electron double slit interference

- Electron interference from passing through a double slit



from Rohlf

FIGURE 5-11 Results from the two-slit experiment using electrons. (*a*) Measured electron intensity as a function of position. From C. Jönsson, Zeit. Phys. **161**, 454 (1961). (*b*) Calculated electron intensity.

Helium diffraction from LiF crystal

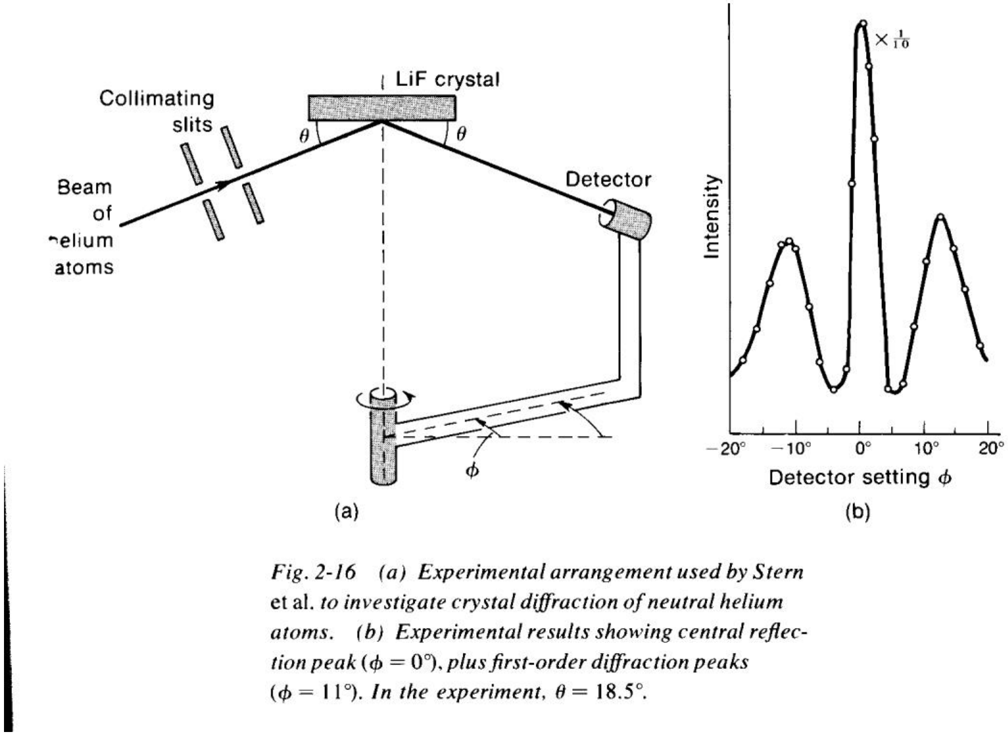
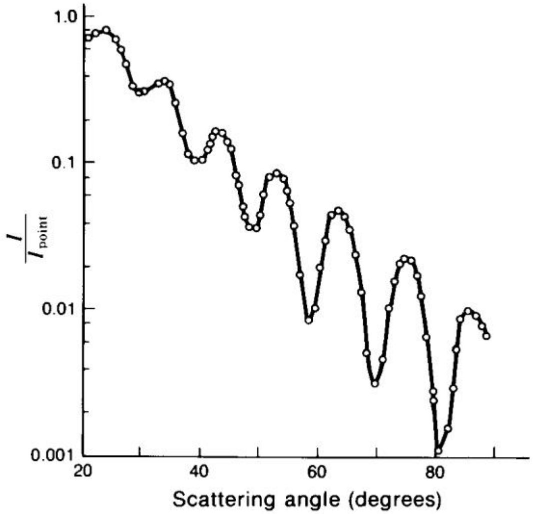


Fig. 2-16 (a) Experimental arrangement used by Stern et al. *to investigate crystal diffraction of neutral helium atoms. (b) Experimental results showing central reflection peak* ($\phi = 0°$), *plus first-order diffraction peaks* ($\phi = 11°$). *In the experiment,* $\theta = 18.5°$.

from French after Estermann and Stern, Z Phys 61, 95 (1930)

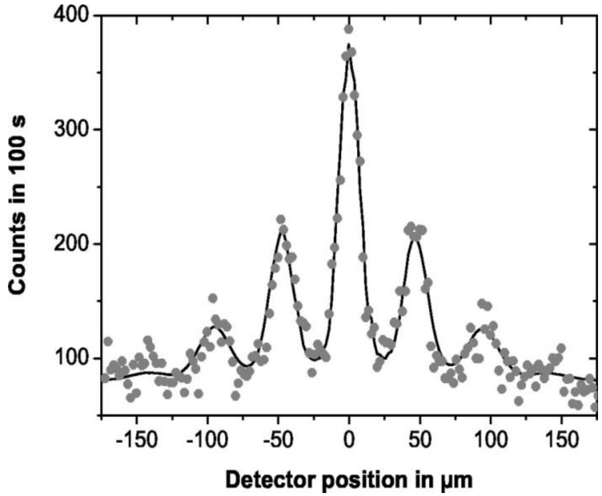
Alpha scattering from niobium nuclei

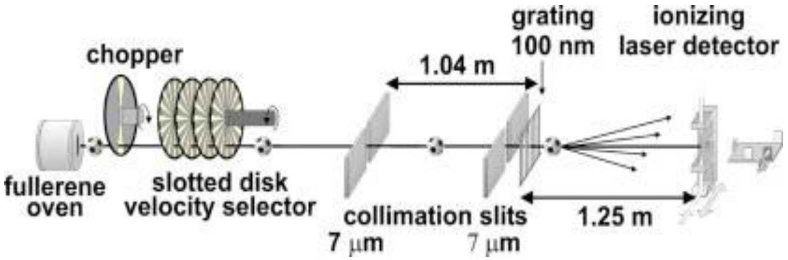


Angular distribution of 40 MeV alpha particles scattered from niobium nuclei.
[from French after G. Igo et al., Phys Rev 101, 1508 (1956)]

Diffraction of Fullerene Molecules (720 amu)

Structure of the fullerene molecule - C60

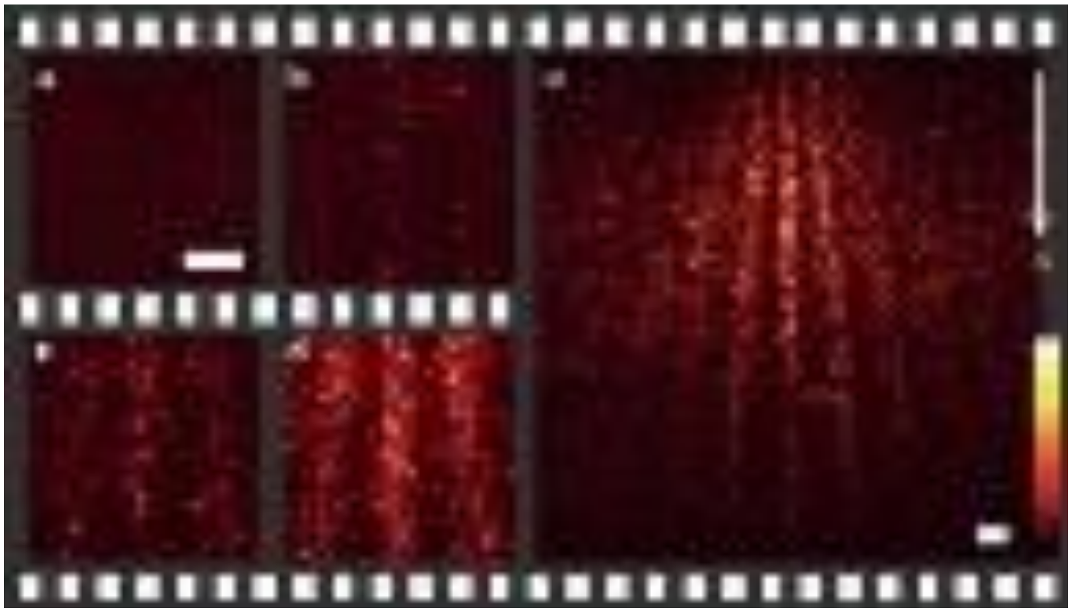


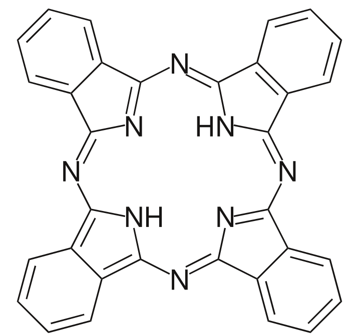


Molecular radius ~ 10^{-8} m
Speed ~ 120 m/s
Wavelength ~ $3\text{x}10^{-12}$ m

from Nairz et al. AJP 2003

Interference of Phthalocyanine (~500 AMU)*





Selected frames from a movie recorded with an EMCCD camera showing the build-up of the quantum interference pattern for PcH_2 molecules.

Nature Nanotechnology 7, 297–300 (2012)

One of the largest molecules exhibiting quantum interference so far (10,000 AMU)

PCCP Paper

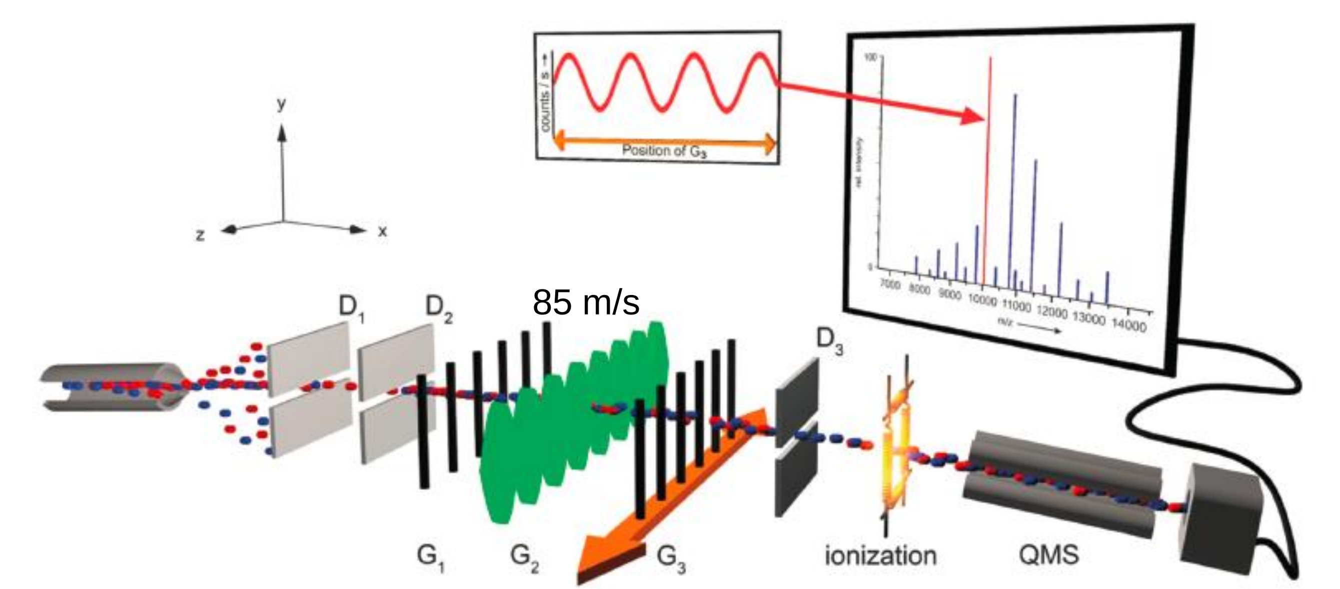


Fig. 2 KDTL interferometer setup: the molecules are evaporated in a furnace. Three height delimiters, D_1–D_3, define the particle velocity by selecting a flight parabola in the gravitational field. The interferometer consists of three gratings with identical periods of $d = 266$ nm. G_1 and G_3 are SiN_x gratings, whereas G_2 is a standing light wave which is produced by retro-reflection of a green laser at a plane mirror. A phase modulation $\Phi \propto (\alpha \cdot P)/(v \cdot w_y)$ is imprinted onto the molecular matter–wave *via* the optical dipole force which is exerted by the light grating onto the molecular optical polarizability α_{opt}. Here P is the laser power, v the molecular velocity, $w_x \simeq 18$ µm and $w_y \simeq 945$ µm the Gaussian laser beam waists. The transmitted molecules are detected using electron ionization quadrupole mass spectrometry after their passage through G_3, which can be shifted along the z-axis to sample the interference pattern.

- Eibenberger et al, Phys Chem Chem Phys 2013, 15, 14696.

The bottom line on particles

- In many experiments, electrons, protons, neutrons, and heavier things act like particles with mass, kinetic energy, and momentum:

$$p = mv \text{ and } K = \frac{1}{2}mv^2 = \frac{p^2}{2m} \text{ for non−relativistic particles}$$

- In many other experiments, electrons, protons, neutrons, and heavier things act like waves with :

$$\boxed{\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK}}}$$

The De Broglie hypothesis

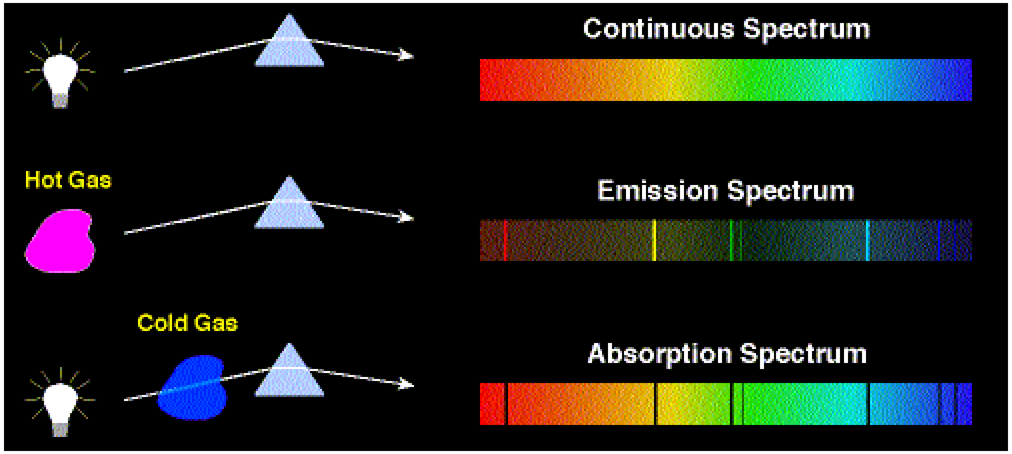
$$\boxed{p = \frac{h}{\lambda}}$$

for everything

Some other well-known historical experiments and observations

- optical emission spectra of atoms are quantized
- the emission spectrum of a hot object (blackbody radiation) cannot be explained with classical theories

Emission spectrum of atoms



from a random astronomy website

For hydrogen:

$$h\nu = -13.6eV\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

Blackbody radiation

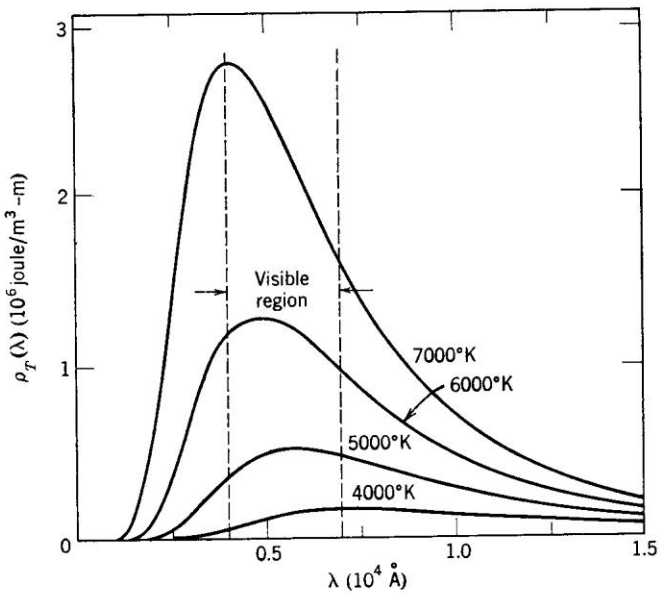


Figure 1-12 Planck's energy density of blackbody radiation at various temperatures as a function of wavelength. Note that the wavelength at which the curve is a maximum decreases as the temperature increases.

Blackbody energy density

$$\rho_T(\lambda)\,d\lambda = \frac{8\pi hc}{\lambda^5}\,\frac{d\lambda}{e^{hc/\lambda kT} - 1}$$

from Eisberg and Resnick

Summary

- Many experiments clearly demonstrate that light and matter both exhibit wave interference behavior consistent with the deBroglie hypothesis:
$$\lambda = h/p$$
where p is the momentum of the particle.
- Many experiments clearly demonstrate that energy in an electromagnetic wave is quantized with a particle kinetic energy of $E = hf$, where f is the oscillation frequency of the wave.
- Electromagnetic particles are massless, yet carry energy and momentum.
- We will further explore the relationship between frequency and energy for particles that have mass.