Problem Chosen	2022 MCM/ICM Summary Sheet	Team Control Number
F		2214494

Asteroid Mining: The Impacts on Global Equity.

(Based on TOPSIS and Backward Propagation Algorithm)

Summary

With the development of human technological progress, mining on asteroids is becoming a reality. But there is still a problem in front of us: how to ensure global equity when mining asteroids, and how to distribute asteroid mineral resources in a fair and reasonable way.

First of all, we give the definition of equity, then we introduce **7** primary indicators and **20** secondary indicators to build a mathematical model to evaluate global equity. Next, we use **Analytic Hierarchy Process (AHP)** to get the weights of each indicator and conduct the **consistency test**. Finally, we establish a formula to measure the global equity score, calculate the global equity score for the last 10 years and compare them.

Then, we describe future scenarios of asteroid mining and validate them. To distribute asteroid resources fairly, we build a **Global Equity Score(GES) Model** with **12** primary indicators and **40** secondary incicators. Next,we use **TOPSIS** method to rank all countries and calculate scores. The weights are assigned by AHP. Then, we simulate the rate of asteroid mining using **t-distribution probability density function**. In order to increase the development rate of poor countries and ensure global equity, poor countries are allocated slightly more mineral resources. Finally, we calculate the number of the mineral resources allocated to each country and the effect of asteroid mining on global equity. The results show that asteroid mining promotes global equity.

In order to quantitatively measure the impact of changing indicators on global equity, using **Backward Propagation Algorithm**, we calculate the impact of indicators on global equity formula and perform an error analysis. Then, we conduct **Pearson correlation coefficient analysis** for each indicator and global equity score and perform **t-test** lastly.

Next, we combine the calculation results with scientific and rational analysis, then propose relevant policies. Next, we increase the mineral tax of developed countries by **10%** and increase the schooling rate of developing countries by **5%**, then we quantitatively calculate their effects on global equity, and the results show that they have a facilitating effect on global equity. Then, we give a schedule of the implementation of each measure.

Finally, we conduct a **sensitivity analysis** and calculate the impact of a **10%** increase and a **10%** decrease in the indicator on global equity. The results show scores vary less than **7%**. We conclude the paper by analyzing the strengths of our **GES** model, demonstrating the robustness and accuracy of our models, and analyzing the weaknesses.

Keywords: AHP; GES Model; t-Distribution Probability Density Function; Backward Propagation Algorithm;

Contents

1 Introduction

1.1 Problem Background and Restatement

With the great progress of science and technology, the asteroid mining is gradually becoming a reality[?]. In recent years, more and more countries have agreed to make outer space benefit whole humanly, so how to ensure the equitable distribution of benefits after mining asteroids has become an issue to be discussed by all allied powers.

Before the asteroid mining project is officially conducted, there are many unsettled issues that deserve to be discussed and resolved, including the feasibility of mining on asteroids and the equitable distribution of benefits. Therefore, it is a critical issue to determine a equitable benefit distribution strategy for the project ensuring the project is carried out while promoting world peace and reducing inequality.

Considering the background, we should address 4 questions in the paper:

Task 1: First, a global equity definition should be developed. Then, find appropriate indicators and build a model to measure global equity. Next, apply the model to a historical or regional analysis to verify its validity.

Task 2: Describe the possible future state and vision of the asteroid mining industry. Then analyze the impact on global equity by using the global equity measurement model developed in Task 1.

Task 3: Improve models to analyze and explore how asteroid mining will affect global equity.

Task 4: Assuming that UN intends to update its Outer Space Treaty to develop asteroid mining and ensure that asteroid mining benefits all of humanity, combine your model and results to propose reasonable policies to ensure that asteroid mining will benefits all of humanity.

1.2 Our Work

First of all ,we give the definition of global equity and introduce 5 primary indicators and 26 secondary indicators to measure the established mathematical model. Next, we use Analytic Hierarchy Process to assign weights to every indicator and performed the consistency test on the weight matrix, and finally the test passed. We calculate the global development balance scores and performe calculations for the last 10 years of data.

Secondly, we combine the data to provide a vision of asteroid mining's future. Next, we improve our model to take into account of the contribution of science and technology to ensure that ”those who contribute more get more”. The model has 10 primary indicators and 51 secondary indicators. We use the t-distribution probability density function to simulate the profitability of asteroid mining, and finally, we use TOPSIS to rank the contribution of each country, and use the contribution of each country to allocate resources.

After that, we calculate the impact of changing indicators on global equity using the back propagation algorithm. Next, we calculate the correlation between each indicator and the global development balance score using Pearson correlation coefficient analysis and pass the t-test.

Finally, in order to ensure that asteroid mining benefits all human beings, we propose policies that is related to asteroid mining and calculate relevant data for validation.

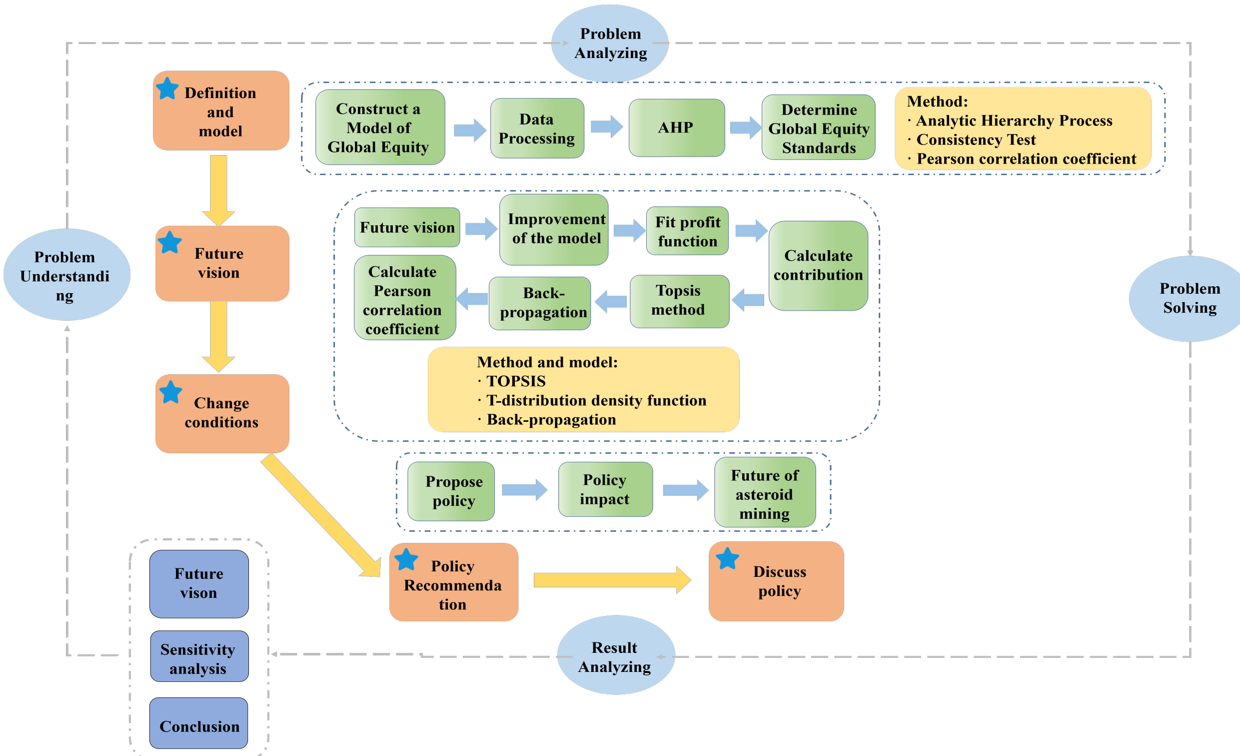

Figure 1: Workflow

2 Assumption and Symbol Explanation

2.1 Assumption

★ Assuming that global equity is influenced by the six indicators including EI, IDG, CEA, MA, HR, ER and SA, and that unexpected factors such as natural disasters do not have obvious impacts on global equity as they may happen anywhere and anytime on the earth.

★ Assuming that our mineral extraction rate rises first and then falls, this means we can fit the mineral extraction rate with a t-distribution probability density function curve.

★ Assuming stable international conditions and a stable development of the space industry. Meanwhile, mankind can achieve asteroid mining in 15 years or so.

★ Assuming that each secondary indicator has a linear effect on the primary indicator, this means that we can easily find the partial derivative of the impact function and analyze the impact of each secondary indicator on global equity accordingly.

2.2 Symbol Explanation

The primary symbols used in our paper are listed in the table below.

Variable	*Meaning*
SA	Sustainability
HR	Human Resources
CEA	Carbon Emission Allocation
EI	Economic Indicator
CR	Consistency Ratio
CI	Consistency Indicator
RI	Stochastic Consistency Index
Eq_k	Development Score of Country k
GE	Global Development Balance Score
$w_{jk}^{(l)}$	The Weights Between the k^{th} Neuron in Layer l-1 and the j^{th} Neuron in Layer l
$\boldsymbol{w}^{(\boldsymbol{l})}$	Weight Matrix for Layer l-1 to Layer l
$b_j^{(l)}$	Bias of the j^{th} Neuron of the l^{th} Layer
$\boldsymbol{b}^{(\boldsymbol{l})}$	Bias Vector of the l^{th} Layer
$z_j^{(l)}$	The Input Value of the j_{th} Neuron of the l^{th} Layer
$\boldsymbol{z}^{(\boldsymbol{l})}$	The Input Vector of the l^{th} Layer
$a_j^{(l)}$	The Activation Value of the j^{th} Neuron of the l^{th} Layer
$\boldsymbol{a}^{(\boldsymbol{l})}$	The Activation Output Vector of the l^{th} Layer
$N^{(l)}$	Number of Neurons in Layer l
$p(y)$	T-distribution Probability Density Function
X	Forwarding Matrix
Z	Normalization Matrix
r	Pearson correlation coefficient

3 Global Eauity Score Model

3.1 Defination of Equity

To address this issue, first we should define what is equity, "equity means equal rights for all people on earth". More specifically, rights include: fair distribution of resources, fair income, fair opportunities, fair carbon emissions, etc.

To measure global equity, we adopt the method of calculating the variance after computing the development equity factor for each country. Then, we combine Zhang's descriptions[?] to establish the following model to measure global equality.

3.2 Global Equity Evaluation System

3.2.1 Establishment of Evaluation Indicators

We use 7 primary indicators and 15 secondary indicators to measure Country Development Score. Country Development Score for each country is the ratio of the country's development score to the average of the development scores of other countries. The primary indicators that affect Country Development Score are shown in Figure **??**:

3.2.2 AHP Method and Model Validation

⋇ AHP Method

Next, we use the Analytic Hierarchy Process (AHP) to calculate the weights[?]. The weight coefficient matrix of the seven primary indicators is established as follows.

Then we obtain the eigenvalues of the matrix and show it in Table 2.

Table 2: The Eigenvalues of the Weight Coefficient Matrix

Indicators	*EI*	*IDG*	*CEA*	*MA*	*HR*	*ER*	*SA*
Eighenvalues	7.7200	1.8900	1.8900	0.2948	0.2948	0.0000	0.0000

⋇ Consistency Check

After getting the eigenvalues of the weight coefficient matrix, we perform a consistency check, the equations are as follow

$$CI = \frac{\lambda_{max} - n}{n - 1} = \frac{7.72 - 7}{7 - 1} = 0.12. \tag{1}$$

Also, RI denotes stochastic consistency index, and its standard values are shown in Table 3.

Table 3: Values of RI

n	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*
RI	0.00	0.00	0.58	0.90	1.12	1.24	1.32	1.41	1.45	1.49

Substituting the data obtained before, we get

$$CR = \frac{CI}{RI} = \frac{0.12}{1.32} = 0.0909 < 0.1. \tag{2}$$

So it pass the consistency check. Then, we use Arithmetic Mean Method, Geometric Mean Method and Eigenvalue Method to calculate the weight of the primary indicators, finally we get the results shown in Table 4.

Table 4: Weight of the First-level Indicators

Indicators	*Arithmetic Mean Method*	*Geometric Mean Method*	*Eigenvalue Method*
EI	0.1831	0.1965	0.1810
IDG	0.3831	0.3965	0.3810
CEA	0.0989	0.0996	0.0921
MA	0.0435	0.0436	0.0438
HR	0.0926	0.0620	0.1027
ER	0.0833	0.0852	0.0808
SA	0.1157	0.1166	0.1187

Next, after calculating the average of the statistics in Table 4, we get Figure **??**.

Finally, we get the formula to measure the national development score(Eq_k):

$$\begin{aligned} Eq_k = {} & 0.187 \times EI + 0.387 \times IDG + 0.097 \times CEA + 0.0436 \times MA \\ & + 0.086 \times HR + 0.0831 \times ER + 0.117 \times SA. \end{aligned} \tag{3}$$

※ Model Validation

Take Income Distribution Gap(IDG) as an example, according to the factors affecting IDG obtained from the previous analysis, the income inequality score of each country is finally shown in Figure 2.

For a certain country, first we de-self the country by comparing its development score(Eq_k) with the average of other countries' development scores(Eq_m)($k \neq m$). Then we subtract it from the mean of all selected countries($\overline{Eq_k}$) and take the mean value. That is, using the following formula

$$GE = \frac{1}{10n} \sum \left(\frac{Eq_k}{\sum\limits_{i \neq k} Eq_i/(n-1)} - \overline{Eq_k} \right)^2. \tag{4}$$

On balance, we derive the degree of inequity over the last 10 years which was shown in Figure 3.

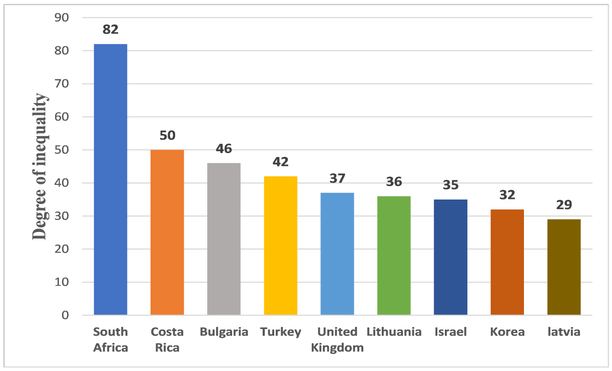

Figure 2: Top 9 Countries with the Highest Degree of Inequity

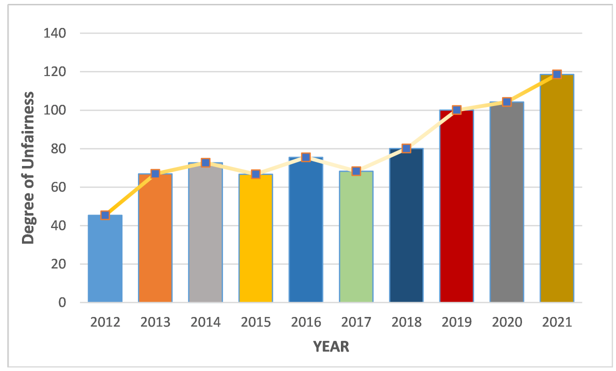

Figure 3: the Global Inequity Index for 2012 - 2021

3.3 Our Conclusion

By calculating the Global Development Imbalance Index (GDI), it can be concluded that global imbalances are tending to accelerate over the past decade due to factors such as uneven economic development.

4 Impacts of Asteroid Mining on Global Equity

To explore the impact asteroid mining can have, we first analyze the value asteroids can bring. Take minerals from asteroids as an example, relevant evidence suggests that the asteroid is indeed rich in minerals.

With the depletion of Earth's resources, people are destined to place goals in outer space, and the first bear to brunt is asteroids. In this task, we have made an outlook on the future of asteroid mining and analyze it.

4.1 Promoting Global Economic Development

Through our research, we found that some of the asteroids have the following commercial valuations shown in Table 5.

Table 5: Commercial Valuations

Asteroid	*Est. Value (US$billion)*	*Est. Profit (US$billion)*	$\Delta V(km/s)$
Didymos	62	16	5.162
Anteros	5570	1250	5.44
2001 CC21	147	30	5.636
1992 TC	84	17	5.648

The growth of global economic has slowed down due to factors such as the COVID-19 outbreak and the declining birth rate of the population. It is speculated that a relatively small metallic asteroid with a diameter of 1.6 km (1 mile) contains more than $20 trillion worth of industrial and precious metals[**?**]. Other studies have shown that rockets can achieve "zero loss" of energy in transit by using solar energy. Other costs are negligible relative to the net profit. The complete utilization of just one 1.6 km asteroid could generate over $20 trillion in net profits. We speculate that after a breakthrough in space technology, the global economy will grow at a rate of at least 9% per year, and the economy will grow by more than $10.17 trillion per year[**?**].

4.2 Space Becomes a Major Battleground for Human Development

The economic appeal of asteroid mining is clear: precious metals such as gold and platinum sell for around US $50,000 per kilogram. Due to the huge potential value of planetary mining, space is becoming a major battleground for various countries. A breakdown of the first successful missions by country is as follows. At the same time, some strong aviation companies (e.g., SPACEX) will also participate, and the booming aviation business will drive the progress of technology and the growth of the job market.

4.3 Improving the Environment

Asteroid mining has equally profound effects on the Earth's environment. Take the mining of the rare earth resource platinum, for example. Space mining would have a lower environmental impact, if the spacecraft is able to return between 0.3% and 7% of its mass in platinum to Earth, assuming 100% primary platinum or 100% secondary platinum, respectively.[**?**]

At the same time, the benefits of asteroid mining are not limited to the replenishment of resources; it also offers a solution to the greenhouse effect of recent years. Here is some data, we compare the amount of CO2 emissions from asteroid mining to Earth-based mining. Let b denotes kg of payload mass launched into space vs. kg of resources delivered to the target destination, then, from Table 6,

Table 6: Comparison of Space and Earth-based platinum mining greenhouse gas emissions

b_{mining}	*CO_2eq /kg Pt*	*Ratio Reference (40 t / kg CO_2eq) Earth vs. Space*	*Ratio Reference (2 t / kg CO_2eq) Earth vs. Space*	*Earth: 33% secondary, 66% primary platinum vs. Space*
10	69	580	29	396
20	65	620	31	424
30	63	635	32	434

40	62	643	32	439

we can see that though asteroid mining account for a lower bound in CO_2eq, compared to Earth-based mining, one order of magnitude higher emissions would lead to one order of magnitude savings[?].

4.4 The Impact of Asteroid Mining

In order to ensure the fairness of resource allocation among different countries in the asteroid mining project, we supplement our model by adding a measure of the scientific and technological contribution of each country. At the same time, in order to narrow the global wealth gap and ensure global equity, we allocate resources slightly more to the extremely poor countries.

4.4.1 Determination of the Total Contribution

Considering the different contributions of different countries in asteroid mining, the countries that contribute more to science and technology deserve to share more resources.

4.4.2 Determination of Scientific and Technological Contribution

We measure the importance of a country in asteroid mining by its scientific and technological contribution. We introduce 5 primary indicators and 21 secondary indicators. We use AHP method to calculate the weight of each indicator and the contribution degree is calculated as follows.

4.4.3 Calculation Total Score by Using TOPSIS

After defining how these variables are determined, we use the TOPSIS method to continue our analysis of the problem[?].

⋇ The basic Steps of TOPSIS

- **Normalize the Original Matrix**

Suppose x_i is a set of intermediate type indicator series and the optimal value is x_{best}, then Equation 5, 6 are the forwarding equations.

$$M = \max |x_i - x_{best}| \tag{5}$$

$$\tilde{x}_i = 1 - \frac{|x_i - x_{best}|}{M} \tag{6}$$

- **Normalize the Normalization Matrix**

If the normalized matrix is noted as Z, then we use Equation 7 to normalize the matrix X.

$$z_{ij} = x_{ij} / \sqrt{\sum_{i=1}^{n} x_{ij}^2}. \tag{7}$$

Assume that there are n objects to be evaluated and a standardized matrix of m evaluation indicators.

$$\boldsymbol{Z} = \begin{bmatrix} z_{11} & z_{12} & \cdots & z_{1m} \\ z_{21} & z_{22} & \cdots & z_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ z_{n1} & z_{n2} & \cdots & z_{nm} \end{bmatrix} = \begin{bmatrix} 0.4056 & 0.2154 & \cdots & 0.2433 \\ 0.3187 & 0.1031 & \cdots & 0.2305 \\ \vdots & \vdots & \ddots & \vdots \\ 0.0290 & 0.1511 & \cdots & 0.2122 \end{bmatrix} \tag{8}$$

Then we define maximum value(Z^+):

$$\begin{aligned} Z^+ &= (Z_1^+, Z_2^+, \cdots Z_m^+) \\ &= (\max\{z_{11}, z_{21}, \cdots, z_{n1}\}, \max\{z_{12}, z_{22}, \cdots, z_{n2}\}, \cdots, \max\{z_{1m}, z_{2m}, \cdots, z_{nm}\}), \end{aligned} \tag{9}$$

minimun value(Z^-):

$$\begin{aligned} Z^- &= (Z_1^-, Z_2^-, \cdots Z_m^-) \\ &= (\min\{z_{11}, z_{21}, \cdots, z_{n1}\}, \min\{z_{12}, z_{22}, \cdots, z_{n2}\}, \cdots, \min\{z_{1m}, z_{2m}, \cdots, z_{nm}\}), \end{aligned} \tag{10}$$

Distance of the $i^{th}(i = 1, 2, \cdots, n)$ evaluation object from the maximum value(D_i^+):

$$D_i^+ = \sqrt{\sum_{j=1}^{m} (Z_j^+ - z_{ij})^2}, \tag{11}$$

Distance of the $i^{th}(i = 1, 2, \cdots, n)$ evaluation object from the minimum value(D_i^-):

$$D_i^- = \sqrt{\sum_{j=1}^{m} (Z_j^- - z_{ij})^2}. \tag{12}$$

Then, we can calculate the un-normalized score of the i^{th} evaluation object:

$$S_i = \frac{D_i^-}{D_i^+ + D_i^-}. \tag{13}$$

Next, we normalize the score using Equation 14:

$$\tilde{S}_i = S_i / \sum_{i=1}^{n} S_i. \tag{14}$$

Finally, we calculated the scores of the top 7 countries which was shown in Table 7.

Table 7: Top 7 Overall Scoring Countries

Countries	*USA*	*China*	*Japan*	*UK*	*France*	*Germany*	*Canada*
Score	298.9914	132.6379	64.8017	60.4439	41.5603	34.6638	32.9397

4.4.4 Determination of Annual Profit

Considering that the technology is not fully mature in the early stage, the mining rate should rise first and then fall. We use the t-distribution probability density function as our mining curve[**?**]. The function is defined as follows.

$$p(t) = \frac{\Gamma(\frac{n+1}{2})}{\sqrt{n\pi}\Gamma(\frac{n}{2})}(1+\frac{y^2}{n})^{-\frac{n+1}{2}}, 0 < t < +\infty \tag{15}$$

With this definition of the mining curve, we then calculate the income.

$$income = \int_{t_1}^{t_2} p'(t) \cdot V dt \tag{16}$$

where V denotes total mineral value and we take V = 70 trillion. Then,

$$Profit = income - cost \tag{17}$$

4.4.5 Determination of Poor Countries

In order to narrow the global wealth gap and ensure global equity, we provide assistance to countries with extreme poverty by giving them slightly more resources. We use GDP as a measure of extreme poverty, and countries in the bottom 20 of the world GDP ranking are considered extremely poor. World GDP per Captica[**?**] is shown below.

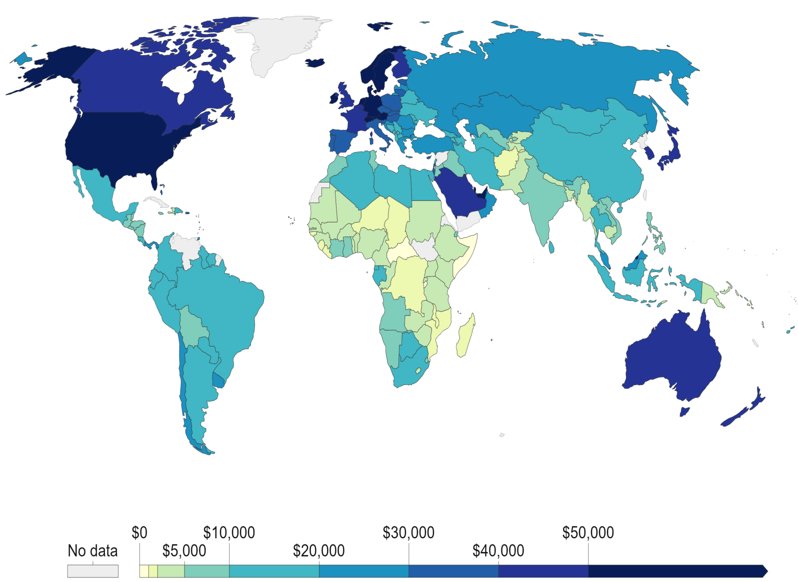

Figure 4: World GDP per Captica

Let γ be the poverty index of the kth country and we specify the value of γ could be calculated by Equation 18.

$$\gamma = \begin{cases} 1.2, \text{ if the country is one of the 20 countries with the lowest GDP,} \\ 1.0, \text{ otherwise.} \end{cases} \tag{18}$$

Then, the formula for total profit is:

$$pro_c = \gamma[(income - cost) \cdot \frac{Eq_k}{\sum Eq_k}] \tag{19}$$

where pro_c represents the total profit of Country C.

After calculating the total profits for each country in turn using Equation 19, we represent the top 5 countries with the highest total profits in Figure 5:

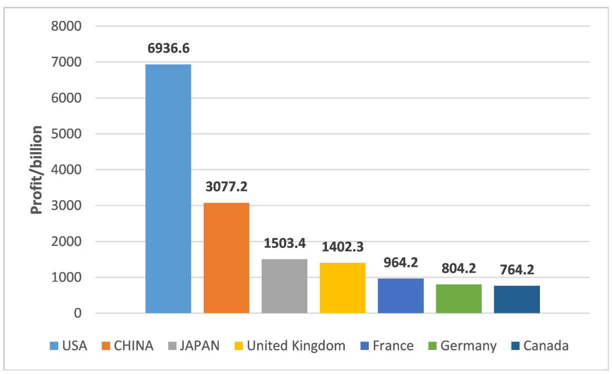

Figure 5: Top 7 Countries with the Highest Total Profits

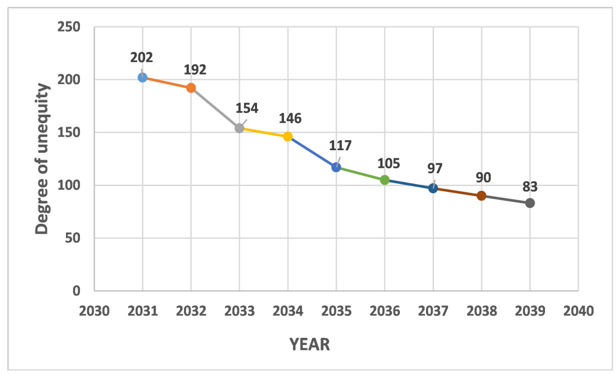

Figure 6: the Global Inequity Index for 2030-2039

Finally, we calculated the Global Inequity Index for 2030-2039 and the results are shown in Figure 6.

5 Impacts of Changing Conditions on Global Equity

To quantitatively measure the impact of each indicator on Country Development Score, we use a back propagation algorithm to calculate it and validate it by Pearson correlation coefficient analysis.

5.1 Backward Propagation Algorithm

5.1.1 Brief Introduction to the Back Propagation Algorithm

The backpropagation algorithm is a supervised learning method used in conjunction with optimization algorithms such as gradient descent[?]. It is a generalization of the Delta rule for multilayer feedforward networks, which allows the gradient to be computed using a chain rule for each layer iteration.

In a word, the following equation holds:

$$\begin{cases} z_1^{(l)} = w_{11}^{(l)} a_1^{(l-1)} + w_{12}^{(l)} a_2^{(l-1)} + \cdots + w_{1N^{(l-1)}}^{(l)} a_{N^{(l-1)}}^{(l-1)} + b_1^{(l)} \\ z_2^{(l)} = w_{21}^{(l)} a_1^{(l-1)} + w_{22}^{(l)} a_2^{(l-1)} + \cdots + w_{2N^{(l-1)}}^{(l)} a_{N^{(l-1)}}^{(l-1)} + b_2^{(l)} \\ \quad\vdots \\ z_{N^{(l)}}^{(l)} = w_{N^{(l)}1}^{(l)} a_1^{(l-1)} + w_{N^{(l)}2}^{(l)} a_2^{(l-1)} + \cdots + w_{N^{(l)}N^{(l-1)}}^{(l)} a_{N^{(l-1)}}^{(l-1)} + b_{N^{(l)}}^{(l)} \end{cases}, \tag{20}$$

written in the form of matrix multiplication:

$$\begin{bmatrix} z_1^{(l)} \\ z_2^{(l)} \\ \vdots \\ z_{N^{(l)}}^{(l)} \end{bmatrix} = \begin{bmatrix} w_{11}^{(l)} & w_{12}^{(l)} \cdots & w_{1N^{(l-1)}}^{(l)} \\ w_{21}^{(l)} & w_{22}^{(l)} \cdots & w_{2N^{(l-1)}}^{(l)} \\ \vdots & & \\ w_{N^{(l)}1}^{(l)} & w_{N^{(l)}2}^{(l)} \cdots & w_{N^{(l)}N^{(l-1)}}^{(l)} \end{bmatrix} \begin{bmatrix} a_1^{(l-1)} \\ a_2^{(l-1)} \\ \vdots \\ a_{N^{(l-1)}}^{(l-1)} \end{bmatrix} + \begin{bmatrix} b_1^{(l)} \\ b_2^{(l)} \\ \vdots \\ b_{N^{(l)}}^{(l)} \end{bmatrix}, \tag{21}$$

that is:

$$\boldsymbol{z}^{(l)} = \boldsymbol{w}^{(l)} \boldsymbol{a}^{(l-1)} + \boldsymbol{b}^{(l)}. \tag{22}$$

Since $\boldsymbol{a}^{(l)} = \sigma(\boldsymbol{z}^{(l)})$, we can conclude that

$$\boldsymbol{a}^{(l)} = \sigma(\boldsymbol{w}^{(l)} \boldsymbol{a}^{(l-1)} + \boldsymbol{b}^{(l)}) \tag{23}$$

where $\sigma(x)$ denotes Activation Function, ususally take $\sigma(x) = \frac{1}{1+e^{-x}}$.

5.1.2 Error Analysis

The back propagation algorithm can be divided into three main layers, namely the input layer, the hidden layer and the output layer, and the relationship between them is shown in Figure 7:

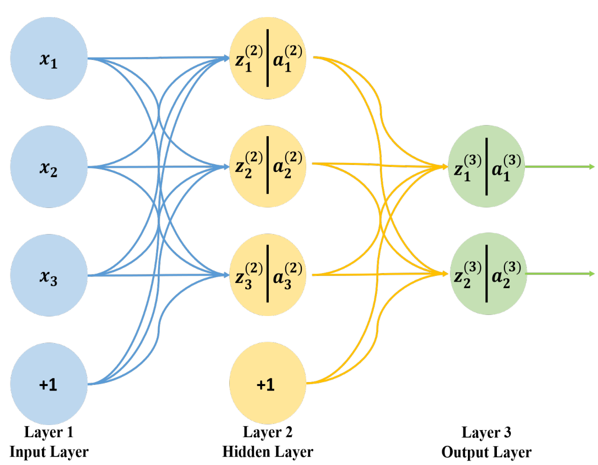

Figure 7: Neural Network Hierarchy

※ **Errors in the output layer**

According to the transmissibility of error propagation $\Delta z_j^{(L)} \rightarrow \Delta a_j^{(L)} \rightarrow \Delta C(\theta)$ and combined with the chain derivative rule, we found the error of the loss function on the output layer neurons $\delta_j^{(L)}$.

$$\begin{aligned}\delta_j^{(L)} = \frac{\partial C(\theta)}{\partial z_j^{(L)}} &= \frac{\partial C(\theta)}{\partial a_j^{(L)}}\frac{\partial a_j^{(L)}}{\partial z_j^{(L)}} \\ &= \frac{\partial C(\theta)}{\partial a_j^{(L)}}\frac{\partial \sigma(z_j^{(L)})}{\partial z_j^{(L)}} \\ &= \frac{\partial C(\theta)}{\partial a_j^{(L)}}\sigma'(z_j^{(L)})\end{aligned} \tag{24}$$

For all neurons on the output layer, this can be represented as a vector form.

$$\begin{aligned}\boldsymbol{\sigma^{(L)}} &= \begin{bmatrix} \sigma_1^{(L)} \\ \sigma_2^{(L)} \\ \vdots \\ \sigma_{N^{(L)}}^{(L)} \end{bmatrix} = \begin{bmatrix} \frac{\partial C(\theta)}{\partial a_1^{(L)}}\sigma'(z_1^{(L)}) \\ \frac{\partial C(\theta)}{\partial a_2^{(L)}}\sigma'(z_2^{(L)}) \\ \vdots \\ \frac{\partial C(\theta)}{\partial a_{N^L}^{(L)}}\sigma'(z_{N^L}^{(L)}) \end{bmatrix} = \begin{bmatrix} \frac{\partial C(\theta)}{\partial a_1^{(L)}} \\ \frac{\partial C(\theta)}{\partial a_2^{(L)}} \\ \vdots \\ \frac{\partial C(\theta)}{\partial a_{N^L}^{(L)}}\sigma'(z_{N^L}^{(L)}) \end{bmatrix} \odot \begin{bmatrix} \sigma'(z_1^{(L)}) \\ \sigma'(z_2^{(L)}) \\ \vdots \\ \sigma'(z_{N^L}^{(L)}) \end{bmatrix} \\ &= \nabla_{\boldsymbol{a}^{(L)}} C(\theta) \odot \sigma'(\boldsymbol{z^{(L)}})\end{aligned} \tag{25}$$

where $\odot$ is the Hadamard Product(the product of the corresponding elements of the two matrices).

※ **Error in the Hidden Layer**

Because the error of the output layer has been found above, according to the principle of error back propagation, the error of the current layer can be understood as a composite function of the error of all neurons in the previous layer(the error of the previous layer is used to represent the error of the current layer, and so on).

$$\begin{aligned}\sigma_j^{(l)} &= \frac{\partial C(\theta)}{\partial z_j^{(l)}} \\ &= \sum_{k=1}^{N^{(l+1)}} \frac{\partial C(\theta)}{\partial z^{(l+1)_k}}\frac{\partial z_k^{(l+1)}}{\partial a_j^{(l)}}\frac{\partial a_j^{(l)}}{\partial z_j^{(l)}} \\ &= \sum_{k=1}^{N^{(l+1)}} \delta_k^{(l+1)} \frac{\partial(\sum_{s=1}^{N^{(l)}} w_{ks}^{(l+1)} a_s^{(l)} + b_k^{(l+1)})}{\partial a_j^{(l)}}\frac{\partial a_j^{(l)}}{\partial z_j^{(l)}} \\ &= \sum_{k=1}^{N^{(l+1)}} \delta_k^{(l+1)} w_{kj}^{(l+1)} \sigma'(z_j^{(l)})\end{aligned}. \tag{26}$$

Similarly for the error of all neurons in the hidden layer, it can be written in vector form.

$$\boldsymbol{\delta}^{(l)} = \begin{bmatrix} \delta_1^{(l)} \\ \delta_2^{(l)} \\ \vdots \\ \delta_{N^{(l)}}^{(l)} \end{bmatrix} = \begin{bmatrix} \sum_{k=1}^{N^{(l+1)}} \delta_k^{(l+1)} w_{k1}^{(l+1)} \sigma'(z_1^{(l)}) \\ \sum_{k=1}^{N^{(l+1)}} \delta_k^{(l+1)} w_{k2}^{(l+1)} \sigma'(z_2^{(l)}) \\ \vdots \\ \sum_{k=1}^{N^{(l+1)}} \delta_k^{(l+1)} w_{kN^{(l)}}^{(l+1)} \sigma'(z_{N^{(l)}}^{(l)}) \end{bmatrix} = \begin{bmatrix} \sum_{k=1}^{N^{(l+1)}} \delta_k^{(l+1)} w_{k1}^{(l+1)} \\ \sum_{k=1}^{N^{(l+1)}} \delta_k^{(l+1)} w_{k2}^{(l+1)} \\ \vdots \\ \sum_{k=1}^{N^{(l+1)}} \delta_k^{(l+1)} w_{kN^{(l)}}^{(l+1)} \end{bmatrix} \odot \begin{bmatrix} \sigma'(z_1^{(l)}) \\ \sigma'(z_2^{(l)}) \\ \vdots \\ \sigma'(z_{N^{(l)}}^{(l)}) \end{bmatrix}$$
$$= (\boldsymbol{w}^{(l+1)})^T \boldsymbol{\delta}^{(l+1)} \odot \sigma'(\boldsymbol{z}^{(l)}) \tag{27}$$

Through the above steps, we get the results and show them in Table 3.

Table 8: Results of Backward Propagation Algorithm

Indicators	*EI*	*IDG*	*CEA*	*MA*	*HR*	*ER*	*SA*
Value	0.042	0.071	0.029	0.013	0.015	0.021	0.019

5.2 Pearson Correlation Analysis

5.2.1 Basic Principles

Pearson correlation analysis is used to explore the correlation between world equity scores and each representative indicator of each country. The Pearson correlation coefficient between the two variables is defined by the Equation 28:

$$\rho_{(Eq_k, GDP)} = \frac{cov(Eq_k, GDP)}{\sigma_{Eq_k}\sigma_{GDP}} = \frac{E[(Eq_k - \overline{Eq_k})(GDP - \overline{GDP})]}{\sigma_{Eq_k}\sigma_{GDP}} \tag{28}$$

Then we use Equation 29 to calculate the correlation coefficient r.

$$r = \frac{\sum_{i=1}^{n}(Eq_k - \overline{Eq_k})(GDP - \overline{GDP})}{\sqrt{\sum_{i=1}^{n}(Eq_k - \overline{Eq_k})^2}\sqrt{\sum_{i=1}^{n}(GDP - \overline{GDP})^2}} \tag{29}$$

Next, the correlation between the two variables is determined based on the r values. And the priciples are shown in Figure **??**.

As can be seen from Figure **??**, the closer the absolute value of r is to 1, the stronger the correlation between the two variables. Meanwhile, the positive or negative of r determines the positive or negative of the correlation.

5.2.2 Calculation Process

※ Calculation of Correlation Coefficient

Take analysis of the correlation between GDP and countries' equity scores as an example, through the analysis of the first question we know that the mean value of Eq_k is 34.7227. Then, we use Equation 30

$$\begin{aligned}\sigma_{Eq_k} &= \sqrt{\sum(Eq_k - \overline{Eq_k})^2} \\ &= \sqrt{\sum(Eq_k - 34.7227)^2} \\ &= 5.7147\end{aligned} \tag{30}$$

to got the variance is 32.6574 and the standard deviation is 5.7147.

Similarly, since the average value of GDP is 11540.8620, we substitute it into Equation 31.

$$\begin{aligned}\sigma_{GDP} &= \sqrt{\sum(GDP - \overline{GDP})^2} \\ &= \sqrt{\sum(GDP - 11540.8620)^2} \\ &= 11.9809\end{aligned} \tag{31}$$

that is, ehile variance is 143.5418, the standard deviation is 11.9809.

Finally, we calculate the sum of the outlying product of the score and GDP by Equation 32.

$$\begin{aligned}cov(Eq_k, GDP) &= \sqrt{\sum(Eq_k - \overline{Eq_k})(GDP - \overline{GDP})} \\ &= \sqrt{\sum(Eq_k - 34.7227)(GDP - 11540.8620)} \\ &= 6.6509.\end{aligned} \tag{32}$$

The sum of the outlying product of the score and GDP is 6.6509. Substitute into Equation 33.

$$\begin{aligned}\rho_{(Eq_k, GDP)} &= \frac{cov(Eq_k, GDP)}{\sigma_{Eq_k}\sigma_{GDP}} = \frac{\sum_{i=1}^{n} \frac{(Eq_k - \overline{Eq_k})}{\sigma_{Eq_k}} \frac{(GDP - \overline{GDP})}{\sigma_{GDP}}}{n} \\ &= \frac{\sum_{i=1}^{n} \frac{(Eq_k - 34.7227)}{\sigma_{Eq_k}} \frac{(GDP - 11540.8620)}{\sigma_{GDP}}}{n} \\ &= 0.871\end{aligned} \tag{33}$$

※ Test of Pearson's Correlation Coefficient

After obtaining the correlation coefficient, we use the Pearson Correlation Coefficient

Method to test it which using Equation 34.

$$r = \frac{\sum_{i=1}^{n}(Eq_k - 34.7227)(GDP - 11540.8620)}{\sqrt{\sum_{i=1}^{n}(Eq_k - 34.7227)^2}\sqrt{\sum_{i=1}^{n}(GDP - 11540.8620)^2}} = 0.78. \tag{34}$$

Next, we propose the hypothesis:

- $H_0 : P = 0$, score is not related to GDP;
- $H_1 : P \neq 0$, score is not related to GDP;

Meanwhile, we determine the corresponding significant level of 0.05.

Using Equation 35, we obtain the value of r.

$$\begin{aligned} t_r &= \frac{|r-0|}{\sqrt{(1-r^2)/(n-2)}} \\ &= \frac{0.78}{\sqrt{(1-0.78^2)/(7-2)}} \\ &= 19.5894 \end{aligned} \tag{35}$$

Since we check the t-test adjacency table, we have obtain the threshold $P = 0.9$ and the linear correlation coefficient is 1.653. As a result, we accept the original hypothesis and deem that GDP has a significant impact on the global equity.

Table 9: T-test Adjacency Table

n \ *P*	*0.25*	*0.1*	*0.05*
100	0.677	1.290	1.660
200	0.676	1.653	1.972
500	0.675	1.283	1.648

POLICY RECOMMENDATION

6 Policies for UN

By calculating the global development score, we found that the global economy gap between rich and poor is becoming larger and larger[?]. We propose policies such as raising taxes on space mining in developed countries that may promote the developing countries' economy. The specific policies are as follows.

(I) Policies for Short Term

▲ Raise mining taxes in developed countries.

Raising mining taxes in developed countries could bring more opportunities to developing countries. Developing countries are generally technologically backward but resource-rich[?]. Lowering mining taxes can promote the development of the mining industry in space, increase per capita income and improve the living standards of the people through the increase in income from mineral exports[?].

▲ Increase Teenager enrollment rate.

Developing countries are supposed to come up with policies that require school-age teenagers to be enrolled in school. Survey results show that developing countries have lower enrollment rates than developed countries. University education enrollment is directly related to enrollment. Increasing enrollment ensures the rate of scientific and technological development in developed countries.

▲ Fair allocation of profits from planetary mining.

Different mineral resources are supposed to be reasonably inclined to countries with different needs to achieve the maximum utilization of resources. By calculating each country's contribution in asteroid mining, the profits of asteroid mining can be scientifically and equally distributed.

▲ Improve the status of scientists.

To encourage young people to study science, countries are encouraged to raise the standard of living of scientists. Incentivize scientists from all countries to advance space exploration technology, establish space intelligence sharing networks in the form of capital, and promote the advancement of space mining technology.

We simulate a 10% increase in mining taxes in developed countries and a 5% increase in school enrollment in developing countries for the period 2030-2039. The result is as follows.

(II) Policies for Long Term

Asteroid mining is still a technical challenge now[?]. In order to ensure global equity when humans enter the asteroid mining era, we should structure the concept of win-win cooperation among countries to prevent a series of problems that asteroid mining may bring.

▶ **Scientific and rational exploitation of the planet.**

Factors such as radiation from mining should be assessed for their impact on the Earth's environment or other planets, and international consultations should be held before work begins. If the majority of countries agree to mining, then mining can begin; otherwise, mining will not be allowed[**?**].

▶ **Improve educational input.**

Science and technology are the first productive force. Increasing investment in education ensures an adequate number of scientists. Calculations show that increasing the number of scientists in each country can increase global equity.

After making policies for UN, the goals for each phase on the timeline is shown below.

7 Sensitivity Analysis

According to the transmissibility of error propagation, $\Delta w_{jk}^{(l)} \rightarrow \Delta z_j^{(j)} \rightarrow \cdots \rightarrow \Delta C(\theta)$, the loss function can be viewed as a composite function of the weights (w). From the chain rule of derivation

$$\begin{aligned}
\frac{\partial C(\theta)}{\partial w_{jk}^{(l)}} &= \frac{\partial C(\theta)}{\partial z_j^{(l)}} \frac{\partial z_j^{(l)}}{\partial w_{jk}^{(l)}} \\
&= \delta_j^{(l)} \frac{\partial \left(\sum_{s=1}^{N^{(l-1)}} w_{js}^{(l)} a_s^{(l-1)} + b_s^{(l)} \right)}{\partial w_{jk}^{(l)}} \\
&= \delta_j^{(l)} a_k^{(l-1)} \\
\Delta Z &= \int_{w_1}^{w_2} \delta_j^{(l)} a_k^{(l-1)} dw,
\end{aligned} \tag{36}$$

we have calculated the results and shown them in Figure 8.

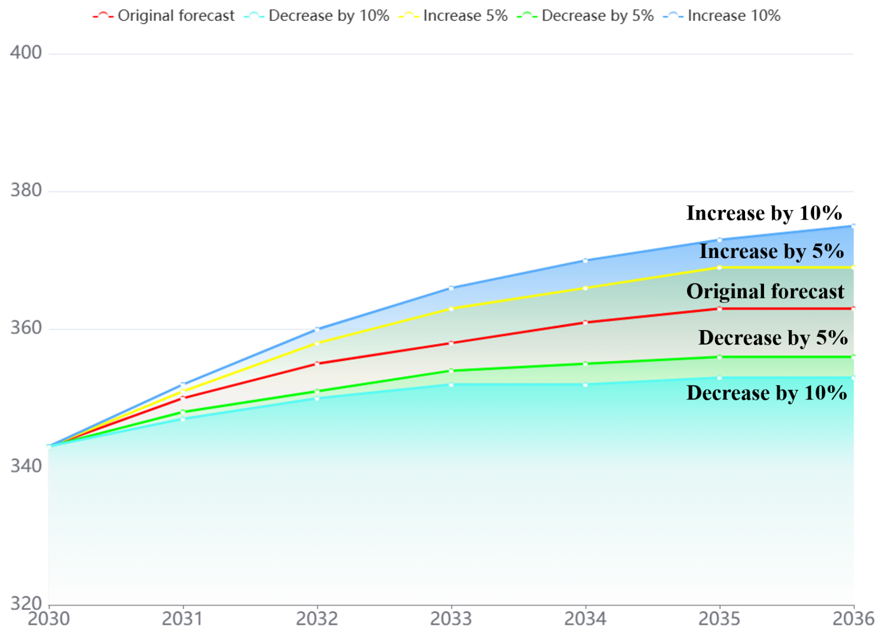

Figure 8: Sensitivity Analysis

The results show that our model is stable, with a variation of no more than 7%.

8 Strengths and Weaknesses

8.1 Strengths

⧫ The development degree of each countryis measured scientifically and comprehensively.

10 primary indicators and 56 secondary indicators are introduced as impact factors, and the weights of each indicator are calculated using the Analytic Hierarchy Process (AHP). Using the topsis method, the score of each country is calculating Finally, the global equity of the last 10 years is calculated through the global development balance score. the model is objective, accurate and convincing.

⧫ An analytical discussion of real data-based decision models is conducted.

The impact of each indicator on the global development balance score is quantitatively analyzed in conjunction with Back-propagation. We calculated the correlation between each indicator and the global development balance score by Pearson correlation coefficient analysis and pass the t-test. Finally, we compare the results with the real situation and discuss the rationality of the model.

⧫ The model is objective and accurate, taking into account that the mineral extraction rate should rise first and then fall, so the t-distribution probability density function is used for fitting.

Higher scores for more contributions to science and technology ensure equity for those who work more. Resources are allocated slightly more to the extremely poor countries, ensuring the high speed of development in the extremely poor countries.

♦ **The information is comprehensive and accurate, and provides detailed and credible predictions about the future of asteroid mining.**

Firstly, it is argued that asteroids are rich in minerals with great potential value and can contribute to global economic development. Then, the future of asteroid mining is discussed, and the TOPSIS method is used to scientifically calculate each country's score, and the total amount of resources allocated to each country is obtained accordingly.

8.2 Weaknesses

■ **Accuracy relies on statistics.**

Although most of the data we selected is from official documents and scientific reports, there are still problems of missing and inaccurate statistics. The models and results rely heavily on the data we use.

■ **The model does not take into account of the impact of unexpected factors such as natural disasters.**

The global development of science and technology may slow down due to unexpected factors such as natural disasters. Since the impact of natural disasters on global science and technology is tiny but difficult to quantify, we ignore the impact of natural disasters on global equity. However, there is no doubt that considering the resilience of each countriy to natural disasters would make our models more accurate.

References